ESSAYS ON RISK MANAGEMENT OF FINANCIAL MARKET WITH BAYESIAN ESTIMATION

by

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ABSTRACT OF THE DISSERTATION

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This dissertation consists of three essays on modeling financial risk under Bayesian framework. The first essay compares the performances of Maximum Likelihood Estimation (MLE), Probability-Weighted Moments (PWM), Maximum Product of Spacings (MPS) and Bayesian estimation by using the Monte Carlo Experiments on simulated data from GEV distribution. I compare not only how close the estimates are to the true parameters, but also how close the combination of the three parameters in terms of estimated Value-at-Risk (VaR) to the true VaR. The Block Maxima Method based on student-t distribution is used for analysis to mimic the real world situation. The Monte Carlo Experiments show that the Bayesian estimation provides the smallest standard deviations of estimates for all cases. VaR estimates of the MLE and the PWM are closer to the true VaR, but we need to choose the initial values carefully for MLE. MPS gives the worst approximation in general.

The second essay analyzes the movement of implied volatility surface from 2005 to

2014. The study period is divided into four sub-periods: Pre-Crisis, Crisis, Adjustment period and Post-Crisis. The Black-Scholes model based daily implied volatility (IV) is constructed and the time series of IV given different moneyness ($m = K/S$) and time to maturity (τ) is fitted into a stochastic differential equation with mean-reverting drift and constant elasticity of variance. After estimating the parameters using a Bayesian Metropolis Hastings algorithm, the comparison across different time periods is conducted. As it is natural to expect abnormality in Crisis and Adjustment period, it is interesting to see the difference between Post-Crisis movement and the Pre-Crisis's. The results reveal that if the catastrophe does not permanently change the investment behavior, the effect from Crisis may last longer than expected. It is unwise to assume the market movement or investment behavior would be identical in Pre-Crisis and Post-Crisis periods. Market participants learn from Crisis and behave differently in Post-Crisis comparing to Pre-Crisis.

The third essay attempts to predict financial stress by identifying leading indicators under a Bayesian variable selection framework. Stochastic search variable selection (SSVS) formulation of George and McCulloch (1993) is used to select more informative variables as leading indicators among a number of financial variables. Both linear model and Probit model under normal error assumption and fat tail assumption are used for analysis. Financial stress indexes issued by Federal Reserve Banks combined with Bloom(2009) and Ng(2015)'s paper are used to identify financial stress. An ex-post approach based on historical perspective and ex ante approach combined with rolling window are used for analysis. The results show promising predictive power and the selection of variables can be used to signal financial crisis period.

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Last but not the least, I would like to thank my parents and my husband for their support throughout my Ph.D study and my life in general.

Dedication

To my parents and my husband.

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Chapter 1

Introduction

Econometric modeling and estimation have made great contributions to the development of risk management in financial markets over the past several decades. This dissertation considers three topics that are crucial in the different sub-fields in financial risk management: left tail risk estimation, implied volatility surface movement and financial stress prediction. My dissertation aims to investigate these topics under a Bayesian framework and provide practical guidance to both policymakers and the private sector in reviewing and developing policies and investment strategies.

Statistical distributions have played an important role in financial modeling and the recent global financial crisis has brought increased attention to the Generalized Extreme Value (GEV) distribution as a way of modeling the extreme observations and the left tail risk in finance. Motivated by this, the question of how well we can estimate the GEV distribution becomes crucial. In the second chapter, I compare the performances of Maximum Likelihood Estimation (MLE), Probability-Weighted Moments (PWM), Maximum Product of Spacings (MPS) and Bayesian estimation by using the Monte Carlo Experiments on simulated data from GEV distribution. I compare not only how close the estimates are to the true parameters, but also how close the combination of the three parameters in terms of estimated Value-at-Risk (VaR) to the true VaR. After estimating the parameters of the GEV distribution, I estimate the VaR at 1%, 5%, 10%, 25% and 50% level, and compare the estimation based on averaging the absolute values of the difference between the estimated VaR and the true VaR. Then the Block Maxima Method is used for analysis because in real data analysis, people use this method to sample the extreme values. To do this, I conduct Monte Carlo experiments of the student-t distribution with 5 degrees of freedom. Then I select the extreme values with sub-group size 5, 10 and 20, and finally compare the MLE, the MPS, the PWM and the Bayesian estimation on these extreme values originated from the student-t distribution. The Monte Carlo Experiments show that the Bayesian estimation provides the smallest standard deviations of estimates for all cases. VaR estimates of the MLE and the PWM are closer to the true VaR, but we need to choose the initial values carefully for MLE. MPS gives the worst approximation in general.

The Black-Scholes-Merton (BSM) model was developed in the early 1970s and implied volatility based on it has been widely studied due to the implications to trading, pricing and risk management. It is widely believed that implied volatility provides important information of the market expectation of future volatility. Moreover, BSM implied volatility has been used as a quoting convention of the option price by practitioners due to historical reasons. Therefore there is a long history of studying the BSM implied volatility surface (see Cont and Fonseca (2002), Szakmary, Ors, Kim and Davidson(2003), Busch, Christensen and Nielsen (2011) and Goncalves and Guidolin (2005)). The third chapter of this dissertation analyzes the movement of implied volatility surface in four time periods: Pre-Crisis, Crisis, Adjustment period and Post-Crisis. I first construct the daily implied volatility surface which is a three-dimensional plot that displays implied volatility given different moneyness ($m = K/S$) and time to maturity (τ). Given each set of (m, τ), the implied volatility time series $IV_t(m, \tau)$ is obtained. The data is then fitted into a stochastic differential equation with mean-reverting drift and constant elasticity of variance. The mean-reverting drift is consistent with the observation and the constant elasticity of variance allows flexibility of modeling the volatility of volatility (vol-of-vol). After estimating the parameters using a Bayesian Metropolis Hastings algorithm, the comparison across different time periods is conducted. I find out that in most scenarios, although the long-run level of implied volatility in Post-Crisis is close to it is in Pre-Crisis, the speed that pulls the implied volatility toward long-run level is much bigger in Post-Crisis. Loosely speaking, the combined effect of volatility parameters: b_1 and b_2 shows the implied volatility of the out-of-the-money put options has bigger conditional vol-of-vol in Post-Crisis than in Pre-Crisis. For at-the-money option the change is more complicated. As it is natural to expect abnormality in

Crisis and Adjustment period, it is interesting to see the difference between Post-Crisis movement and the Pre-Crisis's. The results reveal that if the catastrophe does not permanently change the investment behavior, the effect from Crisis may last longer than expected. It is unwise to assume the market movement or investment behavior would be identical in Pre-Crisis and Post-Crisis periods. Market participants learn from Crisis and behave differently in Post-Crisis comparing to Pre-Crisis.

The fourth chapter of this dissertation attempts to predict financial stress by identifying leading indicators under a Bayesian variable selection framework. While large proportion of the literature in this field focuses on financial crisis, especially for banking crisis, this paper also includes non crisis periods in order to provide more guidance to policy makers and the private sector. To improve the prediction and differentiate my work from others, I use weekly financial variables instead of quarterly macro variables that are used by most of the literature in this strand (see Vasicek et al. (2016) and Slingenberg and de Haan (2011)). A number of financial variables belonging to five categories: interest rate, yield spread, volatility, inflation and market return are used in the analysis. Stochastic search variable selection (SSVS) formulation of George and McCulloch (1993) is used to select more informative variables as leading indicators. Both linear model and Probit model under normal error assumption and fat tail assumption are used for analysis. Three financial stress indexes issued by Federal Reserve Banks are used to identify the level data of financial stress. These indexes together with other papers on financial uncertainty (Bloom(2009) and Ng(2015)) are used to identify binary variable representing the occurrence of financial stress. An ex-post approach based on historical perspective and ex ante approach combined with rolling window are used for analysis. Prediction results are evaluated using predictive likelihoods throughout the sample. The results show that all five variable categories are informative in predicting financial stress. But under normal error assumption less variables are selected compared to fat tail assumption especially for interest rate category. It also shows that none or very few potential indicators are selected when the market is under normal financial stress level. More variables are selected during the 07-09 crisis period. With the impact of economic crisis weakened, few variables are selected. It is also interesting to see that the log return of S&P 500 index is less informative than expected in predicting financial stress level.

Chapter 2

Estimation of Left Tail Risk Using Generalized Extreme Value Distribution and Block Maxima Data

2.1 Introduction

Modeling of tail behavior in statistical distributions have played an important role in financial modeling. The recent recessions have brought increased attention to the Generalized Extreme Value (GEV) distribution as a way of modeling the extreme observations and the left tail risk in finance. As a result, how well we can estimate the GEV distribution becomes crucial.

The GEV distribution was first introduced by Jenkinson (1955) and many papers have been working on analyzing the performance of different estimations for GEV distribution. The Maximum Likelihood Estimation (MLE) is one of the most widely used estimations although it is not favored when applied to small or moderate samples which is the common situation for extreme valued observations. Hosking *et al.* (1985) estimate the GEV distribution by the method of Probability-Weighted Moments (PWM) and conclude that the PWM estimators compare favorably with the MLE estimators. Wong and Li (2006) argue that the MLE may fail to converge due to the unbounded likelihood function. Moreover they argue that the Maximum Product of Spacings (MPS) gives estimators closer to the true values than the MLE and it performs more stable than the MLE and the PWM, especially for small sample size. In this paper, I compare the performances of the MLE, the MPS and the PWM by using the Monte Carlo Experiments on simulated data from GEV distribution and reach the different conclusion from the Wong and Li (2006). The results show that the mean, the median, and the mean absolute error (MAE) of the MLE, the MPS, and the PWM are more or less similar to each other regardless of the number of replications, and the MLE and the PWM perform slightly better than the MPS. Moreover, the MLE provides higher convergence rate than the MPS in all cases I conducted. When the sample size is large, the average runtime of the MLE is smaller than the average runtime of the MPS. Also, I conclude that the PWM estimates are good choices as initial values for the MLE and the MPS.

I compare not only how close the estimates to the true parameters are, but also how close the combination of the three parameters in terms of estimated Value-at-Risk (VaR) to the true VaR. VaR has been widely used in risk management in finance. It measures how much would loss over a defined period for a given probability level. For example if a portfolio has a one week 5% VaR of $100, it means there is a 5% chance that the value of the portfolio will drop more than $100 in a week. In other words, given the distribution and the probability level, we can calculate the VaR. This gives me an idea that instead of comparing the precision of the parameters individually, we should care more about the precision of the combination of all the parameters. In this paper, I use VaR as a model selection criteria. After estimating the parameters of the GEV distribution, I estimate the VaR at 1%, 5%, 10%, 25% and 50% level respectively, and compare the estimation based on averaging the absolute values of the difference between the estimated VaR and the true VaR. The conclusion is that the VaR estimates of the MLE and the PWM are closer to the true VaR than the MPS in general.

GEV distribution is also closely related to the Block Maxima Method which is a method of selecting out extreme observations that follow the GEV distribution. The Block Maxima Method is used to partition the whole sample into groups. According to the Fisher-Tippett-Gnedenko theorem, when the sample size in each group is large enough, as well as the number of groups, the maximum values sampled from each group follow the GEV distribution in limit. I raise the Block Maxima Method in this paper because in real data analysis, people use this method to sample the extreme values. For example, Logan (2000) estimates the GEV distribution of market index returns with r = 5 (5 days), r = 21 (one month), r = 63 (one quarter) and r = 125 (1 semester), where r is the sample size in each group. First, I conduct Monte Carlo experiments of the student-t distribution with 5 degrees of freedom. Then, I select the extreme values based on r = 5, 10 and 20, respectively, and finally compare the MLE, the MPS, the

PWM and the Bayesian estimation on these extreme values originated from the student-t distribution. The Monte Carlo Experiments using Block Maxima method show that the Bayesian estimation provides the smallest standard deviations of estimates for all cases. Based on the VaR estimates, the MPS gives the worst approximation in general.

As to the choice of estimation methods of the GEV parameters, I choose the MLE, the PWM, and the Bayes over the PMS. In using the MLE algorithms, we need to choose the initial values carefully. The PWM procedure does not require initial values and it produces good values of MAE_{VaR}. However, the estimation of the variance matrix of the PWM by the delta-method tends to give large estimates and sometimes it fails to produce an estimate. The Bayesian procedure is free of initial values, since the MCMC draws are burned (i.e. discarded) until the convergence of the MCMC draws is attained.

This paper is organized as follows. Section 2 provides the introduction of the GEV distribution. Section 3 provides the introduction of the MLE, the MPS and the PWM. Section 4 presents the results of the Monte Carlo experiments on simulated data drawn from the GEV. Section 5 presents the results of the Monte Carlo experiments using the Block Maxima Data and the empirical analysis of the Block Maxima Data. Section 6 provides the conclusions and future work.

2.2 Generalized Extreme Distribution (GEV)

Let me present the distribution function (or cumulative density function, cdf) and the probability density function (pdf) of the generalized extreme value distribution (GEV). The cdf of GEV is

$$F(x) = \exp\left[-\left(1 - \gamma\,\frac{x-\mu}{\sigma}\right)^{\frac{1}{\gamma}}\right], \tag{2.1}$$

where

$$1 - \gamma\,\frac{x-\mu}{\sigma} \; > \; 0, \quad \sigma \; > \; 0, \quad \gamma \neq 0, \;\; \text{and} \;\; \mu \in (-\infty,\, \infty).$$

The parameters γ, μ and σ are often labelled as the shape, location, and scale parameters. The pdf is

$$f(x) = \frac{1}{\sigma}\left(1 - \gamma\,\frac{x-\mu}{\sigma}\right)^{\frac{1}{\gamma}-1} \exp\left[-\left(1-\gamma\,\frac{x-\mu}{\sigma}\right)^{\frac{1}{\gamma}}\right]. \tag{2.2}$$

In some text books and papers the cdf and pdf of GEV are given by setting γ as $-\gamma$. Then the GEV distribution gives the cdf and pdf as

$$F(x) = \exp\{-t(x)\} \tag{2.3}$$

and

$$f(x) = \frac{1}{\sigma}\,t(x)^{\xi+1} \exp\{-t(x)\} \tag{2.4}$$

where

$$t(x) = \left(1 + \frac{x-\mu}{\sigma}\,\xi\right)^{-\frac{1}{\xi}}.$$

If I put $\xi = -\gamma$ equations (2.3) and (2.4) become equations (2.1) and (2.2), respectively.

Among the three parameters, the shape parameter γ is the most important: depending on the sign of the shape parameter γ, the GEV is sometimes classified into Type I (Gumbel): $\gamma = 0$, Type II (Frechet): $\gamma < 0$ and Type III (Weibull):$\gamma > 0$.[1]. To get a clear idea of how the GEV pdf's look like, I present three graphs in Figure 2.1.

In the first graph γ is 0 and thus it is a Gumbel pdf (*i.e.* Type I GEV). In the second graph γ is negative and thus it is a Frechet pdf (*i.e.* Type II GEV) In the third graph γ is positive and thus it is a Weibull pdf (*i.e.* Type III GEV). The Frechet and Gumbell pdf's are positively skewed, while the Weibull pdf is negatively skewed. If we let γ grow large the negative skewnesss and kurtosis of Type II Weibbull pdf grows large as shown in Figure 2.1.

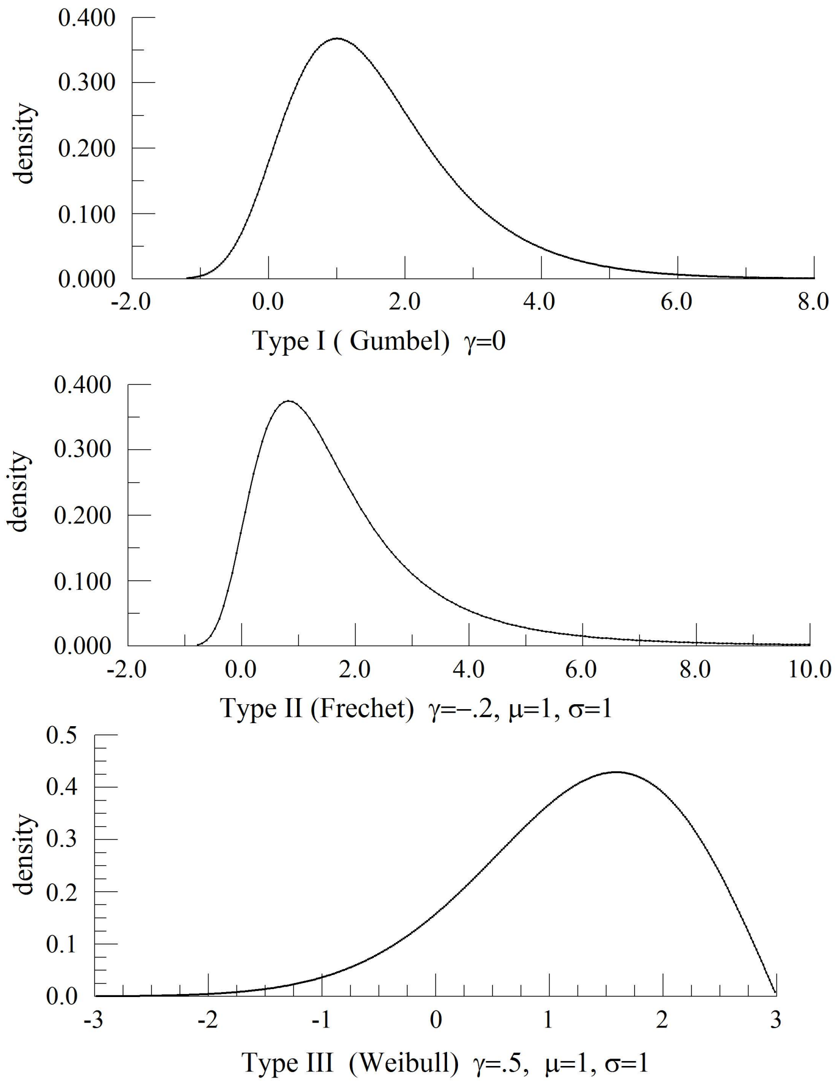
From equation (2.1) by using the probability integral transformation, we can draw the random numbers, x, of the GEV as[2]

$$x = \mu + \frac{\sigma}{\gamma}\left(1 - (-\ln u)^{\gamma}\right) \tag{2.5}$$

[1]The Gumbel, Frechet, and Weibull distributions are given, for example, in *Extreme value distributions*, Mathwave, data analysis and simulation, www.mathwave.com

[2]2.5 is derived based on: Luc Devroye (1986), Non-Uniform Random Variate Generation.

Figure 2.1: Examples of GEV Pdf's



where u is drawn from the uniform distribution over $(0,\ 1)$.

The four moments of the GEV distribution and the domain (or suport) of the GEV variate can be obtained from the probability density function. It is an exercise in integration to obtain the four moments and all four moments involve the gamma function $\Gamma(\cdot)$. The first moment, median, and mode are functions of all the three parameters: γ, μ, and σ. The variance is a function of σ and γ. The skewness and kurtosis are functions only of the shape parameter γ. For example, the skewness is

$$\text{Skewness} = \begin{cases} \dfrac{g_3 - 3g_1\, g_2 + 2g_1^3}{(g_2 - g_1)^{\frac{3}{2}}} & \text{if } \xi \neq 0 \\[2ex] \dfrac{12\sqrt{6}\,\zeta(3)}{\pi^3} & \text{if } \xi = 0 \end{cases}$$

where $g_k = \Gamma(1 - k\,\xi)$ and $\zeta(x)$ is the Riemann zeta function. The negative of ξ is γ:

$$\xi = -\gamma$$

For $\xi < 0$ (or $\gamma > 0$), the sign of the numerator is reversed. Since the argument in the gamma function, $1 - \xi\, k$ needs to be strictly positive the variance does not exist if $\xi \geq \dfrac{1}{2}$ or $\left(\gamma - \dfrac{1}{2}\right)$.

2.3 Three Sample Theory Estimation Procedures for the Parameters of GEV Distribution

Let me discuss the three sample theory estimation procedures for the parameters of the GEV distribution: the maximum likelihood estimation (MLE), the maximum product of spacing estimation (MPS), and the probability-weighted moments estimation (PWM). Among the three sample theory estimation procedure MLE is most frequently used.

2.3.1 Maximum Likelihood Estimation (MLE)

The pdf of GEV is given in equation (2.2). For the independent and identically distributed sample, the joint density function is given as

$$f(x_1, x_2, ..., x_n|\theta) = f(x_1|\theta) \times f(x_2|\theta) \times ... \times f(x_n|\theta)$$

where $\theta = \{\gamma, \mu, \sigma\}$. Consider $x_1, x_2, ..., x_n$ are the observed values and the parameters are the values that are allowed to vary. The likelihood function is given as

$$L(\gamma, \mu, \sigma) = \prod_{i=1}^{n} f(x_i|\theta)$$

Take the the natural logarithm, we derive the log-likelihood function. The maximum-likelihood estimates are obtained by maximizing

$$\text{ln}L(\gamma, \mu, \sigma) = \sum_{i=1}^{n} \left\{ -\text{ln}\sigma + \left(\frac{1}{\gamma} - 1\right) \ln\left(1 - \gamma\frac{x_i - \mu}{\sigma}\right) - \left(1 - \gamma\frac{x_i - \mu}{\sigma}\right)^{\frac{1}{\gamma}} \right\} \tag{2.6}$$

subject to two constraints below

$$1 - \gamma\,\frac{x - \mu}{\sigma} \ > \ 0 \tag{2.7}$$

and

$$\sigma \ > \ 0 \tag{2.8}$$

2.3.2 Maximum Product of Spacing Estimation (MPS)

Let $x_{(1)} < x_{(2)} < ... < x_{(n)}$ be an ordered sample of size n. $D_i(\gamma, \mu, \sigma)$ is define as

$$D_i(\gamma, \mu, \sigma) = F(x_{(i+1)}) - F(x_{(i)}), \ \ i = 1, 2, ..., n \tag{2.9}$$

where $F(x)$ is the cdf of GEV. The Maximum Product of Spacing (MPS) estimates are obtained by maximizing

$$M(\gamma, \mu, \sigma) = \sum_{i=1}^{n} \text{ln}D_i(\gamma, \mu, \sigma) \tag{2.10}$$

or

$$M(\gamma, \mu, \sigma) = \frac{1}{n}\sum_{i=1}^{n} \text{ln}D_i(\gamma, \mu, \sigma) \tag{2.11}$$

subject to equations (2.7) and (2.8). Same as did in MLE, we use CML in Gauss to find the optimal solution.

2.3.3 Probability-Weighted Moments Estimation (PWM)

The probability-weighted moments of a random variable X is defined as

$$Mp, r, s = E[X^p\{F(X)\}^r\{1 - F(X)\}^s] = \int_0^1 X^p\{F(X)\}^r\{1 - F(X)\}^s dF$$

Greenwood *et al.* (1979) favored $M(1,0,s)$ $(s = 0,1,2...)$ for parameter estimation, while Hosking *et al.* (1985) considered $M(1,r,0)$ $(r = 0,1,2...)$ which is also the moments used in this paper. Define moments β_r as

$$\beta_r = M1, r, 0 = E[X\{F(X)\}^r] \quad (r = 0,1,2...)$$

Hosking *et al.* (1985) shows that if β_r is known, the parameters in GEV can be calculated from following equations

$$c = \frac{2\beta_1 - \beta_0}{3\beta_2 - \beta_0} - \frac{\ln 2}{\ln 3} \tag{2.12}$$

$$\hat{\gamma} = 7.8590c + 2.9554c^2 \tag{2.13}$$

$$\hat{\sigma} = \frac{(2\beta_1 - \beta_0)\hat{\gamma}}{\Gamma(1+\hat{\gamma})(1-2^{-\hat{\gamma}})} \tag{2.14}$$

$$\hat{\mu} = \beta_0 + \hat{\sigma}\{\Gamma(1+\hat{\gamma}) - 1\}/\hat{\gamma} \tag{2.15}$$

To estimate the moments β_r, there are two ways: unbiased estimator b_r and plotting-position estimator $\hat{\beta}_r[p_{j,n}]$. The unbiased estimator of β_r was given by Landwehr *et al.* (1979) based on the ordered sample $x_{(1)} < x_{(2)} < ... < x_{(n)}$

$$b_r = n^{-1} \sum_{j=1}^{n} \frac{(j-1)(j-2)...(j-r)}{(n-1)(n-2)...(n-r)} x_{(j)} \quad \text{and} \quad b_0 = n^{-1} \sum x_{(j)}$$

Alternatively, β_r may be estimated by the plotting-position estimator

$$\hat{\beta}_r[p_{j,n}] = n^{-1} \sum_{j=1}^{n} p_{j,n}^r x_{(j)} \tag{2.16}$$

where $p_{j,n}$ is called plotting position. Hosking *et al.* (1985) use $p_{j,n} = (j - .35)/n$ to estimate equation (2.16), then apply equation (2.12), equation (2.13), equation (2.14) and equation (2.15) to estimate parameters in GEV. In this paper I follow the procedure given in Hosking *et al.* (1985).

Let me note that initial values are required for the iterative optimization algorithms of MLE and MPS while the PWM algorithms initial values are not required.

2.4 Monte Carlo Experiments on Simulated Data Drawn from GEV Distribution

Let me compare the performances of the MLE, the MPS, and the PWM by two cases of Monte Carlo experiments. In the first section I examine the Monte Carlo simulations given in Wong and Li (2006) while in the second section I use the value-at-risk (VaR) as the model selection criterion. In the second section the parameters of GEV distributions are set at the typical values that are close to the real data estimates.

2.4.1 Examining the Monte Carlo Experiments of Wong and Li (2006)

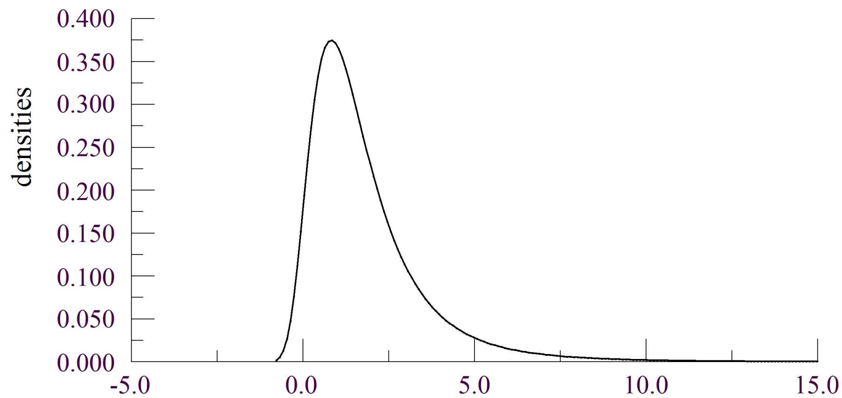
Wong and Li (2006) conducted Monte Carlo experiments to compare the performances of the three sample theory estimation procedures: the MLE, the MPS, and the PWM. Focusing on the small sample sizes of 10, 20, and 50, they make up four parameter settings and concluded that the MPS outperform the MLE and the PWM judged by the mean absolute errors of estimates (MAE). They argue that the MPS provides estimates closer to the true parameters than the MLE. The MPS is also more stable comparing to the PWM and the MLE when sample size is small. The four sets of parameters Wong and Li (2006) evaluated are presented in Table 2.1 together with the support (or domain) of the GEV random variables x. The supports of x are determined by

$$x \in \begin{cases} (\mu + \sigma/\gamma,\ \infty) & \text{if } \gamma < 0 \text{ (Type II: Frechet)} \\ (-\infty,\ \infty) & \text{if } \gamma = 0 \text{ (Type I: Gumbell)} \\ (-\infty,\ (\mu + \sigma/\gamma) & \text{if } \gamma > 0 \text{ (Type III: Weibul)} \end{cases}$$

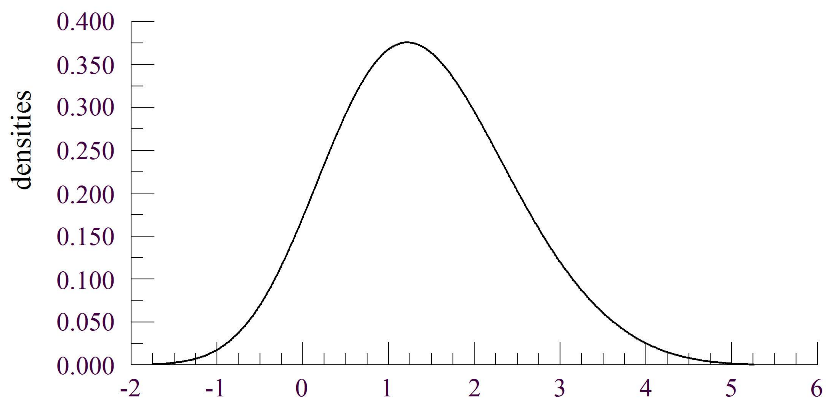
In Figures 2.2 and 2.3, I present the probability density functions (pdf's) of GEV variables for the four cases in Table 2.1 to get a clear idea of how the GEV pdf's look like. We see when γ is negative ($\gamma < 0$) the pdf is skewed to the right and when it is positive ($\gamma > 0$) the pdf is skewed to the left. When $\gamma \geq 1$, the mode of the distribution is at the upper limit of the support of x. Although we do not give the pdf's, the GEV distribution is almost symmetric when $\gamma \in (0.1,\ 0.3)$.

Examining the Monte Carlo experiments Wong and Li (2006) presented in Table 1

Figure 2.2: Exact Densities: Case 1 and Case 2

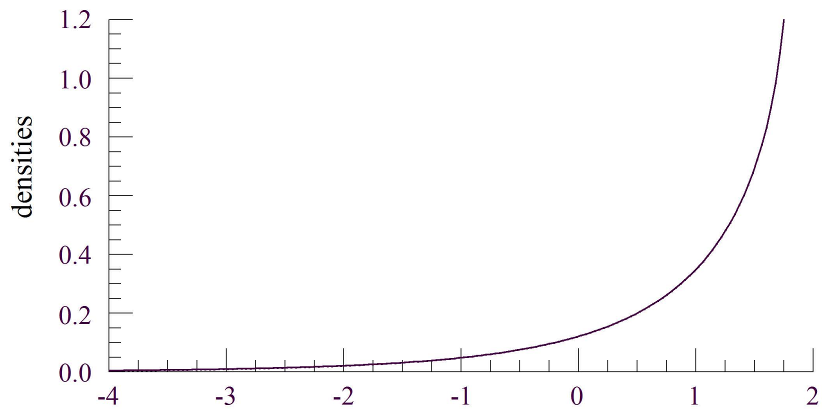


Case 1: x (γ=-.2, μ=1, σ=1)

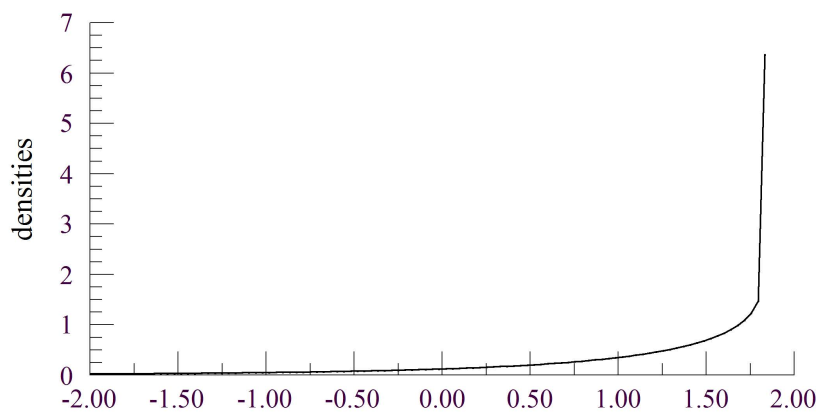


Case 2: x (γ=.2, μ=1, σ=1)

Figure 2.3: Exact Densities: Case 3 and Case 4



Case 3: x (γ=1.0, μ=1.0, σ=1.0)



Case 4: x (γ=1.2, μ=1, σ=1)

Table 2.1: Exact Support of GEV

	True parameters			Exact Support
	γ	μ	σ	x
Case 1	$-.2$	1	1	$(-4^{(1)},\ \infty)$
Case 2	.2	1	1	$(-\infty,\ 6^{(2)})$
Case 3	1.0	1	1	$(-\infty,\ 2^{(2)})$
Case 4	1.2	1	1	$(-\infty,\ 1.8333^{(2)})$

Notes: (1) When $\gamma < 0$ the lower bound is given by $\mu + \sigma/\gamma = 1 - 1/.2 = -4$.
(2) When $\gamma > 0$ the upper bound is given by $\mu + \sigma/\gamma$.
For Case 2: 1+1/.2=6.
For Case 3: 1+1/1=2.
For Case 4: 1+1/1.2=1.8333.

of their paper, we notice that in Cases 2 and 3 the mean absolute errors of estimates (MAE) of the MLE are exceedingly large compared to those of the MPS and PWM. To verify their Monte Carlo experiments, I conducted Monte Carlo experiments for Case 2 where γ is .2 and for Case 4 where γ is 1.0. Tables 2.2 and 2.3 for the sample sizes of 50 and 1,000. The number of replications range from 100 to 1,000.

From the Monte Carlo experiments presented in Tables 2.2 and 2.3 I observe:

1. The mean, median, and MAE of the MLE, the MPS, and the PWM are more or less similar to each other regardless of the number of replications.

2. In footnote (1) of Tables 2.2 and 2.3, I stated that I used CML in GAUSS and put the initial starting values for MLE and MPS. The reasons I put the initial starting values is that the convergence of the MLE and the MPS are extremely sensitive to the choices of the initial values.

3. Let me focus on the results in Table 2.2. For $r = 1,000$. As the sample size increases from 50 to 1,000 the means and medians of estimates of μ and σ get closer to the true values and MAE's get much smaller. However, for the estimates of γ, the MPS estimates deviate slightly further from the true value and the MAE is larger than those of the MLE and the PWM. The MLE and the PWM perform

slightly better than the MPS.

4. In Table 2.3, given the replication number of $r = 1,000$ the MPS estimates are relatively worse than the estimates of other two estimates. When the sample size increases to 1,000 the PWM slightly outperforms the MLE in general.

The reasons why I obtain the Monte Carlo results so different from those of Wong and Lee (2006) seem to lie in the choices of the initial starting values for MLE and MPS and in how the nonlinear constraints are handled. In their paper Wong and Lee do not explicitly state about the initial values and the nonlinear constraint.

The GEV distribution has two constraints on the parameters that are given as equations (2.17) and (2.18):

$$1 - \gamma \frac{x - \mu}{\sigma} > 0 \tag{2.17}$$

and

$$\sigma > 0 \tag{2.18}$$

The positive constraint on σ causes no problem, but the constraint given in constraint (2.17) plays a crucial role in the MLE and MPS algorithms since equation (2.17) shows that the support of the GEV random variable, x, depends on the parameters of the distribution. If we ignore constraint (2.17), the MLE and MPS algorithm do not converge. Also we have often encountered an error message in CML telling us that the Hessian failed to be calculated.

Although the estimates from the Monte Carlo Experiments in Table 2.2 and Table 2.3 show that the mean, the median, and MAE of three estimations give similar estimates and it does not matter much which estimation to use, we can still reach one conclusion that the PWM estimates is good choices as initial values for the MLE and the MPS. This is because first of all the PWM estimates are very close to true values and secondly, the PWM is a point estimation which does not need initial values.

Since the Monte Carlo simulation shows the MLE and the MPS are close to one another in terms of the mean, the median and the MAE, I try to compare the performance of the MLE and the MPS in other ways: the global convergence rate and the runtime. For the Monte Carlo simulation, we know the true parameter values. The idea is to draw initial values qualified with constraints (2.17) and (2.18) from normal distributions with mean set up at the true values. As the standard deviation of the normal distribution gets larger and larger, we collect the average convergence rate and the average runtime for each Monte Carlo Simulation and compare. Below gives the procedure:

1. Draw sets of random values for three parameters (γ, μ, σ) from normal distributions with mean set up at the true parameter values and standard deviation set up at a given value.

2. Plug in the sets random values to the constraints (2.17) and (2.18). Select 100 sets which meet the two constraints and treat them as qualified initial values for the MLE and the MPS.

3. Set the Monte Carlo simulation number to 100. For each set of initial values, calculate the convergence rate and the mean runtime by taking average among 100 Monte Carlo simulations.

4. Given the convergence rate and mean runtime for each set of the initial values, calculate the mean convergence rate and the mean runtime by taking average among 100 sets of the initial value.

For each set of the initial values, I draw random values for γ, μ, and σ separately from each normal distribution with the same standard deviation. I set the standard deviation of the normal distribution as 0.1, 0.5, 1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5 respectively. As you can see, I call this procedure a global measure because the larger the standard deviation is, the more possible the randomly initial value deviates further from the true values. As a result, I am able to compare the convergence rate and the runtime of the MLE and the MPS from a global point of view.

Figure 2.4 and Figure 2.5 provide the easy visual comparison of the MLE and the MPS. Within each figure, the sub-figure a and b show the average convergence rates when the sample size is 50 and 1000, respectively. The sub-figure c and d show the average runtime when the sample size is 50 and 1000, respectively. Although Figure

Figure 2.4: Convergence Analysis: $\gamma = 0.2$

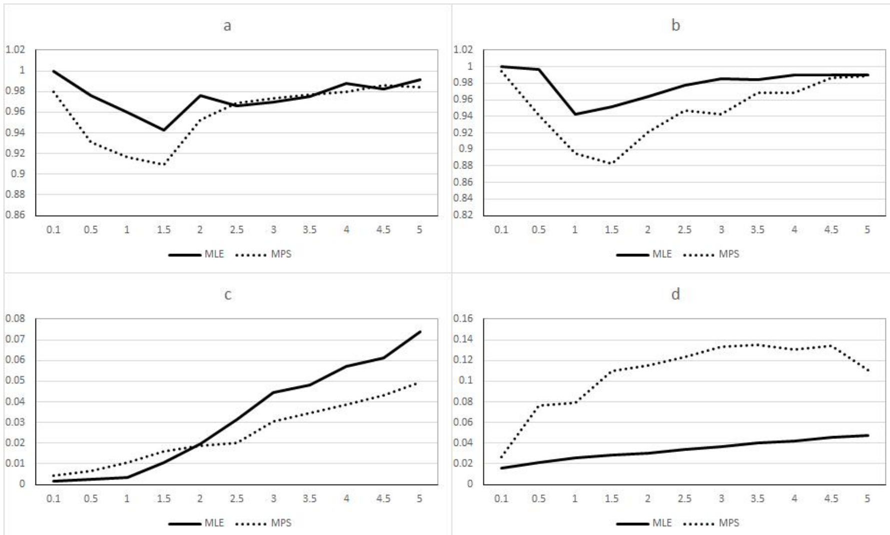
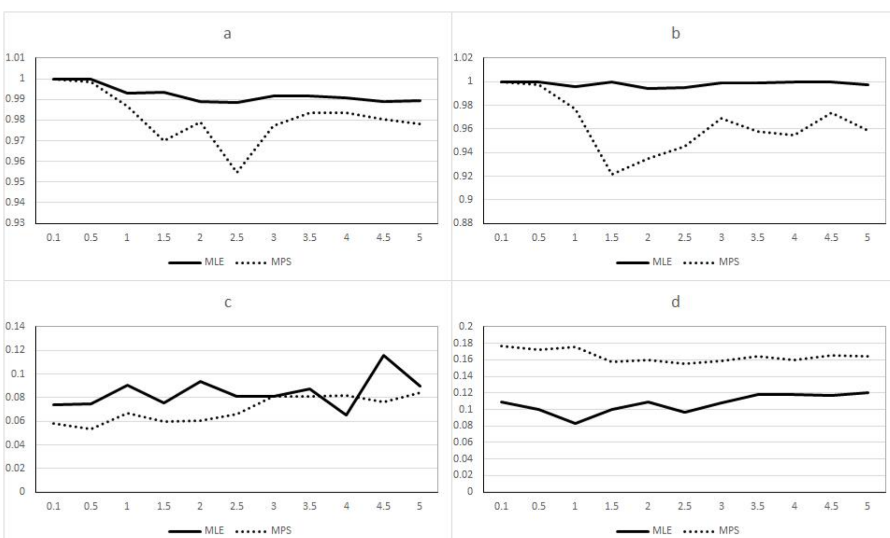


Figure 2.5: Convergence Analysis: $\gamma = 1$



2.4 and Figure 2.5 are based on Monte Carlo Experiments of different true parameter values, they reach the same conclusion. First of all, the MLE show higher convergence rate than the MPS in all cases and different standard deviations, while Wong and Li (2006) find out the MLE has higher rate of failure convergence when $\gamma = 1$ and the sample size is 50. As I mentioned before, this may due to Wong and Li (2006) does not take the nonlinear constraint into the consideration. Secondly, when the sample size is as large as 1000, the average runtime of the MLE is smaller than the average runtime of the MPS for different standard deviations. It means when we need to do large amount of Monte Carlo simulations for large sample size, using the MLE is a big advantage then the MPS. When the sample size is 50, the lines of average runtimes of the two estimations cross each other.

2.4.2 Monte Carlo Experiments Using Value-at-Risk (VaR) as the Model Selection Criteria

In financial time series analysis the GEV distribution is often used to estimate the Value at Risk (VaR) since the GEV distribution can capture the stylized facts that the distributions of the financial returns are skewed and leptokurtic. Also, the sample sizes used in financial time series are often larger than 1,000. Accordingly I conduct Monte Carlo experiments setting the parameter values of the GEV distribution close to estimates of real data (see Table 2.9 and Table 2.11). And I set the sample size at 2,000.

Value-at-Risk (VaR): Before conducting Monte Carlo experiments, let me discuss Value-at-Risk (VaR). According to Holton (2002) the origin of VaR can be traced back to 1922. Since then VaR has been used to measure such risks as market risk, credit risk, operation risk and regulation risk.

Adam *et al.* (2008) analyse the portfolio optimization problem with VaR as one of the risk constraints. Huisman *et al.* (1999) develop an asset allocation model by using US stocks and bonds. The model maximizes the expected return subject to the constraint that the expected maximum loss should be at most of the α-level VaR where α is to be specified apriori. Da Silva *et al.* (2003) compare the VaR estimates using data from the Asian emerging markets. They conclude that the GEV model tends to yield more conservative capital requirements. Gencay and Selcuk (2004) investigate the performance of VaR with the daily stock returns of nine different emerging markets and indicate that VaR estimates based on EVT are more accurate at higher quantiles. Hyung and De Vries (2007) focus on the portfolio selection problem under a downside risk and analyse the sensitivity and convexity of VaR and extend it to the multi assets with dependence. More recently, McGill and Chavez-Demoulin (2012) measure VaR to intra-day high-frequency data since high-frequency data since GEV tends to have fat tails.

The GEV distribution is an appealing candidate to calculate VaR since once the parameters of the GEV distribution are estimated the VaR at the α level can be obtained analytically by the inverse function of the GEV distribution given in equation (2.5) by replacing u by α:

$$\mathrm{VaR}_\alpha = \mu + \left(\frac{\sigma}{\gamma}\right)\left[1 - (-\ln(\alpha))^\gamma\right] \tag{2.19}$$

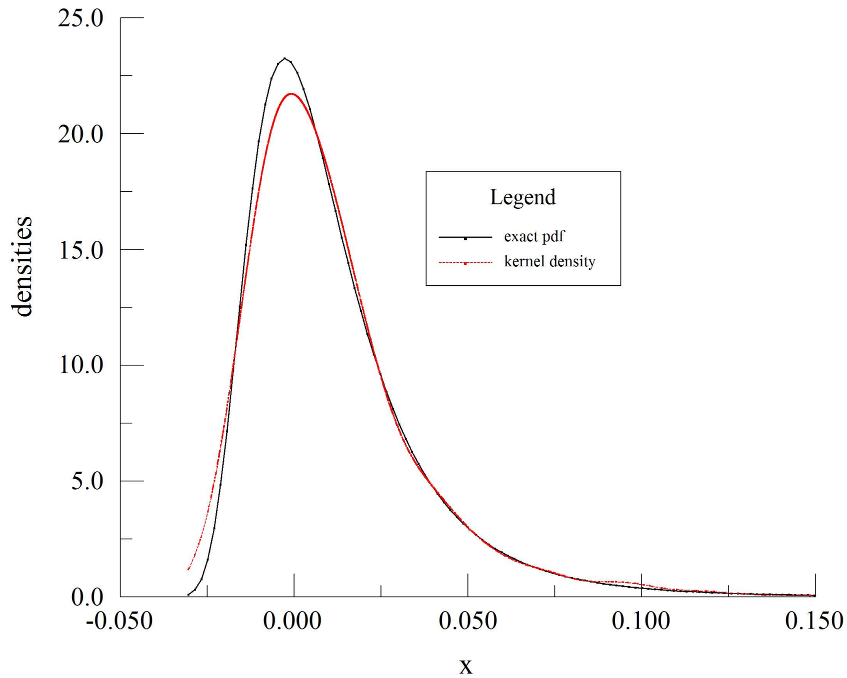
Monte Carlo Experiments: I set the sample size of n is 2000 and make the first 100 Monte Carlo simulations setting the parameter values of GEV at:

$$\gamma = -0.15,\ \mu = -0.0005,\ \sigma = 0.016$$

Figure 2.6 presents the exact GEV pdf and the kernel density that is obtained by drawing 2,000 GEV random variables using the inverse function of equation (2.5).

The exact pdf and the kernel density are close to each other. With the value of γ set at -0.15 the pdf and the kernel density are skewed to the right. In the financial data analysis the loss is often turned into a positive value by multiplying the loss by -1 so that the left hand tail becomes the right hand tail. The financial loss usually has a long fat left tail. As shown in Figures 2.2 and 2.3, the long fat tail of the GEV distribution is more easily captured by a Type II GEV (or Frechet) pdf that has the domain in $(\mu + \sigma/\gamma,\ \infty)$. Hence, the loss of financial return is multiplied by -1 to express it as a positive number. Consequently, the left tail is turned around to be the right hand tail, the VaR at the $\alpha\%$ level, VaR_α, is evaluated at the $(1-\alpha)$-percentage

Figure 2.6: Exact Pdf and Kernel Density of GEV with Parameter Values Set at $\gamma = -.15,\ \ \mu = -.0005,\ \ \sigma = .016$



level:

$$\text{VaR}_\alpha = \mu + \left(\frac{\sigma}{\gamma}\right)\left[1 - (-\ln(1-\alpha))^\gamma\right] \tag{2.20}$$

The model selection measure using the VaR's is based on the difference between the actual and estimated VaR. First I define DIF_α as

$$\text{DIF}_\alpha = \text{true.VaR}_\alpha - \text{dat.VaR}_\alpha \tag{2.21}$$

where true.VaR_α is the VaR from the true GEV distribution at α-percentile given in equation (2.20) and dat.VaR_α is the α-percentile from the simulated data. Choosing five percentile points of $\alpha_1 = .01,\ \alpha_2 = .05,\ \alpha_3 = .1,\ \alpha_4 = .25,\ \alpha_5 = .5$, I obtain the mean absolute error of VaR:

$$\text{MAE}_{VaR} = \frac{1}{5}\sum_{i=1}^{5} |\text{DIF}_{\alpha_i}|. \tag{2.22}$$

Since GEV is often used in financial analysis to examine the left tail risk, I have chosen the five α-percentile points of .01 to .5.

Table 2.4, table 2.5 and Table 2.6 present the estimated parameters and estimated

VaRs using different initial values for the MLE and MPS algorithms to demonstrate the choice of the initial values has a huge impace on the estimates of MLE and of MPS.

The PWM does not require initial values , so the results for the PWM part are the same in these tables. Table 2.4 shows the results based on the initial values closet to the true values. The performance, in terms of the parameter estimates and VaR, from the best to worst are MLE, PWM and MPS. The parameter estimates by MPS are worse than those in Table 2.4. Table 2.6 shows the results based on initial values that are further away from the true parameter values. The estimates of MPS show large deviations from the true ones. The MAE_{VaR}'s of MPS are larger than those of MLE and PWM in Tables 2.4, 2.5 and 2.6.

2.5 Block Maxima Data Analysis

Block maxima data analysis have been widely used in financial risk analysis to check whether financial returns follow a normal distribution or to estimate VaR and the expected Shortfall (ES) and to evaluate the left tails of financial returns. For example, Longin (2005) used the daily returns of S&P500 index from January 1954 to December 2003 for the total of 12,587 observations and concluded that the extreme price changes during the stock market crashes are incompatible with the assumption that the S&P500 returns follow a normal distribution. Da Silva *et al.* (2003) analysed the Asian stock indices by fitting block maxima data to GEV distribution and concluded that the VaR's estimated by GEV are much better fit to actual VaR's than those estimated by using normal distribution. DiTraglia *et al.* (2013) used block maxima data to measure left dependence among the assets. They employed copulas to obtain left dependence measures and used them for portfolio selection.

The statistical justification of using GEV in block data analysis goes back to Fisher and Tippett (1928), but Gnedenko(1943) is often cited as the one who established that the maximum order statistic, under certain assumptions, converge converge to GEV distribution.

Let me first explain how block maxima data are created from n data points, $x_1,\ x_2,\ \cdots,\ x_n$.

We partition x_i's into m blocks with each block containing r number of data points (r is the block size). Then the maximum (or the minimum) of each block is selected. The collection of the maximums (or minimums) is called block maxima (or minima) data.

Assuming that x_i's are independently and identically distributed, Gnedenko (1943) proved that block maxima data, after appropriate scaling, converges to the GEV distribution. Let us present block maxima formally. Suppose there are n iid random variables $x_i, i = 1, ...n$. Divide the whole sample into m blocks (or subsets) with block size r ($i.e.$ r elements in each of them.) Denote the maximum value in each block as $x_{(i)}, i = 1, ...m$. Let F denotes the cumulative distribution of x_i and F^m denotes the cumulative distribution of $x_{(m)}$. The degenerate distribution is:

$$\lim_{m \to +\infty} F^m(x) = \begin{cases} 1 & if \;\; F(x) = 1 \\ 0 & if \;\; F(x) < 1 \end{cases}$$

To obtain the non-generate distribution, assume that there exist sequences $\{a_m\}$ and $\{b_m\}$ and the random variable $x_{(m)}$ can be standardized to $\dfrac{x_{(m)} - a_m}{b_m}$. Gnedenko (1943) proved that as m and $r \to \infty$, the distribution of $x_{(m)}$, after scaling, convergences to the Generalized Extreme Value (GEV) distribution. Let me call this convergence theorem as the Fisher-Tippett-Gnedenko theorem since Fisher and Tippet have their contribution to it.

As an illustration of block maxima data, suppose that 2,400 x_i's are drawn from the student-t distribution from 5 degrees of freedom. We have $n = 2,400$ observations and they are distributed equally into m blocks with each block containing r observations. If we decide on the block size r, then the number of block maxima observations, m, is given by $m = n/r$. We take the maximum from each block. The distribution of m maxima data points no longer follows a symmetric student-t distribution. As m and r grow larger the distribution of the maximums will converge, with appropriate scaling, to a GEV distribution.

In Figure 2.7 the first graph is the student-t pdf with 5 degrees of freedom. The pdf is symmetric. The second graph is the kernel density of m maxima data points with block size r set at 10. Following the convention in financial data analysis the minimum of each block is multiplied by -1 to make it the maximum.

Figure 2.7: Block Maxima and Block Minima Generated from Student-t

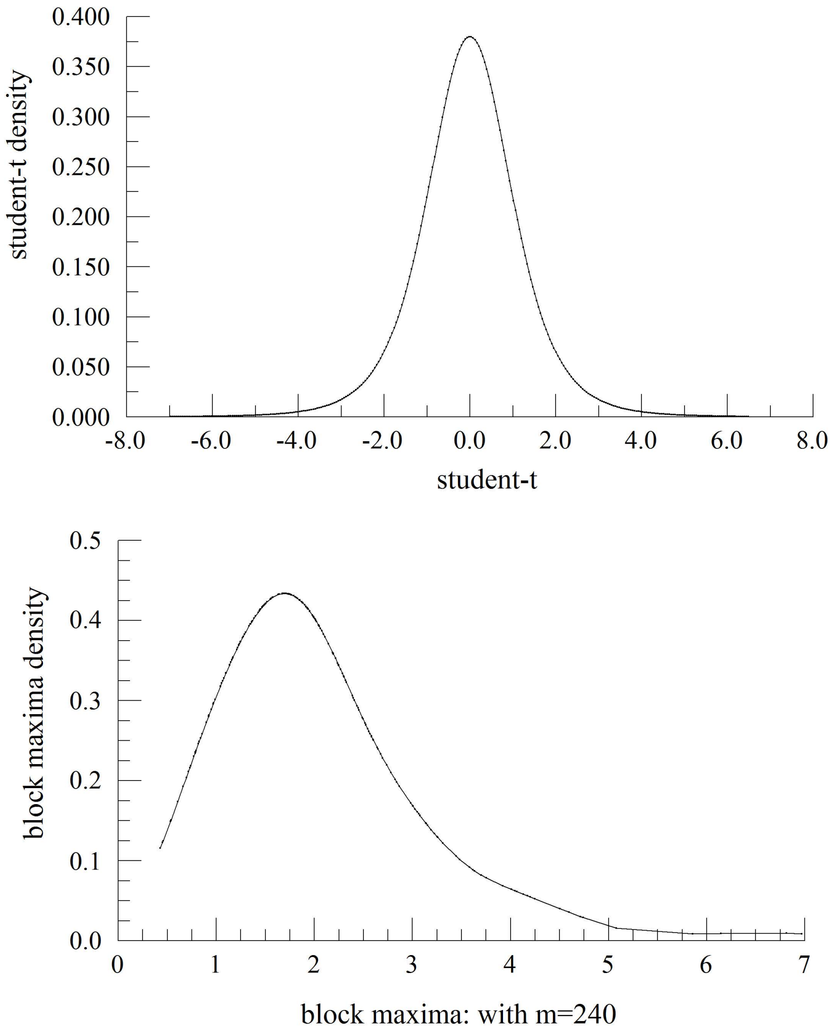


Figure 2.8: Distributions of block maxima data for different number of blocks, m

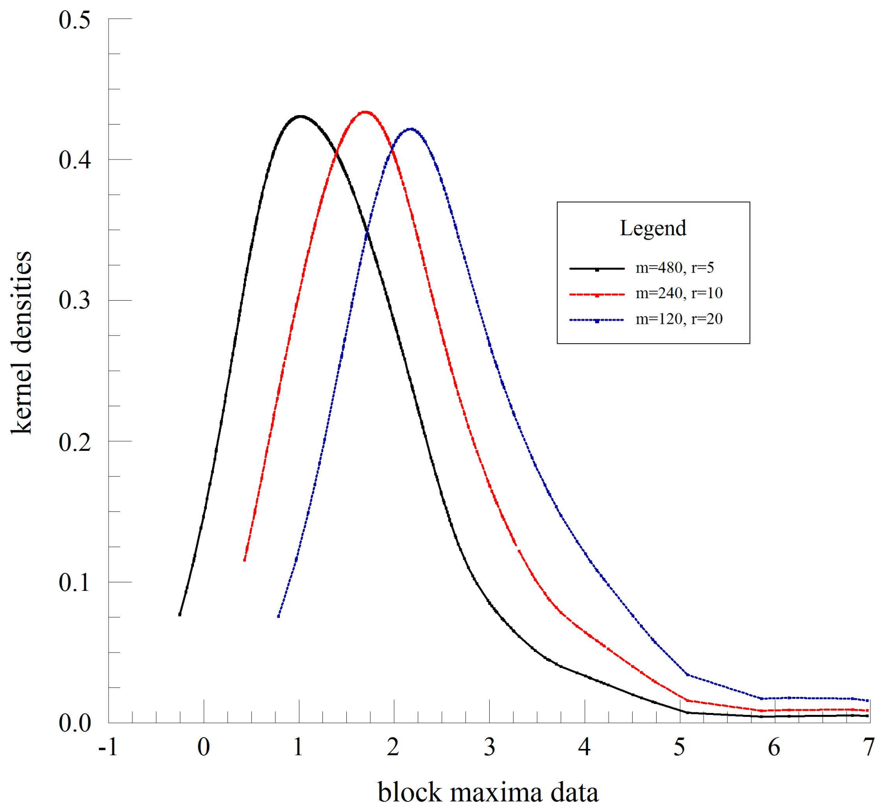


Figure 2.8 shows the kernel densities of the block maxima data as the block size r changes. The black line is the kernel density of the block maxima data obtained by setting the block size at 5 and thus $m = 240$. The red line is the kernel density of the block maxima data with $r = 10,\ m = 240$. The green line is the kernel density of the block maxima data with $r = 20,\ m = 120,\ r = 20$. We observe that all the kernel densities are skewed to the right and that given the sample size of $2,400$ ($n = 2,400$) the kernel densities of block maxima data shift to the right as block size r increases. However, the largest maxima data values are the same for all r, and the tails get fatter and fatter as r increases.

In the financial applications of block maxima data it has been pointed out that block maxima data analysis depends on the choice of block size r. The block size r has to be large enough so that the distribution is close to GEV. But the larger is the block size r, the smaller is the sample size m of the maxima block data, and the larger is the number of discarded observations. Given the original 2,400 observations, we only have to work with 120 maxima data points if we set r=20. In the literature, often more than one block size is chosen: Da Silva *et.al.* (2003) partitioned the Asian stock market indices into one month (block size of 21 days, $r = 21$), two months ($r = 42$), three months ($r = 63$) and six months ($r = 120$.) Logan (2000) estimates the GEV distribution of market index returns with $r = 5$ (5 days), $r = 21$ (one month), $r = 63$ (one quarter) and $r = 125$ (1 semester). DiTraglia *et al.* (2013) choose 22 trading days as the block size.

To sum up, the commonly used block sizes are one month, two months, one quarter, and half a year. However, very few papers justify the choice of the block size. Longin (2000) uses Sherman 's goodness-to-fit statistic that was developed by Sherman (1957) to justify the block size. The Sherman's goodness-to-fit statistic compares how close the estimated and observed distributions are. The method orders the maxima data by $x_i : \ x_1 \leq x_2 \leq ... \leq x_m$. The statistic is:

$$\Omega_m = \frac{1}{2} \sum_{i=0}^{m} |F^{asymp}(x_{i+1}) - F^{asymp}(x_i) - \frac{1}{m+1}|$$

where, F^{asymp} is the estimated asymptotic distribution, $F^{asymp}(x_0) = 0$ and $F^{asymp}(x_{m+1}) = 1$. Ω_m is asymptotically normal with mean $(m/(m+1))^{m+1}$ and an approximated variance $(2e-5)/(e^2 m)$, where e is Napier's constant or the basis of natural logarithm. Longin (2000) uses 5% confidence level to reject/accept the null hypothesis which stands for the adequacy of the asymptotic distribution. The database of Longin (2000) consists of daily S&P 500 returns from Jan 1962 to Dec 1993 (7927 observations). Based on 5% confidence level, the maxima data from the 21-day block, 63-day block and the 125-day block are accepted to obey the null hypothesis that the distributions follow GEV, while the maxima data from the 5-day block is rejected.

2.5.1 Monte Carlo Experiments Using Block Maxima Data

I conducted Monte Carlo experiments using block maxima data first drawing 2,400 random variables from the student-t distribution with 5 degrees of freedom, and made 3 sets of block maxima data by setting $r = 5, \ 10$, and 20. In addition to estimating the parameters of GEV by the three sample theory estimators: the MLE, the MPS, and the PWM I present the Bayesian estimates using the posterior means as the point estimates. I call the Bayesian estimates as Bayes. I use the griddy Gibbs sampler as Bayesian estimation. The number of replications of the Monte Carlo experiments are 100. I present in Table 2.7 the means and standard deviations of the MLE, the MPS, the PWM and the Bayes.

From Table 2.7 we observe that:

1. Bayesian estimation provides the smallest standard deviations for all cases.

2. For $(m = 480,\ r = 5)$ and for $(m = 240, r = 10)$ the mean values of $\hat{\mu}$ are close to each other for all the four estimation methods, and so are the mean values of $\hat{\sigma}$. For the average of $\hat{\gamma}$, the mean values from the PWM and the Bayesian estimations are closer to each other and they are larger than the mean values from the MLE and the MPS estimations.

3. For $m = 240,\ r = 10$, the mean values of both $\hat{\mu}$ and $\hat{\sigma}$, the estimates from the MPS estimation differ most from the estimates of the other three estimations. For the mean values of the $\hat{\gamma}$, the estimates from the MLE estimation differ most from the estimates of the other three estimations.

4. For $(m = 120,\ r = 20)$, the mean values of all three parameters from the MLE are closer to those from the PWM estimations. For the mean values of $\hat{\gamma}$, the estimates from the Bayesian estimation differ most from the estimates from the other three estimations. For the mean values of the $\hat{\mu}$ and $\hat{\sigma}$, the estimates from the MPS estimation differ most from those of the other three estimations.

Unlike the Monte Carlo experiments in the previous sections, we do not know the true parameter values. For that matter, we do not know if the GEV is the better choice to fit the m block maxima data points, but for now let us assume that the GEV distribution is a reasonable choice. We will leave the issue of what distribution fit block maxima data to future work.

Since there is no way of comparing the different estimates to the non-existing unknown true parameters, I will use the MAE_{VaR} criterion that was introduced in the previous section replacing true.VaR$_\alpha$ by dat.VaR$_\alpha$:

$$\mathrm{DIF}_\alpha = \mathrm{est.VaR}_\alpha - \mathrm{dat.VaR}_\alpha \tag{2.23}$$

where est.VaR$_\alpha$ is the VaR from the estimated GEV distribution at α-percentile:

$$\mathrm{est.VaR}_\alpha = \widehat{\mu} + \left(\frac{\widehat{\sigma}}{\widehat{\gamma}}\right)\left[1 - \left(-\ln\left(1-\alpha\right)\right)^{\widehat{\gamma}}\right] \tag{2.24}$$

and dat.VaR$_\alpha$ is the α-percentile from the block maxima data. Choosing five percentile points of $\alpha_1 = .01,\ \alpha_2 = .05,\ \alpha_3 = .1,\ \alpha_4 = .25,\ \alpha_5 = .5$, we take the mean:

$$\mathrm{MAE}_{VaR} = \frac{1}{5}\sum_{i=1}^{5}\left|\mathrm{DIF}_{\alpha_i}\right|. \tag{2.25}$$

The results are presented in Table 2.8. From Table 2.8 we observe

1. Overall, the MPS estimation provides worst approximation of VaR. Especially when $m = 240, r = 10$ and $m = 120, r = 20$, the the absolute value of DIF, |DIF|, from MPS estimation are significantly larger than the DIFs from the other estimations.

2. For the case that $m = 480, r = 5$, when $\alpha = .01$, the PWM estimation provides the smallest DIF. For other values of α, the DIFs of the MLE and the MPS estimations are quite close and the DIFs of the MPS estimation are a little bit smaller than the DIFs of the MLE estimation. None of the estimators dominate the others for all values of α and for all block sizes. The results vary depending on the value of α .

3. For the case that $m = 240, r = 10$, the Bayesian estimation gives the smallest DIFs for all α. In general, the PWM estimation provides smaller DIFs than the MLE estimation, except when $\alpha = .05$.

2.5.2 Empirical Analysis of Block Maxima Data

Let us make an empirical analysis of block maxima data. Two market indices, FTSE100 and SP500, are the most widely used stock indices in the U.K. and U.S., respectively. I downloaded daily data from May 3 2001 to December 31 2012 from Yahoo Finance. The number of observations are 3,001. When the price is missing for one series on a specific day while the other price is not I take the average of the preceding and succeeding prices. For FTSE100, 56 out of 3,000 (1.87%) are calculated by averaging the preceding and succeeding prices, while for SP500, 68 out of 3,001 (2.27%) are computed in the similar manner. Most of the missing prices are due to holidays. The price for 9/11/01 is missing due to the 9-11 incident. The daily return r_t is calculated as

$$r_t = \ln P_t - \ln P_{t-1}.$$

The total number of observations are now 3,000. We multiply the daily return by -1 to express the loss as a positive number.

In conducting Monte Carlo experiments presented in Table 2.7 I have reported the means of the 100 estimates of each parameter as well as its standard deviation. The standard deviation is computed from the 100 estimates of the parameter. In actual empirical data analysis the custom is to report the estimate of a parameter and its standard deviation (or more commonly known as the standard error.) For the MLE and the MPS the standard deviation is computed from the Hessian. For Bayes, the posterior standard deviation is computed from the Markov Chain Monte Carlo (MCMC) draws. For PWM we need to get an estimator of the standard error. I use Hoskin *et al.*'s asymptotic variance-covariance matrix that is given in Appendix A.

Tables 2.9 and 2.10 present the results for FTSE100 while tables 2.11 and 2.12 present the results for SP500. Examining the tables we observe:

1. From Tables 2.9 and 2.11 we see that the four estimators (the MLE, the MPS, the PWM, and the Bayes) yield similar point estimates. The standard deviations of the parameters by the MLE, the MPS, and Bayes are similar. However, the standard deviations of the PWM are much larger than those of the MLE, the MPS, and the Bayes.

2. From Tables 2.10 and 2.12 we see that the MAE_{VaR} of the PWM is consistently the smallest and that of Bayes the largest.

2.6 Conclusions

In this paper I first used three sample theory estimators of the parameters of GEV. The three estimators are the MLE, the MPS, and the PWM. Using Monte Carlo experiments I examined the Monte Carlo experiments of Wong and Li (2006). Contrary to their findings that both the MPS and the PWM estimates perform better than the MLE estimates, I found that the MLE and the PWM outperform the MPS.

In estimating the three parameters of GEV, we need to take care of the two constraints, $1-\gamma\,\dfrac{x-\mu}{\sigma}$ and $\sigma > 0$. The first constraint rises from the fact that the support of GEV variable, x, depends on the parameters of distribution. This first constraint plays a crucial role in the maximization algorithms of the MLE and the MPS. From the paper by Wong and Li (2006) I am unable to find how they handled this constraint. I conjecture the difference between my findings and their findings may be due to how the constraint is handled.

Then I conducted Monte Carlo experiments in the typical settings of the GEV parameters that are found in the empirical studies of financial returns. In addition to the three sample theory parameters I used the MCMC algorithms by Gibbs sampler. I also introduced a model selection criterion, MAE_{VaR},that is the mean absolute error of Value-at-Risk. This model selection criterion works well.

Block maxima data, the major part of this paper, are created by picking the maximum of each block of size r. In creating block maxima data we need to choose the block size r. The larger the block size, the fewer is the number of observations of the block maxima data. I conducted Monte Carlo experiments generating block maxima data from the student-t distribution with 5 degrees of freedom. Then I estimated the block size data created from 3,000 observations of FTSE100 and of SP500 data.

As to the choice of estimation methods of the GEV parameters, I choose the MLE, the PWM, and the Bayes over the PMS. In using the MLE algorithms, we need to choose the initial values carefully. The PWM procedure does not require initial values and it produces good values of MAE_{VaR}. However, the estimation of the variance matrix of the PWM by the delta-method tends to give large estimates and sometimes it fails to produce an estimate. The Bayesian procedure is free of initial values, since the MCMC draws are burned (*i.e.* discarded) until the convergence of the MCMC draws is attained.

Table 2.2: Case 2[1] $\gamma = .2$ $\mu = 1$ $\sigma = 1$

n[2]	r[3]		MLE			MPS			PWM		
			$\hat{\gamma}$	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\gamma}$	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\gamma}$	$\hat{\mu}$	$\hat{\sigma}$
		Mean	0.2013	1.0517	0.9698	0.2205	1.0869	0.9166	0.1991	0.9920	1.0086
50	100	Median	0.1987	1.0517	0.9718	0.2162	1.0918	0.9215	0.1984	0.9967	0.9947
		MAE	0.0821	0.1390	0.0791	0.0916	0.1537	0.1071	0.0773	0.1245	0.0851
		Mean	0.1964	1.0431	0.9493	0.2055	1.0727	0.8959	0.1878	1.0030	0.9942
50	1000	Median	0.1907	1.0442	0.9483	0.2028	1.0744	0.8924	0.1860	1.0031	0.9934
		MAE	0.0887	0.1331	0.0894	0.0895	0.1390	0.1216	0.0805	0.1263	0.0858
		Mean	0.1989	1.0015	0.9955	0.1862	0.9998	0.9779	0.1985	1.0002	0.9978
1000	1000	Median	0.1993	1.0041	0.9955	0.1932	1.0091	0.9786	0.1994	1.0017	0.9988
		MAE	0.0145	0.0282	0.0183	0.0224	0.0370	0.0278	0.0184	0.0280	0.0197

Notes: (1) Programs are written in Gauss and CML library is used for MLE and MPS.
The initial values of CML for n = 50, r = 100 is: $-\gamma = \mu = \sigma = 0.2$.
The initial values of CML for n = 50, r = 1000 is: $-\gamma = \mu = \sigma = 0.2$.
The initial values of CML for n = 1000, r = 1000 is: $\gamma = -0.2$, $\mu = \sigma = 0.4$.
The results are calculated based on replications with converged CML results.
(2) n is sample size in each replication.
(3) r is number of replication.

Table 2.3: Monte Carlo experiments: Case 3[1] with true parameter values of $\gamma = 1$ $\mu = 1$ $\sigma = 1$

n[2]	r[3]		MLE			MPS			PWM		
			$\hat{\gamma}$	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\gamma}$	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\gamma}$	$\hat{\mu}$	$\hat{\sigma}$
		Mean	0.9306	1.1706	0.7518	0.8741	1.1868	0.6909	0.9242	0.9496	1.0302
50	100	Median	0.9458	1.1581	0.7574	0.8841	1.1812	0.6891	0.9181	0.9567	1.0353
		MAE	0.0802	0.1774	0.2482	0.1275	0.1934	0.3091	0.1282	0.1306	0.1283
		Mean	0.9195	1.1648	0.7474	0.8670	1.1761	0.6939	0.9086	0.9632	1.0052
50	1000	Median	0.9459	1.1708	0.7428	0.8754	1.1822	0.6937	0.9054	0.9680	0.9965
		MAE	0.0910	0.1789	0.2548	0.1356	0.1848	0.3061	0.1418	0.1302	0.1258
		Mean	0.958	1.015	0.945	0.9237	1.0498	0.8796	0.9708	0.9890	1.0000
1000	1000	Median	0.976	1.001	0.979	0.9309	1.0307	0.8966	0.9712	0.9901	0.9997
		MAE	0.044	0.047	0.072	0.0779	0.0740	0.1326	0.0382	0.0292	0.0278

Notes: (1) Programs are written in Gauss and CML library is used for MLE and MPS.
The initial values of CML for n = 50, r = 100 is: $-\gamma = \mu = \sigma = 0.2$.
The initial values of CML for n = 50, r = 1000 is: $-\gamma = \mu = \sigma = 0.2$.
The initial values of CML for n = 1000, r = 1000 is: $\gamma = 0.2,\ \mu = \sigma = 0.4$.
The results are calculated based on replications with converged CML results.
(2) n is sample size in each replication.
(3) r is number of replication.

Table 2.4: Monte Carlo Experiments with the Initial Values: $\gamma = -0.1$ $\mu = -0.1$ $\sigma = 0.1$

			MLE[1]			MPS[1]			PWM		
			$\hat{\gamma}$	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\gamma}$	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\gamma}$	$\hat{\mu}$	$\hat{\sigma}$
		Mean	-0.1506	-0.0005	0.0160	-0.1136	-0.0088	0.0314	-0.1318	-0.0003	0.0160
		Median	-0.1472	-0.0006	0.0160	-0.1208	-0.0007	0.0237	-0.1305	-0.0004	0.0160
		MAE	0.0147	0.0003	0.0002	0.0495	0.0131	0.0156	0.0651	0.0013	0.0012

$\alpha^{(2)}$	true.VaR[2]		MLE est.VaR[2]	MLE DIF[3]	MPS est.VaR[2]	MPS DIF[3]	PWM est.VaR[2]	PWM DIF[3]
		Mean	-0.1055	0.0000	-0.1791	-0.0736	-0.1031	0.0024
1%	-0.1055	Median	-0.1049	0.0006	-0.1374	-0.0319	-0.1029	0.0026
		$\text{MAE}_{VaR}^{(4)}$	0.0040		0.0787		0.0152	
		Mean	-0.0593	0.0001	-0.1012	-0.0418	-0.0583	0.0010
5%	-0.0594	Median	-0.0590	0.0004	-0.0820	-0.0226	-0.0592	0.0002
		$\text{MAE}_{VaR}^{(4)}$	0.0016		0.0445		0.0062	
		Mean	-0.0422	0.0001	-0.0711	-0.0288	-0.0418	0.0005
10%	-0.0423	Median	-0.0422	0.0001	-0.0605	-0.0182	-0.0419	0.0005
		$\text{MAE}_{VaR}^{(4)}$	0.0011		0.0309		0.0040	
		Mean	-0.0213	0.0001	-0.0330	-0.0116	-0.0214	0.0000
25%	-0.0214	Median	-0.0213	0.0001	-0.0310	-0.0096	-0.0211	0.0003
		$\text{MAE}_{VaR}^{(4)}$	0.0006		0.0137		0.0024	
		Mean	-0.0055	-0.0001	-0.0030	0.0026	-0.0057	-0.0002
50%	-0.0055	Median	-0.0054	0.0001	-0.0054	0.0002	-0.0056	-0.0001
		$\text{MAE}_{VaR}^{(4)}$	0.0004		0.0101		0.0016	

Notes: (1) True parameter values are: $\gamma = -0.15$ $\mu = -0.0051$ $\sigma = 0.01$
Results are calculated based on Monte Carlo experiments with 100 replications.
(2) true.VaR and est.VaR are computed by equation (2.20): true.VaR uses true γ, μ and σ while est.VaR uses $\widehat{\gamma}, \widehat{\mu}$, and $\widehat{\sigma}$..
(3) DIF = est.VaR - true.VaR.
(4) MAE_{VaR} is given in equaion (2.25).

Table 2.5: Monte Carlo Experiments with the Initial Values: $\gamma = 0.1$ $\mu = 0.1$ $\sigma = 0.1$

			MLE$^{(1)}$			MPS$^{(1)}$			PWM		
			$\hat{\gamma}$	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\gamma}$	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\gamma}$	$\hat{\mu}$	$\hat{\sigma}$
		Mean	-0.1519	-0.0001	0.0162	0.0435	-0.0042	0.0247	-0.1318	-0.0003	0.0160
		Median	-0.1553	-0.0001	0.0162	0.0613	0.0002	0.0205	-0.1305	-0.0004	0.0160
		MAE	0.0153	0.0004	0.0003	0.1945	0.0074	0.0090	0.0651	0.0013	0.0012
$\alpha^{(2)}$	true.VaR$^{(2)}$		est.VaR$^{(2)}$		DIF$^{(3)}$	est.VaR$^{(2)}$		DIF$^{(3)}$	est.VaR$^{(2)}$		DIF$^{(3)}$
		Mean	-0.1079		-0.0024	-0.0963		0.0092	-0.1031		0.0024
1%	-0.1055	Median	-0.1081		-0.0026	-0.0923		0.0132	-0.1029		0.0026
		MAE$_{VaR}^{(4)}$	0.0041			0.0244			0.0152		
		Mean	-0.0607		-0.0014	-0.0631		-0.0037	-0.0583		0.0010
5%	-0.0594	Median	-0.0609		-0.0015	-0.0582		0.0012	-0.0592		0.0002
		MAE$_{VaR}^{(4)}$	0.0017			0.0153			0.0062		
		Mean	-0.0434		-0.0010	-0.0477		-0.0054	-0.0418		0.0005
10%	-0.0423	Median	-0.0434		-0.0010	-0.0421		0.0002	-0.0419		0.0005
		MAE$_{VaR}^{(4)}$	0.0011			0.0124			0.0040		
		Mean	-0.0221		-0.0007	-0.0254		-0.0039	-0.0214		0.0000
25%	-0.0214	Median	-0.0221		-0.0007	-0.0229		-0.0015	-0.0211		0.0003
		MAE$_{VaR}^{(4)}$	0.0007			0.0090			0.0024		
		Mean	-0.0060		-0.0004	-0.0047		0.0008	-0.0057		-0.0002
50%	-0.0055	Median	-0.0060		-0.0005	-0.0069		-0.0014	-0.0056		-0.0001
		MAE$_{VaR}^{(4)}$	0.0005			0.0071			0.0016		

Notes: (1) True parameter values are: $\gamma = -0.15$ $\mu = -0.0051$ $\sigma = 0.01$
Results are calculated based on Monte Carlo experiments with 100 replications.
(2) true.VaR and est.VaR are computed by equation (2.20): true.VaR uses true $\gamma,\ \mu$ and σ while est.VaR uses $\widehat{\gamma}, \widehat{\mu}$, and $\widehat{\sigma}$..
(3) DIF = est.VaR - true.VaR.
(4) MAE$_{VaR}$ is given in equaion (2.25).

Table 2.6: Monte Carlo Experiments with the Initial Values: $\gamma = 0.5$ $\mu = 0.5$ $\sigma = 0.5$

			MLE(1)			MPS(1)			PWM		
			$\hat{\gamma}$	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\gamma}$	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\gamma}$	$\hat{\mu}$	$\hat{\sigma}$
		Mean	-0.0868	-0.0031	0.0247	0.3031	-0.0165	0.0983	-0.1318	-0.0003	0.0160
		Median	-0.1439	-0.0006	0.0160	0.3131	-0.0061	0.0858	-0.1305	-0.0004	0.0160
		MAE	0.0763	0.0047	0.0090	0.4591	0.0261	0.0825	0.0651	0.0013	0.0012
$\alpha^{(2)}$	true.VaR(2)		est.VaR(2)		DIF(3)	est.VaR(2)		DIF(3)	est.VaR(2)		DIF(3)
		Mean	-0.1158		-0.0103	-0.1953		-0.0898	-0.1031		0.0024
1%	-0.1055	Median	-0.1060		-0.0005	-0.1818		-0.0763	-0.1029		0.0026
		$\mathrm{MAE}_{VaR}^{(4)}$	0.0139			0.0958			0.0152		
		Mean	-0.0702		-0.0108	-0.1520		-0.0926	-0.0583		0.0010
5%	-0.0594	Median	-0.0594		0.0000	-0.1468		-0.0874	-0.0592		0.0002
		$\mathrm{MAE}_{VaR}^{(4)}$	0.0124			0.0950			0.0062		
		Mean	-0.0518		-0.0095	-0.1260		-0.0836	-0.0418		0.0005
10%	-0.0423	Median	-0.0425		-0.0002	-0.1227		-0.0804	-0.0419		0.0005
		$\mathrm{MAE}_{VaR}^{(4)}$	0.0106			0.0867			0.0040		
		Mean	-0.0272		-0.0058	-0.0775		-0.0561	-0.0214		0.0000
25%	-0.0214	Median	-0.0214		0.0000	-0.0731		-0.0517	-0.0211		0.0003
		$\mathrm{MAE}_{VaR}^{(4)}$	0.0068			0.0612			0.0024		
		Mean	-0.0059		-0.0003	-0.0166		-0.0111	-0.0057		-0.0002
50%	-0.0055	Median	-0.0054		0.0001	-0.0171		-0.0116	-0.0056		-0.0001
		$\mathrm{MAE}_{VaR}^{(4)}$	0.0041			0.0272			0.0016		

Notes: (1) True parameter values are: $\gamma = -0.15$ $\mu = -0.0051$ $\sigma = 0.01$
Results are calculated based on Monte Carlo experiments with 100 replications.
(2) true.VaR and est.VaR are computed by equation (2.20): true.VaR uses true γ, μ and σ while est.VaR uses $\widehat{\gamma}, \widehat{\mu}$, and $\widehat{\sigma}$..
(3) DIF = est.VaR - true.VaR.
(4) MAE_{VaR} is given in equaion (2.25).

Table 2.7: Monte Carlo Experiments with Block Maxima Data

	MLE[1]			MPS			PWM			Bayes		
	$\hat{\gamma}$	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\gamma}$	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\gamma}$	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\gamma}$	$\hat{\mu}$	$\hat{\sigma}$
						$m = 480$ $(r = 5)$						
Mean	−.0130	.9845	.7868	−.0088	.9900	.7760	−.0336	.9787	.7634	−.0430	.9956	.7788
Std	(.0377)	(.0396)	(.0314)	(.0375)	(.0416)	(.0309)	(.0462)	(.0401)	(.0306)	(.0004)	(.0011)	(.0008)
						$m = 240$ $(r = 10)$						
Mean	−.0632	1.5037	.7581	−.0743	1.1719	.6275	−.0879	1.4945	.7395	−.0813	1.5560	.7678
Std	(.0533)	(.0575)	(.0447)	(.0595)	(.5074)	(.2018)	(.0638)	(.0580)	(.0414)	(.0005)	(.0015)	(.0011)
						$m = 120$ $(r = 20)$						
Mean	−.1061	2.0225	.7668	−.0925	1.8079	0.6873	−.1201	2.0136	.7611	−.0665	2.1251	.8157
Std	(.0772)	(.0765)	(.0679)	(.0805)	(.4990)	(.1622)	(.0832)	(.0797)	(.0723)	(.0006)	(.0022)	(.0015)

Notes: (1) m = number of blocks; r = block size.
(2) For MLE and MPS, the initial values are: $\gamma = -0.1$ $\mu = 0.2$ $\sigma = 0.2$. Results are calculated based on Monte Carlo experiments with 100 replications.
(3) Mean = The average of 100 estimates.
(4) Std = Standard deviation

Table 2.8: Monte Carlo Experiments with Block Maxima Data with Sample Size n set at 2,400

	MLE[1]			MPS			PWM			Bayes		
α	est.VaR_α	Dat.VaR_α	DIF	est.VaR_α	Dat.VaR_α	DIF	est.VaR_α	Dat.VaR_α	DIF	est.VaR_α	Dat.VaR_α	DIF
	$m = 480$ $(r = 5)$											
.01	4.73	4.95	.22	4.65	4.95	.30	4.80	4.95	.15	4.96	4.74	.22
.05	3.37	3.31	.06	3.33	3.32	.01	3.37	3.31	.06	3.46	3.48	.02
.10	2.78	2.69	.09	2.76	2.69	.07	2.77	2.69	.08	2.84	2.74	.10
.25	1.97	1.91	.06	1.96	1.92	.04	1.95	1.91	.04	1.99	1.99	.0
.50	1.27	1.27	.00	1.27	1.27	.00	1.26	1.27	.01	1.28	1.30	.02
	MAE_{VaR}= .086			MAE_{VaR}=.070			MAE_{VaR}= .057			MAE_{VaR}=.060		
	$m = 240$ $(r = 10)$											
.01	5.60	5.72	.12	4.63	5.72	1.09	5.75	5.72	.03	5.84	5.86	.02
.05	4.00	3.97	.03	3.26	3.98	.72	4.02	3.97	.05	4.14	4.13	.01
.10	3.34	3.28	.06	2.71	3.30	.59	3.34	3.28	.06	3.45	3.45	.0
.25	2.49	2.44	.05	2.00	2.45	.45	2.47	2.44	.03	2.56	2.53	.03
.50	1.78	1.78	.00	1.40	1.78	.38	1.77	1.78	.01	1.84	1.85	.01
	MAE_{VaR}= .043			MAE_{VaR}=.538			MAE_{VaR}= .036			MAE_{VaR}=.017		
	$m = 120$ $(r = 20)$											
.01	6.68	6.50	.18	5.84	6.52	.68	6.81	6.50	.31	6.51	6.15	.36
.05	4.73	4.69	.04	4.18	4.69	.51	4.76	4.69	.07	4.80	4.71	.09
.10	3.98	3.94	.04	3.53	3.95	.42	4.00	3.94	.06	4.10	4.13	.03
.25	3.05	3.01	.04	2.72	3.01	.29	3.04	3.01	.03	3.18	3.13	.05
.50	2.31	2.30	.01	2.06	2.30	.24	2.30	2.30	.00	2.43	2.33	.10
	MAE_{VaR}= .052			MAE_{VaR}=.357			MAE_{VaR}= .077			MAE_{VaR}=.105		

Notes: (1) est.VaR_α is the averages of 100 est.VaR_α's.
(2) Dat.VaR_α is the average of 100 Dat.VaR_α's.
(3) DIF is the average of $|\text{VaR}_\alpha - \text{Dat.VaR}|$.
(4) m= number of blocks.
(5) r = number of data points in each blocks.
(6) $n = m \times r$.

Table 2.9: Block Maxima Data: FTSE100

	MLE[1]			MPS			PWM			Bayes		
	$\hat{\gamma}$	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\gamma}$	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\gamma}$	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\gamma}$	$\hat{\mu}$	$\hat{\sigma}$
						$m = 600$ $(r = 5)$						
Est.	$-.1085$	.0074	.0076	$-.1764$	.0072	.0071	$-.1868$	.0071	.0071	$-.1111$	.0778	.0083
Std	(.0234)	(.0003)	(.0003)	(.0326)	(.0003)	(.0003)	(.4233)	(.0105)	(.0160)	(.0254)	(.0068)	(.0138)
						$m = 300$ $(r = 10)$						
Est.	$-.2652$	.0110	.0069	$-.2702$	.0110	.0067	$-.2134$	.0111	.0071	$-.2661$	.0113	.0076
Std	(.0533)	(.0005)	(.0005)	(.0554)	(.0005)	(.0004)	(.0638)	(.0147)	(.0230)	(.0210)	(.0068)	(.0139)
						$m = 150$ $(r = 20)$						
Est.	$-.2316$	.0146	.0079	$-.2145$	.0146	.0078	$-.1780$	.0147	.0082	$-.2297$	.0150	.0087
Std	(.0775)	(.0007)	(.0006)	(.0799)	(.0007)	(.0006)	(.8330)	(.0246)	(.0374)	(.0230)	(.0066)	(.0139)

Notes: (1) m = number of blocks; r = block size.
(2) Est.=Point Estimate
(3) Std = Standard deviation

Table 2.10: VaR: FTSE 100 – Block Maxima Data

	MLE[1]			MPS			PWM			Bayes		
α	est.VaR$_\alpha$	Dat.VaR$_\alpha$	DIF	est.VaR$_\alpha$	Dat.VaR$_\alpha$	DIF	est.VaR$_\alpha$	Dat.VaR$_\alpha$	DIF	est.VaR$_\alpha$	Dat.VaR$_\alpha$	DIF
	$m = 600 \quad (r = 5)$											
.01	.055	.053	.002	.055	.058	.003	.055	.059	.004	.055	.058	.003
.05	.036	.034	.02	.036	.035	.001	.036	.035	.001	.036	.037	.001
.10	.027	.027	0	.027	.027	0	.027	.027	0	.027	.029	.002
.25	.017	.018	.001	.017	.017	0	.017	.017	0	.017	.018	.001
.50	.010	.010	0	.010	.010	0	.010	.010	04	.010	.011	.001
	MAE$_{VaR}$= .001			MAE$_{VaR}$=.001			MAE$_{VaR}$= .001			MAE$_{VaR}$=.002		
	$m = 300 \quad (r = 10)$											
.01	.056	.073	.017	.056	.072	.016	.056	.067	.011	.056	.079	.023
.05	.040	.042	.002	.040	.042	.002	.040	.041	.001	.040	.045	.005
.10	.032	.032	0	.032	.032	0	.032	.035	.003	.032	.035	.003
.25	.022	.021	.001	.022	.021	.001	.022	.021	.001	.022	.023	.001
.50	.014	.014	0	.014	.014	0	.014	0	.01	.014	.014	.0
	MAE$_{VaR}$= .004			MAE$_{VaR}$=.004			MAE$_{VaR}$=.002			MAE$_{VaR}$=.006		
	$m = 150 \quad (r = 20)$											
.01	.056	.079	.023	.056	.076	.019	.056	.073	.017	.056	.086	.030
.05	.049	.048	.001	.049	.047	.002	.049	.047	.002	.049	.052	.003
.10	.039	.038	.001	.039	.037	.002	.039	.037	.002	.039	.040	.001
.25	.026	.026	0	.026	.027	.001	.026	.026	0	.026	.028	.001
.50	.017	.018	.001	.017	.018	.001	.017	.018	.001	.017	.018	.001
	MAE$_{VaR}$= .005			MAE$_{VaR}$=.005			MAE$_{VaR}$= .004			MAE$_{VaR}$=.006		

Notes: (1) est.VaR$_\alpha$ is the VaR estimate using $\widehat{\gamma}$, $\widehat{\mu}$ and $\widehat{\sigma}$.
(2) Dat.VaR$_\alpha$ is the α-percentile of block maxima data.
(3) DIF is $|\text{VaR}_\alpha - \text{Dat.VaR}|$.
(4) m= number of blocks.
(5) r = number of data points in each blocks.
(6) $n = m \times r$.

Table 2.11: Block Maxima Data: SP500

	MLE(1)			MPS			PWM			Bayes		
	$\hat{\gamma}$	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\gamma}$	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\gamma}$	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\gamma}$	$\hat{\mu}$	$\hat{\sigma}$
						$m = 600$ $(r = 5)$						
Est.	$-.2107$	.0073	.0069	$-.2146$	.0073	.0068	$-.1856$	.0074	.0071	$-.2124$	.0077	.0076
Std	(.0234)	(.0003)	(.0003)	(.0326)	(.0003)	(.0003)	(.4233)	(.0105)	(.0160)	(.0219)	(.0068)	(.0139)
						$m = 300$ $(r = 10)$						
Est.	$-.2360$	.0111	.0071	$-.2411$	.0111	.0070	$-.1907$	.0113	.0074	$-.2369$	.0115	.0079
Std	(.0520)	(.0005)	(.0004)	(.0551)	(.0005)	(.0004)	(.6031)	(.0155)	(.0238)	(.0218)	(.0068)	(.0139)
						$m = 150$ $(r = 20)$						
Est.	$-.2705$	.0144	.0074	$-.2536$	.0145	.0074	$-.1937$	.0147	.0080	$-.2707$	.0147	.0082
Std	(.0833)	(.0007)	(.0006)	(.0845)	(.0007)	(.0006)	(.85790)	(.0234)	(.0361)	(.0240)	(.0067)	(.0139)

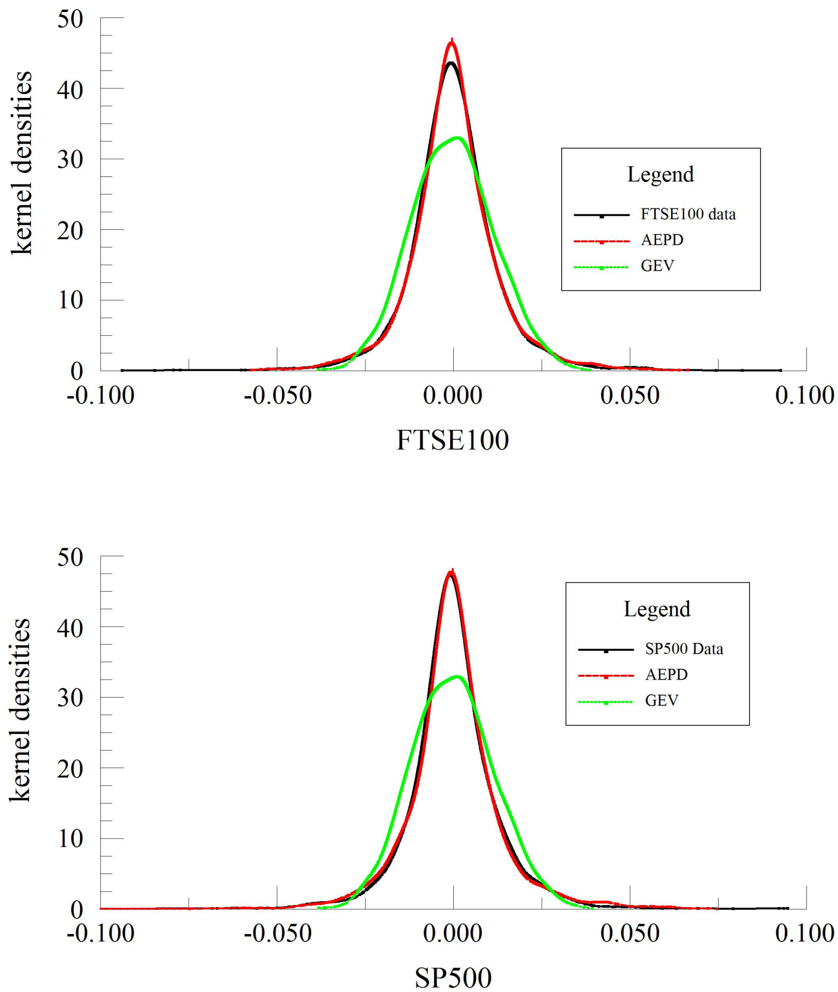
Notes: (1) m = number of blocks; r = block size.
(2) Est.=Point Estimate
(3) Std = Standard deviation

Table 2.12: VaR: SP500 – Block Maxima Data

	MLE[1]			MPS			PWM			Bayes		
α	est.VaR$_\alpha$	Dat.VaR$_\alpha$	DIF	est.VaR$_\alpha$	Dat.VaR$_\alpha$	DIF	est.VaR$_\alpha$	Dat.VaR$_\alpha$	DIF	est.VaR$_\alpha$	Dat.VaR$_\alpha$	DIF
						$m = 600$ $(r = 5)$						
.01	.063	.061	.002	.063	.061	.002	.063	.059	.004	.063	.067	.004
.05	.033	.036	.003	.033	.036	.003	.033	.036	.003	.033	.039	.006
.10	.027	.027	0	.027	.027	0	.027	.027	0	.027	.029	.002
.25	.017	.017	0	.017	.017	0	.017	.017	0	.017	.018	.001
.50	.010	.010	0	.010	.010	0	.010	.010	0	.010	.011	.001
	MAE$_{VaR}$= .001			MAE$_{VaR}$=.001			MAE$_{VaR}$= .001			MAE$_{VaR}$=.003		
						$m = 300$ $(r = 10)$						
.01	.069	.070	.001	.069	.073	.004	.069	.066	.003	.069	.066	.003
.05	.035	.042	.006	.035	.043	.007	.035	.041	.005	.035	.045	.010
.10	.032	.031	.001	.032	.033	.001	.032	.031	.001	.032	.035	.003
.25	.023	.021	.002	.023	.022	.002	.023	.022	.001	.023	.023	0
.50	.014	.014	0	.014	.014	0	.014	.014	0	.014	.014	.0
	MAE$_{VaR}$= .002			MAE$_{VaR}$=.003			MAE$_{VaR}$= .002			MAE$_{VaR}$=.004		
						$m = 150$ $(r = 20)$						
.01	.069	.079	.010	.069	.079	.010	.069	.074	.005	.069	.090	.021
.05	.044	.045	.001	.044	.048	.004	.044	.047	.003	.044	.052	.008
.10	.035	.037	.002	.035	.037	.002	.035	.037	.002	.035	.040	.005
.25	.026	.022	.004	.026	.025	.001	.026	.026	0	.026	.027	.001
.50	.018	.014	.004	.018	.017	.001	.018	.018	0	.018	.018	.0
	MAE$_{VaR}$= .004			MAE$_{VaR}$=.003			MAE$_{VaR}$= .002			MMAE$_{VaR}$=.006		

Notes: (1) est.VaR$_\alpha$ is the VaR estimate using $\widehat{\gamma}$, $\widehat{\mu}$ and $\widehat{\sigma}$.
(2) Dat.VaR$_\alpha$ is the α-percentile of block maxima data.
(3) DIF is $|$VaR$_\alpha-$Dat.VaR$|$.
(4) m= number of blocks.
(5) r = number of data points in each blocks.
(6) $n = m \times r$.

Figure 2.9: Comparison of Kernel Densities of Data with the Estimated Pdf's:FTSE100 and SP500



Chapter 3

Does the 08-09 Crisis Change the Dynamics of Implied Volatility Surface?

3.1 Introduction

The Black-Scholes-Merton (BSM) model was developed in the early 1970s and implied volatility based on it has been widely studied due to the implications to trading, pricing and risk management. In the BSM model, the factors that determine the value of an option P are the moneyness $m = K/S$[1] (S is the current stock price and K is the strike price), time to maturity τ, underlying asset volatility σ and risk free interest rate r. BSM model defines the function of option price as $P(K/S, \tau, \sigma, r)$, and the BSM implied volatility is backed out by treating the price as one of the inputs: $IV(K/S, \tau, P, r)$.

It is widely believed that implied volatility implies the market expectation of future volatility. Future volatility is crucial for option traders because the bigger the volatility, the bigger the magnitude the underlying asset would move towards one's favorable direction within certain time frame. After all, buying an option is no different to buying the volatility of underlying asset. This makes it easier to understand why the BSM implied volatility is used as a quoting convention of the option price. Having said that, different implied volatilities can be derived from different models. Accepting the BSM implied volatility as a communication tool by practitioners is due to historical convention, rather than agreeing with the BMS model assumptions.

Studying the change of implies volatility surface and its relationship to future/realized volatility has a long history. Cont and Fonseca (2002) use time series of option price of

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[1]Some literature defines the moneyness as S/K. The moneyness defines the relationship between strike price and the price of underlying asset.

S&P 500 and FTSE indices, study the deformation of this surface based on a Karhunen-Loeve decomposition and show that it may be represented as a randomly fluctuating surface driven by a small number of orthogonal random factors. Busch, Christensen and Nielsen (2011) find that implied volatility contains incremental information about future volatility in the foreign exchange, stock, and bond markets. The heterogeneous autoregressive (HAR) model is applied and the out-of-sample forecasting experiments confirm that implied volatility is important in forecasting future realized volatility components in all three markets. Szakmary, Ors, Kim and Davidson(2003) test how well the implied volatilities embedded in option prices predict subsequently realized volatility (RV) in 35 futures options markets from eight separate exchanges. They find that for this broad array of futures options, the implied volatilities outperform historical volatility (HV) as a predictor of the subsequently RV in the underlying futures prices over the remaining life of the option. Goncalves and Guidolin (2005) propose a two-stage approach to model and forecast the S&P 500 index options implied volatility surface. They claim that not only the S&P 500 implied volatility surface can be successfully modeled, but also that its movements over time are highly predictable in a statistical sense.

Studying the BSM implied volatility is also due to its implications to trading, pricing and risk management. For example, one of the option trading rules is to buy option at low implied volatility and sell it at high implied volatility. Such strategy requires good understanding of the movement of implied volatility. Measure the movement of the implied volatility conditioned on current implied volatility level is crucial. Like other products, option price is also affected by supply-demand relationship. Theoretical price is not always the price settled by market participants. In reality it is common to observe the price deviate from the theoretical price and fluctuate. The variation of the implied volatility for the same option not only reflects the supply-demand relationship but also suggests the different anticipation from market participants.

This paper analyzes the movement of implied volatility surface in four time periods: Pre-Crisis, Crisis, Adjustment period and Post-Crisis. As it is natural to expect abnormality in Crisis and Adjustment period, it is interesting to see the difference between

Post-Crisis movement and the Pre-Crisis's. The results reveal that if the catastrophe does not permanently change the investment behavior, the effect from Crisis may last longer than expected. It is unwise to assume the market movement or investment behavior would be identical in Pre-Crisis and Post-Crisis periods. Market participants learn from Crisis and behave differently in Post-Crisis comparing to Pre-Crisis.

In this paper, I construct the daily implied volatility surface which is a three-dimensional plot that displays implied volatility given different moneyness ($m = K/S$) and time to maturity (τ). Given each set of (m, τ), the implied volatility time series $IV_t(m, \tau)$ is obtained. The data is then fitted into a stochastic differential equation with mean-reverting drift and constant elasticity of variance. The mean-reverting drift is consistent with the observation and the constant elasticity of variance allows flexibility of modeling the volatility of volatility (vol-of-vol). Another reason to use this model rather than more complicated one is due to its small number of parameters and easy interpretation. Four parameters in this model can be interpreted as the long-run level of implied volatility, the speed that pulls implied volatility towards long run level, the scale of variance and the parameter controlling the relationship between vol-of-vol and level of current implied volatility. After estimating the parameters, the comparison across different time periods are conducted.

The paper is organized as follows. Section 2 describes the methods constructing the implied volatility surface and the data structure. Section 3 explains the model that the data is fitted to. Section 4 analyzes the results. Section 5 concludes.

3.2 Construct Implied Volatility Surface

3.2.1 IVS and Non-parametric Nadaraya-Watson Estimator

At a given trading day, implied volatilities are backed out by plugging all the available put and call option prices into the BSM model. The implied volatility surface IVS : $[m_{min}, m_{max}] \times [\tau_{min}, \tau_{max}] \to (0, \infty)$ can be constructed by using non-parametric kernel smoothing method. Once the implied volatility surfaces are constructed for all the trading days, a time series IV_t can be obtained at each set of (m, τ) and used for further analysis. Different kernel smoothing methods are used in literature for implied volatility surface construction. Audrino and Colangelo(2008) use the non-parametric least square kernel (LSK) smoothing estimator introduced by Gourieroux, Monfort, and Tenreiro (1995). Cont and da Fonseca (2002) use non-parametric Nadaraya-Watson estimator which is also the estimator used in this paper.

Denote the time series of implied volatility measured at specific moneyness m and time to maturity τ given a trading day t as $IV_t(m,\tau)$. The IV_t at (m,τ) is calculated by non-parametric Nadaraya-Watson estimator based on following formula:

$$\hat{IV}_t(m,\tau) = \frac{\sum_{i=1}^{n} IV_t(m_i,\tau_i)\mathbf{K}(\frac{m-m_i}{h_1})\mathbf{K}(\frac{\tau-\tau_i}{h_2})}{\sum_{i=1}^{n}\mathbf{K}(\frac{m-m_i}{h_1})\mathbf{K}(\frac{\tau-\tau_i}{h_2})} \tag{3.1}$$

where $\mathbf{K} = \frac{1}{\sqrt{2\pi}}\exp(-x^2/2)$ is the Gaussian kernel. Here, n is the number of different option series traded on the specific trading day. m_i and τ_i are the moneyness and time to maturity of each option at day t. h_1 and h_2 are the bandwidth parameters. Cont and da Fonseca (2002) claim that the choice of the kernel distribution does not affect the results, but the choice of bandwidths does. Ways of determining the optimal bandwidth are proposed in literature. Here I refer to the normal optimal smoothing bandwidth proposed by Bowman and Azzalini (1997):

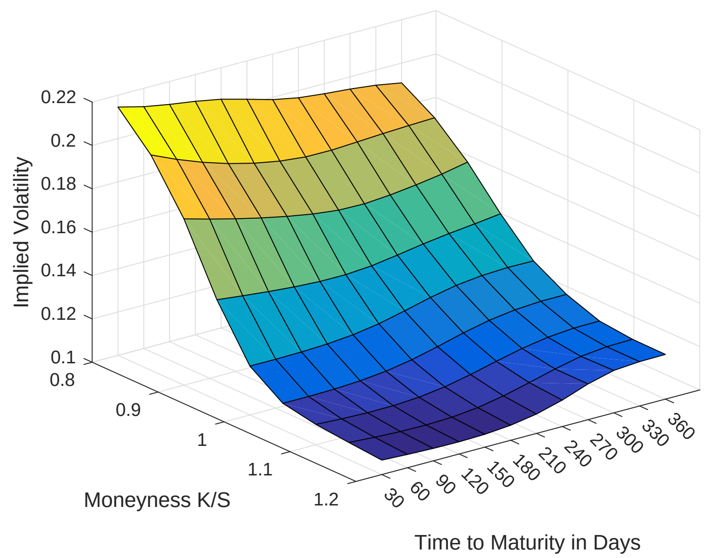
$$h_i = \sigma_i \left\{ \frac{4}{(d+2)n} \right\}^{1/(d+4)} \tag{3.2}$$

where σ_i is the standard deviation of m or τ and $i = 1, 2, ..., d$. d denotes the number of dimensions. In our case, we have m and τ two variables, thus $d = 2$. n denotes the sample size. Here it equals to the number of option series traded on a trading day. Figure 3.1 illustrates the implied volatility surface on 08/01/2006.

3.2.2 Data Structure

The data used in this paper is the daily S&P 500 exchange-traded option data obtained from Market Data Express. The whole period used for analysis is from January 2005 to October 2014. It is divided into four sub-periods: Pre-Crisis, Crisis, Adjustment Period and Post-Crisis. Table 3.1 gives the begin and end dates of each sub-period. While there is no consensus about the beginning and ending of each sub-period, I choose them

Figure 3.1: S&P 500 Implied Volatility Surface on 08/01/2006



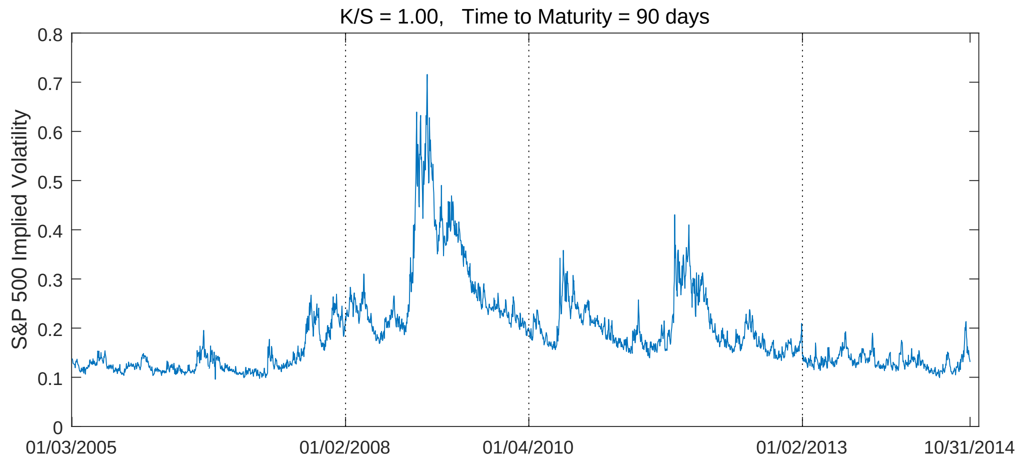
by eyeballing the average level of implied volatilities. Figure 3.2 gives an illustration of implied volatility movement for at-the-money-option with time to maturity equal to 90 days. In general implied volatilities at other moneyness levels and time to maturities display similar shapes across the whole period.

Table 3.1: Period Classification

Period	Begin Date	End Date
Pre-Crisis	01/03/2005	12/31/2007
Crisis	01/02/2008	12/31/2009
Adjustment Period	01/04/2010	12/31/2012
Post-Crisis	01/02/2013	10/31/2014

The data structure of options includes trading date, expiry date, call/put type, strike price, trading volume, high (the highest trade price in this series on this day), low (the lowest trade price in this series on this day), open (the trade price on the first trade in this series on this day), last (the trade price on the last trade in this series on this day), and underlying price (the closing price on the associated underlying instrument on this day). The Market Data Express does not provide BSM implied volatility, but it is easy to calculate it by using the BSM framework. To do so, I use the last trade

Figure 3.2: S&P 500 Implied Volatility



price of options in order to match the stock closing price on the day. Moneyness is defined as the ratio of strike price over stock price. Time to maturity is calculated as the difference between the trading date and the expiry date.

In regard to the risk free rate, I use the treasury yield curve rates published by the US Department of the Treasury. Many literature simply uses a fixed risk-free rate, such as the 3-month US Treasury bill rate. In practice, allowing varying interest rate facilitates accuracy in estimating implied volatility. In this paper, I use 1,3,6 month and 1,2,3 years daily treasury yield curve rates. I use the polynomial interpolation method to approximate the risk-free interest rate for each option series at specific time to maturity for each trading day.

Some criteria are used to filter the data before analysis. First, the options with trading volume smaller than 100 are eliminated. The purpose is to exclude the illiquid options so that to minimize the noise from the meaningful market movements. For the same reason, I exclude options if their time to maturities are less than three days. Third, based on the put-call parity relationship, put options and call options with the same strike price and time to maturity should have the same implied volatility in theory. To avoid conflict, only the out-of-the-money options are included in. Equivalently, let $m = \frac{K}{S}$ denotes the moneyness. Only call options with $m > 1$ and put options with $m < 1$ are used. According to Cont and da Fonseca (2002), these are the options that are believed to contain the most information about the implied volatility surface movements. Out-of-the-money options are less expensive than in-the-money and at-the-money options. They are not only attractive to investors with little capital but also to those who wants to use leverage tool to bet the movement of underlying asset.

Table 3.2: Summary Statistics for Implied Volatilities (Jan 2005 - Oct 2014)

			Short-term $(3, 60)$	Medium-term $[60, 180]$	Long-term $(180, 360]$
	0.75-0.84	Obs.	27087	11615	4674
		Mean IV	0.4188	0.3063	0.2730
		STD IV	0.1286	0.0755	0.0635
	0.85-0.9	Obs.	73355	23514	8294
Put		Mean IV	0.2900	0.2390	0.2324
		STD IV	0.0991	0.0705	0.0597
	0.95-1.00	Obs.	73909	21389	7351
		Mean IV	0.1814	0.1972	0.2059
		STD IV	0.0736	0.0705	0.0586
	1.00-1.04	Obs.	62333	16165	5064
		Mean IV	0.1338	0.1541	0.1659
		STD IV	0.0680	0.0695	0.0570
	1.05-1.14	Obs.	38864	17477	7486
Call		Mean IV	0.1792	0.1535	0.1522
		STD IV	0.0848	0.0671	0.0576
	1.15-1.24	Obs.	3732	2884	2200
		Mean IV	0.3027	0.2024	0.1643
		STD IV	0.1318	0.0802	0.0628

Most literature studying option pricing focuses on the at-the-money options, or those out-of-the money options (in-the-money option) within a small range of deviating from the at-the-money situation. This is simply because those are the most liquid options and have less noise. However in reality the deep-out-of-the money options receive a lot of attention and are actively traded by practitioners. Options belonging to the deep-out-of-the category are those with strike price significantly above or below the underlying asset price. There is no consensus of the threshold separating the at-the-money, out-of-the-money and deep-out-of-the-money options. Goncalves and Guidolin (2005) defines a call option to be deep-out-the-money (DOTM) if the moneyness $m = \frac{K}{S} > 1.06$; out-of-the-money (OTM) if $1.01 < m \leq 1.06$; a put option to be out-of-the-money (OTM) if $0.94 \leq m < 0.99$; and deep-out-of-the-money (DOTM) if $m < 0.94$; and at-the-money (ATM) if $0.99 \leq m \leq 1.01$. Audrino and Colangelo (2008) defines a call option to be out-of-the-money (OTM) if $1.04 < m \leq 1.2$; deep out-of-the-money (DOTM) if $1.2 <$

m; a put option to be out-of-the-money (OTM) if $0.8 < m \leq 0.94$; deep out-of-the-money (DOTM) if $m \leq 0.8$; and at-the-money (ATM) if $0.94 < m \leq 1.04$. In this paper, I do the analysis for options with moneyness at $0.8, 0.85, 0.9, 0.95, 1, 1.05, 1.10, 1.15$ and 1.2 and define the deep-out-the-money if moneyness is $0.8, 0.85, 1.15$ and 1.2, the out-the-money if moneyness is $0.9, 0.95, 1.05$ and 1.10, and at-the-money if the moneyness is 1. Table 3.2 gives the summary statistics of the data.

3.3 Model Specification

A stochastic differential equation is used to capture the movement of the BMS implied volatility. It is, given the moneyness m and the time to maturity τ, $IV_t(m, \tau)$ follows the process as below:

$$dIV_t = \mu(IV_t)dt + \sigma(IV_t)dW_t \tag{3.3}$$

The drift term in this paper is assumed to follow the mean-reverting process. Specifically, the drift term is defined as following:

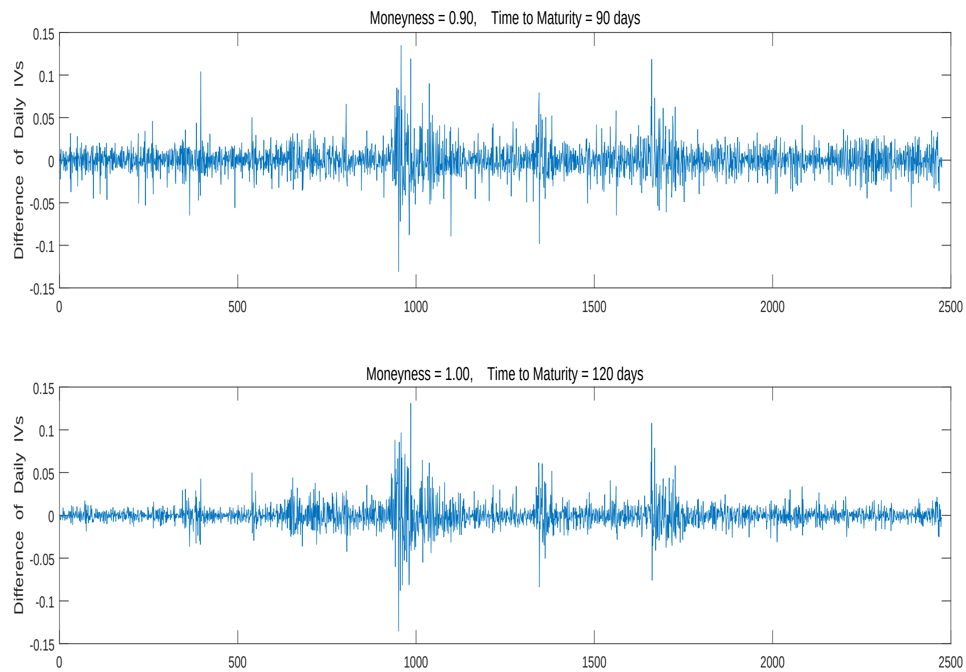
$$\mu(IV_t) = \alpha(\beta - IV_t) \tag{3.4}$$

Cont and da Fonseca (2002) is one of the pioneers analyzing the dynamics of implied volatility surface over time. In that paper, the authors construct implied volatility surface by using the end-of-day prices of European style call and put options on two major indices: the SP500 index and the FTSE 100 index. They find out that the time series of implied volatilities follows a mean-reverting pattern which implies some well-designed diffusion model may be taken into the consideration when tracking the dynamic movement. The data used in this paper also shows the mean-reverting pattern and illustrations are given in figure 3.3 for the sets of moneyness and time to maturity equal to $(0.9, 90)$ and $(1.0, 120)$. In equation (3.4), β is interpreted as the long-run level and α as the speed that pulls IV_t toward long run level. If IV_t does display mean reverting, then we should obtain positive estimates of α and β.

Next, the diffusion term is defined to follow the form of constant elasticity of variance (CEV) model, that is,

$$\sigma(IV_t) = b_1 IV_t^{b_2} \tag{3.5}$$

Figure 3.3: S&P 500 Daily Difference of Implied Volatilities (Jan 2005 - Oct 2014)



Many literature studying equity is based on the square-root stochastic volatility model. However, more and more recent studies find the evidence in favor of constant elasticity of variance (CEV) process due to its flexibility of modeling vol-of-vol. There are other strand of literature specifies volatility process as nonlinear diffusion with more complicated stochastic volatility structure. See Ignatieva et al. (2015) and Bakshi et al. (2006). Although these more complicated models sometimes outperform the others, they are criticized for lacking economic interpretation. In this paper, I adopt the mean-reverting drift with the constant elasticity of variance model to capture the change of implied volatility time series. That is,

$$dIV_t = \alpha(\beta - IV_t)dt + b_1 IV_t^{b_2} dW_t \tag{3.6}$$

where $b_1 > 0$. In this model, β is interpreted as the long-run level of implied volatility. α represents the speed that pulls implied volatility towards long-run level. If the data does exhibit mean-reverting property, the estimates of α and β should be non-negative. b_2 controls the relationship between the volatility of IV_{t+1} and level of IV_t. When b_2 is below 0, the volatility of IV_{t+1} increases as IV_t falls. Conversely, when b_2 is above 0, the volatility of IV_{t+1} increases as IV_t increases. b_1 is another parameter controls the volatility of IV_{t+1}.

To estimate the model, I set the interval $dt = 1$ which corresponds to one day. The Euler discretization version of the model becomes

$$IV_{t+1} - IV_t = \alpha(\beta - IV_t) + b_1 IV_t^{b_2} \varepsilon_t \tag{3.7}$$

where $\varepsilon_t \sim \mathrm{N}(0,1)$. For the estimation convenience, I rewrite the model as following:

$$IV_{t+1} - IV_t = a_1 + a_2 IV_t + b_1 IV_t^{b_2} \varepsilon_t \tag{3.8}$$

If the implied volatility time series displays mean-reverting property, I will get non-negative estimates of a_1 and non-positive estimates of a_2.

The Metropolis Hastings algorithm is used to estimate the model. The prior distribution of a_1 and a_2 are assumed to be $N(0,1)$. b_1 is a positive number. Instead of estimating b_1, I map it to the real line and estimate variable c, where $c = log(b_1)$. Both c and b_2 are assumed to have $N(0,3)$[2] prior distribution. Due to the data filter, there is small amount of missing values in the time series. Here data augmentation is applied to handle the issue. Let X_{mis} represents the missing data, X_{obs} represents the observations and $\theta = \{a_1, a_2, c, b_2\}$ represents the parameters. For each iteration in the Metropolis Hastings algorithm, I draw

$$X_{mis}^{(i)} \sim p(X_{mis}|\theta^{(i-1)}, X_{obs}) \tag{3.9}$$

$$\theta^{(i)} \sim p(\theta|X_{obs}, X_{mis}^{(i)}) \tag{3.10}$$

Drawing the missing values helps to complete the sample that is used to estimate the parameter. Ideally, a function including all observations should be developed as the conditional distribution to draw X_{mis}. Here the nonlinearity of the model with respect to IV_t adds complexity. To simplify the problem, I just draw each missing value at time $t+1$ by plugging the observation at time t into equation (3.8).

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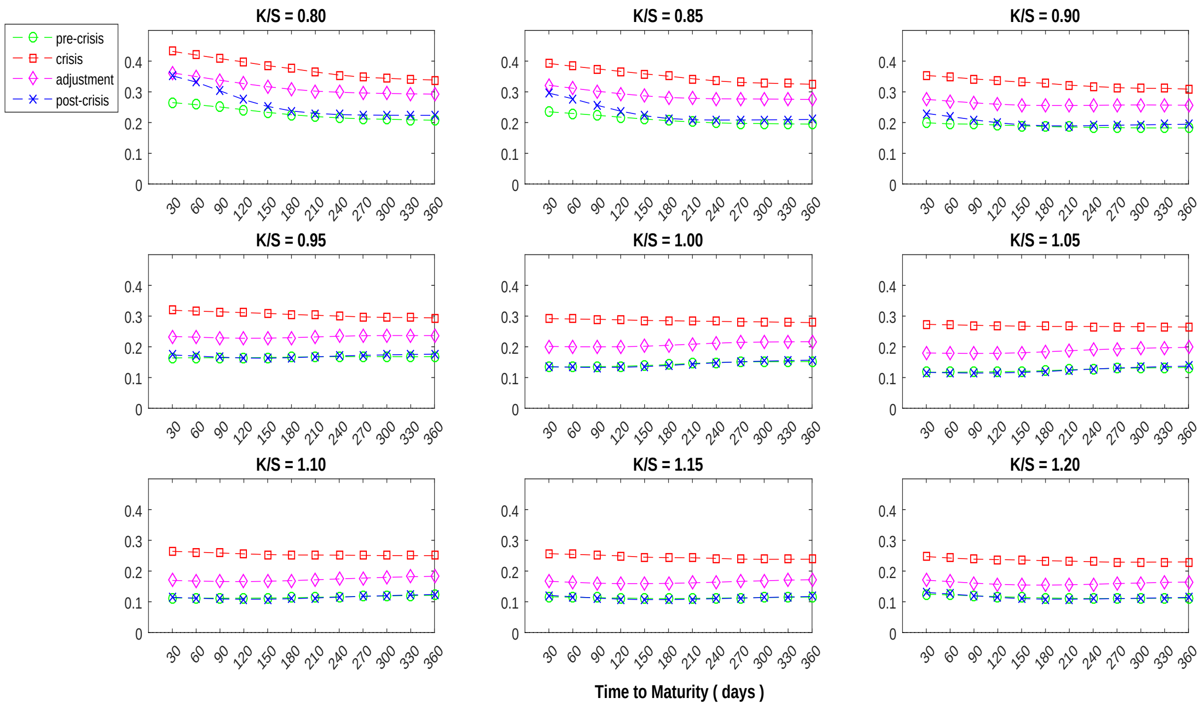
[2] 3 is standard deviation

Figure 3.4: Long-Run Mean (β) given Moneyness

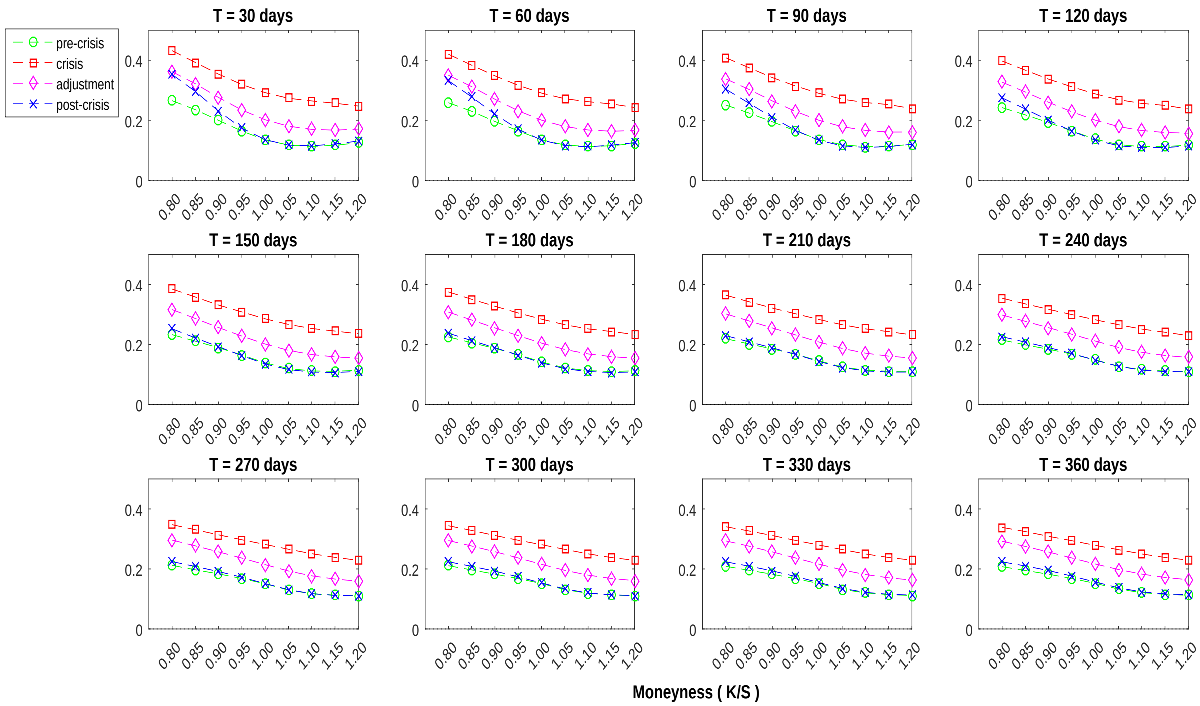


3.4 Results

This part analyze how the financial crisis affects the dynamics of implied volatility and investment behavior.

Figure 3.4 displays the long-run level of implied volatility for each sub-period across all the time to maturities given the moneyness. In general, the Crisis period has the highest long-run mean across all the time to maturities. Lines representing the Adjustment period lie in the middle while the lines of Pre-Crisis and Post-Crisis are in the bottom. This result is consistent with observations. During Crisis the stock market suffers big turmoil and the average market volatility is much bigger than it is in other periods. During the Adjustment period the market is in recovery process. We still can see the market's fluctuation during this period but not as big as it's during the Crisis period. The Pre-Crisis and Post-Crisis periods represent normal or prosperous economic condition. The result is consistent with the findings in other literature, that is, the volatility of equity market decreases as its price increases.

Figure 3.5: Long-Run Mean (β) given Time to Maturity



Several things worth mentioning. First lines are almost flat for all periods when the moneyness (K/S) falls in the range of $[0.95, 1.20]$. This means in those cases, the time to maturity does not affect implied volatility much for out-of-the-money options. Lines representing Pre-Crisis and Post-Crisis are almost overlap except when moneyness in $[0.80, 0.90]$. We observe skewness for moneyness $[0.80, 0.90]$ and lines representing the Post-Crisis period deviate from lines representing the Pre-Crisis as the time to maturity shrinks. Many reasons contribute to the skewness. Firstly, the market participants expect higher volatility in the short-run than in the long-run. In short-run, a news could cause big turmoil of the stock market while in the long-run different effects tend to cancel each other out. On the other hand, the high demand of protecting short-term asset value from bad news pushes short-term out-of-the-money put option price up which in turn increases its implied volatility. The deviation between the lines representing Pre-Crisis and Post-Crisis reveals the market adjustment, that is, market participants increase their estimation of the likelihood of crushes in the short-term and settle at a higher price for short-term out-of-the-money put option after experiencing crisis. Consequently the price increase leads to implied volatility increase. It is interesting to see that implied volatilities from out-of-the-money calls and at-the-money options are not affected much by time to maturity.

Figure 3.5 displays the long-run level of implied volatility for each sub-period across the moneynesses given time to maturities. Again, it shows that the short-term out-of-the-money put options have implied volatility much higher during Post-Crisis than during Pre-Crisis. As the time to maturity increases, the phenomenon disappears.

Figure 3.5 exhibits the well-known option smirk, that is, the implied volatility for option at low moneyness is higher then the implied volatility for option at high moneyness. For Pre-Crisis, Crisis and Adjustment periods, lines almost shift paralleled across all the moneyness. As mentioned previously, implied volatility is used as quoting convention of option price and it is affected by the demand-supply relationship. On the buy-side, majority of the market positions is long positions and the out-of-the-money put options are bought to protect long positions in case the market plunges. Since these options are out of the money, the prices are much cheaper than the at-the-money and in-the-money options. All those factors boost strong demand on the buy-side. On the sell-side, market participants who write and sell out-of-the-money options may suffer severe loss when crisis hits if they underestimate the probability of downside risk and sell the options too cheap. The strong demand and the high asking price in sell-side together raise the price of out-of-the-money option and lead to implied volatility smirk.

Figure 3.6 compares the speed that pulls implied volatility towards long-run level for each sub-period across all the time to maturities given the moneyness. More specifically, $|\beta - IV_t|$ measures the difference between current level of IV_t and the long-run mean. If IV_t lies below the long-run mean β, part of the difference is added to the next period so that the implied volatility is pulled back upwards long-run mean. If IV_t lies above the long-run mean β, part of the difference is subtracted from the next period so that the implied volatility is pulled back downward long-run mean. $IV_t + \alpha(\beta - IV_t)$ represents the expected value of IV_{t+1}. α controls how much of the difference is added to or subtracted from IV_t. Equivalently, it controls how fast the implied volatility is pulled

Figure 3.6: Speed (α) given Moneyness

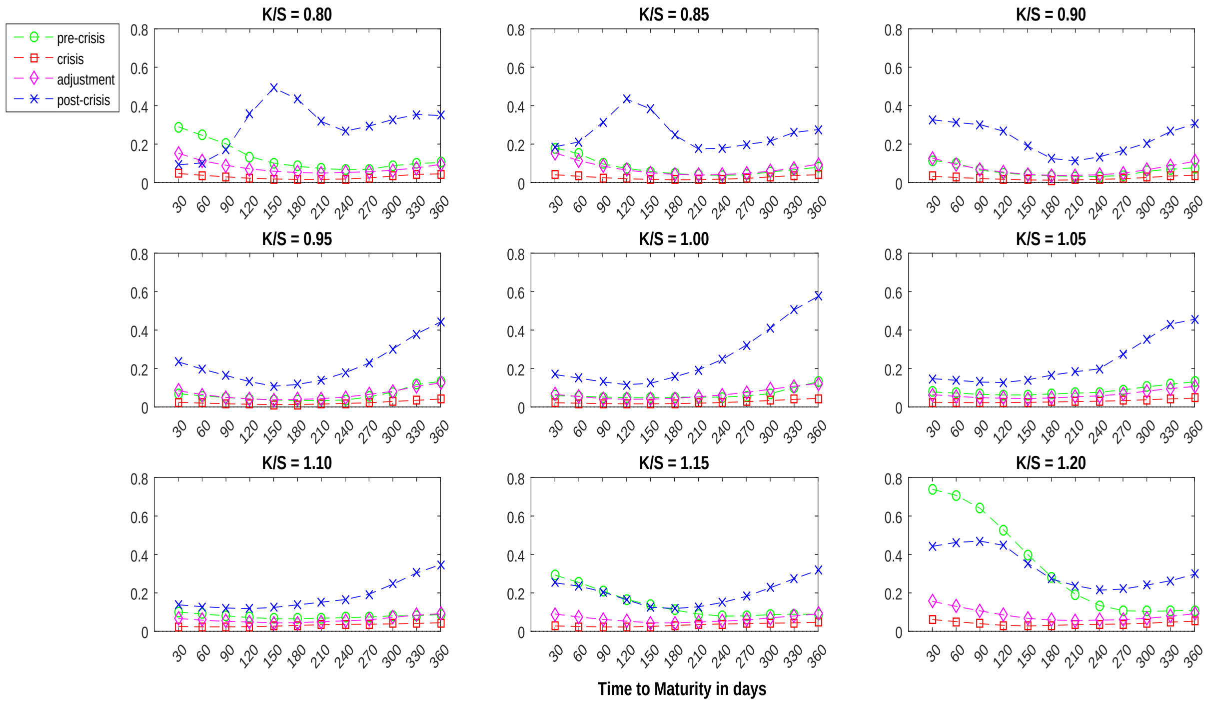
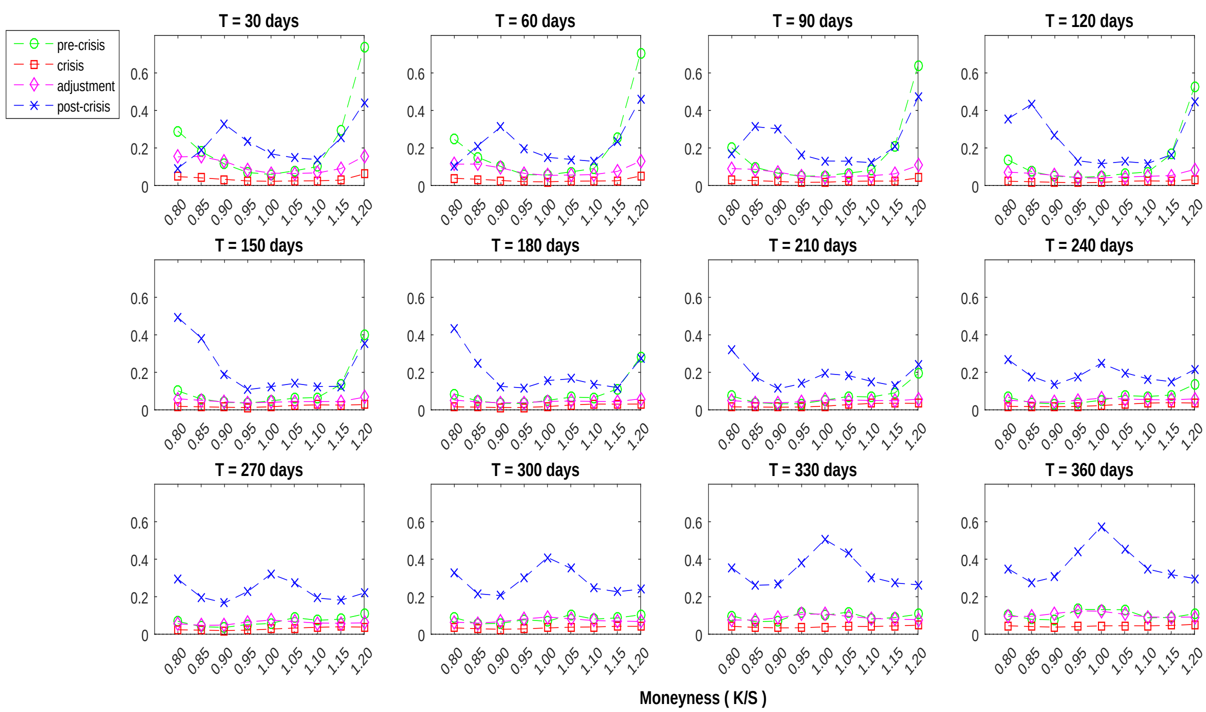


Figure 3.7: Speed (α) given Time to Maturity



towards long-run level.

In general, lines of Pre-Crisis, Crisis and Adjustment periods are quite close except when moneyness is at 0.80, 1.15 and 1.20. Lines of Post-Crisis lie on top. While it is easy to understand that during Crisis it takes longer for market to oscillate back to long term mean, the much greater pulling back powder in the Post-Crisis period than in Pre-Crisis period for most cases indicates the change of investment behavior caused by crisis. That is without take the error term $b_1 IV_t^{b_2}\varepsilon_t$ into the consideration, implied volatility moves back to long term level faster in Post-Crisis than in Pre-Crisis under most scenarios. Another phenomenon is that when the moneyness moves to deep-out-of-the-money scenarios, lines of Post-Crisis and Pre-Crisis cross each other. For time to maturity shorter than the point of intersection, Post-Crisis has α smaller than Pre-Crisis. For time to maturity longer than the point of intersection, Post-Crisis has α bigger than Pre-Crisis.

We can see the same intersections in Figure 3.7 that exhibits the speed that pulls implied volatility toward long-run level for each sub-period across all the time to maturities given the moneyness. For the time to maturity bigger than 120 days, lines from bottom to top represent Crisis, Adjustment, Pre-Crisis and Post-Crisis. For options with time to maturity smaller than 120 days, lines representing Pre-Crisis and Post-Crisis cross each other when the moneyness is far below and above 1. The big gap between the Pre-Crisis and Post-Crisis suggests the change of the speed that pulls the implied volatility towards the long-run level. Figure 3.6 and 3.7 shows for short-term deep-out-of-the-money options with the moneyness at 0.8 and 1.2, the speed in Pre-Crisis is higher than in Post-Crisis.

b_2 controls the relationship between the volatility of IV_{t+1} and level of IV_t. When b_2 is below 0, the volatility of IV_{t+1} increases as the level of IV_t falls. Conversely, when b_2 is above 0, the volatility of IV_{t+1} increases as the level of IV_t increases. The bigger the b_2, the more volatile the IV is in next period given the current level of IV.

Figure 3.8 displays b_2 for each sub-period across all the time to maturities given the moneyness. The big discrepancy between Pre-Crisis and Post-Crisis happens to out-of-the-money put option. And lines represent Crisis and Adjustment period lie in

Figure 3.8: (b_2) given Moneyness

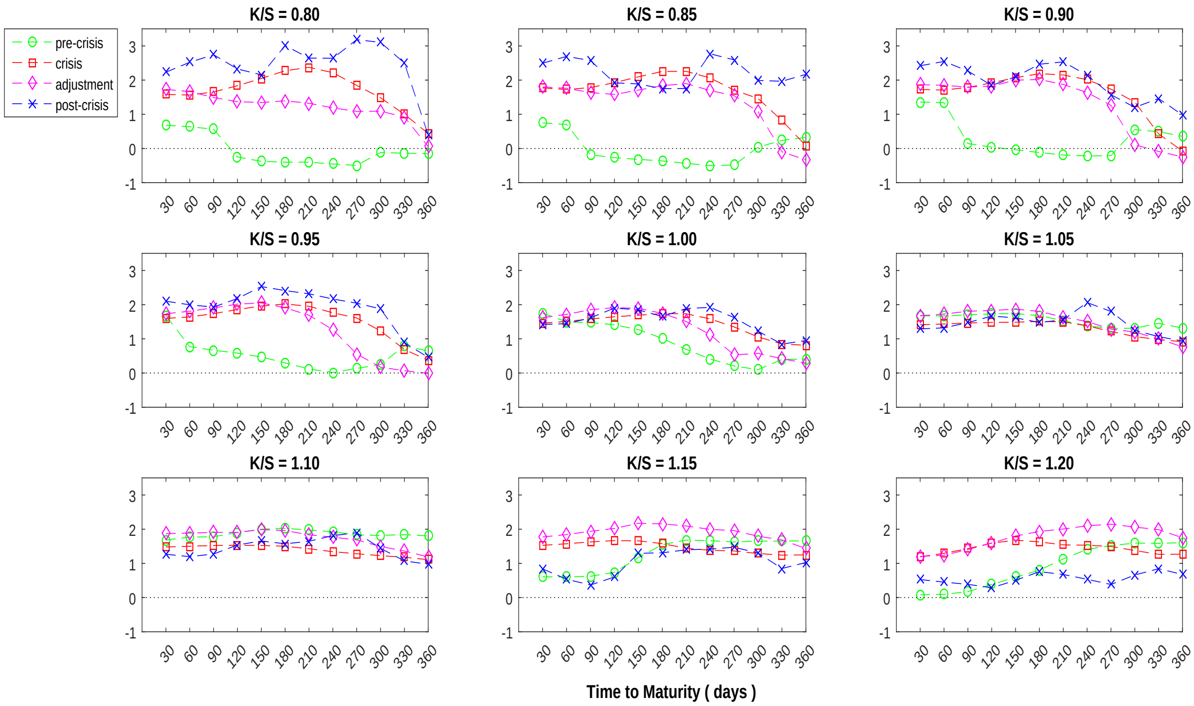
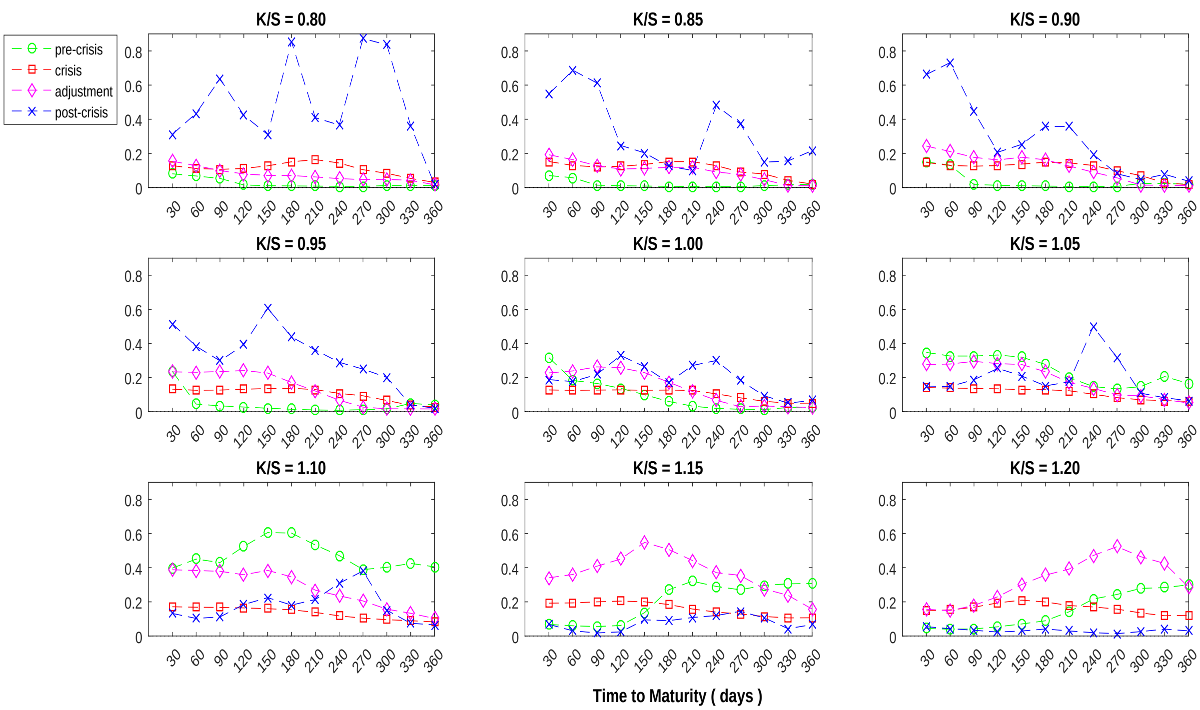


Figure 3.9: (b_1) given Moneyness



between. b_2 is bigger in Post-Crisis than in Pre-Crisis. As the moneyness decreases, four lines tend to get closer. And for deep-out-of-the-money call option, lines represent Post-Crisis moves downward and tend to sit below the Pre-Crisis.

b_1 is a scale parameter controlling the volatility of IV_{t+1}. Figure 3.9 compares b_1 for each sub-period across all the time to maturities given the moneyness. It shows the similar pattern as b_2. For out-of-the-money put options, b_1 of Post-Crisis lies above the lines of other periods. The line representing Pre-Crisis lies at the bottom. As the moneyness moves from 1 to above 1, lines represent Post-Crisis moves downward and tend to sit below the Pre-Crisis.

The combined effect of volatility parameters: b_1 and b_2 shows that in general the implied volatility of the out-of-the-money put options has bigger conditional vol-of-vol in Post-Crisis than in Pre-Crisis. For at-the-money option, conditional vol-of-vol in Post-Crisis is bigger than in Pre-Crisis when time to maturity is bigger than 60 days, and when time to maturity is smaller than 60 days, the relationship reverse. For out-of-the-money call option, when the moneyness is 1.05, conditional vol-of-vol in Post-Crisis is bigger than in Pre-Crisis when time to maturity is between 210 and 300 days. The relationship reverses when time to maturity is shorter than 210 days or longer than 300 days. When moneyness is 1.10, conditional vol-of-vol in Post-Crisis is smaller than in Pre-Crisis. When moneyness is 1.15 and 1.20, conditional vol-of-vol in Post-Crisis is bigger than in Pre-Crisis for mid to long term options.

Figure 3.10 compares b_2 across the moneynesses given the time to maturity. It shows the same results from a different perspective. The lines representing Pre-Crisis and Post-Crisis are not even close, rather they cross each other. In general, on the left side of the intersection point, lines of Post-Crisis lie above the line of Pre-Crisis and the difference between the two gets bigger when the moneyness decreases. On the right side of the intersection point, as the time to maturity gets bigger, lines of Post-Crisis turn to lie below the line of Pre-Crisis and the difference increases as the moneyness increase. Figure 3.11 compares b_1 across the moneynesses given the time to maturity. It shows the similar pattern as it is in Figure 3.10.

Figure 3.10: (b_2) given Time to Maturity

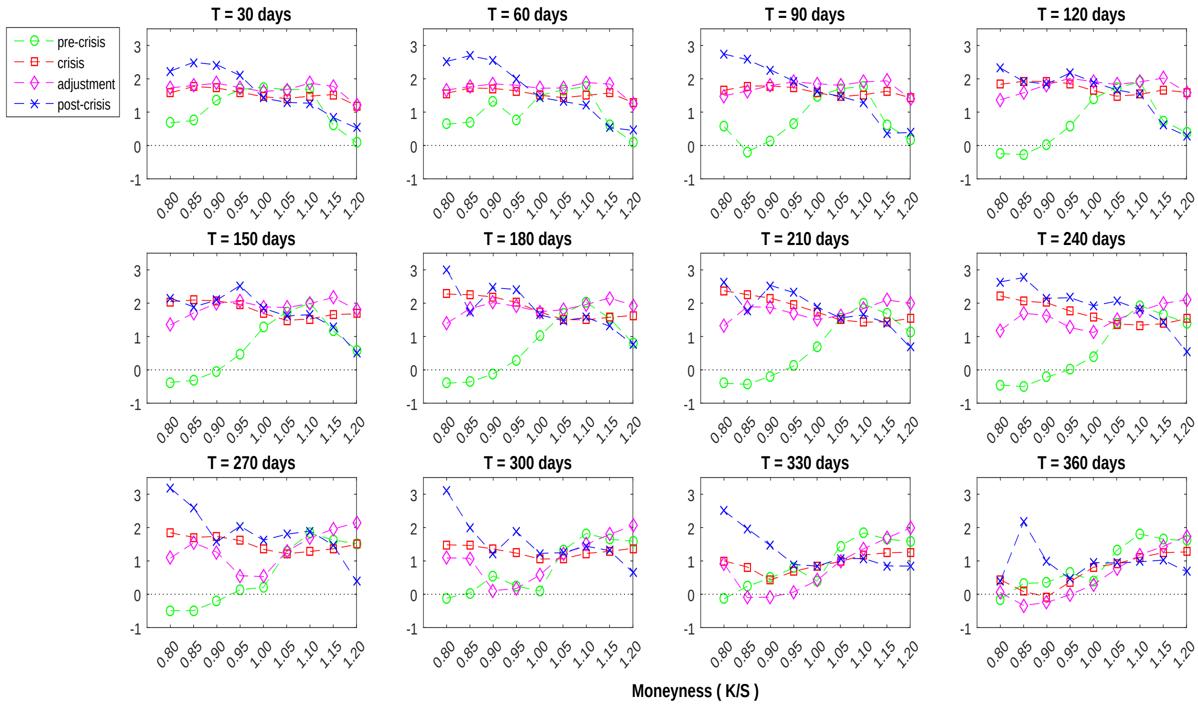
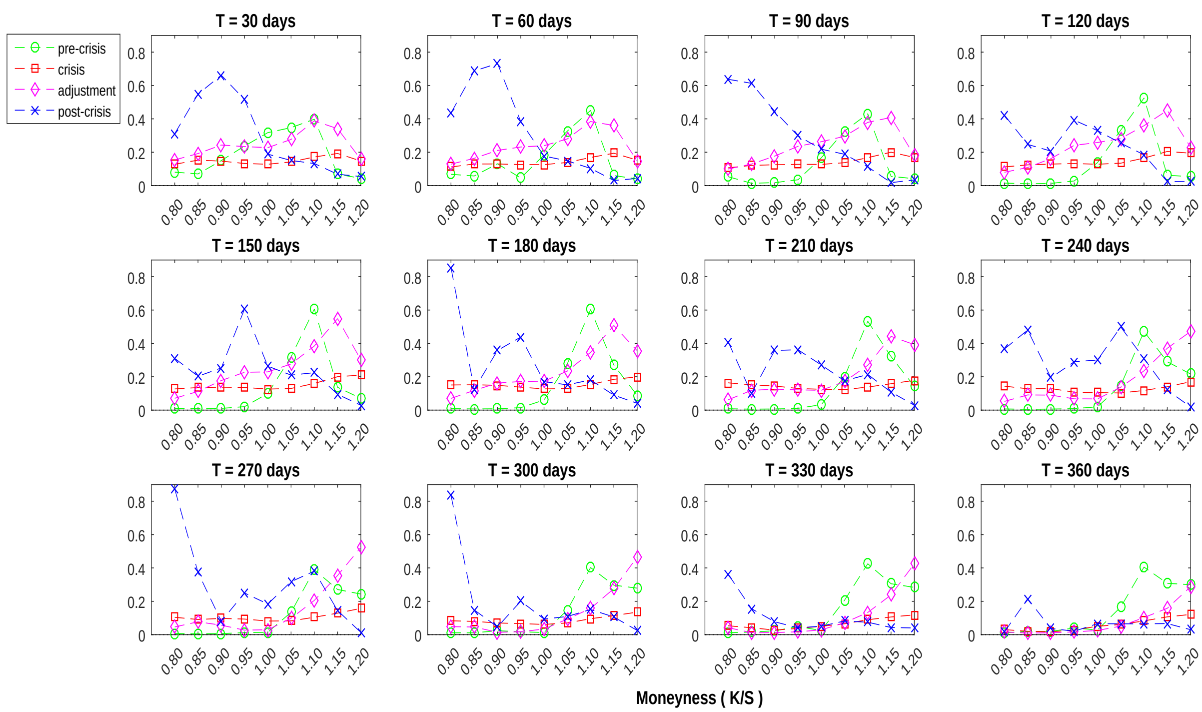


Figure 3.11: (b_1) given Time to Maturity



3.5 Conclusions

In this paper, I construct the implied volatility surface for S&P 500 index options with moneyness from 0.80 to 1.20 and time to maturity from 30 days to 360 days. A stochastic differential model with mean-reverting drift and constant elasticity of variance is used to analyze the time series of the implied volatility at the given moneyness and time to maturity. I find out that in most scenarios although the long-run level of implied volatility in Post-Crisis is close to it is in Pre-Crisis, the speed that pulls the implied volatility toward long-run level is much bigger in Post-Crisis.

Loosely speaking, the combined effect of volatility parameters: b_1 and b_2 shows the implied volatility of the out-of-the-money put options has bigger conditional vol-of-vol in Post-Crisis than in Pre-Crisis. For at-the-money option, conditional vol-of-vol in Post-Crisis is bigger than in Pre-Crisis when time to maturity is bigger than 60 days, and when time to maturity is smaller than 60 days, the relationship reverse. For out-of-the-money call option, when the moneyness is 1.05, conditional vol-of-vol in Post-Crisis is bigger than in Pre-Crisis when time to maturity is between 210 and 300 days. The relationship reverses when time to maturity is shorter than 210 days or longer than 300 days. When moneyness is 1.10, conditional vol-of-vol in Post-Crisis is smaller than in Pre-Crisis. When moneyness is 1.15 and 1.20, conditional vol-of-vol in Post-Crisis is bigger than in Pre-Crisis for mid to long term options.

It is worth noting that although the long-term mean of implied volatilities are close in these two periods, the change of speed and conditional vol-of-vol indicates that the effect from Crisis may last longer than expected if the catastrophe does not permanently change the investment behavior. It is unwise to expect the market movement or investment behavior to be similar in Pre-Crisis and Post-Crisis periods. Market participants learn from Crisis and behave differently before and after Crisis.

Table 3.3: Parameter Estimates - Pre-Crisis Perod

Moneyness	Time to Maturity	a_1	a_2	c	b_2	b_1	α	β
0.8	30	0.0768[3]	-0.2899	-2.5212	0.6766	0.0804	0.2899	0.2650
		(0.0005)[4]	(0.0021)	(0.0051)	(0.0038)			
0.8	60	0.0634	-0.2454	-2.6969	0.6366	0.0674	0.2454	0.2582
		(0.0005)	(0.0019)	(0.0057)	(0.004)			
0.8	90	0.0505	-0.2013	-2.9340	0.5625	0.0532	0.2013	0.2506
		(0.0004)	(0.0018)	(0.0076)	(0.0053)			
0.8	120	0.0328	-0.1354	-4.2083	-0.2485	0.0149	0.1354	0.2419
		(0.0004)	(0.0017)	(0.0074)	(0.0051)			
0.8	150	0.0235	-0.1010	-4.5430	-0.3761	0.0106	0.1010	0.2331
		(0.0004)	(0.0018)	(0.0086)	(0.0057)			
0.8	180	0.0191	-0.0845	-4.6964	-0.3998	0.0091	0.0845	0.2256
		(0.0003)	(0.0013)	(0.006)	(0.004)			
0.8	210	0.0161	-0.0735	-4.8026	-0.4002	0.0082	0.0735	0.2196
		(0.0003)	(0.0012)	(0.0067)	(0.0042)			
0.8	240	0.0143	-0.0666	-4.9639	-0.4553	0.0070	0.0666	0.2152
		(0.0003)	(0.0011)	(0.0064)	(0.0041)			
0.8	270	0.0145	-0.0686	-5.0474	-0.4938	0.0064	0.0686	0.2120
		(0.0003)	(0.0013)	(0.0073)	(0.0046)			
0.8	300	0.0186	-0.0882	-4.4617	-0.1166	0.0115	0.0882	0.2108
		(0.0003)	(0.0016)	(0.0145)	(0.0093)			
0.8	330	0.0205	-0.0983	-4.4744	-0.1385	0.0114	0.0983	0.2089
		(0.0004)	(0.0017)	(0.0156)	(0.0098)			
0.8	360	0.0217	-0.1045	-4.4896	-0.1542	0.0112	0.1045	0.2076
		(0.0004)	(0.0018)	(0.0139)	(0.0087)			
0.85	30	0.0424	-0.1806	-2.6618	0.7483	0.0698	0.1806	0.2350
		(0.0004)	(0.0016)	(0.0067)	(0.0046)			
0.85	60	0.0345	-0.1501	-2.8560	0.7011	0.0575	0.1501	0.2297
		(0.0004)	(0.0017)	(0.0087)	(0.0057)			
0.85	90	0.0217	-0.0969	-4.2854	-0.1845	0.0138	0.0969	0.2238
		(0.0003)	(0.0013)	(0.0065)	(0.0042)			

Continued on next page

[3] Mean of 100 estimates.

[4] Standard error of 100 estimates.

Table 3.3 – continued from previous page

Moneyness	Time to Maturity	a_1	a_2	c	b_2	b_1	α	β
0.85	120	0.0159	-0.0733	-4.5631	-0.2712	0.0104	0.0733	0.2176
		(0.0002)	(0.001)	(0.0056)	(0.0037)			
0.85	150	0.0122	-0.0575	-4.7711	-0.3192	0.0085	0.0575	0.2114
		(0.0002)	(0.001)	(0.006)	(0.0038)			
0.85	180	0.0096	-0.0466	-4.9698	-0.3645	0.0069	0.0466	0.2061
		(0.0002)	(0.0009)	(0.0056)	(0.0035)			
0.85	210	0.0079	-0.0392	-5.1951	-0.4358	0.0055	0.0392	0.2020
		(0.0002)	(0.0009)	(0.0059)	(0.0036)			
0.85	240	0.0074	-0.0374	-5.3562	-0.5014	0.0047	0.0374	0.1989
		(0.0002)	(0.0008)	(0.0058)	(0.0035)			
0.85	270	0.0079	-0.0403	-5.3541	-0.4875	0.0047	0.0403	0.1971
		(0.0002)	(0.001)	(0.0061)	(0.0036)			
0.85	300	0.0108	-0.0549	-4.5141	0.0370	0.0110	0.0549	0.1968
		(0.0002)	(0.0012)	(0.0124)	(0.0074)			
0.85	330	0.0130	-0.0666	-4.1550	0.2372	0.0157	0.0666	0.1956
		(0.0003)	(0.0015)	(0.0153)	(0.0092)			
0.85	360	0.0155	-0.0794	-3.9606	0.3221	0.0191	0.0794	0.1950
		(0.0003)	(0.0017)	(0.0148)	(0.009)			
0.9	30	0.0228	-0.1146	-1.8991	1.3574	0.1497	0.1146	0.1993
		(0.0003)	(0.0017)	(0.0086)	(0.0053)			
0.9	60	0.0197	-0.1007	-2.0351	1.3404	0.1307	0.1007	0.1957
		(0.0003)	(0.0016)	(0.0096)	(0.006)			
0.9	90	0.0126	-0.0649	-4.0448	0.1435	0.0175	0.0649	0.1949
		(0.0002)	(0.001)	(0.0062)	(0.0038)			
0.9	120	0.0096	-0.0500	-4.3407	0.0395	0.0130	0.0500	0.1921
		(0.0002)	(0.0008)	(0.0049)	(0.0028)			
0.9	150	0.0076	-0.0401	-4.5846	-0.0365	0.0102	0.0401	0.1895
		(0.0002)	(0.0008)	(0.0057)	(0.0033)			
0.9	180	0.0065	-0.0350	-4.8083	-0.1195	0.0082	0.0350	0.1872
		(0.0001)	(0.0007)	(0.0057)	(0.0032)			
0.9	210	0.0059	-0.0320	-4.9945	-0.1907	0.0068	0.0320	0.1856
		(0.0001)	(0.0008)	(0.0059)	(0.0034)			
0.9	240	0.0059	-0.0320	-5.0807	-0.2185	0.0062	0.0320	0.1845
		(0.0002)	(0.0008)	(0.0058)	(0.0034)			

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Table 3.3 – continued from previous page

Moneyness	Time to Maturity	a_1	a_2	c	b_2	b_1	α	β
0.9	270	0.0067	-0.0362	-5.0561	-0.2104	0.0064	0.0362	0.1836
		(0.0002)	(0.001)	(0.0066)	(0.0038)			
0.9	300	0.0107	-0.0583	-3.7376	0.5502	0.0238	0.0583	0.1826
		(0.0002)	(0.0013)	(0.0147)	(0.0085)			
0.9	330	0.0127	-0.0698	-3.7672	0.4906	0.0231	0.0698	0.1824
		(0.0002)	(0.0014)	(0.0142)	(0.0084)			
0.9	360	0.0140	-0.0765	-3.9428	0.3545	0.0194	0.0765	0.1824
		(0.0003)	(0.0016)	(0.0172)	(0.01)			
0.95	30	0.0113	-0.0688	-1.4397	1.6793	0.2370	0.0688	0.1637
		(0.0002)	(0.0013)	(0.0098)	(0.0054)			
0.95	60	0.0095	-0.0579	-3.0998	0.7705	0.0451	0.0579	0.1645
		(0.0002)	(0.001)	(0.0063)	(0.0034)			
0.95	90	0.0080	-0.0489	-3.3619	0.6670	0.0347	0.0489	0.1642
		(0.0001)	(0.0008)	(0.0065)	(0.0035)			
0.95	120	0.0070	-0.0424	-3.5894	0.5824	0.0276	0.0424	0.1643
		(0.0001)	(0.0009)	(0.005)	(0.0027)			
0.95	150	0.0059	-0.0359	-3.8785	0.4696	0.0207	0.0359	0.1652
		(0.0001)	(0.0008)	(0.0059)	(0.0032)			
0.95	180	0.0056	-0.0339	-4.2266	0.2963	0.0146	0.0339	0.1664
		(0.0001)	(0.0007)	(0.0064)	(0.0035)			
0.95	210	0.0055	-0.0329	-4.5630	0.1182	0.0104	0.0329	0.1676
		(0.0001)	(0.0008)	(0.0065)	(0.0036)			
0.95	240	0.0060	-0.0360	-4.7266	0.0146	0.0089	0.0360	0.1682
		(0.0001)	(0.0007)	(0.007)	(0.0038)			
0.95	270	0.0087	-0.0519	-4.4119	0.1343	0.0121	0.0519	0.1682
		(0.0002)	(0.0012)	(0.008)	(0.0044)			
0.95	300	0.0130	-0.0774	-4.0748	0.2453	0.0170	0.0774	0.1681
		(0.0002)	(0.0014)	(0.0084)	(0.0046)			
0.95	330	0.0196	-0.1168	-3.0186	0.7968	0.0489	0.1168	0.1676
		(0.0004)	(0.0022)	(0.0159)	(0.0088)			
0.95	360	0.0223	-0.1328	-3.2189	0.6439	0.0400	0.1328	0.1681
		(0.0004)	(0.0022)	(0.0173)	(0.0097)			
1	30	0.0079	-0.0587	-1.1480	1.7499	0.3173	0.0587	0.1343
		(0.0002)	(0.0015)	(0.0089)	(0.0045)			

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Table 3.3 – continued from previous page

Moneyness	Time to Maturity	a_1	a_2	c	b_2	b_1	α	β
1	60	0.0075	-0.0558	-1.6963	1.4980	0.1834	0.0558	0.1346
		(0.0002)	(0.0013)	(0.0083)	(0.0041)			
1	90	0.0070	-0.0514	-1.7963	1.4763	0.1659	0.0514	0.1353
		(0.0002)	(0.0014)	(0.0084)	(0.0041)			
1	120	0.0066	-0.0481	-1.9856	1.4115	0.1373	0.0481	0.1365
		(0.0002)	(0.0013)	(0.0087)	(0.0044)			
1	150	0.0066	-0.0476	-2.3055	1.2694	0.0997	0.0476	0.1388
		(0.0002)	(0.0013)	(0.0085)	(0.0043)			
1	180	0.0072	-0.0505	-2.8052	1.0137	0.0605	0.0505	0.1421
		(0.0002)	(0.0012)	(0.0096)	(0.0048)			
1	210	0.0074	-0.0510	-3.4173	0.6908	0.0328	0.0510	0.1460
		(0.0002)	(0.0011)	(0.0109)	(0.0055)			
1	240	0.0076	-0.0510	-3.9284	0.4101	0.0197	0.0510	0.1490
		(0.0001)	(0.001)	(0.0112)	(0.0057)			
1	270	0.0087	-0.0574	-4.2323	0.2061	0.0145	0.0574	0.1507
		(0.0002)	(0.0011)	(0.0124)	(0.0065)			
1	300	0.0104	-0.0688	-4.3406	0.0985	0.0130	0.0688	0.1515
		(0.0002)	(0.0011)	(0.0126)	(0.0065)			
1	330	0.0154	-0.1014	-3.6900	0.3957	0.0250	0.1014	0.1518
		(0.0003)	(0.0018)	(0.0181)	(0.0095)			
1	360	0.0199	-0.1310	-3.5655	0.4061	0.0283	0.1310	0.1521
		(0.0003)	(0.0023)	(0.0259)	(0.0136)			
1.05	30	0.0091	-0.0779	-1.0574	1.6729	0.3474	0.0779	0.1171
		(0.0002)	(0.0017)	(0.009)	(0.0041)			
1.05	60	0.0085	-0.0722	-1.1194	1.6693	0.3265	0.0722	0.1173
		(0.0002)	(0.0016)	(0.0078)	(0.0035)			
1.05	90	0.0077	-0.0656	-1.1196	1.6985	0.3264	0.0656	0.1179
		(0.0002)	(0.0016)	(0.0088)	(0.004)			
1.05	120	0.0074	-0.0624	-1.1014	1.7321	0.3324	0.0624	0.1187
		(0.0002)	(0.0017)	(0.0097)	(0.0045)			
1.05	150	0.0075	-0.0624	-1.1403	1.7369	0.3197	0.0624	0.1202
		(0.0002)	(0.0015)	(0.0088)	(0.0041)			
1.05	180	0.0082	-0.0674	-1.2829	1.6807	0.2772	0.0674	0.1223
		(0.0002)	(0.0015)	(0.01)	(0.0046)			

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Table 3.3 – continued from previous page

Moneyness	Time to Maturity	a_1	a_2	c	b_2	b_1	α	β
1.05	210	0.0089	-0.0715	-1.6092	1.5304	0.2001	0.0715	0.1251
		(0.0002)	(0.0019)	(0.0128)	(0.0061)			
1.05	240	0.0097	-0.0762	-1.9125	1.3810	0.1477	0.0762	0.1277
		(0.0002)	(0.0015)	(0.0142)	(0.0068)			
1.05	270	0.0115	-0.0886	-2.0099	1.3019	0.1340	0.0886	0.1298
		(0.0002)	(0.0015)	(0.0123)	(0.006)			
1.05	300	0.0137	-0.1047	-1.9158	1.3117	0.1472	0.1047	0.1312
		(0.0002)	(0.002)	(0.0151)	(0.0073)			
1.05	330	0.0155	-0.1178	-1.5788	1.4541	0.2062	0.1178	0.1320
		(0.0003)	(0.0022)	(0.0132)	(0.0065)			
1.05	360	0.0174	-0.1308	-1.7988	1.3151	0.1655	0.1308	0.1330
		(0.0003)	(0.0024)	(0.0175)	(0.0086)			
1.1	30	0.0113	-0.1004	-0.9220	1.6855	0.3977	0.1004	0.1125
		(0.0002)	(0.0019)	(0.008)	(0.0036)			
1.1	60	0.0101	-0.0904	-0.7976	1.7680	0.4504	0.0904	0.1120
		(0.0002)	(0.0018)	(0.0085)	(0.0038)			
1.1	90	0.0090	-0.0805	-0.8430	1.7801	0.4304	0.0805	0.1115
		(0.0002)	(0.0016)	(0.0086)	(0.0039)			
1.1	120	0.0080	-0.0717	-0.6449	1.9021	0.5248	0.0717	0.1116
		(0.0002)	(0.0016)	(0.0097)	(0.0044)			
1.1	150	0.0073	-0.0655	-0.5007	2.0022	0.6061	0.0655	0.1121
		(0.0002)	(0.0016)	(0.0089)	(0.004)			
1.1	180	0.0074	-0.0651	-0.5061	2.0243	0.6029	0.0651	0.1130
		(0.0002)	(0.0019)	(0.011)	(0.005)			
1.1	210	0.0079	-0.0686	-0.6295	1.9759	0.5329	0.0686	0.1145
		(0.0002)	(0.0019)	(0.0094)	(0.0043)			
1.1	240	0.0083	-0.0711	-0.7524	1.9242	0.4713	0.0711	0.1160
		(0.0002)	(0.0016)	(0.0108)	(0.005)			
1.1	270	0.0087	-0.0743	-0.9436	1.8314	0.3892	0.0743	0.1175
		(0.0002)	(0.0018)	(0.0106)	(0.0049)			
1.1	300	0.0096	-0.0806	-0.9105	1.8195	0.4024	0.0806	0.1190
		(0.0002)	(0.0019)	(0.0112)	(0.0052)			
1.1	330	0.0098	-0.0813	-0.8553	1.8398	0.4252	0.0813	0.1203
		(0.0002)	(0.0017)	(0.0102)	(0.0048)			

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Table 3.3 – continued from previous page

Moneyness	Time to Maturity	a_1	a_2	c	b_2	b_1	α	β
1.1	360	0.0106	-0.0877	-0.9035	1.8045	0.4052	0.0877	0.1211
		(0.0002)	(0.002)	(0.0108)	(0.0051)			
1.15	30	0.0341	-0.2928	-2.6944	0.6073	0.0676	0.2928	0.1163
		(0.0002)	(0.0019)	(0.0065)	(0.003)			
1.15	60	0.0292	-0.2544	-2.8165	0.6030	0.0598	0.2544	0.1147
		(0.0002)	(0.002)	(0.0067)	(0.003)			
1.15	90	0.0238	-0.2099	-2.8983	0.6230	0.0551	0.2099	0.1132
		(0.0002)	(0.0018)	(0.008)	(0.0037)			
1.15	120	0.0186	-0.1664	-2.7877	0.7371	0.0616	0.1664	0.1120
		(0.0002)	(0.0019)	(0.0069)	(0.0031)			
1.15	150	0.0151	-0.1358	-1.9897	1.1714	0.1367	0.1358	0.1109
		(0.0002)	(0.0019)	(0.0106)	(0.0047)			
1.15	180	0.0123	-0.1109	-1.3085	1.5474	0.2702	0.1109	0.1105
		(0.0002)	(0.002)	(0.0122)	(0.0055)			
1.15	210	0.0100	-0.0902	-1.1367	1.6807	0.3209	0.0902	0.1109
		(0.0002)	(0.002)	(0.0105)	(0.0048)			
1.15	240	0.0088	-0.0791	-1.2374	1.6621	0.2902	0.0791	0.1118
		(0.0002)	(0.0018)	(0.0115)	(0.0052)			
1.15	270	0.0091	-0.0804	-1.3068	1.6270	0.2707	0.0804	0.1127
		(0.0002)	(0.002)	(0.0122)	(0.0055)			
1.15	300	0.0099	-0.0868	-1.2151	1.6499	0.2967	0.0868	0.1136
		(0.0002)	(0.0017)	(0.0103)	(0.0047)			
1.15	330	0.0102	-0.0887	-1.1865	1.6529	0.3053	0.0887	0.1146
		(0.0002)	(0.0019)	(0.01)	(0.0046)			
1.15	360	0.0103	-0.0889	-1.1779	1.6580	0.3079	0.0889	0.1155
		(0.0002)	(0.0018)	(0.0087)	(0.004)			
1.2	30	0.0922	-0.7389	-3.1223	0.0829	0.0441	0.7389	0.1248
		(0.0003)	(0.0019)	(0.0078)	(0.0037)			
1.2	60	0.0865	-0.7074	-3.2173	0.0941	0.0401	0.7074	0.1222
		(0.0003)	(0.0023)	(0.0074)	(0.0033)			
1.2	90	0.0765	-0.6402	-3.1910	0.1732	0.0411	0.6402	0.1195
		(0.0003)	(0.0025)	(0.0072)	(0.0033)			
1.2	120	0.0615	-0.5267	-2.9204	0.3815	0.0539	0.5267	0.1167

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Table 3.3 – continued from previous page

Moneyness	Time to Maturity	a_1	a_2	c	b_2	b_1	α	β
		(0.0003)	(0.0022)	(0.0066)	(0.003)			
1.2	150	0.0457	-0.3997	-2.6531	0.5969	0.0704	0.3997	0.1143
		(0.0003)	(0.0026)	(0.0086)	(0.0039)			
1.2	180	0.0318	-0.2831	-2.4164	0.8028	0.0893	0.2831	0.1124
		(0.0002)	(0.0019)	(0.0092)	(0.0042)			
1.2	210	0.0214	-0.1929	-1.9496	1.1291	0.1423	0.1929	0.1110
		(0.0003)	(0.0024)	(0.0119)	(0.0054)			
1.2	240	0.0150	-0.1355	-1.5308	1.4102	0.2164	0.1355	0.1105
		(0.0002)	(0.0023)	(0.0127)	(0.0057)			
1.2	270	0.0119	-0.1075	-1.4133	1.5203	0.2434	0.1075	0.1105
		(0.0002)	(0.0019)	(0.012)	(0.0054)			
1.2	300	0.0115	-0.1034	-1.2768	1.5937	0.2789	0.1034	0.1109
		(0.0002)	(0.002)	(0.0119)	(0.0054)			
1.2	330	0.0119	-0.1065	-1.2606	1.5866	0.2835	0.1065	0.1116
		(0.0002)	(0.0019)	(0.0112)	(0.005)			
1.2	360	0.0123	-0.1092	-1.2049	1.6042	0.2997	0.1092	0.1122
		(0.0002)	(0.002)	(0.0105)	(0.0048)			

Table 3.4: Parameter Estimates - Crisis Period

Moneyness	Time to Maturity	a_1	a_2	c	b_2	b_1	α	β
0.8	30	0.0203	-0.0470	-2.0564	1.5782	0.1279	0.0470	0.4318
		(0.0003)	(0.001)	(0.0044)	(0.0046)			
0.8	60	0.0159	-0.0379	-2.1928	1.5644	0.1116	0.0379	0.4199
		(0.0004)	(0.0011)	(0.0043)	(0.0046)			
0.8	90	0.0120	-0.0294	-2.2422	1.6536	0.1062	0.0294	0.4082
		(0.0003)	(0.0009)	(0.0037)	(0.0039)			
0.8	120	0.0089	-0.0223	-2.1915	1.8350	0.1118	0.0223	0.3972
		(0.0003)	(0.0009)	(0.0042)	(0.0042)			
0.8	150	0.0071	-0.0185	-2.0641	2.0483	0.1269	0.0185	0.3852
		(0.0003)	(0.0009)	(0.0039)	(0.0037)			
0.8	180	0.0060	-0.0160	-1.8931	2.2732	0.1506	0.0160	0.3756
		(0.0003)	(0.001)	(0.0045)	(0.0045)			

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Table 3.4 – continued from previous page

Moneyness	Time to Maturity	a_1	a_2	c	b_2	b_1	α	β
0.8	210	0.0058	-0.0159	-1.8111	2.3741	0.1635	0.0159	0.3647
		(0.0003)	(0.001)	(0.0045)	(0.0043)			
0.8	240	0.0066	-0.0185	-1.9410	2.2164	0.1436	0.0185	0.3547
		(0.0004)	(0.0012)	(0.0041)	(0.004)			
0.8	270	0.0083	-0.0238	-2.2614	1.8460	0.1042	0.0238	0.3476
		(0.0003)	(0.001)	(0.0061)	(0.0056)			
0.8	300	0.0117	-0.0339	-2.4876	1.4836	0.0831	0.0339	0.3440
		(0.0004)	(0.0011)	(0.0058)	(0.0052)			
0.8	330	0.0144	-0.0422	-2.9360	1.0001	0.0531	0.0422	0.3408
		(0.0003)	(0.001)	(0.006)	(0.0055)			
0.8	360	0.0153	-0.0451	-3.5021	0.4373	0.0301	0.0451	0.3386
		(0.0004)	(0.0011)	(0.0049)	(0.0044)			
0.85	30	0.0159	-0.0404	-1.8816	1.7708	0.1524	0.0404	0.3924
		(0.0004)	(0.0011)	(0.0043)	(0.0042)			
0.85	60	0.0126	-0.0329	-2.0453	1.7230	0.1293	0.0329	0.3833
		(0.0003)	(0.001)	(0.0039)	(0.004)			
0.85	90	0.0096	-0.0255	-2.1130	1.7820	0.1209	0.0255	0.3749
		(0.0003)	(0.0008)	(0.0046)	(0.0045)			
0.85	120	0.0072	-0.0197	-2.0878	1.9252	0.1240	0.0197	0.3666
		(0.0003)	(0.001)	(0.0045)	(0.0043)			
0.85	150	0.0057	-0.0160	-2.0013	2.0947	0.1352	0.0160	0.3585
		(0.0003)	(0.0008)	(0.0043)	(0.0038)			
0.85	180	0.0050	-0.0143	-1.8837	2.2541	0.1520	0.0143	0.3510
		(0.0003)	(0.001)	(0.0047)	(0.0042)			
0.85	210	0.0051	-0.0148	-1.8861	2.2562	0.1517	0.0148	0.3427
		(0.0003)	(0.0011)	(0.0047)	(0.0042)			
0.85	240	0.0059	-0.0176	-2.0542	2.0633	0.1282	0.0176	0.3355
		(0.0003)	(0.001)	(0.0049)	(0.0043)			
0.85	270	0.0071	-0.0215	-2.3994	1.7066	0.0908	0.0215	0.3307
		(0.0003)	(0.0009)	(0.0053)	(0.0047)			
0.85	300	0.0096	-0.0292	-2.5327	1.4642	0.0794	0.0292	0.3285
		(0.0003)	(0.0011)	(0.0056)	(0.0049)			
0.85	330	0.0118	-0.0362	-3.1782	0.8174	0.0417	0.0362	0.3266
		(0.0003)	(0.001)	(0.0049)	(0.0043)			

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Table 3.4 – continued from previous page

Moneyness	Time to Maturity	a_1	a_2	c	b_2	b_1	α	β
0.85	360	0.0132	-0.0408	-3.8971	0.0810	0.0203	0.0408	0.3246
		(0.0003)	(0.0009)	(0.0041)	(0.0034)			
0.9	30	0.0114	-0.0323	-1.9199	1.7237	0.1466	0.0323	0.3533
		(0.0003)	(0.0009)	(0.0044)	(0.0041)			
0.9	60	0.0092	-0.0264	-2.0441	1.7097	0.1295	0.0264	0.3478
		(0.0003)	(0.001)	(0.0042)	(0.0038)			
0.9	90	0.0072	-0.0210	-2.0810	1.7850	0.1248	0.0210	0.3420
		(0.0002)	(0.0008)	(0.0036)	(0.0032)			
0.9	120	0.0056	-0.0166	-2.0490	1.9221	0.1289	0.0166	0.3367
		(0.0003)	(0.001)	(0.005)	(0.0044)			
0.9	150	0.0045	-0.0136	-1.9859	2.0701	0.1373	0.0136	0.3316
		(0.0003)	(0.001)	(0.0038)	(0.0032)			
0.9	180	0.0040	-0.0123	-1.9070	2.1880	0.1485	0.0123	0.3277
		(0.0003)	(0.001)	(0.0046)	(0.004)			
0.9	210	0.0045	-0.0138	-1.9351	2.1506	0.1444	0.0138	0.3218
		(0.0002)	(0.0009)	(0.005)	(0.0043)			
0.9	240	0.0052	-0.0165	-2.0466	2.0130	0.1292	0.0165	0.3174
		(0.0003)	(0.001)	(0.006)	(0.0049)			
0.9	270	0.0062	-0.0197	-2.3272	1.7354	0.0976	0.0197	0.3137
		(0.0003)	(0.0009)	(0.0056)	(0.0046)			
0.9	300	0.0082	-0.0262	-2.6583	1.3578	0.0701	0.0262	0.3115
		(0.0003)	(0.0009)	(0.0049)	(0.0042)			
0.9	330	0.0105	-0.0338	-3.6036	0.4267	0.0272	0.0338	0.3115
		(0.0002)	(0.0008)	(0.0047)	(0.0038)			
0.9	360	0.0117	-0.0379	-4.0849	-0.0761	0.0168	0.0379	0.3088
		(0.0002)	(0.0008)	(0.0039)	(0.0031)			
0.95	30	0.0077	-0.0240	-2.0218	1.5977	0.1324	0.0240	0.3189
		(0.0002)	(0.0009)	(0.0039)	(0.0033)			
0.95	60	0.0063	-0.0200	-2.0724	1.6403	0.1259	0.0200	0.3161
		(0.0002)	(0.0008)	(0.004)	(0.0033)			
0.95	90	0.0052	-0.0164	-2.0631	1.7378	0.1271	0.0164	0.3139
		(0.0002)	(0.0008)	(0.0042)	(0.0034)			
0.95	120	0.0043	-0.0139	-2.0259	1.8568	0.1319	0.0139	0.3111

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Table 3.4 – continued from previous page

Moneyness	Time to Maturity	a_1	a_2	c	b_2	b_1	α	β
		(0.0002)	(0.0008)	(0.0039)	(0.0033)			
0.95	150	0.0038	-0.0123	-1.9955	1.9608	0.1359	0.0123	0.3088
		(0.0002)	(0.0007)	(0.0045)	(0.0037)			
0.95	180	0.0038	-0.0126	-1.9834	2.0180	0.1376	0.0126	0.3052
		(0.0002)	(0.0007)	(0.0048)	(0.0039)			
0.95	210	0.0045	-0.0148	-2.0431	1.9533	0.1296	0.0148	0.3026
		(0.0002)	(0.0009)	(0.0051)	(0.0041)			
0.95	240	0.0054	-0.0182	-2.2298	1.7614	0.1076	0.0182	0.2999
		(0.0002)	(0.0009)	(0.0051)	(0.0042)			
0.95	270	0.0064	-0.0217	-2.3705	1.6099	0.0934	0.0217	0.2973
		(0.0002)	(0.0006)	(0.0054)	(0.0043)			
0.95	300	0.0085	-0.0286	-2.7153	1.2331	0.0662	0.0286	0.2957
		(0.0002)	(0.0008)	(0.0061)	(0.0048)			
0.95	330	0.0102	-0.0345	-3.2816	0.6883	0.0376	0.0345	0.2952
		(0.0002)	(0.0008)	(0.0056)	(0.0045)			
0.95	360	0.0120	-0.0408	-3.5660	0.3735	0.0283	0.0408	0.2942
		(0.0002)	(0.0008)	(0.005)	(0.0039)			
1	30	0.0064	-0.0219	-2.0577	1.4595	0.1278	0.0219	0.2918
		(0.0002)	(0.0007)	(0.004)	(0.003)			
1	60	0.0056	-0.0193	-2.0753	1.5081	0.1255	0.0193	0.2904
		(0.0002)	(0.0007)	(0.004)	(0.003)			
1	90	0.0050	-0.0172	-2.0639	1.5803	0.1270	0.0172	0.2896
		(0.0002)	(0.0007)	(0.0033)	(0.0026)			
1	120	0.0047	-0.0163	-2.0561	1.6455	0.1280	0.0163	0.2877
		(0.0001)	(0.0006)	(0.0047)	(0.0034)			
1	150	0.0046	-0.0162	-2.0716	1.6909	0.1260	0.0162	0.2858
		(0.0002)	(0.0008)	(0.0043)	(0.0032)			
1	180	0.0049	-0.0171	-2.0625	1.7390	0.1271	0.0171	0.2848
		(0.0002)	(0.0008)	(0.0047)	(0.0036)			
1	210	0.0055	-0.0195	-2.0809	1.7328	0.1248	0.0195	0.2837
		(0.0002)	(0.0009)	(0.0038)	(0.0031)			
1	240	0.0065	-0.0229	-2.2549	1.5825	0.1049	0.0229	0.2827
		(0.0002)	(0.0009)	(0.0053)	(0.0039)			
1	270	0.0077	-0.0275	-2.5057	1.3478	0.0816	0.0275	0.2817

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Table 3.4 – continued from previous page

Moneyness	Time to Maturity	a_1	a_2	c	b_2	b_1	α	β
		(0.0002)	(0.0008)	(0.006)	(0.0045)			
1	300	0.0094	-0.0334	-2.7915	1.0575	0.0613	0.0334	0.2810
		(0.0002)	(0.0008)	(0.0063)	(0.0048)			
1	330	0.0109	-0.0391	-2.9909	0.8497	0.0502	0.0391	0.2799
		(0.0002)	(0.0009)	(0.0069)	(0.0053)			
1	360	0.0125	-0.0447	-2.9873	0.8103	0.0504	0.0447	0.2788
		(0.0003)	(0.001)	(0.0064)	(0.0049)			
1.05	30	0.0066	-0.0241	-1.9546	1.4101	0.1416	0.0241	0.2730
		(0.0002)	(0.0007)	(0.004)	(0.0029)			
1.05	60	0.0061	-0.0226	-1.9765	1.4347	0.1386	0.0226	0.2715
		(0.0001)	(0.0007)	(0.0043)	(0.0029)			
1.05	90	0.0059	-0.0219	-1.9869	1.4633	0.1371	0.0219	0.2700
		(0.0001)	(0.0006)	(0.0041)	(0.0029)			
1.05	120	0.0060	-0.0223	-2.0082	1.4792	0.1342	0.0223	0.2684
		(0.0002)	(0.0007)	(0.0039)	(0.0028)			
1.05	150	0.0063	-0.0236	-2.0413	1.4865	0.1299	0.0236	0.2672
		(0.0002)	(0.0008)	(0.0038)	(0.0028)			
1.05	180	0.0069	-0.0258	-2.0470	1.5095	0.1291	0.0258	0.2661
		(0.0002)	(0.0008)	(0.0041)	(0.0032)			
1.05	210	0.0074	-0.0277	-2.0966	1.4981	0.1229	0.0277	0.2662
		(0.0002)	(0.0009)	(0.0049)	(0.0036)			
1.05	240	0.0079	-0.0298	-2.2764	1.3802	0.1027	0.0298	0.2661
		(0.0002)	(0.0008)	(0.005)	(0.0036)			
1.05	270	0.0087	-0.0327	-2.4923	1.2170	0.0827	0.0327	0.2658
		(0.0002)	(0.0008)	(0.0053)	(0.0039)			
1.05	300	0.0099	-0.0374	-2.6558	1.0640	0.0702	0.0374	0.2654
		(0.0002)	(0.0008)	(0.0061)	(0.0045)			
1.05	330	0.0111	-0.0419	-2.7305	0.9803	0.0652	0.0419	0.2648
		(0.0002)	(0.0009)	(0.0065)	(0.0047)			
1.05	360	0.0120	-0.0454	-2.7957	0.9141	0.0611	0.0454	0.2643
		(0.0002)	(0.0009)	(0.0066)	(0.0048)			
1.1	30	0.0065	-0.0248	-1.7644	1.4798	0.1713	0.0248	0.2642
		(0.0002)	(0.0008)	(0.0038)	(0.0028)			
1.1	60	0.0061	-0.0234	-1.7808	1.5015	0.1685	0.0234	0.2617

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Table 3.4 – continued from previous page

Moneyness	Time to Maturity	a_1	a_2	c	b_2	b_1	α	β
		(0.0001)	(0.0007)	(0.0046)	(0.0032)			
1.1	90	0.0060	-0.0232	-1.7875	1.5225	0.1674	0.0232	0.2587
		(0.0001)	(0.0007)	(0.0041)	(0.0028)			
1.1	120	0.0062	-0.0243	-1.8050	1.5295	0.1645	0.0243	0.2559
		(0.0002)	(0.0008)	(0.004)	(0.0027)			
1.1	150	0.0067	-0.0266	-1.8337	1.5251	0.1598	0.0266	0.2536
		(0.0002)	(0.0008)	(0.004)	(0.0027)			
1.1	180	0.0078	-0.0307	-1.8733	1.5002	0.1536	0.0307	0.2522
		(0.0002)	(0.0009)	(0.0052)	(0.0034)			
1.1	210	0.0087	-0.0343	-1.9789	1.4284	0.1382	0.0343	0.2523
		(0.0002)	(0.001)	(0.005)	(0.0033)			
1.1	240	0.0091	-0.0361	-2.1387	1.3417	0.1178	0.0361	0.2517
		(0.0002)	(0.0008)	(0.0048)	(0.0033)			
1.1	270	0.0093	-0.0369	-2.2603	1.2822	0.1043	0.0369	0.2509
		(0.0002)	(0.001)	(0.0055)	(0.0039)			
1.1	300	0.0097	-0.0387	-2.3479	1.2256	0.0956	0.0387	0.2504
		(0.0002)	(0.001)	(0.0055)	(0.004)			
1.1	330	0.0103	-0.0410	-2.4132	1.1788	0.0895	0.0410	0.2501
		(0.0002)	(0.0009)	(0.0066)	(0.0046)			
1.1	360	0.0112	-0.0448	-2.4932	1.1080	0.0826	0.0448	0.2500
		(0.0002)	(0.0011)	(0.0065)	(0.0045)			
1.15	30	0.0072	-0.0281	-1.6480	1.5273	0.1924	0.0281	0.2565
		(0.0002)	(0.0009)	(0.0037)	(0.0026)			
1.15	60	0.0063	-0.0249	-1.6407	1.5739	0.1938	0.0249	0.2542
		(0.0002)	(0.0008)	(0.0037)	(0.0026)			
1.15	90	0.0058	-0.0230	-1.6096	1.6282	0.2000	0.0230	0.2521
		(0.0001)	(0.0008)	(0.0041)	(0.0027)			
1.15	120	0.0057	-0.0227	-1.5874	1.6670	0.2045	0.0227	0.2493
		(0.0002)	(0.001)	(0.0042)	(0.0029)			
1.15	150	0.0062	-0.0254	-1.6076	1.6601	0.2004	0.0254	0.2450
		(0.0002)	(0.0009)	(0.0046)	(0.003)			
1.15	180	0.0074	-0.0303	-1.6896	1.5777	0.1846	0.0303	0.2435
		(0.0002)	(0.0009)	(0.0047)	(0.0033)			
1.15	210	0.0086	-0.0353	-1.8447	1.4432	0.1581	0.0353	0.2432

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Table 3.4 – continued from previous page

Moneyness	Time to Maturity	a_1	a_2	c	b_2	b_1	α	β
		(0.0002)	(0.001)	(0.0045)	(0.003)			
1.15	240	0.0092	-0.0382	-1.9663	1.3823	0.1400	0.0382	0.2411
		(0.0002)	(0.0009)	(0.0051)	(0.0035)			
1.15	270	0.0095	-0.0398	-2.0479	1.3631	0.1290	0.0398	0.2387
		(0.0002)	(0.001)	(0.0049)	(0.0033)			
1.15	300	0.0101	-0.0422	-2.1687	1.2883	0.1143	0.0422	0.2381
		(0.0002)	(0.0009)	(0.0054)	(0.0037)			
1.15	330	0.0105	-0.0441	-2.2446	1.2413	0.1060	0.0441	0.2383
		(0.0002)	(0.0009)	(0.0057)	(0.0038)			
1.15	360	0.0112	-0.0469	-2.2380	1.2473	0.1067	0.0469	0.2384
		(0.0002)	(0.001)	(0.0058)	(0.0039)			
1.2	30	0.0151	-0.0613	-1.9159	1.1931	0.1472	0.0613	0.2472
		(0.0002)	(0.001)	(0.0041)	(0.0028)			
1.2	60	0.0122	-0.0504	-1.8787	1.2934	0.1528	0.0504	0.2423
		(0.0002)	(0.001)	(0.0042)	(0.0028)			
1.2	90	0.0095	-0.0398	-1.7804	1.4326	0.1686	0.0398	0.2387
		(0.0002)	(0.0008)	(0.0042)	(0.0028)			
1.2	120	0.0074	-0.0311	-1.6398	1.5914	0.1940	0.0311	0.2374
		(0.0002)	(0.0011)	(0.0048)	(0.0033)			
1.2	150	0.0066	-0.0279	-1.5614	1.6792	0.2098	0.0279	0.2360
		(0.0002)	(0.001)	(0.0045)	(0.0029)			
1.2	180	0.0072	-0.0308	-1.6136	1.6290	0.1992	0.0308	0.2337
		(0.0002)	(0.001)	(0.0047)	(0.0031)			
1.2	210	0.0080	-0.0345	-1.7239	1.5409	0.1784	0.0345	0.2323
		(0.0002)	(0.001)	(0.0042)	(0.0027)			
1.2	240	0.0084	-0.0364	-1.7710	1.5321	0.1702	0.0364	0.2301
		(0.0002)	(0.001)	(0.0046)	(0.003)			
1.2	270	0.0087	-0.0380	-1.8478	1.4933	0.1576	0.0380	0.2289
		(0.0002)	(0.0009)	(0.0046)	(0.0031)			
1.2	300	0.0096	-0.0421	-2.0030	1.3758	0.1349	0.0421	0.2289
		(0.0002)	(0.0011)	(0.0049)	(0.0032)			
1.2	330	0.0110	-0.0481	-2.1316	1.2651	0.1186	0.0481	0.2294
		(0.0002)	(0.001)	(0.0055)	(0.0035)			
1.2	360	0.0121	-0.0525	-2.1109	1.2695	0.1211	0.0525	0.2297

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Table 3.4 – continued from previous page

Moneyness	Time to Maturity	a_1	a_2	c	b_2	b_1	α	β
		(0.0002)	(0.0009)	(0.0058)	(0.0039)			

Table 3.5: Parameter Estimates - Adjustment Period

Moneyness	Time to Maturity	a_1	a_2	c	b_2	b_1	α	β
0.8	30	0.0552	-0.1526	-1.8724	1.7275	0.1538	0.1526	0.3617
		(0.0008)	(0.0024)	(0.0081)	(0.0078)			
0.8	60	0.0396	-0.1133	-2.0537	1.6643	0.1283	0.1133	0.3498
		(0.0007)	(0.002)	(0.0083)	(0.0079)			
0.8	90	0.0304	-0.0899	-2.3094	1.4902	0.0993	0.0899	0.3378
		(0.0006)	(0.0018)	(0.0075)	(0.0068)			
0.8	120	0.0232	-0.0710	-2.5292	1.3689	0.0797	0.0710	0.3267
		(0.0005)	(0.0015)	(0.0075)	(0.0066)			
0.8	150	0.0186	-0.0587	-2.6469	1.3436	0.0709	0.0587	0.3167
		(0.0004)	(0.0013)	(0.0063)	(0.0054)			
0.8	180	0.0160	-0.0518	-2.6589	1.3938	0.0700	0.0518	0.3084
		(0.0004)	(0.0013)	(0.0071)	(0.006)			
0.8	210	0.0150	-0.0498	-2.7817	1.3224	0.0619	0.0498	0.3024
		(0.0004)	(0.0012)	(0.0074)	(0.006)			
0.8	240	0.0156	-0.0522	-2.9543	1.1866	0.0521	0.0522	0.2986
		(0.0004)	(0.0012)	(0.0076)	(0.0062)			
0.8	270	0.0166	-0.0562	-3.0718	1.0850	0.0463	0.0562	0.2963
		(0.0004)	(0.0013)	(0.008)	(0.0066)			
0.8	300	0.0189	-0.0642	-3.0321	1.0900	0.0482	0.0642	0.2947
		(0.0004)	(0.0015)	(0.0075)	(0.0062)			
0.8	330	0.0224	-0.0762	-3.1750	0.9210	0.0418	0.0762	0.2936
		(0.0005)	(0.0018)	(0.0076)	(0.0063)			
0.8	360	0.0277	-0.0946	-4.1167	0.0527	0.0163	0.0946	0.2925
		(0.0005)	(0.0017)	(0.0079)	(0.0063)			
0.85	30	0.0497	-0.1545	-1.6508	1.7992	0.1919	0.1545	0.3214
		(0.0007)	(0.0023)	(0.0087)	(0.0075)			
0.85	60	0.0355	-0.1138	-1.8119	1.7614	0.1633	0.1138	0.3118
		(0.0006)	(0.0021)	(0.0086)	(0.0073)			

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Table 3.5 – continued from previous page

Moneyness	Time to Maturity	a_1	a_2	c	b_2	b_1	α	β
0.85	90	0.0259	-0.0856	-2.0505	1.6344	0.1287	0.0856	0.3024
		(0.0005)	(0.0016)	(0.0083)	(0.0068)			
0.85	120	0.0190	-0.0645	-2.2231	1.5808	0.1083	0.0645	0.2939
		(0.0004)	(0.0016)	(0.0075)	(0.006)			
0.85	150	0.0145	-0.0507	-2.1899	1.7067	0.1119	0.0507	0.2869
		(0.0003)	(0.0013)	(0.0057)	(0.0046)			
0.85	180	0.0120	-0.0425	-2.1350	1.8376	0.1182	0.0425	0.2818
		(0.0003)	(0.0013)	(0.0081)	(0.0064)			
0.85	210	0.0109	-0.0392	-2.1339	1.8973	0.1184	0.0392	0.2788
		(0.0004)	(0.0013)	(0.0083)	(0.0064)			
0.85	240	0.0116	-0.0418	-2.3980	1.6942	0.0909	0.0418	0.2775
		(0.0003)	(0.0013)	(0.0086)	(0.0066)			
0.85	270	0.0131	-0.0473	-2.5643	1.5442	0.0770	0.0473	0.2770
		(0.0003)	(0.0013)	(0.0069)	(0.0053)			
0.85	300	0.0164	-0.0592	-3.0832	1.0795	0.0458	0.0592	0.2765
		(0.0004)	(0.0016)	(0.0091)	(0.007)			
0.85	330	0.0207	-0.0749	-4.4315	-0.0979	0.0119	0.0749	0.2760
		(0.0005)	(0.0017)	(0.0065)	(0.0049)			
0.85	360	0.0263	-0.0956	-4.6122	-0.3348	0.0099	0.0956	0.2751
		(0.0005)	(0.0017)	(0.0063)	(0.0049)			
0.9	30	0.0352	-0.1278	-1.4151	1.8670	0.2429	0.1278	0.2755
		(0.0005)	(0.0019)	(0.0078)	(0.006)			
0.9	60	0.0252	-0.0935	-1.5573	1.8463	0.2107	0.0935	0.2698
		(0.0004)	(0.0017)	(0.0079)	(0.006)			
0.9	90	0.0186	-0.0703	-1.7337	1.7958	0.1766	0.0703	0.2642
		(0.0004)	(0.0015)	(0.0067)	(0.0049)			
0.9	120	0.0140	-0.0538	-1.8227	1.8250	0.1616	0.0538	0.2596
		(0.0003)	(0.0012)	(0.0067)	(0.0049)			
0.9	150	0.0109	-0.0423	-1.7298	2.0000	0.1773	0.0423	0.2567
		(0.0003)	(0.0012)	(0.0065)	(0.0047)			
0.9	180	0.0095	-0.0371	-1.8187	2.0166	0.1622	0.0371	0.2555
		(0.0003)	(0.0012)	(0.0078)	(0.0056)			
0.9	210	0.0095	-0.0371	-2.0749	1.8732	0.1256	0.0371	0.2555
		(0.0003)	(0.0012)	(0.0077)	(0.0056)			

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Table 3.5 – continued from previous page

Moneyness	Time to Maturity	a_1	a_2	c	b_2	b_1	α	β
0.9	240	0.0103	-0.0404	-2.4097	1.6367	0.0898	0.0404	0.2562
		(0.0003)	(0.0011)	(0.0071)	(0.0051)			
0.9	270	0.0125	-0.0485	-2.8685	1.2619	0.0568	0.0485	0.2570
		(0.0003)	(0.0012)	(0.0078)	(0.0056)			
0.9	300	0.0172	-0.0666	-4.2443	0.1091	0.0143	0.0666	0.2575
		(0.0003)	(0.0013)	(0.0061)	(0.0045)			
0.9	330	0.0226	-0.0878	-4.3761	-0.0877	0.0126	0.0878	0.2569
		(0.0005)	(0.0019)	(0.0062)	(0.0045)			
0.9	360	0.0282	-0.1101	-4.4771	-0.2458	0.0114	0.1101	0.2565
		(0.0005)	(0.0018)	(0.0072)	(0.0053)			
0.95	30	0.0197	-0.0842	-1.4537	1.7341	0.2337	0.0842	0.2338
		(0.0003)	(0.0014)	(0.0063)	(0.0042)			
0.95	60	0.0149	-0.0644	-1.4641	1.8074	0.2313	0.0644	0.2313
		(0.0003)	(0.0012)	(0.0068)	(0.0045)			
0.95	90	0.0114	-0.0500	-1.4464	1.9079	0.2354	0.0500	0.2291
		(0.0003)	(0.0012)	(0.0057)	(0.0039)			
0.95	120	0.0094	-0.0413	-1.4221	2.0141	0.2412	0.0413	0.2279
		(0.0002)	(0.0011)	(0.0064)	(0.0042)			
0.95	150	0.0085	-0.0373	-1.4835	2.0516	0.2268	0.0373	0.2282
		(0.0003)	(0.0013)	(0.007)	(0.0047)			
0.95	180	0.0089	-0.0388	-1.7635	1.9079	0.1714	0.0388	0.2297
		(0.0002)	(0.001)	(0.0064)	(0.0043)			
0.95	210	0.0100	-0.0431	-2.1078	1.6897	0.1215	0.0431	0.2321
		(0.0003)	(0.0013)	(0.0082)	(0.0056)			
0.95	240	0.0120	-0.0513	-2.6925	1.2688	0.0677	0.0513	0.2344
		(0.0003)	(0.0014)	(0.0081)	(0.0056)			
0.95	270	0.0154	-0.0653	-3.6099	0.5580	0.0271	0.0653	0.2362
		(0.0003)	(0.0012)	(0.008)	(0.0053)			
0.95	300	0.0192	-0.0813	-4.0539	0.1711	0.0174	0.0813	0.2367
		(0.0003)	(0.0014)	(0.0069)	(0.0046)			
0.95	330	0.0252	-0.1067	-4.0734	0.0679	0.0170	0.1067	0.2366
		(0.0004)	(0.0018)	(0.0089)	(0.0061)			
0.95	360	0.0296	-0.1252	-4.1122	-0.0105	0.0164	0.1252	0.2366
		(0.0005)	(0.0019)	(0.0086)	(0.0059)			

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Table 3.5 – continued from previous page

Moneyness	Time to Maturity	a_1	a_2	c	b_2	b_1	α	β
1	30	0.0132	-0.0654	-1.4812	1.6184	0.2274	0.0654	0.2013
		(0.0002)	(0.0014)	(0.0055)	(0.0034)			
1	60	0.0107	-0.0536	-1.4308	1.7157	0.2391	0.0536	0.2002
		(0.0002)	(0.0012)	(0.0056)	(0.0034)			
1	90	0.0087	-0.0438	-1.3363	1.8522	0.2628	0.0438	0.1998
		(0.0002)	(0.0011)	(0.0054)	(0.0032)			
1	120	0.0080	-0.0399	-1.3549	1.9110	0.2580	0.0399	0.2001
		(0.0002)	(0.001)	(0.0059)	(0.0037)			
1	150	0.0081	-0.0404	-1.4737	1.8914	0.2291	0.0404	0.2018
		(0.0002)	(0.0011)	(0.0065)	(0.004)			
1	180	0.0092	-0.0450	-1.7522	1.7462	0.1734	0.0450	0.2049
		(0.0002)	(0.0013)	(0.0074)	(0.0046)			
1	210	0.0110	-0.0530	-2.1324	1.5060	0.1186	0.0530	0.2086
		(0.0002)	(0.0012)	(0.0069)	(0.0045)			
1	240	0.0132	-0.0623	-2.7032	1.1215	0.0670	0.0623	0.2119
		(0.0002)	(0.0011)	(0.0082)	(0.0052)			
1	270	0.0162	-0.0754	-3.5221	0.5290	0.0295	0.0754	0.2145
		(0.0002)	(0.0012)	(0.0083)	(0.0054)			
1	300	0.0198	-0.0917	-3.3841	0.5746	0.0339	0.0917	0.2156
		(0.0003)	(0.0015)	(0.0087)	(0.0056)			
1	330	0.0233	-0.1076	-3.5458	0.4181	0.0288	0.1076	0.2162
		(0.0004)	(0.0016)	(0.0086)	(0.0055)			
1	360	0.0265	-0.1222	-3.6798	0.2879	0.0252	0.1222	0.2167
		(0.0003)	(0.0016)	(0.0098)	(0.0064)			
1.05	30	0.0115	-0.0638	-1.2856	1.6755	0.2765	0.0638	0.1801
		(0.0002)	(0.0014)	(0.0057)	(0.0033)			
1.05	60	0.0099	-0.0552	-1.2744	1.7244	0.2796	0.0552	0.1789
		(0.0002)	(0.0013)	(0.0049)	(0.0028)			
1.05	90	0.0086	-0.0483	-1.2196	1.8069	0.2954	0.0483	0.1783
		(0.0002)	(0.0012)	(0.0053)	(0.0029)			
1.05	120	0.0081	-0.0452	-1.2658	1.8297	0.2820	0.0452	0.1787
		(0.0002)	(0.0012)	(0.0065)	(0.0037)			
1.05	150	0.0079	-0.0437	-1.2845	1.8669	0.2768	0.0437	0.1807
		(0.0002)	(0.0011)	(0.007)	(0.004)			

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Table 3.5 – continued from previous page

Moneyness	Time to Maturity	a_1	a_2	c	b_2	b_1	α	β
1.05	180	0.0085	-0.0465	-1.4524	1.8019	0.2340	0.0465	0.1837
		(0.0002)	(0.0012)	(0.0062)	(0.0036)			
1.05	210	0.0098	-0.0522	-1.7635	1.6346	0.1714	0.0522	0.1872
		(0.0002)	(0.0011)	(0.0068)	(0.004)			
1.05	240	0.0110	-0.0578	-1.9851	1.5081	0.1374	0.0578	0.1905
		(0.0002)	(0.0011)	(0.0074)	(0.0044)			
1.05	270	0.0129	-0.0670	-2.3223	1.2840	0.0980	0.0670	0.1932
		(0.0002)	(0.0013)	(0.0076)	(0.0045)			
1.05	300	0.0161	-0.0823	-2.3751	1.1993	0.0930	0.0823	0.1954
		(0.0003)	(0.0014)	(0.0077)	(0.0046)			
1.05	330	0.0188	-0.0957	-2.6374	0.9981	0.0715	0.0957	0.1969
		(0.0002)	(0.0013)	(0.0076)	(0.0046)			
1.05	360	0.0211	-0.1063	-2.9740	0.7663	0.0511	0.1063	0.1982
		(0.0003)	(0.0015)	(0.0075)	(0.0046)			
1.1	30	0.0113	-0.0668	-0.9466	1.8675	0.3881	0.0668	0.1695
		(0.0002)	(0.0013)	(0.0061)	(0.0034)			
1.1	60	0.0097	-0.0581	-0.9577	1.8881	0.3838	0.0581	0.1673
		(0.0002)	(0.0013)	(0.0066)	(0.0037)			
1.1	90	0.0086	-0.0517	-0.9705	1.9062	0.3789	0.0517	0.1661
		(0.0002)	(0.0012)	(0.0061)	(0.0034)			
1.1	120	0.0080	-0.0485	-1.0250	1.9066	0.3588	0.0485	0.1655
		(0.0002)	(0.0012)	(0.0063)	(0.0033)			
1.1	150	0.0073	-0.0439	-0.9560	1.9885	0.3844	0.0439	0.1668
		(0.0002)	(0.0012)	(0.0065)	(0.0036)			
1.1	180	0.0076	-0.0450	-1.0609	1.9616	0.3461	0.0450	0.1688
		(0.0002)	(0.0012)	(0.007)	(0.0039)			
1.1	210	0.0086	-0.0499	-1.3151	1.8345	0.2684	0.0499	0.1715
		(0.0002)	(0.0011)	(0.0056)	(0.0031)			
1.1	240	0.0095	-0.0543	-1.4466	1.7664	0.2354	0.0543	0.1744
		(0.0002)	(0.0013)	(0.0073)	(0.0041)			
1.1	270	0.0105	-0.0595	-1.5830	1.6867	0.2054	0.0595	0.1771
		(0.0002)	(0.0012)	(0.007)	(0.004)			
1.1	300	0.0131	-0.0727	-1.8509	1.4948	0.1571	0.0727	0.1795
		(0.0002)	(0.0012)	(0.0087)	(0.0051)			

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Table 3.5 – continued from previous page

Moneyness	Time to Maturity	a_1	a_2	c	b_2	b_1	α	β
1.1	330	0.0153	-0.0844	-2.0175	1.3623	0.1330	0.0844	0.1815
		(0.0002)	(0.0014)	(0.0082)	(0.0047)			
1.1	360	0.0167	-0.0915	-2.2777	1.1879	0.1025	0.0915	0.1830
		(0.0002)	(0.0014)	(0.0072)	(0.0042)			
1.15	30	0.0150	-0.0900	-1.0757	1.7737	0.3411	0.0900	0.1667
		(0.0003)	(0.0018)	(0.0074)	(0.0041)			
1.15	60	0.0122	-0.0747	-1.0208	1.8424	0.3603	0.0747	0.1632
		(0.0002)	(0.0015)	(0.0063)	(0.0034)			
1.15	90	0.0100	-0.0622	-0.8951	1.9394	0.4086	0.0622	0.1604
		(0.0002)	(0.0016)	(0.0073)	(0.0039)			
1.15	120	0.0084	-0.0528	-0.7970	2.0220	0.4507	0.0528	0.1589
		(0.0002)	(0.0015)	(0.0074)	(0.0039)			
1.15	150	0.0071	-0.0445	-0.6009	2.1721	0.5483	0.0445	0.1589
		(0.0002)	(0.0013)	(0.0069)	(0.0037)			
1.15	180	0.0070	-0.0439	-0.6786	2.1568	0.5073	0.0439	0.1599
		(0.0002)	(0.0014)	(0.007)	(0.0038)			
1.15	210	0.0078	-0.0486	-0.8160	2.0931	0.4422	0.0486	0.1615
		(0.0002)	(0.0012)	(0.0079)	(0.0043)			
1.15	240	0.0087	-0.0532	-0.9919	1.9943	0.3709	0.0532	0.1638
		(0.0002)	(0.0013)	(0.0076)	(0.004)			
1.15	270	0.0097	-0.0585	-1.0353	1.9594	0.3551	0.0585	0.1664
		(0.0002)	(0.0015)	(0.0075)	(0.0042)			
1.15	300	0.0118	-0.0701	-1.2882	1.7948	0.2758	0.0701	0.1684
		(0.0002)	(0.0013)	(0.0081)	(0.0045)			
1.15	330	0.0136	-0.0799	-1.4347	1.6858	0.2382	0.0799	0.1705
		(0.0002)	(0.0014)	(0.008)	(0.0046)			
1.15	360	0.0156	-0.0907	-1.8449	1.4330	0.1580	0.0907	0.1718
		(0.0002)	(0.0015)	(0.007)	(0.0039)			
1.2	30	0.0266	-0.1560	-1.8563	1.2033	0.1563	0.1560	0.1703
		(0.0003)	(0.0016)	(0.0073)	(0.004)			
1.2	60	0.0215	-0.1299	-1.9091	1.2396	0.1482	0.1299	0.1655
		(0.0003)	(0.0018)	(0.0076)	(0.0042)			
1.2	90	0.0171	-0.1066	-1.7184	1.4046	0.1794	0.1066	0.1607
		(0.0002)	(0.0016)	(0.0088)	(0.0048)			

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Table 3.5 – continued from previous page

Moneyness	Time to Maturity	a_1	a_2	c	b_2	b_1	α	β
1.2	120	0.0133	-0.0850	-1.4692	1.5958	0.2301	0.0850	0.1568
		(0.0002)	(0.0015)	(0.0083)	(0.0044)			
1.2	150	0.0104	-0.0675	-1.1993	1.8010	0.3014	0.0675	0.1546
		(0.0002)	(0.0015)	(0.0071)	(0.0037)			
1.2	180	0.0090	-0.0586	-1.0313	1.9351	0.3566	0.0586	0.1542
		(0.0002)	(0.0014)	(0.0075)	(0.0039)			
1.2	210	0.0088	-0.0564	-0.9351	2.0056	0.3926	0.0564	0.1552
		(0.0002)	(0.0012)	(0.009)	(0.0049)			
1.2	240	0.0091	-0.0581	-0.7523	2.0987	0.4713	0.0581	0.1571
		(0.0002)	(0.0016)	(0.0078)	(0.0041)			
1.2	270	0.0096	-0.0606	-0.6472	2.1449	0.5235	0.0606	0.1593
		(0.0003)	(0.0017)	(0.0078)	(0.0042)			
1.2	300	0.0109	-0.0679	-0.7669	2.0622	0.4645	0.0679	0.1611
		(0.0003)	(0.0018)	(0.0091)	(0.005)			
1.2	330	0.0127	-0.0782	-0.8562	1.9857	0.4248	0.0782	0.1631
		(0.0003)	(0.0019)	(0.0075)	(0.0041)			
1.2	360	0.0150	-0.0915	-1.2491	1.7489	0.2868	0.0915	0.1641
		(0.0003)	(0.0017)	(0.0085)	(0.0047)			

Table 3.6: Parameter Estimates - Post-Crisis Period

Moneyness	Time to Maturity	a_1	a_2	c	b_2	b_1	α	β
0.8	30	0.0324	-0.0921	-1.1667	2.2313	0.3114	0.0921	0.3523
		(0.0009)	(0.0029)	(0.0069)	(0.0063)			
0.8	60	0.0338	-0.1021	-0.8332	2.5210	0.4347	0.1021	0.3316
		(0.0009)	(0.0031)	(0.0096)	(0.0084)			
0.8	90	0.0511	-0.1683	-0.4509	2.7429	0.6371	0.1683	0.3034
		(0.0015)	(0.0053)	(0.0157)	(0.0129)			
0.8	120	0.0977	-0.3561	-0.8610	2.3255	0.4228	0.3561	0.2744
		(0.0018)	(0.0068)	(0.0178)	(0.0137)			
0.8	150	0.1243	-0.4923	-1.1699	2.1548	0.3104	0.4923	0.2525
		(0.0027)	(0.011)	(0.0163)	(0.0118)			
0.8	180	0.1032	-0.4340	-0.1578	3.0116	0.8543	0.4340	0.2379

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Table 3.6 – continued from previous page

Moneyness	Time to Maturity	a_1	a_2	c	b_2	b_1	α	β
		(0.0023)	(0.0099)	(0.0253)	(0.0176)			
0.8	210	0.0729	-0.3174	-0.8958	2.6431	0.4089	0.3174	0.2298
		(0.0023)	(0.01)	(0.0556)	(0.0378)			
0.8	240	0.0604	-0.2674	-1.0077	2.6372	0.3662	0.2674	0.2261
		(0.0024)	(0.0107)	(0.0805)	(0.0541)			
0.8	270	0.0660	-0.2938	-0.1385	3.1837	0.8730	0.2938	0.2245
		(0.0024)	(0.0106)	(0.074)	(0.0496)			
0.8	300	0.0734	-0.3280	-0.1823	3.1020	0.8358	0.3280	0.2239
		(0.0024)	(0.011)	(0.077)	(0.0515)			
0.8	330	0.0793	-0.3543	-1.0177	2.5011	0.3623	0.3543	0.2237
		(0.0023)	(0.0102)	(0.0712)	(0.0474)			
0.8	360	0.0779	-0.3485	-4.1286	0.3908	0.0162	0.3485	0.2236
		(0.0023)	(0.0103)	(0.0764)	(0.0509)			
0.85	30	0.0550	-0.1860	-0.6009	2.4945	0.5484	0.1860	0.2957
		(0.0012)	(0.0043)	(0.0094)	(0.0075)			
0.85	60	0.0586	-0.2113	-0.3755	2.6931	0.6870	0.2113	0.2775
		(0.0012)	(0.0047)	(0.0113)	(0.0087)			
0.85	90	0.0803	-0.3131	-0.4894	2.5723	0.6131	0.3131	0.2565
		(0.0014)	(0.0059)	(0.013)	(0.0095)			
0.85	120	0.1028	-0.4334	-1.4004	1.9194	0.2466	0.4334	0.2372
		(0.0019)	(0.0083)	(0.0204)	(0.0142)			
0.85	150	0.0852	-0.3833	-1.5947	1.8878	0.2030	0.3833	0.2223
		(0.0019)	(0.0088)	(0.0285)	(0.0189)			
0.85	180	0.0526	-0.2467	-2.0822	1.7492	0.1247	0.2467	0.2132
		(0.0017)	(0.0079)	(0.0361)	(0.0234)			
0.85	210	0.0366	-0.1750	-2.2998	1.7542	0.1004	0.1750	0.2092
		(0.0015)	(0.0072)	(0.0402)	(0.0257)			
0.85	240	0.0371	-0.1783	-0.7342	2.7696	0.4801	0.1783	0.2081
		(0.0016)	(0.0079)	(0.0326)	(0.0207)			
0.85	270	0.0411	-0.1973	-0.9866	2.5915	0.3731	0.1973	0.2083
		(0.0017)	(0.0085)	(0.0399)	(0.0254)			
0.85	300	0.0451	-0.2157	-1.9102	1.9902	0.1482	0.2157	0.2089
		(0.0016)	(0.0075)	(0.047)	(0.03)			
0.85	330	0.0546	-0.2605	-1.8606	1.9658	0.1558	0.2605	0.2095

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Table 3.6 – continued from previous page

Moneyness	Time to Maturity	a_1	a_2	c	b_2	b_1	α	β
		(0.0019)	(0.0093)	(0.0531)	(0.034)			
0.85	360	0.0579	-0.2757	-1.5353	2.1674	0.2156	0.2757	0.2101
		(0.0017)	(0.008)	(0.0437)	(0.0281)			
0.9	30	0.0751	-0.3267	-0.4131	2.4196	0.6617	0.3267	0.2298
		(0.0011)	(0.005)	(0.0138)	(0.0093)			
0.9	60	0.0686	-0.3125	-0.3126	2.5460	0.7316	0.3125	0.2197
		(0.0014)	(0.0064)	(0.0177)	(0.0115)			
0.9	90	0.0630	-0.3016	-0.8095	2.2666	0.4452	0.3016	0.2090
		(0.0013)	(0.0062)	(0.0234)	(0.0149)			
0.9	120	0.0532	-0.2667	-1.5770	1.8432	0.2067	0.2667	0.1994
		(0.0012)	(0.0063)	(0.0312)	(0.0194)			
0.9	150	0.0365	-0.1893	-1.3760	2.0956	0.2527	0.1893	0.1926
		(0.001)	(0.0052)	(0.0276)	(0.0167)			
0.9	180	0.0237	-0.1250	-1.0281	2.4646	0.3578	0.1250	0.1895
		(0.001)	(0.0052)	(0.026)	(0.0156)			
0.9	210	0.0213	-0.1125	-1.0283	2.5319	0.3578	0.1125	0.1891
		(0.001)	(0.0055)	(0.0278)	(0.0167)			
0.9	240	0.0254	-0.1336	-1.6425	2.1455	0.1936	0.1336	0.1900
		(0.0011)	(0.0059)	(0.0388)	(0.0233)			
0.9	270	0.0319	-0.1669	-2.5231	1.5729	0.0803	0.1669	0.1913
		(0.0012)	(0.0062)	(0.044)	(0.0265)			
0.9	300	0.0397	-0.2059	-3.0654	1.2030	0.0467	0.2059	0.1926
		(0.0014)	(0.0074)	(0.0555)	(0.0336)			
0.9	330	0.0514	-0.2654	-2.5645	1.4597	0.0772	0.2654	0.1936
		(0.0018)	(0.0096)	(0.0712)	(0.0432)			
0.9	360	0.0594	-0.3054	-3.2948	0.9781	0.0372	0.3054	0.1947
		(0.0022)	(0.0112)	(0.0863)	(0.0525)			
0.95	30	0.0407	-0.2344	-0.6632	2.1007	0.5153	0.2344	0.1736
		(0.0008)	(0.0046)	(0.0177)	(0.0101)			
0.95	60	0.0335	-0.1972	-0.9585	1.9950	0.3835	0.1972	0.1698
		(0.0007)	(0.0042)	(0.017)	(0.0095)			
0.95	90	0.0271	-0.1634	-1.2100	1.9297	0.2982	0.1634	0.1661
		(0.0007)	(0.0042)	(0.0171)	(0.0096)			
0.95	120	0.0215	-0.1316	-0.9364	2.1783	0.3921	0.1316	0.1636

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Table 3.6 – continued from previous page

Moneyness	Time to Maturity	a_1	a_2	c	b_2	b_1	α	β
		(0.0006)	(0.004)	(0.0221)	(0.0121)			
0.95	150	0.0176	-0.1080	-0.5007	2.5297	0.6063	0.1080	0.1631
		(0.0008)	(0.005)	(0.021)	(0.0116)			
0.95	180	0.0192	-0.1167	-0.8260	2.4107	0.4380	0.1167	0.1646
		(0.0008)	(0.0048)	(0.0296)	(0.0164)			
0.95	210	0.0234	-0.1395	-1.0172	2.3173	0.3619	0.1395	0.1673
		(0.001)	(0.0064)	(0.0402)	(0.0224)			
0.95	240	0.0303	-0.1784	-1.2392	2.1651	0.2899	0.1784	0.1701
		(0.0012)	(0.007)	(0.0444)	(0.0251)			
0.95	270	0.0393	-0.2279	-1.3885	2.0431	0.2497	0.2279	0.1723
		(0.0012)	(0.0071)	(0.0429)	(0.0244)			
0.95	300	0.0524	-0.3017	-1.6067	1.8772	0.2010	0.3017	0.1737
		(0.0015)	(0.0086)	(0.0652)	(0.0372)			
0.95	330	0.0665	-0.3803	-3.2002	0.8934	0.0409	0.3803	0.1749
		(0.0019)	(0.0108)	(0.0868)	(0.0497)			
0.95	360	0.0779	-0.4419	-3.8876	0.4555	0.0206	0.4419	0.1762
		(0.0017)	(0.0099)	(0.0914)	(0.0525)			
1	30	0.0227	-0.1684	-1.6696	1.4223	0.1883	0.1684	0.1351
		(0.0004)	(0.003)	(0.0137)	(0.0067)			
1	60	0.0201	-0.1501	-1.7322	1.4418	0.1769	0.1501	0.1337
		(0.0004)	(0.0031)	(0.0145)	(0.0071)			
1	90	0.0173	-0.1302	-1.5230	1.6140	0.2181	0.1302	0.1329
		(0.0004)	(0.0032)	(0.0157)	(0.0078)			
1	120	0.0155	-0.1160	-1.1101	1.8989	0.3296	0.1160	0.1334
		(0.0004)	(0.0034)	(0.0158)	(0.0078)			
1	150	0.0167	-0.1232	-1.3337	1.8496	0.2635	0.1232	0.1355
		(0.0005)	(0.0036)	(0.0181)	(0.0091)			
1	180	0.0217	-0.1560	-1.7674	1.6524	0.1708	0.1560	0.1392
		(0.0006)	(0.0042)	(0.0233)	(0.0118)			
1	210	0.0278	-0.1934	-1.3069	1.8868	0.2708	0.1934	0.1438
		(0.0008)	(0.0053)	(0.0273)	(0.014)			
1	240	0.0367	-0.2477	-1.2037	1.9199	0.3002	0.2477	0.1481
		(0.0009)	(0.0059)	(0.0305)	(0.016)			
1	270	0.0486	-0.3212	-1.6945	1.6221	0.1838	0.3212	0.1513

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Table 3.6 – continued from previous page

Moneyness	Time to Maturity	a_1	a_2	c	b_2	b_1	α	β
		(0.0009)	(0.0058)	(0.0376)	(0.0199)			
1	300	0.0627	-0.4090	-2.3717	1.2251	0.0935	0.4090	0.1534
		(0.0013)	(0.0089)	(0.0602)	(0.0322)			
1	330	0.0782	-0.5050	-2.9800	0.8449	0.0509	0.5050	0.1549
		(0.0012)	(0.0081)	(0.0488)	(0.0261)			
1	360	0.0901	-0.5749	-2.7221	0.9519	0.0658	0.5749	0.1567
		(0.0013)	(0.0087)	(0.0457)	(0.0246)			
1.05	30	0.0172	-0.1470	-1.8952	1.2919	0.1503	0.1470	0.1168
		(0.0003)	(0.0026)	(0.0146)	(0.0068)			
1.05	60	0.0160	-0.1387	-1.9085	1.3212	0.1483	0.1387	0.1154
		(0.0003)	(0.0025)	(0.0157)	(0.0072)			
1.05	90	0.0148	-0.1293	-1.6738	1.4745	0.1877	0.1293	0.1146
		(0.0003)	(0.0031)	(0.0402)	(0.0183)			
1.05	120	0.0146	-0.1270	-1.3701	1.6652	0.2541	0.1270	0.1149
		(0.0003)	(0.0031)	(0.018)	(0.0083)			
1.05	150	0.0164	-0.1409	-1.5621	1.6144	0.2097	0.1409	0.1167
		(0.0004)	(0.0036)	(0.0173)	(0.008)			
1.05	180	0.0198	-0.1653	-1.8879	1.4773	0.1514	0.1653	0.1197
		(0.0005)	(0.0039)	(0.0216)	(0.0101)			
1.05	210	0.0226	-0.1832	-1.7384	1.5576	0.1758	0.1832	0.1235
		(0.0005)	(0.0042)	(0.0204)	(0.0097)			
1.05	240	0.0253	-0.1980	-0.6961	2.0606	0.4987	0.1980	0.1276
		(0.0007)	(0.0053)	(0.0244)	(0.0118)			
1.05	270	0.0359	-0.2742	-1.1465	1.8008	0.3178	0.2742	0.1310
		(0.0007)	(0.0054)	(0.0214)	(0.0105)			
1.05	300	0.0470	-0.3521	-2.2169	1.2486	0.1090	0.3521	0.1334
		(0.0009)	(0.0065)	(0.0269)	(0.0133)			
1.05	330	0.0584	-0.4311	-2.5035	1.0644	0.0818	0.4311	0.1354
		(0.0009)	(0.0068)	(0.0269)	(0.0134)			
1.05	360	0.0626	-0.4561	-2.7335	0.9349	0.0650	0.4561	0.1372
		(0.0009)	(0.0065)	(0.0196)	(0.0098)			
1.1	30	0.0158	-0.1379	-2.0234	1.2673	0.1322	0.1379	0.1145
		(0.0003)	(0.0027)	(0.0171)	(0.0078)			
1.1	60	0.0143	-0.1283	-2.2748	1.1972	0.1028	0.1283	0.1116

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Table 3.6 – continued from previous page

Moneyness	Time to Maturity	a_1	a_2	c	b_2	b_1	α	β
		(0.0003)	(0.0029)	(0.0193)	(0.0087)			
1.1	90	0.0132	-0.1213	-2.1907	1.2747	0.1119	0.1213	0.1092
		(0.0003)	(0.0029)	(0.0185)	(0.0084)			
1.1	120	0.0127	-0.1179	-1.6904	1.5373	0.1845	0.1179	0.1081
		(0.0003)	(0.0029)	(0.0237)	(0.0105)			
1.1	150	0.0135	-0.1244	-1.4941	1.6562	0.2245	0.1244	0.1084
		(0.0003)	(0.003)	(0.021)	(0.0094)			
1.1	180	0.0151	-0.1375	-1.7091	1.5725	0.1811	0.1375	0.1100
		(0.0003)	(0.0032)	(0.0221)	(0.01)			
1.1	210	0.0170	-0.1516	-1.5441	1.6474	0.2136	0.1516	0.1123
		(0.0005)	(0.0043)	(0.0325)	(0.0148)			
1.1	240	0.0190	-0.1652	-1.1754	1.8119	0.3089	0.1652	0.1150
		(0.0005)	(0.0048)	(0.0326)	(0.015)			
1.1	270	0.0227	-0.1930	-0.9655	1.8963	0.3810	0.1930	0.1177
		(0.0006)	(0.0052)	(0.0314)	(0.0147)			
1.1	300	0.0295	-0.2456	-1.8924	1.4417	0.1508	0.2456	0.1199
		(0.0006)	(0.0049)	(0.0236)	(0.011)			
1.1	330	0.0371	-0.3039	-2.5958	1.0721	0.0746	0.3039	0.1219
		(0.0007)	(0.0056)	(0.0233)	(0.0111)			
1.1	360	0.0429	-0.3472	-2.7337	0.9746	0.0650	0.3472	0.1235
		(0.0008)	(0.0063)	(0.0211)	(0.01)			
1.15	30	0.0305	-0.2524	-2.6914	0.8392	0.0678	0.2524	0.1208
		(0.0004)	(0.0034)	(0.0138)	(0.0065)			
1.15	60	0.0274	-0.2353	-3.4913	0.5354	0.0305	0.2353	0.1163
		(0.0004)	(0.0035)	(0.0154)	(0.007)			
1.15	90	0.0230	-0.2057	-4.0315	0.3688	0.0178	0.2057	0.1120
		(0.0004)	(0.0036)	(0.0194)	(0.0088)			
1.15	120	0.0178	-0.1637	-3.7239	0.6000	0.0241	0.1637	0.1086
		(0.0003)	(0.0032)	(0.0194)	(0.0087)			
1.15	150	0.0134	-0.1253	-2.3582	1.2938	0.0946	0.1253	0.1070
		(0.0003)	(0.0031)	(0.027)	(0.0121)			
1.15	180	0.0127	-0.1188	-2.4124	1.3091	0.0896	0.1188	0.1072
		(0.0004)	(0.0037)	(0.025)	(0.0111)			
1.15	210	0.0138	-0.1275	-2.2238	1.3970	0.1082	0.1275	0.1084

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Table 3.6 – continued from previous page

Moneyness	Time to Maturity	a_1	a_2	c	b_2	b_1	α	β
		(0.0004)	(0.004)	(0.0285)	(0.0128)			
1.15	240	0.0165	-0.1502	-2.1220	1.4180	0.1198	0.1502	0.1101
		(0.0004)	(0.0038)	(0.0291)	(0.0132)			
1.15	270	0.0203	-0.1817	-1.9349	1.4720	0.1445	0.1817	0.1120
		(0.0005)	(0.0045)	(0.0258)	(0.0118)			
1.15	300	0.0258	-0.2265	-2.2439	1.3061	0.1061	0.2265	0.1138
		(0.0006)	(0.0051)	(0.0238)	(0.011)			
1.15	330	0.0316	-0.2734	-3.1491	0.8501	0.0429	0.2734	0.1156
		(0.0006)	(0.0048)	(0.025)	(0.0116)			
1.15	360	0.0372	-0.3179	-2.7098	1.0255	0.0666	0.3179	0.1171
		(0.0006)	(0.0054)	(0.0271)	(0.0126)			
1.2	30	0.0576	-0.4423	-2.9135	0.5267	0.0543	0.4423	0.1303
		(0.0004)	(0.0033)	(0.0124)	(0.0059)			
1.2	60	0.0579	-0.4627	-3.1530	0.4523	0.0427	0.4627	0.1252
		(0.0004)	(0.0036)	(0.0134)	(0.0064)			
1.2	90	0.0561	-0.4695	-3.3955	0.3868	0.0335	0.4695	0.1196
		(0.0005)	(0.0039)	(0.016)	(0.0076)			
1.2	120	0.0511	-0.4473	-3.7670	0.2869	0.0231	0.4473	0.1142
		(0.0006)	(0.0052)	(0.026)	(0.0119)			
1.2	150	0.0391	-0.3544	-3.5378	0.5034	0.0291	0.3544	0.1103
		(0.0005)	(0.005)	(0.0298)	(0.0135)			
1.2	180	0.0295	-0.2720	-3.1929	0.7589	0.0411	0.2720	0.1086
		(0.0006)	(0.0055)	(0.0313)	(0.0141)			
1.2	210	0.0258	-0.2380	-3.5275	0.6748	0.0294	0.2380	0.1085
		(0.0005)	(0.0049)	(0.0271)	(0.012)			
1.2	240	0.0234	-0.2143	-3.9444	0.5349	0.0194	0.2143	0.1091
		(0.0005)	(0.0046)	(0.0197)	(0.0089)			
1.2	270	0.0242	-0.2197	-4.2949	0.3824	0.0136	0.2197	0.1102
		(0.0005)	(0.0044)	(0.0235)	(0.0106)			
1.2	300	0.0268	-0.2405	-3.6568	0.6608	0.0258	0.2405	0.1115
		(0.0006)	(0.0053)	(0.0229)	(0.0104)			
1.2	330	0.0297	-0.2635	-3.2143	0.8437	0.0402	0.2635	0.1129
		(0.0006)	(0.0051)	(0.0265)	(0.0121)			
1.2	360	0.0339	-0.2973	-3.5077	0.6762	0.0300	0.2973	0.1142

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Table 3.6 – continued from previous page

Moneyness	Time to Maturity	a_1	a_2	c	b_2	b_1	α	β
		(0.0006)	(0.0053)	(0.0264)	(0.0121)			

Chapter 4
Financial Stress Prediction: A Bayesian Approach

4.1 Introduction

This paper attempts to predict financial stress by identifying leading indicators under a Bayesian variable selection framework. While large amount of the literature in this field focuses on financial crisis, especially for banking crisis, this paper also includes non crisis periods. Since financial crisis can be viewed as severe financial stress period, monitoring the real-time financial stress level as well as predicting it provides more guidance to policy makers and private sectors than aiming at crisis periods only.

So far most papers in this strand studying leading indicators of financial stress conclude that it is hard to predict. Vasicek et al. (2016) identify leading indicators of financial stress, then use those indicators as explanatory variables in both panel model for 25 OECD countries and in models at the individual country level. Their findings suggest that it is hard to predict out-of-sample despite the reasonably good in-sample performance. The similar conclusion is reached by Slingenberg and de Haan (2011) as they conduct the analysis to 13 countries. Not only they find out it is hard to predict financial stress, but also very few variables have predictive power to most countries. To improve the prediction performance, I use weekly financial variables instead of quarterly macro variables that are used by previous literature. These financial variables belong to five categories: interest rate, yield spread, volatility, inflation and market return. Variable selection method is used to select more relevant ones among potential 19 indicators and predictive likelihoods shows optimistic prediction results.

Among the literature predicting crisis referred as the Early Warning System (EWS), there is a great amount uses Logit or Probit model (see Berg and Pattilo (1999), Bussiere and Fratzscher (2006) and Fuertes and Kalotychou (2006)). Basically, a binary variable is defined as one if there is a crisis and zero otherwise. The purpose of these papers is to predict the probability of the occurrence of financial crisis. In this paper, both linear model and Probit model under normal error assumption and fat tail assumption are used for analysis. Three financial stress indexes issued by Federal Reserve Banks are used to identify the level data of financial stress. These indexes together with other papers on financial uncertainty (Bloom(2009) and Ng(2015)) are used to identify binary variable. Linear model and Probit model provide information for decision making of policy makers from different perspectives.

Among literature working on leading indicators there are different model/variable selection approaches. The first approach is the classic model selection approach based on significance tests or stepwise search methods (Forward, Backward Stepwise Procedures). It is based on selection criteria such as R^2, residual sum of squares (RSS), the Akaike Information Criterion (AIC) and the Bayes Information Criterion (BIC). Also t-test can be used to check if the coefficient is significantly different from zero. See Slingenberg and de Haan (2011). The second approach is called signal extraction approach. Kaminsky, Lizondo and Reinhart (1998) is one of the pioneers proposing Early Warning System to predict currency crises. In their paper, the trend of several financial/banking variables which indicating the healthiness of the banking industry are monitored. Whenever an indicator departs from a given threshold level, a warning signal that a crisis might take place within the next 24 months is issued. One of the most recent work belonging to this stand is Christensen and Li (2014). They extend the method by constructing composite indicators and conclude that composite indicator outperforms the others. The third category uses Bayesian approach. Vasicek et al.(2016) is one of the most recent works employing Bayesian model averaging (BMA) to identify leading indicators of financial stress. In this paper, I use stochastic search variable selection (SSVS) formulation of George and McCulloch (1993). SSVS is developed to avoid the overwhelming burden of calculating the posterior probabilities of all 2^p models encountered in BMA. It uses the Gibbs sampler to simulate a sample from the posterior distribution and the beauty of SSVS is the fastness and efficiency. More details are described in Section 4.2.

Overall, this paper differentiates itself from other literature by focusing on predicting financial stress and looking for leading indicator under Bayesian stochastic search variable selection framework.

The paper is organized as follows. Section 2.1 describes Bayesian Stochastic Search Variable Selection process under normal error term assumption, while Section 2.2 introduces formulation under fat tail assumption. Section 3.1 provides data description. Section 3.2 identifies financial stress level and binary variable representing the occurrence of financial stress. Section 3.3 discusses in-sample analysis. Section 3.4 shows out-of-sample predicting results. Section 4 concludes.

4.2 Baysian Stochastic Search Variable Selection

The advantage of SSVS developed by George and McCulloch (1993) comparing to Bayesian model averaging is that it avoids the overwhelming burden of calculating the posterior probabilities of all 2^k models given k potential indicators. The formulation can be found in George and McCulloch (1993) and Koop et al. (2007). In this paper, Both normal assumption and fat tail assumption are considered for the error term.

4.2.1 Normal Error Assumption

Assume financial stress level follows the linear regression model as below:

$$y = X\beta + \epsilon, \quad \epsilon \overset{iid}{\sim} N(0, \sigma^2 I) \tag{4.1}$$

where X is a $n \times (k+1)$ matrix represents the time series of total k potential indicators. β is a $(k+1) \times 1$ vector and includes intercept. The assumed prior distributions are:

$$\begin{aligned} \beta_j | \gamma_j &\sim (1-\gamma_j) N(0, \tau_j^2) + \gamma_j N(0, c_j^2 \tau_j^2), \quad j = 1, 2, ..., k && (4.2) \\ \beta_0 &\sim N(0, \underline{V_0}) && (4.3) \\ \sigma^2 &\sim IG(a, b) && (4.4) \end{aligned}$$

The purpose of Bayesian SSVS is to select informative indicators among all and the key mechanism lies in equation (4.2). The prior distribution of β_j is a mixture of two normal distributions that are both centered at zero but with different variances: one is vary small while the other is vary big. γ_j is a either zero or one. $\gamma_j = 1$ means the normal distribution with big variance is selected. It implies that β_j is most likely different from zero and x_j should be selected. Following Koop et al. (2007), I set $\tau_j^2 = .0000001$ and $c_j^2 \tau_j^2 = 9$. The prior distribution of σ^2 follows a Inverse-Gamma distribution with shape parameter $a = 3$ and scale parameter $b = 1$. The prior distribution of β_0 follows normal distribution with mean 0 and and variance $\underline{V_0} = 1$.

According to Koop et al. (2007), the posterior conditionals can be easily obtained as following:

$$\begin{aligned} \beta | \gamma, \sigma^2, y &\sim N\left(\left[X'X/\sigma^2 + V_\beta^{-1}\right] X'y/\sigma^2, X'y/\sigma^2\right), && (4.5) \\ \sigma^2 | \beta, \gamma, y &\sim IG\left(\frac{n}{2} + a, \left[b^{-1} + 0.5(y - X\beta)'(y - X\beta)\right]^{-1}\right), && (4.6) \\ \gamma_j | \gamma_{-j}, \beta, \sigma^2, y &\sim B\left(1, \frac{p\phi(\beta_j; 0, c_j^2 \tau_j^2)}{p\phi(\beta_j; 0, c_j^2 \tau_j^2) + (1-p)\phi(\beta_j; 0, \tau_j^2)}\right). && (4.7) \end{aligned}$$

where, V_β is a $(k+1) \times (k+1)$ diagonal matrix. Let $V_\beta(i,i)$ represent the i^{th} element on the diagonal. For $i = 1$, $V_\beta(i,i) = \underline{V_0}$. For $i = 2, 3, ..., k+1$, $V_\beta(i,i) = \gamma_i c_i^2 \tau_i^2 + (1-\gamma_i)\tau_i^2$. Here the posterior distribution of γ_j follows Binomial distribution and the hyperparameter $p = 1/2$ and ϕ represents normal probability.

For Probit model, a binary variable is defined as:

$$z = \begin{cases} 1, & \text{if } y \geq 0 \\ 0, & \text{if } y < 0 \end{cases}$$

Thus, the latent variable y can be drawn from truncated normal distribution,

$$y | \beta, z \sim \begin{cases} \mathrm{TN}_{[0,\infty)}(X\beta, \sigma^2) & \text{if } z = 1, \\ \mathrm{TN}_{(\infty,0)}(X\beta, \sigma^2) & \text{if } z = 0 \end{cases}$$

To solve the identification problem in Probit model, I set $\sigma = 1$.

4.2.2 Fat Tail Assumption

Now consider the fail tail assumption for the error term:

$$y_i = \beta_0 + \sum_{j=1}^{k} x_{ji}\beta_j + \epsilon_i, \quad \epsilon_i|\lambda_i, \sigma^2 \overset{iid}{\sim} N(0, \lambda_i\sigma^2) \tag{4.8}$$

with prior distribution of λ_i as:

$$\lambda_i \overset{iid}{\sim} IG\left(\frac{\nu}{2}, \frac{2}{\nu}\right), i = 1, 2, ..., n \tag{4.9}$$

By marginalizing over the mixing variable λ_i, $\epsilon_i \sim t(0, \sigma^2, \nu)$. In other words, I define the student t-distribution as a scale mixture of normal distributions. In this paper I assume $\nu = 5$. Following the notation in Koop et al. (2007), define $\Lambda = \text{diag}\{\lambda_i\}$.

With the same prior distributions for β and σ^2, the posterior conditionals for this model are:

$$\beta|\gamma, \sigma^2, \{\lambda_i\}, y \sim N\left(\left[X'\Lambda^{-1}X/\sigma^2 + V_\beta^{-1}(\gamma)\right] X'\Lambda^{-1}y/\sigma^2, X'\Lambda^{-1}y/\sigma^2\right), \tag{4.10}$$

$$\sigma^2|\beta, \{\lambda_i\}, y \sim IG\left(\frac{n}{2} + a, \left[b^{-1} + 0.5(y - X\beta)'\Lambda^{-1}(y - X\beta)\right]^{-1}\right), \tag{4.11}$$

$$\lambda_i|\beta, \sigma^2, y \overset{iid}{\sim} IG\left(\frac{\nu+1}{2}, \left[\frac{1}{2}\left[\frac{y_i - x_i\beta}{\sigma}\right]^2 + \frac{\nu}{2}\right]^{-1}\right). \tag{4.12}$$

For Probit model, I set $\sigma = 1$. And the latent variable y can be drawn from truncated normal distribution,

$$y_i|\beta, z_i \sim \begin{cases} \text{TN}_{[0,\infty)}(X_i\beta, \lambda_i) & \text{if } z_i = 1, \\ \text{TN}_{(\infty,0)}(X_i\beta, \lambda_i) & \text{if } z_i = 0 \end{cases}$$

Gibbs Sampler is used to draw from the posterior distributions. For in-sample analysis, I set the Markov chain Monte Carlo (MCMC) length as 5,5000 with the first 5000 burn-in. For out-of-sample prediction analysis, I set the MCMC length as 5,2000 with the first 2000 burn-in. The estimates are calculated as the mean of the MCMC chain. The initial values of the parameters for each rolling window are the estimates from previous rolling window.

4.3 Empirical Analysis

4.3.1 Data

Data is obtained from the website of Federal Reserve Bank of St. Louis and Haver database. These are weekly data ranging from 2003-01-03 to 2016-05-20 with total sample size of 698. The potential indicators are classified into five categories: interest rate, yield spread, volatility, inflation and return. For interest rate, I include both level and first difference of the data. All the potential indicators are financial variables. Please see Table 4.1 for the list of the variables.

4.3.2 Identify Financial Stress

Different financial stress indexes (FSI) are issued by Federal reserve banks to measure the financial stress level. Although these indexes are constructed by using different methods and including different

Table 4.1: Potential Indicators

Segment		Variables
Interest Rate	level	**1** effective federal funds rate
		2 30-year treasury interest rate
	first difference	**3** effective federal funds rate
		4 2-year treasury interest rate
		5 10-year treasury interest rate
		6 30-year treasury interest rate
Yield Spread	level	**7** 10-year treasury interest rate - 2-year treasury interest rate
		8 Merrill Lynch Asset Backed Master BBB-AA rated interest rate
		9 10-year treasury - 3-month treasury
		10 Merrill Lynch High Yield corporate master II index - 10-year treasury
		11 Moody's Seasoned Baa Corporate rated bond - 10 year treasury
		12 3-month treasury Eurodollar (TED) spread
		13 3-month AA Financial commercial paper - 3-month treasury
		14 3-month LIBOR - OIS
Volatility	level	**15** VIX
		16 Merrill Lynch 1 month Bond Market Volatility Index
Inflation	level	**17** 10-year nominal treasury - TIPS
Return	level	**18** S&P 500 Index
		19 J.P. Morgan Emerging Markets Bond Index

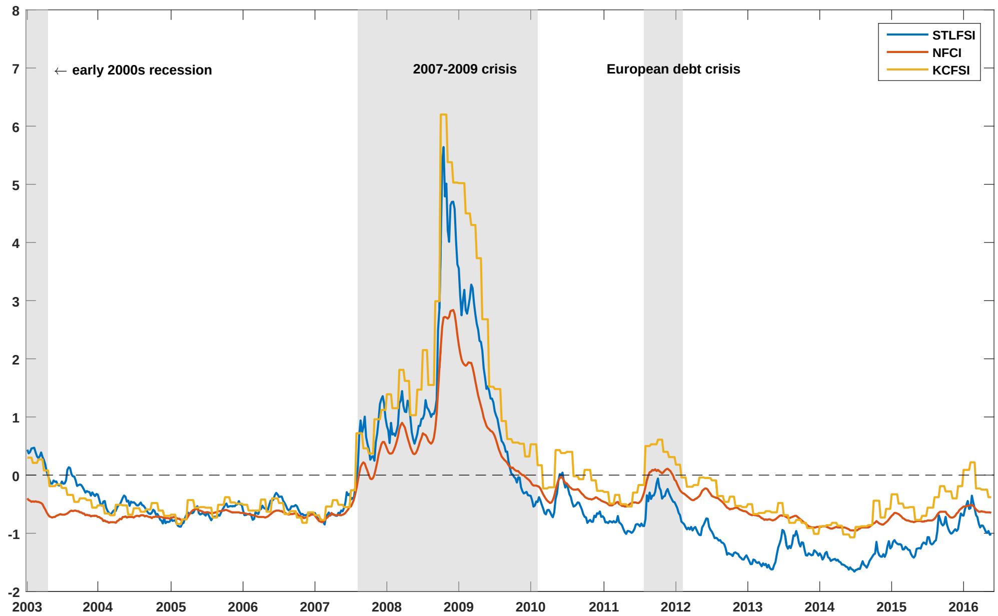
variables, they are good indicators of financial stress level. Manamperi (2013) provides a comparative analysis of the financial stress indexes available for the U.S. and concludes that in general, these financial stress indexes indicates a higher financial risk during the 2008 financial crisis. Kliesen et al. (2012) surveys a number of financial stress indexes and concludes that they are highly correlated but not as high as might be expected.

Three FSIs are used to identify the financial stress level. They are St. Louis Fed Financial Stress Index (STLFSI), The Chicago Fed's National Financial Conditions Index (NFCI) and Kansas City Financial Stress Index (KCFSI). These three indexes are widely used in literature studying financial stress or crisis and they are constructed by including variables belonging to money market, fixed income, equity, foreign exchange and banking sectors. Federal reserve banks provide descriptions on FSIs in terms of the relationship between the index value and the severity of financial stress. All three indexes are standardized with mean 0 and standard deviation 1. If the value is below 0 it indicates below normal financial stress and if value is above zero it indicates above normal financial stress. I take average of three indexes and label it as y. When the averaged value is above 0, binary variable z_{fsi} is identified and assigned as 1. When it is below 0, there is no financial stress and $z_{fsi} = 0$.

Figure 4.1 shows the movement of the FSIs across the whole period studied. There are two major periods that FSI curves bounce around zero. The first one belongs to early 2000 recession and the other is the European debt crisis period. The period with the spike reveals 07-09 crisis.

I also conduct the analysis based on Probit model. To define the binary data, the results from Bloom(2009) and Ng(2015) are used along with three FSIs to identify the occurrence of financial stress.

Figure 4.1: Financial Stress Indexes



Both papers identify big macro uncertainty periods on history and the months are defined according to the NBER Business Cycle Dating Committee. If the period is classified as macro uncertainty by either paper, I assign $z_{bloom} = 1$ or $z_{Ng} = 1$. I define binary variable $z = 1$ as long as one of the three: z_{fsi}, z_{bloom} and z_{Ng}, is 1, otherwise $z = 0$. For the whole period studied, total number of $z = 1$ is 537 and $z = 0$ is 161.

4.3.3 In-Sample Analysis

In-sample analysis shows the variable selection and estimates using the whole sample period. The results are based on 1-week, 1-month, 3-month ahead predictions.

Table 4.2, 4.3, 4.4 and 4.5 show the variable selection and coefficient estimates with 1-week, 1-month and 3-month lags for Linear and Probit model respectively. Variables are selected if $\hat{\gamma} \geq 0.5$. The result shows Different variables are selected according to different models. For Linear model under normal error assumption, interest rates and market returns are less informative predicting financial stress. Inflation is not selected for 1-month and 3-month prediction. For Linear model under student-t assumption with 5 degree of freedom, variables belonging to all categories are selected. For Probit model with normal assumption, inflation and market return variables are also selected along with yield spread and volatility. Interest rate still provides litter information in predicting future. For Probit model with student-t assumption, more variables are selected across all five categories. For 1-week ahead prediction with linear model, the variance of the error term is very small which leads to positive log-likelihood. Also it shows that the bigger the prediction time gap, the bigger the variance of the error term. It is easy to understand that it is more likely to get more accurate prediction for the data in the near future than further.

Results are then evaluated using predictive likelihoods throughout the sample by calculating $\mathrm{E}_{\theta|y}[p(\hat{y}|y,\theta)]$ under Bayesian framework. The probability can be calculated as $N_{cdf}(\hat{y} > 0; \hat{\sigma}^2)$ or $t_{cdf}(\hat{y} > 0; \hat{\sigma}^2, v)$ based on different assumptions of error term. Here N_{cdf} and t_{cdf} represent the probability cumulative functions of normal and student-t distributions. The results are provided in Figure 4.2, 4.3, 4.4 and 4.5. When estimated value of financial stress level is close to 0, the estimated probability of the occurrence of financial stress is close to 0.5. It happens when we suffer mild financial stress, or enter or exist several financial stress period. The results are consistent with the observations.

Figure 4.6 and Figure 4.7 compare the estimated values of financial stress level with the true values. The true value is the average of three indices: STLFSI, NFCI and KCFSI. All these figures suggest that the nearer the predicting period the more precise the estimate is. The deviation is more bigger during the 07-09 crisis period.

The in-sample prediction performances are also presented in two other ways. The first measure is the Mean Squared Error (MSE). Table 4.6 shows the MSE under different scenarios. The MSE gets bigger when the prediction time gap increases for both models. For 1-week and 1-month prediction, normal assumption is slightly better. For 3-month prediction, student-t assumption performances better.

The second way is called signal approach. I define a binary variable Obs to be 1 if $y > 0$ and 0 otherwise. If estimates $\hat{y} > 0$ a financial stress signal S = 1 is assigned, if $\hat{y} < 0$, S = 0. Table 4.7 and 4.8 display the correct ratios under different scenarios and the results are promising. It shows all models show good in-sample performance especially for 1-week prediction. As the prediction time gap increases, the precision decreases.

Table 4.2: In Sample Variable Selection: Linear Model

Variables		Linear Normal			Linear Student-t (5)		
		1-week	1-month	3-month	1-week	1-month	3-month
Interest Rate	effective federal funds rate	**0.69**	0.10	**0.68**	**0.98**	**0.94**	0.45
	30-year treasury interest rate	0.01	0.01	0.01	**0.98**	**0.95**	**0.59**
	effective federal funds rate (first difference)	0.01	0.01	0.01	**0.77**	**0.75**	0.20
	2-year treasury interest rate (first difference)	0.00	0.22	0.03	**0.51**	0.33	0.04
	10-year treasury interest rate (first difference)	0.00	0.01	0.16	**0.54**	0.36	0.15
	30-year treasury interest rate (first difference)	0.00	0.01	0.04	0.42	0.46	0.19
Yield Spread	10-year treasury interest rate - 2-year treasury interest rate	0.00	0.05	0.04	0.14	0.24	0.27
	Merrill Lynch Asset Backed Master BBB-AA rated interest rate	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**
	10-year treasury - 3-month treasury	0.26	0.11	0.12	0.21	0.09	0.45
	Merrill Lynch High Yield corporate master II index - 10-year treasury	**1.00**	**0.99**	0.01	**1.00**	**1.00**	**0.94**
	Moody's Seasoned Baa Corporate rated bond - 10 year treasury	0.01	0.01	0.06	**0.51**	0.12	0.08
	3-month treasury Eurodollar (TED) spread	0.08	**1.00**	0.40	**0.93**	**1.00**	0.07
	3-month AA Financial commercial paper - 3-month treasury	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**
	3-month LIBOR - OIS	**1.00**	**1.00**	0.15	**1.00**	**1.00**	0.10
Volatility	VIX	**1.00**	**1.00**	0.10	**1.00**	**1.00**	**1.00**
	Merrill Lynch 1 month Bond Market Volatility Index	**1.00**	0.00	0.04	**1.00**	**1.00**	**0.88**
Inflation	10-year nominal treasury - TIPS	**0.69**	0.00	0.01	**1.00**	**1.00**	0.07
Return	S&P 500 Index	0.07	0.08	0.02	**0.81**	**0.96**	0.05
	J.P. Morgan Emerging Markets Bond Index	0.02	0.01	0.46	**1.00**	**1.00**	**0.82**

Table 4.3: In Sample Coefficient Estimates: Linear Model

Variables		Linear Normal			Linear Student-t (5)		
		1-week	1-month	3-month	1-week	1-month	3-month
	Intercept	-0.22 (0.01)	-0.23 (0.01)	-0.23 (0.02)	-0.23 (0.01)	-0.26 (0.01)	-0.31 (0.01)
Interest Rate	effective federal funds rate	**0.02** **(0.02)**	0.01 (0.02)	**-0.08** **(0.07)**	**0.07** **(0.03)**	**0.10** **(0.05)**	0.02 (0.06)
	30-year treasury interest rate	0.00 (0.00)	0.00 (0.00)	0.00 (0.02)	**0.04** **(0.02)**	**0.06** **(0.04)**	**0.06** **(0.06)**
	effective federal funds rate (first difference)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	**0.00** **(0.01)**	**-0.01** **(0.01)**	-0.01 (0.01)
	2-year treasury interest rate (first difference)	0.00 (0.00)	-0.01 (0.02)	0.00 (0.01)	**0.00** **(0.01)**	0.00 (0.01)	0.00 (0.00)
	10-year treasury interest rate (first difference)	0.00 (0.00)	0.00 (0.00)	-0.01 (0.02)	**0.00** **(0.01)**	0.00 (0.01)	0.00 (0.01)
	30-year treasury interest rate (first difference)	0.00 (0.00)	0.00 (0.00)	0.00 (0.01)	0.00 (0.01)	0.01 (0.01)	0.00 (0.01)
Yield Spread	10-year treasury interest rate - 2-year treasury interest rate	0.00 (0.00)	0.00 (0.01)	0.00 (0.01)	0.00 (0.01)	-0.02 (0.03)	0.00 (0.03)
	Merrill Lynch Asset Backed Master BBB-AA rated interest rate	**0.33** **(0.02)**	**0.42** **(0.03)**	**0.56** **(0.04)**	**0.27** **(0.02)**	**0.24** **(0.03)**	**0.26** **(0.04)**
	10-year treasury - 3-month treasury	-0.01 (0.01)	0.00 (0.01)	-0.01 (0.04)	0.00 (0.02)	0.00 (0.02)	-0.01 (0.04)
	Merrill Lynch High Yield corporate master II index - 10-year treasury	**0.24** **(0.04)**	**0.14** **(0.04)**	0.00 (0.01)	**0.34** **(0.03)**	**0.30** **(0.05)**	**0.13** **(0.07)**
	Moody's Seasoned Baa Corporate rated bond - 10 year treasury	0.00 (0.00)	0.00 (0.01)	0.01 (0.03)	**0.00** **(0.02)**	-0.01 (0.02)	0.00 (0.02)
	3-month treasury Eurodollar (TED) spread	-0.01 (0.02)	**-0.21** **(0.08)**	-0.11 (0.15)	**-0.02** **(0.04)**	**-0.27** **(0.07)**	-0.01 (0.04)
	3-month AA Financial commercial paper - 3-month treasury	**0.14** **(0.02)**	**0.38** **(0.04)**	**0.59** **(0.09)**	**0.14** **(0.03)**	**0.29** **(0.04)**	**0.38** **(0.04)**
	3-month LIBOR - OIS	**0.21** **(0.02)**	**0.23** **(0.05)**	0.02 (0.06)	**0.17** **(0.03)**	**0.31** **(0.04)**	0.01 (0.03)
Volatility	VIX	**0.20** **(0.02)**	**0.17** **(0.03)**	0.01 (0.03)	**0.18** **(0.01)**	**0.16** **(0.02)**	**0.13** **(0.03)**
	Merrill Lynch 1 month Bond Market Volatility Index	**0.06** **(0.01)**	0.00 (0.00)	0.00 (0.01)	**0.08** **(0.01)**	**0.09** **(0.02)**	**0.05** **(0.03)**
Inflation	10-year nominal treasury - TIPS	**0.03** **(0.02)**	0.00 (0.00)	0.00 (0.00)	**0.05** **(0.01)**	**0.04** **(0.01)**	0.00 (0.01)
Return	S&P 500 Index	0.00 (0.00)	0.00 (0.01)	0.00 (0.00)	**-0.01** **(0.01)**	**-0.02** **(0.01)**	0.00 (0.00)
	J.P. Morgan Emerging Markets Bond Index	0.00 (0.00)	0.00 (0.00)	0.04 (0.05)	**0.05** **(0.02)**	**0.07** **(0.02)**	**0.06** **(0.04)**
	Variance of Error Term	0.02 (0.00)	0.08 (0.00)	0.21 (0.01)	0.01 (0.00)	0.02 (0.00)	0.05 (0.00)
	LogL	544.63	124.66	-300.23	611.05	223.29	-366.54

Table 4.4: In Sample Variable Selection: Probit Model

Variables		Probit Normal			Probit Student-t (5)		
		1-week	1-month	3-month	1-week	1-month	3-month
Interest Rate	effective federal funds rate	0.17	0.68	0.48	0.84	0.33	0.62
	30-year treasury interest rate	0.33	1.00	1.00	0.80	1.00	1.00
	effective federal funds rate (first difference)	0.10	0.05	0.06	0.82	0.30	0.30
	2-year treasury interest rate (first difference)	0.11	0.07	0.07	0.52	0.19	0.29
	10-year treasury interest rate (first difference)	0.13	0.13	0.16	0.35	0.16	0.21
	30-year treasury interest rate (first difference)	0.15	0.27	0.23	0.74	0.32	0.57
Yield Spread	10-year treasury interest rate - 2-year treasury interest rate	0.14	0.28	1.00	0.93	0.59	1.00
	Merrill Lynch Asset Backed Master BBB-AA rated interest rate	1.00	0.95	1.00	1.00	0.84	1.00
	10-year treasury - 3-month treasury	0.37	0.68	1.00	0.79	0.93	1.00
	Merrill Lynch High Yield corporate master II index - 10-year treasury	0.82	0.77	0.96	0.95	0.98	0.96
	Moody's Seasoned Baa Corporate rated bond - 10 year treasury	0.30	0.34	0.31	0.99	0.29	0.33
	3-month treasury Eurodollar (TED) spread	0.69	0.22	1.00	0.52	0.47	0.98
	3-month AA Financial commercial paper - 3-month treasury	0.74	0.42	0.15	1.00	0.58	0.73
	3-month LIBOR - OIS	0.48	0.25	0.52	0.66	0.52	0.90
Volatility	VIX	1.00	1.00	1.00	1.00	1.00	0.90
	Merrill Lynch 1 month Bond Market Volatility Index	0.42	1.00	0.39	1.00	1.00	0.54
Inflation	10-year nominal treasury - TIPS	0.76	0.59	0.11	1.00	0.78	0.30
Return	S&P 500 Index	0.16	0.05	0.08	0.52	0.15	0.17
	J.P. Morgan Emerging Markets Bond Index	0.26	1.00	1.00	1.00	1.00	1.00

Table 4.5: In Sample Coefficient Estimates: Probit Model

Variables		Probit Normal			Probit Student-t (5)		
		1-week	1-month	3-month	1-week	1-month	3-month
	Intercept	-1.60	-1.66	-1.93	-1.81	-1.92	-2.26
		(0.38)	(0.29)	(0.41)	(0.52)	(0.32)	(0.35)
Interest Rate	effective federal funds rate	0.10	0.66	-1.69	2.24	0.59	-1.80
		(0.43)	(1.22)	(1.97)	(2.23)	(1.25)	(1.95)
	30-year treasury interest rate	0.04	2.48	5.37	1.49	3.55	5.89
		(0.66)	(0.96)	(1.14)	(1.68)	(1.08)	(1.19)
	effective federal funds rate (first difference)	0.04	0.00	0.00	0.15	0.00	-0.01
		(0.18)	(0.05)	(0.07)	(0.51)	(0.17)	(0.21)
	2-year treasury interest rate (first difference)	0.01	-0.01	0.01	0.18	-0.03	0.12
		(0.11)	(0.07)	(0.07)	(0.35)	(0.11)	(0.24)
	10-year treasury interest rate (first difference)	0.01	-0.03	-0.03	-0.04	-0.04	-0.05
		(0.21)	(0.11)	(0.16)	(0.43)	(0.12)	(0.18)
	30-year treasury interest rate (first difference)	-0.07	-0.08	-0.13	-0.52	-0.09	-0.20
		(0.25)	(0.15)	(0.21)	(0.48)	(0.17)	(0.23)
Yield Spread	10-year treasury interest rate - 2-year treasury interest rate	-0.01	0.57	2.94	0.98	1.10	3.52
		(0.36)	(1.13)	(0.97)	(2.05)	(1.23)	(1.00)
	Merrill Lynch Asset Backed Master BBB-AA rated interest rate	3.29	1.46	2.04	6.33	1.59	2.73
		(0.94)	(0.85)	(0.74)	(1.55)	(1.04)	(0.99)
	10-year treasury - 3-month treasury	-0.23	-1.00	-4.23	-1.37	-1.90	-4.78
		(0.47)	(1.14)	(1.35)	(1.80)	(1.21)	(1.29)
	Merrill Lynch High Yield corporate master II index - 10-year treasury	1.95	1.86	2.66	2.90	3.05	2.63
		(1.73)	(1.40)	(1.12)	(2.07)	(1.50)	(1.22)
	Moody's Seasoned Baa Corporate rated bond - 10 year treasury	0.05	0.04	-0.45	4.02	-0.16	-0.35
		(0.75)	(0.86)	(0.84)	(1.76)	(0.68)	(0.79)
	3-month treasury Eurodollar (TED) spread	0.36	0.06	3.04	0.21	-0.10	3.72
		(1.56)	(0.61)	(2.29)	(1.37)	(1.07)	(1.50)
	3-month AA Financial commercial paper - 3-month treasury	2.11	0.40	0.80	4.70	0.74	1.04
		(1.78)	(0.71)	(1.16)	(1.56)	(1.12)	(1.25)
	3-month LIBOR - OIS	0.18	0.14	-1.29	0.17	0.43	-2.05
		(0.95)	(0.48)	(1.30)	(1.37)	(0.90)	(1.12)
Volatility	VIX	3.08	1.91	0.53	3.99	2.01	0.88
		(0.65)	(0.44)	(0.56)	(0.97)	(0.49)	(0.50)
	Merrill Lynch 1 month Bond Market Volatility Index	0.58	1.31	-0.16	3.36	1.66	-0.41
		(0.88)	(0.42)	(0.33)	(0.91)	(0.47)	(0.50)
Inflation	10-year nominal treasury - TIPS	1.12	0.50	0.00	3.03	0.73	0.04
		(0.85)	(0.52)	(0.14)	(0.94)	(0.55)	(0.23)
Return	S&P 500 Index	0.03	0.01	0.00	0.05	0.00	0.03
		(0.11)	(0.06)	(0.03)	(0.26)	(0.06)	(0.10)
	J.P. Morgan Emerging Markets Bond Index	0.28	3.09	5.72	3.24	4.08	6.52
		(0.71)	(0.73)	(1.03)	(1.30)	(0.90)	(1.14)
	LogL	-27.13	-49.95	-546.88	-18.85	-61.3863	-205.13

Figure 4.2: Probability of Occurrence of Financial Stress - Linear Model with Normal Assumption

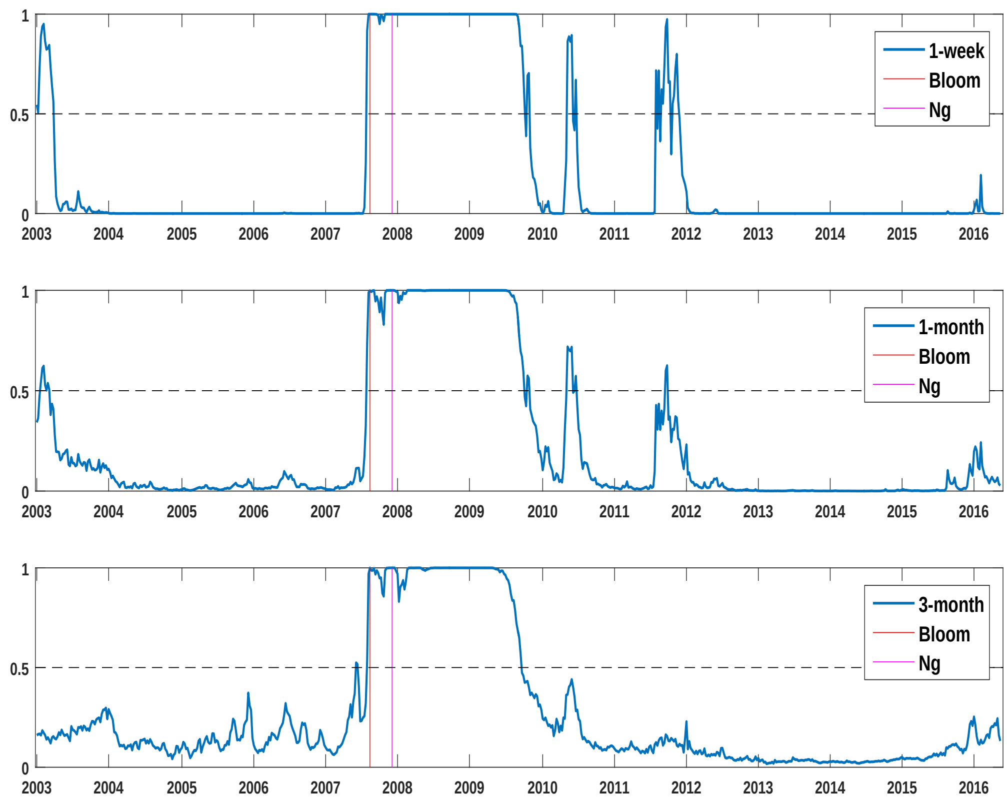


Figure 4.3: Probability of Occurrence of Financial Stress - Linear Model with Student-t Assumption

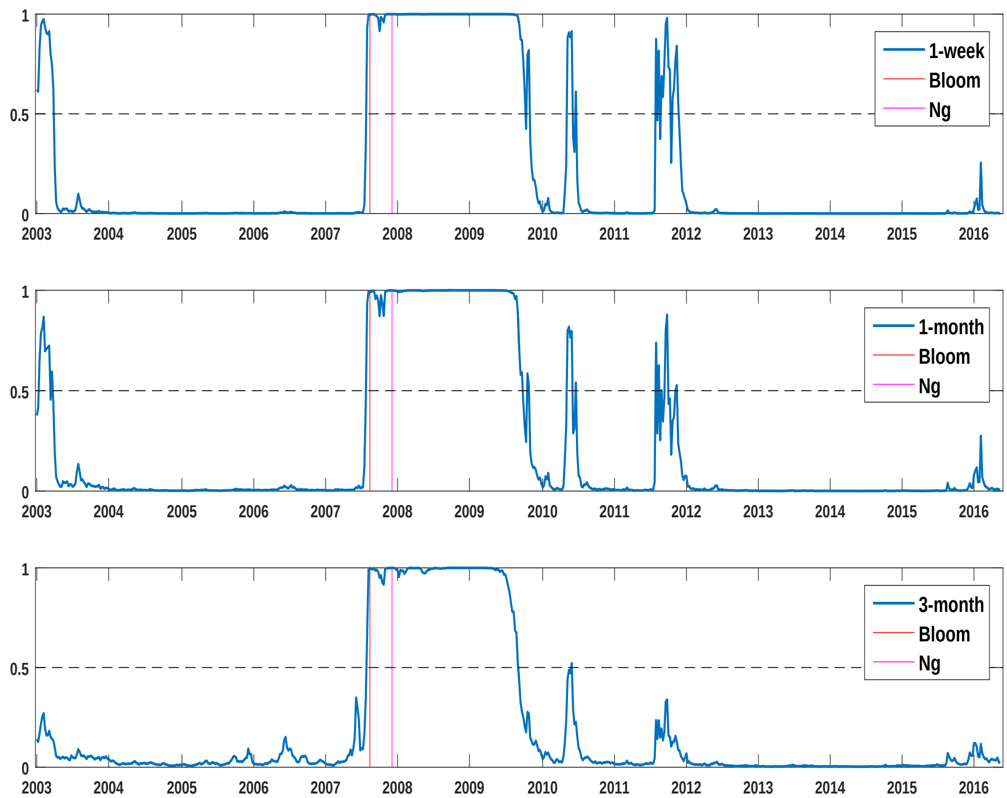


Figure 4.4: Probability of Financial Stress - Probit Model with Normal Assumption

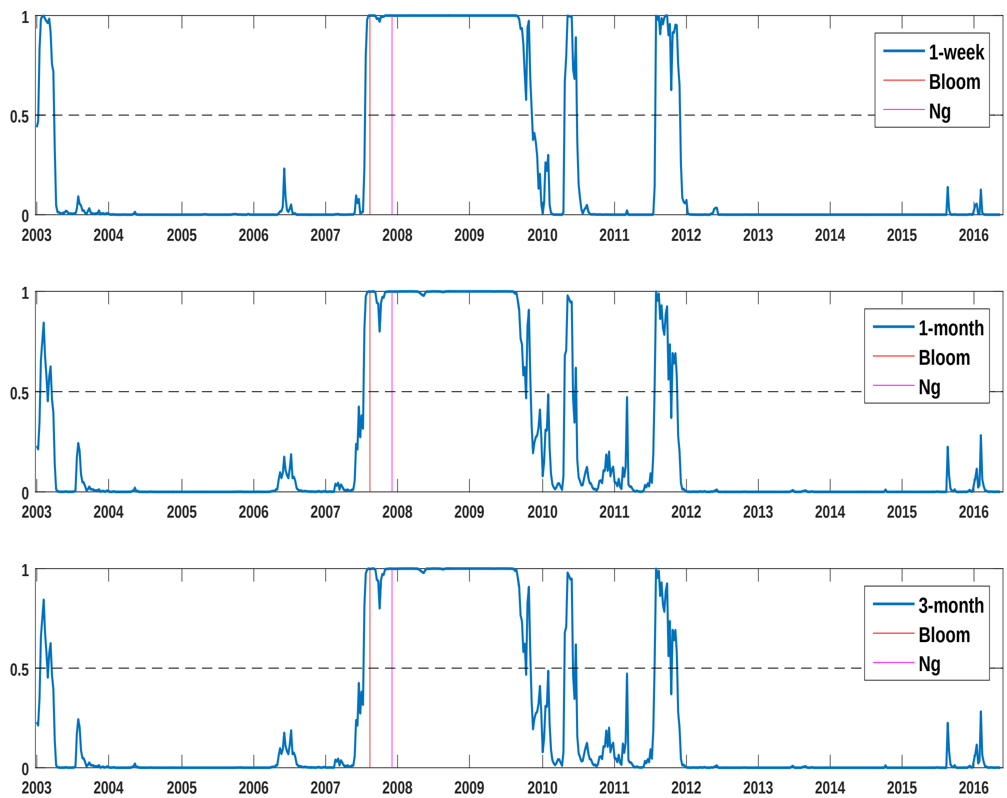


Figure 4.5: Probability of Financial Stress - Probit Model with Student-t Assumption

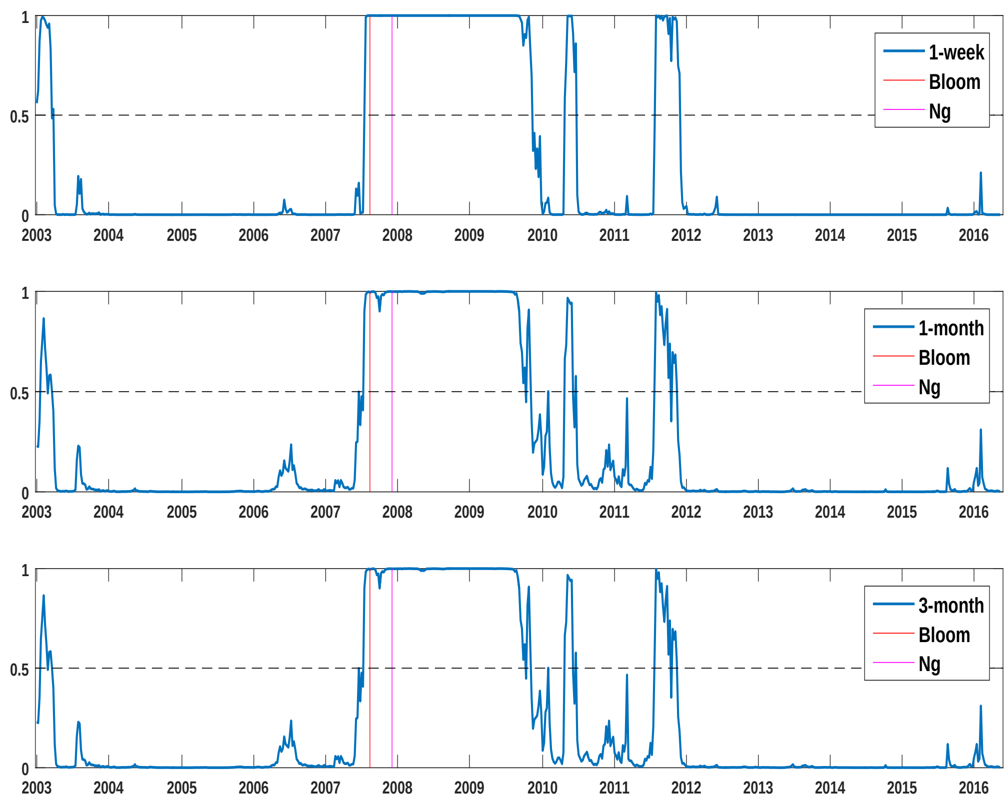
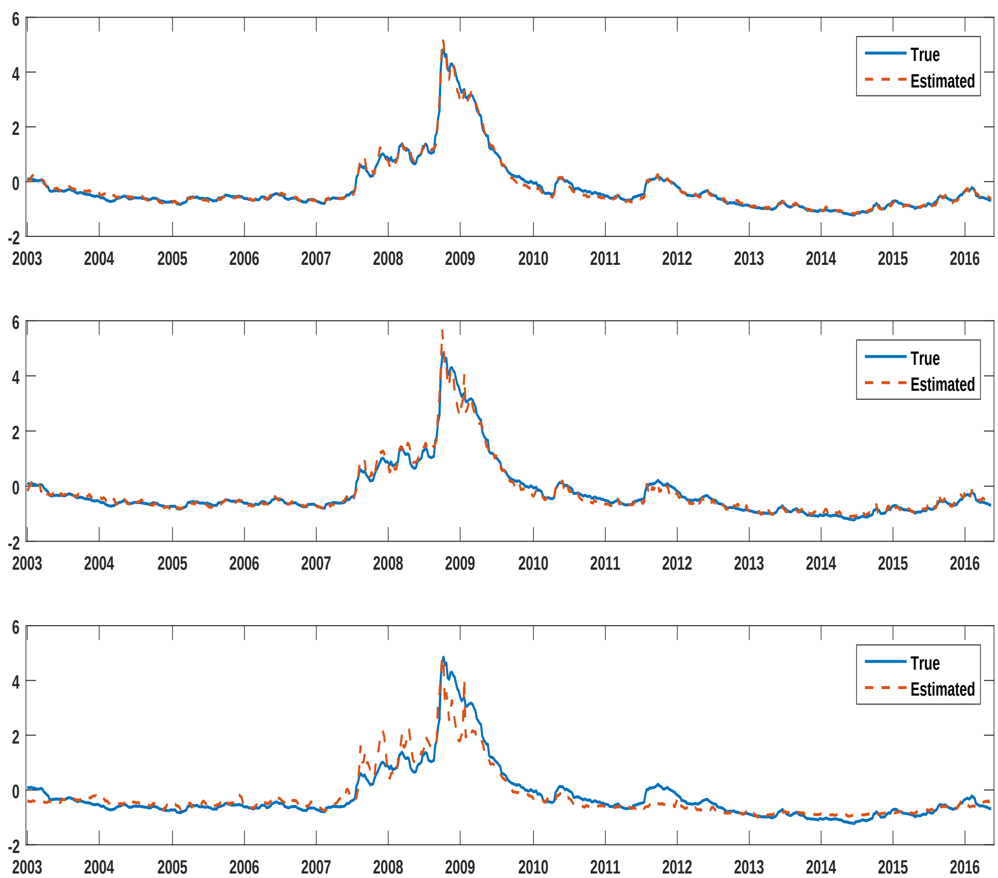
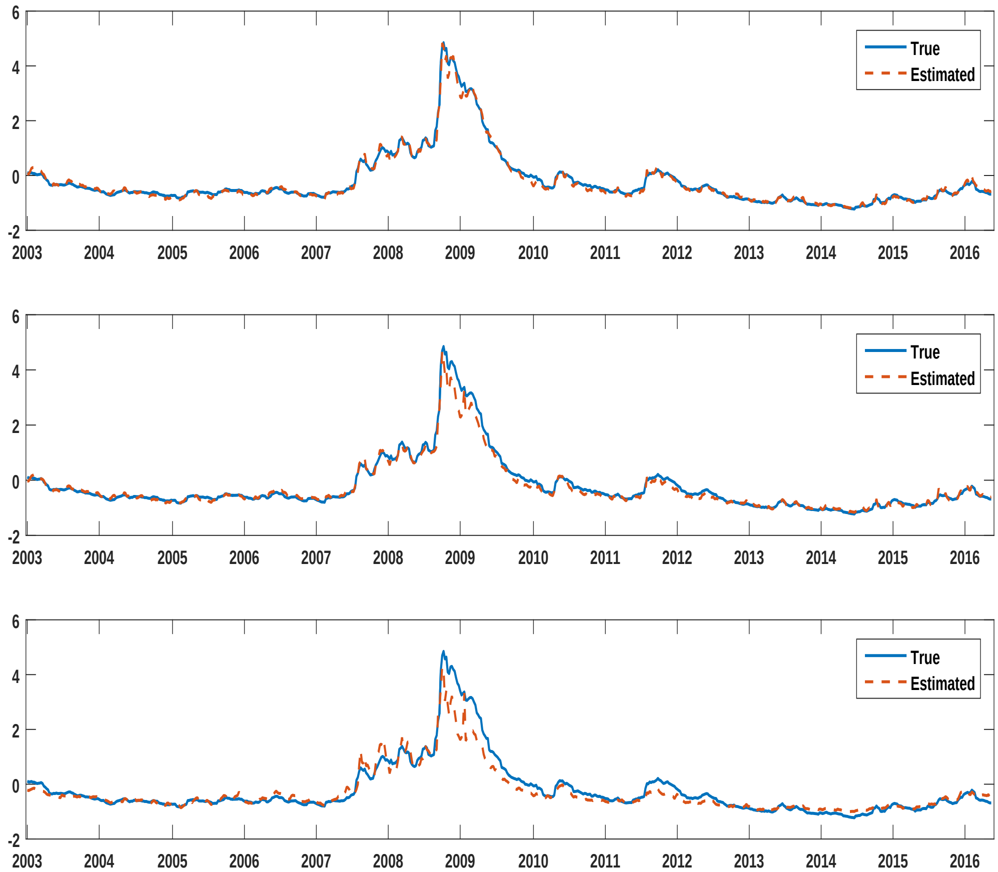


Figure 4.6: Prediction vs True value - Linear Model with Normal Assumption



Note: True = average of three indexes: STLFSI, NFCI and KCFSI

Figure 4.7: Prediction vs True value - Linear Model with Student-t Assumption



Note: True = average of three indexes: STLFSI, NFCI and KCFSI

Table 4.6: Prediction Performance - Linear Model Mean Squared Error

	Normal	Student - t
1-week	0.0098	0.0100
1-month	0.0230	0.0298
3-month	0.1238	0.1077

Table 4.7: Prediction Performance - Financial Stress Signal - Linear Model

1-week

Normal	S = 0	S = 1	Total	% Correct	% Incorrect
Obs = 0	536	1	537	99.81	0.19
Obs = 1	13	148	161	91.93	8.07

Student - t	S = 0	S = 1	Total	% Correct	% Incorrect
Obs = 0	536	1	537	99.81	0.19
Obs = 1	11	150	161	93.17	6.83

1-month

Normal	S = 0	S = 1	Total	% Correct	% Incorrect
Obs = 0	537	0	537	100.00	0.00
Obs = 1	28	133	161	82.61	17.39

Student - t	S = 0	S = 1	Total	% Correct	% Incorrect
Obs = 0	537	0	537	100.00	0.00
Obs = 1	26	135	161	83.85	16.15

3-month

Normal	S = 0	S = 1	Total	% Correct	% Incorrect
Obs = 0	535	2	537	99.63	0.37
Obs = 1	50	111	161	68.94	31.06

Student - t	S = 0	S = 1	Total	% Correct	% Incorrect
Obs = 0	537	0	537	100.00	0.00
Obs = 1	52	109	161	67.70	32.30

Notes: (1) If S = 1, then a financial stress signal is issued.

Table 4.8: Prediction Performance - Financial Stress Signal - Probit Model

1-week

Normal	S = 0	S = 1	Total	% Correct	% Incorrect	Student - t	S = 0	S = 1	Total	% Correct	% Incorrect
Obs = 0	531	6	537	98.88	1.12	Obs = 0	533	4	537	99.26	0.74
Obs = 1	6	155	161	96.27	3.73	Obs = 1	3	158	161	98.14	1.86

1-month

Normal	S = 0	S = 1	Total	% Correct	% Incorrect	Student - t	S = 0	S = 1	Total	% Correct	% Incorrect
Obs = 0	531	6	537	98.88	1.12	Obs = 0	527	10	537	98.14	1.86
Obs = 1	17	144	161	89.44	10.56	Obs = 1	18	143	161	88.82	11.18

3-month

Normal	S = 0	S = 1	Total	% Correct	% Incorrect	Student - t	S = 0	S = 1	Total	% Correct	% Incorrect
Obs = 0	526	11	537	97.95	2.05	Obs = 0	515	22	537	95.90	4.10
Obs = 1	47	114	161	70.81	29.19	Obs = 1	44	117	161	72.67	27.33

Notes: (1) If S = 1, then a financial stress signal is issued.

4.3.4 Out-of-Sample Prediction

The out-of-sample predictions are conducted under linear model by using 52-week rolling window based on both normal error assumption and fat tail assumption. Probit model may have substantial bias when the ratio of event occurrence is too small by using rolling window method. In this part, only linear model is considered for analysis. The reason of using rolling window is that recent data is more informative in predict near future. And excluding earlier period data can reduce the noise. The length of rolling window is quite intuitive and here I use 52-week window which represents one year length.

- 1-week ahead prediction, start predicted date: 1/16/2004;
- 1-month ahead prediction, start predicted date: 2/27/2004;
- 3-month ahead prediction, start predicted date: 6/18/2004

Figure 4.8 and Figure 4.9 show the variable selection results for three prediction gaps under normal and student-t distributions respectively. In each sub-figure, integers 1 to 19 on y axis represent the label of 19 potential variables in Table 4.1. It is interesting to see that before the 07-09 crisis, there is a period that none of the potential indicators is selected. The level of financial stress depends only on the intercept in the linear model. More variables are selected during the 07-09 crisis period. And with the impact of economic crisis weakened, less variables are selected.

Variable 3,4,5 and 6 belong to the interest rate (first difference) category and they are seldom selected especially under 1-week and 1-month scenarios. During the financial stress periods, from 07-09 crisis to European debt crisis period, most of the selected variables belong to yield spread, market volatility, inflation and market return categories. Moreover the bigger the predicting time gap, the more variables are selected as leading indicators for the prediction given a time point. It is also interesting to see that variable 18 which is the log return of S&P 500 index is never selected.

Figure 4.8: Variable Selection under Normal Assumption

(a) 1-week prediction

(b) 1-month prediction

(c) 3-month prediction

Note: total 19 variables. 1 - effective federal funds rate; 2 - 30-year treasury interest rate; 3 - effective federal funds rate (first difference); 4 - 2-year treasury interest rate (first difference); 5 - 10-year treasury interest rate (first difference); 6 - 30-year treasury interest rate (first difference); 7 - 10-year treasury interest rate - 2-year treasury interest rate; 8 - Merrill Lynch Asset Backed Master BBB-AA rated interest rate; 9 - 10-year treasury - 3-month treasury; 10 - Merrill Lynch High Yield corporate master II index - 10-year treasury; 11 - Moody's Seasoned Baa Corporate rated bond - 10 year treasury; 12 - 3-month treasury Eurodollar (TED) spread; 13 - 3-month AA Financial commercial paper - 3-month treasury; 14 - 3-month LIBOR - OIS; 15 - VIX; 16 - Merrill Lynch 1 month Bond Market Volatility Index; 17 - 10-year nominal treasury - TIPS; 18 - S&P 500 Index (return); 19 - J.P. Morgan Emerging Markets Bond Index (return).

Figure 4.9: Variable Selection under Student-t Assumption

(a) 1-week prediction

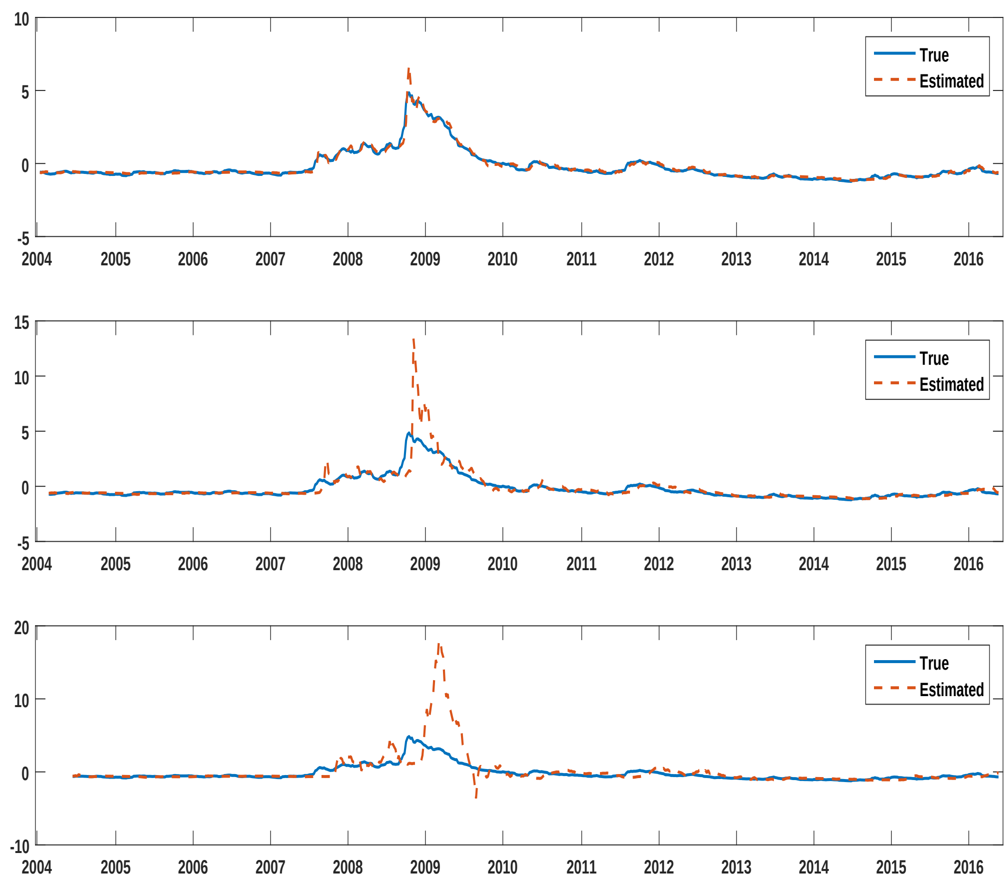
(b) 1-month prediction

(c) 3-month prediction

Note: total 19 variables. 1 - effective federal funds rate; 2 - 30-year treasury interest rate; 3 - effective federal funds rate (first difference); 4 - 2-year treasury interest rate (first difference); 5 - 10-year treasury interest rate (first difference); 6 - 30-year treasury interest rate (first difference); 7 - 10-year treasury interest rate - 2-year treasury interest rate; 8 - Merrill Lynch Asset Backed Master BBB-AA rated interest rate; 9 - 10-year treasury - 3-month treasury; 10 - Merrill Lynch High Yield corporate master II index - 10-year treasury; 11 - Moody's Seasoned Baa Corporate rated bond - 10 year treasury; 12 - 3-month treasury Eurodollar (TED) spread; 13 - 3-month AA Financial commercial paper - 3-month treasury; 14 - 3-month LIBOR - OIS; 15 - VIX; 16 - Merrill Lynch 1 month Bond Market Volatility Index; 17 - 10-year nominal treasury - TIPS; 18 - S&P 500 Index (return); 19 - J.P. Morgan Emerging Markets Bond Index (return).

Figure 4.10 and Figure 4.11 compare the estimated values of financial stress level with the true values, and Figure 4.12 and Figure 4.13 show the absolute errors of estimates. The same as in-sample prediction, all these figures suggest that the nearer the predicting period the more precise the estimate is. The deviation is more bigger during the 07-09 crisis period.

Figure 4.10: Prediction vs True value - Normal Assumption

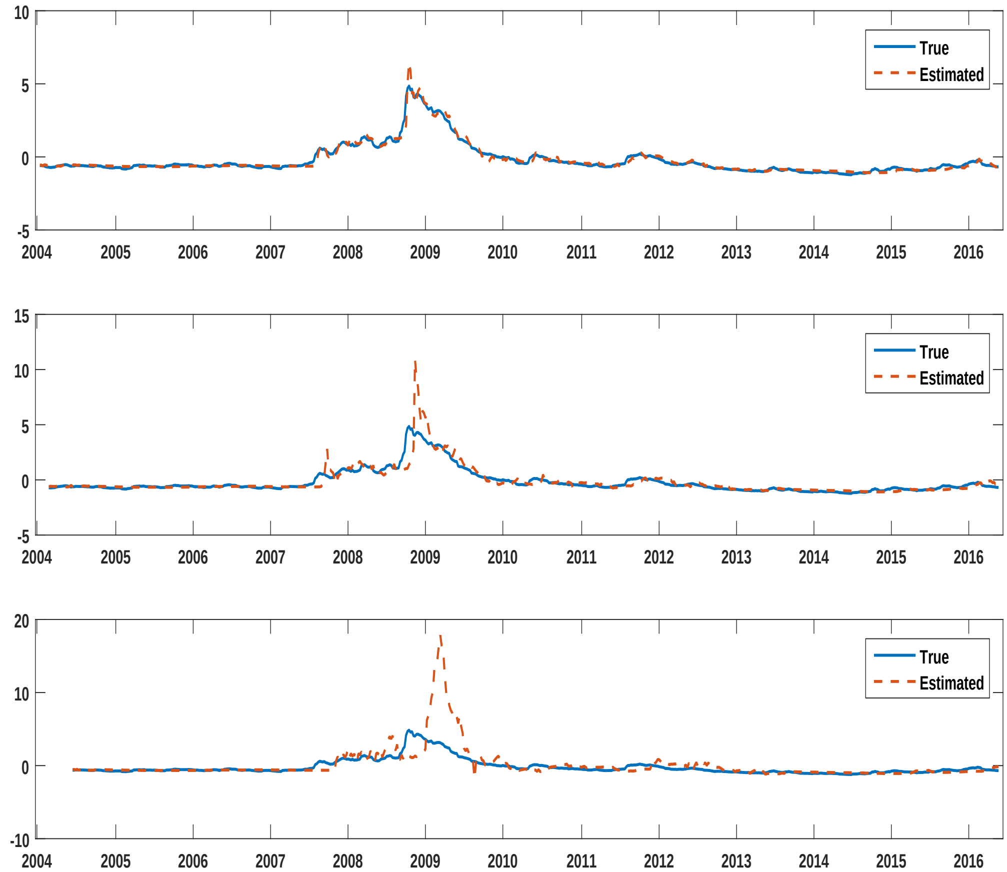


Note: True = average of three indexes: STLFSI, NFCI and KCFSI

The probability of occurrence of financial stress are displayed in Figure 4.14 and 4.15. It is easy to see that during 07-09 financial crisis period, the probability of occurrence of financial stress stay close to 1. The overall results are consistent with the observations.

In this section the out-of-sample prediction performances are also displayed in two ways: MSE snd signal approach. Table 4.9 shows the MSE under different scenarios. The MSE gets bigger when the prediction time gap increases for both models. For 1-week prediction, normal assumption is slightly better. For 1-month and 3-month prediction, student-t assumption performances better.

Figure 4.11: Prediction vs True value - Student-t Assumption



Note: True = average of three indexes: STLFSI, NFCI and KCFSI

Table 4.9: Prediction Performance - Mean Squared Error

	Normal	Student - t
1-week	0.0316	0.0419
1-month	0.4571	0.2509
3-month	2.6027	2.3559

Figure 4.12: Absolute Value of Prediction Errors - Normal Assumption

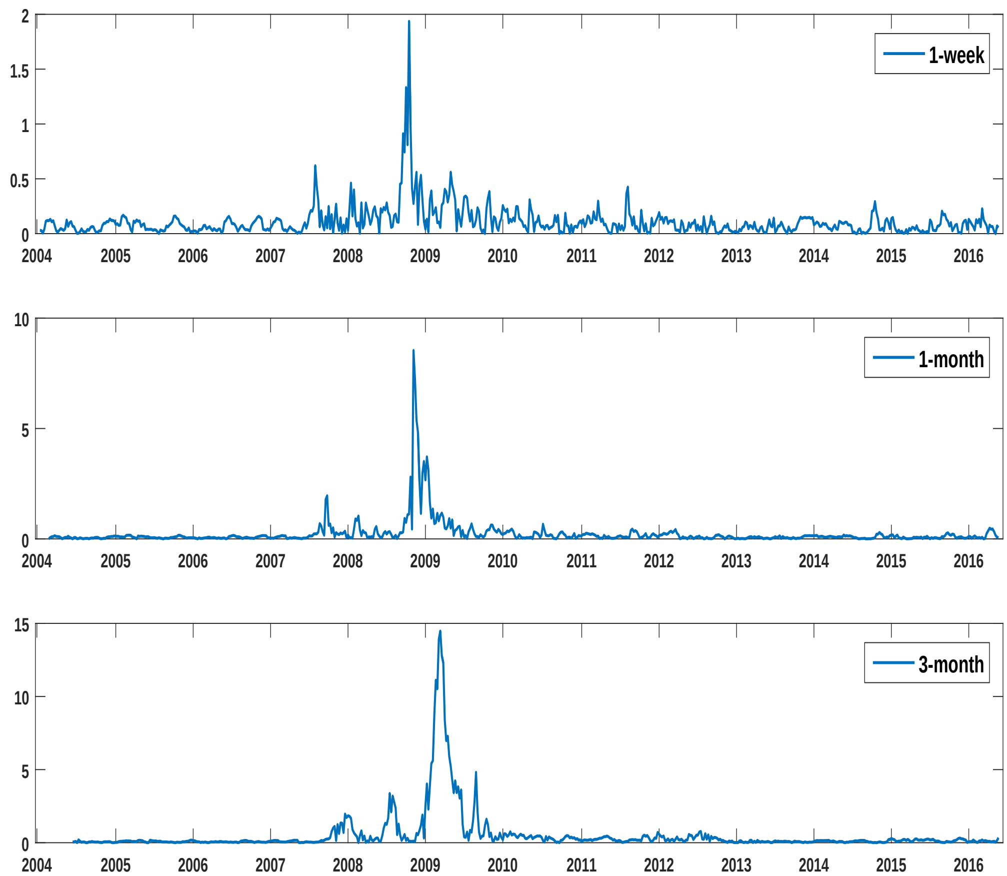


Figure 4.13: Absolute Value of Prediction Errors - Student-t Assumption

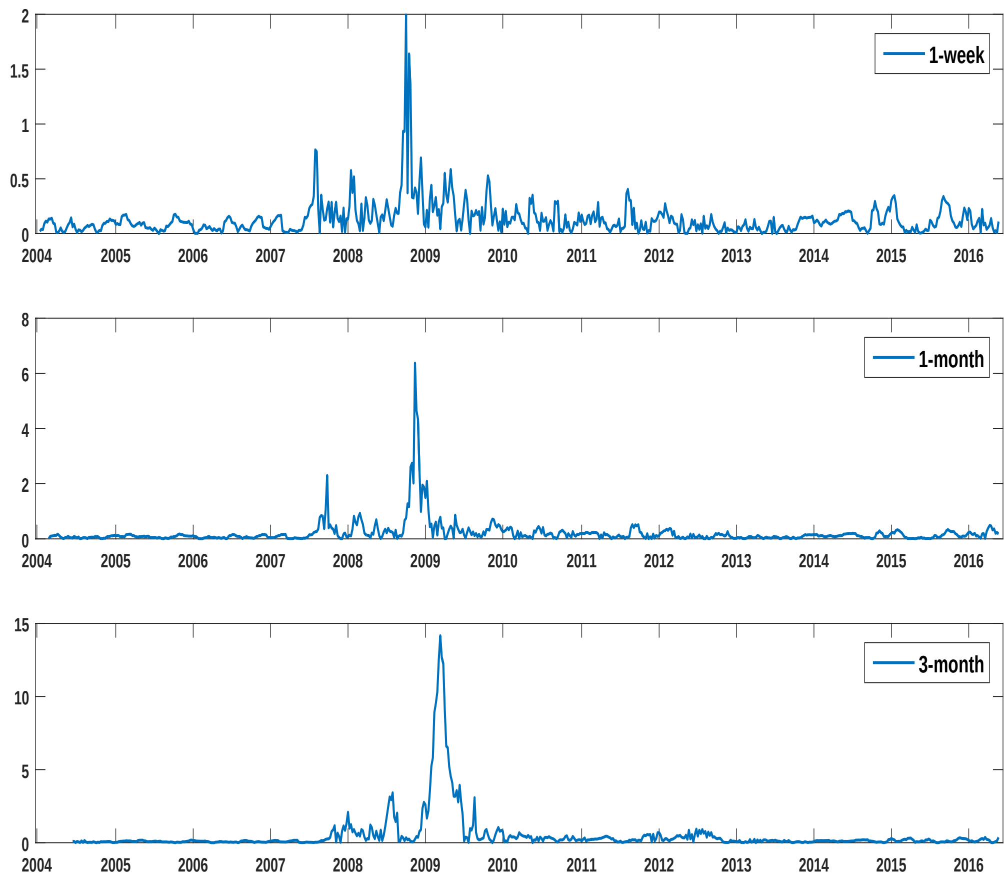


Figure 4.14: Probability of Financial Stress - Normal Assumption

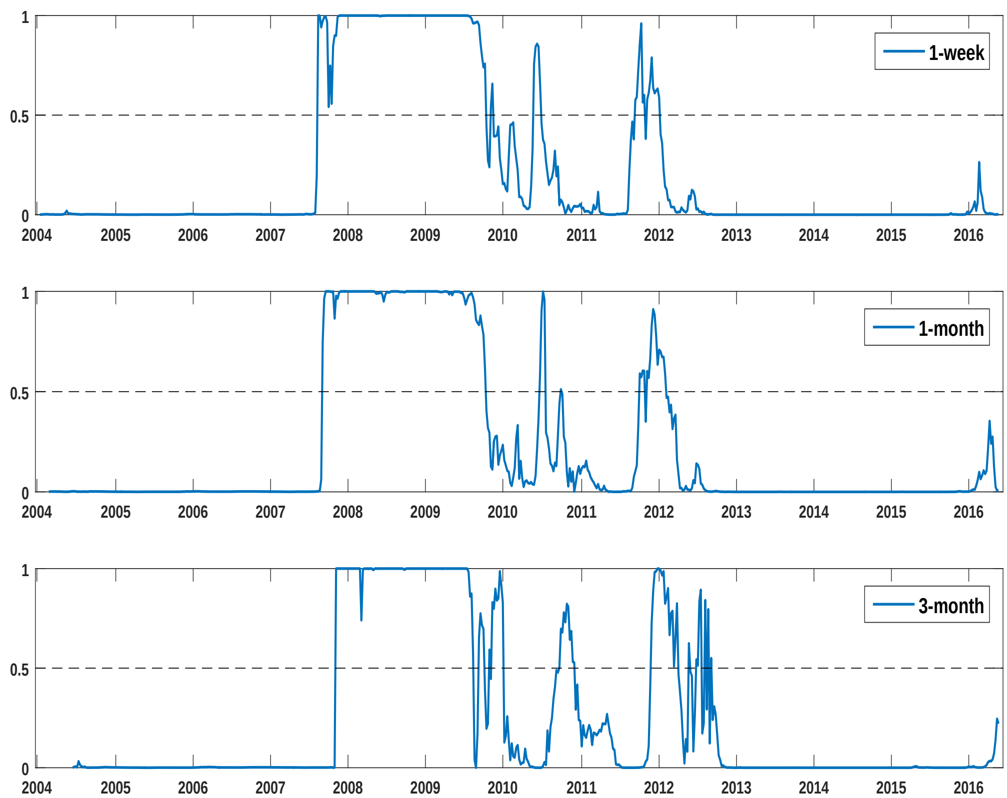


Figure 4.15: Probability of Financial Stress - Student-t Assumption

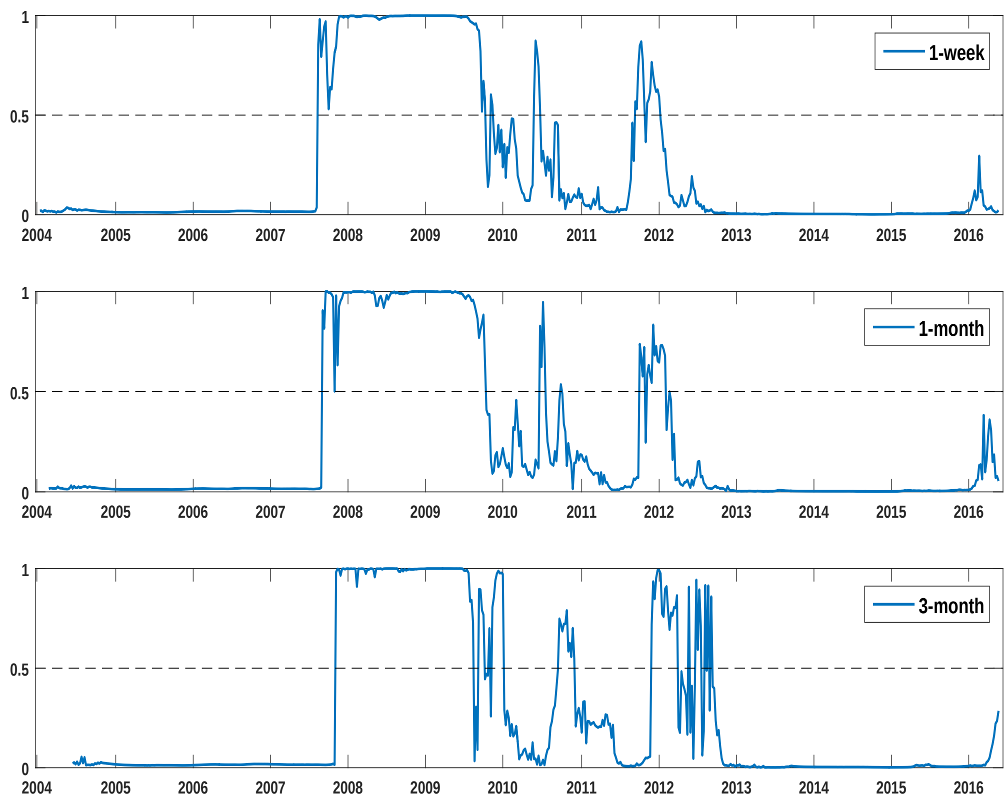


Table 4.10 displays the signal approach results under different scenarios and the results are promising. The correct ratios are quite high for 1-week prediction. As the prediction time gap increases, the precision decreases. In general the normal assumption outperforms the fat tail assumption except for $Obs = 1$ under 3-month case.

Table 4.10: Prediction Performance - Financial Stress Signal

1-week

Normal	S = 0	S = 1	Total	% Correct	% Incorrect	Student - t	S = 0	S = 1	Total	% Correct	% Incorrect
Obs = 0	492	4	496	99.19	0.81	Obs = 0	489	7	496	98.59	1.41
Obs = 1	17	132	149	88.59	11.41	Obs = 1	22	127	149	85.23	14.77

1-month

Normal	S = 0	S = 1	Total	% Correct	% Incorrect	Student - t	S = 0	S = 1	Total	% Correct	% Incorrect
Obs = 0	480	10	490	97.96	2.04	Obs = 0	477	13	490	97.35	2.65
Obs = 1	27	122	149	81.88	18.12	Obs = 1	34	115	149	77.18	22.82

3-month

Normal	S = 0	S = 1	Total	% Correct	% Incorrect	Student - t	S = 0	S = 1	Total	% Correct	% Incorrect
Obs = 0	435	39	474	91.77	8.23	Obs = 0	427	47	474	90.08	9.92
Obs = 1	45	104	149	69.80	30.20	Obs = 1	41	108	149	72.48	27.52

Notes: (1) If S = 1, the a financial stress signal is issued.

4.4 Conclusions

This paper uses a Bayesian variable selection framework to identify leading indicators of financial stress. It contributes to emerging literature on financial stress and the stability of the whole financial system and allows for the calculation of real-time financial stress level. Monitoring and predicting real-time financial stress provides additional guidance to policy makers and the private sector.

The stochastic search variable selection (SSVS) developed in George and McCulloch (1993) is used to find leading indicators. Financial stress indexes issued by Federal Reserve Banks are used to identify the financial stress level. These stress indexes together with Bloom(2009) and Ng(2015) are used to identify the occurrence of major financial stress period. Both linear model and Probit model under normal error assumption and fat tail assumption are used for analysis. Ex-post and ex ante approaches are used for analysis. The first (ex-post) approach allows for the identification of leading indicators from a historical perspective. The second (ex ante) approach allows for the identification of financial stress indicators on a real time basis by using rolling window. The results are then evaluated by predictive likelihoods.

The results show that all five variable categories are informative in predicting financial stress. But under normal error assumption less variables are selected compared to fat tail assumption especially for interest rate category. It also shows that none or very few potential indicators are selected when the market is under normal financial stress level. More variables are selected during the 07-09 crisis period. With the impact of economic crisis weakened, few variables are selected. It is also interesting to see that the log return of S&P 500 index is less informative than expected in predicting financial stress level.

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