Description: Three of 3 Interview with Stephanie about the Towers problem, 03-06-1992 **Location: Kenilworth, NJ** **Researcher: Carolyn Maher** **Video**: https://drive.google.com/open?id=14X2HOXeHBa_nsOpO2sLXj0P71Dlzpp5G	**Transcriber(s): Ella and Victoria Krupnik** **Verifier(s): Adam Smargon, Kara Teehan, Victoria Krupnik** **Date: Summer 2018** **Page:**

Line #	Time stamp	Speaker	utterance
1	0:38	CM	Anyway let's see, where were we the last time?
2		S	Well, when I came here you wanted me to do the towers of four.
3		CM	Oh right, right, and you were going to [mumbling] your new method to show me, and how many towers of four did you tell there were going to be?
4		S	I think I said around twenty.
5		CM	You said around twenty.
6		S	…and I did and when I did it I got twenty. (4 tall)
7		CM	You did. And you were using?
8		S	My method. The one. Alright.
9		CM	Can you tell me about this? Tell me why you need to do this?
10			(Talking others) Can you wait just a minute. Hold on just a second. Make a big difference. How are you
11	1:27	S	Okay. First we have the towers with one white block, and the white block is on the top, and then it's there and then ***(making towers with exactly one white block in an elevator pattern from top to bottom***) it's there -- and then, it's -- there, and then it's… there. And there's our first group of towers.
12		CM	Okay, now remember, I'm supposed to be – Stephen, is it? -- and say to you I think there are more with one white and three black. (points to her towers)
13		S	With one white – no, there's no more with one white…
14		CM	But I'm Stephen, and I say I think there are…
15	2:35	S	Well there's – well, okay. You have – okay. Once you get down to the last one, [mumbling] at the bottom, you can't move the white back up, because then you'll just be repeating these things. But if you move the white down one you'll be missing a space… and if you can only use four blocks, you can't have another one.(***argument by contradiction)***
16		CM	Why can't you move the white on the next position on top?
17		S	Because if you move the white on the next position on top it'll be like this. And you need another -- you'd need another block here, but you can't do that.***(same argument for the top)***
18		CM	Why can't you?
19		S	Because there's only four blocks.(***conditional reasoning)***

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20		CM	Why can't there be five?
21		S	Because well... the assignment said we have to use four. ***conditional reasoning)***
22		CM	Oh ok. So these are. Should we keep a record while we're doing it of how many?
23		S	Okay that's four. Should I draw them?
24		CM	Well you can. Any way you want to record it. You can write a statement to describe it since you have them made already.
25	4:10	S	Alright. [writes "4 one moving down and boxes the statement and puts the group of 4 towers underneath it on top of the paper] Okay and here's the **one moving down**. Cause it starts here and it goes down here. Alright. Then we have **the two. [3 two stuck together moving in an elevator pattern, puts it under the other grou]** And, we go like this. That's one… and that's two, and that's three.
26		CM	Okay and how do you describe this group?
27		S	Well, um, two moving down, two together moving together down?[
28		CM	Okay, so when you wrote "moving down", what do you mean? [S: the one] Okay, why don't you write that there's "exactly one white"… and exactly… how else can we say; there's exactly one white…
29		S	Um…
30		CM	Want to say something about the black?
31		S	With three black. [writing one moving down with 3 black (does not write exactly and does not write white, but has 4 one white moving down still)]
32		CM	Okay…
33	5:50	S	Okay there. And so this one we can go two… two white moving down, down with two black. [writes 2 white moving down with 2 black, boxes it, and puts group of towers under it] And our total there is three.
34		CM	Okay do you want to say anything about those two? They're the only ways you can have two white moving down?
35		S	No they're stuck.[writes also glued together]
36		CM	Stuck.
37		S	[writes 3 glued together two white moving down] Okay there, that way you can't get it mixed up with the ones that were apart. Then you have your three white.[a new group but gets interrupted by researcher]
38		CM	Now hold on. This is the only way to do two white?
39	7:12	S	No, those are not the only way you can do two white but these are the only way you can do **two white together.**
40		CM	Okay so these are exactly one white and these are exactly two white. Want to jump to three white before you finish the two whites?

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41		S	When I did the pattern I did one white, two white stuck together, four white stuck together. then went back and went two white apart.(***describing her method)***
42		CM	I see.
43		S	And then there's the one.
44		CM	I'm sorry to go back but I'm kind of thinking about Stephen again. Suppose he says, "How do you know have all of them?"[referring to two white stuck together]
45		S	Because if you take the last one [BBWW] you can't move them [the two whites] down another one because you're only using four blocks. Okay, here's the first one [referring to WWWB]… and here's the second one. [referring to BWWW of the three stuck together group]
46		CM	Okay.
47		S	That's two, three together [writes "2. 3 together whites with one black"] . Alright there's your next one because you have these two and you have the three at the top, one [black] at the bottom, one on top [black] and three at the bottm [3 whites].
48	9:18	CM	And no more because?
49		S	Because if you take this one [BWWW breaks it apaprat and shows you would need another block to put in between the whites and black]]and you wanted to move the white down another one you'd need another block to put in here. Now four… [drops a block]
50		CM	Don't worry about it.
51		S	Okay, four…
52		CM	Let me just see what you have here. You have **three together whites, three glued together**- Glued together- **Two white glued together.** Three together, okay.
53		S	Okay. And then we have the **[writes "1. 4 whits together"]… okay and that's the last one.** And… now then we go back to the two whites stuck together and we make it… apart. [builds WBWB, WBBW]
54		CM	Here let's look it here next to these twos.
55		S	And we can do it like… this… and that's it. Wait…[pauses and is analyzing the two built so far] hmm, let's see. Um… oh I forgot. You can also go like this. [builds BWBW] That's the reverse.
56		CM	Well let's see.
57		S	That's the reverse of that one.
58	12:18	CM	In what way reversed, Stephanie? what are you reversing?
59		S	Okay well when you show it upside down it's reversed.
60		CM	But you're not, you're standing it up... that's the tower.

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61		S	Well, what I'm saying is I would go and when I'm finished, I'd go I have four here, three here, two here, one here [pointing to the written cases so far 4,3,2,1]… and I would go double that and I'd go I have two here, four here, six here, eight here [doubling from bottom to top of the cases 1,2,3,4]... and I'd add that, but if I went like that…
62		CM	Okay that's… thats how you got your twenty. what was your reasoning for getting the double?
63		S	What it is is this isn't the only way you can get two apart. You can make two apart by taking two black and separating them.
64		CM	Okay but. That's true. That's true. So when you compare these two [alternating color cubes BWBW WBWB], how do you think about them when you use that argument?
65		S	Well, when I use the argument for I double it? I think about them in… well, if I'm doing a pattern where I'm using whites apart then I'm not going to use a pattern where I use blacks apart with it [touching and showing with the alternating color towers even though these towers also have the black apart she doesn't realize this] but when I'm taking this and going and doubling the pattern I just turn it upside down and that's how I get my- my other pattern.
66		CM	And you had a name for that the last time, didn't you?
67		S	This kind of pattern?
68		CM	Yeah you had a name for the other one that went with it
69		S	Yeah pair.. I forget. I forget I think it was um… hm…
70		CM	You use the word pair now. Which means the last time you had this you automatically found the other one. What did the pair look like [WBBB]. Just tell me, you don't have to .
71		S	It looked like black on top and white…[bwww]
72	14:38	CM	Right. That's how you got 20 by that strategy, right? [mhmm]. Just now you didn't do that here. When you just found the ten, then you just found- how many more two more just now, [of two apart whites]
73		S	Two more
74		CM	Two more. You said these were also two whites. is that correct? Exactly two whites. Different than the two whites here? In what way? Just you can tell me.
75		S	These are apart and these are together.
76		CM	Right. But these two are different apart in a certain way. What's the difference between in what way these are apart?[alternating color towers]
77		S	This one has the, uh, black top. This Is the black at the top and this is the white at the top
78		CM	Ok

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79		S	But it's also this pattern because if you look here it's black white black white. which is one of these patterns because the whites are still separated.
80		CM	Let me stop here then. Here we have… let's talk about the strategy you were doing before you did double and pairs. In that strategy you said these are exactly two whites. Now again I'm going to pretend I'm Stephen, and I'm saying you've convinced me there only two together… there's three. Now I'm asking you to convince me when the two whites apart that there are only three [puts th three towers with two white appart saprately from the two whites together towers: WBBW, WBWB, BWBW]; when you have exactly two white and exactly two black when the two whites are apart, they're exactly three. How would you convince me of that? Is this the way you wanted to put them? Is this the order you put it?
81		S	It doesn't matter.
82		CM	That's not how I think you had it I think you had this one first.[rearranges WBWB, WBBW, BWBW]
83		S	This one had white at the top. And that's what I started off with had the white at the top then the blue – the black, then the white, then the black.
84	16:59	CM	This one white on the top then we had a separation then the next white here [referring to WBWB]… okay, this one. [referring to WBBW]
85		S	Then, I made a white at the top.
86		CM	Again white at the top. Okay.
87		S	I had a white at the top. With black in the middle, then a white.
88		CM	Now these two here... are they alike in any way?[comparing wBWB WBBW]
89		S	Yes, they are [yawns]; they both have white separated.
90		CM	Is there any other way that they are alike, or maybe another way of saying it? These all have white separated, are these different from these in any way?[showing WBWB and WBBW and comparing it to BWBW]
91		S	This one is different from this one, because this one has the two black glued together.
92		CM	That's true… that's… but I'm asking is there any way that one of these… these two [the both together] are different.
93		S	No not both of them. Well I guess yes… this has the black at the top but these don't.
94	18:13	CM	If I asked you these two are different than these… these two [referring to WBWB and WBBW] had whites at the top and then you had to skip [referring to the one black skipped in WBWB] you have white here, and now the only way you skip is this white here [pointing to the last W in WBBW] and this starts here [pointing to first W in BWBW] and you had to skip . How do I know there's no more? [Stephanie yawns] You seem tired, Steph…

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95		S	Yeah, I woke up at six. I couldn't sleep.
96	18:58	CM	So now you need a nap, are you getting tired [S: just alittle]… if you want to stop, you decide.
97		S	No, that's okay.
98		CM	I'm Stephen and I want to be sure that when you have exactly two whites separated, that there can't be any others than the ones you have here.
99		S	Okay... okay… the only problem is when you go to make your doubles, you can't make the doubles with this because YOU'VE already made the doubles…[evidence of recognition that the two alternating are opposites of each other]
100		CM	Let's just try not to worry about the made the doubles… let's try to prove…
101		S	It's always harder with the separates, um… oh, okay…
102		CM	I wish Stephen were here. Because I think you'd come up with a way of convincing him.
103		S	Okay. You have the two whites separated by two blacks… you have the white, black, white, black…
104		CM	I'm not convinced… that you're just telling me what you have, that doesn't convince me.
105		S	But then you have the black, white, black, white, and if you want to separate again you'd have to have another piece [using argument by contradiction to add another imaginary black in between the second B and second W of BWBW]. because the white on the bottom.
106		CM	I don't really see, see it. I'm not sure seems to me that if I'm Stephen that you're just trying to make a case of each one of these three why they're there and I'm not really sure that- I'm not quite convinced that you did all of them and that there cant be anymore
107		S	Ok. Well let me put it this way, we know we can't make another one of these [WBBW] because you can't fit any more than two in between these so you know that this one can't be made again
108		CM	So When you have white at the top and white at the bottom
109		S	Yeah you can't fit any more than two in the middle. You know that ones finished… but then these two [alternating color towers] you have white black white black okay and then you move the white down one
110		CM	Mmhmm….
111		S	Okay and if I move the white down again if I were to say let me put the white in the black space and the black... hm… and... if I were to put the white here with the two blacks here I'd be getting.. this [BBWW a duplicate of the two together group]]. If I were to put the white here and the two blacks here and the white here, I'd be getting that [WWBB]. If I were to put the white here and a black here and a white there and a black there you'd be getting this [WBWB]. And if I were to put- I shifted it here [showing a white cube independently of the tower and how she

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			shifted it on every position but stops at last position]- no I didn't , shifted it here, no I didnt shift it there, I would've put this this this [referring to WBBW] and that the other one you'd be getting the same thing [as WBBW]. So I shifted to every single level and you can't make another one.
112		CM	I think I followed everything you said here so there are possible exactly two white you've added how many more.
113		S	Three more.
114		CM	So why don't we get it exactly two white next to it.[pointing to right side of the paper where her orgiinal cases are on the left side of the page.]
115		S	Ok.
116		CM	Let's put it next to it here [showing the two white together case, she wants the separated case next to it t othe right] . so we have all of this row to be exactly two white next to it. You see this box here, why don't we put it right alongside it. This is not exactly two white. These are the three whites… its this box here; these were the two glued together whites, right? and now these are the two whites that are separated. Okay. But you didn't tell me how many whites had to be here.[writes "white separated by black 3.] How many whites exactly are here ?[showing exactly two white group separated][stephanie writes a "2" on top of "white separated by black; 3." . So you've convinced me that you four of exactly one whites, you've convinced me that there are exactly **six** of exactly two whites. And are there **two** of exactly three whites?
117		S	Maybe. maybe. These are separate whites… let's say we'll start at the top. No, we can't start it at the top. Okay we'll start it right here. Okay there's your first one. [ending up starting black at the second poisition instead of at the top and builds WBWW]
118		CM	Oh, now we're talking about **exactly three whites but some separation.**
119		S	Mmhmm…
120		CM	Okay…
121	25:17:00	S	There's your second one [referring to WWBW] because you moved the black down one [showing how the one black cube is moving down. And that's it. You can't make a third one
122		CM	Why?
123		S	Because if you move your black another one and if you move your black down to the last space you would have this.[builds WWWB and shows it's a duplicate of one of the two exactly three white together]
124		CM	Okay I see that but you don't have a black on top here [pointing to the separated group]
125		S	If you have a black on top, then you would have this [referring to BWWW the duplicate of the exactly three white glued together group]

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126		CM	So you have how many extras exactly three whites by having separations?
127		S	Two.
128		CM	Okay so why don't you do that on alongside [pointing again to the right side of the paper to the right of exactly three white glued together][stephanie writes "3 white separated by one black found 2"… so you have exactly three whites… And then this one was exactly four whites... we lost it somehow.
129		S	Oh well. and then you can only make one.[builds WWWW]
130		CM	Wait a minute aren't we doing whites?[builds BBBB] You can only make whites.
131		S	Oh…
132		CM	How many whites are there here exactly now? [S: none]So why don't you tell me how many exactly no whites. [points to paper at the top next to the all white tower case] Why don't you put it down here. How don't you tell me exactly how many. Keep focusing on whites… how many did you get with exactly no whites. Why don't you tell me how many exactly no whites? Keep a record of that.
133		S	mhmm.
134		CM	Zero whites? No whites?
135		S	Okay I got one.[writes "no whites=1]
136		CM	What do we have here?
137		S	So far? [sigh] okay…
138		CM	No whites? How many? [points to the towers built that are organized in a long column starting from no white, one white, two white, three white, and four white]
139		S	One?
140		CM	One white?
141		S	Four.
142		CM	Ok that's Exactly four whites.
143		S	
144		CM	Ok. Exactly two whites.
145		S	Two… three… that's six… that's eleven.
146		CM	Exactly three whites.
147		S	Two, two, That's four… eleven… that's fifteen.

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148		CM	Exactly four whites.
149		S	Sixteen.
150		CM	You told me there was going to be twenty. you just convinced me there can't be any more; I'm very confused.
151		S	We didn't do the opposites.
152		CM	Are you absolutely sure? Ok lets start: What's the opposite of this?[all black and stephanie shows all white tower] Put it back… let's see if all the opposite of these are okay [the one white tower set]… let's just leave here what you have… 'cause you see this. Did you see any of these? This opposite happens to be here[points ;from WBBB to wwwb, THE INCORRECT OPposite]. Isn't that interesting? This is exactly one white and this is exactly three white-
153		S	But you know why.
154		CM	Why; that's very confusing
155		S	This is also three white. [pointing to group of three white both together and separated but also pointing to the group of one white and saying three white-imagining the oposite of those towers where there are 3 blacks would be white so that's why she is saying white to them]] This is also a three white, so these are all going to be the opposite.[identifies the two groups that will be opposites of each other without yet giving an example of how they are oppsotes of each other automatically]
156	28:39:00	CM	What you do you mean, these are all going to be the opposite? What opposite?
157		S	This is the opposite of this.[comparing WBWW and BWBB]
158		CM	Why does that happen?
159		S	Because you're using three whites, this one was the three white column [WBWW] but you're using three whites, this one was one white column but you're using three blacks [BWBB], so it's the opposite, theres your opposites
160		CM	Oh, so is that always going to work? So you're telling me when you're using exactly one white. All of the opposites of the exactly one white turn up where?
161		S	Here…[shows exactly three white group]
162		CM	In the exactly three whites. and that's because these are these opposites. Okay, so I understand how you got these opposites… the no whites with the four whites, and then one white with the three whites. All the opposites in here? for the exactly two writes.
163		S	I thought I saw these ones.
164		CM	All these are, these are exactly two whites. Do you see these opposites any place here? I do.

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165		S	Hm
166		CM	I do I see it here [in the same group]. S:here [shows WWBB BBWW] So that's that opposite. Is the opposite of this any place [BWWB]? S: here [SHOWS IT IN THE SAME GROUP]. That's the opposite of its very interesting. That's exactly two whites. That's very interesting. That's the opposite.
167		S	I wonder. Oh yea there [referring to the last pair that they are also opposites] I guess…
168	30:07:00	CM	You did convince me that everything has an opposite and the opposite of this one turned out here, the opposite of these turned out…[showing each group with each group that is an oppsite]
169		S	These turned out here.
170		CM	…of exactly one white turned out to be with exactly three whites. And the opposites of the exactly two whites
171		S	came out with the exactly two whites separated
172		CM	.Why? It did not come out with the two whites separated…S: oh no
173		CM	Look this is the opposite of these here, somewhere in the exactly two whites you found the opposite of exactly two whites, the exactly two blacks. Why would you find those here when you had to find these here and these here. I wonder why that would be…
174		S	Well that was the only one left.
175		CM	That's a reason. That's not a convincing one; I don't think Stephen would buy that.
176		S	Um…
177		CM	But is there any reason other than that that's true that we have to find them for because we used them all up is there another reason why it would make sense that they would have to be here and couldn't be somewhere else
178		S	Hm I guess. Like um. They have to be here they couldn't be someplace else. They have to be here they couldn't be somewhere else. I have a reason for that. Because if you're using the two, you cant put the two with the three.[referring to two white with three white group]
179		CM	So when you're using exactly two whites, how many blacks are you using?
180		S	Two.
181		CM	Why can't you use more than two blacks when you're using two blacks?
182		S	Because the assignment said to use four and you had to use another one.
183		CM	Now this is the big question. When you started you thought there would be twenty because you found ten by going through a certain plan and you said because of your opposites, that's the word you were using. but

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			now you went through this plan and you convinced me there are no more when there are no whites than this. I believe it, right? There are no more when there's exactly one white. You went through all your arguments where there's exactly two whites, three whites, four whites. How could you solve your problem when you only paying attention to the whites now. Not even worrying about opposites. Come up with sixteen and convince me there are no more. And when you did opposites you ended up with twenty. Do you think that's possible? Do you see how somebody might get confused about that? [S: mhmm] What do you think?
184		S	How could I explain? All right. You have your two [solid towers], right, so there's your opposites. you can't look for any more opposites, 'cause these are solid color. We're gonna say these are solid colors. These have the all black and all white. There's no more of these. Then you have this… now I convinced you that there is no more of these kind [referring to exactly one white]. And you look for the opposites.[finds the set of exactly three white and one black]
185		CM	But you didn't do these by finding opposites remember. You didn't convince me by opposite. Cause if you're gonna use an opposite argument with me, I would say how do you know there are more and more and more of them. you've convinced me because you said there's exactly three whites.
186		S	I convinced you that there were exactly three whites.
187		CM	It Just turned out that there was exactly three whites… that's not how you convinced me.
188		S	Hm I convinced you that there were no more of these [exactly three whites] but then when you put them together you find out that they are opposites and you know that you can't get any more of these.
189		CM	Ok.
190		S	Now that you can't get anymore. Then, you have, the two. And the two you have this one and this one
191		CM	And how did you convince me of the exactly twos
192		S	I convinced you that there exactly, I convinced you of the twos.
193		CM	Right
194	34:22:00	S	Now, these and these are opposites so you know you can't get any more of this kind.
195		CM	Yeh but that's not how you convinced me of more of these - oh I see you want me to accept that you already convinced me. okay why don't you go through that again so you know that Mrs. O'Brien just came in and she made one here, how you convinced me that there are no more than the exactly two whites.
196		S	I convinced you because when I took it, I would go and I compared it for this part of it I compared it. [refering to the alternating towers and

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			showing how white cube cant go anywhere else]. And I can make it this way this way or this way or this way.
197		CM	And how about this part of it.[referring to WWBB, BBWW, BWWB group]
198		S	This part of it. Well you know you can't make any more because when you have this down the bottom wait hold on, yeah this down the bottom, (mutters) if you have this down the bottom. if you move the two down the bottom then you need another one in the middle.[referring to WWBB but not clear what she needs, possibly another space in the tower to move the WW down]Ok. and you find the opposite of this one, so you know you can't make any more of that kind. You have the opposite of this one and the opposite of this one,
199		CM	And more Mrs. O'Brien again, could you tell her why the opposites were exactly two whites you would find the opposites for exactly two blacks in that group?
200		S	This group?[same exactly two group]. Because of what happens is well first of all there's no groups left. And if you um go and you turn this upside down, you have your pair which means if you turn it over again, you have the opposites.
201		CM	But you went from the exactly one white to find an opposite to find three whites, why didn't you go to a different group?
202		S	Well…
203		CM	suppose you started here and you realized there were none-no groups left. Why didn't you- if I said we didn't know this. let's say there was someone doing this for the first time, and they had looked for the opposites of the exactly two whites, they might not go to this same group; they might look for a different group. Why does it make sense, if it does, to find the opposite in this group?
204		S	So for the opposite [yawns] of this and this is in the same group. Because it just cause um cause you're using the same amount of blocks… lets say, ok, here you are using two blocks in the middle and two blocks [whites] separated. Here you are using two blocks in the middle, two blocks separated. And like here [goes to exactly one black group to show another example of a different group that wouldn't be the opppsite of the group being discussed] even if you didn't notice it until the end you're using three blocks and one block and down here you're using three blocks and one block [block is referring to white or black].
205		CM	Okay and now you said sixteen and you convinced me there are sixteen. How do you think… where do you think the twenty come from? Do you think there's sixteen or do you think they're twenty?
206	37:37:00	S	I think there's sixteen. Cause I've went over it and we checked and we made sure there's no duplicates…
207		CM	We did that with twenties and said "See I made these and they have all their opposites."
208		S	Well I think when I did the twenty I must've not checked for the um extras.

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209		CM	Could we have done this problem, instead of looking at exactly no whites and one white and two white and three white and four white, could we have done it by looking at exactly no blacks… and one black, and two blacks...
210		S	Mmhmm that would just be doing the opposite way! You could do that!
211		CM	Does it work?
212		S	Yep. you don't have to use the two whites together, you could go two blacks, but it's gonna be the same thing as using two whites.[shows a tower alterbnating color and refering to the black by pointing to the black cubes and then referring to the white by pointing to the white cubes]
213		CM	How would… sort of… could you… sort of talk about it… how you would explain to someone. Suppose you said to someone, Mrs. Barnes said to Stephen, "I want you to do the same thing Stephanie did, but I don't want you to go through the argument with exactly no whites, one white, two whites, three whites, but I want you to do it with blacks." What would you advise Stephen to do?
214		S	Do it the same way that I did it with whites because it's going to turn out the same way.
215		CM	So I could how would you start if I said exactly no blacks? What would you expect that Stephen would show you?
216		S	Exactly no blacks? He'd show me this.[all white tower]
217		CM	I see, I see… and exactly one black which group would he go to do you think?
218		S	He would show me one black, he would show me this.[exactly three white group]
219		CM	Which was what group for you?
220		S	This was three group.
221		CM	Exactly three whites… what about exactly two blacks? Which group would he be doing?
222		S	He would go to this group. This was my white group; my two white at the bottom.
223		CM	So was it the same group?
224		S	Actually he was just doing the opposite as me. I would put it in the same group.
225		CM	Okay that's very, very interesting. if you had to do another problem, you had two strategies, you had an opposite strategy that gave you twenty, you would expect that you missed something. You were really convinced of the sixteen. Cause you've really gone through all those cases. Are there any other possible cases? One white, two white, three white, four white?
226		S	You can use the black system, but other than that no.

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227		CM	That's a way of doing it but the opposite strategy as me but its that a good way of doing it. But the opposite strategy is that a good way of doing it?
228		S	The opposite strategy can work, but I think it's better to go back and make the opposites.
229		CM	Oh so it's a good thing to double check by finding that you have opposites. Once you've done this you go back… okay, that's very interesting… okay that's very interesting. So you have sixteen… why don't we put these all here for a minute the way we had them in the groups… I'm gonna move these here so we can get a picture of them. Now we had exactly no whites, how was this? …and where's the exactly one white that was this, exactly two whites here, exactly three whites here and exactly no blacks?
230		S	Exactly no blacks.
231	41:14:00	CM	Okay, You really think you feel comfortable with this argument? …and if I asked you -- which I may ask you to do because you've done it so nicely here even though you're tired -- I might ask you to share this with your class… would you be uncomfortable doing that?
232		S	No.
233		CM	And explaining it? So I… What I'm interested in now… you feel comfortable sharing that. I might ask you to come to my graduate class…. I might ask you to do that Stephanie. Do you think this argument would work in the same way, if you had to build towers of five now? I don't want you to do it I want you to talk about what kinds of cases you would have… here you have case one: no whites, case two: exactly one white. Case three: which you did three-a, two whites together… two whites separated by at least one black then you did case four: three whites, might have a separation.. In a sense these cases is a very powerful method of proof… do you know in very advanced math people prove things by cases. And it's a good proof if you consider all cases.
234		S	With this you could probably make an estimate of how many towers you make get with 5
235		CM	How would that work?
236		S	Well okay You know you've done this and you've checked it and you've found that you've got four, with groups of two you've got 3. With groups of- well 6. And with groups of um, but when you started out you got 3.
237		CM	That's interesting…
238		S	And that's how it would work
239		CM	i just noticed something , I just noticed something you've got one 1 4 6 4 1. Isn't that interesting.
240		S	Hm interesting it turns out to kind of like pattern
241		CM	A different kind of pattern huh

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242		S	When you started out you had had one wait you had uh four, you had um three, had two and you had one. And with that you could go and you could say well I'm gonna estimate if this is how many you have but for- with five I might go and say you can go and you can say you had one with the four. With five you had five. Well you'll probably get five.[estimating the 5-tall tower in group of exactly one white] That's what you get with the six that's the same thing.[comparing the 6-tall group which is 6 exactly one white] And with the five, here I might get one more than this [talking about the exactly two white group].
243		CM	Why do you think one more?
244		S	One more, because here you're gonna get one more [the exactly one white group you get one more]… you're gonna get one more because when we did the pattern with six you got one more than when you did the pattern with five
245		CM	What do you think you'd get with exactly one white what do you think you'd get with exactly three white? [trying to ask about a parallel group (opposite group)]
246		S	Not counting the opposites. Probably one white… probably another one.[she refers to one will be added to the exacty three group which she broke apart into two sets of towers: WWWB and WBWW & WWBW and BWWW – not sure why this break up because it should have been broken by together and apart]
247		CM	Probably? You're guessing one more.
248		S	Think this would be five, four, three, two. No… five, four, three, um, one… instead of four, three, two, one.[shows that the concept of opposite is not used, but rather a pattern of the total numbers that are consecutive that Stephanie notices without warrant as to why this might work. She had a warrant for the exactly one white, but then just continues a consecutive pattern because she thinks each group would have an extra one more.]
249		CM	That would be your guess before you start.
250		S	Yea that would be my guess.
251		CM	That's interesting… now let's go back here you think in this group when you're building towers of five you would get one more when you were building towers of 5. [talks to someone else on the side… third person comes in and says AA: "I'm not sure I'm not sure what's in common here.")
252		S	Oh wow I must've separated the cases wrong. Because these two are glued together -- the whites are glued together.
253	45:52:00	CM	You want this one in there too.[fixes her mistake, but S doesn't agree with it yet]
254		S	No that one goes with that. Oh this one doesn't belong…[notices herself the mistake]
255		CM	Those are interesting guesses and let's try not to lose those guesses. But before we do that, I'd like us to try something different. I'd like us to think

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			about something maybe that's not more. But I'd like to think about towers of three.
256		S	Towers of three, okay…
257		CM	And I'd like to think of it if you were using this strategy of towers of three would it work?
258		S	Yeah you might you'd probably get something like this you probably get something with three.
259		CM	Okay these you'd probably- so for towers of three.
260		S	You'd probably get something like three two one because you'd get less than tower four, you know that, so you get probably you're gonna get three here [exactly one white group] And then on the bottom [group of exactly three white] ,when you go for towers of three, you're gonna get one here. Which means you're probably gonna get two in the middle.[the exactly two white group]
261		CM	So how many total?
262		S	Three, two, one… um, six. And then when you do your opposites…
263		CM	But wait a minute… we didn't get extras by opposites before.
264		S	Yeah, that's right.
265		CM	We got extras by cases.
266		S	Just without the opposites, you'd get six.[not enough evidence that she is still not thinking that opposites might still be made]
267		CM	You think six. Okay why don't you show me?
268		S	Okay.
269		CM	Let's not lose these. Why don't we move these over here… and why don't you show me what would happen with towers of three.
270		S	Okay.
271		CM	I'd like you to set up a parallel argument if you don't mind. That's the way I'm thinking now.
272		S	Okay, we're gonna start with one white.
273		CM	No, you're gonna start with no whites.
274		S	Oh no whites. okay so that's three blacks. Then…
275		CM	Any other ways of doing this. (laughs) no.
276		S	No. (laughs) then, oh okay. I'm doing the wrong pattern…. (pause) I have to get the… I have to get it through my head.
277		CM	How many will there be? Don't make them; tell me before you make them.
278		S	Three![one white exactly]

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279		CM	You can make them now. Just wanted to help you…
280		S	Okay (pause) right there…
281		CM	You thought six… do you still think six?
282		S	No I think seven. I think it's gonna..
283		CM	Why do you think seven?
284		S	Because I didn't include the no whites. I forgot to include the no whites.
285		CM	You said in class one time there cant be an odd number. I -- that would mess up the whole opposite thinking. Still think seven?
286		S	Yeah, could be seven.
287		CM	So you think there doesn't always have to have an opposite.
288		S	Uh, yeah, you don't ***always*** have to have an opposite.
289		CM	So we did exactly one white now… what were we doing after exactly one white?
290	49:12:00	S	Exactly two whites.
291		CM	Exactly two whites?
292		S	I would go. [builds WWB, BWW first and then separately WBW later]
293		CM	Glued together ones first…
294		S	Then you have like that and there was my estimate of two…[for glued]
295		CM	Glued together
296		S	…and then you have three so far… you have one, two, three, four, five, six, seven. There's seven.
297		CM	But do the whites always have to be, glued together
298		S	Yeah you don't have to have the whites glued together, you can go like this…[builds WBW now]
299		CM	Is there any other way of not having the whites glued together?
300		S	Mm-mm.[no]
301		CM	So how many did you find?
302		S	Three… okay three and three is six. Eight.
303		CM	Oh. What do you think?
304		S	I think well... um I think. Well -- I think that. That's all you could make with that. That is might... I think that hm... I think that I found the opposite. I think that it's going to turn out the same way that it turned out here with the opposites. [pointing to the 4-talll towers]

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305	50:39:00	CM	What's that same way?
306		S	Well you know how we found the opposites from here to here?[pionting to exactly one white to the exactly three white]
307		CM	Yeah from the… it's exactly one white…to the exactly…[pointing to the 3tall exactly one white]
308		S	Oh exactly two white…[showing the opposites as she speaks]
309		CM	And in this case exactly three white[referring to 4-tall]. The exactly one white The opposite turns out to be in the exactly two white… I wonder why that is? You think that's accidental?
310		S	I don't know.
311		CM	It's very interesting.
312		S	Yeah. And then these two opposites so there you go from sixteen to eight.
313		CM	No you went from six to…
314		S	To seven to eight, yeah…
315		CM	What made you change your mind from the six to the seven again?
316		S	I didn't count the no whites
317		CM	What made you change your mind from the seven to the eight?
318		S	I saw that I forgot about this one… the one with the two separate…[whites]
319		CM	Okay, let's put these back from the way you found them.
320		S	Okay, um well they all go down here so it doesn't matter.[she tries to put the one white and two white groupstgoether similary to the way she did with 4tall possibly not realizing there is amissing case because of the missing height]
321		CM	No because if it's exactly no whites, then it was…
322		S	oh, it was exactly one white…
323		CM	Make sure you have that so Mrs. Austin and Mrs. O'Brien doesn't say there's not more and they can look and be convinced.
324		S	There is exactly one black here.
325		CM	You sure you have them all?
326		S	Yes.
327		CM	And here you are sure you have them all?
328		S	Mm-hmm.
329	53:25	CM	And you did them together and you said you did them separated as you chose to do okay, now exactly three. Oh what the heck let's do exactly...

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			exactly… towers of two. I'm gonna drive Mrs. Demi crazy here with the camera but we'll do that anyway. Exactly towers of two. Same same same way of thinking… same argument. This is a neat way of proof. Exactly no whites [she says after steph builds BB]. How many? would you have guessed they would all be four without doing it?
330		S	I would have guessed that, not exactly four, but probably around it.
331		CM	Okay.
332		S	Because you know it has to be less than eight.
333		CM	Towers of one. Don't make them, guess.
334		S	Towers of one… one!
335		CM	You think there's one way of doing towers of one… okay, let's do towers of one, exactly no whites.
336		S	Actually… there's… okay… there's two. [builds it]
337		CM	Okay let's see then we have exactly no whites. Towers of one. Would be a good idea to write some of this down. okay so we have towers of one, two, three, and four.
338		S	One, two…[she is writing on pice of paper 1. 2. 3. 4. With large spaces and putting in the totals for each height after]
339		CM	Leave some room cause we're gonna go to five…
340		S	…three, four…
341		CM	It's all right. you found how many total?
342		S	Two.
343		CM	And here?
344		S	Four.
345		CM	Okay.
346		S	Eight.
347		CM	…and here.
348		S	Sixteen.
349		CM	Interesting.
350		S	That's weird… look, two times two is four, and four times two is eight, and eight times two is sixteen.
351		CM	So what?
352		S	It goes like in a pattern. two times two is four. and four times four is eight and eight times eight is sixteen.
353		CM	Oh, I wonder why?

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354		S	Well it also turns out that every number is even.
355		CM	Well if this is pattern what would you guess would be with towers of five?
356	55:08:00	S	If I had to guess? Thirty-two.
357	56:18	CM	You would guess thirty-two.
358		S	I would guess thirty-two.
359		CM	Gee, if this works you'd be able to do this pretty fast. Is there a reason why that would work?
360		S	I remember when we did the towers of five in class. It was 32.
361		CM	I wonder why this works. I wonder why this works. Let's look at what we have here. Let's kind of think about what happens when we build towers of one and towers of two. Why do we have twice as many?
362		S	Because you doubled it.
363		CM	What would Mrs. Alston will suggest… let's pretend these are the towers of one… now suppose I start with this and now build towers of two. Why would I expect four rather than two? Why? Now I'm building, starting with my tower of one, which has white on the bottom floor.[s:mm] Why might it be now- Suppose I start with white on the bottom floor. What kind of towers can I build on the bottom floor?
364		S	With one?
365		CM	With two.[s: with two?]
366		CM	We know there's only one I could build with white on the bottom floor with one
367		S	You can build two.
368		Cm:	Why? What could I build?
369		s	You can build the solid white.
		cm	With white on the bottom floor you could do this, right? [referring to the solid white that steph built]…
		s	You could build this [solid] or you could build this.[picks up BW]
370		CM	Ok so, I could either put white on the top floor or black on the top floor, if white is on the bottom floor. Now white doesn't have to be on the bottom floor.
371		S	It could be on the top floor. It could be on the top floor.
372		CM	Right and you told me with white on the bottom floor you could either put a black on the top of it or another white on top of it and there's two of them. but do I have to have only white on the bottom floor?
373		S	No black could be on the bottom floor and theres two
374		CM	And How could I build it with black on the bottom floor?

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375		S	Um, this way? [shows Wb]
376		CM	I could put white on the top floor…
377		S	and this way.[shows BB]
378		CM	That's four. well that helps me see how I could go from towers of two to towers of four. What if I'm going from the top floor… we have twice as many… how could we get twice as many if we start with these towers? We have these four now…[stands up the two tall towers]
379		S	Okay now we're going from here to here… okay. With white on the bottom you have this one and you have one, two, three, four [puts all towers with white on the bottom but not accounting for second floor color, so she gets four of them]… and these all have white on the bottom… and with black on the bottom you have four. [the other four towers not accounting for the second floor]
380		CM	Yeah, but… sort of we are not paying attentin to the second and third floors..how do I know I have the possibilities of that .
381	58:40:00	S	You know what I'm saying, look: with white on the bottom floor, these equal two, but…[one tall] and with white on the bottom floor, you have two. The total here is four. Now the total here is four and you get four. [two tall][she is implying there is a number pattern, from 2 total of 1-tall you got 2 from each, from 4 total of 2-tall you should get 4 from each which she does based on the way she arranged them]
382		CM	Yeah but I could have said in these here [2-tall] you could either have white on the bottom floor or black or white on the top floor or black. It made sense to think of white on the bottom floor or black on the bottom floor because you only started with these two I think.
383		S	Yeah.[incomplete conversation; there is silence for a few seconds and then carolyn moves on to the next question]
384		CM	Do these problems remind you of any other kinds of problems you've ever seen?
385		S	Um these kinds of problems… hm… well, only the one that we did when last time when you came with the um, five.
386		CM	The five.
387		S	Um. usually the problems remind me of the shirts and shorts
388		CM	Right.
389		S	Usually they remind me but this one doesn't not really usually this one reminds me of the shirts and shorts but this one doesn't.
390		CM	What way does it usually remind me of shirts and shorts?
391		S	Well usually you have to pair up.
392		CM	Can you imagine if these could be shirts and pants of some sort an outfit or combination?[shows the two-tall towers]

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393		S	You might be able to… it depends on if you might make it into a problem. You could say Jim has a pair of black pants and a pair of white pants, or white top and a black top…how many outfits can he make? And you can say four.
394		CM	How would you make four?
395		S	Black with black, and white with white, and white with black, and black with white.[pointing to the towers]
396	1:00:57	CM	Is there any way you could show me with a picture or anything?
397		S	Mhm. (draws shirts and pants) Black with black, and white with white, and white with black, and black with white.
398		CM	Okay in a sense this is like the shirts and pants… the building of towers with two… Could you imagine this with shirt and pants?
399		S	Yeah you could go Jim has a black pair of pants and a white top. How many outfits can he make?
400		CM	Well if he doesn't have shirts and pants here he just has…
401		S	You could say well, you could go like this. Jim has a black pair of pants, a white pair of pants, and oh wow it doesn't matter. And a white shirt and then it would make two but…
402		CM	Yeah here he has two articles of clothing, right? A shirt and a pants, here… he only has one article of clothing… yeah, it's a little hard to see…
403		S	Yeah it's a little harder to work with two pieces of clothing…
404		CM	He just has the pants here…
405		S	Yeah
406		CM	Maybe it's summer he walks around without a shirt well Okay it seems like. This really does seem like a outfit problem of 2. Is there any way we could think of the outfits problem with another piece of clothing let's go back to this. Two shirts two pants. Now suppose we had two hats. Black hat and a white hat.
407		S	Okay.
408		CM	Maybe I should have asked you to do it all on another piece of paper. We already did this; we have two hats, two shirts, two pants.
409		S	Alright
410		CM	Black hat, white hat. Learn How many combinations do we have? (long pause)
411		S	Alright so you have white white black black, then you could go black black black, white white white, black white black, that's three, black white white, that's four, black black white, that's five, white white black, that's six, and then you could go do that (white black black) do that… all right makes six.

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412		CM	Want to write out the letters? you've lost me kind of show me as you're doing it
413		S	White hat, white pants. Oh perfect I put white blue.
414		CM	Remember you already have these outfits here. So you think when you have the white outfits here…
415		S	You get two more outfits.
416		S	Black pants black shirt. Black pants black hat black hat white shirt. Black pants white shirt white hat. Black pants white shirt black hat. Wait I did that one. Then you could move on to the what w t b pants. White shirt black hat black pants. No that's the same made that. Black pants. White pants black shirt. White hat. White pants black shirt black hat. And that one the same one that one must be my favorite I keep making it up. One, two, three, four, five, six, seven. Am I sure I did this? I didn't do that one already? No. hm.
417	1:07:23	CM	And you're not satisfied with that? What bothers you about it?
418		s	Here I got six but here I got seven and I'm not sure I think I made one over but I didn't.
419		Cm	Is it possible to have seven?
420		s	No.
421		Cm	Why?
422		s	Because you'd be walking around with a hat and a shirt or a hat and a pants or pants and a shirt.
423		cm	Okay so what do you think?
424		S	I think I messed up someplace.
425		CM	Let's think here, here are your outfits.
426		S	Okay.
427		CM	You made them and you showed me. Here there were four. You were sure of that. And it's like the towers. You believed it. Now here I am. Now imagine you've come home with two really smashing hats. A black one and a white one. What could you do with these four outfits when you have these two hats?
428		S	What can I do with these outfits when you only have four hats?
429		CM	Cause now you have three articles of clothing.
430		S	Combinations… oh, I think you'd still have six.
431		CM	Okay, your black hat.
432		S	My black hat could go here. Could go here. That's eight… no, that's four.
433		CM	Okay keep thinking.

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434		S	Okay then...
435		CM	You've got four more with your black hat. But You came home with two hats.
436		S	That's it you'd have white hat, white hat, white hat, white hat.
437		CM	Well now it's not seven, and it's not six – it's eight.
438		S	Its eight.
439		CM	Well, how about that… here's some towers and now you're making outfits with three articles of clothing. Is there any similarity to this?
440		S	Is there any similarity from these and these?
441		CM	Now you've told me now with three articles of clothing. Black shirt, white pants. You could now make eight outfits. And you're absolutely convinced. Cause you started with the four combinations. And you told me you could put a black hat on top or a white hat on top. Now if I asked you to make them to produce the eight I bet you could make them really easily but you could kind of imagine what they look like, okay like...
442		S	They would look like this.
443		CM	Right.
444		S	And they would look like this one. And they would look like this one.
445		CM	But wait a minute what color hat are you using?
446		S	Oh we're using black hat. This one and this one. This one. And …
447	1:10:49	CM	White on the bottom floor. Black on the middle.
448		S	Yeah this one.
449		CM	You could also give me the other outfits…
450		S	This one. This one. This one does go here. This one. This one. Where'd my last one go?
451		CM	Two whites on the bottom?
452		S	That one must go there.
453		CM	First two floors. Now isn't that interesting?
454		S	Yeah.
455		CM	Now if I gave you two feathers: a black feather and a white feather…
456		S	Ok. then. we go to the fourth blocks
457	1:11:32	CM	Why?
458		S	Because we're adding another piece of clothing.

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459		CM	Lets talk about it.
460		S	Oops… black feathers on the very top…
461		CM	You don't have to do it; you could just tell me what you would do.
462		S	Okay well. Okay.
463		CM	now we have all of these. The w hats on the top of all those outfits. Right. Ok.
464	1:12:05	S	Now we have black hat on top.
465		CM	On top of what where would you be putting the black hats on top of?
466		S	Black hats on top of these?
467		CM	Mhm
468		S	So all of these would now have a black feather on top, that's what you're telling me?
469		CM	Not the right order. I was wondering why I couldn't find the rest.
470		S	I was wondering why I could find it.
471		CM	Started to worry right
472		S	Yea.h.
473		CM	You lose it?
474		S	I'm not sure? Oh.
475		CM	Okay so how many did we get when you put the black on these very outfits?
476		S	You have eight.
477		CM	Now we put white on the very top…
478		S	That's….
479		CM	Okay, you have a white on the very top…
480		S	So. That's sixteen…
481		CM	Does that make sense?
482		S	Wait. We have 9.
483		CM	Ok. These are the ones with the whites on the very top. Let's flatten them out because [inaudible].
484		S	They all they're not the same because does that make you feel better. I was getting worried there too Stephanie. Now I don't want you to do this. I know you're getting tired and I want you to go back. These are your shirts, pants, hats, feathers. And now we're going to have a flower. Black

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			and white flower to go on top or some kind of design (laughs) Without doing it, tell me how many combinations do you get?
485		CM	I would say okay if you get ..probably… twenty?
486		S	Now you have sixteen outfits. Now you're going to put on top of this either a black feather or white flower. Or decoration.
487		CM	Instead of making the towers.
488		S	I'm not even asking you to make them suppose you put a black.
489		CM	On top.
490		S	Ok
491		CM	Just tell me
492		S	What's gonna happen
493		CM	Okay youre gonna get black on these…
494		S	Yeah ok.
495		CM	Do you have black on?
496		S	Okay so You're going to get 32. So it is sort of like the shirts and shorts problem. You could figure out. That's what we got when we multiplied and when we did the problem the last time.
497		CM	What did you have?
498		S	A method.
499		CM	What's the method?
500	1:17:04 (WV)	S	All you have to do is take the last number you had and multiply by two.
501		CM	You're convinced it's always going to work? What I would like you to do is… you've done extraordinary work… and you have really today come up with another method. You have one method you made from last time. Now so if I were to say to you if you were building towers of six, what would you say to me?
502		S	Okay towers of six and towers of five. Thirty-two. Towers of six that's um… sixty-four.
503		CM	Sixty-four, right. And You also have a method of finding also sixty-four… what was it?
504		S	Multiply the last one by two.
505		CM	That was the answer but how would you show me What was your strategy?
506		S	Shirts and shorts problem.
507		CM	That's another way but How would remember when you did towers of four.

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508		S	Um You would go line by line
509		CM	Well you said there were first first case. First cases
510		S	I think there were four in the first case. Four.
511		CM	One in the first case
512		S	Youre talking about the black one.
513		CM	You used all those cases -- one whites, two whites –up to how many?
514	1:17:39	S	With towers of six you could go up to – five, four, six -- to sixty-four.
515		S orcm??	You'd go to 6
516		CM	Exactly 6 w. What id want you to do and You've been wonderful and you've had so little sleep. I just wish you could write me sort of a story about if you were gonna be the teacher or you were gonna explain to a friend. Hey look if were doing these problems Amy might come in again. She may say okay everybody were going to be doing Towers of Six. You'd have all this information and it sort of isn't fair. I'd love if you could do that for me and we'd have to plan some time so you could share it. You just were wonderful. I'd like you to talk to Michelle Ask her say to her tell me Michelle how you did this. I'd like you to come in and tell me how many towers you could build with exactly 6 high that are exactly two colors. How many towers could you build that are exactly 10 high. With exactly two colors. You think you'd know how to do that.
517		s	Yeah
518		cm	Yeah I think you could too.
519		s	All you'd have to do then is we know we have towers of six times it six, seven, eight, nine… actually six, seven, eight, nine, ten… that's four. By four, not by four, by eight because what happens is you multiply by two.
520		S	What would multiply by two what would multiplying by two give you?
521		CM	It would give you.
522		S	Or you could just go like this 64 times two equals um. Then you could just go that would be the seven towers. sixteen one four five two eight that's the eight towers. nine towers. eight towers. Thats twelve one ten eleven one four five five five twelve is the nine towers eighteen nineteen. That's the ten towers. [CM: how many?] one thousand. Wait huh. [cm: Check this]
523		S	That's four, that's two, that's ten one goes on top
524		cm	You wrote one-nine I don't know why
525		s	Yeh but that's ten.
526		S	Oh, that's why, okay so there is 1,024.

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527		CM	Would that be same as you were gonna do multiplying by eight?
528		S	No that's fifty-one – 512, which Gave me eight towers.
529		CM	Wonder why it didn't work multiplying towers by eight. Wonder why that was. The nine towers. Why do you think so?
530		S	I think I it was actually easier going like this trying to figure out
531		CM	I think you had a good idea.
532		S	You had to be sure if you doubled the nmber , tripled the number, that it would absolutely work.
533		CM	What I'd like you to think about I think the idea was very good. You multiplied it by eight. But notice what you did here. You multiplied it by eight. You didn't multiply it by eight. You multiplied it two times two times two times two.
534		S	That's twelve – sixteen..
535		CM	So if you had multiplied by sixteen it would've worked.another two.
536		CM	You gotta think about it why the idea didn't work but almost work. It's nice to have shortcuts if you're not sure they're gonna work then wow. Did you think you learned anything today
537		s	I learned those methods
538		cm	People would love to have those methods. Well again I say thank you I would like to come back and have your essay.
539		S	Okay
540		Cm	I would love to see, if you go back to those towers of six and you could prove all the cases there should be how many
541		S	64
542	1:25:08(wv)	cm	And convince your classmates that you have not left any out because left any out duplicates if you catch them you're okay but if you don't catch them. That's wonderful.