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USING THE REGIONAL SOCIAL ACCOUNTING MATRIX TO FORECAST HOUSEHOLD EXPENDITURES: A FUZZY APPROACH

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Abstract

Using a Social Accounting Matrix (SAM) to forecast the effect of exogenous shocks on the economy should be based on incorporate and error-free information that requires to update data in the SAM cells. Many techniques are available for updating a SAM. Here we use an approach based on the fuzzy set theory. Essentially, we restrict estimates of a matrix of direct input-output coefficients, the core of a matrix of SAM direct coefficients, to just seven possible size categories when they are updates. These rough categorical estimates are transformed to some quantitate functions with domain that reflects of coefficient sizes. These resulting model functions are interpreted as quasi probability density functions so a quasi-stochastic programing problem is used to estimate fuzzy impacts. The fuzzy results are compared to "true" ones, estimated via a classically created SAM, by calculating the estimation error. Finally, we explore factors of the error sensitivity.

In an empirical exercise, we use a 57-industry 2010 SAM for New Jersey and develop a SAM model with fuzzy direct input-output coefficients. Based on this model we forecast effect from relative and absolute changes for three groups of exogenous factors: the elements of the final demand, the institutional incomes and value added. We next extend the initial model by also fuzzifying other structural components of SAM: the matrix of value-added by industry coefficients and the matrix of institutional expenditures by industry coefficients and estimate the same effects. The fuzzy forecasts for the initial and "new" SAM model with fuzzy parameters are compared to results estimated when using a classical approach.

In all cases we obtain the small estimation error that makes it possible to consider a fuzzy approach as an efficient way for forecasting effects of exogenous shocks on the economy using inaccurate and incomplete SAM data

Introduction

A Social Accounting Matrix (SAM) is a conceptual framework to explore the effects of various exogenous shocks on changes in an economy based on its structural characteristics. In this

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article we consider such effects to forecast household expenditures because they represent the largest share of total expenditures for most US regional economies.

One of the main issues when estimating effects of exogenous shocks is the incompleteness and inaccuracy of data in cells of a SAM. Researchers highlight various causes of this issue: updating data irregularly, using different sources to build a SAM, lack of data, sampling and non-sampling measurement errors. Analysis of existing approaches that are used to update SAMs indicates that they do not always yield reasonable results either because they ignore various errors in the SAM, because they do not reconcile different sources of errors, or because they are based on the gross simplifications of reality.

To handle these issues, we propose a fuzzy approach to updating SAMs that permits estimation of impacts with sufficient accuracy based on initial, roughly hewn components of a SAM model. This approach employs fuzzy set theory to take verbal assessment a matrix of direct input-output coefficients, the core of matrix of SAM direct coefficients, and to transform these assessment values into quantitative functions with domain that reflects of coefficient sizes. Interpreting these functions as quasi probability density functions, we employ quasi-stochastic programming procedure to estimate effect of different exogenous shocks via a SAM model with fuzzy parameters. Finally, household spending is projected by using fuzzy changes in household income.

Here we use a 57-industry 2010 SAM for New Jersey to develop a SAM model with fuzzy direct input-output coefficients. Based on this model we forecast effect from relative and absolute changes in different exogenous factors such as the commodity demand, the institutional incomes and the value added. We compare fuzzy result with the forecasts, obtained using a classical approach by calculating the estimation error. Then, we explore factors of the error sensitivity.

Next, we extend the initial model by also fuzzifying other structural components of the SAM: the matrix of institutional expenditures by industry coefficients and the matrix of value-added by industry coefficients and estimate the effects from the same factors. We compare the estimation error for both the initial and the "new" SAM model with fuzzy parameters and analyze the factors of the error sensitivity in this case.

2. Review of existing approaches to SAM estimation

Ideally SAMs are error-free and based on perfect data. Of course, no SAM is. In practice, a number of problems are associated with their construction. SAMs are, thus, prone to inaccuracy and incompleteness. The problems can be grouped into three categories: data accuracy, data actuality, and data completeness.

The problem of *data accuracy* appears because of the use of a variety of data sources to build the SAM. The main data source is the set of surveys that are designed specifically for constructing input-output tables, which comprise the main partition of a SAM. Other statistical sources are national income and product data, e.g., censuses of manufacturing, labor surveys, agricultural data, government accounts, international trade accounts, and household surveys. As a result, the initial SAM is composed of dizzying array of different data sources, each of which contains its own sampling and measurement errors and, hence, inherent reliability. Data accuracy turns out to be a particular problem in international trade accounts kept by different countries; that is, exports of commodity *x* from Country A to Country B do not generally equal the imports of *x* that Country B received from Country A.

The problem of *data actuality* pertains to the lag in time between the date the accounts represent and the data of publication of the accounts. This lag occurs due to the time that is needed to process data from all possible sources into a reliable set of input-output accounts. Each added data source induces reconciliation issues, due to mutual definitional incompatibilities. For example, from survey to publication, in the United States it takes on the order of seven years to produce official quinquennial benchmark input-output accounts. Annual national income and product data are published with a shorter lag but also tend to offer less sector detail as a result. In essence data actuality becomes an issue because additional efforts are required to reconcile the heterogeneous data sets of the SAM, which typically are collected at different time frequencies.

The problem of *data completeness* is associated with the inability of obtaining some official information for a SAM. This can be either for certain cells or entire sections of a SAM; in a fact, it may be that just a marginal amount of information for a SAM. This can be an acute problem in many developing nations (e.g., Haiti, the Gaza Strip, and Afghanistan) as well as at the regional level for many developed nations. In the United States (U.S.), for example, no officially published regional SAMs derived from survey data have ever been published; so collecting and reconciling information to build regional accounts in the U.S. requires specialized inferences and assumptions (Lahr, 2001b). Moreover, even in countries like U.S., which produce many industry-based data series at small areal levels, data for regional industries with a very small but nonzero set of establishments must often be interpolated due to government assurances during data collection that guarantee nonrelease of information that could reveal an establishment's competitive position (so called, "disclosure issues").

Despite the data issues above, builders of economic models continue to seek ever more efficient, cost-effective approaches that yield more accurate estimates of economic and environmental impacts. Such approaches were key in developing several time-series-based multiregional international input-output (world MRIO) databases (Tukker and Dietzenbacher, 2013). During development of these various databases, every one of the research teams dealt with issues of data accuracy, data actuality, and data completeness in different ways. On issues of data completeness, the EORA database made the most heroic leaps, as its national accounts for countries that have not produced a SAM or IO table for decades (e.g., Argentina), if ever (e.g., Haiti). Indeed, the research team has told us that they focused “on standardization, automation, and advance computation” to achieve “labor and cost savings” (Lenzen et al., 2013, p. 39). Moreover, they admit (p. 39) that the “Results will generally be uncertain at the sectoral level and for small sectors, but not necessarily uncertain for small countries, especially not for small countries with high-quality IO data.” In this vein, Lenzen et al. (2006, 2009, 2010) have shown that unknown data are often not entirely unknowable; rather they have ranges and limits and sometimes even known probability distributions.

The above suggests that stochastic approaches might be applied. In particular, the stochastic nature of interindustry transactions, presented in a matrix of direct input-output coefficients as well as in Make and Use Tables, has been studied by Roland-Holst, 1989; Yermoliev and Yastremsky, 1997; Dietzenbacher, 2006 and others. But sufficiently robust results from such an approach require knowledge of the statistical distribution of IO matrix elements. Unfortunately, such distributions are rather expensive and complicated to obtain in practice. Therefore, stochastic methods tend to lean on assumptions that oversimplify reality and, thus data accuracy and data completeness issues remain intractable via application of such methods.

Alternatively, Thorbecke (2003) considered cell estimation error in a SAM to be a combination of both sampling error and bias (systematic measurement error). In this regard, a fuzzy approach that uses “smeared” initial data seems promising. We use fuzzy set theory to estimate cells in SAM, and use a model with a fuzzy SAM to estimate impacts on an economy.

Here, we employ fuzzy set theory (Zadeh, 1965). We enable fuzziness via verbal assessment of direct input-output coefficients, which are key to deriving a SAM’s direct coefficients matrix, and transform each of the assessment values into a quantitate functions with domain which define coefficients sizes. We next develop a model by estimating the effect of exogenous shocks on an economy using a SAM with a fuzzy matrix of direct input-output coefficients. As in the conventional model, the resulting fuzzy direct matrix enables examination of various exogenous factors—such as the final demand, value added, and institutional income—on labor income. Fuzzy changes in household incomes are then translated into household consumption estimates. The fuzzy results are then compared with those obtained via the traditional approach.

In this article we develop a SAM model with fuzzy parameters using a 57-industry 2010 SAM for the economy of the State of New Jersey. We estimate impact of exogenous factors for both a SAM model with fuzzy input-output coefficients and a classical SAM model, calculate the estimation error and explore factors of error sensitivity. Then we extend our model by also fuzzifying the matrix of institutional expenditures by industry coefficients and the matrix of value-added by industry coefficients. We estimate the same impacts using both fuzzy and classical approaches and calculate the estimation error. Finally, we compare error for two models and analyze factors of error sensitivity.

3. The fuzzy approach to forecasting household expenditures using SAM

It is known, that the effect of exogenous shock on changes in output values is calculated by the formula:

$$x = (I - S)^{-1} y, \tag{1}$$

where

x - the composite output vector with dimension ($N_1+N_2+N_3$), which includes the vector of gross output **g**, the vector of value added **v**, and the vector of institutional incomes **u**;

S - the matrix of SAM direct coefficients;

y - the composite exogenous vector with dimension ($N_1+N_2+N_3$) which includes the vector of exogenous commodity demand **d**, the vector of exogenous value added **w**, and the vector of exogenous institutional incomes **e**.

This formula assumes a deterministic matrix of SAM direct coefficients: In reality, this is not the case since, as noted earlier, various errors are inherent to the source data upon which SAMs are based. Thus, even if **y** can, in fact, be perfectly known, **x** cannot be. *So* **x** *is the composite output vector, as obtained via Equation (1), only in exceptional cases*. Thus, it is not possible to get the realistic balance using deterministic **y** and **x**. Still we at least want to find an approach to that minimized the variance inherent with our estimates of **x**. In particular, the output vector can be estimated by minimizing the expected sum of squared residuals:

$$M\left(x - \tilde{S} \cdot x - y\right)^2 \xrightarrow[x]{} \min \tag{2}$$

where $\tilde{S}$ is the nondeterministic matrix of SAM direct coefficients.

Indeed, $\tilde{S}$ is a fuzzy matrix of SAM direct coefficients, so that Equation (2) is a fuzzy interpretation of a stochastic programming problem. To find a solution for this problem the elements of SAM can be reinterpreted as the expectations of a random variable with a certain probability distribution function. In the fuzzy case, each element of the matrix is therefore the expected value of some continuous distribution function (the membership function) that can be interpreted mathematically as some probability density function.

The existence of nonnegative solutions for a similar problem was demonstrated by Yermoliev and Yastremsky (1997), who use a stochastic matrix of input-output coefficients to estimate the influence of an exogenous vector of final demand on the vector of output, given nonnegative values of final demand. If each coefficient of a SAM direct matrix is presented as the expected value of the appropriate membership function and the composite exogenous vector is nonnegative, the existence of a nonnegative solutions of Equation (2) can therefore be assumed by analogy.

Let's consider ***the fuzzy approach to forecasting of household expenditures*** *using the following steps:*

Step 1. The fuzzification of the matrix of direct input-output coefficients as a key sub-matrix of the SAM direct matrix.

Step 2. The estimate of changes in household incomes as a response to exogenous shocks using a SAM model with fuzzy parameters.

Step 3. Forecasting household expenditures from fuzzy changes in household incomes.

Step 4. The comparison of fuzzy forecasts of household expenditures with corresponding ones obtained via the classical approach and the analysis of factors of error sensitivity.

We present these steps using a 2010 SAM for New Jersey (SAMNJ-10) as used in Álvarez-Martínez and Lahr (2016). The SAM's 57 industries are more than sufficient for the present pedagogical purpose of demonstrating a new fuzzy approach for forecasting household expenditures. We now consider each step of the procedure.

Step 1. Using the approach described in Appendix A: We fuzzified the matrix of direct input-output coefficients as a key block of the matrix of SAM direct coefficients. We selected numerical intervals to characterize the domain of each fuzzy variable. To define the intervals, we used cluster analysis.[2] Next, based on the intervals, we identified membership functions for each type of interindustrial relationship. We estimated function parameters using MATLAB's *Fuzzy Logic Tool Box*. From this, we calculated expected value for each function.

As a result of this step, each input-output coefficient was replaced by the expected value of the membership function for its appropriate fuzzy variable α:

$$a_{ij} \rightarrow \tilde{a}_{ij} = M\left(\mu_{\tilde{A}(\alpha)}\right).$$

[2] We used STATISTICA 6.0's Cluster Analysis procedure.

Step 2. We applied the "new" SAM directs coefficients matrix, which includes the fuzzy matrix of direct input-output coefficients, to estimate the vector of gross output of industries **g** and the vector of institutional income **u**.

Let's represent criterion (2) as the combination of the following criteria:

$$\sum_{i=1}^{N_1}\left(\sum_{j=1}^{N_1}\tilde{a}_{ij}g_j+\sum_{m=1}^{N_2}c_{im}u_m+d_i-g_i\right)^2\xrightarrow[g]{}\min \tag{3}$$

$$\sum_{k=1}^{N_2}\left(\sum_{l=1}^{N_3}z_{kl}\left(\sum_{j=1}^{N_1}r_{lj}g_j+w_l\right)+\sum_{m=1}^{N_2}h_{km}u_m+e_k-u_k\right)^2\xrightarrow[u]{}\min \tag{4}$$

where N_1 - the number of industries;
N_2 - the number of institutions;
N_3 - the number of value-added elements.

$\tilde{\mathbf{A}}=(\tilde{a}_{ij}), i\in N_1, j\in N_1$ - fuzzy partition of direct interindustry coefficients;
$\mathbf{R}=(r_{lj}), l\in N_3, j\in N_1$ - partition of value-added by industry coefficients;
$\mathbf{C}=(c_{ik}), i\in N_1, k\in N_2$ - partition of institutional expenditures by industry coefficients;
$\mathbf{Z}=(z_{kl}), k\in N_2, l\in N_3$ - partition of institutional value-added allocation coefficients;

$\mathbf{H}=(h_{km}), k\in N_2, m\in N_2$ - partition of interinstitutional coefficients.
$\mathbf{d}=(d_i), i\in N_1$ - the vector of exogenous final demand;
$\mathbf{e}=(e_k), k\in N_2$ - the vector of exogenous institutional incomes;

$\mathbf{w}=(w_l), l\in N_3$ - the vector of exogenous value added.
$\mathbf{g}=(g_i), i\in N_1$ - the vector of total output by industry;
$\mathbf{u}=(u_k), k\in N_2$ - the vector of total institutional incomes.

To estimate **g** and **u** we differentiate criterion (3) with respect to each g_i, and criterion (4) with respect to each u_k. Setting partial derivatives equal to zero, we have the following system that includes (N_1+N_2) equations:

$$\frac{\partial\left[\sum_{i=1}^{N_1}\left(\sum_{j=1}^{N_1}\tilde{a}_{ij}g_j+\sum_{m=1}^{N_2}c_{im}u_m+d_i-g_i\right)^2\right]}{\partial g_1}=0;$$

$$\frac{\partial\left[\sum_{i=1}^{N_1}\left(\sum_{j=1}^{N_1}\tilde{a}_{ij}g_j+\sum_{m=1}^{N_2}c_{im}u_m+d_i-g_i\right)^2\right]}{\partial g_2}=0;$$

....

$$\frac{\partial\left[\sum_{i=1}^{N_1}\left(\sum_{j=1}^{N_1}\tilde{a}_{ij}g_j+\sum_{m=1}^{N_2}c_{im}u_m+d_i-g_i\right)^2\right]}{\partial g_{N_1}}=0; \quad (5)$$

$$\frac{\partial\left[\sum_{k=1}^{N_2}\left(\sum_{l=1}^{N_3}z_{kl}\left(\sum_{j=1}^{N_1}r_{lj}g_j+w_l\right)+\sum_{m=1}^{N_2}h_{km}u_m+e_k-u_k\right)^2\right]}{\partial u_1}=0;$$

$$\frac{\partial\left[\sum_{k=1}^{N_2}\left(\sum_{l=1}^{N_3}z_{kl}\left(\sum_{j=1}^{N_1}r_{lj}g_j+w_l\right)+\sum_{m=1}^{N_2}h_{km}u_m+e_k-u_k\right)^2\right]}{\partial u_2}=0;$$

....

$$\frac{\partial\left[\sum_{k=1}^{N_2}\left(\sum_{l=1}^{N_3}z_{kl}\left(\sum_{j=1}^{N_1}r_{lj}g_j+w_l\right)+\sum_{m=1}^{N_2}h_{km}u_m+e_k-u_k\right)^2\right]}{\partial u_{N_2}}=0.$$

This system of equations (5) allowed us to estimate the effect of exogenous shocks on changes in gross output of industries as well as institutional incomes.[3] Table 1 shows the solution of (5) for a 10% increment in the first element of the vector of exogenous commodity demand d_1 when a fuzzy $\tilde{\mathbf{A}}$ is used.

[3] To solve (5) we used *Wolfram 8*, which has a very large collection of equation-solving tools. This language's symbolic architecture allows both equations and their solutions to be entered in symbolic form. It then immediately integrates those equations into a set of desired computations and visualizations. We imported the data into Wolfram from MS Excel, specified objective function (3) and (4), found their partial derivatives, built the system of equations (5), and solved it.

Table 1.
The changes in the output vector from the 10% increase of the first element of the exogenous vector of commodity demand

SAM elements	Output values	Fuzzy changes, δ	Classical changes, δ
Industry 1	*g1*	0.0390511	0.0390332
Industry 2	*g2*	0.0001276	0.0000127
Industry 3	*g3*	0.0000193	0.0000131
Industry 4	*g4*	0.0001932	0.0001434
Industry 5	*g5*	0.0003000	0.0000443
Industry 6	*g6*	0.0001616	0.0001428
Industry 7	*g7*	0.0000779	0.0000765
Industry 8	*g8*	0.0001423	0.0001217
Industry 9	*g9*	0.0001205	0.0000956
Industry 10	*g10*	0.0001846	0.0001125
Industry 11	*g11*	0.0001361	0.0001961
Industry 12	*g12*	0.0000772	0.0000627
Industry 13	*g13*	0.0000918	0.0000401
Industry 14	*g14*	0.0001870	0.0002311
Industry 15	*g15*	0.0000857	0.0000889
Industry 16	*g16*	0.0000678	0.0000552
Industry 17	*g17*	0.0001098	0.0000750
Industry 18	*g18*	0.0001176	0.0000888
Industry 19	*g19*	0.0000874	0.0000977
Industry 20	*g20*	0.0000879	0.0000587
Industry 21	*g21*	0.0000310	0.0000105
Industry 22	*g22*	0.0000853	0.0000734
Industry 23	*g23*	0.0000330	0.0000441
Industry 24	*g24*	0.0000291	0.0000233
Industry 25	*g25*	0.0000681	0.0000501
Industry 26	*g26*	0.0000444	0.0000388
Industry 27	*g27*	0.0001193	0.0000765
Industry 28	*g28*	0.0000344	0.0000292
Industry 29	*g29*	0.0000758	0.0000878
Industry 30	*g30*	0.0000799	0.0000746
Industry 31	*g31*	0.0000711	0.0000592
Industry 32	*g32*	0.0002004	0.0002026
Industry 33	*g33*	0.0001164	0.0000635
Industry 34	*g34*	0.0000866	0.0000519
Industry 35	*g35*	0.0001610	0.0001164
Industry 36	*g36*	0.0000711	0.0000706
Industry 37	*g37*	0.0001046	0.0001061
Industry 38	*g38*	0.0000739	0.0000583
Industry 39	*g39*	0.0000679	0.0000635
Industry 40	*g40*	0.0000876	0.0000861
Industry 41	*g41*	0.0000819	0.0000776
Industry 42	*g42*	0.0000884	0.0000621
Industry 43	*g43*	0.0000843	0.0000386
Industry 44	*g44*	0.0001005	0.0000649
Industry 45	*g45*	0.0000000	0.0000522
Industry 46	*g46*	0.0000568	0.0000352
Industry 47	*g47*	0.0000890	0.0000784
Industry 48	*g48*	0.0000997	0.0000876
Industry 49	*g49*	0.0001214	0.0000971
Industry 50	*g50*	0.0001152	0.0001059
Industry 51	*g51*	0.0002079	0.0001053
Industry 52	*g52*	0.0001289	0.0000813
Industry 53	*g53*	0.0002208	0.0008447
Industry 54	*g54*	0.0001074	0.0001071
Industry 55	*g55*	0.0000331	0.0000021
Industry 56	*g56*	0.0000038	0.0000002
Industry 57	*g57*	0.0000027	0.0000059
CORP	*u1*	0.0001760	0.0001617
DIV	*u2*	0.0001723	0.0001617
H	*u3*	0.0001090	0.0001053
NRC	*u4*	0.0000000	0.0000000

The row, highlighted in grey, represents *household incomes*. The next column of this table "Classical changes" contains results obtained when using the regular matrix of direct input-output coefficients **A**. The changes in the vector **g** and in the vector **y** for this column are estimated via Equation (2). As we see, the fuzzy rate of changes in household incomes is very close to this parameter, estimated using the classical approach.

Let's estimate the effect from the 10% increase of each element of the composite exogenous vector **y** using both fuzzy and classical approaches, and compare them. Based on the structure of the vector **y**, there are three groups of exogenous factors:

$\mathbf{d} = (d_i), i \in N_1$ - elements of the final demand;

$\mathbf{e} = (e_k), k \in N_2$ - elements of exogenous institutional incomes;

$\mathbf{w} = (w_l), l \in N_3$ - elements of exogenous value added.

The first group of factors includes the final demand for 48 industries (the final demand for the rest of industries is zero or very small). A second group includes two elements: the resident household consumption and the nonresident household consumption (the third and fourth elements of **e**). A third group is represented by the labor factor (the first element of **w**) — the only element of the value added for which we have knowledge of its origin within the nation but outside of New Jersey.

The results of the estimation are represented in Table 2 and on Picture 1 for more convenient interpretation. As for the first element of the final demand, the fuzzy rate of changes in household incomes is very close to classical one when we change each separate element of the exogenous vector.

Step 3. Household expenditures are forecasted based on the estimation of changes in household incomes:

$$HE = (1+\delta)(1 - MPS - TINS)U_h \qquad (6)$$

where

δ - rate of change in household income;

MPS - marginal propensity to save;

$TINS$ - direct tax rate;

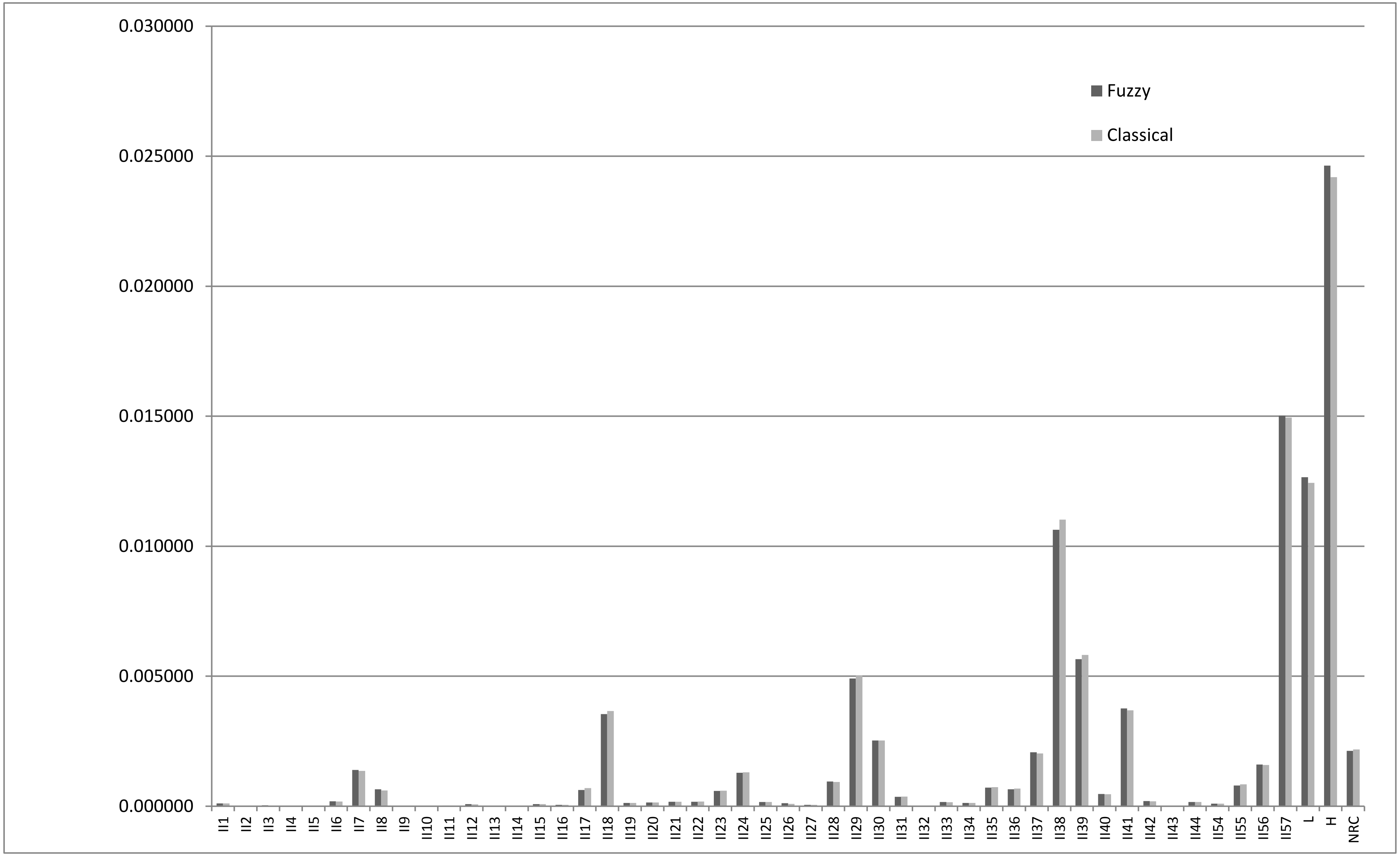
U_h - household income in the SAM.

MPS and $TINS$ are calculated based on household column in the SAM:

MPS = 0.178417; $TINS$ = 0.097340.

Table 2.
The changes in the household income
from the 10% increase of each element of the composite exogenous vector

Exogenous factor	Fuzzy changes, δ	Classical changes, δ
Final demand for industry 1	0.000109	0.000104
Final demand for industry 2	0.000009	0.000007
Final demand for industry 3	0.000035	0.000002
Final demand for industry 4	0.000009	0.000009
Final demand for industry 5	0.000005	0.000005
Final demand for industry 6	0.000188	0.000180
Final demand for industry 7	0.001391	0.001359
Final demand for industry 8	0.000651	0.000606
Final demand for industry 9	0.000012	0.000009
Final demand for industry 10	0.000014	0.000014
Final demand for industry 11	0.000023	0.000023
Final demand for industry 12	0.000079	0.000070
Final demand for industry 13	0.000016	0.000014
Final demand for industry 14	0.000002	0.000002
Final demand for industry 15	0.000083	0.000079
Final demand for industry 16	0.000051	0.000050
Final demand for industry 17	0.000628	0.000694
Final demand for industry 18	0.003540	0.003660
Final demand for industry 19	0.000125	0.000128
Final demand for industry 20	0.000144	0.000142
Final demand for industry 21	0.000169	0.000169
Final demand for industry 22	0.000174	0.000183
Final demand for industry 23	0.000584	0.000599
Final demand for industry 24	0.001287	0.001300
Final demand for industry 25	0.000162	0.000158
Final demand for industry 26	0.000121	0.000088
Final demand for industry 27	0.000051	0.000052
Final demand for industry 28	0.000953	0.000933
Final demand for industry 29	0.004915	0.005023
Final demand for industry 30	0.002532	0.002528
Final demand for industry 31	0.000362	0.000370
Final demand for industry 32	0.000007	0.000005
Final demand for industry 33	0.000160	0.000151
Final demand for industry 34	0.000128	0.000126
Final demand for industry 35	0.000716	0.000732
Final demand for industry 36	0.000651	0.000683
Final demand for industry 37	0.002077	0.002033
Final demand for industry 38	0.010632	0.011024
Final demand for industry 39	0.005655	0.005821
Final demand for industry 40	0.000473	0.000462
Final demand for industry 41	0.003760	0.003689
Final demand for industry 42	0.000199	0.000194
Final demand for industry 43	0.000016	0.000020
Final demand for industry 44	0.000162	0.000160
Final demand for industry 54	0.000095	0.000095
Final demand for industry 55	0.000793	0.000845
Final demand for industry 56	0.001607	0.001584
Final demand for industry 57	0.015023	0.014950
Labor	0.012659	0.012432
Resident household consumption	0.024640	0.024199
Nonresident household consumption	0.002126	0.002188



Picture 1. The fuzzy and classical rate of changes in household income from the 10% increase of each element of the composite exogenous vector

Using the formula (6) and the rate of change in household income estimated by fuzzy model (column 2 of Table 2), we forecast household expenditures from a 10% increase in exogenous factors. The results are shown in column 2 of Table 3.

Step 4. Finally, the household expenditures are forecasted using the classical approach. In this case, formula (6) includes the classical rate of changes in the household income (column 3 of Table 2). The classical result of forecasting is represented in column 3 of Table 3. To compare estimates for fuzzy and classical approaches we calculate the estimation error by dividing the absolute difference between the fuzzy and classical value of household expenditure by the classical value (column 4 of Table 3). The most important achievement is the very small estimation error: its value does not exceed 0.044%, which suggests that the fuzzy approach may well be an effective way to get fairly accurate results.

Let's consider the impact of each factor of these groups on the estimation error. The most significant error takes place when the direct impact of the resident household income (the factor of group 2) on the household expenditures is estimated. The direct impact of the labor income (the factor of the third group) is characterized by significant error as well. For the factors of the first group the estimation error depends on the share of the industry in the value added for the economy of New Jersey: changes in the final demand for the industries with a larger share in the regional value added drive more significant error (the correlation coefficient between the share of industry and the estimation error is $R = 0.82$). The estimation error for this group is also sensitive to the share of the industry in the total gross output: $R = 0.75$.

To get more information about the impact of the first group's factors of on the estimation error, we forecast household expenditures for changes in each element of the exogenous final demand on some constant value. This allows us to estimate error in the case of significant variability of elements of the final demand. In particular, let's implement step 1-4 to examine the fuzzy and classical effect of the increase of each factor of the first group on $100 million and calculate the estimation error (Table 1, Appendix C). The results prove that the estimation error remains remarkably small.

Now let's estimate impacts of a group of such factors on the estimation error. For example, we can calculate impacts for following cases:

Table 3.
The forecast of household expenditures
from the 10% increase of each element of the composite exogenous vector

Exogenous factor	Fuzzy	Classical	Estimation error
Final demand for industry 1	321411.73	321410.02	**0.000530%**
Final demand for industry 2	321379.69	321378.88	**0.000251%**
Final demand for industry 3	321387.88	321377.43	**0.003252%**
Final demand for industry 4	321379.69	321379.61	**0.000026%**
Final demand for industry 5	321378.20	321378.16	**0.000013%**
Final demand for industry 6	321437.06	321434.65	**0.000751%**
Final demand for industry 7	321823.76	321813.43	**0.003211%**
Final demand for industry 8	321586.08	321571.53	**0.004524%**
Final demand for industry 9	321380.43	321379.61	**0.000258%**
Final demand for industry 10	321381.18	321381.05	**0.000039%**
Final demand for industry 11	321384.16	321383.95	**0.000065%**
Final demand for industry 12	321402.04	321399.16	**0.000897%**
Final demand for industry 13	321381.92	321381.05	**0.000271%**
Final demand for industry 14	321377.45	321377.43	**0.000006%**
Final demand for industry 15	321403.53	321402.06	**0.000459%**
Final demand for industry 16	321393.10	321392.64	**0.000143%**
Final demand for industry 17	321578.63	321599.78	**0.006576%**
Final demand for industry 18	322514.46	322552.88	**0.011911%**
Final demand for industry 19	321416.94	321417.99	**0.000326%**
Final demand for industry 20	321422.90	321422.34	**0.000177%**
Final demand for industry 21	321431.10	321431.03	**0.000023%**
Final demand for industry 22	321432.59	321435.37	**0.000865%**
Final demand for industry 23	321564.47	321569.36	**0.001519%**
Final demand for industry 24	321790.23	321794.60	**0.001356%**
Final demand for industry 25	321428.86	321427.41	**0.000454%**
Final demand for industry 26	321415.45	321404.95	**0.003267%**
Final demand for industry 27	321393.10	321393.37	**0.000083%**
Final demand for industry 28	321682.94	321676.54	**0.001988%**
Final demand for industry 29	322956.30	322991.04	**0.010758%**
Final demand for industry 30	322190.34	322189.31	**0.000322%**
Final demand for industry 31	321492.94	321495.48	**0.000791%**
Final demand for industry 32	321378.94	321378.16	**0.000245%**
Final demand for industry 33	321428.12	321425.23	**0.000898%**
Final demand for industry 34	321417.69	321417.27	**0.000131%**
Final demand for industry 35	321606.94	321612.09	**0.001600%**
Final demand for industry 36	321586.08	321596.15	**0.003133%**
Final demand for industry 37	322044.31	322029.98	**0.004451%**
Final demand for industry 38	324793.68	324919.70	**0.038784%**
Final demand for industry 39	323193.98	323247.43	**0.016535%**
Final demand for industry 40	321528.71	321525.18	**0.001097%**
Final demand for industry 41	322585.24	322562.29	**0.007114%**
Final demand for industry 42	321440.79	321438.99	**0.000558%**
Final demand for industry 43	321381.92	321383.23	**0.000405%**
Final demand for industry 44	321428.86	321428.13	**0.000229%**
Final demand for industry 54	321407.26	321407.13	**0.000041%**
Final demand for industry 55	321631.53	321648.30	**0.005214%**
Final demand for industry 56	321893.05	321885.85	**0.002238%**
Final demand for industry 57	326204.88	326181.33	**0.007220%**
Labor	325444.66	325371.97	**0.022333%**
Resident household consumption	329295.27	329153.41	**0.043080%**
Nonresident household consumption	322059.72	322079.65	**0.006187%**

1. 10% increase in the final demand for the transportation industries (industries 31, 32, 33, 34 and 35; the total share for these industries in the regional gross output is about 3%).
2. 10% increase in the final demand for the government services (industries 55, 56 and 57; the total share for these industries in the regional gross output is 9.5%).
3. 10% increase in the resident household consumption and nonresident household consumption and the labor factor.
4. 10% increase in the final demand for all industries.
5. Increase in the final demand by $100 million for all industries.

The results of the estimate are represented in Table 2, Appendix C. The estimation error does not exceed 0.06%, which allows us to consider this fuzzy approach as a promising way of obtaining sufficiently accurate results of forecasting impacts of exogenous factors on household expenditures, using inaccurate and incomplete information about interindustrial transactions.

In practice, the real problem is not so much updating a matrix of direct IO coefficients as updating other structural components of a SAM. Therefore, it makes sense to construct a model that contains a fuzzy interpretation not only of matrix A, but also of a matrix of value-added by industry coefficients R, as well as a matrix of institutional expenditures by industry coefficients C as key structural components of a SAM.

In this case, ***Step 1*** should provide the fuzzification of matrix of value-added by industry coefficients and a matrix of institutional expenditures by industry coefficients. As distinct from a matrix of direct IO coefficients, these matrices have smaller dimensions and not as large a variety in values of their elements. It supposes to describe them by five verbal values. The procedure of the fuzzification for the matrixes R and C is described in Appendix B.

In ***Step 2*** we present the modified model that includes roughly hewn key structural components of a SAM as a combination of the two following criterions:

$$\sum_{i=1}^{N_1}\left(\sum_{j=1}^{N_1}\tilde{a}_{ij}g_j+\sum_{m=1}^{N_2}\tilde{c}_{im}u_m+d_i-g_i\right)^2\xrightarrow[g]{}\min \tag{7}$$

$$\sum_{k=1}^{N_2}\left(\sum_{l=1}^{N_3}z_{kl}\left(\sum_{j=1}^{N_1}\tilde{r}_{lj}g_j+w_l\right)+\sum_{m=1}^{N_2}h_{km}u_m+e_k-u_k\right)^2\xrightarrow[u]{}\min \tag{8}$$

where

N_1 - the number of industries;
N_2 - the number of institutions;
N_3 - the number of value-added elements;

$\tilde{\mathbf{A}} = (\tilde{a}_{ij}), i \in N_1, j \in N_1$ - fuzzy partition of direct interindustry coefficients;

$\tilde{\mathbf{R}} = (\tilde{r}_{lj}), l \in N_3, j \in N_1$ - fuzzy partition of value-added by industry coefficients;

$\tilde{\mathbf{C}} = (\tilde{c}_{ik}), i \in N_1, k \in N_2$ - fuzzy partition of institutional expenditures by industry coefficients;

$\mathbf{Z} = (z_{kl}), k \in N_2, l \in N_3$ - partition of institutional value-added allocation coefficients;

$\mathbf{H} = (h_{km}), k \in N_2, m \in N_2$ - partition of interinstitutional coefficients;

$\mathbf{d} = (d_i), i \in N_1$ - the vector of exogenous final demand;

$\mathbf{e} = (e_k), k \in N_2$ - the vector of exogenous institutional incomes;

$\mathbf{w} = (w_l), l \in N_3$ - the vector of exogenous value added;

$\mathbf{g} = (g_i), i \in N_1$ - the vector of total output by industry;

$\mathbf{u} = (u_k), k \in N_2$ - the vector of total institutional incomes.

By analogy with the model (1), we differentiate criterion (7) with respect to each g_i, and criterion (8) with respect to each u_k and set obtained partial derivatives equal to zero. As a result, we have the new system of (N_1+N_2) equations.

Let's use the modified SAM model for estimating the effect of exogenous shocks on changes in household expenditure and compare results with both classical and fuzzy ones as shown above for the SAM model with a fuzzy matrix of direct IO coefficients A (***Step 2*** – ***Step 4***). Table 3, Appendix C presents household expenditures from the 10% increase in each element of the composite exogenous vector. The estimation error 1 characterizes the absolute value of the relative deviation of a fuzzy result from a corresponding classical one for the SAM model (3) - (4). The estimation error 2 has the same meaning for the SAM model (7) - (8). As expected, the error is more significant for the modified model but it is still quite small (not exceeding 0.15%). Above we have distinguished three groups of exogenous factors for estimating effects of various shocks on the economy and have analyzed the dependence of the estimation error on each group of these factors.

For the modified SAM model the impact of household consumption (group 2) and labor income (group 3) remains significant, but in some cases the more important factors are elements of the final demand. In particular, the most significant estimation error takes place for industry 38 and industry 29, which are among the largest in the regional economy: the share of industry 38 on the total regional output is 9.9% (the second largest industry among 48 industries represented in the SAM for New Jersey) and the share of industry 29 is 4.8% (the eighth largest industry).

Let's analyze the correlation between the estimation error and the industry structure of both the gross output and the value added. For SAM model with three fuzzy matrices we have less significant correlation between these factors. In particular, the correlation coefficient between the estimation error and the industry share in the gross output has decreased by 21% (R = 0.58). The error sensitivity to the industry share in the value added has decreased by 14% (R = 0.70). At the same time, we observe the significant increase of the error sensitivity to such value added component as Labor: the correlation coefficient changed from R = 0.05 for the first model to R = 0.15 for the second one. Moreover, we explored the correlation between the estimation error and the share of the industry consumption in the total consumption for the economy of New Jersey. The correlation is not significant but correlation coefficients for the second model R = 0.24 has increased by 1.52 times as compared with the first model. Even more significant correlation increase takes place for the correlation between error and the industry structure of the resident household consumption: the correlation coefficient has increased by 1.68. Thus, the forecast accuracy depends primarily on the quality of labor and residential household consumption data.

The estimation error for two models when forecasting household expenditures as a result of changes in each element of the final demand on $100 million is presented in Table 4, Appendix C. In this case, the estimation error 2 is also more significant than the estimation error 1 but its values are still quite small.

Finally, let's analyze the impact of groups of exogenous factors on the estimation error using the modified SAM model for 5 previously described cases (Table 5, Appendix C). The estimation error 2 is larger than the estimation error 1. The largest error is obtained for two cases: increase all factors for both group 2 and group 3 and increase of the final demand for all industries in the same proportion. At the same time, error maximum value does not exceed 0.19% that confirms the sufficient accuracy when forecasting impacts of different combinations of exogenous factors on household expenditures using a SAM model with fuzzy structural components. Thus, a fuzzy approach looks like an efficient way to forecast effect of various exogenies factors on household expenditures under condition of lacking or inaccurate SAM data.

Conclusion

A prime challenge when forecasting the influence of exogenous shocks on the socio-economic system is the lack of accurate and complete information on the system's initial state. Existing techniques for updating social accounting matrices (SAMs) either ignore pre-existing error structures in SAM data or are overly constraining given the scarcity of data and their lack of known error structures. We propose a fuzzy approach for updating SAMs that yields sufficient accuracy given a "smear" of SAM data. According to this approach, we convert qualitative assessments into quantitative estimates for direct input-output coefficients – the core of the matrix of SAM direct coefficients. Using expected values of quasi-probability density functions for the quantitative interpretation of coefficient values, we employ quasi-stochastic programming for estimating impacts of different exogenous shocks on household expenditures as it is a major constituent of total expenditures.

We use the following steps: the fuzzification of the matrix of direct input-output coefficients (replace each element by its fuzzy analogue); the estimate of the influence of exogenous factors on the vector of output values, in particular, household incomes using a SAM model with fuzzy input-output coefficients; forecasting household expenditures based on changes in household incomes; the comparison of the fuzzy results with the corresponding ones obtained when using the classical approach and the analysis of factors of error sensitivity.

We applied the approach to a 2010 SAM estimated for the economy of the State New Jersey. We calculated the impact of relative and absolute changes in different exogenous factors such as commodity demand, the value added, and the institutional income on the household expenditures via a SAM model with fuzzy direct input-output coefficients. The estimation error was very small. The maximal error took place as a result of the impact of both the labor factor and residential household consumption factor. When estimating impact of the commodity demand elements, we obtained smaller error that was sensitive to industry structure of both the gross output and the value added.

Then we extended the initial model by also fuzzifying the matrix of value-added by industry coefficients and the matrix of institutional expenditures by industry coefficients. We estimate the same impacts via a "new" SAM model with three fuzzy matrices. The estimation error is still quite small but in this case its maximal value was obtained from impact of changes in the industry final demand. The error sensitivity from the industry structure of value added as well as gross output has decreased but we found two factors, which impact on the estimation error significantly increased when using a modified SAM model: the industry structure of the labor and the industry structure of the resident household consumption. It means that the quality of labor and the resident household consumption data plays the key role for the forecast accuracy of a SAM model.

Thus, a fuzzy approach can be an efficient, cost-effective way for forecasting effects of different exogenous factors on socio-economic system under condition of lacking or inaccurate SAM data.

Appendix A: The fuzzification of the matrix of direct input-output coefficients

The fuzzification of the matrix of direct input-output coefficients means to interpret elements of this matrix as linguistic variables. According to fuzzy set theory, a linguistic variable is defined on some quantitative scale using ordinary words and phrases. The linguistic variable is described by verbal values.

Formally, linguistic variable is characterized by a set:

$$\langle \beta, T(\beta), U \rangle,$$

where β – the name of linguistic variable;

$T(\beta)$ - term-set for linguistic variable β;

$U = \{u\}$ - definitional domain of each verbal value for this linguistic variable.

Each verbal value (term) of the linguistic variable is described by a fuzzy variable, which corresponds to a certain fuzzy subset:

$\tilde{\mathbf{A}}(\alpha) = \{\langle \mu_{\tilde{A}(\alpha)}(u)/u \rangle\}, u \in U$ – fuzzy subset of the set U, which describes the restrictions on possible values of fuzzy variable α;

$\langle \mu_{\tilde{A}(\alpha_q)}(u)/u \rangle$, где $u \in U, \mu_{\tilde{A}(\alpha_q)} \in [0;1]$ – membership function.

For each specific value $u \in U$ the value $\mu_{\tilde{A}(\alpha_q)}(u)$ takes a specific value from a closed interval [0, 1], which is called *the degree of membership* of u to fuzzy set $\tilde{A}(\alpha_q)$.

So, let's define each input-output coefficient as linguistic variable β = "Type of intersectoral relationship." Definitional domain of values for this linguistic variable *U = [0; 8]* is obtained based on the 2010 SAM for New Jersey. This gives the following definition of the linguistic variable:

$$\langle \text{«Type of interindustrial relationship"», } T(\beta), [0;\, 0{,}31] \rangle, U = [\underline{U}, \overline{U}]$$

The term-set for verbal values of the linguistic variable β = "Type of intersectoral relationship" is represented as:

$T(\beta)$ = *{"Very Weak," "Weak, "Below Medium," "Medium," "Above Medium," "Strong," "Very Strong"}.*

To create fuzzy subset $\tilde{A}(\alpha_q)$ for fuzzy variables fuzzy variable α_q for the given type of intersectoral relationship $T(\beta = q)$, it is necessary to select the numerical intervals that characterize the definitional domain of each fuzzy variable using cluster analysis procedure. Next, based on calculated numeric intervals, we determine the related membership functions for each type of intersectoral relationship and calculate the expectation for these functions. In this case, the membership functions are defined as the continuous Gaussian functions:

$$\mu_{\tilde{A}(\alpha)}(u) = e^{-\frac{(u-b)^2}{2c^2}}, u \in U, \alpha_q \in \alpha, \tilde{A}(\alpha_q) \in A \quad (*)$$

The expectation of this membership function are calculated by the formula:

$$M_{\tilde{A}(\alpha)}(u) = \frac{\int_{\underline{U}}^{\overline{U}} u \cdot \exp\left[\frac{-(u-b)^2}{2c^2}\right] du}{\int_{\underline{U}}^{\overline{U}} \exp\left[\frac{-(u-b)^2}{2c^2}\right] du}, u \in U, \alpha_q \in \alpha, \tilde{A}(\alpha_q) \in A \quad (**)$$

The results of the calculations are represented in Table x. Using this table, we replace the elements of the matrix of direct input-output coefficients by the expected values from the membership function for the appropriate fuzzy variable.

Table x.

Cluster	**Terms , which characterize the type of interindustrial relationships**	**The definitional domain of the fuzzy variable**		**The expectation of the membership function for the fuzzy variable**
1	«Very Strong»	0.141323	**0.312829**	0.222000
2	«Strong»	0.047598	0.146627	0.090000
3	«Above Medium»	0.031549	0.049786	0.040000
4	«Medium»	0.019202	0.031972	0.025000
5	«Below Medium»	0.009113	0.019395	0.014000
6	«Weak»	0.002771	0.009341	0.005590
7	«Very Weak»	**0.000000**	0.002800	0.000418

Appendix B: The fuzzification of a matrix of value-added by industry coefficients and a matrix of institutional expenditures by industry coefficients

For fuzzy interpretation we define each element of a matrix of value-added by industry coefficients R as linguistic variable β = "Type of industry expenditure on primary factors". Definitional domain of values for this linguistic variable *U = [0; 0.620677]* is obtained based on the 2010 SAM for New Jersey. This gives the following definition of the linguistic variable:

$$\langle \text{«Type of industry expenditure to primary factors"»}, T(\beta), [0; 0{,}62] \rangle, U = [\underline{U}, \overline{U}]$$

The term-set for verbal values of the linguistic variable β = "Type of intersectoral relationship" is represented as:

$$T(\beta) = \{\text{"Very Weak," "Weak, "Medium," "Strong," "Very Strong"}\}.$$

The procedure of cluster analysis is employed to select the numerical intervals that characterize the definitional domain of each fuzzy variable. Next, the expectation of Gaussian membership functions is calculated based on obtained numeric intervals. Table xx presents the results of calculation.

Table xx.

Cluster	Terms , which characterize the type of industry expenditure on primary factors	The definitional domain of the fuzzy variable		The expectation of the membership function for the fuzzy variable
1	«Very Strong»	0.374776	**0.620677**	0.541000
2	«Strong»	0.176344	0.564830	0.342000
4	«Medium»	0.106893	0.230398	0.162000
6	«Weak»	0.033566	0.115605	0.068000
7	«Very Weak»	**0.000000**	0.034940	0.005041

Using the same idea, each element of a matrix of institutional expenditures by industry coefficients C can be defined as linguistic variable β = "Type of institutional expenditure on industry production". Definitional domain of values for this linguistic variable *U=[0; 0.253962]* is obtained based on the 2010 SAM for New Jersey. This gives the following definition of the linguistic variable for a matrix C:

$$\langle \text{«Type of institutional expenditure on industry production"»}, T(\beta), [0; 0.2540] \rangle, U = [\underline{U}, \overline{U}]$$

This linguistic variable is represented by the same five terms. The expectation of Gaussian membership functions for fuzzy variables of this linguistic variable are shown in Table xxx.

Table xxx.

Cluster	**Terms , which characterize the type of institutional expenditure on industry production**	**The definitional domain of the fuzzy variable**		**The expectation of the membership function for the fuzzy variable**
1	«Very Strong»	0.164961	**0.253962**	0.207000
2	«Strong»	0.101321	0.235066	0.160000
4	«Medium»	0.046693	0.119909	0.079000
6	«Weak»	0.009153	0.064384	0.028000
7	«Very Weak»	**0.000000**	0.011868	0.001475

Appendix C

Table 1.
The forecast of household expenditures from the increase of elements of exogenous final demand on $100 million

Exogenous factor	Fuzzy	Classical	Estimation error
Final demand for industry 1	321390.12	321389.74	**0.000117%**
Final demand for industry 2	321381.18	321379.61	**0.000490%**
Final demand for industry 3	321377.45	321376.71	**0.000232%**
Final demand for industry 4	321398.32	321397.71	**0.000188%**
Final demand for industry 5	321416.94	321421.61	**0.001452%**
Final demand for industry 6	321420.67	321419.44	**0.000383%**
Final demand for industry 7	321405.02	321404.23	**0.000246%**
Final demand for industry 8	321408.00	321405.68	**0.000723%**
Final demand for industry 9	321386.39	321386.12	**0.000084%**
Final demand for industry 10	321400.55	321400.61	**0.000018%**
Final demand for industry 11	321393.10	321392.64	**0.000143%**
Final demand for industry 12	321385.65	321383.95	**0.000529%**
Final demand for industry 13	321380.43	321379.61	**0.000258%**
Final demand for industry 14	321391.61	321391.19	**0.000130%**
Final demand for industry 15	321410.24	321407.85	**0.000743%**
Final demand for industry 16	321403.53	321403.51	**0.000008%**
Final demand for industry 17	321394.59	321396.26	**0.000520%**
Final demand for industry 18	321416.94	321417.99	**0.000326%**
Final demand for industry 19	321406.51	321407.85	**0.000417%**
Final demand for industry 20	321408.00	321407.85	**0.000047%**
Final demand for industry 21	321387.14	321387.57	**0.000135%**
Final demand for industry 22	321413.22	321415.09	**0.000584%**
Final demand for industry 23	321406.51	321407.13	**0.000191%**
Final demand for industry 24	321409.49	321409.30	**0.000060%**
Final demand for industry 25	321398.32	321397.71	**0.000188%**
Final demand for industry 26	321378.94	321378.16	**0.000245%**
Final demand for industry 27	321399.06	321399.16	**0.000031%**
Final demand for industry 28	321408.00	321407.13	**0.000272%**
Final demand for industry 29	321442.28	321443.34	**0.000331%**
Final demand for industry 30	321443.02	321442.61	**0.000127%**
Final demand for industry 31	321437.81	321438.99	**0.000369%**
Final demand for industry 32	321398.32	321396.99	**0.000413%**
Final demand for industry 33	321443.02	321439.72	**0.001028%**
Final demand for industry 34	321448.98	321448.41	**0.000178%**
Final demand for industry 35	321440.04	321441.17	**0.000350%**
Final demand for industry 36	321431.85	321434.65	**0.000872%**
Final demand for industry 37	321448.98	321446.96	**0.000629%**
Final demand for industry 38	321431.10	321433.20	**0.000653%**
Final demand for industry 39	321444.51	321445.51	**0.000311%**
Final demand for industry 40	321441.53	321439.72	**0.000564%**
Final demand for industry 41	321454.20	321452.03	**0.000674%**
Final demand for industry 42	321432.59	321431.03	**0.000486%**
Final demand for industry 43	321424.39	321433.92	**0.002965%**
Final demand for industry 44	321443.77	321443.34	**0.000133%**
Final demand for industry 54	321447.49	321447.68	**0.000060%**
Final demand for industry 55	321510.82	321514.31	**0.001086%**
Final demand for industry 56	321396.83	321396.26	**0.000175%**

Table 2.
The forecast of household expenditures from the increase of group elements of the composite exogenous vector

Cases		Fuzzy	Classical	Estimation error
1.	10% increase in the final demand for transportation industries	321817.06	321822.84	0.001798%
2.	10% increase in the final demand for government services	326976.79	326962.79	0.004283%
3.	10% increase in the institutional incomes and the added value	334046.94	333851.78	0.058455%
4.	10% increase in the final demand for all industries	340844.38	341038.44	0.056903%
5.	Increase in the final demand by on $100 million for all industries	323839.97	323851.44	0.003543%

Table 3.
The forecast of household expenditures from the 10% increase of each element of the composite exogenous vector using two SAM models with fuzzy parameters

Exogenous factor	**Fuzzy matrix A**	**Fuzzy matrixes A, R, C**	**Classic**	**Estimation error 1**	**Estimation error 2**
Final demand for industry 1	321411.73	321420.85	321410.02	**0.000530%**	**0.003368%**
Final demand for industry 2	321379.69	321378.94	321378.88	**0.000251%**	**0.000019%**
Final demand for industry 3	321387.88	321400.18	321377.43	**0.003252%**	**0.007077%**
Final demand for industry 4	321379.69	321379.50	321379.61	**0.000026%**	**0.000032%**
Final demand for industry 5	321378.20	321377.83	321378.16	**0.000013%**	**0.000103%**
Final demand for industry 6	321437.06	321439.85	321434.65	**0.000751%**	**0.001617%**
Final demand for industry 7	321823.76	321858.90	321813.43	**0.003211%**	**0.014130%**
Final demand for industry 8	321586.08	321590.70	321571.53	**0.004524%**	**0.005963%**
Final demand for industry 9	321380.43	321379.50	321379.61	**0.000258%**	**0.000032%**
Final demand for industry 10	321381.18	321380.62	321381.05	**0.000039%**	**0.000135%**
Final demand for industry 11	321384.16	321383.41	321383.95	**0.000065%**	**0.000167%**
Final demand for industry 12	321402.04	321391.24	321399.16	**0.000897%**	**0.002466%**
Final demand for industry 13	321381.92	321380.62	321381.05	**0.000271%**	**0.000135%**
Final demand for industry 14	321377.45	321377.83	321377.43	**0.000006%**	**0.000122%**
Final demand for industry 15	321403.53	321406.88	321402.06	**0.000459%**	**0.001501%**
Final demand for industry 16	321393.10	321392.91	321392.64	**0.000143%**	**0.000084%**
Final demand for industry 17	321578.63	321566.12	321599.78	**0.006576%**	**0.010465%**
Final demand for industry 18	322514.46	322621.01	322552.88	**0.011911%**	**0.021124%**
Final demand for industry 19	321416.94	321408.56	321417.99	**0.000326%**	**0.002935%**
Final demand for industry 20	321422.90	321434.26	321422.34	**0.000177%**	**0.003709%**
Final demand for industry 21	321431.10	321442.64	321431.03	**0.000023%**	**0.003613%**
Final demand for industry 22	321432.59	321436.49	321435.37	**0.000865%**	**0.000349%**
Final demand for industry 23	321564.47	321528.68	321569.36	**0.001519%**	**0.012648%**
Final demand for industry 24	321790.23	321863.93	321794.60	**0.001356%**	**0.021545%**
Final demand for industry 25	321428.86	321425.32	321427.41	**0.000454%**	**0.000649%**
Final demand for industry 26	321415.45	321419.17	321404.95	**0.003267%**	**0.004424%**
Final demand for industry 27	321393.10	321392.35	321393.37	**0.000083%**	**0.000315%**
Final demand for industry 28	321682.94	321651.05	321676.54	**0.001988%**	**0.007926%**
Final demand for industry 29	322956.30	323339.55	322991.04	**0.010758%**	**0.107899%**
Final demand for industry 30	322190.34	322394.17	322189.31	**0.000322%**	**0.063583%**
Final demand for industry 31	321492.94	321511.36	321495.48	**0.000791%**	**0.004939%**
Final demand for industry 32	321378.94	321378.94	321378.16	**0.000245%**	**0.000245%**
Final demand for industry 33	321428.12	321433.70	321425.23	**0.000898%**	**0.002634%**
Final demand for industry 34	321417.69	321426.44	321417.27	**0.000131%**	**0.002853%**
Final demand for industry 35	321606.94	321653.28	321612.09	**0.001600%**	**0.012809%**
Final demand for industry 36	321586.08	321615.29	321596.15	**0.003133%**	**0.005950%**
Final demand for industry 37	322044.31	322271.25	322029.98	**0.004451%**	**0.074922%**
Final demand for industry 38	324793.68	324432.44	324919.70	**0.038784%**	**0.149965%**
Final demand for industry 39	323193.98	323233.39	323247.43	**0.016535%**	**0.004343%**
Final demand for industry 40	321528.71	321533.71	321525.18	**0.001097%**	**0.002655%**
Final demand for industry 41	322585.24	322551.17	322562.29	**0.007114%**	**0.003448%**
Final demand for industry 42	321440.79	321457.17	321438.99	**0.000558%**	**0.005654%**
Final demand for industry 43	321381.92	321382.30	321383.23	**0.000405%**	**0.000290%**
Final demand for industry 44	321428.86	321430.91	321428.13	**0.000229%**	**0.000864%**
Final demand for industry 54	321407.26	321410.23	321407.13	**0.000041%**	**0.000966%**
Final demand for industry 55	321631.53	321723.68	321648.30	**0.005214%**	**0.023437%**
Final demand for industry 56	321893.05	321852.75	321885.85	**0.002238%**	**0.010283%**
Final demand for industry 57	326204.88	326223.19	326181.33	**0.007220%**	**0.012832%**
Labor	325148.18	325444.89	325371.63	**0.022333%**	**0.022516%**
Resident household consumption	328717.94	329295.51	329153.62	**0.043080%**	**0.043106%**
Nonresident household consumption	322110.89	322059.95	322079.22	**0.006187%**	**0.005983%**

Table 4.
The forecast of household expenditures from the increase on $100 million of each element of the composite exogenous vector using two SAM models with fuzzy parameters

Exogenous factor	Fuzzy matrix A	Fuzzy matrixes A, R, C	Classic	Estimation error 1	Estimation error 2
Final demand for industry 1	321390.12	321394.03	321389.74	**0.000117%**	**0.001333%**
Final demand for industry 2	321381.18	321379.50	321379.61	**0.000490%**	**0.000032%**
Final demand for industry 3	321377.45	321379.50	321376.71	**0.000232%**	**0.000869%**
Final demand for industry 4	321398.32	321393.47	321397.71	**0.000188%**	**0.001320%**
Final demand for industry 5	321416.94	321411.91	321421.61	**0.001452%**	**0.003019%**
Final demand for industry 6	321420.67	321423.08	321419.44	**0.000383%**	**0.001134%**
Final demand for industry 7	321405.02	321407.44	321404.23	**0.000246%**	**0.000998%**
Final demand for industry 8	321408.00	321409.12	321405.68	**0.000723%**	**0.001069%**
Final demand for industry 9	321386.39	321383.41	321386.12	**0.000084%**	**0.000843%**
Final demand for industry 10	321400.55	321396.26	321400.61	**0.000018%**	**0.001352%**
Final demand for industry 11	321393.10	321391.24	321392.64	**0.000143%**	**0.000438%**
Final demand for industry 12	321385.65	321381.74	321383.95	**0.000529%**	**0.000689%**
Final demand for industry 13	321380.43	321380.06	321379.61	**0.000258%**	**0.000142%**
Final demand for industry 14	321391.61	321391.79	321391.19	**0.000130%**	**0.000187%**
Final demand for industry 15	321410.24	321414.14	321407.85	**0.000743%**	**0.001958%**
Final demand for industry 16	321403.53	321403.53	321403.51	**0.000008%**	**0.000007%**
Final demand for industry 17	321394.59	321393.47	321396.26	**0.000520%**	**0.000869%**
Final demand for industry 18	321416.94	321420.85	321417.99	**0.000326%**	**0.000889%**
Final demand for industry 19	321406.51	321400.73	321407.85	**0.000417%**	**0.002214%**
Final demand for industry 20	321408.00	321416.38	321407.85	**0.000047%**	**0.002653%**
Final demand for industry 21	321387.14	321390.12	321387.57	**0.000135%**	**0.000792%**
Final demand for industry 22	321413.22	321416.38	321415.09	**0.000584%**	**0.000400%**
Final demand for industry 23	321406.51	321400.73	321407.13	**0.000191%**	**0.001989%**
Final demand for industry 24	321409.49	321415.26	321409.30	**0.000060%**	**0.001855%**
Final demand for industry 25	321398.32	321397.38	321397.71	**0.000188%**	**0.000103%**
Final demand for industry 26	321378.94	321379.50	321378.16	**0.000245%**	**0.000419%**
Final demand for industry 27	321399.06	321398.50	321399.16	**0.000031%**	**0.000206%**
Final demand for industry 28	321408.00	321405.20	321407.13	**0.000272%**	**0.000598%**
Final demand for industry 29	321442.28	321458.84	321443.34	**0.000331%**	**0.004823%**
Final demand for industry 30	321443.02	321459.96	321442.61	**0.000127%**	**0.005396%**
Final demand for industry 31	321437.81	321447.11	321438.99	**0.000369%**	**0.002525%**
Final demand for industry 32	321398.32	321396.26	321396.99	**0.000413%**	**0.000225%**
Final demand for industry 33	321443.02	321450.46	321439.72	**0.001028%**	**0.003342%**
Final demand for industry 34	321448.98	321464.43	321448.41	**0.000178%**	**0.004984%**
Final demand for industry 35	321440.04	321452.70	321441.17	**0.000350%**	**0.003587%**
Final demand for industry 36	321431.85	321439.85	321434.65	**0.000872%**	**0.001617%**
Final demand for industry 37	321448.98	321473.37	321446.96	**0.000629%**	**0.008216%**
Final demand for industry 38	321431.10	321425.88	321433.20	**0.000653%**	**0.002278%**
Final demand for industry 39	321444.51	321445.99	321445.51	**0.000311%**	**0.000149%**
Final demand for industry 40	321441.53	321443.20	321439.72	**0.000564%**	**0.001083%**
Final demand for industry 41	321454.20	321452.14	321452.03	**0.000674%**	**0.000034%**
Final demand for industry 42	321432.59	321446.55	321431.03	**0.000486%**	**0.004830%**
Final demand for industry 43	321424.39	321425.88	321433.92	**0.002965%**	**0.002503%**
Final demand for industry 44	321443.77	321446.55	321443.34	**0.000133%**	**0.000999%**
Final demand for industry 54	321447.49	321454.37	321447.68	**0.000060%**	**0.002081%**
Final demand for industry 55	321510.82	321535.39	321514.31	**0.001086%**	**0.006555%**
Final demand for industry 56	321396.83	321395.15	321396.26	**0.000175%**	**0.000347%**
Final demand for industry 57	321443.02	321443.76	321442.59	**0.000134%**	**0.000363%**

Table 5.
The forecast of household expenditures from the increase of group elements of the composite exogenous vector using two SAM models with fuzzy parameters

Cases		Fuzzy matrix A	Fuzzy matrixes A, R, C	Classic	Estimation error 1	Estimation error 2
1.	10% increase of the final demand for transportation industries	321817.06	321895.77	321822.84	0.001798%	0.022662%
2.	10% increase of the final demand for government services	326976.79	327046.21	326962.79	0.004283%	0.025512%
3.	10% increase of the institutional incomes and the added value	334046.94	333223.59	333851.78	0.058455%	0.188164%
4.	10% increase of the final demand for all industries	340844.38	341668.33	341038.44	0.056903%	0.184699%
5.	Increase of the final demand on $100 million for all industries	323839.97	324033.50	323851.44	0.003543%	0.056216%

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