

Antennas Re-Clustering and Target Handoff for Multiple Radars System

By

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Abstract of the thesis

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Widely distributed multiple radar systems have been shown to offer enhanced localization performance. With smaller radar footprint, the ability to employ larger number of transmit and receive antennas opens new opportunities. In previous research, a subset selection scheme has been proposed for antenna clustering that minimizes the number of transmit and receive antennas required to achieve a preset accuracy performance. The study indicated that some transmit and receive antenna pairs contribute more than others to the localization performance. This thesis concentrates on handoff techniques that enable the transition of target tracking from one antenna cluster to another. As a target moves in an area covered by a grid of multiple radars, its relative position with respect to an existing tracking antenna cluster (or antenna subset) is changing, affecting the accuracy capabilities of the existing antenna cluster. Thus, at some point, there is a need to update the antenna cluster, keeping a useful antenna subset while replacing other antennas with ones that will keep localization accuracy within a given range. Re-clustering methods are proposed to address target handoff within antennas belonging to a larger grid. Low complexity re-

clustering algorithms are proposed for handoff purposes which enable a constrained replacement of antennas. These fast approximation algorithms are based on the optimization of the Cramer Rao bound (CRB) and constrained by the number of antennas that may be replaced at any given time. It is shown that this method performs close to optimal and can be implemented in a decentralized fashion.

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Chapter 1

Introduction

1.1 Radar System Topologies

Radar systems transmit electromagnetic energy in space to search for objects (targets). Existing targets will reflect a portion of this energy (echoes) back to be processed at the receiver to extract information about target position, velocity, and the resolution of target. The transmitter-target-receiver can be static or moving with respect to earth. Radar systems can be classified into different types depending on their design and way of operations for specific application [1][2]. In this chapter we discuss radar architecture as it is important to this thesis. Monostatic radar refers to a radar system that operates with a single antenna that performs the transmission and reception of the radar signal by switching the transmitting-receiving signal through a duplexer and the target location is determined by time of arrival (TOA) estimation at the receiver. Sometimes this definition can be ambiguous using the single antenna for transmit-receive signals, in which case a transmitter antenna and a receiver antenna can be used with a comparable distance with the range such that the system is still considered monostatic radar [3]. Figure 1.1 shows the coordinate of a monostatic radar system. A monostatic radar system can only measure the distance to target if we use an omnidirectional signal. R_T is the range between transmitter and target, and R_R is the range between receiver and target. The range measurement can

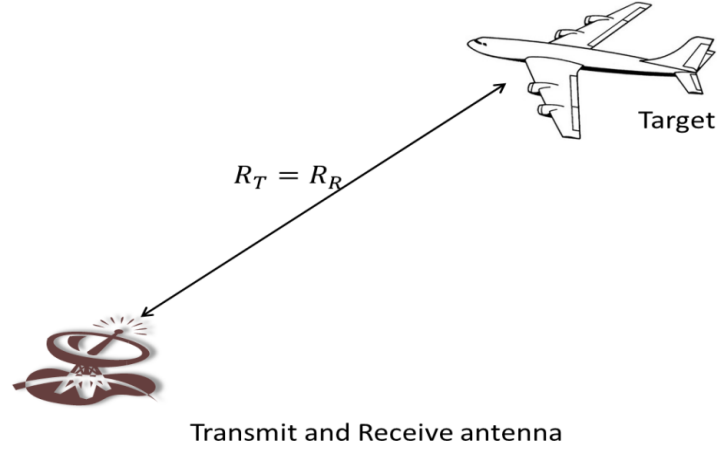


Figure 1.1 Monostatic Radar Architecture

be written as: $R_T = R_R = c\tau/2$, c is the speed of light. Bistatic radar systems operate with transmitting and receiving antennas that are placed in a separated location. The distance L between them is called a baseline range, and the angles θ_T and θ_R are the transmitter and receiver angles respectively, which are also called direction-of-arrival (DOA). The angle between the transmitting antenna and receiving antenna with the vertex at the target is called the bistatic angle or the scattering angle, denote as $\beta = \theta_T - \theta_R$. β and L configure and determine the characteristic of bistatic radar performance. The transmitter-target-receiver triangle is the basic geometrical characteristic of bistatic radar which is called a bistatic triangle [4]. The baseline between the transmitter and receiver is selected to be comparable with the range to the target of interests to achieve a specific operational objective. For example, [5] over the horizon-back-scatter (OTH -B) radars the baseline distance can be 100 - 200 km but the radar is still has monostatic characteristics because the separation is small as compared with the target range which may be up to 4000 km and does not satisfy the separation criteria of

bistatic radar. Bistatic angle β is a principle factor distinguishing bistatic from monostatic radar. If $\beta < 20$ the radar system is monostatic despite the transmitter and receiver antenna located at different sites. Bistatic radar use site separation to achieve some operational objectives as benefits. The process in mono static radar depends on the transmitter-target-receiver propagation time measurements to localize the target. In bistatic radar system the propagation time is converted to range sum which is equal to the transmitter-target time plus target-receiver time. Bistatic radar can use two methods to localize the target, direct and indirect methods. In the direct method, two signals are received at the receiver end, one from the transmitter and the other from the echo. The receiver measures the time intervals ΔT_{rt} between the reception of these two signals, then calculate the range sum as $(R_T + R_R) = c \Delta T_{rt} + L$. This method required adequate Line OF Sight (LOS) between the transmitter and receiver to receive the direct signal from the transmitter. In the indirect method, the receiver calculates the time interval ΔT_{tt} between the transmitted and received signal, then the range sum can be calculated as $(R_T + R_R) = c T_{tt}$ [4]. Figure 1.2 shows the coordinate of bistatic radar system. In bistatic radar system each transmitter-receiver pair forms an ellipse of constant range-sum. These ellipses are intersected to form contours of the target. The cross range does not depend on the angle of received data which gives more accuracy than bistatic, However, this accuracy requires the transmitting and receiving site to locate with the overlapping coverage and estimate the measurements simultaneously, thus the multistatic radar is restricted to short and medium range. This process is similar to multi iteration technique because it depends on the range measurement to determine the target location which will be discussed next. Various types of multi radar systems based on the bistatic have been

developed; the most significant one is multistatic radar [1]. Multistatic radar uses multiple antennas that are located in widely separated sites. Such a system requires at least one transmitting antenna and one receiving antenna, but in order to localize a minimum of four antennas is needed.

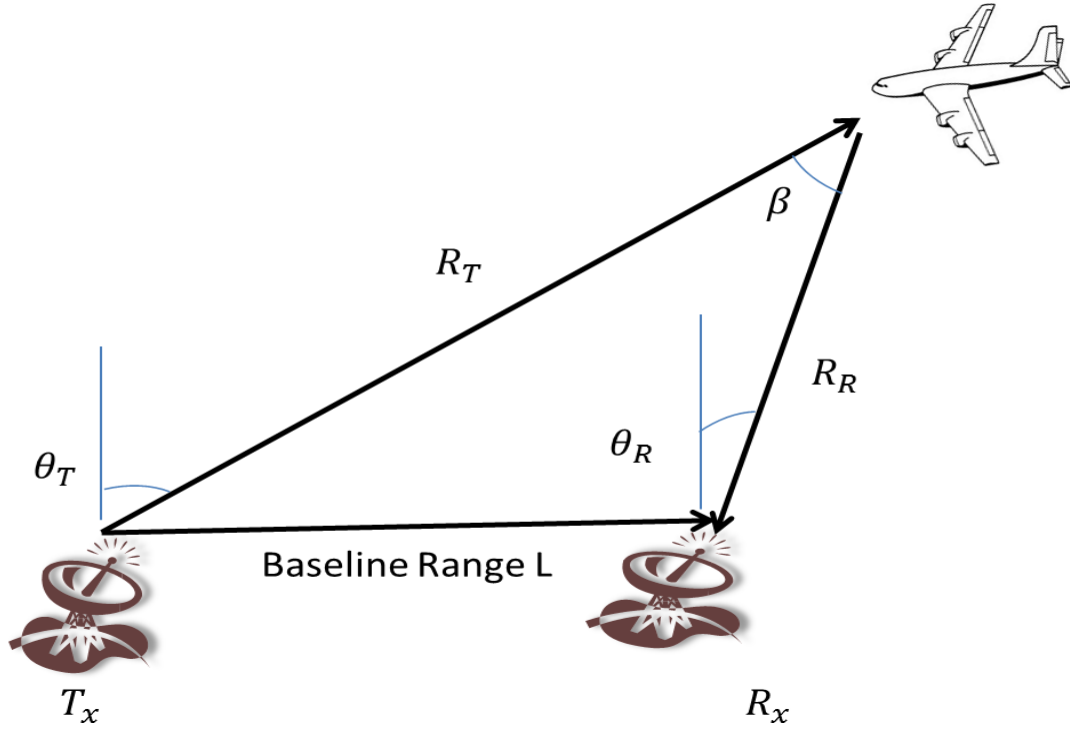


Figure 1.2 Bistatic Radar Architecture

1.2 Positioning System

Recently numbers of geolocation techniques have been developed to determine the location of person or object on earth. It can be relevant to a known location or to a corporate system. Variety of applications of these techniques such as security, road safety, tracking object, and mobile ad hoc network. It can be classified into two categories: Global Positioning System (GPS) and Local Positioning (LP)[6]. GPS is the most widely location-sensing system that provides a reliable and ubiquitous satellite constellation to allow each mobile object to find its own position in the globe. The system

consists of a constellation of 24 satellites (and 3 extra in case of fail) are orbiting about 12,000 miles above the earth, giving the localization in terms of latitude, longitude, and altitude[7]. The GPS receiver estimates the distance from the receiver to the satellites, and compute the position in three dimensions latitude, longitude, and altitude of each satellite signal with the receiver, then process the information to determine the position.

GPS transmits two frequencies of carrier signal, one for civilian use and other for military use. Even though the GPS is widely used in navigation applications it has drawbacks, its performance is limited to outdoor and downtown urban areas [6][7].

Local positioning system is one of the most exciting features of the wireless systems, with this technique the Geographic location of an object can be determined based on a known position or a coordinate system [7][8]. A wireless local positioning system can be formulated by two system components: A measuring unit and signal transmitting and receiving unit. Depending on the functionality of these two components the system is designed [9]. LP can be preliminary classified into two categories self-positioning and remote positioning system [8]. A self-position system allows the mobile object to find its position with respect to static point at a given time, specifically, the measurement unit is mobile and it has the ability to receive the signals from several transmitters at different locations, and then calculate its actual position based on the information of received signals. An example of this system is inertial navigation system (INS) which uses a motion sensing device such as gyroscope to track a moving object or gives the orientation of an object relative to a known point. INS can be integrated with GPS to enable the localization of indoor areas. INS is also used in environmental monitoring applications such as bush fire surveillance and road traffic monitoring since the measurements is

worthless without knowing where the data are obtained [10]. In remote-positing system the transmitter units are mobile and the system allow each unit to find the relative measurement of other units. In such case, several fixed measurement units are collected from the transmitted signals to calculate the instantaneous position of the object. These systems are power efficient and the mobile device is cheap and small. However, the system is complex and requires an expensive infrastructure [9]. Remote-positioning system can be classified into two categories: active target remote positioning and passive target remote positioning [8]. In active target remote positioning system, the target is active and cooperates in the process of positioning, for example wireless local positioning system (WLPS), while in passive target remote positioning system the target is passive without any tag or device, transmitters illuminate the target and the reflective signal are cooperate in the process of positioning to calculate the locations, for example Device-Free passive system, computer vision system, and tracking radar [11].

1.3 Fundamental Techniques of Positioning System

Several measurement principles are used for localization. We discuss some of them as they relate to the work in the next chapters.

1.3.1 Time of Arrival (TOA) estimation

Estimating distance is the main object of many positioning systems such as radar, wireless local positioning system (WLPs), and sonar. There are three approaches to determine the distance between two nodes, direct measurement, time of flight, and iteration [12]. In this thesis we interest in time-of-flight to measure a distance between nodes. TOA technique based on the propagation delay between the transmitter T_x and

receiver R_x . This concept requires precise time synchronization between the fixed and mobile units, For example an error of band $\pm 1\text{m}$ required a time synchronization below 1ns [8] [9]. This technique can be classified into two types: one way ranging which requires common time synchronizations between the transmitter and receiver, and two-way ranging in which the transmitter and receiver nodes exchange timing information through a certain protocol such as in cellular network where the receiving node is synchronized to base station [13].

We can use triangulation location technique to compute the distance of node (target) in this case the technique is called iteration. To compute a position of object in two dimensions required three measurements from coplanar points. Depends on this technique a known position base node gives distance measurement depend on time-of-flight to the target, this measurement is represented in a shape of circle with radius τ where τ the time-of-flight and the base node in the center. The contours of target are located in intersection of the shape of three base nodes as illustrated in figure 1.3.

The measured distance $d=c\tau$. The object (target) may be moving such as airplane travelling with a known velocity, we assume the object is stationary at the observed time interval. TOA has challenge such as multipath signals that are caused by the reflection in the environmental which may be identical to direct signal. These give estimation error in the distance and the shape of circles of different base nodes do not interest at a unique point. Many time delay estimation algorithms are proposed to accurately estimate TOA. TOA receiver may be affected to many other factors such as separate the closely multipath and additive noise. Increasing band width improves the multipath mitigation

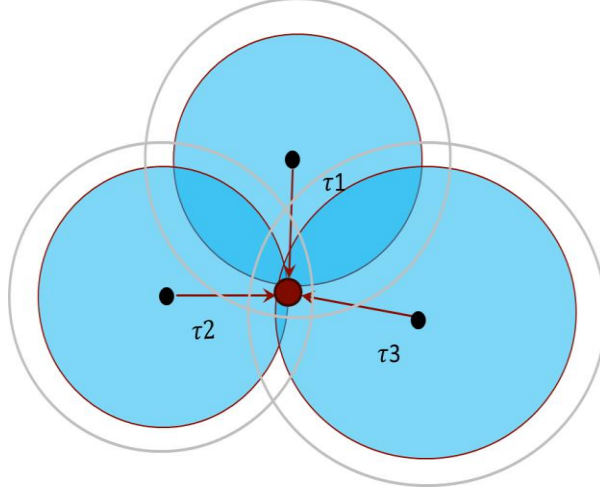


Figure 1.3 Operation of Time of Arrival

algorithm performance. To represent the TOA equation [8], consider the time-of-flight at the base node i_{th} can be written as:

$$\begin{aligned}\tau_i &= \frac{d}{c} \\ &= \frac{\sqrt{(x_i - x)^2 + (y_i - y)^2}}{c}\end{aligned}$$

By squaring the equation we have,

$$\frac{(x_i - x)^2 + (y_i - y)^2}{(c \tau_i)^2} = 1 \quad (1.1)$$

Which is clearly equation of circle and the target lie on its circumference and the center of circle is the i_{th} base node (x_i, y_i) .

1.3.2 Time Difference of Arrival (TDOA) estimation.

The difference between TOA and TDOA that TDOA measuring the difference in time between two base nodes, the TDOA at the base nodes can be represented by a hyperbola.

The first base node that receives the signal from the target will be a reference base

station. Two TDOA measurements are required with respect to base reference to compute the target location this make the technique needs at least three measurements for a coplanar scenario. Each node measurement can be represented by a hyperbola and the target location is located at the intersection of them. The arrival time at node the can be written as $t_i + t_m$, where t_i is time of transmitted signal and t_m is time of flight between the target position in (x,y) and the base node (x_i, y_i) , the position of target (x,y) can be written as[8] :

$$\begin{aligned}
 t_i &= \tau_i + t_m \\
 &= \frac{d_i}{c} + t_m \\
 &= \frac{\sqrt{(x_i - x)^2 + (y_i - y)^2}}{c} + t_m
 \end{aligned} \tag{1.2}$$

Now, we have one equation with three unknown parameters, by taking the difference of two arrival time we obtain:

$$\begin{aligned}
 t_i - t_j &= \tau_i + t_m - (\tau_j + t_m) \\
 &= \tau_i - \tau_j \\
 &= \frac{d_i}{c} - \frac{d_j}{c} \\
 &= \frac{\sqrt{(x_i - x)^2 + (y_i - y)^2} - \sqrt{(x_j - x)^2 + (y_j - y)^2}}{c}
 \end{aligned} \tag{1.3}$$

Since, we have one equation with two unknown parameters, by taking the difference of third measurement we can obtain:

$$t_i - t_k = \frac{\sqrt{(x_i - x)^2 + (y_i - y)^2} - \sqrt{(x_k - x)^2 + (y_k - y)^2}}{c} \quad (1.4)$$

1.3.3 Directional of Arrival (DOA)

Direction of Arrival (DOA) uses angles to determine an objects' position. This technique is called angulation [12]. Determining the position of a target in two dimensions requires at least two angle measurements and the distance between two base nodes. Phase antenna arrays use DOA for positioning, as they use multiple antennas with known position and separation distance. Phase array antennas use directional antennas which propagate energy in specific directions. Phase array systems use a linear array of antennas such that the Angle of Arrival (AOA) relates to the differences in arrival times of an incoming signal can be estimated. The target position is estimated by the intersection of two directional signals [13].

Chapter 2

Target Localization in Multiple Radar System

2.1 Introduction

Multiple-input multiple-output (MIMO) radar employs multiple, spatially distributed or colocated transmitters and receivers [1] [14]. Generally, MIMO radar can be viewed as an emerging concept of multistatic radar, but it can be set apart from multistatic radar system its unique features, the widely separated transmit/receive antenna over the area, and its ability to jointly process received signals at the receiver end. MIMO radar transmits independent, mutually uncorrelated waveforms that achieve broad spatial patterns. Nominally, omnidirectional beam pattern is used for a single transmitted signal, controlling the correlation among transmitted waveforms. These signals are jointly processed at the receiver [15]. MIMO radar has important contributions in radar field. It has more degree of freedom than a system with a single transmit antenna which makes the system support time-energy management modes [16],[17], The high spatial resolution due to the return signals from the target at different locations which are linearly independent of each other, offer excellent interference rejection capabilities[18]. MIMO radar with widely separated antenna improves radar performance by seeking to exploit the target spatial diversity, angular spread (RCS variation as a function of time) [14], and high degree of accuracy of target location [19]. In MIMO radar systems with co-located antennas refer to a linear array based system has a significant parameter identifiability over a phase array by identifying maximum number of targets up to M (number of transmit antennas) times that of its phased array counterpart [20]. It can estimate multiple

targets parameters such as (location and RCS) but the ability to resolve targets location is limited by Rayleigh resolution limit of transmit /receive arrays [14].

In this chapter we present the feature of MIMO radar with widely separated antenna and non-coherent processing which needs time synchronization between transmitting and receiving antennas while in coherent process phase synchronization is needed. The target is static and the lower bound on the attainable accuracy is set by developing the Cramer-Rao lower bound.

2.2 System Model

We assume a widely distributed MIMO radar system with M transmit and N receive antennas. The receiving radars may be co-located with transmitting ones or widely separated. The transmitter and receiver radars are located in two dimension plane (x,y) . The M transmitters are arbitrary located at coordinates $T_k = (x_{tk}, y_{tk})$, $K=1, \dots, M$, and the N receivers are located at coordinates $R_l = (x_{rl}, y_{rl})$, $l = 1, \dots, N$. We assume an extended target consists of many Q isotropic scatters located at coordinates $X_q = (x_q, y_q)$, $q = 1, \dots, Q$, we assume these Q scatterers are concentrated in a circle centered at $X=(x,y)$, within an area smaller than the signal wavelength. The reflectivity of the target is spatially homogenous and it is modeled by a zero-mean, independent and identically distributed (i.i.d) complex random variable $\zeta = \zeta_{re} + j\zeta_{im}$ with variance $E[|\zeta|^2] = 1/Q$. The set of transmitted waveforms in low pass equivalent is

$\sqrt{P_{m_{t_{max}}}} s_k(t)$, $k = 1, \dots, M$, where $P_{m_{t_{max}}}$ is the transmitted power and τ is the common duration of all transmitted waveforms. We introduce a vector

$\mathbf{P}_{t_{max}} = [P_{1_{t_{max}}}, \dots, P_{M_{t_{max}}}]^T$ which represent the waveforms' transmitted power $\mathbf{P}_{t_{max}}$.

Further we define individual effective bandwidth $\beta_k^2 = [(\int_{w_k} f^2 |S_k(f)|^2 df) / (\int_{w_k} |S_k(f)|^2 df) df]$, w_k means the integration over non zero signal content, the average effective bandwidth $\beta^2 = \frac{1}{M} \sum_{k=1}^M \beta_k^2$, and its normalized as $\beta_{R_k} = \beta_k / \beta$. The transmitted waveforms are designed to be orthogonal waveforms to be separated easily at the receiver and its separation may be imposed in the time domain or frequency domain or in signal space. This orthogonality is maintained even for different mutual delays, $\int_{\tau} S_k(t) S_m^*(t - \tau) dt = 0$ for all $k \neq m$, and for all time delays of interest. In our suggesting model, path loss effects are neglected, the sensor/target localization is accounting only through time delays of the signals [21].

In this chapter we present the MIMO radar with non-coherent case, let the propagation time delay between the K_{th} transmitter located at $T_k = (x_{tk}, y_{tk})$, to scatterer at coordinates (x_q, y_q) to the receiver located at $R_l = (x_{rl}, y_{rl})$ can be given as :

$$\tau_{lk} = \frac{\sqrt{(x_{tk} - x_q)^2 + (y_{tk} - y_q)^2}}{c} + \frac{\sqrt{(x_{rl} - x_q)^2 + (y_{rl} - y_q)^2}}{c} \quad (2.1)$$

target location is only determined through time delays of signals, the common path loss of the signals is embedded in ζ , the incoming signal at the receiver end is a mixture of the transmitted signals reflected by the target, it is separated by exploiting the orthogonality between them. The non-coherent process estimate the time delay τ_{lk} of transmitted signal from its variation in the envelope, in this case the transmitting antenna are not phase

synchronized and a common time reference is required for all the sensors of system. It can be shown the baseband of the received signal is given as:

$$r_{m,n}(t) = \sqrt{\alpha_{m,n} p_{m_{t\max}}} h_{m,n} s_m(t - \tau_{m,n}) + w_{m,n}(t) \quad (2.2)$$

Where the term $\alpha_{m,n} \propto \frac{1}{R_{mTx}^2 + R_{nRx}^2}$ represent the variation in the signal strength that caused by the path loss effect. The term $h_{m,n}$ integrates the effect of the phase offset between the transmitting and receiving sources and the target RCS impact on the phase and amplitude of transmitted signals. It is treated as unknown complex amplitude and deterministic. The channel (m,n) refers to the propagation path from transmitter m to the target to the receiver n. $w_{m,n}(t)$ is complex white Gaussian noise, zero mean, circularly symmetric, with auto correlation function $\sigma_w^2 \delta(\tau)$, the noise $\sigma_w^2 = 1/SNR_t$, SNR_t is measured at the transmitter and normalized such that it doesn't depend on the number of the transmitters, The SNR of channel (m,n) can be written as:

$$SNR_{m,n} = \frac{\alpha_{m,n} |h_{m,n}|^2 p_{m_{tx}}}{\sigma_w^2} \quad (2.3)$$

Let define the following vectors for later use, $\boldsymbol{\alpha} = [\alpha_{11}, \alpha_{12}, \dots, \alpha_{lk}, \dots, \alpha_{MN}]^T$, $\boldsymbol{\beta} = [\beta_1, \beta_2, \dots, \beta_M]^T$, $\boldsymbol{\tau} = [\tau_{1,1}, \tau_{1,2}, \dots, \tau_{MN}]^T$, and $\mathbf{P}_{tx} = [p_{1tx}, p_{2tx}, \dots, p_{Mtx}]^T$. We a vector of unknown parameter as:

$$\mathbf{u} = [x, y, \mathbf{h}^T] \quad (2.4)$$

Where $\mathbf{h} = [h_{11}, h_{12}, \dots, h_{MN}]$, and the vector of received signal can be defined as:

$$\mathbf{r}_{m,n} = [r_{1,1}, r_{1,2}, \dots, r_{MN}] \quad (2.5)$$

The radar system can estimate the target location (x,y) directly by using the maximum likelihood estimate (MLE) of the received signal in 2.2, or indirect estimation method by solving 2.1.

2.3 Cramer-Reo Bound (CRB) on Target Localization Estimation

The CRB provides the smallest achievable total variance of any unbiased estimator of deterministic data. Given a deterministic parameter vector \mathbf{u} (as defined in equation 2.4) the relationship between \mathbf{r} and \mathbf{u} is described by the probability density function (pdf) $p(\mathbf{r};\mathbf{u})$ \mathbf{r} parameterized by \mathbf{u} , the variance of any unbiased estimator $\hat{\mathbf{u}}$ is shown to be bounded by CRB and must satisfy the following[23]:

$$\text{var}(\hat{u}_i) \geq [\mathbf{J}(\mathbf{u})]_{ii}^{-1} \quad i=1,2,\dots \quad (2.6)$$

where $[J(\mathbf{u})]_{ii}^{-1}$ is the ii_{th} element of Fisher Information Matrix (FIM), then FIM can be defined as:

$$\begin{aligned} \mathbf{J}_F(\mathbf{u}) &\triangleq E_{\mathbf{r}|\mathbf{u}} \{ [\nabla_{\mathbf{u}} \ln p(\mathbf{r}|\mathbf{u})] [\nabla_{\mathbf{u}} \ln p(\mathbf{r}|\mathbf{u})]^T \} \\ &= -E_{\mathbf{r}|\mathbf{u}} \{ \nabla_{\mathbf{u}} [\nabla_{\mathbf{u}} \ln p(\mathbf{r}|\mathbf{u})]^T \} \end{aligned} \quad (2.7)$$

where $p(r/u)$ is the joint probability density function of \mathbf{r} condition on \mathbf{u} and $E_{r|u}\{\cdot\}$ is the conditional expected value of \mathbf{r} given \mathbf{u} , the CRB is defined as:

$$\text{CRB} \triangleq \mathbf{C}(\mathbf{u}) = [\mathbf{J}(\mathbf{u})]^{-1} \quad (2.8)$$

the conditional pdf $f(\mathbf{r}|\mathbf{u})$ of the observation vector \mathbf{r} has the following form:

$$f(\mathbf{r}|\mathbf{u}) = \frac{1}{(\pi\sigma_w^2)^{\frac{MN}{2}}} \exp\left\{-\frac{1}{\sigma_w^2} \sum_{m=1}^M \sum_{n=1}^N \left| r_{m,n}(t) - \sqrt{\alpha_{m,n} p_{m_{\text{max}}}} h_{m,n} s_m(t - \tau_{m,n}) \right|^2 dt\right\} \quad (2.9)$$

since we receive \mathbf{r}_{MN} vector is statistically independent with identical probability density,

We define the Maximum Likelihood estimator(MLE) as :

$$\hat{\mathbf{u}}_{ML} = \arg \max \{\ln f(\mathbf{r}|\mathbf{u})\} \quad (2.10)$$

The MLE approaches CRB as the SNR becomes large [24]. In [21] it was demonstrated that the MLE is asymptotically tight to the CRB at SNR over 10 dB. Thus, the CRB is used here to represent the localization MSE as a function of the power allocation. The FIM $\mathbf{J}(\mathbf{u})$, is derived in Appendix A. The CRB is the 2×2 upper left block of $\mathbf{J}^{-1}(\mathbf{u})$ which can be represented as:

$$\mathbf{C}_{x,y}(\mathbf{u}, \mathbf{p}_{tx}) = \left\{ \sum_{m=1}^M \sum_{n=1}^N p_{m_{tx}} \begin{bmatrix} u_{a_{m,n}} & u_{c_{m,n}} \\ u_{c_{m,n}} & u_{b_{m,n}} \end{bmatrix} \right\}^{-1} \quad (2.11)$$

Where the terms $u_{a_{m,n}}$, $u_{b_{m,n}}$, and $u_{c_{m,n}}$ have the following expression:

$$\begin{aligned} u_{a_{m,n}} &= \alpha_{m,n} p_{m_{tx}} \frac{8\pi^2 \beta_m^2}{\sigma_w^2 c^2} |h_{m,n}|^2 \left(\frac{x_{m_{Tx}} - x}{R_{m_{Tx}}} + \frac{x_{n_{Rx}} - x}{R_{n_{Rx}}} \right)^2 \\ u_{b_{m,n}} &= \alpha_{m,n} p_{m_{tx}} \frac{8\pi^2 \beta_m^2}{\sigma_w^2 c^2} |h_{m,n}|^2 \left(\frac{y_{m_{Tx}} - y}{R_{m_{Tx}}} + \frac{y_{n_{Rx}} - y}{R_{n_{Rx}}} \right)^2 \\ u_{c_{m,n}} &= \alpha_{m,n} p_{m_{tx}} \frac{8\pi^2 \beta_m^2}{\sigma_w^2 c^2} |h_{m,n}|^2 \left(\frac{x_{m_{Tx}} - x}{R_{m_{Tx}}} + \frac{x_{n_{Rx}} - x}{R_{n_{Rx}}} \right) \times \left(\frac{y_{m_{Tx}} - y}{R_{m_{Tx}}} + \frac{y_{n_{Rx}} - y}{R_{n_{Rx}}} \right) \end{aligned}$$

The trace of the matrix $\mathbf{C}_{x,y}$ represent a lower bound of the sum of the MSEs for target location estimation, such that $tr \mathbf{C}_{x,y} \leq \sigma_x^2 + \sigma_y^2$, where σ_x^2 and σ_y^2 are the target's x and y MSE estimation, respectively. Following some additional matrix manipulations, the trace of CRB can be expressed as:

$$\sigma_{x,y}^2(\mathbf{p}_{tx}) = \text{tr}(\mathbf{C}_{x,y}) = \frac{\mathbf{b}^T \mathbf{P}_{tx}}{\mathbf{P}_{tx}^T \mathbf{A} \mathbf{P}_{tx}} \quad (2.12)$$

where $\mathbf{b} = (\mathbf{u}_a + \mathbf{u}_b)^T$, $\mathbf{A} = \mathbf{u}_a \mathbf{u}_b^T - \mathbf{u}_c \mathbf{u}_c^T$, $\mathbf{u}_a = [u_{a1}, \dots, u_{aM}]^T$, $\mathbf{u}_b = [u_{b1}, \dots, u_{bM}]^T$, and $\mathbf{u}_c = [u_{c1}, \dots, u_{cM}]^T$.

it follows that the lower bound on the variance for the target position of the x coordinate is

$$\sigma_x^2 = \frac{\sigma_w^2 c^2}{8\pi^2 \beta^2} \frac{\mathbf{u}_a}{\mathbf{u}_a \mathbf{u}_b^T - \mathbf{u}_c \mathbf{u}_c^T} \quad (2.13)$$

similarity, for y axis is

$$\sigma_y^2 = \frac{\sigma_w^2 c^2}{8\pi^2 \beta^2} \frac{\mathbf{u}_b}{\mathbf{u}_a \mathbf{u}_b^T - \mathbf{u}_c \mathbf{u}_c^T} \quad (2.14)$$

It is noticed that the variance in (2.13) and (2.14) has the term $\eta = \frac{\sigma_w^2 c^2}{8\pi^2 \beta^2}$ which shows the lower bound on the variance is inversely proportional to the average effective bandwidth as well as to signal to noise ratio $\propto 1/(\text{SNR})$ since $\text{SNR} = 1/\sigma_w^2$. The term $1/\beta_m^2$ means the range estimation between radar and target based on a one-way time delay in single antenna. The other terms in (2.13) and (2.14) show that the CRB depends on the geographical spread of radar sensors and target location.

2.4 Effect of sensor Location

The CRLB is strongly reliant on the geographical position of the sensors, this dependency is incorporated into the terms \mathbf{u}_a , \mathbf{u}_b , and \mathbf{u}_c from those terms we cannot intuitively express the relation between the radar system position and the achievable localization accuracy. The term \mathbf{u}_a is summation of cosine functions and the term \mathbf{u}_b is

summation of sine functions of the same set of angles, so a set of sensors that minimizes the target vertically may result in high horizontal error and vice versa. In order to obtain truly minimum error of localization accuracy in both x and y (reduce error horizontally and vertically) we should minimize the total variance as following:

$$\sigma^2 = (\sigma_{xCRB}^2 + \sigma_{yCRB}^2) \quad (2.15)$$

We need to define a suitable method and to express this relation and set a lower bound on the CRLB over all possible sensor placements. In [21], further analysis is developed by seeking the optimal set of angles $\phi_k = \tan^{-1}(\frac{y-y_{rk}}{x-x_{rk}})$ and $\psi_l = \tan^{-1}(\frac{y-y_{rl}}{x-x_{rl}})$ for which CRB for localization along the x and y axes are jointly minimized. All antennas is symmetrically placed on a circle around the target such that the number of transmitting antennas $M \geq 3$ and the number of receiving antennas $N \geq 3$ have uniform angular spacing $2\pi/M$ and $2\pi/N$ respectively, or any superposition of such sets. This leads to give the minimum value of CRB which is equal to $2\eta/MN$ [20]. For a special case, the CRB for the radar system with single-input multiple-output (SIMO), if the system uses 1 transmitter and $M+N$ receiver the CRB may be equal to $4\eta/MN$. This mean the estimation error is increased by a factor of 2 as compared with MIMO system, and the latter has twice performance. We can address the effect of sensor location on the target position accuracy by using a Geometric Dilution of Precision (GDOP). It is a metric that is commonly used in GPS for mapping the attainable localization precision for a given layout of GPS satellite positions [33][34]. The GDOP metric express the effects of sensor locations on the time-delay estimation errors. It can calculate which sensor is a good selected geometry and the one constitutes a poor choice. For the two dimensions case, GDOP define as:

$$GDOP = \sqrt{\frac{\sigma_x^2 + \sigma_y^2}{c^2 \sigma_\varepsilon^2}} \quad (2.16)$$

Where σ_x^2 denote the variance of location on x axes, σ_y^2 denote the variance of location on y axes, and σ_ε^2 is the time delay variance. GDOP provides normalized measurements that estimate the effect of radars' location on the overall accuracy.

2.5 Physical Interpretation

The localization MSE has a physical connection to the geometry of the sensors location. In Multiple radar system each transmit/receive pair measures a range $R_{m,n} = R_{mTx} + R_{nRx}$, which is equivalent to estimation of time delay $\tau_{m,n}$. This range draws an ellipse with transmitter and receiver positions as the focal points and all possible target positions are located on the surface of ellipse. The range sum is equal to constant $R_{mTx} + R_{nRx} = 2a$, where $2a$ is a major axis of ellipse. In practice the range sum can include an error measurement due to the time delay estimation error with variance defines as:

$$\sigma_{R_{m,n}}^2 \propto \sigma_{\tau_{m,n}}^2 = \frac{\sigma_w^2}{8\pi^2 p_m \alpha_{m,n} |h_{m,n}|^2 \beta_m^2} \quad (2.17)$$

This range variation ΔR draws inner ellipse with a major axis $2a$ and outer ellipse with a major axis $2a + \Delta R$. The separation width between the two ellipses varies depending on target position. It is a minimum on the baseline L and maximum on the perpendicular bisector of the baseline. This is shown in figure 2.1. Multiple radar systems are based on iteration technique that use the range measurements of multiple distributed transmit/receiver pairs. The target is measured by three or more range measurements. Three ellipses can be drawn that intersect at target position. The size of intersection area

of multiple ellipses estimates MSE of target position. This area can be controlled by increasing the power of transmitting antenna or by choosing transmit- receive pair that the target lie in appropriate position as mentioned previously. This reduces uncertain area of target position and gives a better MSE estimation.

In monostatic radar, the range measurement R draws a circle with radius R around the transmit/receive antenna. All possible target positions could be located on the surface of circle as shown in figure 2.2. The variation in range measurements draws an inner circle with radius R and outer circle with range $R+\Delta R$. In netted radar system, three circles or more intersect to give the target position.

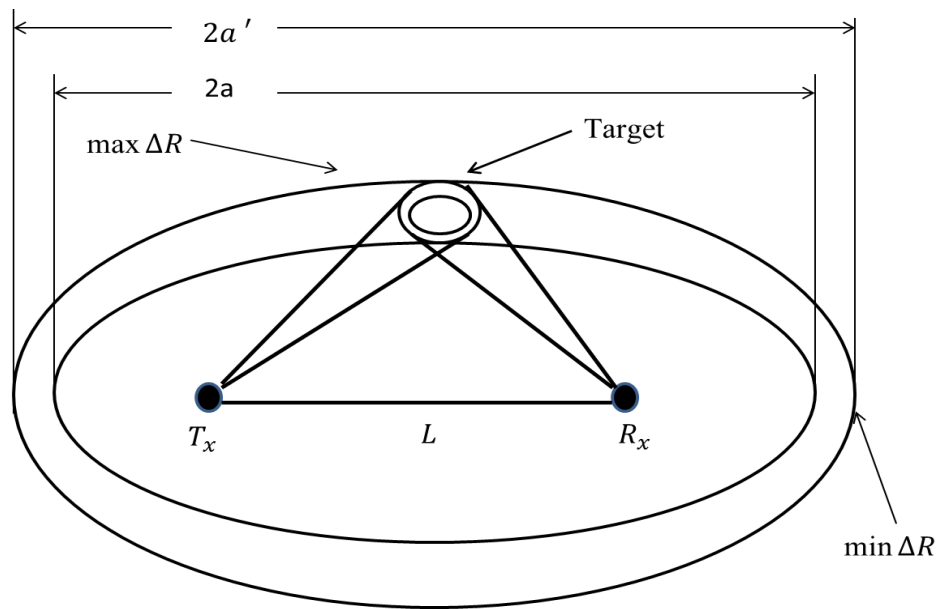


Figure 2.1 Geometry of transmitter /receiver range cell

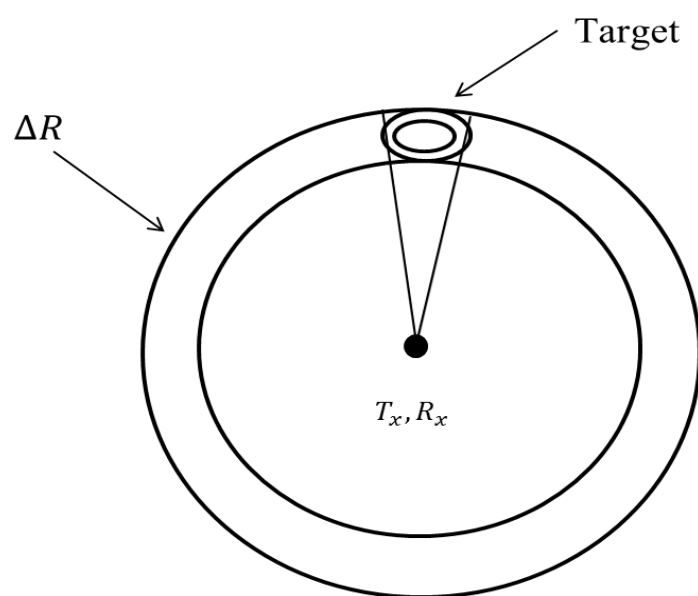


Figure 2.2 Monostatic radar range cell

Chapter 3

Subset Selection

Widely distributed multiple radar systems improve parameter estimation of target localization. For a system with full resource allocation, all available antennas may be used in local process. Thus the number of transmitting and receiving antenna play a very important role in target's parameter estimation; however, many problems arise relating to computational complexity load and efficient communication link with central fusion center. For this reason the concept of resource-aware design is an important one to radar system especially for those that employ mobile station or operate over a prolonged time periods. In [28] a power allocation scheme has developed to support resource-aware design for target localization of multiple radar system. The achievable localization MSE is minimized for a given total energy budget. It has been shown that some transmit/receive antenna pairs that have lower path losses, high target reflectivity, and better angular spread contribute more than those that have high path losses, low target reflectivity, and narrow angular spreads. On the other hand, under some emergency operation conditions, the system is imposed to operate only with a fraction of the antennas at any given time. Thus, select a subset of K sensors out of $M+N$ available ones can offer operational saving, reduce communication needs, and computational complexity.

3.1 Concept of Subset Selection

The problem of selecting an optimal active \mathbf{K} subset can be translated to choosing K_{Tx} transmit antennas out of M , and K_{Rx} receive antennas out of N , where $K = K_{Tx} +$

K_{Rx} is a predetermined size that gives the best estimation performance in terms of localization MSE[25]. This problem can be formulated as a Knapsack problem (KP) [26][27]. Given a set of items, each with a weight (performance level) and a value (cost), KP determine the number of each item to be included in the collection, such that the total performance level is less than or equal to the size of Knapsack (constraint) while optimizing the total cost. In multiple radar systems, we represent the performance level by the estimation MSE and the total cost can be represented by the weighted sum of selected antennas. The CRB may be used to evaluate the temporal performance level for a given subset. This problem may be solved by using an exhaustive search algorithm which gives an optimal solution. This search algorithm is quite simple and always gives the solution if it exists; however, it has a computational complexity on the order of $O(2^{M+N})$, thus it grows up as the size of problem increase. Another algorithm may be implemented such as a heuristic algorithm [25]. This algorithm has a polynomial computational complexity of order $O(KMN (M+N))$ which offers a complex reduction as compared with exhaustive one.

In the next section we will present a system model and mathematical formulation for KP problem, we propose the heuristic algorithm and compare how the algorithm is close enough the optimal solution.

3.2 KP Problem Formulation

We assume multiple radar system with M transmit and N receive antennas. A target is assumed to move through L positions, with the positions denoted by $(x_\ell, y_\ell), \ell = 1, \dots, L$. We assume the target is static at a given ℓ position. The KP problem can be

mathematically formulated by introducing a vector of binary variable having the following:

$$q_{t_{xm}}^\ell = \begin{cases} 1 & \text{if transmit radar } m \text{ is selected} \\ 0 & \text{otherwise, } m = 1, \dots, M \end{cases} \quad (3.1)$$

and

$$q_{r_{xn}}^\ell = \begin{cases} 1 & \text{if receive radar } n \text{ is selected} \\ 0 & \text{otherwise, } n = 1, \dots, N \end{cases} \quad (3.2)$$

Since in our scenario we have a predetermine subset $K = k_{Tx}^\ell + k_{Rx}^\ell$ with the system operation condition $k_{Tx}^\ell \geq 1 \in S_{Tx}^\ell$, and $k_{Rx}^\ell \geq 1 \in S_{Rx}^\ell$. The CRLB for a set $S_A^\ell = \{x_{m_{tx}} \in S_{Tx}^\ell, x_{n_{rx}} \in S_{Rx}^\ell | q_{t_{xm}}^\ell = 1, q_{r_{xn}}^\ell = 1\}$, can be written as:

$$\mathbf{C}_{S_A}^\ell(\mathbf{q}_{tx}^\ell, \mathbf{q}_{rx}^\ell) = \left\{ \sum_{m=1}^M \sum_{n=1}^N q_{t_{xm}}^\ell, q_{r_{xn}}^\ell \mathbf{J}_{m,n} \right\}^{-1} \quad (3.3)$$

where $\mathbf{q}_{tx}^\ell = [q_{t_{x1}}, \dots, q_{t_{xM}}]_{M \times 1}^T$, $\mathbf{q}_{rx}^\ell = [q_{r_{x1}}, \dots, q_{r_{xN}}]_{N \times 1}^T$. The trace of the CRB in 3.3 represents the sum of variances of the estimation error on the target's x and y location

$\sigma_x^2(\mathbf{q}_{tx}, \mathbf{q}_{rx}) + \sigma_y^2(\mathbf{q}_{tx}, \mathbf{q}_{rx})$, this can be written as:

$$\begin{aligned} \text{tr}(\mathbf{C}_{S_A}^\ell(\mathbf{q}_{tx}^\ell, \mathbf{q}_{rx}^\ell)) &= \sum_{m=1}^M \sum_{n=1}^N U_{1,m,n} q_{t_{xm}}^\ell, q_{r_{xn}}^\ell \times \\ &\left[\sum_{m=1}^M \sum_{n=1}^N \sum_{m'=1}^M \sum_{n'=1}^N U_{2,m,n,m',n'} q_{t_{xm}}^\ell, q_{r_{xn}}^\ell q_{t_{xm'}}^\ell, q_{r_{xn'}}^\ell \right]^{-1} \end{aligned} \quad (3.4)$$

The problem arises when a required utilization factor ρ_{max} is imposed on the system operation. The system is restricted to operate on this factor at any given time, and work with a subset $k = [\rho_{max} (M+N)]$ sensors. The problem may be formulated as:

$$\begin{aligned}
& \text{minimize } tr(\mathbf{C}_{S_A}(\mathbf{q}_{tx}, \mathbf{q}_{rx})) \\
& s.t \quad \sum_{m=1}^M q_{tx_m} + \sum_{n=1}^N q_{rx_n} = k \\
& \quad \sum_{m=1}^M q_{tx_m} \geq 1, \sum_{n=1}^N q_{rx_n} \geq 1 \\
& \quad q_{tx_m} \in \{0, 1\}, q_{rx_n} \in \{0, 1\}
\end{aligned} \tag{3.5}$$

This is a quadratic Knapsack problem (QKP). We define a cost objective function $Q_{K-set}(\mathbf{q}_{tx}, \mathbf{q}_{rx}) = \mathbf{C}_{S_A}(\mathbf{q}_{tx}, \mathbf{q}_{rx})$, and the total utilization weight function $W_{K-set}(\mathbf{q}_{tx}, \mathbf{q}_{rx}) = \sum_{m=1}^M v_{tx_m} q_{tx_m} + \sum_{n=1}^N v_{rx_n} q_{rx_n}$. This formulation is appropriate for a unity operational cost, where v_{tx_m} and v_{rx_n} are unit vectors. In case of various operational cost parameter of v_{tx_m} and/or v_{rx_n} are assigned, we add a new inequality constrain $\sum_{m=1}^M q_{tx_m} + \sum_{n=1}^N q_{rx_n} \leq K^v$ to the problem in 3.5 the term K^v refers to the total allowable cost.

The optimal solution for this problem can be obtained through an exhaustive search for all possible chosen sensors of subset K, the drawback of this solution is the heavy computational load which it requires $\sum_{k_o=1}^{K-1} \sum_{k_o \leq M, k-k_o \leq N} \binom{M}{k_o} \binom{N}{K-k_o}$ iterations and has a computational complexity of $O(2^{M+N})$. The heuristic algorithm offers a fast search with lower computational cost. It chooses the combinations of K transmit and receive antennas that minimize the CRLB. This is summarized in table 3.1. We select one transmit antenna and one receive antenna $\{x_{i_{Tx}}^\ell, x_{j_{Rx}}^\ell\}$ out of the original set of set of transmit and receive antennas, S_{Tx} and S_{Rx} the location in the vectors \mathbf{q}_{tx} and \mathbf{q}_{rx} corresponding to $\{x_{i_{Tx}}^\ell, x_{j_{Rx}}^\ell\}$ are set to one and added to active antenna subset $S_{min} = \{\mathbf{q}_{tx}, \mathbf{q}_{rx}\}$ these initial antennas are subtracted from the remaining non-active transmit and receive antenna sets,

$S'_{T_x} = S_{T_x} \setminus x_{i_{T_x}}$ and $S'_{R_x} = S_{R_x} \setminus x_{j_{R_x}}$ respectively. Then, either one transmit antenna or one receive antenna is added to active subset, such that the trace of the temporal CRB matrix is minimized $C_{Tx_temp} = \min_{x_{i_{T_x}} \in S'_{T_x}} \|tr(C_{S_{min} \cup x_{i_{T_x}}})\|$ for adding a transmit antenna and $C_{Rx_temp} = \min_{x_{j_{R_x}} \in S'_{R_x}} \|tr(C_{S_{min} \cup x_{j_{R_x}}})\|$ for adding a receive antenna, then the minimum of C_{Tx_temp} and C_{Rx_temp} is chosen and the values of $S_{min} = \{\mathbf{q}_{tx}, \mathbf{q}_{rx}\}$ is updated to fit the choice of the additional transmit or receive antenna. The search circle stops when reach the KP capacity $\sum_{m=1}^M q_{tx_m} + \sum_{n=1}^N q_{rx_n} = K$. We scan all possible MN pairs $\{x_{i_{T_x}}^\ell, x_{j_{R_x}}^\ell\}_{m=1, \dots, M, n=1, \dots, N}$, thus we get a minimal set S_{min}^{mn} with corresponding sub optimal vectors set $\{\mathbf{q}_{tx}^{mn}, \mathbf{q}_{rx}^{mn}\}$. Then we choose optimal solution set $\{\mathbf{q}_{tx}^*, \mathbf{q}_{rx}^*\}$ out of S_{min}^{mn} . This method offers a polynomial complexity of $O(KMN(M+N))$ which offers a significant computational savings for large numbers of antennas.

3.3 Numerical Analysis

The spatially diverse of multiple radar system have different error characteristics, reliant on the specific path loss, target reflectivity, effective bandwidth, and transmitted power. In this section, numerical analysis is provided for the proposed heuristic subset selection algorithm and exhaustive search. A 9×14 MIMO radar system ($M=9$ and $N=14$) is chosen for this analysis. A transmit radar set is located in $[2 \ 9.5; 1.5 \ 3.5; 3 \ 7; 2.8 \ 2.5; 4.5 \ 8; 5 \ 1.8; 8 \ 6.8; 8 \ 1.2; 9.5 \ 4.6] \times 10^3 \text{m}$ and a receive radar set is located in $[0.57 \ 0.5; 1 \ 5; 3 \ 8.5; 4 \ 7; 5.1 \ 7.5; 3.4 \ 2.8; 4.1 \ 2.8; 6.2 \ 6.8; 6.5 \ 8; 6 \ 2.1; 7 \ 1.9; 7.7 \ 2.9; 8.5 \ 6; 9 \ 1] \times 10^3 \text{m}$. A moving targets' path is assumed to go through nine ($L=9$) positions located in $[5 \ 9.8; 1.1 \ 8.2; 2 \ 6.5; 2.5 \ 5; 3.5 \ 4.5; 4.5 \ 4; 6 \ 5.1; 7 \ 5.6; 8.5 \ 4.5; 9.5 \ 2.5; 9.8 \ 1] \times 10^3 \text{m}$. The target positions, are marked by (x_ℓ, y_ℓ) , $\ell = 1, \dots, L$. Figure 3.1 shows the multiple radar layouts with multiple positions. The transmit and receive radar range are calculated with respect to the

Table 3.1 KP K-Subset selection algorithm

```

1      int :  $q_{tx} = 0, q_{rx} = 0, A_{\min} = \phi, A_{\min}^* = \phi, set = 0$ 
2      for  $m = 1, \dots, M$  and  $n = 1, \dots, N$ 
2.1    select initial subset :  $S_{\min} = \{x_{m_{Tx}}, x_{n_{Rx}}\},$ 
2.2    Update :  $\left\{ \begin{array}{l} S'_{Tx} = S_{Tx} \setminus X_{m_{Tx}}, S'_{Tx} = S_{Rx} \setminus X_{m_{Rx}} \\ q_{txm} = 1, q_{rxn} = 1 \end{array} \right\}$ 
2.3    set :  $count = 2, k = 2$ 
2.4    while  $k \leq K$ 
        if  $S'_{Tx} \neq \text{null}$  then select  $x'_{i_{Tx}} \in S'_{Tx}$  s.t.
             $x'_{i_{Tx}} = \arg \min_{x_{i_{Tx}} \in S'_{Tx}} tr(C_{S_{\min} \cup x_{i_{Tx}}})$ 
2.4.1    update :  $\left\{ \begin{array}{l} X_{Tx\_temp} = x'_{i_{Tx}} \\ C_{Tx\_temp} = tr(C_{S_{\min} \cup x'_{i_{Tx}}}) \end{array} \right\}$ 
            if  $S'_{Rx} \neq \text{null}$  then select  $x'_{j_{Rx}} \in S'_{Rx}$  s.t.
                 $x'_{j_{Rx}} = \arg \min_{x_{j_{Rx}} \in S'_{Rx}} tr(C_{S_{\min} \cup x_{j_{Rx}}})$ 
2.4.1    update :  $\left\{ \begin{array}{l} X_{Rx\_temp} = x'_{j_{Rx}} \\ C_{Rx\_temp} = tr(C_{S_{\min} \cup x'_{j_{Rx}}}) \end{array} \right\}$ 
             $\left\{ \begin{array}{l} \text{Update : } S_{\min} = S_{\min} \cup \{x_{Tx\_temp}\}, \\ S'_{Tx} = S'_{Tx} \setminus \{x_{Tx\_temp}\} \\ \text{set : } k = k + 1, q_{x_{Tx\_temp}} = 1 \end{array} \right\};$ 
        else
             $\left\{ \begin{array}{l} \text{Update : } S_{\min} = S_{\min} \cup \{x_{Rx\_temp}\}, \\ S'_{Rx} = S'_{Rx} \setminus \{x_{Rx\_temp}\} \\ \text{set : } k = k + 1, q_{x_{Rx\_temp}} = 1 \end{array} \right\};$ 
        end while
2.5     $A_{\min} = A_{\min} \cup \{q_{tx}, q_{rx}\};$ 
2.6     $q_{tx} = 0, q_{rx} = 0, set = set + 1;$ 
        end for
3      for index = 1 : set
3.1    select vectors  $\{q_{tx}^*, q_{rx}^*\} \in A_{\min}$  s.t.
             $\{q_{tx}^*, q_{rx}^*\} = \arg \min_{q_{tx}, q_{rx} \in A_{\min}} tr(C_{S_A}(q_{tx}, q_{rx}))$ 
        end for
    end  $\{q_{tx}^*, q_{rx}^*\}$ 

```

target at each given position. $\mathbf{R}_{mTx}^\ell = [R_{mTx}^1, \dots]_{9 \times 1}$ denotes transmitter range set and $\mathbf{R}_{nTx}^\ell = [R_{nTx}^1, \dots]_{14 \times 1}$ denotes receiver range set. A channel model with uniform reflectivity is used and no channel losses are assumed, i.e., $\mathbf{h} = [1, \dots, 1; 1, \dots, 1; 1, \dots, 1]_{9 \times 14}$. All cost factors and \mathbf{v}_{tx} and \mathbf{v}_{rx} are set to be equal and $\mathbf{P}_{txmax} = 100 \times \mathbf{1}_{9 \times 1}$. The subset size is set to $\mathbf{k} = 6$. The heuristic algorithm results are given in Table 3.2. Since we concentrate on the geometric layout of the system, transmitter 3 is selected when the target at position 2, 3, 4, 5, and 7 which is the closed one to the target, same observation for transmitter 7 when the target at position 7, 8, 9, 10, and 11. On the receiver side, receivers 1 and 3 are selected for position 1, 2, and 3 which give the minimum distance to the target and higher angular spread with respect to the target, another observation that receiver 4 is chosen only for positions 3 even though it is closed to position 4, 5, and 6, but it is not selected, the reason that may not give optimum angular of spread if it is chosen with the other sensors of K subset. The exhaustive search result is given in table 3.3 with the same number of subset selection $K=6$ and for all target positions.



Figure 3.1 Multiple Radar Layouts

position	q_{tx}^*	q_{rx}^*	T_{x_set}	R_{x_set}	$MSE_{Heuristic}$
1	1,0,0,0,1,0,0,0,0	1,0,1,0,1,0,0,0,1,0,0,0,0,0,0	2	4	6.605
2	1,0,1,0,1,0,1,0,0	1,0,1,0,0,0,0,0,0,0,0,0,0,0,0	4	2	0.511
3	0,0,1,0,1,0,0,0,0	1,1,1,1,0,0,0,0,0,0,0,0,0,0,0	2	4	2.555
4	0,1,1,1,0,0,0,0,1	0,1,0,0,0,1,0,0,0,0,0,0,0,0,0	4	2	3.230
5	0,1,1,1,0,0,0,1,0	0,0,0,0,0,1,1,0,0,0,0,0,0,0,0	4	2	0.870
6	0,0,1,1,0,1,0,0,0	0,0,0,0,0,1,1,0,0,1,0,0,0,0,0	3	3	0.878
7	0,0,1,0,1,1,1,0,0	0,0,0,0,1,0,0,1,0,0,0,0,0,0,0	4	2	1.328
8	0,0,0,0,1,0,1,0,1	0,0,0,0,0,0,0,1,1,0,0,0,1,0,0	3	3	0.746
9	0,0,0,0,0,0,1,1,1	0,0,0,0,0,0,0,0,1,0,0,1,1,0,0	3	3	0.328
10	0,0,0,0,0,0,1,1,1	0,0,0,0,0,0,0,0,0,0,0,1,1,1,0	3	3	0.767
11	0,0,0,0,0,0,1,1,1	0,0,0,0,0,0,0,0,0,0,0,1,1,1,0	3	3	0.715

Table 3.2 Minimize MSE using a heuristic algorithm

position	q_{tx}^*	q_{rx}^*	T_{x_set}	R_{x_set}	$MSE_{exhaustive}$
1	1,0,0,0,1,0,0,0,0	1,0,1,0,1,0,0,0,1,0,0,0,0,0,0	2	4	6.605
2	1,1,1,0,1,0,0,0,0	1,0,0,0,0,0,0,0,1,0,0,0,0,0,0	4	2	0.511
3	1,1,1,0,1,0,0,0,0	0,1,0,0,0,0,0,0,0,0,0,0,1,0,0	4	2	2.085
4	1,1,1,0,0,1,0,0,0	0,1,0,0,0,0,0,0,0,0,0,0,0,1,0	4	2	2.609
5	1,0,1,1,1,0,0,0,0	0,0,0,0,0,1,0,0,0,0,0,0,0,1,0	4	2	0.849
6	1,0,1,1,0,1,0,0,0	0,0,0,0,0,0,1,0,0,0,0,0,0,1,0	4	2	0.398
7	1,0,0,0,1,1,1,0,0	0,0,0,0,0,0,0,1,0,0,0,0,0,1,0	4	2	0.927
8	1,0,0,0,1,0,1,0,1	0,0,0,0,0,0,0,1,0,0,0,0,0,1,0	4	2	0.456
9	1,0,0,0,1,0,1,0,1	0,0,0,0,0,0,0,0,0,0,0,1,0,1,0	4	2	0.247
10	0,0,0,0,0,0,1,1,1	0,0,0,0,0,0,0,0,0,0,0,1,1,1,0	3	3	0.767
11	0,0,0,0,0,0,1,1,1	0,0,0,0,0,0,0,0,0,0,0,1,1,1,0	3	3	0.715

Table 3.3 Minimize MSE using exhaustive search

Chapter 4

Antenna Re-Clustering for Multiple Radar system

The problem of optimizing localization accuracy while saving on energy use has been evaluated in [28]. The problem of selecting an antennas subset or cluster has been addressed in the literature for passive sensor networks in [29]-[31]. A geometry-based method, minimizing the posterior root mean square error derived from the Kalman filter, is developed in [31] system actively generates energy emission through its transmit antennas for the purpose of observation through its receive antennas. In the previous chapter a subset selection approach is evaluated for multiple radar systems, formulating the problem as a knapsack problem (KP). In this scheme, the number of active radars is minimized, while the system performance goal, in terms of localization accuracy, serves as a constraint.

Given a case where a multiple radar system is spread over a very large area, tracking a target may be effectively performed using an antenna cluster or subset. With the target moving within the radar antennas grid, an effective method for re-clustering a tracking antennas cluster and target handoff between clusters is needed. One way to simplify handoff is to keep a portion of the existing cluster and replace only a preset percentage of an existing cluster. In this chapter we propose four methods for re-clustering and develop fast approximation algorithms to obtain a new antenna cluster that will minimize the localization accuracy while keeping a predetermined number of antennas in the cluster unchanged. In the antennas re-clustering problem the performance level is evaluated by the localization estimation MSE. The CRB is used to evaluate the temporal performance level for a given set.

4.1 Antennas Re-Clustering Algorithms

The subset selection scheme introduced in chapter 3 focuses on antenna subset or cluster selection given a single static target. For the scenario given in Figure 3.1, it is assumed that K antennas are selected at any given time to form a cluster that will track the target position. As localization accuracy degrades with the target moving away from an existing tracking radars cluster, the cluster's antenna selection should be updated to provide a threshold accuracy performance. We refer to the re-clustering as a handoff process. The system has available estimates for unknown parameters, such as the target RCS from previous cycles, and has the same system model characteristics that proposed in chapter 3. for a given cluster size K , as the target moves within a grid of antennas, one looks for an optimal set of transmit and receive antennas that will follow these requirements:

1. Cluster includes K antennas, at least one of them is a receive antenna and one is a transmit antenna.
2. No more than δ antennas should be replaced in re-clustering.
3. Given the two above constraint, re-cluster the antennas such that the localization MSE is minimized.

Next, we propose re-clustering algorithms and discuss how close suboptimal solution is to the optimal one. A moving target's track is assumed to go through L positions, with the positions denoted by $(x_\ell, y_\ell), \ell = 1, \dots, L$.

4.1.1 Case 1

4.1.1.1. A1 At position 1, we choose the optimal set with size K^ℓ , using a heuristic algorithm or exhaustive search, since in our scenario both algorithms give identical

- 1 Using a heuristic algorithm at position (x_ℓ, y_ℓ) , $\ell = 1$,
select the optimal vector $q_{t_x}^*$ and $q_{r_x}^* \in S_{opt}$
- 2 set $S_{sub-set} = S_{opt}$
- 3 for $\ell = 2 : L$
- 4 set target position (x_ℓ, y_ℓ) , $S_{new} = \emptyset$, $CRB_{\ell} = \emptyset$
- 5 Select d_{max} s.t, $d_{max} = \arg \max_{q_{tx}, q_{rx} \in S_{sub-set}} \{\|x_\ell - x_{i_{Tx}}\|_2, \|x_\ell - x_{j_{Rx}}\|_2, \dots\}$
s.t $x_{i_{Tx}}, x_{j_{Rx}} \in S_{sub-set}$.
- 6 Set $S'_{sub-set} = S_{sub-set} \setminus x_{d_{max}}$, $x_{d_{max}}$ is $x_{i_{Tx}}$ or $x_{j_{Rx}}$ give d_{max} in step 5
- 7 Select $S_{new} = S_{new} \cup x_{i_{Tx}}$ s.t $\|x_\ell - x_{i_{Tx}}\|_2 < d_{max}$
 $S_{new} = S_{new} \cup x_{j_{Rx}}$ s.t $\|x_\ell - x_{j_{Rx}}\|_2 < d_{max}$
- 8 Calculate d_{min} s.t $d_{min} = \arg \min_{q_{tx}, q_{rx} \in S_{new}} \{\|x_\ell - x_i\|_2, \|x_\ell - x_j\|_2, \dots\}$, s.t
 $x_{i_{Tx}}, x_{j_{Rx}} \in S_{new}$.
- 9 Set $S'_{sub-set} = S'_{sub-set} \cup x_{d_{min}}$, s.t $x_{d_{min}}$ give d_{min} in step 6
- 10 Find $CRB_{position_\ell} = tr(C_{S'_{sub-set}})$
- 11 end

Table 4.1 Case 1.A1 Add one antenna that has a minimum distance

results when the target in position 1, see table 3.2, 3.3. This set is denoted as $S_{sub-set}$. As target moves to new position $\ell = 2, \dots, L$ at that position select the transmitter or receiver that maximize the distance to the target position $d_{max}^{\ell-1} = \arg \max_{q_{tx}, q_{rx} \in S_{sub-set}} \{\|x_{\ell-1} - x_{i_{Tx}}\|_2, \|x_{\ell-1} - x_{j_{Rx}}\|_2\}$. This draw a circle with radius $d_{max}^{\ell-1}$ around the new target x_ℓ position. The antenna that gives the maximum distance $d_{max}^{\ell-1}$ is subtracted from the sub-set $S'_{sub-set} = S_{sub-set} \setminus x_{m_{Tx}}$ or $S'_{sub-set} = S_{sub-set} \setminus x_{n_{Rx}}$.

Compose a new temporary set of transmitters and receivers that have a distance to the target less than to d_{max} and are not included in $S_{sub-set}$, such that $d_{i_{Tx}} = \|x_\ell - x_{i_{Tx}}\|_2 < d_{max}$ and $d_{j_{Rx}} = \|x_\ell - x_{j_{Rx}}\|_2 < d_{max}$, the location in the vector \mathbf{q}_{tx} and \mathbf{q}_{rx} corresponding to these new sensors are set to one and added to new set of antennas $S_{new}^\ell = \{x_{i_{Tx}}, x_{j_{Rx}}\}$. From S_{new}^ℓ set, select the transmitter or receiver that has the minimum distance to the target and add that antenna to $S_{sub-set}$. Then calculate the CRB for the new position. The proposed search method is illustrated in Table 4.1. Note that in Case A1 one antenna is replaced due to distance from the target and the newly added antenna minimum distance to the target as long as $\mathbf{q}_{tx} \geq 1$ and $\mathbf{q}_{rx} \geq 1$.

4.1.1.2. A2 In case A2, we follow the same steps as in case A1. In step (5), select two antennas that give (d_{max1}, d_{max2}) distance. Subtract them from S_{new} set, select two antennas that have minimum distance to the target and add these minimum antennas to $S_{sub-set}$. This is summarized in Table 4.2. The difference between A1 and A2 is in number of antennas that we replace in each cluster, one for A1 and two for A2.

4.1.1.3. B1 Case B1 is similar to case A1 except we add a new constrain $\mathbf{q}_{tx} \geq 2$ and $\mathbf{q}_{rx} \geq 2$. This constrain restrict the system to have at least two transmitters and two receivers, since we may have SIMO in case A1 and A2.

4.1.1.4. B2 This case is similar to A2 with additional constrain $\mathbf{q}_{tx} \geq 2$ and $\mathbf{q}_{rx} \geq 2$ as in case B1.

4.1.2 Case 2

4.1.2 C1 In this case we follow the same steps as in case1 A1 in choosing d_{max} , subtracting it from subset, and composing $S_{new}^\ell = \{x_{i_{Tx}}, x_{j_{Rx}}\}$. We search in new set

- 1 Using a heuristic algorithm at position (x_ℓ, y_ℓ) , $\ell = 1$,
select the optimal vector q_{tx}^* and $q_{rx}^* \in S_{opt}$
- 2 set $S_{sub-set} = S_{opt}$
- 3 for $\ell = 2 : L$
- 4 set target position (x_ℓ, y_ℓ) , $S_{new} = \emptyset$, $CRB_{\ell} = \emptyset$
- 5 Select $d_{\max 1}, d_{\max 2}$ s.t

$$d_{\max} = \arg \max_{q_{tx}, q_{rx} \in S_{sub-set}} \{\|x_\ell - x_{i_{Tx}}\|_2, \|x_\ell - x_{j_{Rx}}\|_2, \dots\}$$

$$x_{i_{Tx}}, x_{j_{Rx}} \in S_{sub-set}.$$
- 6 Set $S'_{sub-set} = S_{sub-set} \setminus x_{d_{\max 1}}$, $S'_{sub-set} = S_{sub-set} \setminus x_{d_{\max 2}}$
 $x_{d_{\max 1}}, x_{d_{\max 2}}$ give d_{\max} in step 5
- 7 Select $S_{new} = S_{new} \cup x_{i_{Tx}}$ s.t $\|x_\ell - x_{i_{Tx}}\|_2 < d_{\max 1}$
 $S_{new} = S_{new} \cup x_{j_{Rx}}$ s.t $\|x_\ell - x_{j_{Rx}}\|_2 < d_{\max 1}$
- 8 Calculate $d_{\min 1}, d_{\min 2}$ s.t

$$d_{\min} = \arg \min_{q_{tx}, q_{rx} \in S_{new}} \{\|x_\ell - x_i\|_2, \|x_\ell - x_j\|_2, \dots\},$$

$$x_{i_{Tx}}, x_{j_{Rx}} \in S_{new}.$$
- 9 Set $S'_{sub-set} = S'_{sub-set} \cup x_{d_{\min 1}} \cup x_{d_{\min 2}}$
s.t $x_{d_{\min 1}}, x_{d_{\min 2}}$ give d_{\min} in step 8
- 10 Find $CRB_{position_\ell} = tr(C_{S'_{sub-set}})$
- 11 end

Table 4.2 Case 1 A2 Add two antennas that has minimum distance

S_{new}^ℓ for the transmitter or receiver that gives minimum CRB and add it to active subset

$S_{sub-set}$. This case is summarized in table 4.3.

4.1.2.2 C2 This case is different to case C1 by subtracting two antennas that give

- 1 Using a heuristic algorithm at position (x_ℓ, y_ℓ) , $\ell = 1$
select the optimal vector q_{Tx}^* and $q_{Rx}^* \in S_{opt}$;
- 2 set $S_{sub-set} = S_{opt}$;
- 3 for $\ell = 2 : L$
- 4 set target position (x_ℓ, y_ℓ) , $S_{new} = \emptyset$, $CRB_{\ell} = \emptyset$;
- 5 Select d_{max} s.t, $d_{max} = \arg \max_{q_{Tx}, q_{Rx} \in S_{sub-set}} \{\|x_\ell - x_{i_{Tx}}\|_2, \|x_\ell - x_{j_{Rx}}\|_2, \dots\}$
s.t $x_{i_{Tx}}, x_{j_{Rx}} \in S_{sub-set}$;
- 6 Set $S'_{sub-set} = S_{sub-set} \setminus x_{d_{max}}, x_{d_{max}}$ is $x_{i_{Tx}}$ or $x_{j_{Rx}}$ give d_{max} in step 5;
- 7 Select $S_{new} = S_{new} \cup x_{i_{Tx}}$ s.t $\|x_\ell - x_{i_{Tx}}\|_2 < d_{max}$;
 $S_{new} = S_{new} \cup x_{j_{Rx}}$ s.t $\|x_\ell - x_{j_{Rx}}\|_2 < d_{max}$;
- 8 $S_{sub-set}^k = \emptyset$, $CRB_{temp} = \emptyset$;
for $k = 1 : S_{new}$
 $S_{sub-set}^k = S'_{sub-set} \cup x_{\ell_K}$
- 9 $CRB_k = \text{tr}(C_{S_{sub-set}^k})$
 $CRB_{temp} = CRB_{temp} \cup CRB_k$;
end for
- 10 $CRB_{position_\ell} = \arg \min \{CRB_{temp}\}$;
- 11 end

Table 4.3 Case 2 C1 Add one antenna that gives minimum CRB

(d_{max1}, d_{max2}) distance to the target, search in S_{new}^ℓ set to find two antennas that give the minimum CRB and add it to $S_{sub-set}$.

4.1.3 Case 3

In case 3, we choose $d_{max}^{\ell-1}$ as in Case1 A1, compose S_{new}^ℓ , then add the new set to the active subset $S_{sub-set}^\ell = S_{sub-set}^\ell + S_{new}^\ell$. Select the optimal K^ℓ set that minimizes the CRB. Table 4.3 describes this method. This case gives the better approximation to the

- 1 Using a heuristic algorithm at position (x_ℓ, y_ℓ) , $\ell = 1$
select the optimal vector $q_{t_x}^*$ and $q_{r_x}^* \in S_{opt}$;
- 2 set $S_{sub-set} = S_{opt}$;
- 3 for $\ell = 2 : L$
- 4 set target position (x_ℓ, y_ℓ) , $S_{new} = \emptyset, CRB_{\ell} = \emptyset$;
- 5 Select d_{max} s.t, $d_{max} = \arg \max_{q_{t_x}, q_{r_x} \in S_{sub-set}} \{\|x_\ell - x_{i_{Tx}}\|_2, \|x_\ell - x_{j_{Rx}}\|_2, \dots\}$
s.t $x_{i_{Tx}}, x_{j_{Rx}} \in S_{sub-set}$;
- 6 Set $S'_{sub-set} = S_{sub-set} \setminus x_{d_{max}}$, $x_{d_{max}}$ is $x_{i_{Tx}}$ or $x_{j_{Rx}}$ give d_{max} in step 5;
- 7 Select $S_{new} = S_{new} \cup x_{i_{Tx}}$ s.t $\|x_\ell - x_{i_{Tx}}\|_2 < d_{max}$;
 $S_{new} = S_{new} \cup x_{j_{Rx}}$ s.t $\|x_\ell - x_{j_{Rx}}\|_2 < d_{max}$;
- 8 $S'_{sub-set} = S'_{sub-set} \cup S_{new}$;
- 9 $n = \text{size}(S'_{sub-set})$;
for $m = 1 : n$ choose k ;
 $S^k_{sub-set} = S'_{sub-set} \cup x_{\ell_K}$
- 9 $CRB_m = \text{tr}(C_{S^m_{sub-set}})$;
 $CRB_{temp} = CRB_{temp} \cup CRB_m$;
end for
- 10 $CRB_{position_\ell} = \arg \min \{CRB_{temp}\}$;
- 11 end

Table 4.4 Case 3 Select K sensors that give minimum CRB

optimal solution. Table 4.5 gives a summary of all cases and how it is different from each other.

4.2 Numerical Analysis

MIMO radar system ($M = 9$ and $N = 14$) is chosen for this analysis, the same scenario that explained in chapter 3. We suppose the target moves through ($L = 9$) positions.

Case 1 d_{\min}		Case 2 , CRB	Case 3 min CRB
A $T_x, R_x \geq 1$	B $T_x, R_x \geq 2$	C $T_x, R_x \geq 1$ C $T_x, R_x \geq 2$	D $T_x, R_x \geq 1$
A1 switch 1 B1 switch 1 A2 switch 2 B2 switch 2		C1 switch 1 C2 switch 2	Choose sub optimal

Table 4.5 Summarize all cases

We calculate the MSE using all proposed cases. Table 4.6 summarizes the result and gives a comparison with the optimal one. We notice that MSE varies from case1 through case 3 since we developed the performance through these cases depends on computational saving. Case D gives the best approximation to optimal solution. We also observe the gap between MSE of proposed algorithm and optimal one grows up as the target moves away from the first starting position, since the cluster inherits a small error from the previous position and this error is given to next position and so on. We can replace the current cluster with optimal one if MSE a specific threshold. Saving in computational complexity is offered, case 1 and case 2 of $O \{(M+N)-K\}$, and case 3 of $O(2^{\frac{(M+N)-K}{\pi}})$.

4.3 Conclusions

In this paper the idea of target handoff among antenna clusters in a multiple radars systems has been introduced and addressed. Given the large number of available antennas, a simplified algorithm for selecting candidate antennas have been developed performing very close to optimum without the need to consider all antennas. A constraint on antenna replacement enables a smoother handoff process with the best current antennas kept in in the cluster. A subset from the cluster is replaced by antennas from a

location based antenna pool, based on the target estimated trajectory. CRB is used to select the antennas that minimize the localization MSE. Best antennas have a unique relationship with the target in term of distance and angular position. Insight into the best antenna choices enables further improvement of the heuristic algorithms. Comparison between the various methods demonstrates the tradeoff. Case 3 is shown to perform a very close to optimal without need to consider all antennas. Other methods offer very low complexity which results with higher MSE.

Position	MSE A1	MSE A2	MSE B1	MSE B2	MSE C1	MSE C2	MSE D	MSE optimal
1	6.605	6.605	6.605	6.605	6.605	6.605	6.605	6.605
2	1.259	1.724	1.259	1.259	1.260	1.723	1.059	0.511
3	3.266	3.266	3.772	3.772	3.266	3.266	2.466	2.085
4	4.231	3.081	6.809	5.241	4.273	3.9227	2.901	2.609
5	2.986	1.344	2.328	1.212	1.601	1.344	1.100	0.849
6	3.189	2.245	1.131	1.083	2.233	1.067	0.983	0.398
7	5.396	6.170	3.073	5.948	4.577	2.657	1.406	0.927
8	6.793	5.122	4.229	1.422	4.131	1.261	0.792	0.456
9	6.993	0.560	3.510	0.495	2.361	0.560	0.495	0.247
10	5.6303	2.551	0.804	0.767	2.110	0.804	0.767	0.767
11	3.217	2.239	4.273	2.317	3.209	2.317	0.942	0.715

Table 4.6 MSE comparison of all cases

4.4 Future Works

Enabling efficient handoff of targets in densely populated MIMO radar systems is a key to efficiently using the available resources. This operation scheme will enable tracking multiple targets in an efficient manner. The current research is based on the CRB metric. Future research that considers the Bayesian Cramer Rao bound (BCRB) will provide a tighter performance estimate for a moving target. Additionally, understating the

impact of the number of replaced radars on tracking performance with respect to the BCRB is of interest.

Appendix A

The FIM for the vector $\mathbf{u} = [x, y, \mathbf{h}]$ is derived in this Appendix. It is easier to compute FIM with respect to another vector $\gamma = [\tau, h]$, then apply the chain rule to express $\mathbf{J}(\mathbf{u})$ in alternative form [32] as:

$$\mathbf{J}(\mathbf{u}) = \mathbf{Q}\mathbf{J}(\gamma)\mathbf{Q}^T \quad (1)$$

where matrix \mathbf{Q} is the Jacobian

$$\begin{aligned} \mathbf{Q} &= \frac{\partial \gamma}{\partial \mathbf{u}} \\ &= \begin{bmatrix} \frac{\partial}{\partial x} \boldsymbol{\tau}^T & \frac{\partial}{\partial x} h^T \\ \frac{\partial}{\partial y} \boldsymbol{\tau}^T & \frac{\partial}{\partial y} h^T \\ \frac{\partial}{\partial h} \boldsymbol{\tau} & \frac{\partial}{\partial h} h \end{bmatrix} \\ &= \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \end{aligned} \quad (2)$$

where

$$\begin{aligned} G_{11}[i, j] &= \frac{\partial^2 [\ln f(r|\gamma)]}{\partial \tau_{m,n} \partial \tau_{z,q}}, \\ G_{12}[i, j] &= G_{21}[j, i] = \frac{\partial^2 [\ln f(r|\gamma)]}{\partial \tau_{m,n} \partial h_{z,q}}, \\ G_{22}[i, j] &= \frac{\partial^2 [\ln f(r|\gamma)]}{\partial h_{m,n} \partial h_{z,q}}. \\ i &= (n-1) + m, j = (q-1) + z. \end{aligned}$$

$$\mathbf{J}(\gamma) = \alpha_{m,n} p_{m,n} \begin{bmatrix} 8\pi^2 \beta_m^2 |h_{m,n}|^2 & 0 \\ 0 & \mathbf{I} \end{bmatrix}, \quad (3)$$

and

$$\mathbf{Q} = \begin{bmatrix} \mathbf{H}_{M \times N} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \quad (4)$$

where $\mathbf{0}$ is zero matrix and \mathbf{I} is identity matrix. The matrix \mathbf{H} incorporates the derivative of time delay with respect to x and y . The elements of \mathbf{H} are given as:

$$\mathbf{H} = \begin{bmatrix} \frac{x_{1_{Tx}}}{R_{1_{Tx}}} + \frac{x_{1_{Rx}}}{R_{1_{Rx}}} & \dots & \frac{x_{M_{Tx}}}{R_{M_{Tx}}} + \frac{x_{N_{Rx}}}{R_{N_{Rx}}} & 0 \\ \frac{y_{1_{Tx}}}{R_{1_{Tx}}} + \frac{y_{1_{Rx}}}{R_{1_{Rx}}} & \dots & \frac{y_{M_{Tx}}}{R_{M_{Tx}}} + \frac{y_{N_{Rx}}}{R_{N_{Rx}}} & 0 \\ 0 & \dots & 0 & \mathbf{I} \end{bmatrix} \quad (5)$$

Appendix B

Figures show the position of target and handoff of choosing sensors when target moves from position ℓ to $\ell+1$. (position 3,4,5,6,7,8,9, and 10 are chosen)

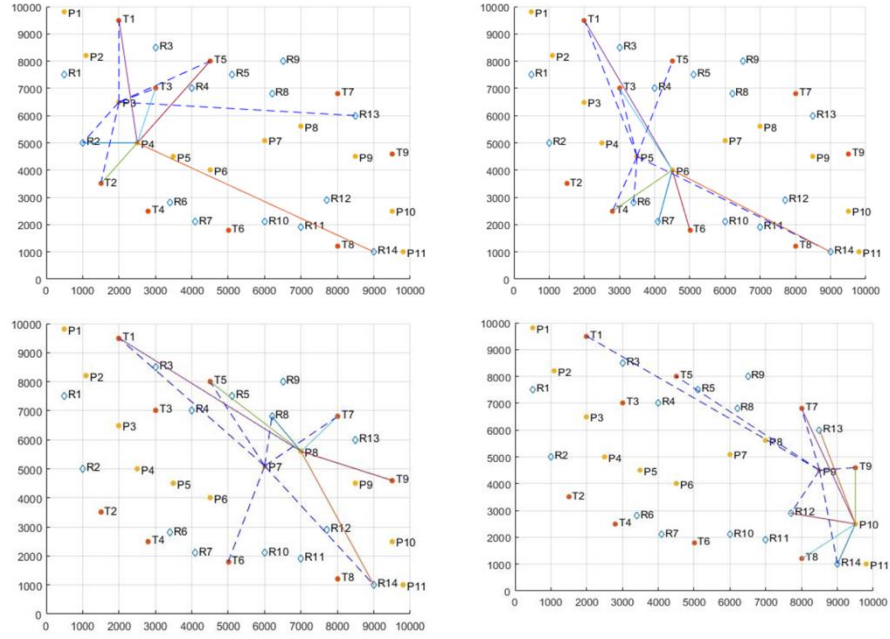


Figure.1 Optimal choosing sensors

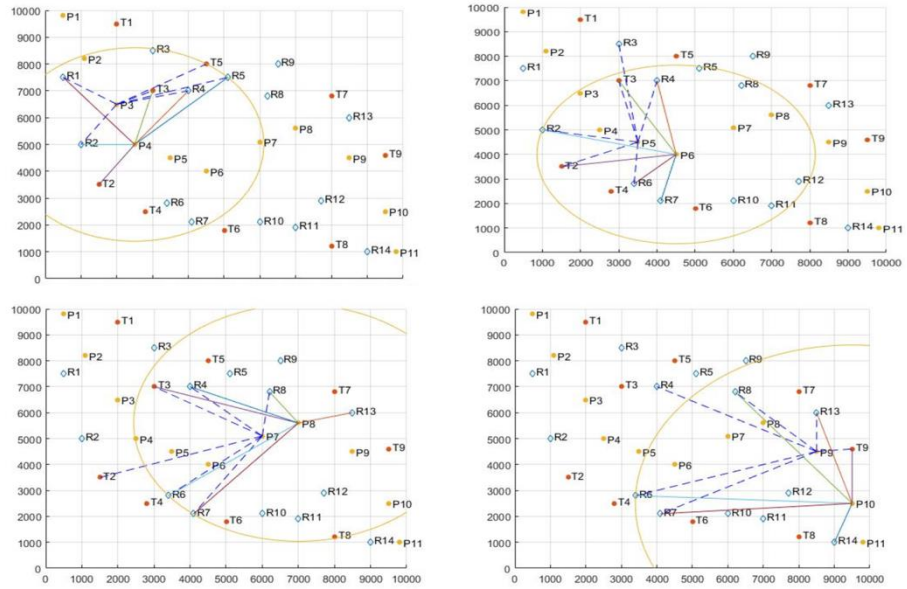


Figure.2 Case 1 A1

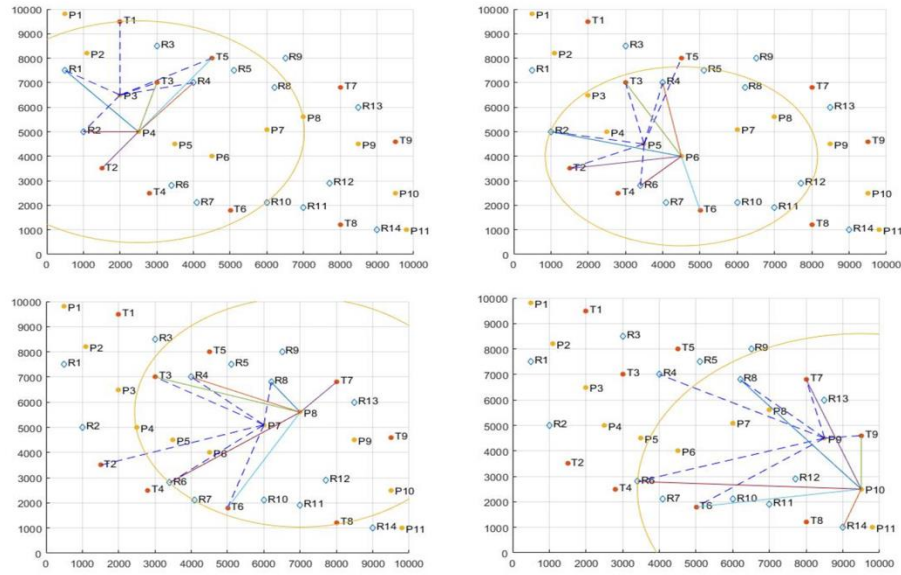


Figure.3 Case 1 B1

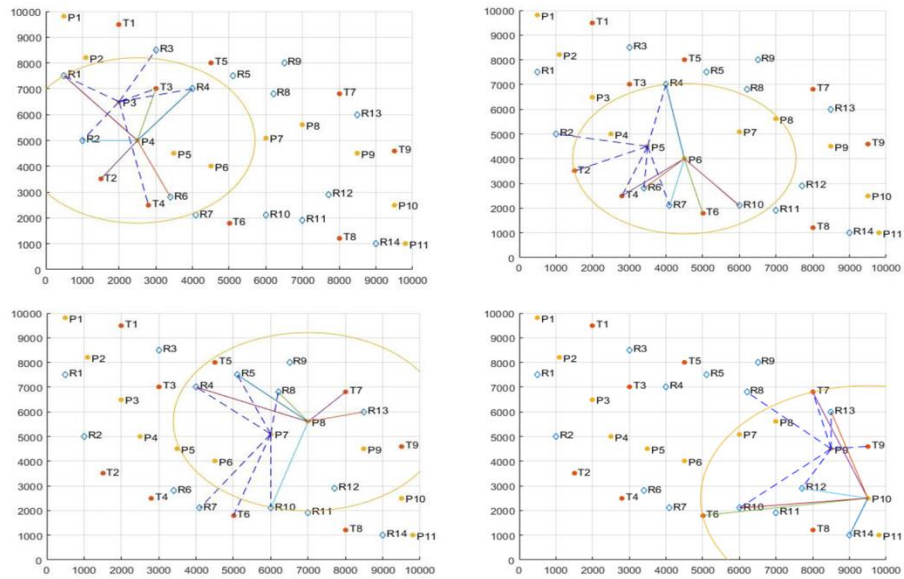


Figure.4 Case 1 A 2

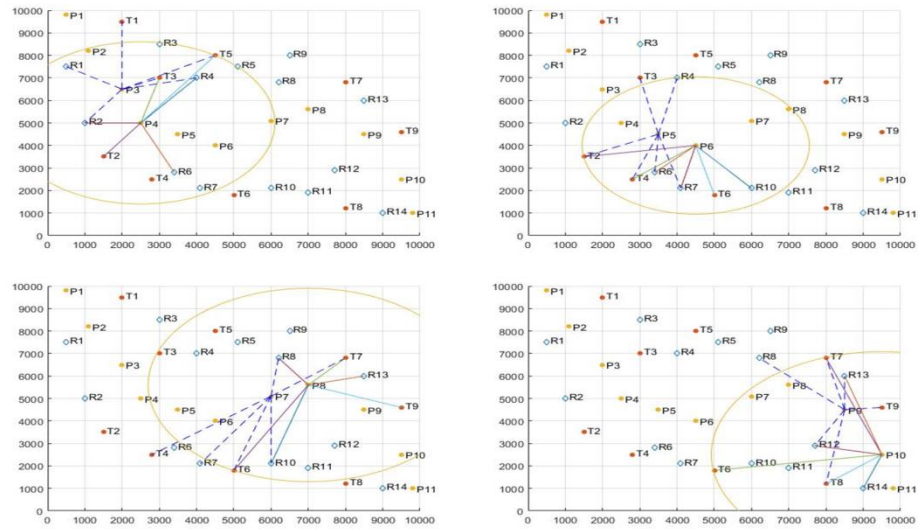


Figure. 5 Case 1 B 2

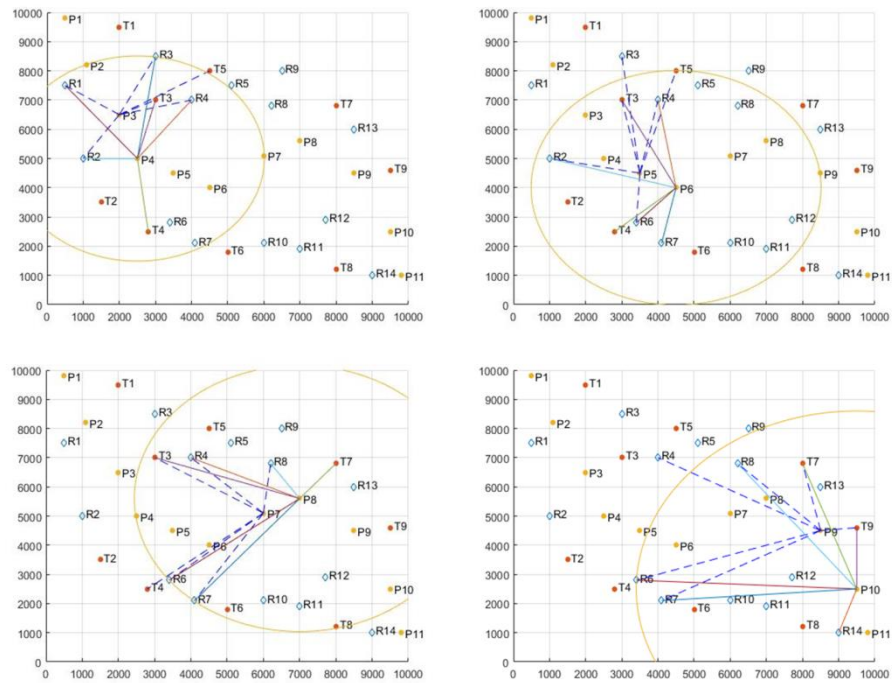


Figure. 6 Case 2 C 1

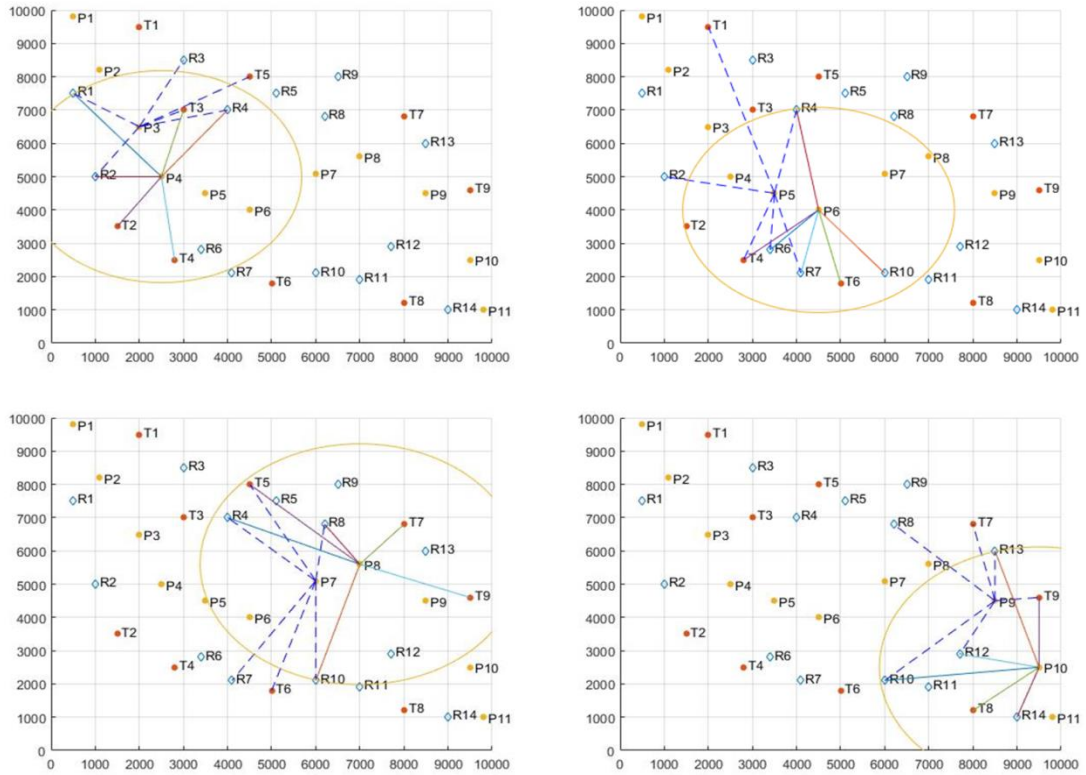


Figure. 7 Case 2 c 2

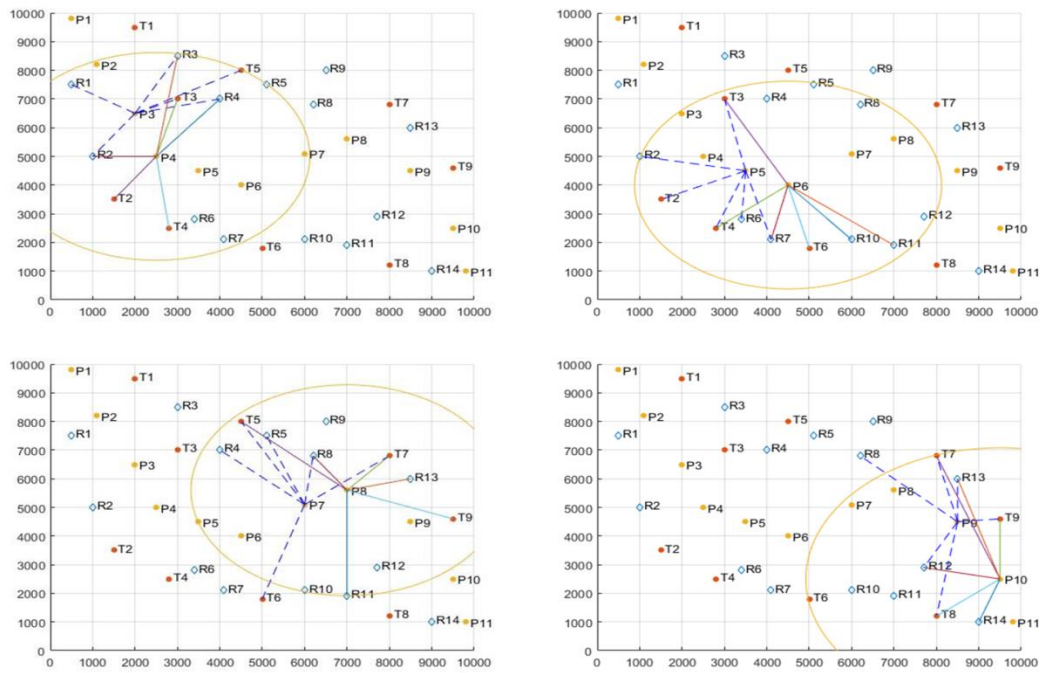


Figure. 8 Case D

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