ASSEMBLY LINE REBALANCING WITH NON-CONSTANT TASK TIME ATTRIBUTE

by

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A dissertation submitted to the

Graduate School-New Brunswick

Rutgers, The State University of New Jersey

In partial fulfillment of the requirements

For the degree of

Doctor of Philosophy

Graduate Program in Industrial and Systems Engineering

written under the direction of

Thomas Boucher

And approved by

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New Brunswick, New Jersey

OCTOBER, 2016

ABSTRACT OF THE DISSERTATION

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Assembly line has been widely used in producing complex items, such as automobiles and other transportation equipment, household appliances and electronic goods. Assembly line balancing is to maximize the efficiency of the assembly line so that the optimal production rate or optimal length of the line is obtained. Since the 1950s there has been a plethora of research studies focusing on the methodologies for assembly line balancing. Methods and algorithms were developed to balance an assembly line, which is operated by human workers, in a fast and efficient fashion. However, more and more assembly lines are incorporating automation in the design of the line, and in that case the line balancing problem structure is altered. For these automated assembly lines, novel algorithms are provided in this dissertation to efficiently solve the automated line balancing problem when the assembly line includes learning automata.

Recent studies show that the task time can be improved during production due to machine learning, which gives the opportunities to rebalance the assembly line as the improvements occur and are observed. The concept of assembly line rebalancing or task reassignment are crucial for the assembly which is designed for small volume production because of the demand variation and rapid innovation of new product. In this dissertation, two forms of rebalancing are provided, forward planning and real time adjustment. The first one is to develop a planning schedule before production begins given the task time improvement is deterministic. The second one is to rebalance the line after the improvements are realized given the task time improvement is random. Algorithms address one sided and two sided assembly lines are proposed. Computation experiments are performed in order to test the performance of the novel algorithms and empirically validate the merit of improvement of production statistics.

ACKNOWLEDGEMENT

The dissertation “assembly line rebalancing with non-constant task time attribute” concludes my career as a Ph.D. student at Rutgers University. It is a great journey that I could never imagine I have finished in such an amazing way. Not only the unanimous passes but also the greatest ever compliments receiving from all committee members of my defense do make me feel all the gritty effort of the past four years finally starts paying off.

My beloved parents, Jin Li and Shangtao Ding provide me untiring, unconditional supports which lead me this far. There are no words to describe how important they are to me. I am so grateful to be their son.

My adviser, Dr. Thomas Boucher, who has been inspiring me from day 1 when I got into the ISE program. He opened the door of my research and help me clean all of the road bumps on the path of completing the dissertation. Thank you, Mr. big picture, I am grateful to be your apprentice.

Now, my friends, who sit tight in the cheering section offering me everything they could possibly offer when I am in dire need. Youhu Zhao comforted me and shared his experience when I hit my bottom and could not escape the trough on my own. Chengguang Lu, Ching Wong, and Yi Zhang are always there to soothe the pain of my loneliness when I need company. Also, special thanks to ChuanChuan (Caroline), who taught me to become a tough person, which I think will benefit me in the future. I am grateful to be your lifelong friends.

I also appreciate my committee members, Sanchoy Das, Susan Albin, Honggang Wang, David Coit, and David Nembhard for their invaluable advice on how to shape the dissertation crystal clear and rock solid.

Ultimately, I wish the dissertation would provide the readers a whole new perspective to the line balancing area. I also ask for your forgiveness in advance for any inaccurate and insufficient statements I made throughout my dissertation.

Yuchen Li at

Library of Science and Medicine café lounge

09/12/2016

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1. Introduction

1.1 Traditional flow assembly line

Flow assembly line is an important process in the manufacturing ecosystem. It is commonly used to produce automobiles, transportation equipment, household appliances and electronic goods. The emergence of the assembly line can be traced back to the 19th century and the first flow assembly line was initiated to produce the steam engines at the factory of Richard Garrett & Sons in 1853. Henry Ford is mostly renowned for his contribution to the rise of mass production in the early 20th century. The assembly line developed for the Ford Model T began operation on December 1, 1913. It had immense influence on the world. In the 1950s, engineers started to experiment with robots in the assembly line as a means of industrial development. Robots have proved to be faster, more cost effective in high wage countries, and more accurate than humans. From late 20th century to the present, with developments in computer science, the innovation of the assembly line reaches a new level where computer programs and algorithms intelligently automate the production process. It gives birth to some novel assembly line systems, such as assembly lines with supervisory control and learning.

The assembly line designs that we address in this thesis can be categorized into the one sided line (OAL) and the two sided line (TAL). For OAL, the assembly line consists of a number of workstations arranged along a transportation device, i.e. conveyer belt, which moves the part (jobs) at same speed in order to pace the line. The speed is determined by a cycle time. The parts are consecutively launched from top to bottom of the line and assembled in workstations after certain operations (tasks) have been done in a certain amount of time. The total sojourn time for a part staying at a workstation should not exceed the cycle time of the assembly line. In order to perform a particular task, machines and workers with certain skills are required in each

workstation. Furthermore, tasks are dependent on each other and the relationships are often characterized in a precedence graph.

For TAL, as opposed to OAL, where workers can only access the tasks from one side of the line, workers can perform the tasks cooperatively from both sides of the line. The basic structure of TAL can be seen in Figure 1. Along the assembly line, there are several positions where parts are processed independently. In each position, there are two stations called mated-stations which are located at different sides of the line. Each mated station is attended by a worker or a robot. Thus, each mated station can work independently. The speed of the line is determined by the cycle time which is the sojourn time of the part in each position. As distinct from OAL, where tasks have two attributes (task times and precedence relations between tasks) the task in TAL has a third attribute which is the operational direction. An individual task can be assigned to either side of the line or only to one side of the line based on the operational direction (left, right or either).

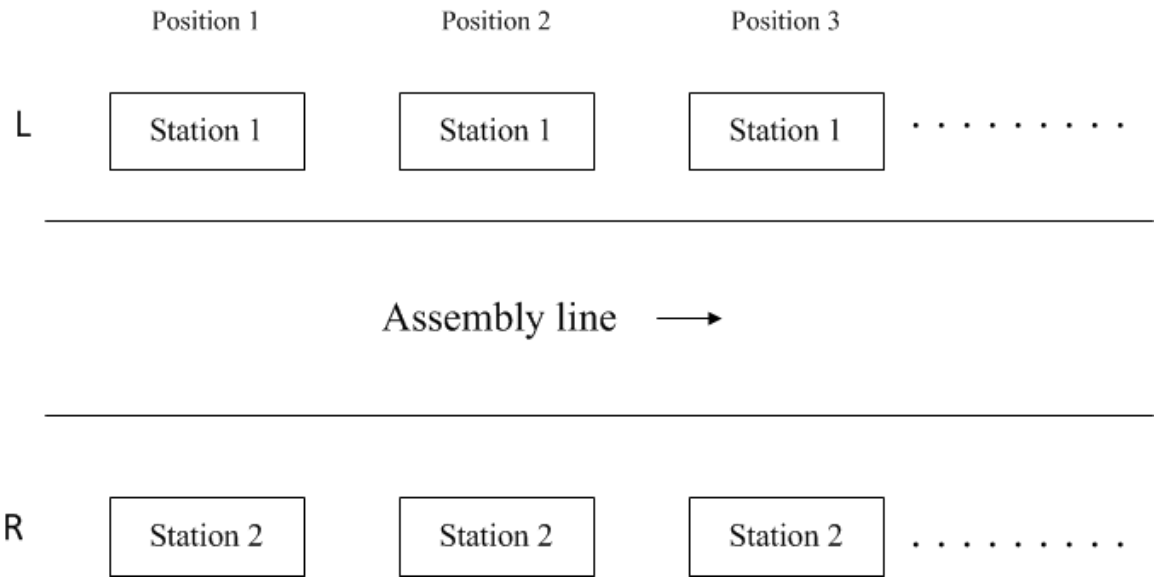

Figure 1 Basic structure of two-sided assembly line

The goal of designing an assembly line is to maximize its production efficiency (E) measured by the product of the cycle time and the number of stations (or positions for TAL) in the assembly line. Mathematically, the efficiency can be quantified by Eq. (1). The assembly line is balanced when the maximal efficiency is achieved.

$$E = \frac{1}{m \times c} \qquad (1)$$

m: the number of stations
c: cycle time of the line

1.2 Automated Flexible Assembly Line

A Highly automated and flexible assembly system brings with it its own set of particular characteristics. Two emerging characteristics that are important to discuss in the context of this thesis are 1) machine learning or learning automata and 2) control architecture and collaborative learning.

It has been shown that robots, as intelligent agents, are able to improve their performance on given tasks. Such improvements result from encoding learning algorithms in the control programs of the agents. For example, Nuttin and von Brussel (1999) have shown that a neural net controller with reinforcement learning can improve the task performance time. They studied the classic “peg-in-a-hole” insertion problem and found that both the insertion time and the amount of contact force required for the insertion (referred to as force quality) improved over the number of executions of the task as shown in Figure 2. The characteristic non-linear relationship between insertion time and the number of insertions attempted is typical of a learning curve function.

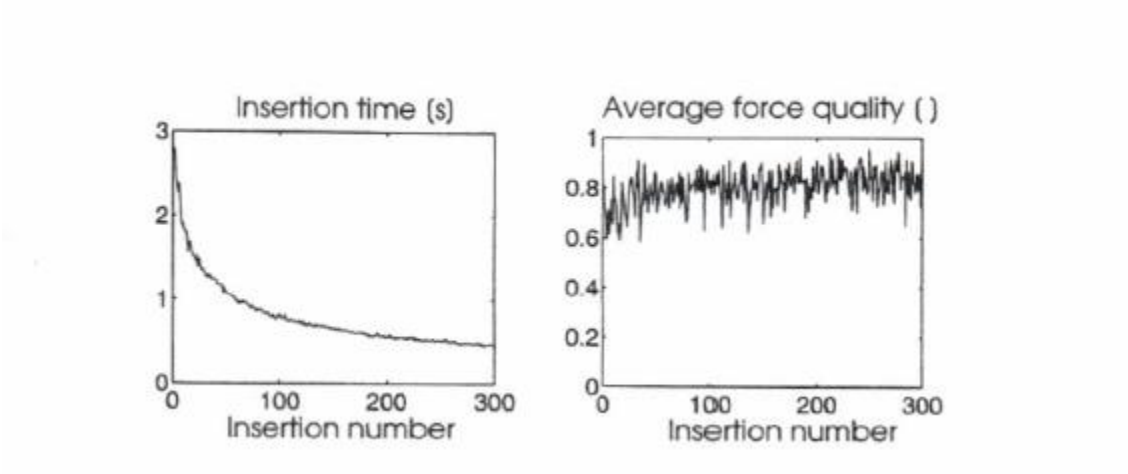

Figure 2 Learning the peg-into-hole assembly operation Left: evolution of the insertion time. Right: evolution of the average quality based on the contact forces (Source: Nuttin and Von Brussel, 1999)

Lopes and Camarinha-Matos (1995) illustrated another aspect of learning in flexible assembly systems. In their work they focus on the detection of errors in the robot assembly task and the design of recovery from error and prevention of reoccurrence of the same errors over time. The improvement comes as a result of building a knowledge base of models for monitoring, diagnosis and recovery through machine learning. In their experiment they study the operation of a pick and place robot. By introducing examples of abnormal conditions they study the sizes of the machine learning branching tree necessary to produce good results in diagnosing and reducing the error rates as shown in Figure 3.

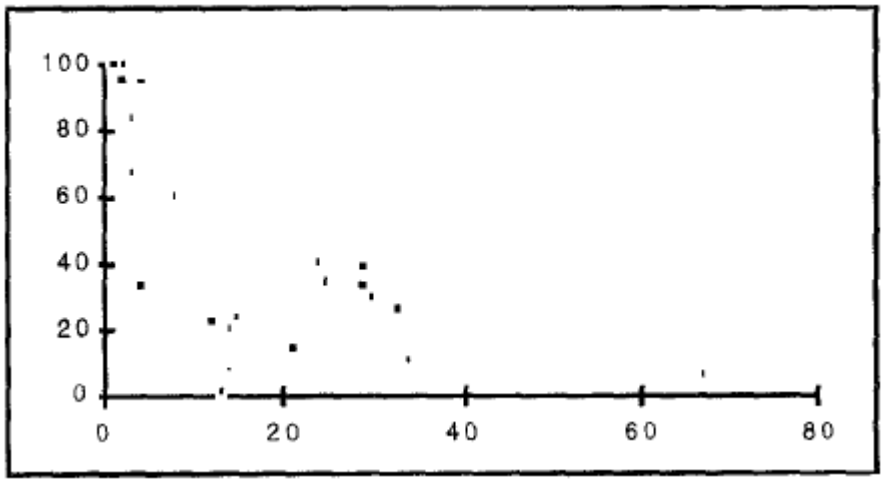

Figure 3 Error rate per attribute value versus the number of examples [Source: Lopes and Camarinha-Matos(1995)]

1.3 Motivation

The two examples show the existence of non-constant task time for the robots when the supervisory control and learning automata are involved. The task time decreases at the beginning, and then approaches a constant (plateau) after producing a number of units, which is the production ramp-up period. In traditional assembly line balancing, for large size production, the line is balanced *once* based on standard time, which is the constant task time that is reached after the production ramp up period. However, this is not applicable to small batch production where the production is still in the ramp-up period. This gives the motivation for considering *rebalancing* the station configurations as a means to reduce the cycle time in each production cycle, i.e. dynamic reassigning of the tasks to workstations. Task reassignment is very common in mixed-model assembly lines in which a common task of different product models can be assigned to different workstations in different production cycles (Boysen et al., 2012). Cost related to task reassignment, e.g. station setup cost and training cost, has to be considered in the optimization

process (Bukchin et al.,2006). However, such cost is negligible in the framework of an automated flexible assembly line when the robot in each station is assumed to be multifunctional, and knowledge regarding the task time improvement can be perfectly shared between robots and workstations. Indeed, Angerer et al. (2010) have demonstrated self-reconfiguration of mobile robots in car manufacturing, including the transfer of skills among robots. Therefore, it may be economic as well as efficient to rebalance the assembly line considering the non-constant task time.

A fundamental difference exists in learning as between an automated assembly line and a worker assembly line. If a task is reassigned among workers, the reduction in the task time is lost because the task skill achieved by one worker cannot be transferred to another worker. In an automated system, such as that which we describe, the improvement is preserved by the supervisory controller that oversees the assembly line and the learned skill (information) can be transferred to other agents on the line.

As such, there is a demand to revisit the classic problem with dynamic task time attribute incorporated. More importantly, fast and efficient algorithms must be provided in order to serve the purpose of rapid responses along the automated assembly system in small batch size production. We study two ways to satisfy the requirement: forward planning and real-time adjustment. As for forward planning, we assume that the pattern of the task time developing in batches is deterministic and assembly line balancing schedule is generated *before* the production process begins. Once the task times change, there is always an optimal solution at hand to adjust the task assignment as a means to rebalance the assembly line. As for real-time adjustment, the pattern of task time evolution is stochastic or random, which requires an instantaneous response *after* the changes of the task times are observed. In such a situation, an efficient algorithm is

necessary to rebalance the assembly line during production. Hence, the objective of this dissertation is to propose methods and algorithms to address the aforementioned problems from the perspectives of planning and real-time adjustment schemes.

1.4 Notations

The following notations are used throughout the paper. The definitions are as follows in Table. 1. Some variables are specific to TAL or OAL which are specified in the parenthesis after the variable definition, and the other variables are used in general.

Table 1 Notations

Variable names	Descriptions
c	Cycle time of the line
n	Number of tasks
t_v	Task processing times, v=1…n
m	Number of stations (OAL)
m*	Optimal number of stations (OAL)
z	Batch size
m_i	Number of stations at production cycle, i=1....z (OAL)
m_i*	Optimal number of stations at production cycle i, i=1...z (OAL)
P	Precedence matrix, P(v,o)=1 iff task v is the direct predecessor of task o.
t_{min}, t_{max}, t_{sum}	Minimum , maximum and total task times
t_v^i:	Task time at production cycle i, v=1...n, i=1,...z
c*	Optimal cycle time of the line (OAL)
c_i	Cycle time of the line at production cycle i, i=1,...z (OAL)
c_i*	Optimal cycle time of the line at production cycle i considering task reassignment, i=1,...z (OAL)
c_i'*	Optimal cycle time of the line at production cycle i without task reassignment, i=1,...z (OAL)
c^{temp}	Temporary upper bound of cycle time during the implementation of ENCORE
N	Total prodution quantities
V	A feasible task sequence (OAL)
V*	Optimal task sequence (OAL)
V^k	A feasible task assignment at station k, k=1,...m (OAL)
V_i	A feasible task sequence at production cycle i, i=1,...z (OAL)
V_i*	Optimal task sequence at production cycle i, i=1,...z (OAL)
NM	Total number of positions (TAL)
NS	Total number of workstations (TAL)
TdL_v	Task v processing time divided by latest position $TdL_v = \frac{t_v}{L_v}$ (2)
TdS_v	Task v processing time divided by slack $TdS_v = \frac{t_v}{L_v - El_v + 1}$ (3)
L_v	Latest possible position to which task v can be assigned (TAL)
El_v	Earliest possible position to which task v can be assigned (TAL)
F_v	Total number of followers of task v
S_v	Priority score of task v
GLB (GUB)	Global lower (upper) bound
LB (UB)	Intermediate lower (upper) bound
LBs (UBs)	Set of intermediate lower (upper) bounds of NM for all developing solutions

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1.5 Assembly Line Balancing Problem

The assembly line balancing problem can be subdivided into simple assembly line balancing (SALBP) and two sided assembly line balancing (TALBP). They correspond to OAL and TAL respectively.

There are two constraints in SALBP, precedence and station cycle time constraints. Firstly, the precedence constraints are always represented in a precedence graph, as shown in Figure 4. A task can only be assigned after all of its *predecessors* are assigned. For instance, in Figure 4, task 5 is available to assign when both task 1 and 4 have been assigned. Secondly, the total task time of one station cannot exceed the cycle time of the line. The following terms are useful to describe the task assignment in relation to constraints.

- The task is *available* when all preceding tasks have been assigned
- The task is *assignable* to a station when the total task time of that station does not exceed the cycle time *c* after the task is assigned.
- The load of the station is *maximal* if no further assignable tasks exists.
- The *bottleneck* station is the station whose total task time is the highest among all stations

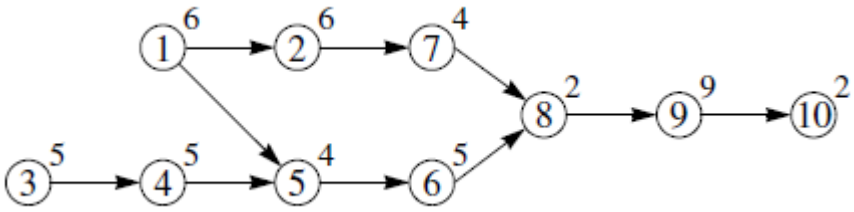

Figure 4 Precedence graph of SALBP

The traditional assembly line balancing problem is categorized into four types according to different inputs and objectives, as are shown in Table. 2.

Table 2 Taxonomy of SALBP

	number of station(*m*)	
cycle time(*c*)	Given	Minimize
Given	SALBPF	SALBP1
Minimize	SALBP2	SALBPE

Similar to SALBP, TALBP also has four types (TALBPF, TALBP1, TALBP2 and TALBPE). The precedence graph of TALBP for an example problem is shown in Figure 5. The task number is stored inside the node. There are two pieces of information stored in the parenthesis associated with each node. The first element is referred to the task time, and the second element designates the sides to which the task can be assigned. The precedence relations between tasks are described in the acyclic graph. For instance, task 3, which requires 1 unit of time to perform and can only be assigned to the right side of the line, has one predecessor (task 1) and one successor (task 5). Each side of a position is attended by one worker or robot doing the tasks assigned to that position/side.

Compared with SALBP, TALBP has an additional characteristic—Delay. We illustrate it with an example. A sample assignment of the problem described in Figure 5 is shown in Figure 6. As can be seen, there are two sources of idle time, namely, remaining capacity and delay. The first source is a common source which occurs in SALBP as well. It indicates that there is no room for the available task with the smallest task time to be assigned. So, if the target cycle time is 5, the example allows that only 4 time units of work can be assigned at position 1. Therefore, there is

some unused capacity. The second source is TALBP specific, it occurs as a task cannot be performed immediately after its previous task in the same station is finished. For instance, in Figure 6, after task 2 is done, task 4 cannot be performed until his predecessor task 1 is finished on the other mated-station. In other words, task 1 creates a mirror image on the other mated-station, and a delay occurs as the end time of the mirror image is later than that of the previous task (task 2) within the station.

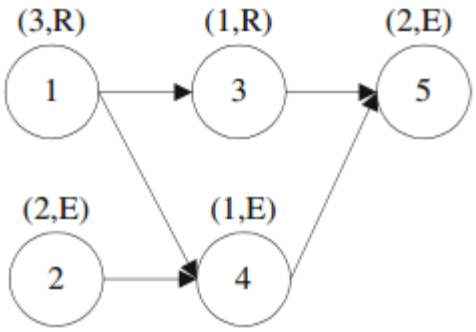

Figure 5 Precedence graph of TALBP

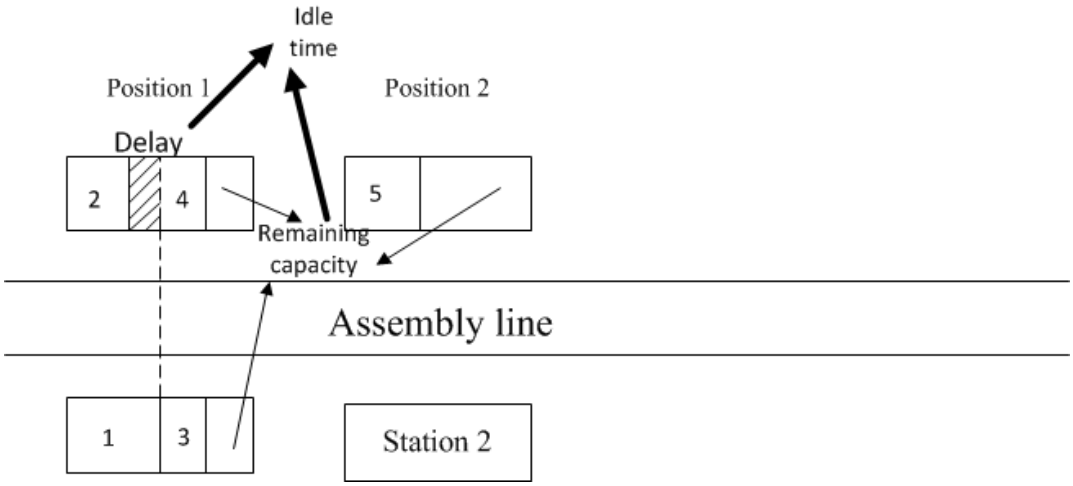

Figure 6 Sample assignment of the example problem

The TALBP is subject to three constraints: cycle time, precedence and operational constraints. The cycle time constraint requires that the finish time of the last task in a station should

be smaller than the cycle time. In SALBP, the finish time of the last task is always *equal* to the total load of the station, which is the sum of the task times at that station. In TALBP, however, the finish time of the last task is always *greater* than or *equal* to the total load of the station because of the delay. The precedence constraint states that a task can be assigned only if all of its predecessors have been finished meaning that the beginning time of a task shall be later than the latest end time of its predecessors. In comparison to the precedence constraint, which only considers whether all predecessors are assigned, the precedence constraint in TALBP takes account of the end time of the predecessors at both mated-stations. The operational constraint guarantees that the task is performed on its designated side.

1.5.1 SALBPF

SALBP-F is a problem which is to establish whether or not a feasible line layout exists for a given combination of m and c. For the example of Figure 4, a feasible sequence of m=7 and c=10 is V^1={1}, V^2={3,4}, V^3={5,6}, V^4={2,7}, V^5={8}, V^6={9}, V^7={10}. This is a NP-complete problem which means no polynomial time solution is known. That is to say, the time required to solve the problem using any currently known algorithm increases very quickly as the size of the problem grows. Hence, whether the problem can be solved in polynomial time depends on the complexities of the problem structure, which will be discussed in the literature review of Chapter 2.

1.5.2 SALBP1

SALBP1 is the most common problem among all versions of the assembly line balancing problem. The objective is to minimize the number of stations for a given cycle time. Equivalently,

the length of the assembly line and the amount of resources are minimized when the number of stations is minimized. Solving the problem is very sophisticated due to many possible station configurations. Indeed, it a NP-Hard problem meaning that solving it in polynomial time is impossible and the solution time is impractical as the size of the problem is relatively large. Mathematically, the SALBP1 line balancing problem can be formulated as follows (Otto and Otto, 2014).

Min m

Decision Variables

$$x_{vk} = \begin{cases} 1 & \text{if task v is assigned to station k} \\ 0 & \text{otherwise} \end{cases}$$

s.t.

$$\sum_{k=1}^{m} x_{vk} = 1 \quad \forall v = 1...n \qquad (4)$$

$$\sum_{k=1}^{m} k \times x_{vk} \leq \sum_{k=1}^{m} k \times x_{ok}, \quad \text{if } P(\mathrm{v},\mathrm{o}) = 1 \ \forall v, o = 1...n \qquad (5)$$

$$\sum_{v=1}^{n} x_{vk} t_v \leq c \qquad \forall k = 1....m \qquad (6)$$

Constraint (4) is the indivisible constraint, it ensures that a task can only be assigned to one workstation; constraint (5) is the precedence constraint, it ensures that a task is only available to a station if and only if all of its predecessors have been assigned; constraint (6) is the speed constraint, it ensures that the tasks at each station of the assembly line can be completed within the cycle time of the line, i.e. the line is paced. It is worth noting that the constraints are identical regardless of the problem types.

The optimal solution of the example in Figure 4 is m=6 given cycle time c= 10. The associated station configuration is V^1={3,4}, V^2={1,5}, V^3={2,7}, V^4={6,8}, V^5={9}, V^6={10}.

1.5.3 SALBP2

SALBP2 is a problem that aims to minimize the cycle time of the assembly line for a given *m*. The cycle time serves as a bound for each workstation and guarantees the line is paced. Moreover, the cycle time is a measure of the productivity of the line. The smaller the cycle time is, the more productive the assembly line will be. It is a dual problem of SALBP1, because SALBP2 minimizes *c* given a fixed *m*, while SALBP1 minimizes *m* given *c*. Like the SALBP1, SALBP2 also falls into the category of NP-Hard problem. The problem can be formulated as follows.

Min c
Decision Variables

$$x_{vk} = \begin{cases} 1 & \text{if task v is assigned to station k} \\ 0 & \text{otherwise} \end{cases}$$

s.t.

$$\sum_{k=1}^{m} x_{vk} = 1 \quad \forall v = 1...n \qquad (7)$$

$$\sum_{k=1}^{m} k \times x_{vk} \le \sum_{k=1}^{m} k \times x_{ok}, \quad \text{if } P(\mathrm{v},\mathrm{o}) = 1 \ \forall v, o = 1...n \qquad (8)$$

$$\sum_{v=1}^{n} x_{vk} t_v \le c \qquad \forall k = 1....m \qquad (9)$$

Given the number of stations m = 5, the optimal solution of the example in Figure 4 is c=11. The associated station configuration is V^1={3,4}, V^2={1,5}, V^3={2,7}, V^4={6,8}, V^5={9,10}.

1.5.4 SALBPE

SALBPE is a more general problem than previous three problems. It is to maximize the efficiency (cf. Section 1.1) of the assembly line by making tradeoffs between minimizing the number of stations and cycle time considering their interrelationships. However, this problem is not as popular as the SALBP1 and SALBP2 because adjusting two variables at the same time is not very practical and sometimes one variable is more attractive than another in industrial problems. Generally speaking, it could be solved by iteratively solving the SALBP1 or SALBP2.

1.5.5 TALBP1

TALBP1 is a more common problem than the other three problems of TALBP in the literature. It is to minimize the number of positions of the assembly line with a fixed cycle time. Because of the additional constraints and the impact of the delay time, it is a more complex problem than SALBP1. Hence, it is a NP-Hard problem as well. Because of its complexity, heuristics and meta-heuristics are always employed to find a good solution in a reasonable amount of time. These methods are introduced in the literature review section. Further, in this dissertation, priority rules- based methods (PRBMs) as well as a bounded dynamic programming (BDP) scheme are proposed to solve the TALBP1 by exploiting the problem specific knowledge.

1.5.6 Forward Planning for SALBP1 under dynamic task attributes

As mentioned before (cf. Section 1.3), there is an existing yet unsolved problem: Dynamic task reassignment under changing task performance times. Task time reduction could be achieved by improvement induced by human or technology in terms of the cumulative proficiency of skills or process innovation. Task reassignment becomes possible in modern assembly systems. For example, the assembly line with supervisory control is capable of changing the production process

between production cycles (repetitions). The supervisory machine will dispatch orders (changes) to the machines in the workstations, and the adjustment will be made accordingly.

A problem arises when the current task assignment may not be optimal given a change in the dynamic task time attribute and the number of workstations can be reduced while maintaining the desired cycle time. Otto and Otto (2014) provides the solution procedures considering the dynamic attributes of task time, when task reassignment is not allowed during the learning period. In this dissertation, we aim to optimize the configuration of the assembly line by considering *reassigning* the tasks in each production cycle as task times are reduced. Figure 7 illustrates the decision making process given a batch size of 30. At the beginning, an optimal solution consisting of the number of stations ($m(1)$) and the task arrangement ($V(1)$) are obtained for the first production cycle. Then, as production cycle increases and task time falls according to a learning function, the current solution is no longer optimal at the uth production cycle. A new solution set ($m(u)$, $V(u)$) has to be searched. Hatched area implies the optimality of solution ($m(1)$, $V(1)$ is maintained from the first production cycle to the (u-1)th production cycle. The above procedure is repeated until the solution to last repetition is found ($m(30)$, $V(30)$). Generally speaking, for the batch size z, the problem we are addressing for SALBP1 can be formulated as follows:

$$Min \sum_{i=1}^{z} m_i$$

Decision Variables

$$x_{vk}^{\ i} = \begin{cases} 1 & \text{if task v is assigned to station k at the ith repetition} \\ 0 & \text{otherwise} \end{cases}$$

s.t.

$$\sum_{k=1}^{m} x_{vk}^{\ i} = 1 \quad \forall v = 1...n \ , \ i = 1...\mathrm{z} \qquad (10)$$

$$\sum_{k=1}^{m} k \times x_{vk}^{\ i} \le \sum_{k=1}^{m} k \times x_{ok}^{\ i}, \quad \text{if } P(\mathrm{v},\mathrm{o}) = 1 \ \forall v, o = 1...n \qquad (11)$$

$$\sum_{v=1}^{n} x_{vk}^{\ i} t_v^{\ i} \le c \qquad \forall k = 1....m \ , \ i = 1...\mathrm{z} \qquad (12)$$

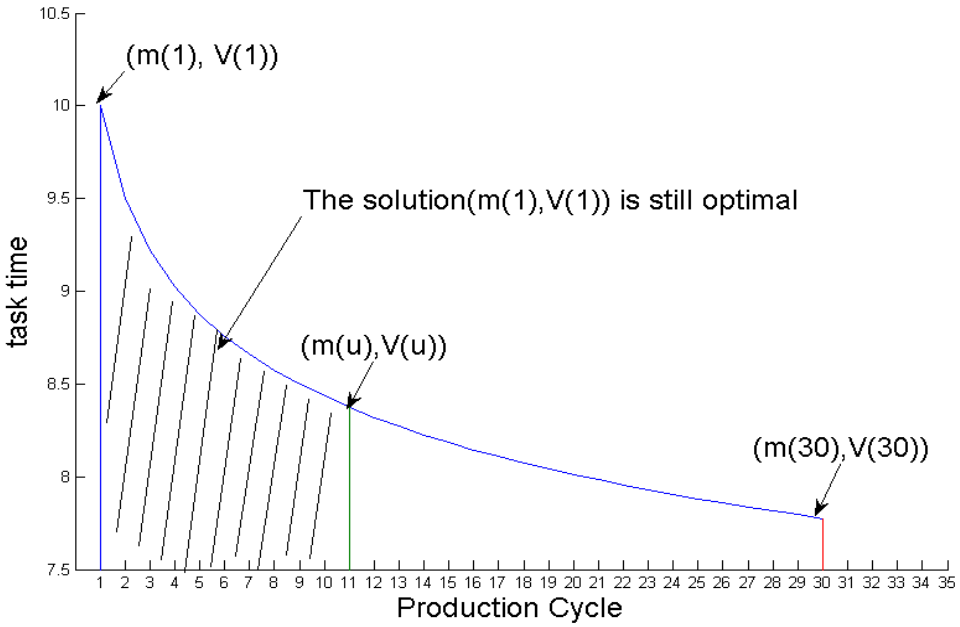

Figure 7 Task reassignment procedure

Our interest is to develop a smart algorithm that will tell us whether or not a new solution is required or whether the current solution is still valid. If the problem could be reduced to determine whether the optimal solution to the current batch is still optimal, the computational effort can be reduced. When a current configuration is no longer optimal, tasks will be dynamically reassigned to stations based on the new optimal design. In automated flexible assembly systems this simply means that the tasks are dynamically reassigned among fewer robots without loss of the information that has led to reduced assembly task times. Robots (agents) that are no longer required in the current assembly configuration can be released for work in other parts of the plant, or on other assemblies, thus maximizing the efficient use of machines. To tackle this problem, a novel algorithm is proposed in this paper concerning task reassignment based on dynamic task assembly times where the learning rate is predetermined. As a result, a planning schedule for the design of the assembly line is generated upfront before production begins in order to better allocate the resources. Comparisons are made with the use of the standard algorithms currently in the literature in terms of the performance.

1.5.7 Task reassignment for SALBP2 under non-constant task attributes

The presence of non-constant task time attribute also provides opportunities for further reducing the cycle time in SALBP2. At times, it is more convenient to adjust the task assignment than to reduce the number of stations. Moreover, the pattern by which the task time progresses could be deterministic or uncertain. In the deterministic case, task time reduction is governed by a predetermined function; in the uncertain case, task time reduction can only be realized *after* a production cycle is finished. The problem can be formulated as follows.

$$Min \sum_{u=1}^{z} c_i$$

Decision Variables

$$x_{vk}^{\ i} = \begin{cases} 1 & \text{if task v is assigned to station k at the ith repetition} \\ 0 & \text{otherwise} \end{cases}$$

s.t.

$$\sum_{k=1}^{m} x_{vk}^{\ i} = 1 \quad \forall v = 1...n \ , \ i = 1...\mathrm{z} \qquad (13)$$

$$\sum_{k=1}^{m} k \times x_{vk}^{\ i} \le \sum_{k=1}^{m} k \times x_{ok}^{\ i}, \quad \text{if } P(\mathrm{v},\mathrm{o}) = 1 \ \forall v, o = 1...n \qquad (14)$$

$$\sum_{v=1}^{n} x_{vk}^{\ i} t_v^{\ i} \le c \qquad \forall k = 1....m \ , \ i = 1...\mathrm{z} \qquad (15)$$

Task reassignment requires rapid and accurate responses to changes in the task times. It is especially useful in practice when there is no knowledge about the future changes of the task time and time limit for making the line rebalancing decision. Compared with production when no rebalancing occurs, the benefit of rebalancing the task assignment among stations during production is the improvement of the cumulative production time. The dissertation provides an efficient algorithm to rebalance the assembly line in a fast manner as well as an implementation of the rebalancing procedure in chapter 4.

1.6 Summary

In this chapter, assembly line and assembly line balancing problem (ALBP) are introduced. Assembly line can be classified into traditional assembly line which is operated by human workers, and automated assembly line which is operated by robot. In comparison to traditional assembly

line, automated assembly is faster, accurate and more standardized in terms of production efficiency. ALBP, which is to maximize the production efficiency by configuring the assembly line, is a classic optimality problem in the area of production and operations management. Four basic types of ALBP are explained and modeled.

Collaborative learning and supervisory control architecture, which are embedded in automated assembly system, enable the assembly line to be reconfigured considering task times improvement in order to achieve overall optimality in production efficiency. Contingent upon the attribute of task time changes, the assembly line can be configured before production begins when the changes is dynamic, or after production begins when the changes is uncertain. These two production situations are investigated in the following chapters.

2. Literature review

In this chapter a comprehensive literature review focusing on solution procedures is conducted. Firstly, the data structure and types of complexity measures are introduced. In what follows, the existing algorithms for the SALBP and TALBP problem types are discussed. In addition, some examples are provided to elaborate on the solution procedures. Lastly, the problem structure under non-constant task time attributes is introduced.

2.1 Literature Review on data structure and complexity

2.1.1 Data structure

In this dissertation, the data sets related to OAL comes from Scholl (1993) and the data sets related to TAL comes from Khorasanian et al. (2013), which encompass almost all popular data sets in the existing literature. The data sets are cataloged in Appendix A. They are used to test and compare the traditional algorithms and the novel algorithms proposed in this thesis. Most of the data sets are collected from industry while a small number are made up for the purpose of testing different facets of the algorithms.

A typical data set is always represented by a precedence graph (Figure 4) which shows the number of tasks, task times and precedence relations among tasks. Following the work of Hoffman (1963), the precedence graph can be transformed into a *precedence matrix* which completely captures the precedence relationships between tasks. In addition to the quantitative benefits, such transformation is also very handy when the algorithm is programmed and executed on a modern computer. A root node, which precedes all tasks, is always added to the precedence matrix. For instance, the precedence matrix for Figure 4 is shown in Table. 3. For the vth row, the value 1 indicates the corresponding column (task) number directly follows task v. For the jth column, the value 1 indicates the corresponding row (task) number directly precedes task j. Zero means the

pair of tasks does not preserve any direct relationships. There are ten ones in the first rows meaning that the root node directly precedes all tasks. There are two ones in the seventh column meaning that task 6 directly succeeds the root task and task 5.

Table 3 The precedence matrix for the example of Figure 4

0	1	1	1	1	1	1	1	1	1	1
0	0	1	0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	1	0	0	0	0	0	0
0	0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0	1
0	0	0	0	0	0	0	0	0	0	0

2.1.2 Complexity of data set

The complexity of a data set or a problem is the computational effort of finding the optimal or near optimal solution by exhaustive enumeration. In the context of assembly line balancing problem, the complexity of a data set can be characterized by the following factors.

Number of tasks (n): Given the number of tasks is *n*, if the precedence graph is ignored, it might be interesting to note there are totally *n*! feasible sequences to SALBP. Therefore, the complexity is expected to grow exponentially in the number of tasks.

Task times and cycle time: The complexity of a data set is partially determined by the ratios between task time and the cycle time. If the task times are relatively large in relation to the cycle

time, it is easier to assign the tasks to a station because there are fewer choices of combining tasks. As a result, the optimum seeking procedure will be advantageous and the computational speed will be fast.

Precedence constraints: The impact of the precedence constraint has two effects. On one hand, it reduces the number of feasible sequences, which may lower the level of complexity. On the other hand, it impedes the process of seeking an optimum because the availability of a task depends on its predecessor and a station configuration is subject to prior stations task assignments. Concretely, any change of task assignments at a station at one point may result in totally different solutions. Therefore, the decision on the current station is more sensitive to the full solution than are the subsequent stations.

Number of stations (m): The influence of the number of stations are twofold. There are more feasible combinations when the number of stations is small in which case the complexity is high. However, the number of stations dealt with is less when the number of stations is small which will lower the complexity. The case in which the number of station is equal to one or to the number of tasks is trivial.

2.2 An overview of algorithms addressing SALBP

Algorithms dealing with SALBP can be classified by different criteria. In terms of the types of solutions, the algorithms could be classified into exact procedures or heuristics. In terms of the optimum seeking procedure, the algorithms could be categorized into dynamic programming (DP) or the Branch and Bound (BnB) procedure. In terms of construction schemes, which defines the procedure of assigning tasks, the algorithms are classified into station-oriented or task-oriented assignment. The heuristics are more favorable than exact solution procedures in dealing with large

size, real-world problems and TALBP because they can find a good quality solution more quickly for complex problem and the programming is easier. The BnB turns out to be more effective than DP empirically (Scholl, 2006). The definitions of station-oriented or task-oriented assignment are as follows.

Station-oriented assignment: In any step of a station-oriented procedure a complete load of assignable tasks is built for a station k, before the next one k + 1 is considered.

Task-oriented assignment: Procedures which are task-oriented iteratively select a single available task and assign it to a station, to which it is assignable.

Either assignment method has its own merit. Compared to the task-oriented assignment, station oriented assignment will take more time to find a feasible solution, but the quality of the solution is better.

2.3 Literature Review on Dynamic Programming

Dynamic programing (DP) has played an important role in the history of TALBP. Jackson (1956) set forth a DP procedure to enumerate the solutions for TALBP. The DP solution approach is to form a graph G(*V, A*) where vertices *V* corresponds to the *states* of the problem solution and arcs *A* are the decisions of transforming one state to another. With DP, TALBP can be treated as a multistage decision process. Held (1963) extended Jackson's pioneered work by subdividing the *stages* into stations. Stage refers to the station number to which DP advances. Each subproblem of TALBP is to search the solution space of task assignment of a station (Scholl and Becker, 2006). The optimal solution is searched in a forward recursion stage-by-stage, i.e., DP enumerates all first station assignments, then considers the loads of the second station for each task sequence of the first station, and so on (breadth-first search). Since the number of sub-solutions grows exponentially as the number of tasks increases, a solution space reduction method is necessary for

the maintenance of the good performance of DP to the large size problem. Two approaches are used to achieve the solution space reduction: 1) Search space reduction techniques, for example, dominance rules (Jackson, 1956) and upper and lower bound (Easton et al., 1989); and 2) Heuristics station enumeration approach. Hoffmann (1963) developed a search method that systematically finds all solutions of a subproblem but only selects the one with the *minimum idle time* to proceed to the next stage. Fleszar and Hindi (2003) extended his work by building solutions from both sides of the precedence graph as well as conjoining tasks and incrementing operation times. In our approach, we modify and improve both of the solution space reduction techniques and incorporate them into our algorithmic design.

DP is an exact solution method based on breadth-first enumeration. However, as previously stated, its efficiency is dampened as the size of the problem increases. Although Hoffmann's approach, in conjunction with solution techniques, can locate a solution quickly, the quality of the solution is not guaranteed. To enhance the solution quality and only sacrifice minimal computational efficiency (computer elapsed time), Bautista and Pereira (2009) developed a BDP and applied it to the SALBP. We highlight their method because it serves as a building block for the novel algorithm proposed in this dissertation optimizing TALBP1. It showed excellent capability of producing good quality solutions with much smaller computational time compared with the exact solution methods. We build on the basic logic behind their work and develop a BDP specifically for TALBP. The BDP can be considered as a modified Hoffmann heuristic with two additional parameters, *window size* and *maximum transitions*. They are both utilized to *reduce* the solution space relatively to DP, but to *increase* the solution space relatively to Hoffmann heuristic. Window size denotes the number of sub-solutions (states) a stage j can keep and uses it to enumerate the next stage $j+1$. Maximum transitions are referred to as the number of states at stage

j+1 that DP can generate from each state at stage j. Hoffmann heuristic is a BDP with window size and maximum transitions both equal to 1. Chapter 5 will show the novel contributions of present work related to BDP.

2.4 Literature Review of the Branch and Bound procedure in Assembly Line Balancing

BnB is a popular procedure to solve combinatorial optimization problems. It is a building block for the research in the dissertation. The solution procedure constructs a tree with *nodes* and *arcs*. In this context, the workstation is represented by the node and the tasks operated in the associated station are listed on the arcs. The station loads are shown in the oval. Figure 8 shows an example data set and a feasible solution. The solution is m=4 and c=9. The station configuration is V^1={1,3}, V^2={2,5},V^3={4,6},V^4={7}. The solution structure is also called a *branch*. A tree consists of many such branches.

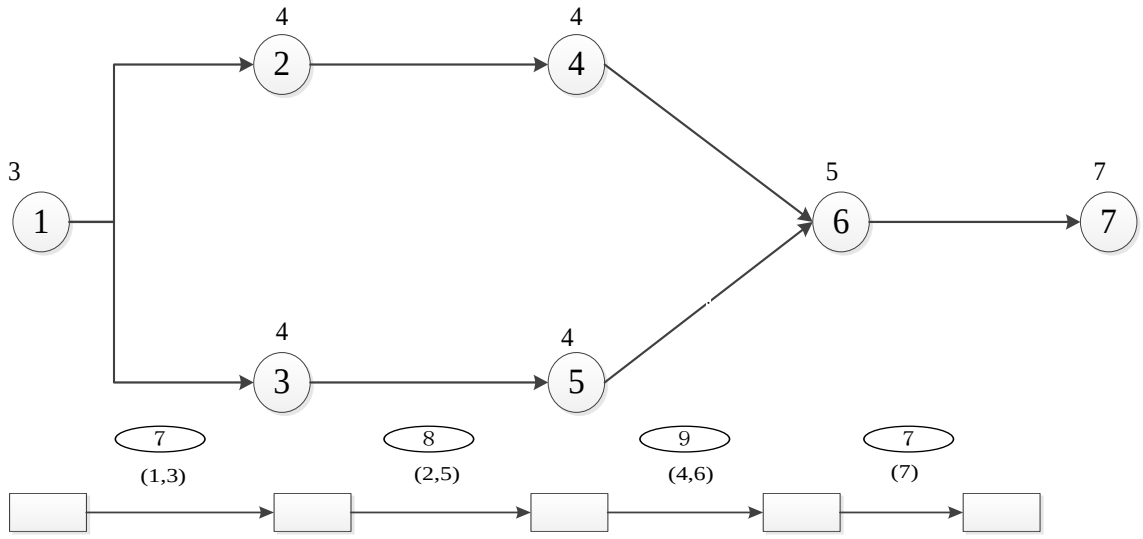

Figure 8 Simple solution to SALBP with cycle time 9

As mentioned in previous sections (cf. 2.1.2), the complexity of the problem will grow exponentially in the number of tasks, indicating the development of the tree needs a lot of memory

to store the branches. Hence, the predicament of the BnB in solving the SALBP is how to reduce the computational effort. There are two basic methods to achieve the effort reduction. 1) Choose a good starting point. 2) Avoid the inferior partial solutions (branches). Hence, one method targets on Bound discovery while the other one focuses on Branch development.

2.4.1 Lower bounds

The first step of the BnB procedure in SALBP is to find a feasible solution. Basically, the feasible solution is found by trying a number of possible solutions, and then verifying their feasibilities. If the trials are selected arbitrarily, it may be time-consuming to find a feasible solution, and the solution may be far away from the optimal solution. This will complicate the subsequent branching process. This is undesirable, but a lower bound construction can provide a reasonable starting point.

2.4.1.1 Lower bounds for SALBP1

SALBP1 aims to minimize the number of stations (m) given the cycle time (c). Due to the integrality of m, only a relatively small number of objective values are possible. A lower bound (LB) close to or even equal to m^* is helpful in solving SALBP1 instances provided that the bounds can be computed efficiently. Hence, if a feasible solution is equal to the LB, the BnB can be terminated and that solution is proven to be optimum.

Bin packing bounds: The SALBP1 will be reduced to a bin packing problem when the precedence relations are ignored. The equivalence of the two problems will make their lower bounds interchangeable. The most obvious lower bound for SALBP1, LB1, is the total capacity

bound invented by Baybars (1986). Based on the fact that the cycle time of the line must be greater than or equal to the station task times, LB1=$\lceil t_{sum}/c \rceil$. Johnson (1988) obtains the simple counting bound considering the relationships between the task time and cycle time. Tasks satisfying $t_v > c/2$ could not share the same station, so those tasks can each be counted as one station. Furthermore, tasks with exactly $c/2$ amount of time are counted as 1/2 station. The lower bound LB2 is based on counting all of the aforementioned tasks.

One-machine scheduling bound: The SALBP1 problem can also be relaxed to a one-machine scheduling problem (Johnson,1988). Tasks are interpreted as jobs $v = 1,\ldots,n$ with ‘‘processing times” $p_v = t_v/c$ which have to be successively performed on a single machine. After processing job v, a certain amount of time called tail jobs of v, tl_v, is necessary before the job is finished. The tail of task v is the lower bound on the number of stations for the successors of task v. It shall be noted that tl_v is not rounded up to the next integer unless it cannot share a station with the task v, i.e. $tl_v + p_v > \lceil tl_v \rceil$. The objective of the one-machine problem is to find a sequence of jobs with which the makespan is minimized, i.e., the time interval from the start of the first job to the end of the last job. The optimum solution is achieved by choosing the maximal processing time because the lead time can be fully utilized.

For the example of Figure 4 with c=10. The above LBs are calculated as follows.

LB1: LB1= $\lceil 48/10 \rceil = 5$.

LB2: There are three task whose task time is greater than 10/2=5 (task 1,2&9) and three tasks whose task time is exactly 10/2=5. Hence, LB2=3+$\lceil 3/2 \rceil$=5

LB3: The calculations of processing time and tails are given in Table 4. LB3=Max$\lceil p_1+tl_1, p_3+p_1+tl_3 \rceil$=6

Table 4 Processing time and tails of Figure 4

v	0	1	2	3	4	5	6	7	8	9	10
$p(v)$	0	0.6	0.6	0.5	0.5	0.4	0.5	0.4	0.2	0.9	0.2
tl(v)	5.7	4.1	3	4	3.4	3	2.2	2.2	2	1	0

2.4.1.2 Lower bounds for SALBP2

SALBP2 aims to minimize cycle time c given the number of stations m. As in the case of SALBP1, the lower bound of SALBP2 is calculated by exploiting the relationship between the cycle time and the task time.

LB1 (McNaughton, 1959): By analogy of LB1for SALBP1, a simple lower bound is calculated by the inequality $mc \geq t_{sum}$. The problem is simplified by assuming that tasks can be divisible. Moreover, for one particular station, the indivisibility of a task yields the inequality $c \geq t_{max}$. Hence, LB1=$\max\{t_{\max}, \lceil t_{\text{sum}}/m \rceil\}$.

2.4.2 Dominance rules

The dominance rules expedite the branching process by eliminating the dominated tasks from the assignable task set. In addition to shrinking the size of the assignable tasks set, it also rules out the incompetent partial solutions, i.e. a particular station configuration. Empirical studies show that the dominance rules can greatly enhance the BnB's efficiency.

Maximum load rule (Jackson, 1956): It excludes each partial solution P2 which contains one or more completed but non-maximal station loads, because there exists at least one other partial solution P1 with the same number of maximally loaded stations, where additional tasks are assigned. In such a case the completion of P1 does not require more stations than that of P2. In the

example of Figure 4, with cycle time c=10, the partial solution refers to the first station's configuration. Given P1={1}, P2={3}, P3={3,4}, P2 is dominated by P3 because the station configuration under P2 is non-maximal. P1 is not dominated by any partial solution because the station configuration under P1 is maximal because there does not exist another task that can be added to P1 without exceeding c.

Jackson's dominance rule (Jackson,1956; Klein and Scholl, 1997): This rule is to eliminate the dominated tasks during the branching process. Prior to branching, a dominated matrix is generated by sorting out the dominated pairs of tasks. Task v is dominated by task j if all of the following conditions are satisfied. 1) task v is not related to task j in terms of precedence. 2) the set containing all followers of task v are a subset of or equal to a set containing the followers of task j, $F_v \subseteq F_j^*$. 3) $t_v \leq t_j$. 4) The station time will not exceed the cycle time after assigning task j. Concretely, in the example of Figure 4, the dominance pair (j,v) are (1,4), (2,6), (3,7),(4,7),(5,7).

2.4.3 Branch and Bound procedure on SALBP1

In this section, the BnB procedure solving SALBP1 is discussed and examples regarding the commonly used algorithms are given for illustrative purposes. They can be perfectly complemented by the lower bounds and dominance rules.

- *Depth-first search* (DFS): By implementing DFS, a feasible solution (developed branch) is found first, and then the output m is set to the current optimum as well as an upper bound for the optimization process. In what follows, the procedure traces back to the node where there are alternative selections of a single task (task-oriented) or a group of tasks (station-oriented) available, and a new branch is started. The new branch is fathomed until the branch is proved to be inferior or renders the same solution, and then the procedure traces

back along the branch again. Otherwise, the current solution is updated to the number of stations underling the new branch. The procedure is repeated until it traces back to the root node. A classic algorithm which bases on DFS is FABLE (Johnson,1988).

Examples: The precedence relation and tasks times are given in Figure 9. Cycle time is 12. For ease of demonstration, only LB1 is applied, LB1=⌈31/12⌉=3.

The DFS firstly develops a feasible branch consists of node 1,2,3,4 and 5 as is shown in Figure 10. The current optimum is *m*=4. Then, the DFS backtracks to node 2 and develops a new branch (1-2-6-7-8). There is no improvement, so the algorithm traces back to node 1. Instead of assigning task 1 and 3 to station 1, the algorithm tries another task assignment set which is {1,2,4}. Then, a new branch (1-9-10-11) is generated with solution equal to 3. Since the current solution is equal to LB1, the algorithm stops and the optimum is found.

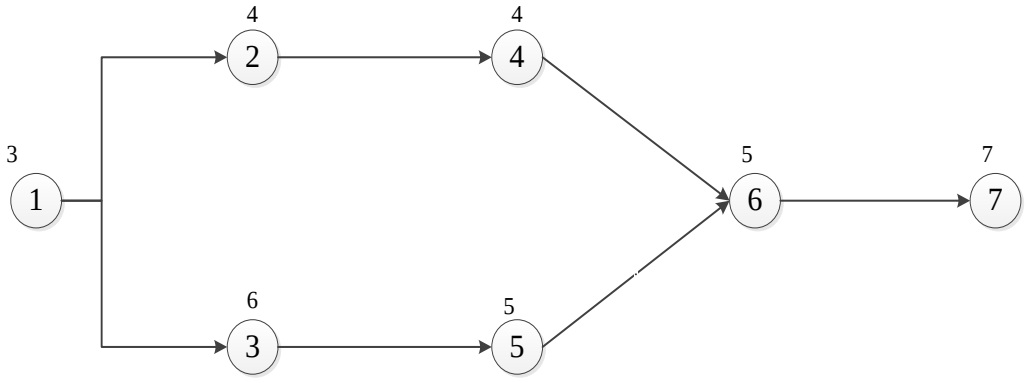

Figure 9 Example for BnB procedure for SALBP1

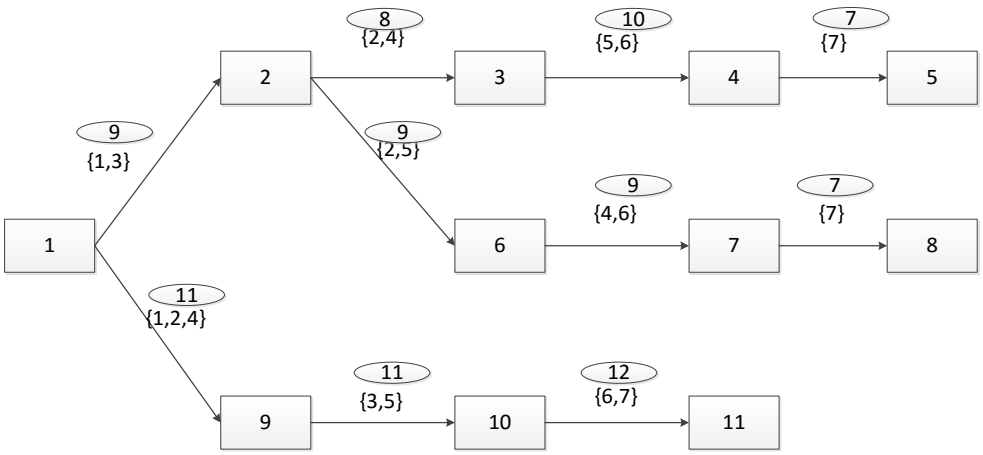

Figure 10 Tree structural solution for SALBP1 (DFS)

- *Bi-directional search method*: At times, the problem is much easier to solve when the precedence graph is reversed. Successive search in both directions, as done in EUREKA (Hoffman,1992), will reduce the size of the tree so that the enumeration effort is alleviated as well. Whether to choose the forward or backward search solely depends on the data. Usually, if the precedence relations are clustered on the right sides of the precedence graph, the forward direction is preferred. The reason is that the more precedence relations, the less available tasks for the stations. Furthermore, the configuration of subsequent stations is contingent upon the prior stations. Hence, it is beneficial to arrange fewer available tasks to the stations at the beginning.
- *Local Lower Bound Method* (LLBM): LLBM is considered as the most efficient algorithm in many situations. It is a foundation for the novel algorithms developed for SALBP in this dissertation. In addition, local lower bound (LLB) will be compared with the novel algorithms when solving the SALBP with dynamic task attributes. LLBM has the advantage of bounding the partial solution before a branch is fully developed. In other words, a partial solution could be proved to be inferior or equal to the current optimal solution before it advances to a full solution. The local lower bound of a station is the

minimum number of stations required given the previous station configurations, i.e. LLB (k)=k+ (LB of the reduced problem). SALOME1 (Scholl and Klein,1997) is an algorithm which utilizes the local lower bounds at length.

Examples: The problem in Figure 9 is revisited and SALOME1 is implemented to seek optimum. The solution tree is displayed in Figure 11. In comparison with DFS, it only takes 9 nodes to find the optimum by conducting SALOME1. The number above the node is the station's local lower bound. Unlike DFS procedure which continues branching after node 6, the branch (1-2-6) is terminated because the LLB (3) (node 6) is equal to the current optimum which is 4.

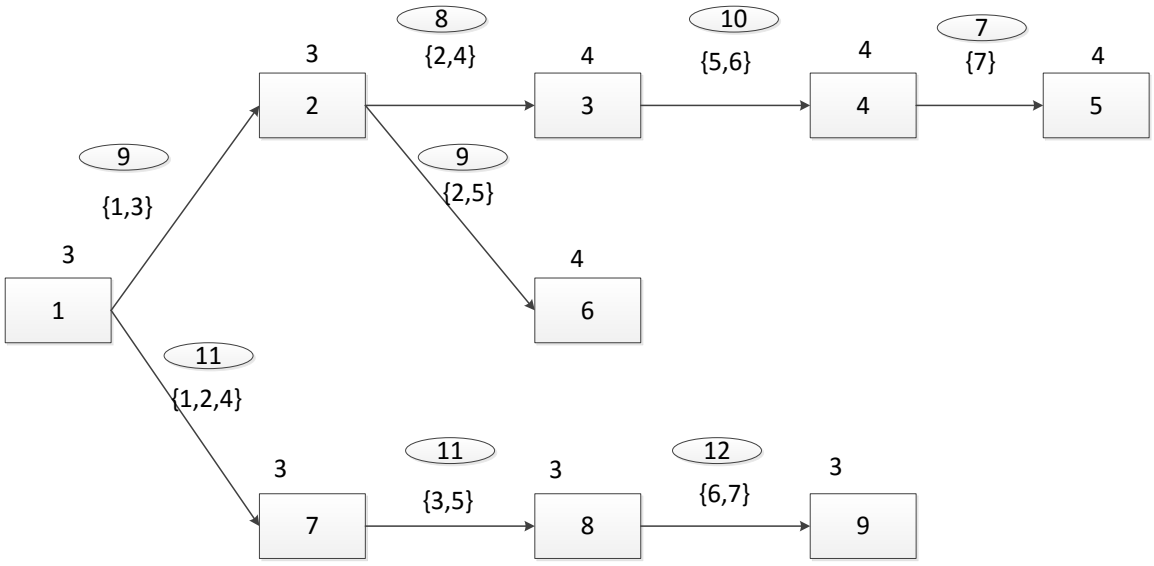

Figure 11 Tree structural solution for SALBP1 (SALOME1)

2.4.4 Branch and Bound procedure on SALBP2

While there is a plethora of literature discussing the exact solutions for SALBP1, only a few algorithms exist to address the SALBP2 problem. Most researches are devoted to solving the SALBP2 by taking the advantage of duality between SALBP2 and SALBP1.

- *Iterative search method*: SALBP2 can be solved by iteratively solving several SALBP1 problems. The basic idea is simply to examine the optimal number of stations for different values of cycle time. The optimal *c* is obtained at the point the m^* changes value. Therefore, several search methods have been proposed and tested [Dar-El and Rubinovitch (1979); Hackman et al. (1989)]. The most commonly used iterative method is *binary search*. The value of cycle time is chosen from an interval [LB, UB]. LB is a lower bound (cf. 2.4.1.2). UB is an upper bound. A simple upper bound for the cycle time is UB=max [t_{max}, $\lfloor t_{sum}/m \rfloor$] (Coffman et al.,1978). In binary search, the interval is subdivided into two subintervals by choosing the c_1=(LB+UB)/2. If m^* is equal to or lower than the number of stations required, the interval [LB, c_1] (UB is set to c_1) is selected for the next iteration. Otherwise, the interval [c_1, UB] (LB is set to c_1) is selected for the next iteration. Such procedure is repeated until the UB-LB=1. Then, examine both LB and UB to determine which one is the optimum. The iterative search method is an *indirect* method to solve SALBP2.
- *Local lower bound method*: By analogy of SALBP1, the enumeration effort can be efficiently reduced by considering the local lower bound. In addition to benefitting internal BnB procedure, LLBM can also be used to *directly* locate the optimum. SALOME2 is an influential algorithm based on the LLBM [Klein and Scholl (1996)]. The essence of the algorithm is to keep track of the total idle time (TOIL) left while fathoming the branching process. The TOIL is an indicator for the feasibility of the current solution. For instance, given t_{sum}=34, m=5, we would like to know the feasibility of a developing branch with c=7. The total idle time is equal to mc-t_{sum}= (5)(7) – 34=1. If the first station is fully loaded (no idle time), the TOIL is 1 for station 2. However, after searching all of the possible combinations for station 2, there exists no assignment with which the idle time of station 2

is equal to or less than 1. In which case, the TOIL turns negative meaning that the current solution is no longer feasible for the current branch, and thereby increasing the cycle time is necessary to regain the feasibility for the current branch.

Examples: For the data in Figure 9, given m=4, the solution tree structure is shown in Figure 12. Above each node, the number on the left side of the slash is the local lower bound, and the number on the right side of the slash is the TOIL. The algorithm starts off with a lower bound 9, and the total idle time is 9×4-34=2. It proceeds to node 2 after assigning task 1 and 2 to station 1. The TOIL for station 2 is zero because station 1 has an idle time of 2 (9-3-4=2). Hence, the rest of the stations have to be fully loaded; otherwise the cycle time 9 is no longer feasible. After searching for all possible selections, there is no combination of tasks whose total task times are equal to 9. The cycle time is increased to 10, which is the minimal value to make the TOIL positive. The loop above a node means the cycle time is increased at the node. After the first branch (node 1-2-3-4-5), the cycle time 10 is the current optimum. Then, backtracking to node 2 which is the first local lower bound whose value is smaller than 10, the procedure assign task 3 and 4 to station 2, and thereby the cycle time is increased to 10. The procedure then traces back to node 1 with a cycle time of 9 and assign task 1&3, 2&4 to station 1 and 2 respectively. However, there is no possible selection to station 3 that will not result in negative TOIL and the cycle time is increased to 10. As a result, the branching process returns to node 6 with a cycle time of 9. The third branch (node 1-6-9-10-11) is fathomed because the best possible solution for the current branch is 10, which is equal to the upper bound. The optimal solution resides in the last branch (node 1-6-9-10-11) and is equal to 9.

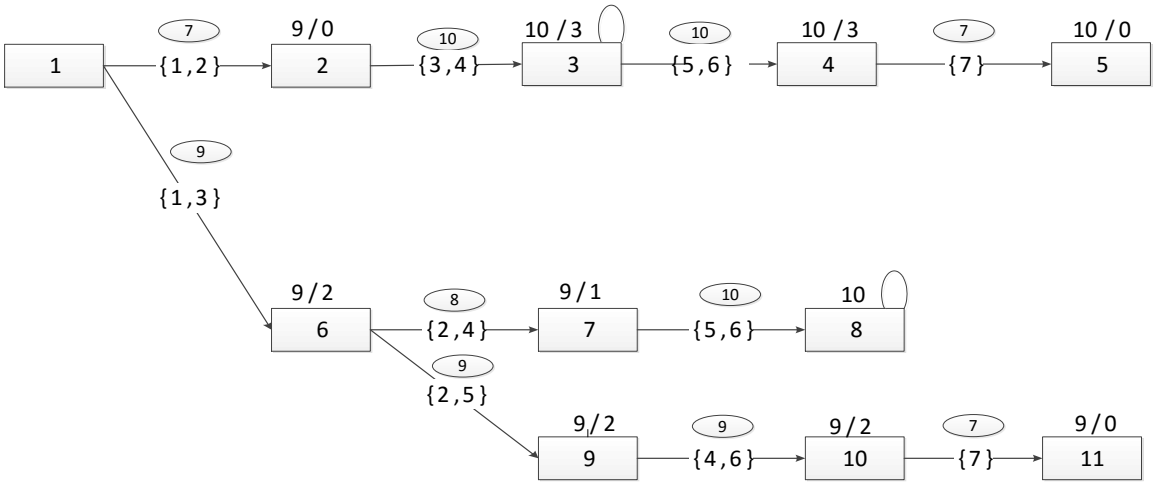

Figure 12 Tree structural solution for SALBP2 (SALOME2)

2.5 Literature Review on Priority-based methods

Priority-based methods (PRBMs) have proved to be efficient and intuitive to obtain good solutions for practical problems in SALBP (Otto and Otto, 2014). PRBMs can be used as stand-alone solution procedures or combined with other techniques, such as branch and bound. When PRBMs are used independently, they are one of the fastest methods because of the low computer memory requirements. They provide a good starting point, deliver reasonable bounds (Otto et al., 2011) and enhance the efficiency of local search (Storer et al., 1992) when used in conjunction with other solution methods (Vance et al. 1994). The first scientific study on using PRBMs to solve the assembly line balancing problem (ALBP) was conducted by Arcus (1966). He used PRBMs to develop a computerized software, COMSOAL, balancing practical problems in assembly lines. Talbot et al. (1986) carried out extensive experiments to evaluate the performance of heuristics and suggested a guideline to choose heuristics under different conditions. Scholl and Voβ (1996) improved their work by extending the heuristic rules as well as combining PRBMs

with Tabu search to overcome local optimality. Otto and Otto (2014) formulated a general design principle on how to apply PRBMs to SALBP. Technical reports of PRBMs can be found in Otto et al. (2011).

Each task is assigned a *priority score S* according to the PRBMs. During the task assignment procedure, the task with the highest priority score is assigned first when all of the constraints are satisfied. In terms of the structure of the PRBMs, it can be distinguished between *elementary* rules and *composite* rules. Elementary rules are constructed by utilizing single attributes of the ALBP. Composite rules are the combination of elementary rules (Haupt, 1989). Elementary priority rules can also be classified according to the type of information they used to calculate the priority scores. When the task time is the variable, the rule is task-oriented. Otherwise, the rule is precedence-oriented (Otto et al., 2011). In this paper, we adopt 5 popular elementary rules, which encompass both precedence-oriented and time-oriented relations, as our building blocks for the algorithmic design. They are maximum task time (*T*), maximum *TdL*, maximum *TdS*, maximum *F* and minimum *L* (see notations in Table 1). Composite rules are a weighted linear sum of elementary rules. Elementary and composite rules can have single-pass or multi-pass attributes in terms of the number of solutions each individual rule generates. The application of PRBMs is to always assign the available task with the highest *S*. In case of a tie (two or more available tasks with the same priority score), another elementary rule or the task's natural order can be used as tie breakers.

Although PRBMs are widely used in the open literature, there is little attention paid to the PRBMs application in TALBP. Lapierre and Ruiz (2004) examined the performance of PRBMs in an industrial assembly line with two-sided and two-heights attributes. The PRBMs are shown to be efficient in solving the practical complex problem. However, the number of priority rules

(only rules related to task time are used) and problem size (only 1 problem) are quite limited to uncover the potential advantages or functionalities of PRBMs, let alone algorithmic design. In chapter 4, more rules (5 elementary rules, 90 composite rules) and problem sizes (34 instances) are involved in the in-depth analysis of PRBMs.

Examples: A 12 tasks example is described in Figure 13, c=7. TALBP1 is the problem to solve. Maximum F rule (choose the task with the maximum number of follower tasks) is used to assign tasks to stations. Initially, 3 tasks are available to assign, 1, 2 and 3. According to rule F, task 2 is assigned first to the right station of position 1 because it has the most number of followers (7). Task 5 (F=6) is assigned to the right station of position 1. In what follows, task 3 (F=4), task 1 (F=3), task 6 (F=3) and task 9 (F=2) are assigned to the left station of position 1. Task 8 is assigned to the right station of position 1. Two stations in position 1 are fully loaded, and position 2 is open. We assign task 4 (F=2), task 7 (F=1) to the left station of position 2. Finally, task 11 (F=1), task 10 (F=0) and task 12 (F=0) are assigned to the left station of position 2. As a result, the optimal number of positions is 2.

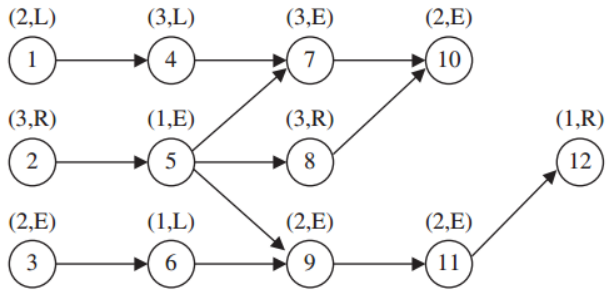

Figure 13 12 Tasks two sided assembly line

2.6 Literature Review on task attributes

In real world applications, task times may not be constant values. Instead, it can follow a function (Dynamic attribute) or can be random (Uncertain attribute) during the production. Even

when task times follow a traditional learning curve there may be considerable variability around the function (Vigil and Sarper, 1994; Globerman and Gold, 1997; Goldberg and Touw, 2003; Boucher and Li, 2016). In this research we consider two cases: 1) deterministic dynamic attribute and 2) random uncertain attribute change.

2.6.1 Dynamic task attributes

For this case, the task times always follow a predetermined function, namely, the learning curve. Such functions (Biskup, 1999; Yelle, 1979) are used especially for the formalization of learning and/or linear deterioration effects observed for workers (Boucher, 1987; Chakravarty, 1988; Cohen and Dar-El, 1998; Digiesi et al., 2009; Toksarı et al., 2008). In this dissertation, only traditional learning curve is considered for assembly line balancing and, therefore, the task improvement process is continuous. The traditional learning curve is formulated as follows.

$$t_v^{\,i} = t_v^{\,1} i^{\beta} \tag{16}$$

t_v^1 is the initial task time of task v, v=1...n

t_v^i is the task time of task v at the ith repetition

β is the learning exponent, $\beta = \dfrac{Log(\text{Learning Rate})}{Log(2)}$ (17)

This model is the dominant one in the cost estimating literature (Ostwald, 2010) and in textbooks on engineering economy (Newman et al., 2009; Sullivan et al., 2015). Therefore, it is the principal guide for practicing engineers to use in forecasting the path of task times when introducing new technology in production.

2.6.2 Uncertain task attributes

At times, the pattern of the changes of task time is unknown at the point the decision has to be made. In manual lines, the effectiveness of operators varies with work rate, skill level, and motivation, which may affect processing times. In automated lines, the technology improvement, process innovation and error detection will lower the task time, but the magnitude may not follow a deterministic function and the timing of events may be stochastic. There exist several ways to model the uncertain task time. Unlike the dynamic case, which the learning is always a continuous process, task learning under uncertain task attributes is sometimes a discrete process.

- *Distribution based modeling*: The task time is treated as a random variable with a given distribution. (Dolgui and Proth, 2010)
- *Fuzzy number based modeling*: Task is modeled as a fussy number with known membership function. This approach is used by Hop (2006), Zacharia et al (2012).
- *Scenario based modeling*: A possible set of scenarios of task time is given over a pre-specified planning horizon. The value of task time can be found in each scenarios. (Battaia and Dolgui, 2012;)
- *Interval based modeling*: The task time falls into an interval defined by the minimal and maximal possible values of the corresponding task attribute. Its definite value can be known only at the moment of line exploitation. Such task processing times were considered by Gurevsky et al.(2012b).

2.6.3 Recent research on ALBP considering non-constant task attribute

The assembly line balancing problem considering learning effect has been studied in the recent literature. Cohen et al. (2006, 2008) considered a work allocation problem which aimed to

minimize the makespan of production under homogeneous and varying learning curves. However, for the purpose of circumventing the combinatorial features of the line balancing problem, the traditional constraints that tasks are indivisible and that precedence relations exist were relaxed. Toksari (2008) presented a simplistic solution procedure to SALBP1 considering the reliability of products. However, task learning only takes place once, which assumes that it is independent of the production quantity. Otto and Otto (2014) provided the solution procedures for a lexicographic problem in order to increase the assembly efficiency and reduce the production ramp-up period considering that the dynamic attribute of task time follows a learning curve. Their optimization process first minimizes the number of stations, which outputs a set of solutions. From that solution set they select the optimal solution which renders the minimal cycle time among the set of solutions. However, task reassignment is not allowed during the production period. Only one task assignment configuration is used throughout the period of production even though task times are changing. Sotskov et al. (2015) performed a stability analysis on the various benchmark data sets that exist in the literature as a means to test the optimality of a solution if task times are subject to small variations. He shows that there exists a lot of unstable optimal solutions, i.e., an optimal solution for a given set of task times is not optimal if task times have changed slightly. Those results shed light on reassigning tasks during the period of batch production when task time improvement occurs in order to obtain optimal conditions for the overall production batch. In the thesis, we take account of this issue, which has not been addressed by the aforementioned papers. We present a production planning algorithm based on finding the optimal number of stations at each production cycle and we address the real-time reassignment of tasks during batch production. We show that the production efficiency is greatly improved compared with the line balancing problem where task reassignment is not considered.

2.7 Summary

In this chapter, a comprehensive literature review is provided. The characteristic of a standard data set and how it can be encoded in a computer program is introduced. The nature of the NP-hard problem structure requires fast and efficient algorithms. The algorithms solving the ALBP are presented and rules which can alleviate the computational burden of the algorithms are also discussed in details. Illustrative examples are given to show the solution procedure of algorithms. Finally, the prior work on ALBP considering non-constant task attribute are reviewed.

3. Simple Assembly Line Balancing Problem-1 with dynamic task time attribute

In this chapter, an efficient algorithm using backward induction is proposed to address the problem defined in section 1.5.6 (Li and Boucher, 2016). The algorithm implements a backward strategy so as to be capable of solving the problem without invoking the conventional Branch and Bound (BnB) procedure in every cycle. Task time will change according to a predetermined function, so that the solution offers a *planning schedule* of task assignments to workstations before the production begins. In what follows, three benchmark data sets are tested for the purpose of comparing the backward induction and conventional BnB procedure.

3.1 Conventional BnB procedure

The problem defined in section 1.5.6 can be addressed by iteratively implementing the conventional BnB procedure (see 2.4.3). The problem can be divided into a number of subproblems with respect to production cycles (repetitions). In each cycle, the input of task time will change according to a traditional learning function while the precedence relations are maintained in every repetition. A traditional learning curve function is described by Eq. (16).

Given that the plan is to produce a batch with total number of items *z*, the BnB algorithm for SALBP1 will be conducted starting from the first repetition and record the first optimal solution set (m_1^*, V_1^*). V^* stands for the optimal branches associated with the optimal solution m^*. The conventional algorithm is repeated until the last optimal solution set (m_N^*, V_N^*) is logged. Hence, the planning schedule for producing N items are (m_1^*, V_1^*) .... (m_N^*, V_N^*).

The formal steps of the conventional algorithm are as follows.

Step 0: Load the data, cycle time *c*, precedence graph *P*, task times of the first repetition t^1,

batch size *z*

Loop from 1 to *z* (*z* is the batch size or the total number of repetitions of the task)

Step 1: In each loop *i*, update the task times, t^i by Eq. (16)

Step 2: Invoke any of the BnB procedures, store the optimal solutions m_i^* and go back to step1.

The conventional algorithm treats each repetition independently and the BnB procedure is conducted in every loop. Although the algorithm exploits the lower bounds rules, dominance rules and reduction rules (Scholl and Becker, 2006) at different degrees as a means to reduce the enumeration effort of BnB, the computational cost is rather expensive as the number of tasks goes beyond a certain number because the complexity of the problem is exponential in the number of tasks (cf. Section 2.1.2). Presumably, the performance also deteriorates in batch sizes. Hence, the conventional algorithm will yield exact solutions to the task reassignment problem but would be very time consuming in large batch size. The conventional algorithm neglects the interrelations between tasks generated by the learning effect in distinctive production cycles. The interrelations underlying the adjacent solution sets (m_i^*, V_i^*) and (m_{i+1}^*, V_{i+1}^*) can be utilized to avoid the repetitive BnB effort, which gives birth to the novel backward induction algorithm developed here.

3.2 Line rebalancing schedule

We propose a rebalancing schedule which considers reassigning the tasks in each repetition in order to reduce the number of workstations. We assume that task times follows a learning curve which is characterized by Eq. (16). When a new solution (task sequence) exists, which results in fewer number of stations (from *m* to *m-x*) after producing *u*th product, *x* workstations are closed as unit *u* to *u+x* leave the assembly line. An implementation example will be shown in Section 3.5.

3.3 Backward induction algorithm

In this section the interrelations between the solutions in adjacent production cycles will be explored, and two versions of the backward induction algorithm regarding the learning curve with single and multiple learning rates will be proposed. As opposed to the conventional

algorithm, the merit of the proposed algorithm is to loop through the production cycles in a backward fashion and avoid invoking the branch and bound. As a result, the solution to the (i+1)th repetition will induce the solution to the ith repetition. Prior to developing the algorithms, two propositions are stated in order to justify the algorithm.

Proposition 1: If an optimal solution to ith repetition m_i is a feasible solution to jth repetition given i>j, it is also an optimal solution to jth repetition, $m_j=m_i$

Proof by contradiction: If $m_j \neq m_i$, it is either $m_j > m_i$ or $m_j < m_i$.

1) Contradiction of $m_j > m_i$

Because m_i is a feasible solution to jth repetition, m_i is also the current upper bound of the solution. The scenario $m_j > m_i$ implies m_j could not be the optimal solution which violates the statement that it is an optimal solution to jth repetition.

2) Contradiction of $m_j < m_i$

Given $i>j$, because of learning effect, the total task time of the ith repetition is always less than the total task time of the jth repetition in each workstation, $\sum_{r \in V^k} t_r^{\,i} \leq \sum_{r \in V^k} t_r^{\,j} \leq c \quad \forall\, k = 1...m_j$. Hence, the optimal solution to the jth repetition is a feasible solution to the ith solution. m_j serves as an upper bound for the ith repetition, which means m_i could not be an optimal solution to the ith repetition. The scenario $m_j < m_i$ contradicts the statement m_i is an optimal solution to the ith repetition. □

Corollary 1 (squeeze theorem): If an optimal solution to the (i+l)th repetition and the (i- l)th repetition is the same, $m_{i+1}=m_{i-1,}$ then it is also an optimal solution to the ith repetition.

Proof: Because the total sum of task times of a station $\sum_{r \in V^k} t_r^{\,i}$ is monotone decreasing in *i*, the optimal solution to the (*i*-1)th repetition m_{i-1} is also a feasible solution to the *i*th repetition. Then, m_{i+1} is also a feasible solution to the *i*th repetition. From proposition 1, m_{i+1} is also an optimal solution to the *i*th repetition. □

Proposition 2: The learning rate for all tasks is the same and m_i is an optimal solution to the *i*th repetition. Let *k* denote the station whose idle time is the least among all of the stations, idle time= $(c - \sum_{r \in V_k} t_r^{\,i})$. *k* is called the bottleneck station. The fact station k is infeasible (idle time <0) in the *j*th repetition is sufficient and necessary condition to render the solution m_i infeasible in the *j*th repetition given $j<i$.

Proof of Sufficiency: When any station, including k, is infeasible, it will result in the fact the current task assignment is infeasible.

Proof of Necessity: If m_i is infeasible in *j*th repetition, there exists at least one station whose idle time is less than zero. It will be shown below that idle time of station k must be less than zero.

Because the learning rate is the same for all tasks

$$\sum_{r \in V^h} t_r^{\,i} = \sum_{r \in V^h} t_r^{\,1}(i^{\beta}) = i^{\beta} \sum_{r \in V^h} t_r^{\,1} \quad \forall h = 1...m \qquad (18)$$

Also, because station *k*'s idle time is the least among all stations in *i*th repetition

$$\sum_{r\in V^k} t_r{}^i \geq \sum_{r\in V^l} t_r{}^i \quad \forall l = 1...m,\ l \neq k \tag{19}$$

$$i^\beta \sum_{r\in V^k} t_r{}^i \geq i^\beta \sum_{r\in V^l} t_r{}^1 \quad \forall l = 1...m,\ l \neq k \tag{20}$$

$$\frac{j^\beta}{i^\beta} \times i^\beta \sum_{r\in V^k} t_r{}^i \geq \frac{j^\beta}{i^\beta} \times i^\beta \sum_{r\in V^l} t_r{}^1 \quad \forall l = 1...m,\ l \neq k \tag{21}$$

$$j^\beta \sum_{r\in V^k} t_r{}^i \geq j^\beta \sum_{r\in V^l} t_r{}^1 \quad \forall l = 1...m,\ l \neq k \tag{22}$$

$$\sum_{r\in V^k} t_r{}^j \geq \sum_{r\in V^l} t_r{}^j \quad \forall l = 1...m,\ l \neq k \tag{23}$$

$$c - \sum_{r\in V^k} t_r{}^j \leq c - \sum_{r\in V^l} t_r{}^j \quad \forall l = 1...m,\ l \neq k \tag{24}$$

□

Hence, station *k*'s idle time is also the least among all stations in the *j*th repetition. As a result, it goes negative earlier than other stations as the repetition descends from *i* to *j* backward.

Proposition 1 implies that as the algorithm loops backward, pursuing optimality is a matter of proving feasibility and the process of checking feasibility is much less complicated compared to the process of searching for optimal solutions using the branch and bound at each cycle. Corollary 1 provides a different approach to solve SALBP1. The BnB procedure would be running very slowly under some task times combinations, i.e. it generates too many branches before the optimal solution is found. In this case, if the process of finding the optimal solution to the previous and next repetition is much quicker than directly pursuing the solution for the current repetition, the current solution should be obtained by corollary 1. Proposition 2 lays down the source of infeasibility and identifies the repetition number where the current solution first become infeasible when the learning rate is the same across all tasks. Since $\sum_{r\in V^k} t_r{}^1 j'^\beta$ is decreasing in *j*, we need to

find the biggest integer for which $\sum_{r\in V^k} t_r^1 j'^{\beta}$ is greater than *c*. Hence, according to Eq. (25), the biggest integer is $j = \lfloor j' \rfloor$; $\lfloor\ \rfloor$ is the floor sign.

By solving $\sum_{r\in V^k} t_r^1 j'^{\beta} = c$

$$j' = \left(\frac{c}{\sum_{r\in V^k} t_r^1}\right)^{\frac{1}{\beta}},\ j = \lfloor j' \rfloor \qquad (25)$$

Therefore, starting from the *i*th repetition, the BnB procedure will not be conducted until the backward loop reaches the *j*th repetition. However, if the assumption that the learning rate for all tasks is the same does not hold, proposition 2 is no longer valid. Actually, evidence shows that the learning parameter, β, increases as the task time increases, i.e. workers learn more slowly on the longer tasks (Boucher,1988; Lyon,1914). Hence, learning rate of a task is affected by its task times. We consider both cases where the learning rates are various or homogeneous and, as a result, a weak (Proposition 2 holds) and a strong version of the backward induction algorithm are proposed to solve the rebalancing problem below respectively.

Backward induction algorithm (Weak form)

Step 0: Load the data, cycle time *c*, precedence graph *P*, task time of the first repetition t^1, batch sizes z

Step 1: At the last repetition *z*, update the t^z and invoke BnB procedure. Store the value m_z and the bottleneck station k. Calculate j using Eq. (12), set *i*=*z*.

Step 2: Set $m_i = m_{i-1} \ldots = m_{j+1}$.Set *i*=*j*. At the *i*th repetition, update t^i by Eq. (16)

Step 3: Invoke BnB procedure and store the value m_i. Update the bottleneck station k. Calculate j using Eq. (12).

Step 4: If j=0, stop, all solutions are generated. Otherwise, go to step 2

Backward induction algorithm (Strong form)

Step 0: Load the data, cycle time *c*, precedence graph P, task time of the first repetition t^1, batch sizes *z*

Step 1: At the last repetition *z*, update t^z and invoke BnB procedure and store the value m_b. Set $i=z$.

Step 2: If $i=1$, stop, all solutions are generated. Otherwise, $i=i-1$, update t^i by Eq. (16).

Step 3: Check the feasibility of each station.

If $c - \sum_{r \in V^l} t_r^{\,i} \geq 0 \quad \forall l = 1....m$ store m_i and go back to step 2. Otherwise, go to step 4.

Step 4: Invoke BnB procedure and store the value m_i, go back to step 2 .

In relation to the conventional algorithm, both strong and weak versions of the backward induction algorithm have the advantage of not having to apply the BnB procedure in every loop. Under the condition that the learning rate for all tasks is the same, the weak version outperforms the strong version in terms of procedures updating all task times and checking the feasibility of all stations in every loop. However, the weak version is not applicable when tasks have different leaning rates. Given that proposition 2 holds, after the initialization which yields the solutions for the last repetition, the complexity of worst case scenarios for three algorithms are the same which is $O(|E_{total}|)$ where $|E_{total}|$ is the total edges of the trees developed by BnB procedure in all repetitions. However, the best case performance, in which case the current solution to the last repetition is optimal for all repetition, are different for three algorithms. The complexity of the

conventional algorithm remains the same, $O(|E_{total}|)$. The best case performance of the strong form backward induction is $z\times(m+n)$, i.e. it is the sum of the number of tasks updates and stations evaluations in all repetitions. The best case performance of the weak form backward induction is $m+1$, i.e. it takes m steps to find the bottleneck station k and 1 step to calculate j by Eq. (25).

3.4 Computational experiments

In this section, the performance of the backward induction algorithm is examined by comparing against the conventional algorithm. The performance measures are the total computer elapsed time, total number of branches developed and total number of BnB procedure skipped. The forward directional SALOME with maximum load rule and Jackson's dominance rule (Jackson,1956) is the BnB procedure invoked in the algorithms. All experiments are conducted on a Dell computer with a 2.5GHz processor. Three computational tests are programmed in Matlab. The codes are available in Appendix C.

3.4.1 Backward Induction (weak) and Conventional Algorithm

The learning rates for all tasks are assumed to be the same in this experimental design, and therefore the weak version of the backward induction algorithm is adopted. The learning rates are chosen for three levels as 0.9, 0.95, and 0.99 respectively. The two algorithms are compared on different batch sizes z=10, 20, 30. The reason for choosing this range of batch size is that the learning curve becomes flat after a certain number of repetitions, in which case the tasks time are approximately stationary. Hence, the optimal solution stays the same after a certain repetition. The results are tabulated for three data sets: Mansoor, 1964; Heskiaoff, 1968; Scholl, 1993. The performance measures are abbreviated as follows.

opt: the number of feasible or partial solutions (Branches developed)

#diff.opt: the difference in the number of branches developed between two algorithms

cpt: elapsed computer time for which an optimal solution is found and proven

#B: number of BnB procedures skipped

As can be seen from Table 5,6&7, the backward induction algorithm clearly outperforms the conventional algorithm in all measures. The results show that the computational effort increases in the size of the data sets, echoing the statement that the complexity of SALBP1 is exponential in section 2.1. It is also noteworthy that differences between the performance of the two algorithms are increasing in batch size. The reason is that as the batch size increases, the tasks times will gradually approach constant values which results in no change in the optimal solution to a big portion of repetitions in the end, e.g. m_{30} = ... =m_{10}=3 in section 2.3. Therefore, the backward induction algorithm does not have to invoke the BnB during these latter cycles.

Table 5 The performance of algorithms on Mansoor's data set (Mansoor,1964, cycle time is 48)

	Learning rate=0.99					
	z=10		z=20		z=30	
Algorithms	Conv.	Backward	Conv.	Backward	Conv.	Backward
#opt	10	3	20	4	30	4
#diff.opt	7		16		26	
cpt(sec)	0.42	0.25	0.55	0.27	0.58	0.29
#B	7		16		26	
	Learning rate=0.95					
	z=10		z=20		z=30	
Algorithms	Conv.	Backward	Conv.	Backward	Conv.	Backward
#opt	10	3	20	4	30	5
#diff.opt	7		16		25	
cpt(sec)	0.41	0.32	0.56	0.33	0.79	0.38
#B	7		16		25	
	Learning rate=0.90					

	z=10		z=20		z=30	
Algorithms	Conv.	Backward	Conv.	Backward	Conv.	Backward
#opt	15	14	25	16	35	17
#diff.opt	1		9		18	
cpt(sec)	0.54	0.54	0.72	0.59	0.73	0.61
#B	1		9		18	

Table 6 The performance of algorithms on Heskiaoff's data set (Heskiaoff,1968, cycle time is168)

	Learning rate=0.99					
	z=10		z=20		z=30	
Algorithms	Conv.	Backward	Conv.	Backward	Conv.	Backward
#opt	2307	1349	6669	2203	8459	2248
#diff.opt	958		4466		6211	
cpt(sec)	800.20	424.60	4396.00	933.37	5815.00	922.74
#B	4		12		21	
	Learning rate=0.95					
	z=10		z=20		z=30	
Algorithms	Conv.	Backward	Conv.	Backward	Conv.	Backward
#opt	54	54	68	67	78	74
#diff.opt	0		1		4	
cpt(sec)	5.70	5.40	6.30	6.23	8.92	6.84
#B	0		1		4	
	Learning rate=0.90					
	z=10		z=20		z=30	
Algorithms	Conv.	Backward	Conv.	Backward	Conv.	Backward
#opt	11	11	22	22	32	32
#diff.opt	0		0		0	
cpt(sec)	1.32	1.34	2.50	2.44	3.20	2.94
#B	0		0		0	

Table 7 The performance of algorithms on Scholl's (Figure 4) data set (cycle time is 10).

	Learning rate=0.99					
	z=10		z=20		z=30	
Algorithms	Conv.	Backward	Conv.	Backward	Conv.	Backward
#opt	40	8	80	8	120	8
#diff.opt	32		72		112	
cpt(sec)	0.74	0.41	1.25	0.42	1.63	0.59
#B	8		18		28	
	Learning rate=0.95					
	z=10		z=20		z=30	
Algorithms	Conv.	Backward	Conv.	Backward	Conv.	Backward
#opt	19	9	35	12	65	12
#diff.opt	10		23		53	
cpt(sec)	0.62	0.46	0.79	0.46	1.13	0.62
#B	7		16		26	
	Learning rate=0.90					
	z=10		z=20		z=30	
Algorithms	Conv.	Backward	Conv.	Backward	Conv.	Backward
#opt	17	10	27	10	53	13
#diff.opt	7		17		40	
cpt(sec)	0.65	0.43	0.69	0.46	1.03	0.53
#B	5		15		24	

3.4.2 Backward Induction (Strong) and Conventional algorithm

When the learning rates are different as among tasks, the strong version of the backward induction algorithm is adopted. The tasks are classified into three groups with regards to tasks time. Each group is given one of the three learning rates (0.9, 0.95, 0.99). The procedures in the experiment is the same as the procedures in section 3.1. The results are tabulated for the three experimental data sets.

The results show that the backward induction algorithm is more efficient than the conventional algorithm in all measures. As can be seen from the Table 10, the largest difference of performance resides in the Scholl's data set.

Table 8 The performance of algorithms on Mansoor's data set (cycle time is 48)

	z=10		z=20		z=30	
Algorithms	Conv.	Backward	Conv.	Backward	Conv.	Backward
#opt	10	3	20	3	30	3
#diff.opt	7		17		27	
cpt(sec)	0.28	0.25	0.35	0.31	0.37	0.33
#B	7		17		27	

Table 9 The performance of algorithms on Heskiaoff's data set (cycle time is 168)

	z=10		z=20		z=30	
Algorithms	Conv.	Backward	Conv.	Backward	Conv.	Backward
#opt	34	34	44	43	54	52
#diff.opt	0		1		2	
cpt(sec)	3.2	3.20	3.72	3.63	4.77	4.77
#B	0		1		2	

Table 10 The performance of algorithms on Scholl's (Figure 4) data set (cycle time is 10)

	z=10		z=20		z=30	
Algorithms	Conv.	Backward	Conv.	Backward	Conv.	Backward
#opt	53	13	86	21	108	23
#diff.opt	40		65		85	
cpt(sec)	0.82	0.40	1.23	0.45	1.77	0.57
#B	7		15		25	

3.5 Case study

In order to demonstrate the industrial application of dynamic reconfiguration as well as the use of the backward induction algorithm, an example is presented in this section. There are many assembly line configurations that can be designed in order to implement the reallocation of agents along an assembly line when learning is taking place and the number of stations is being reduced. In order to place our study within an industrial context, we suggest the configuration given in Figure 14. Here, assembly robots can attend either of two assembly line conveyors and are assigned to work on the product assembly along one assembly line at a time. This case study will assume that six stations are initially needed to make the assembly at the predetermined production rate, or cycle time, *c*. As fewer stations are needed for the particular assembly due to task time improvement, agents can be reallocated to work on another product assembly along the other conveyor.

Instructions (commands) to robots concerning what assembly steps to perform as work assignment and station allocation changes is the responsibility of the supervisory controller, a control program located in the supervisory control and data acquisition (SCADA) node computer. The SCADA node is the repository of learned improvements in task execution and can download all updated task information. The SCADA node is also the gateway to the factory information

system. Such a configuration for the factory network is a common architecture for highly automated manufacturing systems (Boucher, 1996, 2006).

Robots signal when they have completed their set of tasks to the supervisor. When all stations have reported completion, the supervisor signals the conveyor controller to index the line, moving assemblies to the next station and introducing the next assembly into the line. If the next assembly to enter the line requires a reconfiguration of the tasks at each station, the supervisor commands the robot at station 1 to perform the new task set. As this assembly moves along the line the supervisor updates each robot station with its new task set.

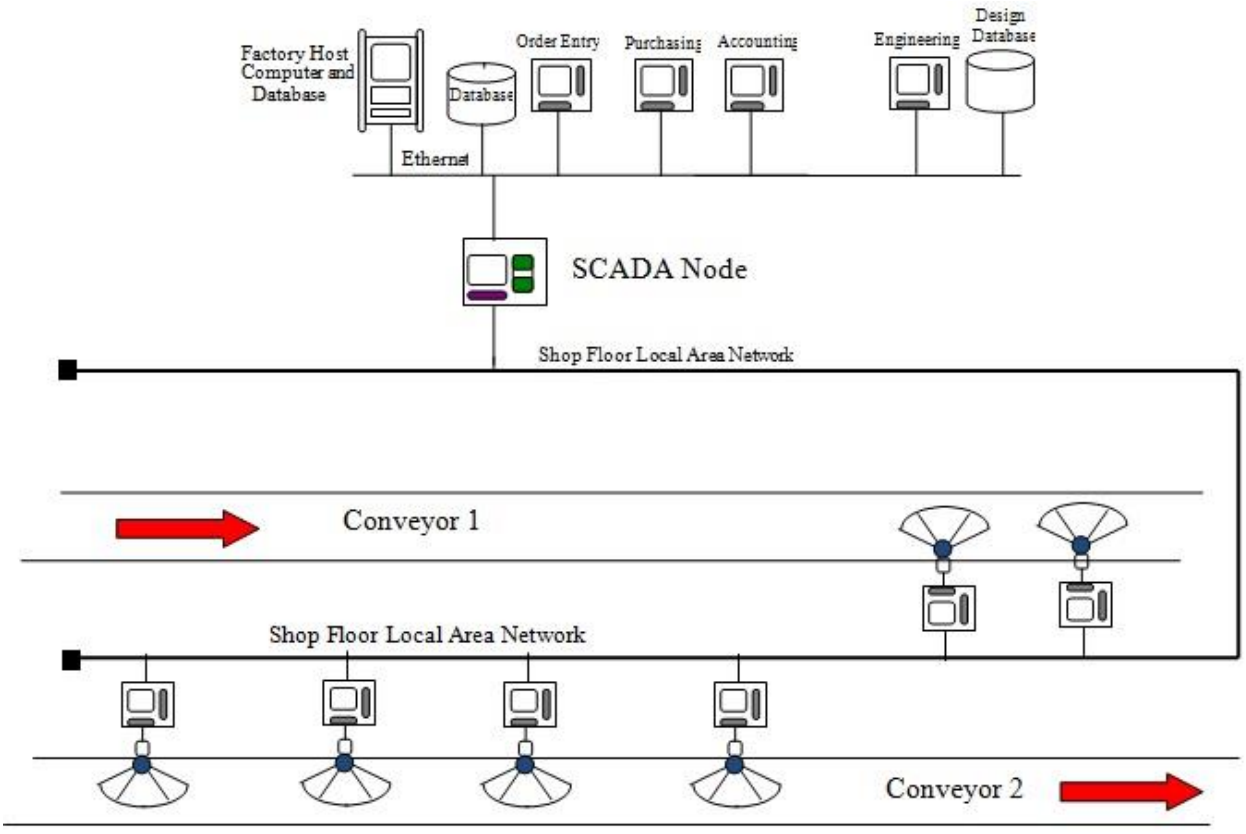

Figure 14 Illustrative Case Study System Architecture

With this physical configuration and industrial context in mind, in this section a simple case study is conducted to illustrate the improvement in production statistics as well as the function

of the backward induction rule (weak form) in reducing the number of optimization runs. The data set used here is from Scholl (2006) and displayed in Figure 14 with batch size b=30, cycle time c=10 and learning rate =0.85. The solution procedure is as follows.

In this section a simple case study is conducted to illustrate the improvement in production statistics as well as the function of the backward induction rule (weak form) in reducing the number of optimization runs. The data set used here is from Scholl (2006) and displayed in Figure 4 with batch size b=30, cycle time *c*=10 and learning rate =0.85. The solution procedure is as follows.

Step 1: It outputs t^{30}=[2.7, 2.7, 2.25, 2.25, 1.8, 2.25, 1.8, 0.9, 4.05, 0.9], m_{30}=3 with the task assignment ({1,3,4},{2,5,6},{7,8,9,10}). The bottleneck station is 3, j=9.

$$\beta = \frac{\log(0.85)}{\log(2)} = -0.234$$

$$j' = \frac{10}{(4+2+9+2)}^{\frac{1}{-0.234}} = 9.64$$

$$j = 9$$

Step 2: m_{30} = ... =m_{10}=3, update t^9=[3.58, 3.58, 2.99, 2.99, 2.39, 2.99, 2.39, 1.19, 5.37, 1.19].

Step 3: Optimization process begins and outputs m_9=4 with task assignment ({1,3,4},{2,5,6},{7,8,9},{10}). The bottleneck station is 1, j=7.

Step 4: m_9 = m_8=4, update t^7=[3.8, 3.8, 3.17, 3.17, 2.53, 3.17, 2.53, 1.27, 5.7, 1.27].

Step 5: Optimization process begins and outputs m_7=4 with task assignment ({1,3},{2,4},{5,6,7},{8,9,10}). The bottleneck station is 4, j=3.

Step 6: m_7= ...=m_4=4, update t^3=[4.64, 4.64, 3.86, 3.86, 3.09, 3.86, 3.09, 1.55, 6.96, 1.55].

Step 7: Optimization process begins and outputs m_3=5 with task assignment ({1,3},{2,4},{5,6},{7,8},{9,10}). The bottleneck station is 1, j=1.

Step 8:$m_3 = m_2$=5, update t^1=[6, 6, 5, 5, 4, 5, 4, 2, 9, 2].

Step 9: Optimization process begins and outputs m_1=6 with task assignment

({3,4},{1,5},{2,7}, {6,8},{9} {10}) and j=0. Stop.

The results state that tasks should be reassigned as the 2nd, 4th and 10th units enter the line. The rebalancing process includes 6 actions as shown in Figure 15. In the first action, the first reassignment occurs when producing the 2nd unit because the task time reduction caused by the production of the 1st unit is big enough to reduce the number of station to 5. In the second action, the second reassignment occurs when producing the 4th unit. In the third action, unit 1 and 2 leave the line simultaneously and the sixth station is closed thereafter. In the fourth action, station 5 is closed as unit 3 and 4 leave the line. Then, in the fourth action the third reassignment occurs when producing the 10th unit. Lastly, station 10 is closed as unit 9 and 10 leave the line. In order to compare the improvement for the production statistics and the performance of the backward induction rule, the following terms are defined.

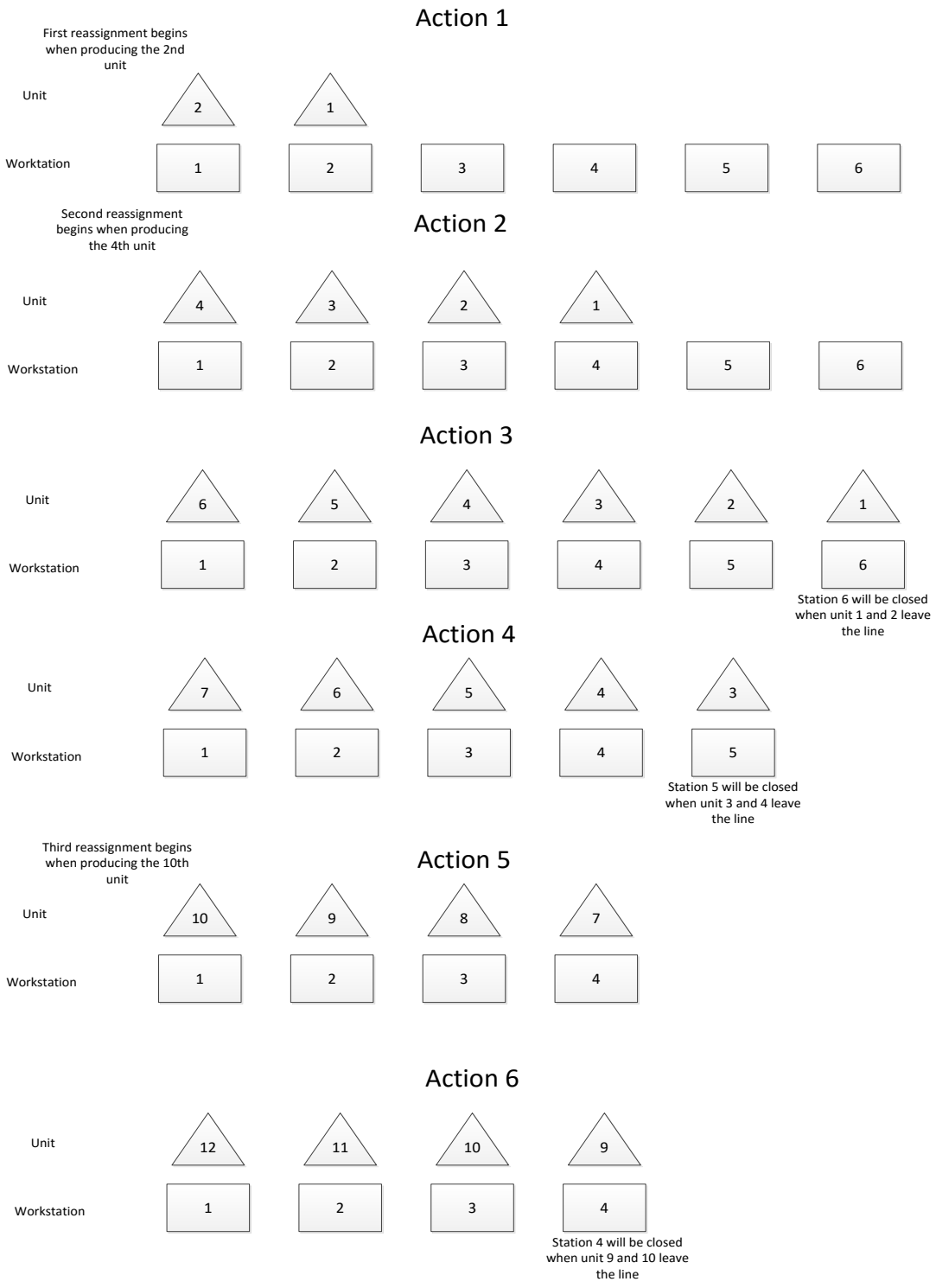

Figure 15 Actions in the rebalancing process

In order to compare the improvement for the production statistics and the performance of the backward induction rule, the following terms are defined.

TI: The Total idle time. It measures the stations utilization.

TM: The total number of operating stations. It measures the reductions of the length of the assembly line which indicates the operational costs. (When a station is closed, the corresponding robot can be switched off). TM= $\sum_{u=1}^{30}$(the number of stations unit u has been through). # O: The number of optimization procedures omitted.

The production statistics are shown in Table. 11. The improvements are very obvious. The total idle time and total number of stations operated have been reduced by 31.96% and 42.78% respectively. The backward induction rule enhances the rebalancing procedure by omitting 25 out of 30 optimization processes. The table is generated by Matlab program and codes are in Appendix D.

Table 11 Production statistics

	Total Idle Time, TI	Total # of Operating Stations,TM
Without rebalancing	348.12	180
With rebalancing	236.87	103
#O	25/30	
Percentage Reductions in TI	31.96%	
Percentage Reductions in TM	42.78%	

Furthermore, we graphically show the evolution of total task times along with the reassignment schedule as we increase the batch size to 1000 in Figure 16. As can be seen, there

are another two opportunities to further reduce *m* to 2 and 1 as 50th and 805th unit leave the line respectively.

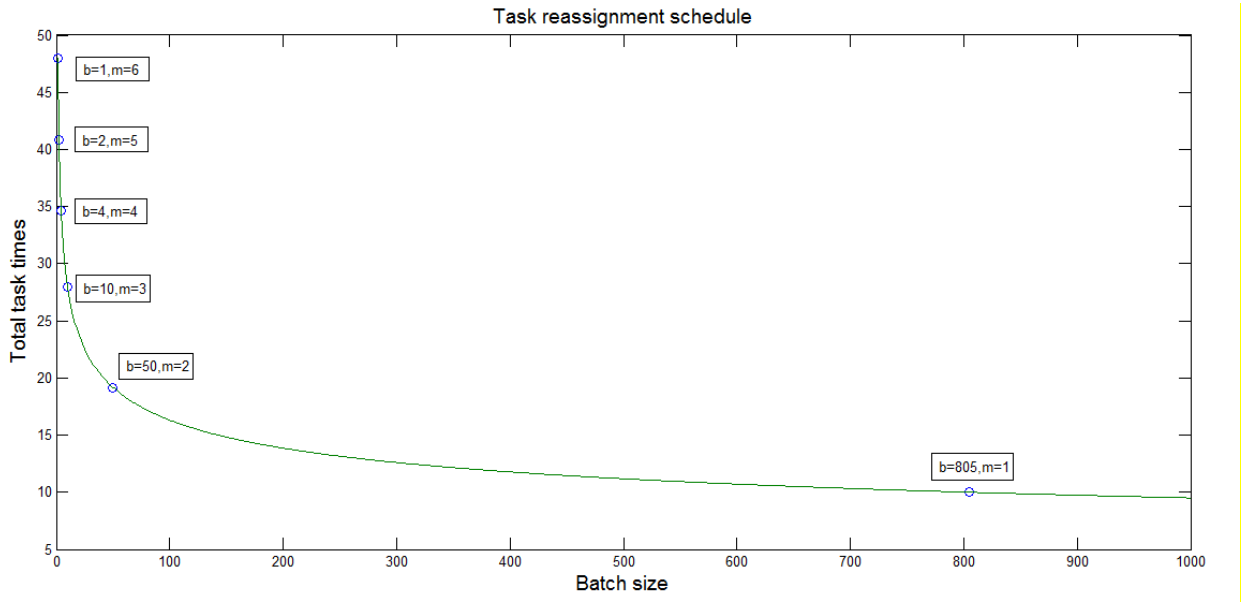

Figure 16 Task reassignment schedule for b=1000

3.6 Summary

In this section, we presented two backward induction algorithms which are capable of providing a production planning scheme by solving the task reassignment problem efficiently in assembly lines in which task times are reduced through learning and learning is conserved when tasks are reassigned amongst stations. The algorithms developed for line balancing type 1 problem under dynamic task time is able to avoid enumeration of the branch and bound by exploiting the properties of the learning curve and the bounds theories of the assembly lines. Computational results show that the backward induction algorithms clearly outperform repetitive use of the conventional BnB algorithm which requires enumeration at all production cycles. Furthermore, the discrepancies between the backward induction algorithms and conventional algorithm are widened by the increasing batch sizes, as the changes in task times are smaller when batch size is

beyond a certain number. A case study is presented to explain the context in which dynamic assembly line reconfiguration would apply and a comparison without reconfiguring the line is made. As research and application on flexible assembly systems with machine learning progresses, these methods will help support the efficient planning of the use of resources by taking into consideration optimal deployment over a range of task time reductions.

4. Simple Assembly Line Balancing Problem-2 with non-constant task time attribute

In this section, Simple Assembly Line Balancing Problem-2 (SALBP2) with non-constant task time attribute is addressed. It focuses on an assembly line where significant reductions in task times occur intermittently and the amount of reduction is random. In these cases, the system may respond by reassigning tasks among workstations if new combinations of tasks reduce the cycle time of the line. Unlike SALBP1, SALBP2 treats the number of stations as given and adjusts the efficiency of the line by reassigning tasks among stations. An efficient algorithm—ENCORE, which leverages the traditional algorithm SALOME2, is proposed to address the problem of computing task reassignment in real time as task times change. The algorithm is designed to rebalance the assembly line during the production once the task time changes are observed. Therefore, the application is not in planning, but in real time response to changing task times.

We assert that the efficiency of the assembly line and the cumulative production time are improved by dynamically reassigning tasks when appropriate. ENCORE is compared with SALOME2 for solving some small and medium data sets in order to demonstrate the improvement in computational efficiency. Then we design an experiment to empirically show the superiority of ENCORE over SALOME2 in different degrees of time limits on large sized benchmark data sets.

4.1 Rebalancing schedule with non-constant task time attributes

Modeling the improvement of the task time is important for creating the rebalancing schedule. Assuming that the task time follows a conventional learning curve as is characterized by Eq. (16), it requires two data points to solve for the two parameters (β and t_v^1), and data points can be collected by running two cycles of production. After the task times for the first and second cycles have been obtained, the parameters can be solved and the task times for the following cycles

can be projected by Eq. (16) and the rebalancing scheme can be made thereafter. If task times for the first two units are used to estimate the learning parameters, the first task reassignment will begin when assembling the third unit. The subsequent task reassignment will begin, when appropriate, in every *m* cycle. The rebalancing process under dynamic task time attribute is described in Figure 17. This is similar to the process described in Chapter 3 except that the reconfiguration involves the reassignment of tasks among stations, not the reduction in stations as was done in Chapter 3.

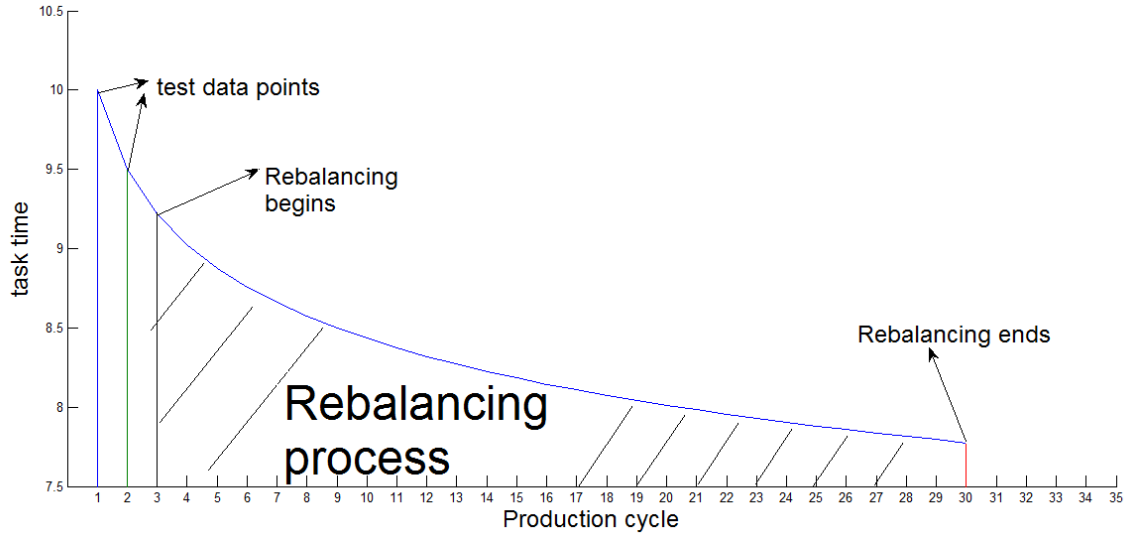

Figure 17 Task reassignment under the dynamic task time attribute

More realistically, the occurrence of task time improvements is uncertain and discrete meaning that the improvement does not always happen in every production cycle. The process of learning automata can result in improvements that are occasional and not consistent in magnitude. In that case, the decision on how to reassign tasks is going to be made right after the task time changes have been realized. We assume the assembly line is paused until a new solution is generated. The rebalancing process is described in Figure 18. When one or more task times change,

the supervisory controller computes the most efficient task assignments during the period of "Decision Making" in Fig. 18.

Efficient solution methods for rebalancing the line must be developed in order to reduce the total production time for a given production quantity. In this research we introduce Efficient Non-COnstant task time REbalancing (ENCORE) which leverages the SALOME2 algorithm by utilizing the solution structure that was optimal immediately before the changes in task times occurred. The ENCORE approach addresses this rebalancing problem under uncertain and discrete task time attribute changes.

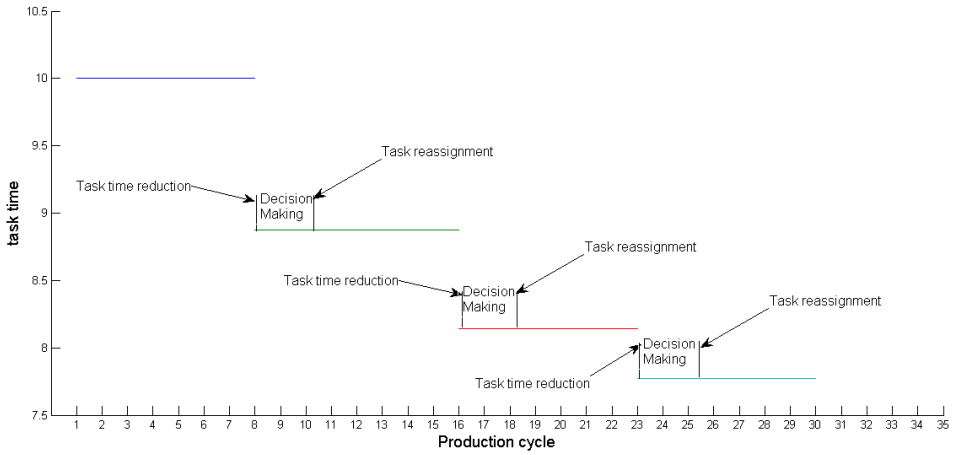

Figure 18 Task reassignment under uncertain and discrete task time improvements

Task reassignment results in reduced assembly line cycle times, which leads to the improvement of the overall production time. Figure 19 illustrates the solution structures of the problem we are addressing and its relationship to the traditional problem in which task times are constant and the cycle time is fixed throughout the production run. As task times change over an assumed lot size of 30, the optimal solution at each cycle may change. In this illustration the optimal cycle time is 5 in all production cycles for the traditional problem based on using the initial

task times. The optimal cycle time is changing as ENCORE uses improved task times to reconfigure the line, and the changes occur when producing 7th, 14th, 24th and 27th unit. The improvement in the solution is depicted in the hatched area.

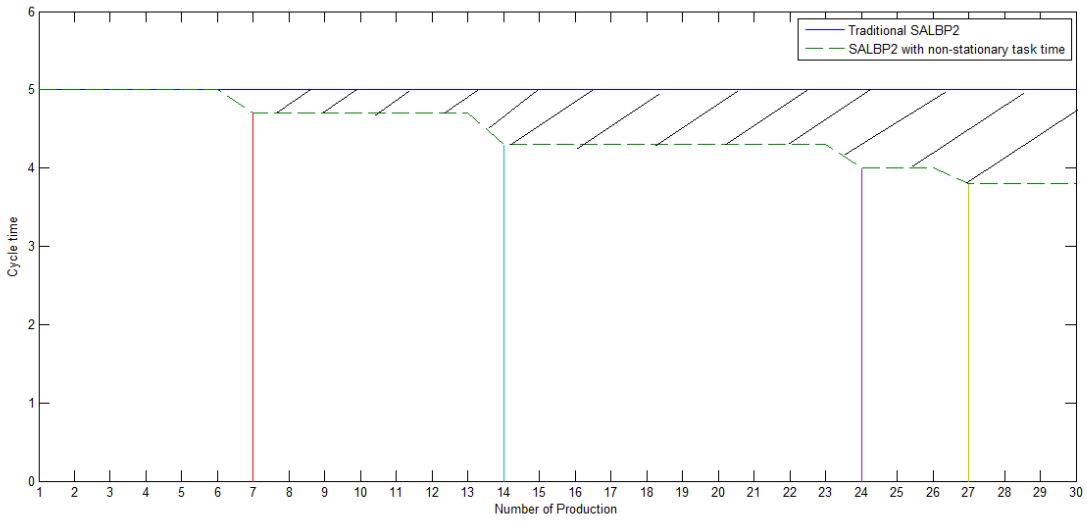

Figure 19 The solution structure to a simple SALBP2 problem with non-constant task time and a lot size of to 30

4.2 A BnB based exact solution procedure to solve SALBP2 with non-constant task time

In this section, ENCORE is developed at length and is compared with the conventional algorithm which solves the SALBP2 problem by iteratively using SALOME2.

4.2.1 A conventional algorithm

From the structure of the new problem as described in Section 1.5.7, it is possible to treat the new problem as *z* independent traditional problems and use a conventional method to solve it. As has been stated in Section 2.4.4, the SALOME2 is the conventional method used to solve the traditional problem. Hence, we adopt it for the iterative solution to our new problem structure.

SALOME2 Conventional Algorithm:

Step 0: Load the data, cycle time c, precedence graph P, task time of the first repetition t^1, lot sizes z (total number of repetitions of the task)

Loop i from 1 to z

Step 1: In each loop i, collect the new task time t_v^i

Step 2: Invoke SALOME2 with the new set of task times, store the optimal solutions c_i^* and go back to step1.

The complexity of the traditional assembly line balancing problem grows exponentially in the number of tasks (Scholl, 1993; Posypkin et al., 2006), which requires fast computational speed and a great amount of memory to generate the exact solution when the number of tasks is beyond a certain number. The exponential complexity of SALOME2 is going to further increase linearly when solving the new problem with the conventional algorithm. That is to say, if the time to solving a single SALBP2 problem is equal to T, the new problem needs approximately $z \times T$ time units to find a solution using the traditional algorithm.

4.2.2 ENCORE

The iterative use of the conventional algorithm simply considers the new problem as multiple independent subproblems, which ignores the link between subproblems. Actually, the structure of the precedence relations between tasks stays constant in every subproblem, so subproblems are not independent. Furthermore, if the task time follows a preordained learning curve, the changes of the task times in a pair of consecutive subproblems might be relatively small. As a result, the solutions for two adjacent subproblems may be similar, which suggests solving the current subproblem by dynamically adjusting the optimal solutions of former subproblems without invoking the SALOME2 algorithm over and over again. Hence, an Efficient Non-COnstant task

time REbalancing algorithm—*ENCORE*, which leverages SALOME2, is proposed to improve the solution time and effort of the conventional algorithm.

Considering the optimal solution V_i^* is solved for the current cycle *i* and a new set of task times are observed, ENCORE will seek the new optimal cycle time for next production cycle *i*+1 by exploiting the current solution in a real-time fashion. There are two phases of cycle time reduction in the optimization procedure of ENCORE: *Line-oriented and Station-oriented cycle time adjustment.*

Line-oriented cycle time adjustment: In the line-oriented phase of ENCORE, the cycle time of the line is instantly reduced *without* reassigning tasks to workstations. Let c'^*denote the optimal solution of SALBP2 when reassignment is not allowed. Let *b* denote the bottleneck station. c'^* is always equal to the total task time of the bottleneck station *b*.

Therefore, as the current optimal solution $V_i^*$is applied to the new set of reduced task time data in production cycle *i*+1, the stations are always idling, and the cycle time c_i^* can be reduced to $\sum_{v \in V_b} t_v = c_{i+1}'^*$ which is the total task time of the bottleneck station. The value of $c_{i+1}'^*$ serves as a current upper bound for (*i*+1)th production cycle.

Station-oriented cycle time adjustment: After the computation of the reduced cycle time in the line-oriented phase, ENCORE will relocate tasks among stations in order to minimize the cycle time. In this phase, the BnB procedure is utilized to ultimately find the optimal c_{i+1}^*. Two essential terms are defined before we explain the BnB procedure.

TOI and $TOIL_k$: TOI, total idle time of stations on the assembly line =mc-t_{sum}. $TOIL_k$, total idle time left at station k=$TOI - \sum_{j=1}^{k-1}(c - \sum_{v \in V_j} t_v)$. TOI is increasing in *c*. Hence, it is an indicator of the amount of idle or slack time of one solution over another solution. TOI also implies how much

idle time is remaining for the current feasible solution. TOIL serves the purpose of verifying the feasibility of a partial solution. If $TOIL_k$ turns negative at some station when assigning tasks under the current cycle time, then the partial solution consumes more idle time than it should in order to maintain feasibility. As a result, current cycle time is no longer feasible and should be abandoned.

Backtracking: There are two types of backtracking contingent upon the outcome of the branching process. The first type of backtracking starts from a developed branch which represents a full solution *V* and traces back to the very *first* bottleneck station. The second type of the backtracking starts from a *partial* solution which is proved to be inferior to the current optimal solution and traces back to the *previous* station *j* where TOIL is still positive. In what follows, all of the stations succeeding *j* (including *j*) are emptied, and the tasks belonging to them are available to be reassigned.

Branching: In the branching process, a task *v* can only be assigned to a station k when all of the following conditions are satisfied: 1) All predecessors of *v* have already been assigned to the current or previous station, 2) The total station time must be *strictly less* than the current upper bound $c_{i+1}'^{*}$ after task *v* is assigned, and 3) If task *v* is the last task of a station k, $TOIL_{k+1}$ must be nonnegative after task *v* is assigned. Condition 1 insures that the precedence relation is not violated. Condition 2 guarantees the improvement of the current upper bound $c_{i+1}'^{*}$ while the new partial solution is being developed. Condition 3 indicates the feasibility of the branching before a partial solution is evolved to a full solution and, therefore, saves some enumeration efforts. If there exist no task combinations because of the violation of condition 3 (condition 1&2 must hold), increasing cycle time should be considered in order to maintain the feasibility of the on-going branch. The trial cycle time is the smallest non-overlapping station load which makes the $TOIL_{k+1}$

nonnegative. If the trial cycle time is less than the current upper bound $c_{i+1}'^{*}$, the trial cycle time is termed the temporary upper bound c^{temp}, and TOI and $TOIL_{k+1}$ are updated accordingly. Otherwise, the branching process should be stopped, and the backtracking process will begin.

Bounding: The branching process will be terminated and return to the backtracking process once the total station times are equal to or greater than the current upper bound. Otherwise, the c^{temp}, TOI, $TOIL_{k+1}$ are updated and the branching process continues to the next station. Finally, a new solution is formed, and the new optimum is set to c^{temp}, $c_{i+1}'^{*}=c^{temp}$,

The BnB procedure will continue until the algorithm traces back to the root node or the current upper bound, $c_{i+1}'^{*}$, is equal to the global lower bound LB. LB=max (t_{max} ,t_{sum}/m) (Klein and Scholl, 1996). Then, $c_{i+1}^{*}=c_{i+1}'^{*}$ is the optimum for the new subproblem. The line-oriented and station-oriented cycle time adjustment procedures are repeated until optimal solutions for all production cycles are found. The formal representation of ENCORE is given as follows.

ENCORE:

Step 0: Initialization. Load the data set and set the lot size equal to z.

Step 1: Find the current optimal solution by the SALOME2.

Loop i from 2 to z.

Step 2: At the ith repetition, adjust all tasks times and set the current upper bound to the total station times of the bottleneck station. Calculate LB=max (t_{max} , t_{sum}/m). Compare the current cycle time with the LB. If they are the same, go to step 7. Otherwise, go to step 3.

Step 3: Trace back to the first bottleneck station (type 1 backtracking), and start the branching process. If a branch is fully developed, the new current upper bound is obtained, go to step 6. Otherwise, go to step 4.

Step 4: Trace back to the previous station (type 2 backtracking). If it traces back to the root node, the current upper bound is the optimum, go to step 7. Otherwise, go to step 5.

Step 5: Start branching process. If a branch is fully developed, the new current upper bound is obtained and go to step 6. Otherwise, go back to step 4.

Step 6: Compare the current upper bound with LB. If they are the same, go to step 7. Otherwise, find the new bottleneck station, then go back to step 3.

Step 7: If $i=z$, stop. Otherwise, $i=i+1$, return to step 2.

The full course of SALOME2 has only been invoked once in the optimization process (Step 1). Step 2 corresponds to the line-oriented cycle time adjustment. Step 3 includes the first type of backtracking process and the branching process. Step 4 is the second type of backtracking process. Step 5 is another branching process. In step 6, a branch is fully developed and a superior solution is generated. The algorithm utilizes the global lower bound twice (Step 2 and 6) which efficiently alleviates or even avoids the BnB procedure. The basic flowchart of the ENCORE is given in the Appendix B.

4.2.3 An illustrative example

An example is provided to demonstrate ENCORE. The data structure of the example is shown in Figure 20. The upper frame of Figure 20 shows the task times prior to improvement. Then, as can be seen in the lower frame of Figure 20, task 4 and 5's task times are adjusted to 3.5

and 3, respectively. It is assumed that those improvements occur on the assembly due to learning automata. The exact solution procedure is given as follows.

Step1) Given m=4, by SALOME2, the optimal cycle time is 9 (step 1 in the algorithm). The optimal task assignment is given in Figure 21. The numbers in the parenthesis represent the tasks which are assigned to the designated station. The station loads (total station task times) are shown in the oval.

Step 2) According to step 2 which is line-oriented cycle time adjustment, the bottleneck station as shown in Figure 22 is station 3 with total task time equal to 8.5. The current upper bound is 8.5. The lower bound is computed as follows: LB=*max* (7, 29.5/4)=7.375, Then, the algorithm goes to step 3.

Steps 3) to 6) is to form a tree structure as shown in Figure 23. Besides the information of task assignment and station load, TOIL is computed and shown on the right side of the task assignment above the arc.

Step 3) According to step 3, the first bottleneck station is station 3, hence the algorithm traces back to station 3 and empties stations 3 and 4. Since task combination 4&6 is the only choice based on precedence requirements and has already been considered in the tree structure, the algorithm goes to step 4.

Step 4) According to step 4, the algorithm traces back to station 2. It is not the root node, so the algorithm goes to step 5.

Step 5) According to step 5, tasks 2 and 4 are assigned to station 2, the bottleneck station is the second station with time 4+3.5=7.5, TOI=4×7.5-29.5=0.5, TOI_3=0.5-(7.5-7)-(7.5-7.5)=0. Then, temporary upper bound is set to 7.5 and station 3 is to be assigned tasks. However, we cannot find any combination of tasks that comply with the three conditions, the increase of the temporary

upper bound shall be considered. The temporary upper bound is increased to 8 by assigning task 5 and 6 to station 3, which is the smallest station load that makes $TOIL_4$ non-negative. Lastly, task 7 is assigned to station 4. A new branch is successfully branched (node 1-2-6-7-8), the algorithm goes to step 6.

Step 6) According to step 6, the bottleneck station is station 3 with total task time 8. The current optimum is set to 8. Since 8 is larger than the LB (7.375), the algorithm goes to step 3. Analogously, two other branches are developed. However, they are proven to be inferior to cycle time 8 during the branching process. In this simple example 4 branches are developed in order to find the optimum equal to 8.

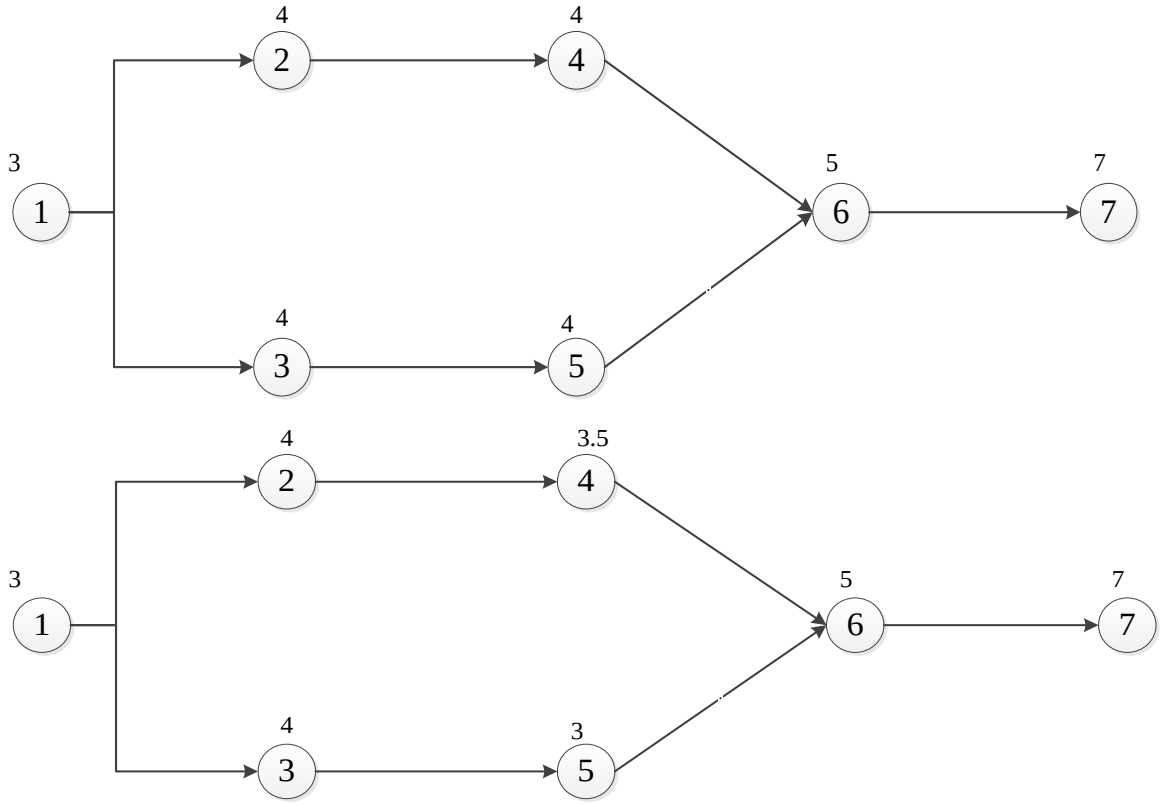

Figure 20 Simple example. Top: original problem. Bottom: new problem

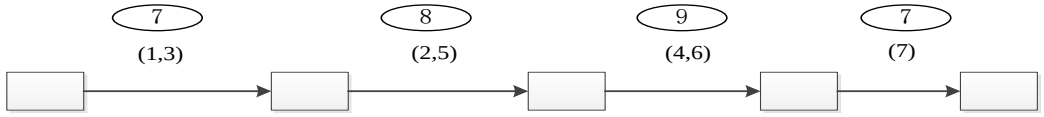

Figure 21 Solution of the original problem (optimal cycle time=9)

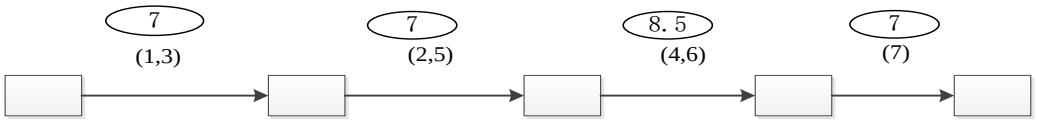

Figure 22 Line-oriented cycle time adjustment procedure (current upper bound=8.5)

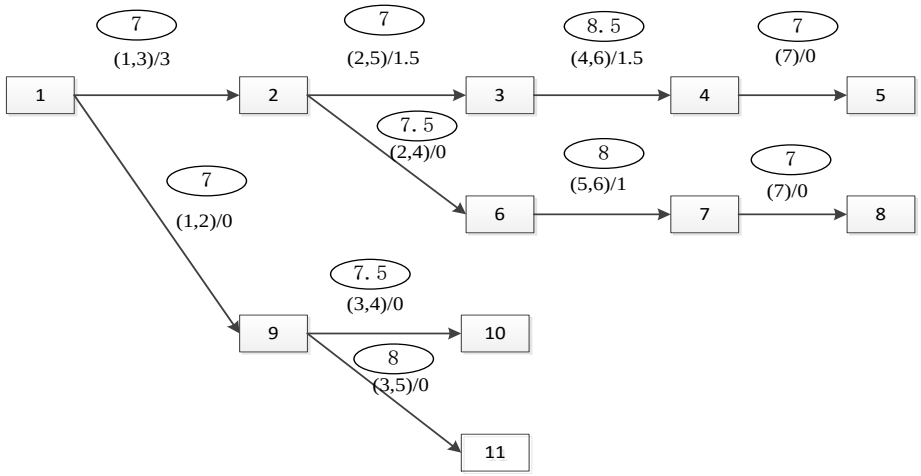

Figure 23 Station-oriented cycle time adjustment procedure (optimal cycle time=8)

4.2.4 The comparison between the conventional solution and ENCORE

The Conventional algorithm and ENCORE share some of the same characteristics. In some sense, ENCORE can be treated as an upgraded version of SALOME2 for solving the line rebalancing problem. The conventional algorithm will invoke the SALOME2 in every subproblem to generate the optimal solution, while SALOME2 will only be invoked once in ENCORE. In addition, they both use the same bounding condition to terminate and verify a solution in the BnB process.

The starting point of the two algorithms are different. The conventional algorithm starts from the root node and begins developing a feasible solution in the forward direction, while ENCORE starts from the leaf node and goes straight to the backtracking phase with a feasible solution which is close to optimal solution in most of the cases in our computational studies.

Within the BnB process both algorithms verify the value of TOIL to determine the feasibility of a developing partial solution. However, SALOME2 is based on the idea of the *local lower bound,* which is the best theoretical value of the cycle time after the current branch has been developed (Klein and Scholl, 1996). The local lower bound is constructed during the branching process and is used as a locator in the backtracking process, i.e. the SALOME2 traces back to the station where its local lower bound is less than the current upper bound. Instead, ENCORE creates a concept of *temporary upper bound*, which is used to determine the superiority of a developing branch. The local lower bound in SALOME2 is constructed based on the task times and the dominance relationship between tasks. Once task times reduction has been realized, the construction scheme is likely to change. As a result, the original local lower bounds will lose their functionality in the BnB process because it no longer indicates the best theoretical cycle time for each station and, therefore, should be abandoned. The complexity of the two algorithms can be demonstrated by their best and worst case scenarios. After the initialization, which yields the solutions for the last repetition, the complexity of the worst case scenario for the two algorithms are the same, which is $O(|E_{total}|)$ where $|E_{total}|$ is the total edges of the trees developed by BnB procedure in all subproblems. However, the complexity of best case scenarios for ENCORE occurs in the cases where the current solution to the last repetition is optimal for all repetitions. The complexity of the conventional algorithm remains the same, $\Theta(|E_{total}|)$. As for ENCORE, it only takes $z\times(m+n)$ steps to verify that the initial solution is the optimal solution, i.e. it is the sum of

the number of task updates and stations evaluations in all repetitions. In actual problems, the advantage of ENCORE should lie somewhere in between. It should be noted that ENCORE is an exponential time algorithm ($O(a^n)$) as well as is SALOME2, but the coefficient a of ENCORE is smaller than that of SALOME2, which reduces the computational time during its implementation. Therefore, ENCORE shall outperform the conventional algorithm in most cases.

4.2.5 Computational experiment

In this section, the performance of ENCORE is examined by comparing it to the conventional algorithm. The performance measures are the total computer elapsed time and the total number of branches developed. The conventional algorithm is the traditional SALOME2 used iteratively over cycles with maximal load rule and Jackson's dominance rule (Jackson,1956) embedded. All experiments are conducted on a Dell personal computer with 2.5GHz processor. The codes are available in Appendix E.

The attribute of the uncertain task time is selected to test the real-time application of the algorithms. The original problem sets are selected from Scholl (1993). The deduction of task times consists of two parts, a fixed part and a random part. The fixed part is equal to 2% of the task times in the previous subproblems, and random part follows a uniform distribution [0.8,1]. Deduction of task times is equal to the product between the fixed part and the random part. The total number of subproblems is 10, 20 and 30, which are the lot sizes for each data set. The randomized reduction of task times is applied at each cycle for every task. The results of comparing algorithm performance are shown in Tables 12, 13 and 14. The performance measures are abbreviated as follows.

opt: the number of full or partial solutions (Branches developed before all optimums are found), which is an indicator of the consumption of computer storage.

#diff.opt: the difference of #opt between algorithms

cpt: elapsed computer time (seconds) for which all optimal solutions are found and proven, which is an indicator of computer speed.

z: the lot size (the number of rebalancing executions)

Table 12 Computational performance between algorithms in Heskiaoff's data set (*n*=28) given *m*=8

	z=10		z=20		z=30	
	Conventional	ENCORE	Conventional	ENCORE	Conventional	ENCORE
#opt	248	63	563	156	843	235
#dif.opt	185		407		608	
cpt(sec)	9.83	3.24	21.32	7.82	33.96	12.75

Table 13 Computational performance between algorithms in Kilbrid's data set (*n*=45) given *m*=8

	z=10		z=20		z=30	
	Conventional	ENCORE	Conventional	ENCORE	Conventional	ENCORE
#opt	194	34	437	69	641	102
#dif.opt	160		368		539	
cpt(sec)	11.92	4.32	22.64	13.36	32.29	15.02

Table 14 Computational performance between algorithms in Arcus's data set (*n*=83) given *m*=8

	z=10		z=20		z=30	
	Conventional	ENCORE	Conventional	ENCORE	Conventional	ENCORE
#opt	3024	150	5087	320	8019	527
#dif.opt	2874		4767		7492	
cpt(sec)	874.26	202.43	1347.87	397.43	2210.43	578.22

ENCORE outperforms the conventional algorithm in computational speed and the computer storage for all three data sets. The cumulative difference in performances are increased as the lot size increases for each data set. Furthermore, the advantage of ENCORE is more obvious in the instances with more tasks (Arcus, *n*=83).

4.3 Design of Experiments

Our interest is to confirm ENCORE's advantage on production statistics over SALOME2 on benchmark data sets. Here we are interested in the industrial merit of the computational efficiency can be transferred into production efficiency. To make the study more practical, the large sized data sets are selected for analysis (Arcus (111 tasks), Bartholdi (148 tasks) and Scholl (297 tasks)). Totally 80 problem instances are available from Scholl (1993). Considering the objective of efficient task reassignment during the production run, a time limit will be posed to both algorithms to restrict the runtime. This constraint is imposed because the context of this problem is assembly line reconfiguration in real time once task time improvements are detected. The best solutions found during the time limit are used in the reassignments. Therefore, given the same amount of time, the better algorithm will produce less cycle time, which should lead to better production statistics.

4.3.1 Components of an experiment

We define the three main components of the experiments—Experimental units, Response and Variables. Each *experimental unit* is a single problem instance from the data sets, and instances are different from each other in at least one of following categories: precedence graph, task times and the number of stations. The intrinsic variance (within group variance) between instances may influence the design of the experiment and is inspected in a pretest to be described in a later section. The *response* is defined as the percentage production time (makespan) improvement in the trials employing ENCORE or SALOME2 to reassign tasks over the case with no reassignment. Eq. (26) shows the calculation of the response. The *variables* are selected algorithms (ENCORE or SALOME2), time limit for the algorithm to run (Rt), the number of learning episodes (Le) and the learning rates (Lr). The goal of this design of experiment is to determine whether the responses resulting from ENCORE are significantly different from the responses resulting from SALOME2 in terms of production makespan performance.

$$Response = \frac{T_{no\ reassignment} - T_{reassignment}}{T_{no\ reassignment}} \qquad (26)$$

Where $T_{no\ reassignment}$ is the total production time of producing *z* items without tasks reassignment

$T_{reassignment}$ is the total production time of producing *z* items considering tasks reassignment with ENCORE or SALOME2

The $T_{no\ reassignment}$ is equal to cycle time based on fixed task times multiplied by *z*. The cycle time can be found from Scholl (1993), so it is a fixed value as *z* is fixed. $T_{reassignment}$ is determined by all of the variables in the experiment, and it can be calculated in Eq. (27). To ensure a small and mass customized production process, we set *z* equal to 400 (Cohen, 2006).

$$T_{reassignment} = \sum_{i=1}^{Le+1} c_i \times PI_i \qquad (27)$$

Where PI_i is the number of cycles with cycle time c_i and L_e is the number of learning episodes.

As Rt increases, the amount of time allowed for the algorithm to find the optimum solution increases, and the current best cycle time decreases. The $T_{reassignment}$ of either algorithm is an aggregation of different cycle times of different learning episodes, so it decreases as well. Le is the number of times that the learning occurs during the production. We assume the occurrence of task learning is evenly distributed throughout the production process (i.e., if Le is 2 and *z* is 400, the task time is improved twice after producing the 134th and 267th unit respectively). For each interval of the production process, there is a different cycle time. Cycle time decreases in Le, and so does the total production time. The learning rate, Lr, is defined as the percentage of improvement of task time. The more Lr is, the less remaining task time is after each learning episode, which leads to less cycle time. Hence, with a selected algorithm, $T_{reassignment}$ decreases in Rt, Lr and Le with other factors fixed.

4.3.2 Pretest

We aim to determine the structure of the experiment which is comprised of the types of the design and levels of each variable. In order to test the impact of the algorithm selection on

performance, the other variables shall be examined as to their effect on performance in a pretest. First, we choose SALOME2 as a control variable and treat all variables as factors. We randomly choose 30 out of 80 problem instances and ran three regressions that pertain to each factor. For the responses that violate the assumptions of a linear regression, transformation techniques are employed in order to make the responses comply with the assumptions.

Time limit

We set Le to 2. To incorporate the heterogeneity of learning, each task has a Lr around 0.15. SALOME2 is used to generate the results. The levels of Rt are [1, 5, 10] seconds, because the real-time performance of the algorithms are our interests. The model results are shown below. As can be seen, Rt is not a significant factor at confidence level 0.05. The sum of squares error (SSE) is 0.1394 and sum of squares due to regression (SSR) is 0.0025. Fitting the data by a linear model is not appropriate because the intrinsic variance is too large relative to the explained variance (within group variance).

```
Linear regression model:
    y ~ 1 + x1

Estimated Coefficients:
                   Estimate    SE           tStat     pValue
    (Intercept)    0.036555    0.0065836    5.5524    2.9462e-07
    x1             0.001379    0.0010873    1.2684       0.20802


Number of observations: 90, Error degrees of freedom: 88
Root Mean Squared Error: 0.0398
R-squared: 0.018,  Adjusted R-Squared 0.00679
F-statistic vs. constant model: 1.61, p-value = 0.208
```

Learning episodes

Lr is chosen as it is in the pretest of time limit. Time limit is set to 1 second. SALOME2 is used to generate the results. The levels of Le are [1, 2, 3]. The model results are shown below. As

can be seen, Le is a significant factor at confidence level 0.05. The sum square of error (SSE) is 0.1189 and sum square due to regression (SSR) is 0.0241.

```
Linear regression model:
    y ~ 1 + x1

Estimated Coefficients:
                   Estimate      SE           tStat       pValue
    (Intercept)    -0.0048489     0.010251    -0.47301       0.63738
    x1               0.020029    0.0047454      4.2207    5.9061e-05


Number of observations: 90, Error degrees of freedom: 88
Root Mean Squared Error: 0.0368
R-squared: 0.168,  Adjusted R-Squared 0.159
F-statistic vs. constant model: 17.8, p-value = 5.91e-05
```

Learning rates

Rt and Le are set to 1 and 2 apiece. SALOME2 is used to generate the results. The Lr for each task is around 0.05, 0.1 and 0.15 respectively. The model results are shown below. As can be seen, Lr is a significant factor at confidence level 0.05. The sum square of error (SSE) is 0.1066 and sum square due to regression (SSR) is 0.1374.

```
Linear regression model:
    y ~ 1 + x1

Estimated Coefficients:
                   Estimate      SE           tStat      pValue
    (Intercept)    -0.013271    0.0097044    -1.3676       0.17493
    x1               0.95722     0.089845     10.654    1.6499e-17


Number of observations: 90, Error degrees of freedom: 88
Root Mean Squared Error: 0.0348
R-squared: 0.563,  Adjusted R-Squared 0.558
F-statistic vs. constant model: 114, p-value = 1.65e-17
```

From the pretest, the variance introduced by the structural differences between data sets (unexplained variance) is so big in magnitude compared with the variance introduced by the factors (Lr, Le and Rt) which dampens the effectiveness of modelling the relationships between the

response and multiple factors. Hence, the linear model shall not be adopted in the formal design of experiment. Instead, the one sample pairwise *t*-test is chosen as we test the two algorithms on the same instance for all problems and control the other 3 variables at different levels. Considering the production and learning process, we assign 3 levels (low, medium and high) to Lr, Le and Rt. Lr is chosen from [0.05, 0.1, 0.15], Le is chosen from [1, 2, 3] and Rt is chosen from [1, 5, 10].

4.3.3 One sample t test

We specify the null hypothesis and alternative hypothesis as follows.

H_0: The difference in the percentage improvement in production time between the two algorithms are the same

H_1: The percentage improvement in production time of ENCORE is greater than the percentage improvement of SALOME2.

We use the power of statistical test to determine the sample size. In order to collect the information to infer the sample size, a preliminary experiment is conducted. 16 problem instances are selected with Rt, *z*, Le and Lr equal to 5, 400, 3 and 0.05 respectively. The mean improvement of SALOME2 is 0.0685 and the standard deviation is 0.039. The mean improvement of ENCORE is 0.0889. Under one sample and one sided *t* test, we conclude that the sample size should be greater than 25 when we require the power to be greater than 0.8 and confidence level to be 0.95. Therefore, 25 out of 80 instances are randomly chosen as experimental units.

There are 27 separate *t* tests for all combinations of levels in Lr, Le and Rt. In each *t* test, two algorithms are examined on 25 instances respectively. Hence, a total of 1350 (27×2×25) runs are performed. We report the *p* value and summary statistics (mean, standard deviation and lower bounds of the 95% confidence interval) in Tables 15 and 16. As can be seen from Table 15, with Rt equal to 1 second, the mean percentage differences in response between ENCORE and

SALOME2 of almost all cases is above 1.5%. However, with Rt equal to 5 and 10 seconds, the improvements are not very obvious for all cases. Furthermore, from Table 16, the *p* value in bold means that the corresponding null hypothesis is rejected under confidence level 0.95. The null hypothesis is rejected 15 out of 27 times. The computer codes used in these experiments are documented in Appendix F.

Table 15 Summary statistics for each t test on response comparisons (Eq. 26)

Difference of improvements between algorithms											
			Time limit (s)								
			1			5			10		
N	Lr	Le	*M*	*SE*	CI	*M*	*SE*	CI	*M*	*SE*	CI
		1	0.0245	0.0087	0.0097	0.0076	0.0051	-0.0011	0.0083	0.0054	-0.0009
		2	0.0298	0.0103	0.0122	0.0084	0.0054	-0.0010	0.0088	0.0058	-0.0011
	0.05	3	0.0296	0.0095	0.0133	0.0119	0.0063	0.0011	0.0094	0.0059	-0.0006
		1	0.0155	0.0060	0.0052	0.0045	0.0026	0.0000	0.0041	0.0028	-0.0008
		2	0.0183	0.0063	0.0074	0.0084	0.0038	0.0019	0.0073	0.0038	0.0009
	0.1	3	0.0179	0.0063	0.0071	0.0107	0.0044	0.0032	0.0107	0.0044	0.0031
		1	0.0256	0.0092	0.0098	0.0061	0.0039	-0.0006	0.0031	0.0022	-0.0007
		2	0.0247	0.0087	0.0098	0.0053	0.0037	-0.0009	0.0041	0.0032	-0.0013
400	0.15	3	0.0214	0.0073	0.0088	0.0065	0.0038	0.0000	0.0060	0.0036	-0.0002

Table 16 p value matrix for each t test

*p*value		Time limit (s)			
			1	5	10
N	Lr	Le			
		1	**0.0047**	0.0733	0.0672
		2	**0.0040**	0.0689	0.0703
	0.05	3	**0.0023**	**0.0352**	0.0610
		1	**0.0083**	**0.0491**	0.0808
		2	**0.0042**	**0.0189**	**0.0316**
	0.1	3	**0.0047**	**0.0114**	**0.0120**
		1	**0.0053**	0.0669	0.0850
		2	**0.0045**	0.0785	0.1008
400	0.15	3	**0.0038**	0.0502	0.0557

The pairwise experiments empirically show that ENCORE has an advantage over SALOME2 in production statistics at low run time level. Practically speaking, low run time or near real time response is desirable in automated systems. The SCADA node and computer network may be controlling different assembly lines and production equipment simultaneously, and there may not be much time allotted for each individual optimization process.

The boundary (improvement difference or Rt) at which the null hypothesis is rejected can be explored by running several t test with different Rt while holding other parameters fixed. With the collected data points of *p* values and mean of improvements, the boundary can be interpolated. For instance, we choose z=400, Lr=0.1 and Le=1 for analysis. We run the experiment with Rt chosen from {1, 1.5, 2, 2.5, 3, 4, 5, 7, 8, 10}. The boundary for the improvement difference is 0.45% and for Rt is 5.24 seconds. We show the relationship of *p* value against improvement difference in Figure 24.

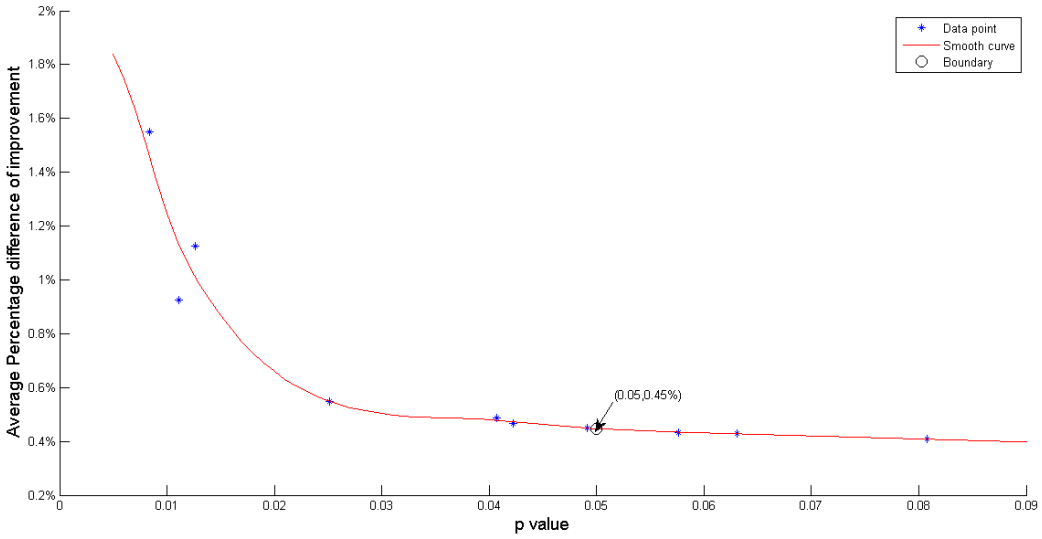

Figure 24 Relationship between *p* value and improvement (Spline interpolation is used to find the boundary)

4.4 Summary

In this chapter we demonstrate the rebalancing of the assembly line during small production run when the task time attribute is non-constant. A novel algorithm—ENCORE is proposed, in order to address the typical SALBP2 with non-constant task time attributes. Computational studies show that ENCORE clearly outperforms the conventional algorithm which iteratively solves the problem, making it a better method for application in real time automated systems.

The implementation of the rebalancing schedule is tested for assembly line problems under uncertain task time attribute. Pairwise *t* tests are conducted to show the significance of superiority of ENCORE over SALOME2 empirically at various computational time limits, appropriate for real time decisions in advanced automated systems.

5. Priority rules-based algorithmic design on two sided assembly line balancing

In this section, we develop a priority rules-based algorithmic design for optimizing a two-sided assembly line. Five elementary rules, ninety composite rules are tested on benchmark data sets and their performances are provided. Two enumerative principles which are specific to two-sided assembly lines are proposed to enhance the performance of the rules. Statistical analysis is provided to complement the studies. Further, priority rules are embedded into a bounded dynamic programing framework to form a novel algorithm where the use of a bound can reduce the solution space as the algorithm is advanced stage-by-stage. A new optimal solution is found and contributed to the existing literature. Furthermore, with the task learning considered, the production statistics generated by each rule are reported.

5.1 The investigation of PRBMs in TALBP

In this section, we examine the performance of 5 elementary rules on benchmark data sets obtained from Khorasanian et al. (2013). Totally 34 problem instances are examined. Then, we design task assignment principles which are TALBP specific to improve the performance of the elementary rules. Statistical analysis is provided to compare the performance of the elementary rule set, as well as the performance enhancements, before and after the principles are incorporated. Five elementary rules are paired with each other with different weights to constitute 90 composite rules which are examined on the benchmark data set as well. We denote Avg_dev, which is the percentage average deviation of the results found by PRBMs from the current best results in Khorasanian et al. (2013), as the performance measure for PRBMs.

5.1.1 Application of elementary rules

The elementary rules selected in this paper are maximum task time (*T*), maximum *TdL*, maximum *TdS*, maximum *F* and minimum *L*. The rules are self-explanatory or can be calculated from the Eq. (2) and (3) in section 1.4. Note, the calculations of the earliest (El_i) and latest (La_i) possible position to which task *i* can be assigned are shown in Eqs. (28) and (29). The enumerative procedure of elementary rules is depicted in the flow diagram (Figure 25). The *assignable task* is defined as the task which conforms to all constraints. In order to increase the efficiency of finding new assignable task sets, *only* the direct followers of the task just assigned to be added to the old assignable task set would be considered. Then, the old assignable task set can be updated by evaluating every task in the old set with regard to the cycle time constraints. When two assignable tasks have the same priority scores, their natural orderings are used to break the order.

$$El_i = \begin{cases} \left\lceil \dfrac{\sum_{r\in Pre^R} t_r + \sum_{l\in Pre^L} t_l + \sum_{e\in Pre^E} t_e}{2c} \right\rceil & if \sum_{e\in Pre^E} t_e > \left| \sum_{l\in Pre^L} t_l - \sum_{r\in Pre^R} t_r \right| \\ \left\lceil \dfrac{\max(\sum_{r\in Pre^R} t_r \ , \sum_{l\in Pre^L} t_l)}{c} \right\rceil & Otherwise \end{cases} \quad (28)$$

$$La_i = \begin{cases} UB + 1 - \left\lceil \dfrac{\sum_{r\in Suc^R} t_r + \sum_{l\in Suc^L} t_l + \sum_{e\in Suc^E} t_e}{2c} \right\rceil & if \sum_{e\in Suc^E} t_e > |\sum_{l\in Suc^L} t_l - \sum_{r\in Suc^R} t_r| \\ UB + 1 - \left\lceil \dfrac{\max(\sum_{r\in Suc^R} t_r , \sum_{l\in Suc^L} t_l)}{c} \right\rceil & Otherwise \end{cases} \quad (29)$$

Where

Suc^R, Suc^L, Suc^E are the sets of task *i's* successors whose operational directions are right, left and either respectively.

Pre^R, Pre^L, Pre^E are the sets of task *i's* predecessors whose operational directions are right, left and either respectively.

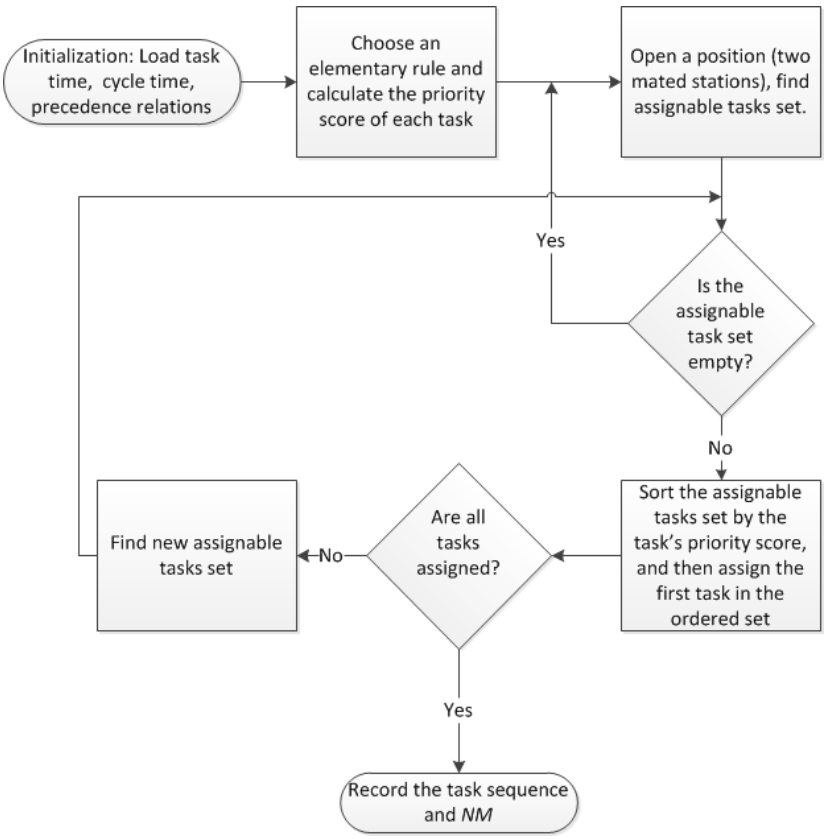

Figure 25 The enumerative procedure of elementary PRBMs

In Table 17 we present the average performance of the 5 elementary rules, where performance is defined as the average percent deviation from the best known solution. As can be seen, maximum *F*, though not obvious, is demonstrated to be the best elementary rule. Paired *t*

tests comparing the Avg_dev are conducted and *p* values are provided in Table 18. In line with the results in Table 17, among 10 paired comparisons, only the performance between rule *F* and *L,* where the most discrepancies occur, shows significant difference statistically. Hence, almost no rule has an obvious edge against others. Equivalently speaking, each rule has its own advantage. The Matlab codes to implement the elementary rules are in Appendix G.

Table 17 Average performance of the elementary rules

Rule	*TDS*	*TDL*	*T*	*L*	*F*
Avg_dev (%)	17.43	16.65	16.55	20.84	12.15

Table 18 p values of paired t tests of Avg_dev of elementary rules (95% confidence level)

	TDS	*TDL*	*T*	*L*	*F*
TDS	**1**	0.828	0.9759	0.2465	0.2186
TDL		**1**	0.8028	0.3548	0.1585
T			**1**	0.2288	0.2232
L				**1**	**0.0224**
F					**1**

As previously stated, the amount of idle time is the determinant of the quality of solutions. We notice that there is a great amount of idle time from delay relative to the idle time from remaining capacity after analyzing the structure of the solutions reported by the elementary rules. Further, we infer that the major cause for the delay is the mated station imbalance where the station load from one mated station is always higher than the other during the enumerative process. A simple example is shown in Figure 26.

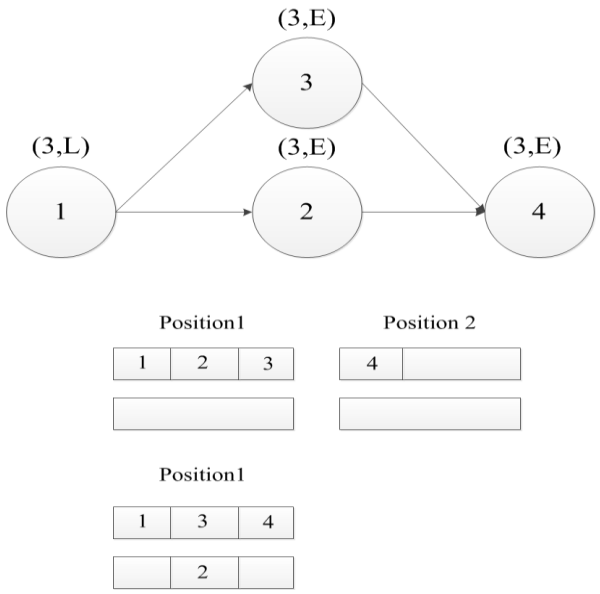

Figure 26 Top: 4 task data set; Middle: Solution 1; Bottom: Solution 2

Two solutions are provided based on two different enumerative principles respectively, which are prior to the PRBMs on determining the order of tasks in the assignable task set. The first principle is to assign the task which will minimize *delay* after the task is assigned (delay-oriented). The second principle is to assign the task to the mated-station whose *load* is smaller than the other (load-oriented). At first glance, the first rule seems superior to the second one because of its direct reduction in the amount of idle time. However, it shows contradictory results. According to the first rule, tasks 1, 2 and 3 are assigned in succession to left station and no delay occurs. When it comes to task 4, however, a new position has to be opened because there is no room for task 4 in position 1. As for the second solution, task 2 is assigned to the right station after task 1 because such action balances the load between mated-stations. 3 units of time is delayed in the right station. Then task 3 is assigned to left station because the load of station 1 (3 units of time) is smaller than that of right station (6 units of time). Finally, task 4 is assigned to the left station. Solution 2 results in less delay overall and, ultimately, generates less *NM* than solution 1 does. Hence, we adopt the

second principle prior to the first one. In other words, the assignable task set is sorted by the load-oriented, delay-oriented and the task priority score in succession. The combinatorial structure of principles is implemented throughout later analysis.

Analogous to the previous analysis, we test the performance of PRBMs with the proposed principles (PRBMs_Plus) as shown in Table. 19. As can be seen, the performance of every elementary rule is improved. Moreover, it is of our interest to test the influence of the principles on the performance of the PRBMs. Table. 20 shows the *p* value of the paired *t*-test of Avg_dev before and after the principles are incorporated. As a result, the performance of rule *TdS* and *TdL* is significantly improved respectively and the improvement for the *T* is marginal. As for rule *L* and *F*, there is no significant difference statistically. The Matlab codes to implement the elementary rules with two principles are in Appendix H.

Table 19 Average performance of the elementary rules with two principles

	TdS_Plus	*TdL_Plus*	*T_Plus*	*L_Plus*	*F_Plus*
Avg_dev (%)	9.88	8.45	9.68	15.00	7.03

Table 20 p values of paired t tests of Avg_dev of elementary rules with and without two principles (95% confidence level)

	Plus
TdS	0.01608
TdL	0.03404
T	0.05262
L	0.1122
F	0.1491

5.1.2 Application of composite rules

We adopt the construction scheme of composite rules in Otto and Otto (2014). Every pair of 5 elementary rules is linearly summed which results in 10 basic composite rules. For each composite rule, two weights are given to the 2 elementary rule respectively. The first weight is chosen from the set {100, 10, 5, 2, 1/2, 1/5, 1/10, 1/100}. The second weight is always equal to 1. Those configurations give a total of 90 composite rules. The enumerative procedure of composite rules is similar to that of the elementary rule. It should be noted that all rules should be scaled in order to be combined on equal footing. The scaled factor is given in Table. 21. The formula to calculate the priority score of task *i* given weight *j* is shown in Eq. (30).

$$S_{i\_comp}^{j} = \frac{Weight\ (j) \times S_{i\_elem1}}{Scale\ factor_{i\_elem1}} + \frac{S_{i\_elem2}}{Scale\ factor_{i\_elem2}} \qquad (30)$$

Table 21 The scaled factor for the application of composite rules

Rule	*TdS*	*TdL*	*T*	*L*	*F*
Scaled factor	*c* for denominator and *GUB* for nominator		*c*	*GUB*	*n*

In Table. 22, we report the results of the best weighted combination for each pairing of rules. For the ease of representation and comparison, the performance of the elementary rule (Table 19) is stored on the diagonal in bold. From the table we see that in all cases composite rules outperform the elementary rule, indicating the combination of different rules provides synergy in solving the TALBP. The performance of each weighted pairing rule in terms of the number of best solutions it generates out of all 34 cases is presented in Table 23. The best weighted pairing rules are {*TdS; F;* 0.01} and {*Tdl; F;* 0.1}, both of which outputs the 30 best results out of 34 problem instances.

Table 22 Average performance of the best weighted combinations for each pairing of rules

	TdS	*TdL*	*T*	*L*	*F*
TdS	**9.88**	8.43	8.01	6.81	4.55
TdL		**8.45**	7.03	5.98	4.64
T			**9.68**	5.56	4.55
L				**15**	4.74
F					**7.03**

Table 23 The number of best solutions of each weighted pairing rules out of 34 cases as measured by Avg_dev

	Weight								
	100	10	5	2	1	0.5	0.2	0.1	0.01
TdS_TdL	20	19	19	18	17	18	19	18	20
TdS_T	20	18	19	15	15	17	19	19	21
TdS_L	19	16	16	18	15	15	16	14	14
TdS_F	20	20	19	20	22	23	25	26	**30**
TdL_T	21	20	20	19	19	19	19	19	22
TdL_L	22	19	19	16	19	17	17	16	16
TdL_F	20	20	20	23	24	25	29	**30**	28
T_L	22	16	17	17	19	18	16	16	16
T_F	21	21	24	21	25	28	29	29	29
L_F	18	18	19	20	19	19	21	24	27

From Tables 17 and 19, we observe that the performance of rule *F* is superior to the other rules. We also notice, from Table 22, that the performance of the other 4 elementary rules are greatly improved when paired with rule *F*. Also, from Table 23, the performance of *F*-related rules improves as the weight that the *F* component receives also increases. Such relations are shown in

the panel plots in Figure 27. The Matlab codes to implement the composite rules are in Appendix I.

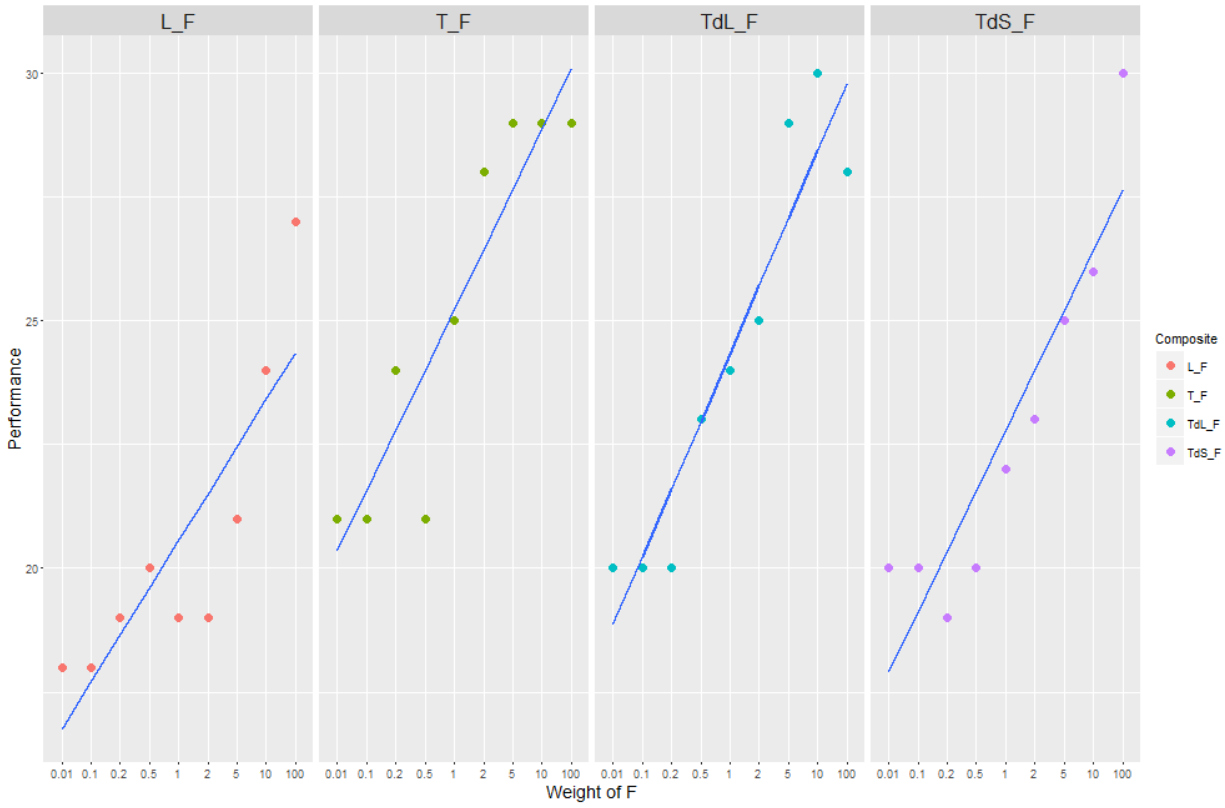

Figure 27 Number of best solutions of composite rules in relation to the weight of F

5.2 Algorithmic design with BDP

In this section, the PRBMs are embedded into a bounded dynamic programming (BDP) aiming to further improve the solution quality of stand-alone PRBMs as well as proposing a design procedure. The novel algorithm (PR_BDP), which is comprised of two main components, enumeration of states and solution space reduction is tested on benchmark data sets in order to compare its performance against the best composite rules.

5.2.1 Enumeration of states

The procedure of enumerating states (positions) is the building block of the BDP. PRBMs are applied to develop the states (partial solutions) at each stage. States which return small idle time are carried forward to develop states at the next stage. The basic idea to use the PRBMs in BDP is to take advantage of several good PRBMs in-stage performance and combine them between the stages. For instance, applying composite rule *L_F* will result the minimum idle time for position 1 and a relatively high idle time for position 2. As for rule *T_F*, the results are just the opposite. If we apply the two rules independently, we obtain two mediocre solutions. However, if we apply *L_F* to position 1 and *T_F* to position 2, we may get a solution that results in the minimum overall idle time. To apply the enumeration procedure effectively, several techniques are applied. These techniques are:

The choices of PRBMs. Instead of choosing all possible rules, we only adopt a few excellent rules to develop the states at each stage for the sake of computational efficiency. Totally, 4 rules are selected. The {*TdS; F;* 0.01} and {*TdL; F;* 0.1} are selected according to Table 23. Moreover, {*L; F;* 0.01} and {*T; L;* 0.01} are added to the set of rules because they can provide better solutions than the first two rules for some problems.

Assignable task set management. The running time during the enumeration procedure and the quality of a partial solution highly depend on the generation of the assignable task set and the order of the task in the set respectively. The assignable task set is updated and ordered based on the following rules.

1) For a data set, the natural number of a task appears after all of its predecessor's natural number.

2) After a task is assigned, its successors along with the rest of the tasks in the previous assignable task set are inspected on the cycle time constraint to constitute the new assignable task set, which largely reduces the effort of reexamining all remaining tasks on all constraints.
3) Tasks in the assignable task set are ordered according to the load-oriented rule, delay-oriented rule, one of the 4 selected PRBMs and their natural number in succession.

5.2.2 Solution space reduction approaches

The solution space shall be relaxed in order to allow for the resolution of a large sized problem as the PR_BDP advances in stages and between stages. For example, consider a problem that requires 10 positions to assign all tasks. The total number of task sequence without space reduction is 4^{10}=1,048,576 which greatly undermines the efficiency of the algorithm. Three approaches are developed to reduce the solution space. 1) *Labeling dominance rule*, 2) *utilization of bounds*, 3) *window size*.

Labeling dominance method. The labeling scheme (Schrage and Baker, 1978) ensures each partial solution gets a unique value by assigning different labels to tasks. The states, developed by different PRBMs, with the same total number of labels are deleted as the algorithm advances to the next stage. As a consequence, the solution space is reduced, so is the computational speed. It is an in-state reduction approach.

Utilization of bounds. The comparisons between different bounds (*GLB, GUB, LB, UB*) can be used to delete the partial solutions whose prospective outputs can be proven to be not superior to the existing best output. The global upper bound (*GLB*) can be calculated by ignoring the precedence and indivisible constraints prior to the optimization process (see Wu, 2008). The

lower bound (*LB*) for each partial solution is equal to the summation of the number of positions already finished and the global lower bound (*GLB*) for the remaining positions to which the rest of tasks are assigned. The upper bound (*UB*) for each partial solution is generated by assigning the first assignable task to the station without enumerating all tasks in the assignable task set. After all tasks are assigned, the *UB* is equal to *NM*. Such a station construction scheme is the fastest method to find a feasible solution meaning that the computational cost for *UB* is negligible. *GUB* is the minimum *UB* among all partial solutions. The following comparisons are conducted and the solution space is reduced accordingly.

1) *UB* and *GLB.* If a *UB* of a partial solution is equal to *GLB* (*UB=GLB)*, the PR_BDP is terminated and the optimum is *GLB.*
2) *LB* and *GUB.* For a partial solution, if its *LB* is not smaller than *GUB* (*LB*≥*GUB*), the partial solution is proved to be dominated and can be deleted from the solution space.
3) *UB* and *GUB.* For a partial solution, if its *UB* is smaller than *GUB, GUB* is set to *UB.*
4) *UB, LB* and *GUB.* For a partial solution, if its *LB* is equal to *UB* but is less than *GUB* (*LB=UB<GUB*), the full solution which results in the *UB* is stored as the best current optimal solution, and then the partial solution is deleted from the solution space.

Comparisons 1 and 3 are conducted within stages during the enumerative process. Comparisons 2 and 4 are conducted between stages after each enumerative process is done. Note, the algorithm is terminated as the solution space is empty and the current best optimal solution is reported.

Window size w. Window size, as defined in section 2.3, restricts a fixed number of states at each state. The states are sorted in a non-descending order regarding the sum of idle times of the assigned stations. Then, the first *w* states are reserved to develop states at next stage.

These approaches are deployed in the following order. The Labeling dominance rule is first used to enumerate a state. Secondly, the comparison 1 is conducted to determine whether the global optimum is found and the algorithm can be terminated. Thirdly, the comparison 3 is assessed to update the *GUB*. Then, the comparisons 2 and 4 are conducted to eliminate the unnecessary states. Finally, if the number of states remaining is greater than *w*, it is downsized to *w*.

5.2.3 The application of PR_BDP

We formally develop the PR_BDP in the flow diagram below (Figure 28). It is graph-based, deterministic and multi-pass solution generating process. It focuses on exploring the states of a stage by exploiting the solution relaxation techniques to reduce the computational effort.

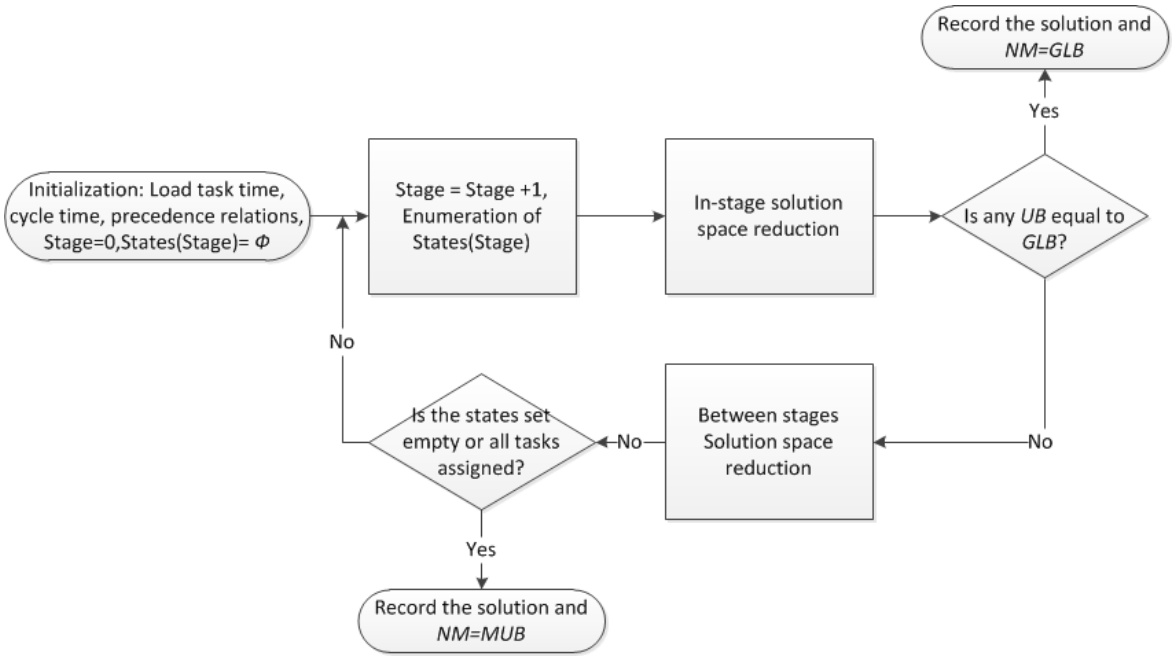

Figure 28 Structure of PR_BDP

We apply the PR_BDP with *w* equal to 5, which is an appropriate number to serve the purposes of maintaining the quality of states as well as computational efficiency. The Avg_dev is 1.11%, as shown in Table 24 which is much improved from the performance of the best elementary and composite rules reported in section 3. In table 25, we present the optimum found by PR_BDP compared with the best *NM* reported in the literature. We also provide the computational times for each instance as a means to gauge the computational efficiency. The CPU time is much smaller than the cycle time of each instance which makes the algorithm applicable when task learning and rebalancing are considered. Furthermore, a new optimum (*NM*=8 for A65, *c*=326 in Khorasanian et al. (2013)) is found and proved. The Matlab codes to implement the PR_BDP are in Appendix J.

Table 24 Comparisons of performance between best PRBMs and PR_BDP

Rule	Best elementary rule	Best composite rules	PR_BDP
Avg_dev (%)	12.15	4.55	1.11

Table 25 Comparisons between the best NM reported and best NM generated by PR_BDP

Data set	*c*	Best reported *NM*	*NM* (PR_BDP)	Computational times (secs)
A12	4	4	4	0.54
	5	3	3	0.11
	6	3	3	0.02
	7	2	2	0.05
A16	15	4	4	0.17
	18	3	3	0.02
	20	3	3	0.03
	22	2	2	0.03
A24	25	3	3	0.31
	30	3	3	0.06
	35	2	2	0.07
	40	2	2	0.09
A65	326	9	**8**	16.59
	381	7	7	10.36
	435	6	6	14.83

	490	6	6	1.53
	544	5	5	1.42
A148	204	13	13	5.33
	255	11	11	5.87
	306	9	9	7.14
	357	8	8	3.91
	408	7	7	2.53
	459	6	6	2.48
	510	6	6	3.15
A205	1133	11	11	5.80
	1322	**9**	10	13.32
	1510	**8**	9	7.56
	1699	**7**	8	5.92
	1888	7	7	3.25
	2077	6	6	4.04
	2266	6	6	3.07
	2454	5	5	4.25
	2643	5	5	2.19
	2832	5	5	0.59

Computational experiments confirm the effectiveness of incorporating PRBMs into BDP. The PR_BDP takes advantage of the capability of different composite rules on solving different problems. The parameters need to be specified are the choice of PRBMs and the associated weights. There are no ubiquitous parameters for all problems in general indicating that the analysis in section 3 has to be performed in order to obtain the parameters. However, they can be found in a short period of time because the search for the performance of each composite rule is a single-pass, forward process, i.e., no backtracking is involved and only the best solution *NM* and score *S* are stored in memory for each rule. Furthermore, the PR_BDP is terminated before enumerating even half of the number of states for most problems. Thereby the assignable task set generation and task allocation process are cut in half relatively to the elementary and composite rules which

require the assignment of all tasks. Such success of computational efficiency is attributed to the relaxation techniques. Finally, with regards to the computational efficiency, the PR_BDP can output the best solutions in a fast manner that it is appealing to deploy the PR_BDP in practice. It is also worth to mention that the computational times of meta-heuristics are also very competitive, e.g. Simulated annealing can output any individual solution within 5 seconds (Khorasanian et al. (2013)). However, the endeavors to tune the parameters for each problem instance, which are not always published in the literature, are a lot more time-consuming for the implementation of meta-heuristics (Hooker, 1995).

5.2.4 Design of Experiments

In this section, some experiments, which are similar to the one in section 4.3, are performed to assess the merit of task reassignment under the regime of two sided assembly system when learning is taking place and it is desirable to reconfigure the assembly line in real time during a production run. Considering learning provides two advantages. For one thing, it generates more data instances with which our algorithm can be further tested. For another, production statistics can be enhanced during the production process if task learning is held accountable. Because it is a type 1 balancing problem, we adopt two performance measures—Average improvement of unit NM (Avg_NM) and Average improvement of unit idle time (Avg_I) to evaluate the production statistics of one problem. Learning period (Le) and learning rates (Lr) are used as control variables as they are in section 4.3 representing the degree of learning. Equations (31) and (32) show the calculations of the two terms.

$$Avg\_NM = \frac{\sum_{i=1}^{Le+1} NM_i \times PI_i}{N} \qquad (31)$$

$$Avg\_I = \frac{\sum_{i=1}^{Le+1} I_i \times PI_i}{N} \qquad (32)$$

Where PI_i is the interval of the production process in which the total positions of the line is NM_i.

Given z=400, Le is selected from [1, 2, 3] and Lr is selected from [0.05, 0.1, 0.15]. The novel algorithm PR_BDP is selected to output the best NM_i in each production interval PI_i. All benchmark data instances (34) are selected for analysis. The summary statistics for the percent improvement in Avg_NM and Avg_I are given below in Table. 26 and 27. The design of experiment are programmed in Matlab and the codes are in Appendix K.

Table 26 Summary statistics for percent reduction in Avg_NM of 34 basic data instances

Avg_NM					
Production quantity	Learning rate	# of task learning	*M*	*SE*	95% CI
400	0.05	1	0.0207	0.0072	0.0081
		2	0.0455	0.0081	0.0313
		3	0.0673	0.0084	0.0526
	0.1	1	0.0475	0.0076	0.0343
		2	0.0904	0.0062	0.0797
		3	0.1298	0.0064	0.1185
	0.15	1	0.0738	0.0062	0.0629
		2	0.1323	0.0068	0.1205
		3	0.1787	0.0081	0.1645

Table 27 Summary statistics for percent reduction in Avg_I of 34 basic data instances

Avg_I					
Production quantity	Learning rate	# of task learning	*M*	*SE*	95% CI
400	0.05	1	0.1412	0.0484	0.0567
		2	0.2874	0.0451	0.2087
		3	0.3858	0.0415	0.3135
	0.1	1	0.3078	0.0447	0.2297
		2	0.4842	0.0279	0.4355
		3	0.5748	0.0231	0.5345
	0.15	1	0.4397	0.0392	0.3713
		2	0.5794	0.0234	0.5386
		3	0.6379	0.0298	0.5858

From the Tables 27 and 28, we can infer that the mean of both performance measures increase in the Le and Lr. This means that, when comparing PR_BDP without dynamic reconfiguration of the line to changing the configuration of the line when learning takes place, Table 26 gives the percent reduction in Avg_NM and Table 27 gives the percent reduction in Avg_I. Unlike the cases in section 4.3, where no obvious pattern can be observed, the positivity correlation between the performance measures and Le or Lr of the current experiment is due to the implementation procedure of algorithms. In the experiment of section 4.3, as for the large sized data sets, we set time limit to restrict the total computational time of algorithm implementation in order to control the cost of experiment because ENCORE and SALOME are exact solution procedures. As such, there are possibilities that some data instances with the same precedence relation but lower task times can produce higher output (cycle time). However, PR_BDP is a heuristics approach which does not require the time limit to manage the overall cost of experiment. In other words, outputs (NM and Idle time) are always smaller for higher degree of learning. In practice, because of the excellent computational efficiency as shown in Table 25, one can use the PR_BDP as an online algorithm to reassign tasks and reduce the number of positions as task learning progresses.

5.3 Summary

In this chapter, we test the effectiveness of several elementary rules on TALBP. Two principles (delay-oriented and load-oriented) are proposed, compared and utilized to minimize the idle time from delay, which is a major negative impact on the performance of PRBMs. The combinations of elementary rules (composite rules) provide synergetic advantage regarding the average quality of solutions. Rule *F* with higher weight is the best partner when combined with

other elementary rules on solving the TALBP. Ultimately, an algorithmic design procedure including selecting and incorporating PRBMs into BDP is proposed (PR_BDP), which is a deterministic and problem specific heuristic procedure. Several important techniques are provided to reduce the solution space significantly as the PR_BDP progresses. With the presence of task learning, the improvement of production statistics is gauged by comparing task reassignment by PR_BDP with the cases of no reassignment. Additionally, average computational times on test problems are a fraction of cycle times, making real time application possible.

The design of algorithms provides distinct advantage over the traditional exact solution procedure and meta-heuristics. Compared with the exact solution procedure, the efficiency is greatly improved as the solution space is largely reduced and the quality of the developing solution is maintained intelligently. Compared with the meta-heuristics, it is a "white box", intuitive process which exploits the problem specific knowledge, e.g., the structure of the line, delay, time and precedence relationships. Furthermore, the optimization process is deterministic which guarantees the reproducibility of the results.

6. Summary

The assembly rebalancing with non-constant task time attribute problem is introduced and solved in this dissertation. The rising problem is motivated by task learning effect during the production process along the assembly line, and is established in the context of automatic assembly line system, where assembly lines are operated by robots. In such a system the task time improvement is preserved by the supervisory controller that oversees the assembly line and the learned skill (information) can be transferred to other agents (robots) on the line. The collaborative learning and structure of the supervisory control enable the line to be rebalanced as task time improvements are realized in such a way that the overall efficiency of the line is optimized.

A total of three problems, residing in one-sided or two-sided assembly line, are addressed by three different algorithms, either exact or heuristics solution procedure. To efficiently implement those algorithms and apply the optimal solutions generated by them to assembly lines, two rebalancing frameworks are proposed; 1) a planning framework, which is suitable for the situation where task time improvements can be well estimated by a learning curve. This framework relies on offline algorithms to generate the optimal solution for every production cycle; 2) a real time framework, which is designed for the situation where task learning is uncertain, employs online algorithms to obtain the optimal solution in real time.

A backward induction algorithm is capable of solving the task reassignment problem efficiently, in which task times are reduced through learning and learning is conserved when tasks are reassigned amongst stations so that the number of stations required in each production cycle is minimized. The advantage of using the backward induction algorithm, is to avoid repetitively using the tradition algorithm, e.g., SALOME, to produce the optimal solution in every production cycle and, therefore, the computational burden is alleviated. Computational experiments are conducted

to validate the superiority of the backward induction algorithm over the conventional algorithm. Furthermore, we demonstrate the industrial application of optimal solutions to the automated assembly line with collaborative learning and supervisory control. It shows that the assembly system is better off in production statistics considering rebalancing in each production cycle.

ENCORE is an online algorithm designed for solving the rebalancing problem with uncertain task learning attribute. It leverages SALOME2 by using a near optimal starting point and utilizing a temporary upper bound instead of the local lower bound, such that the computational efficiency is improved. Pairwise *t* tests are conducted on large sized data instances to show whether ENCORE is superior to SALOME2 empirically at various computational time limits, appropriate for real time decisions in advanced automated systems. We also show the boundary at which ENCORE has no advantage over SALOME2 statistically.

PR_BDP is designed by hybridizing dynamic programming and the priority-based method. It takes its inspiration from the fact that the two-sided line balancing problem can be divided into several subproblems that can be solved more easily than the main problem computationally, in such a way that dynamic programming can be employed. In each subproblems, a solution can be quickly generated by PRBMs, which is based on tasks' priority scores, load-oriented and delay-oriented principles. By piecing solutions of the subproblems together, the solution for the main problem can be found. PR_BDP provides distinct advantage over the traditional exact solution procedure and meta-heuristics. Compared with the exact solution procedure, the efficiency is greatly improved as the solution space is largely reduced and the quality of the developing solution is maintained intelligently. Compared with the meta-heuristics, it is a "white box", intuitive process which exploits the problem specific knowledge. Furthermore, the optimization process is deterministic, which guarantees the reproducibility of the results.

The aforementioned three algorithms and their features and applications are summarized in table 28.

Table 28 Summary of Algorithms

Algorithm	Attribute	Application	Features
Backward induction	Exact	one-sided line (type 1), planning model	Avoid repetitively invoking traditional method
ENCORE	Exact	one-sided line (type 2), real-time model	Use a near optimal starting point and a better upper bound to ease computational burden
PR_BDP	Heuristic	two-sided line (type 1), real-time model	Utilize both PRBMs and DP to divide the problem and produce fast and good solution

With the fast and efficient algorithms proposed, learning effects can be considered during the course of production runs in practical production and operation management. In different production environments (one-sided or two-sided line) with different goals (type 1 or type 2), decision making processes depend on the optimal solutions resulting from different algorithms. Concretely, decision makers should answer the following questions sequentially, and optimal assembly line design and production schedule will be developed automatically.

What is the product?

By answering this question, we can decide which assembly line is suitable for production, one-sided or two sided. If the size of the product is very large and requires workers to access the unit from both sides of the part, two-sided lines shall be selected.

What is the production quantity?

We would like to know whether the dynamic rebalancing is necessary during the production. As suggested in section 1.3, rebalancing during production is only meaningful when the batch size is small, in which case the optimal solution is changed frequently. Otherwise, one can rebalance the line once after the task learning reaches the plateau.

What is the goal?

The goal of designing an efficiently balanced line determines the problem types. If high production rate is to pursue, then it is a type 2 problem in which cycle time is minimized. If the amount of production capacity is the priority, then it is a type 1 problem where the number of stations/positions is minimized.

How does task time behave?

The task learning process can be continuous or discrete in time, certain or uncertain. If the learning process is continuous and almost certain, as in the insertion example of Figure 2. We can approximate the path of task learning by a learning curve and select the planning model to implement the rebalancing schedule. If the timing at which task learning occurs and/or the degree of task time improvement are both uncertain, and they cannot be accurately predicted as using a learning curve, we resort to the real-time rebalancing schedule.

Is there any time constraint on rebalancing the line?

At times there may be a time limit allowed for the algorithm to compute the optimal solution to ensure the continuity of the production. In a planning model, that is not a concern because it uses an offline algorithm. However, in real-time model, it becomes a factor that should be considered in the rebalancing schedule, especially for a complex, large sized problem. We have identified the boundary at which ENCORE is indifferent to SALOME2 in terms of the quality of solutions. Statistically speaking, SALOME2 would generate as good results (cycle time) as

ENCORE does if the time limit is beyond the boundary. Their difference in objective value will converge as the time limit increases, because they both are exact solutions.

In conclusion, to our knowledge this is the first work that addresses the case for dynamically reconfiguring an assembly line during production under conditions where there is learning or task time improvement and the knowledge underlying the improvement is retained as tasks are redistributed among agents. This provides the opportunity to optimize overall production efficiencies while maintaining constraints of the number of stations or the cycle time of the line. This is the first work to focus on algorithmic design for such cases and to demonstrate their value in industrial applications. As research on the application of flexible assembly systems with machine learning progresses, these methods will help support the efficient planning of the use of resources by taking into consideration optimal deployment over a range of task time reductions.

The results of this research can be extended in future research. One open issue includes the global allocation of resources as agents (robots) become available during line reconfiguration. At any time, there are multiple product assemblies to be schedules that will require resources during a day's schedule. Batches of different products are moved in and out of production and require resources. Traditionally a line would be dedicated to one product at a time, though mixed product lines are possible. As improvements take place in the assembly of one product and resources can be reallocated, for example in the configuration of Figure 14, it opens the question of how a full day's production can be scheduled so as to maximize resource utilization overall. This is the question that will be addressed in future research.

References

Arcus, A. L. (1965). A computer method of sequencing operations for assembly lines. *International Journal of Production Research*, 4(4), 259-277.

Angerer, S., Pooley,R. and Aylett, R. (2010). Self-reconfiguration of industrial mobile robots, *Proceeding of Fourth IEEE International Conference on Self-Adaptive and Self-Organizing Systems* (2010), 64 – 73.

Babiceanu, R. and Chen, F. (2006). Development and application of holonic manufacturing systems: A survey, *Journal of Intelligent manufacturing* 17, 111 – 131.

Bartholdi, J. J. (1993). Balancing two-sided assembly lines: A case study. *International Journal of Production Research*, 31(10), 2447–2461.

Battaia, O., and Dolgui,A. (2012).Reduction approaches for a generalized line balancing problem. *Computers&OperationsResearch*39,2337–2345.

Battaïa, O., and Dolgui,A. (2013). A taxonomy of line balancing problems and their solution approaches. *International Journal of Production Economics,142*(2), 259-277.

Bautista, J., and Pereira, J. (2009). A dynamic programming based heuristic for the assembly line balancing problem. *European Journal of Operational Research*, 194(3), 787-794.

Baybars, I (1986). A survey of exact algorithms for the simple assembly line balancing problem. *Management science* ,32.8 , 909-932.

Baykasoglu, A., and Dereli, T. (2008). Two-sided assembly line balancing using an ant-colony-based heuristic. *The International Journal of Advanced Manufacturing Technology*, *36*(5-6), 582-588.

Biskup, D. (1999). Single-machine scheduling with learning considerations. *European Journal of Operational Research*, *115*(1), 173-178.

Boucher, T.O. (1996) *Computer Automation in Manufacturing*. London: Chapman & Hall.

Boucher, T.O. (1987). Choice of assembly line design under task learning. *International Journal of Production Research* ,25.4 , 513-524.

Boucher, T.O. and A. Yalcin (2006) *Design of Industrial Information Systems*. Amsterdam: Elsevier, Academic Press.

Boucher, T., & Li, Y. (2015). Technical note: systematic bias in stochastic learning. *International Journal of Production Research*, 1-12.

Boysen, N., Fliedner, M., & Scholl, A. (2007). A classification of assembly line balancing problems. *European Journal of Operational Research, 183*(2), 674-693.

Boysen, N., Scholl, A., &Wopperer, N. (2012). Resequencing of mixed-model assembly lines: Survey and research agenda. *European Journal of Operational Research, 216*(3), 594-604.

Bryton, B. (1954). Balancing of a Continuous Production Line, M.S. Thesis, Northwestern University, Evanston, IL

Bukchin, J., & Rubinovitz, J. (2003). A weighted approach for assembly line design with station paralleling and equipment selection. *IIE transactions,35*(1), 73-85.

Bukchin, Y., &Rabinowitch, I. (2006). A branch-and-bound based solution approach for the mixed-model assembly line-balancing problem for minimizing stations and task duplication costs. *European Journal of Operational Research,174*(1), 492-508.

Cakir, B., Altiparmak, F., and Dengiz, B. (2011). Multi-objective optimization of a stochastic assembly line balancing: A hybrid simulated annealing algorithm. *Computers& Industrial Engineering*, 60(3), 376-384.

Cavalcante, A., Peixoto,J and Pereira.C. (2012). When agents meet manufacturing: paradigms and implementations, *Anais do XIX CongressoBrasileiro de Automatica,* 957 – 964.

Coffman, Jr, E. G., Garey, M. R., & Johnson, D. S. (1978). An application of bin-packing to multiprocessor scheduling. *SIAM Journal on Computing, 7*(1), 1-17.

Cohen, Y., Dar-El, M. (1998) Optimizing the number of stations in assembly lines under learning for limited production, *Production Planning and Control,* 9 (3), 230–240.

Cohen, Y., Vitner, G., & Sarin, S. C. (2006). Optimal allocation of work in assembly lines for lots with homogenous learning. *European Journal of Operational Research, 168*(3), 922-931.

Cohen, Y., Vitner, G., & Sarin, S. (2008) Work allocation to stations with varying learning slopes and without buffers. *European Journal of Operational Research, 184*(2), 797-801.

Digiesi, S., Kock, A., Mummolo, G., Rooda, J. (2009). The effect of dynamic worker behavior on flow line performance. *International Journal of Production Economics,* 120 (2), 368–377.

Dar-El, E. M. (1975). Solving large single-model assembly line balancing problems—A comparative study. *AIIE Transactions, 7*(3), 302-310.

Dar-El, E.M., Rubinovitch, Y. (1979). MUST—A multiple solutions technique for balancing single model assembly lines. *Management Science* 25, 1105–1114.

Dolgui, A., Finel, B., Vernadat, F., Guschinsky, N., & Levin, G. (2005). A heuristic approach for transfer lines balancing. *Journal of Intelligent Manufacturing, 16*(2), 159-172.

Dolgui, A., Proth, J., (2010). *Supply Chain Engineering: useful methods and techniques*. Springer.

Easton, F., Faaland, B., Klastorin, T.D., Schmitt, T., (1989). Improved network based algorithms for the assembly line balancing problem. *International Journal of Production Research*, 27, 1901–1915.

Fleszar, K., & Hindi, K. S. (2003). An enumerative heuristic and reduction methods for the assembly line balancing problem. *European Journal of Operational Research*, *145*(3), 606-620.

Globerson, S., & Gold, D. (1997). Statistical attributes of the power learning curve model. *International journal of production research*, *35*(3), 699-711.

Goldberg, M. S., & Touw, A. (2003). *Statistical methods for learning curves and cost analysis* (p. 196). Institute for Operations Research and the Management Sciences (INFORMS).

Gurevsky, E., Hazır, Ö., Battaïa, O., &Dolgui, A. (2012). Robust balancing of straight assembly lines with interval task times [star]. *Journal of the Operational Research Society*, *64*(11), 1607-1613.

Hackman, S.T., Magazine, M.J., Wee, T.S. (1989). Fast, effective algorithms for simple assembly line balancing problems. *Operations Research* 37, 916–924.

Haupt, R. (1989): A survey of priority rule-based scheduling. *OR Spectrum*, 11, 3–16.

Held, M., Karp, R. M., &Shareshian, R. (1963). Assembly-line balancing-dynamic programming with precedence constraints. *Operations Research*,*11*(3), 442-459.

Heskiaoff, H. (1968). A heuristic method for balancing assembly line. *The western electric engineering* ,12/3,9-13

Hoffmann, T.R., (1963). Assembly line balancing with a precedence matrix. *Management Science*, 9, 551–562

Hoffmann, T.R. (1992). EUREKA: A hybrid system for assembly line balancing. *Management Science*, 38.1, 39-47.

Hooker, J. N. (1995). Testing heuristics: We have it all wrong. *Journal of heuristics*, 1(1), 33-42.

Hu, X., Wu, E., & Jin, Y. (2008). A station-oriented enumerative algorithm for two-sided assembly line balancing. *European Journal of Operational Research*, *186*(1), 435-440.

Hu, X., Wu, E., Bao, J., & Jin, Y. (2010) A branch-and-bound algorithm to minimize the line length of a two-sided assembly line. *European Journal of Operational Research*, 206(3), 703–707

Jackson, J. R. (1956). A computing procedure for a line balancing problem. *Management Science,* 2.3 , 261-271.

Johnson, Roger V. (1988), Optimally balancing large assembly lines with "FABLE", *Management Science, 34.2* , 240-253.

Klein, R., Scholl, A. (1996). Maximizing the production rate in simple assembly line balancing—A branch and bound procedure. *European Journal of Operational Research* 91, 367–385.

Khorasanian, D., Hejazi, S. R., & Moslehi, G. (2013). Two-sided assembly line balancing considering the relationships between tasks. *Computers & Industrial Engineering*, *66*(4), 1096-1105.

Kim, Y. K., Kim, Y., & Kim, Y. J. (2000). Two-sided assembly line balancing: a genetic algorithm approach. *Production Planning & Control*, *11*(1), 44-53.

Lapierre, S. D., Ruiz, A., & Soriano, P. (2006). Balancing assembly lines with tabu search. *European Journal of Operational Research*, *168*(3), 826-837.

Lapierre, S. D., & Ruiz, A. B. (2004). Balancing assembly lines: an industrial casestudy. *Journal of the Operational Research Society*, 589-597.

Lee, T. O., Kim, Y., & Kim, Y. K. (2001). Two-sided assembly line balancing to maximize work relatedness and slackness. *Computers & Industrial Engineering*, *40*(3), 273-292.

Li, Y., & Boucher, T. O. (2016). Assembly line balancing problem with task learning and dynamic task reassignment. *The International Journal of Advanced Manufacturing Technology*, 1-9.

Lopes, L. S., &Camarinha-Matos, L. M. (1995).A machine learning approach to error detection and recovery in assembly. *IEEE International Conference on Human Robot Interaction and Cooperative Robots* ,3,197-203.

McNaughton, R., 1959. Scheduling with deadlines and loss functions. *Management Science* 6, 1–12.

Mansoor, E.M. (1964), Assembly line balancing-an improvement on the ranked positional weight technique. *Journal of Industrial Engineering* ,15.2 , 73-77.

Mastor, A. A. (1970), An experimental investigation and comparative evaluation of production line balancing techniques. *Management Science* ,16.11, 728-746.

Miltenburg, J. (2001). U-shaped production lines: A review of theory and practice. *International Journal of Production Economics*, 70(3), 201-214.

Nearchou, A. C. (2011). Maximizing production rate and workload smoothing in assembly lines using particle swarm optimization. *International Journal of Production Economics*, *129*(2), 242-250.

Newnan, D.G., J.P. Lavelle, and T.G. Eschenbach (2009) *Engineering Economic Analysis*, (10th ed.). New York: Oxford University press.

Nuttin, M. and van Brussel.H. (1999). Learning sensor assisted assembly operations, *Making Robots Smarter*, K. Morik, M. Kaiser and V. Klingspor, eds. Kluwer Academic Publishers, 45 – 52.

Onori, M., Barata, J. and Frei, R. (2006). Evolvable assembly systems: basic principles, *Information Technology for Balanced Manufacturing Systems*, *International Feceration for Information Processing*, 220, 317-328

Ostwald, P.F. (1992) *Engineering Cost Estimating*, 3rd. Ed. Englewood Cliffs: Prentice Hall.

Otto, A., Otto, C., & Scholl, A. (2011). How to design and analyze priority rules: Example of simple assembly line balancing. Working Papers in Supply Chain Management 3, Friedrich-Schiller-University of Jena.

Otto, A., & Otto, C. (2014). How to design effective priority rules: Example of simple assembly line balancing. *Computers & Industrial Engineering*, *69*, 43-52.

Otto, C., and Otto, A. (2014). Extending assembly line balancing problem by incorporating learning effects. *International Journal of Production Research*, *52*(24), 7193-7208.

Özbakır, L., & Tapkan, P. (2010). Balancing fuzzy multi-objective two-sided assembly lines via Bees Algorithm. *Journal of Intelligent & Fuzzy Systems*,*21*(5), 317-329.

Posypkin, M. A.E and Sigal, I. K. (2006). Speedup estimates for some variants of the parallel implementations of the branch-and-bound method. *Computational Mathematics and Mathematical Physics* ,46.12, 2187-2202.

Ribeiro, L., Barata, J. and Colombo, A. (2008). MAS and SOA: A case study exploring principles and technologies to support self-properties in assembly systems, *Proceedings of Second IEEE Conference on Self-Adaptive and Self-Organizing Systems*, 192-197.

Rekiek, B., Dolgui, A., Delchambre, A., and Bratcu, A. (2002). State of art of optimization methods for assembly line design. *Annual Reviews in Control*,*26*(2), 163-174.

Rubinovitz, J., and Levitin, G. (1995). Genetic algorithm for assembly line balancing. *International Journal of Production Economics*, 41(1), 343-354.

Sanders, D. and Gegovm, A. (2013) AI tools for use in assembly automation and some examples of recent applications, *Assembly Automation*, 33 (2), 184-194.

Salveson, M.E. (1955). The assembly line balancing problem. *Journal of Industrial Engineering*, 6.3, 18-25.

Scholl,A.(1993), data of assembly line balancing problem, http://alb.mansci.de/index.php?content=classview&content=classview&content2=classview&content3=classviewdlfree&content4=classview&classID=44&type=dl

Scholl, A., & Voß, S. (1997). Simple assembly line balancing—Heuristic approaches. *Journal of Heuristics*, 2(3), 217-244.

Scholl, A, and Klein, R. (1997). SALOME: A bidirectional branch-and-bound procedure for assembly line balancing. *INFORMS Journal on Computing*, 9.4 ,319-334.

Scholl, A., & Klein, R. (1999). ULINO: Optimally balancing U-shaped JIT assembly lines. *International Journal of Production Research*, *37*(4), 721-736.

Scholl, A, and Klein, R. (1999). Balancing assembly lines effectively–a computational comparison, *European Journal of Operational Research*, 114.1, 50-58.

Scholl, A and Becker, C. (2006). State-of-the-art exact and heuristic solution procedures for simple assembly line balancing. *European Journal of Operational Research* ,168.3 ,666-693.

Schrage, L., & Baker, K. R. (1978). Dynamic programming solution of sequencing problems with precedence constraints. *Operations research*, *26*(3), 444-449.

Simaria, A. S., & Vilarinho, P. M. (2009). 2-ANTBAL: An ant colony optimisation algorithm for balancing two-sided assembly lines. *Computers & Industrial Engineering*, *56*(2), 489-506.

Sotskov, Y. N., Dolgui, A., Lai, T. C., & Zatsiupa, A. (2015) Enumerations and stability analysis of feasible and optimal line balances for simple assembly lines. *Computers & Industrial Engineering*, *90*, 241-258.

Storer, R. H., Wu, S. D., & Vaccari, R. (1992). New search spaces for sequencing problems with application to job shop scheduling. *Management science*, *38*(10), 1495-1509.

Sullivan, W.G., E.M. Wicks, and C.P. Koelling (2015) Engineering Economy, 16th Ed. Upper Saddle River: Pearson Prentice Hall.

Süer, G. A. (1998). Designing parallel assembly lines. *Computers & industrial engineering*, *35*(3), 467-470.

Suresh, G., and Sahu, S. (1994). Stochastic assembly line balancing using simulated annealing. *The International Journal of Production Research*, *32*(8), 1801-1810.

Talbot, F. B., Patterson, J. H., & Gehrlein, W. V. (1986). A comparative evaluation of heuristic line balancing techniques. *Management science*,*32*(4), 430-454.

Toksarı, M. D., İşleyen, S. K., Güner, E., and Baykoç, Ö. F. (2008). Simple and U-type assembly line balancing problems with a learning effect. *Applied Mathematical Modeling*, 32(12), 2954-2961.

Terwiesch, C., and Bohn, R. E. (2001). Learning and process improvement during production ramp-up. *International Journal of Production Economics* ,70.1, 1-19.

Tuncel, G., & Aydin, D. (2014). Two-sided assembly line balancing using teaching–learning based optimization algorithm. *Computers & Industrial Engineering*, *74*, 291-299.

Ueda, K. (2007). Emergent synthesis approaches to biological manufacturing systems, *Digital Enterprise Technology*, Springer, 25 – 34.

Van Hop, N. (2006). A heuristic solution for fuzzy mixed-model line balancing problem. *European Journal of Operational Research*, *168*(3), 798-810.

Vance, P. H., Barnhart, C., Johnson, E. L., & Nemhauser, G. L. (1994). Solving binary cutting stock problems by column generation and branch-and-bound. *Computational optimization and applications*, *3*(2), 111-130.

Vigil, D. P., & Sarper, H. (1994). Estimating the effects of parameter variability on learning curve model predictions. *International journal of production economics*, *34*(2), 187-200.

Wang, B., Guan, Z., Li, D., Zhang, C., & Chen, L. (2014). Two-sided assembly line balancing with operator number and task constraints: a hybrid imperialist competitive algorithm. *The International Journal of Advanced Manufacturing Technology*, *74*(5-8), 791-805.

Whitney, D.E. (1982). Quasi-static assembly of compliantly supported rigid parts. *Transactions of ASME, Journal of Dynamic Systems Measurement and Control*, 104 (1), 65-77.

Wu, E. F., Jin, Y., Bao, J. S., & Hu, X. F. (2008). A branch-and-bound algorithm for two-sided assembly line balancing. *The International Journal of Advanced Manufacturing Technology*, *39*(9-10), 1009-1015.

Yelle, L. E. (1979). The learning curve: Historical review and comprehensive survey. *Decision Sciences*, *10*(2), 302-328.

Yuan, B., Zhang, C., & Shao, X. (2015). A late acceptance hill-climbing algorithm for balancing two-sided assembly lines with multiple constraints. *Journal of Intelligent Manufacturing*, *26*(1), 159-168.

Zacharia, P. T., & Nearchou, A. C. (2012). Multi-objective fuzzy assembly line balancing using genetic algorithms. *Journal of Intelligent Manufacturing*,*23*(3), 615-627.

Appendix A: Benchmark data sets

Small sized data set are displayed in their precedence graph. Big sized data set in a table consisting two parts, task times (the first column) and direct precedence relations (the second column)

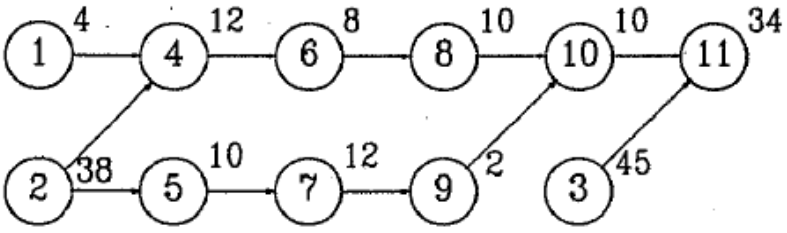

Figure 29 Precedence graph of Mansoor's data set

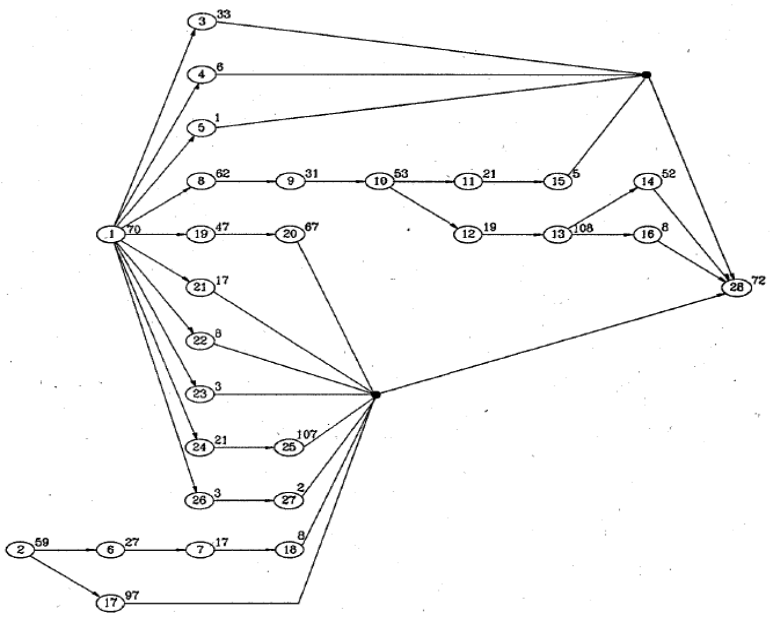

Figure 30 Precedence graph of Heskiaoff's data set

Large sized data sets

Kilbrid		Arcus (83)		Arcus (111)		Bartholdi		Scholl	
45	1,3	83	1,2	111	1,2	148	1,5	297	1,2
9	1,7	1673	2,3	1960	2,3	16	1,6	270	2,3
9	2,4	985	2,4	1715	3,4	30	1,7	270	3,4
10	2,8	1836	2,5	735	4,5	7	1,8	130	4,5
10	3,5	973	3,6	1715	4,6	47	2,3	148	4,22
17	4,6	1700	4,6	490	4,7	29	3,4	190	4,26
17	5,9	2881	4,7	1225	4,8	8	3,5	293	4,27
13	6,10	2231	5,8	169	4,9	39	3,6	348	4,40
13	7,9	1040	6,9	2252	4,10	37	3,7	182	4,48
20	7,14	1793	6,10	1225	5,39	32	4,8	490	4,56
20	8,10	1250	7,11	2319	6,39	29	5,14	212	4,83
10	8,14	700	8,77	1715	7,83	17	6,9	248	4,86
11	9,41	464	8,78	980	8,71	11	7,14	248	4,94
6	10,41	500	9,12	735	9,32	32	8,10	248	4,105
22	11,13	1133	10,13	2281	10,11	15	9,14	248	4,109
11	12,13	577	10,14	2750	10,12	53	10,14	248	4,111
19	12,37	483	10,25	77	11,13	53	11,12	268	4,134
12	13,14	880	11,15	89	11,14	8	12,13	268	4,221
3	13,15	667	12,16	51	11,15	24	14,15	268	4,247
7	14,17	600	13,17	364	11,16	24	14,16	288	4,259
4	14,25	233	13,18	405	11,17	8	15,17	248	5,6
55	14,29	408	13,20	3060	11,18	7	16,17	268	6,7
14	14,30	847	14,19	125	11,19	8	17,18	60	6,8
27	14,31	767	15,20	3429	11,20	14	17,19	268	6,9
29	14,32	850	15,39	43	11,21	13	18,20	240	6,10
26	15,16	780	16,77	3430	12,13	10	19,20	240	7,11
6	15,18	912	16,78	1960	12,14	25	20,21	171	7,12
5	15,23	748	17,21	29	12,15	11	20,22	490	7,13
24	15,24	1863	17,22	27	12,16	25	20,23	182	7,14
4	16,19	714	17,28	15	12,17	11	20,24	170	7,15
5	17,26	1004	18,23	121	12,18	29	21,25	306	7,20
7	17,27	713	19,24	1715	12,19	25	21,26	108	8,11
4	18,19	642	20,26	2127	12,20	10	21,27	248	8,12
15	19,20	629	21,27	1470	12,21	14	21,28	190	8,13
3	19,33	1234	22,27	4037	13,71	41	22,25	240	8,14
7	20,21	1143	23,74	68	14,22	42	22,26	339	8,15
9	21,22	1266	24,28	62	14,23	47	22,27	288	8,20
4	22,28	792	24,29	42	14,24	7	22,28	248	9,11
7	23,33	1251	25,32	364	14,25	80	23,25	455	9,12
5	24,33	1310	26,74	4998	15,26	7	23,26	268	9,13
4	25,26	663	27,69	1470	16,27	41	23,27	270	9,14

21	26,38	494	28,32	2963	17,28	47	23,28	180	9,15
12	27,28	1288	29,30	5689	18,29	16	24,25	121	9,20
6	27,33	792	30,31	68	19,30	32	24,26	270	10,11
5	28,38	578	31,39	18	20,91	66	24,27	440	10,12
5	29,41	594	32,33	10	21,111	80	24,28	249	10,13
	30,41	578	32,34	81	22,31	7	25,29	194	10,14
	31,41	622	32,35	5200	22,83	41	26,29	162	10,15
	32,41	578	32,36	39	23,32	13	27,29	130	10,20
	33,34	564	33,37	67	23,33	47	28,29	388	11,16
	33,35	578	34,77	27	24,69	33	29,31	90	12,17
	33,36	578	34,78	15	24,70	34	31,36	212	13,18
	34,38	578	35,77	121	25,34	11	32,34	246	14,19
	35,40	578	35,78	58	26,82	18	33,35	188	15,21
	36,38	578	36,38	1715	27,35	25	34,36	270	16,23
	37,43	578	36,39	125	28,36	7	35,36	160	17,23
	38,40	578	37,40	4010	29,37	28	36,37	79	18,23
	39,41	578	38,41	1470	30,38	12	37,38	466	19,23
	40,41	578	39,42	1470	31,39	52	37,45	240	20,23
	41,42	578	39,43	2303	32,41	14	38,39	137	21,23
	42,44	578	39,44	1960	33,111	3	39,40	184	22,24
	42,45	578	39,75	2205	34,42	3	40,41	110	22,25
		578	40,77	4018	35,43	8	40,48	275	23,28
		578	40,78	2744	36,44	16	40,54	149	24,29
		578	41,45	2999	36,91	33	42,43	280	25,29
		578	42,77	735	37,45	8	43,44	119	26,30
		578	42,78	735	37,91	18	45,46	184	27,31
		578	43,77	735	38,46	10	46,47	140	28,32
		578	43,78	735	38,91	14	47,48	150	28,37
		467	44,46	545	39,40	28	47,49	190	29,33
		887	45,47	3386	40,111	11	47,54	150	29,44
		396	46,48	3234	41,69	18	50,51	150	29,121
		1296	47,49	2205	41,70	25	51,53	284	30,34
		1100	48,50	2206	42,47	40	51,69	192	30,297
		2543	49,69	490	43,48	40	52,53	347	31,34
		764	50,51	825	43,49	1	54,55	232	31,82
		357	51,52	3528	44,50	5	54,72	140	31,172
		701	52,53	3568	45,51	28	54,76	608	31,179
		1164	53,54	1200	46,52	8	54,89	80	32,36
		286	54,55	618	47,54	81	54,90	40	33,38
		2100	55,56	1470	47,55	7	55,133	130	34,35
		450	56,57	1715	47,56	26	56,73	110	35,42
		1300	57,58	735	47,57	10	57,82	350	36,39
		3691	58,59	1960	47,58	21	58,86	140	37,39

59,60	2889	47,59	26	58,88	240	38,41
60,61	618	47,60	20	59,75	240	39,43
61,62	490	48,53	21	59,89	90	40,44
62,63	735	49,91	47	61,62	54	40,84
63,64	490	50,111	23	62,63	294	40,97
64,65	921	51,111	13	63,67	203	41,45
65,66	326	52,111	19	64,65	150	42,46
66,67	5390	53,111	15	64,71	270	43,47
67,68	243	54,69	35	64,72	155	44,49
68,74	371	54,70	26	65,66	190	45,50
68,75	58	55,61	46	65,99	78	46,51
69,70	5059	55,62	20	66,67	140	46,138
69,71	1225	55,63	31	67,68	241	47,52
70,72	769	56,63	19	68,95	430	48,52
71,73	768	56,64	34	68,98	90	49,53
72,73	1670	57,65	51	69,79	110	50,54
73,74	1670	57,91	39	70,71	9	51,55
73,75	490	58,66	30	72,134	430	51,81
74,76	202	58,91	26	73,86	130	52,57
75,76	203	59,67	13	73,88	289	53,58
76,77	202	59,91	45	73,89	110	54,58
76,78	2744	60,68	58	73,90	160	54,296
77,79	162	60,91	28	73,96	442	55,59
78,79	324	61,69	8	74,75	159	56,60
79,80	162	61,70	83	75,90	250	56,61
79,81	121	62,71	40	75,97	190	57,62
80,82	162	63,111	34	76,77	184	57,63
81,83	91	64,72	23	77,78	690	57,71
82,83		65,111	62	78,82	72	57,76
		66,111	11	79,85	190	58,64
		67,111	19	79,80	190	59,64
		68,111	14	79,143	90	59,99
		69,77	31	79,146	889	59,100
		69,78	32	80,81	170	60,68
		70,73	26	81,82	155	61,65
		71,91	55	82,83	190	62,66
		72,74	31	83,84	130	63,67
		73,75	32	84,106	390	64,72
		74,76	26	86,87	301	65,69
		75,77	19	90,111	54	66,69
		75,78	14	91,105	227	67,70
		75,79	19	92,135	142	68,73
		76,80	48	95,101	184	69,74

76,81	55	96,104	741	70,75
76,82	8	98,101	868	71,77
77,83	11	99,100	230	72,78
78,84	27	100,101	121	73,84
79,85	18	101,102	320	73,97
80,86	36	101,103	126	74,84
80,91	23	102,127	440	74,97
81,87	20	103,127	127	75,84
81,91	46	105,119	134	75,97
82,111	64	106,107	150	76,84
83,91	22	107,108	140	76,97
84,88	15	108,109	110	77,84
84,89	34	109,110	320	77,97
84,91	22	111,112	250	78,79
85,111	51	112,113	232	78,80
86,111	48	113,114	188	78,125
87,90	64	113,116	250	78,192
88,105	70	113,120	377	79,85
89,105	37	113,123	90	80,85
90,111	64	113,128	140	81,87
91,92	78	114,115	90	82,88
91,93	78	115,125	90	82,89
91,94		116,117	70	83,90
92,95		117,118	90	84,91
93,95		118,126	110	85,92
94,95		120,121	150	86,93
95,96		121,122	101	87,99
95,97		122,126	377	87,100
95,98		123,124	118	88,99
95,99		124,125	290	88,100
95,100		128,129	209	89,99
95,104		129,130	150	89,100
96,101		130,131	150	90,95
97,102		130,137	79	91,96
98,103		132,135	150	92,98
99,111		133,135	91	93,98
100,111		134,135	59	94,101
101,105		135,136	218	95,101
102,106		138,139	351	96,101
102,107		139,140	873	97,101
103,107		141,142	130	98,102
103,108		142,143	68	99,103
104,111		142,146	126	100,104

105,111	142,147	120	101,106
106,109	142,148	227	102,107
107,111	144,145	198	103,108
108,110	145,147	132	104,108
109,111	145,148	121	105,110
110,111		150	106,112
		100	107,113
		38	108,114
		70	108,115
		355	108,292
		284	109,119
		122	109,120
		75	110,119
		160	110,120
		140	110,162
		520	111,116
		99	112,117
		182	113,118
		80	114,119
		514	115,120
		96	116,122
		50	117,123
		272	117,124
		226	117,257
		194	118,126
		164	119,127
		96	120,127
		107	120,150
		108	121,128
		167	122,129
		98	123,130
		82	123,145
		482	123,146
		72	123,147
		50	123,148
		130	123,149
		230	124,130
		50	125,130
		240	126,130
		190	127,130
		190	127,157
		240	128,130
		74	129,130

139	129,141
339	130,131
260	130,144
132	131,132
550	131,133
420	132,135
152	133,135
12	133,170
90	134,136
5	135,137
128	136,139
100	137,140
120	138,140
100	138,191
320	139,142
835	139,253
740	140,143
223	140,200
100	141,151
390	142,152
140	143,153
304	143,169
120	144,154
403	145,155
21	146,156
246	147,158
160	148,159
1019	149,160
34	150,161
120	151,163
68	152,164
910	153,165
302	154,166
778	155,166
101	156,166
1310	157,166
20	158,166
278	159,166
81	160,166
290	161,166
100	162,167
372	163,166
72	164,167

28	165,168
90	165,176
250	166,170
144	167,171
303	168,173
220	169,174
58	170,174
224	171,174
211	172,175
99	173,177
44	174,178
120	174,287
70	174,288
421	175,180
231	176,181
214	177,181
196	178,181
280	179,181
398	180,181
72	180,252
280	181,182
356	181,183
193	181,184
140	181,185
130	181,186
300	181,187
456	181,188
7	181,189
170	181,196
252	181,197
210	181,295
308	182,190
308	183,193
121	184,194
52	185,195
426	186,195
104	187,195
1386	188,195
527	189,195
968	190,195
1047	191,200
538	192,201
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255,262
256,263
257,264

258,265
259,266
260,267
261,268
261,269
262,269
263,270
264,271
265,272
266,271
267,273
268,274
269,274
270,274
271,274
272,275
273,276
274,277
274,278
274,282
275,280
276,281
277,283
278,283
279,286
280,290
281,291
282,293
283,293
284,294
285,294
286,294
287,293
288,293
289,294
290,293
291,293
292,293

A65 (Lee et al., 2001), E stands for either side, R and L stand for right and left side respectively.

Task	Side	Task time	Immediate successors
1	E	49	3

2	E	49	3
3	E	71	4,23
4	E	26	5,6,7,9,11,12,25,26,27,41,45,49
5	E	42	14
6	E	30	14
7	R	167	8
8	R	91	14
9	L	52	10
10	L	153	14
11	E	68	14
12	E	52	14
13	E	135	14
14	E	54	15,18,20,22
15	E	57	16
16	L	151	17
17	L	39	31
18	R	194	19
19	R	35	21
20	E	119	21
21	E	34	31
22	E	38	31
23	E	104	24
24	E	84	31
25	L	113	31
26	R	72	31
27	R	62	28
28	R	272	50
29	L	89	50
30	L	49	50
31	E	11	32,36,51,52,53,54,55,56,58,59,60,61,62
32	E	45	33
33	E	54	34
34	E	106	35
35	R	132	50
36	E	52	37
37	E	157	38
38	E	109	39,40
39	L	32	50
40	R	32	50
41	E	52	42
42	E	193	43
43	E	34	62
44	R	34	46
45	L	97	46
46	E	37	47

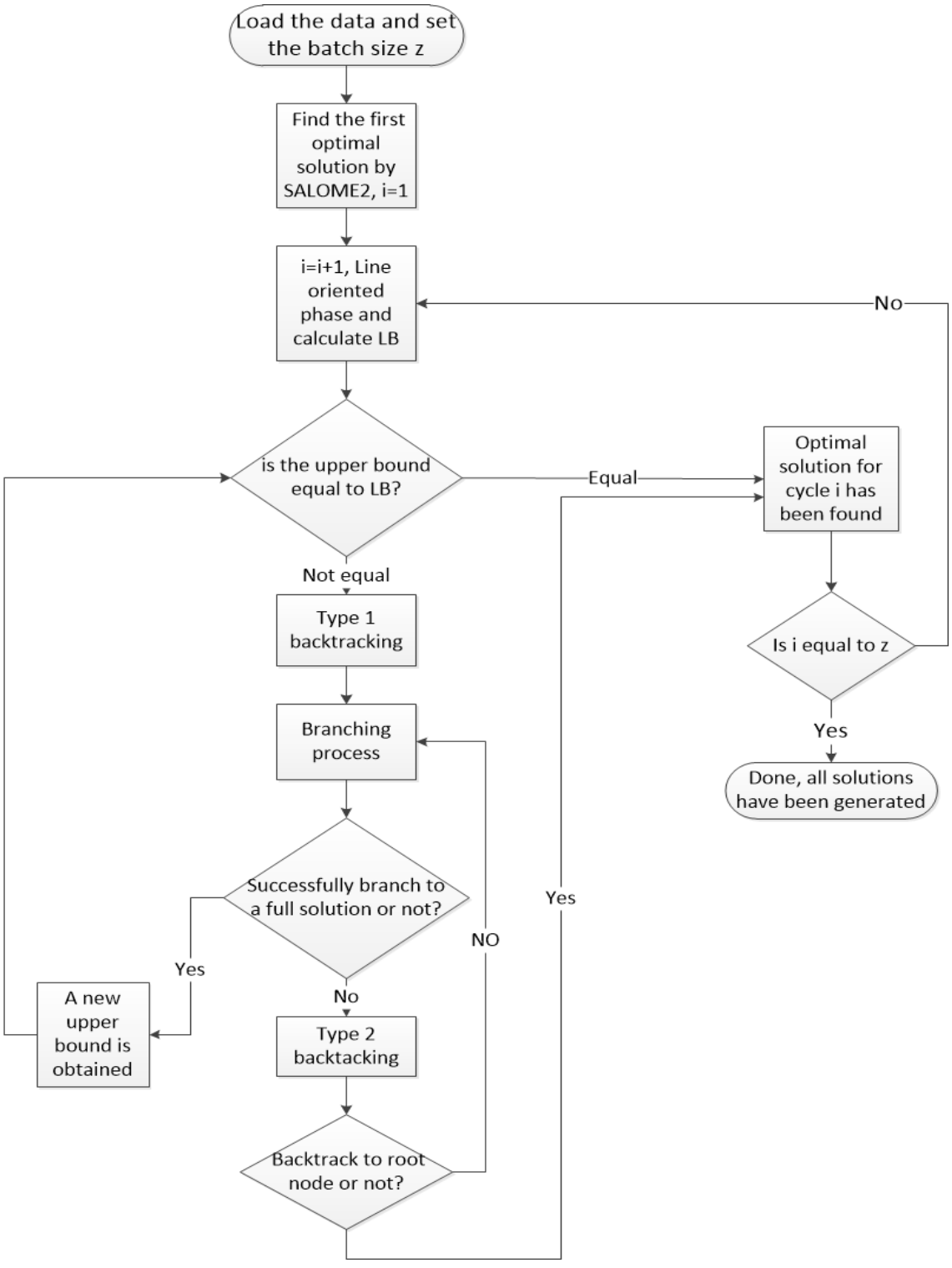
47	L	25	48
48	L	89	50
49	E	27	50
50	E	50	66
51	R	46	65
52	E	46	65
53	L	55	65
54	E	118	65
55	R	47	65
56	E	164	57
57	E	113	65
58	L	69	65
59	R	30	65
60	E	25	65
61	R	106	65
62	E	23	63
63	L	118	64
64	L	155	65
65	E	65	-

New optimal solution. For each position, the first row indicates the mated station to which the task is assigned, 1 is the left station and 2 is the right station; the second row are the task sequences.

Position 1											
1	2	1	2	1	1	2	1	1	2	2	1
1	2	3	13	4	9	11	12	5	6	44	49
Position 2											
1	2	1	1	2	2						
23	7	41	10	8	27						
Position 3											
1	2	1	2	1	2	1					
14	24	20	18	25	19	22					
Position 4											
1	2	1	2	2	1	1	1	2			
15	26	16	21	42	17	31	36	60			
Position 5											
1	2	1	1	2	1	2	1	2	2		
32	37	43	45	33	46	62	38	51	52		
Position 6											
1	2	1	2	1	1	2					
34	56	63	35	47	58	59					
Position 7											
2	1	1	1	2	1						
28	48	64	30	55	39						

Position 8							
1	2	1	2	1	2	2	1
57	54	29	61	53	40	50	65

Appendix B: The basic flow chart of ENCORE



Appendix C: Computational tests for backward induction algorithm

Matlab codes for computational test in section 3.4.1&3.4.2

Tables 5–10 are generated by SALOME and backward induction algorithm.

The steps of execution are as follows.

1) Open the test script

2) Adjust the parameters (learning rate, data set, batch size)

3) Run the script

Sample test script (Backward induction algorithm (weak))

```
clear all
tic
learning=0.95;
alpha=log(learning)/log(2);
c=168;
n=30;
load Heskiaoff
[OS,Sq,m,Tsq,avg_dev,omitted_B]= Algo1_weak(c,P,t0,n,learning);
elapsed_time=toc;
```

Sample test script (Backward induction algorithm (strong))

```
clear all
tic
learning=0.95;
alpha=log(learning)/log(2);
c=168;
n=30;
load Heskiaoff
[OS,Sq,m,Tsq,avg_dev,omitted_B]= Algo1_strong(c,P,t0,n,learning);
elapsed_time=toc;
```

Sample test script (SALOME)

```
clear all
tic;
learning=0.9;
alpha=log(learning)/log(2);
c=168;
n=10;
total_dev=[];
omitted_B=0;
Tsq=0;
load Heskiaoff
```

```
for i=1:n-1
   t(:,i)=t0.*i^alpha;
end
 t(:,n)=t0.*n^alpha;
answer=[];
for i=1:n
    [OS,Sq,m,UM]=mainSALOME(c,P,t(:,i));
    Tsq=Tsq+size(Sq,1);
    total_dev=[total_dev;(UM-min(UM))/(min(UM)-1)*100];
    answer=[answer,m];
end
av_dev=mean(total_dev);

elapsed_time=toc;
```

The main function for SALOME:

```
function [OS,Sq,m,UM]=mainSALOME(c,P,t)
%Jackson dominance rule
%find all followers&direct followers
D_Mat=Jackson_Dominance(P,t);

[S,Sq,UM,LM,cs,Idle]=FinFea(c,P,t);
b=ones(length(t),1);
d=Sq;
Lt=length(d);
OS=S;
GLM=ceil(sum(t)/c)+1;
if UM==GLM
    m=UM;
else
    %whether or not the SALOME is finished
    Fin=0;
    while 1

[S,b,cs,d,Lt,Idle,Fin]=Backtrack(S,b,cs,d,Lt,t,c,LM,Idle,Fin);
        if Fin==1
            break
        end
        %set the uniqueness equal to zero
        Uni=0;
        while 1
            %store states for later retrieval
            b1=b;
            S1=S;
```

```
            d1=d;
            d2=[];

[S,Sq,b,d,Lt,LM,cs,Idle,Uni,d2]=StationLoadsFinder(S,Sq,b,d,cs,L
M,UM,Lt,Idle,Uni,c,P,t,d2,D_Mat);
            if Uni==0
                b=b1;
                S=S1;
                d=d1;
            end
            if Uni==1
                break
            end

[S,b,cs,d,Lt,LM,Idle,Fin]=Backtrack1(S,b,cs,d,LM,t,c,Idle,Fin);
            if Fin==1
                break
            end
        end
        if Fin==1
            break
        end

[OS,S,Sq,b,d,cs,Lt,LM,UM,Idle]=NewSeqGen(OS,S,Sq,b,d,cs,LM,UM,Lt
,Idle,c,P,t,D_Mat);
        if UM==GLM
            break
        end
    end
    m=UM;

end
end
```

The main function for backward induction algorithm (weak):

```
function                          [OS,Sq,m,Tsq,avg_dev,omitted_B]=
Algo1_weak(c,P,t0,n,learning)
%calculate the task time of all repetitions.
alpha=log(learning)/log(2);
omitted_B=0;
for i=1:n
    t(:,i)=t0.*i^alpha;
end
%Initialize the Optimal solution set
OS=zeros(length(t0),length(t0),n);
```

```
m=zeros(1,n);
%find the first optimal solution
total_dev=[];
[OS(:,:,n),Sq,m(n),UM]=mainSALOME(c,P,t(:,n));
total_dev=[total_dev;(UM-min(UM))/(min(UM)-1)*100];
Tsq=size(Sq,1);
%start the explosion theory
i=n;
while 1

    D=OS(:,:,i)*t0;
    e=max(D);
    j=floor((c/e)^(1/alpha));
    if i-j>=2
        %in this case, the deviation to optimality is zero
        total_dev=[total_dev;zeros(i-j-1,1)];
        %total BnB skipped
        omitted_B=omitted_B+i-j-1;
        m(j+1:i-1)=m(i);
        for k=j+1:i-1
            OS(:,:,k)=OS(:,:,i);
        end
    end
    if j==0;
        break
    end
    i=j;
    [OS(:,:,i),Sq,m(i),UM]=mainSALOME(c,P,t(:,i));
    total_dev=[total_dev;(UM-min(UM))/(min(UM)-1)*100];
    Tsq=Tsq+size(Sq,1);
    if i==1
        break
    end

end
avg_dev=mean(total_dev);

end
```

The main function for backward induction algorithm (strong):

```
function [OS,Sq,m,Tsq,omitted_B]= Algo1_strong(c,P,t,n)

%Initialize the Optimal solution set
OS=zeros(size(t,1),size(t,1),n);
m=zeros(1,n);
%find the first optimal solution
```

```
[OS(:,:,n),Sq,m(n),UM]=mainSALOME(c,P,t(:,n));
Tsq=size(Sq,1);
omitted_B=0;
%start the explosion theroy
for i=n-1:-1:1
    %initialize the feasibiliy=1
    fea=1;
    OS(:,:,i)=OS(:,:,i+1);
    m(i)=m(i+1);
    for j=1:m(i)
        if OS(j,:,i)*t(:,i)>c
            fea=0;
            break
        end
    end
    if fea==1
        OS(:,:,i)=OS(:,:,i+1);
        m(i)=m(i+1);
        omitted_B=omitted_B+1;
    else
        [OS(:,:,i),Sq,m(i),UM]=mainSALOME(c,P,t(:,i));
        Tsq=Tsq+size(Sq,1);
    end
end
end
```

The sub-functions called by the SALOME or backward induction algorithm are presented as follows.

The function of finding a feasible solution

```
function [S,Sq,UM,LM,cs,Idle]=FinFea(c,P,t)
%find the lm for station 1 which is loaded by the first node
LM(1)=ceil(sum(t)/c)+1;
%find the available idle time
Idle=(LM(1)-1)*c-sum(t);
%b is the array storing all of the assigned tasks
b=[1;zeros(length(t)-1,1)];
```

```
%initialize station&current station put a dummy task(0) into a
fake station
S=zeros(length(t),length(t));
S(1,1)=1;
cs=2;
%initialize assignable task&sequence
at=[];
Sq=[1 0];
%begin the loop
while 1
%find the assignable task
for i=2:length(t)
   %%three requirements:1 task has not been assigned 2 its
predecessors
   %%have been assigned 3 it could not exceed its cycle time
if b(i)~=1 && b'*P(:,i)==sum(P(:,i)) && c-S(cs,:)*t-t(i)>=0
    %i-1 is actually the task number!
    at=[at,i];
end
end
%assign task
%check if there is any room to assign any task
if isempty(at)
    %calculate the remaining idle time(definition: the remaining
idle time
    %of the station is calculated in absence of any assignment to
that station)
    Idle=Idle-(c-S(cs,:)*t);
    if Idle>=0
        LM(cs)=LM(cs-1);
    else
        LM(cs)=LM(cs-1)+1;
        %one more station time to spare!
        Idle=Idle+c;
    end
    cs=cs+1;
    Sq=[Sq 0];
else
    %find max task time to assign
[a,index]=max(t(at(1:end)));
%retrieve the index of the task
index=at(index);
S(cs,index)=1;
Sq=[Sq,index];
b(index)=1;
end
```

```
if b==ones(length(t),1)
    break
end
% reset assignable task array
at=[];

end
%put an artificial LM for root node, the last LM is the current UM
LM=[LM(1),LM];
%find the current m which is the upper bound of the optimal
solution
for i=1:length(t)
    if sum(S(i,:))==0
    UM=i-1;
    break
    end
end
end
```

Backtrack function

```
function[S,b,cs,d,Lt,Idle,Fin]=Backtrack(S,b,cs,d,Lt,t,c,LM,Idle
,Fin)
%station oriented backtrack
%backtrack process follows the new sequence

%find the last station whose LM is smaller than current station's
LM
k=find(LM==LM(cs)-1,1,'last');
%backtrack to k
if isempty(k)
    %current solution is the optimal solution
    Fin=1;
else
    %empty the stations and sequences from k to current station
    Index2=find(d==0,k-1);
    d(Index2(end):end)=0;
    %release the idle time of the station
    Idle=Idle+((cs-k)*c-sum(S(k:(cs-1),:)*t));
    for j=k:cs
        Index1= S(j,:)~=0;
        b(Index1)=0;
        S(j,:)=0;
    end
```

```
    %reduce one full station time
    Idle=Idle-c;
    cs=k;
    %set the last task: find the (k-1)th zero in the current
sequence, the      last task is next to it.
    E=find(d==0,k-1);
    Lt=E(end)+1;
end
end
```

Station enumeration function

```
function
[S,Sq,b,d,Lt,LM,cs,Idle,Uni,d2]=StationLoadsFinder(S,Sq,b,d,cs,L
M,UM,Lt,Idle,Uni,c,P,t,d2,D_Mat)
at=[];
for i=find(b==0,1):length(t)
    if b(i)~=1&&b'*P(:,i)==sum(P(:,i)) && c-S(cs,:)*t-t(i)>=0
        at=[at,i];
    end
end
%downsize the assignable task by Jackson's dominance rule
if  length(at)>=2
    Dominated_task=[];
    for o=1:length(at)
        for r=(o+1):length(at)
            %is at(o) dominating at(r)?
            if ismember(at(r),D_Mat(at(o),:))
                Dominated_task=[Dominated_task;r];
            elseif ismember(at(o),D_Mat(at(r),:))
                %is at(r) dominating at(o)?
                Dominated_task=[Dominated_task;o];
            end
        end
    end
at(Dominated_task)=[];
end
if isempty(at)
    OL=[];
    for k=1:size(Sq,1)
        if isequal(Sq(k,1:Lt-1),d(1:Lt-1))
            OL=1;
            break
        end
    end
```

```
    %if the new load is of the same local lower bound or branched
before,
    %give it up.
    %TLM:temporary local lower bound
    if Idle-(c-S(cs,:)*t)>=0
        TLM=LM(cs);
    else
        TLM=LM(cs)+1;
    end
    if ~isempty(OL)||TLM>LM(cs+1)|| TLM>=UM
        %give it up, empty the current task in the current station
and sequence(revert to the last state )
        Lt=Lt-1;
        b(d(Lt))=0;
        S(cs,d(Lt))=0;
        d(Lt)=0;
    else
        %calculate the local lower bound
        LM(cs+1)=TLM;
        Uni=1;
        Idle=Idle-(c-S(cs,:)*t);
        if LM(cs+1)>LM(cs)
            Idle=Idle+c;
        end
        cs=cs+1;
        E=find(d==0,cs-1);
        Lt=E(end)+1;
    end
else
    for j=1:length(at)
        b(at(j))=1;
        d(Lt)=at(j);
        S(cs,at(j))=1;
        Lt=Lt+1;
[S,Sq,b,d,Lt,LM,cs,Idle,Uni,d2]=StationLoadsFinder(S,Sq,b,d,cs,L
M,UM,Lt,Idle,Uni,c,P,t,d2,D_Mat);
        if Uni==1
            %already find an unique solution, stop permuting
            break
        end
        % if j is the last choice, but it is still not working,
erase the previous selection when previous selection exists.
        if j==length(at)&&d(Lt-1)~=0
            Lt=Lt-1;
            b(d(Lt))=0;
            S(cs,d(Lt))=0;
```

```
            d(Lt)=0;
        end
    end
end
end
```

Superior solution generation function

```
function
[OS,S,Sq,b,d,cs,Lt,LM,UM,Idle]=NewSeqGen(OS,S,Sq,b,d,cs,LM,UM,Lt
,Idle,c,P,t,D_Mat)
%station oriented BB
at=[];
while 1
%find the assignable task
for i=2:length(t)
   %%three requirements:1 task has not been assigned 2 its
predecessors
   %%have been assigned 3 it could not exceed its cycle time
if b(i)~=1 && b'*P(:,i)==sum(P(:,i)) && c-S(cs,:)*t-t(i)>=0
    %i-1 is actually the task number!
    at=[at,i];
end
if  length(at)>=2
    Dominated_task=[];
    for o=1:length(at)
        for r=(o+1):length(at)
            %is at(o) dominating at(r)?
            if ismember(at(r),D_Mat(at(o),:))
                Dominated_task=[Dominated_task;r];
            elseif ismember(at(o),D_Mat(at(r),:))
                %is at(r) dominating at(o)?
                Dominated_task=[Dominated_task;o];
            end
        end
    end
at(Dominated_task)=[];
end
end
%assign task
%check if there is any room to assign any task
if isempty(at)
    %calculate the remaining idle time(definition: the remaining
idle time
    %of the station is calculated in absence of any assignment to
that station)
    Idle=Idle-(c-S(cs,:)*t);
```

```
    if Idle>=0
        LM(cs+1)=LM(cs);
    else
        LM(cs+1:end)=LM(cs)+1;
        %one more station time to spare!
        Idle=Idle+c;
    end
    %Compare the local LM and the global UM
    if LM(cs+1)>=UM
        %done! stop generating the sequence, backtrack
        Sq=[Sq;d];
        size(Sq,1)
        cs=cs+1;
        break
    else
    cs=cs+1;
    d(Lt)=0;
    Lt=Lt+1;
    end
else
 %find max task time to assign
[a,index]=max(t(at(1:end)));
%retrieve the index of the task
index=at(index);
S(cs,index)=1;
d(Lt)=index;
b(index)=1;
Lt=Lt+1;
end
if b==ones(length(t),1)
     Sq=[Sq;d];
     Idle=Idle-(c-S(cs,:)*t);
     %New UM
     UM=LM(cs);
     LM(cs+1)=LM(cs);
     cs=cs+1;
     OS=S;
    break
end
% reset assignable task array
at=[];

end
end
```

Appendix D: Case study

Matlab codes for the case study in section 3.5

Table 11 is generated by the following script.

```
t0=[6;6;5;5;4;5;4;2;9;2];
learning=0.85;
c=10;
n=30;
t=zeros(10,n);
P =[

     0     1     1     1     1     1     1     1     1     1     1
     0     0     1     0     0     1     0     0     0     0     0
     0     0     0     0     0     0     0     1     0     0     0
     0     0     0     0     1     0     0     0     0     0     0
     0     0     0     0     0     1     0     0     0     0     0
     0     0     0     0     0     0     1     0     0     0     0
     0     0     0     0     0     0     0     0     1     0     0
     0     0     0     0     0     0     0     0     1     0     0
     0     0     0     0     0     0     0     0     0     1     0
     0     0     0     0     0     0     0     0     0     0     1
     0      0      0      0      0      0      0      0      0      0
0];
 alpha=log(learning)/log(2);
for i=1:n
    t(:,i)=t0.*i^alpha;
end
%without rebalancing %({3,4},{1,5},{2,7}, {6,8},{9} {10}) .
TI_wo=0;
for i=1:30
    T1(1)=c-(t(3,i)+t(4,i));
    T1(2)=c-(t(1,i)+t(5,i));
    T1(3)=c-(t(2,i)+t(7,i));
    T1(4)=c-(t(6,i)+t(8,i));
    T1(5)=c-t(9,i);
    T1(6)=c-t(10,i);
    TI_wo=TI_wo+sum(T1);
end
%with rebalacning unit 1 %({3,4},{1,5},{2,7}, {6,8},{9} {10})
T1=[];
TI_w=0;
T1(1)=c-(t(3,1)+t(4,1));
T1(2)=c-(t(1,1)+t(5,1));
T1(3)=c-(t(2,1)+t(7,1));
T1(4)=c-(t(6,1)+t(8,1));
T1(5)=c-t(9,1);
```

```
T1(6)=c-t(10,1);
TI_w=TI_w+sum(T1);
T1=[];
%unit 2,3, ({1,3},{2,4},{5,6},{7,8},{9,10}
for i=2:3
    T1(1)=c-(t(1,i)+t(3,i));
    T1(2)=c-(t(2,i)+t(4,i));
    T1(3)=c-(t(5,i)+t(6,i));
    T1(4)=c-(t(7,i)+t(8,i));
    T1(5)=c-t(9,i)-t(10,i);
    TI_w=TI_w+sum(T1);
end
T1=[];
%unit 4-9, ({1,3},{2,4},{5,6,7},{8,9,10}).
for i=4:9
    T1(1)=c-(t(1,i)+t(3,i));
    T1(2)=c-(t(2,i)+t(4,i));
    T1(3)=c-(t(5,i)+t(6,i)+t(7,i));
    T1(4)=c-(t(8,i)+t(9,i)+t(10,i));
    TI_w=TI_w+sum(T1);
end
T1=[];
%unit 10-30, ({1,3,4},{2,5,6},{7,8,9,10}).
for i=10:30
    T1(1)=c-(t(1,i)+t(3,i)+t(4,i));
    T1(2)=c-(t(2,i)+t(5,i)+t(6,i));
    T1(3)=c-(t(7,i)+t(8,i)+t(9,i)+t(10,i));
    TI_w=TI_w+sum(T1);
end
TS_rebal=1*6+2*5+6*4+21*3
```

Appendix E: Computational tests for ENCORE

Matlab codes for computational test in section 4.2.5

Tables 12–14 are generated by SALOME2 and ENCORE.

The steps of execution are as follows.

1) Open the test script

2) Adjust the parameters (data set, batch size)

3) Run the script

Sample test function (ENCORE)

```
clear all;
m=8;
load Arcus83
%generate random task time matrix
n=30;
t1=zeros(length(t),n);
Q=t;
for i=1:n
    for j=2:length(t)
    t1(j,i)=ceil(Q(j)-random('uni',0.8,1)*t(j)*0.02);
    end
    Q=t1(:,i);
end
tic
[OptiC1,Sq1,OS1]=MainSALOME2(m,P,t);
Tsq3=0;
Tsq=0;
TB=0;
aa=1;
for i=1:n

[OptiC,Sq1,OS1,B]=Algorithm3(Sq1,OS1,m,P,t1(:,i));
Tsq3=Tsq3+size(Sq1,1);
TB=TB+B;
aa=aa+1
end
a=toc;
bb=0;
Tsq=0;
%optimum is not found in current solution
UNF=0;
 for i=1:n

[OptiC,Sq,OS]=MainSALOME2(m,P,t1(:,i));
```

```
if size(Sq,1)>=1000
    UNF=UNF+1;
end

Tsq=Tsq+size(Sq,1);
bb=bb+1
end
b=toc;
c=b-a;
```

Main function for SALOME2

```
function [OptiC,Sq,OS]=MainSALOME2(D_Mat,m,P,t,ct)
%ct computational time requirement
[S,Sq,UB,LM,TOI,TOIL]=FinFea(m,P,t);
if UB==max(ceil(sum(t)/m),max(t))
    OptiC=UB;
    OS=S;
else
    %backtrack
    OS=S;
    d=Sq;
    Lt=length(d);
    b=ones(length(t),1);
    cs=m+1;
    Fin=0;
    %start recording computational time
    tic;
    while 1
        %%enter the first backtrack

[S,d,cs,Lt,b,TOI,TOIL,Fin]=Backtrack(S,b,cs,d,Lt,t,m,LM,TOI,TOIL
,Fin);

        if Fin==1
            break
        end
        %set the uniqueness equal to zero
        Uni=0;
        while 1

[S,b,d,Lt,LM,cs,TOI,TOIL,Uni]=StationLoadsFinder(S,Sq,b,d,cs,LM,
UB,Lt,TOI,TOIL,Uni,m,P,t,D_Mat);
            if Uni==1
                break
```

```
            end
            while 1
                %enter the second backtrack(one station per time)

[S,d,cs,Lt,b,TOI,TOIL,Fin]=Backtrack1(S,b,cs,d,Lt,t,m,LM,TOI,TOI
L,Fin);
                if Fin==1
                    break
                end

[S,b,d,Lt,LM,cs,TOI,TOIL,Uni]=StationLoadsFinder(S,Sq,b,d,cs,LM,
UB,Lt,TOI,TOIL,Uni,m,P,t,D_Mat);
                if Uni==1
                    break
                end
            end
            if Uni==1
                break
            end
            if Fin==1
                break
            end
        end
        if Fin==1
            break
        end

[OS,S,Sq,b,d,cs,Lt,LM,UB,TOI,TOIL,Fin]=NewSeqGen(OS,S,Sq,b,d,cs,
LM,UB,Lt,TOI,TOIL,m,P,t,D_Mat);
        TOC=toc;
        %exit the problem when toc>ct
        if TOC>ct
            OptiC=UB;
        break
        end
    end

end
OptiC=UB;
end
```

The function of finding a feasible solution

```
function [S,Sq,UB,LM,TOI,TOIL]=FinFea(m,P,t)
%Starting from a lower bound
LB=ceil(sum(t)/m);
```

```
%find the lm for station 1 which is loaded by the first node
LM(1)=ceil(sum(t)/m);
TOI=m*LB-sum(t);
TOIL(1)=TOI;
%b is the array storing all of the assigned tasks
b=[1;zeros(length(t)-1,1)];
at=[];
at1=[];
%current station
cs=2;
%initialize station&current station put a dummy task(0) into a
fake station
S=zeros(length(t),length(t));
S(1,1)=1;
Sq=[1 0];
while 1

    for i=2:length(t)
        %%three requirements:1 task has not been assigned 2 its
predecessors
        %%have been assigned 3 it could not exceed its cycle time
        if  b(i)~=1  &&  b'*P(:,i)==sum(P(:,i))  &&  LB-S(cs,:)*t-
t(i)>=0
            %i-1 is actually the task number!
            at=[at,i];
        end
    end
    if isempty(at)
        %calculate the total idle time left
        if TOIL(cs-1)-(LB-S(cs,:)*t)<0
            %find the lowest available task to assign
            for i=2:length(t)
                %%two requirements:1 task has not been assigned 2
its
                predecessors %%have been assigned
                if b(i)~=1 && b'*P(:,i)==sum(P(:,i))
                    %i-1 is actually the task number!
                    at1=[at1,i];
                end
            end
            [a,index1]=min(t(at1(1:end)));
            %retrieve the index of the task
            index1=at1(index1);
            S(cs,index1)=1;
            Sq=[Sq,index1];
            b(index1)=1;
```

```
            %increase the local lower bound
            LM(cs)=S(cs,:)*t;
            LB=LM(cs);
            %update the TOI&TOIL
            TOI=m*LM(cs)-sum(t);
            tempTOIL=TOI;
            for j=2:cs
                tempTOIL=tempTOIL-(LB-S(j,:)*t);
            end
            TOIL(cs)=tempTOIL;
            at1=[];
        else
            TOIL(cs)=TOIL(cs-1)-(LB-S(cs,:)*t);
            LM(cs)=LM(cs-1);
        end
        cs=cs+1;
        Sq=[Sq 0];
    else
        %find max task time to assign
        [a,index]=max(t(at(1:end)));
        %retrieve the index of the task
        index=at(index);
        S(cs,index)=1;
        Sq=[Sq,index];
        b(index)=1;
    end
    if b==ones(length(t),1)
        %at times, there may be one or more station vacant when
the
        %assignment is done!
        if cs==m+1
        TOIL(cs)=TOIL(cs-1)-(LB-S(cs,:)*t);
        LM(cs)=LM(cs-1);
           break
        else
           %take out the last tasks from the last station and fill
them the into to the vacant station.
           for k=1:(m+1-cs)
               cc(k)=Sq(end-k+1);
               Sq(end-k+1)=0;
           end
           S(cs,cc(1:end))=0;
           cc=fliplr(cc);
           l=length(cc);
           for k=1:length(cc)
               Sq(end-l+2)=cc(k);
```

```
                Sq(end-l+3)=0;
                l=l-1;
                TOIL(cs)=TOIL(cs-1)-(LB-S(cs,:)*t);
                LM(cs)=LM(cs-1);
                cs=cs+1;
                S(cs,cc(k))=1;
                TOIL(cs)=TOIL(cs-1)-(LB-S(cs,:)*t);
                LM(cs)=LM(cs-1);
            end
            Sq(length(t)+m+1:end)=[];
            break
        end
    end
    % reset assignable task array
    at=[];
end

UB=LM(end);

end
```

Backtrack function (SALOME2)

```
function
[S,d,cs,Lt,b,TOI,TOIL,Fin]=Backtrack(S,b,cs,d,Lt,t,m,LM,TOI,TOIL
,Fin)
%station oriented backtrack
%backtrack process follows the new sequence

aa=sum(t);
%find the last station whose LM is Equal to current station's
LM,empty it
%out
k=find(LM==LM(cs),1);
if k==1 || cs==1
    %current solution is the optimal solution
    Fin=1;
else
    %empty the stations and sequences from k to current station
    Index2=find(d==0,k-1);
    d(Index2(end):end)=0;
    %recalculate the TOI and TOIL
    TOI=LM(k-1)*m-aa;
    TOIL(1)=TOI;
    for j=2:(k-1)
        TOIL(j)=TOIL(j-1)-(LM(k-1)-S(j,:)*t);
```

```
    end
    for j=k:cs
        Index1= S(j,:)~=0;
        b(Index1)=0;
        S(j,:)=0;
    end
    cs=k;
    %set the last task:find the (k-1)th zero in the current
sequence, the last task
    %is next to it.
    E=find(d==0,k-1);
    Lt=E(end)+1;
end
end
```

Station enumeration function (SALOME2)

```
function
[S,b,d,Lt,LM,cs,TOI,TOIL,Uni]=StationLoadsFinder(S,Sq,b,d,cs,LM,
UB,Lt,TOI,TOIL,Uni,m,P,t,D_Mat)
%%ONLY ONE chance of keep on going to generate new sequence 1)the
%%lower bound is not increasing at next station
at=[];
at1=[];

if d(Lt-1)==0
    for i=find(b==0,1):length(t)
        if b(i)~=1&&b'*P(:,i)==sum(P(:,i)) && UB-S(cs,:)*t-t(i)>0
            at=[at,i];
        end
    end
else
    for i=d(Lt-1)+1:length(t)
        if b(i)~=1&&b'*P(:,i)==sum(P(:,i)) && UB-S(cs,:)*t-t(i)>0
            at=[at,i];
        end
    end
end
%%downsize the assignable task by Jackson's dominance rule
if  length(at)>=2
    Dominated_task=[];
    for o=1:length(at)
        for r=(o+1):length(at)
            %is at(o) dominating at(r)?
            if ismember(at(r),D_Mat(at(o),:))
                Dominated_task=[Dominated_task;r];
            elseif ismember(at(o),D_Mat(at(r),:))
```

```
                %is at(r) dominating at(o)?
                Dominated_task=[Dominated_task;o];
            end
        end
    end
at(Dominated_task)=[];
end
if isempty(at)
    %check the overlapping first 1) if overlap, forfeit the
solution 2)if
    %not, check the feasibility by TOIL
    %if the station is empty now, consider increase
        OL=[];
        for k=1:size(Sq,1)
            if isequal(Sq(k,1:Lt),d(1:Lt))
                OL=1;
                break
            end
        end
        %if the new load is of the same local lower bound or
branched before,
        %give it up.
        if isempty(OL)
            AA=max(LM(cs-1),S(cs,:)*t);
            %check whether the TOIL is less than zero
            tempTOIL=AA*m-sum(t);
            for j=2:cs
                tempTOIL=tempTOIL-(AA-S(j,:)*t);
                if tempTOIL<0
                    break
                end
            end
            if tempTOIL>=0
                %the solution is good
                LM(cs)=AA;
                Uni=1;
                TOI=m*LM(cs)-sum(t);
                tempTOIL=TOI;
                for j=2:cs
                    tempTOIL=tempTOIL-(LM(cs)-S(j,:)*t);
                end
                TOIL(cs)=tempTOIL;
                cs=cs+1;
                E=find(d==0,cs-1);
                Lt=E(end)+1;
            else
```

```
                %forfeit the solution
                Lt=Lt-1;
                b(d(Lt))=0;
                S(cs,d(Lt))=0;
                d(Lt)=0;
            end
        else
            %forfeit the solution
            Lt=Lt-1;
            b(d(Lt))=0;
            S(cs,d(Lt))=0;
            d(Lt)=0;
        end
else
    for j=1:length(at)
        b(at(j))=1;
        d(Lt)=at(j);
        S(cs,at(j))=1;
        Lt=Lt+1;

[S,b,d,Lt,LM,cs,TOI,TOIL,Uni]=StationLoadsFinder(S,Sq,b,d,cs,LM,
UB,Lt,TOI,TOIL,Uni,m,P,t,D_Mat);
        if Uni==1
            %already find an unique solution, stop permuting
            break
        end
        % if j is the last and only choice , but it is still not
working, erase the previous selection(j) when previous selection
exists(has not been erased in the ifempty(at) part).
        if j==length(at)&&d(Lt-1)~=0
            Lt=Lt-1;
            b(d(Lt))=0;
            S(cs,d(Lt))=0;
            d(Lt)=0;
        end
    end

end
end
```

Superior solution generation function (SALOME2)

```
function
[OS,S,Sq,b,d,cs,Lt,LM,UB,TOI,TOIL,Fin]=NewSeqGen(OS,S,Sq,b,d,cs,
LM,UB,Lt,TOI,TOIL,m,P,t,D_Mat)
%station oriented BB
```

```
at=[];
at1=[];
Fin=0;
while 1

%if there is no task left, but it has not reached the last station
%relocate last few tasks to last few stations and recalculate the
UB
if sum(b)==length(t)&&cs<=m
    %the station is full, but cs did not increase by i, which it
should
    cs=cs+1;
    k=m+2-cs;
    %withdraw the tasks
    v=find(d~=0,1,'last');
    aa=d(v-k+1:v);
    %set the S(v) to zero
    for o=1:length(aa)
    S(cs-1,aa(o))=0;
    S(cs+o-1,aa(o))=1;
    end
    %set the d(v)to zero
    d(v-k+1:end)=0;
    j=length(d);
    for l=1:k
        d(j)=aa(k-l+1);
        j=j-2;
    end
    [LL,I]=max(S*t);
    LM(I+1:end)=LL;
    UB=LL;
    OS=S;
    Sq=[Sq;d];
    if UB==max(ceil(sum(t)/m),max(t))
                OptiC=UB;
                Fin=1;
    end
    break
end
%check whether it is the LAST STATION

if cs==(m+1)
    %last station no need to assign
    a1=find(b==0,length(t));
    d(Lt:end)=a1;
    for i=1:length(a1)
```

```
    S(cs,a1(i))=1;
    end
    OS=S;
    Sq=[Sq;d];
    LM(cs)=LM(cs-1);
    b=ones(1,length(t))';
    UB=LM(cs)
    if UB==max(ceil(sum(t)/m),max(t))
                OptiC=UB;
                Fin=1;
    end
    break
end
    %find the assignable task
    for i=find(b==0,1):length(t)
        %%three requirements:1 task has not been assigned 2 its
predecessors
        %%have been assigned 3 it could not exceed its cycle time
        if  b(i)~=1  &&  b'*P(:,i)==sum(P(:,i))  &&  UB-S(cs,:)*t-
t(i)>0
            %i-1 is actually the task number!
            %choose the first number!
            at=i;
            break
        end
    end
    if isempty(at)
            %locate the bottleneck station
            AA=max(S(cs,:)*t,LM(cs-1));
            %calculate the total idle time left
            if TOIL(cs-1)-(AA-S(cs,:)*t)<0&&AA==LM(cs-1)
                %solution is counterfeited
                Sq=[Sq;d];
                size(Sq,1);
                LM
                %backtrack
                break
            else
                Lt=Lt+1;
                LM(cs)=AA;
                %update the TOI&TOIL
                TOI=m*LM(cs)-sum(t);
                tempTOIL=TOI;
                for j=2:cs
                    tempTOIL=tempTOIL-(LM(cs)-S(j,:)*t);
                end
```

```
                TOIL(cs)=tempTOIL;
                at1=[];
                cs=cs+1;
                d(Lt)=0;
            end
    else
        S(cs,at)=1;
        d(Lt)=at;
        Lt=Lt+1;
        b(at)=1;
    end
    at=[];
end
end
```

Main function for ENCORE

```
function [OptiC,Sq,OS,B]=Algorithm3(Sq1,OS1,m,P,t1)
D_Mat=Jackson_Dominance(P,t1);
%update the critical station
T=OS1*t1;
a=max(T);
%set the optimal solution to the critical station
OptiC=a;
%retrieve the optimal sequence
Tot=(1+size(t1,1))*size(t1,1)/2;
SS=sum(Sq1,2);
index=find(SS==Tot,1,'last');
OSq=Sq1(index,:);
B=0;
if OptiC==max(ceil(sum(t1)/m),max(t1))
    OptiC=max(ceil(sum(t1)/m),max(t1));
    Sq=Sq1;
    OS=OS1;
    B=1;
else
    % update TOI and TOIL
    UB=OptiC;
    S=OS1;
    OS=OS1;
    Sq=OSq;
    d=Sq;
    Lt=length(d);
    b=ones(length(t1),1);
    Fin=0;
    TOI=m*UB-sum(t1);
    TOIL(1)=TOI;
```

```
    cs=m+1;
    for i=2:m+1
        TOIL(i)=TOIL(i-1)-(UB-S(i,:)*t1);
    end
    %backtrack
    while 1

[S,d,cs,Lt,b,TOI,TOIL,Fin]=Backtrack3(S,b,cs,d,Lt,t1,TOI,TOIL,UB
,Fin);
        if Fin==1
            break
        end
        Uni=0;
        while 1

[S,b,d,Lt,cs,TOI,TOIL,Uni,tempUB]=StationLoadsFinder3(S,Sq,b,d,c
s,UB,Lt,TOI,TOIL,Uni,m,P,t1,D_Mat);
            if Uni==1
                break
            end
            while 1
                %enter the second backtrack(one station per time)

[S,d,cs,Lt,b,TOI,TOIL,Fin]=Backtrack1_3(S,b,cs,d,Lt,t1,m,TOI,TOI
L,UB,Fin);
                if Fin==1
                    break
                end

[S,b,d,Lt,cs,TOI,TOIL,Uni,tempUB]=StationLoadsFinder3(S,Sq,b,d,c
s,UB,Lt,TOI,TOIL,Uni,m,P,t1,D_Mat);
                if Uni==1
                    break
                end
            end
            if Uni==1
                break
            end
            if Fin==1
                break
            end
        end
        if Fin==1
            break
        end
```

```
[OS,S,Sq,b,d,cs,Lt,UB,TOI,TOIL,Fin]=NewSeqGen3(OS,S,Sq,b,d,cs,UB
,tempUB,Lt,TOI,TOIL,m,P,t1,D_Mat);
    end
    OptiC=UB;
end
end
```

Backtrack function (ENCORE)

```
function
[S,d,cs,Lt,b,TOI,TOIL,Fin]=Backtrack3(S,b,cs,d,Lt,t1,TOI,TOIL,UB
,Fin)
%station oriented backtrack
%backtrack process follows the new sequence
Fin=0;
aa=sum(t1);
%find the first critical station
bb=max(S*t1);
k=find(S*t1==bb,1);
%empty the stations and sequences from k to current station
Index2=find(d==0,k-1);
d(Index2(end):end)=0;
for j=k:cs
    Index1= S(j,:)~=0;
    b(Index1)=0;
    S(j,:)=0;
end
cs=k;
%set the last task:find the (k-1)th zero in the current sequence,
the last task
%is next to it.
E=find(d==0,k-1);
Lt=E(end)+1;
end
```

Station enumeration (ENCORE)

```
function
[S,b,d,Lt,cs,TOI,TOIL,Uni,tempUB]=StationLoadsFinder3(S,Sq,b,d,c
s,UB,Lt,TOI,TOIL,Uni,m,P,t1,D_Mat)
%%the improving cycle time can guarantee uniqueness in the first
backtrack, we do not have to check
%%the uniqueness by comparing with previous branches.

%%ONLY ONE chance of keep on going to generate new sequence 1)the
%%new station time is less than the current upper bound
```

```
at=[];
tempUB=UB;
if d(Lt-1)==0
    for i=find(b==0,1):length(t1)
        %new cycle time has to be less than the critical station
        if b(i)~=1&&b'*P(:,i)==sum(P(:,i)) && S(cs,:)*t1+t1(i)<UB
            at=[at,i];
        end
    end
else
    for i=d(Lt-1)+1:length(t1)
         if          b(i)~=1&&b'*P(:,i)==sum(P(:,i))          &&
S(cs,:)*t1+t1(i)<UB
            at=[at,i];
        end
    end
end
%%downsize the assignable task by Jackson's dominance rule
if  length(at)>=2
    Dominated_task=[];
    for o=1:length(at)
        for r=(o+1):length(at)
            %is at(o) dominating at(r)?
            if ismember(at(r),D_Mat(at(o),:))
                Dominated_task=[Dominated_task;r];
            elseif ismember(at(o),D_Mat(at(r),:))
                %is at(r) dominating at(o)?
                Dominated_task=[Dominated_task;o];
            end
        end
    end
at(Dominated_task)=[];
end
if isempty(at)
        OL=[];
        for k=1:size(Sq,1)
            if isequal(Sq(k,1:Lt-1),d(1:Lt-1))
                OL=1;
                break
            end
        end
        if isempty(OL)
            %%!!find the cycle time!!
            tempUB=max(S*t1);
            %check whether the TOIL is less than zero
            tempTOIL=m*tempUB-sum(t1);
```

```
            for j=2:cs
                tempTOIL=tempTOIL-(tempUB-S(j,:)*t1);
                if tempTOIL<0
                    break
                end
            end
            if tempTOIL>=0
                %the solution is good
                Uni=1;
                TOI=m*tempUB-sum(t1);
                TOIL(1)=TOI;
                for j=2:cs
                    TOIL(j)=TOIL(j-1)-(tempUB-S(j,:)*t1);
                end
                cs=cs+1;
                E=find(d==0,cs-1);
                Lt=E(end)+1;
            else
                %forfeit the solution
                Lt=Lt-1;
                b(d(Lt))=0;
                S(cs,d(Lt))=0;
                d(Lt)=0;
            end
        else
            %forfeit the solution
            Lt=Lt-1;
            b(d(Lt))=0;
            S(cs,d(Lt))=0;
            d(Lt)=0;
        end
else
    for j=1:length(at)
        b(at(j))=1;
        d(Lt)=at(j);
        S(cs,at(j))=1;
        Lt=Lt+1;

[S,b,d,Lt,cs,TOI,TOIL,Uni,tempUB]=StationLoadsFinder3(S,Sq,b,d,c
s,UB,Lt,TOI,TOIL,Uni,m,P,t1,D_Mat);
        if Uni==1
            %already find an unique solution, stop permuting
            break
        end
        %           if j is the last and only choice , but it is
still not working, erase the
```

```
        %           previous selection(j) when previous selection
exists(has not been erased in the ifempty(at) part).
        if j==length(at)&&d(Lt-1)~=0
            Lt=Lt-1;
            b(d(Lt))=0;
            S(cs,d(Lt))=0;
            d(Lt)=0;
        end
    end

end

end
```

Superior solution generation (ENCORE)

```
function
[OS,S,Sq,b,d,cs,Lt,UB,TOI,TOIL,Fin]=NewSeqGen3(OS,S,Sq,b,d,cs,UB
,tempUB,Lt,TOI,TOIL,m,P,t1,D_Mat)
%station oriented BB
at=[];
at1=[];
Fin=0;
while 1
%check whether it is the LAST STATION
if cs==(m+1)
    %last station no need to assign
    a1=find(b==0,length(t1));
    d(Lt:end)=a1;
    for i=1:length(a1)
    S(cs,a1(i))=1;
    end
    OS=S;
    Sq=[Sq;d];
    b=ones(1,length(t1))';
    UB=tempUB;
    if UB==max(ceil(sum(t1)/m),max(t1))
                OptiC=UB;
                Fin=1;
    end
    break
else
    %find the assignable task
    for i=find(b==0,1):length(t1)
        %new cycle time has to be less than the critical station
        if b(i)~=1&&b'*P(:,i)==sum(P(:,i)) && S(cs,:)*t1+t1(i)<UB
            at=i;
```

```
            break
        end
    end
    if isempty(at)
           %locate the bottleneck station
            AA=max(S(cs,:)*t1,tempUB);
            %calculate the total idle time left
            if TOIL(cs-1)-(AA-S(cs,:)*t1)<0&&AA==tempUB
                %solution is conterfeited
                Sq=[Sq;d];
                size(Sq,1);
                %backtrack
                break
            else
                Lt=Lt+1;
                tempUB=AA;
                %update the TOI&TOIL
                TOI=m*tempUB-sum(t1);
                tempTOIL=TOI;
                for j=2:cs
                    tempTOIL=tempTOIL-(tempUB-S(j,:)*t1);
                end
                TOIL(cs)=tempTOIL;
                at1=[];
                cs=cs+1;
                d(Lt)=0;
            end
    else
        S(cs,at)=1;
        d(Lt)=at;
        Lt=Lt+1;
        b(at)=1;
    end
    at=[];
end
end
```

Appendix F: Design of Experiments (ENCORE)

The matlab codes which generate the results in section 4.3.3 (Table 15 and 16) are shown as follows.

The steps of execution

1) Open the script

2) Adjust the learning rate and run time to the specific values (the number of learning epochs is already embedded)

3) Run the script

Some functions which are invoked during the execution are documented in Appendix B

```
clear all;
load learning_rates
x1=x1-0.05;x2=x2-0.05;x3=x3-0.05;
ct=1;
m1=[3 5 8 12 17 20 22 25];
%production quanity
N=400;
%#of learning
ks=[1,2,3];
CI=zeros(3,2);
p1=zeros(3,1);h1=zeros(3,1);M=zeros(3,1);SE=zeros(3,1);
for kk=1:length(ks)
    k=ks(kk);
load Arcus111
%generate random task time matrix
D_Mat=Jackson_Dominance(P,t);
t1=t.*x1';
OptiC_No=[];
%total production time
TP_No=[];AA3=[];
OptiC_Encore=[];
OptiC_old=[];
reassign_status=[];
%total production time
TP_Encore=[];TP_Old=[];AA1=[];AA2=[];
for i=1:size(m1,2)
    [OptiC1,Sq1,OS1]=MainSALOME2(D_Mat,m1(i),P,t,ct);
    t1=t.*x1';
    for j=1:k

[OptiC,Sq1,OS1,B]=Algorithm3(D_Mat,Sq1,OS1,m1(i),P,t1,ct);
        OptiC_Encore(j,i)=OptiC;
```

```
        [OptiC2,Sq,OS]=MainSALOME2(D_Mat,m1(i),P,t1,ct);
        t1=t1.*x1';
        OptiC_old(j,i)=OptiC2;
    end
    A1=[OptiC1;OptiC_Encore(:,i)];
    A2=[OptiC1;OptiC_old(:,i)];
    %calculate the point at which the learning takes place
    pp=round((N-k*m1(i))/(k+1))+m1(i);
    %learning period 0-pp,pp-2pp,2pp-3pp,3pp-80
    p=zeros(1,k+1);
    p(1:end-1)=pp;
    p(end)=N-k*pp;
%     TP_Encore(i)=p*A1;
%     TP_Old(i)=p*A2;
    TP_Old(i)=p*A2;
    TP_Encore(i)=p*A1;
end
for i=1:size(m1,2)
    [OptiC1,Sq1,OS1]=MainSALOME2(D_Mat,m1(i),P,t,ct);
    TP_No(i)=N*OptiC1;
end
AA3=[AA3,TP_No];
AA2=[AA2,TP_Old];
AA1=[AA1,TP_Encore];

%barthodi
m2=[28 32 34 37 41 44 46 48 51];
load Barthodi
%generate random task time matrix
D_Mat=Jackson_Dominance(P,t);
t1=t.*x2';
OptiC_Encore=[];
OptiC_old=[];
reassign_status=[];
%total production time
TP_Encore=[];TP_Old=[];
for i=1:size(m2,2)
    [OptiC1,Sq1,OS1]=MainSALOME2(D_Mat,m2(i),P,t,ct);
    t1=t.*x2';
    for j=1:k

[OptiC,Sq1,OS1,B]=Algorithm3(D_Mat,Sq1,OS1,m2(i),P,t1,ct);
        OptiC_Encore(j,i)=OptiC;
        [OptiC2,Sq,OS]=MainSALOME2(D_Mat,m2(i),P,t1,ct);
        t1=t1.*x2';
        OptiC_old(j,i)=OptiC2;
```

```
    end
    A1=[OptiC1;OptiC_Encore(:,i)];
    A2=[OptiC1;OptiC_old(:,i)];
    %calculate the point at which the learning takes place
    pp=round((N-k*m2(i))/(k+1))+m2(i);
    %learning period 0-pp,pp-2pp,2pp-3pp,3pp-80
    p=zeros(1,k+1);
    p(1:end-1)=pp;
    p(end)=N-k*pp;
%     TP_Encore(i)=p*A1;
%     TP_Old(i)=p*A2;
    TP_Old(i)=p*A2;
    TP_Encore(i)=p*A1;
end
OptiC_No=[];
%total production time
TP_No=[];
for i=1:size(m2,2)
    [OptiC1,Sq1,OS1]=MainSALOME2(D_Mat,m2(i),P,t,ct);
    TP_No(i)=N*OptiC1;
end
AA3=[AA3,TP_No];
AA2=[AA2,TP_Old];
AA1=[AA1,TP_Encore];


%Scholl
m3=[25 28 30 33 35 38 42 45];
load Scholl
%generate random task time matrix
D_Mat=Jackson_Dominance(P,t);
t1=t.*x3';
OptiC_Encore=[];
OptiC_old=[];
reassign_status=[];
%total production time
TP_Encore=[];TP_Old=[];
for i=1:size(m3,2)
    [OptiC1,Sq1,OS1]=MainSALOME2(D_Mat,m3(i),P,t,ct);
    t1=t.*x3';
    for j=1:k

[OptiC,Sq1,OS1,B]=Algorithm3(D_Mat,Sq1,OS1,m3(i),P,t1,ct);
        OptiC_Encore(j,i)=OptiC;
        [OptiC2,Sq,OS]=MainSALOME2(D_Mat,m3(i),P,t1,ct);
        t1=t1.*x3';
```

```
        OptiC_old(j,i)=OptiC2;
    end
    A1=[OptiC1;OptiC_Encore(:,i)];
    A2=[OptiC1;OptiC_old(:,i)];
    %calculate the point at which the learning takes place
    pp=round((N-k*m3(i))/(k+1))+m3(i);
    %learning period 0-pp,pp-2pp,2pp-3pp,3pp-80
    p=zeros(1,k+1);
    p(1:end-1)=pp;
    p(end)=N-k*pp;
%     TP_Encore(i)=p*A1;
%     TP_Old(i)=p*A2;
    TP_Old(i)=p*A2;
    TP_Encore(i)=p*A1;
end
OptiC_No=[];
%total production time
TP_No=[];
for i=1:size(m3,2)
    [OptiC1,Sq1,OS1]=MainSALOME2(D_Mat,m3(i),P,t,ct);
    TP_No(i)=N*OptiC1;
end
AA3=[AA3,TP_No];
AA2=[AA2,TP_Old];
AA1=[AA1,TP_Encore];
%calculate the difference of improvement
ImP_Old=(AA3-AA2)./AA3;
ImP_Encore=(AA3-AA1)./AA3;
%summary statistics
Diff=ImP_Encore-ImP_Old;
M(kk)=mean(Diff);
ts = tinv(0.95,length(Diff)-1);
SE(kk) = std(Diff)/sqrt(length(Diff));
CI(kk,:) = [M(kk) - ts*SE(kk), M(kk) + ts*SE(kk)] ;
%significance test
[h1(kk), p1(kk)]= ttest(ImP_Encore,ImP_Old,'Tail','Right') ;
end
```

Appendix G: Performance of elementary rules

Table 17 and 18 show the average deviation for different elementary rules

Steps of execution

1) Open test script

2) Load the data

3) Run the test script

Test script

```
load('P148.mat')
c=[204,255,306,357,408,459,510];
% w=1;
% max_t=1;
% [D_Mat]=Jackson_Dominance(P,t,D);
% L=Label(t,P);
ms148=[];
for i=1:length(c)
[~,m1]=findOne_TdS(c(i),P,t,D,E148(i,:),L148(i,:));
[~,m2]=findOne_TdL(c(i),P,t,D,E148(i,:),L148(i,:));
[~,m3]=findOne_T(c(i),P,t,D);
[~,m4]=findOne_Latest(c(i),P,t,D,L148(i,:));
[~,m5]=findOne_F(c(i),P,t,D);
ms148(:,i)=[m1;m2;m3;m4;m5];
end
```

Sample elementary rule (Main function)

```
function [Sq,m]=findOne_T(c,P,t,D)
%d(1,Lt)=right or left station
%d(2,lt)=corresponding task number

%one pass, no backtrack
Et=zeros(1,length(t));
b=zeros(1,length(t));
b(1)=1;
d=zeros(2,length(t)+30);
d(1:2,1)=1;
Lt=3;
Sl=[0 0];
cs=1;
while 1
    at=findat_T(b,D,d,P,c,t,Et,Sl,cs);
```

```
    % at first row is the station number to which the task is
assigned, second row is the task number
    if isempty(at)
        %no task availabe to either station, close the position,
release the
        %previously assigned
        %compare the LB and UB of the remaining problem, if equal,
stop the
        %algorithm

        [Sq,UB]= NaturalGen(d,c,P,t,D,cs,b,Et);
        [LB]=LB_UB_Gen(d,c,t,D,cs);
        if UB==LB
            m=LB;
            break
        elseif sum(b)==length(b)
            m=cs;
            break
        end
        Sl=zeros(1,2);
        Sq(:,Lt)=0;
        cs=cs+1;
        Lt=Lt+1;
    else
        %only assign the first one in the list
        b(at(2,1))=1;
        d(1,Lt)=at(1,1);
        d(2,Lt)=at(2,1);
        Lt=Lt+1;
        %task time plus the upfront idle time
        aa=find(P(:,at(2,1))~=0);
        if ~isempty(aa)
            %filter out the predecessors are within the same
            %position
            for kk=1:length(aa)
                pos_in_d=find(d(2,:)==aa(kk));
                Which_Pos(kk)=sum(d(2,1:pos_in_d)==0);
            end
            aa=aa(Which_Pos==cs);
            Which_Pos=[];
            if isempty(aa)
                Et(at(2,1))=Sl(at(1,1))+t(at(2,1));
                Sl(at(1,1))=Et(at(2,1));
            else

Et(at(2,1))=max(Sl(at(1,1)),max(Et(aa)))+t(at(2,1));
```

```
                Sl(at(1,1))=Et(at(2,1));
            end
        else
            Et(at(2,1))=Sl(at(1,1))+t(at(2,1));
            Sl(at(1,1))=Et(at(2,1));
        end
    end
end
end
```

Sample position enumeration rule (rule *L*)

```
function at=findat_Latest(b,D,d,P,c,t,Et,Sl,cs,L)
Delay=[];at=[];Mated_Difference=[];
%if either direction, assign the task to the station whose load is
%smaller (small_I)
for i=find(b==0,1):length(t)
    %check the operational direction
    if D(i)~=3 &&b(i)~=1
        %left or right
        %find its predecessors' ending times
        j=find(P(:,i)~=0);
        if D(i)==1
            cm=2;
        else
            cm=1;
        end
        if isempty(j)
            %no  predecessor  exists  or  all  predecessors  are
assigned
            if Sl(D(i))+t(i)<=c
                %cycle time constraint
                at=[at,[D(i);i]];
                %record its delay time,0
                Delay=[Delay,0];

Mated_Difference=[Mated_Difference,abs(Sl(D(i))+t(i)-Sl(cm))];
            end
        elseif sum(Et(j)>0)==length(j)
            %find out j within the same position
            for kk=1:length(j)
                pos_in_d=find(d(2,:)==j(kk));
                Which_Pos(kk)=sum(d(2,1:pos_in_d)==0);
            end
            j=j(Which_Pos==cs);
            Which_Pos=[];
            if isempty(j) && Sl(D(i))+t(i)<=c
```

```
                %cycle time constraint
                at=[at,[D(i);i]];
                %record its delay time
                Delay=[Delay,0];

Mated_Difference=[Mated_Difference,abs(Sl(D(i))+t(i)-Sl(cm))];
            elseif                  ~isempty(j)                  &&
max(max(Et(j)),Sl(D(i)))+t(i)<=c
                at=[at,[D(i);i]];
                aa=max(max(Et(j))-Sl(D(i)),0);
                Delay=[Delay,aa];

Mated_Difference=[Mated_Difference,abs(Sl(D(i))+t(i)-Sl(cm))];
            end
        end
    elseif b(i)~=1
        %either direction, assign the task to the station whose
load is
        %smaller (small_I)
        j=find(P(:,i)~=0);
        if isempty(j)
            %no  predecessor  exists  or  all  predecessors  are
assigned
            if Sl(1)+t(i)<=c
                %cycle time constraint
                at=[at,[1;i]];
                Delay=[Delay,0];

Mated_Difference=[Mated_Difference,abs(Sl(1)+t(i)-Sl(2))];
            end
            if Sl(2)+t(i)<=c
                %cycle time constraint
                at=[at,[2;i]];
                Delay=[Delay,0];

Mated_Difference=[Mated_Difference,abs(Sl(2)+t(i)-Sl(1))];
            end
        elseif sum(Et(j)>0)==length(j)
            %find out j within the same position
            for kk=1:length(j)
                pos_in_d=find(d(2,:)==j(kk));
                Which_Pos(kk)=sum(d(2,1:pos_in_d)==0);
            end
            j=j(Which_Pos==cs);
            Which_Pos=[];
            if isempty(j)
```

```
                if Sl(1)+t(i)<=c
                    %cycle time constraint
                    at=[at,[1;i]];
                    %record its delay time
                    Delay=[Delay,0];

Mated_Difference=[Mated_Difference,abs(Sl(1)+t(i)-Sl(2))];
                end
                if Sl(2)+t(i)<=c
                    %cycle time constraint
                    at=[at,[2;i]];
                    %record its delay time
                    Delay=[Delay,0];

Mated_Difference=[Mated_Difference,abs(Sl(2)+t(i)-Sl(1))];
                end
            else
                if max(max(Et(j)),Sl(1))+t(i)<=c
                    %cycle time constraint
                    at=[at,[1;i]];
                    aa=max(max(Et(j))-Sl(1),0);
                    Delay=[Delay,aa];

Mated_Difference=[Mated_Difference,abs(Sl(1)+t(i)-Sl(2))];
                end
                if max(max(Et(j)),Sl(2))+t(i)<=c
                    %cycle time constraint
                    at=[at,[2;i]];
                    aa=max(max(Et(j))-Sl(2),0);
                    Delay=[Delay,aa];

Mated_Difference=[Mated_Difference,abs(Sl(2)+t(i)-Sl(1))];
                end
            end
        end
    end
end
    %sort the at in an ascending order of L
        if ~isempty(at)
            Z=L(at(2,:));
            Data_Frame=[at',Delay',Z'];
            Data_Frame=sortrows(Data_Frame,[3,4]);
            at=Data_Frame(:,1:2)';
        end
end
```

Upper bound generation function

```
function [Sq,UB]= NaturalGen(Sq,c,P,t,D,cs,b,Et)
cs=cs+1;
%Delete zero
B=Sq(2,:);
Lt=find(B~=0,1,'last')+2;
%station load
Sl=zeros(1,2);
while 1
    %if either direction, assign the task to the station whose
load is
    %smaller (small_I)
    [~,small_I]=sort(Sl);
    at=[];
    if length(b)==sum(b)
        UB=cs;
        break
    end
    for i=find(b==0,1):length(t)
        %check the operational direction
        if D(i)~=3 &&b(i)~=1
            %left or right
            %find its predecessors' ending times
            j=find(P(:,i)~=0);
            if isempty(j)
                %no predecessor exists or all predecessors are
assigned
                if Sl(D(i))+t(i)<=c
                    %cycle time constraint
                    at=[D(i);i];
                    %record its delay time,0
                    I=0;
                    break;
                end
            elseif sum(Et(j)>0)==length(j)
                %find out j within the same position
                for kk=1:length(j)
                    pos_in_d=find(Sq(2,:)==j(kk));
                    Which_Pos(kk)=sum(Sq(2,1:pos_in_d)==0);
                end
                j=j(Which_Pos==cs);
                Which_Pos=[];
                if isempty(j) && Sl(D(i))+t(i)<=c
                    %cycle time constraint
                    at=[D(i);i];
```

```
                    %record its delay time
                    I=0;
                    break
                elseif                ~isempty(j)                &&
max(max(Et(j)),Sl(D(i)))+t(i)<=c
                    at=[D(i);i];
                    I=max(max(Et(j))-Sl(D(i)),0);
                    break
                end
            end
        elseif b(i)~=1
            %either direction, assign the task to the station
whose load is
            %smaller (small_I)
            j=find(P(:,i)~=0);
            if isempty(j)
                %no predecessor exists or all predecessors are
assigned
                if Sl(small_I(1))+t(i)<=c
                    %cycle time constraint
                    at=[small_I(1);i];
                    I=0;
                    break
                end
                if Sl(small_I(2))+t(i)<=c
                    %cycle time constraint
                    at=[at,[small_I(2);i]];
                    I=0;
                    break
                end
            elseif sum(Et(j)>0)==length(j)
                %find out j within the same position
                for kk=1:length(j)
                    pos_in_d=find(Sq(2,:)==j(kk));
                    Which_Pos(kk)=sum(Sq(2,1:pos_in_d)==0);
                end
                j=j(Which_Pos==cs);
                Which_Pos=[];
                if isempty(j)
                    if Sl(small_I(1))+t(i)<=c
                        %cycle time constraint
                        at=[small_I(1);i];
                        %record its delay time
                        I=0;
                        break
                    end
```

```
                    if Sl(small_I(2))+t(i)<=c
                        %cycle time constraint
                        at=[small_I(2);i];
                        %record its delay time
                        I=0;
                        break
                    end
                else
                    if max(max(Et(j)),Sl(small_I(1)))+t(i)<=c
                        %cycle time constraint
                        at=[small_I(1);i];
                        I=max(max(Et(j))-Sl(small_I(1)),0);
                        break
                    end
                    if max(max(Et(j)),Sl(small_I(2)))+t(i)<=c
                        %cycle time constraint
                        at=[small_I(2);i];
                        I=max(max(Et(j))-Sl(small_I(2)),0);
                        break
                    end
                end
            end
        end
    end
    if isempty(at)
        Sl=zeros(1,2);
        Sq(:,Lt)=0;
        cs=cs+1;
        Lt=Lt+1;
    else
        %symmetric rule does not apply!
        %assign task to the left or right station
        b(at(2))=1;
        Sq(1,Lt)=at(1);
        Sq(2,Lt)=at(2);
        Lt=Lt+1;
        %task time plus the upfront idle time
        t1(at(2))=I+t(at(2));
        Et(at(2))=Sl(at(1))+t1(at(2));
        Sl(at(1))= Et(at(2));
    end
end
end
```

Lower bound generation function

```
function [LB]=LB_UB_Gen(Sq,c,t,D,cs)
A=Sq(2,:);
A=A(A~=0);
r=ones(1,length(t));
r(A(1:end))=0;
t1=r.*t;
T1=sum(t1(D(1,:)==1));
lb1=ceil(T1/c);
%total right task time
T2=sum(t1(D(1,:)==2));
lb2=ceil(T2/c);
%total lateral task time
T3=sum(t1(D(1,:)==3));
lb3=ceil(max(T3-((lb1+lb2)*c-T1-T2),0)/c);
LB=max(lb1,lb2)+ceil(max(lb3-abs(lb1-lb2),0)/2)+cs;
end
```

Appendix H: Performance of enhanced elementary rules

Table 19 and 20 show the average deviation for different elementary rules prioritizing the load-oriented rule

Steps of execution

1) Open test script

2) Load the data

3) Run the test script

Test script

```
load('P16.mat')
c=[15,18,20,22];
% w=1;
% max_t=1;
% [D_Mat]=Jackson_Dominance(P,t,D);
% L=Label(t,P);
ms16=[];
for i=1:length(c)
[~,m1]=findOne_TdS(c(i),P,t,D,E16(i,:),L16(i,:));
[~,m2]=findOne_TdL(c(i),P,t,D,E16(i,:),L16(i,:));
[~,m3]=findOne_T(c(i),P,t,D);
[~,m4]=findOne_Latest(c(i),P,t,D,L16(i,:));
[~,m5]=findOne_F(c(i),P,t,D);
ms16(:,i)=[m1;m2;m3;m4;m5];
end
```

Sample elementary rule embodying the load-oriented rule

```
function [Sq,m]=findOne_TdL(c,P,t,D,E,L)
%d(1,Lt)=right or left station
%d(2,lt)=corresponding task number

%one pass, no backtrack
%time divided by latest task
TdL=t./L;
Et=zeros(1,length(t));
b=zeros(1,length(t));
b(1)=1;
d=zeros(2,length(t)+30);
d(1:2,1)=1;
Lt=3;
Sl=[0 0];
```

```
cs=1;
while 1
    at=findat_TdL(b,D,d,P,c,t,Et,Sl,cs,TdL);
    % at first row is the station number to which the task is
assigned, second row is the task number
    if isempty(at)
        %no task availabe to either station, close the position,
release the
        %previously assigned
        %compare the LB and UB of the remaining problem, if equal,
stop the
        %algorithm

        [Sq,UB]= NaturalGen(d,c,P,t,D,cs,b,Et);
        [LB]=LB_UB_Gen(d,c,t,D,cs);
        if UB==LB
            m=LB;
            break
        elseif sum(b)==length(b)
            m=cs;
            break
        end
        Sl=zeros(1,2);
        Sq(:,Lt)=0;
        cs=cs+1;
        Lt=Lt+1;
    else
        %only assign the first one in the list
        b(at(2,1))=1;
        d(1,Lt)=at(1,1);
        d(2,Lt)=at(2,1);
        Lt=Lt+1;
        %task time plus the upfront idle time
        aa=find(P(:,at(2,1))~=0);
        if ~isempty(aa)
            %filter out the predecessors are within the same
            %position
            for kk=1:length(aa)
                pos_in_d=find(d(2,:)==aa(kk));
                Which_Pos(kk)=sum(d(2,1:pos_in_d)==0);
            end
            aa=aa(Which_Pos==cs);
            Which_Pos=[];
            if isempty(aa)
                Et(at(2,1))=Sl(at(1,1))+t(at(2,1));
                Sl(at(1,1))=Et(at(2,1));
```

```
            else

Et(at(2,1))=max(Sl(at(1,1)),max(Et(aa)))+t(at(2,1));
                Sl(at(1,1))=Et(at(2,1));
            end
        else
            Et(at(2,1))=Sl(at(1,1))+t(at(2,1));
            Sl(at(1,1))=Et(at(2,1));
        end
    end
end
end
```

Position generation function

```
function at=findat_TdL(b,D,d,P,c,t,Et,Sl,cs,TdL)
Delay=[];at=[];Mated_Difference=[];
%if either direction, assign the task to the station whose load is
%smaller (small_I)
for i=find(b==0,1):length(t)
    %check the operational direction
    if D(i)~=3 &&b(i)~=1
        %left or right
        %find its predecessors' ending times
        j=find(P(:,i)~=0);
        if D(i)==1
            cm=2;
        else
            cm=1;
        end
        if isempty(j)
            %no  predecessor  exists  or  all  predecessors  are
assigned
            if Sl(D(i))+t(i)<=c
                %cycle time constraint
                at=[at,[D(i);i]];
                %record its delay time,0
                Delay=[Delay,0];

Mated_Difference=[Mated_Difference,abs(Sl(D(i))+t(i)-Sl(cm))];
            end
        elseif sum(Et(j)>0)==length(j)
            %find out j within the same position
            for kk=1:length(j)
                pos_in_d=find(d(2,:)==j(kk));
```

```
                Which_Pos(kk)=sum(d(2,1:pos_in_d)==0);
            end
            j=j(Which_Pos==cs);
            Which_Pos=[];
            if isempty(j) && Sl(D(i))+t(i)<=c
                %cycle time constraint
                at=[at,[D(i);i]];
                %record its delay time
                Delay=[Delay,0];

Mated_Difference=[Mated_Difference,abs(Sl(D(i))+t(i)-Sl(cm))];
            elseif                  ~isempty(j)                  &&
max(max(Et(j)),Sl(D(i)))+t(i)<=c
                at=[at,[D(i);i]];
                aa=max(max(Et(j))-Sl(D(i)),0);
                Delay=[Delay,aa];

Mated_Difference=[Mated_Difference,abs(Sl(D(i))+t(i)-Sl(cm))];
            end
        end
    elseif b(i)~=1
        %either direction, assign the task to the station whose
load is
        %smaller (small_I)
        j=find(P(:,i)~=0);
        if isempty(j)
            %no  predecessor  exists  or  all  predecessors  are
assigned
            if Sl(1)+t(i)<=c
                %cycle time constraint
                at=[at,[1;i]];
                Delay=[Delay,0];

Mated_Difference=[Mated_Difference,abs(Sl(1)+t(i)-Sl(2))];
            end
            if Sl(2)+t(i)<=c
                %cycle time constraint
                at=[at,[2;i]];
                Delay=[Delay,0];

Mated_Difference=[Mated_Difference,abs(Sl(2)+t(i)-Sl(1))];
            end
        elseif sum(Et(j)>0)==length(j)
            %find out j within the same position
            for kk=1:length(j)
                pos_in_d=find(d(2,:)==j(kk));
```

```
                Which_Pos(kk)=sum(d(2,1:pos_in_d)==0);
            end
            j=j(Which_Pos==cs);
            Which_Pos=[];
            if isempty(j)
                if Sl(1)+t(i)<=c
                    %cycle time constraint
                    at=[at,[1;i]];
                    %record its delay time
                    Delay=[Delay,0];
Mated_Difference=[Mated_Difference,abs(Sl(1)+t(i)-Sl(2))];
                end
                if Sl(2)+t(i)<=c
                    %cycle time constraint
                    at=[at,[2;i]];
                    %record its delay time
                    Delay=[Delay,0];

Mated_Difference=[Mated_Difference,abs(Sl(2)+t(i)-Sl(1))];
                end
            else
                if max(max(Et(j)),Sl(1))+t(i)<=c
                    %cycle time constraint
                    at=[at,[1;i]];
                    aa=max(max(Et(j))-Sl(1),0);
                    Delay=[Delay,aa];

Mated_Difference=[Mated_Difference,abs(Sl(1)+t(i)-Sl(2))];
                end
                if max(max(Et(j)),Sl(2))+t(i)<=c
                    %cycle time constraint
                    at=[at,[2;i]];
                    aa=max(max(Et(j))-Sl(2),0);
                    Delay=[Delay,aa];

Mated_Difference=[Mated_Difference,abs(Sl(2)+t(i)-Sl(1))];
                end
            end
        end
    end
end
    %sort the at in an descending order of TdL
        if ~isempty(at)
            Z=TdL(at(2,:));
            Y=Sl(at(1,:));
            Data_Frame=[at',Delay',Y',Z'];
```

```
            Data_Frame=sortrows(Data_Frame,[3,4,-5]);
            at=Data_Frame(:,1:2)';
        end
end
```

Appendix I: Performance of the composite rules

Table 22 and 23 are generated by the following functions

Test function (All data sets)

```
%record the score
%10 composite ,9 weights
Score=zeros(10,9);
Unique=zeros(10,9);
R=[100,10,5,2,1,1/2,1/5,1/10,1/100];
[MS16, Score]=P16test(Score,R);
[MS12, Score]=P12test(Score,R);
[MS24, Score]=P24test(Score,R);
[MS65, Score]=P65test(Score,R);
[MS148, Score]=P148test(Score,R);
[MS205, Score]=P205test(Score,R);
mss=[MS12, MS16, MS24, MS65, MS148, MS205];
minm=[4 3   3   2   4   3   3   2   3   3   2   2   9   7   6   6
5   13  11  9   8   7   6   6   11  9   8   7   7   6   6   5   5
5];
u = repmat(minm,10,1);
dev=(mss-u)./u*100;
average_dev=sum(dev,2)/34;
```

Sample test function (P16)

```
function [MS16, Score]=P16test(Score,R)
load('P16.mat')
c=[15,18,20,22];
% w=1;
% max_t=1;
% [D_Mat]=Jackson_Dominance(P,t,D);
% L=Label(t,P);
ms16=[];
MS16=[];
[~,All_Follwers,Dir_Follwers]=Jackson_Dominance(P,t,D);
All_Follwers=sort(All_Follwers,2,'descend');
total_follwers=zeros(1,length(t));
for i=1:length(t)
total_follwers(i)=find(All_Follwers(i,:)==0,1)-1;
end


for i=1:length(c)
    for j=1:length(R)
```

```
        W=[R(j),1];
        [~,m1]=findOne_TdS_TdL(c(i),P,t,D,E16(i,:),L16(i,:),W);
        [~,m2]=findOne_TdS_T(c(i),P,t,D,E16(i,:),L16(i,:),W);

[~,m3]=findOne_TdS_Latest(c(i),P,t,D,E16(i,:),L16(i,:),W);

[~,m4]=findOne_TdS_F(c(i),P,t,D,E16(i,:),L16(i,:),W,total_follwe
rs);
        [~,m5]=findOne_TdL_T(c(i),P,t,D,E16(i,:),L16(i,:),W);

[~,m6]=findOne_TdL_Latest(c(i),P,t,D,E16(i,:),L16(i,:),W);

[~,m7]=findOne_TdL_F(c(i),P,t,D,E16(i,:),L16(i,:),W,total_follwe
rs);
        [~,m8]=findOne_T_Latest(c(i),P,t,D,E16(i,:),L16(i,:),W);

[~,m9]=findOne_T_F(c(i),P,t,D,E16(i,:),L16(i,:),W,total_follwers
);

[~,m10]=findOne_Latest_F(c(i),P,t,D,E16(i,:),L16(i,:),W,total_fo
llwers);
        ms16(:,j)=[m1;m2;m3;m4;m5;m6;m7;m8;m9;m10];
    end
    a=min(ms16(:));
    for k=1:10
        I=find(ms16(k,:)==a);
        if ~isempty(I)
        Score(k,I)=Score(k,I)+1;
        end
    end
    MS16(:,i)=min(ms16,[],2);
end
end
```

Sample composite rule (TDS and TDL)

```
function [Sq,m]=findOne_TdS_TdL(c,P,t,D,E,L,W)
%W:weight of each rule
%d(1,Lt)=right or left station
%d(2,lt)=corresponding task number

%one pass, no backtrack
%time divided by slack
TdS=t./(L-E+1);
TdL=t./L;
%normalize
```

```
GUB=max(L);

Composite=W(1)*TdS*GUB/c+W(2)*TdL*GUB/c;

Et=zeros(1,length(t));
b=zeros(1,length(t));
b(1)=1;
d=zeros(2,length(t)+30);
d(1:2,1)=1;
Lt=3;
Sl=[0 0];
cs=1;
while 1
    at=findat_TdS_TdL(b,D,d,P,c,t,Et,Sl,cs,Composite);
    % at first row is the station number to which the task is
assigned, second row is the task number
    if isempty(at)
        %no task available to either station, close the position,
release the
        %previously assigned
        %compare the LB and UB of the remaining problem, if equal,
stop the
        %algorithm

        [Sq,UB]= NaturalGen(d,c,P,t,D,cs,b,Et);
        [LB]=LB_UB_Gen(d,c,t,D,cs);
        if UB==LB
            m=LB;
            break
        elseif sum(b)==length(b)
            m=cs;
            break
        end
        Sl=zeros(1,2);
        Sq(:,Lt)=0;
        cs=cs+1;
        Lt=Lt+1;
    else
        %only assign the first one in the list
        b(at(2,1))=1;
        d(1,Lt)=at(1,1);
        d(2,Lt)=at(2,1);
        Lt=Lt+1;
        %task time plus the upfront idle time
        aa=find(P(:,at(2,1))~=0);
        if ~isempty(aa)
```

```
            %filter out the predecessors are within the same
            %position
            for kk=1:length(aa)
                pos_in_d=find(d(2,:)==aa(kk));
                Which_Pos(kk)=sum(d(2,1:pos_in_d)==0);
            end
            aa=aa(Which_Pos==cs);
            Which_Pos=[];
            if isempty(aa)
                Et(at(2,1))=Sl(at(1,1))+t(at(2,1));
                Sl(at(1,1))=Et(at(2,1));
            else

Et(at(2,1))=max(Sl(at(1,1)),max(Et(aa)))+t(at(2,1));
                Sl(at(1,1))=Et(at(2,1));
            end
        else
            Et(at(2,1))=Sl(at(1,1))+t(at(2,1));
            Sl(at(1,1))=Et(at(2,1));
        end
    end
end
end
```

Appendix J: Performance of the priority-based bounded dynamic programing

Priority-based bounded dynamic programming

Test function (All data sets)

```
%record the score
%10 composite ,9 weights
Score=zeros(10,9);
P12test;
P16test;
P24test;
P65test;
P148test;
P205test;
mss=[MS12, MS16, MS24, MS65, MS148, MS205];
minm=[4 3   3   2   4   3   3   2   3   3   2   2   9   7   6   6
5   13  11  9   8   7   6   6   11  9   8   7   7   6   6   5   5
5];
u = repmat(minm,10,1);
dev=(mss-u)./u*100;
average_dev=sum(dev,2)/34;
```

Sample test function (P16)

```
load('P16.mat')
c=[15,18,20,22];
% w=1;
% max_t=1;
% [D_Mat]=Jackson_Dominance(P,t,D);
% L=Label(t,P);
[~,All_Follwers,Dir_Follwers]=Jackson_Dominance(P,t,D);
All_Follwers=sort(All_Follwers,2,'descend');
total_follwers=zeros(1,length(t));
for i=1:length(t)
total_follwers(i)=find(All_Follwers(i,:)==0,1)-1;
end
W=[0.01,0.1,0.01,0.01];
window_size=5;
ms=[];
a=1;
for i=1:length(c)
[Sq,m]=findOne_BDP(c(i),P,t,D,E16(i,:),L16(i,:),W,total_follwers
,window_size);
```

```
ms=[ms m];
end
```

PR_BDP main function

```
function
[Sq,m]=findOne_BDP(c,P,t,D,E,L,W,total_follwers,window_size)

%initialization

Et=zeros(1,length(t));
b=zeros(1,length(t));
b(1)=1;
d=zeros(2,length(t)+30);
d(1:2,1)=1;
Lt=3;
Sl=[0 0];
cs=1;
%calculate GLB
%total left task time
T1=sum(t(D(1,:)==1));
lb1=ceil(T1/c);
%total right task time
T2=sum(t(D(1,:)==2));
lb2=ceil(T2/c);
%total lateral task time
T3=sum(t(D(1,:)==3));
lb3=ceil(max(T3-((lb1+lb2)*c-T1-T2),0)/c);
GLB=max(lb1,lb2)+ceil(max(lb3-abs(lb1-lb2),0)/2);
%multi pass, no backtrack

TdS=t./(L-E+1);
TdL=t./L;
%normalize
GUB=max(L);
if length(W(1)*TdS*GUB/c)~=length(total_follwers/length(t))
    aaa=1
end
Composite1=W(1)*TdS*GUB/c+total_follwers/length(t);
Composite2=W(2)*TdL*GUB/c+total_follwers/length(t);
Composite3=-W(3)*L/GUB+total_follwers/length(t);
Composite4=W(4)*t/c-L/GUB;


%first station
LBs=[];UBs=[];Idles=[];Sqs=[];
```

```
EnumerateFirstStation;

while 1
    LBs=[];UBs=[];Idles=[];
    cs=cs+1;
    Sq1=Sqs;Sqs=[];
    for i=1:2:size(Sq1,1)
        %retrieve Et,b,d
        Retrieve;

[Sq,LB,UB,Idle]=findOne_TdS_F(d,b,Et,Lt,c,P,t,D,cs,Composite1);

LBs=[LBs,LB];UBs=[UBs,UB];Sqs=[Sqs;Sq];Idles=[Idles,Idle];

[Sq,LB,UB,Idle]=findOne_TdL_F(d,b,Et,Lt,c,P,t,D,cs,Composite2);

LBs=[LBs,LB];UBs=[UBs,UB];Sqs=[Sqs;Sq];Idles=[Idles,Idle];

[Sq,LB,UB,Idle]=findOne_Latest_F(d,b,Et,Lt,c,P,t,D,cs,Composite3
);

LBs=[LBs,LB];UBs=[UBs,UB];Sqs=[Sqs;Sq];Idles=[Idles,Idle];

[Sq,LB,UB,Idle]=findOne_T_Latest(d,b,Et,Lt,c,P,t,D,cs,Composite4
);

LBs=[LBs,LB];UBs=[UBs,UB];Sqs=[Sqs;Sq];Idles=[Idles,Idle];


    end
    if sum(LBs==UBs)==length(LBs)
        [m,I]=min(UBs);
        Sq=Sqs(2*I(1)-1:2*I(1),:);
        break
    end
    if min(UBs)==cs || min(UBs)==GLB
        [m,I]=min(UBs);
        Sq=Sqs(2*I(1)-1:2*I(1),:);
        break
    end

    %using window size to downsize the solution space
    aa=[];
    if size(Sqs,1)/2>window_size
        [~,II]=sort(Idles);
        Idles=Idles(II);LBs=LBs(II);LBs=LBs(II);
```

```
            LBs=LBs(1:window_size); UBs=UBs(1:window_size);
            %take the first w sequences
            II=2*II;
            for jj=1:window_size
                aa=[aa;Sqs(II(jj)-1:II(jj),:)];
            end
            Sqs=aa;
        end
        %delete sequenes where UB=LB;
        A=UBs-LBs;
        Index=find(A==0);
        Index1=[];
        for q=1:length(Index)
            Index1=[Index1,2*Index(q)-1,2*Index(q)];
        end
        Sqs(Index1,:)=[];
        if isempty(Sqs)
            m=min(UBs);
            break
        end


end
end
```

Appendix K: Design of Experiments (PR_BDP)

The following codes are used to assess the improvement of the production statistics considering task learning.

TEST function

```
load('learning rates.mat')
x1=x1-0.1;x2=x2-0.1;
AA2=[13 11  9   8   7   6   6   11  10  9   8   7   6   6   5   5
5];

%learning episodes
ks=[1,2,3];
N=400;
CI=zeros(3,2);
M=zeros(3,1);SE=zeros(3,1);
CI_I=zeros(3,2);
M_I=zeros(3,1);SE_I=zeros(3,1);
% w=1;
% max_t=1;
% [D_Mat]=Jackson_Dominance(P,t,D);
% L=Label(t,P);
W=[0.01,0.1,0.01,0.01];
window_size=5;
Avg_m=[];AA=[];Avg_Is1=[];Avg_Is2=[];AAI1=[];AAI2=[];
for kk=1:length(ks)
    load('P148.mat')
    [~,All_Follwers,Dir_Follwers]=Jackson_Dominance(P,t,D);
    All_Follwers=sort(All_Follwers,2,'descend');
    total_follwers=zeros(1,length(t));
    for i=1:length(t)
        total_follwers(i)=find(All_Follwers(i,:)==0,1)-1;
    end
    c=[204,255,306,357,408,459,510];
    k=ks(kk);
for i=1:length(c)
    ms=[];Is1=[];Is2=[];

[Sq,m]=findOne_BDP(c(i),P,t,D,E148(i,:),L148(i,:),W,total_follwe
rs,window_size);
    I1=2*m*c(i)-sum(t);
    %m1 is the old solution without reassignment
    m1=m;
    Is1=[Is1;I1];
    Is2=[Is2;I1];
```

```
    ms=[ms; m];
    pp=round((N-k*m)/(k+1))+m;
    p=zeros(1,k+1);
    p(1:end-1)=pp;
    p(end)=N-k*pp;
    t1=t;
    for j=1:k
        t1=t1.*x1;

[Sq,m]=findOne_BDP(c(i),P,t1,D,E148(i,:),L148(i,:),W,total_follw
ers,window_size);
        ms=[ms; m];
        Is1=[Is1;2*m1*c(i)-sum(t1)];
        Is2=[Is2;2*m*c(i)-sum(t1)];
    end
    %average position utilization
    Avg_m(i)=p*ms/N;
    %average Idle time
    Avg_Is1(i)=p*Is1/N;
    Avg_Is2(i)=p*Is2/N;
end
AA=[AA Avg_m];
AAI1=[AAI1 Avg_Is1];AAI2=[AAI2 Avg_Is2];
Avg_m=[];Avg_Is1=[];Avg_Is2=[];
load('P205.mat')
c=[1133,1322,1510,1699,1888,2077,2266,2454,2643,2832];
[~,All_Follwers,Dir_Follwers]=Jackson_Dominance(P,t,D);
All_Follwers=sort(All_Follwers,2,'descend');
total_follwers=zeros(1,length(t));
for i=1:length(t)
total_follwers(i)=find(All_Follwers(i,:)==0,1)-1;
end
for i=1:length(c)
    ms=[];Is1=[];Is2=[];

[Sq,m]=findOne_BDP(c(i),P,t,D,E205(i,:),L205(i,:),W,total_follwe
rs,window_size);
    ms=[ms; m];
    I1=2*m*c(i)-sum(t);
    %m1 is the old solution without reassignment
    m1=m;
    Is1=[Is1;I1];
    Is2=[Is2;I1];
    pp=round((N-k*m)/(k+1))+m;
    p=zeros(1,k+1);
    p(1:end-1)=pp;
```

```
    p(end)=N-k*pp;
    t1=t;
    for j=1:k
        t1=t1.*x2;

[Sq,m]=findOne_BDP(c(i),P,t1,D,E205(i,:),L205(i,:),W,total_follw
ers,window_size);
        ms=[ms; m];
        Is1=[Is1;2*m1*c(i)-sum(t1)];
        Is2=[Is2;2*m*c(i)-sum(t1)];
    end
    %average position utilization
    Avg_m(i)=p*ms/N;
    %average Idle time
    Avg_Is1(i)=p*Is1/N;
    Avg_Is2(i)=p*Is2/N;
end
AA=[AA Avg_m];
AAI1=[AAI1 Avg_Is1];AAI2=[AAI2 Avg_Is2];
AA=(AA2-AA)./AA2;
M(kk)=mean(AA);
ts = tinv(0.95,length(AA)-1);
SE(kk) = std(AA)/sqrt(length(AA));
CI(kk,:) = [M(kk) - ts*SE(kk), M(kk) + ts*SE(kk)];
%calculate Idle time improvement
I=(AAI1-AAI2)./AAI1;
M_I(kk)=mean(I);
ts = tinv(0.95,length(I)-1);
SE_I(kk) = std(I)/sqrt(length(I));
CI_I(kk,:) = [M_I(kk) - ts*SE_I(kk), M_I(kk) + ts*SE_I(kk)];
AA=[];AAI1=[];AAI2=[];Avg_m=[];Avg_Is1=[];Avg_Is2=[];
end
```