In my dissertation, I analyze primitive numerical individuation. According to classical Haecceitism, there are non-qualitative facts of the world that are not analyzed, determined, or reduced to qualitative facts of the world. One of the typical non-qualitative facts is an exemplification of a non-qualitative property, a haecceity or thisness. Thisness is often described as the property of being identical with a particular entity. I argue, instead, that there is another way to understand thisness, thisness in terms of numerical individuation, as a property of being numerically individuated as a single individual. Unlike the classical understanding of thisness via identity, this thisness is not an essential property of a particular entity. This thisness purely individuates an entity. It neither constitutes any characteristics of the entity nor explains what the entity is. For an entity to exemplify this property means that it is counted as a single individual. Haecceitism with this thisness implies that the existence of an entity as a single individual and the number of individuals in a world should be primitively and fundamentally given. Numerical individuation minimally requires two processes, singling out an entity from others and avoiding double-counting. There are many ways to avoid double-counting. One of the easiest ways is to eliminate the singled-out entity. We don’t need any
information as to which entity is which when singling it out and/or eliminating it. Numerical individuation can be determined independently of the process of identifying entities.

There is a fact of the matter in a world that can be revealed by the existence of mere numerical individuation, a symmetrical and irreflexive relation: for a relation R, R(x, y) & R(y, x) where x≠y. Consider any indiscernible but numerically distinct entities as standing in the symmetrical and irreflexive relation. This fact of the matter is determined only by the primitive numerical individuation of the relata. One important and opposite theory to this kind of primitive individuation has been a reductive analysis of individuation in terms of the intrinsic qualities of an entity. Recently, another kind of reductive individuation has been proposed, structural individuation. This kind of individuation is developed from a radical version of structuralism that maintains a structure/relation has ontological primacy and objects are ontologically secondary in the sense that an object’s individuation is derived from its standing in the structure/relation. However, I argue that even a purely structuralist entity, such as a graph, requires the enumeration of its nodes/vertices, in addition to the relational descriptions of vertices, in order for the graph to be determined. I also argue that any attempt to establish a metaphysical language without terms, variables, or quantifiers, seems to fail in that it seems impossible to get rid of one of the semantic roles of a variable, its encoding the sameness or distinctness of an individual for predicates to apply. Numerical individuation still appears to be one of the nonreductive fundamental facts of the world.
TABLE OF CONTENTS

Title Page .................................................................................................................. i

Abstract .................................................................................................................... ii

Table of Contents .................................................................................................... iv

List of Figures .......................................................................................................... vi

Introduction .............................................................................................................. 1

Chapter 1. Classical Haecceitism ........................................................................... 13

1.1 Distinction between the qualitative and the non-qualitative ......................... 13

   1.1.1 A Mighty Language ................................................................................ 13

   1.1.2 Thisness: An Initial Account .................................................................. 16

1.2 Transworld Identity .......................................................................................... 20

1.3 De Re Modality and Supervenience ................................................................ 24

1.4 Haecceitistic Intuitions .................................................................................... 29

Chapter 2. Non-Modal Discussions on Haecceitism: Individuation and Identity.... 37

2.1 Thisness ............................................................................................................. 37

   2.1.1 Numerical Individuation ........................................................................ 40

   2.1.2 Primitive Individuation ......................................................................... 41

   2.1.3 Singularity .............................................................................................. 43

2.2 Numerical Individuation: Countability ............................................................ 45

   2.2.1 Counting and Cardinality: An intuitive account .................................. 46
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.2.2</td>
<td>A Minimal Metaphysical Implication of Counting</td>
<td>48</td>
</tr>
<tr>
<td>2.3</td>
<td>Individuation by Kind</td>
<td>52</td>
</tr>
<tr>
<td>2.4</td>
<td>Thisness and Individuation</td>
<td>59</td>
</tr>
<tr>
<td>2.4.1</td>
<td>Individuation and Identity</td>
<td>59</td>
</tr>
<tr>
<td>2.4.2</td>
<td>Primitive Identity and Primitive Individuation</td>
<td>62</td>
</tr>
<tr>
<td>2.5</td>
<td>Advantages of Primitive Individuation</td>
<td>65</td>
</tr>
<tr>
<td>2.5.1</td>
<td>Problem</td>
<td>65</td>
</tr>
<tr>
<td>2.5.2</td>
<td>Symmetrical and Irreflexive Relations</td>
<td>67</td>
</tr>
<tr>
<td>2.5.3</td>
<td>Weak Discernibility: Reduction or Not</td>
<td>69</td>
</tr>
<tr>
<td></td>
<td><strong>Chapter 3. Individuals and Structure</strong></td>
<td>75</td>
</tr>
<tr>
<td>3.1</td>
<td>Ontological Structuralism: General Understanding</td>
<td>75</td>
</tr>
<tr>
<td>3.2</td>
<td>Concrete Structure</td>
<td>81</td>
</tr>
<tr>
<td>3.3</td>
<td>Relations and Their Relata</td>
<td>86</td>
</tr>
<tr>
<td>3.4</td>
<td>Quantitative Facts</td>
<td>91</td>
</tr>
<tr>
<td>3.5</td>
<td>Relation and A Metaphysical Language</td>
<td>98</td>
</tr>
<tr>
<td>References</td>
<td></td>
<td>107</td>
</tr>
</tbody>
</table>
LIST OF FIGURES

Figure 1 .................................................................................................................. 56
Figure 2 .................................................................................................................. 60
Figure 3 .................................................................................................................. 92
Figure 4 .................................................................................................................. 93
Figure 5 .................................................................................................................. 96
Introduction

What are the fundamentals of the world? Are individuals included in the list of fundamentals? Haecceitism deals with one of the fundamentals, non-qualitative properties of individuals. My cat, Nuluk, a particular individual and as such has qualitative features such as shape, size, and mass. But according to the theory of haecceitism there is another property of Nuluk, a non-qualitative property. Even if there is a cat that shares every qualitative feature of Nuluk, it would be a duplicate cat that is still different from Nuluk in its non-qualitative properties. Haecceitists claim that the non-qualitative properties of individuals are fundamental in that they are not analyzed, determined, or reduced by other qualitative properties.

However, it is not easy to explicate the distinction between qualitative and non-qualitative facts. Some philosophers have tried to develop a language to describe the qualitative facts of the world exclusively, which is called the ‘Mighty Language’ by David Lewis (1986) and Robert Stalnaker (2011). The Mighty language is an imaginary one that includes every predicates except purely referential expressions like proper names, indexicals, and derivative proper adjectives or verbs such as ‘Leibnizian’ and ‘Pagasize’. They suggest that this language makes the following distinction: Any fact of the world that is described by this language is a qualitative fact, and any fact that is not describable by this language is non-qualitative. Every qualitative property should be denoted by the predicates in this language, and no non-qualitative property is described
by any predicate in this language. However, there is disagreement over which predicates are included in this language. There is no syntactic distinction between qualitative predicates and referential predicates; for example, ‘flies’ and ‘Pegasizes’ are grammatically indifferent. If the distinction between these two types of predicates is indicated by their meaning (what is denoted by these predicates), then there must be an actual distinction in the properties denoted by these predicates. It is the distinction between an individual’s qualitative and non-qualitative properties that determine the distinction between qualitative and non-qualitative predicates and not vice versa.

Robert Adams (1979) suggests introducing another technical distinction that corresponds to the distinction of qualitative and non-qualitative properties, that is, thisness and suchness. Roughly speaking, ‘thisness’ is the property of being (identical with) a certain entity. Socrates has the property of being identical to Socrates but lacks the property of being identical to Aristotle. ‘Suchness’ is defined relatively to the notion of ‘thisness’ – suchness is not thisness – and not a relational property to a specific individual. And ‘suchness’ is recursively defined in terms of applying operators to basic suchnesses: truth functional operators, intensional operators, etc. However, it is not clear whether this distinction reflects the distinction between qualitative and non-qualitative properties. The qualitative and the non-qualitative are complements to each other, that is, if something is qualitative, then it is not non-qualitative, and if it is non-qualitative, then it is qualitative. However, whether thisness and suchness also have a complementary relation is uncertain.
Consider a modal property, *possibly being a plumber*. Whether this modal property is suchness depends on the understanding of modality. Given the Lewisian counterpart semantic, *possibly being a plumber* is defined by having a counterpart who is a plumber. This property includes a relation to the counterpart who is a plumber. This property is not suchness in that it includes a relation to a specific individual. But this modal property is derived by applying a modal operator to a suchness, *being a plumber*. By the recursive definition of ‘suchness’, it should be a suchness. There is another problem case, a property that is not singular but relational to a specific entity. A property like *being a homeowner* is suchness but the property of owning the house at 1011 Rose Avenue may not be suchness. This property includes a relation to a specific entity, the house, but it can also be multiply instantiated like other suchnesses. It is not certain whether the two properties, suchness and thisness, are exclusive complements to each other. In this dissertation, I focus on the most typical example of non-qualitative properties, thisness. I will try to clarify the notion and examine its implications. Then, I propose an understanding of thisness that makes plausible the haecceitistic claim that thisness constitutes the fundamental fact of the world and individuals are fundamental constituents of the world.

This discussion will not be made in modal contexts. Most of the classical discussions on haecceitism have been made in modal contexts. In the discussion of de re modality, for example, haecceitism is considered to support the Transworld identity theory because thisness is well understood as a Transworld identity condition. An individual can have different qualitative features across possible worlds but its Transworld identity is
guaranteed by its non-qualitative thisness. However, haecceitism can also be consistent with other de re modal theories. For example, if we take the superessentialist view in which every property of an individual is essential to it, then there is no Transworld individual and, accordingly, thisness is not a Transworld identity condition. The notion of thisness does not determine which de re modal theory is right. Regardless of being Transworld or world-bound, individuals will always have their thisness.

Another modal implication of haecceitism is the possibility of indiscernible but distinct worlds. The numerical distinction of indiscernible entities is incompatible with Leibniz’s famous principle, the principle of identity of indiscernibility (PII). According to the PII, for any qualitative property F, an object, call it a, has F if, and only if, an object, call it b, has F, then a is identical to b. Qualitative sameness determines identity. Given the haecceitist thesis that there is a non-qualitative property of thisness over and above the qualitative property, it is possible for two objects to be distinct only in their thisness, and this would debunk the PII. But the rebuttal of the PII is not strong enough to imply the nontrivial type of haecceitism. Suppose that there are two qualitatively indiscernible worlds, say w₁ and w₂, and any individual in w₁, say a₁, and its indiscernible pair in w₂, say a₂. The purely qualitative properties, relations, and anything a₁ has in w₁ are the same as what a₂ has in w₂. The Transworld identity theory takes a₁ as identical to a₂, and in the counterpart theory a₂ is represented as a₁ in w₂ and vice versa. Regardless of the modal theory you ascribe to, a₁’s modal fact based on the facts of a₂ in another possible world is the same as a’s actual fact. No new modal fact is introduced by the existence of
the distinct possible worlds. The only difference is the number of individuals and worlds itself.

In chapter 1, I examine the problems of classical haecceitism and in the following chapters, I suggest a new approach to thisness and haecceitism. When introducing the contemporary discussion of thisness, Robert Adams (1979, p.16) proposes that the numerical individuation of an individual is metaphysically prior to any qualitative and spatiotemporal facts of the individual. I develop this idea to characterize the notion of thisness.

[Characterization of thisness 1] For an individual x, there is a property of being identical to x.
[Characterization of thisness 2] For an individual x, there is a property of numerically being individuated as x.

In chapter 2, I examine the difference of these two characterizations, the thisness based on identity and the thisness based on numerical individuation, and argue that the second characterization gives us a stronger version of haecceitism in that it avoids the primary problem of classical haecceitism – no additional fact of an individual can be given – and is in accordance with the discoveries in contemporary physics such as quantum particles.

According to Characterization of thisness 2, thisness is a property responsible for numerical individuation. To exemplify thisness is to be a single individual. Unlike the characterization of thisness 1, this property is not exemplified by one and the same
individual. This is not an essential property of an individual. This thisness (Thisness 2) only individuates an individual and does not constitute any characteristics of the individual. Thisness does not explain what the individual is but shows that it is counted as a single entity. In order to support haecceitism, this thisness should be a primitive property that is not derived from, reduced by or analyzed in terms of any other properties. That an entity is a single individual should be fundamentally given. Then, the fact of how many individuals there are is determined by an individual’s exemplifying thisness.

The set-theoretical understanding of counting is a cardinality assignment. The standard cardinality assignment is to attribute an ordinal in a certain order to the members of a collection. When counting students in a classroom, for example, we assign an ordinal number such as a natural number to each of the students in the order of natural numbers starting from the smallest. Each ordinal number in this ordinal assignment can be considered as a label or a name of the member of the collection. This label/name takes a metaphysical role to distinguish and also re-identify its individual. It is often asserted that the countable individuals are distinguishable, re-identifiable and labelable things. And haecceitism is often perceived as implying that permuting the labels/names of individuals in descriptions of the world denotes a metaphysically distinct state of the world.

However, there is another way to count individuals. The counting process can be divided into two steps:

(a) Singling-out: a process to differentiate an entity from other entities at one instant in time
(b) Avoiding double-counting: a process to ensure that an entity appears only once in a counting process.

The assignment of ordinal numbers or labels/names do both of these tasks, (a) and (b), at the same time. An ordinal is uniquely assigned to one entity (Process (a)), the ordinal number uniquely distinguishes it from others and is not assigned to other things (Process (b)). But the unique label/ordinal number assignments do much more than these two steps. For (b), it is only required to make a distinction between two groups, a group that is already counted and a group that will be counted. There are many ways to make this distinction. For example, we eliminate entities after singling them out to avoid double-counting. But a label/name enables us not only to make this distinction between two groups in Process (b) but also to identify and re-identify each entity. The minimal condition for numerical individuation is that an individual is this one and not that one at this moment. Numerical individuation can be determined without identifying entities. Mere numerical individuation requires neither the information of which is which nor the labelability of individuals.

There is a fact of the world that can be derived only from mere numerical distinction.

Consider the following relation.

[Symmetrical and Irreflexive Relation] For a relation R, R(x, y) & R(y, x) where x≠y

There are many symmetrical and irreflexive relations between entities, for example, next to, distant from, on the opposite side of, a complement of, etc. This relation can hold even with indiscernible individuals. Two maximally entangled particles such as electrons are indiscernible not only in their intrinsic properties but also in their spatial locations and causal efficacy because they do not have a determinate location or momentum. But the particles do have symmetrical and irreflexive relations such as in an opposite directional
spin of. For each electron, it is indeterminate whether the electron is spinning clockwise or counter-clockwise around the axis. However, it is determinate whether each is spinning in the opposite direction from another. This relation is satisfied by the mere numerical distinction between the particles, that is, their thisness.

The symmetrical and irreflexive relation is sometimes used to distinguish quantum particles: They are indiscernible in their intrinsic properties, spatial locations and causal efficacies, but discernible in their participation in the symmetrical and irreflexive relation. In order for this relation to take a role of individuating properties of the particles, the relation is either: (1) broken down into two distinct monadic properties or (2) somehow individuates its relata. In chapters 2 and 3, I argue that both options are not satisfactory. (1) allows circularity. For the relation of two individuals, a and b, being 2 miles from, a’s monadic individuating property is being 2 miles from b and b’s individuating property is being 2 miles from a. However, these two individuating properties include the distinguished individuals, a and b. Their numerical distinction is required prior to having these two individuating properties. Moreover, the definition of this relation requires the numerical distinction of its relata. If the relata of the relation are identical, this relation cannot be irreflexive. In order to determine whether a relation like being 2 miles from is symmetrical and irreflexive, the numerical distinction of its relata must be given first. Numerical individuation determines the irreflexive relation, and not the other way around.
Considering (2), is it possible for a relation to individuate its relata? If so, then we have a new type of reductive individuation, in addition to the numerical individuation by intrinsic properties of individuals, which is individuation by relation. A radical version of structuralism develops this type of individuation. The thesis of structuralism argues for a structure or a relation that is ontologically fundamental in that the structure/relation is not reduced by the intrinsic and spatiotemporal properties of their relata. The radical version of structuralism claims that a structure/relation has ontological primacy and objects are ontologically secondary in the sense that an object’s individuation is derived from its standing in the structure/relation.

In chapter 3, I consider two issues for discussing structural individuation, an analysis of a mathematical structure and a language for a generalist fact. A mathematical structure such as a graph is the purest structural form that is constituted only by the relation between its nodes that completely lack intrinsic properties. A graph is determined only by how the nodes are related to each other. Some philosophers like Randall Dipert (1997) adopt the graph as a fundamental structure of the world. Others, such as Anjan Chakravartty (2012), argue that the world itself cannot be a graph because an individual is not a relatum of the relation/structure in which it stands. Chakravartty believes that individuals must have intrinsic properties unlike mathematical nodes. I am not interested in the question of whether a graph is a real structure of the world or not. Rather, my interest is in whether or not the numerical individuation of the mathematical node is determined by the relation (vertices) in which it stands. If its individuation is not reductively determined, then it is possible to conclude that the structural individuation
does not hold even for a mathematical structure and the numerical individuation is given as a fundamental fact in addition to structural facts.

Consider an asymmetric graph with each of its nodes related to other nodes in a unique manner. Dipert (1997, p. 349-50) argues that an asymmetric graph individuates nodes purely structurally: each vertex has a unique and purely structural description and each node is individuated by its relation to other nodes in the graph. One of the most simple asymmetric graphs has the following list of relational descriptions of vertices:

- A unique vertex of degree 1 that has only one edge;
- A vertex of degree 4 that has four edges adjacent to a vertex of degree 1;
- A vertex of degree 2 adjacent to two degree 4 vertices;
- A vertex of degree 4 not adjacent to a vertex of degree 1;
- A unique vertex of degree 3;
- A vertex of degree 2 not adjacent to two degree 4 vertices.

In that asymmetric graph, each node has its own unique description. Each description in the above list is assigned to one vertex. So, we have a unique graph from the above descriptions. The descriptions of the vertices, the relations, determine every feature of the graph including every feature of its nodes. However, we need to consider the condition of the asymmetric graph. Without this condition of asymmetric graph, it is not determinate how many vertices would be in a graph. Without a restriction on the number of vertices, there can be many distinct graphs which satisfy all of the above descriptions. In order to have a unique graph, the number of nodes must be given first.
If a relation individuates its relata, the relation is given prior to its relata. What is a relation without its relata? We do not have an appropriate language to express this idea. A structural system, how entities are related, is usually expressed in a quantified formula in the first order language, using predicates, variables and quantifiers. The semantics of variables and quantifiers requires individuals to satisfy them in this language. Individuals, as the relata of a relation, are required to exist for the relation to make sense. Then, we can ask whether there is a metaphysical language that enables us to express a relation without its relata. If not, then we can probably conclude that the relation without relata is not comprehensible.

Shamik Dasgupta (2009, 2011, 2016) suggests a new language for a generalist fact. The language has neither terms nor variables that require individuals for their semantic values. In the last section of chapter 3, I ask the question if Dasgupta’s new language gets rid of variables. Variables take three semantic roles: supplying individuals relative to an assignment; expanding the stock of predicates; and encoding the sameness or distinctness of an individual for predicates to apply. This last semantic role requires numerical individuation. If the new language does not exclude variables, this language requires the addition of numerical distinction to any generalist fact of the world.

In the first order language, an asymmetrical love relation is expressed with the help of variables: \( L(x, y) \land \neg L(y, x) \) where ‘\( L \)’ stands for the predicate, love. In Dasgupta’s language, unrequited love is expressed with two different relations, \( L^2 \land \neg \alpha L^2 \) where ‘\( L^2 \)’ stands for a two-place love relation and ‘\( \alpha \)’ is a functor of permuting the order of
variables of a formula. Here, the meaning of $\alpha L^2$ depends on the meaning of $L^2$. If $\alpha L^2$ can have meaning independently from $L^2$, then $L^2$ and $\alpha L^2$ can express one and the same relation. We do not know whether ‘$L^2$‘ can be translated into a loving relation or a being loved relation. This holds also for the translation of ‘$\alpha L^2$‘. In any case, one and the same relation can be expressed in someone’s loving relation and also someone’s being loved relation. Then, $L^2 \& \neg \alpha L^2$ can result in contradiction.

In order to avoid this unwanted contradiction, the meaning of $\alpha L^2$ should be dependent on the meaning of $L^2$, that is, its conversion. But how can we understand conversion without the order of the relata in $L^2$ and $\alpha L^2$? The order of relata requires the sameness and distinctness between these relata, that is, the basic semantic role of variables. The new language seems to include the role of variables hidden in the meaning of functors. In order to understand the meaning of relations, $L^2$ and $\alpha L^2$, the numerical sameness and distinction of their relata should be given first. Numerical individuation still appears to be a fundamental fact of the world.
Chapter 1. Classical Haecceitism

What is haecceitism? Roughly speaking, there is the non-qualitative – non-qualitative properties, relations, propositions, or facts – over and above the qualitative. In other words, when making a complete and exhaustive description of the world, we need non-qualitative descriptions in addition to qualitative descriptions. Many questions follow. What are the exact meanings of ‘qualitative’ and ‘non-qualitative’? What makes them different? What is the relation expressed by ‘over and above’? What is an example of the non-qualitative? The answers are required to characterize haecceitism. But it is not easy to answer them. In this chapter, I try to find the best way to characterize haecceitism and what we need to discuss in order to understand haecceitism.

1.1 Distinction between the qualitative and the non-qualitative

1.1.1 A Mighty Language

There are two approaches explaining the distinction between the qualitative and the non-qualitative properties: an analysis using a philosophical tool called a ‘Mighty Language’, and an analysis in terms of the technical distinction between thisness and suchness. Suppose there is a language that is rich enough to have every expression except any purely referential expression like proper names, indexicals, and derivative proper adjectives such as ‘Leibnizian’ and proper verbs such as ‘Pegasize’. This language has an infinite resource of qualitative predicates as well as quantifiers, variables, and truth-

---

1 For a detailed discussion of this idea, see Adams (1979) and Lewis (1986).
functional connectives. The mighty language is designed to lack nothing in qualitative predicates while also lacking everything in non-purely qualitative expressions. Every predicate in this language is purely qualitative, every property expressed by this predicate is purely qualitative, and every fact expressed by this language is purely qualitative. On the other hand, non-qualitative facts are the ones that cannot be expressed by the sentences in this language. Then, the core idea of anti-haecceitism is that this language is sufficient to provide a complete description of the world. Accordingly, haecceitism is characterized as a claim of non-qualitativeness such as follows:

[Haecceitistic Thesis 1] There are some facts of the world that cannot be expressed by the mighty language.

According to Robert Stalnaker (2012, pp.61-62), it is circular to analyze qualitative and non-qualitative properties in terms of the mighty language. In this analysis, qualitative properties are defined as the properties expressed by the predicates of this language. But he argues that we cannot know which predicates should be included in this language without knowing what the qualitative properties are. There is no syntactical information to discern proper verbs like ‘Pegasizes’ from qualitative predicates like ‘flies’. Then, we should know the semantic features of ‘Pegasizes’ to exclude it from the language. The semantic information of ‘Pegasizes’ should include what Pegasus is and indicate that its corresponding property is not purely qualitative. Then, an understanding of qualitative and non-qualitative properties is required to determine which predicates are included in this language.
However, Robert Adams (1979, pp.7-11) does not accept this circularity. If we succeed in referring to an individual without knowing its qualitative properties, then we know that any predicates derived from this referential expression are not qualitative. Adams believes that we have this kind of referential expression, such as direct referential terms. But he does not support the attempt to analyze qualitative properties in terms of the mighty language because the linguistic expression does not give us any metaphysical implication.

Several philosophers have made a persuasive case for the view that we often succeed in referring to a particular individual without knowing any clearly qualitative properties, or even any disjunction of such properties, that a thing must possess in order to be that individual. Such direct reference is commonly effected by the use of proper names and indexical expressions, and sometimes by what has been called the “referential” use of descriptions. If these claims are correct (as I believe they are), doesn't it follow that thisnesses are primitive and nonqualitative? Yes and no. It follows that thisnesses are semantically primitive -- that is, that we can express them (and know that we express them) without understanding each thisness (the property of being this or that individual) in terms of some other property or properties, better known to us, into which it can be analyzed or with which it is equivalent. But it does not follow that thisnesses are not analyzable into, equivalent with, or even identical with, purely qualitative properties or suchnesses, as claimed by Leibniz. (ibid., p. 10)

It is possible that what are referred to directly are qualitative entities. If a metaphysical theory that an object is just a bundle of qualitative properties is right, what is directly referred to by a name such as ‘Nuluk’ is just a collection of qualitative properties, being a calico cat, weighing 9 pounds, etc. I can pick out the entity Nuluk only with its name. But the way we refer to something is not necessarily the way the world is. Accordingly, the non-qualitative predicates are not guaranteed to express non-qualitative properties. They could be just a result of our linguistic convention.

The two philosophers’ views about the mighty language analysis do not seem to be opposed to each other, but are rather complementary thus expressing a dilemma. In
Adams’ picture, the mighty language analysis can avoid circularity by adopting a linguistic apparatus, a direct referential expression that does not depend on what properties are expressed by this apparatus. This very feature of the linguistic apparatus loses the connection between the linguistic expression and the properties. The criterion for predicates that are allowed in the mighty language is that they do not include this referential expression. All the predicates allowed in the mighty language are predicates that are not derived from direct referential expressions. Given the lack of correspondence between the referential expressions and properties, we do not know which properties are expressed by the predicates with the direct referential expressions. If we claim a correspondence between the predicates with the direct referential expressions and non-qualitative properties, then we get into Stalnaker’s circularity because this correspondence requires an understanding of what is referred to by the referential expressions, whether qualitative or non-qualitative properties. The analysis of qualitative properties requires an understanding of qualitative properties. But without the correspondence, the predicates of the mighty language are not appropriate for analyzing genuine qualitative properties. We are in a dilemma. The mighty language is not good enough to give us a definition of qualitative properties nor to characterize haecceitism.

1.1.2 Thisness: An Initial Account

The second approach to the analysis of the distinction between the qualitative and the non- qualitative is made in terms of the distinction between thisness and suchness. We have a distinction in properties that is relatively easy to understand, the distinction between general properties and non-general properties. General properties are ones that
are shared by different objects, and non-general properties are ones that an individual in question has by itself\(^2\). Let us call the latter ‘singular properties’. Being general is not necessary and sufficient for being qualitative, but it is often regarded as necessary. Then, what else do we need to define this distinction? The distinction between thisness and suchness\(^3\) has been a good candidate. Adams (1979) defines ‘thisness’ as “the being identical with a certain particular individual”. For example, my cat Nuluk has the thisness of being identical with Nuluk. Thisness is a singular property. The property of being self-identical and the property of being identical to an individual are not thisnesses because they are shared by other individuals, in fact, by every individual. In addition, thisness is a relational property that includes a relation to a particular individual. But note that not every relational property with a particular individual is thisness. The property of living on 29 Redcliffe Ave., Highland Park, NJ, for example, is instantiated by many individuals even though it is a relation to a particular entity, the house located at the address. Adams makes a recursive definition of ‘suchness’, and then defines ‘thisness’ based on suchness.

Jointly sufficient conditions for basic suchness are the following:

(i) It is not a thisness and is not equivalent to one;
(ii) It is not a property of being related in one way or another to one or more particular individuals (or to their thisness);
(iii) It is not a property of being identical with or related in one way or another to an extensionally defined set that has an individual among its members, or among its members’ members, or among its members’ members’ members. (Adams 1979, pp.7-8)

---

\(^2\) We can ask whether the distinction between general and singular properties is a modal notion, a property shared by different things across possible worlds and a property instantiated by only one thing across possible worlds. The latter will constitute an individual essence of the thing. We are not considering such a strong singular property.

\(^3\) The notions of thisness and suchness are introduced by Leibniz. Adams (1979) develops the contemporary discussion of this distinction.
We have a complex suchness in terms of applying suchness operators to the basic suchness. For suchness operators, Adams considers the truth functional operators such as ‘not’ and ‘or’, quantifiers, and propositional attitudes like ‘wishes that p’. Then, we have the following characterization of haecceitism:

**[Haecceitistic Thesis 2]** There are properties that cannot be analyzed, determined, reduced to suchness.

Stalnaker (2012, pp.62-66) doubts that this recursive analysis of suchness is appropriate. Adams does not give us a complete list of suchness operators. He lists some examples: truth functional operators, quantifiers, and an intensional operator, a propositional attitude. Stalnaker adds a modal operator to the list. He does not see any reason to treat as distinct the two intensional operators, a propositional attitude and a modal operator. Then it is at least controversial whether modal operators construct a complex suchness from other suchnesses. If the property of being a lawyer and the property of being a plumber are suchnesses, the property of being a lawyer who might have been a plumber should be suchness. But Stalnaker asks whether this modal property is suchness. It is at least unclear whether this complex property is suchness. Whether or not the complex modal property is suchness will be dependent on the semantics of modal notions. If we take the Lewisian counterpart semantics, possibly *being a plumber* is analyzed into *having a counterpart that is a plumber*. Given a Lewisian understanding of property as a set of objects to instantiate it, the composition of being a lawyer and having a counterpart that is
a plumber is a set of objects that are members of both sets. The modal property includes a
relation to a particular object that is a counterpart, which does not satisfy condition (ii).

We also need to cast doubt on Adams’ notion of basic suchness. Conditions (ii) and (iii)
exclude the relational properties to a specific individual or a set of specific individuals.
Adams (1979, p.8) compares the following two properties: the property of being a
homeowner and the property of owning the house at 1011 Rose Avenue, Ann Arbor,
Michigan. The former is suchness but the latter is not because the latter includes a
relation to a specific object, the house. But this comparison reveals a problem in the
distinction between thisness and suchness. The property of owning the house at 1011
Rose Avenue, Ann Arbor, Michigan is not thisness because it is not singular. There can
be other individuals who own the house. Then this property is neither suchness nor
thisness. Therefore, thisness and suchness are not complementary. Considering that the
qualitative and non-qualitative properties are complementary, the distinction between
thisness and suchness is not equivalent to the distinction between the non-qualitative and
the qualitative. If haecceitism is a claim for non-qualitative properties or non-qualitative
facts, and requires the distinction between the qualitative and the non-qualitative, the
distinction between thisness and suchness is not enough to give a complete analysis. We
have examined two well-known trials to analyze the distinction between the qualitative
and the non-qualitative, in terms of the mighty language and of the distinction between
thisness and suchness. However, these two are not completely satisfactory. Perhaps we
should claim that we have an intuition to discern them if the non-qualitativness is
necessarily required.
1.2 Transworld Identity

In “How to Russell a Frege-Church”, David Kaplan (1975) introduces the term ‘Haecceitism’ into contemporary modal metaphysics. He characterizes anti-haecceitism with his famous metaphor: As individuals might have been different from how they actually are, individuals may be “clothed in attributes which cause them to resemble one another closely. But there is no metaphysical reality of sameness or difference which underlies the clothes.” On the other hand, haecceitism is characterized as claiming that “a common ‘thisness’ may underlie extreme dissimilarity or distinct thisness may underlie great resemblance.” He concludes his characterization with the following puzzling paragraph.

The opposite view, Anti-Haecceitism, holds that for entities of distinct possible worlds, there is no notion of transworld being. They may, of course, be linked by a common concept and distinguished by another concept – as Eisenhower and Nixon are linked across two moments of time by the concept the president of the United States and distinguished, at the same pair of moments, by the concept the most respected member of his party – but there are, in general, many concepts linking any such pair and many distinguishing them. Each, in his own setting, may be clothed in attributes which cause them to resemble one another closely. But there is no metaphysical reality of sameness or difference which underlies the clothes. Our interest may cause us to identify individuals of distinct worlds, but we are then creating something – a transworld continuant – of a kind different from anything given by the metaphysics. Although the Anti-Haecceitism may seem to assert that no possible individual exists in more than one possible world, that view is properly reserved for the Haecceitist who holds to an unusually rigid brand of metaphysical determinism. (Kaplan 1975, pp.722-3)

There are many relevant notions in this paragraph. But most of all, we should disentangle the relation between haecceitism and the notion of transworld identity. Kaplan starts this paragraph with the idea that anti-haecceitism holds a view against the transworld identity. In the last sentence, however, he points out that haecceitists can have the same view when they hold “an unusually rigid brand of metaphysical determinism.”
The basic understanding of transworld identity is that an individual exists in more than one possible world. The opposite of this position is that no individual exists in more than one possible world. That is, everything is world-bound. Does the anti-haecceitism imply the world-bound individual? It is hard to see the relation between the two. Moreover, the discussion on haecceitism is not limited to modality. Historically, this discussion began during the medieval times, without the help of modality.

Kaplan knows that transworld identity is not even a necessary condition for haecceitism. In the last sentence of the quoted paragraph he says that haecceitism is consistent with an “unusually rigid brand of determinism.” As an example of this determinism, Cover and Hawthorne (1999, p.144) and Stalnaker (2012, p.55) take the superessentialist view by Leibniz. According to their view, the possible world talk is legitimate; the transworld identity is false; every property of an individual is essential to it. If you take a possible world as a concrete entity like this actual world, and you believe that every individual is world-bound, then this superessentialist view is inferred. In the possible world analysis of modality, an individual has a property necessarily if and only if it has the property in every possible world in which it exists. If the transworld identity theory is false, then every individual exists in only one world. And every individual has its own actual properties in their world. This implies that an individual has its actual properties across

---

4 Roughly speaking, a possible world is a whole of maximally inclusive situations, one of which is the actual world. The possible world talk is just one of the ways to analyze possibility. There are metaphysical controversies about what the possible worlds are. Some claim that they are abstract objects such as states of affairs; some claim that they are just a set of maximally consistent sentences; others claim that they are real entities like this actual world. In this passage, I take the last position, defending realism about possible worlds. But this is only for convenience. It is easy to visualize and understand the modal situations if we take a possible world like the actual world. I will use modal realists’ descriptions unless the metaphysical view on possible worlds matters.
possible worlds where it is trivial because it exists only on one world. Individuals have all of their properties necessarily. This idea of superesssentialism does not make any distinction or any restriction on what properties an individual has. There is nothing wrong with an individual having non-qualitative properties like thisness. Superessentialism only claims that the properties are essential to the individual in question. Then, the world-bound view in this superessentialism, the opposite of the transworld identity view, is compatible with haecceitistic notions like thisness. The transworld identity view does not imply haecceitistic claims, for example, the existence of non-qualitative thinness.

The notion of transworld identity does not distinguish haecceitism from anti-haecceitism. So why then does Kaplan continue to use the notion of transworld identity in characterizing haecceitism? Perhaps we can find the answer from Lewis. Lewis (1986, pp.222-3) claims that we can make an argument against haecceitism via its relation to the transworld identity view. Let’s analyze the structure of the argument. There are two major rival theories to explain the modality that is predicated of an individual, de re modality: the transworld identity and a counterpart theory. According to the counterpart theory, the relation between different things in different worlds is not identity but a counterpart relation. It is true that I might have different properties and some different things might happen to me. According to the counterpart theory, the de re modal statement is true if and only if I have counterparts in other possible worlds that have those properties and have that experience. Lewis asserts that haecceitist ideas fit well with the transworld identity view while the counterpart theory cannot be compatible with it. However, Lewis’s view on possible worlds does not allow for transworld identity.

---

5 We are going to have a more accurate definition of the counterpart theory in the following section.
According to him, possible worlds are as concrete as the actual world is, and possible
worlds are maximally stretched out in space-time.\(^6\) A possible world is something whose
parts are maximally connected in space-time. Then, distinct worlds do not spatio-
temporally overlap, and other-worldly things are not spatio-temporally related. Given that
a physical entity is spatio-temporally related to oneself, then transworld identity is not
allowed. Without depending on his notion of modality, Lewis offers some reasons for the
implausibility of transworld identity. One of the major reasons is that every accidental
intrinsic property turns out to be a relational property. My being 5.5 feet high is
paraphrased into my being 5.5 feet high in the actual world, which includes a relation I
have to the world. These ideas are not without controversies. But let us just assume that
the transworld identity view is false for the sake of argument. Suppose we have only
these two theories for \textit{de re} modality, the transworld identity theory and the counterpart
theory, they are mutually exclusive, and haecceitism is not considered to fit well with the
counterpart theory. Then, haecceitism can be indirectly refuted by the implausibility of
the transworld identity theory.

If haecceitism is compatible with the counterpart theory, however, this argument does not
stand regardless of whether Lewis’s criticism of transworld identity is right. And the
counterpart theory, at least Lewis’ counterpart theory, can be considered as a kind of rigid
determinism. The metaphysical implication that the counterpart theory gives us has been
controversial. Our intuition says that a \textit{de re} modal statement such as “I might have been
different” is true. The counterpart theory gives us a truth condition for the statement: “an

\(^6\) Lewis (1986, p.2) says “There are countless other worlds, other very inclusive things. Our world consists
of us and all our surroundings, however, remote in time and space; just as it is one big thing having lesser
things as parts, so likewise do other worlds have lesser other-worldly things as parts.”
individual $a$ is possibly $\phi$” is true if and only if there is a possible world in which a counterpart of $a$ is $\phi$. But Lewis does not allow an individual to exist in more than one possible world. If we assume possible worlds are concrete entities like our actual world, as Lewis claims, we can say that every individual lives their life within their world. Even though “I might have been different” is true, I do not live a different life. This metaphysical picture is similar to determinism. Moreover, Lewis accepts haecceity, the non-qualitative property of being identical to a particular individual, for example, being identical to Nuluk.\(^7\) Then, his counterpart theory seems to have a metaphysical implication that every individual has its actual properties in any possible world and some of the properties are non-qualitative. There is no difference in this picture of possible worlds and how the individuals are between this counterpart theory and the “unusually rigid determinism” of theories like Leibniz’s superessentialism. As superessentialism is compatible with haecceitism in this picture of the world, Lewis’s counterpart theory is consistent with haecceitism.

1.3 *De Re* Modality and Supervenience

In contemporary metaphysics, haecceitism is mostly discussed within the topic of *de re* modality. This inclination seems to be affected by Lewis’s distinction between haecceity and haecceitism. Lewis points out that the allowance of haecceity or any kind of non-qualitative property does not imply haecceitism. Anti-haecceitists can accept a non-qualitative property unless it is independent of and not analyzable by any qualitative properties. If the non-qualitative property can be reduced to a qualitative property, there

\(^7\) In Lewis’s understanding of haecceitism, haecceity is not relevant to the characterization of haecceitism. Anti-haecceitists can allow haecceity without any contradiction. This issue will be discussed in the following section.
is no obstacle to being an anti-haecceitist. Lewis characterizes the dependent relation as supervenience that is developed into covariance in a modal context, which leads to the modal discussion of haecceitism.

[Haeccceitist Thesis 3] There is a pair of possible worlds that share the same qualitative facts but differ in non-qualitative facts.

There are several metaphysical analyses of possible worlds, each of which leads to different understandings of what the qualitative facts of a world are. Brad Skow (2008, p.102-4) provides a good example. Suppose we believe that possible worlds are sets of sentence types. If we are serious about this metaphysical analysis of possible worlds, the features of these sets themselves should be one of the facts of the worlds. Suppose two sets of sentences, one of which has a non-qualitative sentence as its member and the other of which does not include it. Suppose this membership is a qualitative relation. Then, these two sets are qualitatively different even though they are different only by having the non-qualitative sentence. This complicated situation does not arise if possible worlds are not analyzed by the membership relation. We want to have a general characterization of haecceitism that can be accepted by any possible world theorists. We need to ignore the ontological features of a world itself. Skow suggests using the notion of representation of a possible world instead of a possible world itself. What a possible world represents is what sentences are true according to the possible world. Then, we have a modified version of [Haecceitistic Thesis 3].

---

8 We have this condition of ‘type’ in order to ignore the difference between sentences only in their syntactical symbols.
[Haecceitistic Thesis 3'] There is a pair of possible worlds according to which all the same qualitative sentences are true but not every same non-qualitative sentence is true.

But I will continue to use the expressions ‘qualitative facts of a possible world’ or ‘qualitative features of a possible world’ for convenience. We can easily interpret the expressions as the facts and the features of what is represented by a world.

There is one more peculiar feature in Lewis’s characterization of haecceitism. He separates haecceitism from the problem of the identity of indiscernibility. The principle of the identity of indiscernibility (PII) claims that two individuals are identical if and only if they share all the same qualitative properties.

[Principle of Identity of Indiscernibility (PII)] For any qualitative property F, object x has F if and only if object y has F, then x is identical to y.

This principle of the identity of indiscernibility has been considered as an extremely strong identity condition. It is at least logically true that we can have numerically distinct but qualitatively similar individuals. This idea holds even for possible worlds.

[Haecceitistic Thesis 4] There are distinct possible worlds that are qualitatively the same.

If [Haecceitistic Thesis 4] is true, there is a non-qualitative fact about the worlds that are not determined by qualitative features of the worlds. Criticisms of PII are often
considered as defenses of haecceitism. However, Lewis claims that this issue of the identity of indiscernibility is not relevant to haecceitism. He says “...anti-haecceitism is neutral about whether there are qualitatively indiscernible worlds: there can be any number of indiscernible worlds, so long as they are alike not only qualitatively but also in representation de re. If worlds are alike in both these respects, perhaps they are altogether indiscernible – how else might they differ?” and then he claims, “For all I know, there are many indiscernible worlds, so that the worlds are even more abundant than we would otherwise think. I see no theoretical benefits to be gained by supposing that there are or that there are not, so on this question I advise that we remain agnostic.” (Lewis 1986, p.224)

Lewis seems to have the following in mind. Suppose two qualitatively indiscernible worlds, say \( w_1 \) and \( w_2 \), and any individual in \( w_1 \), say \( a \). Given Lewis’s modal realism, there must be an individual indiscernible with \( a \), say \( a' \), because these two worlds are indiscernible. Suppose \( a' \) represents \( a \) in \( w_2 \). The purely qualitative properties, relations, and anything \( a' \) has in \( w_2 \) are the same ones \( a \) has in \( w_1 \). If a possible world is accessible to itself, no new facts about \( a \) are added to \( a' \)’s modal predication list from the facts about \( a' \) in \( w_2 \). If \( a \) is a student, \( a \) is possibly a student from the reflexive accessibility and also from the fact that \( a' \) is a student in \( w_2 \). Any modal predication by the facts about \( a' \) in \( w_2 \) is given by the facts of \( a \) in \( w_1 \). The existence of an indiscernible world, in this case, does not give us a new de re modal fact.
What if \( a' \) is not the representation of \( a \) but another individual in \( w_2 \), say \( a* \)? In other words, \( a* \) who is not qualitatively the same as \( a \) is considered to represent \( a' \) in \( w_2 \). \( a* \)'s life is different from a’s, for example, \( a* \) is a teacher in \( w_2 \). This representation is not determined by their qualitative features. Then, a new modal piece of information about \( a \) is added to \( a' \)’s modal list: \( a \) is possibly a teacher.\(^9\) In sum, we need to consider two modal facts separately in characterizing haecceitism: whether there are non-qualitative possible worlds and also how an individual is represented in the indiscernible world. Lewis’s version of haecceitism can be expressed as follows:

[Haecceitistic Thesis 5] There are distinct possible worlds that are qualitatively the same and that differ with respect to how they represent an individual as being in that world.

Lewis’s characterization has been the most promising in that it disentangles relevant issues and sifts the core ideas from them. But it is not an exhaustive characterization. Lewis has shown that the failure of PII is not enough to characterize haecceitism because it does not provide any additional de re modal fact. Not only is the existence of indiscernible but distinct worlds needed, but different de re modal representations are also needed in [Haecceitistic Thesis 5]. Other than the issue of de re modality, however, there are other contexts in which haecceitism is discussed. We have a non-modal tradition of haecceitism. The topic of thisness is an example of this tradition. Another example is an identity condition for particulars, especially whether a particular object is just a bundle of properties or consists of an underlying substratum in addition to properties. This tradition asks how to individuate a world and what is the fundamental structure of the

\(^9\) It will be discussed in the following section whether this representation is plausible.
world. Instead of the relation of supervenience between the qualitative and the non-qualitative, this tradition considers the relation of fundamentality. Whether or not the qualitative provides a complete fundamental fact of the world? Does the fundamental description of the world include referential expressions for particulars? Do we have bare particulars? What is the identity condition for a particular? These issues of fundamentality do not need to be discussed in supervenience or with talk of possible worlds. The PII should be discussed significantly in these debates, especially when finding an identity or an individuation condition for a particular and determining the fundamental structure of the world. This will support the bundle theory of a particular and disprove the indiscernible but distinct worlds view, which, in turn, weakens the non-qualitative fundamental facts of the world.

1.4 Haecceitistic Intuitions

There are many thought experiments that are supposed to support haecceitism. Three of them are intuitively so powerful that anti-haecceitists should have to explain why we have these intuitions.

(a) Max Black’s Symmetrical Globes of Iron

Imagine a universe consisting solely of two large, solid globes of iron. They are always exactly similar in shape, size, chemical composition, and color, in short, in every qualitative respect. They share all their relational qualitative properties like the property of being two diameters from another iron globe similar to itself. That is, we have two distinct globes that share all of their qualitative properties. There is no way to determine
the numerical identity and distinctness of an individual solely based on its qualitative features.

(b) Saul Kripke’s Dice Case
Suppose a toy model of metaphysical possibility in which there exists only two dice and six properties to differentiate the dice. When two dice are thrown, they display two numbers face up. For each die, there are six possible results. Then there are thirty-six possible states of the pair of dice. Only one of these possible states will be actualized. Our basic probability calculation tells us that the probability of having a pair result, say (die A: 5, die B: 6), is 1/36. But some specifications of the possible states like (die A: 5, die B: 6) and (die A: 6, die B: 5) switch the facing up numbers of the two dice. For anti-haecceitists, this switching pair expresses the same possibility, one die landing 5 and another die landing 6. Anti-haecceitists should claim that there are only twenty-one possible states rather than thirty-six possible states by merging the switching cases, which is not consistent with our probability intuition.

(c) Possibility of Repeating History
Imagine that all the events and changes of objects in a specific period of time and a specific region of place occur several or infinite times. This repeating history or repeating world is constituted by several space-temporal epochs that appear so exactly the same as to be indiscernible. What distinguishes each epoch is only the order of their occurrence in this repeating history. Suppose I live in the seventh epoch. It is possible that I live in the eighth epoch. There must be someone, call it a, who represents me in the eighth epoch.
The difference between my living in the seventh epoch and a’s living in the eighth epoch is not explained by the qualitative features of each epoch.

The first case shows a possibility of a symmetric structure, the second shows a possibility of switching individuals, and the last shows a possibility of individuation by ordering. Our haecceitist intuition says that we have these possibilities without any change in qualitative facts. In these cases, (c) does not lead to a serious problem unless what represents an individual in indiscernible but distinct worlds are qualitatively distinct. That is, (c) does not claim [Haecceitist Thesis 5]. However, the switching case (b) and the symmetry world case (a) are different. In this last section, I will consider Lewis’s explanation about these haecceitist intuitions and conclude that this explanation is not sufficient.

Lewis (1986, pp. 230-5) suggests an anti-haecceitist explanation regarding these haecceitist intuitions. Given Lewis’s modal realism, a possible world is the totality of compossible individuals. In following Lewis’s counterpart theory, the possibility of an individual is determined by how its counterpart is in other possible worlds. What possibilities an individual has are determined by what individuals in other possible worlds stand in the counterpart relation to the individual in question. According to Lewis, this counterpart relation is more intricate than the relation between possible worlds. For example, “I could be one of the twins” is true. There is a possible world in which twins exist and I am represented as one of the twins. In this case, I have two counterpart
relations, a relation to one of the twins and another relation to the other of the twins. We have more counterpart relations than the accessibility relation between the worlds.

In order to make sense of haecceitistic intuitions, Lewis makes this counterpart relation more flexible. Consider a switching possibility like Kripke’s dice case. “I could be my sister” or “it is possible that my sister and I switch qualitative roles.” In order to analyze this switching possibility, I am going to introduce two kinds of counterpart relations: an inter-world counterpart relation and an intra-world counterpart relation.

For my sister’s and my qualitative roles in the actual world \( w_1 \), \( Q_1 \) and \( Q_2 \) respectively,

**[Inter-world counterpart relation]** There is another world \( w_2 \) in which my counterpart has my sister’s role \( Q_1 \) and my sister’s counterpart has my role \( Q_2 \).

**[Intra-world counterpart relation]** In the actual world, my counterpart is my sister who has \( Q_1 \) and my sister’s counterpart is me who has \( Q_2 \).

In the inter-counterpart relation, I have a counterpart relation with an individual who has \( Q_1 \) even though there is an individual who shares all of my qualitative roles \( Q_2 \). Then the counterpart relation is not a qualitative one based on similarity. The intra-world counterpart relation is also not qualitatively determined. In the actual world, the most similar individual to me is myself. But my sister is my counterpart in this actual world. This counterpart relation is called a non-qualitative counterpart relation in that the relation is not determined by the qualitative features of the relata. But for anti-haecceitists like Lewis, there is no further analysis of the conditions making the pair related as
counterparts. They are just related. Probably, anything can be related as non-qualitative counterparts. This relation is mysterious to the anti-haecceitists. So, then how can the anti-haecceitists explain away our haecceitistic intuitions?

Some may claim that a non-qualitative counterpart relation is not allowed in the inter-world counterpart relation, but is allowed in the intra-world counterpart relation. It is acceptable for an individual to have anything as its intra-world counterpart. This is what makes our switching possibility intuitive. Lewis seems to take this route. As an anti-haecceitist, he claims that the non-qualitative-inter-world counterpart relation should be prohibited. In order to explain the haecceitist intuition, however, he adopts a different kind of relation, which I call an intra-world counterpart relation. In his words, the qualitative counterpart relation holds between distinct possible worlds, which constitutes a real possibility, but the non-qualitative counterpart relation holds only within one and the same world, which constitutes a mere possibility.

Delia Fara (2008) and Bradford Skow (2008; 2011) cast doubt on whether the non-qualitative-intra-world counterpart relation fits well in our logical and semantic system. Even though the intra-counterpart relation is different in kind from the inter-world counterpart relation, Skow (2008, p.105) takes it into account in determining what a possible world represents. Here is his example: He has a brother who lives in LA. But he himself does not live in LA. So ‘BAS lives in LA’ is false according to this actual world. When his brother is his intra-world counterpart, however, ‘BAS lives in LA’ is true according to this actual world. Then, one and the same sentence has different truth
values. As Skow points out, we make an assumption that if a sentence S is true according to \( w_1 \) and false according to \( w_2 \), then \( w_1 \) and \( w_2 \) are distinct. But this assumption is false here and makes a logically inconsistent possible world representation.

A similar idea is also found in Fara (2008). In this case, the intra-world counterpart relation leads to trouble with a modal operator, an actuality operator, ACT. ACT has the following inference rules as its meaning. (p.293)

\[
(A) \quad \Diamond \text{ACT } P \\
\text{ACT } P
\]

\[
(B) \quad \text{ACT } P & \text{ACT } Q \\
\text{ACT}(P & Q)
\]

Given these inference rules, Skow’s problem can be paraphrased as follows:

i. ACT BAS does not live in LA

ii. \( \Diamond \text{ACT BAS lives in LA} \) (it is possible that Skow lives in LA in this world)

iii. ACT BAS lives in LA (by inference (A))

iv. ACT BAS does not live in LA & ACT BAS lives in LA (by i and iii)

v. ACT (BAS does not live in LA & BAS lives in LA) (by inference (B))

The last sentence allows contradiction in the actual world. In order to avoid this contradiction, we need to abandon at least one of the two inference rules. But this is not
easy. Fara points out that the operator ACT is generally understood as a rigid indexical. The operator picks out any world where it is used, and continues to refer to the same world in any context. Inference (A) implies this understanding of ACT.

Fara (2008, p.294-5) considers whether inference (B) can be rejected. We have similar issues in a context–dependent counterpart relation. Imagine a statue that is made of the lump of clay, call it a. It is definitely possible that the statue is made of other materials like wood. It is also possible that the lump of clay makes up a different thing like a vase. However, the statue cannot be a vase, and the lump of clay cannot be wood. When we consider a as a statue, it is necessarily a statue, and also, when we consider a as a lump of clay, it is necessarily a lump of clay. But we cannot infer that it is necessary that a is both a statue and a lump of clay, which is false. In the context of considering a as a statue, every counterpart of a is a statue, and in considering a as a lump of clay, every counterpart of a is a lump of clay. Because of the context-dependent counterpart relation, the necessity operator cannot commute with other logical operators outside its scope. Can we apply this idea to operator ACT and claim that the intra-world counterpart relation is context dependent, therefore ACT cannot commute with other outside operators? Fara (2008, p.295-6) says no. In the case of a context-dependent counterpart relation, names or variables occur in the argument place of sortal predicates, ‘is a statue’ and ‘is a lump of clay’. The sortal predicate determines the context in which a counterpart relation occurs. But there is no sortal predicate in “BAS lives in LA” to determine the context in which the intra-counterpart relation occurs. It still seems hard to make sense of the intra-world
counterpart relation. Correspondingly, it is hard for the anti-haecceitists to explain the haecceitistic intuitions instead of refusing them.

In the next chapter, I will examine a way for a haecceitist to explain these intuitions, especially (a). The haecceitist explanation should avoid Lewis’ criticism that the haecceitistic difference does not make a real difference in the world.
2.1 Thisness

Haecceitism is basically about the requirement of something non-qualitative. This requirement is expressed in many different contexts. There should be a non-qualitative fact about the world; A non-qualitative substance should be a constituent of the world; there is a non-qualitative property that cannot be determined or grounded by qualitative properties. Most haecceitist discussion has been focused on non-qualitative properties. This is probably because the notion of property is already so well analyzed and accepted.

In this chapter, a property is understood as something expressed by an open formula in a formal first order language. There are various types of non-qualitative properties. The most typical non-qualitative properties are properties concerning particular individuals. The individuals are referred to by direct referential terms such as proper names and indexicals, for example, being a daughter of Aristotle, existing right now, being located in this place, being bound in this world. In addition, there are somewhat controversial cases of non-qualitative properties that do not concern particular individuals. For example, we can consider non-qualitative structural properties or relations such as being next to, being distinct from, being part of. These structural properties or relations are not completely determined by the qualitative properties of the related individuals.

---

10 I am avoiding metaphysical topics what properties are here. Some believe that properties expressed by an open formula are abstract entities like universals (Armstrong 1997, Maurine 2002, Benovsky (2014); Others see properties as some abstract particulars like tropes (Williams 1953, Mulligan, Simon and Smith 1984); Yet another view holds that properties are just sets of concrete particulars (Lewis 1984). I am trying to be neutral to the ontological ideas about properties.
In this chapter, I am going to analyze the fundamental feature of haecceity/thisness\textsuperscript{11} and discuss a non-modal way to defend this property. The notion of thisness is not without its problems. I will try to find the best or an alternative way to understand thisness. This alternative understanding may be different from our ordinary understanding of thisness and similar to some non-qualitative properties that do not concern particular objects.

Since Robert Adams (1979) introduced the notion of thisness in the field of contemporary metaphysics, it has been a typical example of a non-qualitative properties. The notion of thisness is prima facie, simple and clear.

**[General Characterization of Thisness 1]** For an individual x, there is a property of being identical to x.

Every particular individual is expected to have its own thisness: being this chair I am sitting on, being identical to my cat, Nuluk.

But we have another characterization of thisness based on the notion of individuation rather than identity. As in Adams’ (1979) understanding, thisness is a property to *numerically individuate* an entity *primitively* as a *single* individual. A numerically individuated entity is an individual, and the numerical individuation makes it a single individual. A numerically individuated single thing is given prior to any analysis of the individual. For example, my cat Nuluk has many features such as being a cat, being a

\textsuperscript{11} Henceforth, I am going to use only thisness. But I am taking thisness and haecceity as the same property.
calico, being my first cat, etc., but what individuates Nuluk as one individual is just given prior to any analysis of these features. The property of being thus individuated to be my Nuluk is the thisness of Nuluk. This understanding focuses more on numerical individuation rather than identity.

[General Characterization of Thisness 2] For an individual x, there is a property of numerically being individuated as x.

Both characterizations (1) and (2) have been used in our general understanding of thisness. It has not been much discussed whether the two characterizations are distinct or which one is better.

But it is controversial whether identity and numerical individuation coincide. An entity that is not considered to be an individual can have an identity condition. For example, water is matter that is not numerically well individuated.12 There is no standard way to individuate water. But we can have a condition to identify water that has been arbitrarily individuated, for example, the water I have in my cup. Then, does every numerically individuated entity have an identity condition? This will be one of the core issues discussed in this chapter.

Regardless of which characterization is right, moreover, advocates of thisness have the same opponent, a reductive theorist of identity and individuation. The reductive understanding of the identity of an individual, such as Leibniz’s principle of identity of

12 This will be discussed more in the next section.
indiscernibles (henceforth, PII), claims that the property of being Nuluk or the property of being individuated to be Nuluk should be analyzed and broken down into other properties of Nuluk, such as being a cat. In the following sections, I am going to analyze three basic and shared features of thisness: numerical individuation, primitive individuation and singularity. Then I will show how the notion of thisness can be divided.

2.1.1 Numerical Individuation

General characterization of thisness by primitive identity is stronger than the characterization by primitive individuation because the former implies the latter, but not vice versa. Thisness in any characterization is a property responsible for numerical individuation. Any numerically distinct individuals can have their thisnesses. Individuals are just something individuated. If something does not have a thisness, it is not an individual, and if something is an individual, it has its thisness. An analysis of numerical individuation will help us to understand and evaluate thisness. The notion of the individual is broader than the notions of substance and objects. Other than concrete and physical entities, some abstract or some nonphysical entities can be – even though controversially – considered to have a thisness. Richard Swinburne (1995) takes spatio-temporal locations to have thisnesses, and for Joseph Diekemper (2015) an event is a typical case of having a thisness. On the contrary, not every entity is an individual. Universals are individuated by their kind but not numerically individuated. They are shared by multiple individuals rather than by individuate individuals. E. J. Lowe (1989,2003, 2009) claims that entities expressed by some uncountable nouns are not individuals, but rather a quantity of homogeneous matter. For example, water does not
have a principled way of individuation. It is just arbitrarily individuated by objects such as some cups, some bottles, some pails, etc.

What is it to numerically individuate or count\textsuperscript{13} things in a principled way? That a collection of things is counted is understood to be that the collection of things can be assigned a determined cardinality. In addition, a number of things should be the same regardless of who counts, where and when counted if and only if it is numerically individuated in a principled way. Strictly speaking, numerical individuation and counting are different from numerical distinction. In a world where only one individual exists, the individual is countable even though it is (trivially) not distinct from others because there is no other. But in most cases, numerical individuation, countability and numerical distinction coincide. According to proponents of thisness, what makes an individual a countable one and what makes the individual numerically distinct from other individuals are primitively determined. This individuation is sometimes called ‘primitive quantity individuation’ (Schaffer (2001)), and other times ‘transcendental individuation’ (French (2014)).

2.1.2 Primitive Individuation

Thisness is a primitive property, which means it should not be derived from, reduced by, and analyzed in terms of any other properties. My cat Nuluk instantiates some intrinsic properties such as being a cat, being calico, being 9 pounds. The classic discussions of thisness, especially between Duns Scotus and Leibniz, focus on whether the individuation of Nuluk is \textit{reduced} to or \textit{over and above} the conjunction of these intrinsic properties.

\textsuperscript{13} I use two notions, numerical individuation and countability, interchangeably.
[**Reductionism**] The individuation of an individual is entirely determined by and reduced to its intrinsic properties. There is a metaphysical analysis of numerical distinctness.

[**Primitivism**] The individuation of an individual is not reduced but primitively given. There is a brute/primitive fact about numerical distinctness.

Adams (1979, p.7) makes a distinction between constituting properties and individuating properties, and considers thisness as a merely individuating property. Diekemper (2014) explains thisness as an individuating property rather than an organizing property. The constituting properties and the organizing properties are not well defined. But these ideas express one thing: thisness does not characterize an individual. It does not show us how the individual is. This does not imply that there should be a metaphysical individual such as a bare particular that exists without any properties constituting or organizing the individual. Thisness is just the numerical individuation of things. Whether or not individuals cannot exist without their constituting or organizing properties, they are individuated without depending on any properties they have.

There are various types of reductionism. Self-identity and numerical distinctness can be reduced to qualitative properties or to non-qualitative properties other than thisness. If a spatial property, e.g., having a certain spatial locution, individuates entities and the spatial property is not purely qualitative, we have a non-qualitative reductionism.\(^{14}\) In Black’s world, there are two spheres that share not only every constituting property but

---

\(^{14}\) We have a developed version of this non-qualitative reductionism, which is called structuralism. This will be discussed in the next chapter.
also relational spatial properties, e.g., *being 2 miles from another sphere*. In the original form of Black’s world, the numerical distinction of the two spheres is guaranteed because they stand in a spatial relation, *being 2 miles distance from*. Black accepts a principle of space that is called the impenetrability principle which declares that one and the same thing cannot occupy two spatial locutions at once, and he interprets it as the idea that an individual cannot be spatially distant from oneself. The discussion of the possibility of Black’s world has developed into the possibility of its spatial construction: whether or not the space is tightly curved to allow a distance from oneself. As Adams (1979, p.16) points out, however, the issue of thisness is whether the numerical individuation is metaphysically prior to its locution in space or the spatial construction of the world determines the individuation of an individual. He believes that the spatial facts of the world are determined by the number of individuals, their relations, and their movement given the worlds’ physical laws. The primitiveness of thisness refutes not just a reduction to qualitative intrinsic properties but also a reduction to spatial properties or any other property. Primitiveness is a claim stronger than non-qualitative individuation.

2.1.3 Singularity

A thisness is exemplified by only one individual. Only Nuluk can have the property of being identical to Nuluk. Call a property that is exemplified by only one individual a ‘singular property’. Suppose that a property of being a calico cat on 29 Redcliffe Ave., Highland Park, NJ in 2016 is exemplified by only one individual, Nuluk, *accidentally in this world*. Thisness, on the other hand, has been considered to be necessarily a singular property. If thisness is a condition of identity and identity necessarily holds, then thisness
should be a condition for necessary identity. Many philosophers, including Lewis and Stalnaker, take thisness as individual essence and analyze it in a modal context.\textsuperscript{15}

However, the idea of numerical and primitive individuation does not provide a substantial condition to determine a position in the \textit{de re} modality. The idea of thisness can go with any \textit{de re} modal theories such as a counterpart theory. For example, two distinct worlds have their own primitively given individuation, and one of the primitively given individuals in a world can have a counterpart relation with another individual primitively given in another world. If we take a modally neutral notion of numerical and primitive individuality and allow a one-to-one relation, rather than an identity relation, between primitively given individuals in distinct worlds, we can create a model for the counterpart theory. There are many metaphysical theories about what individuals we have and how to individuate the world (or worlds). Thisness theorists can work with any theory that requires individuals in the fundamental description of the world, and then claim that the individuals are given primitively.

We have two options for how to understand this singularity. First, by the singularity of thisness, any identity questions are solved: thisness determines whether individuals are bounded in a world or not, and how to identify temporally extended individuals, etc. If you do not want to solve classical metaphysical problems like transworld identity or personal identity with only thisness, then we need to have an understanding of a weaker version of singularity. One possible understanding is as an individuation into a single

\textsuperscript{15} This issue is discussed in chapter 1. The notion of individual essence is required to make sense of the transworld identity. In the chapter, however, I pointed out that haecceitism and the transworld identity are not in the same boat.
thing. Then, an individual like Nuluk has the property of being individuated to be one thing. Considering that an individual is just something individuated and numerical individuation is being counted into a single thing, then this understanding can be said to hold that each individual is being so individuated. We know that thisness is uniquely instantiated. Here, this is the case not because we find that the property is instantiated by only one *particular* individual but because any individual should be *so individuated an individual*. Singularity seems not a claim of individual essence, but a trivial implication by the meaning of ‘numerical individuation’ and the logical truth of self-identity.

### 2.2 Numerical Individuation: Countability

A particular individual is numerically distinguished and self-identical. These two claims are often believed to be the minimal requirements needed to be an individual.\(^{16}\) Self-identity is relatively well discussed in metaphysics. But the metaphysical implication of countability is not clear. The numerical individuation is often regarded to be a fundamental fact of an individual in order to make sense of its relations to others including their temporal and spatial relations.\(^{17}\) In this section, we are going to analyze the notions of numerical individuation and countability, and examine whether this numerical individuation or counting is given fundamentally and prior to any other features of an individual.

---

\(^{16}\) Check Dorato and Morganti (2012) for a detailed discussion.

\(^{17}\) Adams (1979, p.14-16) uses this idea to criticize Hacking’s response to the dispersal argument for haecceitism. Some philosophers like Lowe do not hold this intuition. We are going to discuss this in a later section.
2.2.1 Counting and Cardinality: An intuitive account

The standard set theoretical understanding of counting is a cardinality assignment: to count a collection of things we point to each of them attributing an ordinal in a certain order.\textsuperscript{18} When counting a number of students in a classroom, for example, we assign an ordinal number, e.g., a natural number, to each of the students in the order of natural numbers from the smallest. In other words, there is a one to one correspondence (or bijection) between a collection of the students to be counted and the set of natural numbers. \textit{All} the entities to be counted get assigned a unique natural number which is the available smallest natural number when counting. The last smallest assigned number is the cardinality of the collection of the students. The cardinality of a collection is the last ordinal in the counting procedure and the counting procedure is to assign a unique ordinal to each member of the collection. Each ordinal number in this unique ordinal assignment can be considered as \textit{a label or as a name} of the student.

How do we consider this label? Is this a rigid designator or merely a technical mark? Redhead and Teller (1992, p.202) say that entities to be counted “are thought of as ‘the first one’, say labeled with the number 1, and another one which can be thought of as ‘the second one’, say labeled with the number 2. In short, whether or not we are able in practice to distinguish, reidentify, and label, tag, or name, objects, they can be conceptualized in all these ways. When one can correctly think of an entity in all these

\textsuperscript{18} According to a standard set theory, Zermelo-Fraenkel, ZF, counting is expressed in Von Neumann cardinality assignment: For a well-ordered set $U$, we define its cardinal number to be the set of smallest ordinal numbers $a$ such that there is a bijection between $U$ and $a$, that is, equinumerous to $U$. This ordinal is called the ‘initial ordinal of the cardinal’.
ways we will refer to it as an *individual.*”¹⁹ Here, the countable entities are
distinguishable, reidentifiable and can be labeled. The things that can be labeled coincide
with things that can be reidentified in any other context. The label here can take a
metaphysical role to identify and reidentify an entity across contexts, that is, a rigid
designator. Many philosophers apply this labeling in counting to the case of a
permutation difference when permuting the labels in a description of the entities creates a
description of another distinct state.

However, it is not true that any entity that can be counted can get a rigid designator. We
have a well-known counterexample against this idea, quantum particles in (non-
relativistic) quantum theory. Quantum particles like electrons are said to be at least
intrinsically and spatially indistinguishable one from another. But they can be counted. It
will be shown in the following sections how to count indistinguishable quantum particles
to which a rigid designator cannot be assigned. Then, do we need to say that countability
does not have any metaphysical significance but merely formal or mathematical
meaning? If so, primitive countability will not support any metaphysical claim including
a thisness theory. This will ruin the argument that the fact of the number of an entity is
metaphysically fundamental and given prior to any qualitative and spatio-temporal facts
about the entity. So we are left with questions, (a) what is the metaphysical implication of
counting? And, (b) whether primitive countability can support thisness?

¹⁹ Redhead and Teller (1998) suggest two types of transcendental individuality, a label transcendental
individuality and a property transcendental individuality. The former is about an individual that is
transcendently determined to be able to be labeled; the latter is about an individual that is
transcendently determined to be able to have properties. For the topic of this chapter, only the label
transcendental individuality is relevant.
2.2.2 A Minimal Metaphysical Implication of Counting

In ordinary situations, there are other ways to count entities. Suppose that you count the number of students coming to a class by clicking a tally counter. You can click the counter whenever one student comes into a classroom, or you can click it after checking whether the student is already counted. We know that the former cannot accurately track the number of students because a student can sneak out of the classroom and keep coming back. In order to have the exact number of students recorded, we need to exclude double-counting. In order to exclude double-counting, you must check whether the student you are about to count is one you already counted before. The method for this counting is a combination of singling out by tally counting and avoiding double-counting.

There are many ways to exclude double-counting. We can attach a number displayed in the tally counter to each student when we count her/him (that is, assigning a unique label/name), or stamp on their wrist a symbol for having been counted (separating a group to be counted from a group already counted by giving some new feature to the students), or keep the counted students in a completely closed room (separating a group to be counted from a group already counted with some external apparatus). In order to avoid double-counting, there should be a distinction between entities to be counted and entities being counted. There are many practical ways to make this distinction like the above examples, whether using a rigid designator, an intrinsic feature of the entities, or some external apparatus. But the tally counting should be required for any counting method. The counting process can be divided into two steps.

(1) Singling-out: single out one entity from other entities at an instant.
(2) Avoiding double-counting: making sure that an entity appears only once in a counting process.

In the classic set theoretical way of counting, the one-to-one correspondence to ordinal numbers or the assignation of labels can do the work of (1) and (2). Because an ordinal is uniquely assigned to one entity, the ordinal number uniquely individuates it from others and is not assigned to two or more things. But the unique label can do much more than these two steps. In order to avoid double-counting, it is only necessary to make a distinction between two groups, a group to be counted and a group to count. But a label enables us not only to make this distinction but also to identify and reidentify each entity. Labeling gives much more detailed information about entities which is over and above the requirement for merely counting them. Labeling is more than just counting.

Domenech and Holik (2007) develop an alternative way to determine the cardinality of a finite set in terms of an extracting procedure. Suppose that for a finite set, one member in the set is extracted at a time successively. After some finite number of applications of this extracting procedure, we get an empty set. The number of members in this counting is the number of times that this extracting procedure is applied. The number of times this extracting procedure is used until no member is left in the set (i.e. an empty set) defines the number of the members of the set. This alternative way of counting orders the application of the extracting procedure instead of ordering the members of the set. This

---

20 Some may say that only the counting process with bijection gives a cardinality to a collection and other alternative counting methods just coincides with the result of the number of members of the collection. Instead of finding a more technical definition of cardinality and a reason why these two counting methods coincide, I will take an intuitive understanding of counting and then an equally good procedure to result in a number of members of a collection.
counting method does not signify which ordinal is assigned to which member in the set and, correspondingly, does not require reidentification of the members.

Domenech and Holik use this alternative counting intuition to count indistinguishable entities like quantum particles. As discussed, we cannot make an assertion about their identity or reidentification, but we can make an assertion about the number of the quantum particles, for example, “a Helium atom has two electrons”. A possible experimental method has been suggested.\(^{21}\)

Put the atom in a cloud chamber and use radiation to ionize it. Then we would observe the tracks of both, an ion and an electron. It is obvious that the electron track represents a system of particle number equal to one and, of course, we cannot ask about the identity of the electron (for it has no identity at all), but the counting process does not depend on this query. The only thing that matters is that we are sure that the track is due to a single electron state, and for that purpose, the identity of the electron does not matter. If we ionize the atom again, we will see the track of a new ion (of charge 2d), and a new electron track. … Now, the counting process has finished, for we cannot extract more electrons. The process finished in two steps, and so we say that an Helium atom has two electrons, (Domenech and Holik, 2007, p. 862)

In this possible experiment procedure, a neutral Helium atom is put in a cloud chamber and ionized using radiation. Then we get the track of an ion and the track of an electron. An electron track is counted as the track of one electron. And by ionizing the atom once again, we can see the track of an ion of charge 2e and of another electron track. We cannot get more electron tracks from the atom because none is available. We conclude that the Helium atom has two electrons. In this procedure, we do not check which electron is which and do not require the reidentification of the counted electrons. It is assumed for an electron to be ionized and eliminated from the atom. Then this electron

---

\(^{21}\) Check Dalla Chiara and Toraldo di Francia (1993), French and Kraus (2006, ch. 5), and Domenech and Holik (2007) for relevant detailed discussions.
merges with others in the experiment. It is supposed that its identity is lost. Because of this identity loss after the measurement, most quantum theorists, including Domenech and Holik, consider that quantum particles do not have identity or identity conditions. But the particles are counted in a principled way. This counting method satisfies the two required steps to count (1) and (2) in a consistent manner. An electron is extracted by ionization and then considered to be eliminated. And the number of this extracting procedure determines the number of electrons. Someone may say that this method still needs labeling, instead of labeling items to be counted, labeling the extracting event. If the event is labeled, can we then say that an item included in the event is also labeled but in an indirect way? No, we cannot. This idea is based on an assumption that if something can be labeled, then its constitutive parts can also be labeled. There should be some metaphysical condition to have a label, for example, existing across some context. But there is no guarantee that if an entity has some metaphysical or other type of feature, then its constitutive parts inherit it. For example, an entity’s mass is not a transitive property to its parts. Given that labelability is a metaphysical feature, a composite entity’s labelability does not guarantee its constituents’ labelability.

Given this alternative way of counting, we can say that the minimal requirement for counting does not require identification and reidentification of an item to be counted. Those are just some of the ways to avoid double-counting. And the standard way of counting by an assignment of ordinal numbers requires more than what is required for counting: the counted entity should discern which is which and be reidentifiable. But any counting process including either a labeling or an extracting procedure to avoid double-
counting should include a singling-out process. The extracting process should take one entity at a time; the labeling process should assign a label to one entity uniquely in any context. In order to be counted, an entity should be individuated as a single entity at an instant. In the above example, the tally counting allowing double-counting does not measure the number of the students, but individuates an entity as a single unit, which is the step of individuating (1). Counting includes numerical (or single) individuation at an instant but does not require identity or reidentification.

This discussion has several implications. The most important implication is that numerical individuation is independently determined from identity. However, identity of an entity requires its numerical individuation. Numerical individuation is considered to be given prior to identification. Numerical individuation in this understanding is so fundamental that it is natural to accept numerical individuation as primitively determined, and so, individuated individuals are primitively given. Because of the distinction between numerical individuation and identification, for a numerically individuated thing, we cannot say which or what it is. Then a question arises. What are entities that are only numerically individuated? Are the entities metaphysically significant entities or merely theoretically or mathematically posited entities? Before getting into this issue, it will be helpful to note some classical arguments against this primitive numerical individuation.

2.3 **Individuation by Kind**

Counting as a numerical individuation metaphysically requires something to be a single thing. Some quantum particles like electrons give us an interesting example of entities
that we can count even though they are indiscernible by their intrinsic features and their spatial locations. An entity is singled out as one individual without depending on any other properties of the entity. Singling-out seems given primitively. But E. J. Lowe thinks that even when this fundamental process of counting requires the analysis of the qualitative structure of an individual, that it is a certain kind or sort of an individual. His arguments are divided into two, one from our counting practice and another from his analysis of having a number.

A sufficient, but not necessary, condition for a general term’s being a sortal is that there should exist some principle for counting or enumerating individual instances falling under it. Thus, there are ways of counting the number of men or tables or books in a given room, but no way of counting the number of red things that there are. And this is not because there is such a number, but one beyond our powers of determining – as in the case of the number of atoms in the room – but rather because it apparently does not even make sense to speak of such a number until the sort or sorts of red thing that one is to count have been specified. Suppose, for example, that the room contained a red table: then that, it might be argued, is clearly one red thing. But what about its red top and its red legs, or the red knob on one of its red drawers? Are these to be counted as different ‘red things’ in the room in addition to the red table itself? And what about, say, the red paint covering one of the table’s legs: is that also to count as a distinct ‘red thing’ in its own right? It rapidly becomes apparent that there is no principled way of deciding these matters, until we are told what sorts of red thing we are supposed to be counting. (Lowe 2009, p.13)

In addition, Lowe often uses Frege’s example, a pack of cards. An entity is categorized as one pack of cards, 52 cards, and also 4 complete suits. When we are told “Count these things” with this entity, we will be confused and unsure what to do. Only when we are asked to count how many cards, or how many packs of cards there are, can we clearly fulfill the demand. But exactly what does this example show? How can Lowe conclude from these cases that an individual should be a thing of a kind in order to be counted?

Frege uses this example to show that a number as a property cannot be assigned to an object because different (inconsistent) numbers cannot be applied to an object. He suggests, rather, that numbers are assigned to concepts. For example, the number 4 is applied to a concept “the complete suits of cards”. His analysis of number is beyond the discussion here.
This case does not imply that it is impossible to count things. Rather, we have many ways to count things but we cannot choose one in a principled way. But there can be many reasons why we can have many ways to count things in this everyday situation. We have many counting ways from our linguistic conventions, or from our cognitive categories, etc. These ways are not guaranteed to reflect a metaphysical individuation. Suppose a person who believes that the fundamental particles of physics are the only legitimate individuals and there is a definite number of the particles in the pack of cards in question. Even this person can be confused by the question ‘how many things are there?’ because she knows that there are many other macro entities in our linguistic conventions or in our visual categories that can be counted. Our confusion when asked to count things does not indicate a metaphysical problem.

Lowe (2009, p. 55-6) makes the metaphysical claim that in order for an entity to be one entity, the entity is required to possess a unity. According to him, there are two kinds of entities, something whose constituents are integrated in a certain principled way and another thing whose constituents are not integrated. And the latter cannot be counted in principle. In order to express the latter kind of entities, he introduces a new ontological notion, dividual. This notion is applied to entities such as quantities of homogeneous and infinitely divisible matter. Dividuals are neither single nor plural entities. They have some quantity of matter in an arbitrary unit.

Both dividuals and individuals may have parts, but the parts of dividuals are further dividuals and need not be unified in any way. In contrast, a composite individual – one that has proper parts – must have parts that are integrated according to some principle that is characteristic of individuals of its kind. For example, an animal, such as a tiger, is a composite individual of such a kind that it must have organic parts that are spatially and causally connected so as to enable them to function
in the right sort of way to sustain the life of the individual animal that they compose. (Lowe, 2009, p.55)

Here Lowe uses his new notion of dividual for an entity whose constituents are not organized to be a (natural) kind and infinitely divided. An entity’s possessing a unity seems to mean that it instantiates a (natural) kind. Here a natural kind is something that appears in the laws of nature. In the above passage, he provides an example of kind, tiger. An entity of the tiger kind is composed of some constituent matter and its (causal and spatial) connection to satisfy the biological definition of tiger. The connection is the condition for being an instance of a kind, tiger, in this case. Lowe’s argument can be stated:

(1) To be countable is to be a unit.
(2) An entity’s being a unit is its being an instance of a kind.

In addition, he believes that an individual is an entity that counts as one entity and only individuals are countable.

(3) Every individual is countable.
(4) Therefore, every individual is an instance of a kind.

(4) is Lowe’s basic understanding of an individual: Individuals are necessarily a kind. (1) can be accepted even by thisness theorists if they take the unit as primitively given without further analysis. As in (2), however, Lowe takes a reductive view: to be a unit is to be organized with some physical constituents, otherwise the constituents can be scattered without making an individual. According to him, an individual/object is an
instance of some organization with physical constituents. This idea is highly influenced by Aristotle’s four-category ontology. According to this ontology, substances, which are (composite) entities, are instantiations of substantial universals, which are kinds in Lowe’s picture.

Accidental universals can be translated into properties that an entity can accidentally have, for example, being red (color), being round (shape), etc. Particular properties are translated into tropes\textsuperscript{23} in the current metaphysics or modes in Lowe’s metaphysics, a particular instantiation of a property. For example, the redness of this pencil is a particular instantiation of being red. A substantial universal organizes some portion of matter and composes a substantial particular, which is a concrete individual. The examples of substantial universals are kinds or sorts in our science or intuition, such as

\textsuperscript{23} Lowe acknowledge that his notion of modes is basically the same as the notion of tropes. But he prefers to use modes because he does not want to get involved in any property nominalism that is supported by some trope theorists.
animal, mammal, cat, atom, proton, etc. One restriction in Lowe’s picture is that kinds should be natural kinds that participate in natural laws. Concrete individuals are an instance of these universals, such as a cat or an animal. Unlike accidental universals, Lowe points out that kinds provide an identity condition for an entity. For this claim he depends on Locke. It is well known that Locke introduced the idea of kinds to solve a problem of personal identity. Two kinds, person and man, are involved in this issue. These two kinds have their own identity and individuate condition: “person” is a kind-term to refer to an intelligent thinking being; “man” to refer to a living body of a particular shape. If a person goes into a state of brain death, then based on these conditions for kinds, the person does not exist anymore, but a man continues to exist.

Individuals should be expressed not by ‘things’ but by ‘a cat’ or ‘an animal’. We do not count things but count the number of cats or animals. Given this idea, an individual cannot be primitively individuated but reductively analyzed. But what makes Lowe believe in this individuation of kind? He relies on metaphysical assumptions. First, he (2009, p. 15-6) makes an assumption about empirical metaphysics. “… particulars cannot even be experienced as being particulars, without being experienced as particulars of some sort”. Second, it will be harder to make sense of kinds sorts in an abstraction from the individuals. These issues are beyond our present purpose. Even without getting into these reasons, I still have a problem with Lowe’s picture.

In Lowe’s picture, individuals are instances of some kind and a numerical individuation requires individuation by kinds. Kinds are just given by our best science or our best
intuition. Lowe knows that kinds overlap. However, not every overlap is allowed in Lowe’s picture. Consider two overlap cases, an instance of cat is also an instance of animal, and an instance of person and an instance of man as in Locke’s case. The two kinds in the first case have an inclusive relation in their characterizations. Every cat is an animal. If something is cat, it is animal. If the individual as cat dies, the individual as animal dies. However, the kinds in the second case do not have the systematic relation in their characterizations. Even if something is a person, this does not imply that it is a man. Even if a person dies, this does not have an implication about the death or survival of a man. The condition to be a person and the condition to be a man are distinct.

Lowe will not accept the second overlap case. The overlap case will be explained in two situations, (a) more than one individual, a man and a person, exist in one spatio-temporal location or (b) the instance of man is identical to the instance of person. Neither of these are satisfactory to Lowe’s picture. Concerning (a), we need to refuse an impenetrability intuition that two physical (macro) entities cannot share the same particular position in space at one time. When Barack Obama waves his hand to me, it is always indeterminate whether it is a man or a person that waves his hands. This allows too many indeterminate facts about the world. (b) is actually a position that gives up individuation by kind. Their identity conditions – then their individuation conditions as well – are not determined by the kinds, man and person.

Lowe should say that when kinds overlap, the persistence conditions for kinds should be the same. If kinds have different persistence conditions, their instances do not overlap
even if they share the same spatiotemporal location. Whether kinds overlap determines the number of instances of the kinds. The overlap of a cat and an animal implies a single instance for two kinds, cat and animal. However, man and person do not overlap implies that when a man and a person exist, there are at least two individuals. What is the condition for kinds to have the same persistence condition? This condition is not determined by the (inclusive) relation of characterizations of kinds. Suppose that a cat can be characterized by a physical entity that has a certain range of size, weight, etc. But two kinds cat and physical entity have distinct persistent conditions. Even if a cat dies, no physical entities disappear. Then what else determines whether kinds share the same persistence condition? It seems hard to find a candidate other than their instances sharing the same persistence condition, which implies that the instances are identical. Then the individuation and identity of instances should be given independently of individuation by kind.

2.4 Thisness and Individuation

2.4.1 Individuation and Identity

In section 2.2.2, we made a distinction between numerical individuation and identity. What is minimally required for entities to be countable is to be singled-out as one entity. But in order for a (finite) set to have cardinality, its member should not be doubly counted. But the double counting can be avoided without identifying which entity is which. An entity has numerical individuality but is not guaranteed to have identity. Quantum mechanics have provided interesting cases that are only numerically individuated without its identity condition.
“No entity without identity” is W.V.O. Quine’s famous slogan. He asks, “what sense can be found in talking of entities which cannot meaningfully be said to be identical with themselves and distinct from one another?” (Quine 1948) His reason here depends on the claim that self-identity and distinction implies identity, and without identity, we cannot claim that an entity\(^{24}\) is self-identical and distinguished. Identity should be given in order to make sense of self-identity and distinction. However, what we have considered is the opposite. We are considering the situation that the numerical distinction of an entity is given first prior to its identity condition. His reasoning is based on what we cast doubt.

Let’s consider quantum particles. They behave differently from individuals in classical physics. Quantum particles have a probability assignment different from the classical one like in Kripke’s dice case. Suppose that two quantum particles in the same kind, two electrons for example, are distributed over two boxes (or energy state), where the particles are indistinguishable in their intrinsic features such that they have the same mass, charge, etc. Then we will have the following arrangements.

\[
\begin{array}{c}
(i) & \bullet & \bullet & \square \\
(ii) & \square & \bullet & \bullet \\
(iii) & \bullet & \bullet & \bullet \\
\end{array}
\]

---

In classical physics, (3) is given a weight of twice that of (1) or (2), which corresponds to the two ways to have (3) by permuting the particles. There are four combinations in

\(^{24}\) His notion of ‘entity’ is different from Lowe’s notion of ‘entity’ that is divided into dividuals and individuals. Entities, objects, and individuals seem interchangeable here.
distributing particles in total. Therefore, the probability to find one particle in each state, represented in (3), is $\frac{1}{2}$. However, in the case of quantum particles like electrons, probability is assigned $\frac{1}{3}$ equally to (1), (2), and (3). The particles in (3) are numerically distinguished but permuting the particles does not generate a distinct new state.

Some people take quantum particles as non-individuals and develop the metaphysical analysis of non-individuals (Darwin 1931, Born 1962, etc) and some take quantum particles to be an example how to modify our notion of individuals (Doranto and Morganti 2013, etc). Some might say that it is not helpful in understanding quantum particles to discuss whether they are individuals (Howard 2011). What a quantum particle it is is beyond our discussion here. From this example, however, it is clear that numerical individuation of particles does not coincide with their identity. The numerical distinction of the particles in (3) constitutes the fact of (3). However, the identity of the particles in (3) do not affect the fact of (3) because the permutation of particles does not change the fact of (3). Numerical individuation and identity do not coincide.

As discussed in Section 2.2.1, a standard way of determining cardinality of a collection of entities is to have a bijection between the entities and ordinal numbers. The assigned numbers through this bijection are considered labels of the entities. This label works as a rigid designator in a counting process that guarantees the identity of the entity to which the label is assigned. In other words, the label determines which entity is which across contexts. The view that an individual should have identity is expressed by this requirement for an entity to be labelable: Individuals are entities that can be labeled. This
analysis fits well with the idea that the facts of the world are what are expressed by the statements in a metaphysically appropriate language. If labels are necessary in the metaphysically appropriate language, the individuals referred by the labels are necessary constituents of the world. When the labels are switched or permuted in some descriptions of the world, then we have a new description of the world and correspondingly a new state of affairs with different entities. In classical physics, the permutation of labels creates a new state of affairs. In quantum mechanics, however, such label is not assigned to entities and such permutation does not obtain. Quantum particles are not rigidly referred and are permutation-invariant. If we allow such entity as an individual in our ontology, we need a new symbol other than labels in our metaphysical language.

2.4.2 Primitive Identity and Primitive Individuation

The idea that individuals are something labelable supports the approach discussing haecceitism in terms of the Kripkean specification and the Mighty Language. The Kripkean specification describes the world by making reference to individuals. The reference in the Kripkean specification refers to an individual that can be labeled and have an identity condition in classical physics. Kripke takes this specification to express the facts of the world. If this Kripkean specification cannot be analyzed to, reduced to or grounded in other specifications without referential terms, then the labeled individuals are fundamental constituents of the facts of the world. The reference in the Kripkean

---

25 Kripke is not completely committed to haecceitism. He says that he does not defend the haecceitistic possibility that worlds may differ in what things they represent without differing qualitatively (Kripke 1980, p. 18, n.17). But in this section, we are working on non-modal discussion of haecceitism and the Kripkean specification implies an haecceitistic intuition.
specification refers to an individual, and the individual is something that can be labeled and have an identity condition.

Haecceitism is often considered to take the Kripkean specification to construct a metaphysical theory of the world. Correspondingly, the non-haecceitistic takes another kind of specification to express the world completely, one in the Mighty Language. The mighty language is an imaginary language including predicates, operators, and quantifiers that express every property and relation in the world, except any directly referential expression. If the Kripkean specifications can be translated completely in this Mighty Language, the referential expressions are not necessary to describe the world. Then the referred individuals are analyzed to, reduced to, or bounded in other non-individual entities as qualitative properties expressed by other non-referential expressions.

In the above analysis of counting, there are two ways to individuate entities numerically, one in terms of the processes of singling-out and labeling and another one in terms of the process of singling-out and eliminating. The counting process with labels metaphysically requires individuals to be identified rigidly across any contexts; the counting process by eliminating metaphysically requires only individuation of entities to be a single individual. If we apply the primitive numerical individuation of thisness to this distinction, we have two cases of primitiveness, \textit{primitively given rigid identity} and \textit{primitively given individuation}.

\textbf{[Primitive Rigid Identity]} It is primitively given that an entity is rigidly individuated as itself in any context.
**[Primitive numerical Individuation]** It is primitively given that an entity is individuated as one individual.

The first individuation gives us the information of which is which, but the second individuation gives us the information of how many there is. Can the things individuated by the primitive numerical individuation have thisness? At the beginning of this chapter, we discussed three features of thisness: numerical individuation, primitive individuation, and singularity. It is trivial that the primitive numerical individuation gives us numerically individuated entity. A merely numerically individuated entity is given even without any identity condition for it. It is natural to have that individual primitively. But does the merely numerically individuated entity have singularity? At least in order for our discussion, we can assume that if an entity is primitively individuated to be one entity, then in any other context, this will be individuated to be one entity, rather than two or more. In different contexts, we don’t know which one is which but we can expect that this one will be a single entity if it exists in another context. The electron now numerically individuated is expected to be counted one electron in another counting if it exists.

Correspondingly, we can have two kinds of thisness, a thisness of an individual rigidly individuated and a thisness of an individual numerically individuated. Let’s call them, *rigid thisness* and *numerical thisness*, respectively. The rigid thisness is similar to the classical notion of thisness: for a rigidly individuated *a*, there is a property of being identical to *a*. However, the mere numerically individuated entity is not labelable. Perhaps we can express this with variables. But it is hard to grasp a property that includes
variables. We just say that the numerically individuated entity has a property of being this one.

Given this primitive individuation and numerical thisness, there are three different facts about individuals, their numerical distinction (thisness), their identity condition (whichness), and then their qualitative features (suchness). If we take thisness as a notion to correspond to the primitive rigid identity, thisness provides whichness, the identity condition and our haecceitistic specification of the world is constituted by labeled individuals and their qualitative properties/relations. In addition to it, there is another understanding of thisness, a property that numerically individuates an entity.

### 2.5 Advantages of Primitive Individuation

#### 2.5.1 Problem

Do we need this weak kind of individuation and thisnesses? Does this distinction help us to understand what is required to describe the world metaphysically or to understand haecceitism? In this section, we are examining if this numerical thisness could help to solve one of the serious problems against haecceitism: There is no fact about the world that cannot be given without thisness, other than some facts directly derived from an assumption of thisness. Lewis points out that the asserted indiscernible individuals are indiscernible in every aspect.

But anti-haecceitism is neutral about whether there are qualitatively discernible worlds: there can be any number of indiscernible worlds, so long as they are alike not only qualitatively but also in representation de re. If worlds are alike in both of these respects, perhaps they are altogether indiscernible – how else might they differ? (Lewis 1983, p. 224)
If there are worlds qualitatively indiscernible from the actual world, the individual in the indiscernible worlds shares all the features of the corresponding individual in the actual world. The individual represented as Humphrey in this world lost the presidential election and the individual represented as Humphrey in the other world also lost the election. Both of them are politicians, the 38th vice-president of the United States, earned a pharmacist license, etc. Other than the numerical distinction between them, there seem to be no difference between them. The difference in thisnesses does not make a difference in representation de re.

Hawthorne and Cover (1997) analyze Lewis’s distinction between haecceity/thisness and haecceitism as follows:

**P1:** A pair of distinct individuals with all the qualitative properties

**P2:** A pair of distinct individuals with all the qualitative properties but having different de re proposition true to them

P1 shows that the qualitative properties of an individual do not determine its numerical individuality, and raises a problem about PII. P2 shows that the numerical distinction between individuals makes a distinction in the facts about them. Hawthorne and Cover (ibid.: 104) argue that P1 does not imply P2. Black’s world has P1 but does not have P2.

---

26 These modal ideas, possible worlds and possible individuals and their relations, must be more carefully expressed. There are two metaphysical theories on possible worlds, concretism and abstractionism. According to concretism, possible worlds are constituted of concrete entities like this actual world. Possible worlds themselves are maximal spatiotemporally connected object. Here, individuals in a world is defined to be part of the world. Abstractionism takes possible worlds as states of affairs that are abstract and intensional entities usually formulated as, for an individual a and property being P, a’s being P and there being P. A possible world is a state of affairs that is possible and total – for any state of affairs t, a possible world includes or precludes t. In this understanding of possible worlds, an individual in a possible world is defined as that the possible world includes the individual’s existing.
In order for haecceitists to avoid this problem, making no difference in representation *de re*, they need to show that a pair of indiscernible but numerically distinct individuals makes a difference in our theory about them or the world. Haecceitists argue for indiscernible individuals. But in order to show the theoretical reason to allow indiscernible individuals, the numerically distinct individuals make something discernible. That is, haecceitists need to show how indiscernibles make something discernible.

2.5.2 Symmetrical and Irreflexive Relations

Our question is whether a fact about a numerical distinction of individuals generates another fact about individuals and the world. What kind of facts can we have only from numerical distinction? Suppose that we have two indiscernible individuals. They share not only their intrinsic properties but also their relations. If there is some relation not shared by both individuals, then the relation, not thisness, individuates them. Some of the shared relations will be a relation to each other. In order for them to be indiscernible, their relation to each other should be symmetrical. On the other hand, the relation should be irreflexive in order to guarantee that numerically distinct individuals stand in this relation. The understanding of irreflexive relations includes numerical distinction.

*Irreflexive Relation* \( R(x, y) \) where \( x \neq y \)
There are many qualitative irreflexive relations such as the “greater-than” relation. But this asymmetrical relation obtains only when the relata have different intrinsic properties. We are considering a relation between indiscernible individuals. The relation should be symmetrical.

**[Symmetrical and Irreflexive relation]** \( R(x, y) \& R(y, x) \) where \( x \neq y \)

We have examples, next to, distant from\(^{27}\), on an opposite side of, a complement of, etc. Two maximally entangled particles such as electrons are indiscernible not only in their intrinsic properties but also in their spatial locations and causal efficacy because they don’t have determinate location or momentum in just the same way. But the particles have symmetrical and irreflexive relations such as in an opposite spin of. For each electron, it is indeterminate whether the electron is spinning clockwise or counterclockwise around the axis. However, it is determinate whether each is spinning in the opposite direction from another. For quantum particles, like electrons, share the same intrinsic properties and relations including a symmetrical and irreflexive relation such as in the opposite-spin-of relation. But only when two indiscernible particles exist, this relation obtains.

There is some relation that is satisfied only by the numerical distinctness of individuals, not by the features each of them has. When some indiscernible distinct individuals satisfy

\(^{27}\) Some may argue that these spatial relations are qualitative. But this is beyond the discussion here. I just take some intuitive non-qualitative relations here.
this relation, this relation is what makes different between two states, the state with a pair of indiscernible individuals and a state with one individual.

**P3:** A pair of distinct individuals with all the same qualitative properties and relations and having a proposition of the pair that is not analyzed by the facts of each individual of the pair

As Lewis, Hawthorne and Cover point out, P1 does not imply P2. But P3 can be derived from P1.

2.5.3 Weak Discernibility: Reduction or Not

This symmetrical and irreflexive relation is first brought up for a reductive approach to identity and individuation. That is, this relation is used to avoid primitive individuation. Simon Saunders (2006) has brought attention to Quine’s third grade of discriminability, weak discernibility.

**[Weak Discernibility]** Individuals are weakly discernible if they stand in some irreflexive relation.

This discernibility is applied to the PII.

**[PII\textsubscript{weak}]** If any two entities share all the same qualitative intrinsic properties, have the same spatio and temporal location, and partake in no irreflexive relation, then they are one and the same.
There are different versions of PII based on what properties and relations are concerned in order to determine indiscernibility. Individuals can be indiscernible in their qualitative intrinsic properties, or in their qualitative intrinsic properties and qualitative relations, or in their qualitative intrinsic facts and their spatial and temporal locations, etc. Based on these different indiscernibility conditions, we can construct different kinds of PII with different strength. The claim that two entities are identical if they are indiscernible in their qualitative intrinsic properties is the strongest version of PII. As more conditions for indiscernibility are added, the corresponding PII gets weaker. It is one of the core issues in the reductionist approach to individuation what strength of PII is appropriate as a condition for individuation and identity. Some reductionists take $\text{PII}_{\text{weak}}$ as a condition for individuation (Saunders 2006, Dorato and Morganti 2013). They can avoid their problem of quantum particles that are numerically distinct but qualitatively and spatio-temporally indiscernible. They are relatively distinguished by participating in the irreflexive relations like in the opposite spin from. Two spheres in Black’s world can be distinguished by the (asserted) irreflexive relation of staying two miles apart.

We have a problem here. For haecceitists, the instantiation of the symmetrical and irreflexive relation is a differential fact that makes distinct two states: two distinct but indiscernible individuals and one identical individual. But reductionists use the irreflexive relation as a condition to make individuals discernible. Does the symmetrical and irreflexive relation support or refute the primitive individuation? Steven French and Decio Krause argue that this relation cannot be a criterion for any PII.
There is the worry that the appeal to irreflexive relations in order to ground the individuality of the objects which bear such relations involves a circularity: in order to appeal to such relations, one has to already individuate the particles which are so related and the numerical diversity of the particles has been presupposed by the relation which hence cannot account for it. (French and Krause 2006, p. 5)

What is their point here? There can be two approaches to the understanding of circularity. First one is found in the discussion whether the relation is grounded\(^{28}\) or dependent on monadic properties. Second one is in the logical definition of this relation. The first approach considers exactly what property only one of the relata, not the other, has. That is, this approach breaks the relation into monadic properties. Katherine Hawley (2009) takes this approach. Concerning the two spheres of Black’s world, call them \(a\) and \(b\), \(a\) and \(b\) are weakly discernible in that they participate in an irreflexive relation, \textit{being two miles from}. Hawley analyzes this weak discernibility as having different monadic properties. The weak discernibility of \(a\) and \(b\) is broken into \(a\)’s having a property of \textit{being two miles from} \(b\) and \(b\)’s having a property of \textit{being two miles from} \(a\). \(a\) and \(b\) are distinct because \(a\), unlike \(b\), has being two miles from \(b\), and \(b\), unlike \(a\), has being two miles from \(a\). Then, she asks what makes these properties distinct.

But what grounds the fact that they are distinct properties? There seem to be two relevant options. The first is that the distinction between the properties is grounded in the distinction between \(a\) and \(b\); that is, that the monadic property \textit{being two miles from} \(b\) depends for its identity upon the two-place relation \textit{being two miles from} and the object \(b\) (and similarly for the property \textit{being two miles from} \(a\)). The second option is that the distinction between the two monadic properties is somehow more fundamental than the distinction between \(a\) and \(b\) themselves. (Hawley 2009, pp.109-110)

The first option is clearly circular. The discernibility of \(a\) and \(b\) is grounded by the above two properties, and the distinction between the two properties are ground by the

\(^{28}\) It is not uncontroversial to adopt a metaphysical notion of ‘grounding’ in this discussion. In this discussion, it won’t be much different if we replace this notion with a metaphysical relation we accept.
distinction between \(a\) and \(b\). But Hawley points out that it is hard take the second option. She gives two reasons. The property \textit{being two miles from} \(b\) is not an intrinsic property of \(a\) or of anything. This property involving a specific individual cannot appear within a law-like generalization. Hawley claims that this property is not a good candidate for a natural property. She considers intrinsic properties as natural properties. Secondly, two instantiations, \(a\)’s having a property \textit{being two miles from} \(b\) and \(b\)’s having a property \textit{being two miles from} \(a\), seem to be explained by one fact that \(b\) and \(c\) collectively instantiate \textit{being two miles apart}. These two properties seem to be interdependent and not totally distinct.

It can be controversial to use the notion of ‘naturalness’ and the ‘collective instantiation’. Even if we accept these ideas, however, the reductionists can avoid these criticisms by not accepting monadic properties, \textit{being two miles from} \(b\) and \textit{being two miles from} \(a\). The weak discernibility is weak in that individuals in question are relationally discernible rather than absolutely discernible in terms of monadic properties. They intend to make a condition for discernibility weaker than having monadic differentiating properties.

Consider the second approach. The reductionists can claim that the irreflexive relation is somehow given first to us, and then when that relation takes place, we know that the individuals that take the role of its relata are distinct.\(^{29}\) The individuals are relatively discernible only when they are put in a pair as participants of the relation. Hawley (2009, p.110) says “the identity of the property \textit{being 2 miles from} \(b\) is grounded by the identity of the \textit{being 2 miles from} relation, and in the identity of \(b\).” The reductionists say that the

\(^{29}\) I am going to discuss this idea more in the next chapter.
property \textit{being 2 miles from} \(b\) and the individual \(b\) are grounded by the relation \textit{being 2 miles from}. If \(a\) and \(b\) do not participate in the relation, they are completely indiscernible. Here the debate is about whether the irreflexive relation grounds its relata or the relata grounds the relation. We will discuss this second approach in chapter 3.

\(\text{PII}_{\text{weak}}\) is based on the irreflexive relation, which is defined as: \(R(x, y)\) where \(x \neq y\). However, sometimes it is hard to determine whether a relation is irreflexive based on the meaning of the predicate to express the relation. Concerning the relation of \textit{being 2 miles from}, for example, it is controversial whether this is irreflexive even though we have all knowledge about how to measure this, what instances of this relation there are, etc. As in Black and Hacking’s disputes on the dispersal argument, this relation won’t be irreflexive if the space is so tightly curved that one can be 2 miles distance from itself. An irreflexive relation is determined by the distinction of its relata. Numerical individuation analyzes, determines, or grounds the irreflexive relation, and not the other way around.

Accordingly, the numerical distinction of the relata should be given prior to the \(\text{PII}_{\text{weak}}\). Rather than \(\text{PII}_{\text{weak}}\) analyzes, determines or grounds individuation, numerical individuation is required to make sense of \(\text{PII}_{\text{weak}}\). If a symmetrical and irreflexive relation takes place, that is a symptom of the existence of numerically distinct individuals. That is, the instantiation of the symmetrical and irreflexive relation is what the existence of indiscernible but distinct individuals makes different.

We are looking for a new fact when there are distinct but indiscernible individuals. I suggest the instantiation of a symmetrical but irreflexive relation as an example of a new
fact. It is a fact about a pair of individuals that cannot be reduced to the facts of each individual. This relation should be dependent only on the numerical distinction of the individuals. If a numerical individuation does not coincide with the identity of the individual, this relation is not determined by identity. In order for this relation to take place, the mere numerical individuation should be given prior to the properties. Primitive numerical individuation seems to be a good standing point for haecceitists. In the next chapter, I am going to discuss more how we can think of this merely numerically individuated entity and the structure of the world.
Chapter 3. Individuals and Structure

In this chapter, we are going to ask a question: is an individual required to make a fundamental description of the world even when individuals are not intrinsically distinguishable? Particularly, we are going to examine the possibility of the numerical individuation of objects in terms of their standing in a relation, that is, their relation. Structuralism develops a theory to claim that there is a fundamental structure of the world. Some radical versions of the theory claim that this structure is the only fundamental constituent of the world. An object in this theory doesn't exist or its existence is derived from the fundamental structure. In the previous chapter, we examined whether the primitive thisness of an object can make a difference in the fact of the world; we also considered a candidate for the difference, obtaining some irreflexive but symmetrical relation. If the radical versions of structuralism succeed, then the completion of the relation will not be the fact derived from the primitive individuation of objects. In this chapter, we are going to see whether a numerical individuation can be derived from a fundamental structure.

3.1 Ontological Structuralism: General Understanding

When John Worrall (1989) first introduced the contemporary version of structuralism, his motivation was to solve the pessimistic meta-induction. Worrall made a distinction between the content and structures of theories, claiming that structures can be retained across theories that have different content. However, he suggests structuralism can be
used to discuss metaphysical issues, especially for an ontology that is appropriate to quantum physics. It has been difficult to individuate and identify quantum particles, space-time points and entanglements in terms of ordinary ways of individuation, for example, an individuation by intrinsic properties. (French & Ladyman, Cassirer, Eddington, Weyl, etc) Structuralism seems to be a good alternative in that structure is a relation that is not determined by the intrinsic nature of the relata, rather, that the relata are individuated by their roles in the structure. Generally speaking, every version of structuralism seems to accept the following thesis.

[The Ontological Thesis of Structuralism] There are structures and relations that are ontologically fundamental in that structures and relations are not dependent on the intrinsic and spatio-temporal properties of their relata.

This thesis seems to go directly against some of our metaphysical intuitions.

A. Humean Supervenience

B. The combinatorial conception of reality (pointed out by Tim Maudlin (1988, p.59-60)): The world is a set of separately existing localized objects, externally related only by space and time.

C. Particularism (introduced by Paul Teller (1989)): A structure is constructed by its participating objects and their intrinsic properties.

Concerning A. Humean Supervenience:

According to Lewis, the world is just a web of intrinsic properties of objects and their spatio-temporal relation.
The world is nothing more than a vast mosaic of local matters of particular fact, just one little thing and then another….We have geometry: a system of external relations of spatio-temporal distance between points (of spacetime, point matter, aether or fields or both). And at these points we have local qualities: perfectly natural intrinsic properties which need nothing bigger than a point at which to be instantiated…All else supervenes on that. (Lewis 1986, p. x)

All facts about the world, including any extrinsic or structural facts about the world, if there are any, are determined and supervenient on the facts about the spatio-temporal distribution of intrinsic properties. But Ontological Structuralism claims that there are facts about the structure, which are not determined by the relata’s intrinsic features.

Concerning B and C:
According to Humean Supervenience, the world is a set of separately existing localized objects, externally related only by space and time (Lewis 1986, p.60). If a structure is constituted by distinct local objects, any fact of the structure should be the local facts of the spatio-temporally related objects. The local facts will be the independent intrinsic properties of each object. Then, we can build the facts of the world, including any structure in the world, from the local facts of the constituent objects, which is the building block depiction of the world. But the non-supervenient relation is not determined by any local facts of the relata.

Most of all, the Ontological Structuralism affects the individual-based ontology. The relata of the non-supervenient relation are not ‘individuals’ as defined by our classic metaphysics. The notion of ‘relatum’ is understood as a group-theoretical construction of an object. A group theory develops the notion of symmetry: ‘symmetry’ being a transformation of some structure or object which leaves it unchanged in some respect.
Especially in quantum mechanics, the objective fact is characterized as the invariance under symmetrical transformation (Weyl 1952, Auyang 1995, etc) and objects are picked out by the identification of invariants with respect to symmetry transformations such as rotational spatio-temporal symmetries and particle permutation symmetries. Even though there are different types of Ontological Structuralism as we are going to see in this section, this group theoretical construction of objects is accepted at least as a way to understand the ontological status of the relata of the structural relation. Many of the early structuralists replaced the classical notion of ‘object’ with this notion of group-theoretical constitution. (Weyl 1952, Castellani 1993, French 1999, 2000, etc)

The group-theoretical notion of object is different from the classical notion of object in many ways. In the former notion, the group structure is given first and then objects are derived from the structures. An object is a mathematical entity, a set of equivalent entities under symmetry transformation. No further individuation of object is offered. (Lyre 2004, 663; Kantorovich 2003, Cassirer 1944; Born 1953, 149; Stein 1989, 59) This group-theoretic object does not satisfy conditions for the classical understanding of individuals as follows:

**D1. Intrinsic Individuation/Distinction**: The identity and distinction between objects are determined by their intrinsic properties.

**D2. Intrinsic Property**: Objects instantiate intrinsic properties.

**D3. Impenetrability**: No two particles have all the same spatio-temporal properties.

**D4. Substantiality**: Objects exist independently from the existence of other objects.

Concerning D1: Intrinsic Individuation/Distinction
The most obvious feature of the group-theoretic objects is that they are not individuated by their intrinsic features but by their role in their participating structure. This is the reason why Structuralism is introduced as the ontology appropriate for tricky contemporary physics such as quantum mechanics. This feature does not imply that the relatum should not be an individual unless the quantum particles and the like are simply presumed to be excluded.

Concerning D2: Intrinsic Properties

According to Russell, individuals should have intrinsic properties even if their individuality is conferred by their relation to other individuals. “If they are to be anything at all, they must be intrinsically something.” (1903, p.249) However, the group-theoretical object exists because of the existence of its structure. Its existence is dependent on the existence of the structure. If the structure is not a concrete entity but an abstract entity, then so are the group-theoretical objects, that is, they are abstract placeholders. There seems to be no reason to think that abstract placeholders should have intrinsic properties. They are similar to mathematical entities like ordinal numbers in that they lack their own intrinsic nature and their nature cannot be described without reference to their participating structure. However, Ontological Structuralism, even some radical version of structuralism as we are going to see in the following section, claims that the structure in question is a concrete entity. And some philosophers like Alan Chakravartty (2012) consider this lack of intrinsic properties as a reason why the mathematically understood structure is not a constituent of the world but just a mathematical model of the world.
It will be beyond our interest to examine whether an individual should have intrinsic properties and whether the relata of a relation are individuals. But we are interested in the question when a structure is given, whether the structure can determine all the facts of the relata. We are going to especially examine whether the fact of a structure determines the numerical individuation of its relata, particularly when our notion of numerical individuation is too weak to be understood as ‘countability’ as in the previous chapter. If the numerical individuation of the relata should be given independently from the fact of a structure, then numerically distinct individuals standing in the concrete structure should be independently given.

Concerning D3: Impenetrability

Given the principle of impenetrability, objects are considered to be individuated by their spatio-temporal trajectories. However, quantum particles are not always assigned well-defined trajectories in space-time. For example, a pair of electrons in the orbital of a helium atom has the same eigenstate and the same position state but cannot be in the same quantum states because their spin state must be opposite. \(^{30}\)

Concerning D4: Substantiality Existence Not as a Substance

At least in our everyday understanding of objects, objects can exist independently. Their existence is not determined by any other object’s existence. This idea is also found in classical metaphysics, as with the idea of substance described by Aristotle, Descartes and others. However, the existence of the group-theoretical object is dependent on the

\(^{30}\) This is called the Pauli exclusion principle.
existence of the structure in which it participates. If we accept the strong structuralist idea that objects are structurally individuated, then objects are merely relata of a relation, and there is no way to individuate and identify objects other than as the relata of a relation, meaning an object is no longer a substance.

3.2 Concrete Structure

What kind of entity is a structure? One way of understanding structure is as an abstract entity that can be instantiated by distinct groups of physical entities in the physical world. The structure itself as an abstract entity exists (or subsists) prior to and independent of its instances. This concept is called an ‘ante rem structuralist’ position in the philosophy of mathematics or called a ‘Platonist conception of structure’ in metaphysics. On the other hand, there is another definition of structure, claiming that structure is not an abstract entity but a physical entity that is instantiated in the physical world. (French and Ladyman, 2003b) The structure is a physical entity in the physical world, a network of concrete physical objects. Structure as a physical entity is something invariant throughout the change of physical objects that participate in the structure. Examples are the physical feature of permutation invariance in many particles quantum theory (Muller 2009), gauge invariance in quantum gauge theories (Lyre 2004), diffeomorphic invariance in the general theory of relativity (Rickles 2006; Esfeld and Lam 2008). In this chapter, we examine the notion of concrete physical structure.
Concrete physical structure can be more carefully characterized based on the relation between a structure and its involved objects. Esfeld and Lam (2010) characterize the relation between a structure and its involved objects in the following five ways.

(1) **A Radical Atomistic Position**: There are only objects; there are no non-supervenience relations (Leibniz 1714). There is no relation and thus no structures in fundamental ontology.

(2) **Ontological Primacy of Objects**: There are non-supervenience relations but there is an ontological primacy of the objects (relata) over the relation.

(3) **Moderate Structural Realism**: Relations and objects are both genuine fundamental ontological entities and they are mutually ontologically dependent. (Esfeld 2004; Esfeld and Lam 2008)

(4) **Ontological Primacy of Relations**: Relations have ontological primacy and objects are ontologically secondary in the sense that they derive their existence from the relations in which they stand and thus from the structures they are part of. Objects are mere nodes within structures.

(5) **A radical Structuralist Position**: There are only relations and no objects, thus no relata, in the domain of fundamental physics. Objects are not genuine fundamental ontological entities. (French and Ladyman 2003, French 2010)

(1) and (5) are Eliminativist views. According to (1), every relation is determined and supervenient on the intrinsic properties of objects that stand in the relation. Some relations such as being heavier than, being lighter than, having the same mass as, are
determined by the intrinsic properties of the related objects, their masses. This position holds that every relation is determined in this way. Consequently, any relation and thus any structures are not included in the ontological list of fundamental entities. This list includes only objects and their intrinsic properties. However, we have several counterexamples, especially in physics. Many discoveries in quantum physics, such as the intrinsically indistinguishable quantum entities, are typical counterexamples. Regardless of contemporary findings in physics, we have a long standing classic counterexample, the spatio-temporal structure of the world. Two well-established theories about the spatio-temporal structure of the world are based on the external relation between objects that is non-supervenient on the intrinsic features of the objects (relationism) and on an object-independent abstract entity that constitutes the spatiotemporal fact (substantivalism). Neither of these theories argue that the spatio-temporal facts of the world are determined by the intrinsic properties of relevant objects. Even if (1) is not refuted completely with counterexamples, this position has many burdensome anomalies to explain, which has not done successfully yet.

A typical example of position (2) is Humean supervenience. According to David Lewis, a proponent of Humean supervenience, all the facts about the world supervene on facts about which individuals instantiate which fundamental properties and relations. The fundamental properties are qualitative intrinsic properties and the only fundamental relation is the spatio-temporal relation. All the facts of the world are determined by the spatio-temporal distribution of the qualitative intrinsic properties. The spatio-temporal relation is the only non-supervenient relation that constitutes the fundamental fact of the

31 See Brian Weatherson (2015) for detailed analyses.
world but does not constitute an object’s individuation, identity, or existence. However, Ontological Structuralism directly opposes this position: there are some cases that should be explained by a certain relation or structure that is not determined by individuals’ intrinsic properties and their spatio-temporal relations. These first two positions compete against ontological structuralism. The other three, (3), (4), and (5), are three different metaphorical explanations of structuralism.

First, consider another eliminative position (5). In this position, there are relations and structures, but no objects, in the fundamental ontology. Objects are not genuine fundamental ontological entities. This position is more radical than the reduction or the dependence of objects to the relations in which they stand. In this position, relations do not need any object-like entity participating in it to make sense or to exist; and, also, object-like entities are eliminated in the ontological inventory. In the case of (4), there are some theories suggesting a new weaker notion of objects, for example, claiming that the intrinsic properties of objects are taken out, or that an object merely acts as a place-holder for a relation. This is controversial, whether this weaker notion of objects appropriately describes an object will be discussed in the following section. However, position (5) does not allow even a weaker notion of object. The existence of objects and any relational role of objects are eliminated from the fundamental ontology. This has also been controversial, the question of whether the notorious notions, “a relation without relata” and “structure is all there is” are intelligible.
One of the main criticisms is that the relation itself requires objects as its relata. Any relation requires at least the concept of its relata, and if the relation is a concrete entity, the relation requires the existence of objects as relata even though the intrinsic properties of the objects do not determine the characteristics of the relation. Moreover, the standard logical systems used by mathematics and physics require the domain of objects, such as the quantification over objects in standard first order language and objects as a member in a standard set theory\textsuperscript{32}. In order to describe the world according to the claims of radical structuralism, our systems of metaphysics and logic would need to be drastically revised. This position needs to pay the price of severely changing some basic philosophical intuitions. Even if we succeed in making sense of the above ideas, accommodating them would require a radical change to our well-established and consistent systems shared by metaphysics, Logic, and many other theories depending on them. These two things, the unintelligibility of the notion of relation without relata and the inevitability of object (or object-like entities) in logical, metaphysical, and physical theories, make us reluctant to accept an eliminative position. However, it is not certain that the weak version of structuralism, such as (4), can be successful. This position also needs to make sense of the relation given prior to its relata and the theory of structure/relation being non-dependent on the domain of objects. Then, we can say that an object is individuated and identified in terms of its structure/relation. Position (3) is not incompatible with a primitive numerical individuation of objects because the facts about objects can be given first while a relation requires the individuation and identity of an object. I am going to examine whether we can metaphysically make sense of a relation without requiring that

\textsuperscript{32} We don’t analyze what the objects are. Regardless of whether the fundamental objects in the world are ordinary objects or something unfamiliar to us, we will take them in our discussion if they are non-qualitatively distinguished entities.
objects be individuated first as its relata and whether we can avoid using the domain of objects in both the standard first-order language and the set-theoretic concepts of physical theories.

3.3 Relations and Their Relata

Structure is understood in terms of relations between objects that are related. The core metaphysical doctrine underlying non-eliminative structuralism is an “ontological priority/primacy” of the relations of objects over both an object itself and its properties. Most Ontological Structuralist theorists, including French and Ladyman, consider structure to be a concrete entity rather than an abstract entity. So, our question is whether we can have a concrete relation in the absence of relata? Some other theorists like Dorato adopt a mathematical analysis of structure for the metaphysical structure of the world. Here we are going to examine how to make sense of the priority/primacy of the structure to its relata in these analyses.

If something takes an ontological priority, its existence is more fundamental than something else’s existence. This fundamentality is determined by the dependence relation in order for them to exist. A sandwich is ontologically dependent on the existence of tomatoes, pieces of bread, etc, that compose the sandwich, and not vice versa. The ingredients or constituents are ontologically prior to their composite. When my cat Nuluk sits under the sunlight, her shadow is ontologically dependent on Nuluk because Nuluk is necessary for the shadow to exist. With respect to some appropriate relations, a thing is ontologically dependent on another.
(P1) $x$ is ontologically prior to $y$ iff $x$ is dependent on $y$ with respect to $R$.

What is an appropriate $R$ for the non-eliminative structuralist theorists who claim that for a relation, the existence of its relata is dependent on the existence of relation? Anjan Chakravartty (2012) formulates this dependence relation based on the determination of identity:

(P2) A relation is ontologically prior to its relata iff the relata depend on their relations for the determination of their identity and not vice versa.$^{33}$

A relation is ontologically prior to its relata in that the identity of relata is determined by the relation. Chakravartty examines what kind of entity has this identity condition using the classic notion of substance applied to a mathematical entity acting as a placeholder. The classic notion of substance cannot satisfy (P2). For the bare substratum, by its definition, properties and relations may come and go, and its identity is determined primitively in terms of its haecceity or primitive thisness. In the case of the bundle theory, an object is simply considered as a group of properties that cohere at locations in space-time. The identity of an object is determined by the identity of the constituting properties. If the constituting properties are intrinsic, then the identity of an object is correspondingly determined by the intrinsic properties it has independently.

$^{33}$ In his original formulation (Chakravartty 2012, p.193), the notion of “less fundamental” is used rather than “ontologically prior”. He understands that the fundamentality and the dependency is relativized by a relation relation, the determination of identity in this case.
Chakravartty (2012, p.195-6) argues that even the most favorable theory of property to structuralism, dispositional essentialism, implies that the identity of property is intrinsically determined. For dispositional essentialism, the identity of a property is determined by its potential for contributing to the causal power, which is the disposition of a thing that has the property.\textsuperscript{34} However, disposition is not just a set of the actual causal relations but also the potentiality to have causal relations. For example, solubility is not the set of relations for an entity to dissolve in water, but the actual relation to the water. Especially considering that our notion of relation is one of concrete entities, this disposition is not a relation itself but an intrinsic property that will be manifested in a certain relation. An object’s possession of this disposition is not determined by the interaction of this object to the environment. Chakravartty (p.203) makes a distinction between possession and manifestation of disposition. An object would possess extrinsic disposition by depending on something external to it.\textsuperscript{35} 36

Chakravartty claims this not only applies to substratum and the bundle of properties, but a notion of any physical concrete object cannot be an object that satisfies the formula of the non-eliminative Ontological Structuralism. He argues that any entity with an intrinsic property will have the property as its identity condition. “So long as the relata have

\textsuperscript{34} See Sydney Shoemaker (1980, p.133).
\textsuperscript{35} In addition to dispositional essentialism, Chakravartty (2012, p.195) considers a categorical theory of properties. According to the theory, a property has a nature of its own independent of how it makes its bearer behave in various circumstances or causally interact with other objects. Categorical properties are just there within objects, which are intrinsic properties. Then the constituting properties of an object in the bundle theory are just intrinsic, which is incompatible with the eliminative OSR.
\textsuperscript{36} Chakravartty (p.203-4) considers the possibility of extrinsic dispositions such as the entanglement of a quantum particle that does not supervene on any intrinsic properties of the particle. He does not consider this possibility seriously because the extrinsic disposition cannot be the identity condition for a quantum particle even if it is its unique feature of a quantum particle. We will discuss his concept of the identity condition in the following paragraphs.
genuinely intrinsic features – qualitative properties, propositions, what have you – this condition remains unsatisfied, because these intrinsic features keep popping up as plausible candidates for determining their identity.” (2012, p.197) It is not clear why intrinsic properties are preferable to relations as the condition for identity. Probably, he depends on the idea that constitution should determine identity.

He believes that what constitute an object itself is something intrinsic. Other than intrinsic entities such as substrata, properties, and primitive identities, nothing could constitute a concrete physical object itself. In order for a concrete entity to exist and hold a relation as its relata, it should have some intrinsic constitution. “… an object with no intrinsic features at all, whether knowable or unknowable in principle, is not an object at all. Lacking anything intrinsic … there is simply nothing left to stand in any sort of relation. … Concrete objects that have no intrinsic features are not anything, and once we have gone this route, we have embraced eliminativism.” It is hard to imagine a physical concrete object without an intrinsic property. To be a concrete object in the physical world, it should have some matter. The characteristic feature of matter is extension, that is, it takes up some location in space and time. Correspondingly, the concrete entity should have a certain size, volume, contour, etc., which will be its intrinsic features. It seems at least intuitive that a concrete object in the physical world is required to have some intrinsic features, its intrinsic physical constitution.

However, the intrinsic constitution does not offer a criterion for an object to be uniquely identified or to be numerically distinctive. Any set of numerically distinct but
This text is a natural representation of the document. It includes discussions on the individuation of entities and the role of relata in structuralism. The text argues that relata still require their primitive numerical individuation that is not given by their involved relation. It considers examples such as mathematical entities and graphs, and discusses the distinction between the identity of a relatum of a relation and the identity of entities to satisfy the role of relatum. The latter entities are what constitutes the physical world and what has intrinsic properties unlike the nodes of a graph. Given the distinction between intrinsic constitution and identity/individuation, a relatum can be individuated or uniquely identified in terms of something other than its intrinsic constitution. The text concludes with the idea that whether a mathematical entity such as a graph constitutes the fact of...
the world or is an analogy for the structure of the world. What I am interested in is that a graph and its nodes are not completely determined by its external relation but require some intrinsic features. The intrinsic features will be interesting to the theorists for primitive thisness.

3.4 Quantitative Facts

A graph is a typical example of a structural entity, all the constituents of which are defined by its relation to others and every feature of the entity is determined by the relations of its constituents. A graph is the purest and simplest form of structure that has the least requirements to be because its constituents have no properties beyond what is required for the graph to be. What is required for a graph to be is required for any type of structure in any field to be.

There are several ways to define a graph. One of the most well-known ways is a set-theoretic definition. A graph is defined set theoretically in terms of points, called “vertices” or “nodes”, and a set of “edges” connecting these points. The graph G is identified as an ordered pair consisting of the vertex set V and the edge set E, G=<V, E>. Every vertex/node is labelled and an edge is either an unordered pair of the labelled vertices or an ordered pair of the labelled vertices, which is a directed graph. However, this way of defining a graph is not appropriate for a metaphysical identification and the distinction of a graph. The labels of vertices make structurally isomorphic graphs distinct when the labels are rearranged.
The two graphs are considered to be distinct based on the above set-theoretic definition of a graph. The left graph includes the set of edges, \{\{A, B\}, \{B, C\}\} while the right graph includes the set of edges, \{\{A, C\}, \{B, C\}\}. If the labels are rearranged, however, one of the graphs will be identical to another. The graphs are isomorphic in that there exists a 1-1 function, \(f\), between the nodes of two graphs that is edge preserving. The isomorphic graphs are structurally identical in that the graphs are distinct only in their labels and not different in the relational facts of their edges. The difference between non-isomorphic graphs is determined in terms of the relational facts of their edges. The non-isomorphic graphs do not have a function, \(f\), that maps vertices onto vertices in an edge-preserving way. Given the set theoretic analysis, there are two kinds of distinction in graphs, a distinction from its label assignments to the nodes (the distinction of isomorphic graphs) and a distinction from its structure of edges (the distinction between non-isomorphic graphs). The structural difference between non-isomorphic graphs is what we are
interested in for the metaphysical analysis of structure. Some of the graphs do not have the function, $f$, and have no isomorphic graphs, and they are called asymmetric graphs.\footnote{Graphs with a small number of nodes always have some isomorphic graphs. There are no asymmetric graphs of orders 2, 3, 4, or 5 that have nodes 2, 3, 4 and 5.}

One of the simplest asymmetric graphs is the following.

This graph is induced by two-place symmetrical relations.\footnote{The edges in this graph are not directional. A graph will be more complicated if it has some directional edges. The direction of an edge can raise a metaphysical problem. In the following section, we are going to discuss what a natural relation will look like and a neutral relation. In analyzing the world as a graph, Dipert (1997, p.355) argues that fundamental relations in the world should not be asymmetrical and the graph that constitutes the world should be non-directional.} Each vertex can be described without a label and the graph that satisfies the descriptions do not have isomorphic graphs. Each vertex can be expressed as followings:

- A unique vertex of degree 1 that has only one edge;
- A vertex of degree 4 that has four edges adjacent to a vertex of degree 1;
- A vertex of degree-2 adjacent to two degree-4 vertices;
A vertex of degree-4 not adjacent to a vertex of degree 1;
A unique vertex of degree-3;
A vertex of degree-2 not adjacent to two degree-4 vertices.\(^{39}\)

Dipert (1997, p.349-50) argues that an asymmetric graph individuates nodes purely structurally. The asymmetric graphs are arranged in such a way that each of their nodes is related to others in a unique manner. In following Dipert’s expression (p.348), each vertex has a unique and purely structural description. The relational descriptions of the nodes (the relational facts of edges and the adjacent relation of each node) determine the graph and individuate its node from other nodes within the graph and from other graphs.

Dipert takes this distinction as a counterexample to Aristotle’s classic idea that a relation requires its relata. In *Categories*, Aristotle asserts that no fundamental constituent of the world is relational and every relation is ultimately reducible to intrinsic properties of substance.\(^{40}\) Aristotle’s reasoning begins from an idea that relations presume the distinctness of the relata. Then he infers that the distinctness of the relata cannot be given by relations but by their own properties, that is, intrinsic properties. Aristotle ultimately concludes that intrinsic properties are more fundamental than relational properties, and the relational properties are reducible to the intrinsic properties. However, Dipert suggests asymmetric graphs as a counterexample to the implication that the distinctness

\(^{39}\) Dipert assigns a label to each node in this description. However, it will be better to avoid using labels in order to avoid controversies relating to the semantic role of labels. If they work as rigid designators, this description is not purely relational but includes particular entities referred by the designators. Dipert and other structuralists like Laydman (2007) suggest asymmetric graphs as cases in which the identities of objects as relata of a relation are determined by the relation in which they stand. In order to accomplish this purpose, it will be better to give the description of a graph without labels.

\(^{40}\) Aristotle uses his technical terms “a primary substance” and “monadic properties” for this claim. These terms can be interpreted as referring to a fundamental constituent of the world and intrinsic properties respectively in this context.
of relata can be given only by their intrinsic properties. However, even if a relation
requires the distinctness of relata, that is, the nodes of an asymmetric graph, the
distinctness can be given by their relation to other nodes. There are two ways for the
nodes of an asymmetric graph to be distinguished, a distinction between nodes within one
and the same graph and a distinction between nodes across graphs. Each node of the
asymmetric graph is distinguished from other nodes in one and the same graph in terms
of their unique relation to the other nodes in the graph. That is, the unique description of
its relation to other nodes individuates each node in the asymmetric graph. The nodes of
an asymmetric graph are also distinguished from other nodes of other graphs because
their distinction is guaranteed by the fact that the graphs they are involved in are
nonisomorphic. In other words, a node is individuated by its relation to other nodes in a
graph and a node is individuated by its involvement in a graph that is distinguished from
other graphs.

However, it is not clear whether an asymmetric graph is defined without depending on
any non-relational fact of its nodes. In order to have a unique graph based on the above
descriptions, the number of nodes, that is, the order of a graph, should be given first. If
we don’t care about the number of nodes of a graph, the above description can be
expressed as some adjacent relations between some nodes as sub-structures:

A degree 1 node structure is adjacent to a degree 4 structure;
There is another degree 4 node structure that is not adjacent to the former degree
1 node;
There is a degree 2 node structure that is adjacent to the above two degree 4 structures;
There is a degree 3 node structure;
There is a degree 2 node structure.

There can be many graphs that satisfy this description, one of which is the following:

![Graph](image)

Figure 5

We can imagine a situation wherein a world that has the above structure or is represented by this structure satisfies the original description of the nodes. But we do not want to allow different graphs like this one to be referred to by the original description. We want to say that the structure of the world is (or represented as) the original graph when it satisfies the description.

Is there a way for the original set of descriptions to determine the number of nodes if we add some conditions to the set of descriptions? For example, we can consider the following two conditions; (i) each node should have its own description and (ii) the graph described by the above set of descriptions has the minimal number of nodes to satisfy the
set of descriptions. Consider condition (i) first. We are considering whether the descriptions determine a graph including its order. The fact that every node has its own description is only known when we know how many nodes there are, that is, the order of the graph. Moreover, we should consider that each node in a graph can be replaced with another graph. This is not only correct for constructing a complex graph, but for having a better representation or a better constitutive structure of the world. Dipert (1997, p.352) considers each node to be occupied by an entity in the world and takes the closest form of the commonsense notion of entity as a sub-graph. Moreover, the idea of subgraph can provide another way to distinguish the nodes. Even in the case wherein a world is constituted by a symmetric graph, its node can be structurally distinguished if the node is an asymmetric graph. If we allow for the replacement of a node for a subgraph, then the way to replace a node in the above graph can be an example of the replacement.

Concerning (ii), we can ask whether this condition is just arbitrarily added in order to determine what is referred by this description or if there is some metaphysical reason that the structure with a minimal number of nodes is preferable. If the structure with a minimal number of nodes is simpler than the others, this means that the number of nodes determine the simplicity or complexity of the structure itself. The number of nodes and the description together determines which structure we are talking about.

We have considered two types of individuation, an individuation by intrinsic features and a numerical individuation. There are cases of entities in which their numerical individuation is given even though there is no distinction in their intrinsic features. In the
graph case, we consider the distinction between an individuation by relations and a numerical individuation. When an object takes a node in an asymmetric graph, we can uniquely individuate its relational roles and relational features from other objects to take other nodes in this graph as well as other nodes in other graphs. However, the fact of how many nodes there are should be given independently in addition to the descriptions of nodes to determine a graph in question. The fact implies that each node is numerically individuated independently without depending on its relational role in the graph.

3.5 Relation and A Metaphysical Language

A structural system of the world, how entities are related, is usually expressed in a quantified formula in first order language, using a predicate for a relation, variables to be satisfied by its relata, and quantifiers. Consider the strong and typical structuralist positions like (4) and (5) in the above list of distinctions in structuralism. If a language is appropriate for the metaphysical positions of structuralism, statements in this language should not semantically require anything that is not allowed in the fundamental constituent of the world. For example, the eliminative structuralists cannot make a statement with direct referential expressions like names because the expressions require particular objects for the statement to have meaning. First order language without names, for example, the Mighty Language by Lewis and Stalnaker, is often used to express the qualitative facts of the world. The quantified sentence does not include the facts about particular objects. Based on the standard understanding of first order language, however, first order language requires a domain of individuals and the quantified sentence requires that some of the individuals in the domain satisfy the sentence. The quantified sentence is
not a fact about a particular individual but a fact about some individuals. The structuralism we are considering now does not allow individuals in the ontological inventory of the fundamental constituents of the world. Nor should the statement about the fundamental facts of the world depend on the existence of individuals. We probably need a language for structuralism rather than the first order language (hereafter FOL).

Shamik Dasgupta (2009, 2011, 2016) develops a new kind of language that gets rid of any expression for individuals. First, he makes a distinction between individualistic facts and generalistic facts.

**Generalist facts**: whether the fact obtains does not depend on how things stand with any particular individuals.

**Individualist facts**: whether the fact obtains depends on how things stand with any particular individuals.

According to the structuralism expressed by (4) and (5), the structural facts should be given without depending on how relata stand. The facts about how the relata stand require the individuations and identities of relata. The structural facts as the fundamental facts of the world should be generalist facts. The language to express the generalist facts will be appropriate for the structuralism of (4) and (5).

Individual facts are usually expressed by sentences with individuals’ names, for example, using the form of ‘\(Fa\)’ or ‘\(Pa \& Qa\)’ where ‘\(a\)’ names an individual. In order to express generalist facts, the quantifiers in the first order language have been used, for example,
‘∃xFx’ or ‘∀y(Py & Qy)’. The claim that all the fundamental facts of the world are or are reduced to the generalist facts imply that the sentences with names are translated into their semantic counterparts of quantified sentences, which is called an existential closure. This claim is called ‘quantifier generalism’: The fundamental facts of the world are the existential closure of true-but-not-fundamental facts of the world.

According to Dasgupta (2009, 2011, 2016), however, quantifier generalism cannot be correct and the fundamental generalist facts should not be quantificational. He argues that the fundamental facts are not grounded to other facts. His notion of ‘grounded’ describes a particular type of explanation: Such as explaining why an event like a certain cricket match occurs, we can give a causal explanation stating the causal sequence of events that lead up to the match, such as two teams agree to play, etc., or we can give a constitutive explanation, what makes it as it is, such as throwing and hitting a ball in accordance with various laws, etc. The second type of explanation shows “in virtue of what” the event in question occurs and what grounds the event. Given this notion, he claims that a quantified sentence, for example, ‘something is red’, is grounded by its instances such as ‘a is red’. Even if it is questionable whether fundamental facts are grounding facts, it is Dasgupta’s argument can be analyzed as follows:

(Q1) Fundamental facts are ungrounded.
(Q2) A quantified sentence is grounded into its instances.
(Q3) What is expressed by a quantified sentence is grounded by what is expressed by its instances.
(Q4) Then the fundamental facts cannot be expressed by a quantified sentence.

This analysis requires more explanation, especially what facts are and what constitution is considered as the grounding constituents of facts. If facts are structured entities including objects, properties, and relations, we can still ask whether any facts about any constituents of objects, properties, or relations will be more fundamental. We probably do not want to allow any physically smaller element to be more fundamental. A fact about a cat is grounded by facts about its physical parts, and grounded by the facts about its parts of the parts, and so on. In addition, it is not always clear that individualist facts are more fundamental than quantificational facts. Compare ‘something is above something’ and ‘a is above b’. ‘a is above b’ implies...
quite intuitive that the quantificational facts, facts expressed by quantified sentences, are grounded into the individual facts, facts expressed by names in our traditional understanding of quantification. The fact that some cats are calico is true in virtue of my calico cat Nuluk, but not vice versa. The fact that some cats are calico does not require that Nuluk exist, but just any calico cat to exist. The quantificational facts are grounded into individual facts but they do not require the specific individual facts to be true. However, the quantificational facts do require the existence of some individuated entities. If facts about structures or relations are expressed in quantified sentences, they will be grounded in individual sentences, and they require the individuation and existence of entities given in the world.

Dasgupta (2009, 2011, 2016) suggests a new language for a generalist fact, without any terms and variables to take objects as their semantic values, which is called Algebraic Generalism. The domain D of this language contains simple qualitative properties and relations whose arity is greater than or equal to 1, which are denoted as the form $P^n$, where $n$ is the number of arity. We can construct a complex property out of these simple ones with property operators/functors. The important functors are $\&, \sim, c$, and $\sigma$.

Suppose we have two 1-place qualitative properties, being red ($R^1$) and being square ($S^1$), and one 2-place property, loving ($L^2$). ‘$\&$’ is a property conjunction functor such that $(R^1 \& S^1)$ expresses a complex property of being both red and square; ‘$\sim$’ is a property negation functor such that $\sim R^1$ is an one-place property of being not red; ‘$\sigma$’ is a functor its converse, ‘$b$ is below $a$’ which is a distinct fact from ‘$a$ is above $b$’. However, we have an intuition that there is only one fact corresponding to ‘something is above something’ and ‘$a$ is above $b$’. Even if ‘something is above something’ is true in virtue of ‘$a$ is above $b$’, ‘something is above something’ does not have any implication of its distinct converse which seems more appropriate to show the fact of the world. An appropriate analysis of relation will be discussed further at the end of this section.
of permuting the order of variables of a formula in FOLs such that when \( \sigma L^2 \) is the 2-place property of being loved; c is known as a cropping functor that takes an n-place property \( P^n \) and yields (n-1)-place property \( cP^n \). For example, \( cL^2 \) is the 1-place property of being loved by someone. We can apply \( c \) to a property repeatedly and then have a 0-place property that is considered as a state of affairs. For example, \( ccL^2 \) describes a state of someone loving someone. As a state of states obtained in the world, we can say that \( P^0 \) obtains where \( P^0 \) is a 0-place property. In addition, Dasgupta considers identity as a simple property, I, added to the domain D. Corresponding to an open formula \( (x=y) \) in the FOL, there is a 2-place property, \( I^2 \). Dasgupta claims that every sentence in FOLs is equivalent to a sentence in this new language with the four fundamental expressions, simple qualitative properties including a property of identity, the property functors, primitive predicate ‘something obtains’ and the normal sentential connectives.

Variables are not notated in this language. But it is not certain whether the semantic role of variables is eliminated in this language. There are at least two roles performed by variables.

\[(V1)\] variables supply objects relative to an assignment
\[(V2)\] variables expand the stock of predicates of a language

Variables range over a domain of individuals or objects, and an assignment is considered a function taking variables of the language into an individual or object from the domain. Relative to an assignment, individuals or objects satisfy a formula. In addition, when we construct a complex condition with plural predicates and operators in the first order
language, we need a notation – variables - to show which object is predicated by the predicates.

In addition, we have two intuitive features of an open formula.

**O1** Fx and Fy for an arbitrary one-place predicate F are semantically the same, and they are just notational variants

**O2** Rxx and Rxy for an arbitrary two-place predicate R are semantically different.

Perhaps someone might question (O1); Is it possible to have an analysis which treats Fx and Fy as distinct? There are two possible ways. First, if we treat variables x and y as having a certain particular denotation, then open formulas with a form Fx and Fy are distinct. But this contravenes our fundamental understanding of a variable, which does not have a denotation but a reference. Second, the sameness and distinction of an open formula is determined by its symbols, the typographic sameness and distinction. Then the semantic difference between Rxx and Rxy will turn out to be a typographic distinction. But as in Teaches (x, x) and Teaches (x, y), their difference cannot be just a matter of their symbols in our ordinary language because they have a different condition for the satisfaction of a sequence of objects. Though a sequence of variables does not denote objects, it gives a certain semantic or at least syntactic relation that is more than just typographic sameness or distinctness. It gives information about the sameness or distinctness of objects assigned to the variables. We seem to have another specification about the semantic role of a variable.

**V3** variables encode the sameness or distinctness of an object for predicates to apply.
(V3) determines how many objects are required for the formula to be satisfied and how these objects are related. (V3) enables us to have the following two formulas distinctively and they will express different states of affairs when satisfied:

For three predicates, F, G, and H, we can distinguish the following states,

\[ Fx \& Gx \quad \text{and} \quad Gx \& Hx; \]

\[ Fx \& Gx \& Hx. \]

This reminds us of traditional disputes between the bundle theory and haecceitism based on the PII. If an object is just a bundle of properties, then two examples wherein an object with F, G, and H, and two objects, one with F and G, and another with G and H, are not distinguishable. In order to distinguish them, we need something to show that there are two distinct instantiations of G in the first cases while there is only one G-instantiation. One of the ways to do this is to introduce individuals that are numerically distinct and instantiate properties, therefore distinct instantiation of properties. (V3) reflects this idea.

A variable should be satisfied by assigning an object to it but it also takes the individuation role of objects that individuates the instantiation of properties to give the FOL its power of expression, as shown in (V2) and (V3).

Now we question whether Dasgupta’s algebraic language can get rid of these roles of variables. Particularly, we should focus on whether the order and number of variables are removed. Their order and number depend on the identity and distinction of variables. Consider whether a permutation functor can be completely defined without depending on the semantic roles of variables. For example, we have a love relation. If someone loves someone, then someone is loved by someone. These are just two expressions for one love
relation, ‘love’ and ‘be loved’. For simplicity, we need only one expression, for example, ‘love’ that is expressed in L(x, y) in the FOL. The asymmetry of a love relation is expressed with the help of variables in the formula: (L(x, y) & ~L(y, x)).

In Dasgupta’s language, unrequited love is expressed with two different relations, L² and σL²: (L² & ~σL²). However, these two are not the same kind of relations in that σL² does not seem to be used independently. It is hard to understand the meaning of σL² without the meaning of L². If the σL² can be an independent relation like ‘be loved’, then L² and σL² will express one and the same relation as do ‘love’ and ‘be loved’. Then when L² is the case, σL² is the case and then ‘L² and σL²’ is also the case. Then we are in trouble. Suppose that (L² & ~σL²). Then by the ordinary meaning of ‘&’, L² is the case. Then we have also (L² & σL²). (L² & ~σL²) implies (L² & σL²), which seems contradictory. In order to avoid this situation, σL² should be dependent on the meaning of L², that is, its conversion. So L² and σL² always express different relations. But in what sense are they different? A reciprocal love between my two cats Nuluk and Miruk will be expressed as (L² & σL²). Their love for each other is equivalent. The only difference between L² and σL² in their case is just the difference between the order of the relata holding the relation. We need an order of relata in a property/relation as well as the relative identity. Then it requires the identity and distinction between the relata, which are the two basic semantic roles of the variable. This new language seems to include the role of variables hidden in the meaning

42 Here ‘&’ and ‘~’ are operators in the FOL. The operators in the FOL should have different operators from property functors in Dasgupta’s language even though I use the symbols ambiguously.
of functors. Accordingly, it is not certain whether the facts expressed in this language is grounded in individualist facts and necessitates the existence of objects.

Designating the order/position of a relation is required to define the relation. The numerical distinction of positions and locutions are required. When the relation obtains, the distinct positions and locutions are satisfied. In order for the distinct positions and locutions to be satisfied, distinct entities are required. It is not the case that entities are individuated because it satisfies the position but the satisfiers should be distinctively individuated in order to satisfy the distinct positions. The distinction between positions is not merely a conceptual distinction because the relation in question is an actual physical entity.

In the traditional understanding of haecceitism, the haecceitistic distinction is much finer than the numerical distinction. Even though there are no differences in numbers of entities in the world, there can be many haecceitic differences. We can create a new haecceitistically distinct possible world by swapping the roles of individuals. However, numerical individuation or primitive thisness understood as primitive numerical individuation is a much weaker claim than this traditional haecceitism but it is harder to defeat.
References


