PROSPECTIVE TEACHERS DEVELOPING FRACTION IDEAS: A CASE STUDY OF INSTRUCTOR'S MOVES

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A dissertation submitted to the

Graduate School of Education

Rutgers, The State University of New Jersey

in partial fulfillment of the requirements

for the degree

Doctor of Education

Graduate Program in Mathematics Education

written under the direction of

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New Brunswick, New Jersey

January 2019

ABSTRACT OF THE DISSERTATION

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Recent data from a cross-national assessment, the Programme for International Student Assessment (PISA), place the United States performance in mathematics at 38 out of 71 countries (OECD, 2016) – one clear indication of the ongoing need for the improvement of mathematics education. This improvement relies, in part, on improving undergraduate mathematics education for prospective teachers of mathematics who should learn mathematics in a manner that encourages active engagement with mathematical ideas (National Research Council, 1989).

Despite the importance of teacher rational number knowledge, the ways in which they successfully acquire that complex body of knowledge are not well understood (e.g. Depaepe et al., 2015; Krauss, Baumert, & Blum, 2008; Newton, 2008; Senk, 2012; Son & Crespo, 2009; Tirosh, 2000). Teachers' capability of building and using different representations of math ideas, including rational number concepts, are considered important areas of mathematical knowledge that must be developed in order to provide meaningful learning experiences for students (National Governors Association for Best Practices & Council of Chief State School Officers, 2010; National Research Council, 2003). Studies on preservice teachers' thinking about fractions have shown that while they bring some knowledge of fractions to their undergraduate

mathematics classes (Mack 1990; Tirosh, 2000**;** Park, Güçler & McCrory, 2012), their misunderstandings are still similar to those reflected in children's fractions learning (e.g. Ball, 1988; Osana & Royea, 2011; Zhou et al., 2006) . Studies have also reported that prospective teachers often enter teacher preparation programs with beliefs inconsistent with the conceptual teaching of mathematics (Ball, Lubienski & Mewborn, 2001; Strohlmann et al., 2015). If improvement in the teaching and learning of mathematics is to be realized, understanding how prospective teachers build and justify their solutions to rational numbers problems will be of importance.

This research, a component of a design study grant funded by the National Science Foundation[1], investigates how prospective teachers extend knowledge of rational number ideas, how they justify solutions and how their beliefs about teaching and learning mathematics evolve. The study also explores the instructor's role and interventions employed within the classroom environment. The students worked on mathematically rich fractions tasks using Cuisenaire rods as they developed representations to understand the concept of unit fraction, to compare fractions, and to build ideas of fraction equivalence. The study is guided by the following research questions:

1. What role does the instructor play in the prospective teachers' building and justification of ideas?
2. What types of interventions does she employ?
3. What changes, if any, in prospective teachers' beliefs about doing, teaching and learning mathematics can be identified over the course of the intervention?

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[1] The Cyber-Enabled Design Research to Enhance Teachers' Critical Thinking Using a Major Video Collection on Children's Mathematical Reasoning is a research and development project sponsored by the National Science Foundation [award DRL-0822204] conducted at Rutgers University and University of Wisconsin, Madison and directed by Dr. Carolyn A. Maher

The videotaped data of six female subjects in a mathematics class at a liberal arts college were captured with two cameras for two 60-minute class sessions. During the sessions, students explored fractions ideas while working with partners in small groups, discussed solutions, and built models to justify solutions. Two sessions of videotaped data, transcripts, student work, beliefs assessments and observation notes were analyzed using the analytical model described by Powell, Francisco, and Maher (2003).

This study contributes to an under-researched body of literature by examining instructor's pedagogical and question moves as prospective teachers build representations of rational number concepts and justifications for solutions to problems within an undergraduate mathematics course. Its findings may be of value to colleges of education as they redesign curricula intended to improve prospective teachers' understanding of and capability for representing rational number ideas.

DEDICATION

- To my committee:
  - Dr. Carolyn A. Maher, for bringing me into the program, for her insight, patience, unwavering belief and constant encouragement. The opportunities, sound advice and constant motivation that she provided throughout my studies provided the necessary springboard for the completion of my dissertation.
  - Dr. Elizabeth Uptegrove for being a wealth of knowledge and an amazing thought partner
  - Dr. Arthur Powell for his encouragement and sharp intellect which simultaneously challenged, intimidated, and inspired me to think more deeply about this work.
- To my children - Ricky, Kyle, and Marcus – for their unconditional love and inspiration, without which I could not have completed this doctoral degree. I am incredibly blessed and proud to be your mom.
- To my family for the countless ways in which they have both knowingly and unknowingly supported me throughout my studies. The support system provided as I dealt with the day-to-day challenges of achieving my goals was invaluable.
- To my former students who have always challenged and inspired me to keep learning and practicing in order to become the best educator that I can possibly be.

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1 THEORETICAL FRAMEWORK

1.1 Understanding Mathematics

Much work has been done on understanding 'mathematical understanding' in an effort to answer questions such as 'How do we come to understand?' and 'What are the conditions for understanding to occur?' (Pirie and Kieren, 1992). According to Davis (1992), understanding a new idea requires that it fit into a "larger framework of previously assembled ideas." Thus, a new idea is constructed and must connect with some prior understanding.

Davis references the work of Pirie and Kieren and their theory of growth of mathematical understanding. Pirie and Kieren offered a model to trace growth in understanding, describing it as a whole dynamic process and not as a single or multi-valued acquisition, nor as a linear sequence of knowledge categories (Pirie & Kieren, 1994). Their theory of growth, constructivist and recursive in nature, attempts to elaborate the constructivist definition of understanding and describes understanding as "the personal building and re-organization of one's knowledge structures" (Pirie & Kieren, 1992, p. 243).

Hiebert and Carpenter (1992) consider a mathematical idea or procedure or fact understood if it is part of an internal network. So, the mathematics is understood if its mental representation is part of a network of representations. Hiebert and Carpenter (1992) also conclude that the degree of understanding is determined by the number and the strength of the connections. The mathematical idea is understood more thoroughly if it is linked to existing networks with stronger or more numerous connections.

Skemp (1976) differentiates between two forms of mathematical knowledge: relational and instrumental understanding. By relational understanding, he refers to a grasp of mathematical concepts as well as an understanding of why the mathematics underlying those concepts works. Instrumental understanding on the other hand, refers to knowledge of rules and procedures. Skemp also opines that, in contrast to instrumental mathematics, relational mathematics is adaptable to new situations and is easier to remember than memorized procedures. Many who study mathematics learning agree that understanding involves recognizing relationships between pieces of information (Hiebert & Carpenter, 1992).

Constructivist theory, grounded in the view that a person's knowledge is composed of building blocks that form mathematical ideas (Davis, 1984), views knowledge construction as contingent on experiences and perception. These building blocks originate in a person's experiences and the mental images derived from previous experiences can be used to build mathematical ideas (Maher, 1998). Davis and Maher (1997) explain that new knowledge is constructed from old knowledge and that by carefully designing students' experiences, new ideas can be integrated accurately into the students' schema. As students create appropriate schemas to make sense of new knowledge, understanding grows out of the formation of connections. Making sense of knowledge is the act of reasoning that derives knowledge from experiences (von Glaserfeld, 1987).

1.1.1 Reasoning

In order to reorganize knowledge, one must reason. Reasoning, broadly defined, is the process of coordinating evidence, beliefs, and ideas to draw conclusions about what is accurate or true (Leighton, 2004). While Rips (1994) describes reasoning as a "mental process that creates new ideas from old ones," Thompson (1996) considers reasoning the 'purposeful inference, deduction, induction, and association in the areas of quantity and structure.' Each recognizes reasoning as a process. However, reasoning is also a tool that is used within the process of understanding that leads to knowing.

Good reasoning ability is prerequisite to understanding. Ball and Bass (2003) discuss the importance of reasoning in school mathematics, positing that mathematical understanding is impossible without reasoning. They assert that without reasoning, understanding mathematics would only be procedural or instrumental. Thus, using mathematical knowledge requires reasoning. Without conceptual understanding, that mathematical knowledge is difficult to use and difficult to apply in new and varied contexts.

Yackel and Hanna (2003) recognize the social aspects of reasoning, describing it as a communal activity that learners participate in as they interact with one another to solve mathematical problems. Skemp (1979) highlights the social construct of convincing others and finds that both the reasoning of justification and logical understanding involve convincing others of the truth of and the rationale supporting the mathematical ideas that one builds. Ball and Bass (2003) describe reasoning as a set of social norms shared by the community. Thus, the ability to convince others through argumentation and justification establishes the foundation of mathematical reasoning (Yankelewitz, 2009).

1.1.2 Representations

Crucial to the study of reasoning are the representations that students create. The term representation refers both to process, the act of capturing a mathematical concept or relationship in some form, and to product, the form itself. Observable processes that encapsulate mathematical concepts and the products of such processes are external representations that can be captured; internal representations are in the minds of the people doing mathematics (Goldin, 2003). As such, when considering issues of representation in mathematics, we must think of both internal and external representations (Hiebert & Carpenter, 1992).

Representations are central to the study of mathematics. Previous views of mathematics have held that mathematics is ultimately about symbols written on a page, while newer views advance the belief that mathematics is a way of thinking that involves mental representations of problem situations and of the relevant knowledge that involves dealing with these mental representations(Davis, 1992). Although it may make use of written symbols, the real essence of mathematics is that which takes place within the mind (Davis, 1992).

1.1.2.1 Mental Representations

Hiebert and Carpenter (1992) establish that to both think about, and ultimately to communicate mathematical ideas, we need to represent them in some way and we necessarily represent them internally. Through a process of constructing internal mental representations, learning – the modification of these mental representations in order to construct mathematical relationships – occurs (Cobb, Yackel and Wood, 1992). Since these internal representations and constructing of relationships are not observable, they can only be inferred (Goldin, 2003).

How do learners build mathematical knowledge? According to Davis (1984), a learner builds mental representational structures that are framed within his/her prior experiences. Davis and Maher (1998) stress that ideas the learner builds through such prior experiences constitute the additional cognitive building blocks for constructing representations. New experiences that create data for the learner to process such that when faced with a mathematical task, a learner first builds mental representations for both the input data and any prior, relevant knowledge. The learner must then construct, evaluate, and possibly modify a mapping between those two mental representations – the input data representation and the existing knowledge representation. (Davis & Maher, 1990). Davis (1984) refers to the process of creating representations from cognitive building blocks as 'assembly' and uses this term to describe "how a new knowledge representation is built up using bits and pieces of previously synthesized knowledge representation structures" (p. 154).

1.1.2.2 External Representations

Lesh, Post and Behr (1987) take the position that some relationship exists between external and internal representations. While building internal mental representations leads to an individually generated external representation of a mathematical idea, and features of those mental representations are made public through external representations, mathematical meaning is not inherent in external representations. The meaning of the external representation is a product of an individual student's interpretation. Thus, absent the student's explanation, any relationship between external and internal representations can only be inferred.

A particular mathematical idea can often be represented in any one form or in multiple forms of representation (Hiebert & Carpenter, 1992). Lesh et al. (1987) identify five types of representation systems: experiential, manipulatable models, pictures or diagrams, spoken language, and written symbols. In experiential representations (or experience based scripts) knowledge is organized around real-world events that are the context for interpreting and solving problems. Manipulatable models – concrete objects such as base ten blocks and Cuisenaire™ rods, have an intuitive appeal and support learning particular ideas. Pictures or diagram representations are static models that can be internalized as images. Spoken language representations and written symbols can refer to specialized languages or sentences, as well as normal English sentences or phrases. While these forms of representation have long been part of school mathematics, unfortunately, they have often been taught and learned as if they were ends in themselves (Goldin, 2003).

To think about and communicate mathematical ideas, we need to represent them in some way (Hiebert & Carpenter, 1992). Communicating math ideas requires that the representations be external. It is important to distinguish external systems of representation from internal, psychological representational systems of individuals. Such internal systems include personal symbolization, personal assignments of meaning to mathematical notation, natural language, visual imagery and spatial representation, problem solving strategies and heuristics, and affect in relation to mathematics (Goldin & Shteingold, 2001). Given the personalization of individual representations, the notion of representation as the ultimate goal of mathematics limits the power and utility of representations as tools for learning and doing mathematics (Goldin, 2003).

1.1.3 Rational Number Ideas

Rational number concepts, while complex, are among the most important mathematical ideas children encounter in the early grades. Rational number ideas are also the arena in which many of the trouble spots in elementary school mathematics arise. Siegler and Lortie-Forgues (2017) report on two main classes of difficulties underlying poor understanding of rational number ideas - inherent and culturally-contingent sources of difficulty. Inherent sources of difficulty are those present regardless of the educational institution. For example, understanding individual rational numbers, one inherent source of difficulty presented by Siegler and Lortie-Forgues (2017), requires distinguishing between rational and whole number representations and relationships. Whole numbers have unique predecessors and successors while between any two rational numbers are an infinite number of other rational numbers. Culturally contingent sources of difficulty, such as teacher knowledge and textbooks, vary with particular students' lives.

Fractions are generally the first experience students have with rational numbers. These early experiences are often meant to develop students' understanding of fractions as numbers.

For example, the fraction "1/4" represents the number that is midway between 0 and 1/2 on a number line. Carraher (1996) contends that viewing a fraction simply as a number is inaccurate. Fractions are also meaningful representations of relationships and understanding them requires understanding relationships between numbers, and the ability to express these relationships in varied ways.

The 1983 work of Behr, Lesh, Post, and Silver asserts that rational numbers can be interpreted in multiple ways; a part-to-whole comparison, a decimal, a ratio, an indicated division (quotient), and an operator exemplify some of the interpretations. "1/4" can represent the equal sharing of 1 candy bar among 4 people, a measurement such as 1/4 mile, a ratio such as 1 out of 4 cupcakes, the quotient of dividend '1' and divisor '4', and as an operator useful for finding 1/4th of the number of 3rd grade students. Post, Behr, Harel, and Lesh (1993) cite these multiple interpretations as contributing to the difficulty that children have in attaining clear understanding of fraction ideas. Further, Freudenthal (1986) posits that learning a new idea with so many different associated meanings presses the student to sort and attach a proper interpretation in each instance before considering any arithmetic approach to a situation.

The traditional way students learn about fractions compounds the complex ideas associated with understanding of fraction. Traditional instruction emphasizes memorization of algorithms and permits insufficient experience with authentic problem solving, thereby detaching learning from sense-making and real-world experiences. Huinker (1998) cautions that a premature introduction of algorithms is damaging to students because the nature of mathematics is distorted. With the imposition of meaningless rules for operating on fractions, a disconnect between understanding of fraction as operator and sense-making of fraction as number occurs. Many researcher studies support the perspective that the operator sense of fractions dominates discussion of the meaning that learners attribute to fraction (Dienes, 1967; Kieren, 1994; Behr et al., 1992; Freudenthal, 1986), while algorithms involving fractions are derived from the concept of a fraction as number (Steencken, 2001).

Units play an important role in understanding fraction concepts and operations. A unit may be a whole – an entity which can be partitioned. A unit may also refer to an amount with which to generate a new amount. These understandings are foundational for defining wholes as well as success with more challenging topics, such as operations (Tobias, 2013).

With fractions, unitizing, a cognitive process for conceptualizing the amount of a given commodity before, during, and after the sharing process, aids students' ability to describe the whole being used in a problem (Tobias, 2013) and to understand fractions as quantities (Lamon, 2002). For example, one third of one whole is not equivalent to one third of another whole when the wholes are different. Unitizing is important for students to understand unit fractions, iterating unit fractions, and composing units (Lamon, 2005).

1.2 Teaching and Learning Mathematics

Improving the teaching and learning of mathematics has been difficult. Ball et al. (2001), having surveyed decades of research on reform efforts, identify five problem areas: (1) the misrepresentations of mathematics that manifests as students are inundated with skills and procedures without developing an interest in and appreciation for the power of mathematics, (2) the resilience of common patterns of instruction reflecting intellectual traditions that expect students to imitate, copy, and memorize knowledge received through transmission, (3) institutional factors such as teacher isolation, time constraints which make taking pedagogical experimentation risky, and preoccupation with standardized test scores that pressures teachers towards a traditional curriculum and a focus on basic skills, (4) the conservative nature of local assessment and curricular materials that often provide inadequately developed concepts, and (5) the weak impact of professional education, particularly preservice teacher education, on teachers' knowledge and beliefs.

Discourse about the desirable ends of mathematics teaching and learning has centered on the development of mathematical power – the capacity to make sense with and about mathematics (Ball, 1990). Sense-making, crucial to learning mathematics, enables the learner to make connections between informal concepts and more formal mathematical ideas.

Learning mathematics takes place over time as a result of repeated experiences that are connected through personal sense-making (Griffin, 1989). Learning includes long-term conceptual development, a learner's shift between attending to relationships and perceiving relationships as properties applicable in other situations (Mason, 2004), and reflects advances in abstract understanding (Watson and Mason, 2006).

Helping students develop this kind of mathematical power requires insightful consideration of both content and learners; careful analysis of the specific content to be learned and understanding of how the students themselves learn particular content is required (Ball, 1990). Therefore, the teacher's role, argues Ball (1990), requires a bifocal perspective - perceiving the mathematics through the mind of the learner while perceiving the mind of the learner through mathematics.

1.2.1 Role of the Instructor

Constructivism, a theory of learning or meaning making, can dictate only guidelines for constructivist pedagogy (Noddings, 1990). Translating a theory of learning into a theory of teaching has proven challenging. In distinguishing between constructivism and constructivist teaching, Maher (1998) theorizes that the constructivist teacher is one who:

encourages children to make conjectures and pursue the reasonableness of their ideas by constructing models, comparing them, developing arguments, discussing ideas, and negotiating conflicts while working on problematic situations that either have been presented to them or that they themselves have initiated and extended. (1998, p. 39)

A necessary component of mathematics instruction, particularly that which supports work on more challenging problems, is attending to the development of student reasoning. Davis (1992) describes teaching mathematics as a matter of guiding student development of a personal repertoire of basic building blocks and helping students develop skill in building and using mental representations.

Effective instruction supports students as they build particular organizational and classification schemes that are necessarily representations of their thinking and understanding. Teachers' awareness of students' thinking and the timely use of questioning are essential to developing mathematical thinking (Maher & Martino, 1999). Additionally, teachers' recognition of and belief that learning is a process of both individual and social construction (Simon, 1995) necessarily informs their pedagogical lens and guides their instructional practice.

1.2.2 Beliefs about Mathematics

A frequently held conception in education is that teachers 'teach they way they were taught.' Research demonstrates the more complex reality that teachers' professional identities are influenced by many factors including their subject matter knowledge, social and political context, family influences, and knowledge developed over time about how to teach particular topics (Shulman 1986; Beijaard et al., 2004). Further, a substantial body of research suggests that teachers' beliefs and values about teaching and learning affect their teaching practices (Clark & Peterson, 1986; Fang, 1996; Kagan, 1992; Thompson, 1992). For example, if a teacher regards mathematics as a set of explicit rules to be followed, classroom practice will tend to focus on memorization, calculation and developing procedural skills. Conversely, if doing mathematics involves complex processes requiring heuristics and analysis, then learning activities that extend beyond memorization and procedural skill, and modes of inquiry are appropriate (Davis, 1990).

All mathematical pedagogy rests on a philosophy of mathematics (Thompson, 1992). While the beliefs upon which a philosophy of mathematics rests may be fairly stable and resistant to change (Brandt et al., 2012), beliefs can also be held with varying degrees of conviction. Thus, an opportunity to shift beliefs about what mathematics is, what value it has, how it is learned, who should learn it, and what mathematical reasoning entails, exists. In order to shift prospective teachers toward adopting teaching practices that are grounded in evidence about how learning occurs, gauging and influencing teachers' beliefs is critical (Stipek et al., 2001).

2 LITERATURE REVIEW

2.1 Introduction

This study situates itself in discourse related to the teaching and learning of rational number ideas. While there are many pedagogical philosophies regarding teaching and learning mathematics in general, this review focuses narrowly on representations elicited by means of particularly sequenced instructional tasks, prospective teachers as learners, and the instructor's role as an intermediary. The goal of this review is to position this study in discussions of interventions for prospective teachers and the instructor moves that undergird those interventions.

The research on prospective teachers' rational number idea development can be organized into three themes. The first theme that will be discussed is the various representations and the sequencing of ideas associated with rational number concepts. A second section discusses the role of the instructor as the facilitator of learning and the moves employed in order to probe students' reasoning and elicit justification. The third theme examines the ways in which mathematical reasoning about rational number ideas is developed in prospective teachers in the context of undergraduate mathematics courses.

2.2 Role of the Instructor

The view of constructivism as a theory of learning guides much of the development of constructivist pedagogy (Richardson, 2003). Maher (1998) describes classrooms that promote 'constructivist teaching' as those that might be characterized by a teacher who (1) provides experiences from which students can build powerful repertoires of mental images to draw upon for the construction of representations of mathematical ideas; (2) assesses the ideas that a student builds by observing their activity (model building) and listening to their explanations; (3)

encourages the students to support ideas with suitable justifications and arguments; (4) works to build a classroom culture that encourages the exchange of ideas; (5) calls differences and disagreements to the attention of students; (6) facilitates the organization and reorganization of student groups to allow for the timely sharing of ideas; (7) encourages student-to-student and student-to-teacher efforts to map representations and develop modes of inquiry that might disclose deeper understanding of discrepancies; (8) provides multiple opportunities for students to talk about and represent ideas; (9) keeps discussion open and revisits ideas over sustained periods of time; and (10) seeks opportunities for generalizations and extensions. These characterizations reflect the non-traditional role of the instructor as an active participant who attends to children's cognitive development and encourages discourse in the classroom community (Maher, 1998).

The instructor's role in task design and selection is crucial in framing desired learning experiences that encourage mathematical reasoning and facilitate student engagement. Doerr and English (2006) assert that tasks should be designed to encourage students to use representations as a window into their thinking which then enables the community of learners to view and understand their ideas. Instructors also facilitate discussions and probe for better understanding of student thinking. These probes manifest through appropriate, timely, purposeful questioning directly related to students' constructions and require an in-depth knowledge of mathematics as well as children's learning of mathematics (Maher, 1998; Smith and Stein, 2011). Yankelewitz et al. (2010) report on two studies in which fourth and sixth grade students investigated a strand of tasks involving Cuisenaire rods and were encouraged to both justify their solutions and question other's explanation. An early task prompted students to find the correct rod that could be called one half when the blue rod was called one. David, a fourth grader, reasoned that there is no such rod. After the instructor questioned his hypothesis, David justified his assertion using an upper and lower bounds argument. Through this task, the instructor provided an experience for building mental images of an idea, in this example a linear representation of one-half, observed the student's model building, and questioned the student's hypothesis as a means of making his reasoning available to the community for questioning (Yankelewitz et al., 2010).

Research by Maher (1998) emphasizes the significance of providing multiple opportunities for students to talk about and represent ideas. Gerstein and Yankelewitz (2017) offer further analysis of the Colts Neck study as students investigate the notion of fraction equivalence. During the fourth session, researcher Martino asks what two white rods would be called if the orange rod were given the number name one (Gerstein and Yankelewitz, 2017). Mark, using an orange, red and two white rods, constructs a model and justifies his solution of one fifth (Gerstein and Yankelewitz, 2017). Researcher Martino provides further opportunities for students to talk about and represent ideas by subsequently asking if there are other solutions. Meredith volunteers a solution of two-tenths and builds a model of one orange rod and ten white rods (Gerstein and Yankelewitz, 2017). Student-to-student efforts to justify and map varying representations ensues as Researcher Martino indicates that she is confused because she believes the various models (Gerstein and Yankelewitz, 2017).

The instructor's role in discourse is also critical. Using intentional teacher moves to promote discourse, the role of instructor is to establish a classroom culture encouraging exchanges of ideas, listen, encourage justification and argumentation, facilitate inquiry and timely sharing of ideas, and provide multiple opportunities to talk about, represent, and revisit ideas. Interactions between instructor and learner that result from teacher moves shape students talk and help to construct understanding. Chapin et al. (2009) emphasize both student-to-student and teacher-student communication in Project Challenge, a four-year study involving approximately 400 Boston school district students in grades 4 through 7. The instructors maintained a consistent focus on explanations of students reasoning while emphasizing communication through support for both lengthy and brief discussions. (Chapin and O'Connor, 2007). Results of the Project Challenge study provide strong evidence that student learning is greatly supported by student engagement in and a sustained emphasis on academically productive talk (Chapin and O'Connor, 2007).

The timing of questions and the pauses between them are also important. It is important not only to wait after a question is posed, but it is equally important to wait after the student responds (Herbel-Eisenmann, 2009). Providing this time allows other students process time during which they determine whether they agree or disagree, and what contributions to make to the discussion (Gronewold, 2009). These subsequent contributions make take the form of questions and situations raised by students, and may be used judiciously to further guide instruction. Decades of research on wait time, defined in terms of the duration of pauses separating utterances during verbal interaction, highlight numerous benefits of pausing for longer periods of time before speaking (Tobin, 1986). Having reviewed studies involving wait time across a range of subjects and grade levels, Tobin (1987) finds that when average wait time was greater than 3 seconds, changes in both teacher and student discourse were observed. Increases in middle school mathematics achievement were also reported. These findings suggest that wait time may facilitate higher cognitive level learning by providing teachers and students with additional time to process information.

2.3 Rational Number Ideas

The study of both the learning and teaching of rational number ideas has been a crucial area of mathematics education research for many years. A review of the research on rational number learning indicates researchers continue to focus on the various aspects of the topic. Considerable research has been conducted focusing on children's learning in the context of experimental instructional materials including physical manipulatives and pictorial representations (Behr, Harel, Post, & Lesh, 1993; Kamii & Kirkland, 2001; Maher & Yankelewitz, 2017; Steencken & Maher, 2003; Schmeelk, 2017). These studies frequently note common misconceptions in children's unsuccessful efforts while using algorithms, and the sense-making void frequently associated with such efforts. Although most students eventually learn the specific algorithms they are taught, retention and conceptual knowledge often remain deficient. Physical manipulatives, particularly linear models, can support the requisite meaning making critical in the acquisition of conceptual understanding of rational number ideas.

The Rational Number Project (RNP), a multi-university NSF funded research effort, developed instructional and assessment materials concerning rational number sub-concepts: part-whole, measure, quotient, decimal, and ratio. The curriculum designed reflected four beliefs: (1) children's learning about fractions can be optimized through active involvement with multiple concrete models, (2) most children need to use concrete models over extended periods of time to develop mental images needed to think conceptually about fractions, (3) children benefit from opportunities to talk to one another and with their teacher about fraction ideas as they construct their own understandings of fraction as a number, and (4) instructional materials for fractions should focus on developing conceptual knowledge prior to formal work with symbols and algorithms (Cramer et al., 2009; Cramer and Henry, 2002). Of particular interest was the role of physical models on the learning of the sub-concepts, as well as the use of math concepts as understanding progressed from concrete to abstract (Behr et al., 1984). The project yielded several long-term studies regarding the teaching and learning of fractions among fourth and fifth grade students (Bezuk & Cramer, 1989; Post et al., 1985).

Research by Post and colleagues (1985) emphasized the significance of physical models and strategies utilized as understanding progressed from concrete to abstract. Part-whole interpretation of rational numbers was facilitated by teachers using both circular and rectangular physical models. Subsequent lessons engaged subjects in modeling solutions with Cuisenaire rods, paper folding, poker chips, and number lines. As the students discussed the solutions to the mathematical tasks, researchers interview questions revealed the strategies that students chose as they participated in the tasks (Post et al., 1985).

The 18-week teaching experiment, conducted in Minnesota and Illinois, included a combination of individual and group work for 12 fourth grade students, six at each site. Before introducing color-coded rectangular models, the teaching experiment introduced color-coded circular models, encouraging students' observation that as size decreases the number to make the whole increases (Post et al., 1985). Students investigated fraction equivalence using paper folding with circles and rectangles, and translated between circular and rectangular models before attaching unit fraction names to models. (Post et al., 1985). Among the tasks students participated in were those requiring use of Cuisenaire rods to name unit fractions, noting fractions as sums of unit fractions, and translating across various physical and pictorial representations (Post et al., 1985).

Each student was interviewed individually on 11 separate occasions throughout the teaching experiment, with each interview audio taped or videotaped. The interviews solicited a verbal explanation or demonstration while administering items that required ordering, assessing the equivalence of or generating equivalent fractions (Post et al., 1985). Results were analyzed according to the three classes of fractions used in the items: fractions with the same numerators, fractions with the same denominators, and fractions with different numerators and denominators. The findings reflect an analysis of students' varied approaches or strategies for comparing fractions (Post et al., 1985). One such strategy, the 'manipulative' strategy in which a student explains his or her response using pictures or manipulative materials, occurred least frequently amongst the valid strategy types for each class of fraction (Post et al., 1985). Considering all three classes of fractions, the manipulative strategy occurred most frequently for the class of fractions that embodied different numerators and denominators – generally a more cognitively demanding task.

Acquisition of quantitative understanding of fractions is based on individual experiences with physical models and on instruction that emphasizes meaning-making rather than procedures (Bezuk & Cramer, 1989). Thus, use of manipulatives is crucial to the development of rational number ideas. Manipulatives aid in the construction of mental images that are essential for meaningfully performing fractions tasks. Among several recommendations that Bezuk and Cramer (1989) offer regarding physical models are the following:

a) use manipulatives at each grade level to introduce all components on fractions
b) delay work with operations to allow necessary time for work on concepts
c) base primary grades instruction on whole-part concepts using first the continuous physical model and then the discrete physical model
d) in primary grades, ask students to name fractions represented by physical models and diagrams
e) use words (two-thirds) initially, then introduce symbols (2/3)

Maher and Yankelewitz (2017) report on a study of fourth grade students investigating fraction ideas under conditions supporting investigation and argumentation. The long-term partnership between teachers in the suburban, public school district of Colts Neck, NJ and the faculty of Rutgers University focused initially on challenging students to construct personal knowledge of fraction concepts such as fraction as number, fraction equivalence, fraction comparison, and operations with fractions (Maher and Yankelewitz, 2017). Steencken and Maher (2003) report on the early investigations, paying particular attention to the flow of the ideas of children whose activities include constructing representations to show part of some finite quantity. In later sessions, students explore fraction properties, perform fraction operations and represent fractions as number. Over the course of these videotaped sessions, the researchers noted that students' language, as they communicate their ideas, becomes increasingly precise (Steencken and Maher, 2003).

An important aspect of the Colts Neck study is the researchers' design of open-ended tasks, monitoring developing ideas of students, and creating new tasks as their judgment suggested (Maher and Yankelewitz, 2017). Researchers designed an adaptive intervention, developing new learning experiences based on the shared ideas of students - a novel approach in studies incorporating experimental instructional materials. The intervention comprised tasks in which learners build models of the fraction ideas that they explored using Cuisenaire rods, attending to the attribute of length. After working on a task or group of tasks, learners were invited to share their solutions by reconstructing earlier models while being encouraged to justify their solutions.

The videotaped sessions of the Colts Neck intervention have been studied by many researchers. Yankelewitz (2009) investigates the forms of argumentation, both its structure and purpose, and forms of reasoning elicited as students work on tasks involving the building of fraction ideas. The study also examines the ways in which student reasoning evolves as students revisit tasks previously introduced. The findings provide insight into students' construction of direct and indirect arguments, as well as justifications and use of counterarguments to refute claims. Further, Yankelewitz (2009) identifies several forms of reasoning elicited as students work on tasks. The forms of reasoning include generic reasoning, reasoning by cases, recursive reasoning, and reasoning by upper and lower bounds. Students were found to spontaneously reason indirectly; a potential indication that indirect reasoning is becoming a way of thinking.

Analysis of the first seven sessions of the Colts Neck intervention by Steencken (2001) evidences the fraction ideas children build, the representations that they use, and how mathematical ideas travel within the classroom. The study finds that children often use different methods to find solutions and often used each other's ideas to assess and/or modify their own thinking. They assisted one another in presenting models and justifying solutions. Children expressed their thinking both verbally and non-verbally, as well as with drawings, constructions and written exchanges. These varying expressions of thought allow mathematical ideas to travel among the community of learners (Steencken, 2001).

One initial goal of Colts Neck intervention was to coordinate students' understanding of fraction as operator with fraction as number as a means of avoiding inappropriate generalizations and in order to annex appropriate extensions of the whole number system to include fractions and their associated ideas (Maher and Yankelewitz, 2017). Reported studies offer evidence that teachers, like children, have similar difficulty conceptualizing fractions and making meaning of fractions in contextualized and decontextualized scenarios (e.g., Lesh & Schultz, 1983; Post et al., 1985).

2.4 Prospective Teacher Education

The urgent need to revitalize mathematics education persists. In *Everybody Counts: A report to the nation on the future of mathematics educ*ation, the National Research Council (1989) reports on a number of challenges to renewing mathematics education. The challenges, among many, include a shortage of qualified mathematics teachers, a need for K-16 curriculum and instruction that demands higher order thinking skills and stimulates students' mathematical interests, and a proliferation of intellectually stagnant undergraduate mathematics courses (NRC, 1989).

The relative impact of colleges and universities on teacher education has received a great deal of attention in the literature (Zeichner & Tabachnick, 1981). Nonetheless, critics cite a weak impact of professional education on teachers as contributing to the difficulty of improving mathematics outcomes; specifically, they observe that preservice teacher education typically has a weak effect on teachers' mathematical knowledge (Ball et al., 2001). The Mathematical Education of Elementary Teachers (MEET) project explored preservice teachers learning in their undergraduate mathematics classes, with a particular focus on fractions. In their analysis of the MEET data, Parke et al. (2013) sought to understand what is taught and learned in undergraduate mathematics courses and to understand the general goals of teaching the course. In a subsequent analysis of MEET video data, observed teaching practices revealed that instructors rarely mentioned fraction-as-number or made explicit connections to the ways that fractions fit into the whole number system (Park et al., 2013). This is consistent with other studies that show teachers tend to overgeneralize their knowledge of whole numbers when working in the domain of fractions (Tirosh et al., 1999).

Many studies have been conducted to better understand prospective elementary teachers' rational number conceptions and misconceptions (Hill et al., 2005). Newton (2008) pursues a comprehensive understanding of elementary teachers' understanding by investigating five aspects of fractions knowledge - computational skill, basic concepts, word problems, flexibility and transfer – across all four operations. With multiple sections of an undergraduate-level elementary school mathematics course as the context for analyzing teacher knowledge and administration of fractions pre and post assessments, the study offers important findings and implications (Newton, 2008). First, because dichotomizing mathematical knowledge into procedures and concepts does not fully account for its complexity, Newton (2008) recommends more studies examine knowledge from multiple perspectives, including the analysis of correct solution methods. Second, studying related topics together (e.g. including all four operations in a study) reveals patterns that would otherwise go unnoticed (Newton, 2008). For example, the misconception that the denominators rather than the operation determined the algorithm was most prevalent misconception in the study (Newton, 2008).

In a similar fashion, Tobias (2013) uses prospective elementary school teachers' work samples and classroom conversations to illustrate difficulties with defining the whole and conceptualizing particular language for describing fractional amounts. In contrast to Newton (2008), Tobias (2013) emphasizes uniquely designed activities, problems focused on part-whole understanding that provide a foundation for language skills to develop, explaining and justifying solutions and solution processes, and the reinforcement of socio-mathematical norms. Taken from a content course focusing on mathematics for teaching elementary school, coding of conversations revealed persistent difficulty using appropriate language for describing the whole. This was noted especially when the problems, which used pizzas as a context, involved more than one pizza but also when pizzas represented fractions less than one. Tobias' results (2013) provide insight into the types of understandings prospective teachers bring to teacher education programs and indicate that when teachers develop understanding of language for fractions less than one, this does not signal understanding of language for fractions greater than one.

Researchers employ specific instructional interventions within teacher education courses in order to study varying aspects of prospective teachers' knowledge of mathematics (Toluk-Uçar, 2009; Osana & Royea, 2011; Lin et al., 2013). Problem posing refers to generating a new problem or question, as well as reformation of a problem, during the problem-solving process (Silver, 1994). Toluk-Uçar (2009), in designing a methods course intervention, limits the notion of problem posing to that of generating an original problem from a given situation. The 2006-2007 study investigated the effect of problem posing as a teaching strategy on pre-service primary teachers and was intended to elucidate their existing understanding of fractions. Teachers' learning experiences focused on discussions of the appropriateness of the word problems generated and justifications of posed problems using different forms of representations.

While external representations can facilitate understanding of mathematical concepts (Janvier, Girardon, and Morand, 1993), a single type of representation does not convey one's understanding of a concept (Stylianou & Pitta-Pantazi, 2002). Lesh et al. (1987) posit that both translations across representation systems as well as transformations within a representation system are important. In the Toluk-Uçar (2009) study, teachers' representations were largely limited to area models, an indication of a lack in flexibility with representational systems.

In a small-scale study of eight undergraduate students, Osana and Royea (2011) implement one-on-one fractions instruction in an elementary teacher training program. The three-week summer intervention, implemented before any participants had taken any of the required mathematics methods courses, had been designed to address specific challenges noted in the university's mathematics methods courses.

The intervention, a replication of the fractions unit from the methods course, required students to solve a series of word problems involving fractional quantities. For each word problem, students were asked to draw a picture that could assist with determining a solution, to write a number sentence for the problem that had been solved. During the problem solving, the instructor highlighted specific foundational fractions concepts that were inherent in the student's solution, and made connections between the student's model and number sentence explicit (Osana & Royea, 2011).

As part of the pretest-posttest design for the study, measures of conceptual and procedural knowledge constituted an attempt to examine effects of the intervention on preservice teacher knowledge and to document the challenges that teachers encounter during the intervention (Osana & Royea, 2011). Included in this assessment was a fractions test designed by Saxe, Gearhart, and Nasir (2001), along with four problem-posing transfer tasks. The problem-posing tasks required teachers to attach meaning to situations by creating word problems for given number sentences. Since they were not a component of the intervention, these tasks were considered transfer tasks.

Consistent with Johnson (1998) who concluded that preservice teachers lack the number sense to solve problems in creative non-algorithmic ways, Osana and Royea (2011) found that reliance on procedures blocked the ability to find mathematical structure in problems and prevented the ability to make sense of word problems and invent meaningful solutions. Further, the researchers found that teachers actively sought to learn procedures that could be applied across problems.

Researchers have continued to find empirical support for the intuitive notion that when elementary teachers possess deep understanding of mathematics, their students learn more (Newton, 2008; Hill et al., 2005). Tirosh et al. (1998), through the conducting of personalized interviews of both mathematics and non-mathematics majors, aim to understand prospective elementary teachers' conceptions of rational numbers and to develop didactic approaches to help them extend (1) their mathematics conceptions and (2) their knowledge of how children think about those concepts. A study by Isiksal and Cakiroglu (2010) similarly focused on prospective teachers pedagogical content knowledge (PCK), analyzes results of a multiplication of fractions questionnaire and results of interviews designed to obtain additional information about each prospective teacher's PCK. This case study on prospective teachers' knowledge of common conceptions and misconceptions held by sixth and seventh grade students about fraction multiplication finds that teachers' perceptions of students' mistakes fall into five categories: algorithmically based mistakes, intuitively based mistakes, mistakes based on formal knowledge of fractions operations, misunderstanding of the symbolism of a fraction, and misunderstanding of the problem. The resulting analysis lead Isiksal and Cakiroglu (2010) to recommend that teacher education programs familiarize prospective teachers with various common types of cognitive processes, including erroneous ones. They further recommend that these programs familiarize teachers with how these cognitive processes may lead to various ways of thinking (Tirosh, 2000).

Teachers' ability to use varying representations of mathematical ideas is deemed an important area of mathematical knowledge to develop in order to provide meaningful learning opportunities for students (National Research Council [NRC], 2003; National Governors Association Center for Best Practices & Council of Chief State School Officers [NGA &

CCSSO], 2010). This mathematical knowledge base includes both subject matter knowledge (SMK) and pedagogical content knowledge (PCK), notions first coined by Lee Shulman (1986). A number of studies (e.g. Tirosh, 2000; Depaepe et al., 2015; Lin et al., 2013) analyze the rational number content knowledge and pedagogical content knowledge of prospective elementary teachers as a means of unearthing gaps in understanding, assessing the impact of particular interventions (e.g. open approach instruction), and generally promoting the need for awareness of likely sources of common misconceptions held by children and prospective teachers.

2.5 Intended Contribution of the Study

In the literature on rational number ideas, rational number knowledge of prospective teachers and the role of the instructor, considerable research focuses on rational number knowledge acquisition of children. While much research has attended to teachers' understanding of fractions operations, prospective teachers' conceptual understanding and representational knowledge of rational number ideas is a burgeoning area of focus. The role that the instructor plays in prospective teachers' mathematical knowledge acquisition is a largely untapped area of study. Specifically, there is a lack of attention to the role of the instructor as prospective teachers reason about rational number concepts, build representations of the associated mathematical ideas, and justify solutions to tasks that elicit rational number idea reasoning. Given this gap in the literature, this study contributes to the literature by examining the following:

1. What role does the instructor play in the students building and justifying of ideas?
2. What types of interventions does she employ?
3. What changes, if any, in prospective teachers' beliefs about doing, teaching and learning mathematics can be identified over the course of the intervention?

3 METHODOLOGY

3.1 Research Context

The Math Reasoning and Assessment course under study took place at a private college in New Jersey during the spring semester of 2011. The course is required for pre-service middle school math teachers and met twice per week for 75 minutes. Six prospective teachers enrolled in the course, all of whom were female, engaged in fractions tasks over the course of five weeks. Data from videotaped problem-solving sessions focusing on rational number ideas was analyzed for this study. The sessions analyzed for this study occurred on April 13, 2011 and April 15, 2011. The table below indicates the organization of the activities in the complete fractions intervention.

Table 3.1 Fractions Intervention Activities

Date	**Topic or Activity**
Wednesday, April 13	Fraction Intervention • Video Analysis Upper and lower bound video • Determine relative number-names of rods • Use rod models to determine which is larger, 3/4 or 2/3.
Friday, April 15	Fraction Intervention • Build rod models to solve word problems, write mathematical sentences for the problems and explain how the rods are related to the mathematical sentences
Wednesday, April 20	Fraction Intervention • Problem solving - sharing pizzas
Wednesday, April 27	Fraction Intervention • Problem solving – products and factors, parts of a whole • Problem solving - measurement
Friday, April 29	Fraction Intervention • Problem solving – represent multiplication of fractions analytically and using either rods or drawing
Wednesday, May 4	Mixed Topics • Signed numbers • Taxicab problems
Friday, May 6	Fraction intervention • What role, if any, can manipulatives in understanding fraction addition/subtraction? multiplication? division? • Why is the result larger when you divide by a fraction less than 1?
Friday, May 13	Final exam, Beliefs Inventory, Fractions post-assessment

3.2 Participants

During the spring semester of 2011, six undergraduate students in their junior year were enrolled in the Math Reasoning and Assessment course instructed by a single instructor at a private College in northern New Jersey. The students in the class were all mathematics majors studying to be teachers. All of the subjects were women. All six prospective teachers agreed to be videotaped and that their work could be used for this study. There was a single classroom instructor.

3.3 Setting

This study is a component of a National Science Foundation (NSF) funded design study in its third year. The National Science Foundation (NSF) grant, conducted at Rutgers University and University of Wisconsin, Madison [award DRL-0822204] and directed by Dr. Carolyn A. Maher, funds the establishment of a repository to store a collection of video data and related metadata from earlier NSF funded projects. The videos and related metadata are being prepared for both pre-service and in-service teacher interventions. By collecting and analyzing video data of students engaged in fractions tasks and studying videos of children reasoning, this study extends the work of the grant.

Throughout the intervention, the participants were seated at two adjoining tables as they engaged in both whole group and small group instruction. The two small groups were self-selected. Each was comprised of three participants.

3.4 Tasks

The intervention studied here is composed of two sessions. Each session consists of the prospective teachers working on a set of mathematically rich fractions tasks. Before the initial session, prospective teachers engaged in preliminary fractions activities that required the use of Cuisenaire rods. Cuisenaire rods – as set of 10 colored rods ranging in length from 1 cm to 10 cm. - enable learners to model mathematical ideas and visualize relationships.

In session 1, the prospective teachers worked on a series of fractions problems requiring building models using Cuisenaire rods. The first problem required that teachers build a model for determining the shortest trains that could be measured by two distinctly colored rods. A second problem required building a model for determining the longest train that measures two distinctly colored rods. Several problems ask prospective teachers to identify a rod having a particular number name or to determine the number name of one or more rods when given the number name for one rod in the set of ten Cuisenaire rods. The final two problems for session 1 ask prospective teachers to create a unique problem that can be answered using Cuisenaire rods, and to build a model that can be used determine which of two fractions is larger and how much larger. The in-class tasks for session 2 on 4/15/11 were real world problem solving tasks that required sharing and/or combining fractional portions of pizzas and candy bars.

3.5 Data Sources

This study draws on multiple sources of data including video data of prospective teachers building solutions, writing solutions, and interacting with each other as well as the instructor. The table below lists the video data pertinent to this study.

Table 3.2 Video Data Sources

Date	**Session/Camera**	**Subjects**
April 13, 2011	Session 1 Camera 1	Group 1 • Fae • Sarah • Kelly
April 13, 2011	Session 1 Camera 2	Group 2 • Janelle • Erika • Darlene
April 15, 2011	Session 2 Camera 1	Group 1 • Fae • Sarah • Kelly
April 15, 2011	Session 2 Camera 2	Group 2 • Janelle • Erika • Darlene

Data also include researcher field notes, prospective teachers' written work such as the belief inventory pre- and post- assessments, and assigned classwork.

3.6 Data Collection

Data directly involving the prospective teachers were collected using two video cameras, one for each of two groups. The data collected include video recordings of the prospective teachers working on the fractions tasks as well as the physical models that were created and their written work. The collected written work is included in Appendix F.

3.7 Methods and Coding

For research questions in this study, a modified coding scheme was designed based on the prior collaborative work of a team of researchers. Details of each coding scheme and relevant definitions are described below. Transcripts of video and prospective teachers' essays were coded using each coding scheme. Beliefs inventory data were aggregated into summary statistics and presented in tabular form.

3.7.1 Framework for Analysis of Video Data

In order to analyze the video data, this study used the method of analysis outlined by Powell, Francisco, and Maher (2003). This model uses a multi-phase process to study video data. The application of each phase within this study is described below.

3.7.1.1 Viewing

Powell et al. (2003) describe the first step as attentively viewing each video several times to become familiar with the content. Multiple viewings of each video allow the researcher to observe and record details in the video that may not have been apparent on the first viewing.

3.7.1.2 Describing

Video data inherently contain enormous amounts of information. After watching each video several times, time-coded objective descriptions of the events in the video are written to allow one to quickly locate particular events in the video. The descriptions contain details of the event, but do not reflect any interpretation by the researcher.

3.7.1.3 Identifying Critical Events

In identifying critical events, the researcher selects events that will be highlighted in the study. Maher and Martino define critical events as those events that provide mathematical insight (1996). The identified events will be any event that is significant to the research agenda of this study and will contain specific representations. Through the identification of the critical events, the full data set for this study takes shape.

3.7.1.4 Transcribing

The video data for each session will be transcribed to provide evidence and a means for detailed analysis. These transcriptions will be verified and as accurate as possible to provide the best possible data for analysis. The purpose of the transcript for this study is simply to transfer to the page sound and sequencing of talk. Although the transcripts will not include any gestic interactions, images of models and written work relevant to the research agenda will be embedded.

3.7.1.5 Coding

Aimed at identifying themes that aid interpretation of data, coding of video data is guided by the theoretical framework and defined relative to the research questions (Powell et al., 2003). For each research question, coding schemes developed collaboratively by teams of researchers were employed. Video transcripts were analyzed and coded using the coding schemes for mathematical representations, teacher moves, and beliefs. Each coding scheme is described below.

3.7.2 Framework for Analysis of Instructor Moves

The instructor moves framework for analysis was used to code the strategies implemented by the instructor to facilitate prospective teachers' building and justification of solutions. This framework, in addition to a framework for the analysis of representations, is used to code the video data of observed instructor moves as prospective teachers worked on mathematical tasks. A coding scheme was developed to describe the types of pedagogical moves employed by the instructor. The codes are organized into two groups: one describing the forms of pedagogical practice; the other describing the type of instructor questioning.

1. **Monitoring:** Checking for teachers' understanding as they work on a task. The instructor monitors for the purpose of making decisions about whether and which strategies and solutions to make available to the class. (Smith & Stein, 2011).
2. **Selecting:** Choosing to share a particular teacher's work. (Smith & Stein, 2011).
3. **Motivating:** Celebrating teachers' work through praise or encouragement. Marzano (2011).
4. **Inviting:** Soliciting multiple solution strategies, often with the goal of "making diverse solutions available for public consideration" or "including multiple students in the discussion. (Herbel-Eisenmann et al., 2013, p. 183).
5. **Revoicing:** "Restating or rephrasing a teacher's contribution." (Herbel-Eisenmann et al., 2013, p. 183).
6. **Creating:** Asking teachers to engage with another teacher's idea. For example, the instructor may ask a teacher to agree or disagree with a solution or to add on to another teacher's explanation or conjecture. (Herbel-Eisenmann, Steele, & Cirillo, 2013).

In addition to the codes characterizing instructor's actions, a set of codes identifying the types of questions the instructor posed reflecting the varying purposes of teacher questioning was developed.

1. **Explanation:** Questions that invite a teacher or group of teachers to describe what they are doing or did. Explanation questions might be used while teachers are working on a task, in contrast to describing a completed task. (Maher & Martino, 1999)
2. **Justification:** Questions that elicit how the teachers are convinced that the solution is correct. (Maher & Martino, 1999) *"questions posed by the teacher which are aimed at justification of an asserted solution can stimulate further thought about the problem situation, and even lead to a reorganization of the student's solution" (Maher et al., 1993). This process of re-organization frequently results in the creation of a more sophisticated form of justification. Questions which encourage mathematical justification include "How did you reach that conclusion?" "Could you explain to me what you did?" and "Can you convince the rest of us that your method works?"*
3. **Probing:** Questions that invite teachers "to elaborate on particular ideas" (Herbel-Eisenmann et al., 2013, p. 183). For the purposes of this study, "probing" will be distinguished from "inviting." "Probing" will refer to situations in which one particular teacher is invited to elaborate on his or her particular idea, whereas "inviting" will refer to situations in which the question is asked in a way to encourage many teachers to respond.
4. **Connecting:** Questions that invite teachers to connect their approach or strategy to the underlying mathematics. (Maher & Martino, 1999; Smith & Stein, 2011).

5. **Sustaining:** Questions designed to sustain the teacher's thought about a mathematical idea or representation that is a component of his/her solution or argument. For example, the instructor may ask "have you considered 'this' possibility?" or "What if we changed the problem to consider "this"?. The purpose of the questioning can be developing a more complete argument or extending thinking about a particular idea. (Maher &Martino, 1999).
6. **Generalization:** Questions that invite teachers to consider a similar problem with the goal of encouraging them to consider patterns that suggest a solution to the original problem. (Maher & Martino, 1999, p. 65).
7. **Other Solution:** Questions that make various solutions public to other teachers. (Maher & Martino, 1999).

3.7.3 Framework for Examination of Beliefs

All participants in the study completed a beliefs inventory prior to and at the end of the fractions intervention. The 34 item inventory, shown in Appendix A, contains some statements presented as inconsistent with the Professional Standards for Teaching Mathematics (NCTM, 1991), while other statements are presented as consistent with those standards. While the inventory included 34 items, 22 items were related to the intervention and linked to changes in teacher beliefs during analyses of intervention models (Maher, Palius, & Mueller, 2010; Maher, Landis, & Palius, 2010). The 22 relevant items were used to track changes in the prospective teachers' beliefs about learning, teaching, and doing mathematics across the intervention.

One of the goals of this study is to examine the participants' beliefs about learning, teaching, and doing mathematics. Data regarding participant beliefs were collected from beliefs inventory assessments, and from participant claims during the intervention. Participants completed two beliefs inventory assessments; one pre-assessment, and one post-assessment. All of the data sources (videos of sessions, final projects) were also analyzed for informing participant beliefs. The methods for analyzing the assessment data, as well as the intervention data are described below.

3.7.3.1 Beliefs Inventory

As indicated earlier, prospective teachers completed a Beliefs Inventory prior to and at the completion of the intervention. The Inventory included 34 items, of which 22 were related to the intervention and linked with changes in teacher beliefs in analyses of the intervention model (Maher, Landis, & Palius 2010; Maher, Palius, & Mueller 2010). These were used to examine the stability of teacher beliefs over time. Some of the belief items were presented as statements consistent with current National Council of Teachers of Mathematics (NCTM) Standards, while others were presented as statements inconsistent with those standards. In the list of questions below, the statements inconsistent with current standards are indicated with an asterisk.

Q1 - Learners generally understand more mathematics than their teachers or parents expect.

Q2 - Teachers should make sure that students know the correct procedure for solving a problem.

Q4 - It's helpful to encourage student-to-student talking during math activities.

*Q5 - Math is primarily about learning the procedures.

*Q6 - Students will get confused if you show them more than one way to solve a problem.

Q7 - All students are capable of working on complex math tasks.

Q9 - If students learn math concepts before they learn the procedures, they are more likely to understand the concepts.

*Q10 - Manipulatives should only be used with students who don't learn from the textbook.

*Q11 - Young children must master math facts before starting to solve problems.

*Q13 - Only really smart students are capable of working on complex math tasks.

Q15 - Learners generally have more flexible solution strategies than their teachers or parents expect.

*Q17 - Manipulatives cannot be used to justify a solution to a problem.

Q18 - Learners can solve problems in novel ways before being taught to solve such problems.

Q19 - Understanding math concepts is more powerful than memorizing procedures.

Q21 - If students learn math concepts before procedures, they are more likely to understand the procedures when they learn them.

*Q23 - Collaborative learning is effective only for those students who actually talk during group work.

Q24 - Students should be corrected by the teacher if their answers are incorrect.

Q28 - Learning a step-by-step approach is helpful for slow learners.

*Q29 - Only the most talented students can learn math with understanding.

*Q30 - The idea that students are responsible for their own learning does not work in practice.

Q31 - Teachers need to adjust math instruction to accommodate a range of student abilities.

*Q32 - Teacher questioning of students' solutions tends to undermine students' confidence.

Some of the questions refer to similar beliefs. For example, questions 10 and 17 relate to beliefs about the use of manipulatives in mathematics classes. For the purposes of analyzing beliefs, the questions were grouped into the following five question categories:

Expectations and Student Abilities: Q1, Q7, *Q13, Q15, Q28, *Q29

Mathematical Discourse: Q4, *Q23

Concepts and Procedures: Q2, *Q5, Q9, *Q11, Q18, Q19, Q21,

Manipulatives: *Q10, *Q17

Student and Teacher Roles: *Q6, Q24, *Q30, Q31, *Q32

Prospective teachers completed the beliefs inventory assessments by rating each statement on a 5-point Likert scale. Responses were recorded as "Consistent", "Inconsistent", or "Undecided" in relation to the educational standard described in each item. Ratings of "3" (neutral) were coded as "Undecided". Ratings expressing agreement with statements consistent with standards, as well as ratings expressing disagreement with statements inconsistent with standards were coded as "Consistent". Ratings expressing disagreement with statements consistent with standards, as well as ratings expressing agreement with statements inconsistent with standards were coded as "Inconsistent". The use of these codes allowed for the exploration of trends in prospective teachers' beliefs relative to the standards expressed in the beliefs assessments.

3.7.3.2 Beliefs Coding

Codes that relate prospective teachers' claims or belief statements made during the intervention to a question category as described in the beliefs inventory were developed. Additional codes identifying beliefs as pertaining to the topics of learning, teaching and doing mathematics were also developed. Prospective teachers' belief statements were coded with both question category codes as well as topic codes.

Each belief statement was coded for its relationship to the standards that are presented by the beliefs inventory assessments. Statements were coded as inconsistent with the standards, consistent with standards, or undecided regarding the standards. The criteria for establishing whether beliefs statements in each question category or topic are consistent or inconsistent with standards presented by the beliefs assessments are described below. Any statement in which teachers references either a topic or question category, but not in a way that clearly aligns or conflicts with the standard was coded as undecided regarding the standards.

Expectations and Student abilities:

Statements indicating lower expectations for some learners, or that only some students are capable of mathematical success will be marked as *inconsistent* with standards.

Statements indicating beliefs that all students are capable of mathematical success will be marked as *consistent* with standards.

Mathematical Discourse:

Statements claiming that student mathematical discourse is not valuable, or that mathematical discourse is only valuable to students actively discussing the mathematics will be marked as i*nconsistent* with standards.

Statements claiming that mathematical discourse is valuable for all students will be marked as *consistent* with standards.

Concepts and Procedures:

Statements claiming that mathematics is more about procedures than concepts will be marked as *inconsistent* with standards.

Statements claiming that concepts and procedures are both important in mathematics will be marked as *consistent* with standards.

Manipulatives:

Statements claiming that <u>manipulatives have a limited value or are only useful for certain learners</u> will be marked as *inconsistent* with standards.

Statements claiming that <u>manipulatives are valuable for all learners, particularly as reasoning and communication tools</u>, will be marked as *consistent* with standards.

Student and Teacher Roles:

Statements claiming that the <u>teacher is the sole authority in the classroom</u> will be marked as *inconsistent* with standards.

Statements claiming that <u>students can have mathematical authority, particularly when making and supporting claims</u>, will be marked as *consistent* with standards.

Learning:

Statements claiming that <u>students learn mathematics through direct instruction as a set of rules or procedures</u> will be marked as *inconsistent* with standards.

Statements claiming that <u>students can take ownership of their learning, or that students can learn from their peers</u> will be marked as *consistent* with standards.

Teaching:

Statements claiming that <u>the teacher must be the authority in the classroom, or that teachers should tell students how to solve problems before students interact with those problems</u> will be marked as *inconsistent* with standards.

Statements claiming that the teacher can assist students in sharing and refining mathematical ideas, without being the sole authority in the classroom will be marked as *consistent* with standards.

Doing Mathematics:

Statements claiming that mathematics is primarily about rules or procedures will be marked as *inconsistent* with standards.

Statements claiming that mathematics is primarily about sense making and justification will be marked as *consistent* with standards.

4 TEACHER JUSTIFICATION NARRATIVES

In this section, prospective teachers' individual justifications that were both supported with a physical model and prompted by an instructor move, specifically a justification question, are presented. Although instructor moves were employed in either small group or whole group settings, opportunities for teachers to offer individual justifications arose. Two of six prospective teachers, one from each of the two small groups, built models in support of their justifications.

4.1 Narratives of Erika (Group 2)

For the beginning session on 04/13, the instructor introduced Cuisenaire rods as the tool prospective teachers would use to construct models of their ideas. After introducing some academic vocabulary essential for effectively engaging in and completing the first tasks (Appendix F), she uses two white rods and a single red rod to show that the white rod 'measures' the read rod. Having been asked to create a model of the shortest train that measures dark green and purple, the prospective teachers work in their small groups to construct models and explain why their models represent the shortest train.

While engaging in this mathematical exploration, Erika builds a model comprised of two dark green rods and three purple rods (figure 4-6). The instructor prompts Erika for justification of her claim that this is the shortest train. In modeling the justification for her claim, Erika removes one dark green rod and one purple rod from her model, revealing that the purples were longer than the dark green rod. Erika then returns the second dark green rod to the model, resulting in two dark green rods longer than the two purple rods.

> T/R: Ok. So any one of those descriptions will be a train that is measured by the dark green and the purple. And the claim is that's the shortest train that you could measure with a dark green and a purple. And how do you know it's the shortest?

> Erika: Um. Well, if I were to use one green the purples are too long. So I needed to add another green , but then the purples are too short. So I grabbed another purple.
> (04/13/11 transcript 1, lines 32-33).

Figure 4-1 Erika's LCM Model

Erika concludes her argument by returning to her original model as shown in figure 6 above.

One of the early tasks on April 13 prompted prospective teachers to determine the number names for each Cuisenare rod when the red rod is called 1. Fae uses numeric pattern recognition to complete the table provided (Appendix G) by first identifying the rods representing whole numbers. She explains that if each rod represented a whole number, then every other rod would represent an odd number; but since the red rod represents one instead of two, the rods increase by one half. As the tasks become more challenging, Fae and her group members begin to use Cuisenaire rods to build models, including those representing mixed fractions for which the unit fraction is one-tenth.

In completing the same task, group 1 members Erika and Darlene use rod models to determine that the light green rod is called one and a half when the red rod is called one. Recognizing that the white rod is called one-half in this case, the pair use numeric pattern recognition to determine the number names for the remaining Cuisenaire rods. For the remaining tasks, the group employs a strategy involving a combination of constructing rod models and analyzing numerical patterns in order to determine their solutions.

As session 1 concludes, the instructor revisits a portion of this task. She facilitates a discussion on the various models of equivalent fractions that the prospective teachers have constructed and connects their representations to the customary strategy for converting mixed numbers to improper fractions. The prospective teachers revisit the task for which the number name for the black rod is determined when the red rod is called one. As they construct models to prove that the black rod would be called both seven halves and three and one-half, Erika is selected to share her proof with the class.

> T/R: So J… has it over here if you don't have enough you can look.
> Fae: I have it too.
> T/R: Oh you've got it too. F… has it over here. Ok and those of you that have enough white cubes have it**.** So, show us your proof.
> Erika: Ok.
> T/R: Tell us about your proof
> Erika: So black is one. Now you said you wanted three
> T/R: No, black is not one.
> Erika: What is it?
> T/R: Red is one.
> Erika: Red's one.
> T/R: And black is …
> Erika: And you want us to prove that black is three and one half.
> T/R: Which … and I want you to show me that three and a half is the same as seven halves.
> Erika: Alright. So, black is three and a half. So, red's one. We've got one, two, three, and a half. Half, half of a red is a white. So that's three and a half. Or, if you wanted … what seven halves?
> T/R: Yeah
> Erika: Since one of these is one, there's two of them for everyone. Alright. So, two times three because we have three reds, is six. Plus the one white we have at the end is seven.
> T/R: Ok. And that was actually… you're sort of giving the proof of the algorithm. Remember three and a half. Remember that rule for converting three and a half to a mixed number. The three times the two plus the numerator. Remember?
> (04/13/11 transcript 2, lines 515-531)

Figure 4-2 Erika's Seven Halves Model

Erika concludes her argument by presenting the model as shown in figure 4-7 above.

4.2 Narratives of Fae (Group 1)

During session 1, Fae, Sarah, and Kelly work collaboratively to construct models that allow them to determine the number name for the red rod when the blue rod is called one. Fae and Sarah build similar models (figure 4-1). The instructor prompts them for justification of their claim that the red rod would be called two-ninths. Fae and Sarah work separately to line up a sufficient number of white rods in when building their models. Each determines that one white rod is called one-ninth and a red rod is the same length as two white rods.

> T/R: Because… Why is red two-ninths?
> Fae: Oh. Because it equals. These are one-ninth each. So, two of them together equals one red. That makes two ninths.
> Sarah: I got two out of nine. It would be like that. Two out of nine.
> (04/13/11 transcript 4, lines 243-245)

Figure 4-3 Blue Rod Model

Distinctly, Sarah uses ratio language – two out of nine – as opposed to fractions language – two ninths - to report her final answer.

The mathematical tasks for the second day of the fractions intervention included problem solving tasks, one of which required sharing fractional portions of a candy bar among friends.

The prospective teachers were instructed to model their solutions with Cuisenaire rods. For the first task, half of one-third of a candy bar is given away. Fae states that her answer, the amount that remains after half is given away, is both two-twelfths and one-sixth. She builds a model containing a train of an orange and red rod, next to a train of four light green rods, next to a train of three purple rods; she then explains that she has done this because it is easy to divide 12 into thirds and halves.

To complete the tasks, students worked together, shared their mathematical ideas with other students, and justified their solutions with a physical model. The instructor observed, facilitated discourse, and employed other pedagogical moves as the prospective teachers, in groups three, sat at a table. The following excerpt (Appendix H) illustrates the instructors simultaneous use of selecting, explanation and justification questions to elicit a physical model and supporting justification from Fae. In response to the instructor moves (04/15/11 transcript 5, lines 287), Fae builds a train using an orange rod and a red rod (figure 4-2) to represent the candy bar that is shared among Pablo, Gordon and Keisha as described in a real-world problem-solving task during the 04/15/11 session (Appendix G). After identifying the train for her model, Fae lines up a sufficient number of white rods that she eventually refers to as twelfths.

> T/R: Two? Ok, now F… over here already has the equation but not the model, so can you explain your model and you see if it agrees with your equation
> Fae: This is half of the candy bar ...
> T/R: But so, what's the whole candy bar?
> Fae: Twelve
> Sarah: Twelve
> T/R: Ok
> Fae: Here I'll move these
> T/R: Ok
> (04/15/11 transcript 5, lines 287-294)

Figure 4-4 Candy Bar Model, part 1

In the excerpt below, Fae explains the fractional relationships in her model, using white rods to represent the unit fraction one-twelfth.

> Fae: Now. Here's the whole candy bar. The orange and the red. Half of it, is two greens. Which if you put them next to the whites, it adds up to six-twelfths or one-half. Um, and then so that's half of it. Now if I put three purples up against it to represent thirds. One third of the candy bar given to Gordon. So there's one third plus a half, which equals ten twelfths. And then …
> T/R: That's what was taken away
> Fae: That's what was taken away. This is Pablo and Gordon. So this is Keisha. The two-twelfths.
> T/R: And you said your answer was?
> Sarah: One-sixth. So two twelfths is one-sixth
> (04/15/11 transcript 5, lines 295-299)

She then presents a model (figure 4-3) to justify naming the green rod one-half and a second model (figure 4-4) to justify calling the purple rod one-third. With Pablo and Gordon's share of the candy bar represented by the green and purple rods of figure 4-5, Fae indicates that the remaining portion of the candy bar would be called two-twelfths.

Figure 4-5 Candy Bar Model, part 2

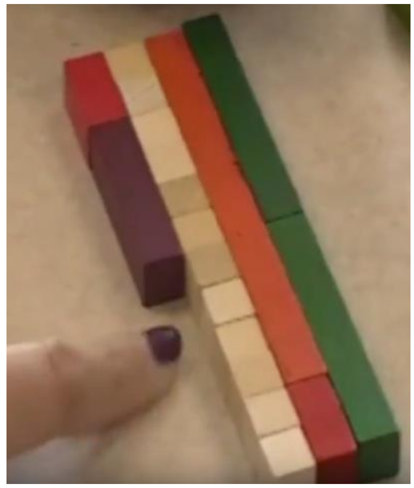

Figure 4-6 Candy Bar Model, part 3

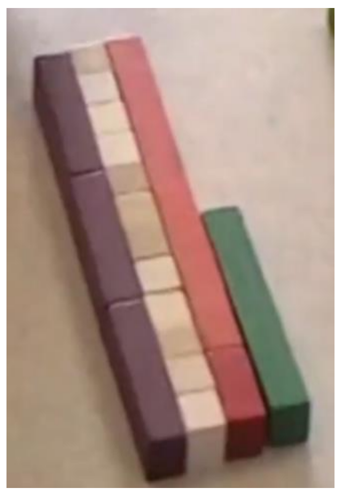

Figure 4-7 Candy Bar Model, part 4

5 INSTRUCTOR MOVES ANALYSIS

This chapter is an analysis of instructor moves for two sessions of the fractions intervention. The instructor moves are examined by session, by task, and by student group; and trends are described within these contexts.

5.1 Instructor Moves by Session

The instructor's use of pedagogical and question moves varied by session, group, and task. The table below summarizes the use of instructor pedagogical practice moves by session. The first number represents the number of moves for each pedagogical practice. The second number represents the percentage of each type of pedagogical practice move relative to the total number of pedagogical practice moves.

Table 5.1 Instructor Practice Moves by Session

Pedagogical Practice	**Session 1**	**Session 2**	**Both Sessions**
Monitoring	45 (33%)	27 (34%)	72 (33%)
Selecting	19 (14%)	10 (13%)	29 (13%)
Motivating	12 (9%)	10 (13%)	22 (10%)
Inviting	13 (9%)	6 (8%)	19 (9%)
Revoicing	38 (28%)	18 (23%)	56 (26%)
Creating	11 (8%)	8 (10%)	19 (9%)
Total Practice Moves	**138 (64%)**	**79 (36%)**	**217 (100%)**

5.1.1 Session 1

During the first session of the fraction's intervention, the most frequently occurring pedagogical move was the practice of monitoring prospective teachers understanding. Forty-five of the instructor's comments were coded as monitoring. The second most frequent pedagogical practice was revoicing a prospective teacher's contribution which occurred 38 times in the session. The third most frequent move was selecting a teacher's contribution for sharing. After selecting Erika to share the type of model she created (04/13/11 transcript 1, line 21), the instructor revoices Erika's contribution when stating "So any one of those descriptions will be a train that is measured by the dark green and the purple; and the claim is that's the shortest train that you could measure with a dark green and a purple" (04/13/11 transcript 1, line 32).

On average, motivating, inviting and creating were used by the instructor twelve times during the first session. The following excerpt illustrates the instructor's simultaneous use of inviting and creating, after selecting Sarah to share her model and explanation:

> Fae: If three of the whites equals one, then it's one plus two extra little ones which is thirds.
> T/R: Now, this F… said the answer was five-thirds. So, show me five-thirds.
> Sarah: Because I counted that this was five whites. Yellow is five whites.
> T/R: OK
> Sarah: So, I said …
> T/R: Yellow is five-thirds.
> Sarah: Yeah.
> T/R: She said yellow is five-thirds. She said yellow is one and two-thirds. Which ones right?
> (04/13/11 transcript 2, lines 488-495)

5.1.2 Session 2

During the second session, practice moves were generally employed less frequently. Seventy-nine practice moves were coded. The types of moves employed were monitoring, 27 times, selecting, 10 times, motivating, 10 times, inviting, six times, revoicing, 18 times, and creating, eight times. Although less frequent as compared to session one, monitoring and revoicing were again the most frequently occurring pedagogical moves in session two. Inviting, soliciting multiple solution strategies, was noted least frequently. After revoicing Fae's conclusion that the number name for a rod is one-third, the class is invited to explain why (04/15/11 transcript 5, lines 197-199). It is possible that the instructor used more pedagogical moves during Session 1 because it was the beginning of the fractions intervention and prospective teachers were being introduced to the use of Cuisenaire rods as representations of rational number ideas.

Throughout each session, the instructor facilitated dialogue with the teachers through questioning. During the discussion, different types of questions were posed. The table below summarizes the use of instructor question moves by session. The first number represents the number of moves for each question type. The second number represents the percentage of each question move relative to the total number of question moves.

Table 5.2 Instructor Question Moves by Session

Question Type	Session 1	Session 2	Both Sessions
Explanation	9 (24%)	10 (45%)	19 (32%)
Justification	12 (32%)	4 (18%)	16 (27%)
Probing	7 (19%)	3 (14%)	10 (17%)
Connecting	2 (5%)	1 (5%)	3 (5%)
Sustaining	4 (11%)	4 (18%)	8 (18%)
Generalization	1 (3%)	0 (0%)	1 (2%)
Other Solution	2 (5%)	0 (0%)	2 (3%)
Total Question Moves	**37 (63%)**	**22 (37%)**	**59 (100%)**

During the first fractions intervention session on 4/13/11, six prospective teachers worked on mathematically rich fractions tasks and built models using Cuisenaire rods. As they worked on the tasks, the instructor asked questions regarding their ideas and their models. Thirty-seven question moves were noted during this first session. The most frequently occurring question move was the practice of asking prospective teachers to justify their solutions. Nine of the instructor's questions were coded as explanation – an invitation for teachers to describe what they are doing. Explanation and justification questions accounted for 56 percent of the session 1 question moves.

The second fractions intervention session occurred on 4/15/11. During this session, the six prospective teachers worked on real world fractions tasks. Mathematical ideas were communicated using physical manipulatives, pictorial representations and symbols as the prospective teachers translated among representations. Twenty-two question moves were noted during this session. The more commonly occurring question move was explanation, 10 times; justification and sustaining – questions designed to sustain a teacher's thought about an idea or representation – were asked frequently. Those question types were the second and third most common, occurring four times each. Probing questions were asked three times. Three question types - probing, explanation and justification - represented 77 percent of the questions in the second session. It is possible that the prevalence of these types of questions reflected the instructors desire to ensure teachers connected the real-world context of the word problems with the underlying mathematical concepts and relationships.

5.2 Instructor Moves by Task

Throughout each session, the instructor employed pedagogical practice moves as teachers worked on tasks. The practice moves were used to support teachers' cognitive engagement and to facilitate teachers' discussions as they worked on mathematically rich tasks requiring teachers engage cognitively with distinct mathematical concepts. Task 1 required physical representations, specifically linear models, of least common multiple and greatest common factor. Task 2 required teachers examine relationships among rods in order to name rods based on relative size. Task 3 required teachers construct models to compare the size of two fractions and identify which was larger and by how much. Teachers engaged with tasks 1 through 3 during session 1. Task 4, solving real-world problems involving the addition or subtraction of fractions, was presented during session 2.

5.2.1 Practice Moves by Task

Table 5.3 Instructor Practice Moves by Task

Pedagogical Practice	Task 1	Task 2	Task 3	Task 4
Monitoring	4 (13%)	23 (38%)	18 (38%)	23 (32%)
Selecting	5 (17%)	4 (7%)	10 (21%)	10 (14%)
Motivating	2 (7%)	9 (15%)	1 (2%)	9 (12%)
Inviting	4 (13%)	6 (10%)	3 (6%)	6 (8%)
Revoicing	14 (47%)	11 (18%)	13 (27%)	17 (23%)
Creating	1 (3%)	7 (12%)	3 (6%)	8 (11%)
Total Practice Moves	**30 (14%)**	**60 (28%)**	**48 (22%)**	**73 (33%)**

Throughout each session, the instructor employed pedagogical practice moves as teachers worked on tasks. Selecting particular teachers to share their models occurred five times and was the second most prevalent pedagogical practice move during task 1. Monitoring and inviting were the next most common, occurring four times each. Motivating and creating, asking teachers to engage with another's ideas, were least frequent of all practice moves. Forty-seven percent of the moves were revoicing moves employed as the teachers worked on this first task. Revoicing may have been the more prevalent practice move because the instructor sought to establish a strong foundational understanding for constructing physical models to represent rational number ideas.

Task 2 was comprised of several questions that, after assigning the number name 'one' to a select rod, require prospective teachers to determine the fractional name for each of the remaining nine rods. Task 2 elicited twenty-three monitoring and eleven revoicing moves from the instructor representing 38 percent and 18 percent of the total practice moves, respectively.

These two moves represented 56 percent of the moves employed during task 2. This task elicited 7 creating moves from the instructor – asking a prospective teacher to engage with the ideas of another teacher. While this represents only 12 percent of the moves employed during this task, this creating move was employed seven times more frequently as compared to task 1 and more than twice as often as compared to task 3.

After constructing physical models to compare the size of two fractions, task 3 required teachers to identify which was larger and by how much. 18 of the practice moves were monitoring prospective teachers' understanding while they worked on the task. This act of monitoring was the most prevalently used move for task 3. Rephrasing teacher ideas was the next most prevalent move. Thirteen moves reflected the instructors rephrasing a teacher's idea. These two moves – monitoring and revoicing – represent 65 percent of the practice moves for this task. Twenty-one percent of the moves, 10 occurrences, reflected the instructor sharing a particular teacher's work. In one instance, the instructor shares Janelle's model with the group (04/13/11 transcript 4, line 517). Subsequently, Erika interprets Janelle's model and identifies a red rod as representing one-twelfth in the model (04/13/11 transcript 4, line 519).

Of the four tasks, the greatest number of pedagogical practice moves is employed during task 4. This task, comprised of three real-word problems, requires that prospective teachers interpret mathematical ideas in context and select a solution strategy. Monitoring and revoicing are again the most prevalent practice moves representing 32 percent and 23 percent respectively. Soliciting multiples solutions and asking teachers to engage with another teacher's idea represent 20 percent of the practice moves. Although soliciting multiple solutions occurred least frequently of all task 4 practice moves, the instructor used the practice move more frequently during task 4 than during task 1 or task 3. Selecting a teacher to share their ideas occurs 10 times, more frequently than during task 1 or task 2.

5.2.2 Question Moves by Task

Table 5.4 Instructor Question Moves by Task

Question Type	Task 1	Task 2	Task 3	Task 4
Explanation	3 (30%)	1 (12.5%)	5 (26%)	10 (53%)
Justification	4 (40%)	4 (50%)	4 (21%)	4 (21%)
Probing	0 (0%)	2 (25%)	5 (26%)	0 (0%)
Connecting	0 (0%)	0 (0%)	2 (11%)	1 (5%)
Sustaining	2 (20%)	1 (12.5%)	1 (9%)	4 (21%)
Generalization	1 (10%)	0 (0%)	0 (0%)	0 (0%)
Other Solution	0 (0%)	0 (0%)	2 (11%)	0 (0%)
Total Question Moves	**10 (17**%)	**8 (14**%)	**19 (32**%)	**19 (32**%)

For task 1, 10 questions were asked by the instructor as teachers constructed physical models. The more common questions were justification, four times, and explanation, three times. The least commonly asked questions were sustaining and generalization. As teachers worked on task 2, fewer questions were asked. Of eight questions asked during this task, half were justification questions. The remaining questions were probing, two times, and sustaining and explanation, one time each.

During task 3 and task 4, approximately twice as many questions were posed when compared to task 1 and task 2. It is possible that task 3 and task 4 were more cognitively demanding tasks for the prospective teachers and, consequently, the instructor posed more questions in order to better understand their thinking throughout the tasks. Explanation and probing were the most commonly used questions during task 3, occurring five times each. Justification questions were asked 4 times; questions that make various solutions public to other teachers and connecting questions were asked two times each; and one question intended to sustain prospective teachers' thought about a representation or idea was asked.

For the fourth task, nineteen questions were asked by the instructor regarding prospective teachers' ideas on the solutions to real-world problems requiring operations on fractions. More than half, 10 questions, sought explanations or descriptions of teachers work. Of the remaining 9 questions, four justification, one connecting, and four sustaining questions were posed. A justification question was asked by the instructor in response to an equation written by Sarah. The instructor noted that Sarah had the symbolic representation of a real-world problem but that she did not yet have a model. Subsequently, the instructor asks Fae to both explain her model and to justify whether or not it agrees with her equation (04/15/11 transcript 5, line 287).

5.3 Instructor Moves by Group

During the first session, the instructor established two small groups each containing three teachers. The members of group 1 (G1) were Kelly, Fae, and Sarah. The members of group 2 (G2) were Darlene, Erika, and Janelle. These small groups remained fixed during the two sessions of this intervention. For each session, the instructor addressed the prospective teachers as a whole group (WG), as well as within each of the smaller groups of three teachers. Pedagogical practice moves and question moves were employed during both types of grouping.

Table 5.5 Instructor Practice Moves by Group

Pedagogical Practice	Group 1	Group 2	WG	Total
Monitoring	49 (43%)	11 (23%)	12 (22%)	72 (33%)
Selecting	9 (8%)	4 (8%)	16 (29%)	29 (13%)
Motivating	18 (16%)	3 (6%)	1 (2%)	22 (10%)
Inviting	6 (5%)	3 (6%)	10 (18%)	19 (9%)
Revoicing	25 (22%)	21 (44%)	10 (18%)	56 (26%)
Creating	7 (6%)	6 (13%)	6 (11%)	19 (9%)
Total Practice Moves	**114 (53%)**	**48 (22%)**	**55 (25%)**	**217 (100%)**

**Note: G1 and G2 refer to group 1 and group 2 small group instruction. WG refers to whole group instruction.*

Table 5.6 Instructor Question Moves by Group

Question Type	Group 1	Group 2	WG	Total
Explanation	4 (20%)	5 (56%)	10 (33%)	19 (32%)
Justification	6 (30%)	0 (0%)	10 (33%)	16 (27%)
Probing	4 (20%)	3 (33%)	3 (10%)	10 (17%)
Connecting	0 (0%)	1 (11%)	2 (7%)	3 (5%)
Sustaining	6 (30%)	0 (0%)	2 (7%)	8 (14%)
Generalization	0 (0%)	0 (0%)	1 (3%)	1 (2%)
Other Solution	0 (0%)	0 (0%)	2 (7%)	2 (3%)
Total Question Moves	**20 (34%)**	**9 (15%)**	**30 (51%)**	**59 (100%)**

**Note: G1 and G2 refer to group 1 and group 2 small group instruction. WG refers to whole group instruction*

5.3.1 Group 1

One hundred fourteen pedagogical practice moves were coded by the researcher for the two sessions of fractions intervention under study. Of those 114 moves, 49 were monitoring moves and 25 were restatements of prospective teachers' ideas by the instructor. Monitoring and revoicing moves were the most prevalent moves for group 1. The next most commonly used practice move was motivating - moves that celebrated or encouraged teachers' work. Selecting, inviting and creating were the least frequently used moves for this small group, occurring nine times, six times, and seven times, respectively.

The instructor used a total of twenty question moves while engaging with group 1 during the two sessions on 4/13/11 and 4/15/11. The most frequently occurring question moves were the practice of sustaining teachers' thinking and asking prospective teachers to justify their solutions. Each move was coded 6 times and accounted for 60% of the question moves employed with group 1. Four of the instructor's questions were coded as explanation and the remaining four questions were probing questions. For both question moves and pedagogical practice moves, the instructor employed more than twice as many moves with group 1 as compared to group 2. It is possible that the instructor's observations led to an intentional use of a greater number of instructor moves with group 1.

As a small group, group 1 experienced 114 practice moves while the whole group experienced 55 practice moves. Although more than double the number of pedagogical practice moves were employed with group 1 as compared to the whole group, selecting and inviting were used more frequently in the whole group setting. Conversely, the whole group experienced more question moves as compared to group 1 independently with explanation and justification being the most frequently occurring question moves within the whole group settings.

5.3.2 Group 2

Of the 48 pedagogical practice moves employed with group 2, revoicing and monitoring were most common, occurring 21 times and 11 times respectively. These two moves account for 67 percent of the practice moves used with group 2. While six opportunities to respond to another teachers' thinking were available, a prospective teacher was selected four times to share their ideas with the group. The least common practice moves were motivating and inviting, occurring three times each. Of the seven types of question moves, the instructor employed only three types with group 2. More than half of the questions, 56 percent, were explanation questions. One connecting and three probing questions were posed. As compared to the whole group, group 2 experienced fewer than one-third the number of question moves.

5.4 Summary of Instructor Moves

Based on the data from this research study, the instructor's moves throughout two sessions of the fractions intervention helped prospective teachers explain, justify and construct representative build models of rational number ideas. The pedagogical practices used and questions asked were analyzed throughout two sessions of the intervention as teachers worked on fractions tasks. Table 5-7 and table 5-8 below summarize the instructor moves analyzed for this study.

Table 5.7 Pedagogical Practice Moves Summary

Pedagogical Practice	Both Sessions
Monitoring	72
Selecting	29
Motivating	22
Inviting	19
Revoicing	56
Creating	19
Total Practice Moves	**217**

Table 5.8 Question Moves Summary

Question Type	Both Sessions
Explanation	19
Justification	16
Probing	10
Connecting	3
Sustaining	8
Generalization	1
Other Solution	2
Total Question Moves	**59**

The researcher coded 276 instructor moves. Of the 276 instructor's moves, 59 were questions posed by the instructor. The most common type of question asked was explanation, 19 times. Other question types frequently employed by the instructor were justification and probing questions.

Of the 276 instructor moves, 79 percent were pedagogical practice moves. The most common practice was monitoring. It is possible that the instructor used monitoring frequently because the mathematical tasks required the prospective teachers to construct models whose meaning could not be inferred or interpreted solely through observation. Other frequently used pedagogical practices were revoicing, 56 times; and selecting, 29 times.

6 TEACHERS' BELIEFS ANALYSIS

A third goal of this study was to determine what changes, if any, in prospective teachers' beliefs about mathematics occurred. The prospective teachers completed the beliefs inventory as a pre-assessment preceding the intervention and as a post-assessment at the conclusion of the intervention. The beliefs pre-assessment offers a baseline for understanding teachers' initial beliefs and allows for later comparison. Pre-assessment data indicate that the prospective teachers agreed with the standard 68.8% of the time, on average - an indication that prospective teachers' beliefs were relatively well aligned with standards. The table below summarizes prospective teachers' pre-assessment and post-assessment scores. In each cell, the first number represents the number of statements for which the prospective teacher's response was consistent, inconsistent or undecided relative to the standard. The second number represents the corresponding percentage of items for which the prospective teacher's response was consistent, inconsistent or undecided relative to the standard.

Table 6.1 Teachers' Scores for Belief Statements by Relation to Standards

Teacher	**Pre-Assessment**			**Post-Assessment**		
	Consistent	*Inconsistent*	*Undecided*	*Consistent*	*Inconsistent*	*Undecided*
Fae	11 (50%)	2 (9%)	9 (41%)	12 (55%)	1 (4.5%)	9 (41%)
Kelly	13 (59%)	6 (27%)	3 (14%)	10 (45%)	7 (32%)	5 (23%)
Erika	18 (81%)	3 (14%)	1 (4.5%)	19 (86%)	0 (0%)	3 (14%)
Janelle	14 (64%)	2 (9%)	6 (27%)	17 (77%)	1 (4.5%)	4 (18%)
Darlene	19 (86%)	1 (4.5%)	2 (9%)	17 (77%)	1 (4.5%)	4 (18%)
Sarah	16 (73%)	3 (14%)	3 (14%)	17 (77%)	1 (4.5%)	4 (18%)

Post-assessment data indicate that the prospective teachers agreed with the standard an average of 69.5% of the time. This suggests that prospective teachers' beliefs remained relatively well aligned with standards. As part of a more granular analysis, data regarding prospective teachers' beliefs will be further examined by beliefs statement category and by prospective teacher.

6.1 Beliefs Assessment Results

Using the beliefs pre-assessment as a baseline for understanding teachers' initial beliefs, the percentage of teacher responses consistent with standards was calculated for each of the 22 beliefs inventory statements. Analysis of post-assessment data, including percentages of teacher responses consistent with standards, reveal a net change for 13 of 22 belief inventory items. This change indicates that teachers' beliefs about the teaching, learning, or doing of mathematics as conveyed by those statements may have changed.

Table 6.2 below presents 7 statements for which the number of prospective teachers indicating beliefs consistent with standards increased and 6 statements for which the number of prospective teachers indicating beliefs consistent with standards decreased.

The concepts and procedures category contains 7 beliefs statements. Post-assessment data analysis indicate that prospective teachers' beliefs may have changed with respect to five of those statements. Of the 7 statements for which growth may have occurred, 4 reflect prospective teacher's beliefs about mathematics concepts and procedures. The beliefs statement within concepts and procedures category for which the greatest change occurred indicated prospective teachers' belief that *young children need not master math facts before starting to solve problems*. There may also have been a change in prospective teachers' beliefs about teachers' and/or parents' expectations of learners understanding and flexibility with solution strategies.

Of the 5 beliefs statements in the student and teacher roles category, data for 3 of those statements suggest that teachers' beliefs may have become inconsistent with standards. This suggests prospective teachers' belief that the teacher is the sole authority in the classroom.

Beliefs statements claiming that mathematical discourse is only valuable to students actively discussing the mathematics were coded as i*nconsistent* with standards. Of 2 statements about mathematical discourse, one statement indicated that prospective teachers' beliefs may have become inconsistent with the standard. Specifically, prospective teachers believed that collaborative learning is effective only for those students who actually talk during group work.

Table 6.2 Summary of Reported Teachers' Beliefs Changes

Beliefs Statement	**Pre-Assessment**	**Post-Assessment**
	CN (CP)	*CN (CP)*
(1) Learners generally understand more mathematics than their teachers or parents expect (E)	3 (50%)	4 (67%)
(2) Teachers should make sure that students know the correct procedure for solving a problem (C)	6 (100%)	2 (33%)
(5) Inverse of: Math is primarily about learning procedures (C)	1 (17%)	3 (50%)
(6) Inverse of: Students will get confused if you show them more than one way to solve a problem (ST)	4 (67%)	3 (50%)
(9) If students learn math concepts before they learn the procedures, they are more likely to understand the concepts (C)	3 (50%)	5 (83%)
(11) Inverse of: Young children must master math facts before starting to solve problems (C)	1 (17%)	4 (67%)
(15) Learners generally have more flexible solution strategies than their teachers or parents expect (E)	3 (50%)	5 (83%)
(18) Learners can solve problems in novel ways before being taught to solve such problems (C)	4 (67%)	5 (83%)
(23) Inverse of: Collaborative learning is effective only for those students who actually talk during group work (MD)	4 (67%)	1 (17%)
(24) Students should be corrected by the teacher if their answers are incorrect (ST)	3 (50%)	2 (33%)
(28) Learning a step-by-step approach is helpful for slow learners (E)	6 (100%)	5 (83%)
(30) Inverse of: The idea that students are responsible for their own learning does not work in practice (ST)	3 (50%)	4 (67%)
(32) Inverse of: Teacher questioning of students' solutions tends to undermine students' confidence (ST)	3 (50%)	2 (33%)

**Note: CN refers to the number of teachers whose responses are consistent with standards. CP refers to the percent of teachers whose responses are consistent with standards.*

6.2 Beliefs by Teacher

Each prospective teacher was administered the beliefs inventory as a pre-assessment and as a post-assessment. The results of those inventories will be described, noting instances of possible change in beliefs.

6.2.1 Fae

Table 6.3 summarizes the pre-assessment and post-assessment data for Fae. The beliefs inventory statements were grouped by category. For each cell, the numbers represent the number of statements, within each category, for which Fae scored consistent with the standard and the percentage of questions in that category for which Fae scored consistent with the standard. Based upon the beliefs inventory, Fae's beliefs regarding mathematical discourse, and concepts and procedures may have changed.

Table 6.3 Beliefs Inventory Results by Statement Category for Fae

Statement Category	Pre-Assessment	Post-Assessment
	CN (CP)	*CN (CP)*
Expectations and Abilities	3 (50%)	3 (50%)
Mathematical Discourse	2 (100%)	1 (50%)
Concepts and Procedures	3 (43%)	5 (71%)
Manipulatives	2 (100%)	2 (100%)
Student and Teacher Roles	1 (20%)	1 (20%)

Claims attesting that mathematics is primarily about sense-making and justification were coded as consistent with standards. During the intervention, Fae made two claims regarding doing mathematics that were consistent with the standards. She also made one claim consistent with the standard for concepts and procedures and one consistent with the standard for manipulatives.

6.2.2 Kelly

Table 6.4 below summarizes the pre-assessment and post-assessment data for Kelly. The beliefs inventory statements were grouped by category. For each cell, the numbers represent the number of statements, within each category, for which Kelly scored consistent with the standard and the percentage of questions in that category for which Kelly scored consistent with the standard. Based upon the beliefs inventory, Kelly's beliefs regarding expectations and abilities, mathematical discourse, and concepts and procedures may have changed. Notably, her beliefs with respect to concepts and procedures may have shifted significantly towards inconsistent with the standard.

Table 6.4 Beliefs Inventory Results by Statement Category for Kelly

Statement Category	**Pre-Assessment**	**Post-Assessment**
	CN (CP)	*CN (CP)*
Expectations and Abilities	3 (50%)	4 (67%)
Mathematical Discourse	2 (100%)	1 (50%)
Concepts and Procedures	5 (71%)	1 (14%)
Manipulatives	2 (100%)	2 (100%)
Student and Teacher Roles	2 (40%)	2 (40%)

Claims suggesting that students can take ownership of their learning, or that students can learn from their peers were coded as consistent with standards for learning mathematics. As part of her end of course essay, Kelly made one claim consistent with standard for learning mathematics. She made two additional claims. Those claims were consistent with the standards for manipulatives and for doing mathematics.

6.2.3 Erika

Table 6-5 summarizes Erika's pre-assessment and post-assessment data.

Table 6.5 Beliefs Inventory Results by Statement Category for Erika

Statement Category	Pre-Assessment	Post-Assessment
	CN (CP)	*CN (CP)*
Expectations and Abilities	3 (50%)	4 (67%)
Mathematical Discourse	2 (100%)	2 (100%)
Concepts and Procedures	4 (57%)	5 (71%)
Manipulatives	2 (100%)	2 (100%)
Student and Teacher Roles	3 (60%)	4 (80%)

The beliefs inventory statements were grouped by category. For each cell, the numbers represent the number of statements, within each category, for which Erika scored consistent with the standard and the percentage of questions in that category for which Erika scored consistent with the standard. Based upon the beliefs inventory, Erika's beliefs regarding expectations and abilities, student and teacher roles, and concepts and procedures may have changed. For each of those categories, Erika's belief may have shifted towards consistent with the standard.

As part of her end of course essay, Erika made a single claim that was inconsistent with the teaching of mathematics. Claims inconsistent with teaching mathematics show the teacher as the authority in the classroom, or that teachers should tell students how to solve problems before students interact with those problems. Erika argues that if a teacher shows students the 'common denominator work', then it will help students excel with equivalent fractions.

6.2.4 Janelle

Table 6.6 below summarizes the pre-assessment and post-assessment data for Janelle. The beliefs inventory statements were grouped by category. For each cell, the numbers represent the number of statements, within each category, for which Janelle scored consistent with the standard and the percentage of questions in that category for which Janelle scored consistent with the standard. Based upon the beliefs inventory, Janelle's beliefs regarding student and teacher roles, and concepts and procedures may have changed. Notably, her beliefs regarding concepts and procedures may have become more consistent with the standard while her beliefs regarding student and teacher roles may have become inconsistent with the standard.

Table 6.6 Beliefs Inventory Results by Statement Category for Janelle

Statement Category	Pre-Assessment	Post-Assessment
	CN (CP)	*CN (CP)*
Expectations and Abilities	6 (100%)	6 (100%)
Mathematical Discourse	1 (50%)	1 (50%)
Concepts and Procedures	4 (57%)	6 (86%)
Manipulatives	2 (100%)	2 (100%)
Student and Teacher Roles	5 (100%)	4 (80%)

Janelle made a total of seven claims regarding manipulatives, doing mathematics, teaching mathematics, learning mathematics, and concepts and procedures. All claims were consistent with the corresponding standard. Three of the claims support the idea that manipulatives are valuable for all learners, particularly as tools for reasoning.

6.2.5 Darlene

Table 6-7 below summarizes the pre-assessment and post-assessment data for Darlene. The beliefs inventory statements were grouped by category. For each cell, the numbers represent the number of statements, within each category, for which Darlene scored consistent with the standard and the percentage of questions in that category for which Darlene scored consistent with the standard. Based upon the beliefs inventory, Darlene's beliefs regarding student and teacher roles, and mathematical discourse may have changed. Notably, her beliefs regarding each of these categories may have become inconsistent with the standard. Further, Darlene is the only teacher whose beliefs regarding concepts and procedures may have remained unchanged.

Table 6.7 Beliefs Inventory Results by Statement Category for Darlene

Statement Category	Pre-Assessment	Post-Assessment
	CN (CP)	*CN (CP)*
Expectations and Abilities	6 (100%)	6 (100%)
Mathematical Discourse	2 (100%)	1 (50%)
Concepts and Procedures	5 (71%)	5 (71%)
Manipulatives	2 (100%)	2 (100%)
Student and Teacher Roles	4 (80%)	3 (60%)

During the intervention, Darlene makes a claim that is consistent with the standard for concepts and procedures. Using the concept of division as an example, she states that when discussing division, understanding that division is the opposite or inverse of multiplication is an important understanding.

6.2.6 Sarah

Table 6-8 summarizes the pre-assessment and post-assessment data for Sarah. The beliefs inventory statements were grouped by category. For each cell, the numbers represent the number of statements, within each category, for which Sarah scored consistent with the standard and the percentage of questions in that category for which Sarah scored consistent with the standard. Based upon the beliefs inventory, Sarah's beliefs regarding student and teacher roles, and concepts and procedures may have changed. Notably, her beliefs regarding student and teacher roles may have become inconsistent with the standard, while her beliefs regarding concepts and procedures may have become consistent with the standard.

Table 6.8 Beliefs Inventory Results by Statement Category for Sarah

Statement Category	Pre-Assessment	Post-Assessment
	CN (CP)	*CN (CP)*
Expectations and Abilities	5 (83%)	5 (83%)
Mathematical Discourse	1 (50%)	1 (50%)
Concepts and Procedures	5 (71%)	7 (100%)
Manipulatives	2 (100%)	2 (100%)
Student and Teacher Roles	3 (60%)	2 (40%)

6.3 Teachers' Beliefs by Statement Category

An analysis of prospective teachers' beliefs data, analyzed by each of the five statement categories, was conducted. The results of this analysis are described, noting instances of possible change in beliefs.

6.3.1 Expectations and Abilities

The standard for the expectations and abilities category reflects the belief that all students are capable of mathematical success. Two prospective teachers' beliefs may have become more consistent with this standard. While four teachers were undecided on at least one of the six expectations and abilities beliefs statements for both the pre- and post-assessments, overall the prospective teachers' beliefs may have become more consistent with the standard for this category.

6.3.2 Mathematical Discourse

The standard for mathematical discourse reflects the belief that mathematical discourse is valuable for all students, as opposed to mathematical discourse as valuable only to students actively discussing the mathematics or not valuable at all. Three prospective teachers' beliefs may have become more inconsistent with this standard. While only one prospective teacher was undecided on one of the two statements in this category for the pre-assessment, four prospective teachers were undecided on a statement regarding mathematical discourse on the post-assessment.

6.3.3 Concepts and Procedures

The standard for concepts and procedures reflects the belief that both concepts and procedures are important in mathematics. Four prospective teachers' beliefs may have become more consistent with this standard, while one prospective teacher's beliefs may have become inconsistent with the standard. While all six prospective teachers were undecided on at least one of the seven statements in this category for the pre-assessment, five were undecided on one or more statements regarding concepts and procedures on the post-assessment.

6.3.4 Manipulatives

The standard for manipulatives reflects the belief that manipulatives are valuable for all learners, particularly as reasoning and communication tools. All prospective teachers' beliefs as reported on the pre- and post-assessment were consistent with this standard.

6.3.5 Student and Teacher Roles

The standard for the student and teacher roles category reflects the belief that students can have mathematical authority, particularly when making and supporting claims. Three prospective teachers' beliefs may have become inconsistent with this standard. While four teachers were undecided on at least one of the five student and teacher roles beliefs inventory statements during the pre-assessment, all six prospective teachers reported being undecided on at least one student and teacher roles belief inventory statements of the post-assessment.

6.4 Summary of Teachers' Beliefs

Based on the data from this research study, prospective teachers' beliefs about the teaching, learning and doing of mathematics may have varied both within and across statement categories. Analysis of pre-and post-assessment beliefs inventory data suggest prospective teachers' beliefs were consistently well aligned with the standards. Of 22 beliefs inventory items, a mean of approximately 15 statements were coded as consistent with standard. The pre-assessment mean for inconsistent and undecided statements was 2.8 and 4, respectively. Analysis of post-assessment data reveal a 1 point reduction in the mean number of items marked inconsistent with the standards. The mean number of post-assessment items coded as undecided increased by .8 points.

Teachers beliefs as evidenced by the end of course essays tended to be consistent with the standard. Of sixteen claims, fifteen claims made by the prospective teachers were consistent with the standards of various statement categories reflecting beliefs about the learning, teaching, and doing of mathematics.

7 FINDINGS

The purpose of this study is to examine the evolving beliefs and pedagogical practices employed during a fractions intervention that was a required undergraduate course for prospective teachers. Specifically, three research questions guided the study:

1. What role does the instructor play in the prospective teachers' building and justification of ideas?
2. What types of interventions does she employ?
3. What changes, if any, in prospective teachers' beliefs about doing, teaching and learning mathematics can be identified over the course of the intervention?

This chapter summarizes the findings relevant to each research question. Video data for this study were analyzed using a multi-phase process developed by Powell, Francisco, and Maher (2003). The critical events identified through this process necessarily provide mathematical insight (Maher & Martino, 1996). The critical events referenced in this research are events where the instructor makes pedagogical moves that prompt the immediate justification of a mathematical idea or solution that is supported by a physical model. Findings regarding instructor's moves are presented first, followed by findings related to prospective teachers' beliefs related to the doing, teaching, and learning mathematics. The findings are discussed through the lens of the relevant literature.

7.1 Instructor Moves

In this section, seminal findings from the instructor moves analysis are reported. The intervention helped prospective teachers to develop and represent rational number ideas, as well as to justify those ideas.

7.1.1 Findings from Instructor Moves Analysis

Many intervention behaviors recommended in the research literature were modeled by the instructor (Martino & Maher, 1999; Smith & Stein, 2011; Herbel-Eisenmann, Steele, and Cirillo, 2013). All of the question moves were employed by the instructor with varying frequency. Discourse that revealed the ways in which prospective teachers' built ideas was facilitated by the instructor by selecting prospective teachers to share their ideas or models, by probing prospective teachers to elaborate on ideas, by soliciting explanations of what prospective teachers were doing as they worked on tasks, and by prompting for justifications of how prospective teachers are convinced that a solution is correct.

While teachers worked on tasks, the instructor observed their physical models, probed for individual ideas of prospective teachers, and encouraged others to respond. The instructor made various solutions and representations available for others to consider as their own ideas were developed. The instructor regularly used revoicing to both check her own understanding of ideas as she heard them, and to allow teachers to confirm their contribution to the discourse.

During whole group discussions, the instructor employed question moves more frequently when compared with small group discussions. Questions that invited prospective teachers to consider similar problems or to make various solutions available for other prospective teachers were employed during whole group discussions only.

Both pedagogical practice moves and question moves were employed to facilitate discourse and the building of mathematical ideas. With few exceptions, the most frequent instructor move, when analyzed by varying contexts (e.g. group, task, session) was that of monitoring prospective teachers' understanding as they worked on tasks. Although monitoring was the most frequent move, its relative frequency varied by task and by session.

Instructor moves also varied by group type. In general, the instructor employed more than twice the number of practice moves with group 1 as compared to group 2. Practice moves were also used more frequently with group 1 as compared to the whole group setting. Irrespective of group type, the prevalent use of monitoring understanding reflects the instructor's attention to building prospective teachers' rational number ideas

7.2 Teachers Beliefs

Seminal findings resulting from instructor moves analysis, along with descriptions of possible relationships among findings, are reported in this section.

7.2.1 Findings from Beliefs Analysis

Over the course of the intervention, it appears that prospective teachers' beliefs in general became less inconsistent with the standards as presented in the beliefs inventory assessment. The percent of beliefs inconsistent with the standards relative to the total number of beliefs statements decreased over the course of the intervention. This is accompanied by an increase in the percent of beliefs for which teachers were undecided about their perspective was noted. Overall, an increase in alignment between prospective teachers' beliefs and the standards, in general, is not reflected in the research data. However, the data do suggest changes in prospective teachers' beliefs. Specifically, (1) prospective teachers' beliefs about concepts and procedures – learning mathematics – became more aligned with the corresponding standard; (2) prospective teachers' beliefs about student and teacher roles – teaching mathematics – became less aligned with the corresponding standard; and (3) prospective teachers no longer espouse beliefs inconsistent with particular standards.

Through the end of course essays, claims were made regarding the learning, teaching, and doing of mathematics. Claims related to manipulatives, concepts and procedures, and student and teacher roles belief categories were also made. Data suggest prospective teachers' beliefs related to the manipulatives and concepts and procedures categories became more aligned with the standard.

7.2.2 Relationships in Findings

Some changes in beliefs may be related to the instructor's moves that prospective teachers experienced throughout the intervention. The instructor regularly modeled posing explanation and justification questions, encouraging prospective teachers to make connections and develop proofs with the support of physical models. One specific instance of this is the whole group discussion in which the instructor asks prospective teachers for a physical model that would be a proof that three and a half and seven halves are equivalent (04/13/11 transcript 2, lines 527-531)

Data suggest prospective teachers' beliefs related to concepts and procedures became more aligned with the standard. During the intervention, instructor moves included questions that invited prospective teachers to connect an approach or strategy to underlying mathematics. As opportunities arose, the instructor employed moves to connect prospective teachers' reasoning about physical models to the underlying mathematics and/or to algorithms. In an end of course written essay, Fae states that she has finally learned the reasoning of equivalent fractions – a belief statement indicating that mathematical reasoning is important for procedural tasks such as adding or subtracting fractions (essay 1, lines 4-6).

8 CONCLUSIONS

In this chapter, implications for instructors moves in the context of undergraduate coursework, an explanation of the limitations of the study, and suggestions for future research are described.

8.1 Implications

The analysis of this intervention demonstrates that particular instructor moves support prospective teachers building and justification of rational number ideas. Specifically, employing combinations of pedagogical practice moves and question moves supporting building rational number ideas. The instructor's use of particular teacher moves reflected current research-based expectations of teachers. Examples of the instructor's interactions with prospective teachers could be used in training instructors of undergraduate mathematics, in training of prospective teachers during undergraduate mathematics courses, or in professional learning for teachers in general.

More specific salient findings for teacher educators include the importance of (1) intently examining teacher justifications alongside the mathematical relationships portrayed by supporting physical models in pursuit of deeper understanding of student reasoning; (2) recognizing and attending to the construction of various solutions and/or strategies in order to seize opportunities for in-the-moment decisions that make them public to the class; and (3) engaging learners in the reconstruction of multiple solutions or representations of a mathematical idea, as well as in the explanation of the relationship between those solutions.

Some change in prospective teachers' beliefs regarding the learning, teaching, concepts and procedures, and student and teacher roles were noted. While the end of course essays captured limited information regarding prospective teachers' beliefs, analysis of the beliefs inventory assessment data revealed shifts in overall beliefs away from perspectives inconsistent with standards.

Physical representations were key aspects to the sequence of tasks that comprise this intervention. Notably, the prospective teachers' beliefs both before and after the intervention were well aligned to the standard indicating that manipulatives are valuable for all learners, particularly as reasoning and communication tools. Consequently, the potential impact of the intervention on prospective teachers whose beliefs are not initially well aligned with this standard was not be examined.

8.2 Limitations

Six prospective teachers enrolled in an undergraduate mathematics course participated in this intervention. The results of a study with such a small sample size are not generalizable. However, a cohort of this size allowed for deep analysis of video data that captured the individual work and discourse of each teacher.

For each session, the prospective teachers worked in two small self-selected groups while sitting at adjoined tables. Two videographers captured the physical models constructed and the rational number ideas communicated. The videographers captured the physical movement and gestures of the instructor only when the instructor happened into view of the camera.

This intervention was the second intervention within this semester-long course. The beliefs inventory conducted as a pre-assessment was administered in advance of a 6-week combinatorics intervention that preceded the fractions intervention. The combinatorics intervention may have impacted the results reported on the post-assessment.

Video data allow for observation of instructors moves and the corresponding reactions of prospective teachers. Video data also record questions and the verbal responses of prospective teachers. Video data do not, however, capture the rationale for particular pedagogical and question moves, thereby limiting the ability to fully describe the dynamics of the intervention.

8.3 Suggestions for Further Study

This study provided detailed information on the instructor's pedagogical and questions moves, as well as on prospective teachers' rational number ideas, physical models, and solutions. However, it might be useful to examine the instructor's non-verbal moves to see what effects those types of moves, not captured in this study, may have on prospective teachers building rational number ideas.

Given that the findings of this study are not generalizable, additional implementations of this intervention might be useful in determining which findings, if any, are independent of the instructor, independent of the cohort of prospective teachers, and therefore durable.

Structured interviews of the instructor designed to assess the intentionality of and rationale for employing particular instructor moves might be useful. Collection and analysis of generalizable data regarding reasoned decision-making when employing particular instructor moves could ultimately be informative in a variety of professional learning contexts for both pre-service and in-service teachers.

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APPENDICES

Appendix A Course Schedule

Math 380 - Mathematics Reasoning and Assessment
Felician College
Class Met W,F from 2:35 – 3:50
Spring 2011

Date	Topic	Attendance
Friday, January 21	First Day - Introductions • Administer beliefs assessment • Administer counting strand pre-assessment • Assigned Gang of Four video – pre-assessment for homework	Fae was absent
Wednesday, January 26	Class Cancelled: Inclement Weather	
Friday, January 28	Mixture of Topics • Collected fraction pre-assessment • Discussed quadratic and exponential functions. • Discussed patterns and deduction. • Discussed triangular and Fibonacci numbers • Worked on the Handshake Problem.	fae was absent
Wednesday, February 2	Class Cancelled: Inclement Weather	
Friday, February 4	Mixture of Topics • Discussed homework questions • Focused on Triangular numbers and the Chessboard problem	Kelly was absent
Wednesday, February 9	Induction • Modeled proofs that demonstrated the steps for induction	All present
Friday, February 11 Videotaped	Combinatorics Intervention • Towers 4-tall choosing from 2 colors • Ankur's Challenge	All present
Wednesday, February 16	Induction • Reviewed Induction Homework	All present
Friday, February 18 Videotaped	Combinatorics Intervention • The towers problem – 4 tall, 2 colors. • The pizza problem – 4 toppings. • Isomorphism between the towers and the pizza problems.	Janelle and Fae. were absent

Date	Topic	Attendance
Wednesday, February 23 Videotaped	Combinatorics Intervention • Discussed the isomorphism between the pizza, the towers, and Pascal's triangle. • Isomorphism between the binomial expansion and the towers and pizza	Darlene and Fae were absent
Friday, February 25	Formal Proofs • The instructor explains proof by contradiction, proof by cases, and induction. • Watched the Brandon video and they were asked to see what types of informal proofs they saw in the video.	All present
Wednesday, March 2	Proofs and Fibonacci numbers	All present
Friday, March 4 Videotaped	Combinatorics Intervention • Addition rule for Pascal's triangle using towers and pizzas. • Taxi Cab Problem.	Darlene, Fae and Janelle were absent
Wednesday, March 9	Spring Break	
Friday, March 11	Spring Break	
Wednesday, March 16 Videotaped	Combinatorics Intervention • Ankur's Challenge • Pascal's Pyramid • Taxi Cab Problem • Isomorphism between the taxicab problem and the towers problem	Darlene was absent
Friday, March 18	Inductive Proofs • Formal algebraic proof for Pascal's Identity	Kelly and Fae were absent
Wednesday, March 23	Inductive Proofs and Number Theory • Completed two inductive proofs together. • Started number theory – discussed divisibility.	Janelle was absent
Friday, March 25	Algebraic Proofs • Assigned gang of four assessment for homework. • Algebraic Proofs	Fae was absent
Wednesday, March 30	Number Theory • Discussed the Golden Ratio and Fibonacci numbers • Discussed a problem from the in-house math contest.	Attendance data not available

Date	**Topic**	**Attendance**
Friday, April 1	Number Theory • Discussed 6 theorems from number theory	Attendance data not available
Wednesday, April 6	Number Theory • Fundamental theorem of arithmetic • Prime Factorization and abundant numbers	Attendance data not available
Friday, April 8	Number Theory/ Introduction to Fraction intervention • Conjectures and proofs • Prime Factorization and abundant numbers. • Introduced Cuisenaire rods	Attendance data not available
Wednesday, April 13 Videotaped	Fraction Intervention • Upper and lower bound video watched	All present
Friday, April 15 Videotaped	Fraction Intervention	All present
Wednesday, April 20	Fraction Intervention	Fae and Kelly absent
Friday, April 22	No Class - holiday	
Wednesday, April 27 Videotaped	Fractions	All present
Friday, April 29 Videotaped	Fraction Intervention	Fae, Sara and Janelle absent
Wednesday, May 4 Videotaped	Mixture of Topics • Signed numbers • Taxicab problems • Some fraction problems	Kelly absent
Friday, May 6 Videotaped	Mixture of Topics • Signed numbers • Taxicab problems • Some fraction problems	Kelly absent
Wednesday, May 11	No Class – reading day	
Friday, May 13	Last day - finals • 2 take home essays • beliefs post-assessment • fractions post-assessment	All present

Appendix B Beliefs Assessment

1. Learners generally understand more mathematics than their teachers or parents expect.

1	2	3	4	5
Strongly Agree				Strongly Disagree

2. Teachers should make sure that students know the correct procedure for solving a problem.

1	2	3	4	5
Strongly Agree				Strongly Disagree

3. Calculators can help students learn math facts.

1	2	3	4	5
Strongly Agree				Strongly Disagree

4. It's helpful to encourage student-to-student talking during math activities.

1	2	3	4	5
Strongly Agree				Strongly Disagree

5. Math is primarily about learning the procedures.

1	2	3	4	5
Strongly Agree				Strongly Disagree

6. Students will get confused if you show them more than one way to solve a problem.

1	2	3	4	5
Strongly Agree				Strongly Disagree

7. All students are capable of working on complex math tasks.

1	2	3	4	5
Strongly Agree				Strongly Disagree

8. Math is primarily about identifying patterns.

1	2	3	4	5
Strongly Agree				Strongly Disagree

9. If students learn math concepts before they learn the procedures, they are more likely to understand the concepts.

1	2	3	4	5
Strongly Agree				Strongly Disagree

10. Manipulatives should only be used with students who don't learn from the textbook.

1	2	3	4	5
Strongly Agree				Strongly Disagree

11. Young children must master math facts before starting to solve problems.

1	2	3	4	5
Strongly Agree				Strongly Disagree

12. Teachers should show students multiple ways of solving a problem.

1	2	3	4	5
Strongly Agree				Strongly Disagree

13. Only really smart students are capable of working on complex math tasks.

1	2	3	4	5
Strongly Agree				Strongly Disagree

14. Calculators should be introduced only after students learn math facts.

1	2	3	4	5
Strongly Agree				Strongly Disagree

15. Learners generally have more flexible solution strategies than their teachers or parents expect.

1	2	3	4	5
Strongly Agree				Strongly Disagree

16. Math is primarily about communication.

1	2	3	4	5
Strongly Agree				Strongly Disagree

17. Manipulatives cannot be used to justify a solution to a problem.

1	2	3	4	5
Strongly Agree				Strongly Disagree

18. Learners can solve problems in novel ways before being taught to solve such problems.

1	2	3	4	5
Strongly Agree				Strongly Disagree

19. Understanding math concepts is more powerful than memorizing procedures.

1	2	3	4	5
Strongly Agree				Strongly Disagree

20. Diagrams are not to be accepted as justifications for procedures.

1	2	3	4	5
Strongly Agree				Strongly Disagree

21. If students learn math concepts before procedures, they are more likely to understand the procedures when they learn them.

1	2	3	4	5
Strongly Agree				Strongly Disagree

22. Students are able to tell when their teacher does not like mathematics.

1	2	3	4	5
Strongly Agree				Strongly Disagree

23. Collaborative learning is effective only for those students who actually talk during group work.

1	2	3	4	5
Strongly Agree				Strongly Disagree

24. Students should be corrected by the teacher if their answers are incorrect.

1	2	3	4	5
Strongly Agree				Strongly Disagree

25. Mixed ability groups are effective organizations for stronger students to help slower learners.

1	2	3	4	5
Strongly Agree				Strongly Disagree

26. Collaborative groups work best if students are grouped according to like abilities.

1	2	3	4	5
Strongly Agree				Strongly Disagree

27. Conflicts in learning arise if teachers facilitate multiple solutions.

1	2	3	4	5
Strongly Agree				Strongly Disagree

28. Learning a step-by-step approach is helpful for slow learners.

1	2	3	4	5
Strongly Agree				Strongly Disagree

29. Only the most talented students can learn math with understanding.

1	2	3	4	5
Strongly Agree				Strongly Disagree

30. The idea that students are responsible for their own learning does not work in practice.

1	2	3	4	5
Strongly Agree				Strongly Disagree

31. Teachers need to adjust math instruction to accommodate a range of student abilities.

1	2	3	4	5
Strongly Agree				Strongly Disagree

32. Teacher questioning of students' solutions tends to undermine students' confidence.

1	2	3	4	5
Strongly Agree				Strongly Disagree

33. Teachers should intervene as little as possible when students are working on open-ended mathematics problems.

1	2	3	4	5
Strongly Agree				Strongly Disagree

34. Students should not be penalized for making a computational error when they use the correct procedures for solving a problem.

1	2	3	4	5
Strongly Agree				Strongly Disagree

Appendix C Beliefs Inventory Statement Data

Overall Beliefs Consistency Results by Belief Statement

Beliefs Statement	**Pre-Assessment**	**Post-Assessment**
	CN (CP)	*CN (CP)*
Learners generally understand more mathematics than their teachers or parents expect (E1)	3 (50%)	4 (67%)
All students are capable of working on complex math tasks (E7)	2 (33%)	2 (33%)
Inverse of: Only really smart students are capable of working on complex math tasks (E13)	6 (100%)	6 (100%)
Learners generally have more flexible solution strategies than their teachers or parents expect (E15)	3 (50%)	5 (83%)
Learning a step-by-step approach is helpful for slow learners (E28)	6 (100%)	5 (83%)
Inverse of: Only the most talented students can learn math with understanding (E29)	6 (100%)	6 (100%)
It's helpful to encourage student-to-student talking during math activities (MD4)	6 (100%)	6 (100%)
Inverse of: Collaborative learning is effective only for those students who actually talk during group work (MD23)	4 (67%)	1 (17%)
Teachers should make sure that students know the correct procedure for solving a problem (C2)	6 (100%)	2 (33%)
Inverse of: Math is primarily about learning procedures (C5)	1 (17%)	3 (50%)
If students learn math concepts before they learn the procedures, they are more likely to understand the concepts (C9)	3 (50%)	5 (83%)
Inverse of: Young children must master math facts before starting to solve problems (C11)	1 (17%)	4 (67%)
Learners can solve problems in novel ways before being taught to solve such problems (C18)	4 (67%)	5 (83%)
Understanding math concepts is more powerful than memorizing procedures (C19)	6 (100%)	6 (100%)
If students learn math concepts before procedures, they are more likely to understand the procedures when they learn them (C21)	4 (67%)	4 (67%)
Inverse of: Manipulatives should only be used with students who don't learn from the textbook (M13)	6 (100%)	6 (100%)

Inverse of: Manipulatives cannot be used to justify a solution to a problem (M17)	6 (100%)	6 (100%)
Inverse of: Students will get confused if you show them more than one way to solve a problem (ST6)	4 (67%)	3 (50%)
Students should be corrected by the teacher if their answers are incorrect (ST24)	3 (50%)	2 (33%)
Inverse of: The idea that students are responsible for their own learning does not work in practice (ST30)	3 (50%)	4 (67%)
Teachers need to adjust math instruction to accommodate a range of student abilities (ST31)	5 (83%)	5 (83%)
Inverse of: Teacher questioning of students' solutions tends to undermine students' confidence (ST32)	3 (50%)	2 (33%)

Appendix D Beliefs Inventory Data by Question Category

Beliefs Consistency Results - Expectations and Abilities Category

Teacher	Pre-Assessment				Post-Assessment			
	CP	*CN*	*IN*	*UN*	*CP*	*CN*	*IN*	*UN*
FC	50	3	0	3	50	3	1	2
KD	50	3	1	2	67	4	1	1
JM	50	3	1	2	67	4	0	2
RH	100	6	0	0	100	6	0	0
JR	100	6	0	0	100	6	0	0
FS	83	5	0	1	83	5	0	1

Beliefs Consistency Results – Mathematical Discourse Category

Teacher	Pre-Assessment				Post-Assessment			
	CP	*CN*	*IN*	*UN*	*CP*	*CN*	*IN*	*UN*
FC	100	2	0	0	50	1	0	1
KD	100	2	0	0	50	1	0	1
JM	100	2	0	0	100	2	0	0
RH	50	1	1	0	50	1	0	1
JR	100	2	0	0	50	1	1	0
FS	50	1	0	1	50	1	0	1

Beliefs Consistency Results – Concepts and Procedures Category

Teacher	Pre-Assessment				Post-Assessment			
	CP	*CN*	*IN*	*UN*	*CP*	*CN*	*IN*	*UN*
FC	43	3	1	3	71	5	0	2
KD	57	4	2	1	14	1	5	1
JM	57	4	1	2	71	5	1	1
RH	57	4	2	1	86	6	0	1
JR	71	5	0	2	71	5	0	2
FS	71	5	1	1	100	7	0	0

Beliefs Consistency Results – Manipulatives Category

Teacher	Pre-Assessment				Post-Assessment			
	CP	*CN*	*IN*	*UN*	*CP*	*CN*	*IN*	*UN*
FC	100	2	0	0	100	2	0	0
KD	100	2	0	0	100	2	0	0
JM	100	2	0	0	100	2	0	0
RH	100	2	0	0	100	2	0	0
JR	100	2	0	0	100	2	0	0
FS	100	2	0	0	100	2	0	0

Beliefs Consistency Results – Student and Teacher Roles Category

Teacher	**Pre-Assessment**				**Post-Assessment**			
	CP	*CN*	*IN*	*UN*	*CP*	*CN*	*IN*	*UN*
FC	20	1	1	3	20	1	0	4
KD	40	2	3	0	40	2	1	2
JM	60	3	0	2	80	4	0	1
RH	100	5	0	0	80	4	0	1
JR	80	4	0	1	60	3	0	2
FS	60	3	0	2	40	2	1	2

Appendix E Final Project Essays

Essay 1
Author: Fae
Topic: The meaning of equivalent fractions and why you need a common denominator when you add or subtract fractions.

Line	Text
1	Learning fractions is a major importance in a student's life. As a student,
2	I never liked solving any math problems that had to do with fractions. I did
3	not like doing fraction equations, or fraction word problems, etc. I would try
4	to avoid fractions in any way possible. During this semester I have finally
5	learned the reasoning of equivalent fractions and why a common
6	denominator is necessary when adding or subtracting fractions.
7	I now know the proper definition of equivalent fractions. Fractions which
8	have the same value even though the numbers are different, is an easy way
9	to understand the meaning of equivalent fractions. The use of manipulatives
10	helped me realize the reasoning of two fractions being equal to one another
11	even though different color rods/different numbers were being used to
12	represent the two fractions. Since I plan to be a future educator I now know
13	a much simpler way of teaching fractions to students. If 1 was able to learn
14	through manipulatives as a college student, students in any grade can be
15	taught through the use of manipulatives to help with the understanding of
16	fractions.
17	Common denominators are used when adding or subtracting fractions
18	because the denominator shows how many equal parts the item is divided

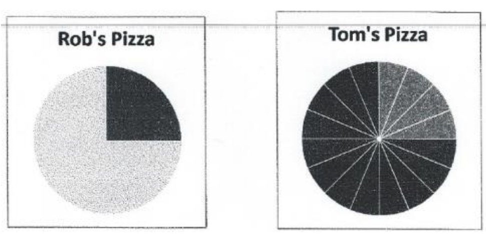
into. In order to add or subtract you need the amount of equal parts to be the same so you know how many pieces of that part you are adding or subtracting from. I always knew I had to find a common denominator in order to add or subtract fractions but never knew why, now I do.
I believe the use of manipulatives makes fractions so much easier and enjoyable to work with. I will no longer mind having to solve equations or word problems with the use of fractions because I can now just draw a picture of the rods or use other sources of manipulatives to help me solve.

Essay 2
Author: Kelly
Topic: Why dividing by two is different from dividing by one-half – why students have trouble with this concept and what you could do to help them increase their understanding.

Text
Dividing by two is different than dividing by one-half because a student can divide a number by two but when he or she is dividing by one-half, the fraction of one-half flips to make the number multiply by two. Students might have trouble with it because when they think of one-half they think of dividing it by two. For example, if the problem was eight divided by 2 (8/2), the answer would be four. If the problem was eight divided by one-half (8/½) the answer would be 16 because there is another bar under the division bar which means that the student has to multiply to get the half from under the fraction bar.
I think I've learned a lot this semester because the fractions make more sense to me. I have a better understanding of how to teach fractions to a group of students. I have more patience for students who do not understand something because 1 know how it feels to get frustrated at something.
Students need manipulatives to help them understand a specific topic because some students might not understand a specific concept just by thinking of it. The student might not understand why the (1/2) is multiplied but I would try to explain using the Cuisenaire rods. I might try to find a video for the students who are better listening to a video on fractions. The students need a bit of everything to practice techniques on how to add

fractions. Some people might explain it better than me and there might be more than one way of explaining it. There could be another way of solving the problem as well. I liked working with other people in case I was not understanding something my partners would try to help me

Essay 3
Author: Janelle
Topic: The meaning of equivalent fractions and why you need a common denominator when you add or subtract fractions.

Line	Text
	A fraction means a part of a whole. Therefore, when you have two-thirds, it means you have two parts out of a whole that consists of three parts. One-half means you have one part out of a whole that consists of two parts. Equivalent fractions mean that two or more fractions have the same value, even if they look different. One-half and two-fourths are equivalent fractions because two-fourths can be reduced to one-half. When you add fractions, you need to have the same number of parts that make up a whole. Having two-eighths and four-sixteenths, you cannot just add the numerator and the denominator together because they are not parts of the same whole.
	An example of equivalent fractions:  Rob has 2/8 of his pizza left over. Tom has 4/16 of his pizza left over. Even though these look different, they are equivalent fractions because they both are the same quantities. Even though Rob has two slices, and Tom has four slices, two of Tom's slices make up one of Rob's slices.
	To add these fractions, you must make the denominators of the fractions the same. Two-eighths is equivalent to four-sixteenths. Therefore, Rob also

Line	Text
18	has four-sixteenths of his pizza left over. When you have the same
19	denominator, you simply add the two numerators together. Therefore, if you
20	put Rob and Tom's left over pizza together, they have eight-sixteenths of
21	pizza between them.
22	$\frac{2}{8}=\frac{4}{16}$
23	$\frac{4}{16}+\frac{4}{16}=\frac{8}{16}=\frac{1}{2}$
24	Manipulatives would be very useful in this area of mathematics. Using
25	slices of pizza or Cuisenaire rods would be excellent manipulatives. Using
26	manipulatives allows students to touch tangible items in order to figure out
27	the fractions. By using pizza, there is a real-world connection that allows the
28	students to realize the importance of mathematics in everyday life. In
29	addition, using tangible items allows basic concepts to be retained quickly
30	and easily. Students are also motived to learn mathematics because they are
31	enjoying it instead of just drilling facts repetitively. Since the Cuisenaire
32	rods come in many different sizes, the fractions can be represented
33	horizontally. For some students, this method may allow fractions to be more
34	easily understood.
35	There are many ways to teach fractions. I believe the best way to
36	introduce fractions to children are with tangible, real-life objects. A pizza
37	would be an excellent way. Since it is a circle, it can but cut in many
38	different ways. You can represent one, one-half, one-third, one-fourth, one-
39	fifth, one-sixth, etc... Any fraction can be represented by a circle. This

Text
allows for tiered lessons. Using manipulatives gives a visual representation of the material instead of just random lines and numbers on a sheet of paper. One method I would avoid is asking students to memorize the relationship of fractions and equivalent fractions. By simply teaching students the methods for solving fraction problems, they will not understand the concept. By allowing them to play with manipulatives and the numbers, they will figure out their own methods to solving problems. In addition, the students will then be able to generalize their methods to continue solving increasingly difficult problems. If the student can figure out the process, the rules can be recreated.

Essay 4
Author: Erika
Topic: The meaning of equivalent fractions and why you need a common denominator when you add or subtract fractions.

Text
Equivalent fractions are fractions that are equal to each other but are written more than one way. (i.e., 3/4 = 6/8) This is an extremely hard concept of children to understand. Many times teachers do not feel like or know how to explain this to children. In order for children to comprehend this topic, they need to know about lowest terms. Therefore, lowest terms must be taught at the same time as equivalent fractions
A common denominator is needed because otherwise it would be almost impossible to add them together. Children may think that all you need to do is add the numerators together and the denominators together to get the correct answer (i.e. 3/4 + 1/2 = 5/4 not 4/6) Those are two very different answers. A common denominator, LCD preferably, will actually help children understand lowest terms as well. So if you show them common denominator work it will help them excel in equivalent fractions. One of the good things about math is that it builds on itself. Teachers that enjoy working with fractions are needed.

Essay 5
Author: Darlene
Topic: What it means to divide by a fraction and why the division algorithm works

Line	Text
	The word "division" is the noun form of "divide" which means to separate into groups, parts or sections. When discussing division, it is also important to understand that division is the opposite or "inverse" of multiplication. To illustrate this concept further, let's suppose that you are having a party for some friends. How do you determine how many guess you can serve if you have 12 large brownies that you are going to split each in half? The brownies are big and each person will eat half of a brownie. In this case, you take 12 but now you have to divide by ½. When you think about it, one large brownie will serve 2 people, since 1/2 plus 1/2 equal one whole. With each guest eating 1/2 a brownie, you can now serve double the amount of people as you have cookies, or in other words, twice the amount.
	When you divide by a fraction, you are essentially asking "How many times will the fraction fit into this number?" For example, 3/½ = 6/1 = 6. 1/2 fits into the number three 6 times. This way of thinking works when both parts of the equation are fractions. In order to make dividing fractions easier is to remember to invert and multiply. For example, if your problem is 2 divided by 1/4 think of this as a big fraction with 2 in the numerator and the fraction 1/4 in the denominator. The invert part of "invert and multiply" means to take the denominator of this big fraction, 1/4, and invert it. In other words, flip it so its numerator becomes its denominator and vice versa. The

inverse of 1/4 is therefore 4/1, or just 4. Now for the multiply part of "invert and multiply": all you need to do is multiply the 2 from the initial problem by the inverted denominator, 4. So, that's 2 times 4, which equals 8. However, now we have an easy method for doing harder problems too. Take 7 divided by 8/9. All we have to do is invert 8/9 to get 9/8, and., then multiply this by 7 (numerator: 7 x 9 = 63; denominator: 1 x 8 = 8) to find that the answer is 63/8, or 7 and 7/8. The division algorithm is a "guarantee" that long division will always work because every number can be written in this form whether it be negative or positive.

Essay 6
Author: Sarah
Topic: The meaning of equivalent fractions and why you need a common denominator when you add or subtract fractions.

Text
It is known in order to add and subtract fractions, you need a common denominator. Since a fraction is actually a division problem not worked out yet, instead of dividing 1 by 2 to get .50, we just say 1/2. I believe it is a lot like algebra, x/y, but we usually don't evaluate it because we don't know the values. They are usually difficult to find especially with two variables. I think the reason we don't think to evaluate fractions is because it is easier to use the fraction as an expression, rather than turn it into a decimal first, which can sometimes be confusing depending on the problem, we can use the distributive property to show this. Therefore, we need a new denominator for the answer. You can use (bd) as a common denominator and convert both fractions by that denominator by multiplying by 1:
(a/b) + (c/d) = (a/b) (1) + (c/d) (1) = (a/b)(d/d) + (c/d)(b/b) = (ad)/(bd) + (bc)/(bd) Then the distributive property shows the common denominator (bd) in a fraction form: = (ad+bc)/(bd) When doing addition, you need a common denominator first so you can factor it out.
In order to do equivalent fractions you need to first start out with a fraction.

For example 1/2. You have to multiply top and bottom by the same number and that is your equivalent fractions. So we can say a/b x d/d is equal to a/b. Let's say we started with the resulting fraction, we can divide d/d by the top and bottom (preferably the GCF) and also get a fraction in its simplest form.

Appendix F Task Statements

Math 380 April 13, 2011

1. What is the shortest train that can be measured by both the dark green and the purple rod?
2. What is the shortest train that can be measured by both the dark green rod and the brown rod?

3. What is the longest train that measures both the dark green rod and the purple rod?
4. What is the longest train that measures both the brown rod and the black rod?

Math 380 Fractions with Cuisenaire Rods April 13, 2011

1. Call the red rod 1. What are the number names for all other rods?

2. Call the orange rod 1. What are the number names for all other rods?

3. Select a different rod to call 1. What are the number names for all other rods?

4. Representing one-half:

a) if you call the brown rod 1, which rod represents one-half?

b) If you call the blue rod 1, which rod represents one half?

5. Call the light green rod 1.

a) What number is represented by the red rod?

b) What number is represented by the dark green rod?

6. Call the white rod one-third.

a) Which rod represents 1?

b) What number does the yellow rod represent?

7. Use Cuisenaire rods to model the following situation and answer the question. Which is larger, 3/4 or 2/3?

8. Make up your own question similar to the one above that can be answered using Cuisenaire rods.

Your name:

Rod that = 1:	
Rod	Fraction
White	
Red	
Lt. Green	
Purple	
Yellow	
Dk. Green	
Black	
Brown	
Blue	
Orange	

Math 380 April 15, 2011

1. Susie has 1/3 of a candy bar. She gives half of what she has to Paul. How much does she give to Paul? How much does she have left?

2. Keisha has a candy bar. She gives 1/2 of a bar to Pablo and 1/3 of a bar to Gordon. What portion of a candy bar does she have left?

3. John has 1/2 of a candy bar. Bill takes 1/3 of a candy bar from John. What portion of a candy bar does John have left?

Use the Cuisenaire rods to answer the above problems.

Math 380 **Problems with Fractions** April 15, 2011

1. Mary, Lisa, and Patricia each sent out for pizza, and they all had some pizza left over. Mary had ¼ of a pizza left over, Lisa had 1/3 of a pizza left over, and Patricia had 1/6 of a pizza left over. If they put all their leftover pizza together, how much pizza would they have?

2. Joe has a piece of wood ¾ meter long. If he cuts off a piece that is 1/6 of a meter, how long a piece of wood does he have left?

Use the Cuisenaire rods to answer the above problems. Then write mathematical sentences for these problems. Explain how the rods are related to the mathematical sentences.

Appendix G 04/13/11 Classwork

FAE

Math 380 Fractions with Cuisenaire Rods April 13, 2011

1. Call the red rod 1. What are the number names for all the other rods?

2. Call the orange rod 1. What are the number names for all the other rods?

3. Select a different rod to call 1. What are the number names for all the other rods?

4. Representing one-half: a) If you call the brown rod 1, which rod represents one-half? b) If you call the blue rod 1, which rod represents one-half?

a) purple = ½ of brown
b) none of them represent ½ of blue

5. Call the light green rod 1. a) What number is represented by the red rod? b) What number is represented by the dark green rod?

a) 2/3 = red
b) 2 = dark green

6. Call the white rod one-third. a) Which rod represents 1? b) What number does the yellow rod represent?

a) Lt green
b) 1 2/3 = yellow

---

7. Use Cuisenaire rods to model the following situations and answer the questions. Which is larger, 3/4 or 2/3?

3/4 is larger by 1/12 b/c the gcd = 12

8. Make up your own question similar to the one above that can be answered using Cuisenaire rods.

Which is larger 5/6 or 2/3

Rod that = 1:					
Rod	**Fraction**				
White	1/2	1/10	1/9		1/3
Red	1	1/5	2/9		2/3
Lt. Green	1 1/2	3/10	1/3		1
Purple	2	2/5	4/9	1/2	1 1/3
Yellow	2 1/2	1/2	5/9		1 2/3
Dk.Green	3	3/5	2/3		2
Black	3 1/2	7/10	7/9		
Brown	4	4/5	8/9	1	
Blue	4 1/2	9/10	1		
Orange	5	1	1 1/9		
	red	orange	Blue	Brown	Lt green

JANELLE

Math 380 Fractions with Cuisenaire Rods April 13, 2011

1. Call the red rod 1. What are the number names for all the other rods?

2. Call the orange rod 1. What are the number names for all the other rods?

3. Select a different rod to call 1. What are the number names for all the other rods?

4. Representing one-half: a) If you call the brown rod 1, which rod represents one-half? b) If you call the blue rod 1, which rod represents one-half?

a) purple
b) there are none b/c it's odd.

5. Call the light green rod 1. a) What number is represented by the red rod? b) What number is represented by the dark green rod?

a) 2/3 b) 2

6. Call the white rod one-third. a) Which rod represents 1? b) What number does the yellow rod represent?

a) light green b) 1 2/3

7. Use Cuisenaire rods to model the following situations and answer the questions. Which is larger, 3/4 or 2/3?

3/4 bigger (purple + light green and white)
3 brown 12 red = bigger by 1/12
4 dark green

8. Make up your own question similar to the one above that can be answered using Cuisenaire rods. Bgg.

Which is larger, 5/6 or 2/3?

Rod that = 1:	red	oran.	yellow
Rod	Fraction		
White	1/2	1/10	1/5
Red	1	1/5	2/5
Lt. Green	1.5	3/10	3/5
Purple	2	2/5	4/5
Yellow	2.5	1/2	1
Dk.Green	3	3/5	1 1/5
Black	3.5	7/10	1 2/5
Brown	4	4/5	1 3/5
Blue	4.5	9/10	1 4/5
Orange	5	1	2

DARLENE

1. Call the red rod 1. What are the number names for all the other rods?

2. Call the orange rod 1. What are the number names for all the other rods?

3. Select a different rod to call 1. What are the number names for all the other rods?

4. Representing one-half: a) If you call the brown rod 1, which rod represents one-half? b) If you call the blue rod 1, which rod represents one-half?

a) Purple
b) None

5. Call the light green rod 1. a) What number is represented by the red rod? b) What number is represented by the dark green rod? a) 2/3 b) 2

6. Call the white rod one-third. a) Which rod represents 1? b) What number does the yellow rod represent? a) Light green b) 1 2/3

---

7. Use Cuisenaire rods to model the following situations and answer the questions. Which is larger, 3/4 or 2/3?

3/4 is larger
Purple and light green rods with white

8. Make up your own question similar to the one above that can be answered using Cuisenaire rods.

Which is larger 5/6 or 2/3?

Rod that = 1:	1 Red	2 Orange	3 Yellow
Rod	Fraction		
White	1/2	1/10	1/5
Red	1	2/10 = 1/5	2/5
Lt. Green	1 1/2	3/10	3/5
Purple	2	4/10 = 2/5	4/5
Yellow	2 1/2	1/2	1
Dk.Green	3	6/10 = 3/5	6/5 = 1 1/5
Black	3 1/2	7/10	7/5
Brown	4	8/10 = 4/5	8/5
Blue	4 1/2	9/10	9/5
Orange	5	1	2

$$4/9 + \frac{1}{18} = \frac{9}{18} = \frac{1}{2}$$

SARA

Math 380 Fractions with Cuisenaire Rods April 13, 2011

1. Call the red rod 1. What are the number names for all the other rods?

2. Call the orange rod 1. What are the number names for all the other rods?

3. Select a different rod to call 1. What are the number names for all the other rods?

4. Representing one-half: a) If you call the brown rod 1, which rod represents one-half?
b) If you call the blue rod 1, which rod represents one-half?

b none = 1

5. Call the light green rod 1. a) What number is represented by the red rod? b) What number is represented by the dark green rod?

a $\frac{2}{3}$

b 2

6. Call the white rod one-third. a) Which rod represents 1? b) What number does the yellow rod represent?

a light green

b $\frac{5}{3}$ or $1\frac{2}{3}$

---

7. Use Cuisenaire rods to model the following situations and answer the questions. Which is larger, 3/4 or 2/3?

larger because of $\frac{1}{12}$

8. Make up your own question similar to the one above that can be answered using Cuisenaire rods.

$\boxed{\frac{5}{6} \text{ or } \frac{2}{3}}$ see sheet

what is larger → $\frac{1}{2}$ or $\frac{3}{4}$

Rod that = 1:	Red	orange	Blue	Brown
Rod	**Fraction**			
White	$\frac{1}{2}$	$\frac{1}{10}$	$\frac{1}{9}$	
Red	1	$\frac{1}{5}$	$\frac{2}{9}$	
Lt. Green	$1\frac{1}{2}$	$\frac{3}{10}$	$\frac{3}{9} = \frac{1}{3}$	
Purple	2	$\frac{4}{10}$	$\frac{4}{9}$	$\frac{1}{2}$
Yellow	$2\frac{1}{2}$	$\frac{1}{2}$	$\frac{5}{9}$	
Dk.Green	3	$\frac{6}{10} = \frac{3}{5}$	$\frac{6}{9} = \frac{2}{3}$	
Black	$3\frac{1}{2}$	$\frac{7}{10}$	$\frac{7}{9}$	
Brown	4	$\frac{8}{10} = \frac{4}{5}$	$\frac{8}{9}$	
Blue	$4\frac{1}{2}$	$\frac{9}{10}$	1	
Orange	5	1	$1\frac{1}{9}$	

KELLY

1. Call the red rod 1. What are the number names for all the other rods?

2. Call the orange rod 1. What are the number names for all the other rods?

3. Select a different rod to call 1. What are the number names for all the other rods?

4. Representing one-half: a) If you call the brown rod 1, which rod represents one-half? b) If you call the blue rod 1, which rod represents one-half?

a) purple is a half

b) there is no half

5. Call the light green rod 1. a) What number is represented by the red rod? b) What number is represented by the dark green rod?

a) $\frac{2}{3}$

b) $\frac{1}{2}$ – 2 light greens = 1 dark green

6. Call the white rod one-third. a) Which rod represents 1? b) What number does the yellow rod represent?

a) light green

b) yellow = $\frac{5}{3}$

---

7. Use Cuisenaire rods to model the following situations and answer the questions. Which is larger 3/4 or 2/3?

It is bigger by $\frac{1}{12}$

LCM = 12

8. Make up your own question similar to the one above that can be answered using Cuisenaire rods.

5/6 or 2/3

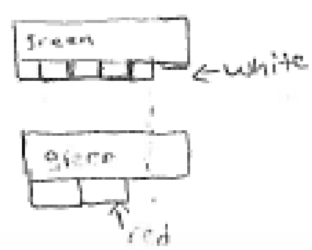
similiar question

$\frac{5}{6}$ is bigger than $\frac{2}{3}$

green = 1

white = $\frac{1}{5}$

red = $\frac{1}{3}$



Your name:	Kim

Rod that = 1:		orange	blue	brown		
Rod	Fraction					
White	$\frac{1}{2}$	$\frac{1}{10}$	$\frac{1}{9}$	$\frac{1}{8}$		
Red	1	$\frac{1}{5}$	$\frac{2}{9}$	$\frac{1}{4}$		
Lt. Green	$1\frac{1}{2}$	$\frac{3}{10}$	$\frac{1}{3}$			
Purple	2	$\frac{2}{5}$	$\frac{4}{9}$	$\frac{1}{2}$		
Yellow	$2\frac{1}{2}$	$\frac{1}{2}$	$\frac{5}{9}$	$\frac{5}{8}$		
Dk.Green	3	$\frac{3}{5}$	$\frac{2}{3}$	$\frac{6}{8}$		
Black	$3\frac{1}{2}$	$\frac{7}{10}$	$\frac{7}{9}$	$\frac{7}{8}$		
Brown	4	$\frac{8}{10}$ $\frac{4}{5}$	$\frac{8}{9}$	1		
Blue	$4\frac{1}{2}$	$\frac{9}{10}$	1	$1\frac{1}{8}$		
Orange	5	1	$1\frac{1}{9}$	$1\frac{2}{8}$		

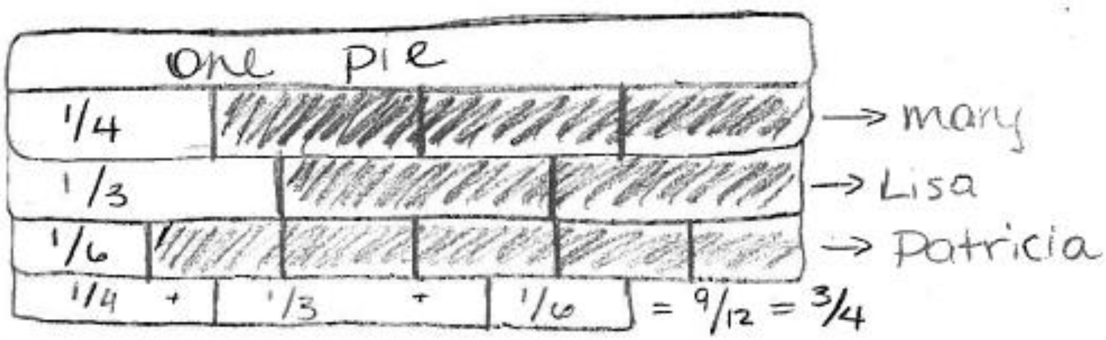
Appendix H 04/15/11 Classwork

FAE

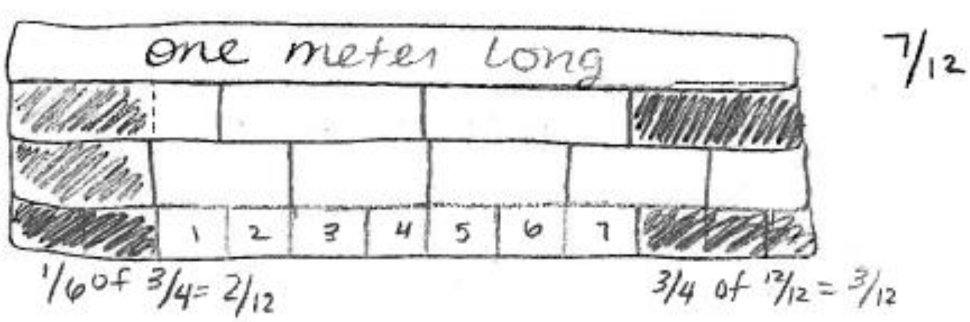
Math 380 April 15, 2011

Problems with Fractions

1. Mary, Lisa, and Patricia each sent out for pizza, and they all had some pizza left over. Mary had 1/4 of a pizza left over, Lisa had 1/3 of a pizza left over, and Patricia had 1/6 of a pizza left over. If they put all their leftover pizza together, how much pizza would they have?



2. Joe has a piece of wood 3/4 meter long. If he cuts off a piece that is 1/6 of a meter, how long a piece of wood does he have left?



Use the Cuisenaire rods to answer the above problems. Then write mathematical sentences for these problems. Explain how the rods are related to the mathematical sentences.

KELLY

Math 380 April 15, 2011

Problems with Fractions

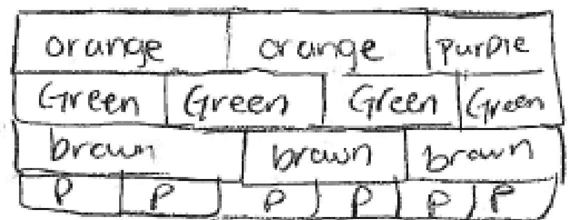
1. Mary, Lisa, and Patricia each sent out for pizza, and they all had some pizza left over. Mary had 1/4 of a pizza left over, Lisa had 1/3 of a pizza left over, and Patricia had 1/6 of a pizza left over. If they put all their leftover pizza together, how much pizza would they have?

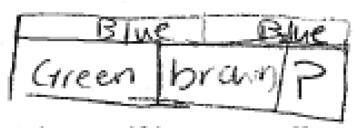
$$\frac{1}{3} = \frac{8}{24}$$

$$\frac{1}{4} = \frac{6}{24}$$

$$\frac{1}{6} = \frac{4}{24}$$

$$\frac{18}{24} = \frac{9}{12} = \frac{3}{4}$$



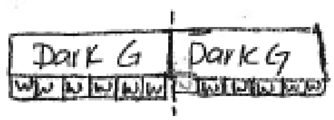


2. Joe has a piece of wood 3/4 meter long. If he cuts off a piece that is 1/6 of a meter, how long a piece of wood does he have left?

$$\frac{3}{4} = \frac{18}{24}$$

$$-\ \frac{1}{6} = \frac{4}{24}$$

$$\frac{14}{24} = \frac{7}{12}$$



6 whites = 1 Dark Green

7 whites / 2 Dark Green = 12

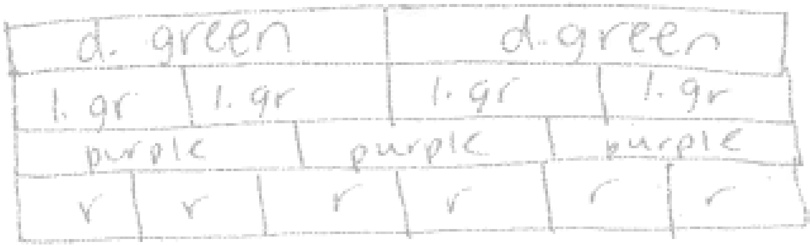
Use the Cuisenaire rods to answer the above problems. Then write mathematical sentences for these problems. Explain how the rods are related to the mathematical sentences.

JANELLE

Math 380 April 15, 2011

Problems with Fractions

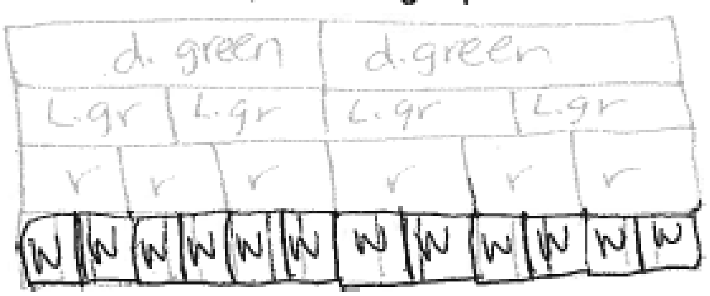
1. Mary, Lisa, and Patricia each sent out for pizza, and they all had some pizza left over. Mary had 1/4 of a pizza left over, Lisa had 1/3 of a pizza left over, and Patricia had 1/6 of a pizza left over. If they put all their leftover pizza together, how much pizza would they have?



3/4

$$\frac{1}{4}x + \frac{1}{3}x + \frac{1}{6}x = 3/4$$

2. Joe has a piece of wood 3/4 meter long. If he cuts off a piece that is 1/6 of a meter, how long a piece of wood does he have left?



$$3/4 - 1/6(1) = 7/12$$

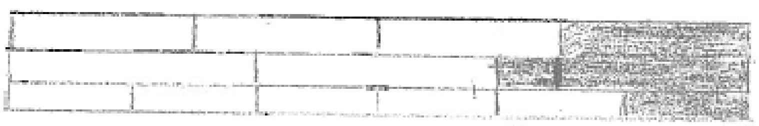
Use the Cuisenaire rods to answer the above problems. Then write mathematical sentences for these problems. Explain how the rods are related to the mathematical sentences.

ERIKA

Math 380 April 15, 2011

Problems with Fractions

1. Mary, Lisa, and Patricia each sent out for pizza, and they all had some pizza left over. Mary had 1/4 of a pizza left over, Lisa had 1/3 of a pizza left over, and Patricia had 1/6 of a pizza left over. If they put all their leftover pizza together, how much pizza would they have?



1/4 is left
1/3 is left
1/6 is left

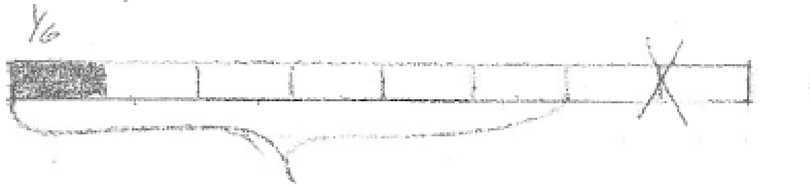
$$\frac{1}{4} + \frac{1}{3} + \frac{1}{6} = \frac{3}{12} + \frac{4}{12} + \frac{2}{12} = \frac{9}{12}$$

2. Joe has a piece of wood 3/4 meter long. If he cuts off a piece that is 1/6 of a meter, how long a piece of wood does he have left?

$$\frac{3}{4} - \frac{1}{6} = \frac{9}{12} - \frac{2}{12} = \frac{7}{12}$$

3/4 of a whole rod minus 1/6.

1/6



6/8 = 3/4

Use the Cuisenaire rods to answer the above problems. Then write mathematical sentences for these problems. Explain how the rods are related to the mathematical sentences.

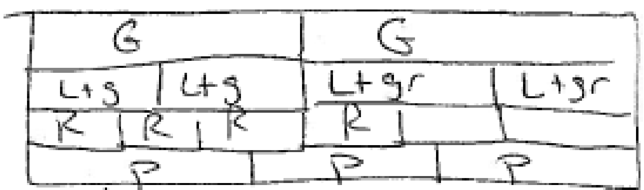
DARLENE

Math 380 April 15, 2011

Problems with Fractions

1. Mary, Lisa, and Patricia each sent out for pizza, and they all had some pizza left over. Mary had 1/4 of a pizza left over, Lisa had 1/3 of a pizza left over, and Patricia had 1/6 of a pizza left over. If they put all their leftover pizza together, how much pizza would they have?

2 Greens (Dark) = 1
4 Lt greens = 1
6 Reds = 1
3 Purples = 1

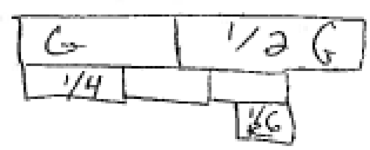


$\frac{1}{4x} + \frac{1}{3x} + \frac{1}{6x} = 3/4$

2. Joe has a piece of wood 3/4 meter long. If he cuts off a piece that is 1/6 of a meter, how long a piece of wood does he have left?

2 Dark greens = 1
3/4 = 3 of lt greens
Red = 1/6

$\frac{3}{4x} - \frac{1}{6x}$ $= \frac{1}{2x} + \frac{1}{12} = \frac{7}{12}$



Use the Cuisenaire rods to answer the above problems. Then write mathematical sentences for these problems. Explain how the rods are related to the mathematical sentences.

SARA

Math 380 April 15, 2011 #10

Problems with Fractions

1. Mary, Lisa, and Patricia each sent out for pizza, and they all had some pizza left over. Mary had 1/4 of a pizza left over, Lisa had 1/3 of a pizza left over, and Patricia had 1/6 of a pizza left over. If they put all their leftover pizza together, how much pizza would they have?

$\frac{1}{4} + \frac{1}{6} + \frac{1}{3}$ $\frac{1}{4}$ $\frac{3}{12}$

1 orange, 1 red

12 whites

12

$+ \frac{1}{6}$ $\frac{2}{12}$ $\frac{9}{12} \div \frac{3}{3} = \frac{3}{4}$

$\frac{1}{3}$ $\frac{4}{12}$

2. Joe has a piece of wood 3/4 meter long. If he cuts off a piece that is 1/6 of a meter, how long a piece of wood does he have left?

1 brown

4 red

8 white

$\frac{3}{4} - \frac{1}{6} =$

$\frac{9}{12} - \frac{2}{12} = \frac{7}{12}$ left

Use the Cuisenaire rods to answer the above problems. Then write mathematical sentences for these problems. Explain how the rods are related to the mathematical sentences.

see above

Appendix I Transcripts

Transcript 1 of 6
Date: 04/13/2011
Length: 00:15:07
Camera 1, Part 1
Transcribed by: Deidre Richardson
Verified by: Mary Huizenga

Line	Time	Speaker	Transcript
1		T/R	So we had definitions. Right? And we're using these things called Cuisenaire rods, and I probably didn't tell you last week about Cuisenaire rods. Did I tell you what they're named for? Ok
2		Erika	No
3		T/R	Yeah, Mr. Cuisenaire, a Belgian mathematician who invented them you know, for teaching various kinds of math. Ok. And notice they're different from the base 10 blocks, which we separated from them because they don't have the little markings like for tens or anything so you can make them equal to anything. And we looked at them last week using them as whole number measures. And so we said, a rod measures another rod, or measures a train if you can line it up. So, for example, thank you. White measures red because white lines up with red evenly whereas, um, red does not measure yellow because you can't line up an even number of reds. So, right that's what we mean by measures
4	1:01		Cool
5		T/R	Ok and a train is just making them, you know end-to-end. So, that's what a train is. So you could have a train be one rod or a whole bunch of any kinds of rods. So, what we said was … and I hope I reconstructed all the questions, we have the dark green rod and the purple rod and we said we want to find a train that both of these measure. And you, three of you guys did it last time. Right you, F… you did it … and you two did it [points to Jaime and Jess]. Ok, so you three here in the middle didn't do this last time. So you [points to F…] can help them [points to K… and F…] reconstruct how you found the train that they both measure, but that's not it! This means something that you can line up, like for example, … uhh shoot uhhh … I can't remember
6	1:52	Erika	This one. This one?

Line	Time	Speaker	Transcript
7		T/R	Ok, yeah but we didn't want to show them right away ?
8		Erika	Oohhh.
9		T/R	Ok, so for example the yellow measures the orange.
10		Fae	Ok
11		T/R	And also, the red measures the orange. Alright. So if I said find something that both yellow and red measure, you could say it measures the orange.
12		Kelly	Ok
13		T/R	That's what I'm doing with this. Something that both this measures [refers to green rod] and this measures [refers to purple rod]. Alright?
14	2:15	Kelly	Uhm?
15	2:16	T/R	And you can't say orange because …
16	2:16	Fae	I do them separately?
17		T/R	You have to do them separately. Right. You can't put them together. You have to line up greens and then you have to line up purples and then you have to get them both to line up to the end. Do you remember how it worked last time?
18	2:30	Sarah	Not the specific colors
19		T/R	Ok, then but you remember how it worked last time …
20	2:34	Erika	Yeah
21	2:34	T/R	…because you were showing us. Ok, so uhm, J… will show us what kind of train she created.
22	2:46	Erika	Since you need the green and the purple, I lined it up to figure out, how like, lined it up so that the purple measured the green. Because you have to make sure they both measure the other.
23		T/R	Ok
24		Erika	One
25		T/R	So does everybody see that? So the train is? Describe the train.
26			Two greens
27	3:04	Erika	What, the two greens? Two, two dark greens.
28	3:04	T/R	Two greens. The train is two dark greens or the train could be called three purples. Either way.

Line	Time	Speaker	Transcript
29		Erika	Yes
30		T/R	Or in fact, I think you could make the train equal to what you just had there.
31		Erika	An orange and a red
32		T/R	Ok. So any one of those descriptions will be a train that is measured by the dark green and the purple. And the claim is that's the shortest train that you could measure with a dark green and a purple. And how do you know it's the shortest?
33	3:31	Erika	Um. Well, if I were to use one green, the purples are too long. So I needed to add another green, but then the purples are too short. So I grabbed another purple.
34		T/R	Right. Ok. So one green doesn't work. And the next thing you tried was two greens and it did work. And there's nothing in between one and two, so we have discrete math. Ok and then we're gonna go to set two. The longest train that measures both the dark green and a purple. So now we're going the other way around. We want a rod that ... a rod that fits into the purples and the same rod fits in evenly into the greens. So a rod that fits in evenly here and a rod that fits in evenly here.
35		Fae	Ok. Like that?
36		T/R	Yeah now does it work for purple too?
37		Fae	You mean for green?
38		T/R	Yeah green sorry.
39	4:28	Fae	No. Yeah.
40		T/R	Yes. Ok, ok, so this is a rod that measures purple and a rod that measures green. Right?
41		Fae	It measures both green and red.
42		T/R	Right. Red measures purple and red measures green. Ok and that's the longest train because if you take anything longer than red, is it gonna work?
43		Fae	Nope
44		T/R	What's the next thing longer than red?
45	4:54	Fae	This one? And it will not work because <inaudible>
46		T/R	Ok, doesn't work for? It works for purple but not for…?
47		Fae	It works for green but not for purple
48		T/R	Right. It works for dark green and not for purple. Ok, so that's

Line	Time	Speaker	Transcript
			the stuff we were doing, um, last time. And then we did, um, the … dark green … The shortest train that can be measured … I'm not sure if this is the exact one we did, but dark green and brown. So you want something … <inaudible>
49	5:26	Kelly	Wait, brown and black? Oh
50	5:30	Erika	We didn't do that one
51		T/R	You didn't do that one. <inaudible> Ok, then why don't you… You stay over here and you come over here.
52	5:48	Fae	Do it like, can we do it like this? With the two greens and three purples? And then, red. Right?
53		FS	Yeah <inaudible>
54		Fae	I think it's this one. Let's try, see if the red one works. Three reds for this one. I don't think the reds are gonna work for this one.
55	6:25	Kelly	They look like bricks
56		Fae	Maybe it does. That's wrong. It's the reds again.
57	6:31	Kelly	Why is this wrong?
58		Fae	Well It's not wrong, but it's not the shortest.
59	6:32	Sarah	This is the one that has two greens and it's only supposed to be one. It's a short, I don't think…<inaudible>
60		Fae	I think it's supposed to be reds.
61	6:43	Erika	We're doing the second part of set one?
62		T/R	We are doing the second part of set one.
63		Erika	Yeah so the shortest train that can be measured by both green and brown
64		T/R	Ok
65		Erika	Oh no, we did
66		T/R	You found one?
67		Erika	But
68		T/R	Ok
69		Fae	I'm confused. Why is it so long?
70		Erika	No no no. We did it the
71		Janelle	Measured… The difference is 'measured by' and 'measures'. You have to realize the difference.

Line	Time	Speaker	Transcript
72		Erika	We did it the
73		Darlene	We did the opposites
74		Erika	We did it the wrong way
75	7:07	T/R	Yes. Well, it's ok. We can save that for later. So you claim that this is the shortest one
76		Janelle	… for green and brown.
77		T/R	Yeah. So lets just pull everything – all the extraneous stuff away.
78		Erika	Yeah. There we go.
79	07:19	T/R	So she said this is the shortest train that is measured by green and brown. And you have the same thing there. You can stay over here and you can go with her. Ok. And how do you know it's the shortest? What happens if you try to make it shorter
80		Janelle	It doesn't work
81		T/R	It doesn't work. And it keeps on not working.
82		Janelle	Um hum
83		T/R	You agree with that too
84		Fae	Yes
85		T/R	If you take one brown away or one green away, they don't line up. And if you take more away, there's nothing that, they don't line up until you get that many of them
86		Fae	This is the shortest it could be, the way it lines up
87	07:48	T/R	Ok. Ok, so that … we didn't do this one last time but that's similar to what we just did.
88		Erika	Um hum
89		T/R	Ok, and then the other thing – the brown rod and the black rod. Now we want the longest one that measures both brown and black. I think this is the one we did last time.
90	08:04	Janelle	So you need smaller ones now.
91		T/R	Right. One that measures brown and black. Right. So that means …
92		Fae	So that's like this
93		T/R	Right. That's like that.
94		Janelle	And you can only use one color right?

Line	Time	Speaker	Transcript
95		Fae	Do the reds line up to the black?
96		Fae	Nope. Do the purple work?
97		Kelly	Right here
98	08:29	Sarah	It might be the white ones.
99	08:32	Fae	I think it is the white ones.
100	08:34	Sarah	It works for that
101	08:35	Fae	Yeah, but purple doesn't work for the black. It has to work for both.
102	08:36	Kelly	Oh
103	08:39	T/R	Yeah. You got that right K…. It has to work for both of them.
104		Kelly	Wait, isn't that the same size?
105		Fae	No
106		Kelly	No
107		Fae	The only one that works is the white ones. Because if you do the red, it lines up. It doesn't line, it doesn't line? Yes it does line up for this one, but it doesn't line up for this one.
108	09:02	T/R	Ok, so you're all … now, you're watching but you don't need to make them right? You get what she's doing?
109	09:06	Sarah	Yeah. I said it was probably the white ones
110	09:08	T/R	OK
111	09:08	Sarah	Because the other ones were too big.
112	09:10	T/R	Ok. And K…you're ok with that too?
113	09:13	Kelly	Um hum. Yeah
114	09:14	T/R	Ok. Ok green ones didn't work for brown, so it doesn't matter whether they work for black or not because they're not gonna work.
115		T/R	So the question for everybody and the question for the group that we had last time is, so what are we doing here? Jaime has an idea. Am I right?
116	9:35	Darlene	Yes. Um
117		T/R	Ok. We could all listen to J… idea.
118		Darlene	What was it? It was either the GCD … or the least common …
119		Erika	Oh! We said this! Oh. It was like. We had said this right when

Line	Time	Speaker	Transcript
			she was dropping me off. Um
120		Darlene	Least common, no.
121		Erika	No.
122		Darlene	Greatest common denominator, no.
123		Erika	It's greatest common factor isn't it? Yeah.
124		Darlene	That's what I thought it was.
125		Erika	That's what it was. We figured out it was greatest common factor.
126		T/R	Ok. So these uh … LCD I guess was the other thing you said but sometimes they call it LCM. Alright so this is greatest common factor. So what were we doing that was the same as the greatest common factor?
127		Erika	We were finding the … the highest, like. Like if these were numbers like one and two, we were finding the highest number that goes evenly into both the black and brown
128		T/R	Ok. So the black represented what number and the brown represented what number?
129		Erika	One, two, three, four, five, six, seven. Black is seven.
130		Janelle	Seven and eight, seven, eight.
131		T/R	So the greatest common factor…
132	10:52	Darlene	Is one
133		T/R	of seven and eight you said
134		Erika	Is one
135		Janelle	Is one
136		T/R	Is one. Which is the …
137		Darlene	We had the right idea
138		Erika	Yeah
139		T/R	… greatest common factor of black and brown is white
140		Fae	Correct
141		T/R	And so the least common multiple… So give me an example of that. What did we do for the brown and uh the purple and the dark green? What numbers did they represent? Purple and dark green.
142		Fae	Two and three?

Line	Time	Speaker	Transcript
143	11:28	Erika	No four
144		Sarah	Purple is four
145		Janelle	Four and six?
146		Erika	Yeah, Four and six.
147		T/R	And the least common – what do they go into?
148		Fae	This, like this represents two.
149		Erika	Uh
150		T/R	And white is one right?
151		Fae	Ok. Right. Right. Sorry
152		T/R	You didn't get that one last time.
153		Erika	I'm missing a color
154		T/R	Ok. So what do you have for the, remember the shortest train that was measured by both the purple
155		Erika	It was three green, no two green and three purple
156	11:55	T/R	Which is, what number would that be?
157		Janelle	Twelve
158		Darlene	Six?
159		T/R	Twelve.
160		Janelle	Six. Twelve
161		Darlene	Twelve
162		Erika	Twelve
163		Janelle	Twelve. Yeah
164		T/R	Are you ok?
165			I'm ok and so is the camera
166	12:06	T/R	Ok. Ok and that's just saying if white is one, that those are the lengths.
167		Fae	Right.
168		T/R	So that's. Yeah, that that's what you would do and that was really fast. You got that. So we're doing greatest common factor and least common multiple just by doing rods representing numbers from one on up and I wanted to use that as an introduction for fractions because we use greatest common factor and least common multiple when we do

Line	Time	Speaker	Transcript
			fractions.
169	12:31	Kelly	My most favorite thing in the world.
170		T/R	You're going to love fractions when we're done with this. [laughter]
171		Erika	K… like oh no, fractions.
172		Janelle	We're going to want to use Cuisenaire rods for the rest of our lives.
173		Erika	That was fun.
174	12:43	T/R	So. Um, I want you to do some stuff with these rods and then we're going to watch a video. And, ok, and the thing with these rods is we give them what we call number-names. And I started out giving white a number-name of one. Right, and then you knew if white was one then the orange was ten and the dark green was six and so on. Right, so if we're going to do them as fractions, we're going to give them number-names – well, for example we could give the orange a number-name of one and then the other things would be fractions. So that's the kind of thing we're gonna look at. If you give them different number-names, what kind of fractions can you represent? And the thing we're gonna start with, um, we'll do more with number-names later, but that's the idea. We're going to start with one half. And here's a simple example. If this was one, the length of this was one, what would represent one half?
175	13:39	Erika	Uh, white.
176		Sarah	white
177		T/R	You're going the other way
178		Fae	Oh, one half of that …
179		T/R	Yeah
180		Fae	… is white
181		Erika	White.
182		T/R	Now you were going the other way. If this was one half, what was one I think. Now white was one, a half because?
183		Janelle	It's half
184		Erika	It's half of the red
185		Fae	It's half the size of the rod
186		T/R	It's half of the size

Line	Time	Speaker	Transcript
187		Erika	Of the red rod
188		T/R	half the length. And another way to look at it is – put another one up here
189		Erika	It takes two
190		T/R	It takes two of them to make the one, so each of them is a half
191		Kelly	Two of them equal
192		Fae	It takes two to equal one
193		T/R	Ok. So, we could find half of various rods uh and in particular, the question we're going to look at is "What's a half of blue?"
194		Fae	Nothing
195		Erika	Wait, what is blue? Oh …
196		Janelle	Nada!
197		Erika	Ahh, you can't!
198		T/R	And how come?
199		Janelle	Because it's an odd number
200		Erika	Well if we were to give it number-names, blue is nine
201	14:32	T/R	It's nine, well …
202		Darlene	Because three …
203		Erika	Well if you count, well if you were to call this one - One, two, three, four, five, six, seven, eight, nine. Nine doesn't have a half, well whole number half.
204	14:43	T/R	Ok, there's no whole number that's half of nine.
205		Janelle	Right.
206		Erika	Exactly
207		T/R	Ok. We'll leave it at that. Ok, and now we're going to watch a video and I have stuff about the video, I hope. Ok so take one and pass them along.

Transcript 2 of 6
Date: 04/13/2011
Length: 00:37:18
Camera 1, Part 2
Transcribed by: Deidre Richardson
Verified by: Mary Huizenga

Line	Time	Speaker	Transcript
1		T/R	Or it doesn't work and purple doesn't work. How does that tell you that nothing else is gonna work?
2	00:04	Erika	Because they said there was nothing in between the purple and yellow.
3		T/R	And how do you know there's nothing in between purple and yellow?
4		Janelle	Because you have – the difference is one.
5		Erika	Yeah. Well yea that
6		T/R	The difference is one and also?
7		Erika	If you line it up like this you can tell that there is nothing in between. There's no other color in between these two.
8		Jaime	So the whites are what fit
9		Erika	The whites are … it goes up by a white every time
10		Jaime	Yeah, so there … just to see
11		Erika	Yeah. Just to show. Yeah. So everything goes up by one.
12	00:32	T/R	Ok. Is that a convincing argument?
13		Darlene	Um hum
14			Um hum
15		T/R	So yellow doesn't work and purple doesn't work and therefore nothing else works.
16		Erika	Yeah
17		Jaime	Yeah
18		T/R	Ok, now they have another thing that they worked on I believe in this class that I want to give you as a homework. And the homework was um, so can you make up a set of Cuisenaire rods so that you have a half of everything? And if you know the answer right now don't tell me just write it up for homework. Ok?
19		Erika	Can we take some home? To try to figure this

Line	Time	Speaker	Transcript
20		T/R	Sure. If anybody wants to take a handful of them home or one of each color. Fine. So that's the question. Make up a set of Cuisenaire rods so that you can always find a half. And now we have, um, some more fraction activities based on the stuff that the kids did in class and I will find my sheets in here. Here they are. Ok. So because we have two videographers, you know, you can be this group of three and you can be this group of three. Here's your stuff.
21	02:11	Erika	Oh. Answer sheet?
22		Janelle	Nooo
23		Erika	Oh. Rod that equals one.
24		Darlene	The answer sheet.
25		Erika	What? What is this sheet?
26		Darlene	I don't know.
27		Erika	What are we doing with this?
28		Darlene	I don't know.
29		T/R	That goes with this.
30		Erika	Oh
31	02:24	Janelle	Write your name down first.
32		T/R	Yes. Right. Put your name.
33		Janelle	Step one
34		Erika	Step one; print your name. Step two; read all questions completely.
35		T/R	Ok. Ok, now, yeah, we can do one. Right? um. Call the red rod one.
36		Erika	Yeah. Red rod one.
37		T/R	Ok. If the red rod is one…
38		Erika	One of these
39		Janelle	So I can write, so you write here one?
40		T/R	Red rod, rod that equals one, you put red. In fact we need more of those sheets, but that's the idea.
41		Erika	Yeah.
42	02:52	T/R	So if the red rod is one, you know what the white rod is what, Right?

Line	Time	Speaker	Transcript
43		Erika	One half
44		Janelle	One half
45		T/R	It's one half. Ok
46		Janelle	And then red is one.
47		T/R	Yeah. Now if we move up, skip something and do purple. If the red is one, what's purple?
48		Erika	Four
49		Janelle	Two
50		T/R	Yeah, four whites but it's
51		Erika	Oh! Sorry.
52		Janelle	Purple is two
53		T/R	Ok. You guys agree with what we were just saying? If the red is one then purple is two.
54		Fae	mmhmm
55		T/R	Yep
56		Kelly	Wait. What?
57		T/R	If red is one, how much is purple?
58		Kelly	Oh. Ohhh ok.
59		T/R	Two. Right? And if red is one and we skip up to orange, tell me what orange is.
60		Kelly	Five
61		T/R	Five. Ok
62		Fae	It's a complete five?
63	03:40	T/R	Right. It's exactly. Right. Ok, and that's the idea. So, what are you going to do for all the colors? And uh, alright. F….. got the hint and she's filling in the even ones. No, you're filling in all of them. So, let's see if I … So, you guys can talk about what she's doing here.
64	03:58	Fae	I said every other one would be the half. This would be the odd numbers. So, like one, three, five, seven, nine. One, three, five, seven, nine. If they were all one.
65		Sarah	Yeah
66		Fae	But due to the fact that red is one, it will go by halves. And we have to answer the questions. Oh, this would be one-fifth.

Line	Time	Speaker	Transcript
			I don't know how many this equals to. One third. Oh. No. I don't know what that one would be called.
67	05:14	Sarah	So, this one is one?
68		T/R	Well, that one was one.
69		Sarah	Oh. We did that already?
70		Fae	It's the chart
71		T/R	Well and she's, and she's saying that if red is one, then light green is one and a half. And I believe that because you know why? Light green is a one plus a half.
72	05:28	Sarah	Yeah. I get it.
73		T/R	Because you already know that the one is a half. And she did that for all of these. Now we're – we've changed our rules. Now, this is one. And she said if this is one, then she gave – then she said red is …. If this is one, then what's red?
74		Kelly	Five
75		Sarah	Five
76		T/R	It takes five reds to make one, but that doesn't mean that red is five. It means?
77		Kelly	One fifth?
78		Sarah	One fifth.
79		T/R	Yeah, but that wasn't a question. Was it?
80	05:52	Kelly	One fifth!
81		T/R	Yes! One fifth! Ok. Right? Because it takes five of them to make one. Ok, so now make a new column. Now orange is one. Rod that equals one, you said red up here.
82		Sarah	Oh. Red. Ok.
83		T/R	Ok. And now you're going to say orange and fill in the other stuff. And now you already know the red. Figure out all the other ones. Together or individually.
84		Sarah	Ok
85		T/R	And, the way you do it is just what you did here; line them up.
86		T/R	So use this little one with orange
87		Fae	Ok, so, white we said white

Line	Time	Speaker	Transcript
88	06:50	T/R	Ok, so yeah. You can do whites. Just line up the whites and see what you get.
89		Fae	It's one tenth.
90		Kelly	True.
91		T/R	Yeah. But go ahead and do it. Right.
92		Kelly	Yeah
93		T/R	Convince yourself
94		Fae	Prove it
95		T/R	That's exactly right.
96		Kelly	Yeah one-tenth
97	07:16	Fae	Then orange with the red would be one fifth because this is double the white.
98		Kelly	Yeah. She just did it right there
99		Fae	Oh yeah. Light green? I don't know. It's three. And then a little bit. What is the little bit though?
100		Kelly	You could put a one. A white one
101		Sarah	Yeah, each of these are three.
102	07:46	Fae	How would you make that a fraction though?
103	07:59	Sarah	<inaudible> one third. We already know it's down by one so
104		Fae	Make it a mixed fraction so its three and one-third
105		Sarah	Yeah. Yeah three and one-third
106	08:07	Fae	Wait. It can't be three and one third. It has to be a mixed fraction.
107		T/R	No. It has to be. It's not a mixed fraction.
108		Fae	I mean
109		T/R	It has to be, you know, the argument that they were making. Right? They all gotta be the same.
110		Fae	That's three and one-third!
111		T/R	Well leave it out. Leave it out for now and work on ones you do know. Unless you want to … Leave it to the side. Um. Alright, so, you couldn't do light green. You did do white though. Right?
112	08:36	Fae	We can't do purple either.

Line	Time	Speaker	Transcript
113		T/R	Well, what else could you do? Leave them sort of to the side.
114		Fae	Yellow is one half
115		T/R	And make a little … show me that yellow is one half. Ok. That's convincing.
116		Sarah	[nods]
117		T/R	Right? OK.
118	08:59	Kelly	Do we have any rulers in here?
119		T/R	Not in here. Use, use the orange rod.
120		Fae	What's the next question?
121		T/R	The next question is pick your own.
122	09:21	Fae	Which one are we picking guys?
123		Sarah	We don't know any more in this column?
124		Fae	No
125		Sarah	Wait, did you get the last… Oh orange is one
126	09:52	Kelly	Did you hear it? thought the furniture wasn't supposed to talk.
127	10:21	T/R	Ok, so down here. Where are you guys at?
128		Sarah	<inaudible>
129		Fae	I'm still trying to figure out how to do light green against the orange. It's three. We know how to do it but we don't know what to call it. What fraction. We realize that the little white ones are three of the green one.
130	10:42	T/R	Ok. You realize that light green is equal to three whites. K…, are you listening?
131		Kelly	Yes.
132		T/R	Ok. Watch what she's doing too. Show her the proof that light green equals three whites.
133		Kelly	Hey <inaudible>
134		T/R	There's the proof. Ok. And what fraction does white equal?
135	10:59	Fae	One tenth
136		T/R	How do you know it's one tenth?
137		Fae	Because we did that first one
138		Kelly	Well if you pull all the greens away, like if you pull it like

Line	Time	Speaker	Transcript
			that, it's like that.
139		T/R	Ok. So, the fraction represented by white is? Say it again.
140		Kelly	One tenth.
141	11:14	T/R	One- tenth. And how long is a light green?
142		Sarah	Three tenths?
143		T/R	That wasn't a question
144		Kelly	Three tenths!
145		T/R	Three tenths! Do you believe that?
146		Fae	Yeah
147		Kelly	Yeah because if you pull the white away and then you pull the three here, you have one left.
148		T/R	Ok.
149		Fae	So, it's three tenths
150		T/R	Ok.
151		Sarah	So three tenths?
152	11:40	Fae	Yeah
153		T/R	You're not asking right? K… knows it for sure so if she explains it to you … Right?
154		Fae	And a purple one is four tenths. Because look
155		Sarah	Yeah
156		T/R	Ok
157		Fae	Dark green. Six tenths or three-fifths
158		Kelly	Yeah three-fifths
159		T/R	Yeah now can you make a model to show that that's ... That, that, model shows that it's six tenths. What's the model that shows that it's three fifths?
160	12:23	Fae	Um. Three
161		T/R	Ok. It's equal… the dark green is equal to three reds. I can see that.
162		Sarah	Right
163		Kelly	Yeah
164		Fae	Three

Line	Time	Speaker	Transcript
165		T/R	And what's a red? What does it say on your sheet a red is?
166		Fae	Alright Five
167		Kelly	One fifth
		T/R	One fifth. Three reds. This is what K… told us before right? Three whites was equal to three… What was three whites equal to?
168		Fae	It's, it's, it's one fifth.
169		Kelly	Wait, what are we on now?
170		Fae	A red equals one fifth.
171		T/R	Yeah
172		Fae	Of the orange
173	12:59	T/R	Right. So, three reds
174		Fae	So, three reds equals three fifths of the orange.
175		T/R	Ok
176		Fae	And a green is equal to three reds.
177		T/R	Ok
178		Fae	That's where my three fifths came from
179		T/R	Do you believe that? That's exactly what you said about whites.
180		Kelly	Yeah.
181		T/R	Ok.
182		Sarah	Yeah
183		T/R	Does that make sense to you too? Ok
184	13:13	Fae	Ok now. Black. <inaudible> None of them equal up to black. Ok. Black equals… We have to go back to tenths now.
185		Kelly	Why?
186		Fae	Because it's odd.
187		Kelly	Is it seven?
188		Fae	Seven tenths. And brown. It would be eight tenths or four fifths. And then blue is nine tenths. And that's it. This one we did four tenths but it can also be two fifths. Got it.
189	14:35	T/R	Ok. So, you're, you're ok with all these answers?

Line	Time	Speaker	Transcript
190		Sarah	Yeah
191		T/R	Am I right? Ok. And you've been writing and you've been making your nice bars but you're ok with the answers you have so far?
192		Kelly	Yeah
193		T/R	Ok
194		Kelly	I don't understand the purple one
195		T/R	Who's got the purple one?
196		Fae	Me. It's two fifths
197		T/R	Purple is two-fifths. Ok
198		Fae	Because here's a purple. Here's an orange.
199		Kelly	Ok, ok
200		Fae	Put the reds next to it. One red is one-fifth of the orange. But one purple is two fifths of the orange. Because its equal to … here's one fifth. Here's two fifths. <inaudible>
201	15:20	Kelly	Ok, so that's
202		Fae	This one is one fifth
203		Kelly	Yeah. And then this one?
204		Fae	Is two-fifths. It's equal to two reds.
205		Kelly	Oh. Yeah. Ok.
206		Fae	Which one do we want to make one?
207		Kelly	Blue
208		Sarah	Light green
209		Fae	Ok
210		T/R	Ok, alright so, you haven't finished the orange, but do the blue altogether. Ok
211	15:53	Fae	Ok. How many white ones make a purple make a blue rather?
212		Kelly	So yeah that is
213		Fae	One, two, three, four, five, six, seven, eight, nine. So that's one ninth. Then the red one is. One red equals…
214		Kelly	What did you get for number one?
215		Fae	What was what?

Line	Time	Speaker	Transcript
216		Kelly	<Inaudible> green
217		Fae	For which one?
218		Kelly	<Inaudible>
219		Fae	Dark green is three fifths because we did the reds again.
220	16:48	Kelly	Oh. Ok
221		Fae	One green equals three of the reds. Oh right. None of them equal the blue. Two. Two, four, six, eight.
222		T/R	So, what have you got here?
223		Sarah	Would that be eight tenths? For red?
224		T/R	Um. Well, let me think
225	17:21	Sarah	I get like confused
226		T/R	Four. Four reds equals eight whites is what you're telling me.
227		Sarah	Yeah
228		Fae	Its two ninths
229		T/R	Yeah but the white is?
230		Sarah	It's like one tenth so
231		T/R	Why is it one tenth?
232		Sarah	Oh. It's one ninth
233		T/R	You wrote
234		Sarah	Oh. It's one ninth
235		T/R	Ok right
236		Sarah	I keep thinking there's ten of them
237		T/R	Right. So, what did you say?
238		Fae	Two ninths
239		T/R	She says
240		Fae	For the red
241		T/R	Red is two ninths
242		Sarah	Yeah
243		T/R	Because… Why is red two-ninths?
244	17:49	Fae	Oh. Because it equals. These are one-ninth each. So, two of them together equals one red. That makes two ninths.

Line	Time	Speaker	Transcript
245		Sarah	I got two out of nine. It would be like that. Two out of nine.
246		Fae	Right
247		T/R	Yeah. Are you good with that K…? What she just said? Did you get what she just said? F….., you say it again. You F… over here. Say it again.
248		Sarah	I just said that like if you take these two out, you can see that this is two ninths. Cause this is like. This whole thing is nine.
249		Kelly	Oh. Because its two out of nine.
250		Sarah	Yeah
251		Kelly	Ok. I got it.
252		Sarah	It's hard to see at first
253		T/R	So, the red is …
254	18:20	Fae	Two ninths
255		Sarah	Two ninths.
256		T/R	Ok
257		Fae	Light green is three-ninths or one third
258		Kelly	Purple.
259		Fae	Did you do…
260		Kelly	Well, there's two reds to a purple. Right?
261		Fae	Four-ninths. Look. Here's the whites …
262		T/R	Now leave that right there. Actually. Because, if I ask you to explain that, you could explain a couple things here. You could explain why this is one third
263		Sarah	Yeah
264		T/R	And you could also explain why it's three ninths and why one third and three ninths are ..
265	19:09	Sarah	Yeah because this is three. If we know that there are nine white.
266		T/R	Yeah
267		Sarah	This is three
268		T/R	Yeah
269		Sarah	White so its three ninths
270		T/R	Yep

Line	Time	Speaker	Transcript
271		Sarah	And one third I mean you could just see it's one third. Like you can just see that it lines up equally
272		T/R	That's right. Ok. Ok. So that's, so let me set this aside in case I ask you to prove this to somebody else and you can go on to another set because you've got extra blues right?
273	19:28	Fae	Yep
274		Sarah	Yep
275		T/R	Ok
276	19:47	Fae	Six ninths or… Two ninths… two, four, six … six ninths, two thirds
277		Sarah	Is purple four ninths?
278		Fae	What's that?
279		Sarah	Is purple four ninths?
280		Fae	Four ninths?
281		Sarah	Mmhmm (yes)
282		Fae	Yeah
283		Sarah	Ok
284		Fae	Do you know why?
285		Sarah	Mmhmm (yes). Cause I did… that one's four so
286		Fae	Yeah it's four whites
287		Sarah	Mmhmm (yes)
288	20:27	Fae	Did you get to yellow yet?
289		Sarah	No, not yet.
290		Fae	Yellow doesn't go into it evenly so it would be five ninths. But then,
291		Kelly	Dark green
292		Fae	For dark green, it's two thirds
293		Sarah	It's six ninths or two thirds. They just like all go up one.
294		Fae	The black would be…
295		Sarah	Seven-ninths
296		Fae	Seven?
297	21:01	Kelly	So now that would be eight ninths?

Line	Time	Speaker	Transcript
298		Fae	The black? Oh, the brown? Yeah
299		Sarah	<inaudible>
300		T/R	You know what the blue is right?
301		Fae	Yeah one.
302		Kelly	And then orange is
303		Fae	One and one tenth
304		Sarah	One and one tenth
305		T/R	One and one ninth.
306		Fae	One and one ninth
307		Sarah	One and one ninth
308		T/R	Yeah, one and a white one. Right?
309		Kelly	Yeah
310		Sarah	Yeah
311		T/R	But with this model the white one is only a ninth.
312	21:23	Fae	Right. So it's one and one ninth.
313		T/R	Ok. Now. I'm gonna... I'll ask you other questions about that later. Ok, so, you're good on these. There's more. Where's the rest of your sheet? Ok. You did three. Ok. Four, five, and six.
314		Fae	Representing one half. If you called the brown rod one, which rod represents one half? So, brown is one. Brown is even so what's one half?
315		Kelly	Purple? I think it's purple. Purple!
316		Fae	Purple
317	22:22	Kelly	Ok, so then half of purple is red. So, red would be…
318		Fae	If you call the blue one rod, which one represents one half. We don't know. None of them.
319		Kelly	Oh, we're going by the sheet now. Never mind.
320		Fae	Yeah
321	23:02	Fae	None of them can be one half.
322		Fae	Call the light green one.
323		Kelly	What number is represented by the red rod?

Line	Time	Speaker	Transcript
324	23:21	Sarah	Two thirds. I think.
325		Fae	Ok. So light green is one. This is three. So red equals what? Two thirds.
326	24:00	Kelly	Two thirds.
327		Fae	Oh. Oops. White is one third. Red is two-thirds. Purple is one and one-third. Yellow is one and two thirds and dark green is two.
328		Kelly	What number is represented by the dark green rod? What are you up to?
329		Sarah	Number five
330		Fae	The red rod is two thirds and
331	24:56	Kelly	I have a question.
332		T/R	Ok
333		Kelly	I don't understand b. What number is represented by the dark green rod?
334		Fae	Two. Look
335		Kelly	Oh we're, but we're still using … Oh
336		T/R	Every question has a different 'one'.
337		Sarah	Yeah
338		T/R	Ok. That make sense to you now?
339		Kelly	Yeah.
340		T/R	Ok
341		Kelly	I didn't read the light green part.
342		T/R	Ok
343		Fae	Call this one third. Which rod represents one? Light green.
344	25:28	T/R	And how do you know that?
345		Fae	Because three thirds would equal one and that adds up to the light green.
346		T/R	Ok. That's a proof. That right there in front of you is a proof. Do you believe that?
347		Sarah	Yeah
348		T/R	Ok
349		Fae	What number does the yellow represent? One and two thirds.

Line	Time	Speaker	Transcript
350		Sarah	So to figure this out would you do like one third plus however many yellows fit? Is that how you want us ..?
351	26:00	T/R	Let me see. What does the yellow represent? You did one thing
352		Fae	One .. plus two thirds. Yeah.
353		Sarah	Oh because you know green is one so
354		T/R	Yes. However, I might do something else. Suppose you didn't do the light green first. Suppose all you knew was the white was one third. Line up the whites and what do you get?
355		Fae	Five-thirds which turns out to one third … one and two thirds.
356		T/R	Alright so, I know we were saying
357		Sarah	That's what I'm saying. Can you add up like five one-thirds.
358		T/R	Sure
359	26:27	Sarah	Yeah that's what I meant
360		T/R	Yeah. So, she says she got one and two thirds, you say you got five thirds, you're both valid. Right?
361		Sarah	Yeah.
362		T/R	Ok so put down your answer and then we can talk about the different ways of representing things. Where are we at? Do we still have enough time? Ok.
363		Kelly	Why is it five ninths?
364		Sarah	Because if this is a third and there's five whites, it would be five thirds.
365		Fae	Four, two
366		Kelly	Which is greater three fourths or two thirds?
367		Fae	Three fourths is larger. I think. See? This is three
368		Kelly	This is four. This is the three. Alright. Two thirds. Two. Do we have any more white ones?
369	27:33	Fae	She left them over here
370		Sarah	Yeah she said to keep them like that.
371		Fae	We'll make it again
372		Kelly	Can we … can I just use those real quick?
373		Fae	We took apart your little thingy. We'll put it back together

Line	Time	Speaker	Transcript
374		T/R	Oh sure. Yeah that's fine. I realize you didn't have enough of the little white ones.
375		T/R	I meant to give you a 4b question that wasn't the blue rod because we already have that one.
376		Erika	Oh
377		T/R	Ok. And these guys got the answer for 4b
378		Erika	How?
379		Jaime	How?
380		T/R	What is your answer for 4b?
381		Erika	I'd love to hear this
382		Fae	None of them
383		Sarah	None
384		Darlene	Oh
385		Erika	Oh well
386		Jaime	We're sitting here …
387		Janelle	Yeah we're sitting here like, if we put three blues together and that's three and what's a half of three and then
388		Erika	Yeah
389		Fae	None of them work. Sorry girls
390		Erika	We didn't know you wanted none. We were trying to find something that wasn't none.
391		Darlene	Because everything else has like a number
392	28:19	T/R	Yeah well you could ... extra credit. Is there anything you can say besides none? When you come up with your new set of rods that can …
393		Fae	No rods but the number is
394		T/R	Yes, ok
395		Fae	Four and a half
396		T/R	Ok. Alright. You got this one already?
397		Sarah	Yeah
398		T/R	What'd you get?
399		Fae	Three fourths

Line	Time	Speaker	Transcript
400		Sarah	Three fourths is bigger.
401		T/R	Yeah
402		Fae	It's right here.
403		T/R	This is the same thing they did
404		Fae	Two thirds. Three fourths
405		T/R	But usually when we do these problems you have to have the same value representing one each time.
406		Sarah	Yeah I did it this way
407		T/R	Yeah but which one is one?
408		Fae	So yeah
409		Sarah	This is … this is the third. This one would be two thirds and I would take out the two
410	28:56	T/R	Yeah
411		Sarah	And then this one would be three fourths and that would make it like that.
412		T/R	However
413		Sarah	I don't know if that's right
414		T/R	Yeah um, in, in this case, this represents one. Right?
415		Sarah	Yeah.
416		T/R	These four together represent one. In this case these three together represent one.
417		Sarah	Yeah
418		T/R	And what I'm saying is that when you have one of these problems, you have to have the same one for both cases. The same thing equals one.
419		Kelly	Oh.
420		T/R	Right
421		Kelly	Got it.
422	29:18	Fae	Yeah but
423		T/R	For example …
424		Kelly	So, if I take two purples and I go like that
425		T/R	Ok

Line	Time	Speaker	Transcript
426		Kelly	That doesn't make sense does it?
427		T/R	Well wait, yeah
428		Kelly	That's three fourths
429		T/R	If you're saying you want purple to be one, then yes you can make two, three fourths. Can you find a rod that represents thirds if purple was one?
430	29:40	Kelly	No
431		T/R	No, so you can't do purple
432		Sarah	Could you
433		Fae	This can be thirds but I don't think it can be fourths.
434		Kelly	Well what about yellow?
435		Fae	I know!
436		T/R	Well remember you're allowed to do trains too
437		Fae	I know. This would
438		T/R	So its… so you always start with what one is. You're telling me this is one?
439		Sarah	Um. I'm not sure. I'm still confused with that.
440		T/R	Ok. Ok. Remember when you're going to show me
441	30:09	Fae	It needs to be twelve. So we do this. The orange plus the red equals one. So there's four. This is three. I know that it needs to be twelve because of the multiples of three and four. So there's the thirds and that's the fourths. So here's two thirds. Here's three fourths
442		Kelly	So, what is orange?
443		Fae	Orange plus the red is twelve
444		Kelly	So, orange, orange plus red equals twelve
445		Fae	Look at the white ones against it.
446	30:49	Kelly	But wait, so when you take away the red, what is it?
447		Fae	Ten. Only because, she said we could make a train. So, I thought of the multiples of four and three.
448		Kelly	Ok
449		Fae	It's twelve.
450		T/R	Right. That's your least

Line	Time	Speaker	Transcript
451		Fae	Least common denominator
452		T/R	Ok, so tell me what represents one. You're doing fractions. You always have to tell me what one is.
453	31:22	Fae	The orange and the red together
454		T/R	The orange and the red together represent one
455		Fae	Which is twelve
456		T/R	Twelve white ones
457		Fae	Yeah
458		T/R	Ok
459		Fae	Twelve twelfths.
460		T/R	Twelve twelfths. Ok, so orange plus red that's your one. So are you going to show me two thirds and show me three fourths.
461		Fae	This green one is a third of twelve. A fourth. I'm sorry
462		T/R	The green one is a fourth. And why is green a fourth?
463		Fae	Because four of the green ones add up to the twelve
464		T/R	Ok
465		Fae	And because one of these is three, and there's four sets of three in twelve.
466	31:56	T/R	Ok. So there's… So, show me what… What's your three fourths
467		Fae	This is fourths. Four fourths equals twelve. Take away one that's three fourths
468		T/R	Ok. Where's your thirds?
469		Fae	Would be the purple because they equal up to four and there's three sets of four in twelve. And there, that represents two thirds.
470		T/R	So, alright. Which is bigger?
471		Kelly	Three fourths is bigger
472		Fae	Three fourths
473		T/R	So K… now tell me how much bigger. Bigger by how much?
474	32:27	Kelly	By one little thingy
475		T/R	By one little thingy but there's a fraction with that one little

Line	Time	Speaker	Transcript
			thingy.
476		Sarah	One twelfth
477		Fae	One twelfth
478		T/R	Right, one twelfth because twelve of those make one so, write all that down now and then they're going to present you with their question and you're going to do something similar with that and we will have just enough time. In fact, maybe before we present you with their problem, we're going to talk about something else. Ok?
479	32:55	T/R	Ok. Group discussion now. Ok. Back to fractions. Um, before we do that last problem, I wanted to talk about some of the other things that you showed me and some of the things that you can prove. For example …
480		Fae	Oh. You want me to show that twelve thing?
481		T/R	Over here. Not yet
482		Fae	Ok
483		T/R	There was the yellow. Back in question, um, five. Do I mean question five? No question six. The white rod is one-third. Ok, and you told me which rod represents one. And F… over here said light blue represents one. Right, now if white is one third then light blue represents one
484		Fae	Light green
485		Erika	Light green
486		Darlene	Light green
487		T/R	Light green. Sorry. Now, this F… said yellow is one and two-thirds. Show me your model for one and two-thirds.
488	33:54	Fae	If three of the white equals one, then it's one plus two extra little ones which is thirds.
489		T/R	Now this F… said the answer was five-thirds. So show me five-thirds.
490		Sarah	Because I counted that this was five whites. Yellow is five whites.
491		T/R	Ok
492		Sarah	So I said
493		T/R	Yellow is five thirds
494		Sarah	Yeah

Line	Time	Speaker	Transcript
495		T/R	She said yellow is five-thirds. She said yellow is one and two-thirds. Which ones right?
496	34:20	Janelle	They both are.
497		Erika	Both are
498		T/R	How come? How do you know?
499		Janelle	Because they're the same number
500		Fae	The fractions are the same
501		T/R	Ok, and you can prove it with the rods. Right?
502		Fae	Yeah
503		T/R	You can prove that three whites is equal to
504		Fae	One and two-thirds
505		T/R	And two-thirds. So, there you go. You know. You have a physical thing that proves that these two fractions are equal. And, you know, think about this as another way to do it besides the numerical things that you learned. That there's actually physical proof and you also showed me in some cases that back when the orange rod was one, some of you said that the red rod was one-fifth. And some of you said that the red rod was
506	34:58	Erika	Two-tenths
507		T/R	Two-tenths. But you could prove to me, right, that one-fifth and two-tenths are actually the same number using these rods. Right?
508		Off screen	Um hum
509		T/R	Ok. So that was some of the kinds of things that I wanted you to think about how you have different answers but they're really the same thing. And when you were doing the first one you said it was um, what, three and a half of the red was one. Something was three and a half, but it could also have been seven halves. Right? You can prove that three and a half is the same as seven halves. In fact, why don't you do that one? If the red is one, the black was three and a half.
510	35:39	Erika	Ok
511		T/R	But the black is also and you should be able to show me that black is also seven halves. So how can you show me, if you have enough room?

Line	Time	Speaker	Transcript
512		Erika	I got it.
513		T/R	… and cubes that the black … that three and a half is the same as seven halves?
514		Erika	It's like this.
515		T/R	So J… has it over here if you don't have enough you can look.
516		Fae	I have it too.
517		T/R	Oh you've got it too. F….. has it over here. Ok and those of you that have enough white cubes have it. So, show us your proof.
518		Erika	Ok
519		T/R	Tell us about your proof.
520		Erika	So black is one. Now you said you wanted three
521		T/R	No, black is not one.
522		Erika	What is it?
523		T/R	Red is one
524		Erika	Red's one.
525		T/R	And black is
526	36:17	Erika	And you want us to prove that black is three and one half.
527		T/R	Which … and I want you to show me that three and a half is the same as seven halves.
528		Erika	Alright. So, black is three and a half. So, red's one. We've got one, two, three, and a half. Half, half of a red is a white. So that's three and a half. Or, if you wanted … what seven halves?
529		T/R	Yeah
530		Erika	Since one of these is one, there's two of them for everyone. Alright. So, two times three because we have three reds, is six. Plus the one white we have at the end is seven.
531	36:55	T/R	Ok. And that was actually… you're sort of giving the proof of the algorithm. Remember three and a half. Remember that rule for converting three and a half to a mixed number. The three times the two plus the numerator. Remember?
532		Fae	Yeah.
533		Erika	Yeah.

Line	Time	Speaker	Transcript
534		Sarah	Um hum.
535		T/R	That's just what you explained. Three of the little reds and there's two white ones in each red. So, there's a model for explaining how you do that. Ok? Ok, I think we're running out of time so

Transcript 3 of 6
Date: 04/13/2011
Length: 00:09:02
Camera 2, Part 1
Transcribed by: Deidre Richardson
Verified by: Mary Huizenga

Line	Time	Speaker	Transcript
1		Erika	… just have a brown rod. That's why it's not working
2		Janelle	But the last one …. what is the longest train that measures the brown rod and the black rod
3		Erika	No she uh … she wanted us to do the second of set one.
4		Janelle	Oh
5		Darlene	There it goes
6		Erika	Yeah, they're the reds because the light greens work for that but not this. And then what's after that? Yellow? Yellow is not gonna do it. So yeah, so the reds
7		Janelle	So why is this wrong?
8		Darlene	Huh?
9		Janelle	I said <inaudible>
10		Erika	Shortest train that can be measured … OH! That can be measured by brown
11		Janelle	Look
12		Erika	We're doing the second part of set one?
13		T/R	We are doing the second part of set one.
14		Erika	Yes, so the shortest train that can be measured by both …
15		Janelle	Like this
16		Erika	… green and brown
17		T/R	Ok.
18		Erika	Oh no, we did
19		T/R	She found one
20		Erika	But
21		T/R	Ok
22		Fae	I'm confused. Why is it so long?
23		Erika	No no no. We did it the

Line	Time	Speaker	Transcript
24	00:53	Janelle	Measured … the the difference is 'measured by' and 'measures'. You have to ...
25		T/R	Ok
26		Erika	Yes, we we did the
27		Janelle	You have to realize the difference
28		Darlene	We did the opposites
29		Erika	We did it the wrong way.
30		T/R	Yes, well … it's ok. We can save that for later. So you claim that this is the shortest one
31		Janelle	… for green and brown
32		Darlene	First we gotta do just green and brown right?
33		Erika	<inaudible>
34		T/R	Yes, so let's just pull everything … all the extraneous stuff away.
35		Erika	Yeah. There we go.
36		T/R	So she said this is the shortest train that is measured by green and brown. And you have the same thing there. So you can stay over here and you can go with her. Ok. And how do you know it's the shortest? What happens if you try to make it shorter?
37		Janelle	It doesn't work
38		T/R	It doesn't work. And it keeps on not working.
39		Janelle	Um hum
40		T/R	Ok. You agree with that too?
41		Fae	Yeah
42	1:34	T/R	If you take one, one brown away or one green away, they don't line up. And if you take more away, there's nothing that, they don't line up until you get that many of them
43		Fae	This is the shortest it could be
44		T/R	Ok
45		Fae	The way it lines up
46		T/R	Ok, so we didn't do this one last time but that's similar to what we just did. Ok, and then the other thing – the brown rod and the black rod. Now we want the longest one that

Line	Time	Speaker	Transcript
			measures both brown and black. I think this is the one we did last time
47		Janelle	So you need smaller ones now.
48		T/R	Right.
49	2:00	Erika	It's not light green <inaudible>
50		T/R	One that measures brown and black. Right. So that means …
51		Fae	So that's like this
52		T/R	Right. That's like that.
53		Janelle	And you can only use one color. Right?
54		Fae	Do the reds line up to the black?
55		T/R	The same color for both of them.
56		Darlene	It's not yellow
57		Erika	It's not light green
58		Darlene	No
59		T/R	Ok
60	02:12	Erika	No
61		T/R	So you, you're doing the same thing. You're sort of working your way down.
62		Erika	Is it purple?
63		Darlene	Is it red?
64		T/R	Ok. Actually you're working your way up.
65	2:21	Erika	Nope. It's not red. Wait, so we … we know it's not yellow. It's not green. It's not purple.
66		Darlene	Maybe it's just the ones. Didn't we do this last time?
67		Erika	I don't know if we did this one last time, but that's all the colors. Yeah it has to be it.
68		Janelle	It's just the ones
69		Erika	yeah
70		Darlene	Just the ones
71		Erika	Alright. You have the brown?
72		Darlene	Yeah
73		Erika	She has it for us.

Line	Time	Speaker	Transcript
			See. I have the black. She has the brown. You have both.
74	2:55	T/R	OK. So you all … now, you're watching, but you don't need to make them. Right? You, you get what she's doing?
75		Sarah	Yeah, I said it was probably the white ones
76		T/R	Ok
77		Sarah	Because the other ones are too big.
78		T/R	Ok. And K… you're ok with that too? Am I right?
79		Kelly	Yeah
80	3:22	T/R	So the question for everybody and the question for the group that we had last time is, so what are we doing here? J… has an idea. Am I right?
81	3:30	Darlene	Yes. Um
82		T/R	Ok. We could all listen to J… idea.
83		Darlene	What was it? It was either the GCD … or the least common …
84		Erika	Oh! We said this! Oh. It was like. We had said this right when she was dropping me off. Um
85		Darlene	Least common, no.
86		Erika	No
87		Darlene	Greatest common denominator, no.
88		Erika	Its greatest common factor isn't it? Yeah.
89		Darlene	That's what I thought it was
90		Erika	Yeah. That's what it was. We figured out it was greatest common factor.
91		T/R	Ok. So these uh … LCD I guess was the other thing you said but sometimes they call it LCM.
92		Erika	Yeah
93		T/R	Alright so this is greatest common factor. So what were we doing that was the same as the greatest common factor?
94	4:17	Erika	We were finding the … the highest, like. Like if these were numbers like one and two,
95		T/R	Yeah
96		Erika	We were finding the highest number that goes evenly into both the black and brown

Line	Time	Speaker	Transcript
97		T/R	Ok. So the black represented what number and the brown represented what number?
98		Erika	One, two, three, four, five, six, seven. Black is seven.
99		Janelle	Seven and eight, seven, eight.
100		T/R	So the greatest common factor …
101		Darlene	Is one
102		T/R	… of seven and eight you said
103		Erika	Is one
104		Janelle	Is one
105		T/R	Is one. Which is the …
106		Darlene	We had the right idea
107		Erika	Yeah
108		T/R	… greatest common factor of black and brown is white
109		Fae	Correct
110	4:58	T/R	And so the least common multiple… So, give me an example of that. What did we do for the, for the brown and uh the purple and the dark green? What numbers did they represent? Purple and dark green.
111		Off camera	What was it?
112		Fae	Two and three?
113		Darlene	No
114		Erika	No four
115		Sarah	Purple is four
116		Janelle	Four and six?
117	5:25	Erika	Yeah, Four and six.
118		T/R	And the least common – and what do they go into?
119		Fae	This, like this represents two.
120		Erika	Uh
121		T/R	And white is one right?
122		Fae	Ok. Right. Right. Sorry
123		T/R	You didn't get that one last time.
124		Erika	I'm missing a color

Line	Time	Speaker	Transcript
125		T/R	Ok. So what do you have for the, remember the shortest train that was measured by both the purple
126		Erika	It was three green, no two green and three purple.
127		Janelle	Mhmm.
128		T/R	Which is, what number would that be?
129		Janelle	Twelve
130		Darlene	Six?
131		T/R	Twelve
132		Janelle	Six. Twelve
133		Darlene	Twelve
134		Erika	Twelve
135		Janelle	Twelve, Yeah
136	5:57	T/R	Are you ok?
137			I'm ok and so is the camera.
138		Erika	Two four. Oh two and six
139		Darlene	Yeah
140		T/R	Ok. Ok and that's just saying if white is one, that those are the lengths.
141		Janelle	Right.
142		Erika	Right
143	6:08	T/R	Alright. So that's. Yeah, that, that's what you were doing, that was really fast. That you got that. So we're doing greatest common factor and least common multiple just by doing rods representing numbers from one on up
144		Janelle	Um hum
145		T/R	And I wanted to use that as an introduction for fractions because we use greatest common factor and least common multiple when we do fractions.
146		Kelly	My most favorite thing in the world.
147		T/R	You're going to love fractions when we're done with this. [laughter]
148		Erika	K…'s like oh no, fractions.
149		Janelle	We're going to want to use Cuisenaire rods for the rest of our

Line	Time	Speaker	Transcript
			lives.
150		Erika	That was fun. Sorry
151		T/R	So. Um, I want you to do some stuff with these rods and then we're going to watch a video. Ok. And the thing with these rods is we give them what we call number-names. And I started out giving white a number-name of one. Right, and then you knew if white was one then the orange was ten and the dark green was six and so on. Right, so if we're going to do them as fractions, we're going to give them number-names – well, for example we could give the orange a number-name of one and then the other things would be fractions. So that's the kind of thing we're gonna look at. If you give them different number-names, what kind of fractions can you represent. And the thing we're gonna start with, um, we'll do more with number-names later, but that's the idea. We're going to start with one half. And here's a simple example. If this was one, if the length of this was one, what would represent one half?
152		Erika	Uh, white.
153		Sarah	white
154		T/R	You're going the other way
155		Fae	Oh, one half of that …
156		T/R	Yeah
157		Fae	… is white
158		Erika	White.
159		T/R	Now you were going the other way. If this was one half, what was one, I think. Now white was one, a half because?
160	7:45	Janelle	It's half
161		Erika	It's half of the red
162		Fae	Its half the size of the rod
163		T/R	It's half of the size
164		Erika	Of the red rod
165		T/R	half the length. And another way to look at it is – put another one up here
166	7:52	Erika	It takes two
167		Janelle	Two of them

Line	Time	Speaker	Transcript
168		T/R	It takes two of them to make the one. Right.
169		Erika	Both of them
170		T/R	Takes two of them to make one so each of them is a half. Okay
171		Kelly	Two of them equal
172		Fae	It takes two to equal one
173		Erika	Ok, yeah.
174		T/R	Ok. So, we could find half of various rods uh and in particular, the question we're going to look at is "What's a half of blue?"
175	8:16	Fae	Nothing
176		Darlene	Yeah. That's what I think
177		Erika	Wait, what is blue? Oh …
178		Janelle	Nada!
179		Erika	Ahh, you can't!
180		T/R	And how come?
181		Janelle	Because it's an odd number
182		Erika	Well if we were to give it number-names, blue is nine
183		T/R	Its nine, well …
184		Darlene	Because three
185		Erika	Well if you count, well if you were to call this one - One, two, three, four, five, six, seven, eight, nine. Nine doesn't have a half, well whole number half.
186		T/R	Ok, there's no whole number that's half of nine.
187		Janelle	Right
188		Erika	Exactly
189		T/R	Ok. Ok.
190		Darlene	That's true.
191		T/R	We'll leave it at that. Ok and now we're going to watch a video and I have stuff about the video I hope. Ok so take one and pass them along.

Transcript 4 of 6
Date: 04/13/2011
Length: 00:33:53
Camera 2, Part 2
Transcribed by: Deidre Richardson
Verified by: Mary Huizenga

Line	**Time**	**Speaker**	**Transcript**
1		Darlene	So the whites are what fit
2		Erika	The whites are … it goes up by a white every time
3		Darlene	Yeah.
4		T/R	Ok
5		Darlene	So there … just to see
6		Erika	Yeah.
7		Darlene	Yeah.
8		Erika	Just to show. Yeah. So everything goes up by one.
9	00:11	T/R	Ok. And is that a convincing argument?
10		Darlene	Um hum
11		T/R	Yellow doesn't work and purple doesn't work and therefore nothing else works.
12		Erika	Yeah
13		Darlene	Yeah
14		T/R	Ok, now they have um another thing that they worked on I believe in this class that I want to give you as a homework. And the homework was um, so can you make up a set of Cuisenaire rods so that you can have a half of everything? And if you know the answer right now don't tell me just write it up for homework. Ok**?**
15	00:47	Erika	Can I take some home? To try to figure this
16		T/R	Sure. Yes. If anybody wants to take a handful of them home or one of each color. Fine. So that's the question. Make up a set of Cuisenaire rods so that you can always find a half. Ok. And now we have, um, some more fraction activities based on the stuff that the kids did in class and I will find my sheets in here. Here they are. So because we have two videographers, you know … you can be this group of three and you can be this group of three. Here's your stuff.
17	1:44	Erika	Oh. Yeah. Oh. Answer sheet?

Line	Time	Speaker	Transcript
18		Janelle	Nooo
19		Erika	Oh. Rod that equals one.
20		Darlene	The answer sheet.
21		Erika	What? What is this?
22		Darlene	I don't know.
23		Erika	What are we doing with this?
24		Darlene	I don't know.
25		T/R	That goes with this.
26		Erika	Oh
27		Janelle	Write your name down first!
28		T/R	Yes. Right. Put your name
29		Janelle	Step one
30		Erika	Step one. Put your name. Step two. Read all questions completely.
31		T/R	Ok. Now. Yeah. We can do one. Right? Um. Call the red rod one
32		Erika	Yeah. Red rod, one.
33		T/R	Ok. If the red rod is one,
34		Erika	One of these
35		Janelle	So I can write … So you write here one?
36		T/R	Red rod … rod that equals one, you put red. In fact we need more of those sheets, but that's the idea.
37		Erika	Yeah
38		T/R	So if the red rod is one, you know what the white rod is what, right?
39		Erika	One half
40		Janelle	One half
41		T/R	It's one half. Ok
42		Janelle	And then red is one.
43		T/R	Yeah. Now if we move up, skip something and do purple. If the red is one, what's purple?
44		Erika	Four

Line	Time	Speaker	Transcript
45		Janelle	Two
46		T/R	Yeah, four whites but its
47		Erika	Oh! Sorry.
48		Janelle	Purple is two
49		Darlene	Yeah.
50		Darlene	Light green. What the heck is the light green?
51		Erika	One and a half
52		Darlene	This one? Oh, the red one. Duhhh
53		Erika	The red one and a half. Because it'd be…look. Just do that.
54		Darlene	Yeah. Gotcha.
55		Erika	So all the odd ones are a halves … have a half of it.
56		Darlene	So that's, one,
57		Erika	So two. Dark green.
58		Darlene	Light green
59		Erika	Light green is one and a half. So that's two and a half. That's three. That's three and a half. That's four. That's four and a half. And orange would be five.
60		T/R	And did you do the same thing?
61		Darlene	Oh we
62	03:38	Erika	Oh, we … we were talking about it
63		Janelle	I did it in decimals instead of fractions.
64		T/R	You did it in decimals.
65		Erika	Well, one half is point five, so
66		T/R	Yes. Decimals are fractions too right?
67		Erika	Yeah
68		T/R	They just don't have a denominator of power of ten. We're probably going to end up doing them as fractions, but …
69		Janelle	Yeah
70		T/R	… you can convert back. Ok. So…
71		Janelle	So if …
72		T/R	So that's the idea. And um, I only gave you one of those sheets, but that's the idea for the other sheets. Right.

Line	Time	Speaker	Transcript
73		Darlene	Ok
74		T/R	So
75		Erika	We could always write it next to it and under it.
76	4:05	T/R	Call the red rod one. What if the orange rod is one and so on. So you guys can … you know …
77		Erika	So we can write on this?
78		T/R	Anywhere you want. But just make sure … well …
79		Darlene	Why don't you just mark next to this?
80		T/R	… it might be easier making another column here.
81		Erika	Yeah, that's what I was gonna …. OCD when I'm trying to draw straight lines
82		T/R	Yeah I should have made more of them
83		Erika	Can't draw straight lines (laughter)
84		Darlene	I'm going to put two colors here. Ok. So which one are we gonna do now?
85		Erika	Um, number two. (laughter) I don't know. We just did number one.
86		Darlene	Orange.
87		Erika	Ok. Alright so orange is one.
88	4:53	Janelle	So yellow is …
89		Erika	Wait. How many whites go into orange? Is it ten? I think its ten.
90		Darlene	I think so
91		Erika	Three
92		Darlene	What'd you say R……?
93		Janelle	Uh, yellow is a half
94		Darlene	Yellow is a half?
95		Janelle	Yeah
96		Darlene	I think its ten
97	5:10	Janelle	Um hum. [agrees]
98		Erika	Yep its ten
99		Darlene	It's ten

Line	Time	Speaker	Transcript
100		Erika	So white is one-tenth
101		Darlene	One-tenth
102		Erika	And we can just work up from there. Two-tenths, three-tenths,
103		Darlene	Um hum. [agrees]
104		Erika	Four tenths, one half, five tenths, six tenths
105		Darlene	Six tenths. Does she want them reduced?
106		Erika	I don't think so. It doesn't … I don't think it really matters. We can write them
107		Darlene	Eight-tenths, nine-tenths
108		Erika	Nine-tenths. So this would be … what? Four-fifths? No
109		Darlene	Which one?
110		Erika	Three, three-fifths.
111		Darlene	Yeah
112		Erika	Six tenths
113		Darlene	Yeah. Eight tenths would be four
114		Erika	Four-fifths. Four-tenths is two fifths and two-tenths is one-fifth.
115		Darlene	Ok
116		Erika	Alright. Alright, so that's number two. Number three.
117	5:59	Darlene	Select a different rod.
118		Janelle	Select any other rod.
119		Darlene	So which one?
120		Erika	Which one do you guys want to call …
121		Janelle	Yellow
122		Erika	Yellow
123		Darlene	Ok. Yellow
124		Erika	So yellow is one.
125		T/R	So you zipped right along with orange.
126		Erika	Which means orange is two remember because you said yellow was a half.
127		Darlene	Yeah.

Line	Time	Speaker	Transcript
128		Erika	So orange is two.
129		Darlene	Yellow is one. Orange is two. So what's white? Let's figure that out? Five?
130		Erika	You made orange … I mean, you made yellow one?
131		Darlene	Yeah, it's five
132	6:26	Erika	Yeah. So white is one-fifth?
133		Darlene	Yes
134		Janelle	So the question is
135		Erika	So everything would follow like that. Two-fifths, three-fifths
136		Janelle	One-fifths, two–fifths, three-fifths
137		Darlene	Four-fifths. Five fifths
138		Erika	Um hum.[agrees]
139		Janelle	Four-fifths, and then one and one-fifth
140		Darlene	Can you write six-fifths?
141		Erika	Well you can either write six-fifths or one and one fifth because it's the same thing.
142		Darlene	True
143		T/R	You're going to explain to me why they're the same thing too. You can do that using the rods.
144		Erika	Ok. Oh that's four-fifths.
145		Darlene	Does it matter which way we write them? Do you want it like mixed or numbers?
146		T/R	Doesn't matter … No, in fact its good if you all do it different ways because then we'll talk about it.
147		Darlene	Ok
148		Janelle	Alright
149		Erika	Um, ok
150		Janelle	So, number four
151		Erika	Four. What does four say?
152		Janelle	You call the brown rod one.
153		Erika	Ok
154		Janelle	What represents one-half?

Line	Time	Speaker	Transcript
155	7:16	Erika	Well we said yellow was for orange. Right? Like this.
156		T/R	Ok you skipped three or you did your own thing for three?
157		Janelle	We did it over here
158		T/R	Ok. Did you all do yellow?
159		Darlene	Yeah
160		Janelle	[agrees]
161		Erika	Oh. Yes. Purple .. is .. one-half
162		T/R	Next time
163		Janelle	Oh, I thought we were still working in groups.
164		T/R	Yeah you are. But yeah fine. Go ahead. If you finish before they finish then I'll ask you to go back to that to pick a different one.
165	07:43	Erika	Yeah. It's um
166		Darlene	What are we doing now? Purple?
167		Erika	Yeah number four, no number four
168		Darlene	Number four is …
169		Erika	You take a brown one and call it one, right?
170		Janelle	Purple
171		Erika	And what would represents one half is purple.
172		Darlene	Yeah
173		Erika	And then if you take a blue rod
174		Janelle	This one's gonna be hard because it doesn't
175		Erika	There's … so … we need to find something that'll combine and call it something else
176		Darlene	Can we write on this?
177		Janelle	Yes
178		Erika	Yeah. Now they said that the purples were too small … like …
179		Darlene	Alright, if you call the blue rod one, what is one half?
180		Erika	Oh. Two purples and a white
181		Darlene	Alright
182		Erika	Because the yellows are too big and so the next one was

Line	Time	Speaker	Transcript
			purple. Soooo
183		Darlene	Wait a minute. There's gotta be a trick to this though.
184		Erika	There's no trick. How can you make a half of a nine?
185		Darlene	No. I know but… a way to represent it though. That's what I'm saying
186		Janelle	It's like, if you have like purple, purple
187		Darlene	Yeah
188		Erika	But, like, so a purple and half of a white is half of blue
189		Janelle	I don't think that's how she wants it though
190		Erika	Well there's no full numbers that go into nine
191		T/R	Make sure you guys make your own
192		Darlene	But there is no number that represents half
193		Erika	Oh wait wait wait wait wait wait wait wait wait. If if blue is one, right, what does four equal? I mean what does purple equal?
194		Janelle	If blue is what?
195	9:12	Erika	If blue is one. Right. What does purple equal? Because then you think of purple, you take that whole number, and then you take whatever white is, and you add them together. Like to get an actual number. Like, here…
196		Darlene	Well blue is down here
197		Erika	If blue is one, right?
198		Darlene	Yeah
199		Erika	So, these, this is all in ninths. Right?
200		Darlene	Yeah
201		Erika	So this is …
202		Janelle	Base nine!
203		Erika	… eight ninths. That's seven-ninths, six-ninths. Why is the furniture talking again?
204		Janelle	It's … we're in base nine.
205		Darlene	Oh god
206		Erika	Don't, no, let's not talk
207		Janelle	That's what it is!

Line	Time	Speaker	Transcript
208		Darlene	… wanna talk about base
209		Janelle	Base nine instead of base ten.
210		Erika	So purple, this is four-ninths. Right? Two-ninths.
211		T/R	You guys are too close together now. But I don't think I can get you far enough apart. So. Talk a little quieter.
212		Erika	Sorry
213	9:59	Janelle	<inaudible> in base nine
214		Erika	So … so two-eighteenths. Oh! So it's um, four-ninths and one-eighteenth. So that's five, six-eighteenths.
215		Janelle	Which is a third
216		Erika	Which would be …
217		Darlene	… should write that down …
218		Janelle	You're supposed to get a half
219		Erika	Well other than … I don't know … well of course it's going to equal one-third because you can't get a half of a ninth.
220		Janelle	So it's a half plus …. it's a third
221		Erika	No, it's two thirds. Isn't it?
222		Janelle	I don't know. You're the one doing the math.
223	10:40	Darlene	It's two thirds
224		Erika	Six eighteenths divide em by three
225		Darlene	It's two thirds.
226		Erika	No. It's not.
227		Darlene	No it's not
228		Erika	No. She's right. It's one-third.
229		Darlene	Two over six. One-third. Yeah
230		Erika	That doesn't equal one-third. Does it? Because one-third is light green. This doesn't equal a third. Because this is a third
231		Darlene	Is this what you're thinking of?
232		Janelle	Yeah
233		Erika	No, see but this is a third right? And we were saying purple and a half of a white should equal half right?

Line	Time	Speaker	Transcript
234		Darlene	Right
235		Janelle	Um hum. [agrees]
236		Erika	So it shouldn't equal one third.
237		Janelle	What shouldn't equal one-third?
238		Darlene	No THIS would equal one third.
239		Erika	Exactly! So I don't know why we're getting one-third here. It's no one-eighteenth
240		Darlene	Maybe, maybe the four-ninths.
241		Janelle	You have to remember, we're not in tens
242		Erika	Oh!! I know what I did! I multiplied
243		Darlene	What'd you do?
244		Erika	I multiplied wrong.
245		Darlene	Ahhhhh.
246		Erika	You have to multiply the top and bottom by two. Which is eight, which is nine-eighteenths which is one-half
247		Darlene	One-half
248		Janelle	Purple is four now.
249	11:38	Erika	Oh no. See I did my math wrong. When I made the denominator the same, I didn't multiply correctly. So, yeah, it's nine-eighteenths which is one-half
250		Darlene	Which is a half. That's why.
251		Janelle	But what colors?
252		Erika	Like purple? I don't know.
253		Janelle	But that's what the problem is.
254		Erika	There's nothing
255		Darlene	You can't represent …
256		Erika	There's no rod to do it so. Other than doing that and explaining it, I don't know what else we can do. Well, let's go on to five. Since four is a problem
257		Darlene	It's probably gonna be the same thing.
258		Erika	Well what's the question?
259		Darlene	Light green

Line	Time	Speaker	Transcript
260		Janelle	Alright. Hold on.
261		Erika	That's an odd number. Oh light green is one? Right?
262		Darlene	Yeah. What number is represented by the red rod?
263	12:34	Janelle	Wait. What are we on now?
264		Erika	Uh number five. We're just doing number five since we can't
265		Darlene	See it's the same kind of thing.
266		Erika	It's the same problem
267		Darlene	It's the same thing.
268		Erika	Well, let's just figure it out. This is what? One, two, three. So, we're in thirds, and so that's …
269		Janelle	Two thirds
270		Erika	Two thirds. And that's one-third. Yeah, red is two-thirds.
271		Janelle	Um hum
272		Erika	And then, what is represented by the dark green rod?
273		Darlene	Dark green.
274		Erika	So that's
275	13:05	Darlene	One-half.
276		Janelle	No
277		Darlene	Or two
278		Erika	Two
279		Darlene	Two
280		T/R	Tell me what these numbers mean up here
281		Erika	Oh that's just the question number
282		T/R	Oh. Ok
283		Erika	So I can follow.
284		T/R	Ok
285		Erika	Yeah. Because I totally went the wrong way.
286		T/R	So, you're actually answering more than I've asked on this question here.
287		Erika	Oh yeah. I like having the whole thing there.

Line	Time	Speaker	Transcript
288		T/R	Ok
289		Erika	I don't like missing parts.
290		T/R	Ok.
291		Erika	Alright, and then …
292		T/R	You're just answering the questions within the questions. Ok
293		Janelle	Yeah
294		Erika	They use … they use the rods and I use the grid. Right?
295		Darlene	Yeah
296		T/R	I just want to see what <inaudible>
297		Darlene	The white one is one-third. Which rod represents the one?
298		Erika	Wait, so white is one third?
299		Darlene	Yeah, so the …
300		Erika	Wait, so if white's one third we already have that answer. What equals one right? Light green
301		Janelle	Oh yeah.[agrees]
302		Erika	Because I have that … see … that's why I filled this out because this thing shows …
303		Darlene	Yeah
304		Erika	… the answers. What number does yellow rod represent? That's one and two thirds. So you need three. You need five of these for it to be… Hmm
305		Janelle	Here. Do it like this.
306		Erika	What? That's … Yeah that's what … ohhh that.
307		Janelle	For seven
308		Darlene	Which did <inaudible>
309		Erika	You have
310		Janelle	Green is… No
311		Erika	One
312		Darlene	Yeah
313		Janelle	It's three
314		Darlene	Um hum. [agrees]
315	14:53	Janelle	No like …

Line	Time	Speaker	Transcript
316		Erika	Oh if you count this as one… yeah one third
317		Janelle	If you have it like that. And then make it three fourths and two thirds. And you see that three fourths is greater.
318		Erika	What? Oh you're doing seven.
319		Darlene	Yeah
320		Erika	I didn't even know what we were doing … what number you guys were doing.
321	15:13	Darlene	Are you <inaudible>
322		Janelle	…light green
323		Erika	Wait, wait, wait! What did you do?
324		Darlene	You set it up so that these are like
325		Janelle	So cause the …
326		Erika	Thirds
327		Janelle	… green, the light green is
328		Darlene	Thirds
329		Janelle	Three-thirds.
330		Darlene	And the purple is four fourths
331		Janelle	Four fourths. So if you take away …
332		Erika	The one, one
333		Janelle	… one of each.
334		Erika	One white
335		Janelle	Yeah, you can see that it's bigger
336		Erika	Yeah
337	15:45	T/R	Ok. Explain to me what you just did there.
338		Janelle	We did…
339		Erika	Number seven.
340		Janelle	We have … you know … that's … that represents three thirds. You know that's one and that's three thirds. And this is three fourths … this is four fourths.
341		T/R	Ok
342		Janelle	So the question is "which is bigger?". So that's now … so if you take away these, that's two thirds and that's three fourths

Line	Time	Speaker	Transcript
343		Erika	Three fourths. Three fourths is bigger than two thirds
344		Janelle	Yeah
345		T/R	However, usually when we do these problems, you have to have the same one
346		Janelle	Oh yeah. That's true.
347		T/R	For example
348		Erika	Oh yeah yeah … that's right. That doesn't make sense then
349	16:21	T/R	Ok
350		Erika	Ok yeah, we know what you mean.
351		T/R	Ok
352		Erika	We need to find something
353		Janelle	Yeah we have to do um…
354		Erika	We need … oh …how about these are fourths, and … what would be thirds? This is a fourth
355		Darlene	Do you have to use the white?
356		Erika	I don't think you have to but … Yeah because
357		Janelle	You know what you have to do? You have to do it like this.
358		Erika	We can't use the white
359		Darlene	Which one is the one?
360		Janelle	This one. The orange.
361		Darlene	The orange. Ok
362		Janelle	Yeah. Do it that way.
363	16:53	Darlene	Yeah. You have to figure out which one is the thirds, isn't the … is it the green? Green is thirds?
364		Erika	No, wouldn't it be yellow?
365		Janelle	Yeah
366		Erika	No yellow is halves.
367		Darlene	Is it green?
368		Janelle	Yeah
369		Darlene	It's green
370		Erika	Which green?

Line	Time	Speaker	Transcript
371		Darlene	Light green
372		Erika	There's two greens
373		Janelle	You're not gonna get thirds from ten
374		Darlene	No, I know that, but, is that the closest way to do it though? Is the light green?
375		Janelle	Yeah
376		Erika	Maybe we should work … well we can't …
377	17:20	Darlene	And then the fourths would be...the red?
378		Erika	Red's not half of green. Does it have to be half of green?
379		Janelle	No, it's bigger than red.
380		Erika	It can't be the reds. Maybe the greens are too big. It's definitely not fourths because three of the reds equals two of the greens. And three fourths does not equal two thirds. So it can't be red and green. Red and light green
381		Darlene	Yeah. Well the green has to be the thirds.
382		Erika	Yes. Because there's nothing bigger than it that fits.
383		Darlene	So why can't it be the red? What's the next thing down?
384		Janelle	I know! I know how to do this.
385		Erika	Good. At least one of us does.
386		Janelle	It's just going to take some trying.
387		Darlene	What's the next color down from green?
388		Janelle	I don't know I just disassembled my thing.
389		Darlene	I thought it was red. Wait. Isn't this in order?
390		Erika	Well, yeah
391		Darlene	Yeah, so it's red.
392	18:36	Erika	But red … but if you line up … look, if you line up
393		Darlene	No, I know … I know what you're saying but I'm saying like logically that should be the answer.
394		Janelle	You have to find … this is the … yeah …
395		Erika	But the thing is, if you take one of those away
396		Darlene	This is three <inaudible> train to … whatever. Alright.
397		Erika	You need to find a train that has twelve

Line	Time	Speaker	Transcript
398		Janelle	You have to find a train…
399		Erika	Look
400		Janelle	…where there's three in one and four in the other. No, like this. Where three of one equals four of the other.
401		Erika	Well the whole thing is, you need to find something that has a base that three and four go into which would be twelve, which would be this.
402		Janelle	But you want the same color. So you can represent …
403	19:17	Erika	Does a train have to be one color? Because …
404		T/R	No, you can put something … as long as you can fit … show me three fourths on that train and show me whatever else you have to show me.
405		Erika	I just have to find the right color for three fourths. I mean, a third. I don't have anything in order anymore
406		Darlene	Why do you have the red there?
407		Erika	Because this is twelve.
408		Darlene	Oh
409		Erika	Because you need something that three. Three…
410		Janelle	Like this look. Look.
411		T/R	Yeah now that's actually two models. This model works and but I think the one you're going for is gonna work too. Tell me what you've got here
412		Janelle	That represents. This represents one third. The brown represents one third and the green represents one fourth. So two-thirds is smaller than three fourths.
413		T/R	Ok, and you have to tell me by how much
414		Janelle	By…
415		T/R	Well not right now, but work it out. But you're … continue with this one because we wanted two models for this.
416		Erika	Yeah I'm trying to find … well we need … you need three items and four items. Well…
417		T/R	So you've got the fourths it looks like. Right?
418		Darlene	Yeah
419		Erika	Yes, she said the greens is the fourths.

Line	Time	Speaker	Transcript
420		Darlene	Green is the …
421		Erika	Light green
422		T/R	Leave the reds there. You might need them for something else.
423		Erika	Alright, so, the greens the fourths. What's smaller than the greens?
424		Darlene	Um, dark green, yellow?
425	20:28	Erika	Yellow? No yellow's bigger than green
426		Darlene	No yellow is the other
427		Erika	Other way.
428		Darlene	Black. No
429		Erika	We're using light green. Red.
430		Darlene	Red.
431		Erika	But does red
432		Darlene	I have red.
433		Erika	Is it four of them?
434		Darlene	Red doesn't work.
435		Erika	No, that's too small. So what can we use that's fourths? Hum. One fourth of twelve
436		Darlene	We're looking for the thirds now. We have four.
437		Erika	Oh, yeah. We're looking for thirds.
438		Janelle	Can we borrow some of your reds?
439		Darlene	It's the yellow isn't it?
440		Erika	So you do need the … Is it … Does yellow…
441		Janelle	Can we borrow some of your reds?
442		Erika	Oh. But yellow is too big
443		Darlene	Um
444		Erika	Purple.
445	21:06	Darlene	Purple should work.
446		Erika	Purple should work. We ruled out just about everything else. Yeah.
447		Darlene	Yeah

Line	Time	Speaker	Transcript
448		Erika	Yeah. Yeah. So it's purple and green.
449		Darlene	Can I use this one? Or are you still using it
450		Erika	Have it.
451		Darlene	Ok.
452		Erika	Because I still have another purple over here.
453		Darlene	Alright
454		Erika	There you go. We did it!
455		Darlene	Got it.
456		Erika	Now, her question is gonna be …
457		Darlene	What?
458		Erika	Because remember the question is two thirds and three fourths and she wants to know by how much is this bigger.
459	21:43	Darlene	So how many
460		Erika	Well
461		Darlene	You use…
462		T/R	You have your answer. It's bigger by…
463		Janelle	By…
464		Erika	Well yeah but …
465		T/R	A red
466		Erika	We need to know what this is called
467		T/R	Ok
468		Erika	You have six reds?
469		Janelle	Hmm?
470		Erika	How many reds do you have?
471		Janelle	Twelve
472		Darlene	Wow you have a lot of reds.
473		Erika	Are you using them as one?
474		T/R	Yeah.
475		Janelle	They're one-twelfth.
476		T/R	Yeah, her model is different from yours. But that was the next question. Make two models of the same thing. So do you have your answers for this one too? So what are you

Line	Time	Speaker	Transcript
			telling me here?
477		Erika	Well the greens, cause she helped. She pulled up the greens.
478	22:18	Darlene	The green is the one fourth.
479		Erika	Yeah
480		T/R	Ok, so show me three fourths. Ok.
481		Darlene	And then… yeah
482		T/R	Ok, and then the other part was? What's the purple?
483		Erika	Two-thirds
484		Janelle	What's two-thirds?
485		Darlene	Two-thirds
486		T/R	Ok. So.
487		Janelle	And what's the difference.
488		T/R	Right. Which is bigger?
489		Erika	The thirds. The, the, fourths. Sorry
490		T/R	Bigger by?
491		Erika	The white but I don't know what the white is.
492		Janelle	Line the whites up.
493		T/R	It's bigger… And you agree with that J…? Its bigger by a white and so you need to know what a white is.
494		Darlene	Yeah, so …
495	22:38	Janelle	Line all the whites up
496		Darlene	This is twelve. Right? The orange and the red together are twelve?
497		Janelle	Yeah.
498		Darlene	Alright.
499		Janelle	So if you line all the whites up
500		Darlene	So, why…
501		Erika	Wait! You know that purple is a third. This is … and there's four of these in here. Right?
502		Darlene	Yeah. I have to see it though.
503		Erika	One fourth of a third is? One twelfth? So it's bigger by one-twelfth?

Line	Time	Speaker	Transcript
504		T/R	Yeah but …
505		Darlene	But if you show
506		T/R	No questions right?
507	23:12	Erika	Yeah. It is. It is bigger by one-twelfth.
508		Darlene	Ok here's a twelfth. And then you take this. Yeah.
509		Erika	Well I didn't even do it that way.
510		Darlene	Yeah I know, but to represent it ... if this is
511		Erika	That's more cleanly represented than mine
512		Darlene	There's twelve here. Then line this one up. And what's missing here is one-twelfth.
513		T/R	Ok, but you had a different argument that's also valid.
514		Erika	Yeah mine was that I knew what this was – that this was one-third. And four of the little whites go into a third. So the white has to be a twelfth. Cause one-third
515		T/R	One fourth of a third
516		Erika	Yeah a fourth of a third is one twelfth.
517		T/R	Ok. Ok. And now did you both see what model R…. has over here?
518		Darlene	Um hum.
519		Erika	She had the reds as twelfth.
520	24:02	T/R	Right. Reds as a twelfth.
521		Darlene	Um hum.
522		T/R	Ok, sooo
523		Erika	So twelve of them is one.
524		T/R	Same thing with a different one. Reds were twelfth and so the three browns made one. Whereas with you, the orange plus red made one.
525		Erika	Yeah
526		T/R	Ok. Ok. So the next question for all three of you since you're done a little bit before them is the last question. Prepare a question for the other group to answer, just like the one you just did. Ok, but you know … no elevenths and seventeenths or anything like that.
527		Erika	Darn. (laughter)

Line	Time	Speaker	Transcript
528		Darlene	Um.
529		Erika	I need to keep this separate for when we talk about number four. Because I want to have a …
530		Darlene	Ok.
531		Erika	Ummm… Do we need to make something like that? She wants a…
532		Darlene	She wants another fraction?
533		Erika	<inaudible> like we need to think of two fractions, and then think of a … that you would have to multiply them together.
534		Janelle	What about this?
535		Darlene	Look at this one.
536		Janelle	Like, which is bigger? You can pick any number.
537		Erika	But … these evenly go into … Oh, like if we did … one, two, three, four, five … like if we did five sixths and two thirds, which one's bigger? Like that?
538		Janelle	Yeah
539		Darlene	Yeah
540		Erika	Hmm … ok, so if blue, if blue is one
541		Darlene	Should I write this down?
542		Janelle	Blue would be half
543		Darlene	I'll just write it down here
544		Erika	Oh, yeah. But … oh, so we just need to write which is larger um five-sixths or two-thirds.
545	25:34	Janelle	Yeah
546		Erika	But we don't have to … but we don't give them the hint if … blue … we just
547		Janelle	Which is larger …
548		Darlene	Five sixths
549		Janelle	Five sixths
550		Darlene	… or two thirds. And then let them … so erase this.
551		Janelle	Yeah
552		Darlene	So … you know …
553		Erika	So … which is larger

Line	Time	Speaker	Transcript
554	26:00	Janelle	So can we go back to this one now?
555		Erika	Which one? Four?
556		Janelle	Yeah
557		Erika	4b?
558		Janelle	If you call the blue one, what represents one half?
559		Erika	Yeah, I just don't know how to go about it other than the way I did it. Like … cause that's … that's an odd
560		Darlene	What is the blue?
561		Janelle	Nine
562		Erika	One. Oh nine. Yeah
563		Darlene	It's nine.
564		Erika	But we're calling it one.
565		Darlene	Ok
566		T/R	Yeah, make sure you've got your notation right.
567		Erika	We went back … yeah we went back to 4b because we were having a problem with that one.
568		T/R	Ok.
569		Erika	I have one solution. But it doesn't work out because you'd have to do …
570		Darlene	Yeah, this is half of it to
571		Erika	… half of it would be four and half of a white.
572		T/R	Ok
573		Erika	Which you can't do
574		T/R	Ok. Well what is 4b? Let me look.
575		Erika	4b is … you get … this is one.
576		T/R	Oh yeah
577		Erika	Find a half of it. Well the …
578		T/R	Oh yeah wait a minute.
579		Erika	Part b
580		T/R	Yeah I know. I didn't think … is that the same 4b that you have?
581		Erika	Yeah

Line	Time	Speaker	Transcript
582		Janelle	We didn't do 4b.
583		T/R	Ok
584	27:07	Darlene	I just thought of a different way to represent it, cause
585		T/R	I thought I had a different 4b question.
586		Erika	I don't know
587		Darlene	Because the purple... is that
588		Erika	Is there something else that we can use evenly that would go
589		Janelle	There are none.
590		Erika	Yeah, there's no even amount.
591		T/R	I meant to give you a 4b question that wasn't the blue rod because we already had that one.
592		Erika	Oh. [laughs]
593	27:33	T/R	And these guys got the answer for 4b.
594		Erika	How?
595		Darlene	How?
596		T/R	What is your answer for 4b?
597		Erika	I'd love to hear this
598		Fae	None of them
599		Sarah	None
600		Darlene	Oh
601		Erika	Oh well
602		Darlene	We're sitting here …
603		Janelle	Yeah we're sitting here like, if we put three blues together and that's three and what's a half of three and then
604		Erika	Yeah
605		Darlene	We did overthink that
606		Fae	None of them work. Sorry girls
607		Erika	We didn't know you just wanted none. We were trying to find something that wasn't none.
608		Darlene	Because everything else has like a number
609		T/R	Yeah well you could ... extra credit. Is there anything you can say besides none?

Line	Time	Speaker	Transcript
610		Erika	So this is a
611		T/R	When you come up with your new set of rods that can …
612		Fae	No rods but the number is
613	28:06	Erika	How many whites go into this?
614		T/R	Yeah, so
615		Fae	Four and a half
616		Darlene	Four
617	28:32	Erika	Oh, they're doing. Oh, they're doing number seven. Oh I didn't even finish writing this. Ummm.
618	28:50	T/R	Ok, for question eight, I see your question. Now you guys have to have a model and an answer for it.
619		Erika	Oh yeah we just
620		Janelle	We did that
621		Erika	We tore it down
622		T/R	Alright … well just … keep it ready.
623		Erika	And what was the other color? Light green? I guess we can just cover it with the paper. So they can't
624		Janelle	You will not cheat.
625	29:23	Erika	You will not be prepared.
626	29:31	T/R	Back to fractions. Um, before we do that last problem, I wanted to talk about some of the other things that you showed me and some of the things that you can prove. For example
627		Fae	Oh you want me to show that twelve thing?
628		T/R	Over here. Not yet
629		Fae	Ok
630		T/R	There was the yellow. Back in question, um, five. Do I mean question five? No question six. The white rod is one-third. Ok, and you told me which rod represents one. And F… over here said light blue represents one. Right, now if white is one third then light blue represents one.
631	30:13	Fae	Light green
632		Erika	Light green
633		Darlene	Light green

Line	Time	Speaker	Transcript
634		T/R	Light green. Sorry. Now, this F… said yellow is one and two-thirds. Show me your model for one and two-thirds.
635		Fae	If three of the white equals one, then its one plus two extra little ones which is thirds.
636	30:35	T/R	Now this F… said the answer was five-thirds. So show me five-thirds.
637		Sarah	Because I counted that this was five whites. Yellow is five whites.
638		T/R	Ok
639		Sarah	So I said
640		Erika	Five thirds.
641		T/R	Yellow is five thirds
642		Sarah	Yeah
643		Erika	One and two-thirds
644		T/R	She said yellow is five-thirds. She said yellow is one and two-thirds. Which ones right?
645		Janelle	They both are.
646		Erika	Both are
647		T/R	How come? How do you know?
648		Janelle	Because they're the same number
649		Fae	The fractions are the same
650		T/R	Ok, and you can prove it with the rods. Right?
651		Fae	Yeah
652		T/R	You can prove that three whites is equal to
653		Fae	One and two-thirds
654		T/R	And two-thirds. So, there you go. You know, you have a physical thing that proves that these two fractions are equal and, you know, think about this as another way to do it besides the numerical things that you learned. That there is actually a physical proof. And you also showed me in some cases that back when the orange rod was one, some of you said that the red rod was one-fifth.
655		Erika	And some said two-tenths.
656		T/R	And some of you said that the red rod was?

Line	Time	Speaker	Transcript
657		Erika	two-tenths
658		T/R	two-tenths. But you could prove to me, right, that one-fifth and two-tenths are actually the same number using these rods. Right? Ok. So that was some of the kinds of things I wanted you to think about how you have different answers but they're really the same thing. And when you were doing the first one you said it was um three and a half of the red was one. And something was three and a half, but it could also have been seven halves. Right? You can prove that three and a half is the same as seven halves. In fact, why don't you do that one?
659		Erika	Three and a half
660		T/R	If the red is one, the black was three and a half.
661		Erika	Ok
662		T/R	But the black is also and you should be able to show me that black is also seven halves. So how can you show me, if you have enough room
663		Erika	I got it.
664		T/R	… and cubes that the black … that three and a half is the same as seven halves?
665		Erika	Like this.
666		T/R	Ok. Alright. So J… has it over here if you don't have enough you can look. Oh, you've got it too. F….. has it over here. Ok and those of you that have enough white cubes have it. So, show us your proof. Tell us about your proof.
667		Erika	Ok. So black is one. Now you said you wanted three
668	32:45	T/R	No, black is not one.
669		Erika	What is it?
670		T/R	Red is one
671		Erika	Red's one.
672		T/R	And black is
673		Erika	And you want us to prove that black is three and one half.
674		T/R	Which … and I want you to show me that three and a half is the same as seven halves.
675		Erika	Alright. So, black is three and a half. So, red's one. We've got one, two, three, and a half. Half, half of a red is a white.

Line	Time	Speaker	Transcript
			So that's three and a half. Or, if you wanted … what seven halves?
676		T/R	Yeah
677		Erika	Since one of these is one, there's two of them for every one. Alright, so two times three, because we have three reds, is six. Plus the one white that we have at the end is seven.
678	33:28	T/R	Ok. And that was actually… you're sort of giving the proof of the algorithm. Remember three and a half. Remember that rule for converting three and a half to a mixed number. The three times the two plus the numerator. Remember?
679		Fae	Yeah
680		Erika	Yeah
681		Sarah	Um hum.
682		T/R	That's just what you explained. Three of the little reds and there's two white ones in each red. So there's a model for explaining how you do that. Ok? Ok, I think we're running out of time so

Transcript 5 of 6
Date: 04/15/2011
Length: 01:00:57
Camera 1
Transcribed by: Deidre Richardson
Verified by: Mary Huizenga

Line	Time	Speaker	Transcript
1		T/R	…take half of everything. Umm
2		Erika	I have an idea
3		T/R	Ok you have a proposal and I ...
4		Janelle	I do too
5		T/R	…I just noticed that F….. had something in her notes about it
6		Erika	Oh she actually like …
7		Janelle	I did.
8		Erika	… did
9		Janelle	I made it all pretty.
10		T/R	Ok
11		Kelly	We're coloring!
12		T/R	So… so go
13		Janelle	Well I did … each … each individual block is two raised to the first power. So, if you have two raised to the first power, then its two raised to the first power plus two raised to the first power
14		T/R	Ok
15		Janelle	Which is two raised to the second power which is four
16		T/R	Ok…
17		Janelle	So then, you just ... each block is one of those so you go all the way up and your last one is ten fifty-four so even though this
18		T/R	Ten twenty-four
19		Janelle	Ten twenty-four. So even though this doesn't look like half of this, it really is half of that.
20	00:46	T/R	Ah. Ok, I see what you're saying.
21		Fae	Interesting
22		T/R	So it's… and that is exactly what you proposed Jess actually

Line	Time	Speaker	Transcript
23	00:51	Erika	Yeah.
24		T/R	Independently
25	00:53	Erika	Oh, well except for I also have this block
26	00:55	T/R	Right you have the …
27		Janelle	Yeah
28		T/R	… two to the zero block
29		Erika	Yeah well I'm just saying yours is red…
30	00:59	T/R	Ok. Right
31		Erika	… mine was …
32		Janelle	Well I did all different colors. I didn't do the same colors as that because this is a different set of
33		Erika	Oh, ok. Yeah, so yeah you did the same thing as me then
34		T/R	So you're saying you can make half of any number with this?
35		Janelle	Yeah. It just does … isn't a visual repre- … proper visual representation
36		Fae	It's a numerical representation
37		Erika	Why not?
38		T/R	How do you make
39		Janelle	Because half of this is that
40		T/R	Well what's half ...
41		Janelle	… but it doesn't …
42		T/R	What's one half of this?
43		Janelle	You would have to have, I don't know, you would just have to start it the other way. I don't know.
44		T/R	Oh. Yeah I can see that half of each one is the previous one, but I don't see a half of this one.
45		Erika	Yeah but we never did a half of this one before.
46		Janelle	Yeah
47		T/R	That's right! And that's why we couldn't do half of blue. So we were trying to do something so that we could do something the equivalent of half of blue.
48	01:48	Erika	Oh see mine is purely uh mathematical.

Line	Time	Speaker	Transcript
49		T/R	Ok
50		Erika	Like mine …
51	01:49	Janelle	I don't know
52		Erika	…mine also includes
53		Janelle	Well mine's wrong
54		Erika	…they can do fractions …
55		T/R	Not wrong. We won't say wrong. We say … So you tell me … so you tell us what the issue …what yours is.
56	02:00	Erika	Mine was that this would be two to the zero.
57		T/R	Ok. That's one
58		Erika	So that's one. And that's two to the first, which is two. But then this is two to the second.
59		T/R	Ok
60		Erika	Which is half.
61		Janelle	So what's green?
62		Erika	There is no green. We're making up completely new ones.
63		T/R	Ok, ok but then
64		Erika	My, my thing took out the odds.
65		T/R	Ok so you're saying the next one, whatever color it is, is twice as long as this one.
66		Erika	Yeah. Um
67		Janelle	So you don't have … you don't go up by one block each time. You go up by two blocks each time
68		Darlene	Here's a brown
69		Erika	Well..
70		T/R	So you're missing
71		Erika	Not two each time
72		T/R	It represents …
73		Erika	Because it was … I went up …
74		Darlene	One
75		Erika	… one here
76		Darlene	Two

Line	Time	Speaker	Transcript
77		Erika	… two here, three here
78		Janelle	Yeah
79		Erika	Four here. Not three here. Four here.
80		Janelle	Yeah
81	02:49	T/R	You're doubling it
82		Erika	Yeah, well basically, yeah.
83		T/R	Ok. So you're doubling the size along with the number and you're doubling the number but not the size
84		Janelle	Yeah.
85		T/R	Correct?
86		Janelle	But I'm not doubling ...well… tech…yeah doubling technically
87		T/R	Well you're doubling the number that goes with these
88		Janelle	Yeah
89		T/R	Ok. I have the same question for you. What's half of this one?
90		Erika	Well theoretically it'd be two to the negative one, but you don't have a block for it
91		T/R	Ok. You don't have a block for it. But, remember that was the deal. If I - I'm supposed to be able to point to any block and you're supposed to be able to tell me what half of it is.
92		Erika	Well you can only do half for so long. There's infinite ...
93		Janelle	Which means you have to start with zero
94		Erika	… infinite amount of numbers.
95	03:28	T/R	Well, ok. If you start with zero does that work? And first off, so I have your two and you had one also. What
96		Kelly	That was my idea. The one that I handed in.
97		Erika	Start with zero
98		T/R	Umm, I can't remember what yours was. It was on the second page.
99		Fae	I don't know what I did
100	03:46	T/R	Ok. So you're saying. What are you saying? You made two browns equal to one

Line	Time	Speaker	Transcript
101		Fae	Right.
102		T/R	Ok.
103		Fae	So that each of those can have... I don't know.
104		Erika	See, we … we couldn't really start with zero. Because you can't …
105		Janelle	Yeah
106		Erika	…show…
107		Janelle	Zero blocks
108		Erika	…zero. That's the problem. You can't show zero blocks.
109		Fae	And this would be one sixteenth of that.
110		T/R	Ok.
111		Fae	I don't know where I was going with this.
112		Janelle	But then how do you find half of that one?
113		Erika	But then … half of the one sixteenth?
114		T/R	Ah. So there's the question.
115		Fae	I don't …
116		Erika	It's infinite. There's no way to just find half of everything because you go on into infinity. It's either positive or negative
117	04:26	T/R	So, what's your … So, what's your answer to the question?
118	04:28	Darlene	No
119	04:28	T/R	Create a rod set …
120	04:29	Janelle	It is impossible.
121	04:30	Erika	There is no way to. There's no way to do it.
122	04:30	T/R	In fact, what was the question? Can you create …
123		Darlene	No
124		T/R	… a set of rods so that everything has a half?
125		Fae	Yeah
126		Janelle	No
127	04:35	Erika	Not physically, no.
128		T/R	Not physically

Line	Time	Speaker	Transcript
129		Erika	Theoretically you can.
130		T/R	Theoretically, yes. Ok.
131	04:41	Erika	That's all I got.
132		T/R	Alright. Now I didn't hear from you two guys. Did you make it … or did you have a rod set … or do you have a discussion
133	04:47	Kelly	I couldn't figure it out. I was sitting there and we were talking about it. I was like, I
134	04:50	Erika	And that's when I told her my idea.
135		T/R	ok
136	04:53	Sarah.	I attempted one.
137		T/R	Ok
138		Sarah.	I just made. I used the same colors and I made white two and then I went, red is four and then six, eight. Like that's what I did all the way up to twenty.
139		T/R	Ok
140		Erika	<inaudible>
141		Janelle	That's what I tried first
142		T/R	So you have two sort of similar. Ok
143		Janelle	But like fourteen, what's seven?
144		Erika	Yeah, you don't have
145		Janelle	What's half of fourteen? How do you represent seven?
146		Erika	Yeah. Yeah, that's the only problem is you have ...
147		T/R	So there … So you found a counterexample and I think you had the right idea. No matter what you pick, I'll pick the smallest one and say 'gimme half of that' and you don't have it. And what you said is, well I'll make half of that. But then I'm going to need half of that next little one.
148	05:27	Erika	Exactly
149		T/R	Ok, So the answer is? Everybody agrees on the answer?
150		Erika	There's no way to do it.
151		T/R	Can't do it
152		Fae.	Not physically

Line	Time	Speaker	Transcript
153		T/R	Not physically
154		Erika	Not physically. We're like-minded
155		T/R	Ok. But theoretically
156		Erika	Theoretically. Yes
157		T/R	Theoretically you could just keep going all the way down. And you're right when you keep going half all the way down … you've done limits right? - at least those of us who've done calculus, you can go down … down to zero. Like you said you want to get to zero. Ok so that was that trick question. Can't be done.
158	05:53	Erika	But we all tried.
159		Janelle	Yeah
160		Erika	All of us tried
161		Janelle	You have no idea how I went through every number until twenty.
162		T/R	So you came up, but you did come up with a nice idea because you both came up with the idea that you can represent any number with those powers of two. Um, but fractions are an issue. So, we're just going to do some fraction activities.
163	06:11	Kelly	Yay!
164	06:11	T/R	I have some papers somewhere that have this exact problem on it. These exact problems and we are going to work on some other problems too, and what we're gonna do is, in your groups you're going to work on them and you're also going to think about the kinds of issues that people have. Kids have or other people have when they work on these types of problems. So, you can look at those … on ones on the sheet …on the board while I look up the sheets. So, your group – you can take this group and you can take this group as they start working on these problems and the bottom of the sheet has the things which is not on here … which is …
165	06:54	Fae	Two three four five six seven eight nine ten. I like Jess's idea since she's talking out loud
166	7:24	Erika	I always talk out loud. I always talk real loud.
167	7:39	Fae	One-third of twelve and also easily find one-half
168	8:12	Fae	She has one third. She has half of this. Two-twelfths. Paul has two-twelfths which would also be one-sixth of the candy

Line	Time	Speaker	Transcript
			bar.
169	08:34	T/R	And you guys can tell me what you are doing now.
170		Fae	Ok. I decided it's easy to cut twelve into one-third as well as one half, so
171		T/R	Ok
172		Fae	… this represents twelve …
173		T/R	Ok
174		Fae	… that's the candy bar…
175		T/R	Ok
176		Fae	… orange and red together...
177		T/R	Ok, and she has one third of the candy bar
178		Fae	…so she has one third, so one third would be this piece.
179		T/R	Ok
180		Fae	She gives Paul half of one-third that would be this piece so two-twelfths or one-sixth
181		T/R	Ok
182	08:59	Fae	Do you want me to represent one-sixth too?
183		T/R	Ok. So alright. So you can tell me that's two-twelfths. In fact that looks like one-sixth because six of these make one.
184	09:10	Fae	Right.
185	09:10	T/R	Right. Ok. But you're also telling me that's two-twelfths.
186		Fae	Correct.
187		T/R	And you can prove that that's two-twelfths right?
188		Fae	Mhmm.
189		T/R	Ok
190		Fae	With these things.
191		T/R	Ok. Because that's … ok
192		Fae	…this many ...I think I know …nope … yeah .. There's twelve of them. This represents two of the twelve.
193	09:35	T/R	Ok. I believe you. That shows that one-sixth equals two-twelfths. So, you started with this. You started out by telling

Line	Time	Speaker	Transcript
			me that the train of orange plus red equals one.
194		Fae	Correct
195	09:47	T/R	Ok. Then you told me … you guys are on board with this too?
196	09:49	Sarah	Yeah
197		T/R	Ok. Alright and then you told me that purple is one-third.
198		Fae	Right
199		T/R	Now somebody besides you tell me why purple is one-third.
200		Kelly	Because three of them makes a whole
201		T/R	Ok. That's a great explanation. Ok. And then she gives half of what she has. So what's that?
202		Fae	Half of the purple
203		Kelly	Because when you take out the purple and then you take out. See?
204		T/R	So the reason ... you know that red is half of the purple …
205		Kelly	Yeah
206		T/R	… because…
207		Kelly	You put two reds to a purple
208	10:19	T/R	… and red is half the purple. Ok. So, how much does she give to Paul? She gives him the red. How much does she have left? You didn't answer that one. How much does she have left?
209		Fae	She gets half of what she gave. She has one-sixth left.
210		T/R	She has…yeah..
211		Fae	Yeah one-sixth
212		T/R	Ok
213		Fae	She gave him one-sixth and she had one-sixth
214		T/R	And so she started out with the purple which is equivalent to this and she gives away one and keeps one
215		Fae	Ok
216		T/R	Ok? Works for me. Write it all down. And the other thing is, the mathematical sentence. You know, so what's the mathematical equation that you write, the sentence that you write that says … gives you one-sixth as the answer. Ok

Line	Time	Speaker	Transcript
217		Fae	To make two-twelfths?
218		T/R	No, well…
219		Fae	Into one-sixth?
220		T/R	No. the equation that says when she has a third of a candy bar and she gives away half of what she has, how do you do that calculation and end up with one-sixth? Which you showed me was the correct answer. Do you know what I'm saying? Half of what she has, she gives away. That's one-sixth. What's the equation? If you were going to do it totally in math … You gotta have paper to write on
221		Kelly	Ohhh
222		T/R	You're going to do it totally with
223		Fae	Oh, okay
224		T/R	…with you know fractions and symbols and so on. How do you end up starting out with one third and ending up with one sixth?
225	14:08	T/R	Now does anybody in this group have a calculator that does fractions? Ok, so you're still writing it up.
226	14:21	Fae	I'm actually incorporating after each sentence how I did it with the blocks and then I'll write the equation.
227		T/R	Ok. Sounds good. Ok. And I see you already have an equation. Wait until the other guys get equations and then we'll … We can move on to the next one. If you're done you can move on to the next one yourself.
228	14:35	Fae	We all did it so differently
229	17:28	T/R	And where are you guys?
230	17:30	Fae	I just finished this one
231	17:31	T/R	Ok. Alright you finished this part but don't forget there's the equation to write too or did you already do that? You wrote it up.
232	17:37	Fae	Well, not really, sort of.
233	17:38	T/R	Ok. That's ok. So. Write the whole thing separately. You did that I know.
234	17:42	Sarah	Yeah.
235		T/R	That's good. Ok. K… still has to do that. Ok. Ok. Great. And so you have the fraction thing here, but you … say the

Line	Time	Speaker	Transcript
			Cuisenaire rod thing that shows this.
236		Sarah	Yeah, I had it for the first one. Well I don't have it anymore. I understood it for the first one but I didn't know how to do it for the second one.
237		T/R	Ok. Well, when they're ready for it, let them … discuss it with your group.
238		Sarah	Ok.
239		T/R	And then, are you on number three yet?
240		Sarah	Yeah, I'm on number three.
241		T/R	Ok.
242	18:29	Fae	What was your equation?
243	18:30	Sarah	For which one? The first one? I did one-third divided by two equals one-third times one-half which equals one half
244	18:39	Fae	Alright. Ok so it's the same thing as one-third minus one-half?
245	18:41	Sarah	Yeah.
246		Fae	Ok. That will make it one-sixth?
247		Sarah	Yeah. That equals one-sixth.
248		Fae	<inaudible>
249		Sarah	Yeah.
250	18:55	T/R	Yeah there's actually two equations for number one. What did she give away? You did that calculation. And then there's a separate calculation for what did she have left. She happens to have the same amount left as she gave away, but that's not necessarily, always the case.
251	20:07	T/R	Are you listening to their …
252		Kelly	I'm like focused on theirs
253		T/R	Ok. But you're still on number two right? So you want to focus back to question two here. I know it's hard when you're so close.
254	20:42	Kelly	Pablo
255	20:49	Fae.	I have a question. How did you guys do this one? Like I'm doing twelve twelfths minus four twelfths because four twelfths is one third
256	21:02	Sarah	Yeah

Line	Time	Speaker	Transcript
257	21:04	Fae.	But I came out with two thirds. That's how much of a candy bar she never even had.
258		Sarah	She never had two-thirds of it?
259	21:13	Fae.	No. She only had one-third.
260	21:15	Sarah	Yeah she… yeah she started with one third. yeah she didn't' have two thirds
261		Fae.	So then that part just leave alone
262		Sarah	Yeah
263	21:22	Fae.	And then <inaudible>. I don't know. I will probably end up figuring it out when I'm doing another problem.
264	21:56	Fae	I feel like I could use twelve again. What number are you up to? Two?
265	21:58	Kelly	Mmhmm
266		Fae	Ok.
267	22:09	T/R	Ok. Ok. Ok. Yeah, this is a tricky one.
268	22:11	Sarah.	Is that right?
269	22:13	T/R	That disagrees with what they have. But, we're having a discussion about right and wrong so we're going to talk about it.
270	22:19	Sarah	I don't know. I just tried to do it with this
271		T/R	But you guys are working on… you're on number two now?
272		Kelly	Yeah
273		T/R	Ok, so well, back up to number two.
274	22:28	Sarah.	Ok.
275		T/R	And ummm, because you have a nice equation but you didn't have the Cuisenaire rods
276		Sarah.	Yeah
277		T/R	And you guys have Cuisenaire rods but no equation. Ok
278	22:44	Kelly	Would it be one-third of just this one?
279	22:47	T/R	You can't do one-third of that one I can tell you because it's ten
280		Kelly	Oh that's right. Duh
281		T/R	But you didn't … you could use this I think if you want to

Line	Time	Speaker	Transcript
			use something smaller.
282	22:57	Fae	She has two-twelfths or one-sixth left
283	22:58	T/R	Ok, and she's doing it. Ok.
284	23:01	Kelly	Oh, I never thought of that
285	23:02	T/R	Which one are you looking at?
286	23:03	Fae	Two.
287	23:03	T/R	Two? Ok, now F… over here already has the equation but not the model, so you explain your model and you see if it agrees with your equation
288		Fae	This is half of the candy bar …
289		T/R	But so, what's the whole candy bar?
290		Fae	Twelve
291		Sarah	Twelve
292		T/R	Ok
293		Fae	Here I'll move these
294		T/R	Ok.
295		Fae	Now. Here's the whole candy bar. The orange and the red. Half of it, is two greens. Which if you put them next to the whites, it adds up to six-twelfths or one-half. Um, and then so that's half of it. Now if I put three purples up against it to represent thirds. One third of the candy bar given to Gordon. So there's one third plus a half, which equals ten twelfths. And then …
296	23:56	T/R	That's what was taken away
297		Fae	That's what was taken away. This is Pablo and Gordon. So this is Keisha. The two-twelfths.
298	24:05	T/R	And you said your answer was?
299	24:08	Sarah	One-sixth. So two twelfths is one-sixth
300	24:08	T/R	One-sixth. So you agree with that?
301		Fae	Yeah
302		T/R	Now you could actually do it with this one I think because these are halves. Right, so if you make your dark green equal to one, you could do just what she did with different colors for halves

Line	Time	Speaker	Transcript
303	24:20	Fae	And it gets one-sixth before this one would
304		T/R	Yeah it gets one-sixth instead of two-twelfths. But you already know that's the same thing from your homework, well you already knew that anyway. You proved it on your homework. And this makes sense to you?
305		Kelly	Uh hun. I'm just really sleepy
306		T/R	Friday afternoons are not good times are they?
307		T/R	I think I saw a different answer else where, but we'll see. I think I want to talk about problem three as a large group.
308	25:32	T/R	So you just finished problem two in fact, just writing
309		Sarah	Is number three two-thirds?
310		T/R	What did you guys get for number three?
311	25:41	Darlene	One-sixth
312	25:41	Janelle	One-sixth
313	25:42	T/R	One-sixth
314		Janelle	I had originally gotten two-thirds, but then you said I was wrong
315		T/R	Ok. Well, we said we need some modification. You don't have to erase.
316		Sarah	No, I had one-third and then I looked and I thought it was two-thirds. But now I think its one-third.
317		T/R	But they said … they didn't have one … two thirds either. Right?
318	26:02	Janelle	They had one third. I thought it was two thirds and then we discussed it and now its one-third.
319		Darlene	One-sixth
320		Erika	One-sixth
321		T/R	One-sixth.
322		Janelle	Or one-sixth I mean.
323		T/R	Ok. But we need to discuss this as a group because we had different ideas so
324		Sarah	No I had one third but then I changed it to two-thirds but now
325		T/R	Yeah, two-thirds is what we got with R….'s interpretation I

Line	Time	Speaker	Transcript
			believe. There is an alternate representation um which maybe means you know we need a different kind of wording for the problem. Some classes have told me that they really don't like candy bars.
326		Erika	I like candy ...
327		Kelly	I like candy bars.
328		T/R	Well the idea … in this problem, and the idea is candy bars are not like standard. Like feet and inches. You know? There is a standard measurement that one foot means something whereas a candy bar doesn't necessarily mean something. And I've had students argue, well when you have a piece of a candy bar you don't know how big the whole was because you don't have the whole one to compare it to.
329		Erika	Well we always, we just used green as the basis of our - what our candy bar size is, so.
330		T/R	Yeah so um, you could do that. Well let's wait and talk about it with everybody. Let's see this group is still
331	27:05	Fae	I'm just writing this last thing and then I'm done
332	28:04	Fae	Ok. I'm up to number three
333		T/R	Ok.
334		Sarah	I actually go the third one. One-sixth
335		T/R	Ok.
336	28:12	Sarah	Not by rods but by doing it.
337		T/R	Ok. I wanted the rods too.
338		Sarah	I tried to do it by rods but I couldn't figure it out.
339		T/R	That's …
340		Fae	I think I got it
341	28:22	T/R	That's perfect. And the thing you did with rods was actually a different question.
342		Fae	So he has half … takes one-third of the candy bar from John
343		Sarah	Yeah
344		T/R	But, um,
345		Sarah	I think it was like … I tried to use six of these
346		T/R	Ok

Line	Time	Speaker	Transcript
347		Sarah	So I was like, he has one-half and then…
348		T/R	You have to go back to remember that it takes six of them to make one.
349	28:38	Sarah	Yeah so you can't use these … you can't just use these as one
350		T/R	Well you could sure, that's one. But then, he only has this but keep in back of your mind that it takes six of them to represent one. So he has this
351		Fae.	One-sixth
352		T/R	Go back to ... how much does it take to … this represents a half.
353		Sarah	Yeah
354		T/R	What represents a third?
355		Sarah	Two
356		T/R	And he takes away two of them
357		Sarah	Oh so then he has one-third
358		T/R	Yeah. What? No
359		Sarah	One-sixth
360		T/R	He has one-sixth left because you have to…
361		Sarah	Oh not one-third. It's one sixth of everything
362		T/R	… the whole thing
363		Sarah	Alright.
364		T/R	Right. You go back. There's always the same thing that equals one. So <inaudible> exactly right.
365		Kelly	Look I figured it out.
366	29:12	T/R	Ok. Go ahead.
367		Kelly	It took me awhile.
368		T/R	So, tell me what you did
369		Kelly	I had, well, it's kind of what she did.
370		T/R	Ok
371		Kelly	Um
372		T/R	Where's your one?

Line	Time	Speaker	Transcript
373		Kelly	Here's my one.
374		T/R	Ok.
375		Kelly	And here's
376		T/R	Ok, there's a half
377		Kelly	So then he's ... she's giving him a half?
378		T/R	Wait, which?
379		Kelly	The second one
380		T/R	Oh ok. Yes, go ahead.
381	29:38	Kelly	She gives him … and then I know, yeah, there's like three
382		T/R	Ok
383		Kelly	Make up that.
384		T/R	Ok
385		Kelly	So then taking away a third. So then these two
386		T/R	That's what was taken away
387		Kelly	Yeah. So that's what she gives both of them. And then she has this one little part left which is one sixth.
388		T/R	Perfect. You're going to write all that down right?
389		Kelly	Yeah, I did.
390		T/R	Ok. Perfect
391		Kelly	Do I have to write it in words?
392		T/R	No, but you did draw the picture? Right?
393		Kelly	Yeah
394	30:04	T/R	Ok.
395	30:05	Kelly	See, that's what they took away and that's what they took away and that's what he has left
396		T/R	That's exactly right. Now there's only one thing I'd add to what you did which is … you converted a half to three-sixths and you converted a third to two-sixths. You can show me on here how you know that a half is three-sixths and a third is two-sixths.
397	30:24	Kelly	Ok. A half. Three equal one
398	30:34	T/R	Yup. Equals one of the halves, yup.

Line	Time	Speaker	Transcript
399		Kelly	So that's one-sixth of what she has left.
400		T/R	Alright, but here. This thing tells me right away that one-half equals three-sixths. This is the proof that three-sixths is the same as a half.
401		Kelly	Ok
402		T/R	Now the proof that two-sixths is the same as a third, you could do something similar. Right?
403		Kelly	One-third. Ok, there we go. No. Wait
404		T/R	Yeah, that's … that's a third. Show me that that's the same as two-sixths.
405		Kelly	Two-sixths.
406		T/R	Ok
407		Kelly	So, if you put these over here. See they're like the same size.
408		T/R	There you go. And we already knew that these were sixths because we already knew that six of these made one.
409		Kelly	Yeah
410		T/R	Ok. So that's the proof
411	31:29	T/R	Now, I think I want to go to a whole class discussion – uh you guys can keep videotaping – of number three because you guys did number three and had some big disagreements about it. And you started … you did number three also and had R….'s … that's ok
412		Erika	That's alright
413		T/R	… you had R….'s issue….
414		Sarah	Yeah
415		T/R	… and I'm not sure that we resolved it or not. And you're not one hundred percent happy with our resolution.
416		Kelly	[laughing] I love your phone
417		Erika	Please continue
418		T/R	Ok. So
419		Janelle	I figured out where my problem lied though
420		T/R	Ok. Now, before you do … you guys read problem three. Am I right K…? I'm not sure you saw it yet

Line	**Time**	**Speaker**	**Transcript**
421		Kelly	No
422		T/R	Alright so read problem three to yourself right now. Ok. Or you can read it out loud if you want
423		Kelly	Ok. John has one-half of a candy bar. Bill takes one-third of a candy bar from John. What portion of a candy bar does not … does John have left?
424		T/R	Ok. Now, let's go with …
425		Fae	The way I thought of it.
426		T/R	Go ahead
427		Fae	Sorry. I just want to explain one thing. The way I thought of it is, because of the wording where it says Bill takes one third of the candy bar **from** [emphasizes 'from'] John …. Because it says 'from John' and John only has half of it, I'm not thinking that John has any of the candy bars. I'm just thinking he has that one half.
428		T/R	Yup
429		Fae	That's why I came up with the one-sixth.
430		T/R	Ok
431		Janelle	And then .. yeah, and then I thought of it where you have ..it says you have **a** [emphasizes 'a'] candy bar
432		Erika	Yeah
433		Janelle	So John has half of **a** candy bar and Bill takes a third of **a** candy bar.
434		Erika	So its two separate candy bars
435		Janelle	If it was the same candy bar, it would be 'the'. John has half of **the** candy bar. Bill takes a third of the candy bar
436		Erika	..of **the**..[emphasizes the]
437		T/R	Ok, and so that was where your two-thirds answer came from
438	33:08	Erika	Yes
439		Janelle	Yeah
440		T/R	And that's sort of – F… – that's where your answer two-thirds came from
441		Sarah	Yeah
442		T/R	Alright now, but, what I wanted was a question that would

Line	Time	Speaker	Transcript
			end up in mathematical terms as one-half minus one-third equals one-sixth
443		Janelle	One-half …
444		T/R	… minus one-third equals one-sixth
445		Fae	Yeah
446		Erika	Yeah
447		Fae	Uh huh
448	33:28	T/R	That's what some of you got for that, but some of you really wanted to say one minus a third equals two-thirds. So because there's some ambiguity about candy bars and how do you know how big the candy bar is, I made some suggestions that I'm not going to repeat, but I want you guys to think about a question where they're going to write down one-half minus a third equals one-sixth and then they're … its not going to be ambiguous. It's not going to be, there's not going to be confusion as to what they're subtracting from what. You know what I'm saying? Like I … because … well you can't say … there's not alike a standard candy bar. People get confused. At least some students have gotten confused when you say a candy bar. So my suggestion was, can you say something else like a foot. Because a foot is always the same size. So if you have something half a foot, you know it's always six inches for example. Can you think of some other way to word a similar problem without using candy bars so we're absolutely positively sure you want to say a half minus a third.
449	34:26	Erika	To get a sixth
450		T/R	To get a sixth
451		Fae	You can just give a measurement to it. Like …
452		T/R	Ok, well. Just talk about it in your group. Ok. And then maybe each group can come up with something.
453	34:38	Fae	Like we could say, a boy ran six miles. So, six miles you know what that measurement is.
454	34:50	Sarah	Yeah
455		Fae	So it's always gonna be the six miles long
456		Sarah	Ok
457		Fae	But all the numbers and all the representations would be the same thing

Line	Time	Speaker	Transcript
458	35:18	Kelly	Why did we need the calculator?
459	35:20	Sarah	I don't know.
460	35:28	Kelly	It won't even turn on
461	35:40	T/R	Now everybody should listen to this so say it a little bit louder.
462		Janelle	Sure. So, I just redid the … the problem three and instead of a candy bar, I did flour. So John has a half a cup of flour. Bill takes a half a cup of flour from John
463		T/R	A third
464		Erika	You mean a third
465		Janelle	A third. Sorry. Yeah. A third of a cup of flour from John.
466		Erika	From John
467		Janelle	What portion of flour does John have left?
468		Erika	Oh, that's a lot easier to understand actually
469		T/R	Ok well first you have a cup of flour
470		Janelle	.. of a cup of flour
471		T/R	Is that? Is that easier?
472		Erika	Yeah that one was actually a lot easier
473		T/R	Does that make sense to you guys too?
474		Janelle	So it's one half
475		T/R	I could visual ... I mean yeah, I tried to use this with the Math114 class and I said, 'you know when you bake stuff' and they said 'we don't bake'. So
476		Erika	You don't even have to bake … well.
477		Fae	The reason why I like the candy bar deal is because these are rods
478		T/R	Yeah
479		Fae	So it's easier to understand …
480		Erika	So it's like the candy bar
481		Fae	… representing this as a candy … you know what I mean?
482		T/R	Yeah
483		Fae	…to break it up into the equal portions

Line	Time	Speaker	Transcript
484		T/R	Yeah. So you're ok with the problem as it was?
485		Fae	I mean …
486		Sarah	Yeah
487		T/R	But, how do you feel about R…..'s proposal?
488		Fae	That works.
489		T/R	Ok
490	36:36	Fae	Numerically that works. Visually, I feel like this works better.
491		T/R	Ok. And did you guys come up with any other wording that you were thinking about?
492		Fae	I was thinking … I don' know. I was gonna say like that ... they're running a … but. No, I don' t know. I was gonna say like they're running a 6 mile race but then how would Bill take anything from them. He's not taking anything.
493		Erika	Yeah
494		T/R	Well, let's see. Six miles
495		Darlene	Taking a lead… Maybe he's like running behind somebody
496		Fae	John has …
497		Janelle	But then you have to do speed
498		Darlene	Yeah
499		Erika	Yeah we're not going to worry about physics at the moment.
500		T/R	But that ... that might work. Let me think about this.
501		Fae	The easiest <inaudible>
502		T/R	Ok, I will think about it., but alright we can move on. And we've already answered some of the questions that I thought about which is … issues. Right? What kinds of issues are there? And, we didn't talk as a group, but I saw individually. In fact, I think I talked about it with your group but I didn't talk about it with your group. Go back to problem one. I think I saw it on your paper K….
503		Kelly	Yeah?
504		T/R	What mathematical sentence did you get for number one?
505	37:42	Kelly	Uhhh one-half minus one-third?
506		T/R	Yes. No.

Line	Time	Speaker	Transcript
507		Erika	You're supposed to have
508		T/R	One half minus one third?
509		Erika	It was you had a third a…
510		Janelle	half minus one third
511		T/R	Yeah
512		Kelly	Oh
513		Erika	…and they're taking a half.
514		T/R	Yeah
515		Kelly	Sorry. One-third minus one-half then.
516		T/R	Yeah. That was the …
517		Erika	But if you do that
518		T/R	Yeah. Right. Do that in your calculator.
519	38:00	Erika	Put that in the calculator. One third minus one half
520		T/R	And is that ... that isn't what you have?
521		Kelly	My calculator is dead.
522		Sarah	I did one third divided by two equals one third times one half equals one sixth.
523		T/R	Yeah. Um is this your calculator?
524		Sarah	Yeah.
525		T/R	Can you do fractions on this calculator? Because …
526		Erika	If you can't, I've got mine
527		T/R	What did you have?
528		Fae	This ones wrong.
529		T/R	Right
530		Fae	I did the one-third minus one-half.
531		T/R	Yes and when you do the one-third minus one-half tell us what you get
532		Erika	You did that too?
533		Fae	On the calculator I don' t know but, like I said with the visual representations, you get one sixth
534		Sarah	It's like. I think it's like one more sixth or something

Line	Time	Speaker	Transcript
535		T/R	Yeah but
536		Erika	Its …
537	38:33	Kelly	I did it wrong
538		T/R	What'd you get?
539		Kelly	A negative number
540		Erika	Yeah. That's right
541		T/R	You got a negative number. One-third minus one-half is a negative number.
542		Erika	Because a third is this size. A half is this size. You can't take more than what you got.
543		Kelly	Oh, yeah. Ok.
544	38:44	T/R	And, but. You guys got it too. And I saw F….. over here had it. You didn't subtract a half.
545		Erika	That's because R….. had it
546		T/R	What did you subtract? You did two
547		Darlene	Multiply
548		Sarah	Yeah I multiplied
549		T/R	You did a third times a half. She did a third times a half. Right? Half of a third means that you're going to multiply it by a half.
550		Sarah.	Yeah
551		T/R	And you got one sixth and that's the thing she subtracted.
552		Fae	Oh.
553		T/R	So the first question was how much did she give away. She gave away half of it which was one-sixth. And the second question. What did she have left? Well that just also happened to be one-sixth but it might not necessarily have been one sixth.
554	39:16	Erika	We … umm she has
555		Janelle	So we did it with x's.
556		T/R	Ok, explain your x's.
557		Janelle	So we had um… you know its for, so for what Paul was getting, she had one-third of x which is the candy bar.

Line	Time	Speaker	Transcript
558		T/R	Ok
559		Janelle	So she had one third x and then multiply it by one half to get what Paul did. And to figure out what she had left, what she had, which was one-third x minus what she gave to Paul equals what she has left.
560		T/R	Ok. I can, I can deal with that. It seems to me that x is going to end up as one though because it's the candy bar.
561	39:44	Janelle	Yeah, it's always gonna be one
562		Erika	Well yeah yeah yeah
563		T/R	So sure. Now, but I would think probably, you know, fourth graders or whatever aren't going to do the x part.
564		Erika	Oh no no no.
565		Janelle	No
566		T/R	But you guys can
567		Erika	Just to take out the x's
568		Janelle	It's the same thing
569		Erika	It's the same thing
570	39:59	T/R	Ok. So ... So we discussed .. ok, issues and the issues, the biggest issues that you guys have tend not to be the issues that students have because they don't know the algorithm. So they don't just jump right in and say one-third minus a half. They just fiddle with these things. Um and I'm also
571		Fae	That was sort of the way I worked.
572		T/R	Right yeah right. There you go. So you can relate. Um, and I think that's a good way. That reminds me of a lecture some of you have heard before. People tend to think that manipulatives are for small children and people who are in remedial developmental or um
573		Janelle	No, I don't think that No. We're talking about …
574		Erika	Definitely not. <inaudible>
575		Fae	I'm a visual learner. Things like this help me
576		Erika	Yeah
577		T/R	And picking things up, some people are tactile. You know?
578		Erika	Yeah I'm one of those that has to do it in order to learn it

Line	Time	Speaker	Transcript
579	40:47	T/R	Yeah. And, in fact there's this great quote that I wanted to use when I was writing a paper and it turns out somebody had already used it … by the famous physicist that none of my other students has ever heard of named Richard Feinman.
580		Janelle	Yeah!
581		T/R	You've heard of him?
582		Janelle	I've heard of him
583		Erika	There ya go. You got one!
584		Janelle	I don't know what he did , but …
585	41:04	T/R	He was the physicist who won the Nobel prize for Physics and he was a very um unusual physicist. He came from Brooklyn and he talked like he came from Brooklyn. (laughter) He also um, some of you may have heard of him. You're too young for this to… the Challenger that exploded, the um thing that exploded.
586		Darlene	That's where I heard it from.
587		Janelle	Mhmm
588		Darlene	Sanford used to talk always talk about it
589		T/R	How long ago was that?
590		Janelle	He's the one that found out why
591		Darlene	Yeah
592		T/R	He's the one that found out about the o-rings and he dipped it in ice water.
593		Darlene	Yeah
594		T/R	They were prepared to sort of…
595		Erika	Sanford talked about this
596		T/R	They were prepared to sort of accept the fact that well it was just one of those things and he went and did the experiment that … it was too cold that day. The o-ring froze.
597		Erika	And then he.. he couldn't tell anybody so he had his friend come look at the car and be like "look! Look what happened!"
598		Kelly	Oh, I remember the. I think um, Dr. Sanford said that
599		Erika	Yeah, he told us about this
600		Janelle	Yeah

Line	Time	Speaker	Transcript
601		T/R	Anyway. Finally, so we admire him because he was a really top guy, plus he was a little weird which we also admire.
602		Kelly	A little weird.
603	41:58	T/R	… and he had this quote that says how he figures things out and its all visualizing things. When he talks about sets, here's a set. He says I think about a ball. Disjoint set. I think about two balls. You know and then you know you talk about it has all these properties and I think about you know he keeps imagining what these two balls look like and then somebody says and therefore here's the conclusion based on the experiments <inaudible> based on the equation and he says no that can't be because it's not true of my fuzzy balls with whatever. So, he was a totally visual learner and he won the Nobel prize in physics. He was really good in Math. So the point is … you can do these anytime and it's not a remedial thing and it's not something that's only for people who have trouble learning. Now it's not necessarily for everybody. I mean we know some members of the math department who don't think this way. But we know some members of the math department who do. Like for example, me. Ok. So. Alright so you're all on board with that.
604		Fae	Mullner always has manipulatives I feel like he's such a visual learner as well as educator.
605		T/R	Yes. So he's really into that kind of stuff like I am too
606		Fae	Yes.
607		T/R	More so probably. He's a little bit better at ...a lot better at relating it to the theoretical. Ok. So. Here's some ideas for 'does this help'. Ok The idea is half of what you have as opposed to half of a candy bar. So how are you going to model these kinds of questions which have whole numbers mixed in with fractions?
608	43:29	Fae	Use six pieces individually. Half of six
609	43:36	T/R	And you have to write the sentences for that too. The number sentences. The equations.
610		Janelle	You don't even have to use green ones.
611		Erika	I just like using green because we've been using them the whole time.
612		Janelle	Are we writing this down too?
613		T/R	Yeah

Line	Time	Speaker	Transcript
614	44:57	T/R	Ok
615		Kelly	So half
616	45:03	Fae	So its six divided by two
617		T/R	Ok. So. A perfect representation. And where are you at? I didn't see what you were doing. Ok
618		Fae	Plus six. So Paul has
619		Sarah	Oh this is three!
620		T/R	Yeah right! That's what I was staring at. Where'd that come from? Ok. And so you're on the next one.
621		Kelly	John has five.
622	45:40		Ok. You got your answers, you got your pictures, you're done.
623			[break]
624	45:45	T/R	You tell me.
625		Sarah	I know each of these are three.
626		T/R	Ok
627		Sarah	And that this whole thing equals fifteen
628		T/R	That's perfect. Ok.
629		Sarah	That's it?
630		T/R	That's it. I think. Now um.
631	46:04	T/R	Ok. Because I did want to have the whole class talk and I want to have F….. over here talk for a minute about her model. Five candy bars …F... So, you wrote it as fifteen-thirds.
632		Sarah	Yeah
633		T/R	So I asked for a model that shows that the five candy bars are equal to fifteen-thirds. So. Explain the model.
634		Sarah	So I just did …I just did um five greens and then you know that each green is equivalent to three white so you get fifteen whites
635		T/R	Ok
636		Erika	Light greens
637		Sarah	So yeah. So one fifth ... one fifth of the green would be

Line	Time	Speaker	Transcript
638		T/R	One third of the green
639		Sarah	Wait one third?
640		T/R	One third of the candy bar
641		Sarah	Oh yeah, yeah, yeah.
642		T/R	Ok
643		Sarah	So, it would be three whites
644		T/R	Right so one-third. So, you got fifteen thirds and you're taking away one third.
645		Sarah	Yeah.
646	46:46	T/R	So take away the one third and you've got … No you're not …
647		Erika	[disagrees]
648		Sarah	No you would take
649		T/R	You're taking away
650		Erika	One third
651		T/R	… the white ones are thirds. So you're taking away one-third
652		Darlene	The white thing
653		Erika	Yeah
654		Darlene	The white
655		T/R	Just the white
656		Janelle	Just one white one
657		Fae	Yeah
658		T/R	Yeah, now.
659		Sarah	Oh a third of this. Sorry
660		T/R	Yeah. And your equation says your answer is…
661		Sarah	Fourteen-thirds
662		T/R	And there's her fourteen-thirds. Now notice how that' often how they tell you …I don't know how you were taught but a lot of times they tell you when you're adding and subtracting fractions you make the whole thing improper fractions.
663		Erika	Oh yeah. Yeah.

Line	Time	Speaker	Transcript
664		Darlene	Yeah
665		Janelle	Right
666		T/R	Which I hate because it's a lot of extra work right? I mean it's true that that's fourteen thirds, but for example if you talk about your model … what did you guys do?
667		Erika	We just set up five of them and for one of the candy bars we have three red ones because three red ones make up a green one ... uh dark green one. So then we did takes away one third, we moved the red. You take away the one red. And take the green and replace it with the two reds that are left. So we went one, two, three, four, and two thirds.
668	47:53	T/R	Which is the same as your answer when you convert it back …
669	47:55	Sarah	Yeah
670		T/R	… but you sort of… you did an extra step your way… which isn't wrong but its an extra step
671	47:59	Sarah	Yeah I did four and two thirds but
672		T/R	Yeah but you had that other model …
673		Sarah	Yeah
674		T/R	…that I wanted you to show.
675		Janelle	It's not fourteen thirds though. It's fourteen fifteenths.
676		T/R	Is it? What's one?
677		Erika	One of them is ... I thought was one third. Oh the ...
678		T/R	No but what represents one in her model?
679		Erika	The green
680		Janelle	The whole … one green thing
681		T/R	One green thing represents the number one
682		Janelle	[agrees]
683		Erika	Yeah
684		T/R	And so the white thing represents
685		Erika	A third … because there's three of them
686		Sarah	One fifteenth
687		T/R	I know you said a third but I want to hear what R… is saying

Line	Time	Speaker	Transcript
688		Janelle	So it represents a third
689		T/R	Ok so she's got fourteen … what color? Fourteen of what color do you have there?
690		Sarah	White
691		T/R	Fourteen whites.. And a white is what fraction?
692		Janelle	A third
693		T/R	So she's got. Why is it fourteen fifteenths then?
694		Janelle	Fourteen-thirds … but then … this is for number two right?
695		Erika	Yeah
696		T/R	Yes
697		Janelle	So … yeah, you're right. I just didn't do my fraction right.
698	48:57	T/R	You're - Well everybody should see
699		Janelle	Yeah
700		T/R	… you're right that it's fourteen-fifteenths if the whole thing is one
701		Janelle	If the whole thing was one
702		Erika	Yeah
703		T/R	See that's why you've got to be careful.
704		Janelle	Yeah
705		T/R	What's one?… and your one doesn't change, you know, throughout the problem .. which is another issue little kids don't necessarily have but people, you know, your age – in math 114 – will have an issue. One keeps changing. Right. And they'll see something like that and they'll think it's a different kind of fraction.
706		Fae	I considered the whole total of five bars four plus three thirds.
707		T/R	Yeah that's a good thing actually… that's sort of what they did here.
708		Erika	That's …yeah … that's … basically ... Yeah that's basically what we did here. Because we just lined them up
709		Fae	And then I converted into the fifteen thirds which would equal up to the five bars and fifteen thirds minus one third is fourteen thirds ... and then converted into four and two-thirds.

Line	Time	Speaker	Transcript
710		Erika	Oh so you …
711		T/R	Oh, if you did all ... you didn't have to do that much. Right? You could have just taken away that one-third the way they did. Converted the one to three thirds, take away one third and you have two thirds left and that's your answer
712		Erika	… that's what she did. Which is really smart. We did the same thing we just put this here and put this next to it, so…
713	49:56	Kelly	Yeah. I just took all of them and just broke them into three parts and then added three. And then when it got to the last one took away one …
714	50:02	T/R	Ok
715	50:02	Kelly	… so I get two. So I added three plus three plus three plus two
716		T/R	Plus one more three
717	50:08	Fae	Which is fourteen thirds
718		Kelly	Yeah
719		Erika	Over three. Yeah. All over three
720		T/R	Yup. Right. So … yeah and notice so we've had at least four different ways among six people that this problem was done and you all got the right answer. Right? So, another thing to remember, the things that we've talked about is there's not only one right way to do it and you definitely don't want your students to come away thinking that there's only one right way to do things. Now, you have to understand what's going on so you can tell whether what they're doing is right. But, there's no reason to insist that they only do it one way. Ok, and I have a couple more and that's …
721	50:47	Kelly	Homework!
722		T/R	Homework. Yes. Homework. We've got ten minutes left and this is the start of a homework. And these are similar, similar to what we've been doing. Just, make some models. Do some um number equations. I think this should be enough for all of you
723		Fae	One more
724		T/R	One more. Ok
725		Erika	A fourth, a third and a sixth
726		Janelle	Can I steal some white one's back?

Line	Time	Speaker	Transcript
727		Erika	A sixth
728		Sarah	Yea
729		Janelle	Thanks.
730		Fae	Nooo
731		Erika	A third
732		Fae	Give them back!
733	51: 57	Kelly	Are they talking about the whole pie?
734		Fae	They each have their own individual pie. That's the way I get it.
735		Sarah	Yeah. Oh. Really?
736		Fae	I think so.
737		Kelly	I have a question.
738		T/R	Yes
739		Kelly	You know how they say that they send out for pizza?
740		T/R	Yeah.
741		Kelly	That's like the whole pie, the pizza?
742		T/R	That's right.
743		Kelly	Like six slices
744		Fae	So, Mary had one-fourth of one pie?
745		T/R	However, we're pretending that the slices can be sliced up any way … I mean they're not always eight slices is what I'm pretending here. So she had a fourth of a pizza left over so it sounds like they cut theirs in fourths right? So. Well that makes more sense. And a third of a pizza, well maybe it was a personal pizza, you know … where they cut it themselves.
746		Fae	That's what I'm thinking. Is it three individual pizzas?
747		T/R	Three pizzas. But then each got leftovers.
748		Fae	Right.
749		T/R	So the idea is they're putting together all their little fractional pieces of pizza and they're seeing what it adds up to.
750		Sarah	So you, what <inaudible>
751		T/R	Yes, you would but I want to see a model for it.
752		Sarah	Its a lot easier just doing it. You know?

Line	Time	Speaker	Transcript
753		T/R	Yes, but
754		Fae	See and for me I'm better with <inaudible>
755		T/R	… if you want to be a teacher you have to know both ways. And you'll also have to, you know, realize that it's easier for some people to do it the way that's hard for you.
756	53:07	Fae	Right.
757		T/R	You know? So…
758		Fae	Mary has one-fourth, so this is fourths. Lisa had one third. one sixth.
759	53:48	T/R	Ok. You can do a model for that or you can do a model for something else maybe.
760		Kelly	Can I … can I reduce this
761		T/R	Yes, you can but, models for everything
762		Kelly	Ok, so I have to do a model. Um, Um I need orange.
763		T/R	Ok, here's orange.
764		Kelly	[laughing]
765		Sarah	Oh my god
766		Fae	What does it say? What'd you do?
767		Kelly	It says hey what up playa
768		T/R	ok
769		Kelly	How many is blue? Nine?
770		T/R	Uh, yeah I think so yeah. Hold it up to the orange. Yeah. Nine.
771		Kelly	Ok. I'm done
772		Sarah	I did twelve …
773		T/R	Ok
774		Sarah	… because I know that <inaudible> thing is twelve.
775		T/R	Yeah… that's … that's really good insight, so
776		Sarah	Yeah
777		T/R	So, hold on to that. Now. Alright. So that's your one. Now. Ok, so. Alright. So you can go from there. Mary has uh …
778		Fae	She ate all these

Line	Time	Speaker	Transcript
779		T/R	… whatever she has. What does she have? A fourth?
780		Sarah	One, one fourth. So do I take that out? Or do I …
781		T/R	Well you… put it somewhere. You're going to take something representing what she's got …
782		Sarah	Ok. So, that's one fourth
783	55:29	T/R	I'm not sure that's one fourth.
784		Sarah	Oh! Wait, ok.
785		T/R	Ok that's one fourth. Ok, and I can believe that because if you take four of those groups …
786		Sarah	Yeah yeah yeah
787		T/R	Ok… and then the one-third is how much?
788		Sarah	One third of this whole thing? Or is it one-third of what's left?
789		T/R	One third of the whole thing. No remember …
790		Fae	They all have their own individual pies
791		T/R	They just … she had her own pizza that she started with
792		Sarah	Ok so that's four
793		T/R	Ok. And then the last person had one-sixth of her own pizza
794		Sarah	So that's two
795		T/R	Ok. So you put them all together and you get how many white ones?
796		Sarah	Nine
797		T/R	Nine white ones. What fraction is nine white?
798		Sarah	Nine-twelfths
799		T/R	And that's what you got over here right?
800		Sarah	Yeah
801		T/R	And then you're going to reduce that
802		Sarah	Yeah so that's …
803		T/R	And then you can also show me …
804		Sarah	So that's three-fourths
805		T/R	Yeah. Now you can put all those nine together and see if you can show me that equals three-fourths by finding a rod that's

Line	Time	Speaker	Transcript
			one-fourth. You know? And then lining it up.
806	56:21	Sarah	Ok.
807		Kelly	Ok
808		T/R	Yes? What've you got?
809		Kelly	That's my eighteen twenty-fourths
810	56:29	T/R	Ok. Yeah. You've got to prove that's the answer though. But that looks really nice. Ok. So you're going to start out saying that Mary has one-fourth of a pizza. So if this is the whole pizza, what's one-fourth of that?
811		Kelly	Um. Good question.
812		T/R	Try that one.
813		Kelly	As the whole pizza?
814		T/R	Try that so, see if that's a fourth.
815		Kelly	Oh. No. Oh
816		T/R	You have to line it up until the whole pizza right?
817		Kelly	Yeah
818		T/R	So this is one fourth.
819		Kelly	Ok
820		T/R	Do you agree with that?
821		Kelly	Yeah
822		T/R	So that's what Mary has.
823		Kelly	Oh
824		T/R	There's Mary's.
825		Kelly	Ok.
826		T/R	Now, Lisa has one-third. Find a different color that's one third of your whole thing.
827		Kelly	Ok. Ummm
828		T/R	And where are you at while she's doing that?
829		Fae	I split one pie into fourths, one into thirds and one into sixths
830		T/R	Ok. And now you're going to line all these up next to each other and see how long it is
831		Fae	Well. There...

Line	Time	Speaker	Transcript
832		T/R	Well no but if they put them all together in the same pizza plate …
833		Fae	Oh, adding it?
834		T/R	Yeah
835		Fae	All together?
836		T/R	Yeah. You're going to add them all up.
837		Fae	Ok
838		Kelly	Wait, what am I finding? One .. one third of it?
839		T/R	Well you already found … what did you … you already found a fourth and now you're finding a third.
840		Fae	One fourth
841		Kelly	Well that's <inaudible> almost have a whole pie.
842		T/R	You're right. So. Almost but not quite
843		Kelly	Brown?
844	58:23	Fae	What are we trying to figure out? How much of the pie they have left altogether?
845		T/R	Yeah, when you put them all together.
846		Fae	One-fourth, one-third plus one-sixth
847		T/R	Yep
848		Kelly	I think the blue is gonna confuse me. I'm taking it out.
849		T/R	Ok. So this is a fourth. And what's the brown?
850		Kelly	That's a third.
851		T/R	Alright. So you've got a fourth, and you're putting it together with a third. Ok. So, put them together.
852		Kelly	What? These two?
853		T/R	No. Well, a fourth and a third. Pick up a fourth.
854		Kelly	Pick up a fourth. And pick up a third.
855		T/R	Yeah. There they are. Together. Now you have to put in …
856	59:00	Kelly	… a sixth.
857		T/R	A sixth. So what what length is a sixth?
858		Fae	I got it!
859		T/R	You got it?

Line	Time	Speaker	Transcript
860		Fae	One-twelfth, and then
861	59:09	T/R	Ok
862		Fae	I didn't mean to write one equals one-fourth because … it multiplies if you divide by three
863		T/R	It doesn't equal one-fourth
864		Kelly	One, two, three
865		Fae	Its one fourth left
866		Kelly	Four
867		T/R	Nine … I don't like the way you wrote that.
868		Fae	I don't know why I divided by three
869		T/R	That's nine-twelfths divided by three is one-fourth, but you're just reducing it. Nine-twelfths is not equal to one fourth. You reduced it. What did you get?
870		Sarah	Three-fourths
871		T/R	Three-fourths. Divide both of them by three
872	59:36	Fae	Oh! How much did they eat? They ate three fourths of the pizza?
873		T/R	No. No, that's how much
874		Fae	I mean how much they have left over is three fourths of the pizza
875		T/R	Yes. They put all their pieces together and they get almost the whole pizza left.
876		Fae	Alright
877		T/R	Ok.
878		Fae	I was saying what was missing.
879		T/R	Yes. And what do you have for your … let's see. What, you were going to show me three-fourths
880		Sarah	Yeah, like I know that all these equal… all these whites equal up to twelve
881		T/R	Yeah
882		Sarah	And I know that three of them equals one-fourth
883	1:00:04	T/R	Yeah
884		Sarah	Like three of them add up

Line	Time	Speaker	Transcript
885		T/R	Yeah. And here's what you want to do I think. See if those make one fourth. Ok
886		Kelly	I have a question
887		T/R	Ok. Ok … just about done. Yes. Ok
888		Kelly	Is this right?
889		T/R	You're talking to me right? You showed me this was a fourth and you showed me this was a third.
890		Kelly	… and that's a sixth.
891		T/R	and how do you know that's a sixth?
892	1:00:27	Kelly	Because here's all the … the one, two, three, four, five. And then if you put another six over here.
893		T/R	Ok. Ok
894		Kelly	And then I took a sixth out
895		T/R	And you told me that equaled eighteen twenty-fourths.
896		Kelly	Yeah
897		T/R	This is nine and this is nine. Does that equal eighteen?
898		Kelly	Yeah
899		T/R	It sure does. There's your proof. You've got to write it all up.
900		Kelly	Ok
901		T/R	…or draw a diagram or something. Ok. Its time. You guys can stop

Transcript 6 of 6
Date: 04/15/2011
Length: 01:03:21
Camera 2
Transcribed by: Deidre Richardson
Verified by: Mary Huizenga

Line	**Time**	**Speaker**	**Transcript**
1		Janelle	… two raised to the first power, then its two raised to the first power plus two raised to the first power which is two raised to the second power which is four
2		T/R	Ok…
3		Janelle	So then, you just .. each block is one of those so you go all the way up and your last one is ten fifty-four so even though
4		T/R	Ten twenty-four
5		Janelle	Ten twenty-four. So even though this doesn't look like half of this, it really is half of that.
6	00:19	T/R	Ah. Ok, I see what you're saying.
7		Fae	Interesting
8		T/R	So its… and that is exactly what you proposed Jess actually
9	00:24	Erika	Yeah.
10		T/R	independently
11	00:25	Erika	Oh, well except for I also have this block
12	00:28	T/R	Right you have the …
13		Janelle	Yeah
14		T/R	… two to the zero block
15		Erika	Yeah well I'm just saying yours is red…
16	00:32	T/R	Ok. Right
17		Erika	… mine was …
18		Janelle	Well I did all different colors. I didn't do the same colors as that because this is a different set of
19		Erika	Oh, ok. Yeah, so yeah you did the same thing as me then
20		T/R	So you're saying you can make half of any number with this?
21		Janelle	Yeah. It just does … isn't a visual repre-… proper visual representation

Line	Time	Speaker	Transcript
22		Fae	It's a numerical representation
23		Erika	Why not?
24		T/R	How do you make
25		Janelle	Because half of this is that
26		T/R	Well what's half ..
27		Janelle	… but it doesn't …
28		T/R	What's one half of this?
29		Janelle	You would have to have, I don't know, you would just have to start it the other way. I don't know.
30		T/R	Oh. Yeah I can see that half of each one is the previous one, but I don't see a half of this one.
31		Erika	Yeah but we never did a half of this one before.
32		Janelle	Yeah
33		T/R	That's right! And that's why we couldn't do half of blue. So we were trying to do something so that we could do something the equivalent of half of blue.
34	01:16	Erika	Oh see mine is purely um mathematical.
35		T/R	Ok
36		Erika	Like mine …
37	01:21	Janelle	I don't know
38		Erika	…mine also includes
39		Janelle	Well mine's wrong
40		Erika	…they can do fractions …
41		T/R	Not wrong. We won't say wrong. We say … So you tell me … so you tell us what the issue … what yours is
42	01:34	Erika	Mine was that this would be two to the zero.
43		T/R	Ok. That's one
44		Erika	So that's one. And that's two to the first, which is two. But then this is two to the second.
45		T/R	Ok
46		Erika	Which is half.
47		Janelle	So what's green?

Line	Time	Speaker	Transcript
48		Erika	There is no green. We're making up completely new ones.
49		T/R	Ok, but then
50		Erika	My, my thing took out the odds.
51		T/R	Ok so you're saying the next one, whatever color it is, is twice as long as this one.
52		Erika	Yeah. Um
53		Janelle	So you don't have … you don't go up by one block each time. You go up by two blocks each time
54		Jaime	Here's a brown
55		Erika	Well..
56		T/R	So you're missing
57		Erika	Not two each time
58		T/R	It represents …
59		Erika	Because it was … I went up …
60		Jaime	One
61		Erika	… one here
62		Jaime	Two
63		Erika	… two here, three here
64		Janelle	Yeah
65		Erika	Four here. Not three here. Four here.
66		Janelle	Yeah
67	02:22	T/R	You're doubling it.
68		Erika	Yeah, well basically, yeah.
69		T/R	Ok. So you're doubling the size along with the number and you're doubling the number but not the size
70		Janelle	Yeah.
71		T/R	Correct?
72		Janelle	But I'm not doubling ..well.. tech…yeah doubling technically
73		T/R	Well you're doubling the number that goes with these
74		Janelle	Yeah
75		T/R	Ok. I have the same question for you. What's half of this

Line	Time	Speaker	Transcript
			one?
76		Erika	Well theoretically it'd be two to the negative one, but you don't have a block for it
77		T/R	Ok. You don't have a block for it. But, remember that was the deal. If I - I'm supposed to be able to point to any block and you're supposed to tell me what half of it is.
78		Erika	Well you can only do half for so long. There's infinite ..
79		Janelle	Which means you have to start with zero
80		Erika	… infinite amount of numbers.
81	03:01	T/R	Well, ok. If you start with zero does that work? And first off, so I have your two and you had one also. What
82		Kelly	That was my idea. The one that I handed in
83		Erika	Start with zero
84		T/R	Umm, I can't remember what yours was. It was on the second page.
85		Fae	I don't know what I did
86	03:17	T/R	Ok. So you're saying. What are you saying? You made two browns equal to one
87		Fae	Right.
88		T/R	Ok.
89		Fae	So that each of those can have... I don't know.
90		Erika	See, we … we couldn't really start with zero. Because you can't …
91		Janelle	Yeah
92		Erika	…show…
93		Janelle	Zero blocks
94		Erika	…zero. That's the problem. You can't show zero blocks.
95		Fae	And this would be one sixteenth of that.
96		T/R	Ok.
97		Fae	I don't know where I was going with this.
98		Janelle	But then how do you find half of that one?
99		Erika	But then … half of the one sixteenth?
100		T/R	Ah. So there's the question.

Line	Time	Speaker	Transcript
101		Fae	I don't …
102		Erika	It's infinite. There's no way to just find half of everything because you go on into infinity. It's either positive or negative
103	03:58	T/R	So what's your … So what's your answer to the question?
104	04:01	Jaime	No
105	04:01	T/R	Create a rod set …
106	04:02	Janelle	It is impossible.
107	04:03	Erika	There is no way to. There's no way to do it.
108	04:03	T/R	In fact, what was the question? Can you create a set of rods…
109		Jaime	No
110		T/R	… so that everything has a half?
111	04:08	Fae	Yeah
112	04:08	Janelle	No
113	04:08	Erika	Not physically, no.
114		T/R	Not physically
115		Erika	Theoretically you can.
116		T/R	Theoretically, yes. Ok.
117	04:15	Erika	That's all I got.
118		T/R	Alright. Now I didn't hear from you two guys. Did you make it … or did you have a rod set … or do you have a discussion
119	04:20	Kelly	I couldn't figure it out. I was sitting there and we were talking about it. I was like, I
120	04:24	Erika	And that's when I told her my idea.
121		T/R	Ok
122	04:26	Sarah.	I attempted one.
123		T/R	Ok
124		Sarah.	I just made. I used the same colors and I made white two and then I went, red is four and then six, eight. Like that's what I did all the way up to twenty.
125		T/R	Ok

Line	Time	Speaker	Transcript
126		Erika	<inaudible>
127		Janelle	That's what I tried first
128		T/R	So you have two sort of similar. Ok
129		Janelle	But like fourteen, what's seven?
130		Erika	Yeah, you don't have
131		Janelle	What's half of fourteen. How do you represent seven?
132		Erika	Yeah. Yeah, that's the only problem is you have ...
133		T/R	So there … So you found a counterexample and I think you had the right idea. No matter what you pick, I'll pick the smallest one and say 'gimme half of that' and you don't have it. And what you said is, well I'll make half of that. But then I'm going to need half of that next little one.
134	05:01	Erika	Exactly
135		T/R	Ok, so the answer is? Everybody agrees on the answer?
136		Erika	There's no way to do it.
137		T/R	Can't do it
138		Fae.	Not physically
139		T/R	Not physically
140		Erika	Not physically. We're like-minded
141		T/R	Ok. But theoretically
142		Erika	Theoretically. Yes
143		T/R	Theoretically you could just keep going all the way down. And you're right when you keep going half all the way down … you've done limits right? - at least those of us who've done calculus, you can go down … down to zero. Like you said you want to get to zero. Ok so that was that trick question. Can't be done.
144	05:27	Erika	But we all tried.
145		Janelle	Yeah
146		Erika	All of us tried
147		Janelle	You have no idea how how I went through every number until twenty.
148		T/R	So you came up, but you did come up with a nice idea because you both came up with the idea that you can

Line	Time	Speaker	Transcript
			represent any number with those powers of two. Um, but fractions are an issue. Ok. So we're just going to do some fraction activities.
149	05:45	Kelly	Yayy!
150	05:45	T/R	I have some papers somewhere that have this exact problem on it. These exact problems and we are going to work on some other problems too, and what we're gonna do is, in your groups you're going to work on them and you're also going to think about the kinds of issues that people have. Kids have or other people have when they work on these kinds of problems. So you can look at those … on ones on the sheet …on the board while I look up the sheets. So your group – you can take this group and you can take this group as they start working on these problems and the bottom of the sheet has the things which is not on here … which is …
151	06:22	Erika	I think it is this …<inaudible> the candy bar is blue. She has one-third of the candy bar.
152		Jaime	Right
153		Erika	But you can't … she gives half of what she has, so we can't use these. What's uh… one up from these? Purple?
154		Janelle	Here, do it like this
155		Erika	Oh! But wait! A candy bar is like a train. A train doesn't have to be one color.
156		Fae	I like J… idea since she's talking out loud.
157		Erika	I always talk out loud. I always talk real loud
158		Janelle	So she has
159		Jaime	Um, what is it? One third
160		Erika	OH! Oh ok.
161		Janelle	She has a third of a candy bar.
162		Erika	She gives half …
163		Janelle	She gives half …
164		Erika	…of what she has …
165		Janelle	… has to Paul. How much does she give to Paul? One-sixth
166		Jaime	[agrees]
167		Erika	And then she has one-sixth left.

Line	Time	Speaker	Transcript
168		Janelle	And then she has one-sixth.
169		Jaime	Yea
170		Erika	Ok
171		T/R	Ok
172		Janelle	Yep
173	07:16	Erika	Ok, so. Second one.
174		T/R	So, you modeled it with the …
175		Erika	The green is the regular size
176		Janelle	The green is the whole candy bar.
177		T/R	Green is one.
178		Janelle	Yeah. The green is the whole candy bar. So, she has …
179		T/R	Not too loud
180		Janelle	Sorry. Sorry
181		Erika	Yeah. Yeah. You put …
182		Janelle	She has … she has one-third of the candy bar … and she gave half to Paul and she has half.
183		T/R	Ok
184		Erika	Of that …of what she started with
185		Janelle	Of one-sixth
186	07:39	T/R	Ok everybody agrees with that? You like that model?
187		Erika	Yeah. I like that one.
188		Jaime	Yeah
189	07:41	T/R	Ok. Write the number sentence. Right? The equation that goes with that model.
190		Jaime	Sure.
191		T/R	You're gonna write that down?
192		Erika	yep
193		T/R	Ok. So, everybody should write that down. Ok.
194		Janelle	Do you have …Oh, you don't have it printed out.
195		T/R	I have them and I think I'm going to have to run downstairs at some point because I don't have all the problems.

Line	Time	Speaker	Transcript
196		Erika	Ok
197		Janelle	So, you want us to just write the equation?
198		T/R	Yes, well draw … write what you did here …
199		Janelle	Ok
200		T/R	… and also write the equation.
201		Erika	So, we're doing the example and the mathematical?
202		Jaime	Yeah. So, are you writing it down first?
203	08:17	Janelle	I'm just drawing the picture.
204		Jaime	Ok
205		Erika	That's what I was gonna do. And what each one of them equals.
206		Janelle	Oh man. I did it wrong.
207		Erika	[laughing] number one for fraction activities. So, you've got … my thirds aren't equal. Alright, so this is one. That's one-third. That's one-half. So, she starts with one-third but then from there she takes one-half away which equals, they each have one-sixth. Ok. So. Yeah. You have the same thing?
208		Janelle	She has one-third of a candy bar and gives half of it away
209		Erika	Oh. See, I just did it as one-third minus one-half.
210		Janelle	It's the same thing
211		Jaime	Yeah
212		Erika	Either way you get the same answer
213		Jaime	Yeah
214		Janelle	Yeah
215		Jaime	So are we supposed to start the next …
216		Erika	Oh yeah, because just with yours x is the candy bar
217		Jaime	Yeah …. She has a candy bar … she gives half … half of the bar to Pablo and
218		Erika	... and a third to
219		Jaime	Ok, um. I'm trying to think
220		Erika	Well, we need to be able to do thirds with it and halves with it
221	10:11	Jaime	So, the one that has six. Right? Can't you do the one that has

Line	Time	Speaker	Transcript
			six?
222		Janelle	No. Yeah.
223		Erika	Sixths.
224		Janelle	Yeah
225		Erika	You need something with sixths.
226		Jaime	Yeah. So, which one was that?
227		Erika	Um, we can use
228		Janelle	The green one again
229		Erika	The light green?
230		Janelle	Like that.
231		Erika	But we need to be able to do
232		Jaime	Yeah
233		Janelle	She has the candy bar.
234		Jaime	Yup
235		Janelle	She gives half to Pablo
236		Erika	She needs to do sixths.
237		Janelle	Yeah. But you can
238		Erika	Sixths.
239		Janelle	This
240		Jaime	Yeah. Then put these there.
241		Janelle	Yeah
242	10:40	Erika	Oh. The whites are gonna be the sixths.
243		Janelle	Yeah. So, she has the candy bar.
244		Erika	The green is the candy bar still?
245		Janelle	Yeah. The dark green
246		Erika	Ok.
247		Janelle	The light green is halves.
248		Jaime	Yep
249		Erika	Halves. And the whites are the sixths?
250		Janelle	The sixths.
251		Jaime	Yeah

Line	Time	Speaker	Transcript
252		Erika	Ok. So
253		Jaime	So. Wait, ok. She has a candy bar. She gives half to Pablo and a third to Gordon.
254		Erika	And what's the question? What portion of the candy bar does she have left?
255		Jaime	…the candy bar does she have left?
256		Janelle	One-sixth
257		Erika	See, that's why you have to keep this here.
258		Jaime	Yeah
259		Erika	Because you go one-third
260		Jaime	Yeah
261		Erika	Two of these little white ones
262		Jaime	Yeah
263		Erika	So, what does she have left? A sixth. That's why you have to use one …
264		Jaime	yeah
265		Erika	Thing and just have them equal. You know what I mean right?
266		Jaime	Yeah
267		Erika	Yeah. Alright so. This is dark green. This was light green. This was white. Is that what the first one was?
268		Jaime	Dark green and red.
269		Erika	Light green. dark
270		Jaime	Red and white
271		Erika	Red and white. Alright, so for this one,
272	11:56	T/R	Anybody here have a calculator?
273		Jaime	I do
274		Erika	I do.
275		T/R	Ok.
276		Jaime	Yeah. I do too.
277		T/R	That does fractions?
278		Jaime	Yeah

Line	Time	Speaker	Transcript
279		Erika	Yeah
280		T/R	Ok
281		Jaime	Do you want it?
282		T/R	No, but I'm going to want you to use it.
283		Jaime	Oh
284		Erika	Alright
285		Jaime	Dark green. Light green
286		Erika	That was halves. Right? Light green
287		Jaime	Yeah.
288	12:20	T/R	Ok. I'll ask you to explain that. Ok?
289		Jaime	And reds …
290		Erika	… was thirds?
291		Jaime	Um hum. And then white is one-sixth
292		Erika	Sixths. Oops. I'm going to take somebody's eye out doing that one day.
293		Jaime	So, this is …
294		Erika	You have one
295		Jaime	… one
296		Erika	And then you give away half
297		Jaime	Minus one half
298		Erika	And then you give away a third
299	12:53	Jaime	Minus one third. So. Take away the half. So, you want to keep this here. Right? And we're just working with these to do it again.
300	13:02	Erika	Yeah
301	13:03	Jaime	Alright
302	13:04	Erika	So, but all we're gonna do is take away the whites …
303		Jaime	Yeah
304		Erika	… that represent …
305		Jaime	Yup. Ok so you have one. You're taking away a half. So that's…
306		Erika	those three are gone

Line	Time	Speaker	Transcript
307		Jaime	…these three and then you're taking away a third…
308		Erika	which is these …
309		Jaime	… and it's those two. So, you've got one-sixth.
310		Erika	Yeah
311	13:20	Jaime	I wish I had these when I was little.
312		Erika	I know right! It would make it so much easier.
313		Jaime	This would have made it so much easier like when we were learning fractions, like ...
314		T/R	Well I think so and then …
315		Erika	It shows it!
316		T/R	.. but you know …it's … I find it hard to use these with the Math114 students because they're so tied to algorithms that they find it hard to think about the meaning of this. So, yeah. I think it's a good idea to get away from the algorithms until they do this kind of stuff
317		Erika	Alright. Now …
318		Jaime	The next one
319	13:51	Erika	For the third one. Here goes a candy bar. Here's halves. Here's thirds.
320		Jaime	Um hum
321		Erika	We're probably going to need the sixths again.
322		Jaime	Yeah might as well put them right in there.
323		Erika	Because … so … he only has half of the candy bar.
324		Jaime	Ok so…
325		Erika	He only has this.
326		Jaime	Ok so …
327		Erika	That's half the candy bar.
328		Jaime	Right
329		Erika	… and someone's taking away …
330		Jaime	… a third
331	14:18	Erika	These have to be the thirds. Now are they taking a third of the candy bar or a third of what he has?
332		Jaime	He has half of a candy bar. Bill takes one third of the candy

Line	Time	Speaker	Transcript
			bar
333		Janelle	Did you guys? The first one … the second one… double check your second one.
334	14:35	Erika	What?
335		Janelle	Because ... she has a candy bar.
336		Erika	Yeah
337		Janelle	Right?
338		Erika	Yeah
339		Janelle	She gives half
340		Erika	Um hum
341		Janelle	Like say this is the candy bar right? She gives …
342		Erika	Half
343		Janelle	… half to Pablo and a third to Gordon. So, what does she have left? Oh no, a third to Gordon is two.
344	14:52	Jaime	Yeah
345		Janelle	Got it.
346		Jaime	That's the way that I thought of it
347		Janelle	Yes
348		Jaime	Ok. What are we doing now?
349		Erika	Um, this one …
350		T/R	So, for the second one, you've got your equation and you've gotten your things with the blocks
351		Janelle	[agrees]
352		T/R	Ok
353		Jaime	I have a question about the third one.
354		T/R	Yes
355		Jaime	Is it a third of the whole candy bar or a third of the half?
356		Erika	Of what she has
357		Janelle	It's what John has.
358		Jaime	Ok
359		Janelle	So, it's a third of the half.
360		T/R	Well, actually I would have argued with … a third of a candy

Line	Time	Speaker	Transcript
			bar…
361	15:21	Jaime	So, it's not a half. It's a whole
362	15:22	Erika	So, a third
363		T/R	It's not … see … It was intended to be different from number one where I said half of what she has
364	15:27	Erika	So, he has …
365	15:28	Janelle	But it says 'from John' and John only has half of a candy bar
366	15:31	Erika	So, he takes a third.
367		Janelle	So, he can't take more than what John has.
368		T/R	That's true. That's true he can't take more than what John has but is one-third of a candy bar more than what John has?
369		Erika	No
370		Jaime	No. Wait a minute
371		Erika	A third of a candy bar that John has.
372		Janelle	John has a half of a candy bar.
373		Erika	Yes. And if you take a third of a half …
374		Janelle	… of John's candy bar.
375		Erika	But you don't want to do that. You want us to have him take a third of what a whole candy bar would have been?
376		Jaime	Yeah
377		T/R	Yeah. And that's … I want you to think of it two different ways … and if that one, the way you started talking about it first and the way I want you to think about it
378		Erika	Like this
379		T/R	… and then, an alternate wording. Suppose I said, I have a half cup of flour and my recipe calls for a third cup of flour. Right. I want to take the third cup of flour away from what I have, what do I have left?
380		Jaime	Oh
381		T/R	Did you hear the whole question?
382		Jaime	Yeah
383		Janelle	Yes
384		T/R	The recipe calls … I have a half cup of flour. The recipe calls

Line	Time	Speaker	Transcript
			for a third cup of flour.
385		Janelle	You want to take it out of the half of cup
386		T/R	That's right. It's not a third of what I have because I don't know what I have when they write the recipe. It's a third of a cup. See what I'm saying? That's the kind of question I meant to be asking here. Maybe
387		Janelle	So then it shouldn't say from John.
388		T/R	Well
389		Erika	Well he does take it from John but he takes a third of the size of the candy bar from John. Alright. Candy bar.
390	16:54	Janelle	I get it. I understand it. I understand it.
391		T/R	Alright. Suppose Johns the one with the half cup of flour.
392		Janelle	I understand. I understand. I understand.
393		T/R	Yeah, but you … but you…
394		Janelle	I just … don't agree.
395			[laughter]
396		T/R	But think of a, kind of a wording that will … that's not too far from this that will totally agree with what you said. I still think I can do it with my half cup of flour. John has a half cup of flour
397		Janelle	Yeah
398		T/R	Bill wants a third of a cup of flour. Well he's going to take it away from John and John is gonna have some left but he's not taking a third of what John has.
399		Janelle	So he's taking a third of a half
400		T/R	No he's not
401		Erika	He's taking a third of the whole.
402		T/R	Well alright. We're gonna talk about this. I don't want to just argue with you because this is a really good thing to discuss.
403		Erika	Oh look. She's … the question …
404		Janelle	<inaudible>
405		Erika	What the question is really saying is that ... who's the second guy? Bill?
406		Janelle	Yeah

Line	Time	Speaker	Transcript
407	17:43	Erika	Bill is taking a third of **a** candy bar. See that's where you have to say "Of **a** candy bar from John". So, he's taking, a third of the total candy bar from John. So, he's taking one of the reds from John. So, John is only left with a sixth.
408	18:01	Jaime	So, it's the same thing? They're all one-sixth?
409		Erika	Wow! They are. Three different ways to get one-sixth.
410		Jaime	Alright, so are you drawing this one?
411		Erika	Yeah
412		Jaime	So, this is what? Green?
413		Janelle	They're all like the same drawing
414		Jaime	Yeah
415		Erika	Yeah, They're all the same. Except for the first one.
416		Janelle	Yeah
417		Erika	The first one's three and the other …
418		Janelle	You didn't need halves on the first one.
419		Erika	Thirds. Sixths. One. Half. Third. Sixth. It's the same as four. Alright.
420	18:57	Erika	I check my answer by looking at the mathematical stuff.
421		Janelle	I figured out what my problem is with number three.
422		T/R	Yes. Ok, tell me
423	19:18	Janelle	If it's the way that you want us to do it, then the size that John has wouldn't change.
424		Erika	Yep
425		T/R	Why not? Oh, what do you mean?
426		Erika	Yes, it would
427		Janelle	Because if he has half of the whole candy bar …
428		Erika	Yeah
429		T/R	Yes, ok
430		Janelle	… and then Bill takes a third of that candy bar,
431		T/R	Yes
432		Erika	Yeah
433		Janelle	John would still have the same amount

Line	Time	Speaker	Transcript
434		Erika	No, he wouldn't. He'd have one sixth. Because …
435		T/R	Uh, I don't want to argue with you. Argue with Jess
436		Janelle	Ok
437		Erika	Argue with Jess. Alright. Alright, so we know that John has that [light green rod]and they're saying that Bill is taking a third of a whole candy bar because it says of **a** candy bar not **the**, so he's taking the red
438		Janelle	Ok, so ... so ... so right now Bill has this and John has that
439		Erika	John is going to be taking this from Bill
440		Janelle	Bill is
441		Erika	I mean, no, Bill is going to be taking this from John. Right?
442		Janelle	From this?
443	20:17	Erika	Yes. So he's taking that much because that's a third of a whole candy bar so this is what …
444		Janelle	... he has left
445		Erika	… he is gonna have left
446		Janelle	It's what John has left.
447		Erika	Yeah. So, it's gonna be one sixth because the white ones in this thing are one-sixth. [laughter] Ok, now I got …
448	20:48	Janelle	It makes sense!
449		Erika	Yeah
450		Janelle	I can just see it both ways
451		Erika	Yeah. So can I, but the wording …
452		T/R	That's good because I want you to argue that other position.
453		Erika	Yeah, the wording ... the wording is what you've gotta pay attention to. Because I was thinking exactly what you were before. I was thinking "he's taking a third of a half??"
454		T/R	So, you guys are good with all your answers
455		Erika	Yes
456		T/R	Ok. Question one. I'm not sure you all have the same equations for question one.
457		Erika	Yes, we do.
458		T/R	I want you to take out your calculator. Ok, and uh R….. you

Line	Time	Speaker	Transcript
			be a little more a part of this group because I want you all to talk about the equations. Type in the equation that you have for question one into your calculator. What'd you get?
459		Jaime	Negative one-sixth
460		T/R	That's not the right answer is it?
461		Jaime	No
462		T/R	Ok
463		Erika	Oh because we took something …
464		Jaime	… bigger … away
465	21:55	T/R	But you didn't have that equation, so three of you work out what's the right equation to have for number one. I'm not sure whether yours is right or not …
466		Janelle	Yeah
467		T/R	… but, work it out.
468		Jaime	Can't you do … wait. Wouldn't it be …
469		Janelle	It has to be the other way
470		Jaime	The opposite
471		Janelle	No because one-half minus one-third …
472		Erika	She didn't start with one …
473		Janelle	…is….
474		Erika	is that division?
475		Jaime	What?
476		Janelle	What?
477		Erika	Let me see the calculator real quick. Oh, yours is different than mine. Darn it. Oh wow. Backspace. One divided by three divided by one divided by two. Yeah, so ours is wrong obviously. But, you're taking one half of one-third.
478		Jaime	She takes half of what she has. Is it something like
479		Janelle	You're taking …
480		Erika	You had … You had one ... one-half times
481	22:59	Janelle	Hold on. Susie has a third. She gives Paul half. So, you're taking half of one-third.
482		Erika	Multiplying

Line	Time	Speaker	Transcript
483		Janelle	So, its one-half times one-third
484		Erika	Yeah you have to multiply. So, hers is right.
485		Janelle	Yeah
486		Jaime	Yeah. That's what it is.
487		Janelle	And then
488		Erika	Alright, so
489		Janelle	She has the candy bar
490		Jaime	We're on the next one
491		Erika	Yes
492		Janelle	She gives half of the bar to Pablo
493		Erika	That's what I'm thinking. And then what? Gives a third to Gordon.
494		Jaime	Yeah
495	23:40	Erika	The only thing I can think of is x for being the whole thing
496		Janelle	Yeah
497		Erika	Minus one-half x minus one-third x
498		Janelle	x minus one-half
499		Jaime	Equals what though?
500		Erika	One-sixth x?
501		Jaime	This is what? Number two?
502		Erika	Yeah. But just …
503	23:57	Jaime	Hey. But how do you know it's one-sixth? We don't know its one-sixth.
504		Erika	One-sixth of the candy bar. Yeah the whole candy bar
505		Janelle	Yeah x represents
506		Erika	The candy bar
507		Jaime	The whole
508		Janelle	If you solve for x it'll just be one.
509		Erika	Yeah
510		Jaime	Yeah
511		Erika	So, you have the whole candy bar. Then you take away half the candy bar. So, you get that left. Then you take a third of

Line	Time	Speaker	Transcript
			that. Wait, you take a third? from the half? Wouldn't you have two-thirds left then?
512		Janelle	What? Sorry.
513	24:30	Erika	You… wait … Keisha has a candy bar. So, she has a candy bar.
514		Janelle	She has x. Ok.
515		Erika	Ok. She gives half of it to Pablo. So then she has this left.
516		Janelle	Minus, is this two? Minus one half of the candy bar.
517		Erika	So she has this left. And then … now … is she giving …
518		Janelle	And then she gives …
519		Erika	… a third of the **whole** candy bar?
520		Janelle	… a third of what's left
521		Erika	Or a third of what's left?
522		Jaime	what's left.
523		Janelle	It's a third of what's left.
524		Erika	It doesn't say that.
525		Janelle	It says **a** bar
526		Erika	Of **a** bar. So it's a third of **the** bar.
527		Jaime	So we did it wrong?
528		Erika	No
529		Janelle	Yeah
530		Erika	We got the same thing still
531		Janelle	Only because it happened to work out that way
532		Jaime	Yeah. It would've been different though if it was
533		Janelle	But if she gives **a** third of the bar instead of a third of …
534	25:16	Erika	Oh, of the … of a **whole** bar … of **a** bar … rather than **the** bar. Ok
535		Janelle	I think it's trying to show that you can do it any way and you'll still get the same answer
536		Erika	I don't think so
537		T/R	I'm not sure about that either. It depends
538		Erika	It depends

Line	Time	Speaker	Transcript
539		Janelle	Well this way, for this problem you get the same answer.
540		T/R	I think I saw a different answer elsewhere but we'll see. I think I want to talk about problem three as a large group.
541		Erika	One third of **a** bar. Yeah, one third of **a** bar is the little red one which is two of the little white ones.
542		Jaime	[agrees]
543		Janelle	[agrees]
544	25:47	T/R	What did you guys get for number three?
545	25:48	Jaime	One-sixth
546	25:48	Janelle	One-sixth
547	25:49	T/R	One-sixth
548		Janelle	I had originally gotten two-thirds, but then you said I was wrong.
549		T/R	Ok. Well, we said we need some modification. You don't have to erase.
550		Sarah	No, I had one-third and then I looked and I thought it was two-thirds. But now I think its one-third.
551		T/R	But they said … they didn't have one … two thirds either. Right?
552	26:06	Janelle	They had one third. I thought it was two thirds and then we discussed it and now its one-third.
553		Jaime	One-sixth
554		Erika	One-sixth
555		T/R	One-sixth.
556		Janelle	Or one-sixth I mean. Sorry.
557		T/R	Ok. But we need to discuss this as a group because we had different ideas so
558		Sarah	No. I had one third but then I changed it to two-thirds but now
559		T/R	Yeah, two-thirds is what we got with R….'s interpretation I believe. There is an alternate representation um which maybe means you know we need a different kind of wording for the problem. Some classes have told me that they really don't like candy bars.
560		Erika	I like candy ..

Line	Time	Speaker	Transcript
561		Kelly	I like candy bars.
562		T/R	Well the idea … in this problem, and the idea is candy bars are not like standard. Like feet and inches. You know? There is a standard measurement that one foot means something whereas a candy bar doesn't necessarily mean something. And I've had students argue, well when you have a piece of a candy bar you don 't know how big the whole was because you don't have the whole one to compare it to.
563		Erika	Well we always, we just used green as the basis of our - what our candy bar size is, so.
564		T/R	Yeah so um, you could do that. Well let's wait and talk about it with everybody. Let's see this group is still
565		Fae	I'm just writing this last thing and then I'm done
566	27:15	T/R	But the last I saw over here was …
567		Erika	We were changing
568		T/R	That the negative one-sixth.
569		Jaime	Yeah, we fixed that.
570		Janelle	We fixed that.
571		T/R	Ok
572		Erika	We, we – we checked with hers and it worked out
573		T/R	Ok. Alright, so what'd you do different now? Multiply …
574		Janelle	We multiplied
575		Jaime	We multiplied them
576		Erika	Multiply
577		T/R	Ok. Half of one-third is one-sixth. Ok
578	27:30	Erika	And x is just a candy bar
579		T/R	Ok, so. So, there's actually two things that are happening here. One is how much are you giving away. And the answer is one-sixth.
580		Janelle and Darlene	Um hum
581		T/R	And the other question is how much do you have left?
582		Janelle	One-sixth
583		Jaime	Also, one-sixth

Line	**Time**	**Speaker**	**Transcript**
584		Erika	It's also one-sixth
585	27:44	T/R	Yeah but there's a different equation that gives you the fact that you have one sixth left. She started with a third of a candy bar. Now she has a sixth of a candy bar. So, I want that equation too.
586	28:10	Jaime	Oh. One third minus one sixth.
587		Erika	Which is two times … two times one third
588	28:30	Jaime	Two times one third is two thirds
589		Erika	Oh. Sorry … ummm …
590		Janelle	So it's like …
591		Erika	… a half times a …what was it?
592		Jaime	Is this for number two or number one?
593		Janelle	Number one. So, she has one-third.
594		Erika	Yeah
595		Janelle	And she gives half of what she has to Paul
596		Erika	Yeah
597		Janelle	So, half of what she has to Paul. And then … So again, half of what she has, this is Paul's. So, the whole thing minus Paul's is hers.
598		Jaime	So that's … <inaudible> ... the answer.
599		Janelle	No half ... it's half of x
600		Jaime	Because there's half …
601		Janelle	Oh, it's a third. I lied
602		Jaime	Yeah.
603		Janelle	So, if a third of x is what she has …
604		Jaime	Because its <inaudible>
605		Erika	Yes
606		Jaime	Yeah
607		Janelle	… minus what she gives to Paul …
608		Erika	Yeah. Yeah. Yeah.
609		Janelle	… equals what she has left over.
610		Erika	A third of the whole thing, minus …

Line	Time	Speaker	Transcript
611		Janelle	… minus what she gives to Paul …
612		Erika	…what she gives to Paul …
613		Janelle	… is what she has left over.
614		Erika	Yeah. Ok.
615	29:25	Janelle	So, then number two.
616		Erika	That one is, she has a whole candy bar…
617		Janelle	She has x.
618		Erika	… and then she gives half of the candy bar…
619		Janelle	So, half of x …
620		Erika	… and then she
621		Janelle	… goes to Pablo
622		Erika	Yeah
623		Janelle	And then a third of x …
624		Erika	Of the whole
625		Janelle	… goes to Gordon.
626		Erika	Yeah
627		Jaime	Equals one-sixth x.
628		Erika	One-sixth is what she has left.
629		Jaime	Yeah, this is what we did already.
630		Erika	Yeah, we did that one.
631		Jaime	So then what portion does she have left?
632		Erika	One-sixth
633		Jaime	One-sixth x
634		Erika	Yeah
635		Janelle	One half x plus one third x. So x minus that.
636	30:04	Erika	Oh, you just combined the two. Ok yeah. To make it easier.
637		Janelle	Number three.
638		Jaime	Ok, so,
639		Janelle	John has one-half x.
640		Jaime	Minus one

Line	Time	Speaker	Transcript
641		Erika	A third x
642		Janelle	Bill has … minus one-third x
643		Jaime	One-third x
644		Erika	Yeah, because we decided it was one-third of the whole candy bar.
645		Jaime	… equals…
646	30:30	Erika	… one-sixth
647		Janelle	… equals John's.
648		Erika	Alright, so. We've got equations now. Now what do we do?
649		Janelle	Just wait
650		Jaime	What time is it? Three twenty
651		Erika	Oh, see if these were… what number was that?
652		Jaime	Oh no, don't Jess.
653		Erika	Eight?
654		Jaime	Jess, please. I don't know
655		Janelle	Orange is ten
656		Jaime	No, we're talking <inaudible>
657		Erika	No no no. I'm talking about uh history of math. If it was straight here, put that there and then there.
658		Jaime	I don't wanna know.
659		Janelle	It's the stage. Six
660		Erika	No six is just the two
661		Janelle	Oh yeah. Yeah.
662		Erika	And then eight is the one with the thing on top.
663		Janelle	Yeah.
664		Jaime	Isn't it ten to one ..
665		T/R	Now, I think I want to go to a whole class discussion – uh you guys can keep videotaping – of number three because you guys did number three and had some big disagreements about it. And you started … you did number three also and had R…..'s … that's ok
666		Erika	That's alright

Line	Time	Speaker	Transcript
667		T/R	… you had R…..'s issue….
668		Sarah	Yeah
669		T/R	… and I'm not sure that we resolved it or not. And you're not one hundred percent happy with our resolution.
670		Kelly	[laughing] I love your phone
671		Erika	Please continue
672		T/R	Ok. So
673		Janelle	I figured out where my problem lied though
674		T/R	Ok. Now, before you do … you guys have read problem three. Am I right K…? I'm not sure you saw it yet
675	32:11	Kelly	No
676		T/R	Alright so read problem three to yourself right now. Ok. Or you can read it out loud if you want
677		Kelly	Ok. John has one-half of a candy bar. Bill takes one-third of a candy bar from John. What portion of a candy bar does not … does John have left?
678		T/R	Ok. Now, let's go with …
679		Fae	The way I thought of it.
680		T/R	Go ahead
681		Fae	Sorry. I just want to explain one thing. The way I thought of it is, because of the wording where it says Bill takes one third of the candy bar **from** [emphasizes 'from'] John …. Because it says 'from John' and John only has half of it, I'm not thinking that John has any of the candy bars. I'm just thinking he has that one half.
682		T/R	Yup
683		Fae	That's why I came up with the one-sixth.
684		T/R	Ok
685		Janelle	And then .. yeah, and then I thought of it where you have ..it says you have **a** [emphasizes 'a'] candy bar
686		Erika	Yeah
687		Janelle	So John has half of **a** candy bar and Bill takes a third of **a** candy bar.
688		Erika	So its two separate candy bars

Line	Time	Speaker	Transcript
689		Janelle	If it was the same candy bar, it would be 'the'. John has half of **the** candy bar. Bill takes a third of the candy bar
690		Erika	..of **the**..[emphasizes the]
691		T/R	Ok, and so that was where your two-thirds answer came from
692	33:09	Erika	Yes
693		Janelle	Yeah
694		T/R	And that's sort of – F…. – that's where your answer two-thirds came from
695		Sarah	Yeah
696		T/R	Alright now, but, what I wanted was a question that would end up in mathematical terms as one-half minus one-third equals one-sixth
697		Janelle	One-half …
698		T/R	… minus one-third equals one-sixth
699		Fae	Yeah
700		Erika	Yeah
701		Fae	Uh huh
702		T/R	That's what some of you got for that, but some of you really wanted to say one minus a third equals two-thirds. So because there's some ambiguity about candy bars and how do you know how big the candy bar is, I made some suggestions that I'm not going to repeat, but I want you guys to think about a question where they're going to write down one-half minus a third equals one-sixth and then they're … its not going to be ambiguous. It's not going to be, there's not going to be confusion as to what they're subtracting from what. You know what I'm saying? Like I … because … well you can't say … there's not alike a standard candy bar. People get confused. At least some students have gotten confused when you say a candy bar. So my suggestion was, can you say something else like a foot. Because a foot is always the same size. So if you have something half a foot, you know it's always six inches for example. Can you think of some other way to word a similar problem without using candy bars so we're absolutely positively sure you want to say a half minus a third.
703		Erika	To get a sixth

Line	Time	Speaker	Transcript
704		T/R	To get a sixth
705		Fae	You can just give a measurement to it. Like …
706	34:32	T/R	Ok, well. Just talk about it in your group. Ok. And then maybe each group can come up with something.
707		Jaime	Yeah. It's the same thing because it's a cup. A cup is like a universal measurement
708		Janelle	Yeah, I did like what you suggested with the flour.
709		Jaime	Isn't it because the cup is like universal, so
710		T/R	Yeah, you know, I still have um – I don't want to take part of the other groups
711		Erika	Yeah
712		T/R	I still have students who argue with me. In fact, I have something like this piece of wood thing and they were still saying its always a third of what he has as opposed to a third of a foot and it was difficult to get that idea across
713		Erika	See. Yeah for this, like, to get the other answer, I thought it should just be worded Bill takes a third of – of John's candy bar, if you wanted to find out the two thirds.
714		T/R	Yeah. Yeah. Ok. That's good. So that's two other questions you've answered. One, how do you get that two-thirds answer and the other, how do you get this answer.
715		Janelle	See. See, I got this literally when you were talking about it. I just recopied the problem with flour instead of a candy bar.
716	35:38	T/R	Ok
717	35:39	Janelle	So, John has a half, a half cup of flour.
718	35:41	T/R	Now everybody should listen to this so say it a little bit louder.
719		Janelle	Sure. So, I just redid the … the problem three and instead of a candy bar, I did flour. So john has a half a cup of flour. Bill takes a half a cup of flour from John
720	35:55	T/R	A third
721	35:55	Erika	You mean a third
722		Janelle	A third. Sorry. Yeah. A third of a cup of flour from John.
723		Erika	From John
724		Janelle	What portion of flour does John have left?

Line	Time	Speaker	Transcript
725		Erika	Oh, that's a lot easier to understand actually
726		T/R	Ok well first you have a cup of flour
727		Janelle	.. of a cup of flour
728		T/R	Is that? Is that easier?
729		Erika	Yeah that one was actually a lot easier
730		T/R	Does that make sense to you guys too?
731		Janelle	So it's one half
732		T/R	I could visual ..I mean yeah, I tried to use this with the Math114 class and I said, 'you know when you bake stuff' and they said 'we don't bake'. So
733		Erika	You don't even have to bake … well I guess
734		Fae.	The reason why I like the candy bar deal is because these are rods
735		T/R	Yeah
736		Fae	So, it's easier to understand …
737		Erika	So, it's like the candy bar
738		Fae	representing this as a candy … you know what I mean?
739		T/R	Yeah
740		Fae	…to break it up into the equal portions
741		T/R	Yeah. So you're ok with the problem as it was?
742		Fae	I mean …
743		Sarah	Yeah
744		T/R	But, how do you feel about R…..'s proposal?
745		Fae	That works.
746		T/R	Ok
747		Fae	Numerically that works. Visually, I feel like this works better.
748		T/R	Ok. And did you guys come up with any other wording that you were thinking about?
749		Fae	I was thinking … I don' know. I was gonna say like that .. they're running a … but. No, I don' t know. I was gonna say like they're running a 6 mile race but then how would Bill take anything from them. He's not taking anything.

Line	Time	Speaker	Transcript
750		Erika	yeah
751		T/R	Well, let's see. Six miles
752		Jaime	Taking a lead… Maybe he's like running behind somebody
753		Fae	John has …
754		Janelle	But then you have to do speed
755		Jaime	Yeah
756		Erika	Yeah we're not going to worry about physics at the moment.
757		T/R	But that... that might work. Let me think about this.
758		Fae	The easiest <inaudible>
759		T/R	Ok, I will think about it. But, alright we can move on. And you've already answered some of the questions that I thought about which is … issues. Right? What kinds of issues are there? And, we didn't talk as a group, but I saw individually. In fact, I think I talked about it with your group but I didn't talk about it with your group. Go back to problem one. I think I saw it on your paper K….
760		Kelly	Yeah?
761		T/R	What mathematical sentence did you get for number one?
762	37:45	Kelly	Uhhh one-half minus one-third?
763		T/R	Yes. No.
764		Erika	You're supposed to have
765		T/R	One half minus one third?
766		Erika	It was you had a third a…
767		Janelle	half minus one third
768		T/R	Yeah
769		Kelly	Oh
770		Erika	…and they're taking a half.
771		T/R	Yeah
772		Kelly	Sorry. One-third minus one-half then.
773		T/R	Yeah. That was the …
774		Erika	But if you do that
775		T/R	Right. Do that in your calculator.

Line	Time	Speaker	Transcript
776		Erika	Put that in the calculator. One third minus one half
777		T/R	And is that .. that isn't what you have?
778		Kelly	My calculator is dead.
779		Sarah	I did one third divided by two equals one third times one half equals one sixth.
780		T/R	Yeah. Um is this your calculator?
781		Sarah	Yeah.
782		T/R	Can you do fractions on this calculator? Because …
783		Erika	If you can't, I've got mine
784		T/R	What did you have?
785		Fae	This one's wrong.
786		T/R	Right
787		Fae	I did the one-third minus one-half.
788		T/R	Yes, and when you do the one-third minus one-half tell us what you get
789		Erika	You did that too?
790		Fae	On the calculator. I don't know but, like I said with the visual representations, you get one sixth
791		Sarah	It's like. I think it's like one more sixth or something
792		T/R	Yeah but
793		Erika	Its …
794	38:33	Kelly	I did it wrong
795		T/R	What'd you get?
796		Kelly	A negative number
797		Erika	Yeah. That's right
798		T/R	You got a negative number. One-third minus one-half is a negative number.
799		Erika	Because a third is this size. A half is this size. You can't take more than what you got.
800		Kelly	Oh, yeah. Ok.
801	38:44	T/R	And, but. You guys got it too. And I saw F….. over here had it. You didn't subtract a half.

Line	Time	Speaker	Transcript
802		Erika	That's because R….. had it
803		T/R	What did you subtract? You did two
804		Jaime	Multiply
805		Sarah	Yeah. I multiplied
806		T/R	You did a third times a half. She did a third times a half. Right? Half of a third means that you're going to multiply it by a half.
807		Sarah	Yeah
808		T/R	And you got one sixth and that's the thing she subtracted.
809		Fae	Oh.
810		T/R	So, the first question was how much did she give away. She gave away half of it which was one-sixth. And the second question. What did she have left? Well that just also happened to be one-sixth but it might not necessarily have been one sixth.
811	39:16	Erika	We … umm she has
812		Janelle	So we did it with x's.
813		T/R	Ok, explain your x's.
814		Janelle	So, we had um… you know it's for, so for what Paul was getting, she had one-third of x which is the candy bar.
815		T/R	Ok
816		Janelle	So, she had one-third x and then multiply it by one half to get what Paul did. And to figure out what she had left, what she had, which was one-third x minus what she gave to Paul equals what she has left.
817		T/R	Ok. I can, I can deal with that. It seems to me that x is going to end up as one though because it's the candy bar.
818	39:44	Janelle	Yeah, it's always gonna be one
819		Erika	Well yeah yeah yeah
820		T/R	So, sure. Now, but I would think probably, you know, fourth graders or whatever aren't going to do the x part.
821		Erika	Oh no no no.
822		Janelle	No
823		T/R	But you guys can

Line	Time	Speaker	Transcript
824		Erika	Just to take out the x's
825		Janelle	It's the same thing
826		Erika	It's the same thing
827	39:59	T/R	Ok. So ..So we discussed .. ok, issues and the issues, the biggest issues that you guys have tend not to be the issues that students have because they don't know the algorithm**s**. So they don't just jump right in and say one-third minus a half. They just fiddle with these things. Um and I'm also
828		Fae	That was sort of the way I worked.
829		T/R	Right yeah right. There you go. So you can relate. Um, and I think that's a good way. That reminds me of a lecture some of you have heard before. People tend to think that manipulatives are for small children and people who are in remedial or developmental or um
830		Janelle	No, I don't think that No. We're talking about …
831		Erika	Definitely not. <inaudible>
832		Fae	I'm a visual learner. Things like this help me
833		Erika	Yeah
834		T/R	And picking things up, some people are tactile. You know?
835		Erika	Yeah, I'm one of those that has to do it in order to learn it
836		T/R	Yeah. And, in fact there's this great quote that I wanted to use when I was writing a paper and it turns out somebody had already used it … by the famous physicist that none of my other students has ever heard of named Richard Feinman.
837		Janelle	Yeah!
838		T/R	You've heard of him?
839		Janelle	I've heard of him
840		Erika	There ya go. You got one!
841		Janelle	I don't know what he did , but …
842		T/R	He was the physicist who won the Nobel prize for Physics and he was a very unusual physicist. He came from Brooklyn and he talked like he came from Brooklyn. And uh, he also um, some of you may have heard of him. You're too young for this to**o**… the Challenger that exploded, the um thing that exploded.

Line	Time	Speaker	Transcript
843		Jaime	That's where I heard it from.
844		Janelle	Mhmm
845		Jaime	Sanford used to talk always talk about it
846		T/R	How long ago was that?
847		Janelle	He's the one that found out why
848		Jaime	Yeah
849		T/R	He's the one that found out about the o-rings and he dipped it in ice water.
850		Jaime	Yeah
851		T/R	They were prepared to sort of…
852		Erika	Sanford talked about this
853		T/R	**…** accept the fact that well it was just one of those things and he went and did the experiment that … it was too cold that day. The o-ring froze.
854		Erika	And then he.. he couldn't tell anybody so he had his friend come look at the car and be like "look! Look what happened!"
855		Kelly	Oh, I remember the. I think um, Dr. Sanford said that
856		Erika	Yeah, he told us about this
857		Janelle	Yeah
858		T/R	Anyway. Finally, so we admire him because he was a really top guy, plus he was a little weird which we also admire.
859		Kelly	A little weird.
860		T/R	… and he had this quote that says how he figures things out and its all visualizing things. When he talks about sets, here's a set. He says I think about a ball. Disjoint set. I think about two balls. You know and then you know you talk about it has all these properties and I think about he keeps imagining what these two balls look like and then somebody says and therefore here's the conclusion based on the experiments err based on the equation and he says no that can't be because it's not true of my fuzzy balls with whatever. So, he was a totally visual learner and he won the Nobel prize in physics. He was really good in Math. So the point is … you can do these anytime and it's not a remedial thing and it's not something that's only for people who have trouble learning. Now it's not necessarily for everybody. I

Line	Time	Speaker	Transcript
			mean we know some members of the math department who don't think this way. But we know some members of the math department who do. Like for example, me. Ok. So. Alright so you're all on board with that.
861		Fae	Mullner always has manipulatives I feel like he's such a visual learner as well as educator.
862		T/R	Yes. So he's really into that stuff like I am too
863		Fae	Yes.
864		T/R	More so probably. He's a little bit better at … a lot better at relating it to the theoretical. Ok. So. Here's some ideas for 'does this help'. The idea is half of what you have as opposed to half of a candy bar. So how are you going to model these kinds of questions which have whole numbers mixed in with fractions?
865		Erika	Six candy bars right?
866		Jaime	Yup
867		Erika	Half
868		Jaime	Half of what she has. There you go
869		Janelle	You don't even have to use green ones.
870		Erika	I just like using green because we were using it the whole time
871		Jaime	How much does she give to Paul?
872		Janelle	Are we writing this down too?
873		T/R	Yeah
874		Jaime	Three. Right?
875		Erika	Yeah, well. Let's see. She has six and she gives him one-half.
876		Jaime	Minus one
877		Erika	So, one-half times six. Which is three. And it's the same exact equation the other way. Of what she has left
878		Jaime	Yeah
879		Erika	Because you, then you just do six minus what she gave him is three
880	44:16	T/R	Yeah, and you see that it's – you can see that it has exactly the same shape as the other equation right. And this is

Line	Time	Speaker	Transcript
			another thing, especially with people who've memorized algorithms have trouble with. It's the same when you do it with fractions as when you do it with whole numbers
881		Erika	… do it with whole numbers.
882		Darlene	Yeah
883		T/R	It's not like the rules change. But they tend to think that the rules do change. Ok.
884		Darlene	Fractions just scare kids.
885		T/R	What'd you say about fractions?
886		Darlene	They scare kids.
887		T/R	Yeah
888		Erika	They scare K… [laughter]
889		Kelly	Yeah, I can't
890		T/R	But not anymore. You're doing great with these things. Right?
891		Erika	Yeah. I think we should just get K… Cuisenaire Rods
892		Kelly	Yeah, if I had this in like fourth grade!
893		Jaime	That's what I said
894		Erika	That would've been great. Umm
895	44:52	Jaime	Which one are we doing now?
896		Erika	He takes a third … Now, I just have a quick question
897		Jaime	<inaudible> model it here
898		T/R	Ok.
899		Erika	That's … the second question is stated correctly?
900	45:04	T/R	Yeah, a third of **a** candy bar
901		Erika	of **a** candy bar
902		T/R	Yeah, just like a third of a cup of flour, the same sort of thing.
903		Erika	So that means if he takes a third. He takes that.
904		T/R	Not a third of what he has. Right.
905		Erika	That's what he has. He has four whole ones and two thirds.
906		Jaime	And two-thirds. Yeah

Line	Time	Speaker	Transcript
907		Erika	So, what it would be is you have five and someone's taking a third of
908		Jaime	A third
909		Erika	A third of one candy bar. So, one-third times one.
910		Jaime	Um hum. Equals
911		Erika	Equals four
912		Jaime	And two thirds
913		Erika	And two thirds. I thought I was trying to find <inaudible>
914	46:00	T/R	Ok, and… You've got your answer you've got your picture. You're good. You guys did the same? Ok. And alright. I don't see a picture but
915		Darlene	No, we don't have a picture
916		Erika	Oh no no. We just did it really quickly right here.
917		T/R	Ok. So you can describe it. Ok and you know, think about what you're doing in terms of algorithms too.
918		Jaime	<inaudible> what?
919		Erika	Oh yeah. I was looking at her thing. She has six times one-third. We have six minus
920		T/R	Six times one-half
921		Erika	Oh that's what I meant. Sorry. Right. Six times one-half.
922		Darlene	Yeah that's the same
923		Erika	It's the same thing. Because what we have is six minus one-half of six. It's the same thing. I just want to make sure we're all on the same page.
924		Janelle	Yeah, it's the same thing.
925		Erika	Alright.
926		T/R	Ok. And you're good here and you're good here.
927		Erika	Right down the middle.
928		T/R	Ok
929		Erika	You know for the first one you had to take a half . You've got six candy bars
930		T/R	There you go. That's half. That's another thing we can talk about.

Line	Time	Speaker	Transcript
931		[break]	[break]
932	48:23	T/R	Ok. Because I did want to have the whole class talk and I want to have F….. over here talk for a minute about her model. Five candy bars … F…. So you wrote it as fifteen-thirds.
933		Sarah	Yeah
934		T/R	So I asked for a model that shows that the five candy bars are equal to fifteen-thirds. So. Explain the model.
935		Sarah	So I just did …I just did um five greens and then you know that each green is equivalent to three white so you get fifteen whites
936		T/R	Ok
937		Erika	Light greens
938		Sarah	So yeah. So one fifth .. one fifth of the green would be
939		T/R	One third of the green
940		Sarah	Wait one third?
941		T/R	One third of the candy bar
942		Sarah	Oh yeah yeah yeah..
943		T/R	Ok
944		Sarah	So it would be three whites
945		T/R	Right so one-third. So you got fifteen thirds and you're taking away one third.
946		Sarah	Yeah.
947	49:06	T/R	So take away the one third and you've got … No you're not …
948		Erika	[disagrees]
949		Sarah	No you would take
950		T/R	You're taking away
951		Erika	One third
952		T/R	… the white ones are thirds. So you're taking away one-third
953		Darlene	The white thing
954		Erika	Yeah

Line	Time	Speaker	Transcript
955		Darlene	The white
956		T/R	Just the white
957		Janelle	Just one white one
958		Fae	Yeah
959		T/R	Yeah, now.
960		Sarah	Oh a third of this. Sorry
961		T/R	Yeah. And your equation says your answer is…
962		Sarah	Fourteen-thirds
963		T/R	And there's her fourteen-thirds. Now notice how that's often how they tell you …I don't know how you were taught but a lot of times they tell you when you're adding and subtracting fractions you make the whole thing improper fractions.
964		Erika	Oh yeah. Yeah.
965		Jaime	Yeah
966		Janelle	Right
967		T/R	Which I hate because it's a lot of extra work right? I mean it's true that that's fourteen thirds, but for example if you talk about your model … what did you guys do?
968		Erika	We just set up five of them and for one of the candy bars we have three red ones because three red ones make up a green one.. uh dark green one. So then we did takes away one third, we moved the red. You take away the one red. And take the green and replace it with the two reds that are left. So we went one, two, three, four, and two thirds.
969		Darlene	And two thirds
970	50:13	T/R	Which is the same as your answer when you convert it back …
971	50:15	Sarah	Yeah
972		T/R	… but you sort of… you did an extra step your way… which isn't wrong but its an extra step
973	50:19	Sarah	Yeah I did four and two thirds but
974		T/R	Yeah but you had that other model …
975		Sarah	Yeah
976		T/R	…that I wanted you to show.

Line	Time	Speaker	Transcript
977		Janelle	It's not fourteen thirds though. It's fourteen fifteenths.
978		T/R	Is it? What's one?
979		Erika	One of them is .. I thought was one third. Oh the ...
980		T/R	No but what represents one in her model?
981		Erika	The green
982		Janelle	The whole … one green thing
983		T/R	One green thing represents the number one
984		Janelle	[agrees]
985		Erika	Yeah
986		T/R	And so the white thing represents
987		Erika	A third … because there's three of them
988		Sarah	One fifteenth
989		T/R	I know you said a third but I want to hear what R….. is saying
990		Janelle	So it represents a third
991		T/R	Ok so she's got fourteen … what color? Fourteen of what color do you have there?
992		Sarah	White
993		T/R	Fourteen whites. And a white is what fraction?
994		Janelle	A third
995		T/R	So she's got. Why is it fourteen fifteenths then?
996		Janelle	Fourteen-thirds … but then … this is for number two right?
997		Erika	Yeah
998		T/R	Yes
999		Janelle	So … yeah you're right. I just didn't do my fraction right.
1000	51:17	T/R	You're - Well everybody should see
1001		Janelle	Yeah
1002		T/R	… you're right that it's fourteen-fifteenths if the whole thing is one
1003		Janelle	If the whole thing was one
1004		Erika	Yeah

Line	Time	Speaker	Transcript
1005		T/R	See that's why you've got to be careful.
1006		Janelle	Yeah
1007		T/R	What's one?… and your one doesn't change, you know, throughout the problem .. which is another issue little kids don't necessarily have but people, you know, your age – in math 114 – will have an issue. One keeps changing. Right. And they'll see something like that and they'll think it's a different kind of fraction.
1008		Fae	I considered the whole total of five bars four plus three thirds.
1009		T/R	Yeah that's a good thing actually… that's sort of what they did here.
1010		Erika	That's …yeah … that's … basically . Yeah that's basically what we did here. Because we just lined them up
1011		Fae	And then I converted into the fifteen thirds which would equal up to the five bars and fifteen thirds minus one third is fourteen thirds .. and then converted into four and two-thirds.
1012	52:05	Erika	Ohm so you …
1013	52:05	T/R	Oh, if you did all .,. you didn't have to do that much. Right? You could have just taken away that one-third the way they did. Converted the one to three thirds, take away one third and you have two thirds left and that's your answer
1014		Erika	… that's what she did. Which is really smart. We did the same thing we just put this here and put this next to it, so…
1015	52:16	Kelly	Yeah. I just took all of them and just broke them into three parts and then added three. And then when it got to the last one took away one …
1016	52:22	T/R	Ok
1017	52:22	Kelly	… so, I get two. So, I added three plus three plus three plus two …
1018		T/R	… plus one more three
1019	52:28	Fae	Which is fourteen thirds
1020		Kelly	Yeah
1021		Erika	Over three. Yeah. All over three
1022		T/R	Yup Right. So … and notice we've had at least four different ways among six people that this problem was done and you

Line	Time	Speaker	Transcript
			all got the right answer. Right? So, another thing to remember, the things that we've talked about is there's not only one right way to do it and you definitely don't want your students to come away thinking that there's only one right way to do things. Now, you have to understand what's going on so you can tell whether what they're doing is right. But, there's no reason to insist that they only do it one way. Ok, and I have a couple more and that's …
1023	53:06	Kelly	Homework!
1024		T/R	Homework. Yes. Homework. We've got ten minutes left and this is the start of a homework. And these are similar, similar to what we've been doing. Just, make some models. Do some number equations. I think this should be enough for all of you
1025		Fae	One more
1026		T/R	One more. Ok
1027		Erika	A fourth, a third and a sixth
1028		Janelle	Can I steal some white one's back?
1029		Erika	A sixth
1030		Sarah	Yea
1031		Janelle	Thanks.
1032		Fae	Nooo
1033		Erika	A third
1034	53:34	Fae	Give them back!
1035		Jaime	And…
1036		Erika	…a fourth.
1037		Jaime	… a fourth is … isn't this the third?
1038		Erika	Well I was saying if we use this as a sixth …
1039		Jaime	Oh
1040		Erika	A third has to be two times the size of a sixth
1041		Jaime	Yeah. And the fourth is …
1042		Erika	Well, why don't we line them up so we can get like one. And then we can use like a that's one-third, that's two-thirds, that's three-thirds so.
1043		Jaime	So that's one.

Line	Time	Speaker	Transcript
1044		Erika	So that's one. We need to find the halves. What's half of a green? A dark green. Is dark green even?
1045		Janelle	Light green.
1046			Light, that's right. Light green. Well look at that. Well because we used it for two of our first equations.
1047	54:17	Jaime	There you go
1048		Erika	Alright, so Mary has a fourth of the pizza. Oh! These are halves
1049		Jaime	There's no quarter
1050		Janelle	Yeah, there's no quarter.
1051		Erika	Yeah, there's no quarters. We can't use these like this
1052		Jaime	Why can't you
1053		Janelle	You have to do
1054		Erika	Because we don't have half of a
1055		Jaime	<inaudible>
1056		Janelle	Can I have a dark, can I have one of your dark green ones?
1057		Jaime	Put the ones here. That makes it into fourths. Look, cause there's two in each of these. Oh there's three
1058		Erika	There's three.
1059	54:50	Jaime	Um, what about the purple? Is the purple bigger than that?
1060		Erika	No because you need to have.. because these are … these are halves
1061		Jaime	Yeah
1062		Erika	You need half of this. There's no half of the green. Remember?
1063		Jaime	Yeah
1064		Erika	We need to use something else. We need to figure out something for fourths. Is there something that four reds equals?
1065		Janelle	Just do two greens as one
1066		Erika	Oh then it works. Yeah! That's right. Multiply it by two.
1067		Janelle	So two greens equals one
1068	55:23	Jaime	Ohh! Ok. Two dark greens right?

Line	Time	Speaker	Transcript
1069		Erika	Yeah. Two dark greens equals one. I just don't have all of my whites. Wait! What do you have purples for?
1070		Janelle	What?
1071		Erika	What are your purples?
1072		Jaime	thirds
1073		Janelle	The thirds
1074		Jaime	Thirds. Because there's a third here.
1075		Erika	Oh. So where we have … hold on. Ok.
1076		Jaime	These we don't need.
1077		Erika	That's one.
1078		Jaime	Right.
1079		Erika	What is our fourth? The green?
1080		Jaime	Yeah. The green are fourths…
1081		Erika	Alright, so this is fourths.
1082		Janelle	[agrees]
1083		Jaime	Yeah. And a red is … sixths
1084		Janelle	Sixths.
1085		Erika	Oh ok. Yeah. She's not using whites. Alright. That actually makes it easier. And these are sixths. Right? Because two of them equals a fourth .. I mean a purple.
1086		Jaime	[agrees]
1087		Erika	Alright. So. Mary has one-fourth. Which is this. Which is half of that green. Lisa has a third. Alright. This is the total
1088		Janelle	Lisa has a third
1089		Erika	She has a fourth
1090		Jaime	Yeah
1091		Erika	That's a fourth
1092		Jaime	She has a third
1093		Erika	She has a third and
1094		Jaime	Patricia has one sixth
1095		Erika	One sixth
1096		Jaime	Ok

Line	Time	Speaker	Transcript
1097		Erika	Ok. So this is what they have in total. What is left over?
1098		Jaime	Whatever this is.
1099		Erika	Well what are those
1100		Jaime	<inaudible>
1101		Erika	There's three of them?
1102		Jaime	Yeah
1103		Erika	So that's … this is a sixth?
1104		Jaime	Yeah
1105		Erika	Right so this is …
1106		Janelle	Here.
1107		Jaime	<inaudible>
1108		Janelle	Why don't you guys …
1109		Erika	Twelfths? No, they're not twelfths.
1110		Janelle	.. look this way? So, Mary has a quarter left over
1111		Erika	Yeah
1112		Janelle	So, take the green out
1113		Erika	That's what we did
1114		Janelle	And then, Lisa has a third left over
1115		Erika	Take the purple
1116		Janelle	Take the purple out. And then Patricia has a sixth, so take a white out
1117		Erika	Yeah
1118		Janelle	So if they put all they're left over pizza together, how much pizza would they have? Line it up and you see three-fourths.
1119		Jaime	Yeah. See the green? It's the same as the green
1120	57:39	Erika	Oh! If you line it up that way … so yeah, three-fourths. Ok. So just draw this one. With the two greens? Or are we drawing the whole thing?
1121		Jaime	I'm drawing the whole thing. So is it one-fourth x minus ... No. Plus. Right? Yeah. One fourth x plus one-third x plus one sixth x.
1122	59:00	Janelle	I need help with this one.

Line	Time	Speaker	Transcript
1123		Darlene	Which one?
1124		Janelle	The second one.
1125		Erika	One third of the pizza and one-sixth of a pizza equals three fourths
1126		Darlene	The next one? Joe has a piece of wood three-fourths meter long. Alright
1127		Erika	So, you start off with three fourths. It's fourths. Do you want to use these as fourths again?
1128		Janelle	Well, yeah. Do it the same way.
1129		Erika	Alright, so, he has three-fourths of the piece of wood.
1130		Janelle	Yeah. So you have … so two greens
1131		Erika	Is one
1132		Janelle	Two dark greens is one.
1133		Erika	Yep
1134		Darlene	[agrees]
1135		Erika	Alright. He cuts off a piece …
1136		Janelle	Off a piece
1137		Erika	… that is one-sixth of a meter which is the ..still … still our reds
1138		Janelle	Yeah, so what he cuts off is …
1139		Darlene	What is the red?
1140		Erika	A sixth
1141		Janelle	Oh, so this is what he cuts off.
1142		Erika	Yeah
1143		Janelle	So, he cuts off this …
1144		Darlene	[agrees]
1145		Janelle	So, he has two and one little piece left
1146	59:59	Erika	Well what is that
1147		Janelle	You have to …
1148		Erika	Three, four, five, six…
1149		Janelle	You have to do it this way
1150		Erika	… and a sixth.

Line	Time	Speaker	Transcript
1151		Janelle	It's less than a sixth.
1152		Darlene	Well you're looking for what? This spot?
1153		Erika	Yeah, I think that the fact that there's two things is screw ...Oh! yeah, because this is a sixth so it's a twelfth.
1154		Janelle	yeah
1155		Darlene	Yeah, that's what it is. So its three-fourths x minus one-sixth x
1156		Erika	It's a half and a twelfth all together. Right? Right. Yeah
1157		T/R	Ok. All you guys are on the second one. Ok.
1158		Erika	Yeah
1159		Darlene	It's a twelfth. Or is it a half? It's a twelfth. One twelfth.
1160		Erika	What is?
1161		Darlene	The answer
1162		Erika	Oh no no no. It's a half plus a twelfth ... is the answer to what he has left.
1163		Darlene	Ok
1164		Erika	Because this whole thing, this is one. And we have three fourths.
1165		Darlene	Three-fourths
1166	1:01:13	Erika	This is a fourth. This is a fourth. This is a fourth. Which makes that a half. That's a half. Alright, and then he took a sixth. Which is… Oh yeah. Ok, I was just making sure that was right.
1167		Darlene	Yeah
1168		Erika	Because like … just having this wasn't helping me at all. It was not. Yeah. So yeah, its uh
1169	1:02:10	Darlene	One ... one-half plus one-twelfth. So, its three-fourths x …
1170		Erika	Oh yeah. One-half
1171		Janelle	Are you sure it's one-twelfth?
1172		Erika	Yeah because like I just did this
1173		Janelle	Yeah, you're right
1174		Erika	… and these are sixths.

Line	Time	Speaker	Transcript
1175		Darlene	Yeah
1176		Erika	Yeah
1177		Darlene	So, its three-fourths x
1178		Janelle	So, its half plus one twelfth
1179		Darlene	Yeah
1180		Erika	Yeah. One-half …
1181		Janelle	Which is seven twelfths
1182		Erika	Yeah. Seven-twelfths. Sorry, I was like what? I had to convert and everything and it wasn't working. Alright.
1183		Janelle	Can I borrow your black pen? Can I borrow it again? Thanks
1184		T/R	Ok. It's time. You guys can stop.