EXTENDING THE ENTANGLEMENT OF RISK AND TIME: RISK WITH MULTIPLE OUTCOMES AFFECTS DISCOUNTING

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ABSTRACT OF THE THESIS

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While many real world decisions involve both risk and delay, relatively little research has investigated the nuanced ways in which they interact. Moreover, those studies which have studied their interaction almost unanimously focus on outcome risk – a 50% chance of $100 in four weeks and a 50% chance of $0. The general finding from these studies is that the combination of risk and time preferences is not a simple combination of each in isolation. This suggests that delay is entangled with risk. In this study, in addition to outcome risk, I investigate how amount risk –risk with multiple outcomes – and combined amount and outcome risk affect discounting. To derive predictions for amount and amount and outcome risk, I compare simulated data from a model which presumes that time and risk are entangled to a model which presumes risk and delay are independent. These predictions are tested in two studies and a within paper meta-analysis. As predicted by the simulations, both outcome and amount risk effect discounting and their effects are more pronounced when the delay to one of the options is in the present. Further, the effect of outcome risk on discounting is attenuated in the presence of amount risk. Taken together, these results show that a nuanced view of risk is required when investigating its entanglement with delay.

Keywords: Decision Making; Intertemporal Choice; Risk.

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1. Introduction

Our decisions often involve a trade-off realized over different time periods. For example, when choosing an age to retire you can wait to receive Social Security payments of $1200 a month or claim early and receive only $1000 a month (Knoll et al. 2015; Schreiber and Weber 2016). Many choices, beyond the delay to receipt, also have an element of risk in the outcome. When investing in your individual retirement account, you know that the total value of this account at the time of your retirement is uncertain. Specifically, there are a multitude of values it could take when you retire, including a small likelihood that it would take a value of $0. While the integration of risk and time preferences has received recent empirical and theoretical consideration (Andreoni and Sprenger 2012; Baucells and Heukamp 2012; Epper, Fehr-Duda, and Bruhin 2011; Hardisty and Pfeffer 2015), it has almost exclusively focused on binary representations of risk (see Hardisty and Pfeffer (2015), and Blackburn and El-Deredy (2013) for counterexamples) -- that is the risk of getting an outcome versus getting nothing (what we will call outcome risk). This paper echoes recent research demonstrating that risk and delay are evaluated jointly (Andreoni and Sprenger 2012; Baucells and Heukamp 2012; Hardisty and Pfeffer 2015) and extends those findings to show that the coupling of delay and risk includes cases where risk has multiple outcomes.

The dominant paradigm for investigating temporal preferences is choices between two certain (i.e., without risk) monetary amounts, each received at a different point in time. For example, participants might be asked to make a choice between receiving $100 today or $110 in 4 weeks. Recent work has attempted to make these choices more realistic by adding risk -- e.g. specifying a 50% probability for all monetary amounts (Ahlbrecht and Weber 1997; Andreoni and Sprenger 2012; Baucells and Heukamp 2012). This increased realism, however, alters discounting. Thus research that considers time in isolation may not be as predictive of real-world scenarios which include both risk and delay.

Consider the immediacy effect, a phenomenon in which people overvalue immediate outcomes. People prefer $100 today to $110 in 4 weeks, but if you push both options back 26 weeks people prefer $110 in 30 weeks to $100 in 26 weeks. However, the power of immediacy diminishes in the presence of risk. For instance, Weber and Chapman (2005) demonstrated that adding outcome risk -- e.g. specifying a 50% probability for all monetary amounts -- attenuates the immediacy effect. In other words, the power of now may in part be due to the certainty of now. Related work has reached a similar conclusion: people discount the future differently if it is risky as compared to if it is certain (Andreoni and Sprenger 2012; Hardisty and Pfeffer 2015). Taken together, these results suggest that risk and time are entangled.

Specifically, we compare the predictions of two models of risky intertemporal prospects for amount risk, outcome risk, and their combination. By amount risk we mean prospects of the following form: a 1/3 chance of $90, 1/3 chance of $100, and a 1/3 chance of $110. One of these models posits that risk and time are entangled[i] and the other posits that they are evaluated independently. Accordingly, these models make identical predictions for atemporal risky prospects as well as arisky intertemporal prospects. For risky intertemporal prospects, however, predictions diverge. Given that most papers which combine risk and time preferences find that time and risk are entangled, we assume that the entangled model describes risky intertemporal choices better than the disentangled model. This assumption allows us to predict the degree to which entanglement affects discounting for different forms of risk. Specifically, it allows us to determine, relative to one another, how much outcome, amount, and combined amount and outcome risk affect intertemporal choices.

Using the two models to compare how much entanglement effects discounting shows that amount, outcome, and combined amount and outcome risk have nearly equivalent effects on discounting. Suggesting a subadditive interaction between amount and outcome risks effects on discounting. Two studies provide empirical tests of the subadditivity for prospects with immediate and non-immediate delays. We find evidence for subaddivity across both types of risk and that these effects are more pronounced for immediate prospects.

1.2 Two Classes of Risky Intertemporal Choice Models

Models of risky intertemporal choices fall into two broad categories. The first category assumes that risk and time are evaluated independently and therefore involves a two stage process: 1) calculate the certainty equivalent of the risky option, then, 2) discount it (Prelec and Loewenstein 1991). Evaluating time and risk separately assumes that they are independent of one another, meaning that, contrary to the evidence presented above, risk should not affect temporal discounting. The other category, which evaluates time and risk together, in a single step, assumes that they entangled and therefore are dependent on one another (Baucells and Heukamp 2012). This combined evaluation assumes that time and risk are evaluated in concert, meaning that the effect of risk on utility is dependent upon the delay to receipt and vice versa.

For the entangled category of models, we focus on a one-stage model, Baucells and Heukamp (2012), which reduces to a common probability weighting model Prelec (1998), for immediate prospects, and a common time discounting model, Ebert and Prelec (2007), for riskless prospects. For the independent category of models, we use a two-stage model that combines the Prelec probability model with the Ebert and Prelec time discounting model and evaluates risk and time independently, i.e. the effect of delay is independent of risk and the effect of risk is independent of delay. While the Baucells and Heukamp model also combines the Prelec model with the Ebert and Prelec model, it evaluates delay and risk together, e.g. the effect of delay on utility depends on how much risk and the effect of risk on utility depends on delay. More colloquially, the Baucells and Heukamp model assumes risk and time are entangled, while the Prelec then Ebert and Prelec model assumes risk and time are disentangled. The two stage model would calculate the utility of a 50% chance of $100 in 10 days as so, the first stage would calculate the value of a 50% chance of $100 via the Prelec probability weighting model then that value would be discounted via the Ebert and Prelec discounting model. The one stage model, on the other hand, would calculate utility in one fell swoop.

Since the Baucells and Heukamp model reduces to the Prelec and Ebert and Prelec models for immediate and riskless prospects, respectively, we can hold parameter values constant across both the one stage and the two stage models. This allows us to determine the predicted utility of intertemporal prospects for both models independent of differing model parameters. For instance, comparing the Baucells and Heukamp model to a two stage model which first calculated a decision weight via Prospect theory's probability weighting function then discounted it via an exponential model (Samuelson 1937), would not allow us to neatly compare the effects of entanglement across types of risk. Importantly, the nested structure allows us to determine how much entanglement alters discounting for amount risk, outcome risk, and combined amount and outcome risk. We derive our hypotheses via the ratio between the predicted utility of the Prelec and Ebert and Prelec model and the Baucells and Heukamp model.

1.3 Prelec Probability Weighting model

The first stage of our two stage model calculates the decision weight of an outcome, given an objective probability. The perceived probability of an unlikely event is generally overweighted, while the perceived probability of a likely event is generally underweighted. Prelec (1998) outlines a probability weighting model which states that the weighted, or perceived, probability w(p) of an objective probability is defined as

$$w(p) = e^{\wedge}((-(-lnp)^{\wedge}\delta))$$

Where p is the probability of the option and the δ parameter is the distortion of probabilities. For δ=1 there is no distortion of probability, but for δ=0.1 probabilities are very distorted, with extreme overweighting of low probabilities, w(.01)= 0.31, extreme underweighting of high probabilities, w(.99)= 0.53. The value of the dollar amount x is then multiplied by w(p) to get the utility, $U(x, \delta)$, of the gamble. The Prelec probability weighting model -- the first stage of our two stage model -- calculates, ignoring delay, the risk adjusted utility of a risky intertemporal prospect. The second stage of our two stage model -- the Ebert and Prelec model of time discounting -- then discounts that risk adjusted utility.

1.4 The Ebert and Prelec Model of Time Discounting

The second stage of our two stage model is the Ebert and Prelec (2007) model of time discounting. People discount the near future differently than the distant future. The Ebert and Prelec (2007) model of time discounting calculates the discount factor -- d(t) -- the factor by which a present value is reduced moving t days into the future by:

$$d(t) = exp(-(r_{daily}\ t)^\wedge\delta\ )$$

Where t is the delay, r_{daily} is the daily discount rate, and δ is the index of time sensitivity. Lower values of δ indicate greater time insensitivity. Time sensitivity "affects discounting in a characteristic way, decreasing discounting for near-future outcomes, but increasing discounting for far-future outcomes" (Ebert and Prelec 2007 pg. 1424). The discount factor from the Ebert and Prelec model is then multiplied by the risk adjusted utility to calculate the utility of the risky intertemporal prospect.

1.5 Our Two-Stage Model: Prelec then Ebert-Prelec

Combining the Prelec Model with the Ebert-Prelec into a two stage model the present utility, U, of a risky intertemporal prospect is simply the product of : $xw(p)d(t)$. This combination of the Prelec probability weighting model and the Ebert-Prelec discounting model is a two stage model of risky intertemporal prospects which we call the PEP model. We will compare the predictions from the PEP model to the predictions of the Baucells and Heukamp Model. These comparisons will allow us to determine if, for varying forms of risk, the two models make divergent predictions. Further, they allow us to determine the degree to which diminishing sensitivity to distance affects discounting for the varying forms of risk. More simply, this comparison demonstrates the degree to which entanglement of risk and time affects utility for different types of risk.

1.6 Our One-Stage Model: Baucells and Heukamp

In contrast to the above two stage model, Baucells and Heukamp (2012) assumes that risk and time are entangled. BH combines the Prelec and Ebert and Prelec models into a one stage model where risk and time into a single distance function, which captures the psychological distance to the outcome. Increasing the amount of time to an option increases psychological distance; similarly, decreasing the probability of receiving an outcome increases psychological distance (Trope and Liberman 2010). Further, this psychological distance function is subadditive, meaning, that the combination of risk and time produces an effect on psychological distance that is less than the linear combination of risk and time by themselves.

The Baucells and Heukamp distance model calculates the utility, U of a risky intertemporal prospect by:

$$U(x,p,t) = e\hat{}(-(-lnp + r\_daily\ t)\hat{}\delta\ )\ x$$

where x is the dollar amount of a gamble, p is the probability with which it will be received and t is the delay until receipt. The parameters for calculating utility are r_{daily} -- the probability discount rate which measures the trade-off between probability and time delay -- and δ -- the sensitivity to psychological distance. Higher δ values indicate less diminishing sensitivity to distance. For lower δ values distance is more subadditive. The value of x is independent of delay and probability, but is scaled by σ the curvature of the monetary value function. The BH model reduces to the Ebert and Prelec time discounting model for certain, i.e. p=1, interetemporal choices and reduces to the Prelec Probability weighting function for atemporal, i.e. t=0, risky choices (Toubia et al. 2012).

While both the Baucells and Heukamp and PEP models were only designed to investigate intertemporal prospects with outcome risk, e.g. a 50% chance of $100 in 10 days, it is easy to extend them to intertemporal prospects with amount risk, e.g. a 1/3 chance of $90 in 10 days, a 1/3 chance of $100 in 10 days, a 1/3 chance of $110 in 10 days. We outline this extension in the Amount Risk section below.

We will now use these models to determine if and, if so by how much, diminishing sensitivity to distance affects outcome risk, amount risk, and their combination. To do this we compare the two stage Prelec then Ebert and Prelec model to the BH model. While many studies show that risk preferences alter discounting (Andreoni & Sprenger; Hardisty & Pfeffer; Baucells & Heukamp; Blackburn & El-dereedy), we do not know how much diminishing sensitivity to distance affects intertemporal prospects with amount risk and those with amount and outcome risk. For instance, in an intertemporal prospect with amount risk, if the utility of the BH model is higher than the PEP model we will conclude that diminishing sensitivity to distance affects how intertemporal prospects with amount risk are discounted. Relatedly by comparing the ratio of the utility, we will be able to determine by how much diminishing sensitivity to distance alters utility across the types or risk. This utility ratio allows us to determine the relative effects of each type of risk. For instance, if relative to our disentangled model, our entangled model predicts increased utility of both amount and outcome risk by the same percentage, then we can conclude that entanglement affects intertemporal prospects with outcome risk similarly to intertemporal prospects with amount risk. We adopt the parameter values used in Toubia et al. (2012) for both the PEP and BH models with one exception, for ease of exposition we assume a linear monetary value function (e.g. for both PEP and BH we set σ=1) while setting r_{daily} =.008 and δ=0.8.

1.7 Two Types of Risk

Using the models outlined above and comparing the predicted utility for each across different forms of risk enables us to determine how much the entanglement between time and different forms of risk alters discounting. Specifically, we investigate intertemporal prospects which are risky in three different ways: outcome risk, amount risk, and their combination. We enter intertemporal prospects with these three types of risk into the models outlined above in order to determine the degree to which the entanglement of risk and time affects utility. Stated briefly, how different are the predictions for the BH compared to the PEP for each type of risk. The comparison between the two, however, is important to determine the degree to which the different types of risk alter discounting. This allows us to determine the relative alteration of utility for much outcome, amount, and outcome combined with amount risk. We assume that risk and time are entangled, this model comparison allows us to determine the degree to which entanglement affects prospects with different forms of risk. These model comparisons show that the relative contribution of entanglement to the utility of risky intertemporal prospects is the same for amount, outcome, and amount and outcome risk.

1.8 Outcome risk

The purpose of this section is show how much diminishing sensitivity to distance changes discounting for different forms of risk. The most common way risk has been added to intertemporal prospects is via Outcome risk -- a 50% chance of $200. Baucells and Heukamp show that diminishing sensitivity to distance can explain why outcome risk affects discounting. Now we will work through a few sample predictions for outcome and amount risk for both the PEP and BH models. For predicting the utility of a risky intertemporal prospect via the PEP model we assume that people first calculate a certainty equivalent of the value, then discount that value. For both the PEP and the BH models, the utility of $100 for certain in 10 days would be 87.58. However, predictions of utility diverge for risky intertemporal prospects. Assume the following prospect: a 50% chance of \$200 in 10 days. For the PEP model we first calculate the utility of a 50% chance of \$200, which using the Prelec weighting function is 94.86. We then use the Ebert-Prelec model to discount that utility: 83.09 However if we use BH then utility of a 50% chance of receiving \$200 in 10 days would be 88.62. The utility for the BH model is higher than the PEP model, because the BH model posits that risk and time both increase distance and this distance is subadditive. This diminishing sensitivity to distance accounts for the fact that risk attenuates the immediacy effect. We compare the utility of the BH model to the PEP model to make predictions. This leads to Hypothesis 1a: *for binary choices, people are, holding everything else constant, more likely to choose the larger later option when there is outcome risk compared to when there is no risk.* While this prediction is not novel – it is entirely consistent with Baucells and Heukamp and other work on risk and delay– the model comparison will be used to show how much diminishing sensitivity to distance affects outcome risk; this will allow us to compare outcome risk to amount risk's effect on discounting.

1.9 Amount Risk

Amount risk refers to the situation when there are multiple outcomes -- a 1/3 chance of \$90, a 1/3 chance of \$100, a 1/3 chance of \$110. Previous research indicates that decisions are influenced by the number of outcomes that each option entails. In atemporal risk, gambles with just two outcomes are treated differently from gambles with multiple outcomes (Luce 1991; Tversky and Kahneman (1992)). Since risk preferences are moderated by the number of outcomes, we predict that risky intertemporal choices with multiple outcomes will differ from those with single outcomes.

For both the PEP and the BH model, there are multiple ways to calculate the utility of risky intertemporal prospects with amount risk. Each outcome could be valued independently and discounted or the multiple outcomes could be combined into 1 outcome and then that single outcome could be discounted. (There are other ways in which risky intertemporal choices with multiple outcomes could be discounted in both the BH and PEP models, but we focus on the aforementioned two for their simplicity and space constraints.) Take the following risky intertemporal prospect 1/3 chance of $90 in 10 days a 1/3 chance of $100 in 10 days a 1/3 chance of $110 in 10 days. Combining multiple outcomes into their mean (100) with a probability of 1, then discounting them: both the PEP and BH models would yield the same utility. If this were the case, amount risk should not alter discounting; however, Hardisty and Pfeffer (2015) and Blackburn and El-Deredy (2013) show that amount risk affects discounting. Accordingly, this method of discounting intertemporal prospects with amount risk does not appear to be psychologically plausible, and we will not focus on it further.

Given that people calculate the utility of each outcome independently, however, the PEP and BH models make divergent predictions for the utility of prospects with amount risk. For PEP each option is valued independently and summed e.g. calculate the utility of a 1/3 chance of $90, 30.62, then add the utility of a 1/3 chance of $100, 34.02, then add the utility of a 1/3 chance of $110, 37.43, yielding a utility of 102.07. (Note because in the Prelec model people overweight low probabilities and underweight high probabilities, and since we are assuming a linear value function, these values are higher than expected values.) Then this utility would be discounted to yield a present utility of 89.39.

The BH model, on the other hand, proceeds as follows: calculate the utility of a 1/3 chance of $90 in 10 days, 28.77, then add the utility of a 1/3 chance of $100 in 10 days, 31.97, then add the utility of a 1/3 chance of $110 in 10 days, 35.16 this would yield a present utility of 95.9. As with outcome risk, the predicted utility of PEP model is less than the predicted utility from the BH model. Since, as with outcome risk, we assume that risk and time are entangled, and the utility of the BH model is higher than the PEP model we predict that amount risk affects discounting similarly to outcome risk. This result leads to *Hypothesis 1b: for binary choices people should be, holding everything else constant, more likely to choose the larger later option when there is amount risk compared to when there is no risk.*

Taking the ratio of utilities (e.g. utility BH/ utility PEP) for outcome risk (1.07) and amount risk (1.07) yields a similar percentage increase in utility. While the prediction for Amount and Outcome is the same, it is of note because, absent this model comparison, it would be easy to predict that amount risk had a smaller effect on discounting than outcome. But comparing the BH and PEP models for risky intertemporal prospects, the two make nearly identical predictions.

1.10 Immediacy Effect for Amount and Outcome Risk

Further, due to diminishing sensitivity of distance, the effects outlined in Hypothesis 1 should be largest for items which involve an immediate outcome. This suggests Hypothesis 2a: as with Weber and Chapman (2005), the effect of outcome risk on intertemporal choices should be largest when the choice involves a comparison between an immediate option and a delayed option (rather than two delayed options -- which both already have distance in the form of delay). It also suggests *Hypothesis 2b: the effect of* *amount risk on intertemporal choices should be largest when the discounting involves a comparison between an immediate option and a delayed option (rather than two delayed options).*

1.11 Amount and Outcome Risk

Combined amount and outcome risk refers to a situation when there are multiple options and some of them happen to be zero: e.g., a 1/4 chance of $220 in 10 days, a 1/4 chance of $180 in 10 days, and a 1/2 chance of $0. For PEP, calculating each option individually yields: the sum of a 1/4 chance of $180: 49.12, plus a 1/4 chance of $220: 60.04, plus a 1/2 chance of $0: $0 or 109.16, which would then be discounted by the Ebert-Prelec model to a utility of 95.61.

For the BH the Utility would be: the sum of a 1/4 chance of $180—46.28, a 1/4 chance of $220—56.56, a 1/2 chance of $0, or 102.85. Similar to outcome risk and amount risk, combined amount outcome risk has a higher utility in the BH than in the PEP.

1.12 Comparing PEP to BH: How much does diminishing sensitivity to distance affect utility for different types of risk

To compare the predictions for the PEP model to those of the BH model, we create an index of how much diminishing sensitivity to distance alters utility. Specifically, for each condition we divide the utility of the BH model by the utility of the PEP model. For Amount risk, the utility of the BH model is 1.07 times that of the PEP model; for outcome risk utility of the BH model is also 1.07 times that of the PEP model; for both amount and outcome the utility of the BH model is 1.08 times the utility of the PEP Model. This example suggests a subadditive interaction between amount and outcome risk. Specifically, in risky intertemporal choices both amount and outcome risk effect the contribution of distance in a similar way, however when combined the effect of amount and outcome on distance is practically the same as amount and outcome alone. Comparing the BH to the PEP the effect of amount risk, outcome risk, and amount and outcome risk are all nearly equivalent This result leads to *Hypothesis 3: there is a subadditive interaction between amount and outcome risk such that when there is amount risk and outcome risk their effects on discounting are smaller than the combination of each alone.* We now present two studies to investigate how amount and outcome risk affect intertemporal choices and to test our three hypotheses.

2. Study 1

The purpose of study 1 was to test if both amount and outcome risk affected discounting and also to test the prediction that these effects would be more pronounced for immediate options. Finally, this study tested the prediction that combined amount and outcome risk would have a smaller effect than the linear combination of amount and outcome risk alone.

2.1 Procedure

2.1.1 Materials

We replicated the 2x2 within subjects design of Weber and Chapman (2005) Study 2 and added 1 additional 2 level factor. The first factor of Weber and Chapman was outcome risk, e.g. a 1/2 chance of receiving $200 (Outcome risk) vs. receiving the $200 for certain. The second factor was the delay to the smaller sooner option: either the smaller sooner outcome was immediate or a delayed by 26 weeks. Our third factor, which was not in Weber and Chapman, was amount risk, e.g. a 1/3rd chance of $90, a 1/3rd chance of $100, and a 1/3rd chance of $110 (Amount risk). Each intertemporal choice entailed a comparison between a smaller sooner and a larger later option, and the interval between these two options was always 4 weeks. Comparison of the two levels of the delay to the smaller sooner factor demonstrates the immediacy effect. The outcome risk factor allows us to test the effect of binary outcome risk on choice of larger later outcome. The interaction between of Outcome risk and delay to the smaller sooner option allows us to test the effect of outcome risk on the immediacy effect; this interaction is a direct replication of Weber and Chapman (2005), which showed that outcome risk attenuates the immediacy effect. The interaction between amount risk and delay to the smaller sooner shows how amount risk attenuates the immediacy effect. The interaction between amount risk outcome risk and the delay to the smaller sooner shows how the effect of outcome risk on the immediacy effect is moderated by amount risk. All items can be seen in Table 1.

Table 1: Table of choices from the 2x2x2 design

Delay	Risk			
	No Outcome		Outcome	
	No Amount	Amount	No Amount	Amount
		1/6 chance to get $90 today	1/6 chance to get $200 today	1/6 chance to get $190 today
		1/6 chance to get $100 today	1/6 chance to get $200 today	1/6 chance to get $195 today
		1/6 chance to get $110 today	1/6 chance to get $200 today	1/6 chance to get $215 today
		1/6 chance to get $80 today	1/6 chance to get $0 today	1/6 chance to get $0 today
		1/6 chance to get $105 today	1/6 chance to get $0 today	1/6 chance to get $0 today
Immediate	get $100 now (No indifference point calculated)	1/6 chance to get $115 today	1/6 chance to get $0 today	1/6 chance to get $0 today
	1/6 chance to get $110 for certain in 4 weeks	1/6 chance to get $100 in 4 weeks	1/6 chance to get $220 in 4 weeks	1/6 chance to get $205 in 4 weeks
	1/6 chance to get $110 for certain in 4 weeks	1/6 chance to get $110 in 4 weeks	1/6 chance to get $220 in 4 weeks	1/6 chance to get $225 in 4 weeks
	1/6 chance to get $110 for certain in 4 weeks	1/6 chance to get $120 in 4 weeks	1/6 chance to get $220 in 4 weeks	1/6 chance to get $230 in 4 weeks
	1/6 chance to get $110 for certain in 4 weeks	1/6 chance to get $90 in 4 weeks	1/6 chance to get $0 in 4 weeks	1/6 chance to get $0 in 4 weeks
	1/6 chance to get $110 for certain in 4 weeks	1/6 chance to get $115 in 4 weeks	1/6 chance to get $0 in 4 weeks	1/6 chance to get $0 in 4 weeks
	1/6 chance to get $110 for certain in 4 weeks	1/6 chance to get $125 in 4 weeks	1/6 chance to get $0 in 4 weeks	1/6 chance to get $0 in 4 weeks
	1/6 chance to get $100 for certain in 26 weeks	1/6 chance to get $90 in 26 weeks	1/6 chance to get $200 in 26 weeks	1/6 chance to get $190 in 26 weeks
	1/6 chance to get $100 for certain in 26 weeks	1/6 chance to get $100 in 26 weeks	1/6 chance to get $200 in 26 weeks	1/6 chance to get $195 in 26 weeks
	1/6 chance to get $100 for certain in 26 weeks	1/6 chance to get $110 in 26 weeks	1/6 chance to get $200 in 26 weeks	1/6 chance to get $215 in 26 weeks
	1/6 chance to get $100 for certain in 26 weeks	1/6 chance to get $80 in 26 weeks	1/6 chance to get $0 in 26 weeks	1/6 chance to get $0 in 26 weeks
	1/6 chance to get $100 for certain in 26 weeks	1/6 chance to get $105 in 26 weeks	1/6 chance to get $0 in 26 weeks	1/6 chance to get $0 in 26 weeks
Remote	1/6 chance to get $100 for certain in 26 weeks	1/6 chance to get $115 in 26 weeks	1/6 chance to get $0 in 26 weeks	1/6 chance to get $0 in 26 weeks
	1/6 chance to get $110 for certain in 30 weeks	1/6 chance to get $100 in 30 weeks	1/6 chance to get $220 in 30 weeks	1/6 chance to get $205 in 30 weeks
	1/6 chance to get $110 for certain in 30 weeks	1/6 chance to get $110 in 30 weeks	1/6 chance to get $220 in 30 weeks	1/6 chance to get $225 in 30 weeks
	1/6 chance to get $110 for certain in 30 weeks	1/6 chance to get $120 in 30 weeks	1/6 chance to get $220 in 30 weeks	1/6 chance to get $230 in 30 weeks
	1/6 chance to get $110 for certain in 30 weeks	1/6 chance to get $90 in 30 weeks	1/6 chance to get $0 in 30 weeks	1/6 chance to get $0 in 30 weeks
	1/6 chance to get $110 for certain in 30 weeks	1/6 chance to get $115 in 30 weeks	1/6 chance to get $0 in 30 weeks	1/6 chance to get $0 in 30 weeks
	1/6 chance to get $110 for certain in 30 weeks	1/6 chance to get $125 in 30 weeks	1/6 chance to get $0 in 30 weeks	1/6 chance to get $0 in 30 weeks

2.1.2 Data Cleaning

In addition to participants removed via check choices, we also removed participants who had a less than 4 unique indifference points. This was done because people who have a small number of indifference points are likely to have just clicked either the certain/now (risky/delayed) option for every choice in the titration, meaning that the estimated indifference points likely are not actual reflections of their underlying preferences.

2.1.2 Participants

We recruited 258 participants from Mechanical Turk for study. Of those 85 missed either one or both of our attention check criteria outlined above and in our study preregistration report and were excluded from analysis [Available on AsPredictied.org]. Of the remaining 173, 93 were female with a mean age of 33.07.

2.2 Results

2.2.1 Main Analysis

We ran a multilevel logistic regression with a varying intercept for each subject which accounts for the correlated errors within each participant's choices. The dependent variable was choice on each question, with smaller sooner set to 0 and larger later set to 1. The independent variables were delay to the smaller sooner option (0 or 26 weeks), outcome risk (present or not), and amount risk (present or not) and all possible interactions between amount and outcome risk.

As seen in Table 2 there is a main effect of delay to the smaller sooner on choice, such that people are more likely to choose the smaller sooner option when the smaller sooner reward was delivered immediately. This indicates the presence of an immediacy effect. Test of Hypothesis 1a, that outcome risk has an effect on choice of the larger later option, is the main effect of outcome risk, which is not significant but in the expected direction. The test of hypothesis 1b, that amount risk has an effect on choice of larger later, is the main effect of amount risk, which is significant and in the expected direction. Test of Hypothesis 2a, that outcome risk has a larger effect on choice of the larger later option when one of the options is immediate, is the interaction between delay now and outcome risk, which is not significant. Test of hypothesis 2b, that amount risk has a larger effect on effect on choice of larger later when one of the options is immediate, is the interaction between delay now and amount risk, which is not significant. Test of hypothesis 3, that there is a subadditive interaction between amount and outcome risk, is the interaction between amount and outcome risk, which is not significant but in the expected direction.

Table 2: The effects of risk on choice of larger later option

	Dependent variable:
	Choice of Larger Later
26 Week Delay to SS	0.713***
	(0.252)
Amount Risk	0.811***
	(0.247)
Outcome Risk	0.461*
	(0.254)
Delay Amount Interaction	-0.313
	(0.343)
Delay Outcome Interaction	-0.241
	(0.352)
Amount Outcome Interaction	-0.561
	(0.346)
Delay Amount Outcome Interaction	-0.109
	(0.486)
Constant	-1.209***
	(0.194)
Observations	1,283
Log Likelihood	-831.445
Akaike Inf. Crit.	1,680.890
Bayesian Inf. Crit.	1,727.302
Note:	*p<0.1;* ***p<0.05;*** p<0.01

How amount risk, outcome risk, and their combination alter the immediacy effect can be seen in Figure 1. People are more likely to choose the larger later option when the delay to the smaller sooner option is 26 weeks as compared to when it is now.

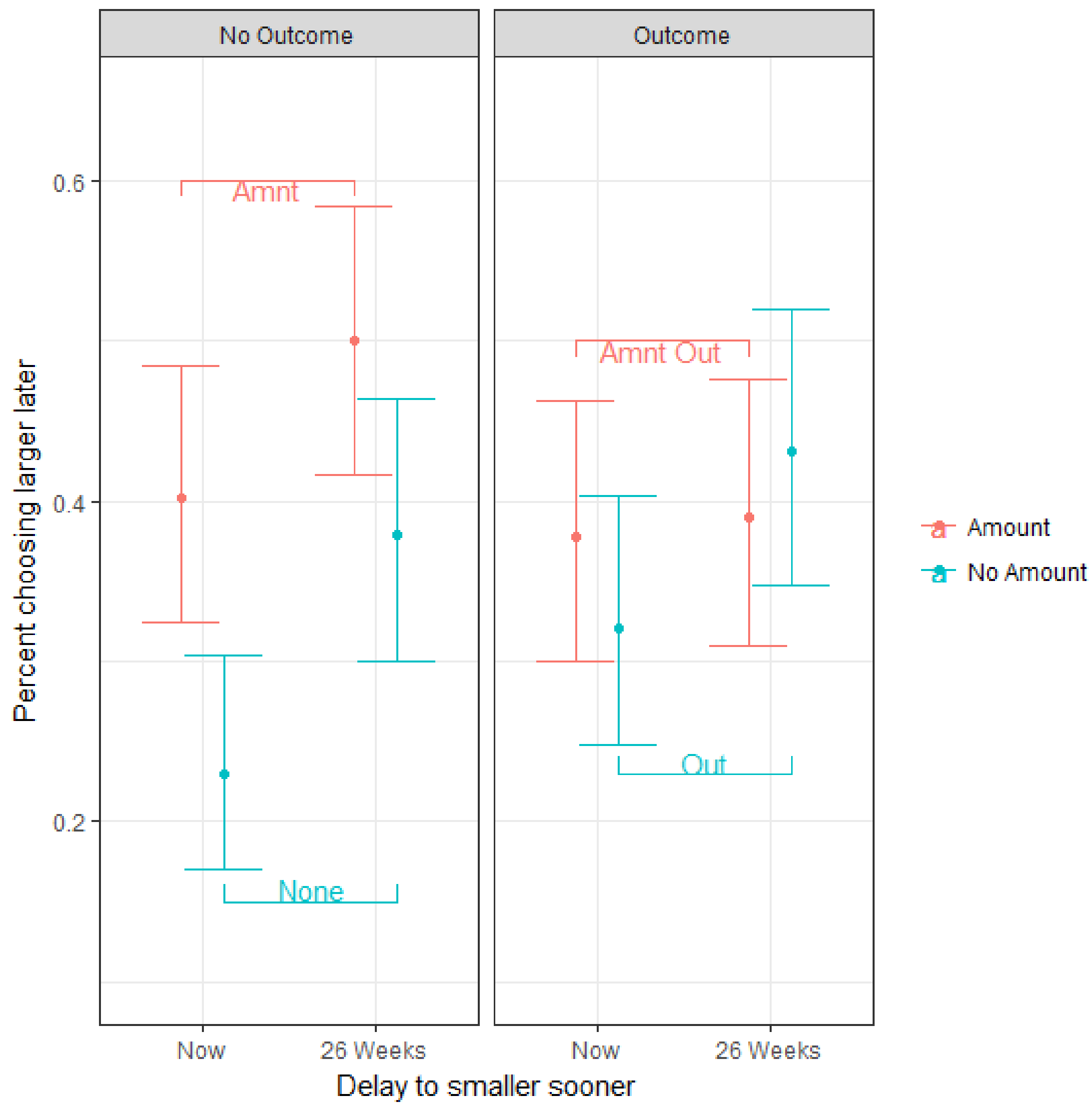

Figure1: Effects of Risk on the Immediacy Effect

Figure 2 shows the effects of amount and outcome risk on choice of the larger later option regardless of the delay to the smaller sooner. The effect of amount risk is larger than the effect of outcome risk Furthermore there is an interaction between amount risk and outcome risk such that the effect of amount risk on choice of the larger later option is attenuated when there is outcome risk as well.

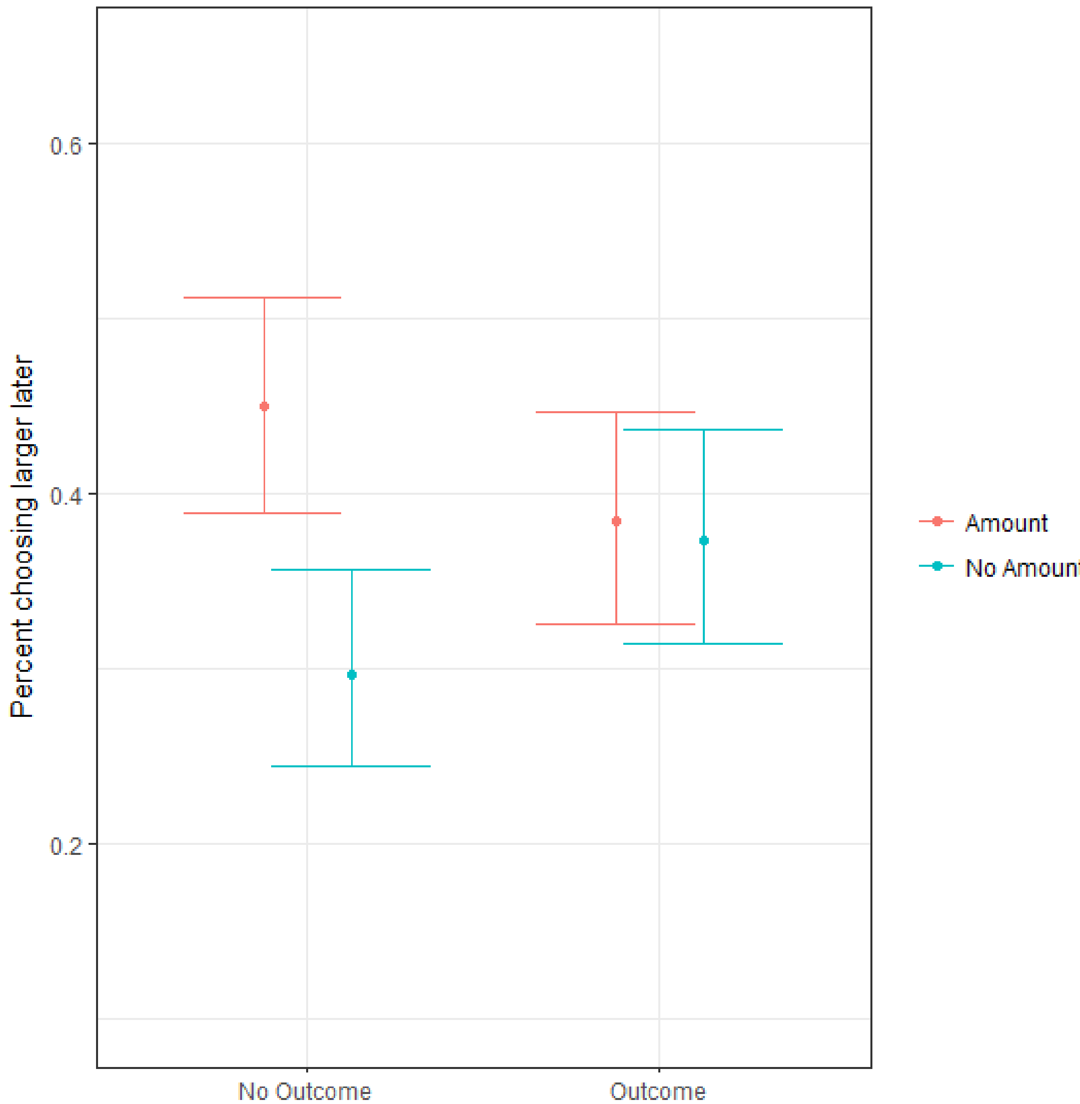

Figure 2: Effects of Risk on Choice of Larger Later

2.2.2 Replication of Weber and Chapman

As a secondary analysis we subsetted the data to be a direct replication of Weber and Chapman (2005). Specifically, we removed all choices with amount risk and focused on those with no risk and those with outcome risk. As seen in Table 3, we ran two regressions. The first regression included main effects of delay to smaller sooner and outcome risk as well as their interaction. Contrary to Weber and Chapman (2005) as evidenced by the nonsignificant interaction we do not find that the addition of outcome risk attenuates the immediacy effect. Further in the second regression, for choices with outcome risk, there is still a significant immediacy effect. This also does not replicate Weber and Chapman (2005) who found that for choices with only outcome risk there was no immediacy effect.

Table 3: Replication of Weber and Chapman Analyses

	Dependent variable:	
	Choice of Larger Later	
	(1)	(2)
26 Week Delay to SS	0.718***	0.453*
	(0.254)	(0.242)
Outcome Risk	0.466*	
	(0.256)	
Delay Amount Interaction	-0.240	
	(0.353)	
Constant	-1.215***	-0.709***
	(0.199)	(0.183)
Observations	636	310
Log Likelihood	-401.852	-204.929
Akaike Inf. Crit.	813.703	415.857
Bayesian Inf. Crit.	835.979	427.067
Note:	*p<0.1;* ***p<0.05;*** p<0.01	

2.3 Study 1 Discussion

The predicted interaction between risk and immediacy was not found. The failure to replicate Weber and Chapman (2005) could be due to the fact that the immediacy effect in the current study was modest in size. Thus, even a manipulation that completely eliminated the immediacy effect would not result in an interaction. As show in Figure 1 and Table 2, however, the simple main effect of the immediacy effect remains significant even in the outcome risk condition. Thus, lack of power to detect an interaction cannot be the entire explanation. Study 1 also demonstrated that participants' choice of larger later options was affected more by amount risk than by outcome risk. This result suggests that psychological distance of intertemporal choices may be affected by the number of possible outcomes. Having multiple possible outcomes may make people attend more to the amounts and neglecting the delay, yielding lower discounting. Specifically, we found support for only Hypothesis 1b, the tests of all other hypotheses, while directionally consistent, were not significant. Study 1 implies, while not all predictions were significant, that risk, regardless of form, has an effect on psychological distance. Since in study 1 the effect of outcome risk was not significant, in Study 2 we attempted to make outcome risk more salient. To do this for outcome risk we used token negative values as opposed to zeros.

3. Study 2

The purpose of study 2 was to create a representation of outcome risk that was more salient. Specifically, we followed Slovic (2004), and added token negative values, which did not alter the expected value, but, we predicted, would make people attend to their value more than zeros. Accordingly, the design of Study 2 was identical to Study 1, except that for outcome risk, instead of a zero value, we used a randomly drawn negative value with a small negative mean value.

Study 2 was preregistered at AsPredicted.org. We recruited 216 participants to take the study. Of those, 22 missed either one or both of our attention check criteria outlined in our preregistration report and were excluded from analyses. Of the remaining 194, 104 were female with a mean age of 34.79.

3.2 Results of Study 2

As with Study 1, we ran a multilevel logistic regression with a varying intercept for each subject. The dependent variable was choice -- smaller sooner option 0, larger later option 1. The results of the regression can be seen in Table 4. Test of Hypothesis 1a, that outcome risk has an effect on choice of the larger later option, is the main effect of outcome risk, which is significant and in the expected direction. The test of hypothesis 1b, that amount risk has an effect on choice of the larger later option, is the main effect of amount risk, which is significant and in the expected direction. Test of Hypothesis 2a, that outcome risk has a larger effect on choice of the larger later option when one of the options is immediate, is the interaction between delay now and outcome risk, which is significant and in the expected direction. Test of hypothesis 2b, that amount risk has a larger effect on effect on choice of the larger later option when one of the options is immediate, is the interaction between delay now and amount risk, which is significant and in the expected direction. Test of hypothesis 3, that there is a subadditive interaction between amount and outcome risk, is the interaction between amount and outcome risk, which is significant and in the expected direction.

Table 4: Logistic regression for effect of outcome risk with negative values on choice.

	Dependent variable:
	Choice of the larger later option
26 Week Delay to SS	0.892***
	(0.248)
Amount Risk	1.235***
	(0.241)
Outcome Risk	0.690***
	(0.249)
Delay Amount Interaction	-0.648*
	(0.331)
Delay Outcome Interaction	-0.761**
	(0.342)
Amount Outcome Interaction	-1.140***
	(0.333)
Delay Amount Outcome Interaction	0.911*
	(0.465)
Constant	-1.380***
	(0.191)
Observations	1,386
Log Likelihood	-896.933
Akaike Inf. Crit.	1,811.867
Bayesian Inf. Crit.	1,858.974
Note:	*p<0.1;* ***p<0.05;*** p<0.01

Figure 4 shows how both amount and outcome risk alter the immediacy effect. Adding outcome risk attenuates the immediacy effect, and adding amount risk marginally attenuates the immediacy effect.

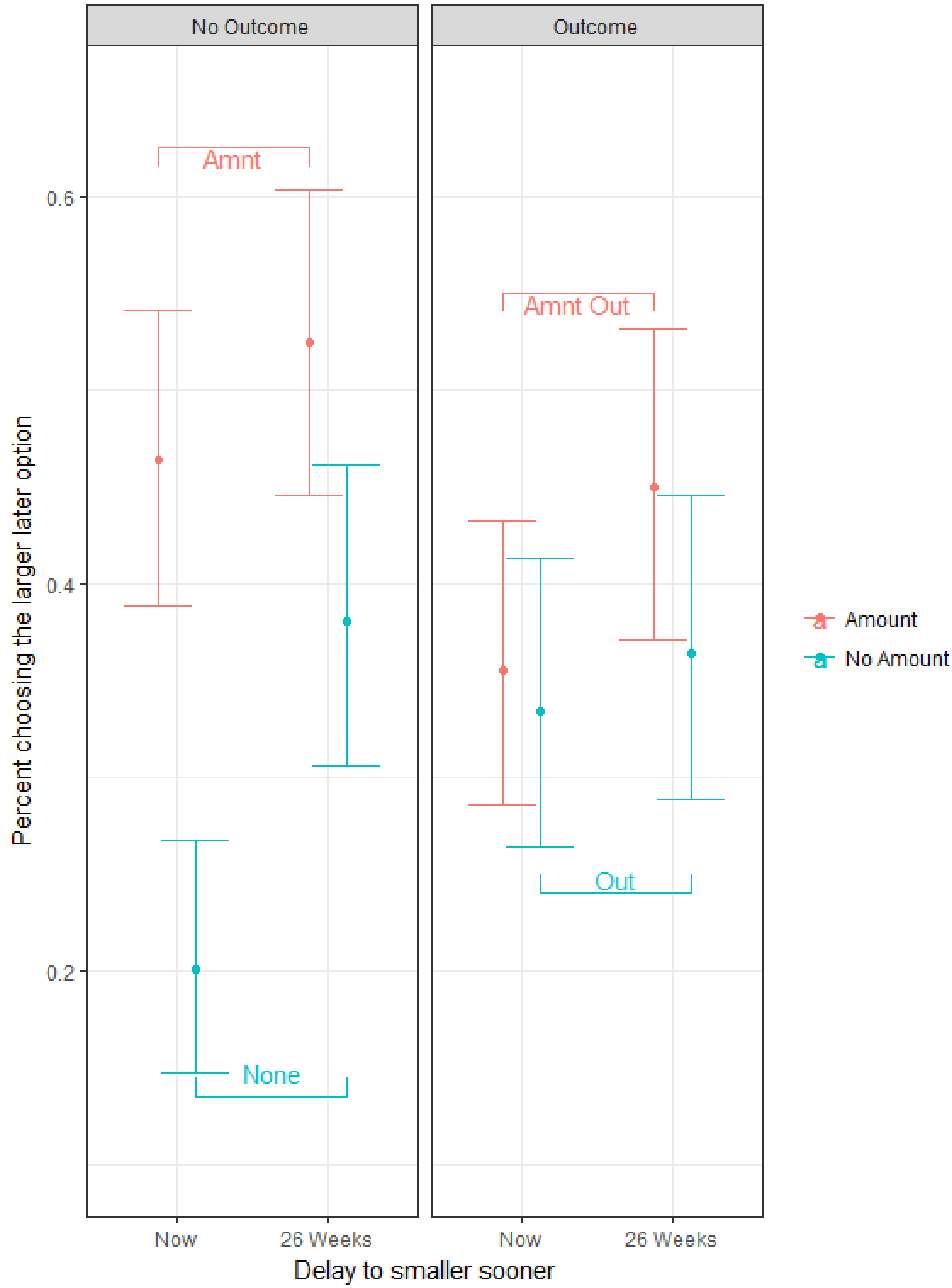

Figure 4: Effects of Risk on the Immediacy Effect

As seen in Figure 5 there is a main effect of amount risk, but not a main effect of outcome risk. Also there is a marginally significant interaction between amount and outcome such that the combination of amount and outcome is closer to outcome alone than amount alone.

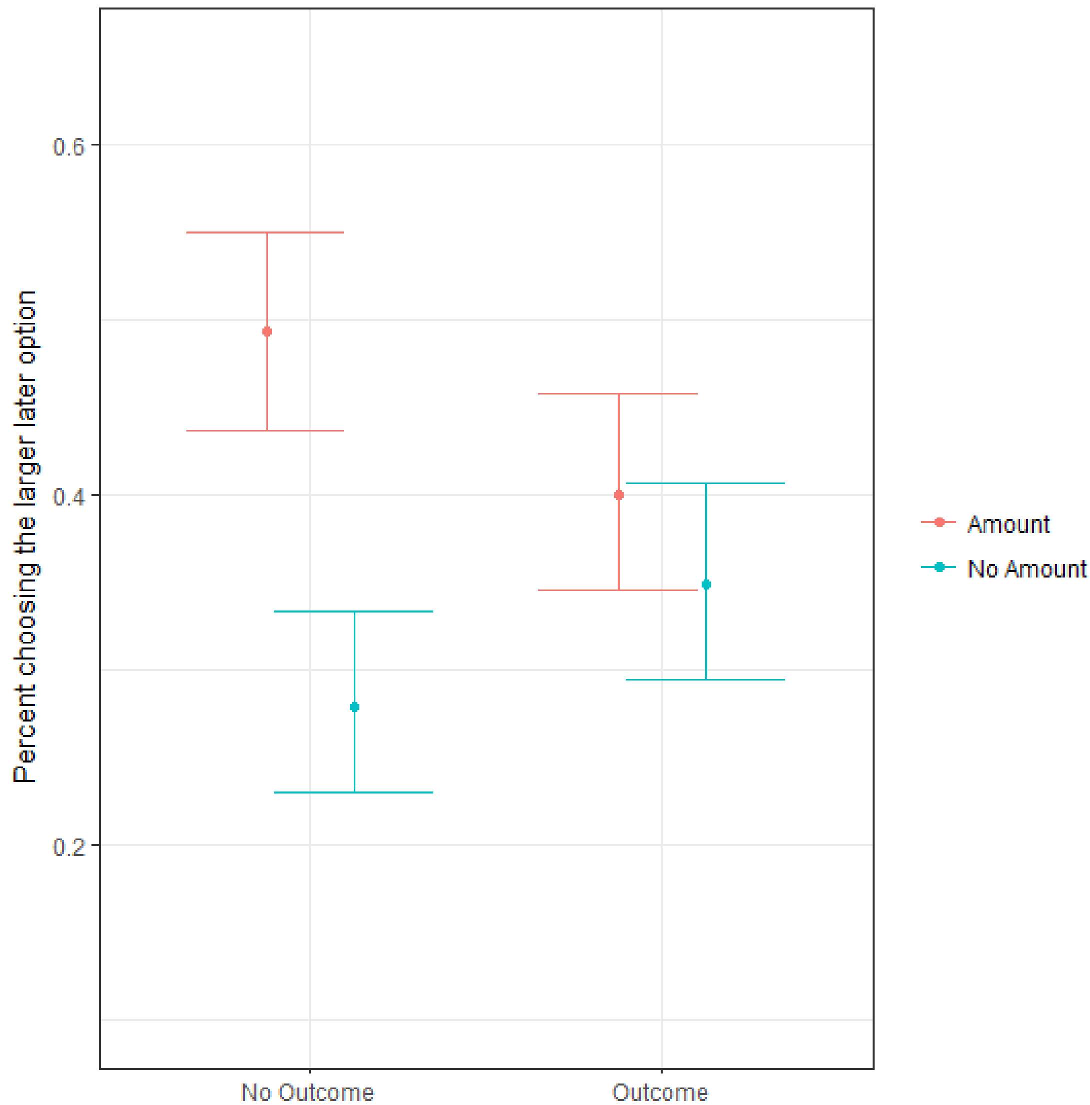

Figure 5: Effects of Risk on Choice of the larger later option

While there is a difference in significance between studies 1 and 2 this does not necessarily indicate that there is a significant difference between the effects observed in studies 1 and

2. To test for this we combined the data from study 1 with the data from study 2 and included a dummy variable to determine if there was an effect of zero versus negative values on choice of the larger later option.

4. Studies 1 and 2 combined

We investigated whether the effect sized observed in study 1 and study 2 were significantly different from one another. The frequentist multilevel modeling framework we used in studies 1 and 2 was unable to handle the complexity of the combined analyses. Therefore, we used a Bayesian Multilevel model, which is able to fit more complicated multilevel models. Specifically, we used the brms package which fits the multilevel model in Stan. We confirmed that our model converged by examining trace plots and checking that all $\hat{R}$ were sufficiently close to 1. Since a Bayesian approach calculates the full posterior distribution of parameter values, it does not lend itself to the use of p values. Accordingly, we follow the tradition of checking to if the 95% Credible Interval includes 0, if it does we conclude that the parameter was not a significant predictor of choice. If, however, the credible interval does *not* include 0, then we conclude that the parameter was a significant predictor of choice.

Table 4 shows the results of the Bayesian Multilevel model. We conclude that a term is significant if the credible interval shown does not include 0. As seen in Table 4 the presentation of outcome risk -- either as zeros or as token negative values -- did not have an effect on choice. This lack of an effect is evidenced by the lack of significant interactions between the type of outcome risk, zero or token negative values, on choice of the larger later option. This suggests that the differences in significance between studies 1 and 2 were not themselves significantly different from one another.

Table 5. Results of the Bayesian Hierarchical model predicting choice for the combined study 1 and 2 data

Parameter	Rhat	mean	sd	2.5%	50%	97.5%	CI incl 0
Intercept	1.00	-1.42	0.19	-1.81	-1.42	-1.05	no
26 Week Delay	1.00	0.92	0.25	0.42	0.92	1.40	no
Amount Risk	1.00	1.27	0.24	0.80	1.27	1.74	no
Outcome Risk	1.01	0.71	0.25	0.22	0.71	1.21	no
Zeros (study 1)	1.01	0.21	0.27	-0.32	0.21	0.74	yes
Delay:Amount	1.00	-0.67	0.33	-1.34	-0.67	-0.04	no
Delay:Outcome	1.00	-0.78	0.35	-1.47	-0.79	-0.09	no
Amount:Outcome	1.00	-1.17	0.34	-1.85	-1.17	-0.51	no
Delay:Zero	1.00	-0.21	0.36	-0.91	-0.21	0.52	yes
Amount:Zero	1.01	-0.46	0.34	-1.12	-0.46	0.24	yes
Outcome:Zero	1.01	-0.25	0.35	-0.94	-0.25	0.46	yes
Delay:Amount:Outcome	1.00	0.94	0.47	0.05	0.94	1.88	no
Delay:Amount:Zero	1.01	0.35	0.49	-0.61	0.36	1.29	yes
Delay:Outcome:Zero	1.01	0.54	0.50	-0.44	0.55	1.51	yes
Amount:Outcome:Zero	1.01	0.62	0.49	-0.35	0.61	1.53	yes
Delay:Amount:Outcome:Zero	1.01	-1.06	0.69	-2.37	-1.07	0.31	yes

Note: The 95% credible intervals are shown above. The "CI incl 0" column indicates if the interval included 0 or not, if it did not, we conclude that the parameter had a significant effect on choice.

As with studies 1 and 2, this combined analysis shows a significant main effect of delay to the smaller sooner, such that people are more likely to choose the larger later option when neither option is immediate -- this is the classic immediacy effect. Test of Hypothesis 1a, that outcome risk has an effect on choice of the larger later option, is the main effect of outcome risk, which is significant and in the expected direction. The test of hypothesis 1b, that amount risk has an effect on choice of the larger later option, is the main effect of amount risk, which is significant and in the expected direction. Test of Hypothesis 2a, that outcome risk has a larger effect on choice of the larger later option when one of the options is immediate, is the interaction between delay now and outcome risk, which is significant and in the expected direction. Test of hypothesis 2b, that amount risk has a larger effect on effect on choice of larger later when one of the options is immediate, is the interaction between delay now and amount risk, which is significant and in the expected direction. Test of hypothesis 3, that there is a subadditive interaction between amount and outcome risk, is the interaction between amount and outcome risk, which is significant and in the expected direction.

4.2 Combined 1 and 2 Discussion

As predicted by our hypotheses, the combined analysis showed that both amount and outcome risk affect discounting, their effects on discounting were more pronounced for immediate options, and that there was a subadditive interaction between amount and outcome risk. However, the combined analysis of studies 1 and 2 did not show a difference between zero values and token negative values on indifference points. Taken together the results of the combined analysis are precisely what the Baucells and Heukamp model predicted and suggest strongly that both outcome and amount risk influences intertemporal choices. Moreover, their effects are largest for choices involving an immediate option and when combined their effects are subadditive

5 General Discussion

In this paper we aimed to test the entanglement of multiple forms of risk and delay. In order to determine if different forms of risk alter the effects of entanglement, we compared predictions from the Baucells and Heukamp model to the Prelec and Ebert and Prelec model to determine how much this entanglement altered utility. By comparing the utility ratio of simulated data for the BH and PEP models we predicted that amount risk would behave similarly to outcome risk for risky intertemporal choices. Namely, we predicted that both amount and outcome risk would increase the likelihood of choosing the larger later option. Outcome risk was found to be a significant predictor of choice in both

Study 2 and the combined analysis. Further amount risk was a significant predictor of choice in both studies as well as the combined analysis. Following from diminishing sensitivity to distance, our second prediction was that the effect of amount and outcome risk on choice of the larger later option would be more pronounced for choices which involve an immediate option. In the combined analysis, we found that this was the case for both amount and outcome risk. Our third prediction, derived from the comparison between the PEP and BH models for all types of risk, was that there would be a subadditive interaction between amount and outcome risk, was supported in the combined analyses. Taken together, these findings support our contention that risk, regardless of form, is entangled with delay.

The current paper replicates and extends previous research on risky intertemporal choices (Andreoni and Sprenger 2012; Baucells and Heukamp 2012; Hardisty and Pfeffer 2015). Specifically, it echoes the finding that risky intertemporal choices are not a simple combination of risky choices with intertemporal choices. It goes beyond prior research by showing that, amount and outcome act similarly for risky intertemporal prospects. Moreover, the current paper elucidates the cognitive mechanisms underlying risky intertemporal choices. While diminishing sensitivity to distance seems to extend across types of risk, it appears that the amount risk's effects on the utility of an immediate prospects do not correspond directly to its effects on the utility of delayed prospects. Time preferences are affected not only by the probability of getting the money, but also by multiple possible monetary values. This suggests that models of risky intertemporal choice need to account for more than outcome risk alone.

6. Limitations and Future Directions

While we found support for our hypotheses based on the comparisons between the BH and PEP models, there is a possible alternative explanation for our findings. Namely that amount risk reduces the relative attention participants are paying to the temporal element of our choices. If relative attention to time is reduced, then the weight of delay in choice would be attenuated. This attenuation would then lead to less discounting because prospect A: a 1/3 chance of $90 today, a 1/3 chance of $100 today, a 1/3 chance of $110 today, would be valued almost the same as prospect B: a 1/3 chance of $90 in 60 days, a 1/3 chance of $100 in 60 days, a 1/3 chance of $110 in 60 days. Our future work will, via process tracing, determine if relative attention to the temporal component of risky intertemporal choices is less for amount risk compared to outcome risk.

Further, a post-hoc investigation of the data uncovered a reason for caution in interpreting the results. Since we assumed that people would value $100 as $100, we did not elicit a participant's certainty equivalent for $100 today. However, eliciting this value would have been advantageous for multiple reasons. First, it would have made identifying participants who responded randomly quite simple: if a person valued $100 today as the equivalent of $50 today they are likely responding randomly. Second, for the choice between $100 today and $110 in 4 weeks, the valuation of $100 today lead to a large percentage of imputed choices being for the smaller sooner option. This high difference for the certain condition may be a driver of some of the interactions we see.

Also, the staircase procedure implicitly assumes that choices are measured without noise. And there are good reasons to believe that choice is stochastic in nature (Luce, 1959). This means that an adaptive choice procedure can, due to random fluctuations, end up in a region which is far from the participant's actual indifference point. While this is a concern, the perturbations arising from the stochastic choice should be normally distributed, and, in the aggregate, should cancel out.

An obvious extension of our modeling is using cumulative prospect theory probability weighting as opposed to the probability weighting from the original prospect theory (Kahneman & Tversky, 1978). Another possible extension of this paper is to more formally model the Prelec then Ebert and Prelec and Baucells and Heukamp models using a Bayesian Hierarchical Model (Lee & Wagenmakers, 2013). This model would allow for a more direct comparison of the models; whichever model fits the data better would be the preferred model. A possible extension of the Bayesian Hierarchical Model is the inclusion of a mixing parameter. This mixing parameter would allow one to determine if people are best fit by different models. For instance, have of the population could be best fit by the BH model while the other half could be best fit by the PEP model. It is possible that for certain people risk and time are entangled, but for other people they are disentangled.

7. Conclusions

Taken together our results suggest that amount risk affects discounting, but when combined with outcome risk, its effects are attenuated. This work also demonstrates that the effects of amount risk and outcome risk are greatest when one of the options is immediate, suggesting that the results of Hardisty and Pfeffer (2015) may exaggerate the effects of risk on ITC. When modeling risky intertemporal choices, considering risk in the amount and its effects on discounting, is an important step towards understanding more naturalistic representations of intertemporal choice.

Returning the example in the introduction, when you are presented a retirement account, you must be mindful of how you interpret both risk and time. Both risk in the eventual worth of the account and the possibility that it could be worthless affect how you discount its future worth.

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FOOTNOTES

[i] In this paper the terms “the interaction between risk and delay” and “risk and delay are entangled” are equivalent. However “diminishing sensitivity to distance” is a specific form of entanglement which is posited by the Baucells and Heukamp model.