RUTGERS UNIVERSITY

DISSERTATION

Accounting Information, Conservatism, Rationally Inattentive Investors, and Market Valuation

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Abstract

My dissertation focuses on relevance of accounting information and models of rational inattention. Rational inattention suggests investors have limited attention spans and need to decide how to allocate their time. In the first part of my dissertation, I suggest that investor inattention provides an indirect incentive to managers for increasing relevance of accounting information. The second part suggests that managers can affect relevance of accounting information through accounting conservatism. In the context of rational inattention, I prove that accounting information more relevant for predicting future earnings. In the third part, I look at how differences in information processing costs of investors in a rational inattention setting impacts information captured by price and I use this to offer an explanation for price movements following financial disclosures and announcements.

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Chapter 1

Literature Review

The usefulness of accounting information has been widely discussed in prior literature. Issues such as the explanation of how accounting choice is related to firms' corporate governance, corporate decisions, agency problems, market valuation and so on have been addressed. This paper argues that the relevance of accounting information should be discussed together with agency problems and corporate decisions. The former can somehow be controlled by the manager's discretion which is generally influenced by the managers' corporate decisions and the agency problems are related to the way in which managers achieve benefits from the contract. The prior literature on the relevance of accounting information and the factors that could affect the role of accounting information in the market will be reviewed in this section.

1.1 Relevance of Accounting Information

Value relevance refers to the ability of information disclosed in a financial statements to capture or summarize the share values of firms (Francis and Schipper, 1999, p.326). According to its definition, relevance of accounting information is determined not only by the magnitude of the information but also by the type of the information. In general, markets will be more likely to react to the information which is of high quality, timely, and meaningful.

Ball and Brown (1968) argue that accounting numbers are aggregated

and thus could be "meaningless". Additionally, accounting numbers are not objective, and could be affected by manager's discretion. Thus it is hard to guarantee the quality of the accounting information all the time. Furthermore, the lag in accounting report will cause the accounting numbers to be less timely. In turn, this will make firm value estimates less timely. However, Ball and Brown (1968) and Beaver (1968) find evidence in their study that accounting numbers could indeed provide information to the market. More studies have been done on the relevance of accounting information during the past three decades (e.g. Bowen et al., 1987; Dechow, 1994; Cheng et al., 1996).

Barth, Li and McClure (2017) recently find that "two new economy-related amounts-recognized intangible assets and research and development expense-increase in value relevance, whereas dividends and, most prominently, earnings decrease."

In theoretical accounting theory, Ohlson (1995) provides a linear relationship between market price and accounting numbers, such as excess earnings and firm's book value.

Besides the positive views of accounting relevance, there is a large literature documenting a decline in earnings' value relevance. It broadly offers two primary explanations for the decline.

The first is the rise of the new economy in which future earnings largely depend on investments in intangible assets and traders will no longer trade based on fundamental information. Lev and Zarowin (1999) find a weaker association between price and earnings for firms with more intangible assets and attribute it to the time mismatch between expenses and revenues associated with such assets.

The second explanation is the presence of more loss firms. Hayn (1995) and Collins, Pincus, and Xie (1999) find earnings are less relevant for loss firms, and Barth, Beaver, and Landsman (1998) find the relevance of net income (equity book value) decreases (increases) as a firm's financial health decreases.

Collins et al. (1997) and Francis and Schipper (1999) find the decline of value relevance in earnings can be offset by increase in relevance of book values. Brown, Lo, and Lys (1999) also find a(n) decline (increase) in value relevance of earnings (equity book value) from the late 1950s to the late 1990s. This is because equity book value can predict future normal earnings and reflect loss firms' abandonment option (Barth et al. 1998a; Collins et al. 1999).

The relevance of accounting information will be affected by other factors too. Therefore, it is necessary to review the possible factors that may influence the relevance of accounting numbers specifically.

1.2 Contracting Costs

Although accounting reporting lags make accounting information less timely, Ball and Brown (1968) and Beaver (1968) provide evidence to show that accounting numbers indeed provide information to the market. However, prior literature has found it difficult to give out an exact hypothesis to explain how accounting choice can affect firm value. Watts and Zimmerman (1990) argue that one possible reason why prior literature fails to find significant relevance of accounting information is the framework which suggests information is costless. However there are a variety of costs such as transaction costs, agency costs, political costs and negotiation costs, which can all be defined as contracting costs. In the presence of such contracting costs, the relevance of accounting information could decrease.

Positive accounting theory posits that the use of accounting information can enhance the efficiency of contracting by lowering agency costs as proposed by Jensen and Meckling (1976). In their view, accounting numbers provide a standard to measure the manager's performance and can serve as a signal to reflect the manager's possible hidden actions which reduce agency costs. Thus, with incentive alignment, accounting information can be treated as a proper proxy for manager's behavior. Once this behavior is associated with the firm's performance, by the theory of Jensen and Meckling (1976), the agency costs will be related to outside financing. Therefore, it is reasonable to say that there is indirect relevance of accounting information.

However, when the accounting information is used for performance evaluation, incentives for earnings management arise (Burgstahler and Dichev, 1997; Graham et al., 2005). Therefore, if the agency costs are increased for earnings management, the relevance of accounting information could be undermined. Though auditing can reduce this problem to some extent, the manager's information advantage over auditors will prevent the problem being eliminated completely. Ronen and Yaari (2008) define the above discretion used by managers for their self-interest as opportunistic earnings management, and define discretion which helps to "establish rapport with owners by signaling value relevant information without getting into too many cumbersome details" as value-enhancing earnings management. By definition, value-enhancing earnings management can help investors generate a more accurate prediction for firm's future performance based on accounting information. Therefore, if there is a contract or accounting policy which can make managers' interest fully consistent with investors' interest, value-enhancing earnings management will not conflict with opportunistic earnings management and increase the usefulness of accounting information.

1.3 Accounting Quality and Voluntary

Disclosure

Similar to earnings management, the manager's accounting choice for accounting quality can also affect the relevance of accounting information. If investors rely on accounting information for pricing of shares, they will reward the companies which provide information of good quality with a lower cost of capital. This is because accounting information with higher quality can not only reduce investor's information costs, but also improve their position in terms of their investment risk management.

Schipper and Vincent (2003) suggest there are three aspects which can be used to measure the quality of accounting information. The first one is the sustainability of accounting information, which means the accounting information is more permanent and less transitory. As discussed earlier, reporting lag could make accounting information less timely; however, if accounting information is sustainable, historical accounting information will become useful for current market valuation. For example, investors view a highly persistent earnings number to be sustainable. Specifically, Lipe (1990) define earnings persistence as the autocorrelation in earnings regardless of the magnitude and sign of an earnings innovation. It is shown that the persistence of reported earnings is associated with larger investor responses to reported earnings (e.g., Kormendi and Lipe 1987).

The second one is the predictability of accounting information. Schipper and Vincent (2003) point out:

"The FASB's Concepts Statement No. 2 (para. 53) refers to predictive ability as an input to an unspecified predictive process. Predictive ability is the capacity of the entire financial reporting package, including earnings components and other disaggregation of the summary earnings number, for improving users' abilities to forecast items of interest. Viewed this way, predictive ability is linked to decision usefulness and is therefore idiosyncratic to a given user's particular prediction process and goal. Researchers, however, sometimes refer to predictive ability specifically as 'the ability of past earnings to predict future earnings' (Lipe 1990)."

Therefore, it is reasonable to argue that the higher the predictability of accounting information, the more relevant the accounting information is to the market. Also, if managers have incentives to increase the accounting quality, they will employ an optimal accounting choice that can increase the predictability of accounting information. In other words, manager's discretion which results in an accurate prediction of accounting numbers can enhance the relevance of accounting information. This is like the value-enhancing earnings management which results in lower contracting costs. More interestingly, earnings that are of high persistence may be have low ability for prediction purposes (Schipper and Vincent, 2003). This is because prediction requires abnormal accounting numbers (predicted number minus the reported number) to be transitory, while persistency requires the shock or abnormal part to be permanent. In the long run, persistence of accounting information will result in a decrease in its predictive power. This fact is also proved later by the example that conservatism which will result in a higher accuracy of earnings prediction, will make earnings less persistent. Furthermore, this paper will later prove the contradiction can be compromised by adopting proper discretion based on certain accounting policy.

The third one is the variability of accounting information. Levitt (1998) suggests that managers tend to smooth earnings because they believe investors prefer smoothly increasing earnings. Moreover, smoothened accounting numbers can decrease the time-series variability and increase prediction power. This seems to be another strategy to increase the predictability of the accounting information. However, it sacrifices firm's long-term value (Graham et al., 2005) Also, it is not a promising one as the results got by Leuz et al. (2003) suggest smoothened earnings are less informative.

To reveal how accounting quality can affect the relevance of accounting information, predictability and persistence of accounting numbers should be included in discussing manager's discretionary accounting choice and disclosure policy.

If the managers are myopic, they would have incentives to hide bad news. However, information disclosure itself could serve as a signal for the investors to tell the type of the news. If managers are confident that the firm will perform consistently well in the future, voluntary disclosure will be a good choice for them; otherwise, it will do no good for the manger's short-term benefits (Verrecchia 1983). Moreover, if the quality of the information, which is known by investors, is higher, the penalty for not disclosing will be also higher (Verrecchia 1989). Thus for voluntary disclosure, there is a tradeoff that managers have to face in choosing the accounting quality when contracting costs are considered.

1.4 Conservatism

Accounting policy plays an important role in making choices to record accounting data. Conservatism, one of the most popular topics discussed in the prior literature, is used to determine effect of accounting choice on market value (Ross L. Watts 2003).

Bliss (1924) defines conservatism as "anticipate no profit but anticipate all loss", while, Basu (1997), interprets conservatism as representing "the accountant's tendency to require a higher degree of verification to recognize good news as gains than to recognize bad news as losses", and measures it as the asymmetric timeliness of earnings where earnings reflects bad news in a timelier way compared to good news. In other words, market reaction for marginal increase in earnings under good news will be higher than market reaction for marginal decrease in earnings under bad news.

The findings seem to suggest the relevance of accounting information will be greater for good news compared to bad news under conservatism.

The indirect impact of conservatism on accounting relevance has been studied using the cost of capital. It is believed that accounting conservatism can mitigate the information asymmetry between investors and managers and decrease the cost of capital. For example, LaFond and Watts (2008) argue that information asymmetry between equity insiders and outsiders will induce conservatism in financial statements because it can reduce the manager's incentives to manipulate accounting numbers and therefore reduce the information asymmetry. Lara, Osma, and Penalva (2010) also suggest conservatism can reduce information asymmetry, and thereby lower the firm's cost of equity.

Moreover, conservatism also has two other indirect influences on the relevance of accounting. Hui, Matsunaga and Morse (2009) suggest conservatism has a negative relationship with mandatory earnings forecast. Lafond and Roychowdhury (2007) also find that the decline of managerial ownership will increase the demand for conservatism.

1.5 Inattention

The prior literature indicates that contracting cost is one of reasons for the low explanatory power of accounting information. Conlisk (1988) assumes information processing is "costly". The higher the information costs, the less likely for investors to collect more information to generate their prediction (information processing costs) even if they know that the accuracy of their prediction can be higher with enough observations. From this perspective, Conlisk (1988), measures such information processing costs by time. Then, investors will optimally choose the time which can minimize the loss function, measured as the possible loss caused by the mispricing from their inaccurate prediction. Thus, the timing for entering the market can be understood as the best position for investors who suffer a tradeoff between increasing accuracy of prediction and lowering information processing costs.

Sim (2003) renders a similar mathematical structure which also results in bounded rationality and uses it in the context of investors' rational inattention to explain why investors sometimes underreact to some news.

In general, because of the optimization costs or inattention, investors want to make their decisions within a limited time and are very likely to accept the information which is accurate and simple enough for quickly understanding their prospects.

Accounting information, which becomes less meaningful as it is a series of aggregated numbers, can lower investors' attention and time for analysis. Accounting information can be useful for investors and thus more relevant.

Chapter 2

Model

2.1 Introduction

There has been much debate in the past few decades about the usefulness of accounting disclosures for asset pricing. Despite the fact that disclosures are widely used by the market, there is little statistical evidence about the explanatory power of accounting numbers.

The purpose of this paper is to establish a theoretical construction to address whether accounting disclosures can be useful for rational investors to arrive at the correct market prices. Though there have been various explanations to interpret the managers' intentions for disclosures, this paper argues that the real reason for managers to publish accounting disclosures should be that they hope their firms could be assessed and treated "properly" (such as the "right" cost of capital).

Since investors' attention is assumed to be limited, the cost of capital would be higher under information asymmetry. From the perspective of rational behavior of inattention, collecting non-financial information, which takes time and effort, will be costly for investors and will result in bounded rationality for investors. In other words, investors will only collect limited samples from the whole population because of information processing costs. Moreover, because time is needed to collect information, bounded rationality caused by inattention may delay time taken by investors to enter the market which increases the cost of capital.

The above assertions have been proved by the rational inattention model constructed by Conlisk (1987). It also suggests that truthful and accurate public disclosures (for example, accounting and financial disclosures) can attract investors to enter the market earlier, provided public disclosures are relatively costless for investors. Such disclosures satisfy the three apsects laid out Schipper Vincent (2003) and allow investors to better predict future firm performance. In addition, under bounded rationality, information bias is acceptable for investors as long as the information can be provided with more accuracy. That is to say, in order to lower the cost of capital, the manager does not have to provide unbiased financial information. He/she only needs to provide financial information which is more accurate compared to nonfinancial information. From this perspective, we can say the usefulness of accounting information is only a relative concept.

Based on prior literature, this paper provides two channels mangers use to reduce the information asymmetry in their disclosures. First, disclosing managerial earnings forecast is a good choice. According to Verrecchia (1983), because managers have incentives to hide bad news, no disclosure itself can signal the type of the news that managers are currently holding. Additionally, Verrecchia (1989) also suggests that the more the information withheld by the manager, the more likely it is that the market will discount the value of the asset. Therefore, there is a tradeoff between disclosing accurate information and the manager's self interest.

Since this paper focuses only on the usefulness of accounting information, the discussion of the optimal policy of disclosing managerial earnings forecasts will be excluded. This paper argues that accounting information provides a better solution than managerial earnings forecasts from the manager's side because accounting reporting lag itself serves as a signal for investors to interpret that the firm has some hidden information. Thus, there will be no penalty for the potential bias generated by the accounting information subject to managerial discretion at the beginning of the accounting period. On the contrary, even though investors know mangers could be myopic and likely to withhold bad news, a lack of signals will certainly reduce investors' ability to infer the type of information that managers are currently holding. This leads the market to react inefficiently (under-react) till investors fully learn the type of the news until the next period. Therefore, the question which arises is: how to make accounting information more accurate for predicting the firm's future performance?

The literature on earnings management suggests that the usefulness of accounting information is largely influenced by manager's discretion. Therefore, it is reasonable to conjecture that the manager's appropriate discretion can make investors better informed.

A more detailed model is established in this paper to explain how the manager could use his or her discretion to lower the prediction bias in value-relevant information. This does not require investors to exert effort (attention) on any cumbersome details such as sample collection. The value-relevant information refers to accounting information which can reflect the manager's decision on how to optimally allocate resource and maximize the firm's future performance (earnings). It is assumed in this paper that the manager has fully captured the future private information and is risk-neutral. Therefore, there will be no agency problem because of which the manager's real operating decisions influence his or her disclosure decision under information asymmetry. So, investors can partly infer the future earnings from value-relevant information. This strategy, by examination, indeed reduces the information asymmetry between managers and investors, though it is quite possible that it results in an underestimation of future earnings prediction over time. It is generally assumed that the manger knows investors' prediction strategy and their prediction bias. Thus, theoretically, managers find it easy to influence the investors' prediction bias by exerting their discretion.

It is noteworthy that to attract investors to enter the market immediately after the public accounting information is released does need the prediction to be fully unbiased. Without further information, the sign of bias is not informative to investors. To know the managers' discretion, I propose that proper accounting policy or incentive contracts providing additional information for investors to further evaluate the accounting information should be released as managers can use their discretion for their self-interest. For example, if the managers are given less equity based incentives and more debt based incentives, they are likely to be conservative. This is to satisfy the demand of the bond holder. This implies that managers will issue conservative estimates of future earnings and triger a positive market reaction when investors later find out that the managers' disclosures underestimated future earnings. Conversely, if managers are given more equity based incentive, they will tend to overstate future firm performance and understate bad news. Consequently, a negative market reactions will occur when the bad news becomes public in the future. In the former case, the investors will underestimate (negative bias) initially, and the opposite will occur in the latter case.

In sum, this paper claims that accounting information can be useful under the investor inattention hypothesis and makes at least five contributions to the accounting theory.

First, this paper further explains how investors' inattention could result in a market underreaction to earnings announcement, which cannot be explained by efficient market hypothesis. Moreover, investors' inattention boosts the manager's incentives to increase the accuracy of public disclosure.

Second, this paper provides a possible explanation for the relation between accounting policy and manager's incentive contracts. This is consistent with the empirical results found in the prior literature for positive accounting which indicates that accounting conservatism is negatively related with the manager's equity based incentives as well as the manager's equity ownership.

Third, the result shows that managers with more debt based incentives are more likely to restrict the standards for recognizing persistent good news and loosen the standards for recognizing persistent bad news. This tends to make the investors believe that current information is more likely to reflect bad news rather than good news. Thus, at the beginning of the accounting period, the prediction for future earnings is more likely to be underestimated. On the earnings announcement date, it is possible that the prediction is higher if earnings are higher than the prior prediction. Then, the market will recognize such persistent good news and react positively to this earnings announcement. The magnitude of earnings response coefficient under accounting conservatism is consistent with Basu(1997)'s empirical findings.

Fourthly, this paper provides an additional explanation for why conservatism can reduce information asymmetry. In contrast to the prior literature which suggests the reason that conservatism can reduce information asymmetry is that it can limit the manger's incentive to manipulate earnings, my findings suggest that conservatism is the consequence of managers applying their discretion negatively (such as deflating the current asset or recognizing more liability than assets). Though information asymmetry will induce the use of accounting conservatism, it does not totally depend on manager's incentives.

Lastly, Watts (2003 II) argue that although "earnings management explanations seem to be consistent with result from the conservatism literature", it "cannot be the general explanation for the systematic long-term evidence." However, the arguments based on the evidence about positive accruals, Basu's results, or earnings-based compensation can be resolved by manager's discretionary accounting choices which, by definition, can be classified as a kind of earnings management although they may not change the actual earnings.

2.2 Rational Inattention (Bounded Rationality)

Conlinsk (1987) argues that investors assumedly suffer some information costs including the effort and time spent on collecting the information required to predict firm performance. Therefore, investors have incentives to minimize the expectation of their loss due to the total information costs.

Loss will arise from the inaccuracy of current information that investors have. This loss can be decreased by collecting more information to increase the total number of observations or the accuracy of information held by them. Therefore, Conlinsk (1987) provides the following expected loss function as an objective function for investors:

$$\min_{\tau} E[(q(\tau) - P)^2] + C\tau$$
(2.1)

P denotes as logarithm of next period's price level; $q(\tau)$ is the investors expectation of P; τ is the amount of time and other resources, where $\tau \in [0, 1)$; C is the cost of the analysis per unit of time; R is the mathematical expectation of next-period price P based on the true model of economy and all information currently available. R can be viewed a rational expectation for future price P, if investors can obtain all available information freely.

Then the loss function can be changed as,

$$\min_{\tau} E[(q(\tau) - R)^2] + C\tau$$
(2.2)

If the investors expectation of P, $q(\tau)$, is an combination of costly estimator $r(\tau)$ and cost-free estimator f or F. I define f as the price estimator only based on firm's management earnings forecasts, while define F as the predicted price solely based on mandatory disclosure of accounting information. Thus $q(\tau)$ is an estimator which only includes one of the

public information. Although investors have incentives to use both public information to generate their estimation, in order to see the marginal benefits of those two kinds of public informations, I seperate the effects of the two public information for investors' analysis into two cases. Then, $q(\tau)$ can be expressed as two kinds of forms, such as,

$$\begin{cases} q(\tau) = \frac{Sf + \tau r(\tau)}{\tau + S} & \text{if manager use f as public information} \\ q(\tau) = \frac{F + \tau r(\tau)}{1 + \tau} & \text{if manager use F as public information} \end{cases}$$
(2.3)

where $r(\tau)$ is as accurate as a sample mean of τ independent observations taken from the whole population of information, its distribution follows mean R and variance σ^2 ; f is as accurate as a sample mean of S independent observations also taken from the whole population of information, whose distribution follows mean R and variance σ^2 ; and F is a prediction suggest or implied by manager's discretionary information, and F - R follows mean μ_{ϵ} and variance σ^2_{ϵ} . F and R are independent.

For the first case, which I only consider the effect of management earnings forecasts on investors' investment decisions. By assumption, investors have incentives to minimize the new loss function based on the optimal choice about τ^* . Thus, by first order condition with respect to investors' choice τ , the solution can be solved as,

$$\tau^* = \max[0, (\frac{C}{\sigma^2})^{-\frac{1}{2}} - S]$$
(2.4)

The solution (2.4) suggests as numbers of management earnings forecasts, S, increases, investors optimal choice for entering the market will decrease to zero. Because S also measures the accuracy of f based on management earnings forecasts, the solution also means if manager discloses enough accurate public information, investors are likely to enter the market earlier. This is reasonable, because if management earnings forecasts provide enough accurate information at this moment, the marginal costs of requiring other information cannot be compensated by additional accuracy (marginal benefits) brought from such other information.

As for managers, they have incentives to lower the optimal τ^* because they want investors to enter the market earlier, which may lower the potential cost of capital, η ,

$$\frac{1}{1+k} = e^{-\eta[t-(t-1+\tau^*)]} = e^{-\eta(1-\tau)}$$

$$\eta = \frac{\ln(1+k)}{1-\tau}, \quad \tau \in [0,1)$$

(2.5)

where k can be thought of as an exogenously fixed hurdle rate for period [t-1, t].

Therefore, for manager, the optimal quality for his or her disclosure shall be determined as follows:

$$S^* \ge (\frac{C}{\sigma^2})^{-\frac{1}{2}}$$
 (2.6)

Further, I assume investors are also likely to accept biased information F provided by manager, because without enough sample size, $r(\tau)$ could generate a very noisy estimate, increase the investors opportunity cost and may result in a gain from the inefficient market (underlying market price does not always reflect firm true value) by other investors who use such estimates. Therefore, as long as the disclosures can reduce the first part in investors' loss function, investors shall use such disclosures, such as,

$$E[(\frac{F+\tau r(\tau)}{1+\tau}-R)^2] < E[(r(\tau)-R)^2]$$

$$\Rightarrow \mu_{\epsilon}^2 + \sigma_{\epsilon}^2 \le 2\sigma^2$$
(2.7)

Proof.

$$\begin{split} & \because \begin{cases} E[(\frac{F+\tau r(\tau)}{1+\tau}-R)^2] = \frac{\mu_{\epsilon}^2 + \sigma_{\epsilon}^2 + \tau^2 \frac{\sigma^2}{\tau}}{(1+\tau)^2} \\ E[(r(\tau)-R)^2] = \frac{\sigma^2}{\tau} \\ & E[(\frac{F+\tau r(\tau)}{1+\tau}-R)^2] < E[(r(\tau)-R)^2] \\ & \therefore \qquad \mu_{\epsilon}^2 + \sigma_{\epsilon}^2 < \frac{1}{\tau} + 2\sigma^2 \\ & \mu_{\epsilon}^2 + \sigma_{\epsilon}^2 \le 2\sigma^2 \end{split}$$

Lemma 1. If investors have limited attention for the market, investors will have incentives to use the disclosure for decision making, which may not help investors generate rational expectations for firm's future performances but help to increase the total accuracy of their prediction, the case when

 $\mu_{\epsilon}^2 + \sigma_{\epsilon}^2 \leq 2\sigma^2$. Otherwise, investors' investment decisions will ignore the disclosure in making their investment decisions.

Then as long as $\mu_{\epsilon}^2 + \sigma_{\epsilon}^2 \leq 2\sigma^2$, investors can have their optimal problem based on new public information F as follows,

$$\min_{\tau} E[(\frac{F + \tau r(\tau)}{1 + \tau} - R)^2] + C\tau$$
(2.8)

and the first order condition for (2.8) follows,

$$(\tau+1)^3 - (\tau+1)\frac{\sigma^2}{C} - \frac{2(\mu_{\epsilon}^2 + \sigma_{\epsilon}^2 - \sigma^2)}{C} = 0$$
(2.9)

This is a cubic function,

$$X^3 + pX + q = 0 (2.10)$$

where, $X = \tau + 1$, $p = -\frac{\sigma^2}{C}$, and $q = -\frac{2(\mu_{\epsilon}^2 + \sigma_{\epsilon}^2 - \sigma^2)}{C}$. From Weiguo Shi (2010), we can see if $\Delta = (\frac{q}{2})^2 + (\frac{p}{3})^3 > 0$, equation (2.10) will have one real root and two conjugate imaginary roots, where the real root is larger than zero if q < 0, while smaller than zero if q > 0; if $\Delta = (\frac{q}{2})^2 + (\frac{p}{3})^3 = 0$, equation (2.10) will have three real roots. And if q < 0, there will be one positive real root and two other same negative real roots; if q > 0, there will be one negative real root and two other same positive real roots. And if q < 0, there will be one negative real roots; if $\Delta = (\frac{q}{2})^2 + (\frac{p}{3})^3 < 0$, equation (2.10) will have three unequal real roots. And if q < 0, there will be one negative real roots. And if q < 0, there will be one negative real roots. And if q < 0, there will be one negative real roots. And if q < 0, there will be one negative real roots. And if q < 0, there will be one negative real roots. And if q < 0, there will be one negative real roots. And if q < 0, there will be one positive real root and two other unequal real roots. And if q < 0, there will be one negative real root and two other unequal and negative real roots; if q > 0, there will be one negative real root and two other unequal and positive real roots.

From above results, we can infer that, investors' optimal timing for entering the market differs based on p and \triangle . It also indicates that investors can enter the market earlier and have some rough information or enter later but with a more accurate analysis generated from more observations. Moreover,

only if q > 0 and $\Delta > 0$, such that,

$$\frac{2(\mu_{\epsilon}^2+\sigma_{\epsilon}^2-\sigma^2)}{C}<0$$

and

$$[\frac{2(\mu_\epsilon^2+\sigma_\epsilon^2-\sigma^2])}{2C}]^2>[\frac{\sigma^2}{3C}]^3$$

Above function (2.10) has only one negative real root and two positive complex roots, otherwise the solution will have at least one positive real root. Therefore, manager has incentive to make root be negative where manager can guarantee the optimal solution can be achieved at the corner for $\tau^* = 0$. Then the optimal F must follow property such as,

$$\mu_{\epsilon}^{2} + \sigma_{\epsilon}^{2} \le \sigma^{2} - (\frac{\sigma^{6}}{27C})^{\frac{1}{2}}$$
(2.11)

This result suggests that in order to make market efficient at the earnings annoucement date, namely, investors who will make their investment decision as soon as they observe the mandatory financial report, investors must believe manager's disclosure can help their prediction become accurate enough such as $\mu_{\epsilon}^2 + \sigma_{\epsilon}^2 \leq \sigma^2 - (\frac{\sigma^6}{27C})^{\frac{1}{2}}$. Thus manager has to make sure the above condition can be achieved everytime to incorporate investors' trust.

2.3 Accounting Information in Feltham and

Ohlson (1995) framework

As we know, Ohlson's linear information dynamics (Ohlson, 1995) and (Feltham and Ohlson, 1995) renders the idea about excess earnings. Therefore, in order to explain the inherent mechanism in Information contained in accounting numbers, This paper also establishes a similar setting to keep the analysis relatively simple.

 $B_t =$ book value of the firm's equity, date t;

 $E_t = \text{earnings}, \text{ period } (t-1,t);$

 d_t =dividends, net of capital contribution, date t;

 i_t =interest revenues, net of interest expenses, period (t-1,t);

 OA_t =operating assets, net of operating liabilities, date t;

 OE_t =operating earnings, period (t-1,t)

 CF_t =cash flows realized from operating activities,

net of investments in those activities, period (t-1,t);

- $c_t =$ dividend ratio, date t;
- $g_t = \text{growth rate of the firm, period } (t 1, t);$
- $k_t = \text{discount rate, period } (t 1, t);$

 P_t =market value of the firm's equity, date t;

 FA_t =financial assets, net of financial liabilities, date t;

 $T_t = \text{net of transitory assets, date } t;$

 $I_t = \text{net of permanent assets, date } t;$

 (δ_t^T, δ_t^I) = marginal cost for transitory and permanent assets respectively, period (t-1, t);

 (α_t, β_t) = the firm future state of performances related with transitory and permanent assets respectively, period (t - 1, t); W_t = total cost budget, date t.

The idea also segregates the firm's activities into financial and operating activities, thus the book value at date t is $B_t = FA_t + OA_t$ and earnings during period (t - 1, t) are $E_t = i_t + OE_t$. Moreover, it requires accounting measurements satisfy the clean surplus relation(CSR), net interest relation(NIR), financial assets relation(FAR) and operating asset relation(OAR),

$$B_t = B_{t-1} + E_t - d_t \tag{CSR}$$

$$i_t = R_f F A_{t-1} \tag{NIR}$$

where R_f denotes cost of debt.

$$FA_t = FA_{t-1} + i_t - (d_t - CF_t) \tag{FAR}$$

$$OA_t = OA_{t-1} + OE_t - CF_t \tag{OAR}$$

For simiplisity, let $i_t = 0$ Then earnings can be replaced by operating earnings, such as

$$E_t = OE_t$$

If future discount rate does not change such as,

$$P_t^b = \sum_{s=1}^{\infty} E_t \left[\frac{d_{t+s}}{1+k_t} \right] = \sum_{s=1}^{\infty} (1+k_t)^{-s} E_t [d_{t+s}]$$

$$k_{t+s} = k_t$$

If firm chooses to have steady dividend policy, make,

$$d_t = c_t E_t = c E_t$$

while growth rate of the firm follows,

$$g_t = (1 - c_t)k_t = (1 - c)k_t$$

Then market price of the firm can be represented as,

$$P_{t-1} = \sum_{s=1}^{\infty} E_t \left[\frac{d_{t+s}}{1+k_t} \right]$$

$$= \frac{cE_t}{k_t - g_t}$$

$$= \frac{cE_t}{k_t - (1-c)k_t}$$

$$= \frac{E_t}{k_t}$$

(2.12)

Compared with fundamental price function derived from Felthem & Ohlson (1995), equation (2.12) is more strict however does not change the conclusion as I later prove.

$$P_t^F = B_t + \sum_{s=1}^{\infty} (1+k_t)^{-s} E_t[E_{t+s}^a]$$

= $B_t + \sum_{s=1}^{\infty} (1+k_t)^{-s} E_t[OE_{t+s}^a]$ (2.13)

Furthermore, I divide the net of operating assets as net of transitory assets T_t and net of permanent assets I_t . This assumption is to cater the idea of earnings persistence. I argue that permanent assets can generate permanent earnings, thus making future earnings more persistent.

$$OA_t = T_t + I_t \tag{2.14}$$

Then based on this idea, I use the assumption made by Clausen and Hirth (2014) that operating income is a Cobb-Douglas production function with factors cumulative transitory assets and permanent assets. Then operating earnings can be expressed as follows,

$$OE_t = f(T_{t-1}, I_{t-1}) - \delta_T T_{t-1} - \delta_I I_{t-1} = K T_{t-1}^{\alpha_t} I_{t-1}^{\beta_t} - \delta_t^T T_{t-1} - \delta_t^I I_{t-1}$$
(2.15)

 (α_t, δ_t^T) and (β_t, δ_t^I) are the firm's future state of performances and marginal costs related with transitory and permanent assets respectively. These parameters varies with the economic environment at period (t-1, t).

However, marginal return and marginal costs of permanent assets are assumed to be more persistent than those of transitory assets, which also means permanent assets are less affected by economic environment. Therefore, it is equivalent to assume,

$$\beta_t = \beta_{t-1} + \epsilon_t^\beta \tag{2.16}$$

$$\delta_t^I = \delta_{t-1}^I + \epsilon_{\delta^I, t} \tag{2.17}$$

And persistence means,

$$E[\beta_t] = \beta_{t-1} + E[\epsilon_{\beta,t}] = \beta_{t-1} \tag{2.18}$$

$$E[\delta_t^I] = \delta_{t-1}^I + E[\epsilon_{\delta^I, t}] = \delta_{t-1}^I$$
(2.19)

Moreover, α_t and β_t are assumed to be independent with each other and have to satisfy the conditions that $0 < \alpha_t < 1$, $0 < \beta_t < 1$ and $0 < \alpha_t + \beta_t \leq 1$, making the operating earnings as a marginally decreasing function of (T_{t-1}, I_{t-1}) . Also it is reasonable to assume W > 0, K > 0 and T > 1 and I > 1, where for simplicity, we redefine (T, I) as (T_{t-1}, I_{t-1}) for the rest part of the paper.

From above construction, the persistence of earnings depends on the persistence of parameter (β_t, δ_t^I) and the proportion of the permanents assets in total operating assets. However, those parameters are assumedly only observable for manager, while unobservable by investors, which means there is an information asymmetry between manager and and outside investors.

To make the study case simple, I assume manager can observe actual future performances of the firm. Manager's information set

 $\mathcal{P}_{t} = \{\alpha_{t}, \beta_{t}, E_{t}, T_{t}, I_{t}, OA_{t}, \delta_{t}^{T}, \delta_{t}^{I}, \delta_{t-1}^{T}, K, W_{t-1}\} \text{ will be more timely than investors', } \mathcal{Q}_{t} = \{E_{t}, T_{t}, I_{t}, OA_{t}, \delta_{t-1}^{T}, \delta_{t-1}^{I}, K, W_{t-1}\}, \text{ which can be collected by mandatory earnings report at time } t.$

Then based on his or her observations, manager will always optimize (T, I) and maximize his or her compensation which is associated firm's future performances, operating earnings. Because there is no uncertain information for the manager, it is reasonable to assume the manager is risk neutral and the fundamental price derived by him is fully efficient.

Additionally, I assume earnings cannot be falsely reported, and the book value of the firm can be reported differently based on different disclosure policy. Therefore, there is no conflict between the manager's objectives between price maximization and earnings maximization, not only because the fundamental price is fully correlated with the future earnings under risk-neutral assumption but also because the market price only conveys the information for the reported numbers. Moreover, I assume the disclosure policy is only available if it can overall reduce the investors' prediction error compared with the fair accounting value policy. Otherwise, the reported numbers under such accounting policy will be considered as a bad signal and investors will not use it. This assumption restricts the manager's intension to inflate market price by inflating the reported numbers.

2.4 Accounting Disclosures of Operating

Earnings in Production Setting

With an information asymmetry between investors and managers, if accounting information at date t can help investors rightly infer firm's performance without other information, it is then reasonable to suggest accounting information can be informative for firm's future performances.

Consider the case that manager at the beginning of the period (t-1,t) can observe the inner information $(\alpha_t, \beta_t, \delta_t^T, \delta_t^I)$ which reflect the marginal return and marginal costs of firm during the period (t-1,t), while investors can only have asymmetric information contained $(E_{t-1}, T_t, I_t, OA_t, \delta_{t-1}^T, \delta_{t-1}^I, W_{t-1})$. It means manager has no uncertainty for firm's future performances, however, accounting reporting lag make manager hard to insert his inner information into the current financial reports.

Therefore, without further information disclosure, such as managerial disclosure about earnings forecasts, investors can hardly infer the right earnings and the correct fundamental price by using the current lagged information. But if investors know manager's objective function, lagged information will be useful, because it reflects manager's timely behavior for the firm's production management and therefore reflects the manager's optimization problem. Based on the assumptions made in last section, investors know the manager will maximize the future operating earnings. Therefore, if investors believe reported (T^*, I^*) is the true optimal choice

made by the manager to maximize OE_t , investors are likely to replicate the following manager's optimal problem,

$$\max_{T,I} E[OE_t|\mathcal{P}] \equiv E[f(T,I)|\mathcal{P}] - \delta_t^T T - \delta_t^I I = KT^{\alpha_t} I^{\beta_t} - \delta_t^T T - \delta_t^I I$$

s.t. $\delta_t^T T + \delta_t^I I = W_{t-1}$ (2.20)

F.O.C.
$$\begin{cases} K\alpha_t T^{\alpha_t} I^{\beta_t} = (1-\lambda)\delta_t^T \\ K\beta_t T^{\alpha_t} I^{\beta_t} = (1-\lambda)\delta_t^I \\ \lambda(\delta_t^T T + \delta_t^I I - W) = 0; \lambda \ge 0 \end{cases}$$
(2.21)

and by solving the equations as follows,

$$\begin{cases} \frac{T^*}{I^*} = \frac{\alpha_t \delta_t^I}{\beta_t \delta_t^T} \\ \delta_t^T T^* + \delta_t^I I^* = W_{t-1} \end{cases}$$
(2.22)

$$\begin{cases} T^* = \frac{W_{t-1}}{(1 + \frac{\beta_t}{\alpha_t})\delta_t^T} = \frac{W_{t-1}}{(1 + \frac{1}{\phi_t})\delta_t^T} \\ I^* = \frac{W_{t-1}}{(1 + \frac{\alpha_t}{\beta_t})\delta_t^I} = \frac{W_{t-1}}{(1 + \phi_t)\delta_t^I} \end{cases}$$
(2.23)

where

$$\phi_t = \frac{\alpha_t}{\beta_t}$$

investors can get the relation between (T^*, I^*) and $(\alpha_t, \beta_t, \delta_t^T, \delta_t^I)$.

$$\frac{\hat{\alpha}_t}{\hat{\beta}_t} = \frac{\delta_t^T T^*}{\delta_t^I I^*} = \hat{\phi}_t \tag{2.24}$$

However, in order to get the accurate predictions for future earnings, investors need further knowledge about the properties about $(\alpha_t, \beta_t, \delta_t^T, \delta_t^I)$. Because investors know $\beta_t \ \delta_t^I$ are more persistent, which means profitability of permanent assets is steady and less risky under information asymmetry, investors can use β_{t-1} and δ_{t-1}^I to estimate $\hat{\beta}_t$ and $\hat{\delta}_t^I$ respectively. This behavior is rational because on average investors' prediction is unbiased based on the definition of persistence.

$$\hat{\beta}_{t} = E[\beta_{t}] = \beta_{t-1} + E[\epsilon_{t}^{\beta}] = \beta_{t-1}$$

$$E[\epsilon_{\beta,t}] = 0$$

$$\hat{\delta}_{t}^{I} = E[\delta_{t}^{I}] = \delta_{t-1}^{I} + E[\epsilon_{\delta^{I},t}] = \delta_{t-1}^{I}$$

$$E[\epsilon_{\delta^{I},t}] = 0$$
(2.25)

The information about β_{t-1} can be obtained by solving equations (15), where historical accounting information $\{OE_{t-1}, \delta_{t-1}^T, \delta_{t-1}^I, T_{t-2}, I_{t-2})\}$ are known to investors.

$$\begin{cases} \frac{T_{t-2}^{*}}{I_{t-2}^{*}} = \frac{\hat{\alpha}_{t-1}\delta_{t-1}^{I}}{\hat{\beta}_{t-1}\delta_{t-1}^{T}} \\ OE_{t-1} = KT_{t-2}^{*\hat{\alpha}_{t-1}}I_{t-2}^{*\hat{\beta}_{t-1}} - \delta_{t-1}^{T}T_{t-2}^{*} - \delta_{t-1}^{I}I_{t-2}^{*} \end{cases}$$
(2.26)

Then by combining the learnt parameter inferred from (2.26) with the reported (T^*, I^*) , outside investors can use prediction process (2.27) to estimate the future performance of the firm more accurately, adjusting their prior belief $(\hat{\alpha}_t, \hat{\beta}_t)$ close enough to (α_t, β_t) .

$$\begin{cases} \hat{\beta}_{t} = \beta_{t-1} \\ \hat{\alpha}_{t} = \hat{\phi}_{t} \hat{\beta}_{t} \\ \hat{\delta}_{t}^{I} = \delta_{t-1}^{I} \\ \hat{\delta}_{t}^{T} = \frac{W_{t-1}}{(1+\frac{1}{\hat{\phi}_{t}})T^{*}} \\ \hat{\phi}_{t} = \frac{W_{t-1}}{\hat{\delta}_{t}^{I}I^{*}} - 1 \\ O\hat{E}_{t} = KT^{*\hat{\alpha}_{t}}I^{*\hat{\beta}_{t}} - W_{t-1} \end{cases}$$

$$(2.27)$$

Thus the prediction process in equation (2.27) suggests reported accounting information can reflect the future earnings more accurately. However, this prediction strategy itself does not meet the requirement of rational expectation even it is driven by a rational behavior. This is because β_t and δ_t^I are not fully stable, there could be some innovation which is only observable for manager to make $\beta_t \neq \beta_{t-1}$ and $\delta_t^I \neq \delta_{t-1}^I$. Then the expectation of the future earnings will be likely larger than the predicted one by Jensen's Inequality.

$$E[OE_{t}] = E[KT^{\alpha_{t}}I^{\beta_{t}} - \delta_{t}^{T}T - \delta_{t}^{I}I]$$

$$\geq KE[T^{\phi_{t}\beta_{t}}]E[I^{\beta_{t}}] - W_{t-1} \quad \text{for } Cov(\phi_{t}\beta_{t},\beta_{t}) > 0$$

$$\geq KT^{E[\phi_{t}\beta_{t}]}I^{E[\beta_{t}]} - W_{t-1} \quad \text{for } \phi_{t} \text{ and } \beta_{t} \text{ are independent}$$

$$\geq KT^{\hat{\phi}_{t}E[\beta_{t}]}I^{E[\beta_{t}]} - W_{t-1} \quad \text{for } E[\phi_{t}] = E[\frac{W_{t-1}}{\delta_{t}^{I}I}] \geq \frac{W_{t-1}}{E[\delta_{t}^{I}]I} = \hat{\phi}_{t}$$

$$= KT^{\hat{\alpha}_{t}}I^{\hat{\beta}_{t}} - W_{t-1}$$

$$= \hat{OE}_{t} \qquad (2.28)$$

This indicates the predicted earnings, which is only partially conditional on accounting information and ignores the possible distribution of the private other information (α_t, β_t) , will be on average lower than the expexcted earnings based on full information. Under Rational Expectation Hypothesis, this biased prediction will be not acceptable for investors. However, Lemma 1 derived by bounded rationality provides another way to make such prediction useful. Namely, as long as the prediction based on the accounting information is accurate enough for investors, even though the prediction process is not right, biased, and does satisfy the Rational Expectation Hypothesis, it is also useful because it reduces the investors' attention and time to collect further information about the acutual distribution of private information (α_t, β_t) .

Thus without further knowledge about information (α_t, β_t) , the price generated by such predicted earnings will be also on average lower than the one by full information, which also equals the fundamental price derived by manager, P^M , then making cost of capital higher.

$$P_{t-1} = \frac{OE_t}{k_t}$$

$$\leq E[\frac{OE_t}{k_t}]$$

$$= E[P_{t-1}^M]$$
(2.29)

This averagely undervaluation of firm is comfirmed by Lev(2004), who suggests the exisiting undervaluation of firms is caused by misvaluation for intangible capital as well as the intangibles-driven earnings other than the operating earnings contributed by physical asset if intangible assets are more persistent than tangible assets (Easton Shroff and Taylor, 2000).

2.5 Informativeness of Accounting

information

There is a large literature indicating that with certain accounting policy, firm can reduce information asymmetry and therefore reduce cost of capital. This paper also believes that this conjecture is true.

From equation (19), if the reporting strategy Q can make prediction higher than the original one derived by (16) through increasing $\hat{\beta}_t^P$ to $\hat{\beta}_t^Q$ or $\hat{\alpha}_t^P$ to $\hat{\alpha}_t^Q$, then earnings's prediction will be less biased.

$$\begin{cases} \hat{\beta}_t^Q = E^Q[\beta_t] = \beta_{t-1} + E^Q[\epsilon_{\beta,t}] \ge \beta_{t-1} = E^P[\beta_t] = \hat{\beta}_t^P \\ \hat{\alpha}_t^Q = \hat{\phi}_t^Q \hat{\beta}_t^Q \ge \hat{\phi}_t^P \hat{\beta}_t^P = \hat{\alpha}_t^P \end{cases}$$
(2.30)

Namely, it means in order to decrease the cost of capital in long term, the manager must unconditionally make one of parameters $\hat{\beta}_t^Q$ and $\hat{\alpha}_t^Q$ higher enough to achieve, $E[OE_t] = OE_t$.

Or equivalently, if manager want investors to use same parameters $\hat{\beta}_t^P$ and $\hat{\alpha}_t^P$, it must make one the following equations hold,

$$\begin{cases} E^{Q}[\epsilon_{t}^{\beta}] \geq 0 \quad \Leftarrow E^{Q}[\beta_{t}] \geq \beta_{t-1} \\ E^{Q}[\epsilon_{\delta^{I},t}] \leq 0 \quad \Leftarrow \delta^{I}_{t-1} + E[\epsilon_{\delta^{I},t}] \leq \delta^{I}_{t-1} \quad \Leftarrow \frac{W_{t-1}}{E^{Q}[\delta^{I}_{t}]I_{t}^{*}} - 1 \leq \frac{W_{t-1}}{E^{P}[\delta^{I}_{t}]I_{t}^{*}} - 1 \end{cases}$$

$$(2.31)$$

Otherwise, investors will on average underestimate the future earnings by using estimator $(\beta_{t-1}, \delta_{t-1}^I)$ to predict (β_t, δ_t^I) .

In detail, there are three reporting strategys that can achieve the condition $E^{Q}[\epsilon_{t}^{\beta}] \geq 0$ and $E^{Q}[\epsilon_{\delta^{I},t}] \leq 0$.

2.5.1 Ideal Situation

The first one is to make prediction error consistently approaching to zero, such that, making $\epsilon_{\beta,t}^Q = 0$ and $\epsilon_{\delta^I,t}^Q = 0$. This method can be achieved only because manager knows exactly the direction of the error term by observing

both (β_t, δ_t^I) and $(\beta_{t-1}, \delta_{t-1}^I)$, then manager could be possible to report the accounting information they want to make investors' prediction of the future earnings unbiased.

Therefore, in this case, manager will find the way to make the prediction useful even permanent information (β_t, δ_t^I) actually changes differently with $(\beta_{t-1}, \delta_{t-1}^I)$ during period (t-1, t).

$$\hat{\beta}_t^Q = E^Q[\beta_t] = \beta_{t-1} = \hat{\beta}_t^P$$

$$\hat{\alpha}_t^Q = \hat{\phi}_t^Q \hat{\alpha}_t^Q = \hat{\phi}_t^P \hat{\alpha}_t^P = \hat{\alpha}_t^P$$
(2.32)

The method is very simple. If the manager observes a positive innovation in β_t , which can be represented as good news, $\beta_t > \beta_{t-1}$, then manager would like to report a higher transitory assets $T^Q > T^*$, by recognizing more assets and less liabilities. In this way, the new predicted earnings based on this new reporting strategy (T^Q, I^Q) will be adjusted higher than the one underestimated by investors, who use the fair value of the accounting information (T, I). Then new predicted earnings can be higher enough to be unbiased with the actual earnings estimated by manager.

Based on this idea, the prediction bias caused by the negative innovation in β_t can also be reduced if manager reports lower transitory assets. Similarly, for δ_t^I , if manager observe $\delta_t^I > \delta_{t-1}^I$, which will make $\hat{\alpha}_t$ overestimated, manager will increase the reported I^Q , to make the prediction error lower, while decrease the I^Q if $\delta_t^I < \delta_{t-1}^I$. However reporting a higher or lower marginal cost of permanent assets without changing the current reporting numbers for permanent assets I is only theoretically possible for manager. This is because reducing the current depreciation rate of permanent assets will also cause current permanent assets to be lower, making prediction $hat\phi_t$ biased if $\delta_{t-1}^I \neq \delta_t^I$. So estimated bias for future earnings generated by prediction error in $\hat{\delta}_t^I$ can only be reduced by changing reporting strategy from T^Q . Therefore, we can conclude the method for manager's the first-best disclosure policy as follows,

$$\begin{cases} \delta_{t-1}^{QT} > \delta_{t-1}^{T} & and \quad T^{Q} < T & \text{if manager observe } \beta_{t} < \beta_{t-1} \\ \delta_{t-1}^{QT} < \delta_{t-1}^{T} & and \quad T^{Q} > T & \text{if manager observe } \beta_{t} > \beta_{t-1} \\ \delta_{t-1}^{QT} > \delta_{t-1}^{T} & and \quad T^{Q} < T & \text{if manager observe } \delta_{t}^{I} > \delta_{t-1}^{I} \\ \delta_{t-1}^{QT} < \delta_{t-1}^{T} & and \quad T^{Q} > T & \text{if manager observe } \delta_{t}^{I} < \delta_{t-1}^{I} \end{cases}$$
(2.33)

Then investors' prediction can beat manager's fundamental price as follows,

$$\begin{cases} P_{t-1}^F = \frac{OE_t}{k_t} = \frac{KT^{\alpha_t}I^{\beta_t} - W_{t-1}}{k_t} = \frac{KT^{Q\phi_t\beta_t}I^{\beta_t} - W_{t-1}}{k_t} = P_{t-1} \qquad (2.34) \end{cases}$$

However, without enough incentive manager does not necessary have to do such reporting strategy and make cost of capital lower. In this case there is no agency cost specifically for management decision because earnings maximization is same as price maximization with risk neutral rate assumption. However, if manager is only assigned with incentive contract based on earnings, the first-best reporting strategy cannot be achieved. This is because manager does not care about the undervalued price at date t - 1, but only cares about the earnings numbers at date t.

2.5.2 Equity based Managerial Contracting

Assigning managers with equity incentive contract is one of the most popular practice for firms. However, it will cause another problem. Managers will tend to hide bad news while making most of the reporting strategy to inform good news to investors. If so, reporting strategy cannot be more efficient than the first best strategy though it can still increase the prediction accuracy by achieving the condition $E^Q[\epsilon_t^\beta] \ge 0$ as follows,

$$\begin{cases} q(\beta_t > \beta_{t-1}) > p(\beta_t > \beta_{t-1}) \\ q(\beta_t < \beta_{t-1}) < p(\beta_t < \beta_{t-1}) \\ p \in P \text{ and } q \in Q \end{cases}$$
(2.35)

Above equation suggests that, if the manager only hides some bad news, making accounting information less likely to reflect bad news, then investors' prediction based on accounting information (T^Q, I^Q) will be more likely to overestimate the future earnings. Then market efficiency can be achieved with the optimal hiding policy Q^* . However, this paper suggest that assigning manager equity incentive contract can only make manager has incentive to achieve the third-best reporting strategy. This is because, manager has incentive to hide all the bad news if he is myopic. Then using the learning process (14) at at next earnings announcement date t, investors will infer a lower $\hat{\beta}_t$ by $\{T^Q, I, \delta^Q_T, \delta_I\}$, making price sufferred a permanent and negative shock after earnings released at this point.

$$\begin{cases} \hat{\beta}_t < \hat{\beta}_{t-1} & \text{if } \hat{\beta}_{t-1} > \beta_t \\ \hat{\beta}_t = \hat{\beta}_{t-1} & \text{if } \hat{\beta}_{t-1} \le \beta_t \\ \hat{OE}_{t+1} | \hat{\beta}_t \le \hat{OE}_{t+1} | \hat{\beta}_{t-1} \end{cases}$$

$$(2.36)$$

2.5.3 Conservative Reporting

This paper believes between the first best and third best reporting strategy, the second best one can be achieved by using accounting conservatism, which means firm's reporting strategy will mandatorily increase the standard of recognizing good news while decrease the standard of recognizing bad news. This is equivalent to having,

$$\begin{cases} \epsilon^{Q}(\beta_{t} > \beta_{t-1}) > \epsilon^{P}(\beta_{t} > \beta_{t-1}) \\ \epsilon^{Q}(\beta_{t} < \beta_{t-1}) < \epsilon^{P}(\beta_{t} < \beta_{t-1}) \\ p \in P \text{ and } p \in Q \end{cases}$$

$$(2.37)$$

The above equations shifts the interval of the error term, making the absolute value of error term by disclosing good news larger than the one by disclosing bad news. General speaking, bad news is more likely to be recognized without bias while good news is likely to be recognized with bias. Therefore, condition $E^Q[\epsilon_t^\beta] \geq 0$ can also be achieved under accounting conservatism. Because reporting bad news is mandatory under conservatism, manager with equity incentive cannot hide bad news for this case. However, the manager's reporting strategy to increase δ_T and lower T is also limited. Thus conservatism makes good news less likely be reflected by accounting information overall. The better thing about this strategy is that at next earnings announcement date, investors are more likely to infer a higher $\hat{\beta}_t$, making firm's future price constantly higher.

$$\begin{cases} \hat{\beta}_t > \hat{\beta}_{t-1} & \text{if } \hat{\beta}_{t-1} < \beta_t \\ \hat{\beta}_t = \hat{\beta}_{t-1} & \text{if } \hat{\beta}_{t-1} \ge \beta_t \\ \hat{OE}_{t+1} | \hat{\beta}_t \ge \hat{OE}_{t+1} | \hat{\beta}_{t-1} \end{cases}$$

$$(2.38)$$

Thus, because of the higher $\hat{\beta}_t$, conservatism makes \hat{OE}_{t+1} also higher for good news thus making the ratio $\frac{OE_t k_{t+1}}{OE_{t+1}}$ lower for good news.

$$E_{t}/P_{t-1} = OE_{t}/P_{t-1} = \frac{OE_{t}k_{t}}{\hat{OE}_{t}} = \frac{OE_{t}}{\hat{OE}_{t}}k_{t}$$

$$P_{t}/P_{t-1} = \frac{\hat{OE}_{t+1}/k_{t+1}}{\hat{OE}_{t}/k_{t}} = \frac{\hat{OE}_{t+1}}{\hat{OE}_{t}}$$

$$E_{t}/P_{t-1} = \frac{OE_{t}k_{t+1}}{\hat{OE}_{t+1}}P_{t}/P_{t-1}$$
(2.39)

This is consistent with Basu(1997)'s results. With a lower ratio $\frac{OE_t k_{t+1}}{OE_{t+1}}$ for good news, Earnings-Price ratio will reflect bad news more than good news.

Proposition 1. If there is an information asymmetry between manager and investors,

a) the earnings estimated by investors with a bounded rationality will be always smaller than or equal to the earnings inferred by the manager with the condition that $0 < \alpha_t < 1$, $0 < \beta_t < 1$, $0 < \alpha_t + \beta_t \le 1$, $0 < \delta_T < 1$, $0 < \delta_I < 1$, W > 0, K > 0 and T > 1 and I > 1.

b) And this estimation bias can be adjusted by accounting conservatism, namely adjust standard for the reported accounting information Q.

2.6 Pricing with Long and Short-term Investors and Rational Inattention

From inattention model in section 2.2, it is reasonable to assume that investors' information processing costs could be different. Moreover, the

Figure 2.1: Market Valuation by Rationally Inattentive Investors



condition (2.11) suggests that investors who have the higher information processing costs C could enter the market earlier, and be close enough to the investment timing where $\tau^* = 0$. Thus, I further define two kinds of investors based on their market entering timing. I define the short-term investors as the investors who enter the market at the timing τ^S and long-term investors as the investors who enter market later than the short-turn investors, such as $\tau^L > \tau^S$. This is because long-term investors are assumed to have a processing cost of C_L , while the short-term investors have a processing cost of C_S , such that $C_L < C_S$. Thus, based on the above idea, I further derive following conditions, if the manager has incentive to make accounting information useful, that is given $\mu_{\epsilon}^2 + \sigma_{\epsilon}^2 \leq 2\sigma^2$, and the following equations hold,

$$\begin{cases} \mu_{\epsilon}^{2} + \sigma_{\epsilon}^{2} \leq \sigma^{2} - (\frac{\sigma^{6}}{27C_{S}})^{\frac{1}{2}} \\ \sigma^{2} - (\frac{\sigma^{6}}{27C_{L}})^{\frac{1}{2}} < \mu_{\epsilon}^{2} + \sigma_{\epsilon}^{2} \leq 2\sigma^{2} \end{cases}$$
(2.40)

then it will tend to attract short-term investors to enter the market earlier and long-term investors later.

Furthermore, because $E[(q(\tau) - R)^2]$ is decreasing with the time τ , long-term investors who later enter the market must hold more accurate information. In other words, the other private information for short-term investors will be less accurate than the one for long-term investors. Thus long-term investors will generate their prediction P_{τ_L} based on the more accurate information for $(\hat{\alpha}_t^L, \hat{\beta}_t^L)$ rather than the naive prediction process used by short-term investors shown in (2.27) which here is denoted as $P_{\tau_L}|(\hat{\alpha}_t^S, \hat{\beta}_t^S)$ in figure (2.1).

As a result, initially price will tend to be biased suggested by Proposition 1, and this bias will correct by itself when the long-term investors come.

This idea is consistent with Ye (2011)'s suggestion that price movement may provide additional information to future earnings prediction. But I give the reason why price will be move to be efficient. Therefore, later in the following sections (2.8, 2.9, and 2.10), I will describe the idea of Ye (2011) and extend to the situation where estimated and realized earnings can be useful for predicting future price.

2.7 Model based on Stationary Abnormal

Earnings Process

Based on Ye's(2011) work, the abnormal earnings as an information can be divided into two parts associated with different level of persistence, I also establish a similar framework which classifies those two types as permanent information and transitory information as shown,

$$E_t^a = OE_t^a = OE - OE_t$$

$$= E[OE_t|\hat{\beta}_t] - E[OE_t|\hat{\alpha}_t, \hat{\beta}_t] + OE_t - E[OE_t|\hat{\beta}_t]$$

$$= (KT^{\alpha_t}I^{\hat{\beta}_t} - KT^{\hat{\alpha}_t}I^{\hat{\beta}_t}) + (KT^{\alpha_t}I^{\beta_t} - KT^{\alpha_t}I^{\hat{\beta}_t})$$

$$= V_{1,t} + V_{2,t}$$
(2.41)

where I define $V_{1,t}$ as transitory information and $V_{2,t}$ as permanent information.

Under the model discussed earlier, $V_{1,t}$ corresponds to the estimate error of $\hat{\alpha}_t$, $V_{2,t}$ corresponds to the estimate error in $\hat{\beta}_t$. In this circumstance, $V_{1,t}$ could be more transitory than $V_{2,t}$ because $\hat{\alpha}_t$ is more transitory than $\hat{\beta}_t$. Based on this intuition, I assume that $V_{1,t}$ follows a low-persistence process and $V_{2,t}$ follows a high-persistence process. The persistence parameters for $V_{1,t}$ and $V_{2,t}$ are ω and γ , respectively, and generalized in the following equations (2.42), The assumptions are shown in the following equations (2.42),

$$V_{1,t+1} = \omega V_{1,t} + \epsilon_{1,t+1} V_{2,t+1} = \gamma V_{2,t} + \epsilon_{2,t+1} 0 \le \omega < \gamma < 1;$$
(2.42)

where ϵ 's are zero-mean random variables. where ϵ 's are zero-mean random variables. While equations (2.42) are not based on the solutions laid out in (2.27), it is needed to develop empirical power. The equations correspond to a linear VAR system; such systems are commonly used in many models that

link earnings evolution with price dynamics (Beaver, Lambert and Morse (1980) and Beaver, Lambert and Ryan (1987)).

Proposition 1 suggests that E_t^a will be negatively biased at the beginning for the unobservable other information, if unbiased accounting is applied. However, this other information can be learnt from markets. Namely, it means outsiders can infer the unobserved components through both stock price and earnings.

Therefore, deducing equation (2.42), we have,

Lemma 2.

$$E_{t+1}^{a} = \omega E_{t}^{a} + V_{t+1} + \epsilon_{t+1}^{E}$$

$$V_{t+1} = \gamma V_{t} + \epsilon_{t+1}^{V}$$
(2.43)

where

$$V_t = (1 - \omega/\gamma)V_{2,t}, \ \epsilon_t^E = \epsilon_{1,t} + \frac{\omega}{\gamma}\epsilon_{2,t}, \ and \ \epsilon_t^V = (1 - \frac{\omega}{\gamma})\epsilon_{2,t}$$

Lemma 2 are similar to what Ohlson (1995) referred to as the linear information dynamics, where V_t is considered to be the 'other information'. Although, $V_t = (1 - \omega/\gamma)V_{2,t}$, it explains r the permanent unobservable-error from information β , based the model from the model (2.15). Moreover, I use V_{t+1} in the first equation instead of V_t in Ohlson (1995). This is reasonable because the current abnormal earnings can only be affected by current other information, while based on (2.15), it is parameter reflect for period (t,t+1) we combine equation (2.13) with (2.43), and then obtain the following equation, slightly different with Feltham and Ohlson (1995),

$$P_t = B_t + \alpha_{1,t} E_t^a + \alpha_{2,t} V_t \tag{2.44}$$

where

$$\alpha_{1,t} = \frac{\omega}{(1+k_t)-\omega}, \ \alpha_{2,t} = \frac{(1+k_t)\gamma}{((1+k_t)-\omega)((1+k_t)-\gamma)},$$

From the results, first we can see that $\alpha_{1,t}$ is increasing in ω ; $\alpha_{2,t}$ is increasing in γ ; as ω increases, $\alpha_{1,t}$ increases faster than $\alpha_{2,t}$, holding fixed

 γ . Second, because information about k_t can make equation (2.42) more accurate. Therefore, it is reasonable to assume,

$$\alpha_{1,s}|t = \alpha_{1,t}, \qquad \alpha_{2,s}|t = \alpha_{2,t} \qquad for \qquad s \le t$$

Models (2.43) and (2.44) pose a number of problems for analysis to proceed. First, a normal error distribution is not suitable for (2.43). This is because a normal error may lead to a negative price, and is not a reasonable approximation for the actual distribution in the data, which is highly skewed (Ye, 2001). More importantly, certain return calculations become non-transparent. The second problem is that a scaling variable is needed. When three equations are involved, scaling become very a difficult issue.

To avoid these problems, Ye (2011) provides two proximations. The first one is a log-linear approximation to the pricing model (2.44). The second approximation is to use the book value of equity to scare abnormal operating earnings. These two approximations together substantially simplify the analysis in the following sections. Note that all the error from approximation can be reclassified into the error terms, if necessary.

First, the empirical observations in Ye(2001) suggests the price error is log-normal, and has a standard deviation that is proportional to the mean,

$$P_{t+1} = (B_{t+1} + \alpha_{1,t} E^a_{t+1} + \alpha_{2,t} V_{t+1})(1 + \epsilon^P_{t+1}),$$

$$log(1 + \epsilon^P_{t+1}) \sim N(0, \sigma^2_P)$$
(2.45)

Second, Ye (2011) re-scale the residual price dynamics as well as earnings dynamics (2.45) by the book value of equity. Assume that $\frac{B_t}{B_t(t+1)} \approx 1$, and the errors are normal after being rescaled by book value of equity.

$$p_{t+1} = b_{t+1} + \log(1 + \frac{B_t}{B_{t+1}}\alpha_{1,t}e^a_{t+1} + \frac{B_t}{B_{t+1}}\alpha_{2,t}v_{t+1}) + \epsilon^p_{t+1},$$

$$\epsilon^p_{t+1} \sim N(0, \sigma^2_p)$$

where

$$p_{t+1} = log(P_{t+1}), \ b_{t+1} = log(B_{t+1}), \ e_{t+1}^a = \frac{E_{t+1}^a}{B_t},$$
$$v_{t+1} = \frac{V_{t+1}}{B_t}, \ \epsilon_{t+1}^p = log(1 + \epsilon_{t+1}^P)$$

BBy using Taylor expansion, $log(1+x) \approx x$, and the assumption that $\frac{B_t}{B_t(t+1)} \approx 1$, we get,

$$p_{t+1} = b_{t+1} + \alpha_{1,t} e^a_{t+1} + \alpha_{2,t} v_{t+1} + \epsilon^p_{t+1}, \ \epsilon^p_{t+1} \sim N(0, \sigma^2_p)$$
(2.46)

For earnings process, we also have,

$$\frac{E_{t+1}^a}{B_t} = \left[\omega \frac{E_t^a}{B_{t-1}} \frac{B_{t-1}}{B_t} + \frac{V_{t+1}}{B_t} + \frac{\epsilon_{t+1}^E}{B_t}\right]$$

The approximation here uses the residual return-on-equity instead of residual income itself. The ROE types of models have also been used extensively in accounting. See Nissim and Penman (2001). similarly,

$$\frac{V_{t+1}^a}{B_t} = \left[\gamma \frac{V_t}{B_{t-1}} \frac{B_{t-1}}{B_t} + \frac{\epsilon_{t+1}^V}{B_t}\right]$$

therefore,

$$e_{t+1}^{a} = \omega e_{t}^{a} + v_{t+1} + \epsilon_{t+1}^{e}, \ \epsilon_{t+1}^{e} \sim N(0, \sigma_{e}^{2})$$
(2.47)

$$v_{t+1} = \gamma v_t + \epsilon_{t+1}^v, \ \epsilon_{t+1}^v \sim N(0, \sigma_v^2)$$
 (2.48)

where

$$\epsilon^e_{t+1} = \frac{\epsilon^E_{t+1}}{B_t}, \ \epsilon^v_{t+1} = \frac{\epsilon^V_{t+1}}{B_t}$$

If the intangible intensive information is ignored in the price model, the esitmated slope of the earnings is biased. I show in the appendix that if $\rho > 0$,

$$Corr(e_t^a, v_t) > 0$$

2.8 Optimal Estimation of Other information

The Kalman filter is a standard approach to latent variables in engineering and statistical literature. In this section, I derive an expression for an optimal estimate of non-accounting information based on observed information which includes both price and earnings.

The essence of the Kalman filter is a recursive optimal estimation of the non-accounting information based on observed information through the system of equations. At time t, we observe (p_t, e_t^a) , while the non-accounting information v_t is not directly observed. The optimal estimation of v_t involves all three equations in (2.46), (2.47) and (2.48), which form a dynamic system,

$$\begin{cases}
e_{t+1}^{a} = \omega e_{t}^{a} + v_{t+1} + \epsilon_{t+1}^{e} \\
v_{t+1} = \gamma v_{t} + \epsilon_{t+1}^{v}, \ \epsilon_{t+1}^{v} \\
p_{t+1} = b_{t+1} + \alpha_{1,t} e_{t+1}^{a} + \alpha_{2,t} v_{t+1} + \epsilon_{t+1}^{p}
\end{cases}$$
(2.49)

For the sake of simplicity, we assume that $(\epsilon_{t+1}^e, \epsilon_{t+1}^v, \epsilon_{t+1}^p)$ is an independently and identically distributed normal random vector with mean (0, 0, 0) and variance-covariance matrix,

$$\Sigma = \begin{pmatrix} \sigma_e^2 & 0 & 0\\ 0 & \sigma_v^2 & 0\\ 0 & 0 & \sigma_p^2 \end{pmatrix}$$

Here the variance σ_e^2 measures the earnings unpredictability. The ratio $\frac{\sigma_v^2}{\sigma_e^2}$ measures the relative amount of other information in earnings. A high value of the ratio indicates that other information is more dominant. The variance σ_p^2 measures the closeness of the price to the fundamentals in e_t^a and v_t . High σ_p^2 implies that price is less related to e_t^a and v_t .

The optimal estimate of v_t is simply an exponential smoothing of the observed information, with the weights dependent on the relative noise level of each information source. The information sources here include historical price and historical earnings. Define the following quantities,

$$c_{1,t} = \frac{\alpha_{2,t}^2}{\sigma_p^2} / (\frac{\alpha_{2,t}^2}{\sigma_p^2} + \frac{1}{\sigma_e^2} + \frac{1}{\gamma^2 \sigma_{\hat{v}_{t-1}}^2 + \sigma_v^2})$$

$$c_{2,t} = \frac{1}{\sigma_e^2} / (\frac{\alpha_{2,t}^2}{\sigma_p^2} + \frac{1}{\sigma_e^2} + \frac{1}{\gamma^2 \sigma_{\hat{v}_{t-1}}^2 + \sigma_v^2})$$

$$c_{3,t} = 1 - c_{1,t} - c_{2,t}$$
(2.50)

Note that $c_i \in [0, 1]$, for i = 1, 2, 3.

Proposition 2. Given e_{t-k}^a and p_{t-k} , k = 0, 1, 2, ...

(a) the optimal estimate of v_t is \hat{v}_t ,

$$v_t \sim N(\hat{v}_t, \sigma_{\hat{v}_t}^2)$$

where $\hat{v_t}$ is

$$\hat{v}_{t} = \sum_{k=0}^{t-1} \frac{c_{1,k} \gamma^{k}}{\alpha_{2,t}} (p_{t-k} - [b_{t-k} + \alpha_{1,t} e_{t-k}^{a}]) + \sum_{k=0}^{t-1} c_{2,k} \gamma^{k} (e_{t-k}^{a} - \omega e_{t-k-1}^{a}) + c_{3,k}^{t} \gamma^{t} v_{0}$$
(2.51)

where $c_{1,k}\gamma^k \to 0$, $c_{2,k}\gamma^k \to 0$, $c_{3,k}\gamma^k \to 0$ except in trivial cases where $\sigma_e = \sigma_p = 0$. The variance $\sigma_{\hat{v}_t}^2$ can be obtained by solving the equation

$$\sigma_{\hat{v}_t}^2 = \left(\frac{\alpha_{2,t}^2}{\sigma_p^2} + \frac{1}{\sigma_e^2} + \frac{1}{\gamma^2 \sigma_{\hat{v}_{t-1}}^2 + \sigma_v^2}\right)^{-1}$$

(b) For the variance $\sigma_{\hat{v}_t}^2$, we also have,

$$\frac{\partial \sigma_{\hat{v}_t}^2}{\partial \sigma_p^2} > 0, \ \frac{\partial \sigma_{\hat{v}_t}^2}{\partial \sigma_e^2} > 0, \ \frac{\partial \sigma_{\hat{v}_t}^2}{\partial \sigma_v^2} > 0, \ \frac{\partial \sigma_{\hat{v}_t}^2}{\partial \omega} < 0.$$

(c) The coefficients c_1 , c_2 , c_3 satisfy the following,

$$\begin{array}{l} \frac{\partial c_1}{\partial \sigma_p^2} > 0, \quad \frac{\partial c_1}{\partial \sigma_e^2} > 0, \quad \frac{\partial c_1}{\partial \sigma_v^2} > 0, \quad \frac{\partial c_1}{\partial \omega} > 0, \quad \frac{\partial c_1}{\partial \gamma} > 0, \\ \\ \frac{\partial c_2}{\partial \sigma_p^2} > 0, \quad \frac{\partial c_2}{\partial \sigma_e^2} < 0, \quad \frac{\partial c_2}{\partial \sigma_v^2} > 0, \quad \frac{\partial c_2}{\partial \omega} < 0, \quad \frac{\partial c_2}{\partial \gamma} < 0, \\ \\ \frac{\partial c_3}{\partial \sigma_p^2} > 0, \quad \frac{\partial c_2}{\partial \sigma_e^2} > 0, \quad \frac{\partial c_2}{\partial \sigma_v^2} < 0, \quad \frac{\partial c_2}{\partial \omega} < 0, \quad \frac{\partial c_2}{\partial \gamma} < 0. \end{array}$$

The first part of the Proposition 2 shows that the optimal estimate of v_t is an exponential smoothing of the historical price and earnings, namely a weighted average of these two kinds of information additional with the part beyond their power of explanation. The second part shows that the accuracy (inverse of variance) of the estimated v_t is positively related to the price informativeness (inverse of σ_p^2) and negatively related to the earnings uncertainty. Holding all the other parameters fixed, the more volatile the earnings and price are, the more difficult it will be to use them as an accurate inference to other information. Lastly, the third part suggest the weights (explanation power) of those two kinds of information variant with change of certain parameters.

From Proposition 2, we can immediately get,

Corollary 2.1. The esimated unobservable accounting information \hat{v}_t depends on recently historical price and earnings (1) if the persistence of tangible-intensive component is higher, (2) if the price is more relevant (lower)

The rest of the paper will be based on \hat{v}_t as given in (2.52). To simplify some of the calculations, we may use the first order approximation by truncating all the terms except the first order lag value. That is,

$$\hat{v}_t = \frac{c_1}{\alpha_{2,t}} (p_t - [b_t + \alpha_{1,t}e_t^a]) + c_2(e_t^a - \omega e_{t-1}^a)$$
(2.52)

2.9 Earnings Predictability

In this section, we consider two applications of the estimated other information for estimating future earnings. The first scenario is to forecast earnings to be announced at time t + 1, based on the information available at time t, we refer the available information sets at t as \mathbb{F}_t , which reflects all the information gathered at and before time t. However, the timeline for different activities suggests a certain order for different kinds of information within a same period, we have to divide information sets into three dimensions, one with the timing t_1 left for analysts whose work is information gathering as well as estimation of v_t , one on the the timing t_2 related with earnings announcement date such as e_t^a and b_t , and another one with the timing t_3 in capital market such as p_t . In order to follow this kind of measurement, we must classify information based on how many dimensions it contains. For example, earnings contains accounting information about itself and the unobservable other information and price contains all the three dimensions. Therefore, it is easy to understand why accounting information is useless if price can hold all the information, indirectly proving only under information asymmetry that make market price is not so much efficient, accounting information can become useful. Therefore, we define information sets $\mathbb{B}(i)$ as *i*-dimensional sets, and the set of all available information sets based on above new time measurement, can be then denoted as where \hat{v}_{t_1} is estimated only by other information $\{v\}$

latest until the time t_1 , b_{t_1,t_2} and $e^a_{t_1,t_2}$ are estimated based on the information set $\{v\}$ latest until t_1 and earnings $\{e^a\}$ latest until the time t_2 , and similarly we have sets $\{v\}$, $\{e^a\}$, and $\{p\}$ for p_{t_1,t_2,t_3} latest until t_1 , t_2 , t_3 respectively. By construction, I assume that information sets belongs $\mathbb{B}(i) \subseteq \mathcal{F}_{t_1,t_2,t_3}$ can be noted as historical or real when standing at time t, if, only if $t \geq t_i$, otherwise the information is just a estimated number or partial learnt from capital market. For example, information $e_{t,t+2} \in \mathbb{B}(2) \subset \mathcal{F}_{t,t+2,t+3}$ is real at time t+3 for $t+3 > t_2 = t+2$. Thus we define this set of historical information sets as,

$$\mathcal{H}_t = \{\mathbb{B}(i) | t_i \le t, \text{ for } i = 1, 2, 3\}$$

Subsequently, we can conclude the relations among \mathbb{F}_t , $\mathcal{F}_{t_1,t_2,t_3}$ and \mathcal{H}_t as,

$$\bigcup_{\substack{0,0,0\\t_a,t_b,t_c}}^{t_1,t_2,t_3 \leq t} \mathcal{F}_{t_1,t_2,t_3} \subseteq \mathbb{F}_t \subseteq \mathcal{H}_t;$$
and
$$\bigcup_{\substack{0,0,0\\0,0,0}}^{t_a,t_b,t_c} \mathcal{F}_{t_1,t_2,t_3} = \mathbb{F}_t = \mathcal{H}_t, \text{ for } t_1 = t_2 = t_3 = t$$
(2.53)

Therefore, based on the above construction, using information at (t, t, t) can predict earnings at (t, t + 1, t).

The second scenario is to forecast earnings just before the time t + 1, where we denoted as (t + 1, t, t). This is because price and other information is more continuous, price change at (t + 1, t, t) reflects the additional information which newly learnt by capital market. At this tricky moment, other information process has already been advanced at time t + 1, however, cannot be fully captured by the market because it is unobservable, while earnings report has not yet been released at the same Therefore, information sets gathered at date (t + 1, t, t) can be time. divided as the information sets $\mathcal{F}_{t,t,t}$ and $p_{t+1,t,t}$ which partly reflected the information at date t+1, other \hat{v}_{t+1} namely, let $\mathcal{F}_{t+1,t,t} = \{ \hat{v}_{t+1}, b_{t+1,t}, e^a_{t+1,t}, p_{t+1,t,t} \}.$ Using such information $\mathcal{F}_{t+1,t,t}$ can predict the earnings $e_{t+1,t+1}^a$. Similarly, in next section, I suggest that as earnings report released at date t + 1, the information sets will updated as $\mathcal{F}_{t+1,t+1,t} = \{\hat{v}_{t+1}, b_{t+1,t+1}, e^a_{t+1,t+1}, p_{t+1,t+1,t}\}, \text{ investors can use such}$ information sets to predict price at date (t + 1, t + 1, t + 1).

2.9.1 Earnings Forecasting

In this section, I explain how to extend Ye(2001) to cases where estimated and realized earnings can be used as predictors of future price. Note that we will estimate $e_{t,t+1}^a$, the residual return-on-equity. We can convert it to earnings by $E_t = B_{t-1}(e_t^a + k_t)$ at date (t, t, t), given $e_{t,t}^a$ and \hat{v}_t , the optimal forecast for v_{t+1} is,

$$\hat{v}_{t+1} = \gamma \hat{v}_t \tag{2.54}$$

This gives a market-based prediction of return on equity for date (t, t+1, t),

$$\hat{e}^{a}_{t,t+1} = \omega e^{a}_{t,t} + \hat{v}_{t+1} = \omega e^{a}_{t,t} + \gamma \hat{v}_{t}$$
(2.55)

Proposition 3. The prediction $\hat{e}^a_{t,t+1}$ is an equilibrium market expectation of next period ROE standing at date (t, t, t) in the sense that if $e^a_{t+1,t+1} = \hat{e}^a_{t,t+1}$, then the expected abnormal return is zero.

This result gives the estimate (2.55) a rational justification. Using the approximation (2.52), we can have,

$$\hat{e}_{t,t+1}^{a} \approx \omega e_{t,t}^{a} + \frac{c_{1}\gamma}{\alpha_{2,t}} (p_{t,t,t} - [b_{t,t} + \alpha_{1,t}e_{t,t}^{a}]) + c_{2}\gamma(e_{t,t}^{a} - \omega e_{t-1,t-1}^{a}) \\ = (\omega - \frac{c_{1}\gamma\alpha_{1,t}}{\alpha_{2,t}})e_{t,t}^{a} + \frac{c_{1}\gamma}{\alpha_{2,t}}log(\frac{P_{t}}{B_{t}}) + c_{2}\gamma(e_{t,t}^{a} - \omega e_{t-1,t-1}^{a}) \\ = \omega(1 - c_{1} - \frac{c_{1}\gamma}{k_{t}})e_{t,t}^{a} + \frac{c_{1}}{k_{t}}(k_{t} - \omega)(k_{t} - \gamma)log(\frac{P_{t}}{B_{t}}) + c_{2}\gamma(e_{t,t}^{a} - \omega e_{t-1,t-1}^{a})$$
(2.56)

This gives rise to a dynamic model for the earnings process. The model suggests that both a high market-to-book ratio and a high profitability growth in the previous period would forecast a relatively high ROE profitability. Moreover, because under unbiased accounting, R_{t+1} are likely to be upward biased, therefore, making prediction of earnings, $\hat{E}_{t+1} = B_t(\hat{e}_t^a + R_{t+1})$ also upward biased as well as a negative surprise in future earnings if market stop acquiring the updated information during this period.

Observation 1. Given the same historical profitability measure, firms with

a higher market-to-book ratio would have higher future profitability.

This result suggests a way to take the market information into consideration in earnings forecast. Fama (1998) found evidence that the market-to-book ratio helps in forecasting earnings, which is consistent with the result here. This is not surprising, since price is a summary of information about future earnings.

Moreover, conservatism which deflates current period of transitory assets discussed earlier may also result in a higher market-to-book ratio. Therefore, it is consistent with the Proposition 1 that conservatism increases the firm' s long-term performances.

Observation 2. The forecasted profitability measure e_{t+1}^a depends both on

the level of current profitability (e_t^a) and the 'surprise', as defined by $e_t^a - e_{t-1}^a$.

Recent empirical literature in accounting assume that residual earnings follow an AR(1) (See for example, Sloan (1988)). Observation 2 shows that the order of auto regression of profitability is higher than 1. Thus using AR(1) for profitability is inadequate.

From Proposition 2(c), c_1 depends negatively on σ_p^2 (the price informativeness) and positively on earnings uncertainty (σ_e^2) . From the coefficients in (2.51), we immediately have the following,

Observation 3. Holding fixed all other parameters, the correlation of future profitability with the market-to-book ratio is higher for firms with higher earnings uncertainty (high σ_e^2), and lower for firms with high price in formativeness (low σ_p^2)

The dependence of the coefficient of the term $log(P_t/B_t)$ on ω and γ is not clear. From calculation of various cases (not given here), I find that the coefficients of is an increasing function of the persistence parameter γ when γ is small, and is decreasing when γ is high. Moreover, from Feltham and Ohlson (1995)'s function of estimation of price based on unbiased accounting, the magnitude of the market-to-book ratio itself is positive autocorrelated with ω and γ . **Observation 4.** Holding fixed all other parameters, the correlation between future profitability and historical profitability is lower for firms with higher earnings uncertainty (σ_e^2). The same is true for earnings surprises ($e_{t,t}^a - \omega e_{t-1,t-1}^a$)

2.9.2 Earnings Revision

Model (2.55) gives an estimate of market expectation of the ROE at time t+1 based on the information up to time t. But with analysts' forecasts and other information sources during period from time t to t + 1, other information can be learnt and by time absorbed by the capital market, then uninformed investors can in turn infer from the price about the other For example, as institutions update their newly information reversely. gathered other information, they are likely to change their optimal investments making price fluctuate. Then price change during this period will in reverse signal uninformed investors, and then update their newly estimation about future earnings at t+1. This process continues until time goes to the date just before the next earnings announcement date (t+1,t,t), and we assume at this moment, the available information about unobservable part of ROE are fully gathered except the actual earnings and actual book value's information that is going to be released for next moment.

If the other information gathered by analysts is estimated without bias and fully absorbed by the capital market just before earnings announcement date, we can have, Then based on the definition about timeline and filtrations denoted by information, we can have,

$$p_{t+1,t,t} = b_{t+1,t+1} + \alpha_{1,t}e^a_{t+1,t+1} + \alpha_{2,t}v_{t+1}|p_{t+1,t,t} + \epsilon^p_{t+1}$$
(2.57)

Then based on the definition about timeline and filtrations denoted by informations, we can have,

$$r_{t+1,t,t}^a = p_{t+1,t,t} - \hat{p}_{t+1,t,t+1}$$

Proposition 4. The conditional expectation $\hat{e}^a_{t+1,t+1}$ given the additional information of $p_{t+1,t,t}$ is

$$\hat{e}^{a}_{t+1,t+1} = E_{t+1,t,t}[e^{a}_{t+1,t+1}] = \hat{e}^{a}_{t,t+1} + \varphi r^{a}_{t+1,t,t}$$
(2.58)

where $0 < \varphi < 1$, and, $\hat{p}_{t+1,t,t+1} = E_{t+1,t,t}[p_{t+1,t+1,t+1}]$

$$\varphi = \frac{(1 + \alpha_{1,t} + \alpha_{2,t})[\sigma_v^2 + \gamma^2 \sigma_{\hat{v}_t}^2] + (1 + \alpha_{1,t})\sigma_e^2}{(1 + \alpha_{1,t} + \alpha_{2,t})^2[\sigma_v^2 + \gamma^2 \sigma_{\hat{v}_t}^2] + (1 + \alpha_{1,t})^2 \sigma_e^2 + \sigma_p^2}$$
(2.59)

Proposition 4 suggests that with market learning, new signal $p_{t+1,t,t}$ can adjusts the bias of the estimation of cost of capital bring from the unbiased accounting policy. If it is true, $\hat{p}_{t+1,t,t+1}$, adjusting $\hat{e}^a_{t,t+1}$ decrease to $\hat{e}^a_{t+1,t+1}$, making capital market efficient again.

Proof. Proposition 4

$$v_{t+1} = \gamma v_t + \epsilon_{t+1}^v = \gamma \hat{v}_t + \epsilon_{t+1} = \hat{v}_{t+1} | p_{t+1,t,t} + \epsilon_{t+1}^{\hat{v}|p_{t+1,t,t}}$$

where

$$\epsilon_{t+1}^{\hat{v}|p_{t+1,t,t}} = \epsilon_{t+1}^v + \gamma(v_t - \hat{v}_t|p_{t+1,t,t}) = \epsilon_{t+1}^v + \gamma(v_t - \hat{v}_t)$$

because the price $p_{t+1,t,t}$ has no effect on the historical estimation of other information at time t. Bassed on earnings process for informations available at (t, t), we can have,

$$e_{t+1,t+1}^{a} = \omega e_{t,t}^{a} + v_{t+1} + \epsilon_{t+1}^{e}$$

$$= (\omega e_{t,t}^{a} - \omega e_{t+1,t}^{a}) + (\omega e_{t+1,t}^{a} + \hat{v}_{t+1} | p_{t+1,t,t})$$

$$+ (v_{t+1} - \hat{v}_{t} | p_{t+1,t,t}) + \epsilon_{t+1}^{e}$$

$$= (\omega e_{t,t}^{a} - \omega e_{t+1,t}^{a}) + \hat{e}_{t+1,t+1}^{a} + (v_{t+1} - \hat{v}_{t+1} | p_{t+1,t,t}) + \epsilon_{t+1}^{e}$$

$$= \hat{e}_{t+1,t+1}^{a} + (v_{t+1} - \hat{v}_{t+1} | p_{t+1,t,t}) + \epsilon_{t+1}^{e}$$

where

$$e_{t,t}^{a} = e_{t+1,t}^{a}, \text{ for } e_{t,t}^{a}, e_{t+1,t}^{a} \in \mathcal{H}_{t}$$
$$\hat{e}_{t+1,t+1}^{a} = E_{(t,t+1)}[e_{t+1,t+1}^{a}] = E_{(t,t+1)}[\omega e_{t+1,t}^{a} + \hat{v}_{t}|p_{t+1,t,t} + \epsilon_{t+1}^{e}]$$

For abnormal return,

$$\begin{aligned} r_{t+1,t,t}^{a} &= p_{t+1,t,t} - E_{t+1,t,t}[p_{t+1,t+1,t+1}] \\ &= (b_{t+1,t+1} - \hat{b}_{t+1,t+1}) + \alpha_1(e_{t+1,t+1}^a - \hat{e}_{t+1,t+1}^a) + \\ &\alpha_2(v_{t+1}|p_{t+1,t,t} - \hat{v}_{t+1}|p_{t+1,t,t}) + \epsilon_{t+1}^p \\ &= (1 + \alpha_1)(e_{t+1,t+1}^a - \hat{e}_{t+1,t+1}^a) + \alpha_2(v_{t+1} - \hat{v}_{t+1}|p_{t+1,t,t}) + \epsilon_{t+1}^p \end{aligned}$$

as $v_{t+1}|p_{t+1,t,t} = v_{t+1}$. Additional with the assumptions we made,

$$b_{t+1,t+1} - \hat{b}_{t+1,t+1} = e^a_{t+1,t+1} - \hat{e}^a_{t+1,t+1}$$
$$p_{t+1,t,t} = b_{t+1,t+1} + \alpha_1 e^a_{t+1,t+1} + \alpha_2 v_{t+1} | p_{t+1,t,t} + \sigma^p_{t+1}$$

we can obtain,

$$\begin{cases} v_{t+1} - \hat{v}_{t+1} | p_{t+1,t,t} &= \epsilon_{t+1}^{\hat{v}|p_{t+1,t,t}} \\ e_{t+1,t+1}^a - \hat{e}_{t+1,t+1}^a &= \epsilon_{t+1}^{\hat{v}|p_{t+1,t,t}} + \epsilon_{t+1}^e \\ r_{t+1,t,t}^a &= (1 + \alpha_{1,t} + \alpha_{2,t}) \epsilon_{t+1}^{\hat{v}|p_{t+1,t,t}} + (1 + \alpha_1) \epsilon_{t+1}^e + \epsilon_{t+1}^p \end{cases}$$

where

$$\sigma_{\hat{v}|p_{t+1,t,t}}^2 = \sigma_v^2 + \gamma^2 \sigma_{\hat{v}_t}^2$$

This gives the variance and covariance of $e^a_{t+1,t+1} - \hat{e}^a_{t+1,t+1}$ and $r_{t+1,t,t}$ as,

$$V(r_{t+1,t,t}) = (1 + \alpha_{1,t} + \alpha_{2,t})^2 [\sigma_v^2 + \gamma^2 \sigma_{\hat{v}_t}^2]$$
$$+ (1 + \alpha_{1,t})^2 \sigma_e^2 + \sigma_p^2$$
$$Cov(r_{t+1,t,t}^a, e_{t+1,t+1}^a - \hat{e}_{t+1,t+1}^a) = (1 + \alpha_{1,t} + \alpha_{2,t}) [\sigma_v^2 + \gamma^2 \sigma_{\hat{v}_t}^2]$$
$$+ (1 + \alpha_{1,t}) \sigma_e^2$$

Note that $e_{t+1,t+1}^a - \hat{e}_{t+1,t+1}^a$ and $r_{t+1,t,t}$ are both zero-random variable at the moment (t + 1, t, t). Because Kalman Filtration suggests that, $E_{t+1,t,t}[e_{t+1,t+1}^a - \hat{e}_{t+1,t+1}^a] = E_{t+1,t,t}[e_{t+1,t+1}^a - \hat{e}_{t,t+1}^a] = 0$, therefore, make a regression $E_{t+1,t,t}[e_{t+1,t+1}^a - \hat{e}_{t,t+1}^a]$ on $E_{t+1,t,t}[r_{t+1,t,t}^a]$ we get intercep to be zero, and the slope to be $\varphi = Cov(r_{t+1,t,t}, e_{t+1,t+1}^a - \hat{e}_{t+1,t+1}^a)/V(r_{t+1,t,t})$. This gives the desired results.

2.10 Earnings Informativeness

In this section, I explore the equilibrium price response after e_{t+1}^a is observed, namely the timing at (t + 1, t + 1, t). I consider two different models. The first model is the total abnormal return between timing (t, t, t)and (t + 1, t + 1, t) based on the timing information at, where long-term investments for certain need of investors focus more on this abnormal return and pay less attentions for price change during this period. For example, for investors invest in the underlying derivatives such as futures are likely to related with the corresponding long-term abnormal stock return, because they cannot change their optimal portfolios based on the additional information at timing (t + 1, t, t) based on long-term contract of the futures. Therefore, for these kinds of investors if they want to hedge their risks by still using long-term investment product, they will pay attention with the long-term abnormal earnings and its coefficients responds to estimated price.

2.10.1 Long-term Informativeness of Earnings

At timing (t+1,t+1,t), financial reports provide the additional information for $e^a_{t+1,t+1}$ and $b^a_{t+1,t+1}$, which also updates investors' belief about v_{t+1} .

Then the optimal estimate of the price after the release of $e^a_{t+1,t+1}$ is given as,

Proposition 5. Given $e_{t+1,t+1}^a$, the long-term expected abnormal stock return from time t to t + 1 is,

$$\hat{r}_{t,t+1,t+1} = \psi[e^a_{t+1,t+1} - (\omega e^a_{t,t} + \gamma \hat{v}_t)], \qquad (2.60)$$

where

$$\psi = (1 + \alpha_1 + \alpha_2 d_1), \text{ and } d_1 = \frac{\sigma_v^2 + \gamma^2 \sigma_{\hat{v}_t}^2}{\sigma_v^2 + \gamma^2 \sigma_{\hat{v}_t}^2 + \sigma_e^2}$$

Proof. Proposition 5

Conditional on $e_{t+1,t+1}^a$, the optimal estimate for v_{t+1} is,

$$v_{t+1} = \gamma v_t + \epsilon_{t+1}^v = \gamma \hat{v}_t + \epsilon_{t+1} = \hat{v}_{t+1} |e_{t+1,t+1}^a + \epsilon_{t+1}^{\hat{v}_{t+1}|e_{t+1,t+1}^a}$$

where

$$\hat{\epsilon}_{t+1}^{\hat{v}_{t+1}|e_{t+1,t+1}^a} = \epsilon_{t+1}^v + \gamma(v_t - \hat{v}_t|e_{t+1,t+1}^a) = \epsilon_{t+1}^v + \gamma(v_t - \hat{v}_t)$$

because the earnings report $e_{t+1,t+1}$ has no effect on the historical estimation of other information at time t. For earnings process,

$$\begin{aligned} e^a_{t+1,t+1} &= \omega e^a_{t,t} + v_{t+1} + \epsilon^e_{t+1} \\ &= (\omega e^a_{t,t} + \hat{v}_{t+1} | e^a_{t+1,t+1}) + (v_{t+1} - \hat{v}_{t+1} | e^a_{t+1,t+1}) + \epsilon^e_{t+1} \\ &= \hat{e}^a_{t,t+1} | e^a_{t+1,t+1} + (v_{t+1} - \hat{v}_{t+1} | e^a_{t+1,t+1}) + \epsilon^e_{t+1} \end{aligned}$$

Then,

$$\begin{cases} v_{t+1} - \hat{v}_{t+1} | e^a_{t+1,t+1} &= \epsilon^{\hat{v}_{t+1} | e^a_{t+1,t+1}}_{t+1} \\ e^a_{t+1,t+1} - \hat{e}^a_{t,t+1} | e^a_{t+1,t+1} &= \epsilon^{\hat{v}_{t+1} | e^a_{t+1,t+1}}_{t+1} + \epsilon^e_{t+1} \end{cases}$$

Also,

$$Var(e^{a}_{t+1,t+1} - \hat{e}^{a}_{t,t+1}|e^{a}_{t+1,t+1}) = \sigma^{2}_{v} + \gamma^{2}\sigma^{2}_{\hat{v}_{t}} + \sigma^{2}_{e}$$
$$Cov(e^{a}_{t+1,t+1} - \hat{e}^{a}_{t,t+1}|e^{a}_{t+1,t+1}, v_{t+1} - \hat{v}_{t+1}|e^{a}_{t+1,t+1}) = \sigma^{2}_{v} + \gamma^{2}\sigma^{2}_{\hat{v}_{t}}$$

Therefore, similarly, using Kalman filtration that, $E_{t,t+1,t}[e_{t+1,t+1}^{a} - \hat{e}_{t,t+1}^{a}|e_{t+1,t+1}^{a}] = E_{t,t+1,t}[e_{t+1,t+1}^{a} - \hat{e}_{t,t+1}^{a}] = 0$, and make a regression $E_{t,t+1,t}[\hat{v}_{t+1}|e_{t+1,t+1}^{a} - v_{t+1}]$ on $E_{t,t+1,t}[e_{t+1,t+1}^{a} - \hat{e}_{t,t+1}^{a}]$, we can have,

$$\begin{split} \hat{v}_{t+1}|e^a_{t+1,t+1} - \hat{v}_{t+1} &= \frac{Cov(e^a_{t+1,t+1} - \hat{e}^a_{t,t+1}|e^a_{t+1,t+1}, v_{t+1} - \hat{v}_{t+1}|e^a_{t+1,t+1})}{Var(e^a_{t+1,t+1} - \hat{e}^a_{t,t+1}|e^a_{t+1,t+1})} \\ &\times (e^a_{t+1,t+1} - \hat{e}^a_{t,t+1}) \\ &= \frac{\sigma^2_v + \gamma^2 \sigma^2_{\hat{v}_t}}{\sigma^2_v + \gamma^2 \sigma^2_{\hat{v}_t} + \sigma^2_e} (e^a_{t+1,t+1} - \hat{e}^a_{t,t+1}) \\ &= d_1(e^a_{t+1,t+1} - \hat{e}^a_{t,t+1}) \end{split}$$

Then the optimal estimate of the price after the release of $e^a_{t+1,t+1}$ is given as,

$$\hat{p}_{t+1,t+1,t+1} = b_{t+1} + \alpha_{1,t} e^a_{t+1,t+1} + \alpha_{2,t} \hat{v}_{t+1} | e^a_{t+1,t+1}$$

Then, the long-term optimal estimate of the abnormal return is,

$$\begin{aligned} \hat{r}_{t,t+1,t+1} &= \hat{p}_{t+1,t+1,t+1} - \hat{p}_{t,t,t+1} \\ &= b_{t+1} - \hat{b}_{t,t+1} + \alpha_{1,t} (e^a_{t+1,t+1} - \hat{e}^a_{t,t+1}) + \alpha_{2,t} (\hat{v}_{t+1} | e^a_{t+1,t+1} - \hat{v}_{t+1}) \\ &= (1 + \alpha_{1,t} + \alpha_{2,t} d_1) (e^a_{t+1,t+1} - \hat{e}^a_{t,t+1}) \\ &= (1 + \alpha_{1,t} + \alpha_{2,t} d_1) [e^a_{t+1,t+1} - (\omega e^a_{t,t} + \hat{v}_{t+1})] \\ &= \psi [e^a_{t+1,t+1} - (\omega e^a_{t,t} + \gamma \hat{v}_t)] \end{aligned}$$

Using the estimate for \hat{v}_t in (2.52), and truncate the terms representing older information, such as $e_t^a - \omega e_{t-1}^a$, and $log(P_{t-1})/B_{t-1}$ in \mathcal{F}_t , we can obtain the following approximation,

$$\hat{r}_{t,t+1,t+1} \approx \psi\{(e^a_{t+1,t+1} - \omega e^a_{t,t}) - \gamma \frac{c_1}{\alpha_{2,t}} [p_t - (b_t + \alpha_{1,t} e^a_t)]\}$$
(2.61)

2.10.2 Short-term Informativeness of Earnings

In empirical accounting research, a popular model is the instantaneous price response at the announcement of the earnings using the short event window. In this subsection, we will obtain the short-window return model based on the analysts forecast of earnings, where this short-term window is the timing from (t + 1, t, t) to (t + 1, t + 1, t) in contrast with the long-term informativeness during the period from (t, t, t) to (t, t + 1, t). Therefore, the short-term expected abnormal stock return is given as,

Proposition 6.

$$\hat{r}_{t+1,t+1,t+1} = \psi[e^a_{t+1,t+1} - (\hat{e}^a_{t,t+1} + \varphi r^a_{t+1,t,t})]$$
(2.62)

Proof. Proposition 6

From proof of proposition 3, and the fact that $e_{t,t}^a = e_{t+1,t}^a$, $e_{t,t}^a, e_{t+1,t}^a \in \mathcal{H}_t$, we can have similar results,

$$\begin{cases} v_{t+1} - \hat{v}_{t+1} | e^a_{t+1,t+1} &= \epsilon^{\hat{v}_{t+1} | e^a_{t+1,t+1}}_{t+1} \\ e^a_{t+1,t+1} - \hat{e}^a_{t+1,t+1} | e^a_{t+1,t+1} &= \epsilon^{\hat{v}_{t+1} | e^a_{t+1,t+1}}_{t+1} + \epsilon^e_{t+1} \end{cases}$$

However, combined with slightly different Kalman filter, $E_{t+1,t+1,t}[e_{t+1,t+1}^a - \hat{e}_{t+1,t+1}^a] = E_{t+1,t+1,t}[e_{t+1,t+1}^a - \hat{e}_{t+1,t+1}^a] = 0$, and make a regression $E_{t+1,t+1,t}[\hat{v}_{t+1}|e_{t+1,t+1}^a - v_{t+1}]$ on $E_{t+1,t+1,t}[e_{t+1,t+1}^a - \hat{e}_{t+1,t+1}^a]$, we can have,

$$\hat{v}_{t+1}|e^a_{t+1,t+1} - \hat{v}_{t+1}|p_{t+1,t,t}|$$

$$= \frac{Cov(e_{t+1,t+1}^{a} - \hat{e}_{t+1,t+1}^{a}|e_{t+1,t+1}^{a}, v_{t+1} - \hat{v}_{t+1}|e_{t+1,t+1}^{a})}{Var(e_{t+1,t+1}^{a} - \hat{e}_{t+1,t+1}^{a}|e_{t+1,t+1}^{a})}(e_{t+1,t+1}^{a} - \hat{e}_{t+1,t+1}^{a})$$
$$= \frac{\sigma_{v}^{2} + \gamma^{2}\sigma_{\hat{v}_{t}}^{2}}{\sigma_{v}^{2} + \gamma^{2}\sigma_{\hat{v}_{t}}^{2} + \sigma_{e}^{2}}(e_{t+1,t+1}^{a} - \hat{e}_{t,t+1}^{a})$$
$$= d_{1}(e_{t+1,t+1}^{a} - \hat{e}_{t+1,t+1}^{a})$$

From Proposition 4, we know,

$$\hat{e}_{t+1,t+1} = \hat{e}_{t+1,t+1}^a + (v_{t+1} - \hat{v}_{t+1}|p_{t+1,t,t}) + \epsilon_{t+1}^e$$

Then, the short-term optimal estimate of the abnormal return is,

$$\begin{split} \hat{r}_{t+1,t+1,t+1} &= \hat{p}_{t+1,t+1,t+1} - \hat{p}_{t+1,t,t+1} \\ &= b_{t+1,t+1} - \hat{b}_{t+1,t+1} + \alpha_{1,t} (e^a_{t+1,t+1} - \hat{e}^a_{t+1,t+1}) \\ &+ \alpha_{2,t} (\hat{v}_{t+1} | e^a_{t+1,t+1} - \hat{v}_{t+1} | p_{t+1,t,t}) \\ &= \psi (e^a_{t+1,t+1} - \hat{e}^a_{t+1,t+1}) \\ &= \psi [e^a_{t+1,t+1} - (\hat{e}^a_{t,t+1} + \varphi r^a_{t+1,t,t})] \end{split}$$

2.10.3 Conservatism Accounting and Price Movement

From Proposition 1, conservatism will underestimate future price and this underestimation will continue until investor fully learns the other information. Thus, the accounting conservatism suggests, the future price will always be higher than the current expectation of future price.

$$r_{t+1,t,t}^a = p_{t+1,t,t} - \hat{p}_{t+1,t,t+1} \ge 0$$

where $\hat{p}_{t+1,t,t+1}$ can be also viewed as the short-term investors' expectation of future price. As time goes by, $r^a_{t+1,t,t}$ increases. Thus, if market price cannot fully capture the private information before earnings annoucement date, $r^a_{t+1,t,t}$ cannot achieve the optimal level,

$$r^a_{t+1,t,t} < r^{a*}_{t+1,t,t}$$

Then, there will be a more positive reaction from the market at the earnings announcement date t + 1 for a lower $r_{t+1,t,t}^a$.

Moreover, under conservatism, short-term investors' abnormal price will be higher that of long-term investors, such that',

$$r_{t+1,t,t}^{Sa} = p_{t+1,t,t} - \hat{p}_{t+1,t,t+1}^S > p_{t+1,t,t} - \hat{p}_{t+1,t,t+1}^L = r_{t+1,t,t}^{La}$$

thus, take more benefit from earnings annoucement, such as,

$$\hat{r}^S_{t+1,t+1,t+1} > \hat{r}^L_{t+1,t+1,t+1}$$

Lastly, based on Proposition 5 and Proposition 6, we can see the strategy, which includes the current price as an inference for future price, reduces the permanent effect by historical earnings annoucement in the context of accounting conservatism.

$$\hat{r}_{t+1,t+1,t+1} - \hat{r}_{t,t+1,t+1} = -\psi\varphi r^a_{t+1,t,t}$$

where $\psi > 0$ and $\varphi > 0$.

In other words, conservatism makes price as a signal more efficient.

2.11 Analysts Forecasts and Auditing Report for Earnings Relevance

Analysts and auditors as other information suppliers can make earnings prediction with more quality, because analysts forecasts and auditing reports can provide two more dimensions of the unobservable part of other information. Therefore, the assumptions about analysts,

$$v_{t+1} = \hat{v}_{t+1}^A + \epsilon_{t+1}^A, \tag{2.63}$$

where,

$$\epsilon^A_{t+1} \sim N(0,\sigma^2_A) \ \& \ Cov(\epsilon^A_{t+1},\epsilon^v_{t+1}) = 0$$

Additionally, We assume earnings qaulity can reflect the earnings uninformed part with less error, namely it means,

$$v_{t+1} = \hat{v}_{t+1} | e^a_{t+1,t+1,t} + \epsilon^Q_{t+1}$$
(2.64)

where

$$\epsilon^Q_{t+1} \sim N(0, \sigma^2_Q) \text{ and } Cov(\epsilon^Q_{t+1}, \epsilon^v_{t+1}) = 0$$

Therefore, earnings prediction totally depends on analysts forecasts, and equation (2.55) will be changed as

$$\hat{e}^a_{t,t+1} = \omega e^a_{t,t} + \hat{v}^A_{t+1} \tag{2.65}$$

Therefore, combined with

$$\hat{e}_{t+1,t+1} = \hat{e}_{t+1,t+1}^a + v_{t+1} - \hat{v}_{t+1} | p_{t+1,t,t} + \epsilon_{t+1}^e$$

For abnormal return,

$$r_{t+1,t,t}^a = (1+\alpha_{1,t})(e_{t+1,t+1}^a - \hat{e}_{t+1,t+1}^a) + \alpha_{2,t}(v_{t+1} - \hat{v}_{t+1}|p_{t+1,t,t}) + \epsilon_{t+1}^p$$

we can obtain,

$$\begin{cases} v_{t+1} - \hat{v}_{t+1} | p_{t+1,t,t} &= \epsilon_{t+1}^{A} \\ e_{t+1,t+1}^{a} - \hat{e}_{t+1,t+1}^{a} &= \epsilon_{t+1}^{A} + \epsilon_{t+1}^{e} \\ r_{t+1,t,t}^{a} &= (1 + \alpha_{1,t} + \alpha_{2,t}) \epsilon_{t+1}^{A} + (1 + \alpha_{1,t}) \epsilon_{t+1}^{e} + \epsilon_{t+1}^{p} \end{cases}$$

for $\hat{v}_{t+1}^A = \hat{v}_{t+1} | p_{t+1,t,t}$ analysts for ecasts for other information are useful if

$$\sigma_A^2 \le \sigma_{\hat{v}|p_{t+1,t,t}}^2 = \sigma_v^2 + \gamma^2 \sigma_{\hat{v}_t}^2$$

If public accounting information cannot reflect other information accurately, $\sigma_{\hat{v}_t}^2$ will be larger, making analysts forecasts more useful. Therefore,

$$V(r_{t+1,t,t}) = (1 + \alpha_{1,t} + \alpha_{2,t})^2 \sigma_A^2 + (1 + \alpha_{1,t})^2 \sigma_e^2 + \sigma_p^2$$
$$Cov(r_{t+1,t,t}^a, e_{t+1,t+1}^a - \hat{e}_{t+1,t+1}^a) = (1 + \alpha_{1,t} + \alpha_{2,t})\sigma_A^2 + (1 + \alpha_{1,t})\sigma_e^2$$

And then, equation (2.58) will change to,

$$\hat{e}_{t+1,t+1} = E_{t+1,t,t}[e^a_{t+1,t+1}] = \hat{e}^a_{t,t+1} + \varphi^A r^a_{t+1,t,t}$$
(2.66)

where $r_{t+1,t,t}^a = p_{t+1,t,t} - \hat{p}_{t+1,t,t+1}$ is the abnormal stock price, $0 < \varphi < 1$, and, $\hat{p}_{t+1,t,t+1} = E_{t+1,t,t}[p_{t+1,t+1,t+1}]$

$$\varphi^{A} = \frac{(1 + \alpha_{1,t} + \alpha_{2,t})\sigma_{A}^{2} + (1 + \alpha_{1,t})\sigma_{e}^{2}}{(1 + \alpha_{1,t} + \alpha_{2,t})^{2}\sigma_{A}^{2} + (1 + \alpha_{1,t})^{2}\sigma_{e}^{2} + \sigma_{p}^{2}}$$
(2.67)

equation (2.65) also makes (2.60) become,

$$\hat{r}_{t,t+1,t+1} = \psi^Q [e^a_{t+1,t+1} - \hat{e}^a_{t,t+1}], \qquad (2.68)$$

where

$$\psi^Q = (1 + \alpha_1 + \alpha_2 d_1^Q), \text{ and } d_1^A = \frac{\sigma_Q^2}{\sigma_Q^2 + \sigma_e^2}$$

Similarly, (2.62) will be also changed as,

$$\hat{r}_{t+1,t+1,t+1} = \psi^Q [e^a_{t+1,t+1} - (\hat{e}^a_{t,t+1} + \varphi^A r^a_{t+1,t,t})]$$
(2.69)

because $\psi^Q < \psi$ and $\varphi^A < \varphi$, absolute value of abnormal return, $\hat{r}_{t+1,t+1,t+1}$, at earnings annoucement date will be lower.

2.12 Conclusion

This paper first proves the reason why managers want to increase accounting predictability under rational inattention model and finds the bounded optimal quality for manager's disclosure of accounting information. Then I prove with some proper discretion, manager can help investors generate more accurate predictions. I discuss three possible strategies that can increase the predictability of accounting information. Conservatism as one of the strategy can result in investors getting long term benefits. Moreover, based on different type of investors who enter the market with different time, I argue there will be a price trend. Short-term investors can benefit more by using such price movement as a signal to re-evaluate future earnings freely. Thus the newly predicted earnings with the realized earnings can generate a price which will reflect the fundamental price more efficiently. Therefore, based on the results, it seems that my paper has also made the following contributions. First, Inattention model can better explain why there will be a price trend after earnings announcement. Second, I provide an example for using conservatism and managers' discretion to achieve relevance of accounting. Third, I explain a reason conservatism can reduce information asymmetry different from the reason suggested by explanations related with earnings management. Fourth, my results are consistent with many results in positive accounting theory.

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