

INTEGRATED OPTIMIZATION METHOD FOR PLASTIC INJECTION
MOLDING

By

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ABSTRACT OF THE THESIS

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This thesis presents an integrated optimization method to find the optimized operational parameters in Plastic Injection Molding (PIM), such as flow rate, melt temperature, mold temperature, pressure holding time and packing pressure that will minimize the shrinkage under the constraints of injection pressure and cooling time. Design of Experiments (DOE) is used to reduce the computational cost for simulations. Furthermore, the possibility value (P-value) is adopted to identify the significant factors among all design variables with respect to each functions. Monotonicity Analysis is then employed to detect the active constraints and to reduce the complexity of the original optimization problem so that the problem can be easily solved by a simple regression. Finally, the responses obtained by the simulation with the optimized operational parameters are used to validate our solutions.

Two design examples are presented in this paper. For both examples, twenty-five initial samples are evaluated using Solidworks Plastic based on the orthogonal array from the DOE with five variables. There are two constraints on injection pressure and cooling time. P-value shows that packing pressure is not a significant factor for shrinkage and two constraints in both examples, then it can be moved out in later optimization. The exact value of flow rate and pressure holding

time can be found out by Monotonicity Analysis. Finally, by solving the regression equations with melt temperature and mold temperature, the optimal parameters combination will be solved. Using the optimized parameters in simulation, the shrinkage for first example and second example are 0.3988mm and 0.0768mm, both of the shrinkage results are smaller than that in initial samples which can satisfy the constraints.

Key words: Plastic injection molding. Design of Experiment. Possibility value. Monotonic analysis.

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CHAPTER 1

INTRODUCTION

Over time, plastic injection molding has become one of the main methods for producing plastic parts. Product quality is one of the most important thing in plastic injection molding(PIM) and plastic shrinkage is always used as an important criterion to evaluate plastic product quality. Moreover, product quality mainly depends on the choice of materials, mold design, and process variables. This paper is aim at finding the optimal set of parameters to minimize plastic shrinkage. The variables are flow rate, melt temperature, mold temperature, pressure holding time and packing time. However,injection pressure and cooling time are also two important role in PIM. Then the research goal is to minimize plastic shrinkage under the constraints of injection pressure and cooling time using design of experiment(DOE),monotonicity analysis and response surface methodology(RSM) to find the optimal parameter settings.

This chapter is mainly introduce some basic concept about plastic injection molding process , injection machine and the research background about plastic injection molding even the goal of this research.

1.1 The introduce about plastic injection molding

1.1.1 The definition of plastic injection molding

Plastic injection molding is a method to obtain molded products by injecting plastic materials molten by heat into a mold, and then cooling and solidifying them.

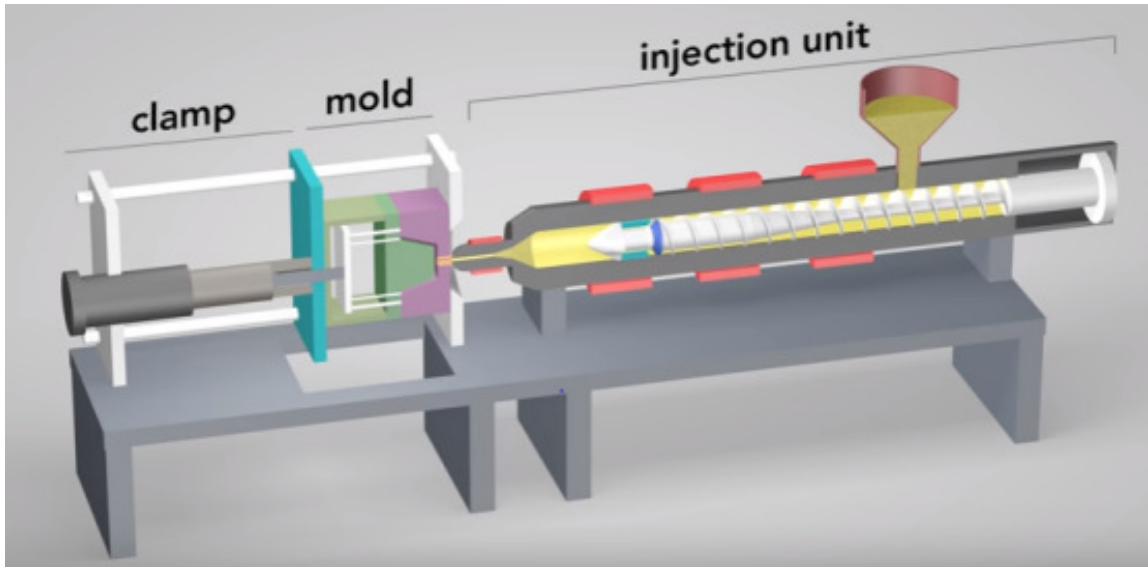


FIGURE 1.1. Injection machine. Introduced from: <https://www.youtube.com/watch?v=RMjtmSr3CqA&t=104s>

1.1.2 The injection machine and working mechanism

The injection machine has three main part: injection unit, mold and clamping unit.

And the injection machine has four main working process: clamping, injection, cooling and ejection.

When clamping begin, the clamping unit is used to close the mold to prepare for the injection process. The clamping unit is usually a four tie-bars link mechanism.

During injection, plastic pellets will put into barrel through hopper. Inside the barrel the screw push the plastic forward, then the heater bands warm up the plastic. There are flights on the screw which can crush the plastic pellets and make them molten, after that the molten plastic will be injected into mold. Then the screw will move back.

Every product has its unique mold based on the shape of product. The cooling process is happen in the mold, different mold has different cooling system, the most economic and popular way is using cool water in the cooling channel to cooling plastic in mold.

After cooling, the mold open and the ejection pin will push the product out of the mold, then the mold close and the process repeat.

1.1.3 The advantages and disadvantages of plastic injection molding

The biggest advantage of plastic injection molding is that the manufacturing process could be suitable for any plastic products with complicated shapes. People make all kinds of products with

different geometry shape by designing different mold. Moreover, PIM has higher efficiency than other type of molding once people have a designed mold. And PIM can also manufacture more than one product in one mold by co-injection. But the shortage is also clear, one mold can just fit one product and some complicated mold may cost a lot.

1.1.4 Challenges On PIM Product Quality

PIM product quality depends on the choice of materials, mold design, and process parameters.

Different materials have different property, so choosing the suitable kind of materials will do good to product quality. But people should do a lot experiment on different type of materials to pick a most suitable one, moreover, the material choose is the most basic step to make sure the product quality, people still can not make the best product quality only by material choose.

Plastic product is shaped by mold, so that a well designed mold is very important for product quality. Also some other details in mold like cooling channel and vent will have effect on product quality. But a well-designed mold may cost a lot time and money, even for some complex product.

Compare to the previous two means, choosing the optimal set of process parameters to improve the product quality can be an efficient and universally applicable way. So this study aim to improve product quality by finding the optimal set of parameters.

1.2 Research background

1.2.1 Research background

Shrinkage is is very important factor to evaluate plastic product quality. Better plastic product usually has smaller shrinkage. The plastic product shrinkage is highly related to the process operation parameters in PIM. Wen-Chin Chen[1] proposed a method to optimize process parameters using Taguchi method, RSM, and hybrid FA-PSO. Melt temperature, Injection velocity, packing pressure, packing time and cooling time are the process parameters in their study. Their study used mechanical device to measure the warpage and shrinkage of the products. Erfan Oloaei[2] optimized warpage and shrinkage in PIM using Taguchi, ANOVA and artificial neural network methods. Ko-Ta Chiang and Fu-Ping Chang[3] using the response surface methodology to analyze the shrinkage and warpage in PIM, the product they focused is thin shell feature. Gao Y and Wang X[4] reduced plastic warpage in PIM by surrogate-based process optimization. In their study, a cellular phone cover is investigated, where mold temperature, melt temperature, injection time, packing time and packing pressure are selected to be the design variables. But if total packing

time is too short, the performance of parts may be affected. Sudsawat, S and Sriseubsai, W[5] optimized warpage and shrinkage in PIM by using response surface methodology with genetic algorithm and firefly algorithm techniques, their study claims that firefly algorithm created better optimal solution than genetic algorithm. Satoshi Kitayama[6] introduced multi-objective optimization to optimize plastic volume shrinkage and cycle time for PIM by using conformal cooling channel. However, the optimized cooling channel may not be suitable for all products with different shapes. Gang Xu and Zhitao[7] Yang used soft computing and grey correlation analysis to optimize the process parameters in PIM, multiple objectives were involved in their optimization. Satoshi Kitayama and Shinji Natsume[8] also introduced multi-objective optimization to optimize plastic volume shrinkage and clamping force for PIM via sequential approximate optimization. In multi-objective optimization, the optimized process parameters combination are one optimal solution chosen among the Pareto-frontier, the selection of optimal solution is based on people's preference.

In this study, the plastic shrinkage is the objective, once the shrinkage in PIM is reduced, the warpage caused by shrinkage will be reduced automatically. On the other hand, compared with multi-objective optimization, the optimization method proposed in this study will provide only one optimal process parameter combination which is more precisely compared with the optimal solution chosen from Pareto-frontier. For the experiment part, Solidworks Plastic simulation was used in this study, not only because computer simulation costs less than experiments with plastic injection machine but also using the results calculated by computer will help people avoid the error in manual measurement. The orthogonal array is then used to help design experiments with five process parameters: flow rate, melt temperature, mold temperature, pressure holding time, packing pressure. The method flow proposed in this study: orthogonal array–P-value–monotonic analysis–RSM can not only optimize the process parameters in PIM but also a general optimization procedure.

1.2.2 Research goal

The research objects are product shrinkage which is caused by stress displacement, injection pressure and cooling time are two constraints. Product shrinkage means the geometric displacement between real product and designed product. So lower product shrinkage means the shape of our product is closer to the design shape, which means that better products always have lower shrinkage. Usually, plastic product warpage is caused by nonuniform shrinkage, taken in this

sense, minimize plastic shrinkage will also be helpful to optimize the plastic product warpage in PIM. Cooling time is mainly influenced by melt temperature and mold temperature, and cooling time is about 70 percent of the whole PIM cycle. So reducing cooling time means the improvement of the PIM process efficiency. As for the injection pressure, it is the pressure at the injection location. Too much pressure at the injection point will do harm to the injection machine, lower injection pressure will prolong the life-span of injection machine. Also, repeated experiments and simulations to find a good set of operation parameters is very expensive. So the research goal is minimizing plastic product shrinkage by finding the good set of parameters efficiently under constraints of injection pressure and cooling time

CHAPTER 2

METHODS

This chapter is going to introduce the method used in the research: Orthogonal arrays ,probability value,monotonic analyze,response surface method and Fmincon in MATLAB and how to use these methods to get the optimal process parameters.

2.1 Orthogonal array

Orthogonal array is a experiment design method pick up from DOE. It represent a experiment data table which conducting the minimal number of experiments which could give the full information of all the factors. This study design experiments based on orthogonal array to can cut down the experiment times and get as much information as possible.

For example, if there are simulation experiments with five input factors and five levels for each input factors. If those experiments done by full factorial experiments, the experiments time will be 3125, but if an L_{25} orthogonal array(table 2.1) were chose to do these experiments ,it will only need 25 times experiments.

Two properties of orthogonal array make sure that orthogonal array can provide the full information within less experiments.First,each level of each factor will appear an equal number of times in orthogonal array,which make sure that people can get same amount of every factor in every level. Secondly,Every kind of level combination between every two input factors can be tested once in the orthogonal arrays, which makes the orthogonal array representative.

2.2 Probability value(P-value)

In statistic testing, null hypothesis is a general statement that there is no relationship between two measured phenomena. And the p-value is the probability that the null hypothesis is true. So that smaller p-value means stronger relationship between factor and response. Usually, suppose if p-value is smaller than 0.05, people can reject null hypothesis then believe that this factor has a strong relationship with response, in other words, it is a significant factor. This study use p-value to find the significant factors for each response and then do specific analyze to those significant factors for each response later.

2.3 Monotonicity analysis

If all of the objective and constraint functions are monotonic increasing or decreasing with respect to the design variables, monotonicity analysis can be used with the optimization model and simplify the optimization models. Figure 2.1 shows the monotonic relationship.

Table 2.1: L_{25} Orthogonal Array

Run	a	b	c	d	e
1	1	1	1	1	1
2	1	2	2	2	2
3	1	3	3	3	3
4	1	4	4	4	4
5	1	5	5	5	5
6	2	1	2	3	4
7	2	2	3	4	5
8	2	3	4	5	1
9	2	4	5	1	2
10	2	5	1	2	3
11	3	1	3	5	2
12	3	2	4	1	3
13	3	3	5	2	4
14	3	4	1	3	5
15	3	5	2	4	1
16	4	1	4	2	5
17	4	2	5	3	1
18	4	3	1	4	2
19	4	4	2	5	3
20	4	5	3	1	4
21	5	1	5	4	3
22	5	2	1	5	4
23	5	3	2	1	5
24	5	4	3	2	1
25	5	5	4	3	2

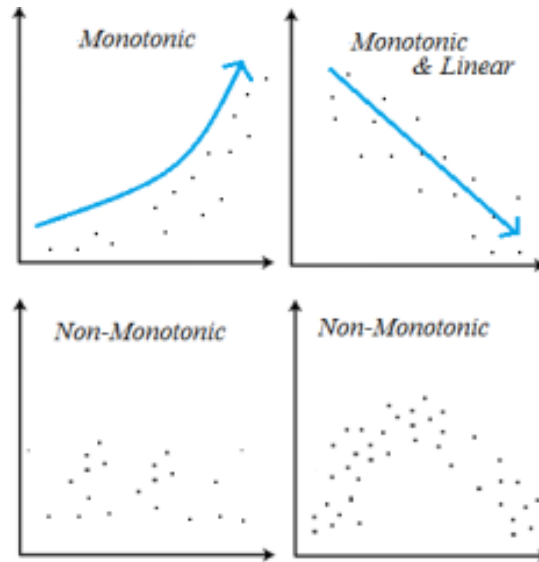


FIGURE 2.1. Monotonic Feature. Introduced from: <http://www.statisticshowto.com/monotonic-relationship/>

Monotonicity analysis is based on two simple principles: First Monotonicity Principle (MP1): In a well-constrained objective function every increasing (decreasing) variable is bounded below (above) by at least one active constraint.

Second Monotonicity Principle (MP2): Every monotonic variable not occurring in a wellconstrained objective function is either irrelevant and can be deleted from the problem together with all constraints in which it occurs, or relevant and bounded by two active constraints, one from above and one from below.

Once figure out the significant factors for each response, monotonicity analysis can be introduced to find out the active constraints and then simply the optimization model.

2.4 Regression Analysis

After using monotonic analyze simplify the optimization model, regression analysis is used to build up regression equations with those significant and monotonic variables.

Regression analysis is used for process optimization and drawing the empirical relationship between independent variables and the response of the system using the data collected from experiment.

Different model will need different equation order. Figure 2.2 is an example of regression analysis, Y is response and x_1, x_2 are two factors. The relationship of independent variables and

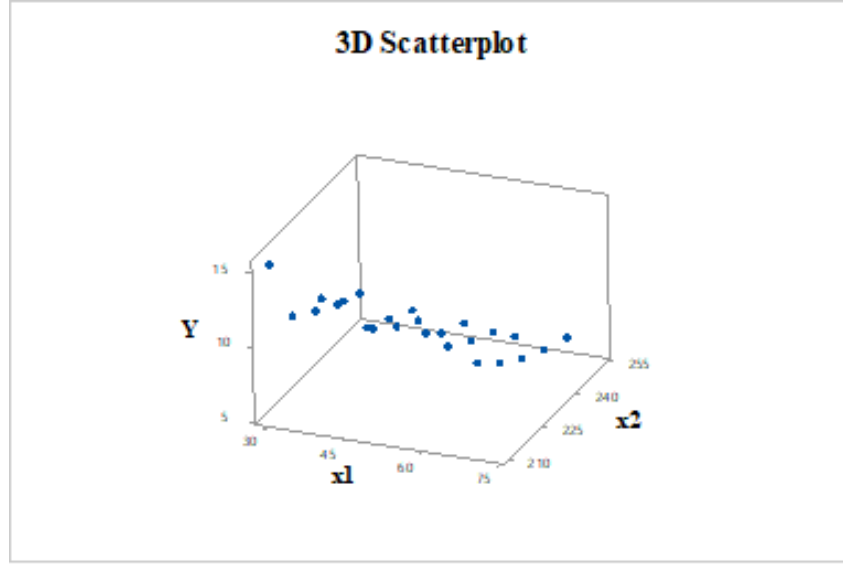


FIGURE 2.2. RSM Plot.

response in figure 2.2 can be calculated using following first-order polynomial equation (2.1).

$$(2.1) \quad y = ax_1 + bx_2$$

2.5 Fmincon

Fmincon is a Nonlinear programming solver in MATLAB. We use it to help us optimize the objective function. 'F' means the objective function, 'min' means minimize objective function, and 'con' means constraints. Fmincon can find the minimum of a problem specified by

$$(2.2) \quad \min f(x) \quad \text{such that} \quad \begin{cases} c(x) \leq 0 \\ ceq(x) = 0 \\ A \cdot x \leq b \\ Aeq \cdot x = beq \\ lb \leq x \leq ub \end{cases}$$

$c(x)$ is inactive constraint, $ceq(x)$ is active constraint, "A", "Aeq", "b" and "beq" are used to express linear constraints, the lb and ub are vectors which means low boundary and up boundary.

2.6 Method flowchart

In the method flow, after identify experiment factors and levels of the factors, This research will do simulation experiments based on orthogonal table, then identify significant factors and do

monotonic analyze to simplify optimization model. Then regression equations from RSM can be build up and optimal parameter setting can be solved out by Fmincon or other optimization method. The proposed method flowchart is shown in figure 2.3.

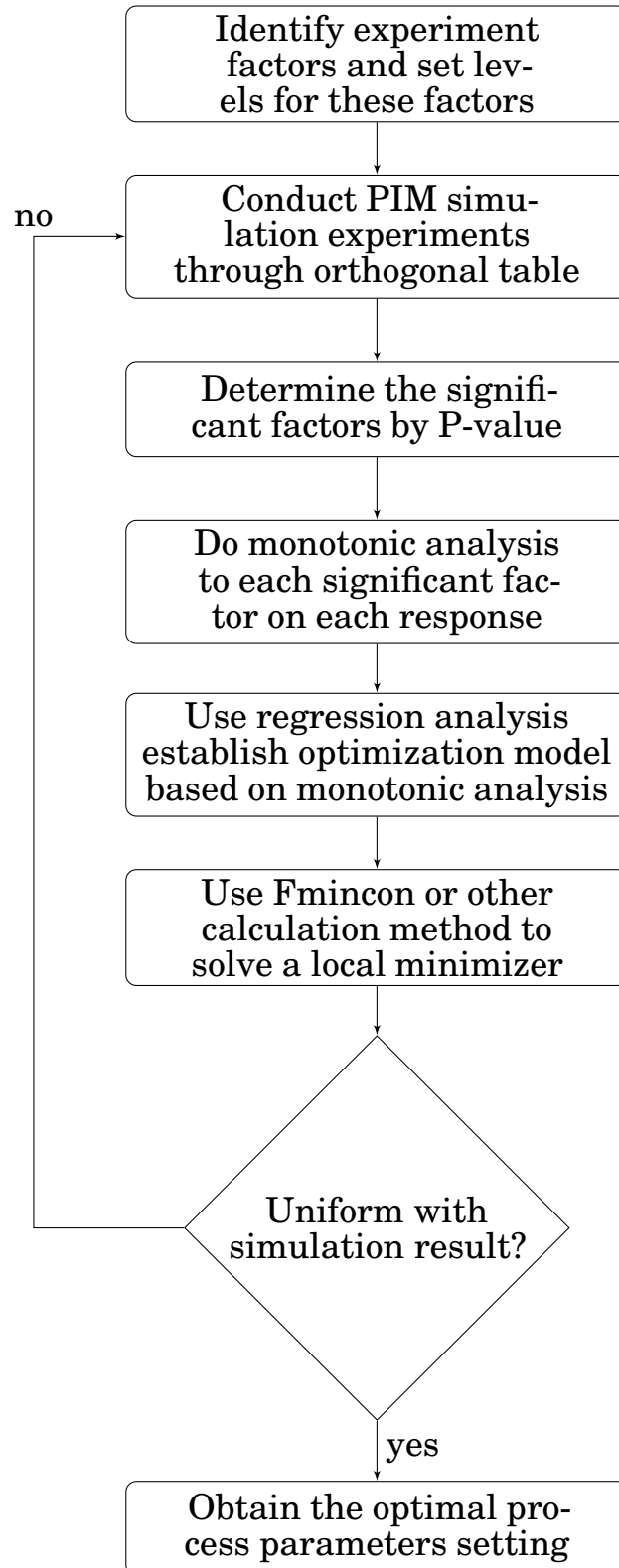


Figure 2.3: Method Flowchart

CHAPTER 3

OPTIMIZATION OF PLASTIC SHRINKAGE EXAMPLE I

In this chapter we are going to introduce an example about optimization of plastic shrinkage using the methods combination in last chapter. As in first chapter, shrinkage is a important role to estimate plastic quality, and smaller shrinkage of plastic means better quality. The first model in this chapter is a small circular packaging box. The optimized shrinkage of this model is the biggest shrinkage part in the model.

3.1 Model geometric property

The simulation model here is a small plastic bowl shown in figure 3.1. The outside radius and inside radius are 1.1 inch and 1.0 inch, the total height is 1.0 inch, outside fillet and inside fillet are 0.15 inch and 0.05 inch.

3.2 Material Choose

Acrylonitrile butadiene styrene (ABS) is chose as the plastic polymer. ABS is a kind of polymer witch is tough, hard and rigid and has good chemical resistance and dimensional stability. The temperature feature of ABS is shown in table 3.1.

For the mold material we choose steel- 420SS. The mold material parameters are shown in table 3.2.

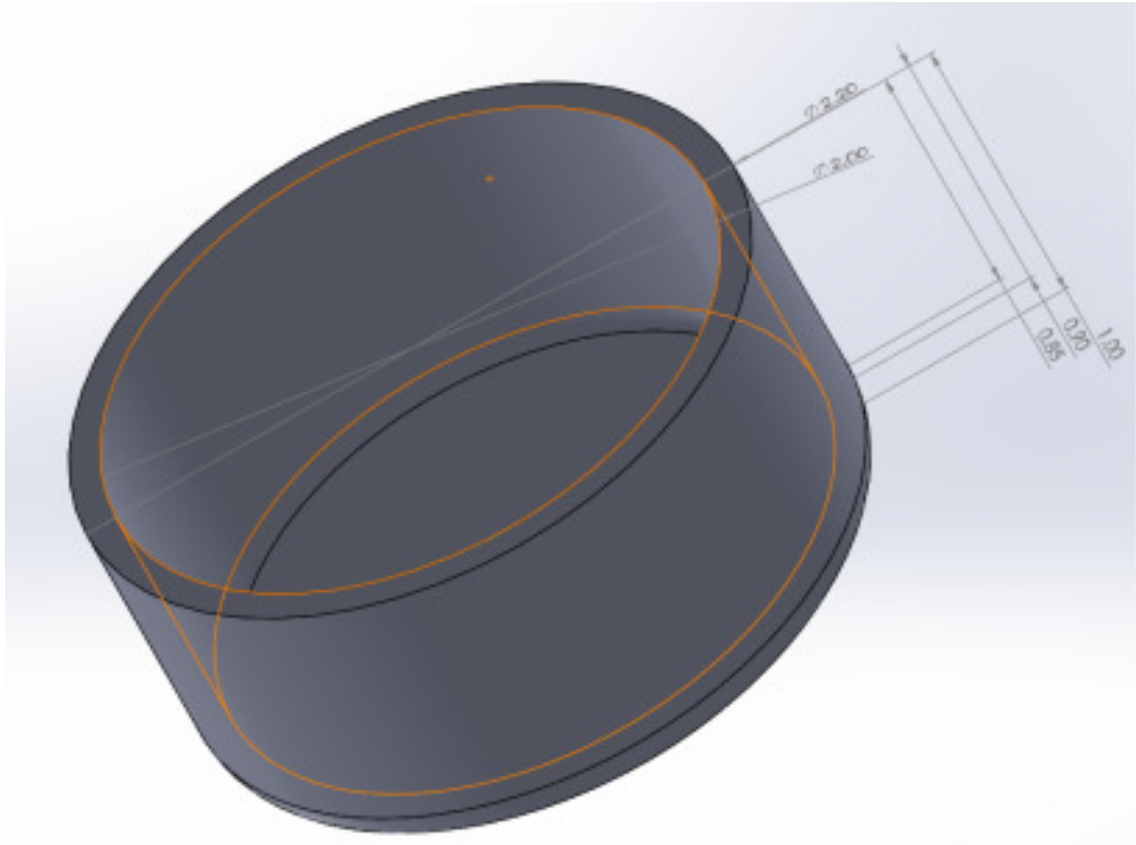


FIGURE 3.1. Simulation Model.

Table 3.1: ABS Temperature Feature

ABS: General material of ABS	
Melt Temperature	230°C
Max.Melt Temperature	280°C
Min.Melt Temperature	200°C
Mold Temperature	50°C
Mix.Mold Temperature	80°C
Min.Mold Temperature	25°C
Ejection Temperature	90°C
Glass Transition Temperature	100°C

Table 3.2: Mold Material Parameters

Steel-420SS	
Specific Heat: Constant	4.62e+006 erg/(g-C)
Thermal Conductivity: Constant	2.5e+006 erg/(sec-cm-K)
Density: Constant	7.73 g/cm3
Shear Modulus: Constant	7.9e+011 dyne/cm2
Thermal Expansion Coefficient: Constant	1.3e-005 1/°C
Young Modulus: Constant	2.1e+012 dyne/cm2
Poisson's Ratio: Constant	0.28

3.3 Response data from simulation

3.3.1 Input variables and responses

This simulation has five input factors: flow rate(used to control injection speed), melt temperature(temperature of the polymer at the injection location),mold temperature(temperature the mold is heated to),pressure holding time(the interval between the packing switch point and the packing end point), packing pressure(the holding pressure during pressure holding time) and three responses injection pressure, cooling time and displacement.

This study aim to minimize shrinkage under the constraints of injection pressure and cooling time by finding a optimal set of combination of input factors.

3.3.2 Level setting of variables

There are 5 levels for each of the input variables. The level setting of the inputs variables are shown in table 3.3.

Table 3.3: Input variables Level Setting

	x_1 :Flow Rate(cc/s)	x_2 :Melt Tempera- ture($^{\circ}$ C)	x_3 :Mold Tempera- ture($^{\circ}$ C)	x_4 : Pressure Holding Time(s)	x_5 :Packing Pres- sure(MPa)
Level 1	20	210	30	3	7
Level 2	25	220	40	4	8
Level 3	30	230	50	5	9
Level 4	35	240	60	6	10
Level 5	40	250	70	7	11

3.3.3 Orthogonal array and simulation result

After level setting a $L_{25}(5^5)$ orthogonal array of these input factors can be build up, L_{25} means 25 experiment times in Latin square,(5^5) means there are 5 input variables and 5 levels for each variable. The $L_{25}(5^5)$ orthogonal array and the simulation result of the three responses are shown in table 3.4.

3.4 Significant factors

This study will decide which factors are significant factors for each response based on the simulation result and P-value of each factor for each response.

Based on the simulation result shown in table 3.4 there are response plots and P-value of each input factors for each response in following images. For wide consideration, suppose factors whose P-value less than 0.15 will be assumed as significant factors.

Figure 3.2 is the shrinkage response plot. It shows that x_3 (melt temperature) and x_3 (mold temperature) are monotonic increase for shrinkage and x_3 (pressure holding time) is monotonic decrease. x_3 (flow rate) and x_3 (packing pressure) do not have strong relationship with shrinkage. The P-value for each input factors are shown in table 3.5, its shows that the P-value of x_3 (melt temperature), x_3 (mold temperature) and x_4 (pressure holding time) are less than 0.1, which means that x_2, x_3 and x_4 are significant factors for shrinkage.

Figure 3.3 is the injection pressure response plot. It shows that x_1 (flow rate), x_2 (melt temperature) and x_3 (mold temperature) are monotonic decrease for injection pressure. x_4 (pressure holding time) and x_5 (packing pressure) do not have strong relationship with displacement. The

Table 3.4: L_{25} Orthogonal Array Simulation Result

Run	x_1 :Flow Rate(cc/s)	x_2 :Melt Temp($^{\circ}$ C)	x_3 :Mold Temp($^{\circ}$ C)	x_4 : Pressure Holding Time(s)	x_5 :Packing Pres- sure(MPa)	Pressure (MPa)	Cooling Time(s)	Shrinkage (mm)
1	20	210	30	3	7	15.37	12.9620	0.5329
2	20	220	40	4	8	12.00	14.7024	0.5437
3	20	230	50	5	9	9.82	17.1122	0.5630
4	20	240	60	6	10	8.05	20.0615	0.6813
5	20	250	70	7	11	6.72	24.1086	0.7508
6	25	210	40	5	10	13.63	14.1119	0.4553
7	25	220	50	6	11	10.85	16.6980	0.4654
8	25	230	60	7	7	8.91	19.6145	0.5065
9	25	240	70	3	8	7.45	23.5249	0.7904
10	25	250	30	4	9	7.46	14.5765	0.7984
11	30	210	50	7	8	12.20	15.8575	0.4323
12	30	220	60	3	9	10.01	18.8078	0.6928
13	30	230	70	4	10	8.34	22.8522	0.7336
14	30	240	30	5	11	8.32	14.1129	0.5846
15	30	250	40	6	7	6.92	16.0987	0.5513
16	35	210	60	4	11	12.45	18.4390	0.5570
17	35	220	70	5	7	9.50	22.4391	0.5843
18	35	230	30	6	8	9.28	13.6833	0.4393
19	35	240	40	7	9	7.80	15.6246	0.4769
20	35	250	50	3	10	6.61	17.8409	0.8488
21	40	210	70	6	9	10.97	21.6792	0.4990
22	40	220	30	7	10	10.49	13.2208	0.3705
23	40	230	40	3	11	8.78	15.1929	0.7268
24	40	240	50	4	7	7.37	17.5117	0.7851
25	40	250	60	5	8	6.33	20.5609	0.8123

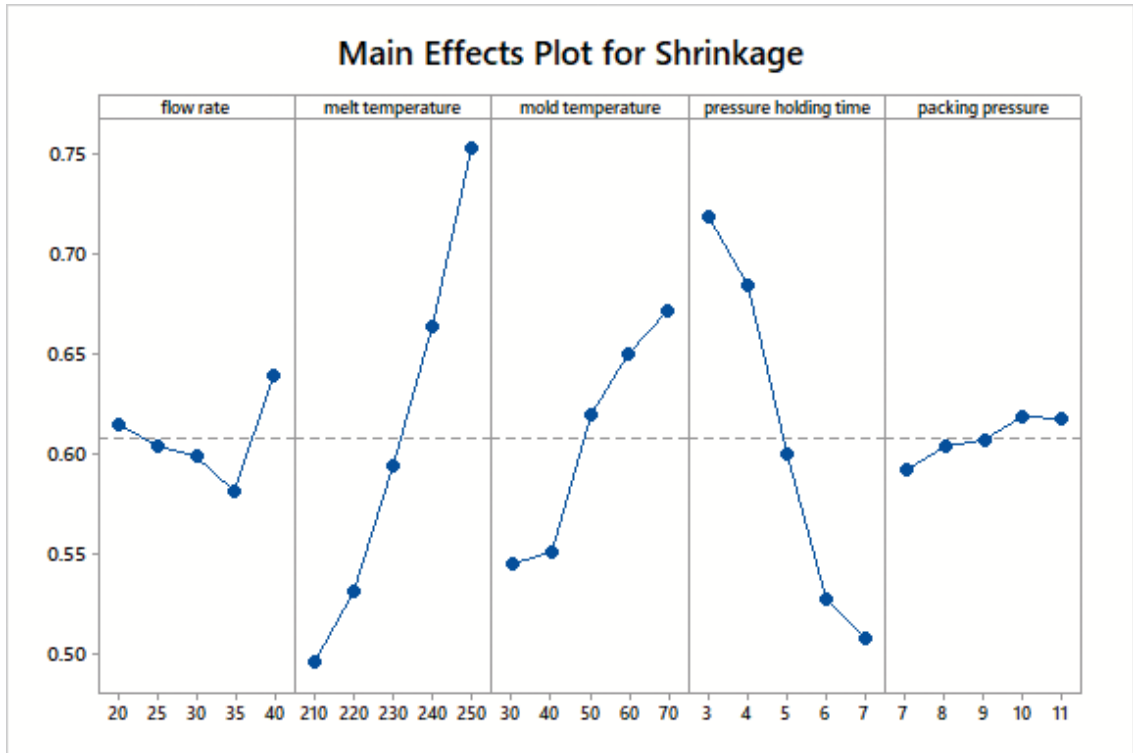


FIGURE 3.2. Shrinkage Response Plot.

P-value for each input factors are shown in table 3.6, its shows that the P-value of x_1 (flow rate), x_2 (melt temperature) and x_3 (mold temperature) are less than 0.15, which means that x_1, x_2 and x_3 are significant factors for injection pressure.

Figure 3.4 is the cooling time response plot. It shows that x_2 (melt temperature) and x_3 (mold temperature) are monotonic increase for cooling time. x_1 (flow rate), x_4 (pressure holding time) and x_5 (packing pressure) do not have strong relationship with cooling time. The P-value for each

Table 3.5: P-value For Shrinkage

Factor	P-value($P \leq 0.15$)
Flow Rate	0.731
Melt Temperature	0.016
Mold Temperature	0.113
Pressure Holding Time	0.024
Packing Pressure	0.963

Table 3.6: P-value For Pressure

Factor	P-value($P \leq 0.15$)
Flow Rate	0.023
Melt Temperature	0.000
Mold Temperature	0.024
Pressure Holding Time	0.506
Packing Pressure	0.722

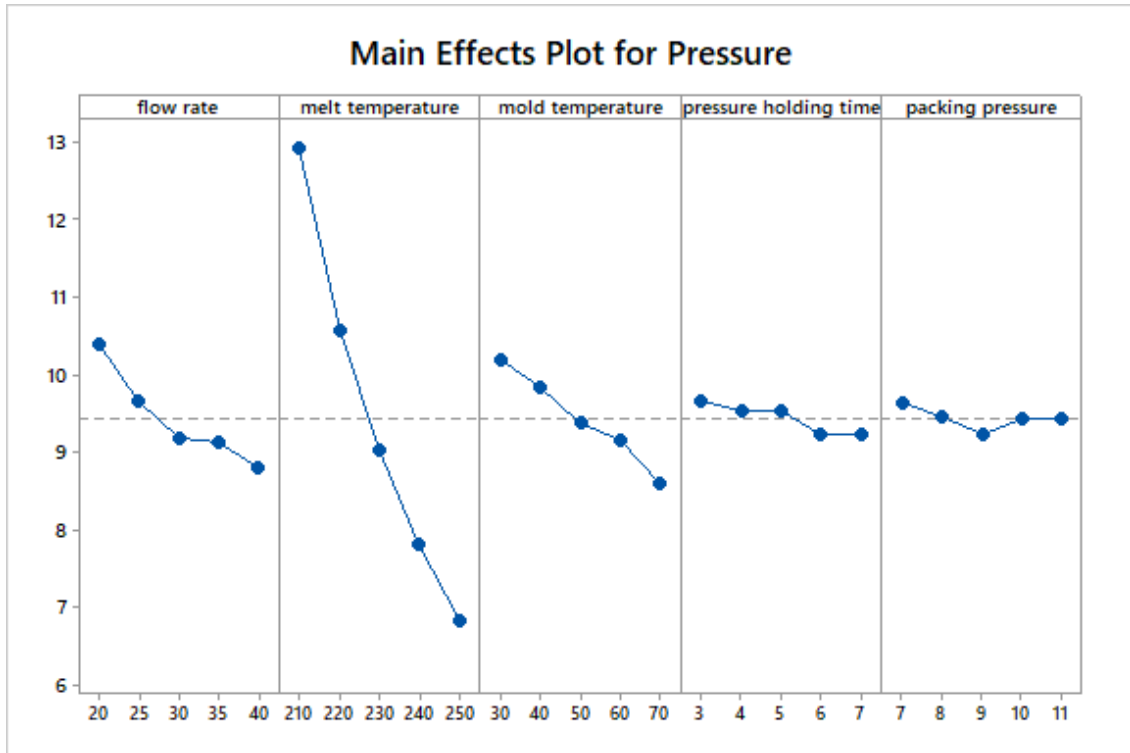


FIGURE 3.3. Pressure Response Plot.

input factors are shown in table 3.7, it shows that the P-value of x_2 (melt temperature) and x_3 (mold temperature) are less than 0.15, which means that x_2 and x_3 are significant factors for cooling time.

Table 3.7: P-value For Cooling Time

Factor	P-value($P \leq 0.15$)
Flow Rate	0.176
Melt Temperature	0.000
Mold Temperature	0.000
Pressure Holding Time	0.917
Packing Pressure	0.368

3.5 Monotonic analysis

3.5.1 Optimization model

Based on the response plots and significant factors analysis, x_5 (packing pressure) out of the response model can be removed, because x_5 is not the significant factor for each response. Then use monotonicity analysis with the rest four variables on each response. In the optimization model, the $S(x^*)$ (Shrinkage function according to its significant factors) is the objective function and

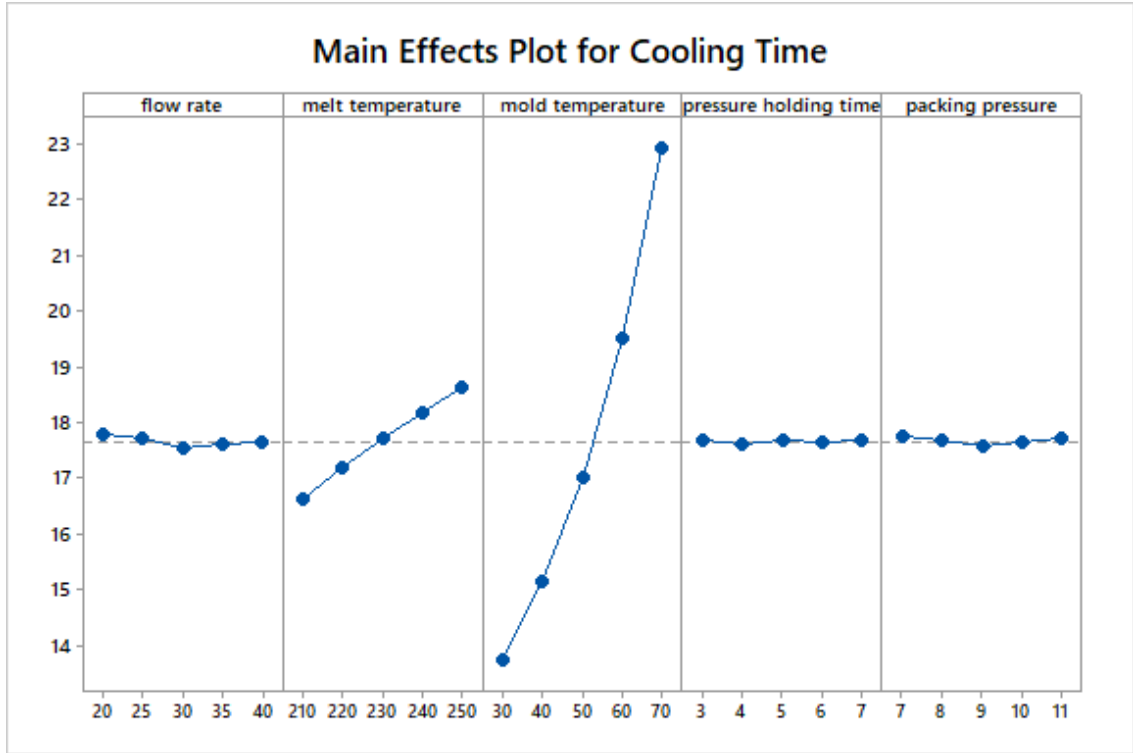


FIGURE 3.4. Cooling Time Response Plot.

$P(x^*)$ (pressure function according to its significant factors), $C(x^*)$ (cooling time function according to its significant factors) become two constraints g_1, g_2 which represent injection pressure can not beyond 9MPa and cooling time can not beyond 15s. " x^* " represent the vector $[x_1, x_2, x_3, x_4, x_5]$. Considering about the low boundary and up boundary of the four variables, there are eight more constraints from g_3 to g_{10} .

The optimization model with objective function and constraints are shown as follow:

$$\text{minimize } f(x^*) = S(x^*)$$

subject to:

$$g_1(x^*) = P(x^*) - 9 \leq 0$$

$$g_2(x^*) = C(x^*) - 15 \leq 0$$

$$g_3(x^*) = x_1 - 40 \leq 0$$

$$g_4(x^*) = -x_1 + 20 \leq 0$$

$$g_5(x^*) = x_2 - 250 \leq 0$$

$$g_6(x^*) = -x_2 + 210 \leq 0$$

$$g_7(x^*) = x_3 - 70 \leq 0$$

$$g_8(x^*) = -x_3 + 30 \leq 0$$

$$g_9(x^*) = x_4 - 7 \leq 0$$

$$g_{10}(x^*) = -x_4 + 3 \leq 0$$

3.5.2 Monotonic table

Based on the response plots in figure 3.2, 3.3, 3.4 and the optimization model, a monotonic table can be build up shown as table 3.8

Table 3.8: Monotonic Table

	x_1	x_2	x_3	x_4
f		+	+	-
g_1	-	-	-	
g_2		+	+	
g_3	+			
g_4	-			
g_5		+		
g_6		-		
g_7			+	
g_8			-	
g_9				+
g_{10}				-

“+” means that variable is monotonic increasing on the objective function or constraint, “-” means that variable is monotonic decreasing on the objective function or constraint.

From the response plots, x_2 and x_3 are monotonic increasing on objective function, x_4 is monotonic decreasing on objective function. x_1, x_2 and x_3 are is monotonic decreasing on g_1 . x_2 and x_3 are monotonic increasing on g_2 . The following monotonic relationship are establish from the optimization model.

3.5.3 Monotonicity analysis

Then do monotonicity analysis with the monotonic table to simplify the optimization model based on monotonicity principle1(MP1) and monotonicity principle2(MP2).

First Monotonicity Principle (MP1): In a well-constrained objective function every increasing (decreasing) variable is bounded below (above) by at least one active constraint. x_2 and x_3 are monotonic increasing on objective function, x_4 is monotonic decreasing on objective function.

For x_2 : x_2 is monotonic decreasing on g_1 and g_6 , which means there is at least one active constraint from g_1 and g_6 .

For x_3 : x_3 is monotonic decreasing on g_1 and g_8 , which means there is at least one active constraint from g_1 and g_8 .

For x_4 : x_4 is only monotonic increasing on g_9 , so g_9 is one of a active constraint, which means:

$$(3.1) \quad g_9(x^*) = x_4 - 7 = 0$$

Then we have five active constraints combination possibility:

Condition 1: g_1 and g_9 are active constraints.

Condition 2: g_6, g_8 and g_9 are active constraints.

Condition 3: g_1, g_6 and g_9 are active constraints.

Condition 4: g_1, g_8 and g_9 are active constraints.

Condition 5: g_1, g_6, g_8 and g_9 are active constraints.

Second Monotonic Principle(MP2): Every monotonic variable not occurring in a well-constrained objective function is either irrelevant and can be deleted , or relevant and bounded by two active constraints, one from above and one from below. Based on MP2, x_1 is not occurring in objective function so it may irrelevant and can be deleted. On the other hand, x_1 could be relevant and bounded by two active constraints.

When condition 2: g_6, g_8 and g_9 are active constraints, x_1 is irrelevant, which means:

$$(3.2) \quad \begin{cases} g_6(x^*) = -x_2 + 210 = 0 \\ g_8(x^*) = -x_3 + 30 = 0 \\ g_9(x^*) = x_4 - 7 = 0 \end{cases}$$

Check this condition by simulation, test x_1 from 20 to 40 cc/s the result of pressure is 15.37 to 12.31, which means the pressure is always higher than 9 MPa, so if g_6 and g_8 are active constraints, g_1 can not be satisfied. Then the condition 2 and condition 5 in MP1 can not be true. No matter which is true between condition 1, condition 3 and condition 4, there is a conclusion that as least g_1 and g_9 are active constraints.

Based on MP2, When g_1 is an active constraint, x_1 is monotonic decreasing on g_1 while x_1 is monotonic increasing on g_3 then g_3 is another active constraint, which means:

$$(3.3) \quad g_3(x^*) = x_1 - 40 = 0$$

By monotonicity analysis, there is equation(3.4), only x_2 and x_3 need to be optimized in later steps.

$$(3.4) \quad \begin{cases} x_1 = 40 \\ x_4 = 7 \end{cases}$$

3.6 Regression equations

Then build up the regression equation for each response only with their significant factors. From the response plot in section 3.4: x_2, x_3 and x_4 are the significant factors for shrinkage, then the regression equation of shrinkage $S(x)$ is:

$$(3.5) \quad S(x^*) = -0.766 + 0.006464x_2 + 0.003521x_3 - 0.05782x_4$$

For injection pressure, x_1, x_2 and x_3 are significant factors, the regression equation of injection pressure $P(x)$ is:

$$(3.6) \quad P(x^*) = 48.10 - 0.0748x_1 - 0.15004x_2 - 0.03852x_3$$

For cooling time, x_2 and x_3 are significant factors, the regression equation of cooling time $C(x)$ is:

$$(3.7) \quad C(x^*) = -5.34 + 0.05048x_2 + 0.2277x_3$$

Then we put the monotonic analyze result (3.4) into (3.5), (3.6) and (3.7), the regression equations become:

$$(3.8) \quad \begin{cases} S(x^*) = -1.17074 + 0.006464x_2 + 0.003521x_3 \\ P(x^*) = 45.108 - 0.15004x_2 - 0.03852x_3 \\ C(x^*) = -5.34 + 0.05048x_2 + 0.2277x_3 \end{cases}$$

3.7 Optimization based on Regression equations

3.7.1 First order regression equations optimization

After build up regression equations then objective function and constraints can be established and optimal setting of x_2 and x_3 can be solved out. After monotonic analyze, in the first optimization model, g_1 is a active constraint, g_4 and g_{10} can be remove because g_3 and g_9 are active constraints. Then there is a new optimization model II as follow:

Table 3.9: Good Operation Parameter Setting

x_1 :Flow Rate(cc/s)	x_2 :Melt Temperature($^{\circ}$ C)	x_3 :Mold Temperature($^{\circ}$ C)	x_4 :Pressure Holding Time(s)
50	231.7628	30	5

minimize $f(x^*)=S(x^*)=-1.17074 + 0.006464x_2 + 0.003521x_3$

subject to:

$$g_1(x)=P(x)-9=36.108 -0.15004x_2 -0.03852x_3= 0$$

$$g_2(x)=C(x)-15=-20.34 +0.05048x_2 +0.2277x_3 \leq 0$$

$$g_3(x)=x_2-250 \leq 0$$

$$g_4(x)=-x_2+210 \leq 0$$

$$g_5(x)=x_3-70 \leq 0$$

$$g_6(x)=-x_3+30 \leq 0$$

By solving the active constraint g_1 , x_3 can be used to express x_2 as follow:

$$(3.9) \quad g_1 : x_2 = -0.2567x_3 + 240.6558$$

Then remove the active constraint and put the equation(3.9) into the optimization model II to have a new optimization model III:

minimize $f(x^*)=S(x^*)=0.001861x_3+0.3849$

subject to:

$$g_1(x^*)=x_3-38.15 \leq 0$$

$$g_2(x^*)=-x_3+30 \leq 0$$

In optimization model III, low boundary of x_3 : $x_3=30$ picked to get the minimum $f(x^*)$. When $x_3=30$, solve equation (3.9) can have a result $x_2=229.5972$, which can satisfy the constraint g_1 and g_2 .

From now on we have the good plastic operation parameter setting to minimize plastic shrinkage under the constraints of injection pressure and cooling time shown in table 3.9. And the results shown by simulation with the good operation parameter setting are shown in table 3.10.

Table 3.10: Result Verification

simulation result	Shrinkage(mm)	Pressure(MPa)	Cooling Time(s)
Optimal parameter setting	0.3988	8.6	13.7199

3.7.2 Second order regression equations optimization

Then the study try higher order regression equations to do optimization. Here second order equations is used to do the optimization in Fmincon. The second order regression equations are shown in equation (3.11).

$$(3.10) \quad \begin{cases} S(x^*) = 3.53 - 0.0305x_2 + 0.00389x_3 - 0.087x_4 + 0.00008x_2^2 - 0.000004x_3^2 + 0.00292x_4^2 \\ P(x^*) = 165.5 - 0.29X_1 - 1.15x_2 - 0.0274x_3 + 0.00359x_1^2 + 0.002174x_2^2 - 0.000111x_3^2 = 0 \\ C(x^*) = -6.37 + 0.1256x_2 - 0.1018x_3 - 0.000163x_2^2 + 0.003295x_3^2 \leq 0 \end{cases}$$

Then we put the monotonic analyze result equation (3.5) into the equation (3.11), and the optimization model become equation(3.12).

$$(3.11) \quad \begin{cases} f(x^*) = S(x^*) = 3.06408 - 0.0305x_2 + 0.00389x_3 + 0.00008x_2^2 - 0.000004x_3^2 \\ g_1(x^*) = P(x^*) - 9 = 150.644 - 1.15x_2 - 0.0274x_3 + 0.002174x_2^2 - 0.000111x_3^2 = 0 \\ g_2(x^*) = C(x^*) - 15 = -21.37 + 0.1256x_2 - 0.1018x_3 - 0.000163x_2^2 + 0.003295x_3^2 \leq 0 \\ g_3(x^*) = x_2 - 250 \leq 0 \\ g_4(x^*) = -x_2 + 210 \leq 0 \\ g_5(x^*) = x_3 - 70 \leq 0 \\ g_6(x^*) = -x_3 + 30 \leq 0 \end{cases}$$

By solving the optimization model in Fmincon we can get a optimal setting of x_2 and x_3 , where $x_2=231.5464, x_3=30$.

Then the good plastic operation parameter setting to minimize plastic shrinkage under the constraints of injection pressure and cooling time are shown in table 3.11. And the results are shown in table 3.12. Compare with the result from first order regression equations, second order regression equations will provide smaller shrinkage because of the higher equation accuracy. However, this shrinkage have not reduced too much compared with the result from first order equation, so that the first order regression equation is enough to get a good operation parameter setting.

Table 3.11: Good Operation Parameter Setting

x_1 :Flow Rate(cc/s)	x_2 :Melt Temperature($^{\circ}$ C)	x_3 :Mold Temperature($^{\circ}$ C)	x_4 :Pressure Holding Time(s)
40	231.5464	30	7

Table 3.12: Result Verification

Result verification	Shrinkage(mm)	Pressure(MPa)	Cooling Time(s)
simulation result	0.3950	8.78	13.7201

3.8 Result verification

Checking all the simulation results in the table 3.4, only no. 22 has the smaller shrinkage than the shrinkage get from the good plastic operation parameter setting, but the pressure of no. 22 is 10.49 which beyond the constraint. Then there is a conclusion that the optimal parameters setting from the optimization can provide a good plastic operation parameter setting to minimize plastic shrinkage under the constraints of pressure and cooling time.

OPTIMIZATION OF PLASTIC SHRINKAGE EXAMPLE II

In this chapter another packing model is introduced use the method in the flowchart to optimize the plastic shrinkage under the constraint of injection pressure and cooling time. The simulation model in the chapter is a rectangular plastic box which use the same material as last model. Also, the optimized shrinkage of this model is the biggest shrinkage part in the model.

4.1 Model property

The simulation model here is a rectangular plastic box shown in figure 4.1. The length is 3 inch, width is 1.5 inch and height is 1 inch, the coordinate is also shown in figure 4.1.

The same model material Acrylonitrile butadiene styrene (ABS) and mold material steel-420SS are chose as last example. The material parameters are also same as in table 3.1 and table 3.2.

4.2 Response data from simulation

4.2.1 Input variables and responses

The same five input control variables in this example: flow rate, melt temperature, mold temperature, pressure holding time, packing pressure. The responses are still injection pressure, cooling time and shrinkage. The study goal is to minimize shrinkage on on x direction under the constraints of injection pressure and cooling time by finding a optimal set of combination of input factors.

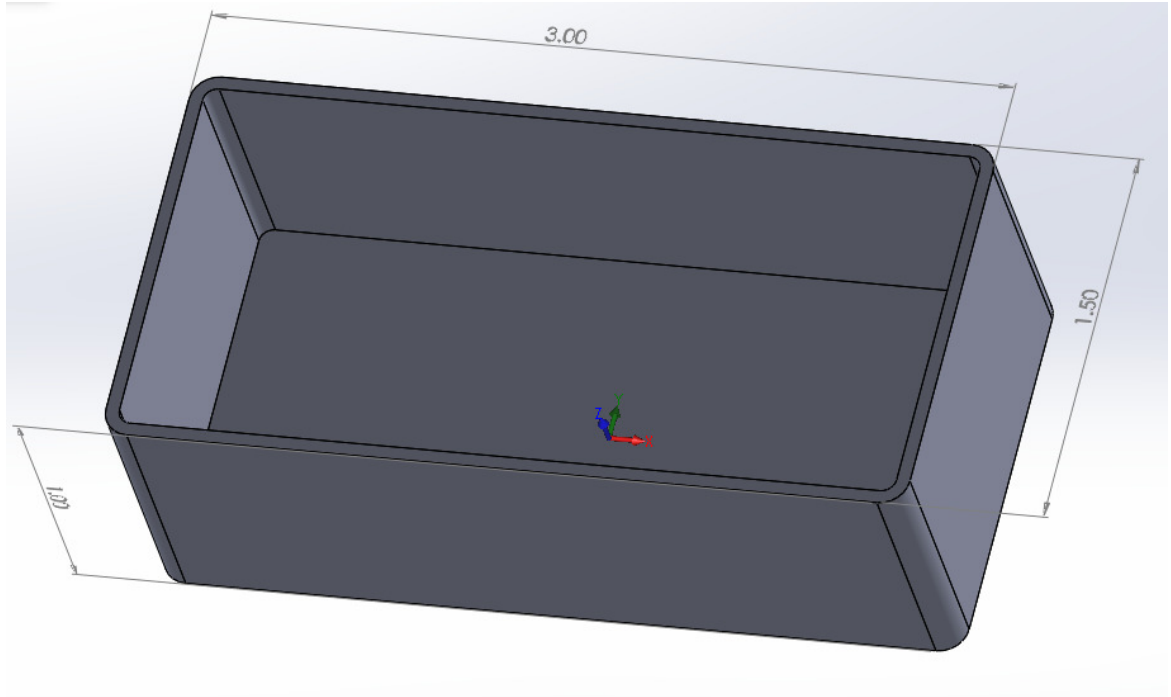


FIGURE 4.1. Simulation Model II.

Table 4.1: Input variables Level Setting II

	x_1 :Flow rate(cc/s)	x_2 :Melt Tempera- ture($^{\circ}$ C)	x_3 :Mold Tempera- ture($^{\circ}$ C)	x_4 : Pressure Holding Time(s)	x_5 :Packing pres- sure(MPa)
Level 1	10	210	30	1	20
Level 2	20	220	40	2	25
Level 3	30	230	50	3	30
Level 4	40	240	60	4	35
Level 5	50	250	70	5	40

4.2.2 Level setting of variables

There are 5 levels for each of the input variables. The level setting of the inputs variables are shown in table 4.1.

4.2.3 Simulation result

After level setting the orthogonal arrays get be build up of these input factors. In this example, $L_5^5(5^5)$ orthogonal array still be used and the simulation result of the three response are shown in table 4.2.

4.3 Significant factors

The study decide which factors are significant factors for each response based on the simulation result and P-value of each factor for each response.

Based on the simulation result shown in table 4.2 there are response plots and P-value of each input factors for each response in following images. Suppose variables whose P-value less than 0.15 will be assumed as significant factors and calculated in later regression equations.

Figure 4.2 is the shrinkage response plot. The P-values in table 4.3 shows that the P-value of x_2 (melt temperature), x_3 (mold temperature) and x_4 (pressure holding time) are smaller than 0.15, which means that x_2 (melt temperature), x_3 (mold temperature) and x_4 (pressure holding time) are the significant factor for shrinkage, x_1 (filling time) and x_5 (packing pressure) do not have strong relationship with shrinkage. The response plot show that x_2 (melt temperature) and x_3 (mold

Table 4.2: L_{25} Orthogonal Array Simulation Result II

Run	x_1 :Flow rate(cc/s)	x_2 :Melt Temper- a- ture($^{\circ}$ C)	x_3 :Mold Temper- a- ture($^{\circ}$ C)	x_4 : Pressure Holding Time(s)	x_5 :Packing pres- sure(MPa)	Pressure (MPa)	Cooling Time(s)	Shrinkage (mm)
1	10	210	30	1	20	47.88	3.2989	0.3099
2	10	220	40	2	25	43.44	3.6525	0.2281
3	10	230	50	3	30	39.86	4.3638	0.2044
4	10	240	60	4	35	36.05	4.8000	0.1959
5	10	250	70	5	40	32.34	5.7906	0.2158
6	20	210	40	3	35	46.47	3.0402	0.1660
7	20	220	50	4	40	41.55	3.6065	0.1582
8	20	230	60	5	20	36.61	4.2694	0.1776
9	20	240	70	1	25	32.18	5.3105	0.4396
10	20	250	30	2	30	32.10	3.2343	0.2329
11	30	210	50	5	25	45.09	3.3287	0.1377
12	30	220	60	1	30	39.11	3.9993	0.3936
13	30	230	70	2	35	33.97	5.0398	0.3271
14	30	240	30	3	40	33.85	2.9680	0.1561
15	30	250	40	4	20	29.93	3.4335	0.1374
16	40	210	60	2	40	42.76	3.8085	0.2821
17	40	220	70	3	20	36.94	4.8506	0.2664
18	40	230	30	4	25	36.47	2.7751	0.0967
19	40	240	40	5	30	31.99	3.2499	0.1138
20	40	250	50	1	35	28.41	3.8186	0.4255
21	50	210	70	4	30	41.11	4.6069	0.2231
22	50	220	30	5	35	39.90	2.6261	0.0732
23	50	230	40	1	40	34.93	3.0135	0.3750
24	50	240	50	2	20	30.93	3.5822	0.2846
25	50	250	60	3	25	27.54	4.3344	0.2571

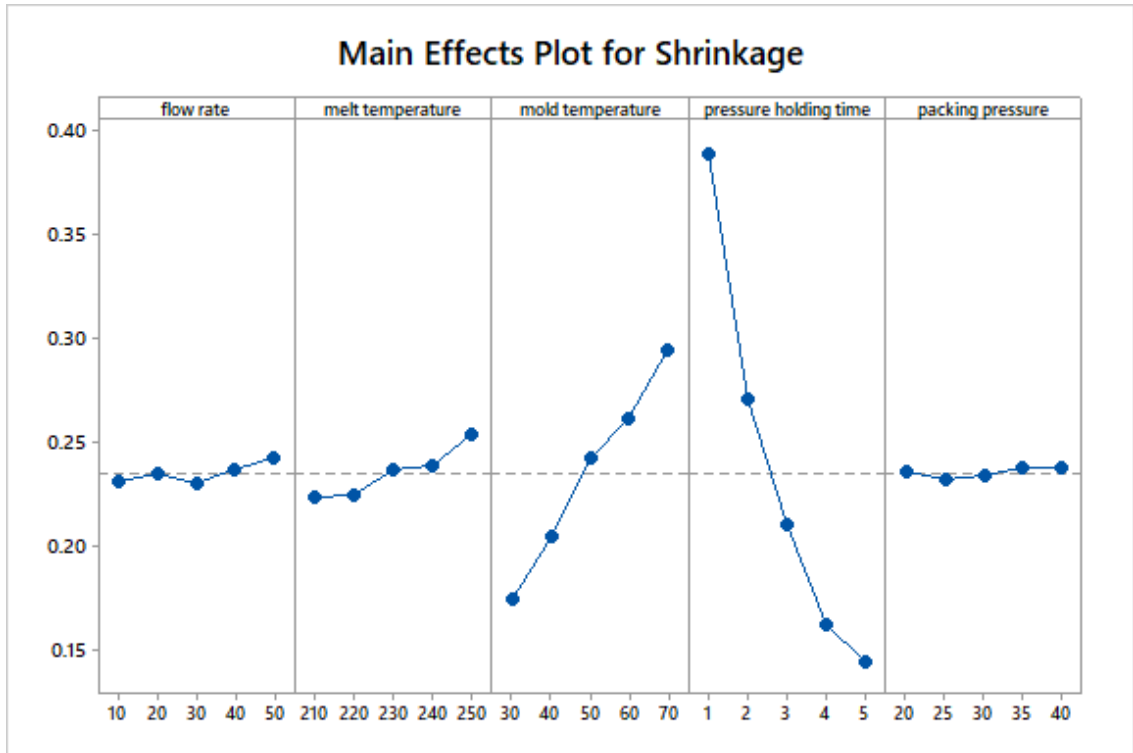


FIGURE 4.2. Shrinkage Response Plot II.

temperature) are monotonic increasing on shrinkage, x_4 (pressure holding time) is monotonic decreasing on shrinkage.

Table 4.3: P-value For Shrinkage II

Factor	P-value($P \leq 0.1$)
Filling Time	0.687
Melt Temperature	0.120
Mold Temperature	0.001
Pressure Holding Time	0.000
Ambient Temperature	0.956

Figure 4.3 is the injection pressure response plot. The P-values in table 4.4 shows that the P-value of x_1 (flow rate), x_2 (melt temperature) and x_3 (mold temperature) are smaller than 0.05, which means that x_1 (flow rate), x_2 (melt temperature) and x_3 (mold temperature) are the significant factors for injection pressure, and x_4 (pressure holding time), x_5 (packing pressure) do not have strong relationship with injection pressure. The response plot show that x_1 (flow rate), x_2 (melt temperature) and x_3 (mold temperature) are monotonic decreasing on injection pressure.

Figure 4.4 is the cooling time response plot. The P-values in table 4.5 shows that only the P-value of x_1 (flow rate), x_2 (melt temperature) and x_3 (mold temperature) are smaller than 0.05,

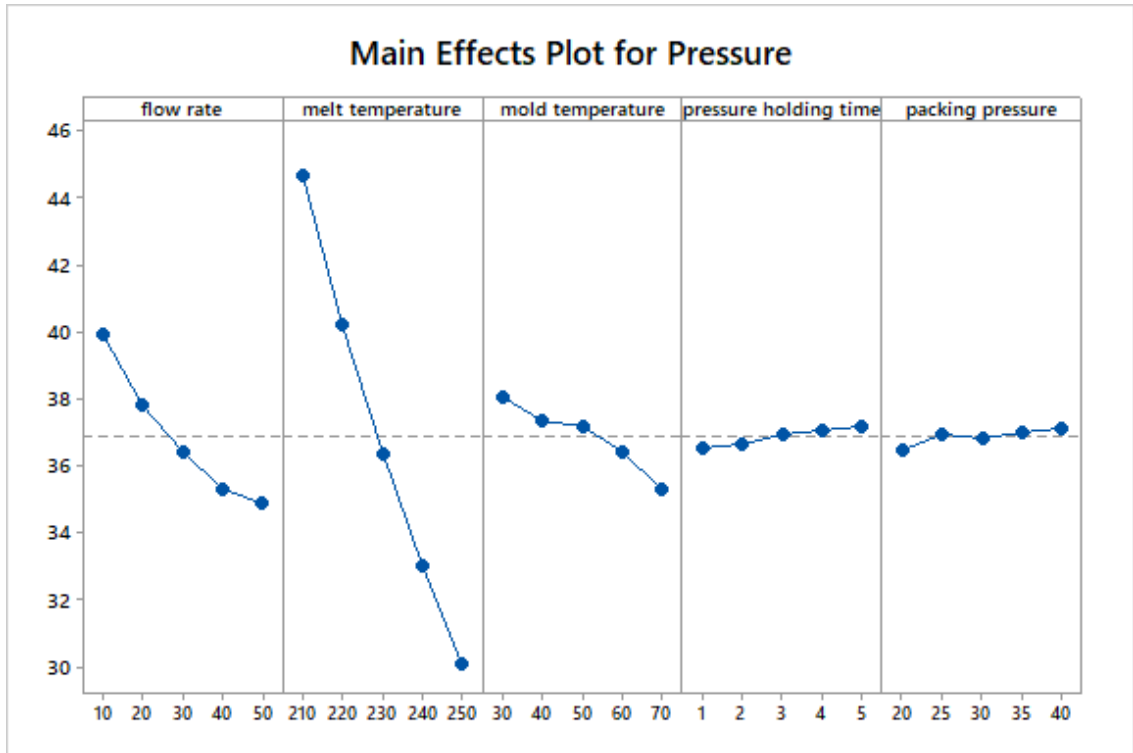


FIGURE 4.3. Pressure Plot II.

which means that x_1 (flow rate), x_2 (melt temperature) and x_3 (mold temperature) are the significant factors for cooling time, x_4 (pressure holding time) and x_5 (packing pressure) do not have strong relationship with cooling time. The response plot show that x_1 (flow rate) is monotonic decreasing on cooling time while x_2 (melt temperature) and x_3 (mold temperature) are monotonic increasing on cooling time.

Table 4.4: P-value For Pressure Plot II

Factor	P-value($P \leq 0.1$)
Filling Time	0.000
Melt Temperature	0.000
Mold Temperature	0.005
Pressure Holding Time	0.298
Ambient Temperature	0.405

Table 4.5: P-value For Cooling Time II

Factor	P-value($P \leq 0.1$)
Filling Time	0.000
Melt Temperature	0.002
Mold Temperature	0.000
Pressure Holding Time	0.643
Ambient Temperature	0.787

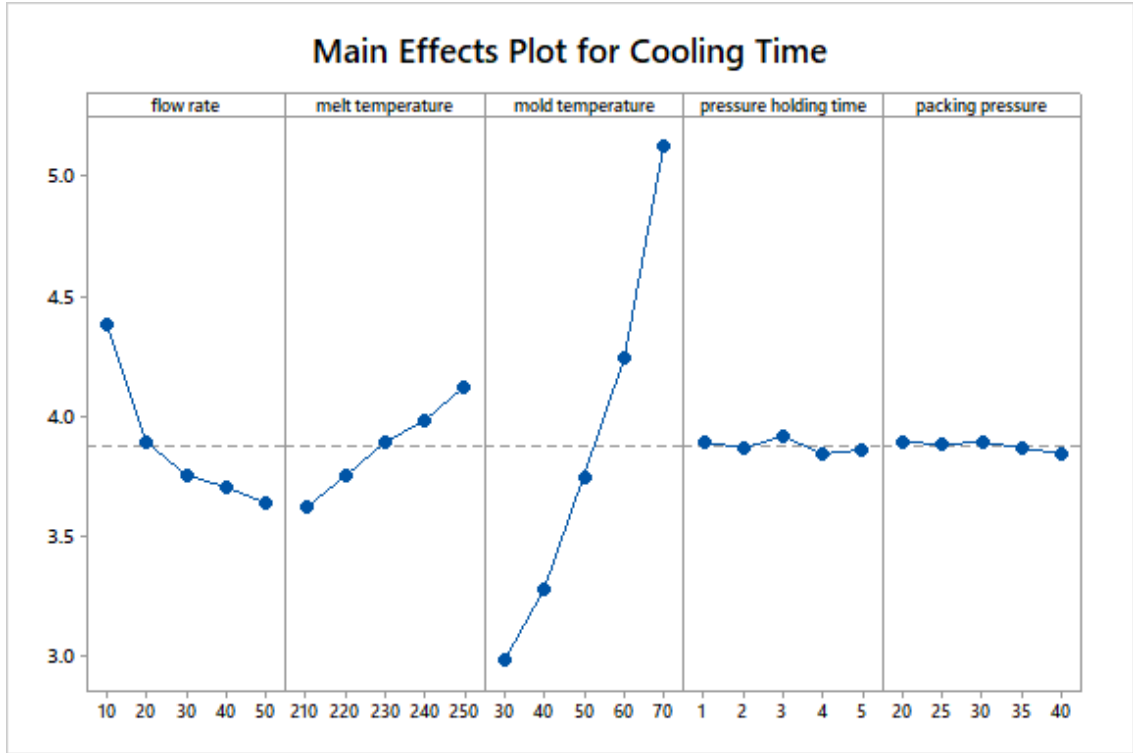


FIGURE 4.4. Cooling Time Plot II.

4.4 Monotonic analysis

4.4.1 Optimization model

Based on the response plots and significant factors analysis, x_5 (packing pressure) can be removed out of the optimization, because x_5 is not the significant factor for each response. Then introduce monotonicity analysis with the rest four variables on each response. In the optimization model, the $S(x^*)$ (Shrinkage function according to its significant factors) is the objective function and $P(x^*)$ (pressure function according to its significant factors), $C(x^*)$ (cooling time function according to its significant factors) become two constraints g_1, g_2 which represent injection pressure can not beyond 35MPa and cooling time can not beyond 4s. " x^* " represent the vector $[x_1, x_2, x_3, x_4]$. Based on the low boundary and up boundary of the four variables, there are eight more constraints from g_3 to g_{10} .

The optimization model with objective function and constraints are shown as follow:

$$\text{minimize } f(x^*) = S(x^*)$$

subject to:

$$g_1(x^*) = P(x^*) - 35 \leq 0$$

$$g_2(x^*) = C(x^*) - 4 \leq 0$$

$$g_3(x^*)=x_1-50 \leq 0$$

$$g_4(x^*)=-x_1+10 \leq 0$$

$$g_5(x^*)=x_2-250 \leq 0$$

$$g_6(x^*)=-x_2+210 \leq 0$$

$$g_7(x^*)=x_3-70 \leq 0$$

$$g_8(x^*)=-x_3+30 \leq 0$$

$$g_9(x^*)=x_4-5 \leq 0$$

$$g_{10}(x^*)=-x_4+1 \leq 0$$

4.4.2 Monotonic table

Based on the response plots in figure 4.2, 4.3, 4.4 and the optimization model, we can build up a monotonic table shown as table 4.6.

Table 4.6: Monotonic Table

	x_1	x_2	x_3	x_4
f		+	+	-
g_1	-	-	-	
g_2	-	+	+	
g_3	+			
g_4	-			
g_5		+		
g_6		-		
g_7			+	
g_8			-	
g_9				+
g_{10}				-

“+” means that variable is monotonic increasing on the objective function or constraint, “-” means that variable is monotonic decreasing on the objective function or constraint.

From the response plots, x_2 and x_3 are monotonic increasing on objective function, x_4 is monotonic decreasing on objective function. x_1, x_2 and x_3 are monotonic decreasing on g_1 . x_1 is monotonic decreasing on g_2 while x_2 and x_3 are monotonic increasing on g_2 . The following monotonic relationship are established from the optimization model.

4.4.3 Monotonicity analysis

Then introduce monotonicity analysis with the monotonic table to simplify the optimization model based on monotonicity principle1(MP1) and monotonicity principle2(MP2).

First Monotonicity Principle (MP1): In a well-constrained objective function every increasing (decreasing) variable is bounded below (above) by at least one active constraint. x_2 and x_3 are monotonic increasing on objective function, x_4 is monotonic decreasing on objective function.

For x_2 : x_2 is monotonic decreasing on g_1 and g_6 , which means there is at least one active constraint from g_1 and g_6 .

For x_3 : x_3 is monotonic decreasing on g_1 and g_8 , which means there is at least one active constraint from g_1 and g_8 .

For x_4 : x_4 is only monotonic increasing on g_9 , so g_9 is one of a active constraint, which means:

$$(4.1) \quad g_9(x^*) = x_4 - 5 = 0$$

Then there are five active constraints combination possibility:

Condition 1: g_1 and g_9 are active constraints.

Condition 2: g_6, g_8 and g_9 are active constraints.

Condition 3: g_1, g_6 and g_9 are active constraints.

Condition 4: g_1, g_8 and g_9 are active constraints.

Condition 5: g_1, g_6, g_8 and g_9 are active constraints.

Second Monotonic Principle (MP2): Every monotonic variable not occurring in a well-constrained objective function is either irrelevant and can be deleted, or relevant and bounded by two active constraints, one from above and one from below. Based on MP2, x_1 is not occurring in objective function so it may irrelevant and can be deleted. On the other hand, x_1 could be relevant and bounded by two active constraints.

When condition 2: g_6, g_8 and g_9 are active constraints, x_1 is irrelevant, which means:

$$(4.2) \quad \begin{cases} g_6(x^*) = -x_2 + 210 = 0 \\ g_8(x^*) = -x_3 + 30 = 0 \\ g_9(x^*) = x_4 - 5 = 0 \end{cases}$$

Check this condition by simulation, even choose up boundary for x_1 which means $x_1=50$ the result of pressure is 44.84MPa which is still higher than 35MPa, so if g_6 and g_8 are active constraints, g_1 can not be satisfied. Then the condition 2 and condition 5 in MP1 are not true. No matter which is true between condition 1, condition 3 and condition 4, there is a conclusion that as least g_1 and g_9 are active constraints.

Based on MP2, When g_1 is an active constraint, x_1 is monotonic decreasing on g_1 and only monotonic increasing on g_3 then g_3 is another active constraint, which means:

$$(4.3) \quad g_3(x^*) = x_1 - 50 = 0$$

By monotonicity analysis there is equation (4.4), only x_2 and x_3 need to be optimized in later steps.

$$(4.4) \quad \begin{cases} x_1 = 50 \\ x_4 = 5 \end{cases}$$

4.5 Regression equations

The regression equation for each response only with their significant factors can be build up. From the response plot in section 4.3: x_2, x_3 and x_4 are the significant factors for shrinkage, then the regression equation of shrinkage $S(x)$ is:

$$(4.5) \quad S(x^*) = 0.095 + 0.000741x_2 + 0.002985x_3 - 0.05989x_4$$

For injection pressure, x_1, x_2 and x_3 are significant factors, the regression equation of injection pressure $P(x)$ is:

$$(4.6) \quad P(x^*) = 127.5 - 0.1253x_1 - 0.3638x_2 - 0.064x_3$$

For cooling time, x_1, x_2 and x_3 are significant factors, the regression equation of cooling time $C(x)$ is:

$$(4.7) \quad C(x^*) = -1.109 - 0.01689x_1 + 0.01246x_2 + 0.05243x_3$$

Then put the monotonic analyze result (4.4) into (4.5), (4.6) and (4.7), the regression equations become:

$$(4.8) \quad \begin{cases} S(x^*) = -0.20445 + 0.000741x_2 + 0.002985x_3 \\ P(x^*) = 121.235 - 0.3638x_2 - 0.064x_3 \\ C(x^*) = -1.9535 + 0.01246x_2 + 0.05243x_3 \end{cases}$$

4.6 Optimization based on Regression equations

Once regression equations build up ,the new optimization model can be build up.By solving objective function and constraints the optimal setting of x_2 and x_3 can be solve out. After monotonic analysis for optimization I, g_1 is a active constraint, g_4 and g_{10} can be remove because g_3 and g_9 are active constraints. Then there is a new optimization model II as follow:

$$\text{minimize } f(x^*)=S(x^*)=-0.20445 +0.00741x_2+0.002985x_3$$

subject to:

$$g_1(x^*)=P(x^*)-35=86.235 -0.3638x_2 -0.064x_3= 0$$

$$g_2(x^*)=C(x^*)-4=-5.9535+0.05048x_2 +0.02277x_3 \leq 0$$

$$g_3(x^*)=x_2-250 \leq 0$$

$$g_4(x^*)=-x_2+210 \leq 0$$

$$g_5(x^*)=x_3-70 \leq 0$$

$$g_6(x^*)=-x_3+30 \leq 0$$

By solving the active constraint g_1 , x_2 can be expressed by x_3 as follow:

$$(4.9) \quad x_2 = -0.1759x_3 + 237.0396$$

Then remove the active constraint and put the equation(18) into the optimization model II to have a new optimization model III:

$$\text{minimize } f(x^*)=S(x^*)=0.002855x_3-0.0288$$

subject to:

$$g_1(x^*)=x_3-59.99 \leq 0$$

$$g_2(x^*)=-x_3+30 \leq 0$$

In optimization model III, low boundary of x_3 : $x_3=30$ are picked to get the minimum $f(x^*)$. When $x_3=30$, solve equation 4.9 can have a result $x_2=231.7628$,which can satisfy the constraint

Table 4.7: Optimal Process Parameter Setting

x_1 :Flow Rate(cc/s)	x_2 :Melt Temperature($^{\circ}$ C)	x_3 :Mold Temperature($^{\circ}$ C)	x_4 :Pressure Holding Time(s)
50	231.7628	30	5

Table 4.8: Result Verification

Result verification	Shrinkage(mm)	Pressure(MPa)	Cooling Time(s)
simulation result	0.0768	34.96	2.73

g_1 and g_2 .

Now the solved operation parameter setting are shown as table 4.7 and the simulation result are shown as table 4.8. Checking all the simulation results in the table 4.2, only no. 22 has the smaller shrinkage than the shrinkage get from the good plastic operation parameter setting, but the pressure of no. 22 is 39.90 which beyond the constraint. Then there is a conclusion that the optimal parameters setting from the optimization can provide a good plastic operation parameter setting to minimize plastic shrinkage under the constraints of pressure and cooling time.

CONCLUSION

In this study, an integrated optimization method is proposed to minimize plastic product shrinkage in plastic injection molding. The method flow as: orthogonal array – P-value calculation – monotonic analysis – regression analysis. In the two simulation examples, flow rate, melt temperature, mold temperature, pressure holding time and packing pressure are taken as input variables, shrinkage is the objective, cooling time and injection pressure are two constraints. Monotonic analysis in the two examples shows that shrinkage is monotonic increasing with melt temperature and mold temperature, decreasing with pressure. Injection pressure is monotonic decreasing with melt temperature and mold temperature while cooling time is monotonic increasing with them. The results in the examples shows that the proposed method can be used to help designer find a good operation parameter setting efficiently to minimize plastic shrinkage under constraints. Moreover, this systematic optimization method can not only be used in PIM, but also a widely range area.

CHAPTER 6

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