ESSAYS ON SUPPLY CHAIN COORDINATION AND OPTIMIZATION

BY JU MYUNG SONG

A dissertation submitted to the
Graduate School—Newark
Rutgers, The State University of New Jersey
in partial fulfillment of the requirements
for the degree of
Doctor of Philosophy
Graduate Program in Management

Written under the direction of
Dr. Yao Zhao and Dr. Xiaowei Xu
and approved by

Newark, New Jersey
May, 2017
ABSTRACT OF THE DISSERTATION

Essays on Supply Chain Coordination and Optimization

by Ju Myung Song

Dissertation Director: Dr. Yao Zhao and Dr. Xiaowei Xu

The dissertation comprises of three essays that 1) shipping for online sellers to meet the peak demand, 2) incentives and gaming in collaborative projects, and 3) emergency operations optimization under the demand uncertainty.

The online retailing is changing the landscape of retail industry as Amazon’s market cap has recently doubled that of Wal-Mart in the US. Different from brick and mortar, online sellers rely on 3rd party logistics (3PLs) for the delivery of the goods but the hugely spiked demand during holiday seasons (Cyber Monday, Black Friday, Christmas in the US, or Singles day in China) poses a substantial challenge for the 3PLs to deliver on time. To better manage demand, 3PLs such as UPS, require the sellers to make reservation and to pay a surcharge for extra work. In the first essay, we discuss how these shipping arrangements may affect the online sellers’ inventory decisions, and how to coordinate the channel for the sellers and shippers to win-win.

In the second essay, we study gaming and incentive issues in collaborative projects. Driven by the needs of technology, finance, and marketing, many projects in diverse industries are moving irreversibly from the one-firm-does-all model to outsourcing and
collaboration. However, collaborative projects often ran into delays and cost overruns with the Boeing 787 Dreamliner being one of the latest examples. Applying game theoretical models and economics theory of teamwork to project management, we study how the popular risk sharing partnership may change firms incentives in project execution and affect project performance in cost and time for various project networks (serial vs. parallel), cost structures (time independent vs. dependent), and information status (symmetry vs. asymmetry).

In the third essay, we study the emergency response operations, which are critical to save humans lives and properties after a disaster happens. The limited resources and time requirements call for cooperated supply chain operations. We examine a coordinated production and distribution network for rescue kits in disaster reliefs and develop an optimization model to minimize the total tardiness and peak tardiness over the planning horizon. Proposing to deploy supply chain flexibility to cope with the uncertain demand, we show the effectiveness of increasing supply chain flexibility and suggest some managerial insights on configuring such flexibility in emergency operations.

In this dissertation, the three essays are structured to form a coherent body as described above on the topic of the coordination and optimization of a supply chain considering different ways to achieve it.
Acknowledgements

I would never have been able to finish my dissertation without the guidance of my committee members, assistance from colleagues, and support from my family.

I would like to express my deepest gratitude to my co-advisors, Dr. Yao Zhao and Dr. Xiaowei Xu. They brought me to academia and encouraged me to explore on my own. They patiently supported my development and exploration of research ideas, which are a significant basis for my dissertation. Without their trust and consistent support, I could not overcome the many crisis situations during my PhD studies.

I would like to thank to Dr. S. Chan Choi, who continuously found time to discuss the work with me and patiently guided me. He also helped me for the first chapter in this dissertation.

I am also very grateful to Dr. Weiwei Chen. I would like to express my warmest appreciation for his help for my fourth chapter. I am deeply grateful to him for helping me and he has been always there to help and provide suggestions.

I would like to thank Dr. Junmin Shi for serving on the committee and supporting me in improving my dissertation with his insightful comments.

I deeply appreciate Dr. Lei Lei for her help for my study. I would like to acknowledge my gratitude to Dr. Alok Baveja for his consistent support as department chair. Also, I am grateful to Dr. Xin Xu for his early work for my third chapter.

Most importantly, none of my achievement would have been possible without the love and patience of my family. Particularly, for my wife, Kyunghee Yoon, and for my parents, Kiheung Song and Soonhee Lee, words cannot describe how grateful I am for
their support and love.
# Table of Contents

Abstract ......................................................... ii

Acknowledgements ................................................ iv

List of Tables ....................................................... ix

List of Figures ....................................................... x

1. Introduction ................................................... 1

2. Shipping Peak Demand for Online Sellers:
   Penalty vs. Flat Rate ........................................ 5
     2.1. Introduction ........................................... 5
     2.2. Literature Review ..................................... 10
     2.3. The Model and Seller’s Problem ....................... 14
     2.4. The Shipper’s Problem ................................ 20
     2.5. The Channel’s Problem (The First-Best Solution) .... 22
     2.6. The Channel Coordination .............................. 24
     2.7. Numerical Results .................................... 26
     2.8. Conclusion ............................................ 28
     2.9. Appendix: Proofs and Technical Details ............. 29

3. Incentives and Gaming in Collaborative Projects - Risk Sharing Partnership ........................................ 56
3.1. Introduction ...................................................... 56
3.2. Literature Review .............................................. 61
3.3. The Model and Preliminaries ................................. 65
3.4. Information Symmetry .......................................... 68
  3.4.1. The Base Model - The Prisoners’ Dilemma in Project Execution 69
  3.4.2. Time-dependent Costs - The Supplier’s Dilemma .............. 72
  3.4.3. Expediting and Reward - The Coauthors’ Dilemma .......... 76
  3.4.4. Multiple Suppliers - The Worst Supplier Dominance ........ 80
3.5. Information Asymmetry ......................................... 82
  3.5.1. Time-dependent Cost - Revisited ......................... 84
  3.5.2. Expediting and Reward - Revisited ....................... 85
  3.5.3. Multiple Suppliers - Revisited .......................... 89
3.6. Linking Theory to Practice .................................... 91
3.7. Conclusion ..................................................... 94
3.8. Appendix: Proofs and Technical Details ...................... 95

4. Flexibility in Emergency Supply Chain Operations: Coping with Demand Uncertainty ................................. 105
  4.1. Introduction .................................................. 105
  4.2. Literature Review ........................................... 107
  4.3. Problem Description and Model Framework ................. 109
    4.3.1. Problem Description .................................. 109
    4.3.2. Model Framework ...................................... 112
  4.4. Optimization Model for Base Demand ....................... 115
    4.4.1. Single-Period Optimization Model ..................... 115
    4.4.2. Optimizing Inventory Policy across Periods .......... 121
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.5</td>
<td>Supply Chain Flexibility for Coping with Demand Surges</td>
<td>123</td>
</tr>
<tr>
<td>4.5.1</td>
<td>Scenario Description</td>
<td>124</td>
</tr>
<tr>
<td>4.5.2</td>
<td>Flexibility Effects</td>
<td>126</td>
</tr>
<tr>
<td>4.6</td>
<td>Conclusion</td>
<td>131</td>
</tr>
</tbody>
</table>
List of Tables

4.1. Parameter settings of test cases. . . . . . . . . . . . . . . . . . . . . . . . . . . 125
4.2. Comparison of metrics for uniform-shaped demand. . . . . . . . . . . . . 127
4.3. Comparison of metrics for normal-shaped demand. . . . . . . . . . . . . 128
4.4. Comparison of metrics for triangular-shaped demand. . . . . . . . . . . . 129
List of Figures

2.1. e-Commerce and Small Package Volume. 6
2.2. Online Shopping and Shipping Rush in Holidays. 6
2.3. The seller’s optimal inventory and reservation decisions as a function of $p_u$. 18
2.4. The channel’s optimal inventory and capacity decisions as a function of $c_o$. 24
2.5. Comparison of Average Profits. 27
3.1. One-Firm-Does-All vs. Risk Sharing. 59
3.2. Project Cost Structure. 66
3.3. A typical collaborative project. 67
3.4. Sequence of Events. 68
3.5. The extensive form of the game in the base model. 71
3.6. An example for the base model with time-dependent costs and its pay-off matrix. 73
3.7. The extensive form of the game in the base model with time-dependent costs. 74
3.8. The extensive form of the game in the base model with expediting and reward. 77
3.9. The extensive form of the game in the base model with multiple suppliers. 102
4.1. The full network structure of an emergency supply chain for rescue kits. 110
4.2. Demand can be decomposed into a base component and unpredictable surges. 114
4.3. The full supply chain network of the test case. . . . . . . . . . . . . . 125
Chapter 1
Introduction

Coordinating activities across multiple players in a distribution channel or supply chain has been a subject of numerous studies in the marketing and supply chain literature (Choi, Song, Zhao, and Xu (2016)).

When two or more independent companies are involved in a distribution channel, a solution that integrates their activities for the channel-level optimality requires that (a) the modeler has a complete knowledge of the channel partners costs and capacities, and (b) all involved channel members implement respective actions as prescribed by the solution. The first requirement becomes increasingly realistic as the companies integrate their information technology on various operational levels (e.g., Wal-Mart and its suppliers; Procter and Gamble and its retailers). However, the second requirement would be difficult to enforce unless (i) there is a binding agreement among the channel members, (ii) the actions are policed with a threat of retaliation, or (iii) the prescribed solution is incentive compatible.

Depending on the assumptions on these three conditions, the channel coordination literature can be divided largely in two classes. The first group, which is relatively simpler in methodology, seeks to find system-level optimal decisions in pricing, advertising, and promotion. The coordination is enforced by either condition (i) or (ii). That is, the channel members would agree (or be forced) to maximize the systems joint profit through a partnership or a binding contract. The main agenda of this research stream is to find a sharing mechanism that makes all channel members not worse-off
and at least one better-off with coordination than without. This line of research is more prevalent in the recent supply chain literature. While the concept of joint-profit maximization is neither new nor challenging as a coordination mechanism, this approach can accommodate multiple decision variables such as price and advertising.

The objective of the second group of research is for one member of the channel (usually the supplier) to find a mechanism that induces the other member (usually the buyer), who has its own profit motivation, to make a decision (pricing, advertising, promotion, service, or order quantity) that also maximizes the system profit (i.e., condition (iii)). These incentive-compatible mechanisms do not need an assumption on compliance or a binding contract because the objectives of all channel members are aligned with the system-optimum. These models are normally set as Stackelberg games with an upstream channel member (a supplier or a manufacturer) as the leader and the downstream member (a buyer or a retailer) as the follower. By definition of the game, the leader is likely to gain more than a Nash outcome. However, depending on the underlying demand function, there are instances in which the follower can benefit more than the leader does (Choi (1991)). Regardless, a conventional Stackelberg game is not likely to result in a 100% coordination efficiency (an integrated channels output). On the contrary, the coordination efficiency may even be negative, which means the retailer profit can be decreased more than the manufacturer gains from the Stackelberg leadership. However, a more sophisticated mechanism can be designed in the form of quantity discount, two-part tariff, or advertising subsidy such that the retailers profit incentive is aligned with the whole channels profit (e.g., McGuire and Staelin (1986)). It is a more attractive mechanism of channel coordination that benefits the whole system, and it does not require the retailers cooperation.

In this dissertation, we research various models of channel coordination and optimization which can correspond with above conditions to achieve channel coordination.
These studies are organized in the following chapters based on how to lead uncoordinated decisions to channel efficiency and what tools are used to bring that incentive.

In Chapter 2, we study supply chain contracts to coordinate online sellers inventory decision and 3rd party shippers capacity decision to meet the unpredictable demand in the peak season. Online sellers are quickly changing the landscape of the retail industry in many countries. In the US, Amazon’s market cap has recently doubled that of Wal-Mart. Different from brick and mortar, online sellers rely on 3rd party logistics (3PLs) for the delivery of the goods to the consumers but the hugely spiked demand during holiday seasons (Cyber Monday, Black Friday, Christmas in the US; Singles day in China) poses a substantial challenge for the 3PLs to make deliveries on time. To handle this challenge, 3PLs such as UPS or FedEx initiated different shipping contracts with the sellers to better manage the demand. UPS proposed that the sellers make forecast (i.e., reservation) and pay surcharges as penalties for deviations; FedEx, on the other hand, only raised the shipping (flat) rates. In this chapter, we study these two shipping contracts (penalty and flat rate) and discuss how these shipping arrangements may affect the online sellers’ inventory decisions, the 3PL’s capacity decisions, and the performance of the supply chain (of the seller and shipper) as a whole. We also discuss coordination strategies for both parties to win-win. In this chapter, the coordination is enforced by the condition (i).

In Chapter 3, we examine incentives and gaming behaviors in collaborative projects under the popular risk sharing partnership. Collaboration prevails in projects in diverse industries, and the risk sharing partnership, where each partner pays for its own cost and shares the outcome (reward / loss) upon project completion, is one of the most popular ways to manage collaboration in practice. However, collaboration under the risk sharing may lead to project failures in the forms of excessive delays and cost overruns, as shown by recent real-life examples. In this chapter, we characterize the strategic behaviors of
firms under the risk sharing partnership in a project management context. Relative to one-firm-does-all, we study how the risk sharing partnership changes firms’ incentives in project execution and affect project metrics (cost and duration) for various network topologies (serial vs. parallel), cost structures (time independent vs. dependent), and information status (symmetry vs. asymmetry). In this chapter, the coordination is implemented by the condition (iii).

In Chapter 4, we work on the supply chain operations to bring emergency rescue kits to the victims after disasters, so that the total tardiness and the peak tardiness over the planning horizon are minimized. After a disaster happens, emergency response operations are critical to save humans’ lives and properties. The limited resources and time requirements call for coordinated supply chain operations. This chapter studies a supply chain network for rescue kits in disaster reliefs, motivated by a real-world application. The objective is to minimize the total tardiness and peak tardiness of product delivery over the multiple-period planning horizon. One major challenge is the lack of reliable prediction of customer demand in disasters. In order to cope with demand uncertainty while maintaining the tractability of the optimization model, we decompose the demand into two components: a relatively stable base demand predicted by historical data, and unpredictable demand surges. For the base demand, an optimization model is developed to optimize the production and distribution operations, as well as the inventory replenishment policy for manufacturers and distribution centers, so as to minimize the total tardiness. For the demand surges, we propose to deploy supply chain flexibility to cope with the uncertainty. In this chapter, we develop an optimization model for coordinated supply chain operations.
Chapter 2

Shipping Peak Demand for Online Sellers: Penalty vs. Flat Rate

2.1 Introduction

The retail industries in many countries are under a drastic transition from brick and mortar to e-commerce. Forrester Research Online Retail Forecast ("How UPS Plans to Deliver Your Christmas Packages On Time", Fortune, 12/18/2015) shows that from 2008 to 2015, the US online sales has nearly doubled from $152 billion to $279 billion. According to US Census Department (U.S. Census Bureau News, U.S. Department of Commerce, 2/17/2017), e-commerce sales as a percentage of total applicable retail sales have grown from 5.4% in 2008 to 10.9% in 2015. Because online sellers rely on shippers for delivery to complete the sales transactions, the booming online sales boosts the small package shipping volume, which continues to rise, mirroring the growth of e-commerce (see Figure 2.1).

Online shopping holidays present a significant challenge for the shipping industry due to the (hard to predict) hugely spiked demand and high customer expectations for on-time delivery. For instance, Cyber Monday was America’s biggest e-commerce sales day with online-only deals. In 2015, the one-day sales in the US amounts to $3.07 billion, which is more than three times the average daily online sales. In 2014, Peak Day [Black Friday, Cyber Monday, Christmas] scheduled deliveries exceeded 35 million packages, more than 100% above an average day. These huge spikes of sales for
online shopping holidays in US and China (e.g., the singles’ day of 11/11, see Figure 2.2) bring huge problems for shippers, such as UPS, FedEx and Cainiao, to deliver the goods on time as promised by the sellers within tight time windows. Quoted by The Wall Street Journal (“UPS, FedEx Want Retailers to Get Real on Holiday Shipping”, 10/2/2014): “The sheer volume of packages overwhelmed the system over past years, as more consumers opted to shop from home amid nasty weather, and retailers egged them on with 11th-hour delivery guarantees.”
The unexpectedly high shipping volume inevitably led to delays. In 2013, “an estimated 2 million express packages due to be delivered Christmas Eve were left stranded on trailers and delivery trucks across the nation.” (UPS). FedEx Corp experienced similar problems on a smaller scale. To please upset customers, online sellers and shippers issued gift cards and refunds related to missing deliveries in the amount of millions of dollars (From “UPS, FedEx Want Retailers to Get Real on Holiday Shipping.” 10/3/2014, The Wall Street Journal).

In order to meet the demand and ensure successful peak season operations, shippers invested significantly to expand their labor forces and equipment. For instance, UPS hired 100,000 temporary employees and invested over $1 billion in facility expansions and equipment modernization to prepare for the upcoming peak seasons (UPS 10-K, 12/31/2015). However, the unpredictable holiday shipping volume may fail to meet the shippers’ expectations and brought down their earnings. Quoted by the article (“UPS hikes fuel surcharges despite dramatic declines in oil, fuel prices”, DC Velocity, 2/2/2015), “UPS last week warned that fourth-quarter earnings would be lower than expected due to higher-than-expected costs during the fourth-quarter peak season [of 2014] to handle an expected volume of holiday merchandise that never materialized.”

To manage such a unpredictable spike in demand, shippers started to engage online sellers with holiday pricing initiatives in 2015. Some of them (FedEx) raised the shipping rates (see “UPS: Prepare for holiday shipping surcharges”, CBS MoneyWatch, CBS Interactive Inc, 1/4/2015, namely, the flat rate strategy), UPS, in addition to raising the peak holiday shipping rates, rolled out new charges. “UPS recently announced that this holiday season, the company will be dish out a surcharge to retailers that cannot accurately forecast how much merchandise they plan on moving.”, “UPS will be penalizing retailers that cannot accurately forecast their demand with surcharges.” (“UPS to surcharge retailers for unexpected peak order volume”, Shipping Management,
SalesWarp, 8/12/2015, namely, the penalty strategy). These holiday pricing initiatives will have a significant impact on the online sellers, the shippers and the supply chain as a whole as “These surcharges could affect a large portion of online retailers, with Internet Retailer reporting that nearly 200 of the top 500 eCommerce sites utilize UPS as their primary carrier.” (Shipping Management, SalesWarp, 8/12/2015).

What happened in the US is nothing as compared to that in China. As reported by Fortune on 11/11/2015 (“Alibaba Rings Up a Record $14.3 Billion in Sales for Singles Day”, Fortune, 11/11/2015), “Alibaba’s next big challenge following a record-breaking Singles Day will be fulfilling the orders. Already China’s post office estimates that Singles Day will lead to nearly 800 million packages being shipped. Alibaba’s logistics company Cainiao will likely have to coordinate ten times their average daily volume of packages.” Comparatively “Sales in China in this one 24-hour period are expected to be larger than the Black Friday weekend and Cyber Monday combined, ... It occurs mostly online.” (“Alibaba leads world’s biggest online shopping spree”, USATODAY, 11/11/2015). Shipping is one of the biggest challenges because “Delays are the biggest problem, accounting for 44 percent of consumer complaints.” (“Record Alibaba Singles Day sales reveal China delivery challenges”, JOC, 11/12/14).

Clearly, one fundamental problem in resolving the shipping challenges of online shopping holidays lies in the coordination between the online seller and the shipper to meet the unpredictable external demand. The recent practice inspires the following research questions:

• What is the impact of these holiday pricing initiatives on the sellers’ inventory and forecast (reservation) decisions?

• How to set the penalties and flat rate from the shipper’s or the channel’s perspective?
• Which initiative is more effective, penalty vs. flat rate? For whom?

Because inventory and shipping are controlled by different companies (the seller and shipper, respectively), the key question is how to get them to work together to win-win?

The supply chain contracts and coordination literature focuses on the relationships between suppliers (or manufacturers) and retailers but not those between retailers and shippers, which have distinct features (see a detailed discussion in §2.2). The shipping management literature for online sellers considers optimal shipping strategies for either the seller or the shipper but not the coordination issues of both. In this chapter, we consider a novel supply chain model consisting of an online seller and a shipper where the seller makes forecast and inventory decisions and the shipper makes shipping capacity decisions in a Newsvendor setting. We first consider the seller’s problem under the holiday pricing initiatives (penalty vs. flat rate) to characterize their impact on the sellers’ inventory and forecast (reservation) decisions. We then study the shipper’s and channel’s problems by determining the optimal and channel coordinating decisions in these initiatives for the shipper and the channel respectively. Finally, we compare the effectiveness between the penalty and flat rate contracts for both the shipper and the channel. Our analysis leads to the following main insights:

• The holiday pricing initiatives may have a significant impact on the seller’s inventory and forecast (reservation) decisions.

• Surprisingly, the penalty contract can coordinate channel only if the shipper makes zero profit while flat rate can coordinate the channel and yield positive profit for both the seller and the shipper.

• For the shipper who optimizes the contract parameters for its own profit, the penalty contract is at best marginally better than the flat rate contract despite the seemingly flexibility and advantage of the former.
This chapter is organized as follows: In §2.2, we review the related literature and elaborate on our contributions. In §2.3, we present the model and key assumptions, and study the seller’s problem under the holiday pricing initiatives. In §2.4, we consider the shipper’s problem, and in §2.5 - §2.6, we analyze the channel’s problem and show how the penalty and flat rate contracts may coordinate the channel. In §2.7, we conduct a numerical study to quantify the performance difference between centralized and decentralized controls for the channel, and the performance difference between penalty and flat rate contracts for the shipper. §2.8 concludes the chapter.

2.2 Literature Review

This chapter is related to three streams of literature: Supply chain contracts and coordination, optimal shipping-fee schedules for online sellers, and gaming among online sellers, suppliers and/or shippers. We shall review related work in each stream and point out our contributions.

Supply chain contracts and coordination. This literature is voluminous, with excellent reviews provided by Lariviere (1999), Cachon (2003), and Cachon and Netessine (2006). Most of this literature studies supply chain contracts and coordination issues between a supplier (or manufacturer or wholesaler) and a retailer. Various types of contracts are considered, for instance, the wholesale price only contract (resembling the flat rate shipping contract) is studied by Lariviere and Porteus (2001), which demonstrates the double marginalization effect. It is well known that this contract cannot achieve channel coordination (it can achieve only in special cases, e.g., when the supplier makes zero profit).

Various types of risk sharing contracts (resembling the penalty shipping contract) are studied where the supplier and retailer share the risk caused by the uncertain demand, for example, buy-back contracts by Pasternack (1985), revenue sharing contracts by
Cachon and Lariviere (2005), and two wholesale price contracts by Cachon (2004) and Dong and Zhu (2007). It is shown that the risk sharing contracts can coordinate the channel and provide a positive profit for each player. Other types of supply chain contracts are studied in the literature, see Tsay (1999) for quantity flexibility contracts, and Taylor (2002) for sales rebate contracts.

Tomlin (2003) investigates price-only contracts in supply chain capacity procurement games. For a two-party supply chain, comprising a manufacturer and a supplier that both invest in capacity, they prove the existence of a class of coordinating price-only contracts that arbitrarily allocate the supply chain profit. Martínez-de Albéniz and Simchi-Levi (2005) studies option contracts in a general framework whose portfolios of contracts can be analyzed and optimized. Özer and Wei (2006) studies the problem of how to assure credible forecast information sharing between a supplier and a manufacturer.

The literature has yet considered the coordination issue in a supply chain of an online seller and a shipper facing unpredictable demand. In fact, shipping is often ignored in this literature, which, however, can interact strongly with the supply chain’s inventory decisions. The retailer-shipper supply chain model of this chapter explicitly considers this interaction through a unique feature where the retailer (or seller) requires the availability of both inventory and shipping capacity to complete the sales transaction; while in a traditional supply chain of a supplier and a retailer, shipping is not a concern and the retailer requires the availability of only inventory (or supplier capacity). Thus, in the retailer-shipper supply chain under the penalty contracts, the seller faces two inter-dependent decisions: how much inventory to stock up, and how much shipping capacity to reserve. Unlike assembly systems where the supplies of components need to be synchronized, the inventory decision of the seller may be made intentionally different from the shipping capacity (reservation) decision. Since the seller’s problem lies at the
heart of all subsequent problems (e.g., the shipper’s problem, the channel’s problem),
this new feature requires new analysis and solutions, and leads to new and surprising
insights.

**Optimal shipping-fee schedules for online sellers.** This literature studies the
shipping charge strategies for online sellers. For instance, Jiang, Shang, and Liu (2013)
considers the selling prices as a base and the shipping fees as an add-on (that is, the par-
titioned strategy), and investigate how customers process the such prices, how the par-
titioned prices impact purchase intention, and whether the online seller should choose
the partitioned strategy or the combined strategy, in which, the customers are offered
free shipping by charging a total price including the product price and shipping fee.
Jiang, Shang, and Liu (2013) provides two nonlinear mixed-integer programming mod-
els for the online sellers to optimize shipping-fee charges for single and multiple product
transactions.

Burman and Biswas (2007) identifies boundary conditions for the effectiveness of
partitioned pricing by examining the role of the reasonableness of a penalty and the
need for cognition in consumers’ processing of pricing information. It shows that for
customers with high need for cognition, partitioned strategy has a significantly favorable
effect on customers’ purchase when penalty is reasonable and the effects reverse when
penalty is unreasonable. Zhou, Katehakis, and Zhao (2009) considers a free shipping
option where the shipping charge reduces to zero when the shipping quantity exceeds a
certain limit, and characterizes the optimal and/or effective heuristic inventory policies
for a retailer facing the free shipping option from a supplier. For other related work in
literature, we refer to Hamilton and Srivastava (2008); Lewis (2006).

Our work differs from this stream of literature because they focus on the optimiza-
tion issues of the seller, e.g., setting the optimal order quantity and pricing decisions,
or the customer, e.g., inventory management strategies, with exogenous shippers, but
we study contracts and coordination issues between the online seller and the shipper.

**Gaming among online sellers, suppliers and/or shippers.** There is a limited but growing body of literature studying gaming issues among online sellers, their suppliers and shippers. Mutlu and Çetinkaya (2011) studies channel coordination issues for a seller-carrier supply chain in a deterministic quantity discount model where the external demand is a decreasing function of the retail price. The carrier sets the freight rate whereas the seller sets the retail price for the product and buys the supply at regular intervals in fixed quantities. While this work considers seller-shipper coordination issues under the assumptions of the classical economic order quantity (EOQ) model, we consider the framework of newsvendor model because the main challenge for online shopping holidays that we hope to address in this chapter is shipping to meet the random external demand.

Yao, Kurata, and Mukhopadhyay (2008) analyzes the interaction between revenue sharing and quality of order fulfillment that occurs in an Internet drop-shipping distribution system comprising an online tailer and a supplier. The online seller manages sales activities and sends customer orders to the supplier that fulfills the order. Using a Stackelberg game, they explore how the seller (leader) can give the supplier (follower) appropriate incentive to improve the level of delivery reliability. Gan, Sethi, and Zhou (2010) studies a drop-shipping supply chain where the supplier keeps inventory and bears inventory risks, and the seller focuses on marketing and customer acquisition. They propose a menu of commitment-penalty contracts to reduce demand and supply uncertainties. In these drop-shipping supply chains, the seller is not asset-based and make no inventory decisions. In contrast, we consider asset-based sellers that face both inventory and shipping capacity (reservation) decisions.


2.3 The Model and Seller’s Problem

We consider a single seller and a single shipper in a newsvendor model where the external customer demand is random with known distribution, $D$ follows $f(x)$ as p.d.f and $F(x)$ as c.d.f., which are continuous and differentiable. The seller is a newsvendor with a selling price of $p$, a cost for inventory procurement of $c$ and a salvage value of $s$ after the sales event (the shopping holiday).

We consider both the penalty and flat rate contracts between the seller and the shipper. In the penalty contract, the shipper sets a base shipping rate of $\nu$ and ask the seller to provide a forecast, i.e., the reservation. After demand, $D$, is realized, the shipper charges the seller $p_o$ per unit for over-forecast, and $p_u$ per unit for under-forecast. In the flat rate contract, the shipper only charges a base shipping rate of $\nu$.

The sequence of events works as follows: the shipper first announces the contract terms (e.g., $\nu, p_o, p_u$), the seller then makes inventory and forecast (reservation) decisions, $Q$ and $R$. Upon observing the seller’s decisions, the shipper builds shipping capacity, $C$. Then the demand is realized, the shipper builds emergency capacity if needed to guarantee the shipments. The game played by the seller and the shipper is setup as follows: It is a Stackelberg game where the shipper takes the lead, online seller follows. We assume information symmetry. Both players are rational, risk neutral and profit maximizing.

To construct the model, we define the following notation:

- $D$: external demand. $D \sim F(D)$ as c.d.f., $f(D)$ as p.d.f.. it is continuous and $F(D)$ is differentiable.

- $Q$: the seller’s inventory decision (and order quantity, we assume the seller starts with zero inventory).
• $R$: the seller’s forecast (reservation) to the shipper.

• $p_o$: the penalty that the shipper charges the seller for over-forecast.

• $p_u$: the surcharge that the shipper charges seller for under-forecast.

• $\nu$: the base rate of shipping charged by the shipper on the seller.

• $c_o$: the overage cost borne by the seller.

• $c_u$: the underage cost borne by the seller.

Please note $c_o = c - s$ and $c_u = p - c$ (where $p > c > s$ and $p$ is product price, $c$ is product cost, $s$ is salvage value).

We first consider a non-asset based seller as a benchmark. Here, the seller carries no inventory and drop-ships from an exogenous supplier with abundant supply. Suppose $p - c - \nu \geq p_u$ (a regularity assumption to ensure that the seller is profitable when it under-forecasts). Assuming the penalty contract and the seller can always ship $D$, the seller’s expected profit can be written as follows:

$$
\pi_{Sellr}(R) = E(D)(p - c) - E(D)\nu - E[p_o \max(R - D, 0) + p_u \max(D - R, 0)]
$$

$$
= E(D)(p - c - \nu) - E[p_o \max(R - D, 0) + p_u \max(D - R, 0)]
$$

(2.1)

The second term, $p_o \max(R - D, 0) + p_u \max(D - R, 0)$, represents the mis-match cost of shipping. By the newsvendor model,

$$
R^* = R^0 = F^{-1}\left(\frac{p_u}{p_u + p_o}\right)
$$

For non-asset based sellers, the penalty contract puts on a limit on their forecast (reservation of the shipping capacity).
If the seller is asset-based, the seller carries inventory and plans inventory and shipping capacity together. The analysis is more complex due to the interaction between the shipping and inventory decisions. Assuming that the seller can always ship \( S = \min(Q, D) \), the seller’s expected profit is,

\[
\pi_{Seller}(R, Q) = E(D)(p - c - \nu) - G_I(Q) - G_S(R, Q),
\]

(2.2)

where \( G_I(Q) \) \((G_S(R, Q))\) represents the mis-match cost of inventory (shipping, respectively), and \( G_I(Q) = E[c_o \max(Q - D, 0) + (c_u - \nu) \max(D - Q, 0)] \) and \( G_S(R, Q) = E[p_o \max\{R - S, 0\} + p_u \max\{S - R, 0\}] \). For the ease of exposition, we define \( G(R, Q) = G_I(Q) + G_S(R, Q) \) to be the total mis-match cost.

\[
G(R, Q) = E[c_o \max(Q - D, 0) + (c_u - \nu) \max(D - Q, 0)]
+ E[p_o \max\{R - S, 0\} + p_u \max\{S - R, 0\}]
\]

(2.3)

We make the following regularity assumptions for the seller to avoid trivial cases.

**Assumption 1** Regularity assumptions: (1) \( c_u \geq \nu + p_u \), (2) \( \nu \geq p_o \).

We need the first assumption because \( c_u = p - c \) is the profit margin, it should be greater than \( \nu + p_u \) for the seller to ship more than its forecast (or reservation) should \( D > R \). Second, \( p_o \) should be at most \( \nu \) because otherwise, the seller just needs to pay the shipper \( \nu \) rather than \( p_o \) in case she reserves more capacity than needed.

Clearly, the profit function in Eq. (2.2) is convex in \( R \), but not necessarily in \( Q \) because of the minimization operations. Despite the complexity, we can characterize the seller’s optimal decisions in closed-form. For this purpose, we first establish the following proposition and lemma. All proofs are included in the appendix.

**Proposition 1** \( R^* \leq Q^* \) and \( G(R, Q) \) is convex in \( R \) (given \( Q \)).
Intuitively, the reseller should not reserve more shipping capacity than its inventory because it would never be able to sell more than its inventory.

Although $G(R, Q)$ is not jointly convex in $R$ and $Q$ across the entire feasible region, we prove the joint convexity in certain regions and some monotonicity results.

**Lemma 1** Convexity properties of $G(R, Q)$:

- $G(R, Q)$ is jointly convex in $Q$ and $R$ for $R \leq Q$,
- $G(R, Q)$ is jointly convex in $Q$ and $R$ for $R > Q$,
- $G(R, Q)$ is monotonically increasing in $R$ for $R > Q$.

Now we are ready to establish the main theorem that characterizes the seller’s best response in closed-form under the penalty contract.

**Theorem 1** Under the penalty contract, the seller’s optimal decisions $(R^*, Q^*)$ is given by,

- **Case a:** If $p_u < \frac{p_o}{p_u + c_0} (c_u - \nu)$,
  
  $$Q^* = Q_{PL}^1 = F^{-1}(\frac{(c_u - \nu - p_u)}{(c_u - \nu - p_u) + c_0}),$$
  
  $$R^* = R^0 = F^{-1}(\frac{p_u}{p_u + p_o}).$$

- **Case b:** If $p_u \geq \frac{p_o}{p_u + c_0} (c_u - \nu)$,
  
  $$Q^* = Q_{PL}^2 = F^{-1}(\frac{(c_u - \nu)}{(c_u - \nu) + p_o + c_0}),$$
  
  $$R^* = Q^*.$$

Clearly, when $p_u = \frac{p_o}{p_u + c_0} (c_u - \nu)$, we must have $Q_{PL}^1 = Q_{PL}^2 = R^0$. Theorem 1 shows that the seller’s optimal decisions can be expressed in closed-form, although not as simple
as a newsvendor solution, but as two newsvendor solutions under a switch condition. Note that the optimal forecast (reservation), $R$, can reduce to $R^0$ for a drop-ship seller in a special case. Figure 2.3 illustrates the seller’s optimal decisions as a function of $p_u$.

Figure 2.3: The seller’s optimal inventory and reservation decisions as a function of $p_u$.

Theorem 1 and Figure 2.3 provide the following insights:

- Shipping charges can affect the seller’s optimal inventory decision – shipping is connected with ordering, the shipper can use shipping charges to influence the seller’s order quantity. Although the shipping charges always have an impact on the inventory decision, the inventory factors may not always have an impact on the shipping (reservation) decision. The switch condition on $p_u, p_o$ is important because it specifies when the seller can or cannot make shipping (reservation) decisions independently of the inventory factors.
• When \( p_u \) is small relative to \( p_o \) \( (p_u \leq \frac{p_o}{p_o+c_o}(c_u-\nu)) \), the seller is mainly concerned about the penalty of reserving too much, so \( R^* < Q^* \). Intuitively, the seller can make the shipping (reservation) and inventory decisions separately (or independently of each other), where \( R^* \) reduces to the non-asset based seller’s best \( R \), independent of the inventory factors. Because \( R^* < Q^* \) and \( p_o \) only applies to \( R^* - D \), \( Q^* \) is not affected by \( p_o \). But \( p_u \) still affects \( Q^* \) and \( Q^* \) decreases in \( p_u \).

• When \( p_u \) is big relative to \( p_o \) \( (p_u > \frac{p_o}{p_o+c_o}(c_u-\nu)) \), the seller fears the penalty of not reserving enough shipping capacity, so sets \( R^* = Q^* \). \( Q^* \) is independent of \( p_u \) because \( p_u \) is paid only when \( R^* < Q^* \). Because \( D < Q \) is likely, \( p_o \) still affects \( Q^* \) which decreases in \( p_o \).

The following special cases deserve some attentions. We first consider one-sided penalty. If \( p_o > 0 \) but \( p_u = 0 \), then the seller loses nothing if she reserves zero shipping capacity, but suffers a loss if she reserves too much, thus \( R^* = 0 \) consistent to the result of Theorem 1. If \( p_o = 0 \) but \( p_u > 0 \), then the seller loses nothing if she reserves too much, but suffers a loss if she does not reserve enough, clearly, \( R^* = Q^* \).

Under the flat rate contract, we have \( p_o = p_u = 0 \). Theorem 1 implies that

\[
Q^* = Q_S = F^{-1}\left(\frac{(c_u-\nu)}{(c_u-\nu)+c_o}\right),
\]

and \( R^* \) is irrelevant.

The following proposition characterizes the relationships among various inventory decisions. Let \( Q_{NS} = F^{-1}(\frac{c_o}{c_u+c_o}) \) be the classical newsvendor solution without the shipping consideration, and \( Q_{PL}^1 \) and \( Q_{PL}^2 \) are defined in Theorem 1.

**Proposition 2**

\[
max\{Q_{PL}^1, Q_{PL}^2\} < Q_S < Q_{NS}.
\]
Proposition 2 shows that the seller stocks less under flat rate relative to the classical newsvendor solution without shipping consideration, and even less under penalty. By Theorem 1, the inventory decision decreases in the penalties, \( p_o \) and \( p_u \).

### 2.4 The Shipper’s Problem

We make the following assumptions for the shipper to avoid trivial cases.

**Assumption 2**

1. \( u \): unit cost of building regular shipping capacity before the sales event
2. \( u' \): unit cost of building emergency shipping capacity (\( u' > u \), incur during the sales event)

First, let’s consider the shipper’s problem under the penalty contract. The decision variables for shipper are \( p_o, p_u, \nu \), and the shipping capacity, \( C \). Let \( S^* = \min(Q^*, D) \) be the actual shipment. Then, the shipper’s problem becomes

\[
\max_{C, \nu, p_o, p_u} \pi_{\text{Shipper}} = \max_{C, \nu, p_o, p_u} \nu \cdot E[S^*] + E[p_o \max\{R^* - S^*, 0\}] + p_u \max\{S^* - R^*, 0\} - \text{Cost},
\]

where \( \text{Cost} = u \cdot C + u' E[\max\{S^* - C, 0\}] \), \( Q^* \), and \( R^* \) are the seller’s best response to \( \nu, p_u, p_o \) from Theorem 1. The shipper is subject to the following constraints:

\[
u' > u,\]

\[
p_u \leq c_u - \nu,\]

\[
p_o \leq \nu.\]

The last two constraints come from the regularity conditions to guarantee non-negative profit for seller, see Assumption 1.
Let \( C^0 = F^{-1}(\frac{u'}{u}) \), we can characterize the optimal shipping capacity decisions as follows,

**Proposition 3** If \( C^0 \leq Q^* \), Then \( C^*(Q^*) = C^0 \); otherwise, \( C^* = Q^* \).

This is important because it can show that the inventory decision, \( Q^* \), can affect the shipping decision, \( C^* \).

Next we consider the shipper’s problem under the flat rate contract. The decision variables for the shipper under the flat rate contract are \( \nu \) and capacity, \( C \). Let \( S^* = \min(Q^*, D) \) be the actual shipment. the shipper’s problem becomes

\[
\max_{C, \nu} \pi_{\text{Shipper}} = \nu \cdot E[S^*] - \text{Cost}, \tag{2.5}
\]

where \( \text{Cost} = u \cdot C + u' E[\max\{S^* - C, 0\}] \), and \( Q^* \) is the seller’s best response to the flat rate contract, that is,

\[ Q^* = F^{-1}(\frac{c_u - \nu}{(c_u - \nu) + c_o}). \]

The shipper is subject to the regularity conditions to guarantee non-negative profit for seller, as specified in Assumption 1:

\[ u' \leq \nu, \]

\[ \nu \leq c_u. \]

Clearly, the shipper’s problem under the flat rate contract is a special case of its problem under the penalty contract (i.e., \( p_o = 0 \) and \( p_u = 0 \)). It is difficult to characterize the shipper’s optimal decisions, \( p_o \), \( p_u \) and \( \nu \), in closed-form, however, we can establish the following partial equivalence theorem for the shipper’s problem between the penalty and flat rate contracts.
Theorem 2 At $p_o = 0$ or $p_u = 0$, the shipper’s problem under the penalty contract is equivalent to the shipper’s problem under the flat rate contract.

This theorem decreases the value of the penalty contract to the shipper. The implication is that the shipper has to set $p_o > 0$ and $p_u > 0$ in order to improve its profit over the flat rate contract.

2.5 The Channel’s Problem (The First-Best Solution)

In this section, we consider the integrated channel’s problem such as in vertical integration (e.g., Amazon or Wal-Mart.com) of both seller and shipper. Now the decision variables are only $C$ and $Q$ and channel’s problem is

$$\max_{C, Q} \pi_c = \max_{C, Q} E(D)(p-c) - E[c_o \max(Q-D,0) + c_u \max(D-Q,0)] - \text{Cost}, \text{ (2.6)}$$

where $\text{Cost} = u \cdot C + u'E[\max\{\min(D,Q) - C, 0\}]$. The constraints are $u' > u$ and $c_u - u' > 0$ (to guarantee non-negative profit for the channel).

Let’s define

$$H(C, Q) \equiv E[c_o \max(Q-D,0) + c_u \max(D-Q,0)] + \text{Cost}. \text{ (2.7)}$$

Then the channel’s problem is equivalent to minimizing $H(C, Q)$ subject to the same constraints. Due to the minimization operation, $H(C, Q)$ may not be convex in $Q$. We shall first establish the following properties for $H(C, Q)$.

Lemma 2 (Convexity of $H(C, Q)$ in $C$ and $Q$)

- $H(C, Q)$ is joint convex in $C$ and $Q$ for $C \leq Q$,

- $H(C, Q)$ is joint convex in $C$ and $Q$ for $C > Q$,.
• \( H(C, Q) \) is an monotonically increasing function in \( C \) for \( C > Q \).

We are ready to characterize the channel’s optimal decisions in closed-form.

**Theorem 3** (\( C^* \) and \( Q^* \) in Channel’s Problem)

Case a: If \( c_o < \frac{c_u - u'}{u'' - u} \cdot u \),

\[ Q^* = Q^2_{FB} = F^{-1}\left(\frac{c_u - u'}{c_o + (c_u - u')}\right), \]

\[ C^* = C^0 = F^{-1}\left(\frac{u'' - u}{u'}\right). \]

Case b: If \( c_o \geq \frac{c_u - u'}{u'' - u} \cdot u \),

\[ Q^* = Q^1_{FB} = F^{-1}\left(\frac{c_u - u}{c_o + c_u}\right), \]

\[ C^* = Q^*. \]

The channel’s problem is similar to the seller’s problem structurally. We can find closed-form optimal decisions which are in the form of two newsvendor solutions under a certain switch condition. If \( u' \) is close to \( u \), then the channel’s problem is likely to be the Case a. If \( u' \) is much bigger than \( u \), then the channel’s problem is likely to be the Case b. Note that the optimal \( C \) for the channel can be \( C^0 \) for the shipper/ channel without inventory considerations. Figure 2.4 illustrates the channel’s optimal decisions as a function of \( c_o \).

Theorem 3 and Figure 2.4 provide the following insights:

• If \( u' \) is close to \( u \), then \( C^* < Q^* \), intuitively, we can make the shipping reservation and inventory decisions separately (or independently of each other), where \( C^* \) is the best \( C \) for the shipper w/o inventory, independent of the inventory factors.

• If \( u' >> u \), then the channel should build \( Q^* \) shipping capacity, i.e., \( C^* = Q^* \).
The condition on $u'$ is important because it tells us when the channel can make capacity (for shipping) decisions independently of the inventory factors. Note that shipping factors always affect inventory decisions.

Figure 2.4: The channel’s optimal inventory and capacity decisions as a function of $c_o$.

$Q^2_{FB} = F^{-1}\left(\frac{(c_o - u')}{c_o + (c_u - u')}\right)$

$Q^1_{FB} = F^{-1}\left(\frac{(c_o - u)}{(c_o + u) + (c_u - u)}\right)$

$C^0 = F^{-1}\left(\frac{u' - u}{u'}\right)$

2.6 The Channel Coordination

The theorem below shows that penalty can coordinate the channel.

**Theorem 4** If and only if $p_u = u' - u$, $p_o = u$, $v = u$, penalty coordinates the channel, that is, the shipper (or seller) chooses the first best $C$ (or $Q$) to maximize its profit.

**Theorem 5** (Shipper’s expected profit under channel coordination in penalty model)

Under the channel coordinating penalty contract, the shippers profit is always zero.
Despite the conceptual similarity between penalty and risk sharing contracts, Theorems 4-5 correspond to the those of a single wholesale price contract in the supply chain contract and coordination literature.

Now we analyze channel coordination for the flat rate contract.

**Theorem 6** *(Coordination conditions in Flat Rate Model)*

- when \( c_o < \frac{c_u - u'}{u' - u} \cdot u \), flat-rate coordinates the channel if and only if \( \nu = u' \)
- when \( c_o \geq \frac{c_u - u'}{u' - u} \cdot u \), flat-rate coordinates the channel if and only if \( \nu = \frac{c_o + c_u}{c_o + u} \cdot u \)

**Theorem 7** *(Shipper and Seller’s expected profit under channel coordination in flat rate model)*

- **Shipper’s expected profit in the flat-rate model under the channel coordination conditions** (i.e., \( c_o < \frac{c_u - u'}{u' - u} \cdot u \), \( \nu = u' \) and when \( c_o \geq \frac{c_u - u'}{u' - u} \cdot u \), \( \nu = \frac{c_o + c_u}{c_o + u} \cdot u \) is greater than or equal to zero.

- **Seller’s expected profit in the flat-rate model under the channel coordination conditions** (i.e., \( c_o < \frac{c_u - u'}{u' - u} \cdot u \), \( \nu = u' \) and when \( c_o \geq \frac{c_u - u'}{u' - u} \cdot u \), \( \nu = \frac{c_o + c_u}{c_o + u} \cdot u \) is greater than or equal to zero.

Despite the conceptual similarity between flat-rate and a single wholesale price contracts, Theorems 6-7 correspond to the those of risk sharing contracts in the supply chain contract and coordination literature.

When we compare our model with classic supply chain contracts literature, we found interesting implications. For the seller-shipper supply chain, the channel coordination under the flat rate contract can bring profits to both parties, but the channel coordination under the penalty contract can be achieved only when the shipper makes zero profit. This is in sharp contrast to the classical results in the supplier-retailer supply
chain that the wholesale price only contract (conceptually similar to flat rate) can co-or-
ordinate the supply chain only when the manufacturer makes zero profit, and the risk
sharing contracts (conceptually similar to penalty) can coordinate the supply chain and
yield positive profits for everyone.

This is quite surprising as the flat rate contract resembles the single wholesale
price contract and the penalty contract resembles the risk sharing contracts. This
is true because in flat rate, all risk is taken by the shipper, as in the single wholesale
price contract where all risk is taken by the retailer. In penalty, the risk is shared
between the seller and shipper, as in risk sharing contracts. Our results show that
double marginalization effect (of the single wholesale price contract) and the risk sharing
contract does not work in the same way in the seller-shipper supply chain as in the
supplier-retailer supply chain.

2.7 Numerical Results

From our analysis in previous sections, we can see that, clearly, the shipper makes more
profit in the decentralized setting (both penalty and flat rate) than in the coordinated
ones; seller makes more profit in the coordinated setting than the decentralized ones.
What is unclear is loss of decentralization, shipper’s preference in the decentralized
setting (flat rate vs. penalty), and how the profits of decentralized and coordinated split
among the seller and the shipper. To answer these questions, we perform a numerical
study to quantify the loss and preferences.

In this study, we first choose parameters following multiple demand distributions
such as normal and gamma and also set various means and coefficients of variation
for the study. Then we measure the three things: loss of decentralization (the ratio
of decentralized channel profit / channel optimum), shipper’s preference by comparing
their optimal profit between penalty and flat rate, and the profit split (the ratio of the
shipper (seller)’s profit / channel optimum).

We conducted a sample study and set $D$ to follow a normal distribution, $N(50,25^2)$. Then we normalize $c_u$ as 1.0 and run the loops of other parameters as $c_o=[0.1,0.2,...,1.0]$, $u=[0.1,0.2,...,1.0]$, $u’=[0.1,0.2,...,1.0]$ (subject to the regularity conditions). To find the optimal solutions, we enumerated $\nu$ (For flat rate) and $\nu$, $p_u$, and $p_o$ (for penalty) in an increment of 0.01.

Figure 2.5 shows the average profits over all parameters. We can find that both contracts can achieve the coordination. The shipper has no motivation to coordinate the channel, but the seller has a motivation to build up shipping capacity.

Figure 2.5: Comparison of Average Profits.

For the shipper, penalty is, at most, slightly better than flat rate. Loss of decentralization looks significant. On average, loss is 13% of channel’s optimal profit for both flat rate and penalty under decentralized control, while the worst case’s loss is 42%. Profit split is much different from each policy. For flat rate, from decentralized to coordinated, the shipper’s profit decreases significantly, but the seller’s profit increases significantly. For penalty, the shipper makes zero profit in the coordinated setting.

In the coordinating settings, the shipper sacrifices itself for the channel (and seller).
So the seller should compensate the shipper. The compensation (e.g., side-payment) should depend on demand to be fair, e.g., we cannot give the shipper a big payment if demand is zero. For any realization of $D$, the side payment is a part of the negotiable amount. The side-payment can be easily administrated here because both parties know the demand precisely.

The shipper has no incentive to achieve channel coordination because it loses relative to the decentralized cases. The seller, on the other hand, has a substantial incentive to move from the decentralized cases to channel coordination. That implies that many large sellers should eventually build up their own shipping capacity.

2.8 Conclusion

In this chapter, we consider the coordination issues in a seller-shipper supply chain under the practically proposed flat rate and penalty contracts with the objective of developing win-win strategies to meet unpredictable demand in online shopping holidays. We find that Shipping contracts and terms can have a significant impact on the seller’s forecast and inventory decisions. When the shipper optimizes for itself, penalty does not offer much advantage over flat rate. When the shipper optimizes for itself, both penalty and flat rate can result in a sizable loss of efficiency for the channel, and side payment contracts may work here. Surprisingly, both penalty and flat rate can coordinate the channel but they yield different profits for the seller and shipper. While flat rate can bring profit to both, penalty brings profit only to the seller. Finally, the shipper has no incentive to coordinate the channel, but the seller has a substantial incentive to build up their own shipping capacity whenever possible.

Many questions are left unanswered for the online seller and shipper supply chains. For instance, we consider exogenous demand but the sales effort can make the demand endogenous. For endogenous demand, would penalty be more effective than flat rate?
Should the shipper penalize the seller’s sales effort at all? Would our analysis and results hold if there are multiple sellers? If not, how would it be changed?

2.9 Appendix: Proofs and Technical Details

Proof of Proposition 1

We prove $R^* \leq Q^*$ by showing that if $R > Q$, then reducing $R$ by $\Delta$ (so that $R - \Delta \geq Q$) is always better.

$$
\pi_{Retailer}(R, Q) = E(D)(p - c - \nu) - E[c_o \max(Q - D, 0) + (c_u - \nu) \max(D - Q, 0)]
- E[p_o \max\{R - \min(Q, D), 0\} + p_u \max\{\min(Q, D) - R, 0\}]
$$

$$
\pi_{Retailer}(R - \Delta, Q) = E(D)(p - c - \nu) - E[c_o \max(Q - D, 0) + (c_u - \nu) \max(D - Q, 0)]
- E[p_o \max\{(R - \Delta) - \min(Q, D), 0\} + p_u \max\{\min(Q, D) - (R - \Delta), 0\}]
$$

We want to show $\pi_{Retailer}(R, Q) \leq \pi_{Retailer}(R - \Delta, Q)$.

1) If $D < Q < R - \Delta < R$, we can show it by comparing only different parts for each profit, $\pi(R, Q)$ and $\pi(R - \Delta, Q)$;

$$
\pi_{Retailer}(R, Q) = -p_o (R - D) < \pi(R - \Delta, Q) = -p_o (R - \Delta - D)
$$

2) Otherwise,

$$
\pi_{Retailer}(R, Q) = -p_o (R - Q) < \pi(R - \Delta, Q) = -p_o (R - \Delta - Q)
$$

To prove $G(R, Q)$ is convex in $R$ (given $Q$), let’s define $D' \equiv \min(Q, D)$. Only the last two terms of $G(R, Q)$ have $R$ and it follows traditional newsvendor model. Therefore,
\[ R^* = F'^{-1}(\frac{p_o}{p_o + p_u}), \text{ where } F'^{-1} \text{ is the c.d.f. of } D'. \]

\[ \square \]

**Proof of Lemma 1**

*Proof of Claim 1 of Lemma 1*)

From Eq. (2.3),

\[ G(R, Q) \equiv E[c_o \max(Q - D, 0) + (c_u - \nu) \max(D - Q, 0)] \]

\[ + E[p_o \max\{R - \min(Q, D), 0\} + p_u \max\{\min(Q, D) - R, 0\}] \]

We have the following 3 cases:

1) \( D < R \leq Q \)

\[ G(R, Q, D) = c_o(Q - D) + p_o(R - D) \]

2) \( R \leq D \leq Q \)

\[ G(R, Q, D) = c_o(Q - D) + p_u(D - R) \]

3) \( R \leq Q < D \)

\[ G(R, Q, D) = (c_u - \nu)(D - Q) + p_u(Q - R) \]

\[ G(R, Q) = \int_0^R [c_o(Q - D) + p_o(R - D)]f(D)dD \]

\[ + \int_R^Q [c_o(Q - D) + p_u(D - R)]f(D)dD \]

\[ + \int_Q^\infty [(c_u - \nu)(D - Q) + p_u(Q - R)]f(D)dD \]

Using Leibniz rule,

\[ \frac{\partial G(R, Q)}{\partial R} = (p_o + p_u)F(R) + (-p_u) \]
\[ \frac{\partial G(R, Q)}{\partial Q} = (c_o + c_u - \nu - p_u)F(Q) - (c_u - \nu - p_u) \]

Take the second derivatives,

\[ \frac{\partial^2 G(R, Q)}{\partial Q^2} = (c_u - \nu - p_u) + c_o \]
\[ \frac{\partial^2 G(R, Q)}{\partial R^2} = p_o + p_u \]
\[ \frac{\partial^2 G(R, Q)}{\partial Q \partial R} = \frac{\partial^2 G(R, Q)}{\partial R \partial Q} = 0 \]

So, the Hessian of \( G(R, Q) \) is

\[ \nabla^2 G(R, Q) = \begin{bmatrix} (c_u - \nu - p_u) + c_o & 0 \\ 0 & p_o + p_u \end{bmatrix} \]

\( G(R, Q) \) is joint convex in \( Q \) and \( R \) if and only if \( \nabla^2 G(R, Q) \) is positive semidefinite for all \( Q \) and \( R \) (Chiang (1984)).

Since \( (c_u - \nu - p_u) + c_o > 0 \) (by Assumption 2), \( p_o + p_u > 0 \), and \( |\nabla^2 G(R, Q)| > 0 \), \( \nabla^2 G(R, Q) \) is positive semidefinite. Therefore, \( G(R, Q) \) is joint convex for \( R \leq Q \).

**Proof of Claim 2 and 3 of Lemma1**

Recall Eq. (2.3),

\[ G(R, Q) \equiv E[c_o \max(Q - D, 0) + (c_u - \nu) \max(D - Q, 0)] + E[p_o \max\{R - \min(Q, D), 0\}] + p_u \max\{\min(Q, D) - R, 0\} \]

Thus we have the following 3 cases:

1) \( D < Q < R \)

\[ G(R, Q, D) = c_o(Q - D) + p_o(R - D) \]
2) $Q < D < R$

$$G(R, Q, D) = (c_u - \nu)(D - Q) + p_o(R - Q)$$

3) $Q < R < D$

$$G(R, Q, D) = (c_u - \nu)(D - Q) + p_o(R - Q)$$

$$G(R, Q) = \int_0^Q [c_o(Q - D) + p_o(R - D)] f(D) dD$$

$$+ \int_Q^R [(c_u - \nu)(D - Q) + p_o(R - Q)] f(D) dD$$

$$+ \int_R^\infty [(c_u - \nu)(D - Q) + p_o(R - Q)] f(D) dD$$

Using Leibniz rule,

$$\frac{\partial G(R, Q)}{\partial R} = p_o$$

$$(2.8)$$

$$\frac{\partial G(R, Q)}{\partial Q} = (c_o + c_u - \nu + p_o) F(Q) - (c_u - \nu + p_o)$$

Take the second derivatives

$$\frac{\partial^2 G(R, Q)}{\partial Q^2} = (c_u - \nu + p_o) + c_o$$

$$\frac{\partial^2 G(R, Q)}{\partial R^2} = 0$$

$$\frac{\partial^2 G(R, Q)}{\partial Q \partial R} = \frac{\partial^2 G(R, Q)}{\partial R \partial Q} = 0$$

So, the Hessian of $G(R, Q)$ is

$$\nabla^2 G(R, Q) = \begin{bmatrix} \frac{\partial^2 G(R, Q)}{\partial Q^2} & \frac{\partial^2 G(R, Q)}{\partial Q \partial R} \\ \frac{\partial^2 G(R, Q)}{\partial R \partial Q} & \frac{\partial^2 G(R, Q)}{\partial R^2} \end{bmatrix} = \begin{bmatrix} (c_u - \nu - p_o) + c_o & 0 \\ 0 & 0 \end{bmatrix}$$

$G(R, Q)$ is jointly convex in $Q$ and $R$ if and only if $\nabla^2 G(R, Q)$ is positive semidefinite for all $Q$ and $R$. Since $(c_u - \nu + p_o) + c_o > 0$ (by Assumption 2) and $|\nabla^2 G(R, Q)| > 0$, ...
\[ \nabla^2 G(R, Q) \text{ is positive semidefinite.} \]

Therefore, \( G(R, Q) \) is joint convex for \( R > Q \) and Eq. (2.8) implies that \( G(R, Q) \) is a monotonically increase function in \( R \) for \( R > Q \). \( \square \)

**Proof of Theorem 1**

1) To find \( R^*(Q) \).

For any given \( Q \), if \( R < Q \), from Lemma 1,

\[
\frac{\partial G(R, Q)}{\partial R} = (p_o + p_u) F(R) + (-p_u)
\]

Implies \( R^0 = F^{-1}(\frac{p_u}{p_u + p_o}) \), independent of \( Q \).

If \( R > Q \), from Lemma 1,

\[
\frac{\partial G(R, Q)}{\partial R} = p_o > 0
\]

Implies that \( G(R, Q) \) is an monotonically increase function in \( R \) for \( R > Q \).

Now we discuss two cases: \( R^0 < Q \) and \( R^0 \geq Q \)

In the first case, it is clearly seen that \( R^*(Q) = R^0 \). In the second case, \( R^*(Q) = Q \).

2) To find \( Q^* \)

First consider \( Q \leq R^0, R^*(Q) = Q \) from the above finding. From Eq. (2.3),

\[
G(R, Q) = E[c_o \max(Q - D, 0) + (c_u - \nu) \max(D - Q, 0)] + \\
E[p_o \max\{R - \min(Q, D), 0\}] + p_u \max\{\min(Q, D) - R, 0\}
\]

Plug \( R^*(Q) = Q \) in \( R \), then

\[
G(R^*(Q), Q) = G_1(Q)
\]

\[
= E[c_o \max(Q - D, 0) + (c_u - \nu) \max(D - Q, 0)] + E[p_o \max(Q - D, 0)]
\]

\[
= E[(c_o + p_o) \max(Q - D, 0) + (c_u - \nu) \max(D - Q, 0)]
\]
From the solution of the traditional Newsvendor model, \( G_1(Q) \) is convex and

\[
Q_{PL}^2 = F^{-1}\left(\frac{c_u - \nu}{c_u - \nu + c_o + p_o}\right).
\]

Note that \( Q_{PL}^2 \leq R^0 \) if and only if \( p_u \geq \frac{p_o}{p_o + c_o}(c_u - \nu) \).

If \( p_u < \frac{p_o}{p_o + c_o}(c_u - \nu) \), then \( Q_{PL}^2 > R^0 \), so in the region where \( Q \leq R^0 \), \( G_1(Q) \) is monotonically decreasing.

Second consider \( Q > R^0 \), we know \( R^*(Q) = R^0 \). So, Plug \( R^*(Q) = R^0 \) in \( R \), then \( G(R^*(Q), Q) = G_2(Q) \)

\[
G(R^*(Q), Q) = \begin{bmatrix}
E\left[\max\{Q - D, 0\}\right] + \max\{c_o(Q - D)\}
+ \max\{(c_u - \nu)(D - Q), 0\}
+ \max\{p_o(D - R^0), 0\}
+ \max\{p_u\}
\end{bmatrix}
\]

(Actually, this part can be drawn from Lemma 1 with replacing \( R \) by \( R^0 \))

Consider different scenario of \( D \),

\[
G(R, Q) = \int_0^{R^0} [c_o(Q - D) + p_o(R^0 - D)]f(D) \, dD
+ \int_{R^0}^{Q} [c_o(Q - D) + p_u(D - R^0)]f(D) \, dD
+ \int_{Q}^{\infty} [(c_u - \nu)(D - Q) + p_u(Q - R^0)]f(D) \, dD
\]

Using Leibniz rule,

\[
\frac{\partial G(R^*(Q), Q)}{\partial Q} = c_oF(R^0) + \int_{R^0}^{Q} c_o f(D) \, dD + [c_o(Q - Q) + p_u(Q - R^0)]
+ \int_{Q}^{\infty} \left(- (c_u - \nu) + p_u\right) f(D) \, dD
+ [(c_u - \nu)(Q - Q) + p_u(Q - R^0)]
\]

\[
= c_oF(R^0) + [c_o(F(Q) - F(R^0))] + p_u(Q - R^0)\]

+ \left\{-(c_u - \nu) + p_u\right\}\left(1 - F(Q)\right) - \left[p_u(Q - R^0)\right]
By taking first-order condition, $Q_{PL}^1 = F^{-1}\left(\frac{(c_u - \nu - p_u)}{(c_u - \nu - p_u) + c_o}\right)$ and $G_2(Q)$ is convex ($\therefore \frac{\partial^2 G_2(Q)}{\partial Q^2} \geq 0$).

Note that $Q_{PL}^1 > R^0$ if and only if $p_u < \frac{p_o}{p_o + c_o}(c_u - \nu)$.

If $p_u \geq \frac{p_o}{p_o + c_o}(c_u - \nu)$, then $Q_{PL}^1 \leq R^0$. So in region $Q > R^0$, $G_2(Q)$ is monotonically increasing.

Therefore, if $p_u \geq \frac{p_o}{p_o + c_o}(c_u - \nu)$ or equivalently, $\frac{p_u}{c_u - \nu} \geq \frac{p_o}{p_o + c_o}$,

So, $G(R, Q)$ is convex in the region where $Q \leq R^0$, and monotonically increasing where $Q > R^0$,

$$Q^* = Q_{PL}^2 = F^{-1}\left(\frac{(c_u - \nu)}{(c_u - \nu) + c_o + p_o}\right)$$

$$R^* = Q^*$$

This proves Case b.

Please note that $G(R, Q)$ may not be convex in $Q$. But as long as, in the region $Q > R^0$, $G(R, Q)$ is monotonically increasing, we can still prove $Q_{PL}^1$ is the minimum.

Otherwise, if $p_u < \frac{p_o}{p_o + c_o}(c_u - \nu)$ or equivalently, $\frac{p_u}{c_u - \nu} < \frac{p_o}{p_o + c_o}$,

So, $G(R, Q)$ is convex in the region where $Q > R^0$, and monotonically decreasing where $Q \leq R^0$,

$$Q^* = Q_{PL}^1 = F^{-1}\left(\frac{(c_u - \nu - p_u)}{(c_u - \nu - p_u) + c_o}\right)$$

$$R^* = R^0 = F^{-1}\left(\frac{p_u}{p_u + p_o}\right)$$

This proves Case a. \[\square\]

**Proof of Proposition 3**

To find $C^*(Q^*)$
From shipper’s problem, Eq. (2.4),

\[
\max_{C, \nu, p_o, p_u} \pi_{Shipper} = \max_{C, \nu, p_o, p_u} \nu \cdot E[\min(D, Q^*)] \\
+ E[p_o \max\{R^* - \min(Q^*, D), 0\}] \\
+ p_u \max\{\min(Q^*, D) - R^*, 0\}] - \text{Cost}
\]

(where \(\text{Cost} = u \cdot C + u' E[\max\{\min(D, Q^*) - C, 0\}]\) and \(u' > u\))

And only \(\text{Cost}\) function has \(C\) so we only minimize \(\text{Cost}\) function to find \(C^*(Q^*)\).

If \(C < Q^*\),
1) \(D < C \leq Q\)

\[
\text{Cost}(C, Q, D) = u \cdot C
\]

2) \(C \leq D \leq Q\)

\[
\text{Cost}(C, Q, D) = u \cdot C + u'(D - C)
\]

3) \(C \leq Q < D\)

\[
\text{Cost}(C, Q, D) = u \cdot C + u'(Q^* - C)
\]

\[
\text{Cost}(C, Q, D) = \int_0^C [u \cdot C] f(D) dD + \int_C^Q [u \cdot C + u'(D - C)] f(D) dD \\
+ \int_Q^\infty [u \cdot C + u'(Q^* - C)] f(D) dD
\]

Using Leibniz rule,

\[
\frac{\partial \text{Cost}(C, Q)}{\partial C} = u' F(C) + (u' - u)
\]

Implies \(C^0 = F^{-1}\left(\frac{u' - u}{u'}\right)\), independent of \(Q^*\).
If $C > Q^*$, from the same analysis,

$$\frac{\partial \text{Cost}(C, Q^*)}{\partial C} = u > 0$$

Implies that $\text{Cost}(C, Q)$ is an monotonically increase function in $C$ for $C > Q^*$. □

Proof of Theorem 2

Recall the shipper’s problem, Eq. (2.4),

$$\max_{C, \nu, p_o, p_u} \pi_{\text{Shipper}} = \max_{C, \nu, p_o, p_u} \nu \cdot E[\min(D, Q^*)] + E[p_o \max\{\min(Q^*, D) - R^*, 0\}] - \text{Cost}$$

(where $\text{Cost} = u \cdot C + u' E[\max\{\min(D, Q^*) - C, 0\}]$ and $u' > u$)

where

if $\frac{p_u}{c_u - \nu} < \frac{p_o}{p_o + c_o}$,

$$Q^* = F^{-1}\left(\frac{(c_u - \nu - p_u)}{(c_u - \nu - p_u) + c_o}\right)$$

$$R^* = F^{-1}\left(\frac{p_u}{p_u + p_o}\right)$$

if $\frac{p_u}{c_u - \nu} \geq \frac{p_o}{p_o + c_o}$,

$$Q^* = F^{-1}\left(\frac{(c_u - \nu)}{(c_u - \nu) + p_o + c_o}\right)$$

$$R^* = Q^* = F^{-1}\left(\frac{(c_u - \nu)}{(c_u - \nu) + p_o + c_o}\right)$$

1) $p_o = 0$

$$\pi_{\text{Shipper}} = \max_{C, \nu, p_o, p_u} \nu \cdot E[\min(D, Q^*)] + E[p_u \max\{\min(Q^*, D) - R^*, 0\}] - \text{Cost}$$

(where $\text{Cost} = u \cdot C + u' E[\max\{\min(D, Q^*) - C, 0\}]$ and $u' > u$)
Because \( \frac{p_u}{c_u - \nu} \geq \frac{p_o}{p_o + c_o} \),

\[
Q^\ast = F^{-1}\left( \frac{(c_u - \nu)}{(c_u - \nu) + c_o} \right)
\]

\[
R^\ast = Q^\ast = F^{-1}\left( \frac{(c_u - \nu)}{(c_u - \nu) + c_o} \right)
\]

Since \( Q^\ast = R^\ast \), \( p_u \max\{\min(Q^\ast, D) - R^\ast, 0\} \) is zero.

Hence,

\[
\pi_{\text{Shipper}} = \max_{C, \nu, p_o, p_u} \nu \cdot E[\min(D, Q^\ast)] - \text{Cost}
\]

, which is the same as the flat rate model.

2) \( p_u = 0 \)

\[
\pi_{\text{Shipper}} = \max_{C, \nu, p_o, p_u} \nu \cdot E[\min(D, Q^\ast)] + E[p_o \max\{R^\ast - \min(Q^\ast, D), 0\}] - \text{Cost}
\]

(where \( \text{Cost} = u \cdot C + u' E[\max\{\min(D, Q^\ast) - C, 0\}] \) and \( u' > u \))

If \( \frac{p_u}{c_u - \nu} < \frac{p_o}{p_o + c_o} \),

\[
Q^\ast = F^{-1}\left( \frac{(c_u - \nu)}{(c_u - \nu) + c_o} \right)
\]

\[
R^\ast = F^{-1}\left( \frac{p_u}{p_u + p_o} \right) = 0
\]

Since \( R^\ast = 0 \), \( p_o \max\{R^\ast - \min(Q^\ast, D), 0\} \) is zero.

Hence,

\[
\pi_{\text{Shipper}} = \max_{C, \nu, p_o, p_u} \nu \cdot E[\min(D, Q^\ast)] - \text{Cost}
\]

, which is the same as the flat rate model.

\[ \square \]

**Proof of Lemma 2**

*Proof of Claim 1 of Lemma 2)*

We have the following 3 cases:

1) \( D < C \leq Q \)

\[
H(C, Q, D) = c_o(Q - D) + u \cdot C
\]
2) $C \leq D \leq Q$

$$H(C, Q, D) = c_o(Q - D) + u \cdot C + u'(D - C)$$

3) $C \leq Q < D$

$$H(C, Q, D) = c_u(D - Q) + u \cdot C + u'(Q - C)$$

$$H(C, Q) = \int_0^C [c_o(Q - D) + u \cdot C]f(D)dD + \int_C^Q [c_o(Q - D) + u \cdot C + u'(D - C)]f(D)dD + \int_Q^\infty [c_u(D - Q) + u \cdot C + u'(Q - C)]f(D)dD$$

Using Leibniz rule,

$$\frac{\partial H(C, Q)}{\partial C} = u'F(C) + (u - u')$$

$$C^* = F^{-1}\left(\frac{u' - u}{u'}\right)$$

(The interpretation is $C^* = F^{-1}\left(\frac{u' - u}{u'}\right) = F^{-1}\left(\frac{u' - u}{u + (u' - u)}\right)$

The overage cost is $u$ and the underage cost for $C$ is $u' - u$ because $u' - u$ is additional cost for uncut capacity.)

$$\frac{\partial H(C, Q)}{\partial Q} = (c_o + c_u - u')F(Q) - (c_u - u')$$

$$Q^* = F^{-1}\left(\frac{c_u - u'}{c_o + (c_u - u')}\right)$$

(The interpretation is that the overage cost is $c_o$ and the underage cost for $Q$ is $c_u - u'$ because we can save $u'$ under $C \leq Q$ assumption.)

Take the second derivatives

$$\frac{\partial^2 H(C, R)}{\partial C^2} = u'$$
\[
\frac{\partial^2 H(C, R)}{\partial Q^2} = c_o + c_u - u'
\]
\[
\frac{\partial^2 H(C, R)}{\partial Q \partial C} = \frac{\partial^2 H(C, R)}{\partial C \partial Q} = 0
\]

So, the Hessian of \(G(R, Q)\) is
\[
\nabla^2 G(R, Q) = \begin{bmatrix}
\frac{\partial^2 G(R, Q)}{\partial Q^2} & \frac{\partial^2 G(R, Q)}{\partial Q \partial R} \\
\frac{\partial^2 G(R, Q)}{\partial R \partial Q} & \frac{\partial^2 G(R, Q)}{\partial R^2}
\end{bmatrix} = \begin{bmatrix}
(c_u - \nu - p_u) + c_o & 0 \\
0 & 0
\end{bmatrix}
\]

\(H(C, Q)\) is joint convex in \(C\) and \(Q\) if and only if \(\nabla^2 H(C, Q)\) is positive semidefinite for all \(C\) and \(Q\). Since \(c_u - u' > 0\) (by the Assumption in channel’s problem), \(u' > 0\), and \(|\nabla^2 H(C, Q)| > 0\), \(\nabla^2 H(C, Q)\) is positive semidefinite. Therefore, \(H(C, Q)\) is joint convex for \(C \leq Q\).

**Proof of Claim 2 and 3 of Lemma 2**

We have the following 3 cases:

1) \(D < Q \leq C\)

\[H(C, Q, D) = c_o(Q - D) + u \cdot C\]

2) \(Q \leq D \leq C\)

\[H(C, Q, D) = c_o(Q - D) + u \cdot C\]

3) \(Q \leq C < D\)

\[H(C, Q, D) = c_u(D - Q) + u \cdot C\]

Note \(u'\) is disappeared for all three cases.

\[H(C, Q) = \int_0^\infty [c_o(Q - D) + u \cdot C] f(D)dD\]
Using Leibniz rule,
\[
\frac{\partial H(C, Q)}{\partial C} = u
\]
\[
\frac{\partial H(C, Q)}{\partial Q} = c_o
\]
This implies that $H(C, Q)$ is an monotonically increase function in $Q$ and $R$, and joint convex in $Q$ and $R$ for $R > Q$.

**Proof of Theorem 3**

To find $C^*(Q)$.

For any given $Q$, if $C < Q$, from Lemma 2,
\[
\frac{\partial H(C, Q)}{\partial C} = u' F(C) + (u' - u)
\]
Implies $C^0 = F^{-1}(\frac{u' - u}{u'})$, independent of $Q$.

If $C > Q$, from Lemma 2,
\[
\frac{\partial H(C, Q)}{\partial C} = u > 0
\]
Implies that $H(C, Q)$ is an monotonically increase function in $C$ for $C > Q$.

Now we discuss two cases: $C^0 < Q$ and $C^0 \geq Q$

In case 1, it is clearly seen that $C^*(Q) = C^0$. In case 2, $C^*(Q) = Q$.

To find $Q^*$.

First consider $Q \leq C^0$, $C^*(Q) = Q$ from above.

From Eq. (2.7),
\[
H(C, Q) = E[c_o max(Q - D, 0) + c_u max(D - Q, 0)] + u \cdot C 
\]
\[
+ u' E[max\{min(D, Q) - C, 0\}]
\]
Plug $C^*(Q) = Q$ in $C$, then

$$H(C^*(Q), Q) = H_1(Q) = E[c_o \max(Q - D, 0) + c_u \max(D - Q, 0)]$$

$$+ u \cdot Q + u' E[\max\{\min(D, Q) - Q, 0\}]$$

$$\frac{\partial H(C, Q)}{\partial Q} = c_o F(Q) - c_u (1 - F(Q)) + u = 0$$

$$Q_{FB}^1 = F^{-1}(\frac{c_u - u}{c_o + c_u})$$

(The interpretation is $Q_{FB}^1 = F^{-1}(\frac{c_o - u}{c_o + c_u}) = F^{-1}(\frac{c_u - u}{(c_o + u) + (c_u - u)})$)

The overage cost is $c_o + u$ and the underage cost for $Q$ is $c_u - u$. Since $C^* = Q^*$, the overage cost increase $C^*$ and the underage cost reduces $C^*$ as well.)

Note that $Q_{FB}^1 \leq C^0$ if and only if $c_o \geq \frac{c_u - u'}{u' - u} \cdot u$.

If $c_o < \frac{c_u - u'}{u' - u} \cdot u$, then $Q_{FB}^1 > C^0$, so in the region where $Q \leq C^0$, $H_1(Q)$ is monotonically decreasing.

Second consider $Q > C^0$, we know $C^*(Q) = C^0$. So, Plug $C^*(Q) = C^0$ in $C$, then

$$H(C^*(Q), Q) = H_2(Q) = E[c_o \max(Q - D, 0) + c_u \max(D - Q, 0)]$$

$$+ u \cdot C^o + u' \cdot E[\max\{\min(D, Q) - C^o, 0\}]$$

From Lemma 2,

$$Q_{FB}^2 = F^{-1}(\frac{c_u - u'}{c_o + (c_u - u')})$$

Note that $Q_{FB}^2 > C^0$ if and only if $c_o < \frac{c_u - u'}{u' - u} \cdot u$

If $c_o \geq \frac{c_u - u'}{u' - u} \cdot u$, then $Q_{FB}^2 \leq C^0$. So in region $Q > C^0$, $G_2(Q)$ is monotonically increasing.

Therefore, if $c_o \geq \frac{c_u - u'}{u' - u} \cdot u$
\[ Q^* = Q_{FB}^* = F^{-1}\left(\frac{(c_u - u)}{(c_o + u) + (c_u - u)}\right) \]

\[ C^* = Q \]

This proves Case b.

Otherwise, if \( c_o < \frac{c_u - u'}{u' - u} \cdot u \)

\[ Q^* = Q_{FB}^2 = F^{-1}\left(\frac{(c_u - u')}{c_o + (c_u - u')}\right) \]

\[ C^* = C^0 = F^{-1}\left(\frac{u' - u}{u'}\right) \]

This proves Case a. \( \square \)

**Proof of Theorem 4**

(The logic: we set \( p_o, p_u \) so that the seller’s self-best Q is the First-Best Q, the shipper’s self-best C is the first best C).

a) when \( c_o < \frac{c_u - u'}{u' - u} \cdot u \),

\[ Q^* = Q_{FB}^2 = F^{-1}\left(\frac{(c_u - u)}{c_o + (c_u - u')}\right) \]

\[ C^* = C^0 = F^{-1}\left(\frac{u' - u}{u'}\right) \]

b) when \( c_o \geq \frac{c_u - u'}{u' - u} \cdot u \),

\[ Q^* = Q_{FB}^1 = F^{-1}\left(\frac{(c_u - u)}{(c_u - u) + (c_o + u)}\right) \]

\[ C^* = Q^* = F^{-1}\left(\frac{(c_u - u)}{(c_u - u) + (c_o + u)}\right) \]

(1) If

First, we equal the solutions on \( Q \) between the global optimum and shipper optimum
to get the channel coordination conditions. Then use the conditions found, we find the seller’s optimal \( Q \) (Theorem 1). Using both (conditions and optimal \( Q \)), we find the shipper’s optimal \( C \) in the shipper’s optimal problem under penalty model, which we prove to be the same as the channel optimal \( C \).

Apply the conditions of this theorem; If \( p_u = u' - u \), \( p_o = u \), \( \nu = u \),

a) when \( c_o < \frac{c_u - u'}{u - u} \cdot u \Leftrightarrow p_u < \frac{p_o}{p_o + c_o} (c_u - \nu) \)

\[
Q^* = Q_{FB}^2 = F^{-1}(\frac{c_u - u'}{c_o + (c_u - u')}) = Q_{PL}^1 = F^{-1}(\frac{(c_u - \nu - p_u)}{(c_u - \nu) + p_o + c_o})
\]

So, Penalty model and the First-Best model have the same \( Q^* \) with the same condition.

Also we can see \( C^0 < Q^* \) because \( c_o < \frac{c_u - u'}{u - u} \cdot u \Leftrightarrow C^0 < Q^* \)

Therefore, applying Proposition 3, it is clearly seen that \( C^*(Q^*) = C^0 = F^{-1}(\frac{u' - u}{u}) \).

Next,

b) when \( c_o \geq \frac{c_u - u'}{u - u} \cdot u \Leftrightarrow p_u \geq \frac{p_o}{p_o + c_o} (c_u - \nu) \)

\[
Q^* = Q_{FB}^1 = F^{-1}(\frac{c_u - u}{(c_u - u) + (c_o + u)}) = Q_{PL}^2 = F^{-1}(\frac{c_u - \nu}{(c_u - \nu) + p_o + c_o})
\]

So, Penalty model and the first-best model have the same \( Q^* \) with the same condition.

Applying Proposition 3 again, \( C^0 \geq Q^* \) because \( c_o \geq \frac{c_u - u'}{u - u} \cdot u \Leftrightarrow C^0 \geq Q^* \)

Therefore, it is clearly seen that \( C^*(Q^*) = Q^* = F^{-1}(\frac{(c_u - u)}{(c_u - u) + (c_o + u)}) \).

(2) Only if

There are two possibilities that \( Q_{FB}^2 = Q_{PL}^2 \) (and \( Q_{FB}^1 = Q_{PL}^1 \)) or \( Q_{FB}^2 = Q_{PL}^1 \)

(and \( Q_{FB}^1 = Q_{PL}^2 \)) to be coordinated. (the first possible works out, but the second doesn’t - if we focus the values to be the same then the conditions conflict).

First, consider \( Q_{FB}^2 = Q_{PL}^2 \) (and \( Q_{FB}^1 = Q_{PL}^1 \))
a) If $Q_{FB}^2 = Q_{PL}^2$, 

$$F^{-1}\left(\frac{(c_u - u')}{(c_u - u') + c_o}\right) = F^{-1}\left(\frac{(c_u - \nu - p_u)}{(c_u - \nu - p_u) + c_o}\right)$$

$$(\nu + p_u - u')c_o = 0$$

$$\therefore p_u = u' - \nu$$

b) If $Q_{FB}^1 = Q_{PL}^1$, 

$$F^{-1}\left(\frac{c_u - u}{c_u + c_o}\right) = F^{-1}\left(\frac{c_u - \nu}{(c_u - \nu) + c_o + p_o}\right)$$

$$(p_o - u)c_u + (\nu - u)c_o + (\nu - p_o)u = 0$$

This equation must be hold no matter what $c_u$, $c_o$, and $u$, so by the identical equation property,

$$\therefore p_o = u , \nu = u = p_o$$

Also, from above proof for “ (1) If ”, it can be easily shown that $c_o < \frac{c_u - u'}{u - u} \cdot u \iff p_u < \frac{p_o - c_o}{p_o + c_o}(c_u - \nu)$. 

Second, consider the other case, $Q_{FB}^2 = Q_{PL}^1$ (and $Q_{FB}^1 = Q_{PL}^2$).

a) If $Q_{FB}^2 = Q_{PL}^1$, 

$$F^{-1}\left(\frac{(c_u - u')}{(c_u - u') + c_o}\right) = F^{-1}\left(\frac{(c_u - \nu)}{(c_u - \nu) + c_o + p_o}\right)$$

$$(p_o)c_u + (\nu)c_o + (-p_o)u = 0$$

From the same identical equation property,

$$\therefore p_o = 0 , \nu = 0$$
b) If $Q_{FB}^{1} = Q_{PL}^{2}$,

$$F^{-1}\left(\frac{c_u - u}{c_u + c_o}\right) = F^{-1}\left(\frac{(c_u - \nu - p_u)}{(c_u - \nu - p_u) + c_o}\right)$$

$$(-u)c_u + (\nu + p_u - u)c_o + (\nu + p_u)u = 0$$

Again, this equation must be hold no matter what $c_u$, $c_o$, and $u$, by the identical equation property,

$$\therefore u = 0, \quad \nu = 0, \quad p_u = 0$$

But from these condition, it’s not hold that $c_o < \frac{c_u - u'}{u - u} \cdot u \Leftrightarrow p_u \geq \frac{p_o}{p_o + c_o}(c_u - \nu)$, which is required to verify that $Q_{FB}^{1} = Q_{PL}^{1}$ (and $Q_{FB}^{2} = Q_{PL}^{2}$). So, the second case ($Q_{FB}^{2} = Q_{PL}^{1}$ (and $Q_{FB}^{1} = Q_{PL}^{2}$)) cannot be hold.

In sum, only if $Q_{FB}^{2} = Q_{PL}^{2}$ (and $Q_{FB}^{1} = Q_{PL}^{1}$), i.e., the coordination is achieved,

$p_u = u' - u, \quad p_o = u, \quad \nu = u.$

\[\square\]

Declaration of Theorem 5

1. when $c_o < \frac{c_u - u'}{u - u} \cdot u$, then from Theorem 3,

$$Q^* = Q_{FB}^{2} = F^{-1}(\frac{(c_u - u')}{(c_u - u') + c_o})$$

$$C^* = C^0 = F^{-1}(\frac{u' - u}{u'})$$

The shipper’s profit in the penalty model is

$$\pi_{Shipper} = \nu \cdot E[min(D, Q^*)] + E[p_o \max\{R^* - min(Q^*, D), 0\}]$$

$$+ p_u \max\{min(Q^*, D) - R^*, 0\}$$

$$- u \cdot C - u'E[max\{min(D, Q^*) - C, 0\}]$$
Plug in $p_u = u' - u$, $p_o = u$, $\nu = u$, then

$$\pi_{\text{Shipper}} = u \cdot E[\min(D, Q^*)] + E[u \cdot \max\{R^* - \min(Q^*, D), 0\}$$

$$+ (u' - u) \max\{\min(Q^*, D) - R^*, 0\} - u \cdot C^0$$

$$- u' E[\max\{\min(D, Q^*) - C^0, 0\}]$$

From the two properties:

$$\max(R^* - \min(Q^*, D), 0) = R^* - \min(R^*, \min(Q^*, D))$$

$$\max(\min(Q^*, D) - R^*, 0) = \min(Q^*, D) - \min(R^*, \min(Q^*, D))$$

$$\pi_{\text{Shipper}} = u \cdot \left[R^* - C^0\right] + u' \left[\min\{C^0, \min(Q^*, D)\}\right] - \min\{R^*, \min(Q^*, D)\}$$

Note that, from Theorem 4,

$$c_o < \frac{c_o - u'}{u' - u} \cdot u \iff p_u < \frac{p_o}{p_o + c_o} (c_o - \nu)$$

when $p_u = u' - u$, $p_o = u$, $\nu = u$, then

By theorem 1, $R^* = F^{-1}(\frac{p_u}{p_u + p_o}) = F^{-1}(\frac{u' - u}{u})$

Therefore, we can compare $C^0$, $Q^*$ and $R^*$:

1) $D < C^0 = R^* \leq Q$

$$\pi_{\text{Shipper}} = 0$$

2) $C^0 = R^* \leq D \leq Q$

$$\pi_{\text{Shipper}} = 0$$

3) $C^0 = R^* \leq Q < D$

$$\pi_{\text{Shipper}} = 0$$
2. when \( c_o > \frac{c_u - u'}{u' - u} \cdot u \), then from Theorem 3,

\[
Q^* = Q^1_{FB} = F^{-1}\left( \frac{c_u - u}{c_o + c_u} \right)
\]

Also, from Theorem 4,

\[
c_o > \frac{c_u - u'}{u' - u} \cdot u \iff p_u > \frac{p_o}{p_o + c_o}(c_u - \nu)
\]

when \( p_u = u' - u \), \( p_o = u \), \( \nu = u \), then \( R^* = Q^* \) (from Theorem 1)

1) \( D < Q^* = R^* = C^* \)

\[\pi_{Shipper} = 0\]

2) \( D \geq Q^* = R^* = C^* \)

\[\pi_{Shipper} = 0\]

Therefore, in all cases, the shipper’s profit is zero under the coordination condition. □

**Proof of Theorem 6**

(1) If

(The idea is the same as the proof for Theorem 4).

1) when \( c_o < \frac{c_u - u'}{u' - u} \cdot u \),

It’s easy to show

If \( \nu = u' \), \( Q^* = F^{-1}\left( \frac{c_u - u'}{(c_u - u') + c_o} \right) = Q^2_{FB} = F^{-1}\left( \frac{c_o - u'}{c_o + (c_u - u')} \right) \)

Using the \( Q^* \), find the \( C^* \) which optimizes the shipper’s expected profit.

\( C^0 < Q^* (= Q^2_{FB}) \) because \( c_o < \frac{c_u - u'}{u' - u} \cdot u \iff C^0 < Q^* (= Q^2_{FB}) \)

Therefore, it is clearly seen that \( C^* = C^0 = F^{-1}\left( \frac{u' - u}{u'} \right) \).

2) when \( c_o \geq \frac{c_u - u'}{u' - u} \cdot u \),

It’s shown that,
If $\nu = \frac{c_o + c_u}{c_o + u} \cdot u$, $Q^* = Q^1_{FB} = F^{-1}\left(\frac{(c_u - u)}{(c_u - u) + (c_o + u)}\right)$

Also, $Q^* (= Q^1_{FB}) \leq C^0$ because $c_o \geq \frac{c_u - u'}{u - u} \cdot u \iff Q^* (= Q^1_{FB}) \leq C^0$

Therefore, it is clearly seen that $C^* = Q^* = F^{-1}\left(\frac{(c_u - u)}{(c_u - u) + (c_o + u)}\right)$

(2) Only if

1) when $c_o < \frac{c_u - u'}{u - u} \cdot u$,

It’s easy to show

If $Q^* = F^{-1}\left(\frac{(c_u - u')}{(c_u - u') + c_o}\right) = Q^2_{FB} = F^{-1}\left(\frac{(c_u - u')}{c_o + (c_u - u')}\right)$, $\nu = u'$

2) when $c_o \geq \frac{c_u - u'}{u - u} \cdot u$,

It’s shown that,

If $Q^* = F^{-1}\left(\frac{(c_u - u')}{(c_u - u') + c_o}\right) = Q^1_{FB} = F^{-1}\left(\frac{(c_u - u)}{(c_u - u) + (c_o + u)}\right)$, $\nu = \frac{c_o + c_u}{c_o + u} \cdot u$ \hfill \square

\textbf{Proof of Theorem 7}

\textit{Claim 1 of Proof of Theorem 7)}

1. when $c_o < \frac{c_u - u'}{u - u} \cdot u$, then from Theorem 3,

$$Q^* = Q^2_{FB} = F^{-1}\left(\frac{(c_u - u')}{(c_u - u') + c_o}\right)$$

$$C^* = C^0 = F^{-1}\left(\frac{u' - u}{u'}\right)$$

The shipper’s expected profit in the flat rate model is

$$\pi_{Shipper} = \nu \cdot E[min(D, Q^*)] - u \cdot C - u' E[max\{min(D, Q^*) - C, 0\}]$$

Plug in $\nu = u'$ then

$$\pi_{Shipper} = u' \cdot E[min(D, Q^*)] - u \cdot C^0 - u' E[max\{min(D, Q^*) - C^0, 0\}]$$
From the property:

$$\max\{\min(Q^*, D) - C^0, 0\} = \min(Q^*, D) - \min\{C^0, \min(Q^*, D)\}$$

$$\pi_{\text{Shipper}} = u' \cdot E[\min\{C^0, \min(D, Q^*)\}] - u \cdot C^0$$

Therefore, we can compare $C^0$ and $Q^*$:

1) $D < C^0 \leq Q^*$

$$\pi_{\text{Shipper}} = u' \cdot D - u \cdot C^0$$

2) $C^0 \leq D \leq Q$

$$\pi_{\text{Shipper}} = u' \cdot C^0 - u \cdot C^0$$

3) $C^0 \leq Q < D$

$$\pi_{\text{Shipper}} = u' \cdot C^0 - u \cdot C^0$$

Therefore the shipper’s expected profit is

$$\pi_{\text{Shipper}} = \int_0^{C^0} [u' \cdot D - u \cdot C^0] f(D) dD + \int_{C^0}^\infty [(u' - u)C^0] f(D) dD$$

We add and subtract

$$\int_0^{C^0} [(u' - u)C^0] f(D) dD$$

, then

$$= \int_0^{C^0} [u' \cdot D - u' \cdot C^0] f(D) dD + (u' - u)C^0$$

By using integration by parts,

$$= u'(D - C^0) F(D)|_0^{C^0} - u' \int_0^{C^0} F(D) dD + (u' - u)C^0$$

$$= 0 - u' \int_0^{C^0} F(D) dD + (u' - u)C^0$$

$$\geq -u' C^0 F(C^0) + (u' - u)C^0 \text{ (since } F(\cdot) \text{ is a monotonically increasing function)}$$
= 0 (since \( F(C^0) = \frac{u' - u}{u'} \))

2. when \( c_o \geq \frac{c_u - u'}{u - u'} \cdot u \), then from Theorem 7,

\[
Q^* = Q_{FB}^* = F^{-1}(\frac{(c_u - u)}{(c_u - u) + (c_o + u)})
\]

\[
C^* = Q^* = F^{-1}(\frac{(c_u - u)}{(c_u - u) + (c_o + u)})
\]

The shipper’s expected profit in the flat rate model is

\[
\pi_{\text{Shipper}} = \nu \cdot E[\min(D, Q^*)] - u \cdot C - u'E[\max\{\min(D, Q^*) - C, 0\}]
\]

Plug in \( \nu = \frac{c_o + c_u}{c_o + u} \cdot u \) then

\[
\pi_{\text{Shipper}} = \left(\frac{c_o + c_u}{c_o + u} \cdot u\right) \cdot E[\min(D, Q^*)] - u \cdot Q^* - u'E[\max\{\min(D, Q^*) - Q^*, 0\}]
\]

Since \( u'E[\max\{\min(D, Q^*) - Q^*, 0\}] \) is always zero,

\[
\pi_{\text{Shipper}} = \left(\frac{c_o + c_u}{c_o + u} \cdot u\right) \cdot E[\min(D, Q^*)] - u \cdot Q^*
\]

1) \( D < Q^* \)

\[
\pi_{\text{Shipper}} = \left(\frac{c_o + c_u}{c_o + u} \cdot u\right) \cdot D - u \cdot Q^*
\]

2) \( Q^* \leq D \)

\[
\pi_{\text{Shipper}} = \left(\frac{c_o + c_u}{c_o + u} \cdot u\right) \cdot Q^* - u \cdot Q^*
\]
Therefore the shipper’s expected profit is

\[
\pi_{\text{Shipper}} = \int_0^{Q^*} \left( \frac{c_o + c_u}{c_o + u} \cdot u \right) \cdot D - u \cdot Q^* f(D) dD + \int_{Q^*}^{\infty} \left( \frac{c_o + c_u}{c_o + u} \cdot u \right) \cdot Q^* - u \cdot Q^* f(D) dD
\]

\[
= \left( \frac{c_o + c_u}{c_o + u} \cdot u \right) \left( \int_0^{Q^*} D \cdot f(D) dD + Q^*(1 - F(Q^*)) \right) - u \cdot Q^*
\]

By using integration by parts,

\[
= \left( \frac{c_o + c_u}{c_o + u} \cdot u \right) \left( \int_0^{Q^*} F(D) dD - Q^* \right) - u \cdot Q^*
\]

\[
= \left( \frac{c_o + c_u}{c_o + u} \cdot u \right) \left( Q^* - \int_0^{Q^*} F(D) dD ight) - u \cdot Q^*
\]

\[
\geq - \left( \frac{c_o + c_u}{c_o + u} \cdot u \right) F(Q^*) Q^* + \left( \frac{c_o + c_u}{c_o + u} \cdot u \right) Q^*
\]

(since \( F(\cdot) \) is a monotonically increasing function)

\[
= - \left( \frac{c_o - c_u}{c_o + u} \cdot u \right) Q^* + \left( \frac{c_u - u}{c_o + u} \cdot u \right) Q^*
\]

(since \( F(Q^*) = \frac{c_u - u}{c_o + u} \))

\[
= 0
\]

\[
\square
\]

Claim 2 of Proof of Theorem 7)

1. when \( c_o < \frac{c_u - u'}{u - u} \cdot u \), then from Theorem 5,

\[
Q^* = Q^2_{FB} = F^{-1} \left( \frac{c_u - u'}{c_u - u'} + c_o \right)
\]

The seller’s profit is

\[
\pi_{\text{Retailer}} = E(D)(c_u - \nu) - E[c_o \max(Q - D, 0) + (c_u - \nu) \max(D - Q, 0)]
\]
Plug in $\nu = u'$ then,

$$\pi_{Retailer} = E(D)(c_u - u') - E[c_o \max(Q - D, 0) + (c_u - u')\max(D - Q, 0)]$$

From the property: $\max(0, Q - D) = Q - \min(Q, D)$

$$\pi_{Retailer} = (c_o + c_u - u') \cdot E[\min(D, Q^\star)] - c_o \cdot Q^\star$$

1) $D < Q^\star$

$$\pi_{Retailer} = (c_o + c_u - u')D - c_o \cdot Q^\star$$

2) $Q^\star \leq D$

$$\pi_{Retailer} = (c_o + c_u - u')Q^\star - c_o \cdot Q^\star$$

Therefore the seller’s expected profit is

$$\pi_{Retailer} = \int_0^{Q^\star} [(c_o + c_u - u')D - c_o \cdot Q^\star]f(D)dD$$

$$+ \int_{Q^\star}^{\infty} [(c_o + c_u - u')Q^\star - c_o \cdot Q^\star]f(D)dD$$

$$= (c_o + c_u - u')(\int_0^{Q^\star} D \cdot f(D)dD + Q^\star(1 - F(Q^\star))) - c_o \cdot Q^\star$$

By using integration by parts,

$$= (c_o + c_u - u')(D \cdot F(D)|_0^{Q^\star} - \int_0^{Q^\star} F(D)dD + Q^\star(1 - F(Q^\star))) - c_o \cdot Q^\star$$

$$= (c_o + c_u - u')(Q^\star \cdot F(Q^\star) - \int_0^{Q^\star} F(D)dD + Q^\star(1 - F(Q^\star))) - c_o \cdot Q^\star$$

$$= (c_o + c_u - u')(Q^\star - \int_0^{Q^\star} F(D)dD) - c_o \cdot Q^\star$$

$\geq -(c_o + c_u - u')F(Q^\star)Q^\star + (c_o + c_u - u' - c_o)Q^\star$ (since $F(\cdot)$ is a monotonically increasing function)

$$= -(c_o + c_u - u')(\frac{(c_u - u')}{c_u - u' + c_o})Q^\star + (c_u - u')Q^\star \quad \text{(since } F(Q^\star) = \frac{(c_u - u')}{c_u - u' + c_o})$$
2. when \( c_o \geq \frac{c_u - u'}{u - u} \cdot u \), then from Theorem 3,

\[
Q^* = Q^*_FB = F^{-1}(\frac{(c_u - u)}{(c_u - u) + (c_o + u)})
\]

The seller’s profit is

\[
\pi_{Retailer} = E(D)(c_u - \nu) - E[c_o \max(Q - D, 0) + (c_u - \nu)\max(D - Q, 0)]
\]

Plug in \( \nu = \frac{c_o + c_u}{c_o + u} \cdot u \) then

\[
\pi_{Retailer} = E(D)(c_u - \nu) - E[c_o \max(Q - D, 0) + (c_u - \nu) \max(D - Q, 0)]
\]

\[
= \left(\frac{c_o + c_u}{c_o + u} \cdot c_o\right) \cdot E[\min(D, Q^*)] - c_o \cdot Q^*
\]

1) \( D < Q^* \)

\[
\pi_{Retailer} = \left(\frac{c_o + c_u}{c_o + u} \cdot c_o\right) \cdot D - c_o \cdot Q^*
\]

2) \( Q^* \leq D \)

\[
\pi_{Retailer} = \left(\frac{c_o + c_u}{c_o + u} \cdot c_o\right) \cdot Q^* - c_o \cdot Q^*
\]

Therefore the seller’s expected profit is

\[
\pi_{Retailer} = \int_0^{Q^*} \left[\left(\frac{c_o + c_u}{c_o + u} \cdot c_o\right) \cdot D - c_o \cdot Q^*\right] f(D) dD
\]

\[
+ \int_{Q^*}^{\infty} \left[\left(\frac{c_o + c_u}{c_o + u} \cdot u\right) \cdot Q^* - c_o \cdot Q^*\right] f(D) dD
\]

\[
= \left(\frac{c_o + c_u}{c_o + u} \cdot c_o\right) \left(\int_0^{Q^*} D \cdot f(D) dD + Q^* (1 - F(Q^*))\right) - c_o \cdot Q^*
\]
By using integration by parts,

\[
\begin{align*}
&= \left(\frac{c_o+cu}{c_o+u} \cdot c_o\right) \left(\frac{D}{0}^{Q^*} F(x) dx - \int_0^{Q^*} F(D) dD + Q^* (1 - F(Q^*))\right) - c_o \cdot Q^* \\
&= \left(\frac{c_o+cu}{c_o+u} \cdot c_o\right) \left(Q^* \cdot F(x) - \int_0^{Q^*} F(D) dD + Q^* (1 - F(Q^*))\right) - c_o \cdot Q^* \\
&= \left(\frac{c_o+cu}{c_o+u} \cdot c_o\right) \left(Q^* - \int_0^{Q^*} F(D) dD\right) - c_o \cdot Q^* \\
&\geq -\left(\frac{c_o+cu}{c_o+u} \cdot c_o\right) F(Q^*) Q^* + \left(\frac{c_o+cu}{c_o+u} \cdot c_o - c_o\right) Q^* \quad \text{(since } F(\cdot) \text{ is a monotonically increasing function)} \\
&= -\left(\frac{cu - u}{c_o+u} \cdot c_o\right) Q^* + \left(\frac{cu - u}{c_o+u} \cdot c_o\right) Q^* \quad \text{(since } F(Q^*) = \frac{cu - u}{c_o+u}\right) \\
&= 0
\end{align*}
\]
Chapter 3
Incentives and Gaming in Collaborative Projects - Risk Sharing Partnership

3.1 Introduction

Over the last three to four decades, advances in technology and the networked economy have led the business models in many project industries to evolve from “one-firm-does-all” to outsourcing and collaboration. Risk sharing partnership, where all partners pay for their own costs to make contributions and share the reward / loss of the final outcome, is a predominant way used in practice to manage collaborative projects. Examples can be found in many industries, such as engineering-procurement-construction (EPC), commercial aerospace, and book publishing.

In the EPC industries, the $150 billion international space station (ISS) is a representative example where the design and construction of ISS are spread out to multiple countries around the world. The elements of ISS are launched from different countries and mated together on orbit. Each country is financially responsible for designing, making and maintaining its elements (see NASA, 2013).

In the commercial aerospace industry, suppliers are playing an increasingly important role in the development of new aircrafts. Recent examples are Boeing 787 Dreamliner, Airbus 380, China Comac C919. In particular, the Boeing 787 Dreamliner outsourced 65% of the development work to more than 100 suppliers from 12 countries (see Horng (2006) and Exostar (2007)). Tier 1 suppliers design and fabricate 11 major
subassemblies, Boeing integrates and assembles the airplane. To manage the collaboration, Boeing made the suppliers stakeholders of the program by engaging them in a risk sharing partnership where the suppliers were responsible for more than 50% of the non-recurring development cost and must wait until the completion of the project to get paid (see Xu and Zhao, 2011). In return, the suppliers own the intellectual property (IP) of their work.

The most down-to-the-earth example of collaboration under risk sharing is perhaps the coauthorship in book publishing, where every author contributes his/her own time and knowledge, and is rewarded after the book is published. Projects across these industries vary significantly in content and scale. However, they face common challenges: First, they require diverse knowledge and expertise that few companies or individuals possess them all; Second, they demand a significant investment of capital / time up front; Third, they face huge market risks as demand is hardly predicable. Collaboration under risk sharing offers a solution to handle all these challenges: First, it allows the project to utilize the best in-class expertise and knowledge; Second, it reduces the upfront non-recurring investment of time and cost for each partner; Third, it motivates all partners to expand the market so everyone can benefit. The benefits are best summarized by McNerney, Boeing’s CEO, in April 2008: “The global partnership model of the 787 remains a fundamentally sound strategy. It makes sense to utilize technology and technical talent from around the world. It makes sense to be involved with the industrial bases of counties that also support big customers of ours.”

From a project management perspective, Boeing and the industry also believed that collaboration under risk sharing would work on both cost and time metrics. As quoted by Kotha, Olesen, Nolan, and Condit (2005): “Boeing had asked its structural suppliers to fund their own research and development (a first for a Boeing project) for the 787 project. This way, Boeing believed suppliers were likely to have a greater
financial incentive to minimize their cost and, at the same time, assist Boeing market the new plane.” Pat Russell, director of global supply at Vought Aircraft, commented in 2007 about the collaborative strategy of the Dreamliner 787: “What Boeing is trying to do will really set the standard for how you reduce time to market, from design to implementation.” The logic seems simple: suppliers may have incentives to be on-time because they share the delay damage; suppliers may also be cost effective because they are spending their own money. Believing that the partners are properly motivated, Boeing left the selection and control of 2nd tier supplier to its risk sharing partners, and abandoned its practice (used previously in 777) of sending engineers and inspectors to suppliers’ sites.

In reality, however, the development of 787 was a disaster - the 1st delivery of 787 Dreamliner was delayed by 40 months with a total cost overrun of at least $11 billion, including, write-offs (about $2.5 billion) due to defects, excessive R&D costs (about $3.5 billion), and customer contract penalty (about $5 Billion). In addition, 7% orders were canceled. This is in sharp contrast to Boeing 777 which had the same planned duration and was delivered on time. An inside look shows that a majority of the 787 delays were caused by non-technical but managerial slips of Boeing and its suppliers, such as low-wage, train-on-the-job workers, student inspectors, lack of Q/A equipment and testing, poorly written instruction for installation, and poor documentation (incomplete or lost in transfer), etc. (see Zhao, 2016). To combat the delays, Boeing had to take back the control of the supply chain, and resume the practice of sending its people to inspecting suppliers’ sites, even bought out a few suppliers.

The structural difference between one-firm-does-all and collaboration under risk sharing can be illustrated by a simple model. Consider a project with two sequential tasks, A and B. According to the project management theory on time-cost trade-offs
(and the 787 example), delaying tasks A and B by one period saves $s_A$ and $s_B$ respectively in direct cost (cost contributing directly to the task, e.g., management and processes, labor, equipment, material and shipping). However, delaying the completion of the project by one period incurs a penalty of $p$ in indirect cost (costs contributing indirectly to tasks, e.g., overhead, capital cost, contract penalty and order cancelation).

In the one-firm-does-all model (Figure 3.1a), the firm is responsible for all savings and costs. In the collaboration under risk sharing (Figure 3.1b), tasks A and B are performed by firms A and B respectively, who receive the corresponding saving, $s_A$ or $s_B$, and share the delay penalty, $p_A$ or $p_B$, and $p_A + p_B = p$. The key questions are, how would the firms behave in the collaboration under risk sharing? What is the impact on time and cost?

![Figure 3.1: One-Firm-Does-All vs. Risk Sharing](image)

The simple model is easy to analyze (see Section 3.4.1 for details and Zhao (2016) for an example); however, real-life collaborative projects can be much more intriguing because of their general network topologies (combination of serial and/or parallel networks), cost structure (expedition offsets delay, time independent vs. dependent reward and penalty), and information status (information symmetry vs. asymmetry). In this
chapter, we raise the following research questions: (1) Relative to one-firm-does-all, 
how does the risk sharing partnership change firms’ incentives in project execution and 
affect project metrics (cost and duration) for various project networks, cost structures, 
and information status? (2) Bridge theory and practice - what are the practical im-
plications of the theory? How to improve project outcome for collaboration under risk 
sharing?

Applying the economic theory of teamwork to project operations, we characterize 
equilibrium decisions and project outcomes for the risk sharing partnership under both 
information symmetry and asymmetry. Our analysis reveals a few new mechanisms 
unique in a project management setting which may cause serious incentive issues, such 
as the prisoners’ dilemma, the supplier’s dilemma, the coauthors’ dilemma, and worst 
supplier dominance. We also find that information asymmetry may either prolong or 
shorten the project duration relative to information symmetry, contingent on network 
topology and cost structure.

Applying the theory to practice, we identify potential gaming behaviors in collabor-
native projects, suggest strategies that may help to regulate such behaviors, and specify 
conditions under which the partnership may work well. The results can be used to 
better predict the strategic behaviors and project performance in collaborative projects 
under the risk sharing partnership; aid in partner selection (who to collaborate with); 
verify the importance of Boeing’s practice of sending inspectors to supplier sites; and 
show how the supplier’s belief may affect project performance (and so what belief that 
the manufacturer should help the suppliers to shape up).

The chapter is organized as follows. In §3.2, we review the related literature and 
elaborate on our contributions; which is followed by §3.3 where we introduce our models 
and methodology. In §3.4, we consider information symmetry and establish the theory 
on firms’ incentive and strategic gaming behaviors for various network topologies and
cost structures. In §3.5, we consider information asymmetry and highlight the impact of information status. We link theory to practice in §3.6 with an extensive discussion on practical implications of the theory and suggestions on improvement strategies. We conclude the chapter in §3.7.

3.2 Literature Review

This chapter is related to the project management literature, such as, scheduling, bidding and subcontracting, economics theory of moral hazard and teamwork, and most recent work on partnerships in project management. We shall review related results in each area and point out the contribution of our work.

Project management. The most well known results in this literature include the critical path method (CPM), project evaluation and review techniques (PERT), time-cost analysis (TCA), and resource constrained project scheduling (RCPS). This literature studies the scheduling and planning of project(s) for a single firm (as in one-firm-does-all) and thus the main issue is on optimization. We refer the reader to Elmaghraby (1977), Nahmias and Cheng (1993), Józefowska and Weglarz (2006), and Klastorin (2004) for reviews of this literature. Our chapter draws project management specifics, e.g., cost structure, project network and time-cost trade-off, from this literature but studies incentive issues and gaming behaviors in collaborative projects.

This literature also studies project bidding and subcontracting where multiple firms are involved. Elmaghraby (1990) studies project bidding from the contractor’s perspective, and Gutierrez and Paul (2000) compares fixed price contracts, cost-plus contracts and menu contracts in project bidding from the project owner’s perspective. Paul and Gutierrez (2005) studies how to assign tasks to contractors for projects with parallel or serial tasks. Szmerkovsky (2005) studies the impact of payment schedule on contractors’ performance. In this model, the owner first selects the payment terms, the
contractor then decides the schedule to maximize its net present value. Aydinliyim and Vairaktarakis (2010) considers multiple manufacturers which outsource certain operations to a third party by booking its capacity, and the third party identifies a schedule to minimize the total cost for all manufacturers.

Bayiz and Corbett (2005) considers projects with sequential or parallel tasks in a subcontracting arrangement, and compares the effectiveness between the fixed-price contracts and linear incentive contracts in a principal-multi-agent model under both information symmetry and asymmetry. The chapter derives the optimal incentive contracts under information asymmetry and shows that project performs better under the symmetric information than under asymmetric information. Kwon, Lippman, and Tang (2010b) is the first chapter that studies nonlinear time-based and cost-based incentive contracts in project subcontracting. It finds that a fixed-price and cost-plus contract cannot coordinate the project, but with carefully chosen parameters, time-based and cost-sharing contracts can. Kwon, Lippman, and Tang (2011) analyzes the trade-off between efficiency (out-sourced tasks) and control (tasks performed in-house) for projects with both parallel and serial structures, and shows that outsourcing tasks can generate a higher operating profit for the owner when the project size is intermediate. Chen and Lee (2016) studies the problem of how a manufacturer may coordinate the material delivery schedule with the suppliers’ production schedules for a sequence of tasks. It shows that a delivery schedule-based contract can attain channel coordination, which involves a fixed price, a targeted delivery date, and a bonus/penalty contingent on the supplier’s delivery performance.

Our work differs from this literature because the risk sharing partnership is structurally different from subcontracting. In subcontracting, a supplier is only responsible for its own actions and its interest is tied only to its tasks. However, in partnerships, a supplier may be responsible for others’ actions because its interest is tied to the
Thus, partnerships may completely alter firms incentives relative to subcontracting, hence require different models, analyzes and insights.

**Moral hazard and teamwork.** The economics literature of teams discusses incentives and gaming behaviors in general teamwork settings. This literature is vast, we refer the reader to several seminal chapters, e.g., Holmstrom (1982), Demski and Sappington (1984), McAfee and McMillan (1986), and Holmstrom and Milgrom (1991), for principal-agent models and moral hazard games; and Bhattacharyya and Lafontaine (1995), Kim and Wang (1998), and Al-Najjar (1997) for the double moral hazard games.

More recently, economic theory of teams finds its applications in operations management issues, such as joint product development, joint services and production, please see Plambeck and Taylor (2006), Bhaskaran and Krishnan (2009), Roels, Karmarkar, and Carr (2010), Ülkü and Schmidt (2011), Rahman, Roels, and Karmarkar (2013) and reference therein.

These studies consider partnerships and gaming but not in a project management setting. In this chapter, we integrate the economics theory of teams with project management specifics to understand how the unique features and metrics of project operations may affect firms' incentives in collaborative projects under the risk sharing partnership.

**Partnerships in project management.** The study of partnerships in a project management setting is mostly related to our work and has just begun.

Kwon, Lippman, McCardle, and Tang (2010a) is the first chapter that studies a delayed payment contract in a project with parallel tasks, where the contractors get paid after the all tasks are completed. The contract shares certain similarities as partnerships because one contractor’s profit may be affected by other contractors’ actions. The chapter examines, from the project owner’s perspective, how a delayed payment affects
each contractor’s effort level and the project’s net profit in equilibrium relative to non-
delayed payment (subcontracting). It identifies conditions under which the project is
worse off under the delayed payment, and shows how these conditions depend on the
revenue, the number of suppliers, and the supplier’s capability to adjust their work
rates dynamically.

Chen, Klastorin, and Wagner (2015) studies projects with sequential tasks under
both delayed and non-delayed payment contracts, where each task is outsourced to a
contractor. It proposes a nonlinear (exponential) incentive payment contract to mo-
tivate the contractors. It finds that the proposed contract dominates a fixed price
contract for the project as a whole on both profit and schedule, regardless of payment
timing considerations.

Inspired by real-life cases, our study identifies incentive issues / gaming behaviors
and predicts outcomes in collaborative projects under the risk sharing partnership. This
chapter contributes in the following ways:

• We are first to study the risk sharing partnership, where all partners co-own
the project, under information asymmetry. Comparing the results between infor-
mation symmetry and asymmetry sheds new insights on the incentive issues in
collaboration and the impact of information.

• We consider a general project network with both sequential and parallel tasks
(resembling some practice), and discover new mechanisms (e.g., the Supplier’s
and Coauthors’ Dilemmas) unique in the project management setting through
which the firms may behave against the best interest of the project.

• We apply the theoretical results to practice and develop insights on gaming be-
haviors in real-life examples. We also discuss practical improvement strategies.
3.3 The Model and Preliminaries

In this section, we first present the project management specifics and key assumptions on network topologies, cost structure and information status. Second, we introduce the risk sharing partnership and the one-firm-does-all benchmark. Finally, we outline our approach and the game theoretical models.

Deterministic Task Durations. We exclude the environmental uncertainty and consider deterministic task durations. Doing so allows us to focus on the human factors such as incentive issues and gaming behaviors (managerial risk). It can be a good approximation in projects without significantly technical risks and natural disaster threats, such as textbooks and projects of upgrading, extension and new combination of existing technologies, which account for a vast majority of development projects. If environmental uncertainty is high but we can distinguish it from human errors (true for the 787 case as many of the management issues are known to the public), then the model of deterministic duration can still be useful.

Project Cost Structure. We classify project costs into two categories (Nahmias and Cheng (1993)): direct cost and indirect cost. Direct cost includes all spending directly contributing to a task, such as the cost of management, labor, material and shipping. Normally, a longer task duration is coupled with a lower direct cost. Indirect cost includes all spending not directly contributing to tasks but depending on the project duration, such as the overhead (e.g., rent, utilities, benefits), interests and financial costs, delay penalty and order cancelation loss. Normally, a longer project duration is coupled with a higher indirect cost. We refer the reader to Nahmias and Cheng (1993) for more details.

Consistent to a majority of practical situations, we assume that direct cost is convex and decreasing as task duration increases and indirect cost is convex and increasing as
project duration increases (Figure 3.2, Nahmias and Cheng (1993)). If task \(i\) is delayed by one period, firm \(i\) saves \(s_i\) in the direct cost. If the project is delayed by one period, it suffers a penalty \(p\) in the indirect cost. Conversely, if task \(i\) is expedited by one period, firm \(i\) incurs a cost \(c_i\) for expediting. If the project is completed one period earlier, it receives a reward \(r\).

**Project Network.** We consider projects with a network (precedence) structure shown in Figure 3.3. It has two stages: At stage 1, there are several tasks to be completed in parallel, similar to the design and fabrication of subsystems in the 787 Dreamliner program, the writing of individual chapters in a coauthored book, and the development of subsystems and components of the International Space Station (ISS). At stage 2, there is the task of integration and assembly of all parts completed in stage 1, similar
to the system integration task in the 787 Dreamliner program, the integration and proofreading of a coauthored book, and the final assembly and testing of the ISS. Clearly the task at stage 2 cannot start until all tasks at stage 1 are completed.

Figure 3.3 shows the general project network; two special cases deserve some attentions: $n = 1$ denotes the case with only one task at stage 1, and thus the project network reduces to two sequential tasks. When $n \geq 2$, there are two parallel tasks at stage 1.

Figure 3.3: A typical collaborative project.

**The Risk-Sharing Partnership.** In this partnership, each firm pays for the direct costs of its own task(s), and shares the reward or loss of the project. Intuitively, if a firm delays its task, it saves on its direct cost but everyone (including the delayed firm) suffers an increase in indirect cost (a penalty) if the firm’s delay results in a project delay. Thus one firm’s delay can affect other firms, and this delayed firm is not fully responsible for all the consequences of its action as the penalty is shared among all firms.

**The Benchmark: One-Firm-Does-All.** In the benchmark, one-firm-does-all tasks of the project and follows the optimal project schedule under its centralized control. For the ease of comparison, we start the project with the optimal project schedule under the centralized control and analyze how the risk sharing partnership may (or may not) motivate firms to deviate from the benchmark.
**Game Theoretical Framework.** We assume that each task in the 2-stage project network is assigned to a different firm. For the ease of exposition, we use “supplier(s)” to name the firm(s) responsible for the task(s) at stage 1 and “manufacturer” to name the firm responsible for the task at stage 2. Each firm maximizes its own payoff (its direct and the shared indirect costs) by adjusting the duration of its own task.

### 3.4 Information Symmetry

In this section, we study firms’ incentive issues and strategic behaviors under information symmetry where the direct and indirect cost functions of all firms are public knowledge. The sequence of events is described as follows (see also Figure 3.4): At the beginning of phase 1, suppliers choose task durations and carry out their tasks. After all suppliers complete their tasks, phase 1 is concluded. At the beginning of phase 2, the manufacturer observes the suppliers’ actions and chooses its task duration. When the manufacturer completes its task, the project is completed and all costs are realized for each firm.

Figure 3.4: Sequence of Events.
By the structure of the project network, a two-phase game theoretic model is appropriate for predicting the behaviors of the supplier(s) and the manufacturer in equilibrium. In this game, the suppliers take the lead by playing a simultaneous game among themselves (anticipating the manufacturer’s response) and the manufacturer follows by responding to suppliers’ actions accordingly. We shall derive subgame perfect Nash equilibrium (SPNE) for each case considered below and compare the resulting project performance to that of one-firm-does-all. If the SPNE is not unique, we shall compare different SPNEs and report the Pareto or strong equilibrium.

To derive managerial insights, we start our analysis by a base model in §3.4.1 with one supplier and time independent cost. In this model, each firm can either “keep” the optimal task duration or “delay” it by one period. In §3.4.2, we relax the time-independent cost assumption in the base model to study the impact of time-dependent costs, for instance, penalty per period may increase as the project delays more. In §3.4.3, we extend the base model to allow each firm an additional option of “expedite” its task by one period. In §3.4.4, we extend the base model to include multiple suppliers.

3.4.1 The Base Model - The Prisoners’ Dilemma in Project Execution

In this section, we consider the base model (defined by Assumption 3). Our objective is to understand the impact of risk sharing on project duration and cost.

**Assumption 3** At stage 1 of the project network, there is only one task. Each task cannot be expedited but can be delayed by at most one period relative to the optimal duration. If the project is delayed, it is subject to a penalty which is time independent.

In this model, the supplier and manufacturer only have two options (actions) available: “keep” (keeping the original task duration) or “delay” (delaying it by one period). We use K for “keep” and D for “delay” for simplicity. We assume that firm i is responsible for task i for i = 0, 1 where firm 1 (or 0) refers to the supplier (or manufacturer,
respectively). The action set, [supplier’s action, manufacturer’s action], is \{[K, K], [D, D], [K, D], [D, K]\}. When task \(i\) is delayed, firm \(i\) receives a saving of \(s_i\) in terms of its direct cost. When the project is delayed, a penalty of \(p\) per period (the additional indirect cost) is shared by all firms, where firm \(i\) pays \(p_i\) and \(p_0 + p_1 = p\).

To facilitate the comparison between risk sharing and the benchmark, we assume

**Condition 1** Global Optimum - Base Model: \(s_1 < p, s_0 < p\).

Under Condition 1, it is easy to verify that the optimal schedule under the centralized control (in one-firm-does-all model) is [K, K]. This is true because at [D, K], we receive a saving of \(s_1\) from task 1 but must pay a penalty of \(p\). Since \(s_1 < p\), [K, K] outperforms [D, K]. The same logic can be applied to [K, D]. At [D, D], the total saving is \(s_1 + s_2\) but the total penalty is \(2 \times p\). Thus, [K, K] is the optimal solution.

Now we are ready to study the firms’ strategic behaviors under the risk-sharing partnership and their impact on project performance. Note that in this game, the supplier leads and the manufacturer follows. If the project is finished on time, there is no penalty. For every period of the project delay, the supplier pays a penalty of \(p_1\) and the manufacturer pays the rest which is \(p_0\). The firm whichever delays obtains a saving from the direct cost of its own task. For example, if the supplier delays but the manufacturer keeps the original duration of its task, the supplier saves \(s_1\) from its direct cost which brings its pay-off to be \(s_1 - p_1\), and the manufacturer bears a pure penalty of \(p_0\). Figure 3.5 shows the extensive form of the game in the base model.

We derive the following results on the dominant strategies and equilibrium (all proofs are presented in the Appendix unless otherwise mentioned).

**Lemma 3 (Dominant Strategy):** Under Condition 1, when \(s_i < p_i\), “keep” is the dominant strategy for firm \(i, i = 0,1\); when \(s_i > p_i\), “delay” is the dominant strategy for firm \(i, i = 0,1\).
Figure 3.5: The extensive form of the game in the base model.

For simplicity, we use “S” (“M”) to denote the supplier (the manufacturer, respectively).

**Theorem 8 (Equilibrium):** For the base model, under Condition 1, the subgame perfect Nash equilibrium (SPNE) is given by,

<table>
<thead>
<tr>
<th>Case</th>
<th>Condition on S</th>
<th>Condition on M</th>
<th>Optimal strategy for S</th>
<th>M’s best response</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$s_1 &lt; p_1$</td>
<td>$s_0 &lt; p_0$</td>
<td>K</td>
<td>K</td>
</tr>
<tr>
<td>2</td>
<td>$s_1 &gt; p_1$</td>
<td>$s_0 &lt; p_0$</td>
<td>D</td>
<td>K</td>
</tr>
<tr>
<td>3</td>
<td>$s_1 &lt; p_1$</td>
<td>$s_0 &gt; p_0$</td>
<td>K</td>
<td>D</td>
</tr>
<tr>
<td>4</td>
<td>$s_1 &gt; p_1$</td>
<td>$s_0 &gt; p_0$</td>
<td>D</td>
<td>D</td>
</tr>
</tbody>
</table>

Note that by Theorem 8, the optimal solution under one-firm-does-all, [K, K], may still work, that is, be the SPNE under risk sharing. However, for this result to hold, we require a much stronger condition, $s_1 < p_1$ and $s_0 < p_0$ where $p_1 + p_0 = p$, than $s_1 < p$ and $s_0 < p$ (Condition 1). Based on these results, we present the following key insight...
for the base model under the risk-sharing partnership:

**The Prisoners’ Dilemma in Collaborative Projects:** *if \( p_1 < s_1 < p, p_0 < s_0 < p, \) it is in each firm’s best interests to delay although keep benefits the entire project.*

### 3.4.2 Time-dependent Costs - The Supplier’s Dilemma

In this section, we relax the “time-independent cost” assumption in the base model to study the impact of time-dependent penalty costs on the dominant strategies and the Prisoners’ Dilemma. We define the model by Assumption 4.

**Assumption 4** Assumption 3 holds here except that project delay penalties are time dependent, that is, the penalty can be different for different periods of project delay.

Let \( p^1 \) (or \( p^2 \)) be the penalty for the 1\(^{st} \) (the 2\(^{nd} \), respectively) period of project delay; and let \( p^1_i \) and \( p^2_i \) be the penalties shared by firm \( i \), where \( p^1_i + p^0_0 = p^1 \) and \( p^2_i + p^0_0 = p^2 \). We assume the following conditions:

**Condition 2** (1) Global Optimum - Time-Dependent: \( s_1 < p^1, s_0 < p^1 \). (2) Monotonicity - Time-Dependent: \( p^1 < p^2, p^1_1 < p^2_1, p^1_0 < p^2_0 \).

The first condition ensures that \([K, K]\) is the optimal schedule under one-firm-does-all. The second condition comes from the convex and increasing cost functions (see §3.3).

To develop intuitions on the impact of time-dependent penalty costs, we first study an example (see Figure 3.6), where the saving per period is \( s_1 = $600 \) for task 1 and \( s_0 = $1200 \) for task 0. The first (second) period project delay penalty, \( p^1 \) (\( p^2 \)), is \$1600 \((\$2500,\) respectively), where the supplier bears \( p^1_1 = $750 \) and \( p^2_1 = $1100 \), and the manufacturer pays \( p^1_0 = $850 \) and \( p^2_0 = $1400 \). Clearly, Condition 2 is satisfied in this example, and it is in the project’s best interests to keep the original schedule.
We consider the following scenarios under the risk-sharing partnership,

- **Delay-Keep**: firm 1 (the supplier) delays but firm 0 (the manufacturer) keeps. In this scenario, firm 1 saves $600 but must pay $750 with a net loss of $150, while firm 0 must pay $850. Relative to the optimal schedule, the firms’ pay-offs are $(-150, -850)$ and the project’s pay-off is $-1000$.

- **Keep-Delay**: firm 1 keeps but firm 0 delays. In this scenario, the firms’ pay-offs are $(-750, 350)$ and the project’s pay-off is $-400$.

- **Delay-Delay**: both firms delays. In this scenario, the project is delayed by two periods and the firms’ pay-offs are $(-1250, -1050)$. This is the worst scenario for the project as a whole with a total loss of $2300$.

- **Keep-Keep**: both firms keep. The firms’ pay-offs are $(0, 0)$ relative to the optimal schedule.
Figure 3.6 summarizes the action set and the pay-off matrix. Clearly, if the supplier (firm 1) keeps, the manufacturer’s best response is “delay” because its saving exceeds its share of the penalty of the 1st period project delay. However, if the supplier delays, the manufacturer’s best response is “keep” because now its share of the penalty of the 2nd period project delay exceeds its saving. Thus the supplier has to delay (even at a loss) because otherwise, the manufacturer will delay and bring a greater loss to the supplier. We call such a phenomenon the “Supplier’s Dilemma”. It is easy to verify that the SPNE in this example is [D, K].

We now study the model in general. We note that the only difference between this model and the base model in §3.4.1 is that when both firms delay, the delay penalty is $p_1^i + p_2^i$ for firm $i$. Figure 3.7 shows the extensive form of the game between the supplier and the manufacturer.

Figure 3.7: The extensive form of the game in the base model with time-dependent costs.

<table>
<thead>
<tr>
<th>Payoff</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Supplier</td>
</tr>
<tr>
<td>K</td>
</tr>
<tr>
<td>K</td>
</tr>
<tr>
<td>D</td>
</tr>
<tr>
<td>D</td>
</tr>
<tr>
<td>D</td>
</tr>
</tbody>
</table>

We can derive the following results on the dominant strategies and equilibrium.
Lemma 4 (Dominant Strategy): In the base model with time-dependent costs, under Condition 2, when \( s_0 < p_0^1 \), “Keep” is the dominant strategy for the manufacturer; when \( p_0^2 < s_0 \), “Delay” is the dominant strategy for the manufacturer; when \( p_1^1 < p_1^2 < s_1 \), “Delay” is the dominant strategy for the supplier.

Theorem 9 (Equilibrium): For the base model with time-dependent costs, under Condition 2, the subgame perfect Nash equilibrium is given by:

<table>
<thead>
<tr>
<th>Case</th>
<th>Condition on S</th>
<th>Condition on M</th>
<th>Optimal strategy</th>
<th>M’s best response for S</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( s_1 &lt; p_1^1 )</td>
<td>( s_0 &lt; p_0^1 )</td>
<td>( K )</td>
<td>( K )</td>
</tr>
<tr>
<td>2</td>
<td>( s_1 &gt; p_1^1 )</td>
<td>( s_0 &lt; p_0^1 )</td>
<td>( D )</td>
<td>( K )</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>( p_0^1 &lt; s_0 &lt; p_0^2 )</td>
<td>( D )</td>
<td>( K )</td>
</tr>
<tr>
<td>4</td>
<td>( s_1 &lt; p_1^2 )</td>
<td>( s_0 &gt; p_0^2 )</td>
<td>( K )</td>
<td>( D )</td>
</tr>
<tr>
<td>5</td>
<td>( s_1 &gt; p_1^2 )</td>
<td>( s_0 &gt; p_0^2 )</td>
<td>( D )</td>
<td>( D )</td>
</tr>
</tbody>
</table>

Cases 1-2 and 4-5 of Theorem 9 resemble those of Theorem 8 where the firms have dominant strategies. However, case 3 of Theorem 9 is new and unique to time-dependent cost structure: when \( p_0^1 < s_0 < p_0^2 \) (the manufacturer’s saving is in between its penalty of the 1st period project delay and its penalty of the 2nd period project delay, as illustrated in the example), the manufacturer’s best strategy depends on the supplier’s action. If the supplier keeps, the manufacturer will delay; otherwise, the manufacturer will keep. Thus, in this case, the supplier must take the manufacturer’s response into account in making its own decision.

Based on these results, we present the following key insight for the base model with time-dependent costs under the risk-sharing partnership:

The Supplier’s Dilemma: if \( p_0^1 < s_0 < p_0^2 \), the supplier has to delay (even at a loss) to raise the penalty too high for the manufacturer to delay, to avoid a greater loss.
3.4.3 Expediting and Reward - The Coauthors’ Dilemma

In this section, we relax the base model by allowing each firm an additional option: expediting by one period (see Assumption 5). With the new action of “expedite”, the project could be completed earlier than the optimal schedule. The question is, will this happen in equilibrium under risk sharing?

Assumption 5 Assumption 3 holds here except that each task can be expedited by at most one period, and there is a reward per period if the project is expedited.

We use “E” to denote “expedite”. Let $c_0$ (or $c_1$) be the cost of expediting (i.e., the additional direct cost) for task 0 (or 1, respectively). Let $r$ be the reward for the project per period expedited, and $r_0$ and $r_1$ be rewards shared by the firms where $r_1 + r_0 = r$.

With expediting, firms’ pay-off functions are different from previous sections without expediting. Specifically, if the supplier expedites, the action set $[E, K]$ yields $-c_1 + r_1$ for the supplier and $r_0$ for the manufacturer, $[E, D]$ yields $-c_1$ for the supplier and $s_0$ for the manufacturer, and $[E, E]$ yields $-c_1 + 2r_1$ for the supplier and $-c_0 + 2r_0$ for the manufacturer. If the manufacturer expedites, the pay-off functions could be derived in a similar way.

Similar to previous sections, we assume the following conditions:

Condition 3 (1) Global Optimum - Expediting: $s_1 < p$, $s_0 < p$; $r < c_1$, $r < c_0$; $s_1 < c_0$, $s_0 < c_1$. (2) Monotonicity - Expediting: $r < p$; $s_1 < c_1$, $s_0 < c_0$. (3) Risk Sharing - Expediting: $r_1 < p_1$, $r_0 < p_0$.

Condition 3 (Global Optimum) provides a necessary condition for $[K, K]$ to be the optimal schedule under one-firm-does-all. For instance, $[E, K]$ should yield less profit for the entire project than $[K, K]$, which requires $-c_1 + r_1 + r_0 < 0$, and $[E, D]$ should yield less profit for the project than $[K, K]$, which requires $s_0 < c_1$. Condition 3 (Monotonicity) comes from the assumption of convex and increasing indirect cost and
convex and decreasing direct cost (see §3.3). Condition 3 (Risk Sharing) indicates that
the monotonicity condition on the project’s reward and penalty also applies to each
firm’s share of the reward and penalty.

The extensive form of the game is shown in Figure 3.8. For instance, if the supplier
expedites but the manufacturer keeps, the supplier gets an award of $r_1$ but must pay
an expediting cost of $c_1$; the manufacturer gets an award of $r_0$.

Figure 3.8: The extensive form of the game in the base model with expediting and
reward.

We can derive the following results on the dominant strategies and equilibrium.

**Lemma 5 (Dominant Strategy):** In the base model with expediting and reward,
under Condition 3, when $p_i < s_i$, “delay” is the dominant strategy for firm $i$, $i = 1, 0$;
when $s_0 < r_0 < p_0 < c_0$, “keep” is the dominant strategy for the manufacturer.
Lemma 5 differs from Lemma 3 on the conditions for “keep” because we must consider not only “delay” but also “expedite” in this model.

**Theorem 10 (Equilibrium):** For the base model with expediting and reward, under Condition 3, the subgame perfect Nash equilibrium is given by,

<table>
<thead>
<tr>
<th>Case</th>
<th>Condition on S</th>
<th>Condition on M</th>
<th>Optimal strategy M’s best response</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$c_0 &lt; p_0$</td>
<td>$D$</td>
<td>$E$</td>
</tr>
<tr>
<td>2</td>
<td>$s_1 &lt; p_1$</td>
<td>$s_0 &lt; p_0 &lt; c_0$</td>
<td>$K$</td>
</tr>
<tr>
<td>3</td>
<td>$s_1 &gt; p_1$</td>
<td>$s_0 &lt; p_0 &lt; c_0$</td>
<td>$D$</td>
</tr>
<tr>
<td>4</td>
<td>$c_1 &lt; p_1$</td>
<td>$s_0 &gt; p_0$</td>
<td>$E$</td>
</tr>
<tr>
<td>5</td>
<td>$s_1 &lt; p_1 &lt; c_1$</td>
<td>$s_0 &gt; p_0$</td>
<td>$K$</td>
</tr>
<tr>
<td>6</td>
<td>$s_1 &gt; p_1$</td>
<td>$s_0 &gt; p_0$</td>
<td>$D$</td>
</tr>
</tbody>
</table>

Cases 2-3 and 5-6 of Theorem 10 resembles Theorem 8 because in these cases, the expediting cost is greater than the delay penalty either for the manufacturer or for the supplier. Thus, the option of expediting is excluded and the model returns to the base model. When the expediting cost is smaller than the delay penalty, then we have two new cases: the 1st (equilibrium: $[D, E]$) and 4th (equilibrium: $[E, D]$) cases that involve expediting. We shall explain the intuition behind these two new cases as follows.

- **1st case**, $c_0 < p_0$, $[D, E]$ is the equilibrium: In this case, the manufacturer faces a delay penalty that is greater than its expediting cost, and so it would do anything to prevent the project from being delayed. Due to information symmetry, the supplier knows the manufacturer’s weakness, and so would delay regardless of its own cost structure, and earn a net saving without any penalty. Thus, even if the manufacturer expedites its task, the project will not be expedited because the supplier will delay.
An example in the book publishing industry: Let’s consider a coauthor and a lead author working sequentially on a textbook. The coauthor writes a part of the book and must pass on the manuscript to the lead author to complete. The lead author is responsible for the delivery and is very concerned about the deadline. Thus the lead author will do anything possible to finish the book on time. Knowing this, the coauthor will delay as much as what the lead author can catch up without a penalty.

- 4th case, $c_1 < p_1$ and $p_0 < s_0$, [E, D] is the equilibrium: In this case, “delay” is the dominant strategy for the manufacturer (by Lemma 5). In addition, the supplier faces a delay penalty that is greater than its expediting cost, and so the supplier will have to expedite to prevent the project from being delayed.

An example of the academic thesis completion: Let’s consider a PhD student and his/her advisor. The student shall write the PhD thesis and handle it over to the advisor to read and approve. The student needs to graduate and will do anything possible to complete his/her thesis on time. The advisor, on the other hand, is well established and much less concerned. Knowing that the advisor is the bottleneck, the student has to work extra hard in the hope of getting the thesis done on time.

Theorem 10 implies that in the base model with expediting and reward, the project will never be expedited in the equilibrium under the risk-sharing partnership as compared to the optimal schedule. We summarize the results in this section by the following dilemma:

The Coauthors’ Dilemma: Risk sharing cannot expedite the project relative to one-firm-does-all because even if some firms expedite their tasks, other will delay.
Although it is obvious that one-firm-does-all should outperform risk sharing in cost, it is not clear how risk sharing may affect the project duration. Theorem 10 and the Coauthors’ Dilemma reveal a non-trivial and interesting insight, that is, risk sharing tends to increase the project duration relative to one-firm-does-all rather than shortening it. The coauthors’ dilemma holds in the general multi-period setting with time dependent cost structure (Xu, 2014).

### 3.4.4 Multiple Suppliers - The Worst Supplier Dominance

In this section, we extend the base model to include two suppliers at stage 1 to study the impact of parallel tasks. The analysis of a N-supplier system is similar. The model is defined in Assumption 6 where the suppliers play a simultaneous game among themselves in phase 1 anticipating the manufacturer’s response in phase 2 to their aggregated actions. The question is, how does the parallel project network affect the results?

**Assumption 6** *Assumption 3 holds here except that stage 1 has two tasks each conducted by a unique supplier, and the manufacturer can only start its task after both suppliers complete their work.*

We denote supplier 1 (2)’s saving in the direct cost from delay to be $s_1$ ($s_2$) per period. The project penalty shared by the supplier 1 (or 2) is $p_1$ (or $p_2$ respectively) where $p_1 + p_2 + p_0 = p$. We assume the following conditions which are necessary for “keep” to be the optimal solution for all tasks under one-firm-does-all,

**Condition 4** *Global Optimum - Two Suppliers: $s_1 + s_2 < p$, $s_0 < p$.*

Without the loss of generality, we assume that the optimal durations of tasks 1 and 2 are identical (otherwise, the system reduces to the base model as we can ignore the supplier with a shorter duration).
We have the following results on the dominant strategies and equilibrium.

**Lemma 6 (Dominant Strategy):** In the base model with two suppliers, under Condition 4, when \( s_0 < p_0 \), “keep” is the dominant strategy for the manufacturer; when \( s_0 > p_0 \), “delay” is the dominant strategy for the manufacturer. When \( s_i > p_i \), “delay” is the dominant strategy for supplier \( i \).

Lemma 6 differs from Lemma 3 because of the parallel project structure at stage 1 – there is no unilateral condition for a supplier to keep the original duration of its task as the stage 1’s on time performance depends on both suppliers’ actions.

**Theorem 11 (Equilibrium):** For the base model with two suppliers, under Condition 4, the subgame perfect Nash equilibrium is given by,

<table>
<thead>
<tr>
<th>Case</th>
<th>Condition on S</th>
<th>Condition on M</th>
<th>Optimal strategy for S1, S2</th>
<th>M’s best response</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( s_1 &lt; p_1 ) and ( s_2 &lt; p_2 )</td>
<td>( s_0 &lt; p_0 )</td>
<td>K, K</td>
<td>K</td>
</tr>
<tr>
<td>2</td>
<td>( s_1 &gt; p_1 ) or ( s_2 &gt; p_2 )</td>
<td>( s_0 &lt; p_0 )</td>
<td>D, D</td>
<td>K</td>
</tr>
<tr>
<td>3</td>
<td>( s_1 &lt; p_1 ) and ( s_2 &lt; p_2 )</td>
<td>( s_0 &gt; p_0 )</td>
<td>K, K</td>
<td>D</td>
</tr>
<tr>
<td>4</td>
<td>( s_1 &gt; p_1 ) or ( s_2 &gt; p_2 )</td>
<td>( s_0 &gt; p_0 )</td>
<td>D, D</td>
<td>D</td>
</tr>
</tbody>
</table>

Note that with multiple suppliers, the SPNE is no longer unique due to the simultaneous game played among the suppliers in phase 1. We only report Pareto optimum equilibrium here.

Theorem 11 illustrates the impact of the parallel project network on the equilibrium and project performance, that is, the project is more likely to be delayed with multiple suppliers. For the optimal schedule to be the SPNE, we require \( s_1 < p_1 \) and \( s_2 < p_2 \) (i.e., penalty exceeds saving for both suppliers) and \( s_0 < p_0 \). If the saving exceeds penalty for any supplier, all suppliers will have to delay in equilibrium. This observation gives rise to the following key insight:
The Worst Supplier Dominance: if one supplier delays, the other supplier(s) have to follow.

3.5 Information Asymmetry

In this section, we consider information asymmetry for the two-stage project under the risk sharing partnership, where a partner doesn’t know the cost structure of others. Following Harsanyi (2004), we assume that nature draws a type of supplier and manufacturer about their cost structure. More specifically, each player’s type is determined by the realization of a random variable following an exogenous probability distribution, known to all the players. By Gibbons (1992), we further assume that the partners’ types are independent, which means, a partner knows others’ believes about its types and their believes are identical.

In this sequential game under information asymmetry, we have the following sequences of events: Anticipating the manufacturer’s reaction and other suppliers’ actions, the suppliers choose their best actions based on their believes of others’ cost structures. Observing the suppliers’ actions, the manufacturer chooses its action. Note that the manufacturer makes a decision after observing the suppliers’ actions, so effectively, it enjoys an information advantage over the suppliers. We shall identify Bayesian Nash Equilibrium (BNE) - a Nash Equilibrium in a Bayesian game among suppliers.

The impact of information asymmetry is widely studied in the literature, and many previous works show a disadvantage relative to information symmetry. Schieg et al. (2008) comments: “The situation in which one of the two co-operation partners is better informed than the other one is described as asymmetric information. Problems resulting from this are economic disadvantages for one of the parties, the inefficient use of resources, and the resulting losses of welfare.” By Xiang, Zhou, Zhou, and Ye (2012),
“Asymmetric information means that some participants possess information that others do not.” “From the perspective of information economics, asymmetric information gives rise to opportunistic behavior, namely, adverse selection and moral hazards, which is the primary cause of breaking faith in construction market and essentially drives construction project risk.” Bayiz and Corbett (2005) shows that the project performance under the symmetric information is superior to that under asymmetric information with either optimal linear contracts and fixed-price contracts in both serial and parallel cases. Thus, it confirms the prior literature on the potential loss to the project under information asymmetry relative to information symmetry.

In this chapter, we raise the following research questions:

- Comparing to information symmetry, how does information asymmetry affect the partners’ decisions in equilibrium?

- Does information asymmetry always prolong (worsen) the project’s duration relative to information symmetry?

It is easy to see that in the basic model (Section 3.4.1), the dominant strategies and the equilibrium remain the same under information asymmetry because there is a dominant strategy for both supplier and manufacturer in every possible case. Intuitively, in this model, information asymmetry and symmetry yield the same equilibrium for both the supplier and manufacturer, and thus the same project outcome on cost and time. However, the analysis of information asymmetry of the other models in Section 3.4 are much more involved. In the following sections, we shall analyze these models but under information asymmetry, and compare the results to those under information symmetry.
3.5.1 Time-dependent Cost - Revisited

The cost and saving information of each firm is the firm’s private information. We first note that the supplier has a dominant strategy of “delay” when the saving, $s_1$, exceeds the highest penalty, $p_1^2$; that is, $p_1^1 \leq p_1^2 < s_1$ (high saving). If either $s_1 < p_1^1 < p_1^2$ (low saving) or $p_1^1 < s_1 < p_1^2$ (medium saving), the supplier does not have a dominant strategy, then its action may be affected by information asymmetry.

We make the following assumption on the supplier’s belief.

**Assumption 7** The supplier does not know $s_0, p_0^1, p_0^2$, but holds the following believes on the manufacturer’s cost structure,

1. $s_0 < p_0^1 < p_0^2$ with probability $q_0^l$ (superscript “$l$” indicates low saving relative to $p_0^1$ and $p_0^2$).
2. $p_0^1 < s_0 < p_0^2$ with probability $q_0^m$.
3. $p_0^1 < p_0^2 < s_0$ with probability $q_0^h$.

Note $q_0^l + q_0^m + q_0^h = 1$.

The following proposition characterizes the supplier’s equilibrium decision under information asymmetry.

**Proposition 4** In both cases (a) $s_1 < p_1^1 < p_1^2$ and (b) $p_1^1 < s_1 < p_1^2$, the supplier’s equilibrium decision is “delay” if $s_1 > p_1^1 q_0^l + p_1^2 q_0^h$, “keep” otherwise.

Proposition 4 implies that the supplier is more likely to delay in case (b) than (a). This is true because as supplier’s saving increases, it is more likely to delay with the same believes ($q_0^l$ and $q_0^h$) on the manufacturer’s cost structure.

The manufacturer doesn’t have a dominant strategy only when $p_0^1 < s_0 < p_0^2$ (its dominant strategies in other cases are presented in Lemma 4). In this case, the manufacturer can make a decision after observing the supplier’s action; its optimal solution is
“delay” if the supplier keeps; “keep” if the supplier delays, as in information symmetry.

To reconcile the results between information symmetry and asymmetry, we consider a few special cases. When \( q_{10}^m = 1 \), that is, the supplier knows for sure that \( p_{10}^1 < s_0 < p_{20}^2 \), by Theorem 9, supplier’s equilibrium is “delay”. This is consistent to the result of Proposition 4 because \( q_{10}^l = q_{10}^h = 0 \) and thus \( s_1 > p_{10}^l q_{10}^l + p_{10}^h q_{10}^h \). When \( q_{10}^l = 1 \) (\( q_{10}^h = 1 \)), then we return to cases 1 and 2 (4 and 5, respectively) of Theorem 9.

We now compare information symmetry and asymmetry on the project duration. We first note that information asymmetry can prolong project’s duration relative to information symmetry. For instance, in case 4 of Theorem 9 where \( p_{10}^1 < s_1 < p_{10}^2 \) and \( p_{00}^1 < p_{00}^2 < s_0 \), the equilibrium is [K, D] under information symmetry. However, under information asymmetry, supplier does not know \( p_{00}^1 < p_{00}^2 < s_0 \) for sure. By Proposition 4, supplier may choose “delay”. The manufacturer chooses “delay” in this case because it is the dominant strategy. Thus, the project duration could be delayed by one more period in information asymmetry relative to information symmetry.

Interestingly, we also note that information asymmetry can shorten project’s duration relative to information symmetry. For instance, in case 2 of Theorem 9 where \( p_{10}^1 < s_1 < p_{10}^2 \) and \( s_0 < p_{00}^1 < p_{00}^2 \) the equilibrium is [D, K] under information symmetry. However, under information asymmetry, supplier does not know \( s_0 < p_{00}^1 < p_{00}^2 \) for sure. By Proposition 4, supplier may choose “keep”. The manufacturer chooses “keep” in this case because it is the dominant strategy. Thus, the project duration could be shortened in information asymmetry relative to information symmetry.

### 3.5.2 Expediting and Reward - Revisited

The supplier has a dominant strategy of “delay” when \( p_1 < s_1 < c_1 \) (low penalty, high expediting cost) which holds regardless of information status. Information asymmetry may have an impact on the supplier’s decision when (a) \( s_1 < c_1 < p_1 \) (low saving, high
penalty), and (b) \( s_1 < p_1 < c_1 \) (low saving, high expediting cost). These cases exhaust all possible scenarios because \( s_1 < c_1 \) by Condition 3.

We make the following assumption on the supplier’s belief.

**Assumption 8** The supplier does not know \( s_0, r_0, c_0, p_0 \), but holds the following believes on the manufacturer’s cost structure,

1. \( s_0 < r_0 < c_0 < p_0 \) with probability \( q_{l0}^{l1} \).
2. \( r_0 < s_0 < c_0 < p_0 \) with probability \( q_{l0}^{l2} \).
3. \( s_0 < r_0 < p_0 < c_0 \) with probability \( q_{l0}^{m1} \).
4. \( r_0 < s_0 < p_0 < c_0 \) with probability \( q_{l0}^{m2} \).
5. \( p_0 < s_0 < c_0 \) with probability \( q_{l0}^{h} \).

Note \( q_{l0}^{l1} + q_{l0}^{l2} + q_{l0}^{m1} + q_{l0}^{m2} + q_{l0}^{h} = 1 \).

For the ease of exposition, let \( q_{l0}^{l1} + q_{l0}^{l2} = q_{l0}^{l} \) and \( q_{l0}^{m1} + q_{l0}^{m2} = q_{l0}^{m} \).

We have the following proposition on the supplier’s equilibrium decision under information asymmetry.

**Proposition 5** A pairwise comparison of the supplier’s payoff functions among all decisions shows:

1. The supplier chooses “delay” over “keep” if \( s_1 > p_1 (1 - q_{l0}^{l}) \), “keep” over “delay” otherwise;
2. It chooses “expedite” over “delay” if \( r_1 (q_{l0}^{l1} + q_{l0}^{m1}) + p_1 (1 + q_{l0}^{h} - q_{l0}^{l}) > c_1 + s_1 \), “delay” over “expedite” otherwise;
3. It chooses “expedite” over “keep” if \( c_1 < r_1 (q_{l0}^{l1} + q_{l0}^{m1}) + p_1 q_{l0}^{h} \), “keep” over “expedite” otherwise.
Claim 2 of Proposition 5 is less likely to hold in case (b) than in case (a) because of the larger $c_1$ in (b) than in (a). In case (b), $s_1 < p_1 < c_1$ implies $c_1 > r_1 q_0^{d1} + r_1 q_0^{m1} + p_1 q_0^h$.

By Claim 3 of Proposition 5, “keep” is always preferred by the supplier over “expedite”. Thus, in case (b), the supplier can choose either “keep” or “delay” but not “expedite”, while in case (a), the supplier can choose all three options.

The manufacturer has a dominant strategy of “keep” (“delay”) when $s_0 < r_0 < p_0 < c_0$ ($r_0 < p_0 < s_0 < c_0$, respectively) regardless of the information status. In other cases, the manufacturer does not have a dominant strategy but make decisions accordingly after observing the supplier’s action. We consider three cases and make the following observations (the analysis is straightforward, proof is omitted):

- $s_0 < r_0 < c_0 < p_0$: if the supplier expedites (or keeps or delays), the manufacturer should keep (or keep or expedite, respectively).
- $r_0 < s_0 < c_0 < p_0$: if the supplier expedites (or keeps or delays), the manufacturer should delay (or keep or expedite, respectively).
- $r_0 < s_0 < p_0 < c_0$: if the supplier expedites (or keeps or delays), the manufacturer should delay (or keep or keep, respectively).

We now reconcile the results under information symmetry and asymmetry. We observe that for the extreme value of the belief probabilities, the results under information asymmetry return to those under symmetry. Let’s consider the extreme probability of 1 for $q_0^{l1}, q_0^{l2}, q_0^{m1}, q_0^{m2}$, and $q_0^h$. When $q_0^l = q_0^{l1} + q_0^{l2} = 1$, we can easily show that the supplier should choose “delay” by Proposition 5 which matches the result of case 1 in Theorem 10. When $q_0^m = q_0^{m1} + q_0^{m2} = 1$, it can be shown that the supplier should choose “keep” (or “delay”) depending on $s_1 < (>)p_1$ by Proposition 5, which matches the results of cases 2 and 3 in Theorem 10. Lastly, when $q_0^h = 1$, it’s the best for the supplier to choose “keep”, “expedite” or “delay” depending on $s_1 < (>)p_1 < (>)c_1$.
which matches the results of cases 4, 5 and 6 in Theorem 10.

Similar to the model in Section 3.5.1, we find that information asymmetry may either prolong or shorten the project duration relative to information symmetry. For instance, in case 4 of Theorem 10 where \( s_1 < c_1 < p_1 \) and \( p_0 < s_0 < c_0 \), the equilibrium is \([E, D]\) under information symmetry. However, under information asymmetry, supplier does not know \( p_0 < s_0 < c_0 \) for sure and so may choose “keep” or “delay” fantasizing that the manufacturer may expedite or keep, but manufacturer actually chooses “delay” (the dominant strategy). Thus, the project duration would be delayed in information asymmetry relative to information symmetry.

Information asymmetry can maintain or even shorten project’s duration relative to information symmetry. For instance, in case 1 of Theorem 10 where \( s_0 < r_0 < c_0 < p_0 \), the equilibrium is \([D, E]\) under information symmetry. Under information asymmetry, however, the supplier does not know that the high penalty cost of the manufacturer, i.e., \( s_0 < r_0 < c_0 < p_0 \), for sure. Supplier may choose “keep” or “expedite” guessing that the manufacturer doesn’t care. Observing these actions of the supplier, the manufacturer chooses “keep”. Thus, the project duration would be maintained or reduced under information asymmetry relative to information asymmetry. Note that with the right belief, the project may be even completed earlier than the optimal schedule under information asymmetry.

It is now perhaps adequate to compare our results with that of Bayiz and Corbett (2005) which considers the impact of information asymmetry in project subcontracting. It shows that information asymmetry between the client and contractors always introduces inefficiency to the project as compared to information symmetry. In particular, if the client has a better information about the contractors, s/he can better optimize for the project by ruling out the opportunistic behaviors of the contractors due to hidden information. In contrast, we consider collaboration and partnership where information
asymmetry among the partners does not always introduce inefficiencies to the project as compared to information symmetry. Specifically, if a partner has better information about others, s/he can take advantage of others to maximize its own benefits, which may or may not be in the best interest of the project.

3.5.3 Multiple Suppliers - Revisited

As in Section 3.4.4, the manufacturer has dominant strategies of “keep” (or “delay”) if \( s_0 < p_0 \) (or \( p_0 < s_0 \), respectively). The suppliers play a simultaneous game under information asymmetry. We shall characterize the Bayesian Nash Equilibrium (BNE) of this game.

We make the following assumption on the suppliers’ believes on each others’ cost structures.

**Assumption 9** Supplier 1 (2) does not know \( s_2, p_2 \) \((s_1, p_1)\), but holds the following believes on the other supplier. The belief on supplier 1:

1. \( s_1 < p_1 \) with probability \( q_1^l \).
2. \( s_0 > p_1 \) with probability \( q_1^h \).

The belief on supplier 2:

1. \( s_2 < p_2 \) with probability \( q_2^l \).
2. \( s_2 > p_2 \) with probability \( q_2^h \).

Note \( q_1^l + q_1^h = 1, q_2^l + q_2^h = 1 \), the believes on supplier 1 are identical among all other suppliers, and also known to supplier 1.

Bayesian Nash Equilibrium is simply a Nash Equilibrium in a Bayesian game. We can think of each type of player \( i \) as a separate player. From supplier 1’s perspective,
the supplier 2 has two types: low type \((s_i < p_i)\) and high type \((s_i > p_i)\). Each type can take two actions: “keep” and “delay”, which means that each supplier’s strategy is a function of its type (low, high). Note that a high type supplier takes only delay as a dominant strategy. Hence, each supplier now has two possible pure strategies: \(\begin{bmatrix} K \\ D \end{bmatrix}\) and \(\begin{bmatrix} K \\ K \end{bmatrix}\), where the upper (lower) argument in the bricks represents the action taken if the supplier is a low (high) type.

We have the following proposition on the Bayesian Nash Equilibrium (BNE) among the suppliers.

**Proposition 6** If \(q_{l1} \neq 0\) and \(q_{l2} \neq 0\), the conditions for \{\(\begin{bmatrix} K \\ D \end{bmatrix}\), \(\begin{bmatrix} K \\ K \end{bmatrix}\)\} to be BNE are

\[
\frac{p_{1}}{s_{1}} > \frac{q_{l1}}{q_{l2}} = \frac{1}{q_{l2}} \quad \text{and} \quad \frac{p_{2}}{s_{2}} > \frac{1}{q_{l1}}.
\]

In comparison, the conditions for \(\begin{bmatrix} K \\ K \end{bmatrix}\) to be equilibrium under information symmetry are \(\frac{p_{1}}{s_{1}} > 1\) and \(\frac{p_{2}}{s_{2}} > 1\). Because \(\frac{1}{q_{l2}}\) and \(\frac{1}{q_{l1}}\) are greater than 1, the conditions specified in Proposition 6 for information asymmetry are stronger than the conditions for information symmetry. Thus information asymmetry trends to prolong the project duration relative to information symmetry with the same cost structures \((s_i, p_i, i = 1, 2)\). In other words, to keep the project duration under information asymmetry the same as information symmetry, we need the penalty cost to be much greater than the saving.

Intuitively, in the case of the suppliers being a low type and a high type, the low type may choose “keep” based on its false belief that the other supplier is also low type. But this does not shorten the project duration relative to information symmetry because the other supplier chooses “delay”. In case of both suppliers being low type, a supplier may choose “delay” based on its false belief that the other supplier is high type. That is why information asymmetry may prolong project duration relative to information symmetry.

Information asymmetry can reduce to information symmetry in some special cases.
For instance, if \( q_1^l = q_2^l = 1 \), then \( \begin{bmatrix} K & K \\ D & D \end{bmatrix} \) must be a BNE and Pareto because 
\(-s_1 q_1^l + p_1 q_1^l q_2^l = -s_1 + p_1 > 0 \) and 
\(-s_2 q_2^l + p_2 q_1^l q_2^l = -s_2 + p_2 > 0 \) (by Proposition 6), which reduces to case 1 in Theorem 11. If \( q_1^l = 0 \), supplier 2 chooses \( \begin{bmatrix} D \\ D \end{bmatrix} \) because 
\(-s_2 q_2^l + p_2 q_1^l q_2^l < 0 \). Similar argument applies to the case of \( q_2^l = 0 \). Hence if \( q_1^l = 0 \) or \( q_2^l = 0 \), \( \begin{bmatrix} K & K \\ D & D \end{bmatrix} \) cannot be BNE but \( \begin{bmatrix} D \\ D \end{bmatrix} \) is the only BNE, which reduces to case 2 in Theorem 11.

The manufacturer’s equilibrium decisions are the same regardless of the information status. By Lemma 3, the manufacturer has a dominant strategy of “keep” when \( s_0 < p_0 \) and “delay” when \( p_0 < s_0 \). Thus, it doesn’t need to know the suppliers’ cost structures.

3.6 Linking Theory to Practice

As mentioned in §3.1, Kotha, Olesen, Nolan, and Condit (2005) made the following comments on 787, “Boeing had asked its structural suppliers to fund their own research and development (a first for a Boeing project) for the 787 project. This way, Boeing believed suppliers were likely to have a greater financial incentive to minimize their cost and, at the same time, assist Boeing market the new plane.”

Our theoretical results confirm “the incentive to minimize cost”, and take one step further to show that the incentive of cost minimization can be so much stronger than Boeing’s anticipation as to be against the best interest of the project. In fact, the risk sharing partnership can create an incentive trap by encouraging the firms to save their own cost at the expense of the project! This chapter characterizes some of the mechanisms for such wrong incentives depending on project networks, cost structures and information status; of which, the Supplier’s Dilemma represents a surprising result: the supplier may delay even if \( s_1 < p_1 \) (the saving on the direct cost from delay is less than the penalty on the indirect cost of delay)!
Linking theory to practice, we can see, much clearly, what went wrong in the development of 787. Some management issues of the suppliers in the 787 development are summarize below (Zhao (2016)).

- Lack of testing and Q/A equipment and personnel.
- Use low-wage, train-on-the-job workers.
- Inability to attract competent technicians, have to use novice student inspectors.
- Workers lack of training & FAA compliance.
- Incomplete documentation or lost in transit.

Clearly, these actions (intended or not) helped the suppliers to save their own cost but contributed significantly to the delay, which led to huge losses for the program and all firms. Thus, the 787 program serves as an example of the Prisoners’ Dilemma in the base model (§3.4.1).

The “Worst Supplier Dominance” insight of the base model with multiple suppliers (§3.4.4) is manifested in 787’s Ramp-up Issues. By Xu and Zhao (2011): “Boeing’s second FAL was encouraged by customers, but the real bottleneck seemed to be the supply chain. While Alenia and Kawasaki were investing in new factory and/or production equipment, Spirit, Vought and Global Aeronautica showed no investment in facilities or equipment for the ramp-up up-to the point of October 2008.” Obviously, some firms were waiting for others to take actions first before starting to build up capacity in parallel for the 787 program.

Theoretical results in §3.5.2 under information asymmetry provide a plausible explanation for the extensive traveled work of the suppliers and Boeing’s slow progress to fix them at the final assembly line (FAL) in the 787 development. Suppliers may falsely believe that Boeing cares about the program delay (because ultimately, it is Boeing’s
plane), they delayed to save their cost and hoped Boeing will catch up (which didn’t happen, at least initially); this resulted in prolonged delays. If the supplier believed the opposite (Boeing won’t make up their delays), they may have worked much harder and avoid a majority of 787 delays. Clearly, the suppliers’ believes have a significant impact; and Boeing should know what believes to shape up its suppliers.

Despite the disasters in the 787 program, the trend of outsourcing and collaboration is irreversible in the aerospace and defense industries. In April, 2008, after the first few major delays were announced, McNerney, Boeing’s CEO, admitted that Boeing has had problems executing its new strategy (outsourcing and risk sharing) but he sees no reason for change. He reiterated his commitment to Boeing’s global sourcing approach: “The global partnership model of the 787 remains a fundamentally sound strategy. It makes sense to utilize technology and technical talent from around the world. It makes sense to be involved with the industrial bases of counties that also support big customers of ours. But we may have gone little too far, too fast ...” Thus one key question is, how to manage collaborative projects under the risk sharing partnership?

Knowing the incentive issues and mechanisms in collaborative projects under risk sharing (either co-developing a plane or coauthoring a book) can help the firms to take proactive measures in practice to improve the project outcome. One such measure is the practice of sending on-site teams (consisting of liaison and industrial engineers, inspectors, tooling and manufacturing experts) to suppliers’ sites - if you cannot motivate them, you have to watch them closely. This measure is supported by Scott Carson, who heads Boeing’s commercial airplane unit, after the disasters took place in 787: “In addition to oversight, you need insight into what’s actually going on in those factories.”

Kotha, Olesen, Nolan, and Condit (2005) provides more details: “Pat Shanahan [head of 787 program] took action to comply with McNerney’s insistence that the 787-team be more aggressive with suppliers by sticking their noses into suppliers’ operations,
including stationing Boeing employees in every major supplier’s factory.”

The second practical measure to battle against the incentive issues is to select the right partner in the first place. As demonstrated by our theoretical results, risk sharing may work as well as the benchmark of one-firm-does-all under certain conditions. The theory is supported by the 787 practice as not all partners of this program delayed because of information asymmetry and the delay penalties being higher than savings for them. In the applications of coauthorship, the theory implies that it is wise to collaborate with tenure-track faculty and admit PhD students who are not only talented but also motivated.

The third practical measure is to help your partners to shape up proper believes. For example, as a system integrator (or book editor), the believes that you want to help your suppliers (or coauthors) to shape up may be that you care about the project but won’t make up their delays, as shown by our theoretical results in Proposition 8 under information asymmetry in §3.5.2.

3.7 Conclusion

In this chapter, we consider collaborative projects for which the workload and outcome are shared by multiple firms. Despite the “positive” connotation, we show that collaboration under the risk-sharing partnership may negatively affect project performance by distorting the firms’ incentives and encouraging deliberate delays and cost overruns, contrary to popular believes in practice. Linking theory to practice, we point out the practical implications of the theory in examples of airplane development and coauthorships. Understanding the incentive issues and the mechanisms through which the incentive issues lead to suboptimal project performance, we make practical suggestions in the areas of supplier control and monitoring, partner selection, and believe shaping, which may help to regulate the firms’ gaming behaviors and achieve better outcomes.
for the project as a whole.

To combat the incentive issues, some chapters suggested that the manufacturer should charge penalty costs for suppliers’ delays. While the suggestion sounds intuitive, it arouses two problems: (1) if the manufacturer asks the suppliers to pay for their delays, then the manufacturer should also pay the suppliers for its own delay. (2) How much to pay to ensure fairness and efficiency under information asymmetry is a challenging and open question.

The research bridging supply chain, economics and project management literature promises to be fruitful to both practitioners and academicians because of the high impact on practice, and the potential of making exciting theoretical discoveries by integrating these rich bodies of literature. There are many questions left to be answered in this area, for instance, signaling - when would a partner reveal its type to others prior to choosing their actions? How would environmental uncertainty affect the results? What if some partners (e.g., the manufacturer) has more bargaining power than others? These questions are outside of the scope of this chapter, we shall leave them to future studies.

3.8 Appendix: Proofs and Technical Details

Proof of Lemma 3

For the supplier with $s_1 < p_1$, if the manufacturer chooses “keep”, then $0 > s_1 - p_1$ and so the supplier will choose “keep”; if the manufacturer chooses “delay”, then $-p_1 > s_1 - 2p_1$ so that the supplier will choose “keep” as well. Thus, the supplier has a dominant strategy of “keep” when $s_1 < p_1$. Similarly, we can prove that when $s_1 > p_1$, “delay” is the dominant strategy for the supplier.

For the manufacturer with $s_0 < p_0$, if the supplier chooses “keep”, then $0 > s_0 - p_0$ and so the manufacturer will choose “keep”; if the supplier chooses “delay”, then $-p_0 >$
$s_0 - 2p_0$ so that the manufacturer will choose “keep” as well. Thus, the manufacturer has a dominant strategy of “keep” when $s_0 < p_0$. Similarly, we can prove that when $s_0 > p_0$, “delay” is the dominant strategy for the manufacturer. □

**Proof of Theorem 8**

This theorem is a straightforward result of Lemma 3. □

**Proof of Lemma 4**

For the manufacturer with $s_0 < p_0^1$, if the supplier chooses “keep”, then $0 > s_0 - p_0^1$ and so the manufacturer will choose “keep”; if the supplier chooses “delay”, then $-p_0^1 > s_0 - p_0^1 - p_0^2$ and so the manufacturer will choose “keep” as well. Thus, the manufacturer has a dominant strategy of “keep” when $s_0 < p_0^1$. Similarly, we can prove that when $s_0 > p_0^2$, “delay” is the dominant strategy for the manufacturer.

When the supplier’s cost structure lies in $p_1^1 < p_1^2 < s_1$, the manufacturer can be one of the three cases:

1. $s_0 < p_0^1 < p_0^2$
2. $p_0^1 < s_0 < p_0^2$
3. $p_0^1 < p_0^2 < s_0$

In case 1, when the supplier chooses “keep”, the manufacturer chooses “keep” since $s_0 < p_0^1$, and when the supplier chooses “delay”, the manufacturer chooses “keep” as well since $s_0 < p_0^2$, so based on the manufacturer’s corresponding reactions, the supplier choose “delay” since $p_1^1 < s_1$.

In case 2, when the supplier chooses “keep”, now the manufacturer chooses “delay” since $p_0^1 < s_0$, and when the supplier chooses “delay”, the manufacturer chooses “keep” since $s_0 < p_0^2$, so based on the manufacturer’s corresponding reactions, the supplier choose “delay” since $0 < s_1$. 
In case 3, when the supplier chooses “keep”, the manufacturer chooses “delay” since $p_{10} < s_0$, and when the supplier chooses “delay”, the manufacturer chooses “delay” as well since $p_{20}^2 < s_0$, so based on the manufacturer’s corresponding reactions, the supplier choose “delay” since $p_{11}^2 < s_1$.

In summary, when the supplier has the cost structure as $p_{11}^1 < p_{11}^2 < s_1$, the supplier’s best strategy is to choose “delay” regardless of the manufacturer’s cost structure, so it’s the dominant strategy for the supplier. □

**Proof of Theorem 9**

Lemma 4 implies,

- when $s_1 > p_{11}^1$ and $s_0 < p_{10}^1$, the supplier has a dominant strategy of “delay” and the manufacturer has a dominant strategy of “keep”.
- when $s_1 > p_{12}^2$ and $s_0 > p_{20}^2$, the supplier has a dominant strategy of “delay” and the manufacturer has a dominant strategy of “delay”.

When $s_1 < p_{11}^1$ and $s_0 < p_{10}^1$, if the supplier chooses “keep”, then the manufacturer will choose “keep” as $0 > s_0 - p_{10}^1$; if the supplier chooses “delay”, then the manufacturer will choose “keep” as $-p_{10}^1 > s_0 - p_{10}^1 - p_{20}^2$. The former strategy gives the supplier a higher pay-off (0) than the latter strategy ($s_1 - p_{11}^1$) and thus the supplier will choose “keep” and then the manufacturer will choose “keep”.

When $p_{10}^1 < s_0 < p_{20}^2$, if the supplier chooses “keep”, then the manufacturer will choose “delay” as $s_0 > p_{10}^1$; if the supplier chooses “delay”, then the manufacturer will choose “keep” as $s_0 < p_{20}^2$. The latter strategy gives the supplier a higher pay-off ($-p_{11}^1$) than the former strategy ($s_1 - p_{11}^1$) and thus the supplier will choose “delay” and then the manufacturer will choose “keep”.

When $p_{10}^1 < s_0 < p_{20}^2$, the manufacturer has the dominant strategy of “delay”. Since $-p_{11}^1 > s_1 - p_{11}^1 - p_{12}^2$, the supplier will choose “keep”. □
**Proof of Lemma 5**

When \( s_0 > p_0 \), we know that \( s_0 > r_0 \) and \( r_0 < c_0 \) from Condition 3. If the supplier chooses “expedite” or “keep”, the manufacturer always gets the highest pay-off if it delays. If the supplier chooses “delay”, because \( p_0 < s_0 < c_0 \), “delay” yields the highest pay-off for the manufacturer. Thus, the manufacturer has a dominant strategy of “delay” in this scenario. By a similar logic, we can prove that when \( s_0 < r_0 < p_0 < c_0 \), “keep” is the dominant strategy for the manufacturer.

When the supplier’s cost structure lies in \( r_1 < p_1 < s_1 < c_1 \), the manufacturer can be one of the five cases:

1. \( s_0 < r_0 < c_0 < p_0 \)
2. \( r_0 < s_0 < c_0 < p_0 \)
3. \( s_0 < r_0 < p_0 < c_0 \)
4. \( r_0 < s_0 < p_0 < c_0 \)
5. \( r_0 < p_0 < s_0 < c_0 \)

In case 1, when the supplier chooses “expedite”, the manufacturer chooses “keep”, and when the supplier chooses “keep”, the manufacturer chooses “keep” as well, and lastly, when the supplier chooses “delay”, the manufacturer chooses “expedite”. So, based on the manufacturer’s corresponding reactions, the supplier choose “delay”.

In case 2, when the supplier chooses “expedite”, the manufacturer chooses “delay”, and when the supplier chooses “keep”, the manufacturer chooses “keep”, and lastly, when the supplier chooses “delay”, the manufacturer chooses “expedite”. So, based on the manufacturer’s corresponding reactions, the supplier choose “delay”.

In case 3, when the supplier chooses “expedite”, the manufacturer chooses “keep”, and when the supplier chooses “keep”, the manufacturer chooses “keep” as well, and
lastly, when the supplier chooses “delay”, the manufacturer also chooses “keep”. So, based on the manufacturer’s corresponding reactions, the supplier choose “delay”.

In case 4, when the supplier chooses “expedite”, the manufacturer chooses “delay”, and when the supplier chooses “keep”, the manufacturer chooses “keep”, and lastly, when the supplier chooses “delay”, the manufacturer chooses “keep”. So, based on the manufacturer’s corresponding reactions, the supplier choose “delay”.

In case 5, when the supplier chooses “expedite”, the manufacturer chooses “delay”, and when the supplier chooses “keep”, the manufacturer chooses “delay” as well, and lastly, when the supplier chooses “delay”, the manufacturer also chooses “delay”. So, based on the manufacturer’s corresponding reactions, the supplier choose “delay”.

In summary, when the supplier has the cost structure as \( r_1 < p_1 < s_1 < c_1 \), the supplier’s best strategy is to choose “delay” regardless of the manufacturer’s cost structure, so it is the dominant strategy for the supplier.

Proof of Theorem 10

All potential actions are listed below:

<table>
<thead>
<tr>
<th>S</th>
<th>M</th>
<th>S’s Pay-off</th>
<th>Conditions</th>
<th>M’s Best Response</th>
<th>M’s Pay-off</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>E</td>
<td>2r_0 - c_0</td>
<td></td>
<td>K</td>
<td>r_1 - c_1</td>
</tr>
<tr>
<td></td>
<td>K</td>
<td>r_0</td>
<td>if ( r_0 &gt; s_0 )</td>
<td>K</td>
<td>r_1 - c_1</td>
</tr>
<tr>
<td></td>
<td>D</td>
<td>s_0</td>
<td>if ( r_0 &lt; s_0 )</td>
<td>D</td>
<td>-c_1</td>
</tr>
<tr>
<td></td>
<td>E</td>
<td>r_0 - c_0</td>
<td></td>
<td>K</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>K</td>
<td>0</td>
<td>if ( p_0 &gt; s_0 )</td>
<td>K</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>D</td>
<td>s_0 - p_0</td>
<td>if ( p_0 &lt; s_0 )</td>
<td>D</td>
<td>-p_1</td>
</tr>
<tr>
<td></td>
<td>E</td>
<td>-c_0</td>
<td>if ( p_0 &gt; c_0 )</td>
<td>K</td>
<td>s_1</td>
</tr>
<tr>
<td></td>
<td>K</td>
<td>-p_0</td>
<td>if ( p_0 &lt; c_0 )</td>
<td>K</td>
<td>s_1 - p_1</td>
</tr>
<tr>
<td></td>
<td>D</td>
<td>s_0 - 2p_0</td>
<td>if ( p_0 &lt; s_0 )</td>
<td>D</td>
<td>s_1 - 2p_1</td>
</tr>
</tbody>
</table>

- When \( p_0 < s_0 \), “delay” is the dominant strategy for the manufacturer by Lemma 5. The supplier’s pay-off is \(-c_1\) with “expedite”, \(-p_1\) with “keep”, and \(s_1 - 2p_1\) with “delay”. We consider three cases:
- (a) When $p_1 > c_1$, the supplier’s optimal strategy is “expedite” because $c_1 > s_1$ by Condition 3(2) and so $-c_1$ is the largest payoff.

- (b) When $s_1 < p_1 < c_1$, the supplier’s optimal strategy is “keep”.

- (c) When $p_1 > c_1$, the supplier’s optimal strategy is “delay”.

- When $s_0 < p_0 < c_0$ and $r_0 > s_0$, “keep” is the dominant strategy for the manufacturer by Lemma 5. The supplier’s pay-off is $r_1 - c_1$ with “expedite”, 0 with “keep”, and $s_1 - p_1$ with “delay”. We consider two cases:

  - (a) When $p_1 > s_1$, the supplier’s optimal strategy is “keep” because $r_1 < c_1$ by Condition 3(1).

  - (b) When $p_1 < s_1$, the supplier’s optimal strategy is “delay” because $r_1 < c_1$.

- When $s_0 < p_0 < c_0$ and $r_0 < s_0$, there is no dominant strategy for the manufacturer. If the supplier chooses “expedite”, the manufacturer will choose “delay”. If the supplier chooses “keep” or “delay”, the manufacturer will choose “keep”. Thus, the supplier’s pay-off is $-c_1$ with “expedite”, 0 with “keep”, and $s_1 - p_1$ with “delay”.

  - (a) When $p_1 > s_1$, the supplier’s optimal strategy is “keep”.

  - (b) When $p_1 < s_1$, the supplier’s optimal strategy is “delay”.

- When $p_0 > c_0$ and $r_0 > s_0$, by $c_0 > s_0$ (Condition 3(2)) we obtain $p_0 > s_0$. If the supplier chooses “expedite”, the manufacturer will choose “keep”. If the supplier chooses “keep”, the manufacturer will choose “keep”. If the supplier chooses “delay”, the manufacturer will choose “expedite”. (Note: the manufacturer will do whatever it could to prevent project delay.) Given the manufacturer’s optimal response, the supplier’s pay-off is $r_1 - c_1$ with “expedite”, 0 with “keep”, and $s_1$
with “delay”. Since \( r_1 < c_1 \) by Condition 3(1), the supplier’s optimal strategy is “delay”.

- When \( p_0 > c_0 \) and \( r_0 < s_0 \), by \( c_0 > s_0 \) (Condition 3(2)) we obtain \( p_0 > s_0 \). If the supplier chooses “expedite”, the manufacturer will choose “delay”. If the supplier chooses “keep”, the manufacturer will choose “keep”. If the supplier chooses “delay”, the manufacturer will choose “expedite”. (Note: the manufacturer will do whatever he could to prevent delay.) Given the manufacturer’s optimal response, the supplier’s pay-off is \(-c_1\) with “expedite”, \(0\) with “keep”, and \(s_1\) with “delay”. Clearly, the supplier’s optimal strategy is “delay”.

Summarizing all cases, we have proved the theorem. \(\square\)

**Proof of Lemma 6**

The extensive form of the game is shown in Figure 3.9.

By Lemma 3, the first two results are immediate, that is, when \( s_0 < p_0 \), “keep” is the dominant strategy for the manufacturer; when \( s_0 > p_0 \), “delay” is the dominant strategy for the manufacturer.

When \( s_1 > p_1 \), an enumerating over all options of supplier 2 and the manufacturer finds that supplier 1 archives the highest pay-off when it delays. \(\square\)

**Proof of Theorem 11**

By Lemma 6, as long as one of the suppliers has a dominant strategy of “delay”, the other has to delay as well. Otherwise, it suffers a pure penalty. Combining the dominant strategies leads to the theorem. \(\square\)

**Remarks**: With two suppliers, the SPNE is no longer unique due to the simultaneous game played among the suppliers in phase 1. For instance, when \( s_0 < p_0 \), the manufacturer keeps its original task duration, and the pay-off matrix for suppliers 1 and 2 is
Figure 3.9: The extensive form of the game in the base model with multiple suppliers.

\[
\begin{array}{ccc}
\text{Payoff} & \text{Supplier 1} & \text{Supplier 2} & \text{Manufacturer} \\
\hline \\
\text{K} & 0 & 0 & 0 \\
\text{D} & -p_1 & -p_2 & s_0 - p_0 \\
\text{K} & -p_1 & s_2 - p_2 & -p_0 \\
\text{D} & -2p_1 & s_2 - 2p_2 & s_0 - 2p_0 \\
\text{K} & s_1 - p_1 & -p_2 & -p_0 \\
\text{D} & s_1 - 2p_1 & -2p_2 & s_0 - 2p_0 \\
\text{D} & s_1 - p_1 & s_2 - p_2 & -p_0 \\
\text{D} & s_1 - 2p_1 & s_2 - 2p_2 & s_0 - 2p_0 \\
\end{array}
\]

Clearly, if \( s_1 < p_1 \) and \( s_2 < p_2 \), both \([K, K]\) and \([D, D]\) are SPNE. We only report \([K, K]\) here because it is Pareto optimal but \([D, D]\) is not.

**Proof of Proposition 4**

For the supplier with (a) \( s_1 < p_1^1 < p_1^2 \) and (b) \( p_1^1 < s_1 < p_1^2 \), if supplier chooses “keep”, then the manufacturer has two options: “keep” or “delay”.

If \( s_0 < p_0^1 < p_0^2 \) case applies to the manufacturer, then the manufacturer chooses “keep”, and the supplier’s payoff is 0. If \( p_0^1 < s_0 < p_0^2 \) case applies to manufacturer, the manufacturer chooses “delay”, and the supplier’s payoff is \(-p_1^1\). If \( p_0^1 < p_0^2 < s_0 \)
case applies to the manufacturer, the manufacturer chooses “delay”, and the supplier’s payoff is $-p_1^1$.

In the supplier’s view, her expected payoff under her believes is $-p_1^1(q_0^m + q_0^h)$. If supplier choose “keep”, its expected pay-off is $(s_1 - p_1^1)(q_0^d + q_0^m) + (s_1 - p_1^1 - p_1^2)q_0^h$ by a similar analysis.

Thus, the supplier must choose “delay” over “keep” when $s_1 > p_1^1 q_0^h + p_1^2 q_0^h$. □

Proof of Proposition 5

Similar to the Proposition 4, if supplier chooses “keep”, the supplier’s expected payoff is $(-p_1)q_0^h$. If supplier chooses “delay”, the supplier’s expected payoff is $(s_1 q_0^d + (s_1 - p_1)q_0^m + (s_1 - 2p_1)q_0^h$. If supplier chooses “expedite”, the supplier’s expected payoff is $(r_1 - c_1)q_0^1 + (r_1 - c_1)q_0^m + (r_1 - c_1)q_0^h$.

Thus the condition that the supplier must choose “delay” over “keep” is $s_1 > p_1(1 - q_0^d)$. Similarly, the condition that the supplier must choose “expedite” over “delay” is $r_1(q_0^d + q_0^m) + p_1(1 + q_0^h - q_0^d) > c_1 + s_1$. Lastly, the condition that the supplier must choose “expedite” over “keep” is $c_1 < r_1(q_0^d + q_0^m) + p_1 q_0^d$. □

Proof of Proposition 6

If manufacturer chooses “keep”, the payoffs of supplier 1 and 2 are given by:

$$\begin{bmatrix} D \\ D \end{bmatrix}$$

If manufacturer chooses “delay”, the payoffs of supplier 1 and 2 are given by:

$$\begin{bmatrix} K \\ D \end{bmatrix}$$

$$\begin{bmatrix} D \\ D \end{bmatrix}$$

$$\begin{bmatrix} D \\ D \end{bmatrix}$$

{ } is always a BNE because $-s_1 q_1^d < 0$. \{ } can be a BNE when $-s_1 q_1^d + p_1 q_1^1 q_2^d > 0$ and $-s_2 q_2^d + p_2 q_1^1 q_2^d > 0$.

Thus, if $q_1^d \neq 0$ and $q_2^d \neq 0$, the conditions for \{ } to be BNE are $\frac{p_1}{s_1} > \frac{q_1^d}{q_1^1 q_2^d} = \frac{1}{q_2^d}$ and $\frac{p_2}{s_2} > \frac{1}{q_1^d}$.

If manufacturer chooses “delay”, the payoffs of supplier 1 and 2 are given by:
<table>
<thead>
<tr>
<th>Supplier 1 \ Supplier 2</th>
<th>( K )</th>
<th>( D )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K )</td>
<td>(-s_1 q_1^1 + p_1 q_1^1 q_2^2 + (s_1 - 2p_1))</td>
<td>(-s_1 q_1^1 + (s_1 - 2p_1), s_1 - 2p_2)</td>
</tr>
<tr>
<td>( D )</td>
<td>((-s_2 q_2^1 + p_2 q_2^1 q_2^2 + (s_2 - 2p_2))</td>
<td>((-s_2 q_2^1 + (s_2 - 2p_2))</td>
</tr>
</tbody>
</table>

So BNE is the same regardless of manufacturer’s action. □
Chapter 4

Flexibility in Emergency Supply Chain Operations: 
Coping with Demand Uncertainty

4.1 Introduction

Natural disasters such as hurricanes, tornadoes, tsunamis, floods, earthquakes, etc. have happened frequently across the world and caused significant damages. According to a report from National Climatic Data Center\(^1\), there were 9 weather/climate disaster events with losses exceeding $1 billion each in the U.S. in 2013. In October 2012, Hurricane Sandy affected 24 states in the U.S. with an estimated damage of $65 billion. Millions of people were affected and many lost their homes. The frequency and severity of disasters call for increasing attentions to emergency management.

The emergency management in response to disasters can be typically classified into four phases: mitigation, preparedness, response, and recovery (Altay and Green, 2006). Mitigation is the long-term efforts to reduce the chance of a disaster happening and mitigate the impacts should one occur, such as building dams and land use controls in hazard areas. Preparedness consists of the activities to prepare and plan for a disaster, such as personnel training, evacuation planning, and construction of an emergency center. Response is to deploy plans and procedures during and immediately after a disaster to meet urgent needs. This involves search and rescue, collection and distribution of food, medical care and other related products to victims. Recovery covers the long-term

\(^1\)http://www.ncdc.noaa.gov/billions
actions to restore the communities, including rebuilding facilities and restoring services.

The topic of interest in this chapter is emergency response operations, which is critical to save people’s lives and protecting properties. In particular, we study the problem of supply chain operations to produce and distribute emergency rescue kits to hospitals and shelters. A disaster rescue kit, or an emergency pack, is a collection of basic items needed in the event of an emergency. It typically includes water, first aid medicine and hygiene supplies (e.g., pain killers, bandages, gloves, soap, hand wipes), lighting, sheltering materials, etc. While there are rescue kits assembled for common purposes, the components most desired after a disaster may vary and be dependent on type, severity, time and geographical location of the disaster. Regions impacted by the same disaster may demand different components due to various levels of damage and population compositions. Hence, both standard and customized rescue kits are desired in order to achieve the highest level of rescue-and-relief effectiveness. The integrated supply chain operations studied in this chapter originates from a real-world project. The chain of bringing rescue kits to a disaster scene includes shipping components, assembling components into standard and/or customized packages, and distributing them to the demand zones.

Due to the complicated situations and limited resources during and after disasters, the production and distribution of rescue kits remains a challenging problem in emergency response. Coordinated operations among supply chain players and centralized decision making in such scenarios have been proven effective and capable of offering higher operational efficiency (Balcik et al., 2010). Tremendous efforts have been invested by federal and state governments to standardize the operational procedures and improve coordination and information sharing among different players. The problem under study assumes coordinated and optimized supply chain operations given known information. However, the presence of demand uncertainty, which remains one of the
main challenges in disasters (Holguín-Veras et al., 2012) and is out of control by human efforts, calls for a study on the effects of different types of supply chain flexibility.

The remainder of this chapter is organized as follows. Related literature is reviewed in Section 4.2 and the problem of interest is described in Section 4.3. Section 4.4 introduces an optimization model of the emergency supply chain operations given the forecasted base demand. Unpredictable portion of demand is handled by adding supply chain flexibility, which is investigated in Section 4.5 via an empirical study. Section 4.6 concludes the chapter.

4.2 Literature Review

There has been active research during the past thirty years in an effort to improve efficiency of emergency supply chain operations, though the literature is still limited compared to its commercial counterpart. A significant amount of attention has been paid to the logistics and transportation problems that emerged in distribution. In Haghani and Oh (1996), a multi-commodity, multi-modal network flow problem with time windows is formulated to minimize the costs of transporting relief commodities to the disrupted area using different transportation modes. Özdamar et al. (2004) formulates a dynamic time-dependent vehicle pickup and delivery problem in emergency dispatch and distribution, with the objective of minimizing the total unsatisfied demand throughout the planning horizon. Yi and Kumar (2007) minimizes the total delay in the coordinated transportation of commodities from suppliers to distribution centers and injured people from affected areas to medical centers. In Balcik et al. (2008), a two-phase route generation and selection model is developed to minimize the transportation cost of last mile distribution in humanitarian relief.

A few chapters took a further step to handle distribution integrally with planning and production. Barbarosoğlu et al. (2002) develops a two-level model for helicopter
planning and scheduling during a disaster relief, where the top level model makes tactical decisions (e.g., helicopter fleet composition, pilot assignment and high-level route planning), and the base level model minimizes the makespan of the operations (e.g., vehicle routing and re-fueling schedule for each helicopter). In Lei et al. (2016), a three-echelon supply chain for producing and distributing emergency rescue kits is considered. A mixed integer program is modeled to minimize the total tardiness in fulfilling customer orders, and a heuristic algorithm is developed to solve the problem effectively.

The above literature focuses on deterministic settings, while a few others have considered uncertainty in disaster reliefs. Barbarosoğlu and Arda (2004) proposes a two-stage stochastic programming model to deal with transportation planning of first-aid commodities and emergency personnel after earthquakes, capturing uncertainties arising from demand, supply and route capacity. In Chang et al. (2007), a flood emergency logistics problem is modeled as a two-stage stochastic program, in order to determine the locations of warehouses, resource allocation and distribution, in presence of stochastic rescue demand. Najafi et al. (2013) proposes a multi-objective, multi-period robust optimization model to manage the logistics of relief commodities and injured people in an earthquake, considering supply and demand uncertainties. For comprehensive literature surveys on emergency management, we refer to Altay and Green (2006), Galindo and Batta (2013). It is worth mentioning that supply chain design and planning activities, such as facility locations (Jia et al., 2007) and pre-positioning of emergency supplies (Rawls and Turnquist, 2010), are considered in emergency preparedness and taken as inputs in this study.

The major contributions of this chapter are threefold. First, an optimization model for a four-echelon supply chain for rescue kits is developed extending the model in Lei et al. (2016) to multi-periods, which originates from a real-world emergency case and has practical significance. The objectives of the model are to minimize the total tardiness
and peak tardiness of demand fulfillment over the planning horizon. Second, solving the multi-period model with demand uncertainty is challenging. While most existing chapters handle such uncertainty using known probability distributions or disaster-specific sample scenarios, in many practical situations the demand can hardly be reliably predicted. In this chapter, we first optimize certain parameters of the model based on the predictable portion of the demand, and then deploy supply chain flexibility in case of unexpected demand surges. Finally, we investigate the effects of different supply chain flexibility types in terms of defined metrics, and provide managerial insights via an empirical study.

4.3 Problem Description and Model Framework

4.3.1 Problem Description

The supply chain players for rescue kits include component suppliers, manufacturers, distribution centers (DCs) and customers. The demand is initialized by customers, such as hospitals, temporary medical facilities, and regional emergency shelters. Depending on the severity of damage, composition and situation of victims, customers may order standard kits or customized kits with a desired quantity and preferred delivery time. Upon receiving the demand, manufacturers and/or DCs schedule shipment and/or production depending on the availability of inventory. Each manufacturer and DC maintains an inbound inventory of components and an outbound inventory of standard kits. Due to the varied requests, customized kits are not built to stock. Production follows specific bill of material (BOM) based on the received order. Standard kits and customized kits have different BOMs. A manufacturer only produces standard kits, while a DC assembles customized kits, but not standard kits due to the limited production capacity and much lower production rate compared to manufacturers. A manufacturer may choose to fulfill orders for standard kits from existing inventory or new production.
A DC can fulfill standard kits from inventory and customized kits from production. When there are no sufficient components for production, manufacturers and DCs can request replenishment from suppliers. A DC may decide to disassemble standard kits in inventory for additional components. Each supplier has a stable and fixed capacity during each period. We assume that there is only one supplier for each component. The products are always shipped in a batch (e.g., by trucks) between two locations, subject to available shipping capacity. Figure 4.1 shows a full network of the aforementioned emergency supply chain. Note that in practice, only part of these connections are available, e.g., a customer may only procure from one dedicated manufacturer/DC.

Figure 4.1: The full network structure of an emergency supply chain for rescue kits.
The notation used in the model is listed below.

**Sets & Indices:**

- $S$: Set of suppliers, $s \in S$
- $M$: Set of manufacturers, $m \in M$
- $K$: Set of regional distribution centers, $k \in K$
- $H$: Set of customers, $h \in H$
- $T$: Set of time periods, $t \in T = \{1, \ldots, |T|\}$

**Parameters:**

- $r^P_m$: Production rate of standard kits at manufacturer $m$
- $r^D_k$: Dissembling rate of standard kits into components at DC $k$
- $r^P_{kh}$: Production rate of customized kits for customer $h$ at DC $k$
- $v^O_{st}$: Available inventory of component $s$ at supplier $s$ in period $t$
- $v^O_{mst}, v^O_{kst}$: Initial inventory of component $s$ at manufacturer $m$ and DC $k$ in period $t$, respectively
- $v^S_{mt}, v^S_{kt}$: Initial inventory of standard kits at manufacturer $m$ and DC $k$ in period $t$, respectively
- $B^S_s$: BOM of standard kits for component $s$
- $B^C_{hs}$: BOM of customized kits from customer $h$ for component $s$
- $T^0_{mt}, T^0_{kt}$: Earliest possible production start time at manufacturer $m$ and DC $k$ in period $t$, respectively.
- $T^e_t$: End time of time period $t$
- $T^S_{ht}, T^C_{ht}$: Order due time of standard kits and customized kits for customer $h$ in period $t$, respectively
- $D^S_{ht}, D^C_{ht}$: Demand of standard kits and customized kits by customer $h$ in period $t$, respectively
- \( \tau_{sm}, \tau_{sk}, \tau_{mk}, \tau_{mh}, \tau_{kh} \): Shipping time between each pair of locations
- \( F_{sm}, F_{sk}, F_{mk}, F_{mh}, F_{kh} \): Shipping capacity between each pair of locations
- \( G^O_{ms}, G^O_{ks} \): Inbound inventory capacity of component \( s \) at manufacturer \( m \) and DC \( k \), respectively
- \( G^S_m, G^S_k \): Outbound inventory capacity of standard kits at manufacturer \( m \) and DC \( k \), respectively

### 4.3.2 Model Framework

One difficulty of the model is its rolling and dynamic nature. Some existing models, such as the one in Lei et al. (2016), optimize the operations given current demand for a single time period. In real-world practices, decision makers typically need to take into consideration the entire planning horizon, \( T \). Further assuming that a decision is made at the beginning of each period, the primary objective is to minimize the total tardiness of both standard and customized orders from all customers across all time periods, given by

\[
\min_{\omega \in \Omega} \mathbb{E}_{\omega} \left[ \sum_{t \in T} \sum_{h \in H} \left( TD^S_{ht}(\omega) \cdot D^S_{ht}(\omega) + TD^C_{ht}(\omega) \cdot D^C_{ht}(\omega) \right) \right], \tag{4.1}
\]

where \( \omega \) represents a random scenario, and \( TD^S_{ht} \) and \( TD^C_{ht} \) represent the tardiness of delivering standard kits and customized kits to customer \( h \) in period \( t \), respectively.

In addition, the late delivery of medical supplies is often directly correlated to the survival rate of victims. Fatalities may grow rapidly if victims are lack of treatment after certain period of time (Fiedrich et al., 2000). Hence, to avoid peak growth of victims, it is also important to control the peakiness of total tardiness across all time
periods,

$$\min_{\omega \in \Omega} \mathbb{E} \left[ \max_{t \in T} \sum_{h \in H} \left( TD_{ht}^S(\omega) \cdot D_{ht}^S(\omega) + TD_{ht}^C(\omega) \cdot D_{ht}^C(\omega) \right) \right], \quad (4.2)$$

as well as the peak tardiness among all customers across all time periods,

$$\min_{\omega \in \Omega} \mathbb{E} \left[ \max_{t \in T} \max_{h \in H} \left( TD_{ht}^S(\omega) \cdot D_{ht}^S(\omega) + TD_{ht}^C(\omega) \cdot D_{ht}^C(\omega) \right) \right]. \quad (4.3)$$

Our objectives in (4.1), (4.2) and (4.3) are consistent with the findings in Campbell et al. (2008), which shows that service quality in humanitarian relief distribution can be improved by minimizing maximum arrival time and average arrival time.

Due to the existence of random variable $\omega$ in objectives (4.1) and (4.2), the problem becomes a multi-period stochastic optimization problem. There are several technical difficulties for solving this problem using traditional approaches. The dominant challenge is the availability and reliability of future demand prediction, including locations, times and quantities. Existing literature treats such uncertainties as probability distributions (Najafi et al., 2013) or using representative, disaster-dependent sample scenarios (Barbarosoðlu and Arda, 2004, Chang et al., 2007). Unfortunately, many complex humanitarian emergencies have unpredictable demand patterns (Holguín-Veras et al., 2012). These characteristics pose technical difficulties in using stochastic optimization techniques adopted in the aforementioned literature. Even if it is possible to generate sample scenarios to represent the reality, it heavily relies on specific disaster models, e.g., earthquake or hurricane models, and the resulting model using sample average approximation can become intractable with even a moderate number of sample scenarios.

To tackle this problem, we propose a two-stage approach, by decomposing the uncertain demand into two components: a relatively stable demand forecast over the entire planning horizon (called base demand), and unpredictable demand surges on top of the
forecast (called demand surge). Figure 4.2 illustrates this idea, where the base demand has a normal shape, and the intensity and frequency of demand surges are random. Here, the base demand is typically forecasted from historical data. Subsequently, two approaches are used for these two demand components. For the base demand component, we optimize the system design and operations of the emergency supply chain in Section 4.4, so as to minimize the total tardiness in Eq. (4.1). Then, given the optimized system parameters, we deploy supply chain flexibility to cope with the unpredictable demand surges. For the latter, we further study the effects of different flexibility types in our case in Section 4.5.

Figure 4.2: Demand can be decomposed into a base component and unpredictable surges.
4.4 Optimization Model for Base Demand

In this section, given the base demand, we present the model to optimize the system designs and operations so as to minimize the total tardiness in Eq. (4.1).

4.4.1 Single-Period Optimization Model

Minimizing the total tardiness in Eq. (4.1) requires a multi-period optimization model, which can be very large in size and thus difficult to solve. Therefore, we decompose this problem into $|\mathcal{T}|$ single-period optimization problems, where in each time period $t$, the model minimizes the total tardiness in time $t$ subject to the base demand in the same period. At the end of period $t$, each manufacturer/DC reviews its inventory level, and applies an $(s, S)$ policy to determine whether the inventory needs to be replenished and the amount of replenishment. In other words, when the inventory level $v$ drops below $s$, a replenishment amount of $(S - v)$ is requested and fulfilled in the next period, where $S$ is the inventory capacity.

To this end, we present the optimization model for any given period $t \in \mathcal{T}$, with the objective of minimizing the total tardiness in Eq. (4.1) in period $t$. The decision maker needs to determine the following decision variables in the coordinated emergency supply chain operations.

**Continuous Variables:**

- $q_{smt}, q_{skt}$: Quantity of component $s$ shipped from supplier $s$ to manufacturer $m$ and DC $k$ in period $t$, respectively
- $q_{mkt}, q_{mht}$: Quantity of standard kits shipped from the inventory of manufacturer $m$ to DC $k$ and customer $h$ in period $t$, respectively
- $p_{mt}^s$: Quantity of standard kits produced at manufacturer $m$ in period $t$
- $q^N_{mkt}, q^N_{mht}$: Quantity of standard kits shipped from the new production of manufacturer $m$ to DC $k$ and customer $h$ in period $t$, respectively.
- $d^C_{kt}$: Quantity of standard kits from the inventory dissembled at DC $k$ in period $t$.
- $q^C_{kht}$: Quantity of customized kits produced and shipped from DC $k$ to customer $h$ in period $t$.
- $w^O_{mst}, w^O_{kst}$: End inventory of component $s$ at manufacturer $m$ and DC $k$ in period $t$, respectively.
- $w^S_{mst}, w^S_{kst}$: End inventory of standard kits at manufacturer $m$ and DC $k$ in period $t$, respectively.
- $ST^P_{mt}, ST^P_{klt}$: Production start time at manufacturer $m$ and DC $k$ in period $t$, respectively.
- $TD^S_{ht}, TD^C_{ht}$: Tardiness of delivering standard kits and customized kits to customer $h$ in period $t$, respectively.

**Binary Variables:**

- $y^S_{mst}, y^S_{kst}$: Binary variable indicating whether replenishment of component $s$ is needed for production at manufacturer $m$ and DC $k$ in period $t$, respectively.
- $y^I_{mkt}, y^I_{mht}$: Binary variable indicating whether standard kits from inventory of manufacturer $m$ are shipped to DC $k$ and customer $h$ in period $t$, respectively.
- $y^N_{mkt}, y^N_{mht}$: Binary variable indicating whether newly produced standard kits at manufacturer $m$ are shipped to DC $k$ and customer $h$ in period $t$, respectively.
- $y^C_{kht}, y^C_{kht}$: Binary variable indicating whether standard kits and customized kits are shipped from DC $k$ to customer $h$ in period $t$, respectively.

We first consider product balance constraints. Specifically, the shipping quantity from each supplier cannot exceed its capacity; For each manufacturer, the shipping quantity from inventory and new production cannot exceed the specific amount of each source; For each DC, the quantity of shipped and dissembled standard kits cannot
exceed its initial inventory; The total amount of products received by each customer should cover the desired demand.

$$\sum_{m} q_{smt}^{O} + \sum_{k} q_{skt}^{O} \leq v_{st}^{O} \quad \forall s$$  \hspace{1cm} (4.4)

$$\sum_{k} q_{mkt}^{I} + \sum_{h} q_{mht}^{I} \leq v_{mt}^{S} \quad \forall m$$  \hspace{1cm} (4.5)

$$\sum_{k} q_{mkt}^{N} + \sum_{h} q_{mht}^{N} \leq P_{mt}^{S} \quad \forall m$$  \hspace{1cm} (4.6)

$$\sum_{h} q_{kht}^{I} + d_{kt}^{I} \leq v_{kt}^{S} \quad \forall k$$  \hspace{1cm} (4.7)

$$\sum_{m} (q_{mht}^{I} + q_{mht}^{N}) + \sum_{k} q_{kht}^{I} \geq D_{ht}^{S} \quad \forall h$$  \hspace{1cm} (4.8)

$$\sum_{k} q_{kht}^{C} \geq D_{ht}^{C} \quad \forall h$$  \hspace{1cm} (4.9)

For each manufacturer, the new production amount is bounded by the amount of components in inventory as specified by BOMs. If extra components are needed in production, the shipping quantity from the supplier should also be considered. Hence, one of constraints (4.10) and (4.11) is effective, where $\eta$ is a large positive number. Similar constraints (4.12) and (4.13) apply to each DC with disassembling considered.

$$B_{s}^{S} \cdot p_{mt}^{S} \leq v_{mst}^{O} + \eta \cdot y_{smt} \quad \forall s, m$$  \hspace{1cm} (4.10)

$$B_{s}^{S} \cdot p_{mt}^{S} \leq v_{mst}^{O} + q_{smt}^{O} + \eta \cdot (1 - y_{smt}) \quad \forall s, m$$  \hspace{1cm} (4.11)

$$\sum_{h} B_{hs}^{C} \cdot q_{kht}^{C} \leq v_{kst}^{O} + B_{s}^{S} \cdot d_{kt}^{I} + \eta \cdot y_{skt} \quad \forall s, k$$  \hspace{1cm} (4.12)

$$\sum_{h} B_{hs}^{C} \cdot q_{kht}^{C} \leq v_{kst}^{O} + q_{skt}^{O} + B_{s}^{S} \cdot d_{kt}^{I} + \eta \cdot (1 - y_{skt}) \quad \forall s, k$$  \hspace{1cm} (4.13)
The end inbound and outbound inventory levels at each manufacturer and DC depend on the initial inventory level, production quantity and shipping quantity. Meanwhile, the inventory level cannot exceed the corresponding capacity.

\[
v_{mst}^O + q_{smt}^O - B_s^S \cdot p_{mt}^S = w_{mst}^O \quad \forall s, m \quad (4.14)
\]

\[
v_{mt}^S + p_{mt}^S - \sum_k (q_{imkt}^I + q_{imkt}^N) - \sum_h (q_{imht}^I + q_{imht}^N) = w_{mt}^S \quad \forall m \quad (4.15)
\]

\[
v_{kst}^O + q_{skt}^O + B_s^S \cdot d_{kt}^I - \sum_h B_{hs}^C \cdot q_{kht}^C = w_{kst}^O \quad \forall s, k \quad (4.16)
\]

\[
v_{kt}^S + \sum_m (q_{imkt}^I + q_{imkt}^N) - \sum_h q_{kht}^I = w_{kt}^S \quad \forall k \quad (4.17)
\]

\[
w_{mst}^O \leq G_{ms}^O \quad \forall s, m \quad (4.18)
\]

\[
w_{mt}^S \leq G_m^S \quad \forall m \quad (4.19)
\]

\[
w_{kst}^O \leq G_{ks}^O \quad \forall s, k \quad (4.20)
\]

\[
w_{kt}^S \leq G_k^S \quad \forall k \quad (4.21)
\]

The shipping quantity from each supplier, manufacturer and DC is constrained by
the shipping capacity between each pair of locations, if shipping is needed.

\[ q_{smt}^O \leq F_{sm} \quad \forall s, m \quad (4.22) \]
\[ q_{skt}^O \leq F_{sk} \quad \forall s, k \quad (4.23) \]
\[ q_{mkt}^I + q_{mkt}^N \leq F_{mk} \quad \forall m, k \quad (4.24) \]
\[ q_{mht}^I + q_{mht}^N \leq F_{mh} \quad \forall m, h \quad (4.25) \]
\[ q_{kht}^I + q_{kht}^C \leq F_{kh} \quad \forall k, h \quad (4.26) \]
\[ q_{mkt}^I \leq F_{mk} \cdot y_{mkt}^I \quad \forall m, k \quad (4.27) \]
\[ q_{mkt}^N \leq F_{mk} \cdot y_{mkt}^N \quad \forall m, k \quad (4.28) \]
\[ q_{mht}^I \leq F_{mh} \cdot y_{mht}^I \quad \forall m, h \quad (4.29) \]
\[ q_{mht}^N \leq F_{mh} \cdot y_{mht}^N \quad \forall m, h \quad (4.30) \]
\[ q_{kht}^I \leq F_{kh} \cdot y_{kht}^I \quad \forall k, h \quad (4.31) \]
\[ q_{kht}^C \leq F_{kh} \cdot y_{kht}^C \quad \forall k, h \quad (4.32) \]

A critical part of the model are the time constraints. At each manufacturer and DC, production cannot start before the earliest production available time. If additional components are needed in production, production start time is no earlier than the arrival time of supply. For each DC, the production start time is further constrained by the time spent on dissembling standard kits into components if needed. The tardiness of delivering standard kits depends on manufacturers and DCs involved. For each manufacturer, constraint (4.40) is effective if it ships from inventory only; otherwise, constraint (4.41) considers both production and shipping times. Similar constraints hold for each DC. When a DC ships both standard and customized kits to a customer, both products are bundled in a single shipment to achieve economic of scale. Lastly,
domain of all variables are specified in Eq. (4.45).

\[
T^0_{mt} \leq ST^P_{mt} \quad \forall m \tag{4.33}
\]

\[
\tau_{sm} \cdot y_{smt} \leq ST^P_{mt} \quad \forall s, m \tag{4.34}
\]

\[
T^0_{kt} \leq ST^P_{kt} \quad \forall k \tag{4.35}
\]

\[
\tau_{sk} \cdot y_{skt} \leq ST^P_{kt} \quad \forall s, k \tag{4.36}
\]

\[
d^I_{kt} \leq ST^P_{kt} \quad \forall k \tag{4.37}
\]

\[
\tau_{mk} \cdot y_{mkt} - \eta \cdot y^N_{mkt} \leq T^e_k \quad \forall m, k \tag{4.38}
\]

\[
ST^P_{mt} + \frac{p^{S}_{mt}}{r^P_m} + \tau_{mk} - \eta \cdot (1 - y^N_{mkt}) \leq T^e_k \quad \forall m, k \tag{4.39}
\]

\[
\tau_{mh} \cdot y_{mht} - T^S_{ht} - \eta \cdot y^N_{mht} \leq TD^S_{ht} \quad \forall m, h \tag{4.40}
\]

\[
ST^P_{mt} + \frac{p^{S}_{mt}}{r^P_m} + \tau_{mh} - T^S_{ht} - \eta \cdot (1 - y^N_{mht}) \leq TD^S_{ht} \quad \forall m, h \tag{4.41}
\]

\[
\tau_{kh} \cdot y_{kht} - T^S_{kht} - \eta \cdot y^C_{kht} \leq TD^S_{kht} \quad \forall k, h \tag{4.42}
\]

\[
ST^P_{kt} + \frac{q^{C}_{kht}}{r^P_{kh}} + \tau_{kh} - T^C_{kht} - \eta \cdot (2 - y^I_{kht} - y^C_{kht}) \leq TD^C_{kht} \quad \forall k, h \tag{4.43}
\]

\[
ST^P_{kt} + \frac{q^{C}_{kht}}{r^P_{kh}} + \tau_{kh} - T^C_{kht} - \eta \cdot (1 - y^C_{kht}) \leq TD^C_{kht} \quad \forall k, h \tag{4.44}
\]

All variables are non-negative, and y’s are binary

Finally, in addition to the demand from customers, the replenishment order for each manufacturer and DC should be also considered in the production plan.

\[
p^{S}_{mt} - \sum_k q^{N}_{mkt} - \sum_h q^{N}_{mht} \geq (G^S_m - v^S_{mt}) \cdot I_{\{v^S_{mt} \leq \epsilon \cdot G^S_m\}} \quad \forall m \tag{4.46}
\]

\[
\sum_m (q^{N}_{mkt} + q^{N}_{mk}) \geq (G^S_k - v^S_{kt}) \cdot I_{\{v^S_{kt} \leq \epsilon \cdot G^S_k\}} \quad \forall k \tag{4.47}
\]

\[
q^{O}_{smt} \geq (G^O_{ms} - v^O_{mst}) \cdot I_{\{v^O_{mst} \leq \epsilon \cdot G^O_{ms}\}} \quad \forall s, m \tag{4.48}
\]

\[
q^{O}_{skt} \geq (G^O_{ks} - v^O_{kst}) \cdot I_{\{v^O_{kst} \leq \epsilon \cdot G^O_{ks}\}} \quad \forall s, k \tag{4.49}
\]
where $I_{\{A\}}$ is an indicator function equaling 1 if $A$ is true and 0 otherwise, and $0 \leq \epsilon \leq 1$ is the minimum inventory level to replenish.

Given any period $t$ and the corresponding base demand, the resulting optimization model in Eqs. (4.1), (4.4)–(4.45) is a mixed integer program (MIP), denoted as $P^S$. When the problem size is small to medium (e.g., 5-10 suppliers, 2-5 manufacturers, 2-5 DCs, and 5-20 customers), the model can typically be solved to optimality within reasonable amount of time (e.g., from a few seconds to 30 minutes) using a commercial MIP solver, such as Gurobi or CPLEX. However, for solving a large-scale problem, a heuristic can be faster without sacrificing too much of the solution quality. The major difficulty of our model is the existence of binary variables. With these binary variables relaxed, the problem becomes a linear program (LP) which is much faster to solve. Based on this special structure, we use a partial LP-relaxation based heuristic for solving large problems. Such heuristics have been proven effective in solving large-scale MIP models, such as in logistics problems (Chen et al., 2009) and supply chain optimization (Lei et al., 2016). Since the main purpose of this chapter is not an algorithm development, we leave the details of the heuristic for future reference. Note that the empirical tests in Section 4.5 are solvable cases using Gurobi MIP solver.

4.4.2 Optimizing Inventory Policy across Periods

At the beginning of each time period, the single-period optimization problem, $P^S$, is solved. The optimal solution is then deployed. The end state of the supply chain for the current period becomes the initial state in the next period. The process repeats until the end of the planning horizon. The state transitions from period $t$ to period
$t + 1$ are specified as follows.

$$v_{m,t+1}^S = w_{mt}^S \quad \forall m$$

(4.50)

$$v_{m,s,t+1}^O = w_{mst}^O \quad \forall s, m$$

(4.51)

$$v_{k,t+1}^S = w_{kt}^S \quad \forall k$$

(4.52)

$$v_{k,s,t+1}^O = w_{kst}^O \quad \forall s, k$$

(4.53)

$$T_{m,t+1}^0 = \max \left\{ 0, ST_{mt}^P + \frac{w_{mt}^S}{r_{mt}^t} - T_{t}^e \right\} \quad \forall m$$

(4.54)

$$T_{k,t+1}^0 = \max \left\{ 0, \max_h \left\{ ST_{kt}^P + \frac{w_{kt}^S}{r_{kt}^t} - T_{t}^e \right\} \right\} \quad \forall k$$

(4.55)

Eqs. (4.50)–(4.53) mean that the end inventory of each manufacturer/DC in period $t$ becomes the initial inventory in the next period. Eqs. (4.54) and (4.55) specify the earliest production start time for each manufacturer/DC. If the production scheduled for period $t$ completes before the end of the period ($T_{t}^e$), production can start immediately in the next period; otherwise, the new production has to wait till the production for period $t$ finishes.

Note that in problem $\mathbf{PS}$, model parameters, such as demand, shipping time, inventory capacities, and production rates, are known in advance. However, inventory policy, i.e., the minimum inventory level to replenish, $\epsilon$, can be chosen by the decision maker. If the replenishment level is too high, manufacturers/DCs may not have sufficient products in inventory when the demand has unexpected increase. On the other hand, if the replenishment level is too low, the manufacturers may spend too much production time on products that are not immediately needed, and thus slows down the delivery of products to customers. Consequently, it is necessary to optimize the replenishment level by minimizing the total tardiness for all time periods in Eq. (4.1).

To this end, we optimize the following objective given the base demand across the
periods:

\[
\min_{\epsilon} \left[ \sum_{t \in T} \sum_{h \in H} \left( T D^S_{ht} \cdot D^S_{ht} + T D^C_{ht} \cdot D^C_{ht} \right) \right]. \tag{4.56}
\]

Problem above is an optimization problem with a continuous decision variable, \(0 \leq \epsilon \leq 1\). In this chapter, we simply the problem by discretizing the solution space of \([0, 1]\). For each discrete value of \(\epsilon\), we solve \(|T|\) problems \(P^S\) sequentially for each time period \(t\), and make state transitions to period \(t + 1\) using Eqs. (4.50)–(4.53). The optimal replenishment level, \(\epsilon^*\), is the one that yields the minimal value in Eq. (4.56) among this discrete set.

### 4.5 Supply Chain Flexibility for Coping with Demand Surges

Given the base demand, we optimize the supply chain operations in each single time period in Section 4.4.1, and choose the optimal inventory policy in Section 4.4.2. However, the unpredictable demand surges might disrupt the optimal solutions obtained, and hence we rely on enhancing supply chain flexibility to cope with such unexpected demand.

The concept of flexibility has emerged in manufacturing systems (Gerwin, 1993). More recently, flexibility has attracted increasing attention and become one of the strategic goals in commercial supply chains. Flexibility is generally described as the ability of a supply chain to react and adapt to variations (demand in our case), with little penalty in time, effort, cost or performance (Gosling et al., 2010). In this section, we will show that flexibility can be properly managed to improve overall responsiveness when demand surges happen.

To this end, we study the following supply chain flexibility types.

- **Procurement flexibility:** Figure 4.1 shows the full supply chain network, where each customer (hospital) is connected to all manufacturers and DCs. However,
in reality, a customer typically procures from a dedicated manufacturer or DC, which will serve as our baseline.

- **Inventory capacity**: Each manufacturer/DC has an inventory capacity, which determines the level of flexibility to respond to urgent demand increases. Increasing inventory capacities hence increases the level of flexibility.

- **DC redundancy**: More DCs may provide more flexibility to respond to demand surges. Our baseline setting has two DCs as shown in Figure 4.3.

### 4.5.1 Scenario Description

We study the effects of supply chain flexibility in a disaster scenario based on Hurricane Sandy in 2012, which severely damaged New Jersey and New York areas. The full emergency supply chain network includes $|S| = 4$ suppliers, $|M| = 2$ manufacturers, $|K| = 2$ DCs and $|H| = 5$ demand zones. The five demand zones locate at New Jersey and New York City and are shown in Figure 4.3. It also shows the locations of manufacturers, DCs and suppliers. Note that the connections from suppliers to DCs and from manufacturers to customers are not shown in the figure to avoid a messy presentation. The shipping time between each pair of locations is obtained using Google Map and weighted by a factor considering the prolonged travel time during the disaster.

The demand zones are estimated based on the amount of emergency aid reported\(^2\). Totally 11 time periods is considered. The base demand portion is forecasted by historical data and pre-generated in advance, while the additional demand surges are randomly generated from probability distributions. We varied the shape of the base demand to be uniform-shaped, triangular-shaped, and normal-shaped, respectively (Huang et al.,

\(^2\)http://www.americares.org/map/Hurricane-Sandy.html
The means of demand distributions are customer dependent and are set to be proportional to the population of each demand zone as given in Table 4.1. The demand surge is generated with random intensity and frequency. The production rate of each manufacturer is set to 1,000 units per hour, and that of each DC is set to 200 units per hour. The dissembling rate of each DC is 200 units per hour. The due date of all demands are 12 hours.

### Table 4.1: Parameter settings of test cases.

<table>
<thead>
<tr>
<th>$h$</th>
<th>Mean of $D_{h}^{S}$</th>
<th>Mean of $D_{h}^{C}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>26,925</td>
<td>5,385</td>
</tr>
<tr>
<td>2</td>
<td>30,662</td>
<td>6,132</td>
</tr>
<tr>
<td>3</td>
<td>27,667</td>
<td>5,533</td>
</tr>
<tr>
<td>4</td>
<td>19,887</td>
<td>3,977</td>
</tr>
<tr>
<td>5</td>
<td>25,804</td>
<td>5,161</td>
</tr>
</tbody>
</table>
4.5.2 Flexibility Effects

In this subsection, we varied the aforementioned flexibility types, and fed them as parameters into the multi-period optimization problem in Section 4.4. Note that the resulting single-period optimization model are small in scale and solvable using the Gurobi MIP solver on a computer with 2.60GHz CPU.

Specifically, we first compute three metrics, i.e., the total tardiness in Eq. 4.1 (TTD), the peak tardiness across all time periods in Eq. 4.2 (PTT), and the peak tardiness among all customers across all time periods in Eq. 4.3 (PTH), for three shapes of demand forecast respectively, using the baseline flexibility level. Here, the baseline flexibility configuration has the dedicated procurement setting (where each customer uses only one manufacturer/DC for procurement), default inventory capacity, and two DCs. Then, we computed the same metrics for varied flexibility configurations, and compared to the baseline metrics. Totally ten flexibility settings were compared, including the baseline: (a1) full procurement flexibility from manufacturers (MFs), that is, each customer can procure from any MF, everything else being equal to the baseline (same below); (a2) full procurement flexibility from DCs; (a3) full procurement flexibility from MFs and DCs; (b1) doubling inventory capacities in MFs; (b2) doubling inventory capacities in DCs; (b3) doubling inventory capacities in MFs and DCs; (c1) 3 DCs; (c2) 4 DCs; and (d) combination with full procurement flexibility and doubling inventory capacities in MFs and DCs.

Tables 4.2 – 4.4 show the relative improvement percentage of each flexibility configuration compared to the baseline configuration, where each result is the average of 20 random replications. Note that a negative percentage means that the performance of that flexibility configuration is worse than the baseline in terms of the selected metric. Further, the number in parenthesis beside each percentage value is the rank of each flexibility configuration for the corresponding metric.
Table 4.2: Comparison of metrics for uniform-shaped demand.

(a) Varied procurement flexibility level

<table>
<thead>
<tr>
<th>Metric</th>
<th>(a1) Full MF+Dedicated DC</th>
<th>(a2) Dedicated MF+Full DC</th>
<th>(a3) Full MF+Full DC</th>
</tr>
</thead>
<tbody>
<tr>
<td>TTD</td>
<td>3.799% (9)</td>
<td>11.159% (5)</td>
<td>13.984% (4)</td>
</tr>
<tr>
<td>PTT</td>
<td>-1.633% (8)</td>
<td>1.937% (6)</td>
<td>4.051% (5)</td>
</tr>
<tr>
<td>PTH</td>
<td>2.583% (6)</td>
<td>16.449% (3)</td>
<td>18.872% (2)</td>
</tr>
</tbody>
</table>

(b) Varied inventory capacities

<table>
<thead>
<tr>
<th>Metric</th>
<th>(b1) 200% MF Capacities</th>
<th>(b2) 200% DC Capacities</th>
<th>(b3) 200% MF&amp;DC Capacities</th>
</tr>
</thead>
<tbody>
<tr>
<td>TTD</td>
<td>24.703% (3)</td>
<td>6.228% (7)</td>
<td>25.575% (2)</td>
</tr>
<tr>
<td>PTT</td>
<td>18.842% (2)</td>
<td>-14.472% (9)</td>
<td>1.736% (7)</td>
</tr>
<tr>
<td>PTH</td>
<td>2.236% (7)</td>
<td>0.392% (8)</td>
<td>-22.752% (9)</td>
</tr>
</tbody>
</table>

(c) Varied DC redundancies

<table>
<thead>
<tr>
<th>Metric</th>
<th>(c1) 3 DCs</th>
<th>(c2) 4 DCs</th>
<th>(d) Full Flexibility in (a3)+(b3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TTD</td>
<td>9.242% (6)</td>
<td>4.583% (8)</td>
<td>82.343% (1)</td>
</tr>
<tr>
<td>PTT</td>
<td>5.841% (4)</td>
<td>11.035% (3)</td>
<td>78.303% (1)</td>
</tr>
<tr>
<td>PTH</td>
<td>9.020% (5)</td>
<td>13.821% (4)</td>
<td>58.258% (1)</td>
</tr>
</tbody>
</table>
Table 4.3: Comparison of metrics for normal-shaped demand.

(a) Varied procurement flexibility levels

<table>
<thead>
<tr>
<th>Metric</th>
<th>(a1) Full MF+Dedicated DC</th>
<th>(a2) Dedicated MF+Full DC</th>
<th>(a3) Full MF+Full DC</th>
</tr>
</thead>
<tbody>
<tr>
<td>TTD</td>
<td>5.137% (7)</td>
<td>15.289% (4)</td>
<td>19.605% (3)</td>
</tr>
<tr>
<td>PTT</td>
<td>1.561% (7)</td>
<td>6.950% (4)</td>
<td>7.663% (3)</td>
</tr>
<tr>
<td>PTH</td>
<td>4.703% (5)</td>
<td>6.992% (3)</td>
<td>6.946% (4)</td>
</tr>
</tbody>
</table>

(b) Varied inventory capacities

<table>
<thead>
<tr>
<th>Metric</th>
<th>(b1) 200% MF Capacities</th>
<th>(b2) 200% DC Capacities</th>
<th>(b3) 200% MF&amp;DC Capacities</th>
</tr>
</thead>
<tbody>
<tr>
<td>TTD</td>
<td>8.837% (6)</td>
<td>13.713% (5)</td>
<td>19.866% (2)</td>
</tr>
<tr>
<td>PTT</td>
<td>-8.697% (9)</td>
<td>9.267% (2)</td>
<td>-0.957% (8)</td>
</tr>
<tr>
<td>PTH</td>
<td>-22.787% (9)</td>
<td>10.903% (2)</td>
<td>-18.985% (8)</td>
</tr>
</tbody>
</table>

(c) Varied DC redundancies

<table>
<thead>
<tr>
<th>Metric</th>
<th>(c1) 3 DCs</th>
<th>(c2) 4 DCs</th>
<th>(d) Full Flexibility in (a3)+(b3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TTD</td>
<td>1.618% (9)</td>
<td>3.258% (8)</td>
<td>51.173% (1)</td>
</tr>
<tr>
<td>PTT</td>
<td>3.354% (6)</td>
<td>5.821% (5)</td>
<td>36.597% (1)</td>
</tr>
<tr>
<td>PTH</td>
<td>1.852% (7)</td>
<td>4.249% (6)</td>
<td>17.294% (1)</td>
</tr>
</tbody>
</table>
Table 4.4: Comparison of metrics for triangular-shaped demand.

(a) Varied procurement flexibility levels

<table>
<thead>
<tr>
<th>Metric</th>
<th>(a1) Full MF+Dedicated DC</th>
<th>(a2) Dedicated MF+Full DC</th>
<th>(a3) Full MF+Full DC</th>
</tr>
</thead>
<tbody>
<tr>
<td>TTD</td>
<td>8.105% (6)</td>
<td>11.440% (3)</td>
<td>20.383% (2)</td>
</tr>
<tr>
<td>PTT</td>
<td>2.095% (8)</td>
<td>5.500% (7)</td>
<td>18.392% (2)</td>
</tr>
<tr>
<td>PTH</td>
<td>2.547% (6)</td>
<td>17.387% (3)</td>
<td>20.629% (1)</td>
</tr>
</tbody>
</table>

(b) Varied inventory capacities

<table>
<thead>
<tr>
<th>Metric</th>
<th>(b1) 200% MF Capacities</th>
<th>(b2) 200% DC Capacities</th>
<th>(b3) 200% MF&amp;DC Capacities</th>
</tr>
</thead>
<tbody>
<tr>
<td>TTD</td>
<td>2.472% (9)</td>
<td>8.324% (5)</td>
<td>9.711% (4)</td>
</tr>
<tr>
<td>PTT</td>
<td>7.624% (5)</td>
<td>-7.888% (9)</td>
<td>7.772% (4)</td>
</tr>
<tr>
<td>PTH</td>
<td>-21.912% (9)</td>
<td>0.000% (7)</td>
<td>-21.841% (8)</td>
</tr>
</tbody>
</table>

(c) Varied DC redundancies

<table>
<thead>
<tr>
<th>Metric</th>
<th>(c1) 3 DCs</th>
<th>(c2) 4 DCs</th>
<th>(d) Full Flexibility in (a3)+(b3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TTD</td>
<td>3.526% (8)</td>
<td>3.737% (7)</td>
<td>54.037% (1)</td>
</tr>
<tr>
<td>PTT</td>
<td>7.354% (6)</td>
<td>11.413% (3)</td>
<td>32.363% (1)</td>
</tr>
<tr>
<td>PTH</td>
<td>7.040% (5)</td>
<td>10.507% (4)</td>
<td>20.053% (2)</td>
</tr>
</tbody>
</table>
More specifically, Table 4.2 shows the results for the uniform-shaped base demand and random demand surges. It is seen that imposing full procurement flexibility and doubling the inventory capacities for both manufacturers and DCs substantially reduces total tardiness and peak tardiness, compared to the baseline configuration. The second most effective flexibility is to double the inventory capacities, especially the capacities at manufacturers. Although doubling capacities at both manufacturers and DCs effectively reduces the total tardiness (TTD), it increases the peak tardiness (PTH), meaning that the worst case becomes worse. This is because minimizing TTD in problem $P^S$ may cause the resources to be unevenly distributed among customers, and the larger inventory capacity requires more unused products to be produced before demand is materialized; recall that $(s,S)$ policy is used for replenishment. Further, increasing procurement flexibility, especially between customers and DCs, is effective in reducing all three metrics. Finally, adding more DCs definitely reduces tardiness, but the marginal effect diminishes.

For the normal-shaped base demand, Table 4.3 shows similar observations as seen in Table 4.2. These are (1) increasing procurement flexibility reduces all three metrics, while increasing this flexibility at DCs is much more effective than at manufacturers; (2) increasing inventory capacities effectively reduces total tardiness, but increases peak tardiness, since more production efforts need to be spent at producing products that are not immediately needed; and (3) increasing number of DCs has diminishing effects. Different from the observations in uniform-shaped demand case, for the normal-shaped demand, increasing inventory capacities at DCs substantially reduces peak tardiness (PTT and PTH). This is probably because the demand is increasing initially for the normal-shaped forecast, and thus inventories in DCs provide rapid coverage of demand surges. Further, it is seen that the percentage reductions on three metrics are smaller than those observed in Table 4.2, even for the full flexibility configuration of setting
(d).

Table 4.4 shows the results for the triangular-shaped base demand. The observations are similar to those in Table 4.3 for the normal-shaped base demand. A noticeable difference is that the effects of adding inventory capacities are further reduced, while the effects of increasing procurement flexibility are more substantial.

Summarizing the above observations, we have the following recommendations.

- The unexpected demand surges in emergency response can cause disruption to the existing production and distribution plans. But increasing supply chain flexibility, such as procurement flexibility, inventory capacity, and more DCs, can substantially reduce the tardiness caused to the delivery of rescue kits.

- Allowing customers to source from all available manufacturers and DCs can effectively reduce both total tardiness and peak tardiness. Typically, increasing this flexibility from DCs is more effective than from manufacturers.

- Increasing inventory capacities has mixing effects: it can effectively reduce total tardiness (our primary objective in the optimization model), but may increase peak tardiness simultaneously. Therefore, too much redundancy in inventory is not recommended.

- Adding DCs has a diminishing effect in reducing tardiness. Considering the setup costs of adding a DC, it may not be the top flexibility type to consider in emergency supply chain operations.

4.6 Conclusion

Emergency response operations is a crucial segment in disaster reliefs due to the urgency and importance of saving victims’ lives and properties. Emergency rescue kits, consisting of necessary items desired during and after disasters, need to be efficiently
produced and distributed to hospitals and shelters. The coordinated operations among emergency supply chain players, including suppliers, manufacturers and DCs, are critical in order to provide a prompt response to customer demands. In this chapter, we study an emergency supply chain for the aforesaid rescue kits, with the objective of minimizing the total tardiness and peak tardiness during the planning horizon with multiple time periods. The main difficulty is the uncertainty in demand, which we decompose into a relatively stable base demand forecast, and unpredictable demand surges. For the former, we develop an optimization model to minimize the total tardiness of delivery rescue kits; for the latter, we investigate the effects of increasing supply chain flexibility to cope with the uncertainty. Flexibility has been used as a strategy to cope with changing environment in manufacturing systems and commercial supply chains, but its adoption in emergency operations has not been studied to the best of our knowledge. Based on the Hurricane Sandy case, we ran experiments and provide managerial insights on how to effectively increase the aforementioned supply chain flexibility.

Although we provide some recommendations on how to deploy supply chain flexibility using the case study, the conclusions may change as the scenarios change, e.g., for a different base demand pattern. However, the two-stage approach proposed can be generalized in all such analysis. Finally, it would be interesting to study other types of supply chain flexibility in addition to the three types considered in this chapter.
Bibliography


Choi, S. Chan, Ju Myung Song, Yao Zhao, Xiaowei Xu. 2016. Chapter x: Models of channel coordination. Handbook of Research in Distribution Channels.


Rahmani, Morvarid, Guillaume Roels, Uday S Karmarkar. 2013. Contracting and work dynamics in collaborative projects.


