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ESSAYS ON SOVEREIGN DEBT IN THE EURO AREA

By

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ABSTRACT OF THE DISSERTATION

Essays on sovereign debt in the euro area

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This dissertation explores the interaction between sovereign debt and investor preferences in the euro area during the recent crisis from both a theoretical and an empirical perspective. From an empirical perspective, in Chapter 2 (written with Marco Pagano and Saverio Simonelli), we investigate the relationship between the divergence of sovereign yields and CDS premia and the rise in banks' home bias, as well as its rationale. Our approach is based on (i) the decomposition of yield differentials and CDS spreads in a country-specific and a common risk component via a dynamic factor model and (ii) the estimation of a vector error-correction model on 2008-12 monthly data. We find that (i) in euro area periphery countries, banks increase their domestic exposure in response to increases in country risk and (ii) in most euro area countries, banks respond to an increase in the common risk factor by raising their domestic exposures. Finding (i) suggests distorted incentives in periphery banks' response to changes in their own sovereign's risk. Finding (ii) indicates that, when systemic risk increases, all banks tend to increase the home bias of their portfolios, making the euro area sovereign market more segmented. Policy implications are finally drawn from these findings.

From a theoretical perspective, Chapters 3 and 4 analyze the interactions between macroeconomic fundamentals and debtor and creditor incentives through the lenses of dynamic general equilibrium models with strategic sovereign default. In Chapter 3, I provide a theoretical framework to understand three phenomena occurred at the onset of the recent

sovereign debt crisis: (1) the increase in investor risk aversion, (2) the reversal in the process of union-wide financial integration and (3) the rise in the perceived imperfect substitutability of government bonds. Advancing a novel approach to modelling optimal portfolio strategy, the model assumes investors to exhibit preferences with a constant elasticity of substitution between bonds, which is inversely related to the degree of risk aversion, perceived financial segmentation and imperfect asset substitutability. Consistently with empirical evidence, a low elasticity, representing adverse market sentiments, implies a high sensitivity of yields to macroeconomic fundamentals. In an empirical assessment, the model captures several features of Greek sovereign yields, debt and default before and during the crisis.

Finally, Chapter 4 proposes a comprehensive analytical framework for the assessment of fiscal sustainability in the euro area. The standard Eaton-Gersovitz model is enriched with two novel features to reflect salient features of euro area economies. First, the presence of domestically held debt implies that a sovereign default, through lower repayments on sovereign debt, determines both benefits for the public sector and costs for private domestic investors. As the sovereign seeks to maximize domestic welfare, a higher domestic share of debt increases the government's incentives to honor its obligations and, thus, expands its borrowing opportunities. Second, the introduction of credible supranational fiscal rules creates the possibility for self-fulfilling debt crises and, accordingly, increases the borrowing costs for the government. In this way, fiscal rules reduce the sovereign's optimal level of debt and foster market-based fiscal discipline. In an empirical application, the calibrated average euro area country faces a low risk of default, but it can reap both welfare and sustainability gains through fiscal consolidation.

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Contents

Abstract of the dissertation	ii
Acknowledgements	iv
List of Tables	ix
List of Figures	x
1 Introduction	1
2 Systemic risk, sovereign yields and bank exposures in the euro crisis	6
2.1 Introduction	6
2.2 Euro area sovereign yields, CDS premia and bank exposures: data description	11
2.3 Sovereign yields, country-specific risk and systemic risk	18
2.3.1 Data	21
2.3.2 Methodology	22
2.3.3 Results	23
2.4 Home bias in banks' sovereign exposures, yield differentials and systemic risk	31
2.4.1 Data and methodology	32
2.4.2 Results	35
2.5 Summary and policy implications	46
2.5.1 Dealing with “moral suasion by regulators”	47
2.5.2 Dealing with “search for yield by banks”	48

2.5.3	Dealing with the fallout of redenomination risk	50
3	Sovereign default, market sentiments and financial segmentation	52
3.1	Introduction	52
3.2	Related literature	56
3.3	The model	58
3.3.1	International investors	61
3.3.2	The domestic economy	63
3.3.3	Default sets, default incentives and recursive equilibrium	65
3.3.4	The bond price schedule	68
3.3.5	Sustainable debt and price	71
3.4	Quantitative assessment of the model	73
3.4.1	Data	73
3.4.2	Calibration and functional forms	75
3.4.3	Numerical analysis	77
3.4.4	Empirical analysis	78
3.5	Conclusions	81
4	Debt limits and sovereign default in the euro area	83
4.1	Introduction	83
4.2	Related literature	89
4.3	The model	92
4.3.1	Households	93
4.3.2	Firms	94
4.3.3	The government	95
4.3.4	Lenders	101
4.3.5	General equilibrium and debt limit	106
4.3.6	Default incentives with domestic sovereign debt	109
4.4	Fiscal rules and credibility	111

4.5	Quantitative assessment	119
4.5.1	Calibration	120
4.5.2	Model mechanics	123
4.5.3	Macroeconomic stabilization and debt sustainability	131
4.6	Conclusions	134
Bibliography		135
Appendices		153
A	Appendix to Chapter 2	154
A.1	Preliminary data analysis	154
B	Appendix to Chapter 3	158
B.1	Theoretical framework	158
B.2	Proofs of theorems	171
B.3	Computational algorithm	173
C	Appendix to Chapter 4	175
C.1	Proof of theorem	175

List of Tables

2.1	Dynamic factor model estimation: variance decomposition	26
2.2	VECM estimates for the response of banks' domestic sovereign exposures to yield differentials and their components	36
3.1	Greece Business Cycle Statistics, 2002.1-2012.1	75
3.2	Specification of Model Parameters	76
3.3	Model Simulations	80
4.1	Calibration of model parameters in average euro area country	120
4.2	Debt limit and fiscal space at different horizons and risk levels	133
A.1	ADF tests (H_0 : Unit root): p -values	155
A.2	Johansen's trace test ($H_0 : r^* \leq r; H_1 : r^* = n$): p -values	155
A.3	VECM specification: deterministic terms and lag order	156

List of Figures

2.1	Eurozone 5-year benchmark government bond yields (monthly, percent) . .	12
2.2	Eurozone 5-year government CDS premia (monthly, basis points)	13
2.3	Sovereign yield differentials and CDS premia, by country	14
2.4	Domestic sovereign debt holdings of periphery versus core-country banks as proportion of the total assets of banks	15
2.5	Domestic sovereign debt holdings of periphery versus core-country banks . .	17
2.6	Non-domestic eurozone sovereign debt holdings of periphery versus core- country banks	18
2.7	Common factor of yield differentials and CDS premia (left axis) and Google trend indicator of eurozone break-up risk (right axis)	24
2.8	Common factor of yield differentials and CDS premia (left axis) and Intrade- based probability of euro break-up (right axis)	25
2.9	Common and country components of the German yield differential and CDS premium (first differences)	28
2.10	Common and country components of the Italian yield differential and CDS premium (first differences)	29
2.11	Common and country components of the Spanish yield differential and CDS premium (first differences)	30
2.12	IRFs of sovereign exposures to shocks in yield differentials: periphery countries	38
2.13	IRFs of sovereign exposures to shocks in yield differentials: core countries .	39
2.14	IRFs of yield differentials to shocks in sovereign exposures: periphery countries	41

2.15	IRFs of yield differentials to shocks in sovereign exposures: core countries .	42
2.16	IRFs of sovereign exposures to shocks in the common and country components of yield differentials: periphery countries	44
2.17	IRFs of sovereign exposures to shocks in the common and country components of yield differentials: core countries	45
3.1	Patterns in the euro area sovereign debt markets	53
3.2	Timing of events in country i at period t	59
3.3	Maximum sustainable interest rate and debt-to-output ratio	72
3.4	Model bond price, government debt and probability of default	78
3.5	Model bond price and government debt dynamics	79
3.6	Greece and model time series	81
4.1	Public finances and real economy in the euro area between 2000 and 2015 .	85
4.2	Debt ratio and domestic share of debt	86
4.3	Estimates of debt and productivity elasticities of tax rates	98
4.4	Occurrence of repayment, compliance, deviation and default in the state space	118
4.5	Price and spread on secondary markets and spread between primary market bids	124
4.6	Default probabilities at different time horizons	125
4.7	Sensitivity analysis of the probability of default	127
4.8	Sensitivity analysis of the probability of default (continued)	128
A.1	Sovereign debt portfolio diversification in Italian banks before the crisis . .	169
A.2	Sovereign debt portfolio diversification in Italian banks during the crisis . .	170

Chapter 1

Introduction

The recent sovereign debt crisis confronted the European Economic and Monetary Union (EMU), both at the supranational and the national level, with unprecedented challenges. At the turn of the decade, large contractions in output, consumption and employment throughout the euro area were associated with deteriorating fiscal fundamentals, soaring sovereign debt levels and increasing volatility and widening cross-country differentials of government bond yields across the board. At the same time, the receding economic activity was coupled with, and amplified by, a pervasive turbulence in financial markets: interbank markets collapsed and were replaced by the European Central Bank (ECB) as the main source of liquidity; bank interest rates systematically diverged across countries; portfolios of financial intermediaries became increasingly biased towards domestic – especially sovereign – securities, thus creating a vicious loop between weak governments and weak banks. As a result, extensive empirical evidence identified three concomitant events in the wake of the crisis: (1) international investors shifted their pricing paradigm and exhibited a rising risk aversion; (2) the process of financial integration in euro area sovereign debt markets incurred a dramatic reversal; (3) the perceived imperfect substitutability between bonds issued by different euro area governments sharply increased. These events raised concerns about the possibility for euro area sovereigns to roll over their debts and avoid default. Sovereign solvency in a monetary union, even one of advanced economies, was then at the

centre of the debate in academic and policy circles alike. Sovereign default, a phenomenon previously considered as mostly confined to emerging countries, became in fact a concrete risk in one of the most developed economic regions, as well. Finally, when hefty haircuts on Greek government debt were imposed on investors in early 2012, sovereign default in developed economies left the domain of speculation to become a real event.

Departing from these observations, this dissertation investigates the relationship among sovereign debt, default incentives and investor sentiments during the recent euro area sovereign debt crisis along three main dimensions. In Chapter 2, entitled “**Systemic risk, sovereign yields and bank exposures in the euro crisis**”, together with my co-authors, I conduct an empirical analysis on euro area government bond yield dynamics and their interaction with bank sovereign portfolio strategies from both a positive and a normative perspective. Our methodology, based on a dynamic factor model and a vector error-correction model, allows us to discriminate to some extent between three different reasons why banks may change their domestic sovereign exposures in response to a widening yield differential. First, according to the “moral suasion” hypothesis, high-risk sovereign issuers may exert pressures on the banks in their jurisdiction to increase their domestic sovereign holdings in order to support demand for sovereign debt when demand is low and therefore yields are comparatively high. Second, the “carry trades” hypothesis posits that undercapitalized banks may bet for resurrection by shifting the composition of their sovereign portfolios towards high-risk, high-yield sovereign debt, away from low-yield debt. Third, the “comparative advantage” hypothesis entails that, in the event of a collapse of the euro, domestic banks can hedge better than foreign ones against the redenomination risk of domestic sovereign debt, as their liabilities (e.g., their deposits) and assets (e.g., their domestic sovereign debt holdings) would be simultaneously redenominated into new national currencies. Although all three hypotheses share a common prediction, that is the positive correlation between domestic banks’ home bias and domestic sovereign yield differentials, the first two hypotheses predict that this correlation should arise irrespective of whether changes in yields are generated by country-level or common risk, which we find in periphery-

country banks. In contrast, the third predicts that this correlation should arise *only* from changes in common risk, e.g. the risk of collapse of the euro, which we find especially in core-country banks. Our results have several implications for policy makers. The increase of periphery banks' home bias cannot be explained entirely as a response to greater systemic euro area risk and it must have been also induced to some extent by national regulators' moral suasion or by banks' opportunistic carry trades. If due to moral suasion by national regulators, periphery banks' behavior indicates that these regulators tended to induce risk-taking by banks in a context where government solvency was at danger, thus enhancing the vicious loop between fiscal solvency and bank solvency deterioration. If due to opportunistic carry trades by banks, it raises concerns about the appropriateness of macro-prudential regulation.

In Chapter 3, entitled **“Sovereign default, market sentiments and financial segmentation”**, I lay out a theoretical framework aimed at understanding the empirical regularities described in Chapter 2, thus contributing to the literature extant in three ways. First, this study contributes to the empirical literature on euro area sovereign debt markets by proposing a model able to explain three typical patterns identified in euro area sovereign debt markets, namely the increase in investor risk aversion, the reversal in the process of financial integration and the sharp increase in the perceived imperfect substitutability between euro area government bonds observed in the wake of the recent crisis. Second, the theoretical analysis conducted in this chapter bridges a gap of general equilibrium models of unsecured sovereign debt and default by incorporating market sentiments – notably, in the form of investor risk aversion, perceived financial segmentation, and imperfect asset substitutability – within a multi-country model *à la* [Eaton and Gersovitz \(1981\)](#). Third, the model posits a simple portfolio selection strategy alternative to the mean-variance theory commonly used in the literature on open economy financial macroeconomics. The main point of departure of this model concerns the assumption on investors' optimal portfolio strategy, whereby international investors perceive different sovereign debt securities as imperfect substitutes and compose their sovereign debt portfolios by maximizing a utility

function exhibiting constant elasticity of substitution (CES) between bonds. Crucially, the elasticity of substitution between bonds may be interpreted as (negatively) related to the degree of investor risk aversion, perceived financial segmentation, and imperfect asset substitutability at the same time. The implications of this modelling choice on equilibrium dynamics are explored analytically, numerically and empirically. As described in the analytical solution of the model, the bond demand function establishes a direct link between equilibrium quantities and prices for domestic sovereign debt, whereby a low price elasticity of bonds entails large movements of interest rates in response to small variations in the amount of optimal debt chosen by the government to insure consumption against volatility in the stochastic endowment process of the domestic economy. In other words, adverse market sentiments amplify the impact of macroeconomic fluctuations on government bond yields. The numerical evaluation of the model confirms this result. Further, in an empirical assessment, the model is evaluated against Greek empirical dynamics in two sub-periods, namely the pre-crisis and the crisis periods. Different values in the elasticity of substitution explain the large discrepancy in the standard deviation of government bond yields and their sensitivity to macroeconomic fluctuations before and during the crisis, hence increasing the explanatory power of standard quantitative models of strategic default.

In Chapter 4, entitled “**Debt limits and sovereign default in the euro area**”, I take a step forward and draw policy implications from the analysis of the interaction between macroeconomic fundamentals, sovereign default incentives, investor behavior and supranational fiscal governance through the lens of a DSGE model with strategic sovereign default. This study aims at providing a comprehensive framework particularly suitable for the assessment of fiscal space in advanced economies, such as euro area countries. With this goal in mind, I then propose a probabilistic, forward-looking notion of fiscal space, defined as the distance of the current debt level from its state-contingent debt limit distribution evaluated at different time horizons. Compared to standard models *à la* [Eaton and Gersovitz \(1981\)](#), the economic environment is enriched with several features, such as risk averse investors, long-duration bonds, recovery rates over defaulted debt, as well as

taxes on consumption and labor income. Moreover, the standard Eaton-Gersovitz model is endowed with two novel features aimed at reflecting important structural characteristics of euro area economies, namely domestic debt and supranational fiscal rules. As regards the first novel feature of the model, in a first stage, domestic investors are assumed to interact with foreign lenders to bargain the optimal domestic share of total debt on primary markets and post their bids in a binding auction with the government; in a second stage, trades between lenders and the government take place on perfectly competitive secondary markets. Importantly, this approach avoids the proliferation of state variables with significant benefits in terms of computational time. As regards the second novel feature of the model, the government is assumed to interact with supranational fiscal authorities. The latter require compliance with fiscal rules, but, in the absence of any enforcement power and commitment technology, the government may optimally choose to deviate but, at the same time, send lenders a negative signal about its credibility and trigger a sudden stop in capital flows. The numerical solution of the model shows the effects of different structural characteristics (as represented by specific parameter values) on the sovereign's incentives to default and the associated debt limit distributions. Most prominently, a higher domestic share of total debt entails higher default costs, due to the losses incurred by domestic investors, thus increasing the sovereign's incentives to repay and expanding its borrowing opportunities. Moreover, given the possibility of self-fulfilling roll-over crises via a credibility channel, fiscal rules introduce a market-based mechanism to reduce the sovereign's incentives to borrow, hence decreasing its optimal debt levels and fostering fiscal discipline. Finally, in an empirical assessment, the model is calibrated to the average euro area country and evaluates the optimal course for fiscal policy in light of its dual objective. Interestingly, the average euro area economy does not face the conventional trade-off between macroeconomic stabilization and debt sustainability. While the domestic government is subject to a low risk of default given the current state of the economy, it finds it optimal to reduce its borrowing level in order to maximize the overall welfare for the economy. Hence, the average euro area country can simultaneously reap welfare and sustainability gains through fiscal consolidation.

Chapter 2

Systemic risk, sovereign yields and bank exposures in the euro crisis^{1,2}

2.1 Introduction

Starting from late 2008, the euro area has experienced turmoil in financial markets: inter-bank markets have virtually frozen, and have been replaced by the European Central Bank (ECB) as the main source of liquidity for banks; sovereign debt yields of peripheral euro area countries have repeatedly spiked above those of core countries; bank interest rates have also started to differ systematically across countries; portfolios of financial intermediaries and households have become increasingly biased towards domestic securities. Hence, most of the indicators traditionally considered as gauges of financial integration have started to point towards a reversal in the process of integration that initiated before the inception of the European Monetary Union (EMU), and proceeded in the first seven years of its life.

This paper analyses both the dynamics of sovereign yields and the concomitant changes in banks' sovereign portfolios, and explores how the two are related. Our starting

¹This is a pre-copyedited, author-produced PDF of an article accepted for publication in *Economic Policy* following peer review. The version of record, Battistini, Niccolò, Marco Pagano, and Saverio Simonelli. 2014. "Systemic risk, sovereign yields and bank exposures in the euro crisis." *Economic Policy*, 29(78): 203-251, is available online at: <http://economicpolicy.oxfordjournals.org/content/29/78/203>.

²This chapter is co-authored by Marco Pagano and Saverio Simonelli.

point is that euro area sovereign yield differentials may reflect both differences in sovereign default risk and in countries' exposures to common (or systemic) risk, arising from the danger of euro area breakup and the implied currency redenomination. Especially since 2010, the budgetary crisis of Greece and its eventual default have obviously refocused investors' minds on the solvency risks of euro area countries, especially periphery ones. But at the same time media, companies, investors and academics repeatedly have voiced concerns about the possible breakup of the euro area. Between late 2010 and 2011 four issues of *The Economist* featured cover illustrations referring to the breakup of the euro.³ In November 2011 the managers of several multinational companies disclosed euro-breakup contingency plans.⁴ In January 2012, the newsletter of global institutional investor PIMCO contemplated several breakup scenarios, the mildest one being the exit by Greece, possibly followed by Portugal and Ireland, intermediate ones being the exit of all periphery or all core countries, and the extreme scenario being the abandonment of the euro by all 17 member countries.⁵ Economists were no less explicit. Between April 2010 and July 2012, Paul Krugman regularly prognosticated the collapse of the euro from his columns in *The New York Times*. At the 2012 World Economic Forum meeting in Davos, Nouriel Roubini predicted that Greece would leave the euro area in the subsequent 12 months, followed by Portugal, and assessed at 50% the chance that the euro area would break up in the subsequent three to five years.⁶ Even ECB President Mario Draghi pointed to the effect of redenomination risk on sovereign yield differentials when he stated in a speech at the Global Investment Conference in London on 26 July 2012 that "the premia that are being charged on sovereign states borrowings... have to do more and more with convertibility, with the risk of convertibility".⁷

³The issues are those of November 20 and December 4, 2010, and of July 16 and September 17, 2011.

⁴"Businesses plan for possible end of euro," *The Financial Times*, November 29, 2011.

⁵"Thinking about the Implications of Rising Euro-exit Risks", *European Perspectives*, Pimco, January 2012.

⁶"Eurozone will collapse this year, says Nouriel Roubini", *The Daily Telegraph*, January 28, 2012.

⁷Kenneth Rogoff sums it all up very effectively: "From early 2010 until quite recently, there was every reason to worry about a disorderly exit from the Eurozone potentially blowing up the whole thing. This was the big call – the one that everyone was focusing on." ("Britain should not take its credit status for granted", *The Financial Times*, 3 October 2013, p. 9).

Hence, in this paper we proceed in two steps. The first is to decompose sovereign yield differentials relative to the euro area swap rate in a country-specific component due to sovereign default risk and a common component arising from redenomination risk. To this purpose, we estimate a dynamic two-factor model for euro area sovereign debt. We validate the interpretation of the common factor as arising from the risk of euro collapse by correlating it with indicators of investors' expectations of the euro breakup based on Google searches and on prediction markets.

Our second step is to explore how these two estimated components of yield differentials contribute to explain changes in the sovereign debt portfolios of euro area banks. This allows us to discriminate to some extent between three different reasons why banks may change their domestic sovereign exposures in response to a widening differential between the domestic yield and the euro area swap rate:

- (i) High-risk sovereign issuers may exert “moral suasion” on the banks in their jurisdiction to increase their domestic sovereign holdings, in order to support demand for sovereign debt when demand is low and therefore yields are comparatively high.
- (ii) Undercapitalized banks may bet for resurrection by engaging in “carry trades” whereby they go long on high-risk, high-yield sovereign debt, and fund such exposures either by going short on low-yield debt or by borrowing from the ECB, as suggested by the bank-level evidence in [Acharya and Steffen \(2015\)](#) and [Drechsler et al. \(2013\)](#): insofar as most undercapitalized banks are in periphery countries, this may result in a home bias in the sovereign portfolios of periphery-country banks.
- (iii) In the event of a collapse of the euro, the liabilities of banks in each country (e.g., their deposits) would be redenominated into new national currencies, at the same time as their holdings of domestic sovereign debt. Hence, domestic banks are better hedged than foreign ones against the redenomination risk of domestic sovereign debt: they have a “comparative advantage” in bearing the systemic component of its risk.⁸

⁸In the case of core-country banks, this response may have been amplified by national prudential regulators' recommendations to domestic banks to reduce the risk of their sovereign portfolios.

Thus banks' home bias should be correlated with the systemic component of sovereign risk, but not with its purely country-specific component, which instead should equally affect domestic and foreign investors.

All three stories – the “moral suasion”, the “carry-trade” and the “comparative advantage” hypothesis – share a common prediction: the home bias in banks' sovereign portfolios should be positively correlated with sovereign yield differentials. However, the first two hypotheses predict that this correlation should arise irrespective of whether changes in yields are generated by country-level or common risk; in contrast, the third predicts that this correlation should arise *only* from changes in common risk, e.g. the risk of collapse of the euro. Moreover, since in our sample period sovereign risk and yields increased appreciably only in the euro area periphery, the first two hypotheses can only apply to periphery-country banks, while the third may also apply to core countries.

We explore the response of euro area domestic sovereign exposures to their respective yields and their components, obtained from our dynamic factor model, by estimating a vector error-correction model (VECM) on 2007-13 monthly data for ten euro area countries.⁹ When the model is estimated using actual yields, the sovereign exposures of euro area banks are seen to respond positively to increases in yields in most countries, except Belgium, France and the Netherlands. But this pattern stems from a very different response of sovereign exposures to the country risk factor in the core and in the periphery: (i) in most periphery countries banks respond to increases in the country risk factor by *raising* their domestic exposure, while in core countries they do not; (ii) in contrast, in almost all countries banks increase their domestic exposures in response to an increase in the common risk factor.

Finding (i) suggests that, for periphery-country banks, and only for those, there is evidence in support of the “moral suasion” and/or the “carry-trade” hypothesis, since these banks increase their exposures in response to increases in country-level sovereign risk, not

⁹The countries in our sample are Austria, Belgium, Germany, France, and the Netherlands (henceforth, the euro area core countries), and Spain, Greece, Ireland, Italy, and Portugal (henceforth, the euro area periphery countries).

just in response to systemic euro area risk. It is worth noting that in equilibrium an increase in country-specific sovereign risk needs not result either in an increase or a decrease of domestic banks' exposures, unless these banks are either less or more risk averse than the others. In our data, periphery banks appear to behave as if they were less risk averse than other investors, reflecting either government-dictated or opportunistic risk-taking incentives. The resulting increase in the home bias of their portfolios can be attributed to such distorted incentives, rather than to the increase in country-specific risk *per se*. Even though our evidence is compatible with the “carry trade” hypothesis only for periphery banks, we cannot rule out that this hypothesis also holds for core-country banks. Testing it would require data on core-country banks' holdings of periphery debt: if they engage in carry trades, these banks should respond to higher yields on periphery debt by increasing their exposure to periphery sovereigns. Unfortunately our data do not allow us to perform this test, since a two-entry matrix of euro area banks' aggregate sovereign portfolios by holding and issuing countries is currently unavailable. However, using bank-level data on bank borrowing from the ECB, [Drechsler et al. \(2013\)](#) find that, during the euro crisis, banks from both core and periphery countries engaged in risk shifting (akin to our “carry trade” hypothesis): in both groups of countries weakly-capitalized banks borrowed more and pledged riskier collateral to the ECB over time. Actually, according to their estimates in core countries risk-shifting can explain the entire variation in banks' collateral risk, while in periphery countries this variation is partly to be attributed also to other factors, including political economy motives (similar to our “moral suasion” hypothesis).

Finding (ii) indicates that, when systemic risk increases, most banks both in core and in periphery countries “turn back home”, by increasing their domestic sovereign holdings. This suggests that increased risk of euro collapse and currency redenomination has led to greater home bias of banks' portfolios, especially in core countries. It is worth noticing that these results can be detected only as a result of the decomposition between the country and the common risk factors: they cannot be deduced only from the regressions based on the actual sovereign yields.

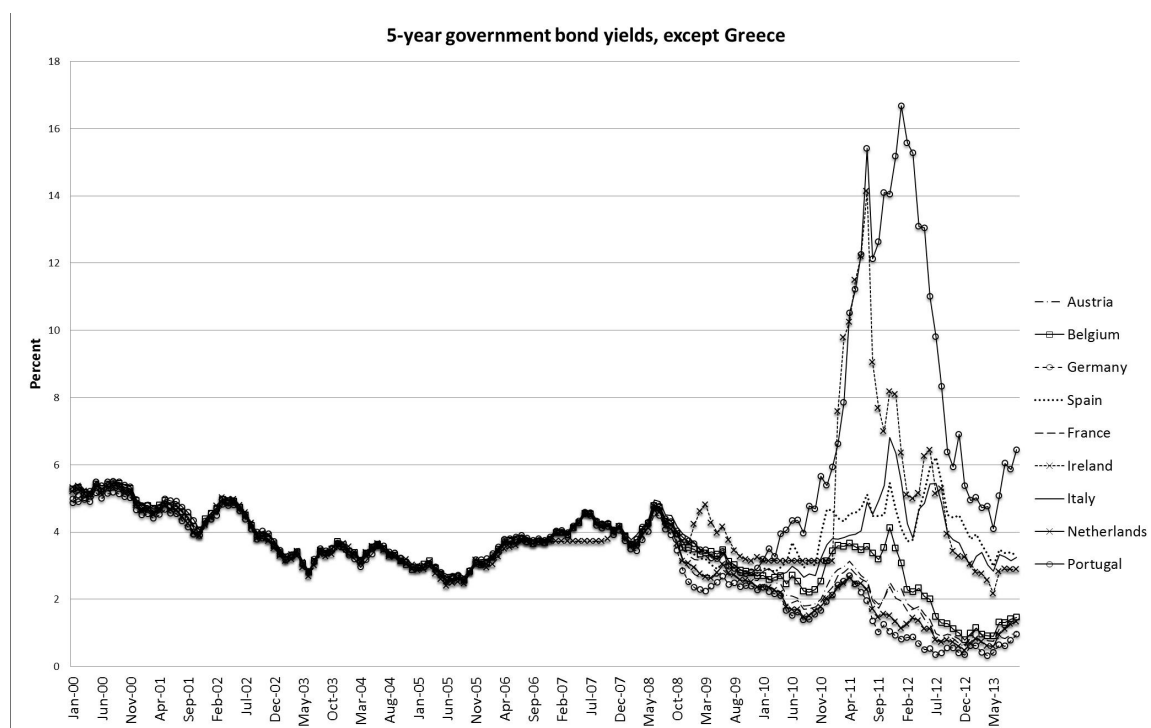
The results of our analysis have several implications for policy. First, decomposing sovereign risk into a country-specific and a systemic component allows a better understanding of the motives behind changes in the home bias in the sovereign debt market. As explained above, the increase of banks' sovereign holdings in the periphery cannot be explained entirely as a response to greater systemic euro area risk, since this increase was associated mostly with greater country-specific sovereign risk. In other words, it cannot be attributed only to periphery banks' comparative advantage in hedging systemic risk: it must have been also induced to some extent by national regulators' moral suasion or by banks' opportunistic carry trades. We cannot distinguish between these two motives, but in either case the behaviour of periphery banks should be regarded as problematic from the standpoint of a policy-maker. If due to moral suasion by national regulators, it indicates that these regulators tended to induce risk-taking by banks in a context where government solvency was at danger, thus enhancing the “diabolic loop” between fiscal solvency and bank solvency deterioration. If due to opportunistic carry trades by banks, it raises concerns about the appropriateness of banks' prudential regulation.

The paper is structured as follows. Section 2.2 illustrates the recent dynamics of yield differentials, CDS premia and bank sovereign exposures in the euro area. Section 2.3 uses dynamic factor analysis to decompose euro area sovereign yield differentials in their country and common components. Section 2.4 investigates how the home bias of banks' sovereign portfolios is related to the components of yield differentials, by estimating a vector error-correction model. Section 2.5 explores the policy implications of our results.

2.2 Euro area sovereign yields, CDS premia and bank exposures: data description

euro area sovereign yields, which had converged dramatically right before the inception of the euro, have diverged equally dramatically starting from late 2008: as illustrated by Figure 2.1, the cross-country dispersion of interest rates on 5-year benchmark bonds increased

Figure 2.1: Eurozone 5-year benchmark government bond yields (monthly, percent)

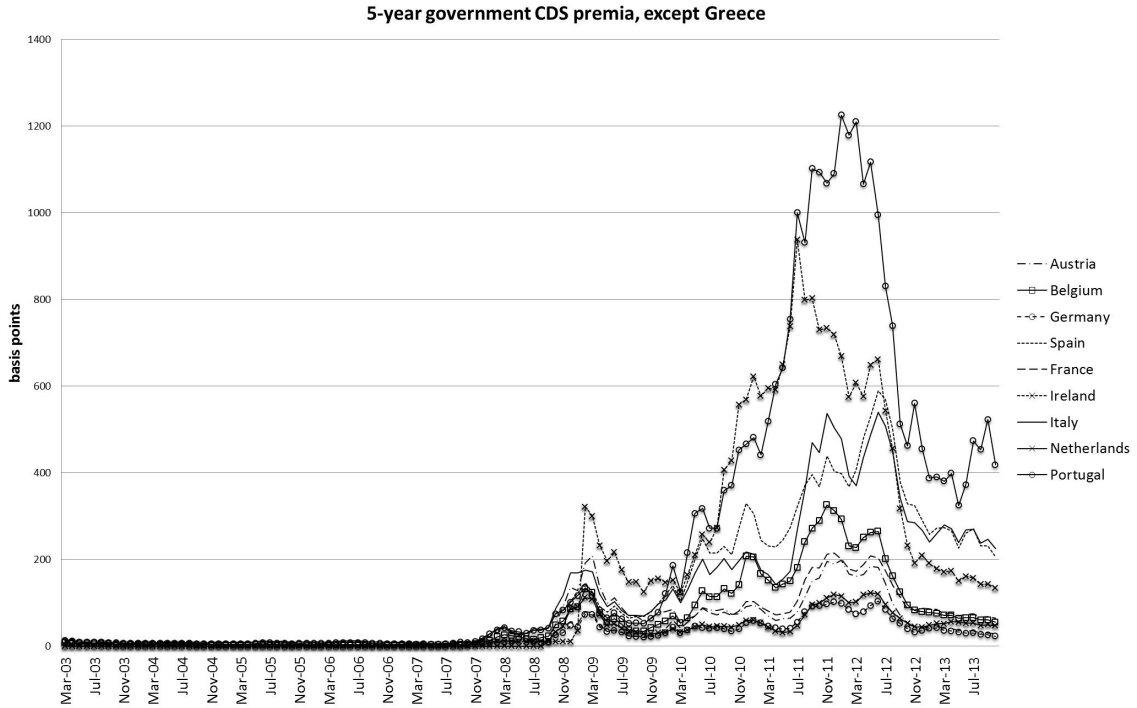


Sources: Bloomberg.

steadily, especially in 2010-11, and peaked in late 2011, before abating somewhat in 2012. The figure shows that the increase in dispersion in 2010 arose mainly from the pattern of sovereign yields in Ireland and Portugal, while in 2011 also the sovereign yields of Spain and Italy rose well above those of the core countries (Greece is omitted to reduce the scale of the vertical axis).

The increase in the dispersion of sovereign yields in 2010 and 2011 is paralleled by that of CDS premia on sovereign debt, as shown by Figure 2.2: the increases in Irish, Portuguese, Italian and Spanish CDS premia in 2011 and 2012 largely coincided with the respective yield increases. But it is worth noticing that CDS premia already diverged to some extent in late 2008 and early 2009, that is, during the subprime financial crisis, even though at that time yields did not appear to react to them almost at all, except for Ireland. Hence, for the more stressed countries the CDS market appears to have been a more sensitive gauge of sovereign risk than the underlying bond market, in line with [Fontana and Scheicher \(2010\)](#),

Figure 2.2: Eurozone 5-year government CDS premia (monthly, basis points)

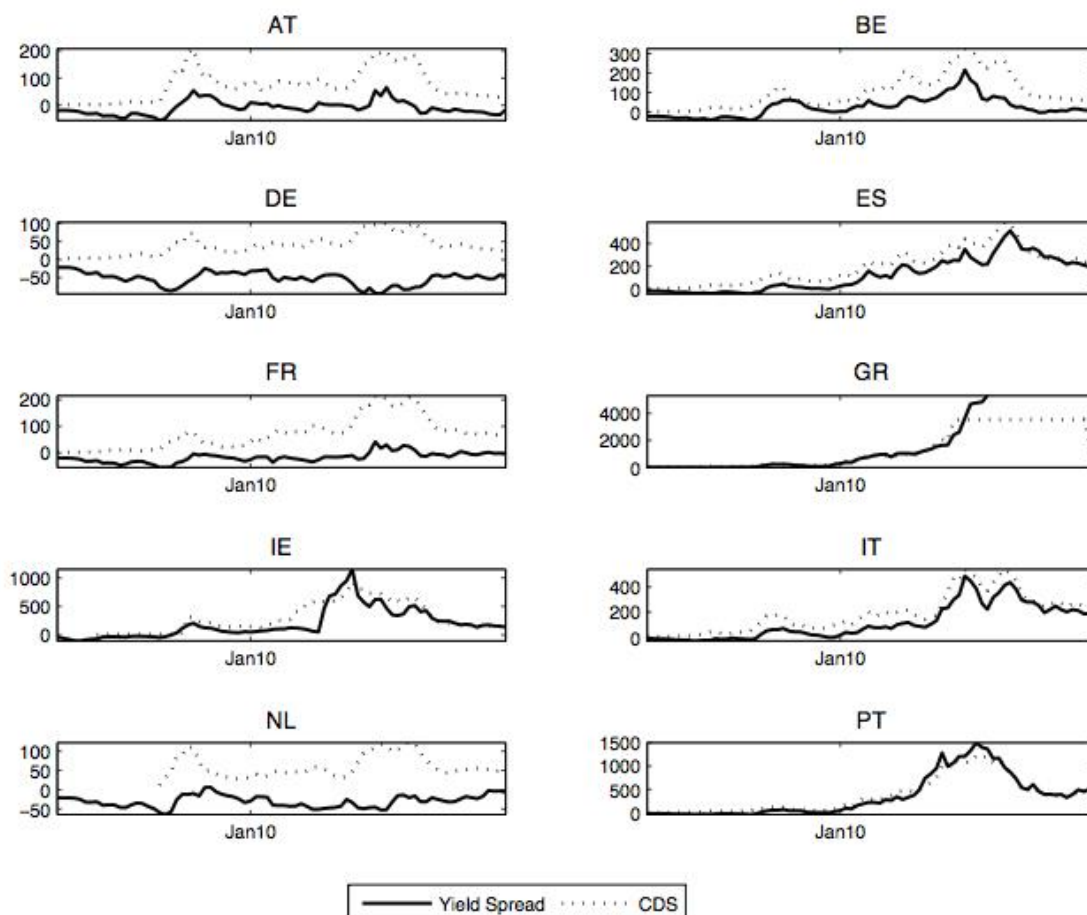


Sources: Bloomberg.

who find that since 2008 price discovery takes place in the CDS markets for Italy, Ireland, Spain, Greece and Portugal, and in the bond market for the core countries. Even though in principle a CDS can be replicated by a short position in the underlying risky bond and a long position in a safe bond of the same maturity, its arbitrage relationship with the underlying bond may break down due to short-sales constraints in the cash market, especially at times of great market stress. In these situations, the CDS become the cheapest way to trade credit risk, because of their synthetic nature, and therefore they also become more sensitive to changes in such risk.

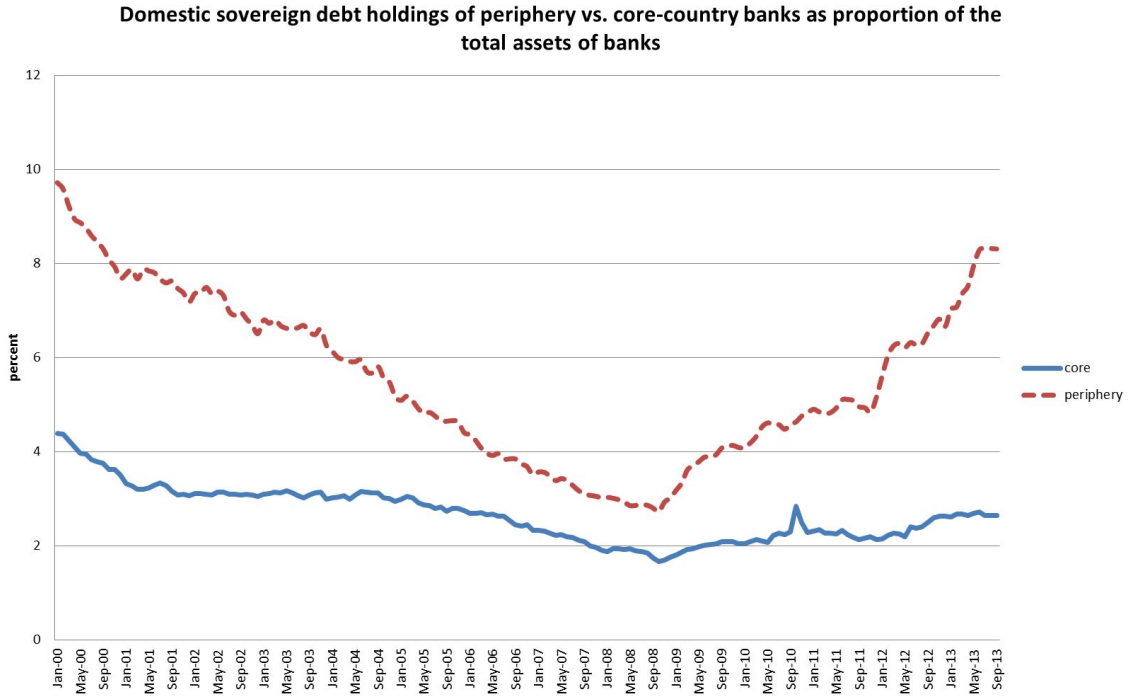
Figure 2.3 allows to compare the time series behaviour of monthly sovereign yields and CDS premia on a country-by-country basis, from March 2007 to October 2013: for each country, it plots the difference between the 5-year sovereign yield and the swap rate for the 5-year maturity, together with the CDS premia for the same maturity. The two series grow over time and are very closely correlated for periphery euro area countries and Belgium, for

Figure 2.3: Sovereign yield differentials and CDS premia, by country



which it is close to 1. The correlation between them is still positive but weaker for Austria and France, is close to zero for the Netherlands, and is negative and significantly different from zero for Germany (-0.68). This striking difference can be interpreted as follows: when the risk of sovereign debt increases throughout the euro area, it triggers a “flight to safety” from periphery issuers towards core ones, and especially towards Germany, and therefore it increases the yields of periphery countries while compresses the Bund yield, even though credit risk increases in Germany too. Hence, while the yield differentials of all other euro area issuers are positively correlated with their respective CDS premia, the German yield

Figure 2.4: Domestic sovereign debt holdings of periphery versus core-country banks as proportion of the total assets of banks



Sources: ECB and authors' calculations.

end up being negatively correlated with the German CDS premium, whose increase signals greater credit risk for the euro area as a whole – including Germany. Of course, the premise of this argument is that to some extent changes in euro area sovereign risk have a common component, captured by correlated movements in CDS premia across the euro area. As we shall see in the econometric analysis of Section 2.3, this is indeed consistent with the data.

Over the same period, the sovereign debt portfolios of euro area banks have featured an increasing degree of home bias. Figure 2.4 shows the time series of the domestic euro area sovereign exposure of banks in euro area core and periphery countries. Specifically, it plots the sum of the monthly values of the euro area sovereign debt holdings of the banks from each of these two groups scaled by the total assets of those banks.¹⁰ The figure shows

¹⁰For the purpose of Figures 2.4, 2.5 and 2.6 we define Austria, Belgium, Finland, France, Germany, and the Netherlands as “core countries”, and Greece, Ireland, Portugal, Spain and Italy as “periphery countries” of the euro area. In the econometric analysis of the subsequent Sections, however, Finland is not included due to data availability problems, and the set of “core countries” is redefined accordingly. Our monthly data for banks' sovereign debt holdings are drawn from the ECB Statistical Data Warehouse (SDW), where they

that, in both groups of countries, banks' sovereign exposures were considerably larger at the inception of the European Monetary Union than they are now. However, while in both groups of countries banks reduced their domestic sovereign debt exposures until 2008, with periphery banks reducing their domestic exposures proportionately more, they both started increasing it again after 2008, with periphery banks increasing it by more than core-country banks.

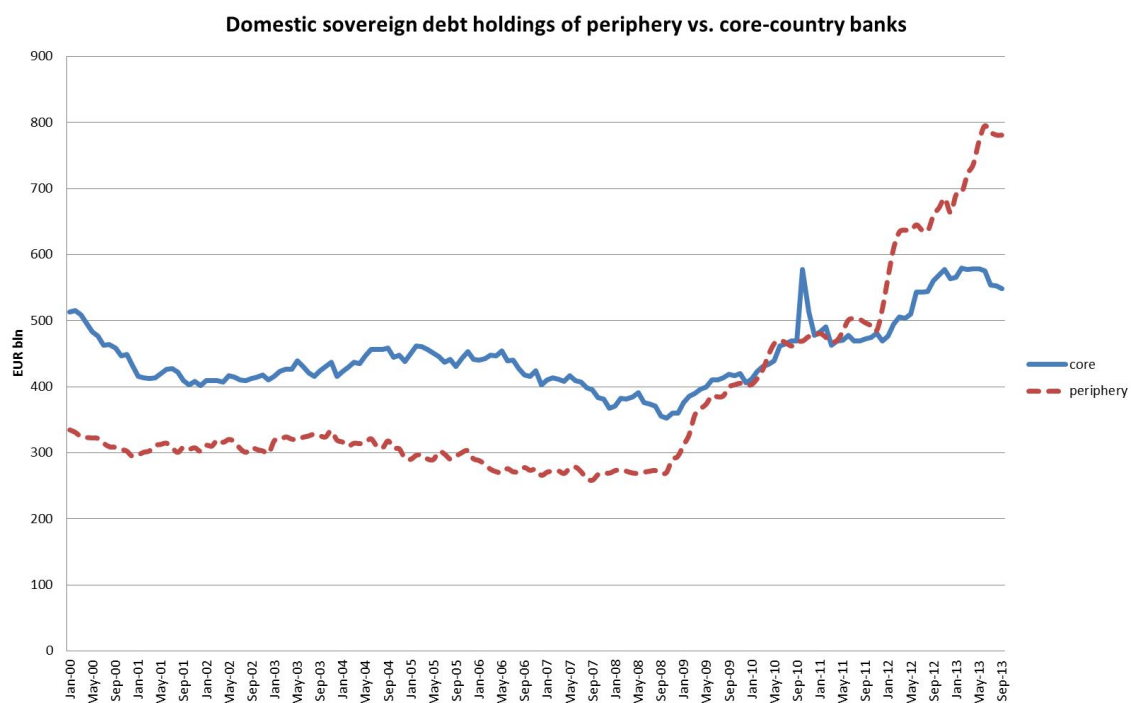
One may suspect that the behavior of the time series for the domestic sovereign exposures in periphery and core-country banks illustrated in Figure 2.4 is driven more by the denominator than by the numerator, namely, is dominated by the time pattern in banks' total assets, rather than by that of their sovereign holdings. To investigate this point, Figures 2.5 and 2.6 plot the time series of the *level* of the domestic and non-domestic euro area debt holdings of banks in periphery and core countries (in billion euro). The two figures show that also the *levels* of banks' sovereign debt holdings – not just their ratio to total assets – have a turning point in 2008, and that they behaved quite differently in the two groups of countries starting in the last part of that year.

Specifically, Figure 2.5 shows that, while after 2008 banks have increased their domestic sovereign debt holdings in both groups of countries, they have done so to a much greater extent in periphery than in core countries: the domestic sovereign debt holdings of periphery banks rose from €270 to €781 billion between October 2008 and September 2013, while those of core-country banks rose from €352 to €548 billion: a 189% increase in the former versus a 56 increase in the latter!

Taken together, Figures 2.5 and 2.6 indicate that, at least partly, the recent increase in banks' holdings of domestic sovereign debt has resulted from a substitution away from the debt issued by foreign euro area sovereigns: starting from 2006, banks in each group of countries have reduced their holdings of sovereign debt issued by non-domestic sovereigns, and therefore have increased the home bias of their sovereign debt portfolios. This realloca-

appear under the name of “Balance sheet item: Securities other than shares of MFIs (excluding ESCB)”, for securities issued by the General Government of all euro-area countries. These data contain the holdings by the banks in each euro-area country of (i) debt issued by all euro-area governments and (ii) domestic government debt, from September 1997 onwards.

Figure 2.5: Domestic sovereign debt holdings of periphery versus core-country banks

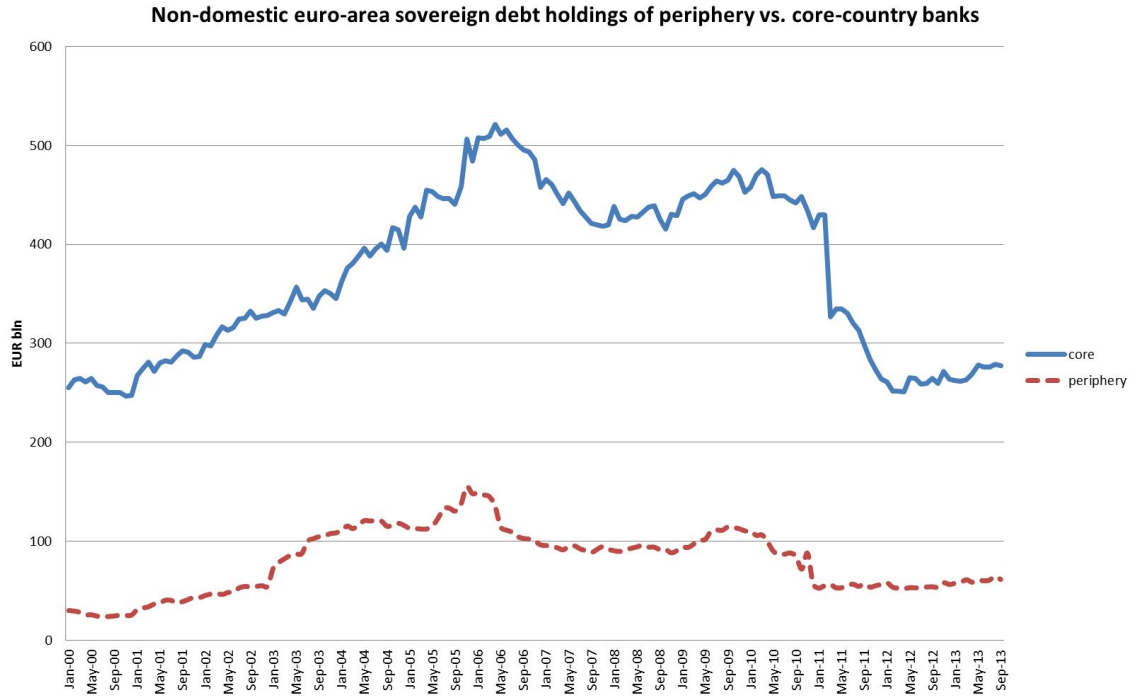


Sources: ECB and authors' calculations.

tion has been relatively modest for banks in the periphery, but very sharp in core-country banks, which have reduced their holdings of non-domestic sovereign debt from €430 billion in February 2011 to €277 billion in September 2013. Hence the overall picture is that of core-country banks reallocating their portfolios away from non-domestic sovereign debt and towards the debt issued by their domestic governments. Indeed, their shift away from non-domestic sovereign debt has been so large as to exceed their investment in domestic public debt, so that their euro area sovereign holdings have decreased since late 2010. This has not been the case for banks in periphery countries, whose total holdings of euro area sovereign debt have sharply increased.

Incidentally, this reshuffling of banks' sovereign portfolios towards domestic public debt is part of an increase in the home bias of their overall portfolios: during the euro area crisis banks have also raised the fraction of domestic loans in their total lending, a “flight-home” phenomenon that appears regularly in financial crises. [Giannetti and Laeven](#)

Figure 2.6: Non-domestic eurozone sovereign debt holdings of periphery versus core-country banks



Sources: ECB and authors' calculations.

(2012) document that the collapse of the global market for syndicated loans during the financial crises that occurred from 1997 to 2009 is partly due to lenders rebalancing their loan portfolios in favor of domestic borrowers. Similarly, De Haas and Van Horen (2013) show that after the collapse of Lehman Brothers large international banks reduced their cross-border lending, especially to clients located far away.

2.3 Sovereign yields, country-specific risk and systemic risk

The dynamics of sovereign yield differentials illustrated in Section 2.2 suggest that since 2008 investors have dramatically reassessed the risk of euro area sovereign issuers, especially those of periphery countries. However, in principle, this reassessment may have concerned either one or both of two different risks: the default risk of individual sovereign issuers or the currency redenomination risk stemming from the collapse of the euro. While sovereign

default risk should reflect mainly country-specific factors, redenomination risk should stem from common threats to the survival of the monetary union, even though exposure to this common risk may differ across countries depending on their different expected exchange rate adjustment in a post-euro regime (as argued by [Di Cesare et al., 2012](#)). As highlighted in the introduction, this source of common risk loomed large on investors' horizon between 2010 and 2012.

We propose to identify these two components of sovereign risk – a country-specific and a common or systemic one – by estimating a dynamic latent factor model, which partitions the shocks driving the sovereign yields of each euro area issuer in three components: (i) a common factor, capturing world and euro area shocks; (ii) a country factor, reflecting shocks to that country's credit risk; (iii) an unexplained idiosyncratic shock.¹¹ Of these three components, the country factor captures the shocks that affect only the yield, CDS premium and financial variables of a specific country, and therefore can be interpreted as the credit risk that concerns only the country itself, without spreading to other countries. The common factor is instead supposed to capture common shocks as well as country-level shocks whose effects spread beyond a specific country, such as those capable of destabilizing the euro area as a whole: for instance, a statement by the Prime Minister of a major euro area country that raises the likelihood of sovereign default by that country might lead investors to reassess the likelihood of collapse of the monetary union, and thereby contribute to the common factor. Importantly, the model allows the same common shock to elicit responses in yields and CDS premia that are completely different in sign and magnitude across countries: hence, the same perceived risk of collapse of the euro may have widely different impacts on different countries.

Our study is related to [Ang and Longstaff \(2013\)](#), who use CDS spreads to study the

¹¹Dynamic factor models were originally proposed as a time-series extension of factor models previously developed for cross-sectional data. They have the ability to model simultaneously and consistently data in which the number of series exceeds the number of time-series observations. The assumption of a dynamic factor model is that a few latent dynamic factors drive the comovements of a high-dimensional vector of time-series variables, which is also affected by a vector of mean-zero idiosyncratic disturbances. These idiosyncratic disturbances arise from measurement error or from the intrinsic characteristics of an individual series. The empirical evidence shows that these assumptions are appropriate for many macroeconomic series (see for instance [Giannone, Reichlin and Sala, 2005](#), and [Watson, 2005](#)).

nature of sovereign credit risk for the U.S. Treasury, individual U.S. states, and major European countries. They use a multifactor affine framework that allows for both systemic and sovereign-specific credit shocks, and find that the sensitivity to systemic risk differs considerably across U.S. and European issuers, which parallels our findings for euro area countries. Interestingly, Ang and Longstaff document that the highly integrated U.S. sovereign debt market features far less systemic risk than its European counterpart. This is in line with the view that the systemic component reflects mainly the danger of collapse of the common currency in the euro area, a danger clearly absent in the U.S. Many other studies have analyzed the determinants of sovereign yield spreads and CDS premia. A first strand of the literature has explored the role of country-level variables such as the debt-GDP ratio, the projected fiscal balance and other macro fundamentals, attributing the unexplained component of yield spreads or/and CDS premia to the mispricing of risk due to panic or contagion effects or, in the context of the euro crisis, to the perceived risk of breakup of the common currency ([Aizenman, Hutchison and Jinjarak, 2011](#), and [Di Cesare et al., 2012](#)). Another strand of the literature allows for both country-specific and common factors in the determination of sovereign yield spreads, by regressing spreads on a vector of country-specific variables (especially fiscal and macroeconomic variables) and one that is common across countries, aimed at capturing time-varying global risk aversion or contagion effects. [Attinasi, Checherita-Westphal and Nickel \(2009\)](#) and [De Santis \(2012\)](#) proxy risk aversion by the spread between the US AAA corporate bonds and the US 10-year sovereign bonds, [Caceres, Guzzo and Segoviano Basurto \(2010\)](#) estimate it as the market price of risk of a stress event, and [Sgherri and Zoli \(2009\)](#) measure it as a latent common factor in spreads by estimating a first-stage regression. [Giordano, Pericoli and Tommasino \(2013\)](#) not only include country-level and common risk variables, but also attempt to capture contagion by interacting these variables with a post-Greek-crisis dummy variable, and find evidence that country-level fundamentals have a greater impact after the Greek crisis (“wake-up call” contagion), while common factors do not (no “pure contagion”).

A possible pitfall of these studies is that they ignore that in some circumstances,

country-specific shocks can have effects on several countries, and therefore turn into common shocks: for instance, a fiscal imbalance in a distressed country such as Italy can be perceived as a possible threat to the survival of the euro, and therefore affect yield spreads not only in Italy but also in other periphery countries of the euro area. Our methodology avoids this pitfall by decomposing yield spreads via a latent factor approach that identifies a country-specific and a common component. This allows to quantify the role played by each of these two components without relying on an assumed relation between them and a set of observables, as in the studies discussed above.

2.3.1 Data

Monthly sovereign yields differentials and CDS premia are the main inputs of our dynamic factor model. Data for both are drawn from the Bloomberg database. For each country, we compute the difference between the 5-year sovereign yield and the 5-year euro swap rate (referring to a swap between a 5-year bond and 12-month Euribor). CDS premia also refer to the 5-year maturity. The dynamic factor model includes 15 countries, 10 of which belong to the euro area (Austria, Belgium, France, Germany, Ireland, Italy, Netherlands, Portugal and Spain) and 5 do not (Denmark, Japan, Sweden, United Kingdom, and United States).

The yield and CDS series are non-stationary, and therefore they are all differenced in the estimation of the dynamic factor model. However, the correlation pattern just described for their levels is similar when computed on the first differences of both variables.

To proxy for the conditions of the financial system in each country, we use the percentage change in the national stock market indices of all the 15 countries present in our sample. We also include variables intended to capture global risk: (i) measures of the “appetite for risk” at the global and European level, namely the percentage change of the VIX and VSTOXX indices; (ii) measures of the possible concerns for the stability of the euro, namely the percentage change of the euro-dollar exchange rate and of the effective exchange rate of the euro.¹²

¹²Stock market price, VIX, and VSTOXX indices are drawn from Bloomberg. The euro-dollar exchange rate and the effective exchange rate are drawn from the ECB database.

2.3.2 Methodology

To identify the different factors, we impose appropriate zero restrictions in the factor loading matrix. Formally, let Δy_c denote the first difference of the government bond yield of country c relative to the swap rate, Δp_c the percentage change in its sovereign CDS premium, and z_c its stock market return. Moreover, let $(x_1, \dots, x_n)'$ be a vector of the variables capturing world risk, namely the percentage change in the VIX index, the VSTOXX index, the euro-dollar exchange rate, and the effective euro exchange rate.

To give an idea of the restrictions imposed in the estimation, consider (for simplicity) the case of two countries ($c = \{1, 2\}$). Then, the dynamic factor model would be as follows:

$$\begin{bmatrix} \Delta y_1 \\ \Delta p_1 \\ z_1 \\ \Delta y_2 \\ \Delta p_2 \\ z_2 \\ x_1 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} \alpha_{1G} & \alpha_{1C} & 0 \\ \alpha_{2G} & \alpha_{2C} & 0 \\ \alpha_{3G} & \alpha_{3C} & 0 \\ \alpha_{4G} & 0 & \alpha_{4C} \\ \alpha_{5G} & 0 & \alpha_{5C} \\ \alpha_{6G} & 0 & \alpha_{6C} \\ \alpha_1 & 0 & 0 \\ \vdots & \vdots & \vdots \\ \alpha_n & 0 & 0 \end{bmatrix} \begin{bmatrix} f_G \\ f_1 \\ f_2 \end{bmatrix} + \xi = \Lambda \mathbf{f} + \xi, \quad (2.1)$$

where f_G is a global common factor, f_1 and f_2 are the country-specific factors, Λ is the matrix of factor loadings, and ξ is the vector of idiosyncratic errors. The latent factors – whether common or country-specific – are assumed to have an autoregressive structure:

$$\begin{bmatrix} f_G \\ f_1 \\ f_2 \end{bmatrix} = \mathbf{f}_t = A(L)\mathbf{f}_{t-1} + u_t, \quad (2.2)$$

where $A(L)$ is diagonal with two lags, so that the factors are orthogonal, and the errors are

modelled as AR(1). The factors are estimated via a two-step procedure: in the first step, they are estimated by principal components and, in the second, by the Kalman filter. The asymptotic justification for this procedure is given in [Doz, Giannone and Reichlin \(2011\)](#).¹³

2.3.3 Results

We now present the results of the estimation of the dynamic factor model just described over the interval from March 2007 to October 2013. First, we show that the common latent factor arising from our estimates can be interpreted as the time-varying redenomination risk arising from the potential collapse of the euro. Second, we assess the relative importance of the common and country factors in explaining the dynamics of yield differentials and CDS premia in different countries, by looking at their variance decomposition and by illustrating how the dynamics of the two components differ across countries.

Interpreting the common factor as Euro collapse risk

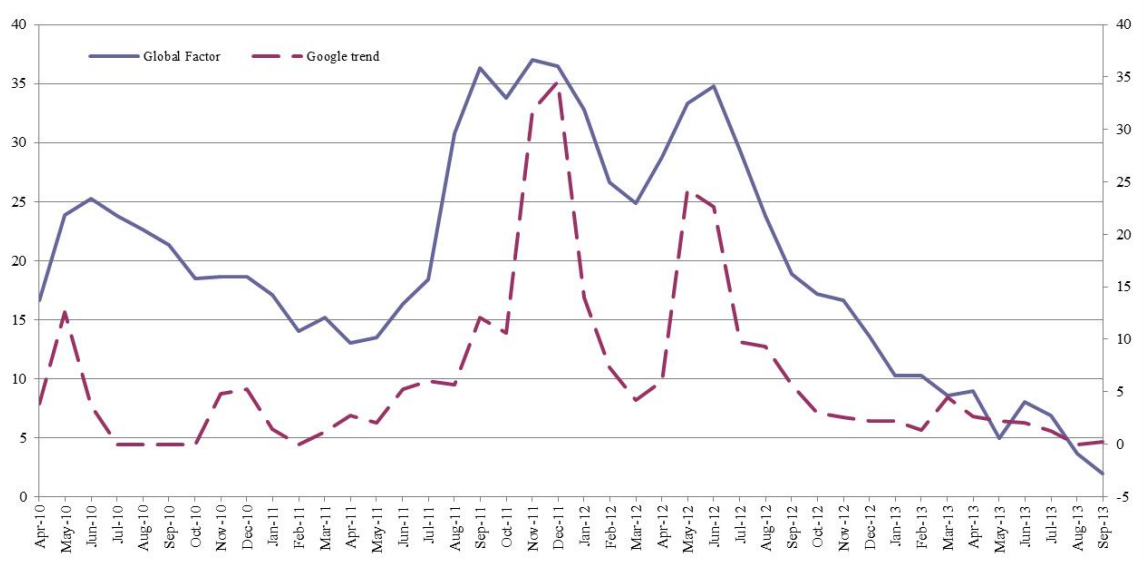
Figures [2.7](#) and [2.8](#) show that the time series of the common factor estimated by our model correlates closely with two estimates of the risk of euro collapse between April 2010 and September 2013.

One way to gauge the concern of investors about the risk of euro breakup is to look at the intensity with which such concern translated in their Google clicks, as captured by a Google Trends index that measures how often search-terms related to the collapse of the euro were entered in the Google search engine, relative to the total worldwide search-volume.¹⁴ In Figure [2.7](#), we plot this search frequency index together with the estimated

¹³This maximum likelihood approach differs from the principal component (PC) analysis for three reasons. First, it allows imposing over-identifying restrictions on the factor model to capture the presence of common and country-specific factors. Second, it may lead to efficiency improvements over the principal component method. Finally, once we have a parametric model estimated by likelihood methods, it is possible to handle missing data. The latter feature is important in our case, because we have an unbalanced panel due to the missing observations for CDS premia and sovereign yield spreads, both at the beginning and at the end of the sample. Hence, compared to PC analysis, our maximum likelihood approach allows us to estimate factors over a longer time interval, which also includes the sub-periods from March 2007 to September 2008 and from February 2012 to October 2013.

¹⁴The search-terms are: “end of euro”, “end of the euro”, “euro break-up”, “euro break up”, “euro breakup”, “euro exit”, “euro collapse”, “collapse of the euro”. We specifically exclude all searches containing the words “euro20” and “euro cup” to avoid contaminating the data with searches related to the UEFA

Figure 2.7: Common factor of yield differentials and CDS premia (left axis) and Google trend indicator of eurozone break-up risk (right axis)



Sources: Google website and authors' calculations.

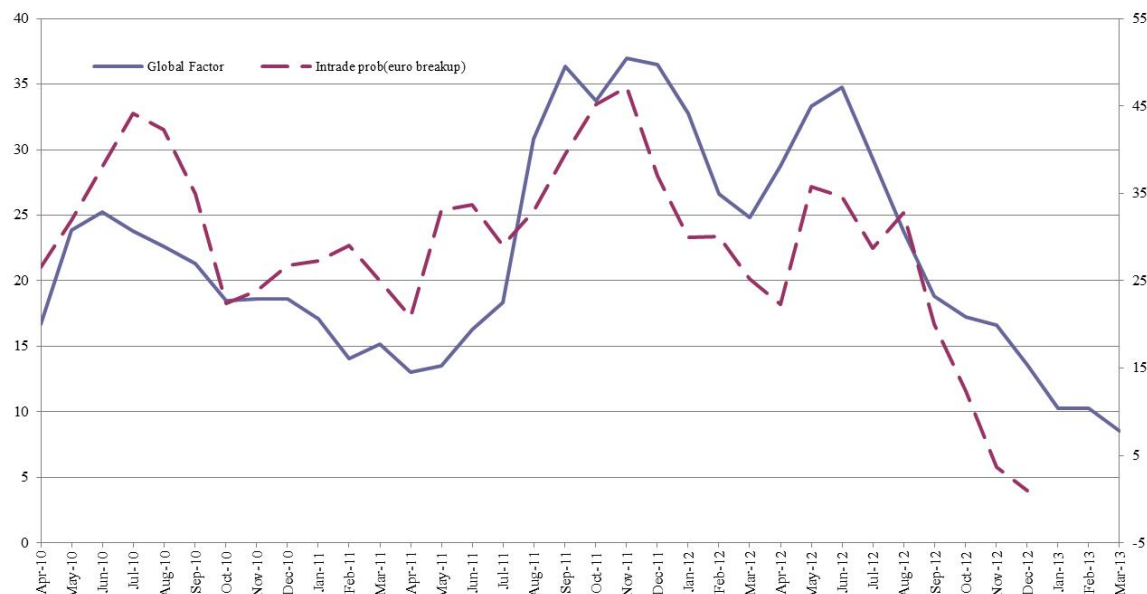
common factor: the correlation between the two series is 0.73, and their turning points coincide.

The perceived risk of exit of member countries from the euro can also be gauged from prediction markets. We look at data drawn from the Intrade online exchange, where individuals can take positions (trade “contracts”) on whether (non-sports-related) future events will or will not occur. The exit of member countries from the euro area is one such event, and the price of the corresponding contract (relative to its payoff if the event occurs) is an estimate of its probability. Figure 2.8 plots our common factor together with the probability that any country that used the euro as of March 12th, 2008 would announce its intention to drop the euro as its national currency or would be expelled from the euro area before the end of 2012, based on Intrade data.¹⁵ The correlation coefficient with our common factor is 0.60 and again the two series' turning points are synchronized.

Championships from 2000 onwards.

¹⁵The market is settled when an announcement is made: the euro does not actually have to be dropped as a national currency by the date specified in the contract. For example, if there is an announcement on December 1, 2012 that the euro will be dropped in June 2013 the market will be settled at \$10.00 (the contract's notional settlement value) on the date of the announcement (December 1, 2012) and not the date the euro will no longer be used (June 2013).

Figure 2.8: Common factor of yield differentials and CDS premia (left axis) and Intrade-based probability of euro break-up (right axis)



Sources: Google website and authors' calculations.

Interestingly, our common factor peaks at times when the media expressed particular concern about the sustainability of the euro. In particular, it peaks in October and November 2011, when the Greek prime minister proposed a referendum for the euro, and then resigned to be replaced by Papademos. In that period, German officials approached Greek ones with proposals about a Greek orderly exit from the euro.¹⁶ Indeed in November 2011 *The Financial Times* reported of multinational companies' preparations for the possible euro breakup. The common factor peaks again in May and June 2012, a time of considerable political uncertainty in Greece, which led to two successive general political elections in essentially a month's time. Coincidentally, in May 2012 *The Sunday Telegraph* published an interview with Lloyds' CEO Richard Ward describing his company's preparations for euro collapse. Conversely, our common factor declined after ECB President Draghi delivered his famous "whatever-it-takes" speech on July 26, 2012, which laid out the basis for the ECB's Outright Monetary Transaction (OMT) policy.¹⁷

¹⁶See the statements by former ECB Board member Lorenzo Bini-Smaghi reported in

Table 2.1: Dynamic factor model estimation: variance decomposition

Austria	Δ Sovereign yield	0.05	0.86	0.09
Belgium	Δ Sovereign yield	0.17	0.45	0.38
Germany	Δ Sovereign yield	0.36	0.32	0.32
Spain	Δ Sovereign yield	0.21	0.72	0.07
France	Δ Sovereign yield	0.01	0.95	0.04
Greece	Δ Sovereign yield	0.03	0.27	0.69
Ireland	Δ Sovereign yield	0	0.34	0.66
Italy	Δ Sovereign yield	0.29	0.74	-0.02
Netherlands	Δ Sovereign yield	0.04	0.92	0.04
Portugal	Δ Sovereign yield	0.01	0.83	0.16
Denmark	Δ Sovereign yield	0.02	0.97	0.01
U.K.	Δ Sovereign yield	0.11	0.81	0.08
Sweden	Δ Sovereign yield	0.1	0.89	0.01
U.S.	Δ Sovereign yield	0.14	0.62	0.24
Japan	Δ Sovereign yield	0.01	0.89	0.1
Austria	Δ CDS premium	0.67	0.15	0.19
Belgium	Δ CDS premium	0.56	0.17	0.27
Germany	Δ CDS premium	0.6	0.07	0.33
Spain	Δ CDS premium	0.43	0.47	0.1
France	Δ CDS premium	0.65	0.04	0.31
Greece	Δ CDS premium	0.05	0.61	0.34
Ireland	Δ CDS premium	0.13	0.21	0.66
Italy	Δ CDS premium	0.59	0.35	0.06
Netherlands	Δ CDS premium	0.49	0.06	0.45
Portugal	Δ CDS premium	0.11	0.73	0.16
Denmark	Δ CDS premium	0.56	0.01	0.43
U.K.	Δ CDS premium	0.41	0.01	0.58
Sweden	Δ CDS premium	0.58	0.03	0.39
U.S.	Δ CDS premium	0.24	0.09	0.67
Japan	Δ CDS premium	0.43	0.07	0.5
Austria	Stock market return (%)	0.79	0.02	0.19
Belgium	Stock market return (%)	0.8	0.02	0.18
Germany	Stock market return (%)	0.75	0	0.25
Spain	Stock market return (%)	0.69	0.03	0.28
France	Stock market return (%)	0.81	0.02	0.18
Greece	Stock market return (%)	0.72	0.02	0.26
Ireland	Stock market return (%)	0.6	0.01	0.39
Italy	Stock market return (%)	0.83	0	0.17
Netherlands	Stock market return (%)	0.71	0	0.29
Portugal	Stock market return (%)	0.68	0	0.32
Denmark	Stock market return (%)	0.62	0	0.38
U.K.	Stock market return (%)	0.67	0.08	0.25
Sweden	Stock market return (%)	0.62	0	0.38
U.S.	Stock market return (%)	0.71	0	0.29
Japan	Stock market return (%)	0.53	0	0.47
	Δ VIX (%)	0.26		0.74
	Δ VSTOXX (%)	0.31		0.69
	Δ effective exchange rate (%)	0.26		0.74
	Δ euro-dollar exchange rate (%)	0.08		0.92

The relative importance of the common and country risk factors

Identifying the common and country-specific factors allows to estimate the fraction of the variance in the yield differentials relative to the swap rate that can be attributed to each of them: the resulting variance decomposition is shown in Table 2.1. Three main results emerge from it.

First, country risk plays a dominant role in explaining yield differentials relative to the swap rate, with the exception of Greece and Ireland, whose yield are mainly idiosyncratic, and of Germany, whose yield is equally explained by the common factor.¹⁸ Indeed, the common factor affects mainly the German yield, a fact that can reflect the “flight to quality” by investors when they become more concerned about the survival of the euro.

Second, the variance decomposition for CDS premia indicates that common risk is important for all euro area countries, but that its role differs greatly across countries, in line with what is found by Ang and Longstaff (2013) with a different methodology. In particular, common risk plays a minor role in countries that have been involved in a sovereign bailout programme by the EFSF/ESM (Greece, Ireland and Portugal). But for most of the euro area periphery, country-specific risk is also important: this is the case for Italy, Portugal and Spain, and to a more limited extent for Ireland.

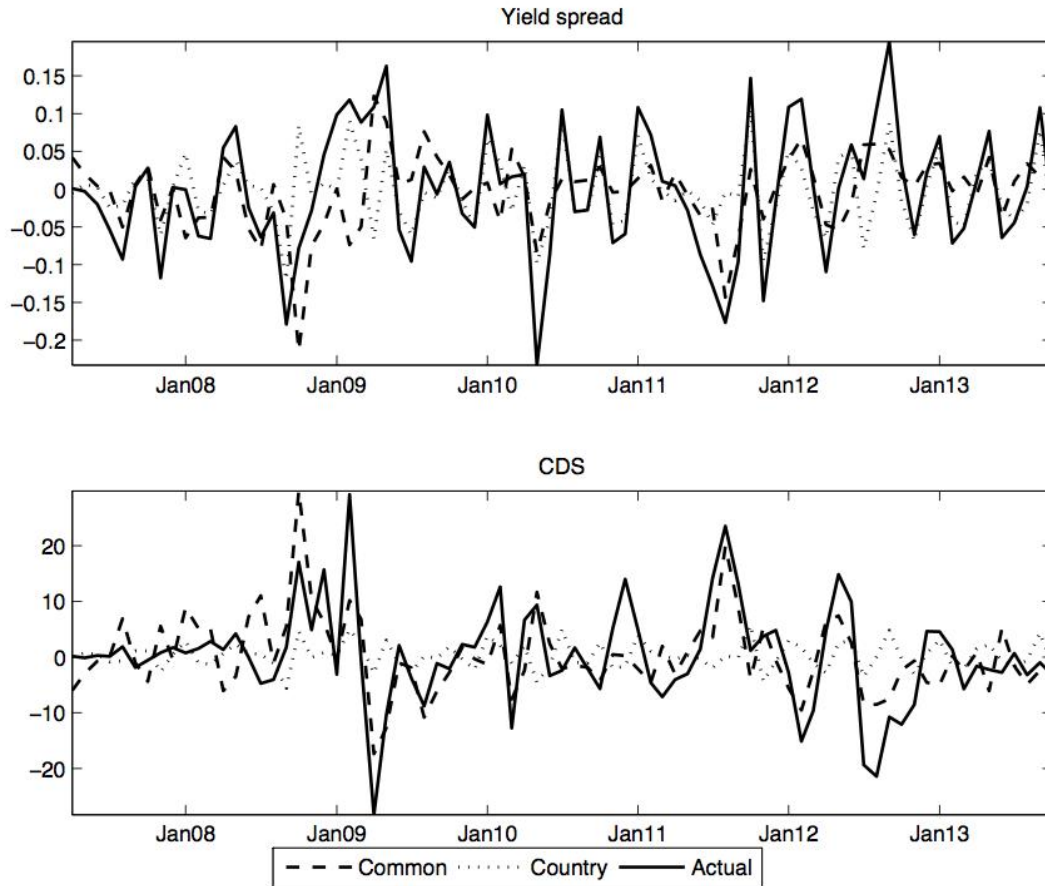
Third, common risk appears to explain the bulk of the variability in financial variables: the stock returns in the third block, and the volatility and exchange rate measures in the fourth block of Table 2.1. In particular, it accounts for over 60% of the variability in the stock returns of almost all euro area countries, and for over one fourth of the variation in

www.cnbc.com/id/101031815.

¹⁷On that date, Mario Draghi stated in a speech at the Global Investment Conference in London: “Within our mandate, the ECB is ready to do whatever it takes to preserve the euro. And believe me, it will be enough.” The OMT policy is a program under which the ECB makes secondary market purchases (“outright transactions”) of euro area sovereign bonds, once a euro area government asks for financial assistance.

¹⁸For Greece, the idiosyncratic component is particularly large, while the country component is modest. This is explained by the fact that in October 2011 investors agreed a “haircut” of 50% in converting their existing bonds into new loans, leading to a freeze of the Greek CDS market: in our data, the Greek sovereign CDS price becomes constant from October 2011 onwards. Since in our dynamic factor model the country component is driven by the country-level correlation between CDS and yield spreads, the constancy of the CDS premium in 1/4 of the sample considerably reduces the variance explained by the country risk component, and raises that explained by the idiosyncratic component.

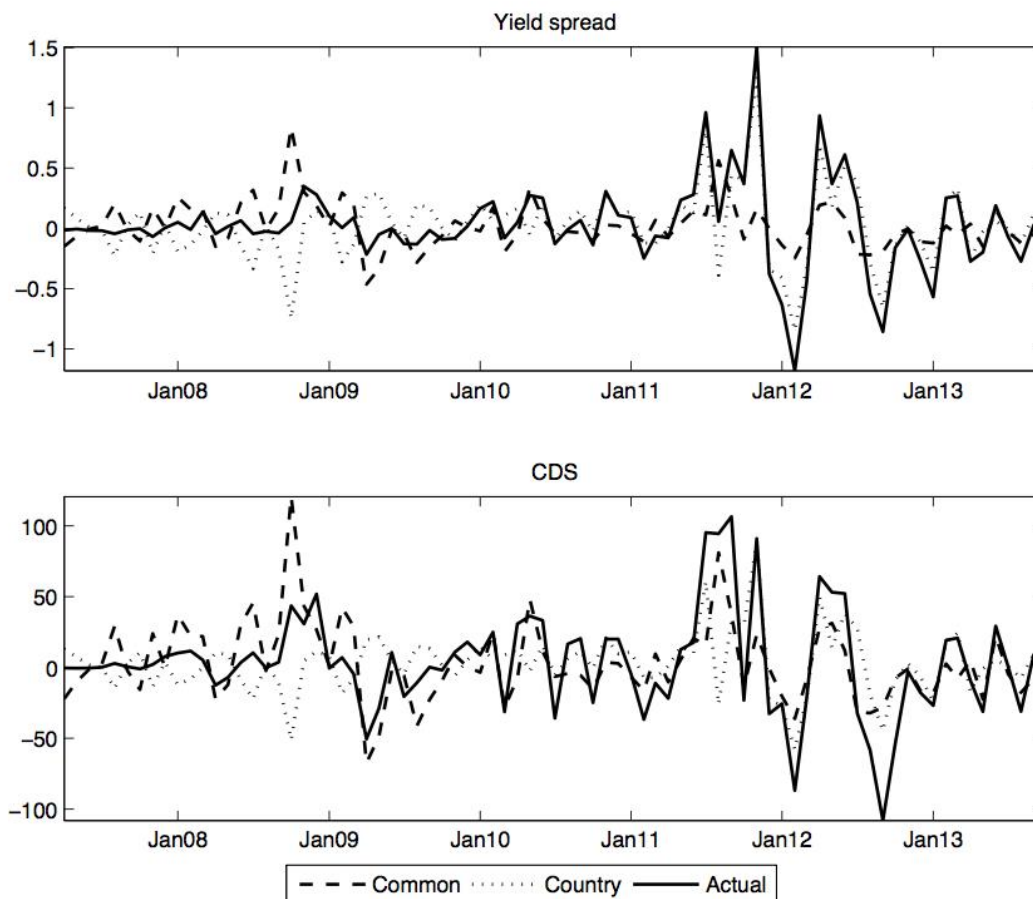
Figure 2.9: Common and country components of the German yield differential and CDS premium (first differences)



the VIX index.

To interpret these results, it is worth looking at Figures 2.9, 2.10 and 2.11, which show the time patterns of the common and country components of the yield differential and the CDS premium for Germany, Italy and Spain. In all three figures, the solid line shows the actual series (yield differential or CDS premium), the dashed line plots the common component of the series, and the dotted one the country component. Figure 2.9 shows that the common component explains most of the movement of the German CDS premium and to some extent also of the German yield. In contrast, Figures 2.10 and 2.11 show that in Italy and Spain the country component explains most of the yield pattern, while for their CDS premium both the common and the country component play a role. It is worth

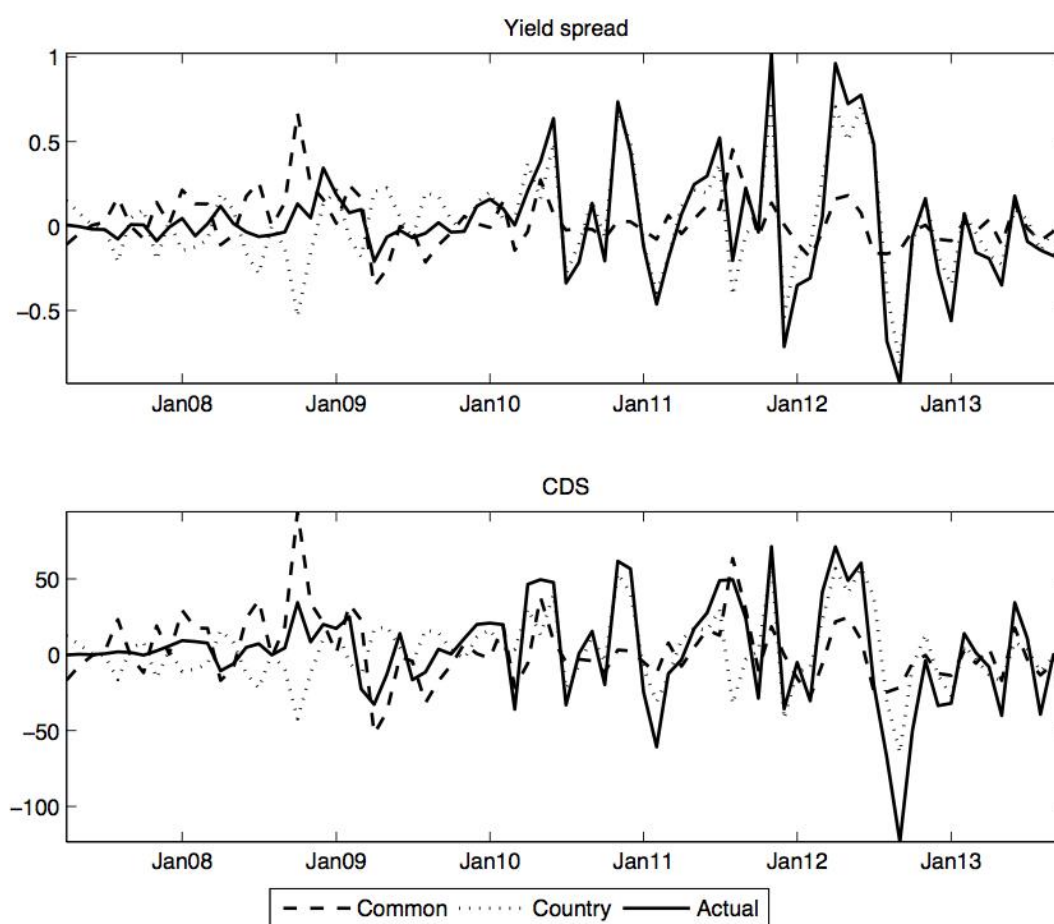
Figure 2.10: Common and country components of the Italian yield differential and CDS premium (first differences)



considering how a rise in the common risk factor affects CDS premia and yield differentials in the three countries in late 2011. Their response is captured by the respective common components (the dashed lines): CDS premia rise in all three countries, but while both the Italian and Spanish yield differentials increase, the German one drops sharply.¹⁹ The opposite happens towards the second half of 2012, when both common and country risks recede in Italy and Spain: all CDS premia decline, and the Italian and Spanish yields also drop, while the German one rises.

¹⁹Interestingly, in Figure 2.10 the estimated country-specific component of the yield spread for Italy falls sharply at the end of December 2011 and beginning of January 2012, exactly at the time when the newly appointed Monti government announced and started implementing its new agenda and passed emergency economic legislation, thus calming the Italian public debt market.

Figure 2.11: Common and country components of the Spanish yield differential and CDS premium (first differences)



The interpretation of these patterns is that common shocks induce generalized changes in CDS premia, including those of core countries (though more so in the periphery), while they push bond yields in opposite directions, as investors flee from periphery bond markets towards the core of the euro area, or vice versa.

2.4 Home bias in banks' sovereign exposures, yield differentials and systemic risk

Section 2.2 documents two aggregate patterns in the euro area market for sovereign debt: (i) the home bias of banks' sovereign debt portfolios decreased until 2008, and then started increasing; (ii) sovereign yield differentials were close to zero until the same date, and then started widening. In this section, we investigate whether these two facts are related, namely, whether banks' home bias (a quantity-based measure of segmentation) is related to domestic yield differentials (a price-based measure of segmentation). As explained in the introduction, a positive correlation between domestic sovereign exposures and yield differentials might arise from three different (not mutually exclusive) reasons:

- (i) the “moral suasion” exerted by national regulators on the banks in their jurisdiction to purchase domestic debt when the sovereign experiences difficulties in its placement, i.e. at times when its yield is relatively high;
- (ii) the tendency by undercapitalized banks, which are mostly located in the euro area periphery, to bet for resurrection by engaging in “carry trades” in high-yield sovereign debt, i.e. by buying periphery debt at times of market stress;
- (iii) the “comparative advantage” of each country's banks in bearing the currency redenomination risk of their country's sovereign debt, arising from the potential breakup of the euro area.

The first two motivations are compatible with banks increasing their domestic exposures not only in response to greater systemic euro area risk but also in response to increased country-specific risk; in contrast, the third motivation implies that banks should increase their domestic exposures only in response to greater systemic euro area risk, as they have no comparative advantage in hedging against country-specific risk.²⁰ Hence, in this Section

²⁰Incidentally, these three reasons may also contribute to explain the increased home bias of banks' loan portfolios: banks may redirect their lending towards domestic companies because (i) this increases the

we also investigate how domestic sovereign exposures respond to the common and country risk factors that drive yield differentials, so as to shed some light on the mechanisms that have driven the response of banks' domestic exposures during the euro crisis.

2.4.1 Data and methodology

Our analysis proceeds in two steps. First, we estimate a baseline model, where we investigate the dynamic relationships between banks' domestic sovereign exposures and yield differentials between the domestic 5-year government bond yield and the 5-year annual euro swap rate. Second, we estimate a factor-based model, where the yield differential is replaced by the country and common risk components estimated in Section 2.3. Beside the 5-year yield differentials relative to the euro swap rate used in Section 2.3, the data used in the estimation include monthly values of aggregate euro area banks exposures to domestic sovereign debt, drawn from the ECB SDW.²¹ The sample period ranges from April 2007 to September 2013 for all countries except Greece, Ireland and Portugal, for which the sample ends in April 2011, December 2010 and April 2011 respectively, since we exclude observations after the inception of the IMF/ECB bailout programs implemented in those countries.²²

To select the econometric model most suitable for the analysis of the dynamic relationships between banks' sovereign exposures and yield differentials (and their components), we consider several features of the relevant time series. First, although we are particularly interested in the response of sovereign exposures to the sovereign yield differentials, feed-

probability of a bailout by domestic authorities ("moral suasion"), (ii) they wish to earn the differential between the interest charged to domestic companies and their funding rate ("carry trade"), or (iii) they are better hedged against redenomination of their loans than foreign banks ("comparative advantage"). An additional reason for the increased home bias of bank loans in a crisis is that in turbulent times asymmetric information problems become more acute, so that banks prefer to revert to more familiar borrowers, who typically are domestic ones.

²¹For further details about our data on sovereign exposures, see footnote 10 above.

²²The choice of the sample period is mainly driven by the fact that the dynamics of the domestic sovereign exposures and of the sovereign yields spreads during the euro debt crisis are considerably different from the previous years, showing a shift to a "new regime" after 2007. Moreover, we do not have data for CDS premia before 2007. In order to avoid small sample bias, we estimate the VECM adopting the 2-step procedure described by Lütkepohl and Krätzig (2004). Stock (1987) presents Monte Carlo examples where the OLS estimates are biased, while these biases disappear adopting the 2-step procedure that we use. This is also highlighted by Engle and Granger (1987).

back effects from banks' sovereign exposures to interest rate spreads cannot be ruled out. Second, the model should be dynamic, so as to allow for the possibility of gradual short-run adjustment of banks' sovereign portfolios towards their long-run desired composition, due to adjustment costs deriving from illiquidity, uncertainty about the persistence of yield differentials, etc. Finally, in order to have a correctly specified model, we must account for the non-stationarity of all the series in our data sample.

All these motivations lead us to estimate a vector error-correction model (VECM) for each country in order to analyze the joint determination of its banks' domestic sovereign exposure and yield differential, since this model (i) allows for all possible patterns of time-precedence among variables, (ii) can capture the gradual adjustment of sovereign exposures to long-run equilibrium levels determined by movements in yield differentials, and (iii) can deal with non-stationarity in the data generating process. The preliminary analysis of the data and the specification search (see the Appendix) lead us to the following VECM(p) in reduced-form representation, where p denotes the number of lags:

$$\Delta y_t = \alpha(\beta' y_{t-1} + \gamma d_{t-1}) + \Theta_1 \Delta y_{t-1} + \cdots + \Theta_p \Delta y_{t-p} + \Gamma D_t + u_t \quad (2.3)$$

In this expression, y_t is a $n \times 1$ vector, n being the number of endogenous variables, defined as the 2-element vector $y_t = [\textit{spread}_t \textit{soveexp}_t]'$ in the baseline model and the 3-element vector $y_t = [\textit{common}_t \textit{country}_t \textit{soveexp}_t]'$ in the factor-based model, where \textit{spread}_t is the domestic sovereign debt yield differential (with respect to the euro swap rate), $\textit{soveexp}_t$ denotes the domestic sovereign exposures of banks as a fraction of their total assets, and \textit{common}_t and $\textit{country}_t$ denote the common and the country components of the yield differential in month t , respectively. Moreover, d_t and D_t are $m \times 1$ and $M \times 1$ vectors, referring to the restricted and unrestricted deterministic terms (or dummy variables) included in each country's specification, respectively; the $n \times 1$ vector u_t denotes the reduced form residuals. Finally, α is the $n \times r$ matrix of adjustment parameters, β is the $n \times r$ matrix of cointegrating parameters, Θ_j is the $n \times n$ matrix of short-run parameters referring to lag j , and γ and

Γ are the $r \times m$ and $n \times M$ matrices of coefficients associated with the restricted and unrestricted deterministic terms, respectively; r is the cointegrating rank (i.e. the number of cointegration relations) of the system. As usual, our analysis focuses on the coefficients in α , which capture the adjustment of each variable in response to shocks (towards the long-run equilibrium if the coefficient is negative, and away from it if positive), and β , which indicate the long-run relationship between variables (positive if the coefficient is negative, and vice versa).

As described in the Appendix, the cointegrating rank of the model in equation (3) is identified through Johansen’s trace test for cointegration. This step is crucial to impose the most suitable restrictions and identify the parameters α and β of the error-correction term, which capture the adjustment of the differenced dependent variables towards their long-run equilibrium levels in response to shocks in the levels of the same variables. Our preliminary analysis supports setting $r = 1$ for all countries in the baseline model; Johansen’s trace test reveals that $r = 2$ is more suitable to investigate the factor-based model.

The reduced-form VECM in equation (3) is estimated using [Johansen \(1995\)](#) maximum likelihood method. Accordingly, restrictions on the cointegrating parameters in β are imposed following Johansen’s strategy, whereby in the cointegrating equation(s) we impose a unit restriction on the coefficient(s) on *spread_t* (*common_t* and *country_t*) in the baseline (factor-based) model and the coefficients on *soverxp_t* are estimated for each cointegration relation. In the specification of the model for all countries, we also include dummy variables in order to account for two of the most important events in the recent chronicles of the euro crisis: (i) the long-term refinancing operations (LTROs) executed by the ECB since December 2011 (henceforth, the *ltro* dummy), and (ii) the speech by ECB President Mario Draghi at the Global Investment Conference in London on 26 July 2012 where he committed to do “whatever it takes to preserve the euro” (henceforth, the *wit* dummy – a mnemonic for whatever-it-takes).²³ The rationale for the inclusion of these dummy variables is the impact of both events on the conditions of euro area financial markets and on investors’ behavior:

²³The *ltro* and *wit* dummies equal one after December 2011 and June 2012, respectively, and zero otherwise.

(i) the LTROs changed the conditions at which euro area banks could obtain liquidity from the central bank, so that they may have affected their portfolio decisions; (ii) by stating the commitment of the ECB to the survival of the euro, President Draghi’s speech dampened financial market volatility and eased financing conditions for governments in the euro area periphery, and thus generated a remarkable reversal in the patterns of their sovereign bond yields.²⁴ These dummies are irrelevant for Greece, Ireland and Portugal, as both of these events occurred after the start of the respective bailout programs, which mark the end of the sample for these countries.

2.4.2 Results

Table 2.2 reports the results of the estimation of the baseline (columns 1 and 2) and factor-based (columns 3 to 6) VECMs for all countries. First, column 1 (baseline model) and columns 3 and 5 (factor-based model) show the cointegrating parameters (β) obtained by normalizing the estimated coefficient on $soverxp_t$ to unity in each cointegration relation. More specifically, column 1 refers to the cointegrating relationship between sovereign exposures and yield differentials, and shows the normalized coefficient on $spread_t$; column 3 refers to the cointegrating relationship between sovereign exposures and the common factor, and shows the normalized coefficient of $common_t$; column 5 refers to the cointegrating relationship between sovereign exposures and the country factor, and shows the normalized coefficient of $country_t$.²⁵ Second, column 2 (baseline model) and columns 4 and 6 (factor-based model) report the adjustment parameters (α) for domestic sovereign exposures (i.e. the estimated coefficients of the $soverxp_t$ equation).²⁶ The long-run parameters

²⁴ “Measuring Mario Draghi’s Promises”, *The Wall Street Journal*, July 26, 2013.

²⁵ In order to interpret the results, let the relevant cointegration relation in normalized form (disregarding deterministic terms) be $soverxp_t = -\beta_x x_t + z_t$, where x_t denotes yield differentials, the common factor or the country factor, depending on the model and the cointegration relation of interest, and z_t represents the error-correction term. Then, if the normalized cointegrating parameter β_x is negative (positive) and significantly different from zero, we infer the existence of a positive (negative) long-run equilibrium relationship between $soverxp_t$ and x_t , i.e. sovereign exposures tend to increase (decrease) towards their equilibrium level in response to an increase in x_t .

²⁶ A negative and statistically significant adjustment parameter α indicates that, whenever the error-correction term z_t is different from zero, the dependent variable of the corresponding equation of the VECM adjusts towards its equilibrium level. If instead α is positive and/or statistically insignificant, then the process for the dependent variable does not converge to its equilibrium level.

Table 2.2: VECM estimates for the response of banks' domestic sovereign exposures to yield differentials and their components

Country	Baseline model		Factor-based model			
	β	α	β	α	β	α
	(1)	(2)	(3)	(4)	(5)	(6)
Spain	-0.906*** (0.000)	-0.026* (0.088)	-3.001* (0.072)	-0.039*** (0.000)	-0.693*** (0.000)	-0.010 (0.566)
Greece	-1.299*** (0.000)	-0.366*** (0.000)	0.739*** (0.001)	0.465*** (0.000)	-0.603*** (0.000)	-0.653*** (0.000)
Ireland	-0.524* (0.097)	-0.046** (0.03)	0.073*** (0.000)	1.974*** (0.001)	-0.009*** (0.000)	-2.092*** (0.001)
Italy	-0.762*** (0.003)	-0.077*** (0.000)	-8.128 (0.71)	-0.009*** (0.003)	-0.408*** (0.000)	-0.066* (0.054)
Portugal	-0.240*** (0.001)	-0.144** (0.02)	1.113*** (0.000)	-0.195*** (0.006)	-0.790** (0.041)	-0.095** (0.036)
Austria	-3.065*** (0.008)	-0.031*** (0.000)	5.106* (0.06)	0.046*** (0.009)	2.023** (0.022)	-0.032* (0.079)
Belgium	-12.904 (0.74)	0.001 (0.816)	-0.612*** (0.002)	-0.14** (0.037)	1.978 (0.106)	-0.013 (0.715)
Germany	-0.499*** (0.000)	-0.343*** (0.000)	2.374* (0.093)	-0.052 (0.228)	7.741*** (0.000)	-0.097 (0.129)
France	0.590*** (0.002)	-0.080* (0.086)	-3.411*** (0.001)	-0.261*** (0.000)	1.799** (0.018)	0.032 (0.39)
Netherlands	2.193** (0.016)	-0.064*** (0.001)	-3.538*** (0.000)	-0.082 (0.193)	1.214*** (0.000)	-0.116** (0.014)

The table reports the results of the estimation of the baseline (columns 1 and 2) and factor-based (columns 3-6) VECM's for all countries. First, column 1 (baseline model) and columns 3 and 5 (factor-based model) show the cointegrating parameters (β) obtained by normalizing the estimated coefficient of $soverp_t$ to unity in each cointegration relation. More specifically, column 1 refers to the cointegrating relationship between sovereign exposures and yield differentials, and shows the normalized coefficient of $spread_t$; column 3 refers to the cointegrating relationship between sovereign exposures and the common factor, and shows the normalized coefficient of $common_t$; column 5 refers to the cointegrating relationship between sovereign exposures and the country factor, and shows the normalized coefficient of $country_t$. Second, column 2 (baseline model) and columns 4 and 6 (factor-based model) report the adjustment parameters (α) for domestic sovereign exposures (i.e. the estimated coefficients of the $soverp_t$ equation). The long-run parameters can be computed as $\alpha\beta'$. The sample ranges from April 2007 through September 2013 for all countries, except Greece, Ireland and Portugal (whose end dates are April 2010, December 2010 and April 2011, respectively). The coefficients of restricted and unrestricted deterministic terms are not reported. One, two or three asterisks denote significance at the 10%, 5% or 1% significance level, respectively. Numbers in parentheses are p -values.

can be computed as $\alpha\beta'$.

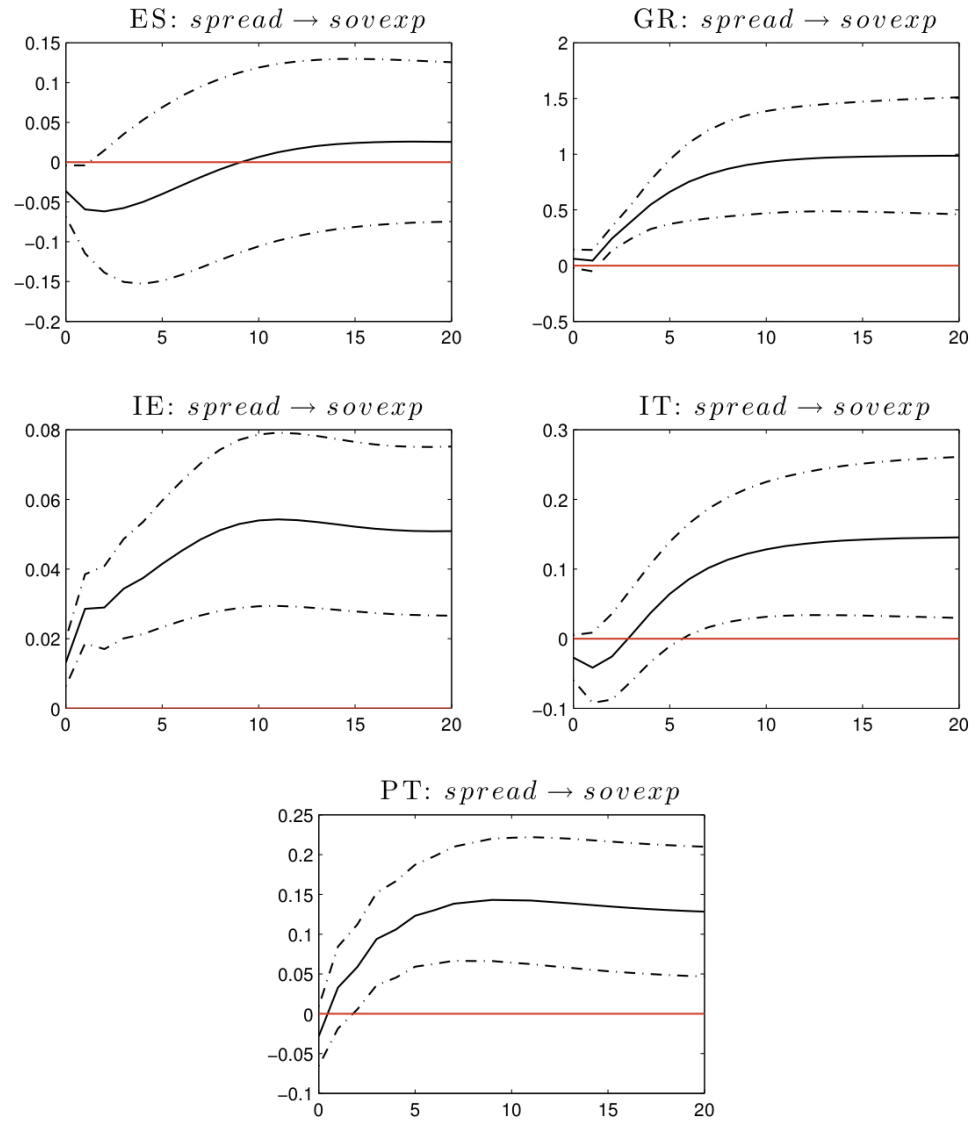
The estimated cointegrating parameter β in the baseline model (column 1 of Table 2.2) is negative and significant in all countries except Belgium, where it is negative but not significant, and France and the Netherlands, where it is positive and significant. This indicates that for most countries in the long run a higher yield spread is associated with a greater sovereign domestic exposure of banks. It is interesting to notice that evidence for a positive long-run correlation is stronger for the periphery countries than for the core countries. The estimated adjustment parameter α in column 2 is negative and significant at the 5 percent level in all countries, except France and the Netherlands, where it is significant at the 10 percent level, and Belgium, where it is not significantly different from zero.²⁷ Finally, the long-run effect of a shock to the yield differential on sovereign exposures is given by the product of the vectors α and β , and is positive for all countries except Belgium, France and the Netherlands: in all countries except these three, a rise in the domestic yield differential prompts an increase of the domestic sovereign exposure of local banks, and their gradual adjustment to a higher steady-state level.

These results are consistent with the impulse response functions (IRFs) of the domestic sovereign exposure to a shock in the yield differential shown in Figures 2.12 and 2.13. The IRFs are obtained from a structural VECM specification of the baseline model, in which we impose the restriction that a shock to exposures cannot determine a contemporaneous effect on the yield differential, while the change in the domestic sovereign yields can immediately affect the corresponding domestic sovereign exposures.²⁸ The economic rationale of this identifying assumption is that, since domestic exposures are measured at market values, they immediately reflect changes in the yield of domestic sovereign debt, even if banks do not react to the yield change by restructuring their portfolio. Instead, changes in the amount of domestic sovereign debt owned by banks affect sovereign yields

²⁷The estimates indicate that domestic sovereign exposures adjust faster in response to shocks in program countries: *soverp_t* adjusts by more than 37 and 14 percent towards its equilibrium level within a month in Greece and Portugal, respectively. Though Germany also features a high speed of adjustment, most core countries have a slower adjustment than periphery countries.

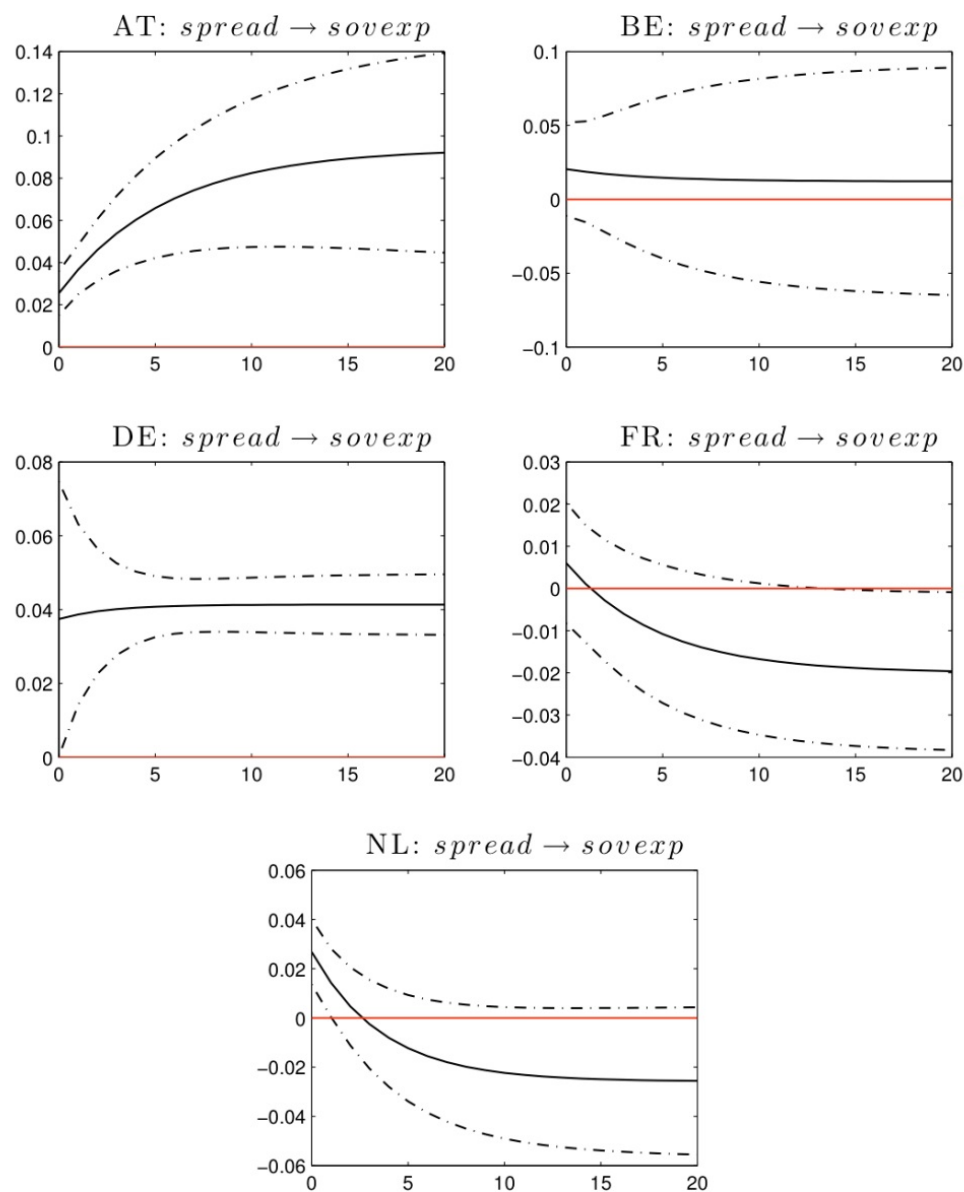
²⁸Short-run and long-run linear restrictions are imposed following the methodology described, for instance, in Vlaar (2004), based on the scoring algorithm originally proposed by Amisano and Giannini (1997).

Figure 2.12: IRFs of sovereign exposures to shocks in yield differentials: periphery countries



Sources: Each chart reports point estimates (solid line) and 90% studentized bootstrap confidence intervals (dashed lines) of the respective IRFs.

Figure 2.13: IRFs of sovereign exposures to shocks in yield differentials: core countries



Sources: Each chart reports point estimates (solid line) and 90% studentized bootstrap confidence intervals (dashed lines) of the respective IRFs.

only gradually.²⁹

In Figure 2.12 and 2.13, the solid line indicates the predicted response, while the dashed lines plot the respective 90% studentized bootstrap confidence bounds.³⁰ In the long run, in periphery countries domestic sovereign exposures respond positively to an increase in the yield differential, the response being statistically significant for all countries except Spain.³¹ In core countries, the response is positive for Austria and Germany, whereas it is negative for France and insignificant for Belgium and the Netherlands. In Italy and Spain, the response features a small initial drop in exposures, which is reversed within a few months. This initial dip may reflect the mechanical impact of an increase in domestic yields, which is equivalent to a drop in the price of domestic debt: such a price drop, if not sufficiently compensated by a buildup in exposures, mechanically translates into a drop in the market value of sovereign exposures. The much smaller response for Ireland is probably explained by the fact that the Irish banking sector is dominated by the offshore activities of global banks, Ireland being a giant offshore centre whose aggregate financial sector is detached from the local economy; but unfortunately separate data for local Irish banks are not available.

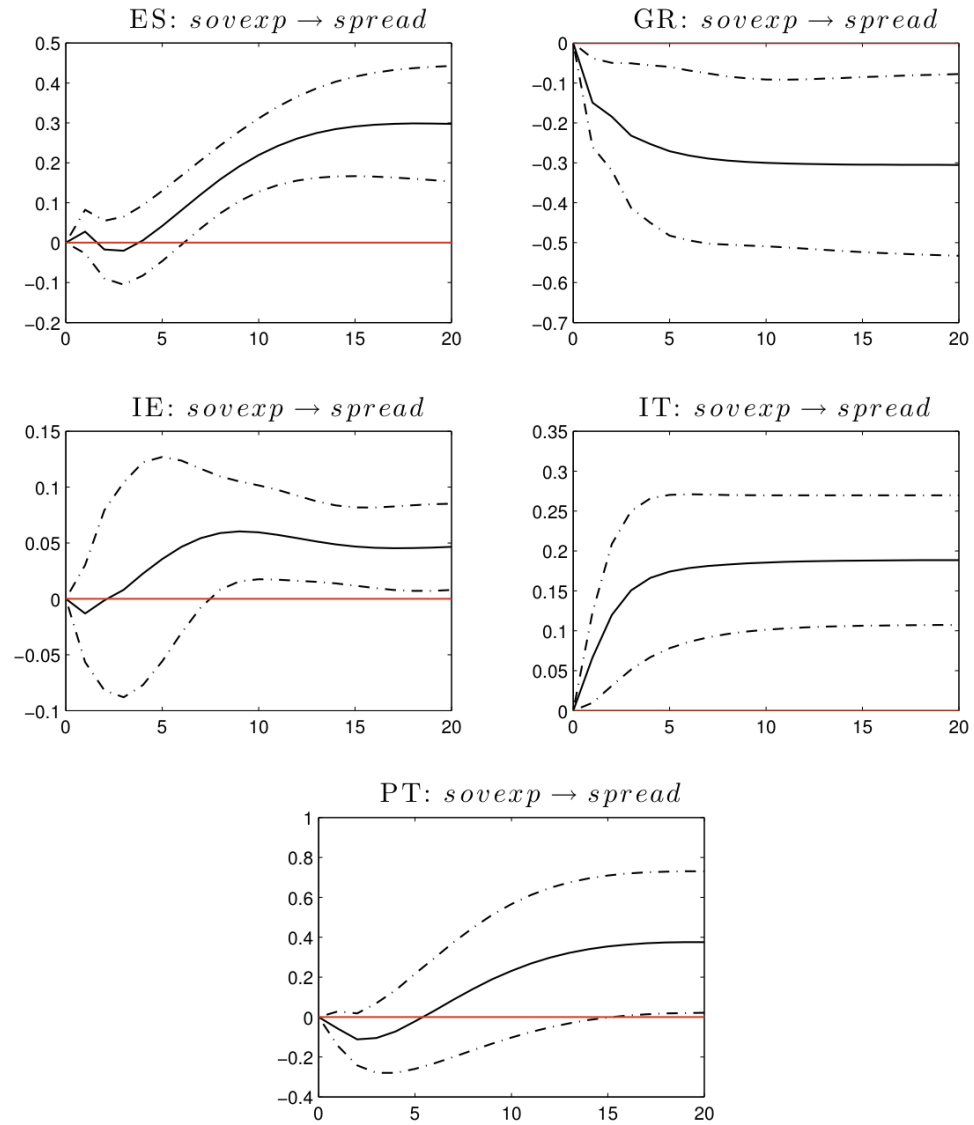
Further, we investigate the effect of domestic sovereign exposures on yield differentials by looking at the IRFs of the yield differential to a shock in domestic exposures. As illustrated in Figures 2.14 and 2.15, core countries (except Austria and Belgium), together with Greece, show a negative long-run response of their domestic differentials to an increase in domestic exposures. Hence, in these countries, increases in banks' domestic exposures effectively curb investors' concerns over sovereign solvency and contribute to tightening yield differentials. However, in periphery countries (except Greece) as well as Austria and

²⁹ Although euro area banks are important players in the market for domestic sovereign debt, their holdings typically do not exceed one fourth of the total stock of debt. Between the third quarter of 2010 and 2011, banks' average holdings of domestic euro-area sovereign debt, as a percent of the corresponding country's sovereign debt, were 13.32% for Austria, 25.73% for Belgium, 27.98% for Germany, 22.96% for France, 21.15% for Ireland, 21.15% for Italy, 10.65% for the Netherlands, and 23.02% for Portugal.

³⁰ Studentized bootstrap confidence intervals are computed with 2,000 replications. Results do not change when the number of replications is either smaller (1,000) or larger (3,000).

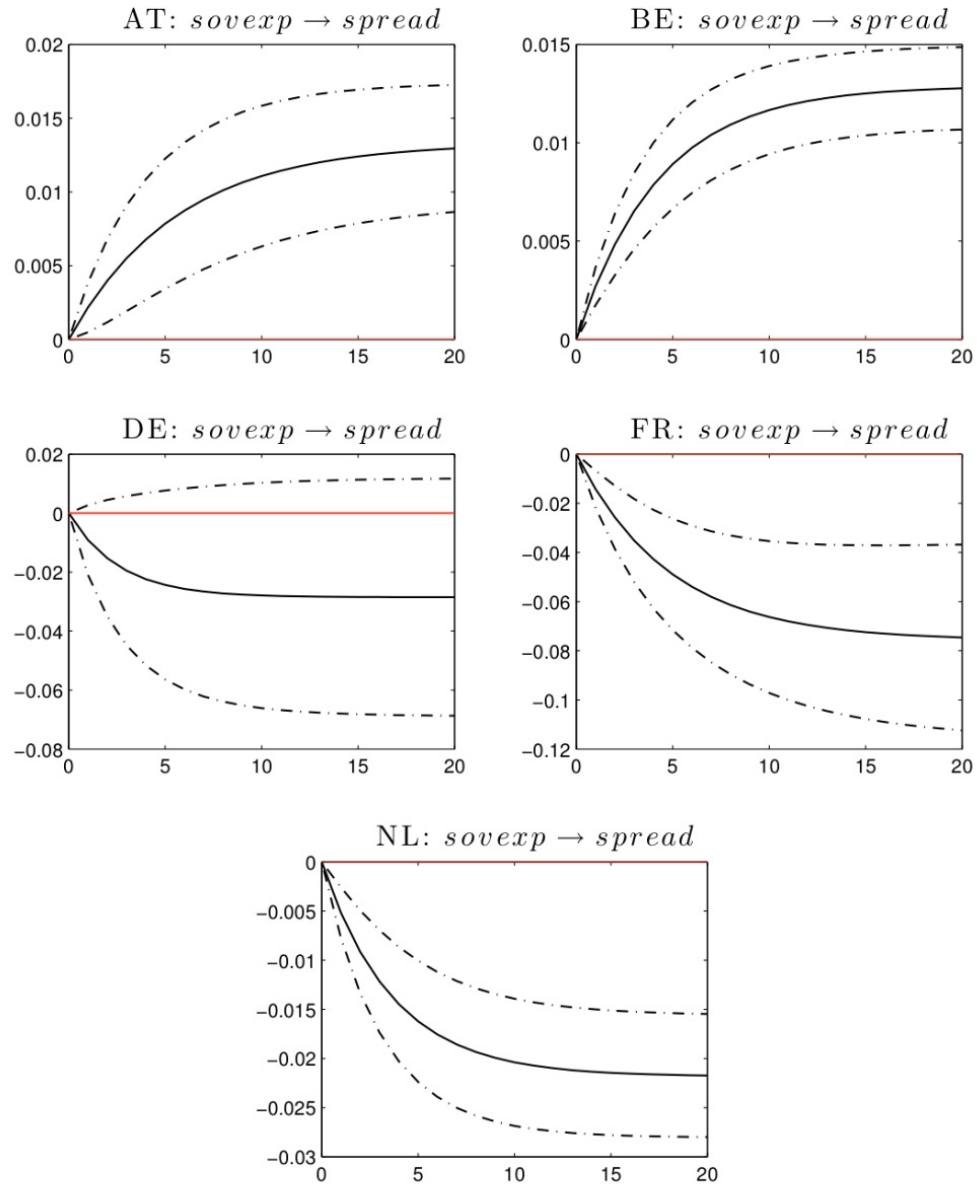
³¹ However, the IRF for Spain is not only positive but also significant if the *wit* dummy is excluded from the VECM.

Figure 2.14: IRFs of yield differentials to shocks in sovereign exposures: periphery countries



Sources: Each chart reports point estimates (solid line) and 90% studentized bootstrap confidence intervals (dashed lines) of the respective IRFs.

Figure 2.15: IRFs of yield differentials to shocks in sovereign exposures: core countries



Sources: Each chart reports point estimates (solid line) and 90% studentized bootstrap confidence intervals (dashed lines) of the respective IRFs.

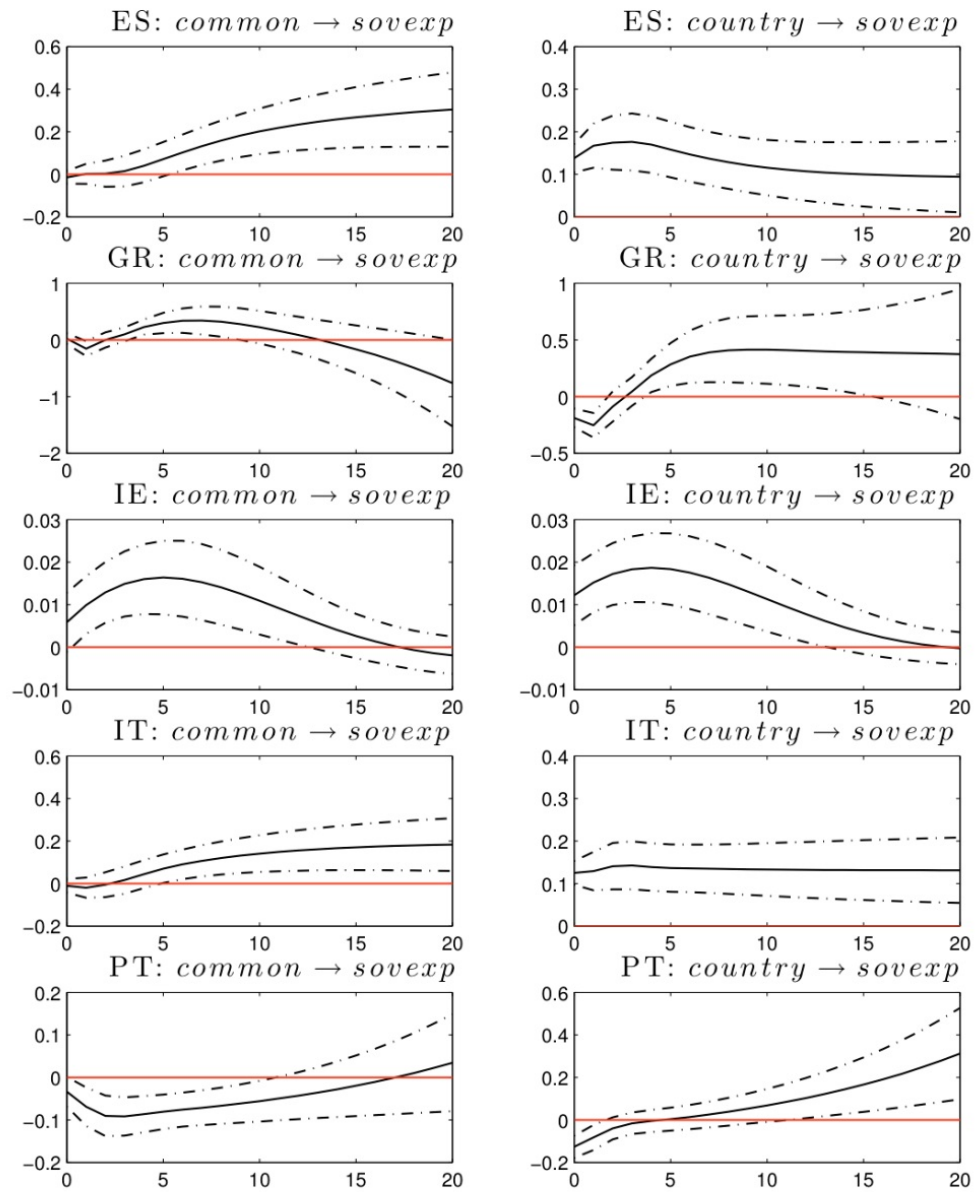
Belgium, a shock in banks' sovereign exposures appears to trigger an increase of the domestic yield differentials. A possible interpretation is that a greater bank exposure to sovereign risk increases investors' concerns about the solvency of the banks themselves and therefore about their eventual bailout by the respective government, thus prompting the market to require a higher yield on domestic sovereign debt.

Turning to the factor-based model (whose estimates are shown in columns 3-6 in Table 2.2), for the sake of brevity it is worth focusing directly on the product of the coefficient vectors α and β , which captures the dynamic response of domestic sovereign exposures to the common component (columns 3-4) and to the country component (columns 5-6) of the yield differential. The response to the common risk factor is positive and significant for all countries except Italy and the Netherlands (where it is not significant but still positive) and Portugal and Germany (where it is negative but not significant). This indicates that for most countries when there is an increase in common risk, local banks increase the home bias of their sovereign debt portfolios, consistently with the “comparative advantage” hypothesis. In contrast, the response to the country risk factor differs considerably across countries: in core countries (except France), an increase in country risk prompts local banks to reduce their domestic exposures, while in periphery countries it leads local banks to increase their domestic exposures.

However, the product of the coefficients α and β does not provide a full account of the dynamic response of domestic sovereign exposures to shocks in the common and country components of the yield spread. To this purpose, we identify structural IRFs by imposing the following restrictions:

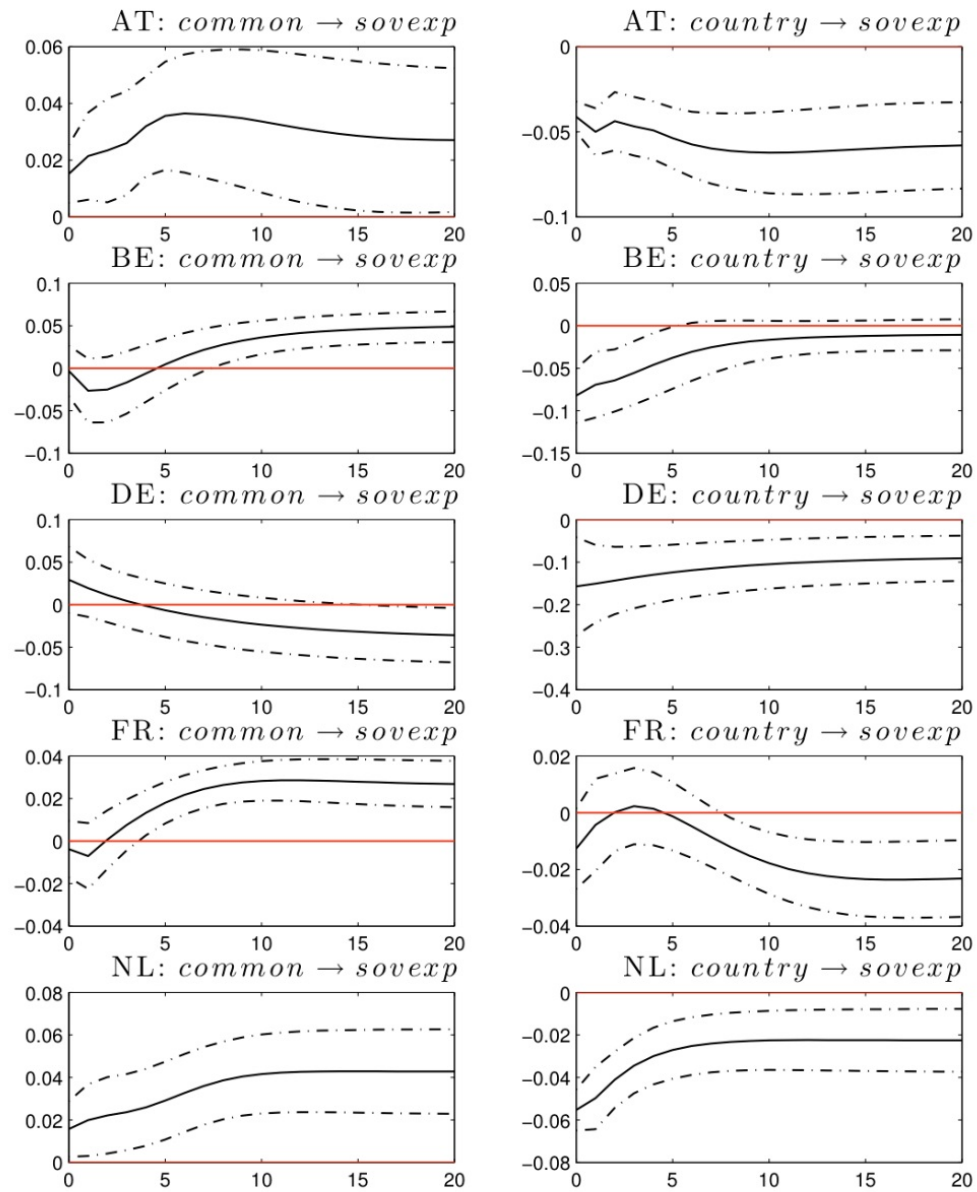
- (i) only the common and the country shocks may have a permanent effect on sovereign exposures;
- (ii) the common and the country shocks do not contemporaneously affect each other;
- (iii) a shock in the domestic sovereign exposure has no contemporaneous impact on the common factor.

Figure 2.16: IRFs of sovereign exposures to shocks in the common and country components of yield differentials: periphery countries



Sources: Each chart reports point estimates (solid line) and 90% studentized bootstrap confidence intervals (dashed lines) of the respective IRFs.

Figure 2.17: IRFs of sovereign exposures to shocks in the common and country components of yield differentials: core countries



Sources: Each chart reports point estimates (solid line) and 90% studentized bootstrap confidence intervals (dashed lines) of the respective IRFs.

The resulting IRFs are shown in Figures 2.16 and 2.17, where the graphs on the left show the response to a shock in the common factor, and those on the right the response to the country factor. The common risk factor leads to a significant increase in domestic sovereign exposures in all the core countries (except Germany, where the response is negative but not statistically significant). The same applies to periphery countries (except Portugal), although initially Greek and Italian banks feature a dip in their domestic sovereign exposure (again, possibly explained by the mechanical impact of the drop in price on the value of their exposures). Hence the IRFs confirm that in most countries an increase in systemic risk leads to an increase in domestic exposures.

The country risk factor prompts domestic sovereign exposures to decrease significantly in the core countries (except Belgium, where the response is also negative but not significant), and to increase in the periphery. Hence, for the periphery countries the evidence cannot be explained only by the “comparative advantage” hypothesis, which predicts a positive response of exposures only to the common factor. Since exposures appear to increase also in response to increases in country-specific risk, in the euro area periphery the “moral suasion” or/and the “carry trade” hypotheses must have played a role.

2.5 Summary and policy implications

This paper analyses the dynamics of sovereign yields in the euro area crisis that unfolded since 2007 and the concomitant reshuffling of banks’ sovereign debt portfolios, and the relationship between these two phenomena. We proceed in two steps.

First, using a dynamic factor model we decompose yield differentials in a country-specific and a common (or systemic) risk component, in order to assess to what extent the increase in euro area yield differentials is a reward for differential default risk as opposed to a reflection of the differential exposure to common risk. Our estimate of the common risk factor correlates closely with two indicators of investors’ concerns about the danger of breakup of the euro area, one being the frequency of relevant terms searches in Google and the other being the euro area breakup probability drawn from a prediction market.

Next, we investigate how the changes in the exposures of banks to domestic sovereign risk is related to the changes in yield differentials and in their two components, as estimated in the previous step. We perform this second step by estimating a vector error-correction model on 2007-13 monthly data. The domestic sovereign exposures of banks in most euro area countries turn out to respond positively to increases in yields, especially in periphery countries. When yield differentials are decomposed in their country-risk and common-risk components, we find that: (i) in all periphery countries, banks respond to increases in country risk by increasing their domestic exposure, while in core countries they do not; (ii) in contrast, in most euro area countries banks react to an increase in the common risk factor by raising their domestic exposures.

Finding (i) indicates that, in the euro area periphery, banks responded to increases in their own sovereign's risk by increasing even further their exposure to such risk, in line with the "moral suasion" and the "carry trade" hypotheses. Finding (ii) indicates that most euro area banks have responded to greater systemic risk by increasing the home bias of their portfolios, consistently with the "comparative advantage" hypothesis. Each of these findings is problematic from a policy standpoint and, also depending on its interpretation, has different implications for policy.

2.5.1 Dealing with "moral suasion by regulators"

Suppose that our finding (i) – namely, that periphery banks have increased their domestic sovereign exposures in response to a rise in their relative yield – is due to moral suasion by their regulator, concerned by the distressed state of the domestic sovereign's finances – consistently with the findings by [Drechsler et al. \(2013\)](#) for periphery countries. Under this interpretation, regulators themselves prompted banks to increase their domestic sovereign exposures in situations where government solvency was already at danger, thus enhancing the "diabolic loop" between fiscal solvency and bank solvency deterioration. This problem, if present, should be eliminated or at least mitigated by the introduction of the planned euro area banking union: the ECB acting as "single supervisor" would likely be more

insulated from the pressures of national governments than national banking supervisors. The rationale for this impending policy change is reinforced by the fact that it is becoming increasingly clear that, when euro area governments are fiscally distressed, they are no longer the only ultimate backstops of their domestic banks, as illustrated by the contribution of the European Stability Mechanism (ESM) to the recapitalization of Spanish banks since late 2012: it is then consistent that, *ex ante*, a euro area bank supervisor should constrain the bets that euro area banks, especially distressed ones, can take on the bonds issued by their equally distressed sovereign.

2.5.2 Dealing with “search for yield by banks”

Our finding (i) could equally well be interpreted as the result of periphery banks increasing their sovereign exposures to search for yield, especially considering that many of these banks were undercapitalized and could borrow cheaply from the ECB: if successful, their sovereign-debt carry trades would help them to shore up their capital ratios. Indeed, [Acharya and Steffen \(2015\)](#) and [Buch, Koetter and Ohls \(2013\)](#) provide evidence that banks that are less capitalized banks and depend more on wholesale funding invest more in sovereign debt than others. A variant of this “carry trade” story, which is popular among euro area bankers, goes as follows: “if my sovereign defaults, also my bank goes under, so I can ignore the default risk of my own sovereign”. This argument may contribute to explain why carry trades by banks have been far more prevalent in fiscally distressed countries than in fiscally sound countries. While such behaviour may appear rational from a bank’s individual standpoint, it is no less inefficient for society than if it were motivated by plain moral hazard: since it leads the banks of the fiscally distressed country to overexpose themselves to sovereign risk, it also makes them more likely to require a bailout in the event of an increase in domestic yields. Insofar as this increases their demands on the public finances of their country in bad states of the world, it also exacerbates the chances that their sovereign will be distressed. In other words, however motivated, banks’ carry trades strengthen the diabolic loop between financial instability and fiscal distress. These carry trades also have severe implications

for the real economy: banks with sizeable exposures to impaired sovereign debt have been forced to curtail their lending to firms and households in 2010 and 2011 (Bofondi, Carpinelli and Sette, 2013, De Marco, 2013, and Popov and Van Horen, 2013), in turn leading firms to significantly reduce their investment, employment and sales growth (Acharya et al., 2013).

Discouraging carry trades would require revising the prudential regulation of sovereign exposures in the euro area, by scrapping the current preferential treatment of sovereign exposures: currently, euro area banks face no capital requirement (a “zero risk weight”) for holdings of sovereign euro area debt, irrespective of its issuer;³² moreover, sovereign holdings are exempted from the “large exposures regime”, which limits exposures to a single counterparty to a quarter of their eligible capital. Such regulation makes it particularly attractive for euro area banks to invest in high-yield euro-denominated sovereign debt, especially considering that they can fund such investments by borrowing at low rates from the ECB. This problem is acutely perceived by policy makers, as witnessed by ECB President Draghi’s statement on 5 December 2013: “If we do operations similar to LTRO, we want to make sure this is being used for the economy. We want to make sure that this operation is not going to be used for subsidizing capital formation by the banking system under these carry-trade operations.”³³

In principle, such carry trades can be discouraged either by imposing positive risk weights on sovereign debt in computing banks’ capital or by imposing limits on banks’ exposure towards each single sovereign issuer, hence requiring them to diversify their sovereign portfolios. Each of these two choices is not without problems: on one hand, the responsiveness of banks’ portfolio choices to the level of risk weights on sovereign exposures is unknown, and in practice may be quite low in the presence of very profitable carry trades, so that risk weights could prove ineffective; on the other hand, setting limits to exposures *vis-à-vis* each single sovereign issuer would require most euro area banks to undertake very

³²Specifically, euro area sovereign debt carries a zero risk weight in the computation of the “risk-weighted assets” that are used to determine the capital required from a bank for prudential purposes according to the so-called “standardized approach”. Alternatively, banks can opt for the “internal ratings-based approach”, namely construct an internal risk model to determine the risk weight that they wish to attach to each type of sovereign debt in computing their risk-weighted assets.

³³*Bloomberg News*, “Draghi Hints Any New Liquidity Tools Will Be Conditional”, December 5, 2013.

substantial portfolio adjustments, which may result in gyrations in relative yields in the euro area sovereign debt market.

However, there are ways to guide the banks' portfolio reallocation process smoothly in the direction of greater diversification: for instance, the limit on sovereign exposures could be phased in very gradually; moreover, euro area banks may be exempted from this limit altogether insofar as they were to invest in a well-diversified portfolio of euro area sovereign bonds rather than in those issued by a specific sovereign. In this respect, the portfolio reallocation process could be made smoother by the introduction of European Safe Bonds, as proposed by the Euro-nomics Group: a European Debt Agency (EDA) could buy a GDP-weighted portfolio of bonds from euro area sovereigns, and use them as collateral to issue two securities. The first security, European Safe Bonds or ESBies, would be a senior claim on the payments from the sovereign bonds held in the portfolio. The second security, European Junior Bonds, would have a junior claim on these payments – that is, it would be first in line to absorb whatever loss is realized in the pool of sovereign bonds that serve as collateral for these issues. That is, any failure by a sovereign state to honour in full its debts would be absorbed by the holders of the junior tranche security, not by the EDA, any euro area entity or the European Union. Owing to the diversification of country-specific risk and to their seniority, ESBies would have virtually no exposure to sovereign risk, and therefore would be an ideal asset for euro area banks to diversify their sovereign portfolios.³⁴

2.5.3 Dealing with the fallout of redenomination risk

What about the policy implications of our finding (ii) – namely, that even in core countries euro area banks have responded to greater systemic (or redenomination) risk by increasing the home bias of their sovereign portfolios? As already mentioned, this response would appear completely consistent with economic rationality and market equilibrium: in the event of euro breakup, the banks of each country would be better positioned to bear the brunt of redenomination of domestic sovereign debt in the new national currency, as their deposits

³⁴See <http://euro-nomics.com/http://euro-nomics.com/2011/european-safe-bonds/> for a more detailed description of this policy proposal.

would also be redenominated in the new currency. Insofar as redenomination risk gives them a “comparative advantage” in holding domestic debt relative to foreign banks, home bias in the euro area sovereign debt market is an equilibrium phenomenon. Incidentally, such an outcome has probably been reinforced by “ring-fencing” by the regulators of core countries, who are often reported to have pressured the banks under their supervision to shed periphery-country debt in favor of core-country debt, in late 2010 and 2011.

The only way to address this source of segmentation of euro area sovereign bond markets – and more generally of euro area debt markets – is to address the credibility issue, as was done by Draghi’s “whatever-it-takes” July 2012 speech and subsequent inception of the Outright Monetary Transactions (OMT) program: by creating the credible threat that the ECB could buy the sovereign debt of distressed euro area countries, the ECB reduced investors’ estimate of the probability of a possible euro breakup. Nevertheless, the degree of segmentation of euro area debt markets remains high: in each member country, banks are still the almost exclusive source of funding for both the domestic sovereign and the local private sector, so that their private-sector lending tends to be more severely crowded-out in countries with larger stocks of public debt such as Italy and Greece. At the same time, even though cross-country differences between domestic interest rates have considerably abated, at the time of writing they are still non-negligible, and may spike again if investors’ concerns about the survival of the euro were to reignite.

Chapter 3

Sovereign default, market sentiments and financial segmentation

3.1 Introduction

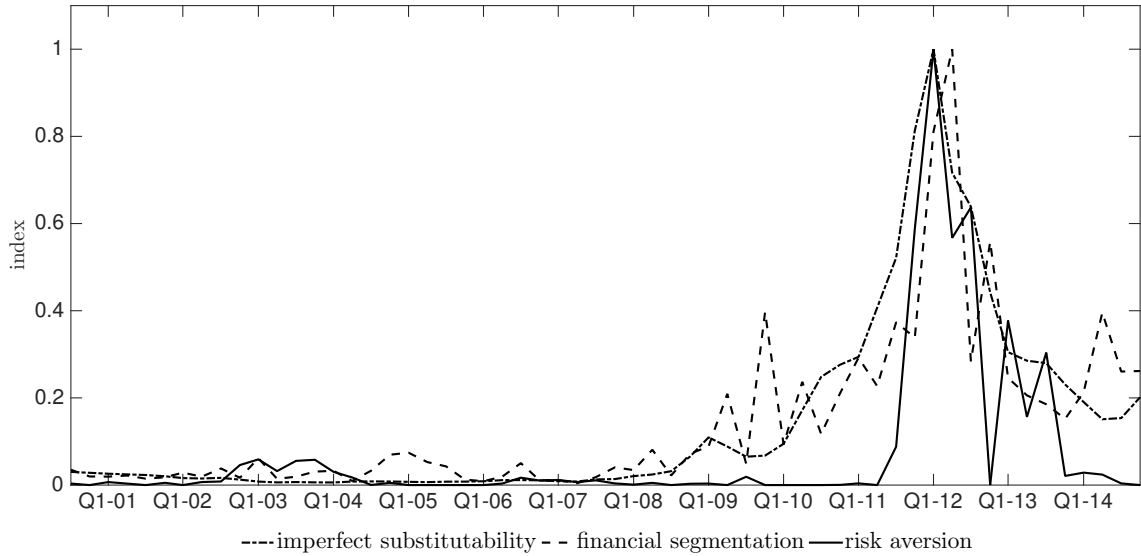
Market sentiments and financial integration play a crucial role in informing the pricing paradigm applied by investors to macroeconomic fundamentals. Extensive empirical evidence has shown three concomitant patterns occurred in the euro area sovereign debt markets since the inception of the European Monetary Union (EMU).¹

Pattern 1. International investors changed their pricing paradigm over time, whereby their risk aversion rose in the aftermath of the crisis. These developments led to a higher sensitivity of bond yields to macroeconomic fluctuations during the crisis.

Pattern 2. The process of financial integration in euro area sovereign debt markets incurred a dramatic reversal at the onset of the crisis, as bond yields increasingly reacted to country-specific factors.

¹A thorough discussion of the empirical literature on euro area sovereign debt markets and its relationship with the theoretical framework presented in this paper is provided in Appendix [B.1](#).

Figure 3.1: Patterns in the euro area sovereign debt markets



Sources: European Central Bank, DataStream and author's calculations.

Note: The indicators are rescaled so as to lie within the unit interval.

Pattern 3. Debt securities issued by different sovereign governments were persistently perceived as imperfect substitutes since the creation of the EMU. However, in the wake of the crisis, the perceived imperfect substitutability of government bonds sharply increased, as witnessed by the rising dispersion of cross-country differentials.

Figure 3.1 illustrates the dynamics for investor risk aversion, financial segmentation and imperfect asset substitutability in euro area sovereign debt markets in the period between 2001 and 2014.² During the first years of the EMU, sovereign debt markets featured low levels of risk aversion, financial segmentation and imperfect substitutability. Market sentiments and financial integration started worsening as the global financial crisis erupted in 2008 and further deteriorated at the inception of the euro area sovereign debt crisis in 2009. Eventually, euro area sovereign debt markets gradually stabilized after the heights of

²Following the European Central Bank (2006), the indicator of imperfect substitutability is computed as the cross-country standard deviation of government bond yield spreads, whereas the indicator of financial segmentation is calculated as the average distance of a linear regression coefficient from the values implied by complete integration. As described in Meucci (2007), the indicator of risk aversion is constructed as the weight placed on the covariance matrix by a representative investor solving a mean-variance portfolio optimization problem, given the historical time series for quantities, prices and residual maturities of euro area government bonds.

the crisis in the course of 2012. This account of the events surrounding the crisis reflects a broad consensus in the literature.³

However, the theoretical literature on international lending and sovereign default has mostly neglected the role of market sentiments and financial structure as determinants of bond yields dynamics.⁴ Models failing to account for these features of the sovereign debt market may not effectively inform the discussion on policy measures aimed at moderating financial turmoil and its backlash on the real economy.

The contribution of this paper to the literature extant is threefold. First, this paper bridges a gap of general equilibrium models of unsecured sovereign debt and default by incorporating market sentiments – notably, in the form of investor risk aversion, perceived financial segmentation, and imperfect asset substitutability – within a multi-country model *à la* [Eaton and Gersovitz \(1981\)](#). Second, this paper contributes to the empirical literature on the euro area sovereign debt markets by proposing a theoretical framework able to explain the three typical patterns identified in euro area sovereign debt markets. Third, the model posits a simple portfolio selection strategy alternative to the mean-variance theory commonly used in the literature on open economy financial macroeconomics.

The main point of departure of this model from the standard literature on endogenous sovereign default concerns the assumption on investors’ optimal portfolio strategy. In the multi-country (or union) economy, capital markets are incomplete and government bonds issued by different sovereign governments represent future claims on heterogeneous goods. Hence, international investors perceive different sovereign debt securities as imperfect substitutes and compose their sovereign debt portfolios by maximizing a utility function exhibiting constant elasticity of substitution (CES) between bonds. The optimal solution of investors’ portfolio composition problem determines a downward-sloping demand function for each debt security. As standard in the macroeconomic literature, a unique param-

³For an encompassing description of the early stages of the euro area sovereign debt crisis, see [Lane \(2012\)](#) among many others.

⁴The bulk of theoretical models characterizes investors as risk neutral agents in perfectly competitive capital markets. One exception is the work by [Lizarazo \(2009, 2013\)](#), who considers risk averse investors but does not account for alternative financial market structures or imperfect asset substitutability, as discussed below in Section [3.2](#).

ter identifies the elasticity of substitution between bonds as well as the elasticity of the demand for each individual bond with respect to its (relative) price, that is the price elasticity of bonds. Crucially, the same parameter may be interpreted as (negatively) related to the degree of investor risk aversion, perceived financial segmentation, and imperfect asset substitutability at the same time.

The implications of this modelling choice on equilibrium dynamics may be understood by focusing on an environment with unfavorable market sentiments, thus characterized by a low elasticity of substitution between bonds. Notice that the bond demand function establishes a direct link between equilibrium quantities and prices for domestic sovereign debt: a low price elasticity of bonds corresponds to a high bond elasticity of prices. Therefore, a low price elasticity of bonds (i.e., a high bond elasticity of prices) entails large movements of interest rates in response to small variations in the amount of optimal debt chosen by the government to insure consumption against volatility in the stochastic endowment process of the domestic economy. In other words, adverse market sentiments amplify the impact of macroeconomic fluctuations on government bond yields.

The numerical evaluation of the model confirms these results. Monte Carlo simulations show that a low elasticity of substitution is associated with highly volatile and counter-cyclical interest rates. Further, a low elasticity generates low debt levels and high interest rates and default probabilities on average. This result is due to the upward rigidity of bond prices: as the latter are prevented from exceeding unity (i.e., negative interest rates are not allowed), when the elasticity is low, the government cannot freely adjust the marginal cost of an additional unit of debt, so that in the long run it faces tight borrowing constraints and high incentives to default. In this light, the amplification mechanism induced by adverse market sentiments acts in an asymmetric fashion.

Finally, in an empirical assessment, the baseline calibration of the model targets first and second moments of Greek data between 1999Q1 and 2014Q4. Further, the model is evaluated against Greek empirical dynamics in two sub-periods, namely the pre-crisis (1999Q1-2009Q3) and the crisis (2009Q4-2014Q4) periods. The separation of the full sam-

ple into two sub-periods is based on the widespread view that identifies the unexpected announcement by the Greek government of an upwards revision in the headline deficit in 2009Q3 as the event marking the shift in market sentiments that triggered the euro area sovereign debt crisis. In the baseline calibration, the model successfully captures the mean and the standard deviation of Greek spreads, their correlation with real GDP, as well as the frequency of default. In addition, different values in the elasticity of substitution explain the large discrepancy in the standard deviation of government bond yields and their sensitivity to macroeconomic fluctuations before and during the crisis without compromising other desirable empirical features.

The remainder of the paper is organized as follows. Section 3.2 reviews the related literature on models of sovereign default and international lending. Section 3.3 presents the model and discusses its analytical features. Section 3.4 contains numerical and empirical assessments of the model. Section 3.5 provides some concluding remarks.

3.2 Related literature

This paper contributes to the literature on sovereign default and international lending, and extends the models described in the seminal works by [Eaton and Gersovitz \(1981\)](#) and [Arellano \(2008\)](#). In the baseline framework, the sovereign government of a small open economy determines its borrowing and default policies on the basis of the realization of a stochastic stream of income and the level of outstanding debt seeking to smooth the domestic household's consumption. A positive feature of this class of models is its fair ability to replicate several dynamics and moments of macroeconomic variables, notably in emerging countries.⁵ Numerous extensions have been proposed in order to enrich the description of the economic environment and overcome some theoretical flaws and empirical shortcomings of the baseline model.

A strand of studies has developed on the basis of the analyses by [Bulow and Rogoff \(1989a\)](#) and, more recently, [Schumacher, Trebesch and Enderlein \(2013\)](#), who argue that

⁵See, for instance, [Neumeyer and Perri \(2005\)](#).

direct sanctions available to creditors (e.g., legal procedures and seizure of assets held abroad), compared to the loss of reputation, provide better incentives for the repayment of delinquent debt. Hence, the design of debt contracts has been modified so as to include the expectation of bailouts ([Aguiar and Gopinath, 2006](#)), renegotiation procedures ([Yue, 2010](#), [Arellano and Bai, 2014b](#)), long-term borrowing ([Hatchondo and Martinez, 2009](#), [Chatterjee and Eyigungor, 2012](#), [Arellano and Ramanarayanan, 2012](#)), domestic private debt ([Arellano and Kocherlakota, 2014](#)), voluntary debt exchanges ([Hatchondo, Martinez and Sosa Padilla, 2014](#)), and information revelation ([Sandleris, 2008](#)) as mechanisms defining incentives for the repayment of non-enforceable foreign debt.

Moreover, a limitation of standard models lies in the assumption of exogenous output costs of default, as argued by [Bulow and Rogoff \(1989b\)](#), [Borensztein and Panizza \(2009\)](#), and [Levy-Yeyati and Panizza \(2011\)](#). Along these lines, recent models incorporate endogenous output costs of default triggered by an inefficient substitution of imported inputs ([Mendoza and Yue, 2012](#)), an optimal devaluation ([Na et al., 2015](#)), an interbank market freeze ([Engler and Große Steffen, 2016](#)), and a private credit contraction ([Sosa Padilla, 2013](#)). These models are typically based on two features – namely, the link between the financial and the real economy through working capital financing and high costs of default – with desirable implications – namely, the containment of the number of state variables and high debt-to-output ratios, respectively.⁶

Further, a growing body of work has explored fiscal policy responses to adverse shocks ([Cuadra, Sanchez and Sapriza, 2010](#), [Gonçalves and Guimaraes, 2015](#)), investigated the effects of political instability ([Cuadra and Sapriza, 2008](#)), news shocks ([Durdu, Nunes and Sapriza, 2013](#)), and investor risk aversion ([Lizarazo, 2013](#)), as well as extended the scope of the analysis to a multi-country framework ([Arellano and Bai, 2014a](#))

This paper relates specifically to [Lizarazo \(2009\)](#), who studies risk averse investors in a multi-country framework. Some of the empirical features are shared by the two frameworks: notably, a higher risk aversion implies larger volatilities in the price of debt and the current

⁶On the relevance of computational efficiency, see [Hatchondo, Martinez and Sapriza \(2010\)](#).

account. However, the models differ in at least two related aspects. In [Lizarazo \(2009\)](#), financial markets are assumed to be perfectly competitive and thus create the usual credit supply schedule with infinite elasticity. As a consequence, investors' preferences generate an excess risk premium that compensates them for holding a disproportionate amount of non-enforceable debt. In contrast, this model advances an alternative design for the process of quantity and price formation, whereby a regime of monopolistic competition among sovereign governments implies a downward sloping demand curve for bonds. By internalizing the effects of its default option and exerting its market power, the sovereign can impose a “negative” risk premium, thus adding upward pressure on the bond price. Hence, compared to the standard analytical framework, this model provides an explanation for the concomitant dynamics of market sentiments and financial integration identified by the empirical literature.

3.3 The model

Consider a large region (or *union*) composed of an infinite number of small open economies. The size of the union is normalized to unity. Every economy receives a stochastic stream of income and endowments are heterogeneous across countries. In each country, the sovereign trades one-period discount bonds with risk-averse perfectly-competitive international investors, in order to insure domestic households against volatile income shocks and smooth their consumption patterns over time. Since each debt security is a non-contingent claim to one unit of output, union-wide financial markets are incomplete.

Two types of financial frictions affect the economic region's market for government bonds. First, debt contracts are not enforceable and governments lack any commitment technology. Thus, a sovereign may default on the total amount of outstanding debt at any time. Second, government bonds represent future claims on differentiated goods, so that they are perceived as imperfect substitutes. As investors are risk averse, they want to hold a positive quantity of all the available securities, and thus seek to diversify their sovereign debt portfolios, allocating their intratemporal expenditures according to downward-sloping

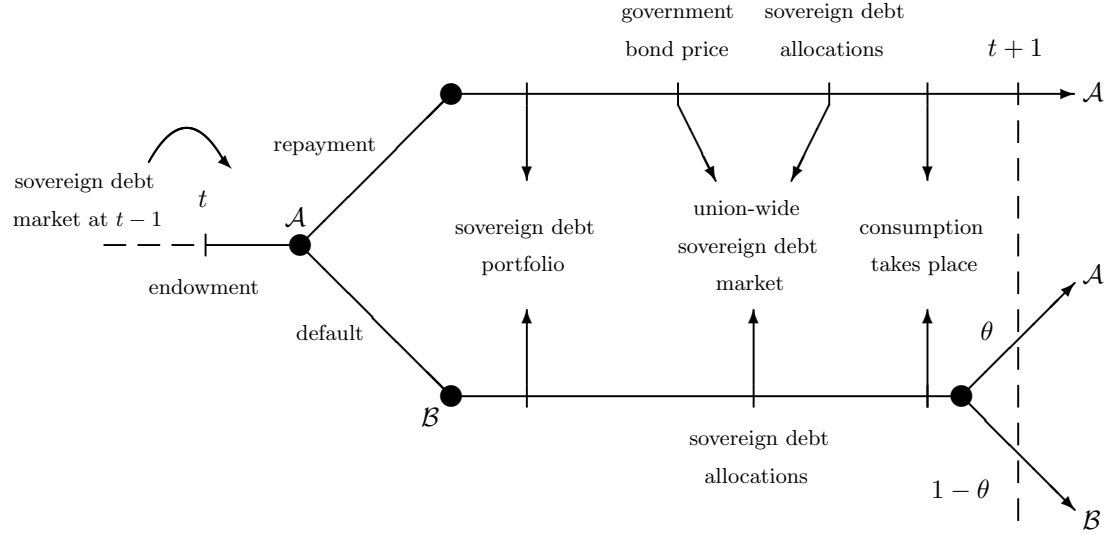


Figure 3.2: Timing of events in country i at period t .

demand functions for government bonds. Hence, risk aversion affects lenders' behavior along the intratemporal dimension, whereas along the intertemporal dimension investors are assumed to be risk neutral (namely, there are no profit smoothing motives driving investors' optimal decisions).⁷

In this section, I first lay out the timing of events; then, I describe the optimal decisions for international lenders in the union-wide sovereign debt market and for the domestic household and government in every country; finally, I define the equilibrium of the union economy.

The timing of events is illustrated in Figure 3.2. Events simultaneously occur in every country at each stage. At the beginning of period $t \in [1, \infty)$, the state of the economy in country $i \in [0, 1]$ is characterized by the sovereign debt market conditions (namely, bond prices and quantities) determined at $t-1$. Following the endowment shock, country i 's sovereign stands at node A and decides whether to repay or default on its outstanding debt.⁸ If the sovereign opts for repayment, the cross-border market for government bonds

⁷A model with intertemporal risk aversion in investors' preferences is presented by Lizarazo (2013).

⁸As recent episodes witness, sovereign credit events usually entail defaults on both foreign and domestic debt. However, a complete interruption of financial relationships between the government and domestic investors is not frequent: domestic investors usually suffer write-offs in old contracts and underwrite new

opens: first, international lenders choose the optimal amount of aggregate investments in government bonds (namely, their sovereign debt portfolios); second, taking investors' aggregate investments as given, the sovereign determines the optimal bond price; finally, a set of bilateral demand functions for bonds issued by every sovereign determines pairwise (namely, investors-to-sovereign) allocations and clears the union-wide sovereign debt market. Subsequently, the domestic household consumes the current endowment plus net capital inflows (namely, net proceedings of the domestic sovereign's trades in bonds with international creditors). Eventually, agents reach node \mathcal{A} next period and face the same problems.

If the sovereign opts for default, then it reaches node \mathcal{B} and does not participate in the union-wide sovereign debt market. Hence, no trades of domestic government bonds occur at time t and international investors purchase bonds issued by every other sovereign. Once transfers of bonds have realized, the domestic household consumes country i 's endowment net of exogenous costs of default. Eventually, agents in the defaulting country either face the same problems at period $t + 1$ as at period t and reach node \mathcal{A} with constant probability θ or they return to node \mathcal{B} with probability $1 - \theta$.

In this sense, the union-wide sovereign debt market features a Stackelberg game between governments and investors. Due to the cross-country heterogeneity of endowments and investors' appetite for diversification, sovereigns feature market power, thus acting as *leaders* and determining the optimal prices, taking into account their effects on investors' demand for government bonds. Given government bond prices, creditors can finally choose the amount of bonds purchased from each sovereign on the basis of bilateral demand functions. In what follows, I characterize the equilibrium strategic incentives implied by the framework *à la* Stackelberg by describing the problems of international investors, the household and the sovereign government.

contracts with their sovereign. Such feature of the relationship between domestic investors and government often implies dramatic consequences in terms of production and welfare. In the model presented herein, output costs of default are modeled as exogenous ([Arellano, 2008](#), [Aguar and Gopinath, 2006](#)).

3.3.1 International investors

At time t , international investors purchase one-period discount bonds $b_{t+1}^i \geq 0$ issued by sovereign in country $i \in [0, 1]$ at price $q_t^i \in (0, 1]$. In equilibrium, investors follow a two-stage budgeting approach: first, they determine the optimal intertemporal choice on the aggregate amount of government bonds; second, they choose the optimal intratemporal allocation of expenditures among different government bonds. The two stages of the investor's problem are presented proceeding backwards.

International investors perceive bonds issued by different governments as imperfect substitutes, and diversify their sovereign debt portfolios according to the following CES preferences:

$$b_{t+1}^* = \left[\left(\frac{1}{\rho_t^*} \right)^{\frac{1}{\eta}} \int_0^1 (1 - d_t^i) h_t^i (b_{t+1}^i)^{\frac{\eta-1}{\eta}} di \right]^{\frac{\eta}{\eta-1}}, \quad (3.1)$$

where b_{t+1}^* is an aggregate index representing foreign lenders' sovereign debt portfolio, which combines their holdings of bonds issued by every sovereign i , whereas d_t^i and h_t^i are two indicator functions taking on value one if sovereign i defaults and has a good credit history, respectively, and zero otherwise. Formally,

$$h_t^i = \begin{cases} 1 & \text{if either (i) } d_{t-1}^i = 0 \text{ or (ii) } d_{t-1}^i = 1 \text{ with probability } \theta, \\ 0 & \text{if } d_{t-1}^i = 1 \text{ with probability } 1 - \theta. \end{cases}$$

Hence, investors account for the possibility of exclusion of country i 's government from the union-wide sovereign debt market, which occurs if the product $(1 - d_t^i)h_t^i$ is equal to zero. Finally, $\rho_t^* \equiv \int_0^1 (1 - d_t^i)h_t^i di$ is the tally of participating sovereigns and thus measures the size of the union's capital markets.

In order to determine their optimal allocation of expenditures, $\int_0^1 (1 - d_t^i)h_t^i q_t^i b_{t+1}^i di$, international lenders compose their equilibrium sovereign debt portfolios according to the

demand function below:

$$b_{t+1}^i = \left(\frac{q_t^i}{q_t^*} \right)^{-\eta} \frac{b_{t+1}^*}{\rho_t^*}, \quad (3.2)$$

where

$$q_t^* = \left[\frac{1}{\rho_t^*} \int_0^1 (1 - d_t^i) h_t^i (q_t^i)^{1-\eta} di \right]^{\frac{1}{1-\eta}} \quad (3.3)$$

is the union-wide aggregate bond price index.

Three considerations about investors' preferences are appropriate. First, any pair of bonds features the same constant elasticity of substitution $\eta > 1$. Parameter η , as a measure for the degree of substitutability between bonds, represents a gauge of international lenders' risk aversion and, thus, perceived segmentation of union-wide financial markets: the lower the η , the higher imperfect substitutability, risk aversion, and financial segmentation. Second, since income distribution problems among investors are neglected, b_{t+1}^* can be regarded as a multiple of a representative investor's portfolio (assuming the appropriate aggregation conditions to be fulfilled). Sovereign debt diversity can then be interpreted either as different investors purchasing different bonds, or as diversification on the part of each investor. Third, since imperfect substitutability between government bonds implies a downward-sloping demand function, an increase in the price of a government bond determines only a sizable change in its demand. In contrast, a raise in the price of one of two bonds that were perfect substitutes (for instance, due to perfect correlation) would result in the demand shifting entirely to the security whose price has not changed (as the demand function for each bond would be perfectly elastic).

In order to determine their optimal intertemporal choice on the aggregate amount of bonds b_{t+1}^* , international investors solve the recursive maximization problem below:

$$\Pi(s_t^*) = \max_{b_{t+1}^*} \left\{ \pi_t + \frac{1}{1+r} E_t[\Pi(s_{t+1}^*)] \right\} \quad (3.4)$$

where π_t denotes profits from trades in government bonds and, combining equations (3.1)-(3.3), can be written as

$$\begin{aligned}\pi_t &= \int_0^1 (1 - d_t^i) [(1 - d_{t-1}^i) h_{t-1}^i b_t^i - h_t^i q_t^i b_{t+1}^i] di \\ &= r_t^* b_t^* - q_t^* b_{t+1}^*,\end{aligned}\tag{3.5}$$

with $r_t^* \equiv (1/\rho_{t-1}^*) \int_0^1 (1 - d_t^i)(1 - d_{t-1}^i) h_{t-1}^i (q_{t-1}^i/q_{t-1}^*)^{-\eta} di > 1$ denoting the union-wide aggregate index for bond returns, and r the (net) risk-free interest rate. Notice that r_t^* moves inversely with government bond prices, namely, higher bond prices determine a higher bond price index and a lower bond returns index. Further, s_t^* denotes the state of the domestic economy at the beginning of period t . Since lenders are risk neutral along the intertemporal dimension, the first-order condition for problem (3.4) is the following zero expected-profit condition:

$$q_t^* = \frac{E_t r_{t+1}^*}{1 + r},\tag{3.6}$$

which equates the aggregate bond price index to the aggregate discounted expected return on union-wide government bonds.

3.3.2 The domestic economy

In country i and at period t , domestic households are identical and risk averse, and share the same preferences, given by

$$E_0 \sum_{t=0}^{\infty} \beta_i^t u_i(c_t^i),\tag{3.7}$$

where E_0 denotes the expectations operator, β_i the household's subjective discount factor, $u_i(\cdot)$ the strictly increasing and concave current-period utility function, and c_t^i consumption.

The government is benevolent and its objective is to maximize the utility of households. The sovereign determines its optimal policies by deciding (i) whether to repay or

default on the total amount of its outstanding debt and (ii) if repayment is optimal, the bond price. Let y_t^i denote a stochastic endowment, which follows a Markov process with transition probability function $p(y_t^i|y_{t-1}^i)$, and s_t^i the state of the economy when the country i 's credit history is in good standing (namely, $(1 - d_t^i)h_t^i = 1$). If country i can access union-wide capital markets, the domestic government determines the optimal bond price by solving the following problem:

$$U_i^R(s_t^i) = \max_{q_t^i} \{u_i(c_t^i) + \beta_i E_t[U_i(s_{t+1}^i)]\} \quad (3.8)$$

subject to the flow budget constraint

$$\begin{aligned} c_t^i &= y_t^i + q_t^i b_{t+1}^i - b_t^i \\ &= y_t^i + q_t^i \left(\frac{q_t^i}{q_t^*} \right)^{-\eta} \frac{b_{t+1}^*}{\rho_t^*} - \left(\frac{q_{t-1}^i}{q_{t-1}^*} \right)^{-\eta} \frac{b_t^*}{\rho_{t-1}^*}, \end{aligned} \quad (3.9)$$

where the second equality is obtained by substituting for b_{t+1}^i as given by equation (3.2). The government also faces a non-negativity constraint for consumption, $c_t^i \geq 0$, and two boundary conditions for the bond price, $0 < q_t^i \leq 1$. An upper bound on debt, namely, $b_{t+1}^i \leq B^i$, prevents the government from engaging in Ponzi schemes but does not bind in equilibrium. By means of equation (3.2), the latter condition translates into a constraint on the minimum value of the domestic bond price, namely, $q_t^i \geq Q^i$, where Q^i is a function of the aggregate variables and B^i .

If sovereign i defaults on its outstanding debt and cannot access capital markets (namely, $(1 - d_t^i)h_t^i = 0$), its continuation value after default is given by:

$$U_i^D(y_t^i) = u_i(c_{Dt}^i) + \beta_i E_t [\theta U_i(s_{0t+1}^i) + (1 - \theta) U_i^D(y_{t+1}^i)], \quad (3.10)$$

where $c_{Dt}^i = y_t^i - \phi(y_t^i)$ denotes household consumption under default, $\phi(y_t^i)$ the loss function of output under financial autarky, and s_{0t}^i the subset of states in s_t^i such that the domestic government's outstanding debt at period t equals zero. As argued by [Arellano \(2008\)](#), the

specification for consumption under default c_{Dt}^i should define a nonlinear relationship with output aimed at reducing the sensitivity of the value of financial autarky to shocks, thus increasing the menu of bond prices with positive but finite default probability.⁹ Following Chatterjee and Eyigungor (2012), the loss function is specified as $\phi(y_t^i) = \max\{0, \omega_1^i y_t^i + \omega_2^i y_t^{i^2}\}$; also, in the numerical evaluation of the model, only calibrations with $\omega_1^i < 0$ and $\omega_2^i > 0$ are considered, so that the cost is zero for $0 \leq y_t^i \leq -\omega_1^i/\omega_2^i$ and rises more than proportionately with output for $y_t^i > -\omega_1^i/\omega_2^i$.¹⁰

Completing the description of the model, at the beginning of time t , the domestic sovereign chooses whether to exert the default option. Hence, the sovereign solves the problem below:

$$U_i(s_t^i) = \max_{d_t^i} \{(1 - d_t^i)U_i^R(s_t^i) + d_t^i U_i^D(y_t^i)\}, \quad (3.11)$$

which denotes the general continuation value of the government before the determination of a default policy.

3.3.3 Default sets, default incentives and recursive equilibrium

As emerges from the model described above, the state of international investors (s_t^*) differs from the state of the domestic economy, whether in good credit standing (s_t^i), financial autarky (y_t^i) or fresh access to capital markets after default (s_{0t}^i). Formal definitions of the states for the different agents are provided below.

Definition 1 (State of the economy). *(i) Define the state of the union economy (namely, the relevant state for international investors' problem (3.4)) s_t^* as the collection of every country's optimal decisions on government bond prices and default options at $t-1$, as well as credit histories at $t-1$ and t , namely, $s_t^* = \{(q_{t-1}^i, d_{t-1}^i, h_{t-1}^i, h_t^i)_{i \in [0,1]}\} = \{q_{t-1}^*, b_t^*, \rho_{t-1}^*\}$.*

(ii) Moreover, define the state of the domestic economy in good credit standing (namely,

⁹This property also derives from Proposition 1 below.

¹⁰This specification can imply negative consumption under autarky for large y_t^i . This situation never arises in the numerical solution of the model and is formally ruled out by setting $c_{Dt}^i = \max\{0, y_t^i - \phi(y_t^i)\}$.

the relevant state for the domestic sovereign's problem (3.8)) s_t^i as the collection of country i 's income realization, government bond price as well as the aggregate sovereign debt portfolios, the aggregate bond price index, and the size of the union sovereign debt market, namely, $s_t^i = \{y_t^i, q_{t-1}^i, s_t^*\}$.

(iii) Finally, define the state of the domestic economy with access to capital markets after default (namely, the relevant state for the domestic sovereign's problem (3.10)) s_{0t}^i as the subset of states in s_t^i such that the domestic government's share of union-wide portfolios at the beginning of period t equals zero, namely, $s_{0t}^i = \{y_t^i, \infty, s_t^*\}$.

From Definition 1, it is clear that the problems of international investors and domestic sovereigns ultimately depend on every country's default policies, credit histories, and bond prices and quantities. However, international investors' problem does not depend on any country-specific variable, but only on aggregate conditions; thus, aggregation significantly simplifies the analysis, as the state of the union economy actually depends on union-wide indexes only. Conversely, the state of each domestic economy depends on domestic variables and accounts for the effects of every foreign country's variables through aggregate variables. As regards Definition 1(i), including credit histories at $t-1$ and t is necessary because, given a default at $t-1$, a country's access to capital markets at t is the realization of a time-invariant stochastic process. Thus, the set of default options and credit histories at time $t-1$ is not sufficient to well define the state of the union economy. As regards Definition 1(iii), fixing the domestic bond price equal to infinity is equivalent to setting the domestic amount of sovereign debt equal to zero.

After the definition of the state of the union and domestic economies, it is possible to identify default sets conditional on the state at time t in country i . The following definition is consistent with the sovereign debt literature (see, for instance, Arellano, 2008, Mendoza and Yue, 2012, Lizarazo, 2013).

Definition 2 (Default set). *Define country i 's default set $\mathcal{D}_i(q_{t-1}^i, s_t^*)$ as the set of endowments y_t^i for which contracts in the union economy's sovereign debt market make it optimal*

to default for sovereign i , namely,

$$\mathcal{D}_i(q_{t-1}^i, s_t^*) = \{y_t^i : U_i^R(s_t^i) < U_i^D(s_{Dt}^i)\}. \quad (3.12)$$

Hence, the default set at $t + 1$, $\mathcal{D}_i(q_t^i, s_{t+1}^*)$, and the transition probability function of endowments, $p(y_{t+1}^i | y_t^i)$, can be used to compute the expected value of any variable x_{t+1}^i dependent of country i 's endowment only.

The following proposition determines the dynamic interactions between default incentives and government bond prices within the domestic economy.

Proposition 1. *Let $\hat{q}_{t-1}^i > \bar{q}_{t-1}^i$, $\hat{s}_t^i = (y_t^i, \hat{q}_{t-1}^i, s_t^*)$, and $\bar{s}_t = (y_t^i, \bar{q}_{t-1}^i, s_t^*)$. If default is optimal in state \hat{s}_t^i , then it will be optimal in state \bar{s}_t^i , namely, $\mathcal{D}_i(\hat{q}_{t-1}^i, s_t^*) \subseteq \mathcal{D}_i(\bar{q}_{t-1}^i, s_t^*)$.*

Proof. See Appendix B.2. □

Proposition 1 states that a lower government bond price increases the probability of the domestic sovereign defaulting on its outstanding debt. Analogous conclusions were reached by Eaton and Gersovitz (1981), Chatterjee et al. (2007) and Arellano (2008), who observed that the value of staying in the contract decreases with the level of outstanding debt. The same result occurs in this model since the value of repayment (default) is increasing in (independent of) the government bond price: through the CES demand function for domestic government bonds, a higher bond price generates a lower level of debt, hence improving the fiscal stance (namely, the wealth) of the domestic sovereign under repayment relative to default. As a consequence, Proposition 1 can be extended so as to analyze the effect of aggregate variables on the domestic sovereign's default incentives. Since aggregate indexes b_t^* and q_{t-1}^* (ρ_{t-1}^*) are positively (negatively) related to the amount of domestic outstanding debt b_t^i , it then follows that an increase in b_t^* or q_{t-1}^* (ρ_{t-1}^*) induces a higher (lower) probability of default.¹¹

¹¹Further, assuming that (i) the domestic stochastic endowment is an i.i.d. process, (ii) there exist no exogenous output costs of default (namely, $\omega_1^i = \omega_2^i = 0$), and (iii) the sovereign can never re-access international capital markets after default (namely, $\theta = 0$), a necessary condition for default is the absence of contracts that allow the borrowing government to roll over its debt. In other words, sovereign default

Definition 3 (General equilibrium). *Given state $s_t^i = \{y_t^i, q_{t-1}^i, s_t^*\}$ and union-wide variables $\{q_t^*, r_t^*, b_{t+1}^*, \rho_t^*\}$ at period t , the recursive equilibrium of country i is defined as the set of policy functions for (i) consumption c_t^i ; (ii) international investors' sovereign debt portfolio allocations b_{t+1}^i ; (iii) the sovereign's bond price q_t^i , default option d_t^i and default sets $\mathcal{D}_i(q_{t-1}^i, s_t^*)$; and (iv) value functions $\{U_i(s_t^i), U_i^R(s_t^i), U_i^D(y_t^i)\}$, such that:*

- *Taking as given the domestic government's default and bond price policies as well as international investors' sovereign debt portfolio allocations, c_t^i satisfies the domestic household's budget constraint (3.9);*
- *Taking as given the domestic government's default and bond price policies, b_{t+1}^i satisfies international investors' demand function for government bonds (3.2);*
- *Taking as given the domestic government default policy, q_t^i and d_t^i are consistent with the domestic sovereign's maximization problems, and $\mathcal{D}_i(q_{t-1}^i, s_t^*)$ is given by condition (3.12).*
- *Value functions $\{U_i(s_t^i), U_i^R(s_t^i), U_i^D(y_t^i)\}$ solve problems (3.11), (3.8), and (3.10).*
- *The union-wide market for sovereign debt clears, so that union-wide demand equals supply of government bonds; by Walras law, union-wide consumption equals union-wide endowments at each point in time.*

3.3.4 The bond price schedule

Given the definition of equilibrium in the domestic economy above, an analysis of the Euler equation for the sovereign can provide further insight into the main features of the model.

The differentiability of the value function is not claimed or proved. However, as known from

arises only when all contracts available generate fiscal deficits. This result, together with the monotonicity and concavity of the utility function $u_i(\cdot)$, implies that default incentives decrease in present wealth. Thus, in times of adverse output shocks, the available contracts cannot offset the weak state of the economy or provide insurance for a highly indebted sovereign because none can increase consumption relative to wealth. For a proof of these results, see [Arellano \(2008\)](#).

the literature (see [Aguar and Gopinath, 2006](#), and [Cuadra, Sanchez and Sapriza, 2010](#)), the quantitative results do not depend on differentiability.¹²

When country i 's sovereign can access union-wide capital markets, the domestic government bond price is determined as follows:

$$q_t^i = \mu \beta_i \frac{E_t[(1 - d_{t+1}^i) \lambda_{t+1}^i]}{\lambda_t^i}, \quad (3.13)$$

where $\mu \equiv \frac{\eta}{\eta-1}$ and λ_t^i is the marginal utility of household consumption under repayment. The Euler equation has a standard interpretation: the current price of borrowing one unit of future consumption is equal to the discounted marginal rate of substitution between future and current consumption. Compared to a standard Euler equation with international borrowing, the assumption of a monopolistically competitive sovereign debt market introduces two new elements in condition (3.13). First, the discounted marginal rate of substitution is augmented by the union-wide mark-up μ over the marginal cost of debt that would prevail under perfect competition. Hence, a constant upward shift on the bond price is determined by the sovereign's market power, in turn due to investors' intratemporal risk aversion, asset imperfect substitutability and perceived financial segmentation. Second, since the government retains market power and can choose the price of additional borrowing accounting for the household's intertemporal risk aversion, consumption smoothing motives affect the bond price schedule.

A manipulation of the right-hand side of condition (3.13) can shed further light on the price effects of the interaction between household preferences and sovereign default risk. The term in expectation can be separated into two components as follows:

$$\begin{aligned} q_t^i &= \mu \left\{ \beta_i \frac{E_t[1 - d_{t+1}^i] E_t[\lambda_{t+1}^i]}{\lambda_t^i} + \beta_i \frac{Cov_t[(1 - d_{t+1}^i), \lambda_{t+1}^i]}{\lambda_t^i} \right\} \\ &= \mu (q_{RNt}^i + q_{RAt}^i). \end{aligned} \quad (3.14)$$

¹²In order to focus on the main determinants of the bond price schedule, the first-order condition for the domestic sovereign's problem disregards the upper boundary on the bond price and, thus, the Lagrange multiplier of the associated constraint. The numerical evaluation of the model ensures that such constraint is neither violated nor binding, so that the multiplier equals zero.

The first term in brackets refers to the price that would prevail with a risk neutral household: q_{Rnt}^i compensates the borrower for the cost of repayment when preserving access to capital markets is optimal. The second term represents the risk premium charged by the government and associated with household preferences: q_{RAnt}^i internalizes the detrimental effects of high levels of borrowing on the debtor's future wealth and adds upward pressure on the bond price. The following proposition formalizes this result.

Proposition 2. *Let the debt-to-output ratio be larger than the exogenous costs of default per unit of output, namely, $b_{t+1}^i/y_{t+1}^i > \omega_1^i + \omega_2^i y_{t+1}^i > 0$. Then, default is inversely related to the marginal utility of consumption next period, namely, $Cov_t[(1 - d_{t+1}^i), \lambda_{t+1}^i] \geq 0$.*

Proof. See Appendix B.2. □

According to Proposition 2, the covariance term is non-negative when the government's future wealth is lower under repayment than under default, which occurs when the cost of repayment (i.e., the face value of the bonds) is larger than the cost of default (i.e., the exogenous output costs), that is, when the debt level is sufficiently high. Intuitively, when the country is highly indebted, the sovereign seeks to smooth household consumption by charging a non-negative risk premium on the risk neutral price paid by international lenders. Mechanically, when the price increases, the face value of the bonds (and, thus, the cost of repayment) decreases via the demand function; as a result, the difference between future wealth under repayment and under default decreases, as the former approaches the latter, thus reducing the *ex-ante* volatility of consumption.¹³ This result contrasts with previous models focusing on investor risk aversion. In particular, Lizarazo (2013) shows that a sovereign debtor owes a premium (leading to lower bond prices) to risk averse investors aiming to smooth consumption. In contrast, this model assumes that risk averse investors aiming to diversify their portfolios give substantial leeway to borrowing governments, who

¹³As stated in the proof of Proposition 2 (see Appendix B.2), the opposite is true when the cost of repayment is lower than the cost of default. When the debt level is low, the sovereign seeks to reduce the volatility of consumption by decreasing its price, hence attracting a higher demand for bonds, increasing the future cost of borrowing, and narrowing the gap between consumption under repayment and under default. Therefore, for a given probability of default, bond prices may be higher or lower than those observed in a model with a risk neutral household, depending on the indebtedness of the borrowing country.

can then impose a mark-up and seek to smooth household consumption (implying higher bond prices, especially for high levels of debt).

3.3.5 Sustainable debt and price

Back-of-the-envelope calculations *à la* Lucas (1985) and Aguiar and Gopinath (2006) show how market sentiments influence debt and price sustainability through the endogenous bond demand function. A comparison between the present discounted values of capital market access and autarky reveals the sustainable amount of debt (price level) for a given price level (amount of debt). On the one hand, autarky is assumed to be an absorbing state (i.e., autarky lasts forever), featuring *iid* output shocks and no domestic savings. In particular, assume that $y_t^i = y^i e^{z_t^i} e^{-\frac{1}{2}\sigma_y^{i2}}$ with $z_t^i \sim N(0, \sigma_y^{i2})$, so that $E y_t^i = y^i$ does not depend on the volatility of the shocks; then,

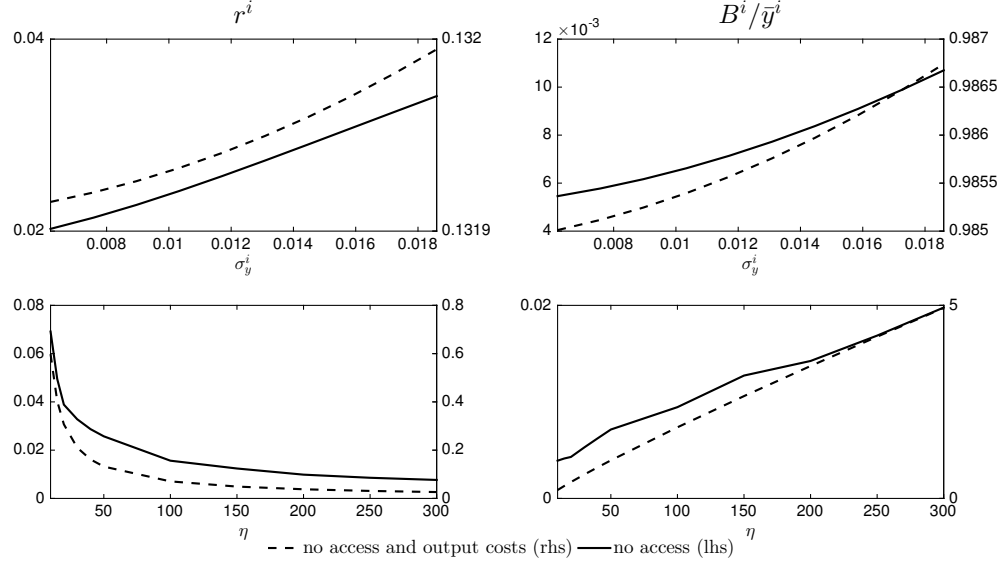
$$U_i^D = E \sum_{t=0}^{\infty} \beta_i^t \frac{y_t^{i1-\sigma_i}}{1-\sigma_i} = \frac{(y^i e^{-\frac{1}{2}\sigma_y^{i2}\sigma_i})^{1-\sigma_i}}{(1-\sigma_i)(1-\beta_i)}.$$

On the other hand, financial integration promises perfect insurance (i.e., a constant stream of income) in exchange of period-by-period interest payments κ^i . Hence, a higher debt service κ makes autarky relatively more attractive than international borrowing. Formally,

$$U_i^R = E \sum_{t=0}^{\infty} \beta_i^t \frac{c_t^{i1-\sigma_i}}{1-\sigma_i} = \frac{(y^i - \kappa^i)^{1-\sigma_i}}{(1-\sigma_i)(1-\beta_i)}.$$

Now, borrowing is preferred to autarky as long as $U_i^R \geq U_i^D$, which occurs whenever capital outflows are below a certain share of output, i.e., $\kappa^i/y^i \leq 1 - e^{-\frac{1}{2}\sigma_y^{i2}\sigma_i}$. Debt service may be separated into two components as $\kappa^i = r^i B^i$, where $r^i \equiv 1/q^i - 1$ denotes the maximum sustainable interest rate and $B^i \equiv q^{i-\eta} s^* = q^{i-\eta} q^{*\eta} b^*/\rho^*$ the associated maximum sustainable debt level, which may be interpreted as the natural debt limit. Both r^i and B^i are inversely related to the minimum sustainable price level Q^i . For a given calibration of the

Figure 3.3: Maximum sustainable interest rate and debt-to-output ratio



relevant parameters, Q^i can be computed by solving the following nonlinear equation

$$\left(\frac{1}{q^i} - 1\right) \frac{q^{i-\eta} s^*}{y^i} = 1 - e^{-\frac{1}{2}\sigma_y^{i2}\sigma_i} \quad (3.15)$$

for q^i . If an additional output loss is introduced in autarky, the maximum sustainable debt service-to-output ratio can be computed as $\kappa^i/y^i \leq 1 - [1 - \phi(y^i)/y^i]e^{-\frac{1}{2}\sigma_y^{i2}\sigma_i}$ and Equation (3.15) should be modified accordingly.

Assuming the calibration for the baseline model reported in Table 3.2, Figure 3.3 illustrates the maximum sustainable interest rate r^i and debt-to-output ratio B^i/y^i as a function of σ_y^i and η and under different assumptions regarding the costs of default. Three main results can be identified. First, a higher volatility of output determines a lower value of default for risk averse households seeking to smooth their consumption. Hence, as preserving access to capital markets becomes relatively more attractive, the government is willing to sustain higher borrowing costs, stemming from higher interest rates and debt ratios. Second, as the elasticity of substitution between bonds increases, the maximum sustainable interest rate and debt ratio move in opposite directions, whereby the former decreases and the latter increases. This result can be explained by the asymmetric effect of the bond price

on interest rates and debt levels. For a given $q^i \in [0, 1]$, a higher η implies a higher B^i via the bond demand function, thus increasing the right-hand side of Equation (3.15). To maintain equality and keep κ^i constant, q^i increases, hence putting downward pressure on both r^i and B^i . However, the reduction in B^i need not fully offset its initial increase, as it is accompanied by the decrease in r^i . Eventually, the debt level rises in response to a higher elasticity of substitution. Therefore, as the sovereign debt market becomes more segmented and investors more risk averse, the borrowing government is willing to pay higher yields on lower debt levels, so as to maintain its debt service constant. Third, the government can sustain higher interest rates and debt ratios if repudiation triggers an economic contraction besides financial autarky, that is, if default is more costly. Under the proposed calibration, assuming concomitant autarky and recessions supports debt-to-output ratios above unity for relatively low elasticity levels (below 100).

3.4 Quantitative assessment of the model

3.4.1 Data

This section documents business cycle features of the Greek economy leading to the default event at the turn of this decade.¹⁴ In March 2012, after a lengthy bargaining process, the majority of private investors of Greek sovereign debt agreed to a bond swap, thus accepting a reduction in the face value of their bonds by more than half, an extension of the maturity of their holdings, and a decrease in the interest rate paid by the government.¹⁵ This event marked the largest sovereign-debt restructuring in history, as it allowed Greece to write off

¹⁴Zettelmeyer, Trebesch and Gulati (2013) and Ardagna and Caselli (2014) provide a detailed analysis of the Greek sovereign default and its economic costs, gone down to history as the largest debt restructuring ever arranged.

¹⁵In February 2012, in order to prevent a disorderly default and a possible exit of Greece from the euro area, an IMF/ECB bailout package included a debt restructure agreement with private investors affecting more than €200 billion of Greek government bonds (see Reuters, February 29th 2012, “Insight: How the Greek debt puzzle was solved”). Following the use of collective action clauses by the Greek government to settle its obligations with private investors, the International Swaps and Derivatives Association (ISDA) declared the debt restructuring as a credit event (namely, a “technical” default), hence triggering swaps (namely, payouts on the underlying CDS contracts) on about \$3 billion of default insurance at an auction held on March 19th 2012 (see Bloomberg, March 9th 2012, “Greek Credit Swaps Payouts to Be Expedited After Trigger Ruling”).

about €100 billion (\$130 billion), accounting for 29 percent of its outstanding debt and 48 percent of its 2011 GDP.¹⁶ Greece also experienced a severe economic crisis, with a cumulative output loss of about 9 percent in the two quarters before the default.¹⁷

Table 3.1 reports business cycle statistics for Greece between the first quarter of 2002 and the first quarter of 2012. The first column reports the standard deviations and correlations (with output and spread) of the relevant variables in the full sample, whereas the second and the third columns report the same statistics for the pre-crisis (2002.1-2008.2) and the crisis periods (2008.3-2012.1), respectively.¹⁸ Finally, the fourth column shows the p -values of the Levene's test for the standard deviations of the pre-crisis and crisis periods. This test is a useful alternative to Bartlett's test when the sample distributions are not normal, and especially when they are prone to outliers.

The sample statistics for Greece are consistent with well-known results reported in the literature on sovereign default in emerging economies (see, for instance, Neumeyer and Perri, 2005, Aguiar and Gopinath, 2006, Arellano, 2008): consumption has a higher volatility than output and both the current account and the yield spread are countercyclical (albeit the negative correlation with output of the spread is not statistically significant). However, unlike a typical emerging economy, Greece does not display positively correlated current account and spread in any considered period. Interestingly, according to Levene's test, the volatility of both the current account and the interest rate spread experienced a significant shift in the crisis period; whereas the standard deviation of the current account decreased from 1.5 to 0.8 percent, the same statistic for the spread dramatically increased from 0.1 to 15.9 percent. Interestingly, Levene's test reveals that the volatility of fundamentals (i.e. real

¹⁶ *The Economist*, March 17th 2012, "Greece's default: The wait is over".

¹⁷ The decline in output is computed as the sum of the deviations of (log) real GDP from its HP filter trend in 2011Q4 and 2012Q1.

¹⁸ The separation between pre-crisis and crisis period aims at increasing the number of observations in each period as much as possible, in order to maximize the power of the sample statistics. Although the narrative in the paper identifies the revision of the deficit expectations by the Greek government in the fourth quarter of 2009 as the clear start of the shift in government bond yield dynamics, an early signal was the collapse of Lehman Brothers in the third quarter of 2008, which arguably affected investors in euro-area financial markets, as well (see, for instance, Bernoth, von Hagen and Schuknecht, 2012, Christiansen, 2014). This timing choice is thus consistent with the main focus of the paper, i.e. the effects of investors' sentiments on sovereign yield dynamics.

Table 3.1: Greece Business Cycle Statistics, 2002.1-2012.1

	Full	Pre-Crisis	Crisis	Levene's test
	[1]	[2]	[3]	[4]
$\sigma(y)$	2.199	1.145	1.733	0.455
$\sigma(c)/\sigma(y)$	1.296	0.505	1.752	0.552
$\sigma(ca/y)$	1.635	1.490	0.810	0.045
$\sigma(r)$	9.603	0.075	15.932	0.096
$\rho(c, y)$	0.754 (0.000)	-0.091 (0.66)	0.894 (0.000)	— —
$\rho(ca/y, y)$	-0.580 (0.000)	0.009 (0.966)	-0.439 (0.133)	— —
$\rho(r, y)$	-0.185 (0.26)	-0.208 (0.309)	-0.091 (0.769)	— —
$\rho(ca/y, r)$	0.095 (0.564)	-0.240 (0.237)	-0.051 (0.869)	— —

Notes: The series for real GDP (y), consumption (c), and the current account-to-GDP ratio (ca/y) were first seasonally adjusted. After logging y and c , and computing the interest rate spread (r) as the difference between each country and the Eurozone's (changing composition) euro spot interest rate on 1-year benchmark government bonds, all series were HP filtered with a smoothing parameter of 1600. Levene's test (H_0 : no difference in standard deviations between samples) refers to the corresponding p -values for sample standard deviations (σ); Pearson's p -values are reported in parentheses for sample correlations (ρ). Full sample refers to the period 2002.1-2012.1; pre-crisis sample refers to 2002.1-2008.2; crisis sample refers to 2008.3-2012.1. The two quarters 2011.4-2012.1 for government bond yields have been excluded from computations in order to net out the effects of outliers in the historical series.

Sources: ECB, Datastream, author's calculations.

GDP and consumption) did not significantly change in the wake of the financial crisis. This result supports the view according to which the shift in market agents' pricing paradigm was not driven by changes in country-specific risk characteristics, but rather by exogenous events (e.g. the Lehman collapse and/or the upward revision by the Greek government of its expected fiscal deficit).

3.4.2 Calibration and functional forms

In this section, the benchmark calibration and results from the numerical solution and simulation of the model are presented. The functional form for the utility function of domestic households is:

$$u_i(c_t^i) = \frac{c_t^{i1-\sigma_i}}{1-\sigma_i}.$$

Table 3.2: Specification of Model Parameters

parameter	value	target
θ	0.125	$\simeq 2$ years of exclusion
σ_i	2	
β_i	0.99	4% annual rate
q^*	0.9943	Euro-area 1-year gov.t bond yield (2002.1-2011.4)
b^*	0.001	Average investor's exposure to gov.t bonds
ω_i^1	0.048	Greece's output costs of default (2011.4-2012.1)
σ_y^i	0.0135	Greece's log output (2002.1-2013.4)
ρ_y^i	0.8549	Greece's log output (2002.1-2013.4)
η	80, 150, 2000	model gov.t bond yield standard deviation

The domestic endowment is assumed to follow a first-order Markov process with transition probability $p(y_t^i | y_{t-1}^i)$. The stochastic process for income then approximates the continuous AR(1) process around a long run mean μ_y^i below:¹⁹

$$y_t^i = \mu_y^i(1 - \rho_y) + \rho_y^i y_{t-1}^i + \varepsilon_t^i, \quad (3.16)$$

where $0 < \rho_y^i < 1$, $\varepsilon_t^i \sim N(0, \sigma_y^{i2})$. In order to solve the model, the state space is discretized. The support for the domestic endowment spans 21 equally spaced grids of the original processes steady state distribution. The state space for the government bond price includes 300 grid points.²⁰

Table 3.2 reports the values of the calibrated parameters. The AR(1) process for income is fitted to the Greek detrended real GDP between the first quarter of 2002 and the first quarter of 2012. Autoregressive coefficient $\rho_y^i = 0.8549$, standard deviation for output shocks $\sigma_y^i = 0.0135$, whereas long run mean μ_y^i is standardized to one. The coefficient for the output loss function $\omega_1^i = 0.048$ (and, consequently, ω_2^i) is obtained as the (percentage) deviation of output from its trend in the first quarter of 2012. International investors' sovereign debt portfolio index b^* is calibrated to match the average exposure of euro-area

¹⁹The values of the Markov transition matrix are then computed by integrating the underlying normal density over each interval (see Aguiar and Gopinath, 2006).

²⁰An additional grid point for the price equal to infinity is included in order to compute the value function of the government with access to capital markets in the period after default.

monetary and financial institutions to euro-area non-domestic sovereign debt.²¹ The aggregate bond price index q^* matches the long run average of the euro-area spot interest rate on 1-year benchmark government bonds (rescaled to a quarterly frequency). Parameter θ is set to 0.125, which corresponds to an expected duration of financial autarky of two years. This value is in line with the Greek experience, since the government could not tap international capital markets from the default event in March 2012 to April 2014.²²

Different specifications for the intratemporal risk aversion of the domestic investor are compared: notably, the model is solved for η equal to 80, 150, and 2000, corresponding to the low, baseline, and high elasticity scenarios, respectively. The choice for the remaining parameters is in line with the standard real business cycle literature: the coefficient of intertemporal risk aversion σ_i is set to 2 and the discount factor β_i corresponds to an annualized rate of 4 percent. However, each specification of η is associated with different maximum and minimum levels of debt-to-mean income b_{Gy} ; hence, in order to preserve comparability between different specifications, I define the state space for the bond price q_t (which is linked to b_{Gy} through the aggregate CES demand for domestic sovereign debt) so that in every specification the set of debt-to-mean income between $100\omega_i$ and 50 percent is analyzed.

3.4.3 Numerical analysis

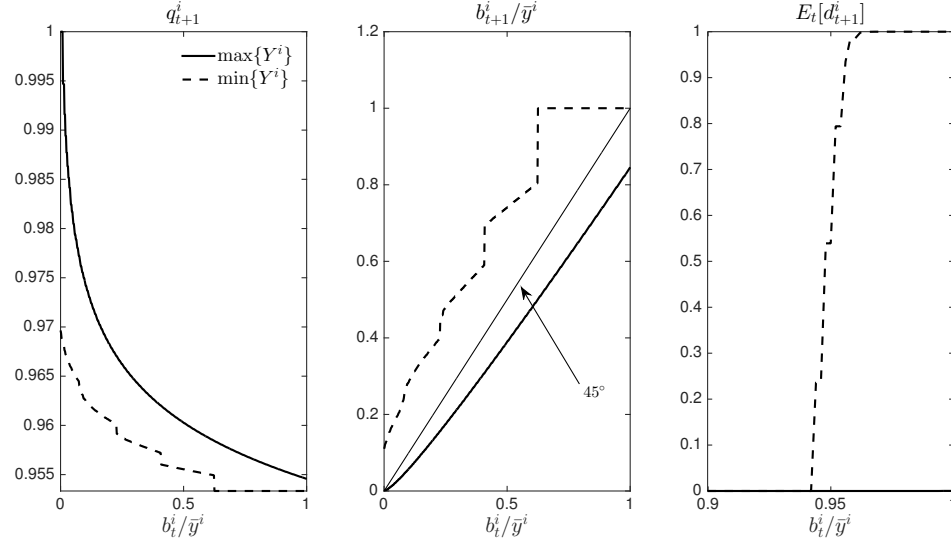
The results of the numerical evaluation of the model with the baseline specification are showed in Figure 3.4. The optimal choice for the government bond price and the default probability are positively and negatively related to the state of the country's endowment today and the current government bond price, respectively.

In the left and right upper panels, Figure 3.5 shows the optimal government bond price and the optimal level of borrowing relative to mean output as a function of the current price and the ratio between outstanding debt and mean output, respectively, at the

²¹The data is collected from the ECB database; exposures refer to securities other than shares held by monetary and financial institutions other than the ECB and the European System of Central Banks.

²²*The Financial Times*, April 9th 2014, "Greece: Gre-entry".

Figure 3.4: Model bond price, government debt and probability of default

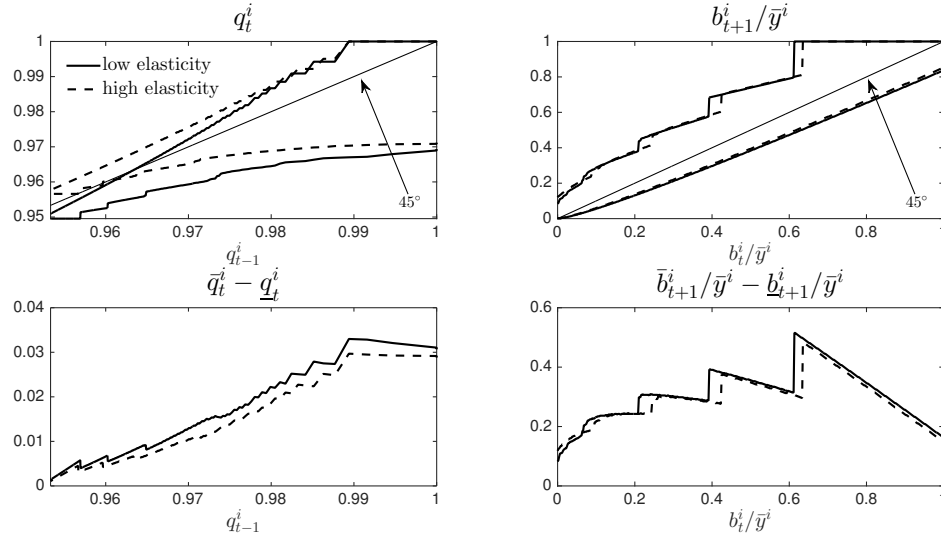


extreme levels of the current endowment and for alternative specifications of parameter η , as reported in Table 3.1. As in the baseline calibration, both the high and the low elasticity scenarios allow prices and debt ratios to follow an upwards or downward paths depending on the income realization in almost any state for current prices and debt ratios, respectively. Further, the left and right lower panels show the difference between the prices and the debt ratios, respectively, implied by the maximum and the minimum endowment shocks for different values of η . In virtually every state for current government bond markets, a low elasticity entails a wider range of possible values for both prices and debt ratios. Therefore, a low elasticity of substitution, representing adverse market sentiments, is associated with a higher sensitivity of government bond yields to macroeconomic fluctuations.

3.4.4 Empirical analysis

The empirical results reported in this section are obtained by simulating the model for 10,000 periods. From these 10,000 periods, sub-samples that have the economy stay in the credit market for 40 periods before going into a default are taken to compute the economy's business cycle statistics. Hence, these statistics are comparable with actual sample statistics

Figure 3.5: Model bond price and government debt dynamics



on 10 years of Greek data. This process is repeated 500 times, and the cycle statistics are the average of the statistics derived from each of these repetitions.

Table 3.3 reports results from the simulation of the model. As expected, the standard deviation of the simulated series for the government bond price decreases with the elasticity of substitution. More specifically, the standard deviation of the government bond spread when $\eta = 2000$ is just 0.888 percent and it sharply increases to 6.762 percent when $\eta = 80$.²³ Hence, the model correctly predicts that in times of financial distress, when investors are relatively more risk averse and perceive bonds as imperfect substitutes, sovereign debt yields become more sensitive to macroeconomic fundamentals, so that their volatility increases. The model also delivers a higher volatility of consumption relative to output and countercyclical interest rates. Interestingly, as government bonds become more perfect substitutes (i.e. as η increases) and the perceived segmentation of the financial market shrinks, the level of countercyclicality of interest rates decreases; a number of studies (e.g. Neumeyer and Perri, 2005) reports that acyclical or mildly procyclical interest rates characterize business cycles in developed and financially integrated economies. However, the

²³The spread is calculated as $1/q_t^i - 1/q^*$.

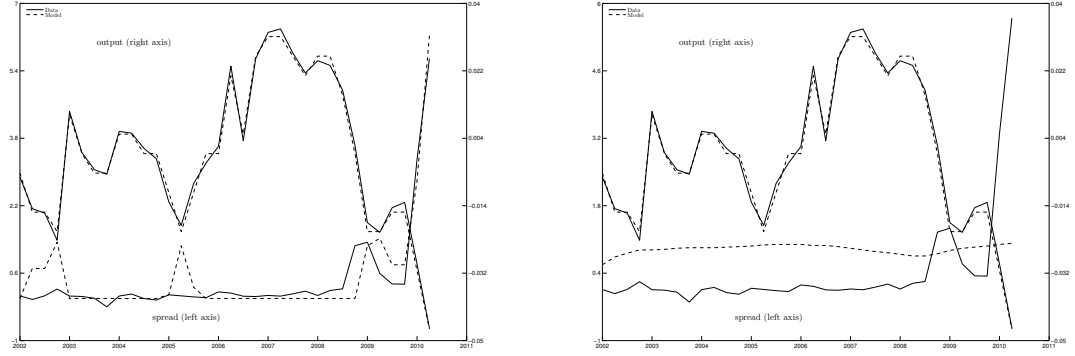
Table 3.3: Model Simulations

	Baseline	High elasticity	Low elasticity
	[1]	[2]	[3]
$\sigma(y)$	1.595	1.583	1.657
$\sigma(c)/\sigma(y)$	1.674	1.635	5.461
$\sigma(ca/y)$	1.601	1.103	6.762
$\sigma(r)$	4.056	0.888	6.315
$\rho(c, y)$	0.548	0.767	-0.066
	(0.000)	(0.000)	(0.008)
$\rho(ca/y, y)$	0.434	0.304	0.347
	(0.000)	(0.001)	(0.000)
$\rho(r, y)$	-0.391	0.113	-0.452
	(0.000)	(0.14)	(0.000)
$\rho(ca/y, r)$	-0.340	-0.295	-0.422
	(0.000)	(0.001)	(0.000)
mean debt-to-GDP	1.118	4.775	2.371
mean default	11.067	12.230	8.373

model cannot capture the countercyclicality of the current account. This result stems from the specification of the demand function for government bonds: since $\eta > 1$, an increase in the price translates into a decrease of a larger magnitude in the demand for bonds. As the government chooses high values in the bond price menu during good states (due to the persistence of the autoregressive process for output), the aggregate effect is a low borrowing level, so that output, which is higher than consumption, is used to repay outstanding debt and the country experiences net capital outflows, i.e. a current account surplus.

Figure 3.6 compares time series for output and government bond spread under the baseline (left-hand chart) and the high elasticity (right-hand chart) specification. I feed the model with the series for Greek (log) detrended output from the first quarter of 2012 up to the second quarter of 2010, when the first bailout package was extended to Greece. In the high elasticity scenario, the model dynamics of the spread resemble the data in the pre-crisis period, characterized by smooth changes and a very low sensitivity to movements in fundamentals. Conversely, in the baseline scenario, the model can successfully replicate data during the crisis, when the decline in output generated a sharp increase in the spread. However, the high degree of investors' risk aversion induces considerable movements in the spread before the crisis as well. This result is consistent with empirical evidence on the mis-

Figure 3.6: Greece and model time series



alignment between country-specific risk characteristics and investors' pricing paradigm.²⁴

3.5 Conclusions

This paper lays out a stochastic dynamic general equilibrium model of endogenous sovereign default in a large economic region composed of an infinite number of small open economies. International investors are risk averse as they perceive sovereign debt markets as segmented and bonds issued by different governments as imperfect substitutes. The paper shows that such a specification for investor preferences is essential in order to replicate three salient empirical regularities observed in euro area sovereign debt markets, namely, the increase in investor risk aversion, the reversal in the process of financial integration and the sharp increase in the perceived imperfect substitutability between euro area government bonds observed in the wake of the recent crisis. Moreover, the assumption on investors' portfolio strategy allows the model to replicate key stylized facts on the allocation of sovereign debt holdings in banks before and during the crisis. In the numerical evaluation of the model, different specifications of the elasticity of substitution are analyzed: a high degree of investor risk aversion, financial segmentation and imperfect asset substitutability determines a high

²⁴In the literature on the determinants of euro-area government bond yields, the non-fundamental component of sovereign bond spreads, i.e. the part of bond spreads not explained by differences in fiscal and macroeconomic fundamentals have been often designated with the term mispricing (see, for instance, De Grauwe and Ji, 2012, Di Cesare et al., 2012, Dewachter et al., 2014).

sensitivity of yields to movements of macroeconomic fundamentals. Finally, in an empirical assessment, the model can replicate several features of Greek sovereign debt, yields and default dynamics in the periods preceding and following the eruption of the recent sovereign debt crisis.

Chapter 4

Debt limits and sovereign default in the euro area

4.1 Introduction

Since the beginning of the recent sovereign debt crisis, euro area economies have confronted exceptional fiscal challenges. Dramatic recessions have exerted considerable pressures on euro area public finances across the board: large contractions in real output and consumption, coupled with rising unemployment rates, have led to persistent fiscal deficits, soaring sovereign debt levels as well as widening cross-country government bond differentials, as shown in Figure 4.1.¹ In this context, the debate on the measurement and assessment of the fiscal space available to euro area governments has gained ever more prominence in policy circles.² The notion of fiscal space crucially hinges on the dual objective of fiscal

¹The median euro area country experienced its largest contraction in real GDP and real private consumption in 2009; at the same time, the median unemployment rate soared as the crisis unfolded up to 2015 without ever returning to its pre-crisis levels. Between 2007 and 2015, the receding real economy took its toll on fiscal balances, whereby the median debt-to-GDP ratio almost doubled and an initial median balanced budget was followed by repeated deficits. Concomitantly, a dramatic increase in the dispersion of cross-country sovereign spreads signaled a shift in market sentiments, as investors perceived bonds issued by different governments as imperfect substitutes and thus diversified between core and highly-indebted peripheral countries. For an encompassing description of the early stages of the euro area sovereign debt crisis, see Lane (2012).

²In the flagship document of European Union (EU) institutions, also known as the “Five Presidents’ Report”, Juncker et al. (2015) deem the presence of fiscal space as a necessary condition for the achievement of the optimal level of economic stabilization. Further, they recognize unsustainable fiscal policies as a major

policy, namely ensuring macroeconomic stability and, *at the same time*, preserving debt sustainability.³ The accurate identification of fiscal space is then essential to evaluating the ability of fiscal policy to stabilize macroeconomic fluctuations without endangering the sustainability of public finances.

This paper contributes to the current debate by measuring and assessing fiscal space through the lenses of a dynamic stochastic general equilibrium model with strategic sovereign default à la [Eaton and Gersovitz \(1981\)](#). This paper puts forward a probabilistic, forward-looking notion of fiscal space, thus addressing the concerns of policy makers seeking to evaluate debt sustainability at different time horizons. More specifically, fiscal space is defined as the distance of the current debt level from its state-contingent debt limit, which is in turn considered as the maximum level of borrowing that the government is willing to honor within a certain time horizon. In this light, the model studies the implications of macroeconomic fundamentals and fiscal policy on the sovereign's incentives to default and the associated distributions of debt limits and fiscal space. Compared to the standard Eaton-Gersovitz framework, the model is endowed with several features aimed at reflecting important structural characteristics of euro area economies, such as risk averse investors, long-duration bonds, recovery rates over defaulted debt, taxes on consumption and labor income as well as, via novel applications, domestic debt and supranational fiscal rules.

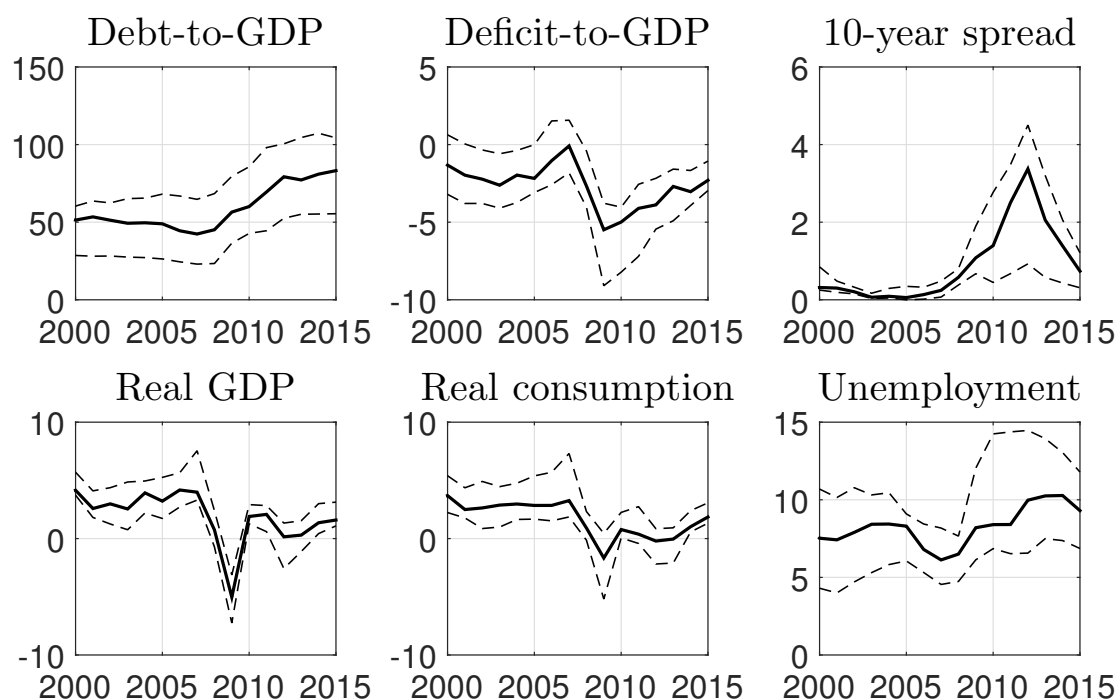
As regards the first novel feature of the model, domestic debt is introduced via a two-stage process. In the first stage, domestic investors interact with foreign lenders to bargain the optimal domestic share of total debt on primary markets and post their bids in a binding auction with the government. In the second stage, trades between lenders and the government take place on perfectly competitive secondary markets, given the overall price for debt.

The inclusion of domestic debt is essential to investigating sovereign risk and its

obstacle in the creation of a European fiscal governance framework and, generally, a full-fledged Economic and Monetary Union (EMU).

³In general, all definitions refer to a government's room of manoeuvre to provide resources for a desired purpose without endangering the sustainability of its financial position or the stability of the economy. The [International Monetary Fund \(2010\)](#) succinctly defines fiscal space as "the scope for further increases in public debt without undermining sustainability".

Figure 4.1: Public finances and real economy in the euro area between 2000 and 2015

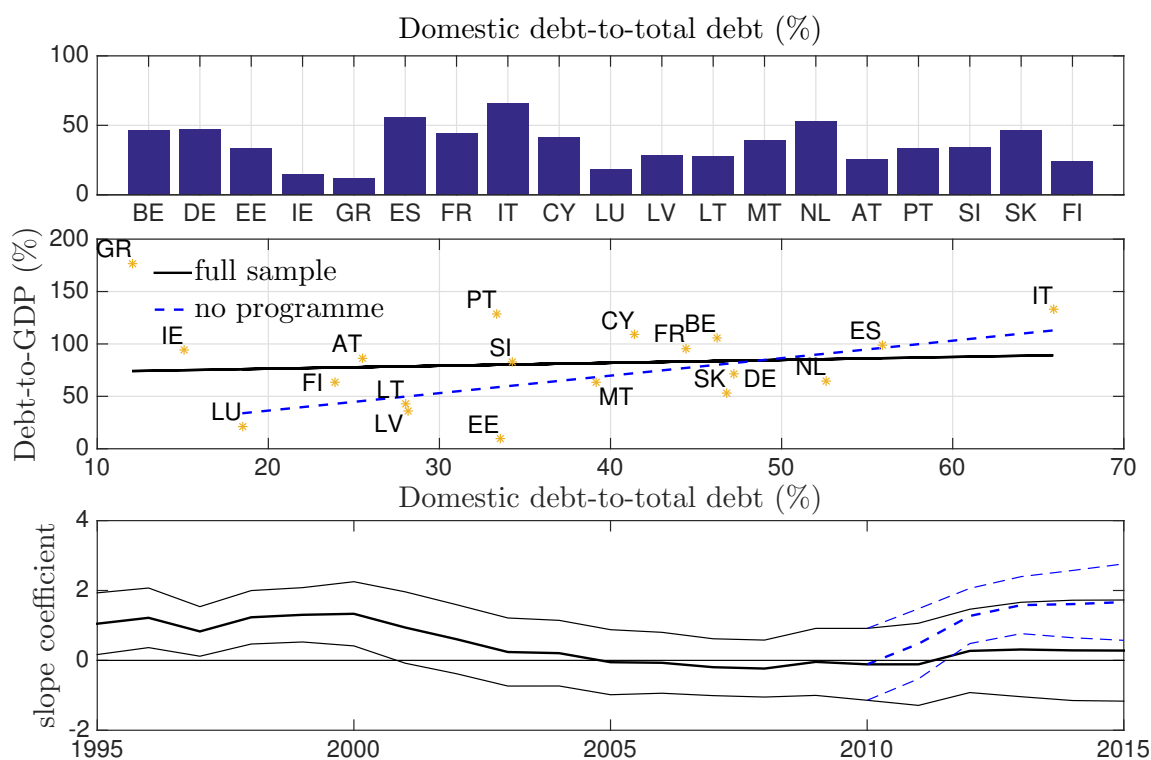


Sources: Eurosystem.

Notes: Proceeding leftwards and downwards, the charts show the debt-to-GDP ratio, the deficit-to-GDP ratio, the 10-year government bond spread (relative to the Bund yield), the real GDP growth, the real private consumption growth, the unemployment rate in 19 euro area countries. Each chart shows the median (solid line) and the interquartile range (area between the dashed lines) of the cross-sectional distribution. All variables are reported in percentage terms.

implications for debt sustainability and welfare in advanced economies and, especially, in the euro area. The importance of domestic debt stems from four empirical regularities. First, domestic debt constitutes a considerable fraction of government debt. In a panel of 43 emerging and advanced economies in 2011, the median domestic share of total debt exceeds 50% (Mallucci, 2015), whereas the same figure for euro area countries in 2015 is a still sizeable 34% (with a peak of 66% for Italy), as shown in the first chart of Figure 4.2. Second, domestic default occurs frequently, implying that, against conventional wisdom, domestic lenders are not senior to external lenders: in a panel of 30 developing countries, domestic default occurs in 48% of crises episodes between 1980 and 2005, whereas the figure increases to 73% between 1990 and 2005 (Mallucci, 2015; see also Reinhart and

Figure 4.2: Debt ratio and domestic share of debt



Sources: Eurosystem.

Notes: The first chart shows the share of total government debt held by domestic residents in 19 euro area countries. In the second chart, the solid (black) line is the regression line obtained from regressing the debt-to-GDP ratio on the domestic-to-total debt ratio for the full sample of 19 euro area countries at the end of 2015; the dashed (blue) line refers to the same regression for the full sample except the (ex-)programme countries (i.e. Greece, Ireland, Portugal and Cyprus). In the third chart, the same regression is repeated for every year from 1995 to 2015 and the same line convention is used; further, the thick lines represent the point estimates, whereas the thin lines show the 95% confidence intervals.

Rogoff, 2011). Third, mainly through a reduction in external and domestic credit to the domestic non-financial private sector, output contractions are significantly more pronounced when the sovereign reneges debt contracts that are also held domestically, as opposed to defaults on debt exclusively held by foreign creditors (Arteta and Hale, 2008, Sandleris, 2015, and Gennaioli, Martin and Rossi, 2014; for the case of a euro area country, see Albertazzi et al., 2014). Fourth, in euro area economies, higher domestic holdings of sovereign debt are associated with higher debt ratios. As shown in the second chart of Figure 4.2, regressing the debt-to-GDP ratio on the domestic share of debt for a cross-section of euro area countries in 2015 yields a positive slope coefficient of 0.3, which is not statistically significant at the 10%

significance level. Yet, this result may stem from the fact that macroeconomic adjustment programmes have biased the correlation between domestic and total debt, as supranational or bilateral cross-country financial assistance considerably reduced the share of debt held domestically with negligible effects on the amount of total debt. When the same regression is run excluding (ex-)programme countries (i.e. Greece, Ireland, Portugal and Cyprus), the slope coefficient increases to 1.7 (thus, a more than one-to-one relationship) and becomes significant. The third chart of Figure 4.2 depicts the series of slope coefficients obtained through the same regression repeated each year from 1995 to 2015 with a full cross-sectional sample (solid line) and excluding programme countries as of the beginning of their financial assistance (dashed line). Clearly, between the inception of the EMU and the start of the crisis, increases in domestic debt were not accompanied by increases in total debt. However, the positive relationship between domestic and total debt emerges before the creation of the EMU (for the full sample) and after the start of the crisis (excluding programme countries). Ultimately, this evidence hints at the presence of incentives towards debt accumulation in countries featuring large shares of total debt held by domestic residents, as long as no distortions affect the pricing of country-specific default risk.

As regards the second novel feature of the model, the government is assumed to interact with supranational fiscal authorities. The latter require compliance with fiscal rules, but, in the absence of any enforcement power and commitment technology, the government may optimally choose to deviate. If the government faces tight borrowing constraints and deviates from fiscal rules, lenders receive a negative signal about its credibility and trigger a sudden stop in capital flows. Given the possibility of self-fulfilling roll-over crises via a credibility channel, fiscal rules reduce the sovereign's incentives to borrow, hence decreasing its optimal debt levels and fostering market-based fiscal discipline.

The inclusion of fiscal rules is motivated by their increased importance as means to foster fiscal discipline in recent years. Over the last decades, an increasing number of countries has relied on fiscal rules to guide policy.⁴ In the European context, the recent sovereign

⁴According to the [International Monetary Fund \(2009\)](#), 80 countries had national and/or supranational rules in place as of 2009 compared to only 7 countries in 1990. According to the database on numerical fiscal

debt crisis paved the way for the inclusion of fiscal rules within the EU fiscal governance framework as well as their anchoring in national legislation.⁵ Notwithstanding difficulties in evaluating the desirability of fiscal rules (see, for instance, [Afonso and Hauptmeier, 2009](#)), several studies investigate their impact on a government's fiscal space. In their empirical investigation of national fiscal governance frameworks in the EU, [Nerlich and Reuter \(2015\)](#) show that national fiscal rules contribute to increasing the available fiscal space while reducing the degree of pro-cyclicality of national fiscal policies. In an analysis of European bond spreads before the financial crisis, [Heinemann, Osterloh and Kalb \(2014\)](#) find that the interaction of stability preferences and fiscal rules points to a particular potential for the latter to restore market confidence in countries with a historical lack of stability culture.⁶

As regard the results under the benchmark calibration, a sensitivity analysis shows the effects of different structural characteristics (as represented by specific parameter values) on the sovereign's incentives to default and the associated debt limit distributions. First, a higher domestic share of total debt entails higher default costs, due to the losses incurred by domestic investors, thus increasing the sovereign's incentives to repay and expanding its borrowing opportunities. Second, a higher risk aversion for the private sector implies that domestic investors value relatively more the benefits of consumption smoothing; then, default is relatively more costly, and the sovereign has lower incentives to renege its obligations and faces higher debt limit distributions. Third, when the volatility of productivity shocks increases, the debt limit distribution becomes more disperse and borrowing opportunities shrink as the economy may be subject to deeper recessions. Fourth, as the costs of default for the overall economy increase, either by decreasing the probability of receiving a settlement offer or increasing the exogenous productivity loss, the sovereign has higher incentives to honor its obligations. Finally, as the steady-state marginal utility of the pri-

rules prepared by European Commission, 25 EU countries had some kind of fiscal rule in place in 2014, up from only 6 countries in 1990.

⁵The so-called "Two-Pack" regulation required Member States to transpose the fiscal rules envisaged under the Stability and Growth Pact into national legislation, as well as the set-up of independent national bodies monitoring compliance with the fiscal rules.

⁶Further theoretical and empirical evidence on the effects of fiscal rules in the context of European Union countries, see [Pappa and Vassilatos \(2007\)](#) and [Plödt and Reicher \(2014\)](#).

vate (relative to the public) sector increases, for instance through higher tax rates or lower public-to-private consumption ratios, the sovereign finds default less attractive and hence faces a looser borrowing constraint.

As regards the analysis of fiscal space in light of the dual objective of fiscal policy, macroeconomic stabilization and debt sustainability are assessed under different configurations of the model. First, the presence of fiscal rules does not worsen sustainability concerns as it does not affect the government's fiscal space. Yet, the possibility of self-fulfilling roll-over crises fosters market-based fiscal discipline by reducing the sovereign's optimal level of debt. In addition, the average euro area country is not subject to the conventional trade-off between macroeconomic stabilization and debt sustainability. On the one hand, the benchmark economy faces a low risk of default, since at the current debt level, the associated default probability stands below the minimum threshold associated with an investment-grade government bond. On the other hand, the domestic government finds it optimal to reduce its borrowing level in order to maximize the overall welfare for the economy. Hence, in the particular case of the benchmark calibration, the average euro area country can simultaneously reap welfare and sustainability gains through fiscal consolidation.

The remainder of the paper is organized as follows. Section 4.2 reviews the related literature. Section 4.3 presents and analyzes the theoretical features of the baseline model. Section ?? extends the baseline model to allow for the presence of a government (1) without commitment to fiscal rules and (2) maximizing its tax receipts. Section 4.5 contains the quantitative assessment, including the numerical solution, sensitivity analysis and empirical application of the model for the average euro area country. Section 4.6 puts forward concluding remarks.

4.2 Related literature

This paper is at the cross-way between two main strands in literature extant on sovereign debt. First, as regards the literature on fiscal sustainability, a number of studies has advanced different measures of fiscal space. The methodological approaches to quantify fiscal

space may be grouped in three broad categories. First, rules-based fiscal frameworks measure fiscal space as the distance of a budgetary indicator from a medium-term objective (European Commission, 2016). Second, a comprehensive framework for debt sustainability analysis (DSA) evaluates the overall sustainability of public finances through a large set of significant indicators (e.g. DSA toolboxes developed at several international organizations). Third, model-based approaches typically gauge fiscal space as the distance of the debt-to-GDP ratio from the debt limit, defined as the maximum amount of debt a government is able or willing to honor (e.g. International Monetary Fund, 2010, and Moody's, 2016). Model-based frameworks may analyze a government's debt limit via either a reduced-form (e.g. Ghosh et al., 2013 and Collard, Habib and Rochet, 2015) or a structural-form approach, depending on whether the sovereign's option to default is explicitly modeled. Among structural frameworks, a class of models conceives default as a random event linked to the government's *ability* to service its debt (e.g. Davig, Leeper and Walker, 2011, Bi, 2012 and Polito and Wickens, 2014);⁷ a different class of models posits default as a strategic event based on the government's *willingness* to honor its obligations (e.g. Aguiar and Gopinath, 2006, and Arellano, 2008).⁸

Although the implementation of policies aimed at deciding the optimal use of fiscal space ultimately depends on political and institutional factors, methodological approaches for fiscal sustainability may effectively guide policy. For instance, they may anchor decisions to consistent targets (rules-based frameworks) or show the effects of diverse scenarios on the future path of the debt level (DSA-based frameworks and reduced-form model-based frameworks). In this light, structural model-based frameworks are particularly suitable for policy evaluation, as forward-looking agents define their optimal policies based on clearly

⁷In these models, sovereign default is ultimately the result of a “divine coincidence”, or *bad luck*, occurring when the debt ratio is above a threshold randomly drawn from the debt limit distribution, constructed as the discounted sum of expected budgetary outcomes. Polito and Wickens (2014) describe alternative approaches (see also Sachs, 1989, and Aiyagari, 1994).

⁸In this class of models, sovereign default occurs as an outcome of a strategic decision by the government weighing the utility of repaying against the utility of reneging its debt. The notion of debt limit hence derived in the framework studied by Eaton and Gersovitz (1981) retains similarities with the concepts studied, for instance, by Lucas (1985) and Zhang (1997).

discernible assumptions, thus being immune from the [Lucas \(1976\)](#) critique.⁹ Finally, structural model-based frameworks related to the government's willingness to repay involve aggregate welfare considerations. Hence, they may inform the policy debate on fiscal space by assessing not only the feasibility of specific fiscal policies, but also about their desirability.

As this paper lays out a model based on the latter class of models, it is related to the literature on unsecured sovereign debt and default. In the baseline version of quantitative models with endogenous sovereign default, pioneered by [Eaton and Gersovitz \(1981\)](#), [Aguiar and Gopinath \(2006\)](#) and [Arellano \(2008\)](#), a small open economy can access incomplete international financial markets where only state non-contingent assets are available. In the absence of any commitment technology, the government determines its default policy on outstanding debt on the basis of the future discounted values of repayment and default. Default may endogenously occur in equilibrium depending on the trade-off between the benefits from debt relief and the costs from temporary financial autarky and loss in output. In turn, international lenders charge risk premia on sovereign borrowers, thus pricing in the probability of default. A positive feature of this class of models is their fair ability to replicate several dynamics and moments of macroeconomic variables, notably in emerging countries.¹⁰

The contribution of this paper to the second strand of literature is threefold. First, as regards domestic debt, this paper mainly relates to [Mallucci \(2015\)](#) and [Engler and Große Steffen \(2016\)](#), who introduce domestic debt as part of domestic intermediaries' asset holdings to explain the contraction of credit determined by a sovereign default. Both studies need two different state variables for domestic debt and total or foreign debt linked to domestic intermediaries' net worth. The comparative advantage of this paper stems from the assumption that domestic debt is held by domestic agents for simple consumption smoothing purposes, so that domestic investors' net worth is implicitly represented by output. So, the model includes domestic debt in a parsimonious way, as no additional state

⁹However, prudence requires distinguishing debt limits from optimal debt levels, whereby the latter are typically well below the former (see, for instance, the discussions in [Davig, Leeper and Walker, 2011](#), and [Ghosh et al., 2013](#)).

¹⁰See, for instance, [Neumeyer and Perri \(2005\)](#).

variable is required.

Second, this paper introduces fiscal rules imposed by supranational fiscal authorities. In this regard, this model relates to the works by [Hatchondo, Martinez and Roch \(2015\)](#) and [Arellano and Bai \(2016\)](#). Similarly to both studies, this paper measures the desirability of fiscal rules via welfare gains. In contrast to them, this model does not assume the presence of a perfect commitment technology that enforces compliance of budgetary policies with fiscal rules. Conversely, the government is allowed to deviate from fiscal rules and thus, conditional on facing a binding borrowing constraint, trigger self-fulfilling roll-over crises.

Third, this paper advances a novel measure of debt limit and proposes economically meaningful thresholds relevant for policy makers seeking to address sustainability concerns in the short-to-medium term. In particular, this paper advances a forward-looking, probabilistic notion of debt limit contingent on the current state of the economy.¹¹ Further, three thresholds are put forward to pin down meaningful points in the spectrum of sovereign borrowing opportunities, thus identifying debt limits at different levels of riskiness and time horizons.

4.3 The model

In a small open economy, households, firms and the government interact with a pool of domestic and foreign lenders. Households have preferences defined over private and public consumption and labor. They consume, work and are entitled to the entire profits of firms and domestic lenders. Firms produce consumption goods combining labor and total factor productivity (TFP) via a production function with constant returns to scale. The government is benevolent and seeks to insure households' welfare against TFP volatility. It finances public expenditures by borrowing from abroad and taxing households' consumption and labor income. Financial markets are incomplete, as the only traded assets are non-contingent long-term bonds that mature probabilistically. Debt contracts are not en-

¹¹In contrast, deterministic debt limits in the context of sovereign default may be found in [Aguiar and Gopinath \(2006\)](#), on the basis of computations à la [Lucas \(1985\)](#), and [Arellano \(2008\)](#), on the basis of considerations in [Zhang \(1997\)](#)

forceable, since the government has the option to default on their total amount. When the government repudiates its outstanding debt, the economy is temporarily in financial autarky and incurs an exogenous TFP loss. Domestic and foreign lenders bargain the optimal domestic share of total sovereign debt on primary markets and post their bids in a binding auction with the government, which in turn determines the issuance of total debt given the overall price prevailing in secondary markets.

4.3.1 Households

Time is discrete and denoted as $t \in \{0, 1, 2, \dots, \infty\}$. Infinitely-lived identical households feature the following preferences:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, h_t, g_t), \quad (4.1)$$

where E_0 denotes the expectations operator at time zero, β households' subjective discount factor, $u(c_t, h_t, g_t)$ the strictly increasing, concave and twice differentiable instantaneous utility function, c_t private consumption, h_t hours worked and g_t public consumption at time t . Households take other agents' choices as given and determine their optimal level of private consumption and working hours subject to the flow budget constraint below:

$$(1 + \tau_t^c)c_t = (1 - \tau_t^h)w_t h_t + p_t^E + p_t^H, \quad (4.2)$$

where τ_t^c and τ_t^h indicate the tax rates on consumption and labor income, respectively, while w_t per-hour wages and p_t^E and p_t^H profits received from firms and domestic lenders owned by domestic households, respectively, in units of consumption goods. The first-order conditions with respect to c_t and h_t then determine the solution for the households' problem:

$$-\frac{u_c(c_t, h_t, g_t)}{u_h(c_t, h_t, g_t)} = \frac{1 + \tau_t^c}{(1 - \tau_t^h)w_t}, \quad (4.3)$$

where $u_x(c_t, h_t, g_t)$ is defined as the derivative of $u(c_t, h_t, g_t)$ with respect to variable x_t . Equation (4.3) determines the equilibrium labor supply function.

4.3.2 Firms

Firms seek to maximize their profits, which are given by:

$$p_t^E = y_t - w_t h_t, \quad (4.4)$$

where output y_t is produced by combining labor services h_t and TFP a_t via a production function $f(h_t)$ with constant returns to scale:

$$y_t = a_t f(h_t). \quad (4.5)$$

The stochastic process for a_t is assumed to follow a first-order Markov sequence with transition probability function $F(a_t|a_{t-1})$, which is defined over the finite set $A = \{a^1, \dots, a^N\} \subset \mathbb{R}_{++}$ and approximates a first-order autoregressive (AR) process:

$$\log a_t = (1 - \rho_a) \log \bar{a} + \rho_a \log a_{t-1} + \varepsilon_t^a, \quad (4.6)$$

with ρ_a and \bar{a} denoting the AR coefficient and the long-run mean of the TFP process, respectively, $\varepsilon_t^a \sim N(0, \sigma_a^2)$ TFP shocks and σ_a their standard deviation. The first-order condition with respect to h_t solves the firms' problem and yields the optimal condition below:

$$w_t = a_t f_h(h_t), \quad (4.7)$$

which defines the equilibrium labor demand function.

4.3.3 The government

The sovereign government is benevolent and its objective is to maximize households' welfare. The sovereign determines its optimal policies by deciding (1) whether to repay or default on the total amount of its outstanding debt, (2) its fiscal policy, including public expenditures and taxes on consumption and labor income and, if repayment is optimal, (3) the amount of debt to issue or purchase on capital markets.

Debt and default policy

Each period, conditional on having access to credit markets, the government exerts its option to default d_t deciding whether to honor or default on the total amount of its outstanding debt b_t by comparing the net benefits of the two options.¹²

On the one hand, if default is optimal ($d_t = 1$), the defaulting government suffers an exogenous cost in productivity $\omega(a_t)$.¹³ In addition, the government foregoes the benefits of consumption smoothing due to the economy's exclusion from credit markets. However, financial autarky is temporary as the economy can return to international credit markets. Following [Hatchondo, Martinez and Sosa Padilla \(2014\)](#) and [Hatchondo, Martinez and Roch \(2015\)](#), as of the period after default, the government has the opportunity to end the default with a constant probability θ . In this case, the government may end default by exchanging delinquent debt with bonds promising to pay $1 - \delta \in (0, 1)$ times the payments promised by the exchanged debt. If the government rejects the deal and continues in default, its debt level remains $1 - \delta$ times the debt level before the restructuring opportunity. Hence, the

¹²As noted in previous studies with long-duration bonds (see, for instance, [Hatchondo and Martinez, 2009](#), [Chatterjee and Eyigungor, 2012](#), and [Arellano and Ramanarayanan, 2012](#)), the assumption of repudiation of all current and future debt obligations is consistent with the actual behavior of defaulting governments. Sovereign debt contracts often contain two types of clauses. The acceleration clause allows all creditors to call their debt in case the government defaults on a payment. The cross-default clause implies that, after a default event, future debt obligations become current, since a default on any government obligation also constitutes a default on the contracts containing that clause.

¹³Following [Chatterjee and Eyigungor \(2012\)](#), the loss function is assumed to be quadratic in TFP, i.e. $\omega(a_t) = \min\{a_t, \max\{0, \omega_1 a_t + \omega_2 a_t^2\}\}$ so that $a_t - \omega(a_t)$ is greater than zero. Moreover, $\omega_2 < \frac{1-\omega_1}{2a^N}$ so that $a_t - \omega(a_t)$ is strictly increasing in a_t for any $a_t \in A = \{a^1, \dots, a^N\}$. Finally, $\omega_1 < 0$ and $\omega_2 > 0$ so that the cost is nil for $0 \leq a_t \leq -\frac{\omega_1}{\omega_2}$ and rises more than proportionately with TFP for $a_t > -\frac{\omega_1}{\omega_2}$, resembling the asymmetric loss function in [Arellano \(2008\)](#). [Mendoza and Yue \(2012\)](#) show that this type of loss function may arise endogenously due to the reduction in international trade in the aftermath of a sovereign default.

recovery rate gradually declines with the time spent by the country in financial autarky. However, during default, the government's payment obligations grow at the risk-free interest rate r . Notice that δ may be interpreted as the government's haircut, or actual default rate, on the net present value of its total liabilities;¹⁴ further, the expected duration of financial autarky without restructuring opportunities is $1/\theta$.

On the other hand, if repayment is optimal ($d_t = 0$), the government weighs the costs of lowering public consumption to repay the non-contingent loan against the benefits of increasing private consumption by servicing its debt. Further, the government retains the option to borrow or lend in international credit markets by selling or buying bonds, respectively. Financial markets are incomplete, since debt contracts are state non-contingent claims to future units of consumption goods. Long-term bonds are assumed to mature probabilistically, as in [Chatterjee and Eyigungor \(2012\)](#). Each unit of debt matures next period with probability λ and gives out a coupon payment z with probability $1 - \lambda$. Thus, the expected duration of bonds can be computed as $1/\lambda$. Let the total amount of bonds issued at t be denoted by b_{t+1} . Since unit bonds are assumed to be infinitesimally small, the issuer's coupon and principal obligations next period will be $z(1 - \lambda)b_{t+1}$ and λb_{t+1} , respectively, with certainty.¹⁵ Every period the government can choose its optimal debt level b_{t+1} for the following period, anticipating that the price q_t for selling or purchasing bonds is such that lenders make zero profits in expectation. As only λb_t debt matures every period, the government can repurchase non-maturing bonds $(1 - \lambda)b_t$ and issue new bonds b_{t+1} at the zero expected-profit price on secondary markets.

¹⁴In this way, the model introduces a concept of recovery rate of debt in default similar to those studied by, for instance, [Sturzenegger and Zettelmeyer \(2008\)](#) and [Cruces and Trebesch \(2013\)](#).

¹⁵Notice that unit bonds of type (z, λ) issued at different periods in the past have exactly the same payoff structure. This formulation implies that bonds are "memoryless", whereby one needs to keep track of the total number of bonds only, thus limiting the number of state variables. Alternative formulations of long-term debt in the literature extant have used assumptions with similar implications (see, for instance, [Hatchondo and Martinez, 2009](#) and [Arellano and Ramanarayanan, 2012](#)).

Fiscal policy

Following its optimal default and debt decisions, the sovereign determines its fiscal policy. Public spending can be financed through foreign debt and domestic taxes on households' consumption and labor income. As shown by [Cuadra, Sanchez and Sapriza \(2010\)](#), in a model with unenforceable sovereign debt contracts, optimal taxation engenders a pro-cyclical fiscal policy. Since the repayment of non-contingent loans is more costly in recessions, the incentives to default are higher in bad times. Thus, in recessions the government faces higher opportunity cost of borrowing due to higher default risk and finds it optimal to rely more heavily on taxation to finance public expenditures. Conversely, in expansions cheaper credit determines an increase in financing through borrowing, while taxes play a lesser role. Thus, tax rates are pro-cyclical, since lower (higher) tax rates occur during good (bad) times.

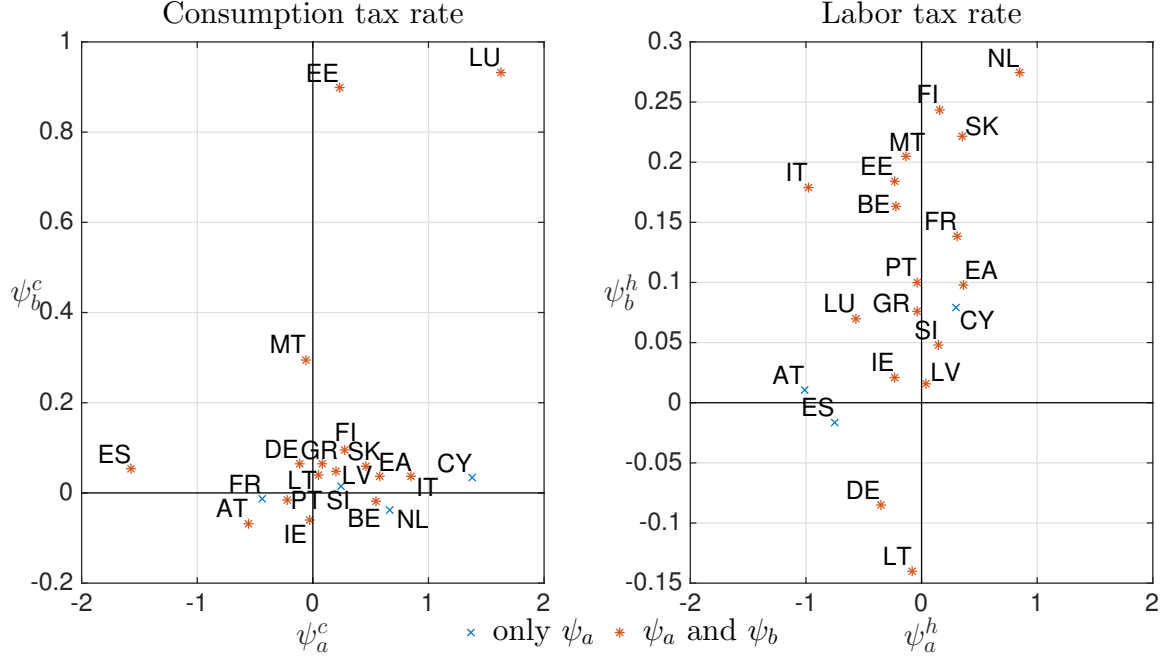
However, several empirical studies observe how fiscal policies vary across countries featuring heterogeneous economic and institutional characteristics. For instance, [Talvi and Végh \(2000\)](#) observe a-cyclical and pro-cyclical fiscal policies in G7 and developing economies, respectively, and explain the difference as due to political failures typical of emerging countries in insuring welfare against volatile macroeconomic fluctuations. Further, [Kaminsky, Reinhart and Végh \(2004\)](#) associate similar patterns with capital flow cycles, exchange rate regimes and financial market integration. Therefore, a quantitative model seeking to replicate the features of actual fiscal policies across countries with different degrees of economic and institutional development as well as their implications for debt sustainability and welfare should allow for a flexible specification.

Hence, in the baseline model, the government is assumed to tax consumption and labor income according to the targeting rules below:

$$\tau_t^c = \bar{\tau}^c + \psi_a^c(a_t - \bar{a}) + (1 - d_t)\psi_b^c(b_t - \bar{b}) \quad (4.8a)$$

$$\tau_t^h = \bar{\tau}^h + \psi_a^h(a_t - \bar{a}) + (1 - d_t)\psi_b^h(b_t - \bar{b}), \quad (4.8b)$$

Figure 4.3: Estimates of debt and productivity elasticities of tax rates



Sources: Eurostat, Eurosystem.

Notes: Markers for each point in the scatter plot refer to the statistical significance of the parameters as identified in the legend, on the basis of two-tail tests at the 10% significance level.

where τ_t^c and τ_t^h refer to the tax rates on consumption and labor income, $\bar{\tau}^c$ and $\bar{\tau}^h$ denote the tax rate on consumption, the tax rate on labor income and the amount of debt, respectively, in the risk-free (default-free) deterministic steady state of the model economy, and \bar{b} is an economic or institutional constant critical level for debt.¹⁶ Further, ψ_j^l indicates the elasticity of the tax rate on variable $l \in \{c, h\}$ with respect to the deviation of variable $j \in \{a, b\}$ from its constant critical level. Figure 4.3 plots the estimates of the elasticities ψ_j^l of tax rates with respect to TFP and debt in all euro area countries according to Equations (4.8).¹⁷ Notice that a significantly positive (negative) ψ_a^l for $l \in \{c, h\}$ may be interpreted as representing a counter-cyclical (pro-cyclical) tax policy, while a significantly positive (negative) ψ_b^l for $l \in \{c, h\}$ may be considered as associated with a generally sustainable

¹⁶Details on the identification of the critical level \bar{b} are provided in Section 4.5.1 below.

¹⁷Details on the estimation sample and the model specification can be found in Section 4.5.1 below.

(unsustainable) tax policy.¹⁸ In line with aforementioned empirical evidence, a diverse picture emerges from the estimation of the responses of governments to economic fluctuations and debt levels across the euro area. For instance, economic and institutional differences may explain pro-cyclical and counter-cyclical indirect taxation in two often-thought similar countries such as Spain and Italy, respectively. Likewise, France and Germany, typically belonging to the host of euro area core countries, exhibit sustainable and unsustainable direct taxation patterns, respectively.

Finally, given τ_t^c and τ_t^h , the government's flow budget constraint determines public spending g_t as follows:

$$g_t = \tau_t^c c_t + \tau_t^h w_t h_t + (1 - d_t)\{q_t[b_{t+1} - (1 - \lambda)b_t] - [\lambda + (1 - \lambda)z]b_t\}. \quad (4.9)$$

As in [Cuadra, Sanchez and Sapriza \(2010\)](#), public spending provides direct utility to the private sector, which prefers a smooth path of public spending over a volatile one. Yet, as non-contingent bonds are not good instruments for consumption smoothing purposes, the government is not able to smooth public spending, so that public expenditures are pro-cyclical. Therefore, the government optimally implements a pro-cyclical expenditure policy.

Recursive formulation

The intertemporal problem of the government can be expressed in a recursive dynamic programming form. This model focuses on Markov perfect general equilibria: in each period,

¹⁸[Leeper and Leith \(2016\)](#) show that in a simple model, where the government follows a debt-targeting surplus rule, a non-explosive equilibrium exists as long as the primary surplus has a sufficiently positive response to increases in real debt. When this condition is not satisfied, solutions with unbounded debt inevitably rely on non-distorting taxes, which permit revenues to grow forever at the same rate as interest receipts on government bond holdings. However, in this model, a negative (or not sufficiently positive) ψ_b^l need not lead to unbounded equilibria for two main reasons. First, unsustainable tax rates on either tax base may be more than compensated by sustainable tax rates on the other tax base. Second, the pricing of default risk entails higher costs and lower borrowing opportunities for the government when its outstanding debt is high; thus, the possibility of default naturally eliminates the possibility of unbounded debt in equilibrium. In the rare event of an unbounded solution, a negative ψ_b^l for $l \in \{c, h\}$ is set to zero to ensure convergence of the bond price/value function iterations. In any case, tax policies are dubbed (un)sustainable for the sake of convention, although tempered by the term “generally”.

the government's equilibrium strategies depend only on payoff-relevant state variables.¹⁹ The government makes all its decisions given the state of the economy, i.e. TFP a_t , the amount of outstanding foreign debt b_t and the credit situation of the country d_t . Conditional on having access to credit markets, the sovereign decides whether to honor its debt, then determining the amount of borrowing b_{t+1} , or default. In either case, after choosing its optimal default strategy, the government defines its fiscal policy, including the tax rates on consumption τ_t^c and labor income τ_t^h as well as the level of public expenditures g_t .

Conditional on a good credit standing, the government exerts its default option by solving the following problem:

$$V(a_t, b_t) = \max_{d_t} \{(1 - d_t)V^R(a_t, b_t) + d_t V^D(a_t)\}, \quad (4.10)$$

which denotes the general continuation value of the government before the determination of a default policy. If the government decides to honor its debt, then it determines the optimal borrowing level as follows:

$$V^R(a_t, b_t) = \max_{b_{t+1} \in B} \{u(c_t, h_t, g_t) + \beta E_t[V(a_{t+1}, b_{t+1})]\} \quad (4.11)$$

subject to the equilibrium conditions (4.2), (4.3), (4.4) and (4.7) for the private sector as well as (4.8) and (4.9) for fiscal policy. Notice that the borrowing decision is defined over a finite set $B = \{b^1, \dots, b^M\}$, whereby an upper bound on debt, i.e. $b_{t+1} \leq b^M$, prevents the government from engaging in Ponzi schemes but does not bind in equilibrium. If the

¹⁹As discussed by [Krusell and Smith \(2003\)](#), infinite-horizon economies may feature multiple Markov perfect equilibria. In order to avoid this problem, the numerical solution of the model entails a backward procedure simulating a finite-horizon economy. As the number of periods increases until the bond price function and the value function for the first and second periods of this economy converge, the first-period equilibrium functions are considered equilibrium functions of the infinite-horizon-economy. The outline for the solution strategy is sketched in Section 4.5.2.

government reneges its debt, then it faces the following lifetime utility:

$$V^D(a_t, b_t) = u(c_t, h_t, g_t) + \beta E_t[(\theta V(a_{t+1}, (1 - \delta)(1 + r)b_t) + (1 - \theta)V^D(a_{t+1}, (1 + r)b_t)], \quad (4.12)$$

where the same equilibrium conditions for the private sector and fiscal policy hold and TFP under default is determined as $a_t - \omega(a_t)$. The government's continuation value under default reflects the possibility to return to international credit markets with recovered debt the following period with probability θ . The presence of recovered debt (growing each period at the risk-free rate) implies that the value of default depends on outstanding debt as well as the current productivity level.

4.3.4 Lenders

A pool of domestic and foreign lenders interact on international capital markets with the domestic sovereign and have perfect information regarding the economy's realized TFP shocks. The pool of lenders can coordinate so that the government must satisfy the conditions of both groups of creditors in order to access domestic and international capital markets. The two groups of lenders interact with each other and with the sovereign in two stages.

First stage: Primary markets

In the first stage, primary markets open and an auction takes place where both domestic and foreign lenders post their respective bids on the share of total sovereign debt they are willing to purchase. At the same time, the two groups choose their optimal share of sovereign debt by engaging in a Nash bargaining game in which each counterpart's bargaining power is constant over time. Domestic lenders determine the domestic fraction b_{t+1}^H of total sovereign debt b_{t+1} issued by the government *vis-à-vis* foreign lenders by solving the

following problem:

$$\max_{b_{t+1}^H \in (0,1)} \{(E_t p_{t+1}^H)^{1-\zeta} (E_t p_{t+1}^F)^\zeta\}, \quad (4.13)$$

subject to

$$E_t p_{t+1}^H \geq 0 \quad (4.14a)$$

$$E_t p_{t+1}^F \geq 0, \quad (4.14b)$$

where $\zeta \in [0, 1]$ denotes foreign lenders' bargaining power, and $E_t p_{t+1}^H$ and $E_t p_{t+1}^F$ are the expected profits of domestic and foreign lenders. Equations (4.14) ensure that each group in the pool of lenders is willing to participate in the bargaining game. Hence, lenders are willing to purchase their respective share of government bonds only if their expected profits satisfy the conditions of each individual type.

As customary in the literature, foreign lenders can trade riskless assets on international capital markets at the risk-free rate r and are assumed to be risk neutral. In contrast, domestic lenders can only trade domestic sovereign bonds. Since they are owned by domestic households, domestic lenders are assumed to be risk averse, thus charging a premium on the actuarially fair price. As argued, for instance, by [Attinasi, Checherita-Westphal and Nickel \(2009\)](#), [Longstaff et al. \(2011\)](#), [Kennedy and Palerm \(2014\)](#) and [Cimadomo, Claeys and Poplawski-Ribeiro \(2016\)](#), investor risk aversion is a major driver of the dynamics of government bond yields across emerging and advanced economies. Following [Arellano and Ramanarayanan \(2012\)](#), the pricing kernel is modeled as a function of the borrower's income.²⁰ Domestic lenders' bond price kernel is given by $M(a_{t+1}|a_t) \equiv \exp(-(1-\beta)/\beta - \sigma \varepsilon_{t+1}^a - \frac{1}{2} \sigma^2 \sigma_a^2)$, where ε_{t+1}^a is defined by Equation (4.6). This definition of

²⁰The pricing kernel is a function of only the borrower's income because it is a parsimonious way to model risk premia that vary with the probability of default. This method has the advantage of avoiding (1) the introduction of an additional exogenous state variable into the model and (2) the computational burden of non-linear numerical solutions for each grid point of the state space and at each bond price/value function iteration. Although theoretically appealing, modeling the discount factor as the marginal rate of substitution between consumption today and tomorrow would inevitably forgo these benefits.

the stochastic discount factor is a special case of the discrete-time version of the one-factor model of the term structure proposed by [Vasicek \(1977\)](#) and [Backus, Foresi and Telmer \(1998\)](#). As households own domestic lenders, they share the same preferences, whereby the subjective discount rate is given by $(1 - \beta)/\beta$ and the market price of risk by σ .²¹ This specification implies that $M(a_{t+1}|a_t)$ is negatively correlated with domestic lenders' payoff next period and bond prices reflect compensation for the risk of a sovereign default in states when investors have high marginal utility.²² The risk premium then comes from the interaction of the lenders' pricing kernel with default outcomes and future bond prices.

Domestic and foreign lenders' expected profits, respectively, are given by

$$E_t p_{t+1}^H = E_t \{ M(a_{t+1}|a_t) \{ d_{t+1} q_{t+1}^D + (1 - d_{t+1}) [\lambda + (1 - \lambda)(z + q_{t+1})] \} \} b_{t+1}^H b_{t+1} - q_t^H b_{t+1}^H b_{t+1} \quad (4.15a)$$

$$E_t p_{t+1}^F = E_t \{ (1 + r)^{-1} \{ d_{t+1} q_{t+1}^D + (1 - d_{t+1}) [\lambda + (1 - \lambda)(z + q_{t+1})] \} \} (1 - b_{t+1}^H) b_{t+1} - q_t^F (1 - b_{t+1}^H) b_{t+1}, \quad (4.15b)$$

where q_t , q_t^H and q_t^F denote the unique price prevailing in secondary markets, the bid price for domestic lenders and the bid price for foreign lenders, respectively. Equations (4.15) show the relevant expected profits for both types of investors before participating in the auction. In period $t + 1$, in the event of repayment, lenders get fraction λ of a maturing bond and, on the remaining fraction $1 - \lambda$, they receive the coupon payment z . In addition, the fraction that remains outstanding is traded in secondary markets and its value q_{t+1} depends on the persistent component of the TFP shock next period and on the sovereign's outstanding debt next period. Notice that, given the recursive formulation of the government's problem, q_{t+1} depends on the same policy functions used by the government at period t for q_t . Moreover, in the event of default, lenders' payoff is equal to the bond

²¹In a similar environment, [Lintner \(1970\)](#) shows that in purely competitive markets in which investors with given constant risk aversion choose their investment positions on the basis of identical Normal distributions over end-of-period outcome, the markets' risk aversion is the market price of risk.

²²The negative correlation between $M(a_{t+1}|a_t)$ and foreign lenders' payoff is ensured as long as $\sigma > 0$; in this case, a negative shock ε_{t+1}^a to future income decreases both the repayment probability and future prices, whereas it increases $M(a_{t+1}|a_t)$.

price under default q_t^D , defined below. Finally, next period payoff is discounted by the discount rates $M(a_{t+1}|a_t)$ and $(1+r)^{-1}$, which represent the relevant bond pricing kernels for domestic and foreign lenders, respectively.

As both types of lenders are perfectly competitive, both constraints in Equations (4.14) are satisfied with equality at the same time. Hence, both domestic and foreign lenders are subject to a zero-expected profit condition:

$$q_t^H = E_t\{M(a_{t+1}|a_t)\{d_{t+1}q_{t+1}^D + (1-d_{t+1})[\lambda + (1-\lambda)(z + q_{t+1})]\}\} \quad (4.16a)$$

$$q_t^F = E_t\{(1+r)^{-1}\{d_{t+1}q_{t+1}^D + (1-d_{t+1})[\lambda + (1-\lambda)(z + q_{t+1})]\}\}, \quad (4.16b)$$

thereby ensuring that both the auction and the bargaining process always occur in equilibrium.

Finally, the solution to the Nash bargaining game yields

$$b_{t+1}^H = 1 - \zeta, \quad (4.17)$$

which links the optimal share of sovereign debt captured by financial intermediaries to their bargaining power. This specification then allows the parameter for foreign lenders' bargaining power ζ to explicitly target the average domestic share of sovereign debt.²³

Second stage: Secondary markets

In the second stage, as type-specific bond prices satisfy the zero-expected profit conditions and domestic and foreign lenders bargain the optimal allocation of sovereign debt, secondary markets open and the exchange of bonds between the pool of lenders and the government occurs. Since the bonds issued by the government represent a homogenous good, the law of one price holds in equilibrium. Given the regime of perfect competition in international capital markets, the overall market price of bonds must satisfy a zero-expected profit con-

²³Notice that this formulation is not subject to the curse of dimensionality, since it introduces domestic debt without requiring the introduction of an additional state variable.

dition for the entire pool of lenders. Hence, the bond price prevailing in secondary markets converges towards a weighted average of the prices for the two groups of lenders:

$$q_t = b_{t+1}^H q_t^H + (1 - b_{t+1}^H) q_t^F, \quad (4.18)$$

where the weight on each price is the respective share of total sovereign debt purchased by the individual group of lenders.²⁴ Ultimately, the overall price q_t represents the average cost of borrowing and becomes the relevant price for the government's borrowing decisions. Notice that this specification for q_t closely reflects the process of price formation in actual issuances of sovereign bonds on primary and secondary markets. On primary markets, auctions typically start with different investors posting their individual profit-maximizing bids; thereafter, the sovereign issues the total amount of debt on the basis of the overall price, a weighted average of the different bid prices, which becomes the relevant price on secondary markets. Secondary markets also form the bond price under default q_t^D , which is given by

$$\begin{aligned} q_t^D = & q_t E_t \{ (1 - \theta)(1 + r) q_{t+1}^{DD} \} + \\ & q_t E_t \{ \theta(1 - \delta) [d_{t+1} q_{t+1}^{DR} + (1 - d_{t+1}) [\lambda + (1 - \lambda)(z + q_{t+1})]] \}, \end{aligned}$$

where, given b_{t+1} , q_{t+1}^{DD} is equal to q_{t+1}^D for $(1 + r)b_{t+1}$ and q_{t+1}^{DR} is equal to q_{t+1}^D for $(1 - \delta)b_{t+1}$.

Two considerations suggest that the price schedule should be bound. First, [Hatchondo, Martinez and Sosa Padilla \(2014\)](#) argue that, in a model with long-duration bonds with a positive recovery rate, the government may have incentives to issue large amounts of debt, hence strongly expanding consumption, before defaulting. Following [Hatchondo, Martinez and Padilla \(forthcoming\)](#) and [Hatchondo, Martinez and Roch \(2015\)](#), in order to avoid this

²⁴This result follows from the observation that the aggregate zero-expected profit condition for the pool of lenders is given by

$$\begin{aligned} b_{t+1}^H q_t^H + (1 - b_{t+1}^H) q_t^F = \\ E_t \{ [d_{t+1} q_{t+1}^D + (1 - d_{t+1}) [\lambda + (1 - \lambda)(z + q_{t+1})]] [b_{t+1}^H M(a_{t+1} | a_t) + (1 - b_{t+1}^H)(1 + r)^{-1}] \}. \end{aligned}$$

and that, by the law of one price, $q_t = q_t^H = q_t^F$.

problem, the sovereign is assumed to face a lower bound \underline{q} on the issuing prices of bonds q_t^H and q_t^F , while the price of government debt can be lower than \underline{q} on secondary markets. The value for \underline{q} is chosen so as to avoid consumption booms before defaults.²⁵ Second, the sovereign faces an upper bound \bar{q} on the secondary market bond price q_t . The value for \bar{q} matches the maximum bond price available in an economy inhabited by perfectly-competitive risk-neutral international lenders, that is the inverse of the (gross) risk-free rate $1 + r$. If there is no possibility of default, the unit price would be a constant \bar{q} such that $\bar{q} = [\lambda + (1 - \lambda)(z + \bar{q})]/[1 + r]$, which implies $\bar{q} = [\lambda + (1 - \lambda)z]/[\lambda + r]$. Since $q_t \leq \bar{q}$, it follows that an internal rate of return r_t^* which makes the present discounted value of the promised sequence of future payments on a unit bond equal to the unit price is never below r . Finally, since the model is solved at annual frequency, the difference between the per-period yield r_t^* , such that $q_t = [\lambda + (1 - \lambda)z]/[\lambda + r_t^*]$, and r is the annualized government bond yield spread r_t .

4.3.5 General equilibrium and debt limit

The following definition formally establishes the (Markov perfect) general equilibrium of this model.

Definition 4. *Given any state (a_t, b_t) at period t , the recursive general equilibrium is defined as the set of policy functions for (i) households' consumption and labor supply, (ii) firms' labor demand, (iii) financial intermediaries' share of domestic debt, (iv) the government's debt supply, default option and tax rates, (v) foreign lenders' debt demand and (vi) value functions $\{V(a_t, b_t), V^R(a_t, b_t)$ and $V^D(a_t)\}$, such that:*

- *Taking as given the government and lenders' policies, c_t satisfies households' budget constraint (4.2), h_t satisfies households' labor supply function (4.3) and w_t satisfies firms' labor demand function (4.7);*
- *Taking as given the private sector and foreign lenders' policies, d_t and b_{t+1} solve*

²⁵The yield to maturity implied by \underline{q} is higher than the maximum yield to maturity at which any European government issued debt since 2008 (Trebesch and Wright, 2013) and is never binding in simulations.

problems (4.10)-(4.11), and τ_t^c , τ_t^h and g_t satisfy the government's targeting rules and budget constraint (4.8a), (4.8b) and (4.9), respectively;

- Taking as given the government and the non-financial private sector's policies, b_{t+1}^H satisfies the solution to the Nash bargaining game (4.17), q_t^H , q_t^F and q_t are consistent with investors' zero-expected profit conditions (4.16) and (4.18).

The definition below identifies the default set as a function of the outstanding amount of total sovereign debt.

Definition 5. For any given amount of outstanding debt, the default set $\mathcal{D}(b_t)$ is defined as the set of TFP shocks a_t for which available debt contracts in international capital markets make it optimal for the sovereign to default. Formally,

$$\mathcal{D}(b_t) = \{a_t : V^R(a_t, b_t) < V^D(a_t)\}. \quad (4.19)$$

The following definition establishes the forward-looking, probabilistic notion of debt limit contingent on the current state of the economy.

Definition 6. The probability- ϕ debt limit $b_{t+1}(\phi)$ is defined as the maximum level of borrowing today that makes it optimal for the sovereign to repay the total amount of its outstanding debt tomorrow with minimum probability $1 - \phi$ for any given choice of borrowing tomorrow. Formally,

$$b_{t+1}(\phi) = \sup\{b_{t+1} : \Pr[V^R(a_{t+1}, b_{t+1}) < V^D(a_{t+1})] \leq \phi\} \quad (4.20a)$$

$$= \sup\{b_{t+1} : E_t \mathcal{D}(b_{t+1}) \leq \phi\}. \quad (4.20b)$$

According to Definition 6, the probability- ϕ debt limit $b_{t+1}(\phi)$ is the maximum level of borrowing today associated with a probability of default tomorrow smaller than or equal to ϕ . This definition of debt limit is state contingent, due to the persistence of the TFP process as well as the debt and default policy functions, which link the current state of the

economy to next period's TFP shock, optimal borrowing level and default option. Moreover, this definition of debt limit is forward looking and probabilistic, as $b_{t+1}(\phi)$ depends on the expected realization for the TFP shock and the choice of default tomorrow. Finally, notice that this definition explicitly links the government's value associated with both options $V^R(a_{t+1}, b_{t+1})$ and $V^D(a_{t+1})$ to the maximum amount of borrowing it is committed to repay. Hence, the notion of debt limit herein proposed is inherently related to the government's *willingness* to repay. As such, this notion differs from other definitions used in the literature on debt limits and sovereign creditworthiness, mostly hinging on the government's *ability* to repay.²⁶

From the probability- ϕ debt limit, it is possible to derive the current fiscal space associated with a probability ϕ of future default. Notice that, disregarding stock-flow adjustments, the government's debt adjustment corresponds to the decrease (increase) in the face value of debt from the current to the next period, that is $\Delta b_t = b_t - b_{t+1}$. Let the probability- ϕ debt adjustment $\Delta b_t(\phi) = b_t - b_{t+1}(\phi)$ denote the minimum adjustment ensuring a default probability smaller than or equal to ϕ . Then, the relevant measure of fiscal space may be computed as

$$f_t(\phi) = \Delta b_t - \Delta b_t(\phi), \quad (4.21a)$$

$$= b_{t+1}(\phi) - b_{t+1}. \quad (4.21b)$$

The probability- ϕ fiscal space $f_t(\phi)$ is the maximum (absolute) amount available to the government for expanding its borrowing level while maintaining the probability of default tomorrow below or at ϕ . Hence, $f_t(\phi)$ gauges the sovereign's leeway in maximizing households' welfare while ensuring debt sustainability.

Given the recursive formulation of the government's policy functions for default, borrowing as well as taxation and public consumption, it is then possible to calculate the probability of default at different time horizons. Let ϕ_t^n denote the probability of default in

²⁶See, for instance, the works by [Ghosh et al. \(2013\)](#), [Bi \(2012\)](#) and [Polito and Wickens \(2015\)](#).

any period from $t + 1$ to $t + n$ without defaulting in any previous period and conditional on information available at t . Formally,

$$\phi_t^n \equiv \sum_{s=1}^n E_t(d_{t+s} | d^{t+s} = 0) \quad (4.22)$$

$$= \sum_{s=1}^n E_t(d_{t+s}) \prod_{r=0}^{s-1} [1 - E_t(d_{t+r})], \quad (4.23)$$

where d^{t+s} refers to the history of default decisions from t to $t + s - 1$. Hence, $b_{t+1}(\phi_t^n)$ and $f_t(\phi_t^n)$ are the relevant measures of debt limit and fiscal space for forward-looking policy makers seeking to evaluate the sustainability of debt at different time horizons.

4.3.6 Default incentives with domestic sovereign debt

In a model with domestic sovereign debt, the government internalizes the effects of its debt and default policies on households' welfare by solving problems (4.10)-(4.12). To see how the presence of domestic debt affects the government's incentives to default, consider the case of one-period bonds (i.e. $\lambda = 1$). The proposition below formalizes the main result.

Proposition 3. *For all $b_t^1 \leq b_t^2$, if default is optimal for b_t^1 , then it will be optimal for b_t^2 , namely $\mathcal{D}(b_t^1) \subseteq \mathcal{D}(b_t^2)$, if and only if $\frac{1-\zeta}{1+\tau_t^c} \leq \frac{u_g(c_t, h_t, g_t)}{u_c(c_t, h_t, g_t)}$. Conversely, if default is optimal for b_t^2 , then it will be optimal for b_t^1 , namely $\mathcal{D}(b_t^2) \subseteq \mathcal{D}(b_t^1)$, if and only if $\frac{1-\zeta}{1+\tau_t^c} \geq \frac{u_g(c_t, h_t, g_t)}{u_c(c_t, h_t, g_t)}$.*

Proof. See Appendix C.1. □

The first part of Proposition 3 states that, due to the monotonicity of the utility function, the sovereign faces higher incentives to default for higher levels of aggregate debt, but, due to the concavity of the utility function, this result holds only if the share of debt held by the domestic private sector is smaller than the marginal rate of substitution between public and private consumption. In other words, rising amounts of aggregate debt imply an increasing number of states for a_t in which the value of default is higher than the value of repayment only for sufficiently high levels of private (relative to public) consumption or, equivalently, for sufficiently low levels of marginal utility of private (relative

to public) consumption. In contrast, the second part of Proposition 3 states the opposite result according to which the value of staying in the contract increases with aggregate sovereign debt as long as the share of debt held domestically is greater than the marginal rate of substitution between public and private consumption.

As observed by Arellano (2008), higher incentives to default are associated with higher levels of debt because the value of staying in the contract is decreasing in b_t , whereas the value of default is independent of b_t . If default is preferred in a given state a_t for some level of debt b_t , the value of the contract is less than the value of default. As b_t increases, the value of the contract monotonically decreases, becomes even lower than before and so default will continue to be preferred, since the value of default remains constant. Nevertheless, differently from Arellano (2008), this result does not hold for any level of private (relative to public) consumption. To see this, notice that, to the extent that government bonds are held domestically, public liabilities represent private assets. Hence, delinquent debt affects private and public consumption in different ways: default on a unit of debt raises public consumption by $u_g(c_t, h_t, g_t)$ whereas it lowers private consumption by $\frac{1-\zeta}{1+\tau_t^c} u_c(c_t, h_t, g_t)$ units. Hence, default incentives increase with debt as long as the positive effect of default on public consumption is larger than its negative effect on private consumption. Notice that, due to the concavity of the utility function, this condition holds when private consumption is sufficiently high relative to public consumption. In this case, the net effect of default on aggregate welfare is positive since the marginal benefit on public consumption more than compensates its marginal cost on private consumption.

Therefore, the relationship between default incentives and additional borrowing essentially depends on the distribution of domestic resources between the private and the public sectors. Given households' preferences as well as a path of borrowing decisions and TFP shocks, the distribution of the economy's output between private and public consumption is essentially determined by the government's tax rates on consumption and labor income. *Ceteris paribus*, as tax rates decrease, public consumption declines compared to private consumption; hence, the marginal benefit of default for the former increases compared to

its marginal cost for the latter. Therefore, additional borrowing raises default incentives as long as tax rates are sufficiently low. In previous literature (see, for instance, [Cuadra, Sanchez and Sapriz, 2010](#)), taxation contributes to debt sustainability only by increasing public consumption and thus reducing the marginal cost of honoring the government's obligations. In this model, domestic debt amplifies the positive externality of taxation on debt sustainability: the government's incentives to stay in the contract also increase because taxes decrease private consumption and, thus, increase the overall marginal cost of default. Importantly, the model may produce the counterfactual result according to which the marginal cost of default may offset its marginal benefit, thus determining negative net welfare effects of default: in this case, the probability of default *decreases* with debt.²⁷ Hence, an economy with high tax rates features a higher (lower) public (private) consumption, a lower (higher) marginal benefit (cost) of default on public (private) consumption, so that the government faces lower incentives to default and, thus, higher levels of sustainable debt compared to an economy with low tax rates. Finally, the relationship between default incentives and additional borrowing also depends on the domestic share of aggregate sovereign debt: a higher share $1 - \zeta$ reduces default incentives as it increases the marginal private cost of default relative to its marginal public benefit. The exact quantitative implications of domestic sovereign debt on default incentives require knowledge of the specific functional forms and are then analyzed in the numerical assessment of the model.

4.4 Fiscal rules and credibility

Extending the baseline model, the government is assumed to interact with supranational fiscal authorities requiring compliance with fiscal rules. Fiscal rules are typically defined as numerical targets on fiscal aggregates expected to be in place over a long period, aimed at correcting distorted incentives in policy making by binding national fiscal authorities to medium-term objectives. In this extension of the model, the presence of credible fiscal rules imposed by supranational fiscal authorities allows for the possibility of roll-over crises,

²⁷Although possible in theory, this result never occurs with the plausible calibrations studied in this paper.

namely self-fulfilling defaults, via a signaling mechanism. The government's compliance with fiscal rules informs market expectations about sovereign credibility: as a signal of an imminent sovereign default, a deviation from fiscal rules triggers a run by creditors on the government's debt and causes its anticipated default, thus realizing market expectations. This signaling mechanism shows how supranational fiscal authorities and international investors may interact through fiscal rules and market perceptions and contribute to an effective market-based fiscal discipline.

Fiscal rules are typically considered as institutional mechanisms aimed at supporting fiscal credibility and discipline (see, for instance, [Giavazzi and Pagano, 1990](#)). Difficulties in identifying the effects of fiscal rules are well documented (see, for instance, [Poterba, 1996](#) and [Afonso and Hauptmeier, 2009](#)). However, several studies on the effects of fiscal rules in the EU find that rules are essential means to inform market expectations about sovereign credibility, interpreted as the *perceived* probability of a government servicing its debt (see, for instance, [Heinemann, Osterloh and Kalb, 2014](#), and [Nerlich and Reuter, 2015](#)). The credibility channel of fiscal rules may play a major role especially in institutional and economic communities where supranational fiscal authorities may not credibly enforce compliance through alternative methods, such as sanctions or outright exclusion of borrowing opportunities deviating from the applicable fiscal rules. In this case, the possibility of fiscal slippage is particularly relevant, as governments can *de facto* expand their liabilities beyond the levels implied by fiscal rules.

This observation carries important implications for the model. First, in contrast to previous studies on sovereign debt and default with fiscal austerity plans ([Hatchondo, Martinez and Roch, 2015](#), and [Arellano and Bai, 2016](#)), the government is not assumed to be endowed with a commitment technology that forces compliance with the fiscal rules imposed by supranational fiscal authorities.²⁸ Moreover, similarly to [Chatterjee and Eyigungor \(2012\)](#), this extension allows for the possibility of self-fulfilling debt crises in a model with

²⁸When comparing model predictions with past experiences with fiscal rules, one should consider that previous studies are assuming certainty about the government's ability to commit to enforcing a rule, but such certainty has often been lacking in the past.

long-term debt. Yet, while their focus is on exogenous “sunspot run equilibria”, in this model roll-over crises are endogenously triggered by a deviation from fiscal rules via a signaling mechanism that informs market perceptions about the government’s credibility. More specifically, if the government is subject to credit constraints – i.e. it would rather default than buy back debt from the market – and deviates from the applicable fiscal rules, foreign lenders receive a negative signal on the sovereign’s credibility and fret over its willingness or ability to honor its obligations. As a result, creditors run on the government’s liabilities and cause its default, thus realizing their own expectations. When “run equilibria” become more likely with higher deficits or debt levels, lower government bond prices reflect the increased probability of an imminent default. Therefore, as the sovereign seeks to stave off the risk of self-fulfilling debt crises and thus decrease its marginal cost of borrowing, fiscal rules foster market-based fiscal discipline via the credibility channel.²⁹

On the basis of these observations, this model devises a signaling mechanisms linking a government’s deviation from fiscal rules to increased market pressure via the credibility channel. Similarly to previous studies on sovereign debt and default with fiscal austerity plans, fiscal rules assume that the correction occurs through the government’s debt policy, rather than its fiscal policy.³⁰ More specifically, the model investigates two rules, namely a deficit rule and a debt rule, closely reflecting the deficit criterion and the debt reduction rule, respectively, enshrined in the defining text of the EU fiscal governance framework, the Stability and Growth Pact (see [European Commission, 2016](#)).

First, the deficit rule requires the surplus s_t to be above a certain threshold:

$$s_t \geq \hat{s}y_t, \quad (4.24)$$

²⁹Notice that, in a model with long-term debt, this mechanism rewards longer over shorter maturities through lower borrowing costs. In the context of sovereign borrowing, [Cole and Kehoe \(2000\)](#) argue that “run equilibria” are less likely if the sovereign issues long-term debt. With a large stock of long-term debt, the maturing portion of debt can be small, so that lenders’ refusal to roll over does not significantly affect the borrower. Knowledge of this informs lenders’ expectations and runs fail to be an equilibrium outcome.

³⁰This assumption implies that the government may alter its issuance of debt more flexibly than its tax rates. However, the government cannot freely adjust its budget: if the required correction entails negative (public) consumption, then the government is assumed to default and suffer a TFP loss.

whereby the government commits to run a deficit-to-output ratio below the deficit ceiling $-\hat{s} \in \mathbb{R}$ every period. The measure of surplus in the model needs to be consistent with surplus actually targeted by governments. To extract the model equivalent of the headline budget balance, or net government lending, considered by the EU fiscal governance framework, notice that, in a model with long-duration bonds and primary and secondary markets, the (negative) change in debt can be decomposed as follows from Equation (4.9):

$$b_t - b_{t+1} = \underbrace{\tau_t^c c_t + \tau_t^h w_t h_t - g_t}_{\text{primary surplus}} - \underbrace{(1 - \lambda) z b_t}_{\text{interest payments}} + \underbrace{(1 - q_t)[(1 - \lambda)b_t - b_{t+1}]}_{\text{deficit-debt adjustments}}$$

headline surplus (s_t)

where the first term corresponds to primary surplus (tax receipts minus public consumption), the second term refers to interest payments and the third term indicates deficit-debt (or stock-flow) adjustments due to market-to-face-value corrections.³¹ Hence, the model equivalent of the headline budget balance is given by the difference between the first and the second term.

Second, conditional on the debt-to-output ratio exceeding the debt target \hat{b} , the debt rule requires that

$$b_{t+1} \leq b_t - \frac{b_t - \hat{b}y_t}{\hat{T}}, \quad (4.25)$$

whereby the government commits to reduce the part of its debt-to-output ratio in excess of $\hat{b} \in \mathbb{R}_+$ so as to reach the debt target within $\hat{T} - 1$ periods from t . Hence, $\hat{T} \in \{1, 2, \dots, \infty\}$ gauges the degree of backloading required by supranational fiscal authorities, with $\hat{T} = 1$ implying a complete frontloading of the debt adjustment in the current period.³² The

³¹According to the [European Central Bank \(2014b\)](#), general government debt (and therefore the change in debt) is recorded at face value, whereas financial transactions in the ESA 2010 are recorded at market value including accrued interest. In order to compensate for this difference in valuation, the deficit-debt adjustment includes the market-to-face-value adjustment as one of the items contributing to valuation effects and other change in debt excluded from EDP deficit. The adjustment is calculated as face values minus market values and applies only to transactions – that is, to new borrowings and repayment or buying-in of debt at prices which differ from nominal value (issuances and redemptions below or above par).

³²Further, notice that the convergence path required by the debt rule does not entail the actual achievement of the debt target, but only a gradual (and endless) adjustment towards it.

two constraints (4.24) and (4.25) restrict the government's set of available debt contracts depending on the current state of the economy (a_t, b_t) and a set of “institutional” parameters $(\hat{s}, \hat{T}, \hat{b})$.

In what follows, the signaling mechanism is described. Consider a static coordination game played by the sovereign and foreign lenders at the start of any period in which the sovereign has positive outstanding debt and, conditional on honoring its obligations, desires to issue new bonds. The columns give the strategies of the sovereign and the rows give the strategies of lenders. If lenders purchase the new bond (B) and the sovereign repays its existing debt (R), the sovereign receives the payoff from repaying the loan and borrowing, denoted $V^+(a_t, b_t)$, and lenders earn a net return of 0 (i.e. lenders earn the risk-free return $M(a_{t+1}|a_t)$ which is also the opportunity cost of their funds). If lenders purchase (B) and the sovereign defaults (D), the new bond is returned to the issuer and earns no interest, whereby lenders incur the (discounted) loss of interest earnings $r\hat{M}_{t+1}\Delta_t$, where \hat{M}_{t+1} is the weighted average of domestic and foreign lenders' discount factors and Δ_t is the amount of new lending, and the sovereign receives $\bar{V}^D(a_t, b_t)$. If lenders do not purchase the new bond (N) even if the sovereign repays (R), the sovereign receives $V^-(a_t, b_t) \leq V^+(a_t, b_t)$ and lenders earn 0. Finally, if lenders do not lend (N) and the sovereign defaults (D), the payoffs are 0 and $\bar{V}^D(a_t, b_t)$ for lenders and the sovereign, respectively.

	R	D
B	$0, V^+(a_t, b_t)$	$-r\hat{M}_{t+1}\Delta_t, \bar{V}^D(a_t, b_t)$
N	$0, V^-(a_t, b_t)$	$0, \bar{V}^D(a_t, b_t)$

Further, assume that the sovereign always repays if it is indifferent between repaying and defaulting and lenders always purchase if they are indifferent between purchasing and not purchasing. Hence, this game has the following set of Nash equilibria, depending on the value of $\bar{V}^D(a_t)$. When $\bar{V}^D(a_t) \leq V^-(a_t, b_t) \leq V^+(a_t, b_t)$, the unique equilibrium is (B, R) ; if $V^-(a_t, b_t) \leq V^+(a_t, b_t) < \bar{V}^D(a_t)$, the unique equilibrium is (N, D) ; and if $V^-(a_t, b_t) < \bar{V}^D(a_t) \leq V^+(a_t, b_t)$, both (B, R) and (N, D) are equilibria of the game. In

this case, the realized equilibrium depends on the government's compliance with the fiscal rules in force. If the sovereign complies with the fiscal rule, the (B, R) equilibrium occurs; otherwise, the (N, D) equilibrium is selected. The latter case corresponds to a self-fulfilling debt crisis, whereby the government is credit constrained and deviates from fiscal rules, so that lenders receive a negative signal about the sovereign's credibility and run on its outstanding obligations.

This game requires the modification of the recursive equilibrium described above as follows. Let $U(a_t, b_t)$ denote the lifetime utility of the sovereign, depending on the latter's compliance with the applicable fiscal rules. Then,

$$V^+(a_t, b_t) = \max_{b_{t+1}} \{u(c_t, h_t, g_t) + \beta E_t[U(a_{t+1}, b_{t+1})]\}$$

subject to the same equilibrium conditions as problem (4.11). If there is no b_{t+1} such that the arguments of the current utility functions are nonnegative, then $V^+(a_t, b_t)$ is assumed to be equal to $-\infty$. Moreover,

$$V^-(a_t, b_t) = \max_{b_{t+1}} \{u(c_t, h_t, g_t) + \beta E_t[U(a_{t+1}, b_{t+1})]\}$$

subject to the same equilibrium conditions as problem (4.11) and $b_{t+1} \leq (1 - \lambda)b_t$, so that only nonpositive net issuance of new bonds is considered. Again, if there is no b_{t+1} such that the arguments of the current utility functions are nonnegative, then $V^-(a_t, b_t)$ is assumed to be equal to $-\infty$. Notice that $V^-(a_t, b_t) \leq V^+(a_t, b_t)$, unless the government finds it optimal to issue new bonds, in which case $V^-(a_t, b_t) < V^+(a_t, b_t)$. In addition, the value under default is similarly defined as

$$\bar{V}^D(a_t, b_t) = u(c_t, h_t, g_t) + \beta E_t[(\theta U(a_{t+1}, (1 - \delta)(1 + r)b_t) + (1 - \theta)\bar{V}^D(a_{t+1}, (1 + r)b_t)].$$

Finally, the continuation value for the sovereign is given by

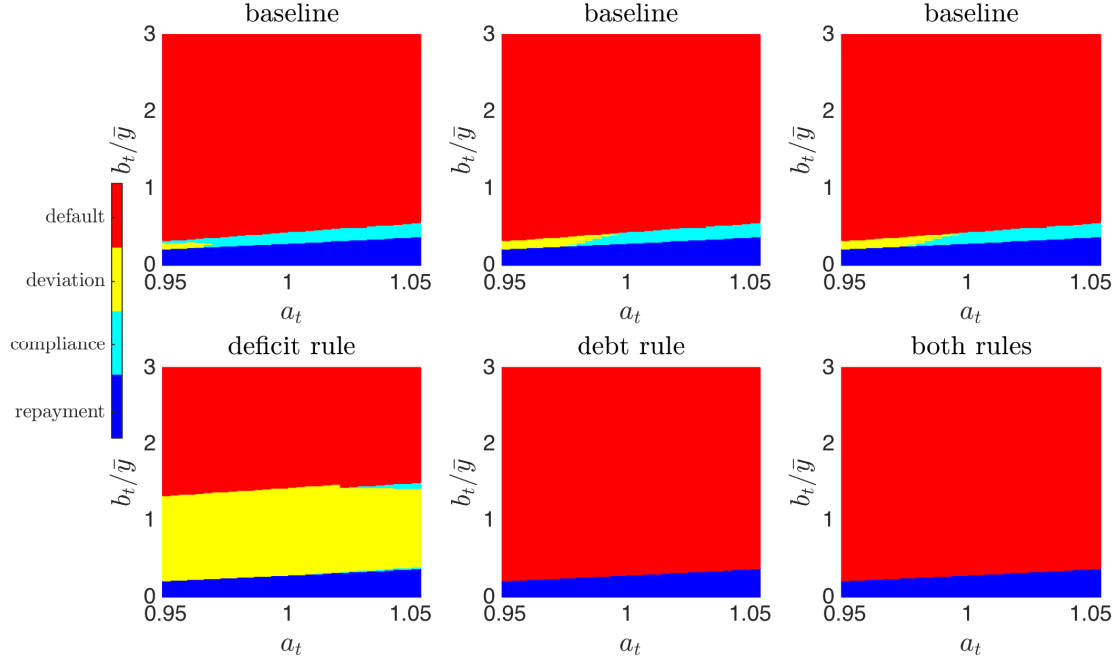
$$U(a_t, b_t) = \begin{cases} V^+(a_t, b_t) & \text{if } \bar{V}^D(a_t) \leq V^-(a_t, b_t) \quad (a) \\ \bar{V}^D(a_t) & \text{if } V^+(a_t, b_t) < \bar{V}^D(a_t) \quad (b) \\ V^+(a_t, b_t) & \text{if } V^-(a_t, b_t) < \bar{V}^D(a_t) \leq V^+(a_t, b_t) \text{ and } k_t = 1 \quad (c) \\ \bar{V}^D(a_t) & \text{if } V^-(a_t, b_t) < \bar{V}^D(a_t) \leq V^+(a_t, b_t) \text{ and } k_t = 0, \quad (d) \end{cases} \quad (4.26)$$

where $k_t = 1$ if the government complies with all the applicable fiscal rules (4.24) and/or (4.25) and $k_t = 0$ otherwise. As a consequence, the policy functions for the borrowing level b_{t+1} and the zero-expected profit condition for the bond price q_t need to be updated consistently with problem (4.26). Notice that the probability of a future deviation from the fiscal rule $E_t(1 - k_{t+1})$ depends on the current borrowing decision as well as the future TFP shock.³³ Hence, *ex ante* compliance with or deviation from the fiscal rule need not coincide with its *ex post* outcome. In other words, the government does not retain full control over the outcome of its decisions: anticipating this, when deficits or debt levels (and, hence, the risk of roll-over crises) are high, the sovereign may prefer larger fiscal consolidations compared to a situation without self-fulfilling defaults in order to reduce the cost of servicing its debt.

The numerical solution of the model sheds further light into the quantitative implications of the credibility channel of fiscal rules. Figure 4.4 obtains by solving the model with the calibration for the average euro area country reported in Table 4.1, while assuming impatient agents ($\beta = 0.85$), a high coupon rate ($z = 0.1$), one-period bonds ($\lambda = 1$) and no domestic debt ($\zeta = 1$) in order to create noticeable effects on the agents' optimal policies. Figure 4.4 shows the occurrence of repayment (i.e. case (a) in Equation (4.26) above), compliance (i.e. case (c) above), deviation (i.e. case (d) above) and outright default (i.e. case (b) above) in the state space for TFP and debt. Each chart on the upper row refers to

³³In Chatterjee and Eyigungor (2012), the occurrence of self-fulfilling roll-over crises depends on the realization of an i.i.d. sunspot variable, so that “run equilibria” realize with a constant probability independent of income fluctuations.

Figure 4.4: Occurrence of repayment, compliance, deviation and default in the state space



Notes: These charts are produced with the calibration reported in Table 4.1 except for four parameters, namely $\beta = 0.85$, $z = 0.1$, $\lambda = 1$ and $\zeta = 1$.

the baseline model, where only outright default affects the solution, and shows the states where roll-over crises would occur in case the government deviated from the fiscal rule(s) applicable in the model reflected in the adjacent chart on the lower row. The charts on the lower row refer, from left to right, to the extension of the baseline model where only the deficit rule, only the debt rule and both rules are enforced. Under this specification of model parameters, fiscal rules tighten the borrowing constraint faced by the government as states with the possibility of a roll-over crisis in the baseline model transform into states with realized roll-over crises – in the model with only the deficit rule – or outright defaults – in the models with only the debt rule and both rules.

However, the benchmark calibration for the average euro area country does not exhibit such noticeable effects.³⁴ The credibility channel of fiscal rules may have little additional impact because the gap between $V^+(a_t, b_t)$ and $V^-(a_t, b_t)$ is positive only when the sovereign wishes to increase its debt b_{t+1} to more than $(1 - \lambda)b_t$. In general, the gap is positive when

³⁴Results for different models with the benchmark calibration are reported in Section 4.5 below.

borrowing costs are low and the government has an incentive to issue new debt, which happens when output is high and outstanding debt is low. Since during such times the default cost is high and $\bar{V}^D(a_t, b_t)$ is low, the conditions for a roll-over crisis (which requires that $V^-(a_t, b_t) < \bar{V}^D(a_t) \leq V^+(a_t, b_t)$) rarely occur and the randomness introduced by fiscal rules bears little consequence. In any case, as argued by [Chatterjee and Eyigungor \(2012\)](#), the introduction of stochastic roll-over crises is essential to explain the benefits of long-term debt relative to short-term debt. If the sovereign is carrying a large amount of one-period debt, the gap between $V^+(a_t, b_t)$ and $V^-(a_t, b_t)$ is large since the full payment of a large amount of debt is very costly. Thus, it is much more likely that the conditions for a default triggered by a deviation from fiscal rules will be satisfied.

4.5 Quantitative assessment

In this section, the benchmark calibration and results from the numerical evaluation of the baseline and extended models with policy experiments are presented. The functional form for households' momentary utility function is assumed to be

$$u(c_t, h_t, g_t) = \kappa \frac{\left(c_t - \frac{h_t^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}}\right)^{1-\sigma}}{1-\sigma} + (1-\kappa) \frac{g_t^{1-\sigma}}{1-\sigma},$$

where κ denotes the relative weight placed by households on private consumption, σ the coefficient of relative risk aversion (or the reciprocal of the intertemporal elasticity of substitution), ν is the Frisch elasticity of working hours. In order to solve the model, the state space is discretized. The support for TFP shocks spans 31 grid points around the long-run mean of the original process. The state space for the government bond price includes 300 equally-spaced grid points between 0 and $3\bar{y}$, so that implications can be drawn for debt-to-mean output ratios up to 300%.

Table 4.1: Calibration of model parameters in average euro area country

parameter	value	target
σ	2	standard macroeconomic literature
\bar{a}	1	normalization
\bar{h}	0.33	standard macroeconomic literature
\hat{s}	-0.03	EU fiscal governance framework
\hat{b}	0.60	EU fiscal governance framework
\hat{T}	20	EU fiscal governance framework
ρ_a	0.38	TFP AR coefficient
σ_a	0.0163	TFP shock standard deviation
β	0.96	private sector's average credit rate
r	0.03	1-year Euribor
θ	0.33	expected duration of economic adjustment programme
ω_1	-0.80	1% TFP loss at lower bound
ω_2	0.88	maximum TFP loss at upper bound
$\bar{\tau}^c$	0.27	tax rate on total consumption
$\bar{\tau}^h$	0.33	tax rate on total labor income
\bar{g}/\bar{c}	0.36	public-to-private consumption ratio
λ	0.17	average maturity of government liabilities
z	0.06	government implicit tax rate
δ	0.65	haircut on net present value of sovereign liabilities
ζ	0.55	share of total debt held by domestic residents
\bar{r}	1.00	highest observed interest rate
ψ_b^c	0.13	elasticity of consumption tax rate to debt
ψ_a^c	0.19	elasticity of consumption tax rate to TFP
ψ_b^h	0.09	elasticity of labor tax rate to debt
ψ_a^h	-0.13	elasticity of labor tax rate to TFP

4.5.1 Calibration

Table 4.1 reports the values of the model parameters calibrated at an annual frequency. The sample covers 19 euro area countries between 1995 and 2015. Data are drawn from the Eurosystem (Government Finance Statistics), Eurostat (National Accounts) and DataStream. Parameters for the representative euro area country are computed by taking a simple average over the estimated country-specific parameters. Values for 25 parameters need to be determined, whereas the other parameters are derived from the equilibrium conditions in the default-free deterministic steady state of the model. The inverse of the intertemporal elasticity of substitution σ and the steady state level of labor supply \bar{h} are set to 2 and 0.33, respectively, which are standard values in the macroeconomic literature. The long-run mean of the TFP process is normalized to 1. Further, the parameters defining the fiscal

rules \hat{s} , \hat{b} and \hat{T} take on values -3%, 60% and 20 to reflect the deficit and debt criteria envisaged in the EU Stability and Growth Pact. Defining TFP as the detrended ratio of real GDP to hours worked, the AR(1) process is fitted to Equation (4.6), determining an AR coefficient $\rho_a = 0.38$ and the standard deviation for TFP shocks $\sigma_a = 0.0163$. Households' subjective discount factor β matches the non-financial private sector's average credit rate, defined as the cost of borrowing for new short-term loans on households and non-financial corporations. The rate of return r on riskless assets is set to 3% due to the long-run average value for the 1-year Euribor. The probability θ of receiving an offer to settle for recovered debt is set to 0.33, which corresponds to the expected duration of macroeconomic adjustment programmes implemented in Europe since the start of the sovereign debt crisis.³⁵ The output cost of default $\omega_1 = -0.80$ targets a 1% TFP loss at the lower end of the state space, as this loss is estimated by [Borensztein and Panizza \(2009\)](#) to be the *direct* real output cost of default; ω_2 is then set to 0.88 so as to ensure that $\omega(a_t)$ is increasing in a_t and reaches a maximum at the upper end of the state space. Similarly to [Mendoza, Razin and Tesar \(1994\)](#) and [Reicher \(2014\)](#), the steady-state tax rate on consumption $\bar{\tau}^c = 27\%$ is calculated as the long-run average of the ratio between indirect taxes and the sum of private consumption and government intermediate consumption net of indirect taxes, while the steady-state tax rate on labor income $\bar{\tau}^h = 33\%$ is computed as the long-run average of the ratio between the sum of direct taxes on households and enterprises and social contributions net of subsidies and the sum of compensation of employees in the total economy and the gross operating surplus net of the numerator. The gross operating surplus is in turn defined as nominal GDP minus compensation of employees in the total economy minus indirect taxes plus subsidies. The steady-state public-to-private consumption ratio \bar{g}/\bar{c} is set to 0.36 as

³⁵In the context of EU financial assistance to requesting countries, economic adjustment programmes typically last roughly three years, as in the case of Ireland (2010-2013), Portugal (2011-2013) and Cyprus (2013-2016); the third programme for Greece started in 2015 and was set to end in 2018. The duration of financial autarky and debt renegotiations after sovereign defaults, in particular on external debt, has received considerable attention in the literature. For instance, for Argentina's default in 2001 the settlement with the majority of the creditors was reached in 2005. In the default episodes of Russia (1998), Ecuador (1999) and Ukraine (1998), the renegotiation process lasted 2.3, 1.7 and 1.4 years, respectively, according to [Benjamin and Wright \(2009\)](#). In general, domestic debt restructuring periods tend to be not as long as in the case of external debt. For example, as documented by [Sturzenegger and Zettelmeyer \(2007\)](#), after the default by Russia in 1998 it took six months to restructure the domestic GKO bonds.

the long-run average of the ratio of government consumption to private consumption. The probability λ of debt obligations maturing is 17%, targeting a long-run average maturity of total debt of 5.97 years. The coupon rate z is defined as the implicit tax rate on non-matured debt, that is the long-run average of the ratio of interest payments to $(1 - \lambda)$ times outstanding debt at the end of the previous period. Following the empirical methodology in [Sturzenegger and Zettelmeyer \(2008\)](#), the actual default rate δ on the net present value of the government's total liabilities is set to 65%. Foreign lenders' bargaining power $\zeta = 0.55$ is determined as 1 minus the share of total debt held by domestic residents.³⁶ On the basis of arguments in the study by [Trebesch and Wright \(2013\)](#), the maximum interest rate charged on the sovereign takes on value 100%, higher than any interest rate actually ever observed in the data for euro area countries on relevant maturities. The elasticities ψ_j^l of tax rates for $l \in \{c, h\}$ and for $l \in \{c, h\}$ are fitted to Equations (4.8), where the constant critical level \bar{b} for debt is set to the default-free steady-state level of debt.³⁷ Given the estimates of these 25 parameters, the remaining parameters obtain via the default-free deterministic steady-state of the model economy. Most notably, the values for the Frisch elasticity of labor supply ν and the weight of private (relative to public) consumption κ are 1.70 and 0.26, respectively, within the range of values commonly found in the macroeconomic literature. Furthermore, the maximum price $\bar{q} = 1.11$ implies a negative risk-free yield to maturity of -9.6% on 6-year government bonds,³⁸ the minimum price $\underline{q} = 0.19$ implies a maximum

³⁶When time series for the share of total government debt held by domestic residents are not available (as in the case for Ireland, Luxembourg and Malta), they are proxied by the share of total securities other than shares issued by the government held by domestic monetary and financial institutions.

³⁷Manipulating the government's budget constraint (4.9) yields the steady-state condition $\bar{b} = [\bar{\tau}^c \bar{c} + \bar{\tau}^h \bar{a} \bar{h} - \bar{g}] / [(1 + \bar{q} - z)\lambda + z]$. Alternatively, the critical level \bar{b} could be equal to $\bar{b}\bar{y}$ (60% of GDP) following extensive empirical research on targeting rules for fiscal policy in the European context. In a DSGE model for fiscal policy evaluation, [Pappa and Vassilatos \(2007\)](#) set the steady-state level of debt-to-GDP ratio to 60% to match the long-run average ratio for France and Germany between 2000 and 2005. In their empirical investigation of the determinants of government's fiscal behaviour for EU countries, [Afonso and Hauptmeier \(2009\)](#) evaluate the effects of fiscal rules against relevant benchmarks and find that, when the debt-to-GDP ratio is below the debt thresholds of 60%, 70% or 80%, a stronger overall fiscal rule contributes to improve the primary budget balance. In their estimates of fiscal reaction functions for the EU, [Plödt and Reicher \(2014\)](#), set the critical level for debt to reflect the 60% debt limit laid out by the Stability and Growth Pact and find that a fiscal rule that encourages a strong reduction in debt levels within twenty years would result in substantial pressure to run large primary surpluses for some countries.

³⁸This result stems from the relationship between the coupon rate z and the risk-free interest rate r , whereby high values for the former (due to high interest payments) and low values for the latter (due to monetary policy rates around the zero lower bound) entail negative interest rates paid on riskless long-

yield to maturity of 434%, which rarely binds in numerical solutions but effectively limits the occurrence of consumption booms before default.

4.5.2 Model mechanics

The model is solved numerically via simultaneous value function and bond price function iterations.³⁹ The convergence issues due to the presence of multiple Markov perfect equilibria reported by Chatterjee and Eyigungor (2012) are negligible once a tie-break rule for the optimal debt policy is applied. According to this rule, the optimal debt policy b_{t+1}^i at iteration i cannot be higher than b_{t+1}^{i-1} . As iterations are initialized assuming a nil value function next period and a price function at its risk-free level, this rule may be interpreted as a backward solution in a finite-horizon environment from the last period, in which the highest amount of debt is chosen (as no default risk is charged on the sovereign borrower), to the current period, in which the sovereign internalizes the costs of default and opts for lower debt levels. This approach retains considerable computational advantages, as it does not entail the introduction of an additional state variable to allow for the “randomization” of the optimal debt policy.⁴⁰

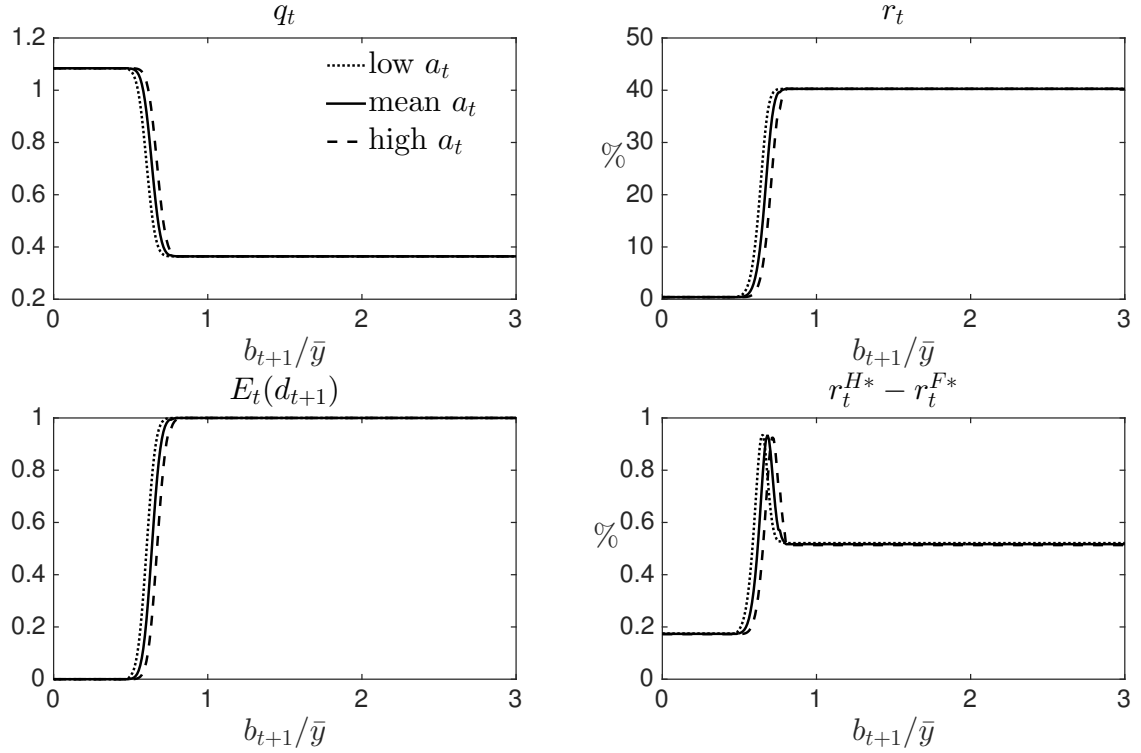
As shown in Figure 4.5, the bond price schedule q_t and the associated per-period spread r_t exhibit the common patterns observed in models with both strategic and non-strategic sovereign default (see, Arellano, 2008, and Bi, 2012). The price (spread) decreases (increases) as borrowing levels rise and productivity shrinks (top charts). This pattern is typically associated with the probability of default next period $E_t(d_{t+1})$, which increases as the government builds up leverage and during recessions (bottom left-hand chart). However, three additional factors drive interest rate dynamics. First, the presence of long-duration bonds drives a wedge between interest rates and default probabilities, as lenders understand

duration bonds, consistently with empirical evidence (see, for instance, *Wall Street Journal*, June 14, 2016: German 10-Year Government Bond Yields Dip Below Zero as Brexit Fears Hit Market).

³⁹This approach is shown by Hatchondo, Martinez and Sapriz (2010) to have considerable advantages over a solution algorithm with separate iterations for the value function and the bond price function in terms of computational time.

⁴⁰This solution strategy bears similarities with several approaches in the literature on quantitative sovereign default (see, for instance, Arellano and Bai, 2014a, and Hatchondo, Martinez and Roch, 2015).

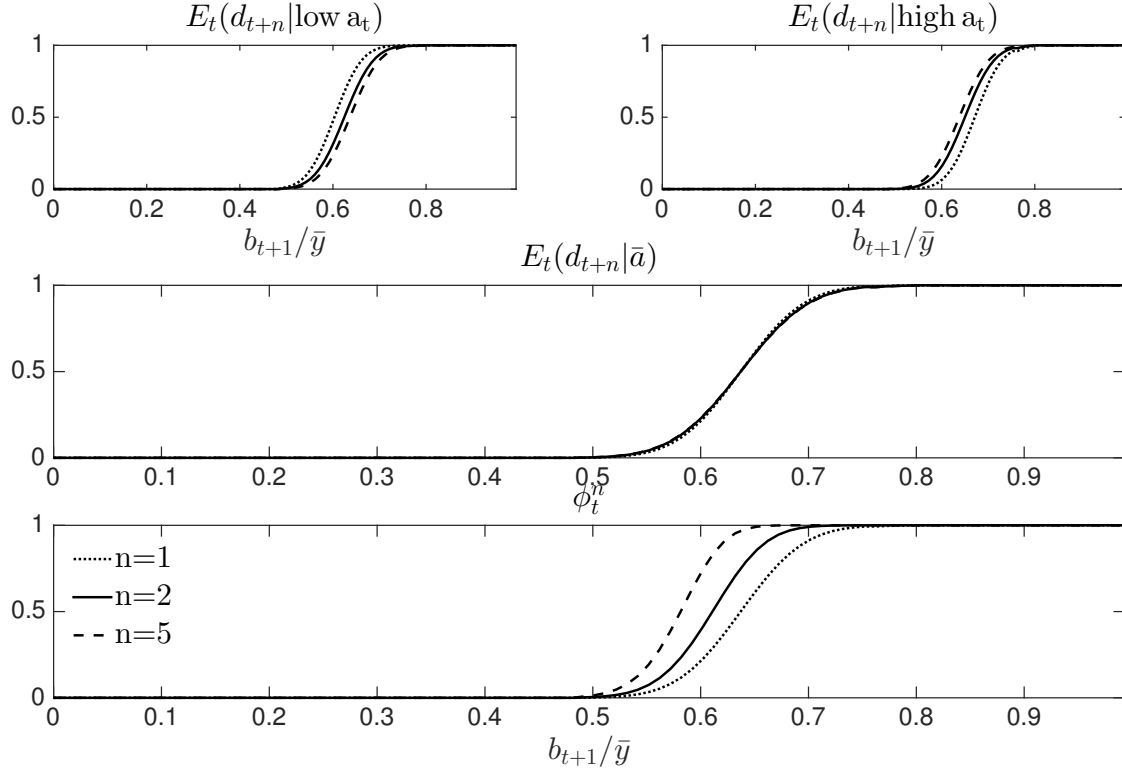
Figure 4.5: Price and spread on secondary markets and spread between primary market bids



Notes: The variables are shown at the steady-state level for a_t and two standard deviations on each side of \bar{a} .

that the sovereign's optimal decision next period is to take on a significant amount of debt even when the sovereign borrows a very small amount in the current period. Therefore, lenders' rational expectation to suffer a capital loss on the non-maturing portion of the debt further depresses prices even when the default probability is zero. Second, the existence of a recovery rate for debt implies that government bonds can be traded at a positive price q_t^D under default, as debt obligations grow at the risk-free rate while the country is in financial autarky, thus putting upward pressure on the secondary market price q_t . Third, spreads and default probabilities diverge due to the presence of a pool of lenders composed of risk neutral foreign investors and risk averse domestic investors. Hence, risk aversion drives a wedge between the actuarially fair price charged by foreign lenders and the price requested by domestic investors. The spread between the interest rate charged by domestic

Figure 4.6: Default probabilities at different time horizons



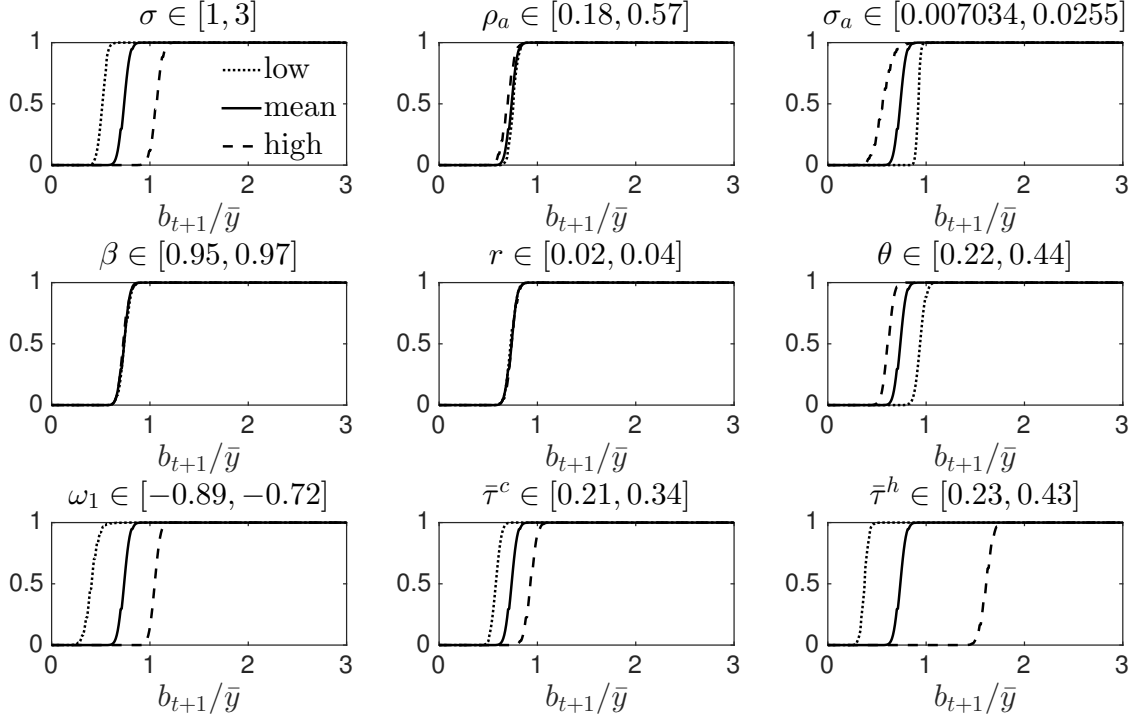
and foreign lenders r_t^{H*} and r_t^{F*} , calculated as r_t^* but replacing q_t with q_t^H and q_t^F is depicted in the bottom right-hand chart. A persistent spread of 0.2% is charged as long as debt is perceived as safe, but it quickly rises up to almost 1% as borrowing becomes risky and then decreases to below 0.6% as the steepness of the price schedule becomes smaller. Notice that productivity significantly affects domestic investors' pricing paradigm only in the risky part of the sovereign's borrowing opportunities, whereby low TFP levels are associated with faster increases in interest rates as the sovereign borrows more compared to high TFP levels, since domestic investors factor in the negative effects of default on their expected income.

This model produces state-contingent estimates of debt limit distributions in an economy with stylized fiscal policy and strategic sovereign default decisions. Debt limit distributions essentially depend on two factors, namely the current state of the economy – via the equilibrium conditions of the model – and the default-free deterministic steady state of the economy – via the calibrated parameters. As regards the current state of the economy,

the distributions of the debt limit $b_{t+1}(\phi_t^n)$ as a ratio of mean output can be observed from the probability of default ϕ_t^n at different horizons and its determinants are shown in Figure 4.6. In the top charts, the probability $E_t(d_{t+n})$ of a default occurring exactly at time $t + n$ given information at t has different behaviors as the time horizon n increases depending on whether the economy is below or above its default-free steady-state TFP level \bar{a} at t . Due to the mean reversion property of the stochastic process for TFP, negative (positive) shocks are expected when TFP is above (below) \bar{a} , so that TFP is expected to return to its long-run mean. Hence, $E_t(d_{t+n})$ decreases at longer horizons during recessions due to the expectation of an economic recovery (whereas the opposite is true when the economy is growing). In the case the economy is close to its long-run mean (second row in Figure 4.6), the distribution becomes slightly more disperse without noticeable effects on its expected value reflecting increased uncertainty as agents look farther into the future. As expected, the probability ϕ_t^n of default between $t + 1$ and $t + n$ given no default in any period $s \in \{t, \dots, t + n - 1\}$ rises with n for any borrowing level, due to the increasing odds of a sovereign reneging its debt contracts as the time horizon expands.

As regards the deterministic steady state of the economy, a sensitivity analysis reveals the crucial role of some parameters in determining the results of the model. Figures 4.7 and 4.8 show the effect of different structural conditions on the debt limit distribution $b_{t+1}(\phi_t^n)$ for $n = 1$. The range of values under consideration spans two standard deviations around the average of the cross section of parameter estimates for the countries in the sample, except for parameters common across countries, whose range is defined as the chosen value plus or minus one third. Hence, all the values studied in the sensitivity analysis lie within a set of plausible estimates. Most importantly, results should be interpreted in view of their significance for the selected sample of countries, while bearing in mind that setting parameters to values beyond the investigated range – some of which are often found in the related literature – may produce further changes to the debt limit distributions. The distributions for the benchmark calibration are compared against those implied by a different calibration of the model, where only one parameter is changed over the grid of plausible

Figure 4.7: Sensitivity analysis of the probability of default



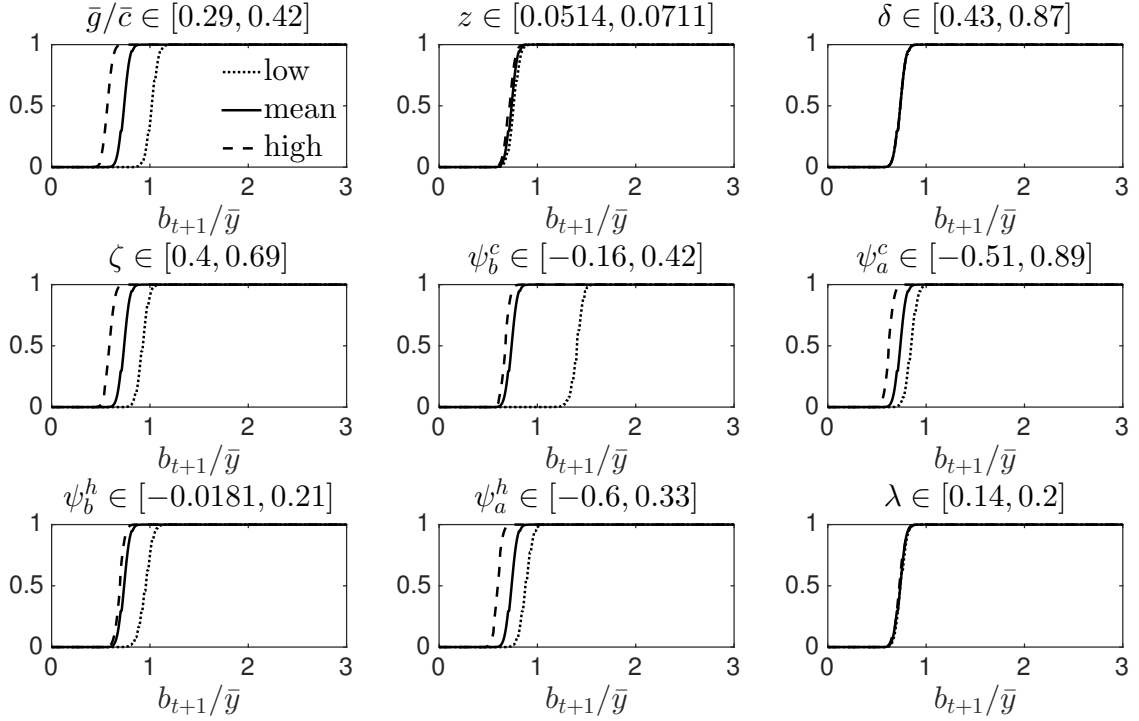
Notes: The default probabilities are shown at the steady-state level for a_t . The charts show debt limit distributions for the benchmark calibration (mean) and those implied by a calibration where only the analyzed parameter is changed over the grid of plausible values.

values.

As shown in Figures 4.7 and 4.8, some parameters do not have a sizeable impact on the distribution of $b_{t+1}(\phi_t^1)$, notably ρ_a , β , r , z , δ and λ . The most relevant parameters may be grouped as follows.

Riskiness and uncertainty. As domestic households' risk aversion σ increases, the distribution for the debt limit shifts rightward: since households become more risk averse, consumption smoothing through sovereign debt is valued more and, in turn, the government's borrowing opportunities increase. Furthermore, given a certain degree of risk aversion, a higher standard deviation of TFP shocks σ_a increases uncertainty; as the likelihood of tail events increases, higher TFP volatility makes default more frequent, especially at relatively low debt levels, and it translates into lower and more

Figure 4.8: Sensitivity analysis of the probability of default (continued)



Notes: The default probabilities are shown at the steady-state level for a_t . The charts show debt limit distributions for the benchmark calibration (mean) and those implied by a calibration where only the analyzed parameter is changed over the grid of plausible values.

disperse debt limit distributions.

Cost of default. Parameters linked to the direct cost of default, such as the probability θ of receiving a settlement offer over recovered debt and the outright TFP loss of default ω_1 , have similar effects: if θ increases or ω_1 decreases, then default becomes relatively less costly, as the economy spends shorter spells under financial autarky or it experiences a smaller contraction in productivity, respectively; hence, the sovereign faces lower incentives to honor its obligations and its debt limit distribution shifts leftward.⁴¹

⁴¹To see why lower values for ω_1 are associated with lower costs of default, notice that $\omega_1 = (\iota - a^1/2a^N)/(1 - a^1/2a^N)$, so as to target a $\iota\%$ TFP loss at the lower end of the state space for $a_t \in A = \{a^1, \dots, a^N\}$. Clearly, ι is directly related to ω_1 . Then, since $\omega_2 = (1 - \omega_1)/2a^N$, the loss function $\omega(a_t)$ is increasing in its argument and reaches a maximum at a^N , so that a higher cost at a^1 translates into a higher cost at all $a_t \in A$.

Aggregate welfare. A set of parameters refers to the weight of the public sector's utility relative to the weight of the private sector's utility on aggregate welfare, namely $\bar{\tau}^c$, $\bar{\tau}^h$, \bar{g}/\bar{c} , ζ and ψ_j^l for $j \in \{a, b\}$ and for $l \in \{c, h\}$. The effects of different parameter values can be understood in light of Proposition 3 and its implications for the aggregate welfare of the economy by comparing the marginal private cost and public benefit implied by the sovereign's enforcement of its option to default.

Steady-state tax rates. Higher $\bar{\tau}^c$ and $\bar{\tau}^h$ increase the resources available to the government to repay its debt and, thus, expand its borrowing capacity; yet, this effect may hold only as long as tax rates $\bar{\tau}^c$ and $\bar{\tau}^h$ are below the values corresponding to the peak of the Laffer curve; most importantly, higher tax rates imply a transfer of resources from the private to the public sector and, accordingly, an increase in the marginal utility of private consumption relative to public consumption; thus, the marginal cost of default on private consumption becomes higher, as opposed to the marginal benefit of default on public consumption; eventually, the sovereign's default incentives decrease and its borrowing opportunities rise.

Public-to-private consumption ratio. As \bar{g}/\bar{c} increases, the steady-state marginal rate of substitution between public and private consumption decreases and so does the relative weight κ on the utility of private consumption, which is negatively affected by a domestic default; thus, as incentives to repudiate debt increase, the government faces tighter borrowing constraint.

Foreign lenders' bargaining power. A lower ζ determines a higher domestic share of total debt, so that a sovereign default imposes a relatively higher loss on the private sector's consumption, thus becoming relatively more costly for aggregate welfare; hence, the reduction in default incentives translates into higher borrowing opportunities.

Debt elasticities of tax rates. Lower (or even negative) ψ_b^l for $l \in \{c, h\}$, corre-

sponding to a generally more unsustainable tax policy, deteriorate the credit conditions for the sovereign borrower, but only at sufficiently high debt levels; conversely, the government's borrowing capacity increases at low debt levels since a lower (or less positive) elasticity implies a lower decrease (or a larger increase) in the tax rate for debt b_t *below* the critical level \bar{b} and a higher transfer of resources from the private to the public sector; as usual, a lower private (relative to public) consumption implies a higher marginal private cost (relative to the marginal public benefit) of default and higher incentives for the government to service its debt for sufficiently low debt levels; in the explored deviations from the benchmark calibration, this improvement outweighs the tightening of the borrowing constraint at relatively high debt levels, so that the overall effect is a rightward shift in the debt limit distribution.

TFP elasticities of tax rates. Higher ψ_a^l for $l \in \{c, h\}$, corresponding to a more counter-cyclical (or less pro-cyclical) tax policy, decrease the set of borrowing opportunities available to the sovereign; this tightening of the government's credit constraint occurs as a higher (or less negative) elasticity implies a larger decrease (or a lower increase) in the tax rate for TFP a_t *below* the long-run mean \bar{a} and a lower transfer of resources from the private to the public sector during recessions; again, a higher private (relative to public) consumption implies a lower marginal private cost (relative to the marginal public benefit) of default and a higher default probability during economic contractions; in the explored deviations from the benchmark calibration, this deterioration offsets the loosening of the credit constraint during economic expansions, so that the overall effect is a leftward shift in the debt limit distribution.

Ultimately, although results warrant caution in the calibration methodology, this analysis suggests that the model is sufficiently flexible, so that it can explain a wide range of economic conditions. A proper calibration may yield significant results through the estimation of realistic debt limit distributions for a specific country with specific structural

features.

4.5.3 Macroeconomic stabilization and debt sustainability

This structural approach allows for the accurate assessment of debt limit distributions and associated fiscal space, so as to evaluate the ability of fiscal policy in acting as a stabilizer of economic fluctuations and, thus, cushioning the economy against shocks. This section presents a yardstick approach useful for policy makers seeking to identify meaningful thresholds for the riskiness of different levels of debt and a measure for welfare associated with them.

First, in order to assess the riskiness implied by borrowing levels in different sectors of the debt limit distribution, significant thresholds for ϕ_t^n should be identified. To this purpose, three levels for the default probability bear a particular significance. First, according to the Eurosystem credit assessment framework (so-called ECAF), a regulatory text aimed at mitigating the credit risk of collateral used in monetary policy operations, all assets accepted by the Eurosystem as eligible collateral must meet the minimum requirement of an assessment of a certain credit quality on a harmonised rating scale, corresponding to a probability of default over a one-year horizon of up to 0.40%. Second, credit ratings are perceived by investors and policy makers as suitable proxies for the probability of default. Assuming that a credit rating reflects exclusively the ability of a government to repay debt, [Polito and Wickens \(2014\)](#) use interpolation techniques to map credit ratings into default probabilities at different time horizons. On the basis of credit ratings and default probabilities available in [Moody's \(2011\)](#), they find that the minimum default probability associated with speculative grade securities issues by sovereign borrowers is 41.5% at the one-year horizon – and increasing at longer maturities. Updating their results on the basis of [Moody's \(2015\)](#), the corresponding figure decreases to 34.5%, mainly reflecting normalizing market conditions from the heights of the crisis. Hence, a conservative assumption is to consider sovereign debt securities featuring a speculative grade whenever the probability of default crosses the 30% threshold. Third, the natural benchmark to assess debt sustainability is the

case in which the government is expected to repudiate its obligations with certainty, that is a 100% probability of default. Accordingly, the analysis below refers to the probability- ϕ debt limit and fiscal space associated with a certain horizon n and risk threshold w given information at t as $b_{t+n}^w(\phi_t^n)$ and $f_{t+n}^w(\phi_t^n)$, respectively, for $w \in \{L, M, H\}$, where L , M and H denote a low, medium and high level of riskiness depending on whether ϕ_t^n is below or equal to 0.40%, 30% or 100%, respectively.

Second, it is essential to identify a measure of the welfare gains from adjusting away from the current debt level within the spectrum of borrowing opportunities. With this purpose in mind, welfare gains are gauged as the constant proportional change in units of consumption that would leave a consumer indifferent between preserving the current debt level in the following period, such that $b_{t+1} = b_t$, and borrowing a different amount.⁴² Consider the model economy subject to the possibility of a roll-over crisis triggered by the credit-constrained sovereign deviating from the either fiscal rule, so that the violation of either constraint (4.24) or (4.25) may trigger run equilibria. Let $U(a_t, b_t, b_{t+1})$ denote the value function given the current state of the economy (a_t, b_t) and a borrowing decision b_{t+1} when both rules are enforced. The welfare gain $W(b_{t+1})$ of adjusting away from b_t towards b_{t+1} is computed as follows:

$$W(b_{t+1}) = \left(\frac{U(a_t, b_t, b_{t+1})}{U(a_t, b_t, b_t)} \right)^{\frac{1}{1-\sigma}} - 1. \quad (4.27)$$

By definition, the welfare gain $W(\tilde{b}_{t+1})$ under the optimal debt policy \tilde{b}_{t+1} exceeds the welfare gain $W(b_{t+1})$ implied by any other borrowing decision. Moreover, notice that, as the specification of the utility function values consumption smoothing, the proposed measure of welfare is inherently linked to the benefits of macroeconomic stabilization on the overall economy.

Table 4.2 reports debt limits $b_{t+n}^w(\phi_t^n)$ and fiscal spaces $f_{t+n}^w(\phi_t^n)$ for $w \in \{L, M, H\}$ and $n \in \{1, 2, 5\}$ years in Panels A and B, respectively; Panel C shows the associated

⁴²This approach is common in the literature on quantitative sovereign debt and default. See, for instance, the works by [Arellano and Bai \(2016\)](#) and [Hatchondo, Martinez and Roch \(2015\)](#).

Table 4.2: Debt limit and fiscal space at different horizons and risk levels

Panel A. Debt limit – $b_{t+n}^w(\phi_t^n)$ (% of GDP)											
	2015	Optimal	L	1 year M	H	L	2 years M	H	L	5 years M	H
Baseline	81.44	65.59	122.21	135.94	161.55	120.54	133.35	161.55	119.83	130.04	161.49
Deficit rule	81.44	65.25	122.21	135.94	161.55	120.54	133.35	161.55	119.83	130.04	161.49
Debt rule	81.44	64.78	122.21	135.94	161.55	120.54	133.35	161.55	119.83	130.04	161.49
Both rules	81.44	64.78	122.21	135.94	161.55	120.54	133.35	161.55	119.83	130.04	161.49
Panel B. Fiscal margin – $f_{t+n}^w(\phi_t^n)$ (% of GDP)											
	Excess	L	1 year M	H	L	2 years M	H	L	5 years M	H	
Baseline	15.85	40.77	54.5	80.11	39.1	51.91	80.11	38.39	48.6	80.05	
Deficit rule	16.19	40.77	54.5	80.11	39.1	51.91	80.11	38.39	48.6	80.05	
Debt rule	16.66	40.77	54.5	80.11	39.1	51.91	80.11	38.39	48.6	80.05	
Both rules	16.66	40.77	54.5	80.11	39.1	51.91	80.11	38.39	48.6	80.05	
Panel C. Effects of adjustment from current debt level under both rules (%)											
	Optimal	L	1 year M	H	L	2 years M	H	L	5 years M	H	
Sustainability gain	0.00	-0.40	-30.00	-100.00	-0.40	-30.00	-100.00	-0.40	-30.00	-100.00	
Welfare gain	0.38	-9.51	-62.88	-116.25	-8.17	-44.31	-80.44	-7.69	-24.43	-41.17	

welfare gains $W(b_{t+n}^w(\phi_t^n))$ and sustainability gains, defined as the difference in the default probability implied by opting to remain at the current level of debt, so that $b_{t+1} = b_t$, and the default probability implied by moving to a different amount of borrowing, so that $b_{t+1} = b_{t+n}^w(\phi_t^n)$ for $n = 1$. The first column reports the optimal borrowing level \tilde{b}_{t+1} (Panel A) and excess debt $b_t - \tilde{b}_{t+1}$ (Panel B) which separates the economy from reaching its welfare-maximizing level of debt. The figures in Table 4.2 refer to the benchmark calibration of the average economy.

As regards debt limits, Panel A shows how $b_{t+n}^w(\phi_t^n)$ (as a share of steady-state GDP) increases with the level of riskiness w from $b_{t+1}^L(\phi_t^1) = 122\%$ to $b_{t+1}^H(\phi_t^1) = 162\%$ and decreases with the number of years n to $b_{t+5}^L(\phi_t^5) = 119\%$ and $b_{t+5}^H(\phi_t^5) = 161\%$. Although the presence of fiscal rules does not significantly affect the debt limit distribution, the optimal debt is lower whenever the model economy is subject to any fiscal rule compared to the baseline economy, with the debt rule implying the largest reduction in the government's optimal debt level (from 66% to 65% of GDP). Thus, in the average euro area country, the possibility of rule-driven market crises deters the credit-constrained government from increasing its borrowing levels compared to a situation where no rules are enforced.

As regards fiscal space, Panel B shows that $f_{t+n}^w(\phi_t^n)$ is positive for any risk level w at any horizon n and under any specification for fiscal rules, so that the debt level in the average euro area economy $b_t = 81\%$ of GDP exhibits a low risk of default regardless of the presence of supranational fiscal authorities. However, consistently with the implications from the optimal debt policy, the government exhibits an excess debt under the baseline model and under any arrangement with fiscal rules.

Finally, Panel C confirms the conventional wisdom suggesting that sustainability gains are typically associated with welfare losses from adjusting away from the current debt level. In the benchmark calibration of the average euro area country under both fiscal rules, as welfare gains decrease from $W(b_{t+1}^L(\phi_t^1)) = -9.51\%$ to $W(b_{t+1}^H(\phi_t^1)) = -116.25\%$, sustainability gains decline from -0.4% to -100% . Welfare gains $W(b_{t+1})$ reach their peak with an increase by 0.38% in aggregate consumption when the government reduces its debt ratio from $b_t = 81\%$ to the optimal borrowing ratio $\tilde{b}_{t+1} = 65\%$. At the same time, this adjustment produces a negligible sustainability gain as the government already faces a low risk of default at the current debt level. Therefore, when either fiscal rule may trigger a roll-over crisis for a credit-constrained sovereign borrower, the model economy may reap the benefits of debt reduction without forgoing welfare. Ultimately, as observed through the lenses of a general equilibrium model with strategic sovereign default, the average euro area country does not face a trade-off between macroeconomic stabilization and sustainability, as a debt-driven fiscal consolidation can reduce the risk of default while increasing welfare at the same time.

4.6 Conclusions

This paper analyzes fiscal space through the lenses of a dynamic general equilibrium model with strategic sovereign default. This paper contributes to the current debate on fiscal space in the euro area by analyzing and quantifying the implications of macroeconomic fundamentals and fiscal policy on a government's incentives to default and the implied debt limit distributions. Crucially, besides the quantification of debt limits, this approach enables

welfare considerations and seeks to complement alternative non-structural approaches to analyzing fiscal sustainability, such as rules-based and DSA-based fiscal frameworks. The baseline model includes several features, such as fiscal policy instruments, risk averse investors, long-duration bonds with recovered debt as well as, via novel applications, domestic debt and fiscal rules. The analytical and numerical solution of the model shows that aggregate welfare, weighing the marginal private cost against the marginal public benefit of default, a major driver of a sovereign's incentives to honor its obligations. A sensitivity analysis within a range of plausible calibrations confirms this result and highlights the effects of parameters linked to uncertainty and default costs. In the benchmark calibration, the presence of fiscal rules does not worsen sustainability concerns as it does not affect the government's fiscal space; yet, the possibility of roll-over crises fosters market-based fiscal discipline by reducing the sovereign's optimal level of debt. The empirical application of the model finds that the average euro area country is currently facing low risk of default. However, the economy is not subject to the conventional trade-off between macroeconomic stabilization and fiscal sustainability. By implementing a debt-based fiscal consolidation, the average euro area country can simultaneously increase its welfare and reduce its sustainability concerns.

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Appendices

A Appendix to Chapter 2

A.1 Preliminary data analysis and specification search for the regressions of Table 2.2

This appendix presents the preliminary steps leading to the specification of the vector error-correction model (VECM) whose estimates are presented in Table 2.2.

The first step is to control for the presence of unit roots in the data: we perform Augmented Dickey-Fuller (ADF) tests for all the time series and sampled countries in regressions with a constant drift and four lags (assuming that a quarterly information set contains the relevant information on the considered time series). This is a conservative choice aimed at reducing the autocorrelation in the residuals: for some series, optimal lag order selection criteria (such as the Schwarz-Bayes Information Criterion, SBIC, or the Hannan-Quinn Information Criterion, HQIC) would suggest even smaller lag orders, which would however increase the autocorrelation of residuals. The results, reported in Table A.1, indicate the presence of unit roots at the 5% significance level in all countries' time series for domestic sovereign exposures (except for France), in domestic yield differentials (except for Austria, Germany and the Netherlands), in the common component of domestic yield differentials (except for Austria, Belgium and Germany), and in the country component of domestic yield differentials (except for Austria and the Netherlands). This indicates the presence of non-stationarity in the data.

The second preliminary step focuses on the determination of the cointegrating rank,

Table A.1: ADF tests (H_0 : Unit root): p -values

	<i>soverp</i>	<i>spread</i>	<i>common</i>	<i>country</i>
Austria	0.972	0.002	0.012	0.001
Belgium	0.766	0.330	0.026	0.584
Germany	0.792	0.009	0.041	0.549
Spain	0.995	0.763	0.323	0.812
France	0.001	0.342	0.415	0.395
Greece	0.505	0.976	0.909	0.991
Ireland	0.955	0.291	0.773	0.987
Italy	0.998	0.661	0.144	0.853
Netherlands	0.954	0.022	0.154	0.024
Portugal	0.991	0.999	0.770	0.999

Table A.2: Johansen's trace test ($H_0 : r^* \leq r; H_1 : r^* = n$): p -values

	Baseline model		Factor-based model		
	$r = 0$	$r = 1$	$r = 0$	$r = 1$	$r = 2$
Austria	0.006	0.724	0.001	0.004	0.442
Belgium	0.983	0.936	0.629	0.700	0.642
Germany	0.730	0.579	0.001	0.708	0.577
Spain	0.013	0.753	0.004	0.580	0.755
France	0.308	0.165	0.005	0.120	0.185
Greece	0.004	0.745	0.001	0.020	0.131
Ireland	0.004	0.486	0.001	0.068	0.557
Italy	0.085	0.806	0.041	0.642	0.564
Netherlands	0.867	0.718	0.567	0.757	0.710
Portugal	0.001	0.893	0.001	0.042	0.301

i.e. the number of cointegration relations: we wish to verify whether the time series are tied by long-run relationships. Hence, we carry out a trace test (see [Johansen, 1995](#)) to verify the cointegrating rank of the time series included in our analysis. The trace test verifies the null hypothesis of the cointegrating rank being $r^* \leq r$, for $r = 0, 1, \dots, n - 1$, where n denotes the number of time series, against the alternative of $r^* = n$ (which would entail that a VAR model in levels could be used to capture the dynamic interactions between time series). Table [A.2](#) reports p -values for trace tests considering the time series included in the baseline model and the factor-based model for every country in our sample. Taking a conservative approach, in order to limit the number of parameters to be estimated

Table A.3: VECM specification: deterministic terms and lag order

	Deterministic terms		Lag order (p)	
	Baseline model	Factor-based model	Baseline model	Factor-based model
Spain	<i>rc, rltro, rwit</i>	<i>rc, rltro, rwit</i>	2	2
Greece	<i>uc, ut</i>	<i>uc, ut</i>	1	0
Ireland	<i>uc, ut</i>	<i>uc, ut</i>	2	0
Italy	<i>rc, rltro, rwit</i>	<i>rc, rltro, rwit</i>	1	2
Portugal	<i>uc, ut</i>	<i>rc, ut</i>	2	1
Austria	<i>rc, rltro, rwit</i>	<i>rc, rltro, rwit</i>	0	1
Belgium	<i>rc, ut, rltro, rwit</i>	<i>uc, rt, rltro, rwit</i>	0	1
Germany	<i>rc, rt, rltro, rwit</i>	<i>rc, rltro, rwit</i>	0	0
France	<i>rc, rltro, rwit</i>	<i>rc, rt, rltro, uwit</i>	0	1
Netherlands	<i>rc, rltro, rwit</i>	<i>rc, rt, rltro, uwit</i>	0	1

Note: The acronyms in the table should be interpreted as follows. A specification with restricted constant (*rc*), trend (*rt*) and/or dummies (*rltro* and *rwit*) excludes the constant, linear trends and/or dummies from the term ΓD_T in model 2.3, by an appropriate choice of the matrix Γ : intuitively, such deterministic terms have an effect on the long-term relation among the variables but not on their adjustment dynamics. Conversely, a model with unrestricted constant (*uc*), trend (*ut*) and/or dummies (*uwit*) includes the constant, linear trends and/or dummies in the term ΓD_T in the model 2.3, so that such deterministic terms have an effect on the adjustment dynamics of the variables but not on their long-term relation. The lag orders (p) reported in the table refer to the $\text{VECM}(p) = \text{VAR}(p+1)$ representation of the corresponding model.

and to preserve comparability between countries, we include only a constant term in the cointegration relations and zero lagged differences. As regards the baseline model, our results support the presence of (at most) one cointegration relation, i.e. $r^* = 1$, in every country and rule out the possibility that $r^* = 0$ in most countries (the exceptions are the Belgium, Germany, France, and the Netherlands) at the 10% significance level. Evidence of the presence of cointegration is even stronger when the trace test is applied to the time series considered in the factor-based model. In this case, the trace test rejects the null hypothesis of no cointegration at the 10% significance level for every country, except Belgium and the Netherlands. Furthermore, in several countries, notably Austria, Greece, Ireland, and Portugal (as well as France, to a lesser extent), we find evidence in favor of $r^* = 2$, whereas $r^* = 1$ is rejected at the 10% significance level. Also, for every country in the sample, the trace test reveals that a VECM with two cointegration relations is to be preferred to a VAR model in levels.

Based on these results, we choose a VECM specification with one and two cointegration relations in the baseline and the factor-based model, respectively: this choice is consistent with the presence of cointegration among the time series, and enables us to identify long-run interactions of sovereign exposures with domestic yield differentials (in the baseline model) and with the two components of these differentials (in the factor-based model).

Finally, in order to determine the lag structure of the VECM, we perform both a pre-estimation and a post-estimation analysis: in particular, we consider (i) SBIC and HQIC, (ii) a stability analysis (control of eigenvalues, obtained from the estimation with all sampled residuals) and (iii) a residual analysis (Portmanteau and Lagrange Multiplier tests for autocorrelation in the residuals at different lag lengths and Lomnicki-Jarque-Bera test for non-normality). Our results (not reported) indicate that the VECMs for different countries should include up to two lagged differences of the endogenous variables, and lead us to opt for different lag structures across countries, as shown in Table [A.3](#).

B Appendix to Chapter 3

B.1 Theoretical framework

In order to capture the main features of the euro area sovereign debt market – namely the time-varying pricing paradigm applied to fundamentals, the process of financial integration, and the persistent imperfect substitutability of government bonds – the theoretical framework of the model is based on three major assumptions. This section investigates the rationale of these assumptions, followed by the motivation of the choice of the modelling framework and the analysis of novel key stylized facts on the dynamics of market sentiments in the euro area.

Imperfect asset substitutability

First, bonds issued by different borrowing sovereign governments are assumed to be imperfect substitutes. Traditional portfolio theory posits that two securities may be interpreted as imperfect substitutes for each other if (i) their prices reflect the stochastic streams of income received by their respective issuers and (ii) such streams of income are imperfectly correlated, so that the income patterns of the two issuers (and the returns of the associated securities) differ in at least some state of nature. In other words, if two risky securities are not perfectly correlated, then they are imperfect substitutes for each other.⁴³

From a general economic perspective, [Christiano \(2006\)](#) observes that different assets, even if with the same expected return, are only imperfectly substitutable, since they have different inherent risks. Moreover, according to [Stiglitz \(1975\)](#), although asset diversity may be small and merely perceived, it should not be considered any less real as long as individuals are willing to pay a price to have one security rather than another.

Findings from a number of studies on the euro area sovereign debt market warrant modelling government bonds as imperfect substitutes. Documenting the increasing financial integration in the euro area sovereign debt market in the wake of the monetary unification,

⁴³This definition is consistent with those in standard textbook treatments of financial markets and portfolio decisions (see, for instance, [Mishkin, 2004](#)).

Hartmann, Maddaloni and Manganelli (2003) and Oliveira, Dias Curto and Nunes (2012) argue that the observed convergence of government bond yields occurred since investors perceived euro area government bonds as an ever more homogeneous class of financial products. However, Pagano and von Thadden (2004) and Haselmann and Herwartz (2008) note that, even in the transition to monetary unification, small but sizeable yield differentials suggested that euro area bonds were still not considered as perfect substitutes, due to modest but persistent differentials in fundamental risk across countries. As reported by the European Central Bank (2013, 2014a), with the start of the financial crisis, a reversal in the convergence process led market participants to perceive bonds issued by different sovereigns as inherently different. Battistini, Pagano and Simonelli (2014) explain this phenomenon with the apparent increase in home bias in euro area sovereign debt portfolios and the cross-country dispersion of yield differentials.

Incompleteness of capital markets

The relevance of imperfect asset substitutability for investors' portfolio decisions crucially depends on the second major assumption of the model, namely capital markets incompleteness. When capital markets are incomplete, the presence of financial frictions prevents investors from getting fully insured against adverse shocks and thus increases the attractiveness of assets providing hedging benefits, namely assets that are imperfect substitutes.⁴⁴ Financial frictions can be separated into three main categories.

First, transaction costs may affect investors' propensity to diversify their portfolios. For instance, Rowland (1999) argues that transaction costs thwart incentives towards an active portfolio management, whereas, in the models by Michaelides (2003) and Geromichalos and Simonovska (2014), they foster home bias when foreign assets are affected by relatively

⁴⁴On the relevance of financial frictions for modelling portfolio choice, Maskin (2002) argues that, even in the absence of a complete menu of state-contingent contracts, risk averse investors can attain the same welfare as when financial markets are complete under certain assumptions. However, his conclusions heavily rely on agents' ability to foresee future payoffs, whereas bounded rationality arguments warrant the consideration of non-diversifiable risks and call for the treatment of financial markets as incomplete. Further, in their thorough analysis of open economy financial macroeconomics, Coeurdacier and Rey (2013) claim that transaction costs and informational frictions should be included in any proper modelling of portfolio decisions in an open economy.

higher liquidity constraints. However, evidence on the role of transaction costs and liquidity in the euro area is mixed. On the one hand, [Favero and Missale \(2012\)](#), and [Oliveira, Dias Curto and Nunes \(2012\)](#) observe that liquidity differentials play at most a minor role in explaining yield spreads, both before and during the sovereign debt crisis. On the other hand, the [European Central Bank \(2014a\)](#) reports that in several market segments, such as the fixed-income and equity markets, the fragmentation of the underlying market infrastructure has persistently resulted in complex processes and considerable costs and can still be observed in the trading, clearing and settlement layers. In line with this observation, [Manganelli and Wolswijk \(2009\)](#), [Favero, Pagano and von Thadden \(2010\)](#), and [Monfort and Renne \(2014\)](#) find that spreads between euro area government bond yields are related to market liquidity, cyclical conditions, and an aggregate risk factor.

Second, information costs may also reduce investors' ability to optimally diversify their portfolios, thus rendering concentrated portfolios significantly more attractive. For instance, in the rational expectation models by [Srivastava, McInish and Price \(1984\)](#) and, more recently, [Barron and Ni \(2008\)](#), if issuers are not identical and information is costly, agents may find it optimal to collect information on a limited number of borrowers and allocate their funds to those on which they are well informed. As regards the euro area, on the one hand, recent developments in regulation have brought about relevant improvements in several market segments in terms of transparency and standardization of financial markets.⁴⁵ On the other hand, several structural features of sovereign debt markets across the board, such as institutional settings (e.g., related to political instability risk; see [Huang et al., 2015](#)), legal arrangements (e.g., concerning standard clauses and debt restructuring procedures; see [Panizza, Sturzenegger and Zettelmeyer, 2009](#)) and tax-related issues (e.g. in relation to distortionary capital income taxation; see [Berriel and Bhattacharai, 2013](#)), typically affect portfolio decisions by increasing investors' uncertainty over the pattern of future returns.

⁴⁵Some examples of regulatory reforms aimed at fostering quality, transparency, standardization, and simplicity in EU capital markets are the "Alternative Investment Funds Managers Directive (AIFMD)", the "Undertakings for Collective Investment in Transferable Securities (UCITS V) Directive", the "Regulation on key information documents for investment products", the "European Market Infrastructure Regulation (EMIR)", as well as the definition of "Prime Collateralised Securities" and the development of a legal entity identifier (see [European Central Bank, 2013](#)).

Third, fixed set-up costs may create barriers to entry for new issuers, thus limiting the number of available securities in financial markets. This is typically the case in the market for corporate bonds, where non-convexities in production processes may limit the number of competing firms, and even more so in the sovereign debt market, where additional participants, namely new governments, may only originate because of dramatic political and social events ([Ardagna and Caselli, 2014](#)).

In conclusion, although financial frictions are certainly decreasing in the euro area sovereign debt market, both theoretical and empirical arguments justify the assumption of capital market incompleteness in view of an accurate analysis of investors' portfolio decisions.

Investor risk aversion

Asset imperfect substitutability and capital markets incompleteness are closely related to the third major assumption of the model, which concerns international investors' risk attitude. In the presence of incomplete capital markets and imperfectly substitutable securities, risk averse international lenders assign a positive value to each additional investment opportunity, as each asset offers a sizeable gain by insuring investors against adverse shocks affecting other assets. If there are many such securities, they will be close substitutes for one another, as there is only a small (but positive) marginal benefit from investing in an additional unit of any security. Therefore, when different risky securities are (close but) imperfect substitutes for one another, risk averse investors seek to diversify their portfolios by holding a positive amount of all the available securities, regardless of their number.

The literature has extensively explored the rationale behind investor risk aversion. In the context of models with endogenous sovereign default, [Lizarazo \(2013\)](#) argues that several factors help explain why institutional (besides individual) investors may feature risk aversion.⁴⁶ These factors fall into two broad categories: (i) regulations over the composition

⁴⁶In this model, international investors' preferences exhibit an intratemporal form of risk aversion, which differs from the intertemporal form of risk aversion studied by [Lizarazo \(2013\)](#). In her model, international investors' preferences exhibit constant relative risk aversion (CRRA) and seek to smooth their consumption; hence, investments in different periods (rather than securities) have a different marginal utility, since they

of investments portfolios and (ii) characteristics of the institutions' management.⁴⁷

As regards the euro area sovereign debt market, most empirical evidence shows that fundamentals (e.g., subdued economic activity, fiscal imbalances and current account deficits), explain a sizeable share of the dynamics of euro area government bond yields, especially during the sovereign debt crisis (Corsetti et al., 2014, Costantini, Fragetta and Melina, 2014). Further, considering the effects of news shocks, Beetsma et al. (2013) highlight the role of market sentiments in driving sovereign interest rate spreads. In line with these findings, a broad consensus view explains this finding as the effect of a dramatic shift in market sentiments and global risk aversion (Sgherri and Zoli, 2009, Oliveira, Dias Curto and Nunes, 2012, Cimadomo, Claeys and Poplawski-Ribeiro, 2016) or investors' pricing paradigm (Attinasi, Checherita-Westphal and Nickel, 2009, Favero and Missale, 2012, De Grauwe and Ji, 2012, Beirne and Fratzscher, 2013) as their perception of fundamentals and the implied default risk changes over time. On the basis of a semiparametric time-varying coefficient models and non-linear GVAR approach applied to euro area countries, Bernoth and Erdogan (2012), Bernoth, von Hagen and Schuknecht (2012), and Favero (2012) study the impact of a country's growth prospects and relative fiscal position on interest rate differentials; they find that the cost of loose fiscal policy and deteriorating macroeconomic conditions has considerably increased in the wake of the financial crisis, as the sensitivity of spreads to fundamentals has inverted its previous downward trend.

Moreover, the empirical literature has linked investor risk aversion and portfolio diversification, stressing how the latter has been one of the main drivers for investors' investment strategies (see, e.g., De Santis and Gérard, 2009). More specifically, investors' propensity to diversification may arise because of benefits from hedging against liquidity risk (e.g., con-

affect agents' consumption pattern across time. In contrast, in this model, investors aim at an optimal portfolio composition (i.e., allocation of expenditures) taking into account the imperfect substitutability of their different investment options as well as the financial frictions hindering the efficient functioning of capital markets.

⁴⁷ As regards regulations, institutional investors typically face capital adequacy ratios, leverage restrictions, as well as strict limits on exposures to some asset classes. As regards management characteristics, any class of institutional investors ultimately assigns discretionary powers on portfolio decisions to managers; the latter can also be treated as risk averse individuals, since they might face mandates from the providers of funds and the performance of their portfolios affects their compensation.

cerning the effects of transaction costs; see [Fratzscher and Imbs, 2009](#)), default risk (e.g., in regard of country-specific sovereign risk; see [Bessler and Wolff, 2014](#)), and systemic risk (e.g., related to the risk of a euro area break-up; see [Battistini, Pagano and Simonelli, 2014](#)).

Financial integration in the sovereign debt market

In this theoretical framework, imperfect asset substitutability and incompleteness of capital markets combine so as to influence risk averse investors' portfolio decisions. To the extent that financial frictions depend on locational differences, the degree of imperfect substitutability and investor risk aversion is related to the level of financial integration of the aggregate market for government bonds ([Bai and Zhang, 2012](#), [Yu, 2015](#)).

From a theoretical perspective, the market for a given class of financial instruments in a specific region can be defined as integrated if (i) potential market participants across the region face a single set of rules and receive equal treatment in their operations on the market for such class of financial instruments, and (ii) there exist negligible financial frictions affecting the allocation of investments on the basis of their location within the region ([Baele et al., 2004](#)). In other words, in a financially integrated region no barriers discriminate economic agents and their investments in a particular set of financial instruments because of locational differences. Hence, financial integration implies the absence of systematic differences in the portfolio allocations and sources of funding of individual economic agents inside the region ([Hartmann, Maddaloni and Manganelli, 2003](#)).

A detailed account of the process of integration in euro area financial markets is reported in a number of studies, and is beyond the scope of this study.⁴⁸ According to the typical narrative, during the first decade of the European monetary union, decreasing barriers to exchange have boosted the process of convergence and gradually reduced financial segmentation. The presence of a single currency, a unique monetary policy, and a union-wide wholesale payment system have been the essential conditions for the creation of deeply

⁴⁸ Among others, see [Adam et al. \(2002\)](#), [Baele et al. \(2004\)](#), [Baele and Inghelbrecht \(2008\)](#), [Abad, Chuliá and Gómez-Puig \(2010\)](#), [Wagenvoort, Ebner and Morgese Borys \(2011\)](#), [Bai and Zhang \(2012\)](#), [Christiansen \(2014\)](#), and [Sehgal, Gupta and Deisting \(forthcoming\)](#).

integrated continental financial markets. However, even during the process of convergence, financial markets have never been fully integrated, because of cross-country differences in the rules governing trades and transaction costs affecting several classes of financial instruments. Eventually, although financial markets have stabilized across the board in the euro area, in the wake of the euro area sovereign debt crisis, conditions have remained fragile.

Hence, empirical evidence shows that the presence of significant financial frictions has contributed to impede the smooth allocation of capitals towards diversified portfolios. Some of these frictions are not necessarily associated with locational differences. In particular, transaction costs may stem from technical issues as well as business practices (e.g., bid-ask spreads, brokerage commissions, and fees associated with trading and post-trading activities) specific to certain sectors (or even individual financial firms within those sectors). However, information and fixed set-up costs typically differ across countries: national differences in taxation and legal arrangements, as well as political events tied to the social structure of a particular country, are likely to play a major role in informing investors' portfolio decisions.⁴⁹ Hence, these frictions are strictly dependent on locational differences and, as such, represent the foremost determinants of the persistent financial segmentation affecting the euro area sovereign debt market.

The choice of a theoretical framework

A major theoretical challenge lies in the selection of a modelling approach. The choice of a preferred theoretical framework may ultimately fall on either implicit modelling (i.e., reduced-form or utility approach) or explicit modelling (i.e., structural form approach).

⁵⁰ In modern macroeconomic theory, the utility approach has served the micro-founded

⁴⁹See, for instance, the discussion in [La Porta et al. \(1997\)](#) and, with a particular focus on the Greek debt crisis, [Ardagna and Caselli \(2014\)](#).

⁵⁰From an economic perspective, implicit modelling relies on a reduced-form approach which defines, restricts, and interprets an agent's decision problem without the need to explore its explicit structure. From a technical point of view, the reduced-form approach assumes that the relevant decision variables directly enter the agent's utility function (hence, utility approach); in contrast, the structural-form approach defines the underlying mechanisms at work in the agent's decision-making process while assuming a minimal configuration of preferences. Therefore, the utility approach results into a comprehensive modelling framework encompassing the various components of the agent's decision problem, while foregoes an insightful (but cumbersome) investigation of their detailed and precise structure.

investigation of optimal decisions under the assumption of product diversity (at least) since the [Lucas \(1976\)](#) critique. Despite the numerous merits of the structural form approach, conventional models have used the utility approach as the workhorse framework to construct aggregate demand and supply functions determined on the basis of consumer and producer preferences defined over a basket of real goods and factors.

In the context of models with financial asset markets, the utility approach has never gained a widespread favor from academics or practitioners.⁵¹ In particular, the vast majority of macroeconomic models of international portfolio selection is based on the concept of mean variance efficiency, which is the cornerstone of modern portfolio theory as formulated in the seminal works by [Markowitz \(1952, 1959\)](#). Mean variance portfolio theory is a well-developed and widely known paradigm, which appeals to both scholars and professionals, as (i) it gives an intuitive solution to the fundamental problem of the allocation of wealth among alternative assets, and (ii) it is verifiable empirically.⁵² Further, as [Coourdacier and Rey \(2013\)](#) document, the literature on open economy financial macroeconomics counts numerous extensions of the baseline mean variance portfolio choice model embedding realistic features, such as heterogeneous preferences and incomplete financial markets, within conventional macroeconomic models.⁵³

However, the selected theoretical framework should include two features essential in a thorough macroeconomic analysis of portfolio decisions in the presence of imperfect asset substitutability. First, the preferred modelling approach should be able to shed light on the fundamental problem of the optimal scale and allocation of investments as well as their effects on aggregate economic fluctuations. In their influential contribution to the study of

⁵¹One of the few examples of macroeconomic models where the demand for financial assets is implicitly based on the agent's utility is an early work by [Obstfeld \(1980\)](#) on imperfect asset substitutability.

⁵²See, for instance, the discussion in [Constantinides and Malliaris \(1995\)](#).

⁵³Several recent methodological advances have allowed a thorough analytical representation of international portfolio decisions with relatively flexible solution methods. [Devereux and Sutherland \(2010, 2011\)](#) and [Tille and van Wincoop \(2010\)](#) develop solution methods based on standard linear solution techniques for macroeconomic models. The main difference is that the latter method uses numerical iterations to compute first-order approximations of steady-state portfolios, whereas the former has full analytical characterization of portfolios up to the second order. Other approaches that can be applied to very general classes of models have been proposed by [Evans and Hnatkovska \(2007, 2012\)](#) and [Judd, Kubler and Schmedders \(2002\)](#), but their solution methods depart from standard DSGE models.

optimum product diversity, [Dixit and Stiglitz \(1977\)](#) recognize that the reliance on a method based on correlations among securities, augmented with several types of financial frictions, may hamper the interpretation of results derived from the mean variance portfolio theory in general terms. Hence, posing the question as a problem of quantity versus diversity, the authors advocate the use of a different approach to modelling the desirability of variety, a ‘direct route’ suitable to evaluate the economic effects of decisions concerning the optimal amount and distribution of expenditures among imperfectly substitutable goods.

Second, from a macroeconomic perspective, a model of international portfolio selection requires the use of aggregate measures, first and foremost an economically significant index of investors’ total holdings of financial assets. In line with the original observation by [Fisher \(1921\)](#), an aggregate portfolio index and the optimal quantities of the individual financial assets measured by such index should be economically consistent and thus reflect the same preference structure. As [Barnett, Offenbacher and Spindt \(1981\)](#) argue, a simple unweighted summation form does not provide a meaningful index of optimal portfolio allocations among imperfectly substitutable assets.

These arguments have identical implications as regards the choice of a theoretical framework suitable for modelling international portfolio decisions. A direct approach to analyze decisions of quantity and diversity and their effects on the aggregate economy, as well as devise an economically meaningful functional quantity index entails preferences specified according to a conventional (i.e., increasing and concave) utility function defined over the quantities of all available financial assets.⁵⁴ First, building on the long-established approach of traditional macroeconomic theory originated from the influential studies by [Armington \(1969\)](#) and [Dixit and Stiglitz \(1977\)](#), the aggregation in composite indexes is suitable to investigate the macroeconomic effects of agents’ preferences in the presence of imperfectly substitutable commodities and frictions to the smooth allocation of resources. Second, a portfolio index based on a utility function with constant elasticity of substitution (CES)

⁵⁴A neoclassical utility function features convexity in its indifference surfaces and implicitly incorporates agents’ propensity to diversification: a risk averse investor who is indifferent between two assets promising the same expected return always prefers a portfolio composed of both assets to either extreme, as long as they are not perceived as perfectly fungible.

would feature the desired properties of an economically meaningful aggregate measure of international investors' total holdings of a diverse set of financial assets.⁵⁵

In conclusion, notice that the CES specification of investors' preferences is flexible and integrates all relevant assumptions within a comprehensive, consistent framework. In fact, the parameter relating to the intratemporal elasticity of substitution between any pair of government bonds is a direct gauge of the degree of imperfect asset substitutability and, at the same time, can be interpreted as a proxy for the level of investor risk aversion and financial segmentation of the aggregate sovereign debt market. Moreover, the CES parameter can be calibrated to match different conditions in market sentiments. Hence, any quantitative and empirical assessment applies common solution methods typical of standard macroeconomic models (i.e., first-order approximation of equilibrium dynamics around a non-stochastic steady state) and does not necessitate the involved techniques (e.g., local perturbation methods, global methods) usually required by structural-form approaches.

The implications of the utility-based modelling approach

The choice of the particular theoretical framework, based on the determination of international portfolio decisions according to the utility approach, has three main implications. First, the configuration of international lenders' preferences CES utility function entails a market structure of monopolistic competition among borrowing sovereign governments. On the basis of a mean variance portfolio selection approach, [Stiglitz \(1975\)](#) studies the rationale for the definition of capital markets as monopolistically competitive. His analysis suggests that the peculiar features of the market for sovereign debt warrant a monopolistically competitive structure and hints at the importance to further investigate the economic effects of monopolistic competition.

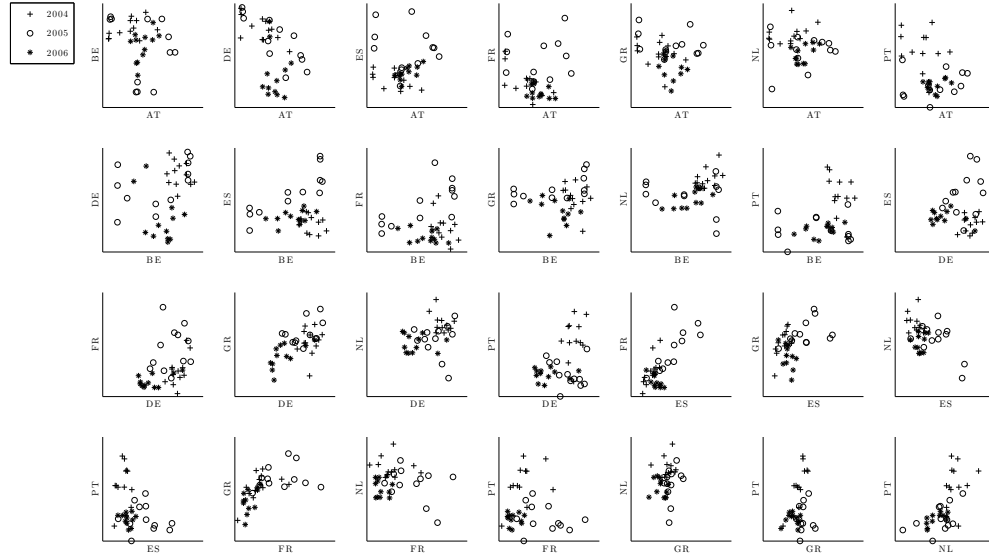
⁵⁵On the basis of aggregation and index number theory as well as direct empirical evidence, [Barnett \(1980\)](#) notes that any functional quantity aggregate with economic relevance should exhibit the properties of a neoclassical utility function, notably homotheticity and weak separable nesting within the agent's full utility function. A conventional CES function features such properties and represents the basis for the formulation of the so-called Monetary Services Indexes currently in use at several central banks, such as the U.S. Federal Reserve and the European Central Bank, to measure monetary aggregates and originally envisaged by [Divisia \(1925\)](#).

Second, the structure of the sovereign debt market deriving from the CES representation of investors' preferences implies that monopolistically competitive governments face a downward sloping demand curve for debt securities. The presence of a government bond demand curve obtains from the major assumptions of this theoretical framework: a risk averse investor receives a sizeable benefit from holding a positive amount of every security, as the number of securities is not unlimited and they are imperfect substitutes for one another; thus, any variation in the price of an asset may only induce a finite change in its demand.⁵⁶

Third, in a regime of monopolistic competition, sovereigns determine the optimal bond price by virtue of the market power allowed by investors' perception of government bonds as imperfect substitutes. This framework contrasts with traditional models of international lending, where borrowers pick the amount of debt given the price level fixed on perfectly competitive markets. However, from a theoretical perspective, this model is consistent with general equilibrium theory, since every agent's optimal strategy takes into account the strategies followed by all other agents at each point in time. Further, any model of sovereign borrowing can only reflect certain aspects of the auction process observed on the actual market for newly issued government bonds. In this light, traditional models focus on the fact that governments usually post the amount of debt to be assigned and the final price is determined as a weighted average of investors' bids. In contrast, this model inspects the mechanism of quantity and price formation at a deeper level: the set of individual pairs of offered amounts and bidden prices implicitly creates a downward sloping demand curve, whose curvature may represent a measure of the current state of market sentiments.

⁵⁶Notice that the government bond demand schedule derived in this model through optimal portfolio decisions has properties similar to those of the demand functions for money and bonds denominated in domestic and foreign currency defined in Obstfeld (1980). In particular, the functional form of investors' preferences implies that the demand for a government bond issued by one sovereign depends on the price of that bond relative to the aggregate price level of other sovereigns' bonds. Hence, the demand functions of this model are also consistent with the fundamental motivation of the mean variance portfolio theory, as highlighted by Elton and Gruber (1997): portfolio allocations should consider not only asset-specific characteristics, but also the co-movements of every asset with all other assets. In the context of models of endogenous sovereign default, Chatterjee and Eyigungor (2012) prove the existence of a downward sloping equilibrium price function, but their result depends on the introduction of long-term debt and is applied to an environment with a single borrowing government.

Figure A.1: Sovereign debt portfolio diversification in Italian banks before the crisis



Source: Banca d'Italia.

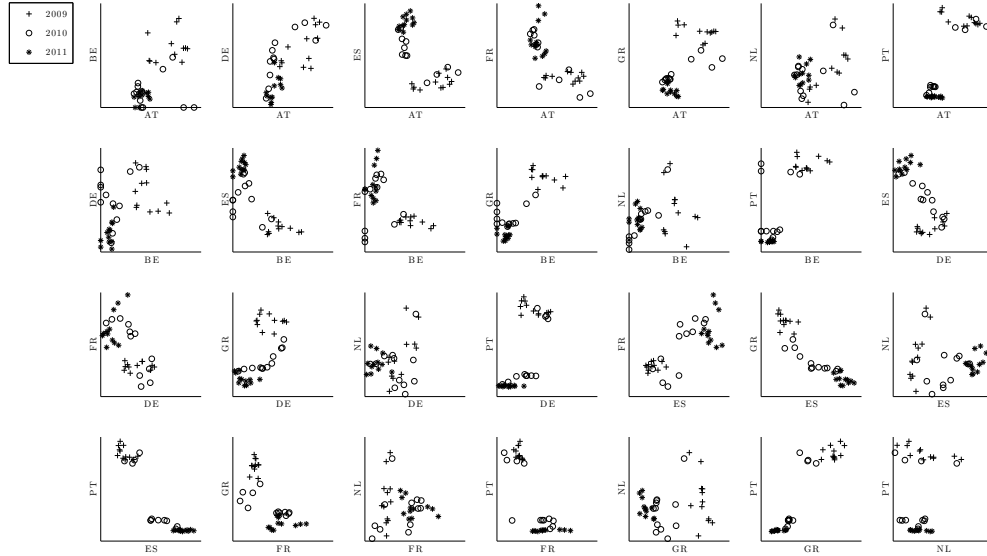
Notes: Each data point refers to Italian banks' holdings (expressed in EUR millions) of general government securities other than shares (total maturity) issued by a single pair of countries in a given month. Axis ticks are not reported due to confidentiality of data.

Key stylized facts: The crisis and market sentiments

The definition of the theoretical framework for investors' sovereign debt portfolio decisions may support the analysis of some key stylized facts and shed further light on the dynamics of market sentiments occurred in the wake of the euro area sovereign debt crisis.

Figures A.1 and A.2 show the composition of Italian banks' sovereign debt portfolios in the periods 2004-2006 (henceforth, pre-crisis period) and 2009-2011 (henceforth, crisis period), respectively; each panel depicts the holdings (in million euros) of government bonds issued by a pair of sovereigns (indicated on the two axes); each point corresponds to a realization in particular month. In order to interpret these figures, notice that sovereign debt portfolios (consisting of holdings of pairs of government bonds) observed at different points in time represent a sequence of equilibrium allocations. If substitution effects dominated, movements in relative prices (as proxied by inverse yields) and income (as proxied by total assets) would generate a sequence of equilibria lying on the same indifference curve.

Figure A.2: Sovereign debt portfolio diversification in Italian banks during the crisis



Source: Banca d'Italia.

Notes: See Figure A.1.

As shown in Figure A.1, no particular pattern emerges in the pre-crisis period: only the allocation of funds between Spanish and Portuguese sovereign debt seems to lie on a unique indifference curve for investors with standard preferences across the whole period (except for short periods of some months). However, as emerges from Figure A.2, optimal sovereign debt portfolios seem to lie on downward sloping, convex indifference curves for the whole crisis period: this is the case for several pairs of sovereigns, notably Austria with Spain and France, Belgium with Spain and France, Spain with Greece and Portugal, France with Greece and Portugal and, to a lesser extent, Germany with France. Again, the same pattern emerges for numerous sub-periods of shorter duration.

The patterns emerging from the pre-crisis and the crisis period may have two explanations. On the one hand, the difference in the relative size of income and substitution effects in the two periods may be the main mechanism at work. Before the crisis, large income effects might occur due to large changes in total assets, so that the equilibrium points actually lie on distinct indifference curves: the average (absolute) annual growth

rate of total assets for Italian banks in the pre-crisis period is 7.54%, almost twice the same figure during the crisis (4.18%), where incidentally total assets remain roughly constant (indicating the presence of positive as well as negative growth rates). On the other hand, the difference in patterns may be the consequence of a marked convexity of indifference curves in times of financial distress, pointing at a higher level of financial segmentation, investor risk aversion and imperfect substitutability of euro area government bonds. In fact, the two explanations need not be mutually exclusive: indeed, both substitution effects and propensity to diversification (driven by risk aversion, financial segmentation or imperfect asset substitutability) are associated with investors' portfolio rebalancing, a phenomenon typically observed during crises. Therefore, in line with the bulk of empirical evidence, this model provides a theoretical framework to interpret key stylized facts on the dynamics of sovereign debt portfolios as a consequence of a deterioration in market sentiments occurred at the onset of the euro area sovereign debt crisis.

B.2 Proofs of theorems

In the following proofs, I assume that $h_t^i = 1$, for all i , namely, every sovereign repaid its debt at $t - 1$ and is in good standing, and can choose whether to repay or default on its outstanding debt at t . Also, country i 's state at time t is given by $s_t^i = \{y_t^i, q_{t-1}^i, s_t^*\}$. As a reminder, domestic households' budget constraint under repayment and default is given by the conditions below, respectively,

$$c_t^i = y_t^i + q_t^i \left(\frac{q_t^i}{q_t^*} \right)^{-\eta} \frac{b_{t+1}^*}{\rho_t^*} - \left(\frac{q_{t-1}^i}{q_{t-1}^*} \right)^{-\eta} \frac{b_t^*}{\rho_{t-1}^*} \quad \text{and} \quad c_{Dt}^i = y_t^i - \phi(y_t^i).$$

Proposition 1. *Let $\hat{q}_{t-1}^i > \bar{q}_{t-1}^i$, $\hat{s}_t^i = (y_t^i, \hat{q}_{t-1}^i, s_t^*)$, and $\bar{s}_t = (y_t^i, \bar{q}_{t-1}^i, s_t^*)$. If default is optimal in state \hat{s}_t^i , then it will be optimal in state \bar{s}_t^i , namely, $\mathcal{D}_i(\hat{q}_{t-1}^i, s_t^*) \subseteq \mathcal{D}_i(\bar{q}_{t-1}^i, s_t^*)$.*

Proof. Suppose that $\hat{q}_{t-1}^i > \bar{q}_{t-1}^i$, so that the corresponding states of the union economy are

$\hat{s}_t^i = (y_t^i, \hat{q}_{t-1}^i, s_t^*)$, and $\bar{s}_t^i = (y_t^i, \bar{q}_{t-1}^i, s_t^*)$, respectively. Notice that

$$\frac{\partial c_t^i}{\partial q_{t-1}^i} = \eta(q_{t-1}^i)^{-\eta-1}(q_{t-1}^*)^\eta b_t^* > 0 \quad \text{and} \quad \frac{\partial c_{Dt}^i}{\partial q_{t-1}^i} = 0,$$

since $0 < q_{t-1}^i \leq 1$, for all i , so that $\hat{c}_t^i > \bar{c}_t^i$ and c_{Dt}^i is independent of q_{t-1}^i ; thus,

$$\begin{aligned} U_i^R(\hat{s}_t^i) &= u(\hat{c}_t^i) + \beta_i E_t [U_i^R(s_{t+1}^i)] \\ &> u(\bar{c}_t^i) + \beta_i E_t [U_i^R(s_{t+1}^i)] \\ &= U_i^R(\bar{s}_t^i), \end{aligned}$$

since $u'_i(\cdot) > 0$, which implies that the value of the debt contract under repayment is increasing in previous period's domestic bond price q_{t-1}^i . Now, suppose that $y_t^i \in \mathcal{D}_i(b_t^*, \hat{q}_t^i)$; then, by definition, $U_i^D(s_{Dt}^i) > U_i^R(\hat{s}_t^i)$. Therefore, $U_i^D(s_{Dt}^i) > U_i^R(\bar{s}_t^i)$ and $y_t^i \in \mathcal{D}_i(b_t^*, \bar{q}_t^i)$. \square

Proposition 2. *Let the debt-to-output ratio be larger than the exogenous costs of default per unit of output, namely, $b_{t+1}^i/y_{t+1}^i > \omega_1^i + \omega_2^i y_{t+1}^i > 0$. Then, default is inversely related to the marginal utility of consumption next period, namely, $Cov_t[(1 - d_{t+1}^i), \lambda_{t+1}^i] \geq 0$.*

Proof. Assume that the sovereign does not default next period in any state of the domestic economy (namely, $\mathcal{D}(q_t^i, s_{t+1}^*) = \emptyset$) given some bond price q_t^i ; then $d_{t+1}^i = 0$, so that $Cov_t[(1 - d_{t+1}^i), \lambda_{t+1}^i] = 0$.

Similarly, assume that the sovereign defaults next period in every state of the domestic economy (namely, $\mathcal{D}(q_t^i, s_{t+1}^*) = Y^i$, with Y^i denoting the whole set of realizations y_t^i) given some bond price q_t^i ; then $d_{t+1}^i = 1$, so that $Cov_t[(1 - d_{t+1}^i), \lambda_{t+1}^i] = 0$.

In contrast, if the sovereign defaults next period only in some states of the domestic economy (namely, $\mathcal{D}(q_t^i, s_{t+1}^*) \subset Y^i$), then either

- (i) the sovereign repays, namely, $d_{t+1}^i = 0$, and its wealth is $w_{Rt+1}^i = y_{t+1}^i - b_{t+1}^i$; or
- (ii) the sovereign defaults, namely, $d_{t+1}^i = 1$, and its wealth is $w_{Dt+1}^i = y_t^i - \phi(y_t^i)$.

Now, given realizations of the aggregate variables s_{t+1}^* and some choice of the bond price next period q_{t+1}^i , the government's wealth is larger under default than under repayment, namely, $w_{Rt+1}^i < w_{Dt+1}^i$, as long as the debt-to-output ratio is larger than the costs of default per unit of output, namely, $b_{t+1}^i/y_{t+1}^i > \omega_1^i + \omega_2^i y_{t+1}^i > 0$. Then, due to the concavity of the household's utility function $u_i(\cdot)$, the marginal utility of consumption is larger under repayment than under default, namely, $\lambda_{Rt+1}^i > \lambda_{Dt+1}^i$. Therefore, a higher probability of default is associated with a lower marginal utility of consumption, so that $Cov_t[(1 - d_{t+1}^i), \lambda_{t+1}^i] \geq 0$. Conversely, when $\omega_1^i + \omega_2^i y_{t+1}^i > b_{t+1}^i/y_{t+1}^i > 0$, the opposite is true and $Cov_t[(1 - d_{t+1}^i), \lambda_{t+1}^i] \leq 0$. \square

B.3 Computational algorithm

In order to reproduce the analytical solution of the model, the numerical solution of the model is implemented as follows. After calibrating the model parameters (including aggregate variables):

1. Define states $s = \{y, q\}$, as well as probability distribution $p(y'|y)$.
2. Guess initial values for value functions U^R and U^D .
3. Value function iteration:
 - (a) Find optimal policy for q' .
 - (b) Find optimal policy for d .
 - (c) Use the value functions associated with the optimal policies as new guesses for U^R and U^D .
4. Update.

Finally, once convergence is achieved, it is possible to simulate the model and compute the relevant statistics predicted by the model for a given calibration. Notice that the algorithm above includes a unique iteration for the policy function q_I , as the level of debt is simultaneously identified via the endogenous demand for government bonds. This method

innovates on the algorithms used in most of the sovereign default models; also, it is essential in order to guarantee an efficient computation of the model.

C Appendix to Chapter 4

C.1 Proof of theorem

In the following proofs, consider the case of one-period bonds (i.e. $\lambda = 1$) and let x_t^D denote the value of variable x_t under financial autarky.

Proposition 1. *For all $b_t^1 \leq b_t^2$, if default is optimal for b_t^1 , then it will be optimal for b_t^2 , namely $\mathcal{D}(b_t^1) \subseteq \mathcal{D}(b_t^2)$, if and only if $\frac{1-\zeta}{1+\tau_t^c} \leq \frac{u_g(c_t, h_t, g_t)}{u_c(c_t, h_t, g_t)}$. Conversely, if default is optimal for b_t^2 , then it will be optimal for b_t^1 , namely $\mathcal{D}(b_t^2) \subseteq \mathcal{D}(b_t^1)$, if and only if $\frac{1-\zeta}{1+\tau_t^c} \geq \frac{u_g(c_t, h_t, g_t)}{u_c(c_t, h_t, g_t)}$.*

Proof. The first part of Proposition 1 is proven as follows. For all $a_t \in \mathcal{D}(b_t^1)$, $V^D(a_t) = u(c_t^D, h_t^D, g_t^D) + \beta E_t[(\theta V(a_{t+1}, 0) + (1-\theta)V^D(a_{t+1}))] > u(c_t, h_t, g_t|b_t^1) + \beta E_t[V(a_{t+1}, b_{t+1})] = V^R(a_t, b_t^1)$, given any b_{t+1} . Notice that

$$\frac{(1 - \tau_t^h)w_t h_t + p_t^F + (1 - \zeta)(b_t^1 - q_t b_{t+1})}{(1 + \tau_t^c)} \leq \frac{(1 - \tau_t^h)w_t h_t + p_t^F + (1 - \zeta)(b_t^2 - q_t b_{t+1})}{(1 + \tau_t^c)},$$

whereas

$$\tau_t^c c_t + \tau_t^h w_t h_t + q_t b_{t+1} - b_t^1 \geq \tau_t^c c_t + \tau_t^h w_t h_t + q_t b_{t+1} - b_t^2.$$

Hence, if and only if $\frac{1-\zeta}{1+\tau_t^c} \leq \frac{u_g(c_t, h_t, g_t)}{u_c(c_t, h_t, g_t)}$, then

$$\frac{\partial u(c_t, h_t, g_t)}{\partial b_t} = \frac{1 - \zeta}{1 + \tau_t^c} u_c(c_t, h_t, g_t) - u_g(c_t, h_t, g_t) \leq 0$$

so that the flow utility function is decreasing in b_t and

$$\begin{aligned} V^R(a_t, b_t^1) &= u(c_t, h_t, g_t|b_t^1) + \beta E_t[V(a_{t+1}, b_{t+1})] \\ &\geq u(c_t, h_t, g_t|b_t^2) + \beta E_t[V(a_{t+1}, b_{t+1})] \\ &= V^R(a_t, b_t^2). \end{aligned}$$

Thus, the value of the contract under repayment is decreasing in aggregate sovereign debt.

Hence, $V^D(a_t) = u(c_t^D, h_t^D, g_t^D) + \beta E_t[(\theta V(a_{t+1}, 0) + (1 - \theta)V^D(a_{t+1}))] > u(c_t, h_t, g_t | b_t^2) + \beta E_t[V(a_{t+1}, b_{t+1})] = V^R(a_t, b_t^2)$, which implies that $a_t \in \mathcal{D}(b_t^2)$.

The second part of Proposition 1 is proven as follows. For all $a_t \in \mathcal{D}(b_t^2)$, $V^D(a_t) > V^R(a_t, b_t^2)$, given any b_{t+1} . If and only if $\frac{1-\zeta}{1+\tau_t^c} \geq \frac{u_g(c_t, h_t, g_t)}{u_c(c_t, h_t, g_t)}$, then $\frac{\partial u(c_t, h_t, g_t)}{\partial b_t} \geq 0$ so that the flow utility function is increasing in b_t and $V^R(a_t, b_t^2) \geq V^R(a_t, b_t^1)$. Thus, the value of the contract under repayment is increasing in aggregate sovereign debt. Hence, $V^D(a_t) > V^R(a_t, b_t^1)$, which implies that $a_t \in \mathcal{D}(b_t^1)$. \square