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THE EFFECTS OF VARYING DIAMETER ON COAXIAL PROPELLERS  
FOR THE PROPULSION OF MULTIROTOR SYSTEMS

By

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ABSTRACT OF THE THESIS

The Effects of Varying Diameter on Coaxial Propellers  
for the Propulsion of Multirotor Systems

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Rotorcraft are in use today as both unmanned and manned aerial vehicles, and are limited in performance by the size constraints on their propellers. In order to expand the nature and scope of missions that unmanned aerial vehicles can be used for, said performance limitations must be mitigated. As the payload requirements for rotorcraft increase, the thrust necessary to operate must also increase, which can be achieved to a certain extent by increasing the size of the propellers on the craft. However, large propellers increase the footprint of unmanned aerial vehicles, limiting their mobility in flight, their options for landing sites, and their transportability. When the use of increasingly larger propellers on the aircraft is no longer viable, an increase in the number of propellers is the next logical step. To increase the number of propellers without increasing the footprint of the craft, the propellers can be stacked so that they rotate coaxially. This investigation explains the reduction in the performance of downstream propellers in comparison to upstream and to single propeller arrangements. The loss of thrust is due to the wake of the upstream propeller applying a counter thrust

to the downstream propeller. This study investigates methods to mitigate these thrust losses by alternating rotational direction, increasing separation distance, changing the balance of electrical power input, and varying the diameter of the propellers. Experiments were performed to compare the thrust output of contrarotating propellers and corotating propellers. Separation distance between the propellers was increased from the minimum possible up to a single diameter length and the effect on output was measured. The balance of electrical power input between the propellers was varied to determine which propeller should be powered first. The thrust and efficiency of systems of propellers with constant and increasing diameter are experimentally measured and compared to single propellers. Therefore, systems of propellers of increasing diameter may be viable for unmanned aerial vehicles whose mission requirements include maneuverability, transportability, and greater payload while accepting shorter flight times.

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## **1. Introduction**

Unmanned multirotor vehicles are becoming increasingly prevalent due to their decreasing cost of manufacturing and operation. The various applications of these multirotor vehicles are numerous among hobbyists, photographers, environmental surveyors, and the military, owing to their high controllability and ability to move with six degrees of freedom. Multirotor technology is continuing to be investigated and developed beyond the standard quadrotor configurations of past unmanned aerial vehicles. To overcome the limitations on payload capacity, vehicle operation time and endurance, and stability issues due to wind gusts, engineers are continuously investigating variations on multirotor configurations and developing novel solutions for each application's requirements. Payload capacity in particular is limited by the thrust output achieved by the propellers in each system. Achieving greater thrust with a vehicle would allow such a vehicle to accelerate more quickly and therefore achieve faster movement, and it would allow such a vehicle to carry greater payloads, expanding the scope of available missions. Commercially available products are limited in their thrust capacity, and designing and manufacturing custom parts and vehicles is cost prohibitive to all but the largest of specialty projects. Less costly solutions must be achieved to advance the state of the art.

To increase the thrust capacity of an aerial vehicle, larger propellers and motors could be used instead of comparatively smaller propellers, but such systems require high voltage batteries to operate which decreases safety for the vehicle operator. Instead, increasing the number of smaller propellers instead of increasing the size of

the propellers requires battery capacity to be added to the system, but this can be achieved in a simple and safe way by adding additional, low voltage batteries.

One solution previously presented to achieve greater thrust from a single aerial vehicle is to expand the system horizontally outwards to add additional propellers. While this approach is intuitive, in that more propellers operating will increase the total thrust capacity of the vehicle, the large footprint of such a vehicle would be impractical for many of the applications sought by hobbyists and professionals alike. Difficulty in transport, reduced maneuverability, and limited landing pads and flight paths are all consequences of expanding an aerial vehicle horizontally. Figure 1 shows an example of a horizontally expanded vehicle able to lift a man.



**Figure 1: This vehicle uses 54 propellers in order to be able to produce sufficient thrust to lift a man. Even with efficient packing, the area covered by the vehicle is about seven times that of the man.<sup>1</sup>**

Instead of adding additional propellers to an aerial vehicle in a horizontal geometry, the vehicle can be expanded vertically, allowing the same propellers and motors to operate while allowing the vehicle to have a smaller footprint. The vehicle would then have a greater payload capacity than that of a vehicle with the same

footprint and a single layer of propellers while only requiring the same size landing and operating spaces, such as narrow hallways. In addition, a vertically expanded vehicle can be designed to reduce the size of the support structures connecting the motors to the payload, decreasing the material cost and weight of the aerial vehicle, increasing flight time.

Systems with coaxially rotating propellers exist, but are limited to designs with two layers of propellers. As the propellers rotate coaxially within the same airstream, there are losses associated with operating the downstream propellers in the expanded and turbulent wake of the upstream propellers.

## **1.1 Objective**

The objective of this research is to investigate the gains and losses resulting from adding two or more propellers operating coaxially in series, to determine whether those losses can be mitigated by varying the diameter of the propellers, and to characterize the thrust capacity of a propeller based on the cross-section of its operating area being obscured in the wake of an upstream propeller.

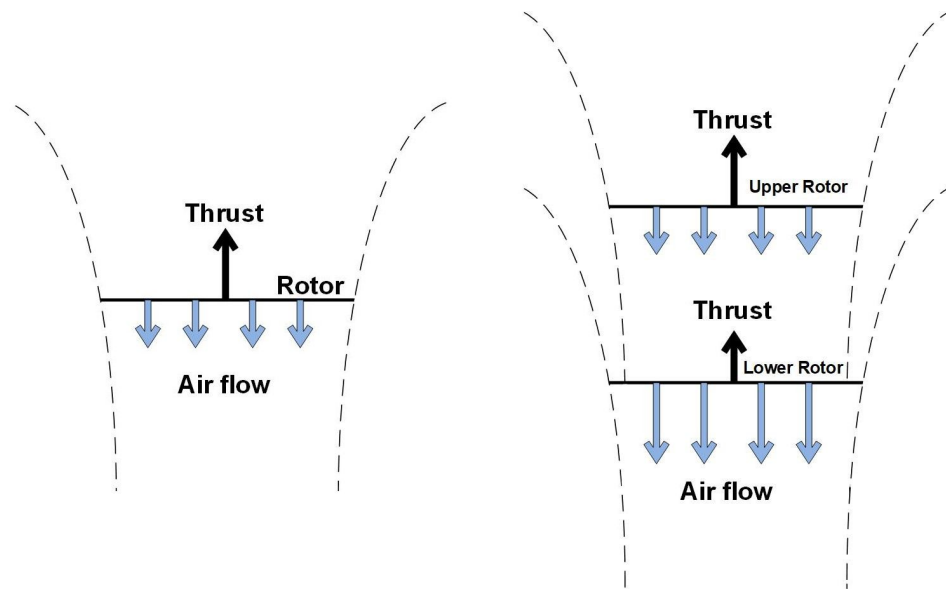
## **1.2 Literature Review**

The investigation into the viability of coaxial rotors dates back to 1754, when Mikhail Lomonosov presented a small helicopter model with coaxial rotors to the Russian Academy of Sciences. It developed from there, with Henry Bright receiving the first British patent for a helicopter with coaxial rotors in 1859. Further designs developed from there, with many iterations of coaxial helicopters developed for use in professional and hobby applications.

The research into multirotor systems was summarized by Coleman<sup>2</sup> (1997) in a survey of theoretical and experimental research conducted globally on coaxial rotor vehicles. The research was limited to helicopter rotors, not propeller rotors, leaving a need for additional research to be conducted on coaxial propellers, as performed in the following sections of this thesis. A general summary of the coaxial rotor configuration for aerial vehicles was given, including the advantages and applications of the coaxial configuration. A coaxial rotor helicopter is defined as having an upper and lower rotor stacked vertically on the same central axis but rotating opposite directions, or contrarotating. The specific issues with coaxial rotors discussed here are separation distance, load sharing between the rotors, wake structure, solidity effects, and swirl recovery. It was concluded both theoretically and experimentally that coaxial rotors required more power to operate than an equivalent single rotor, due to the downstream rotor having to operate in the wake of the upstream rotor. This is the main obstacle to coaxial rotor systems, and what this research seeks to overcome by varying the diameter of the rotors being studied.

A number of researchers have sought to characterize the flow of air through rotor blades, including their wakes where a second rotor would operate, by using a variety of analytical and simulation techniques. In 2000, Brown<sup>3</sup> used a computational model to find a numerical solution to the unsteady-fluid dynamic equations governing the vortices seen in helicopter rotor wakes. Later, in 2006, Leishman and Anathan<sup>4</sup> characterized and visualized the flow through coaxial proprotors using blade-element momentum theory as shown in Figure 2. This technique models each rotor's air flow

volume as contracting, with the air going into the flow being pulled from a larger cross-sectional area than the air behind the rotor. Therefore, when a second downstream rotor is introduced into the system, it pulls air from beyond the wake of the previous rotor as well as the narrow channel of accelerated air. The second rotor experiences a greater volume of air flow, but produces less thrust than the first rotor.



**Figure 2: A simplified view of the air flow through a single rotor and a coaxial rotor system, adapted from Leishman and Anathan (2006)**

Lim, McAlister, and Johnson<sup>5</sup> used experimental data to relate the performance of full-scale and model-scale coaxial helicopter rotors when hovering. They investigated the effects of separation distance between the rotors on the output of each. It was noted that there was a greater scatter in the power data for the system for separation distances between 15% and 20% of the diameter of the rotor, and that there were no major changes in behavior of the rotors for separation distances between 20% and 80% of the rotor diameters. Also, the rotors maintained a high level of thrust capacity

in comparison to a single rotor of the same type, with the upstream rotor retaining 90% and the downstream rotor 81% the thrust capacity of an unimpeded rotor.

In addition, many computational studies have been conducted investigating coaxial rotors whose results need to be verified using experimental methods. Lakshminarayan and Baeder<sup>6</sup> used a compressible Reynolds-averaged Navier-Stokes solver to investigate aerodynamics of a hovering coaxial rotor configuration in order to characterize the unsteadiness of the aerodynamic flow field, implement trimming for detailed thrust and yaw, and observe how the wakes contract at varying rates. They again used the solver to investigate aerodynamics of hovering micro-scale coaxial rotors and to evaluate the accuracy of said computational approach. It was determined that as rotor spacing increased, the thrust output of the front rotor increased and the back rotor decreased, and the total thrust output of the pair remained mostly constant.<sup>7</sup> They also used the solver on micro air vehicles, which typically operate at low Reynolds number and low Mach flows.<sup>8</sup> The tip vortex flow field was shown to be very complicated, with the presence of secondary vortices and additional vortices near the trailing edge of the rotors. These results were validated with testing on a micro hovering rotor.

Rajmohan, Zhao, and He<sup>9</sup> developed a methodology for analyzing multirotor aircraft based on first principles called the Viscous Vortex Particle Method. They also developed a computational model to simulate coaxial rotor aerodynamics. They built two hybrid models, one between a Viscous Vortex Particle Method and Computational Fluid Dynamics approach and one between a Viscous Vortex Particle

Method and a Lifting Line coupled solution, and determined that the VPM/CFD model was the most accurate against available experimental data.<sup>10</sup>

Another solution to increase thrust without significantly increasing the size of the aerial vehicle was proposed by Otsuka, Hikaru, and Keiji Nagatani,<sup>11</sup> and consisted of an octocopter with overlapping rotor wakes, as opposed to coaxial rotors. It was shown to improve thrust output over a traditional coaxial rotor system. Another way to improve the output of a UAV system is by decreasing its energy consumption, as shown by Penkov and Alekzandrov<sup>12</sup> when applying a shroud to the propellers.

It should be noted, however, that the majority of research into and industrial applications of coaxial rotors has focused on helicopter blades, which differ in important ways from propellers. Helicopter blades are designed with high aspect ratios, and typically have the same cross-sectional shape throughout the length of the blade. Propellers are designed with lower aspect ratios, use additional blades per rotor to make up for the reduced area of the wings, and use an airfoil shape that changes

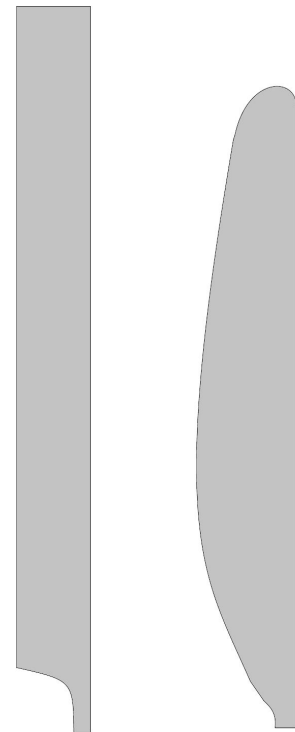


Figure 3: A comparison between the shape of a helicopter blade (left) and a propeller blade (right)

along the length of the blade, as shown in Figure 3.<sup>13</sup> Therefore, additional research must be conducted into the effects of

coaxial propeller rotor systems to investigate how the change in blades affects the wake and overall system performance of aerial vehicles with coaxial propellers.

Holzager<sup>14</sup> performed an investigation into the effects of stacking up to four propellers in a coaxial system. Using experimental data and varying separation distance, pitch, and number of propellers in the airstream, he concluded that a system of four propellers could ideally produce 401% of the thrust of a single propeller, eliminating the thrust losses associated with the downstream propellers operating in the wake of the upstream propellers. He achieved this by increasing the pitch to a greater angle of attack in the downstream propellers, while keeping separation distance constant.

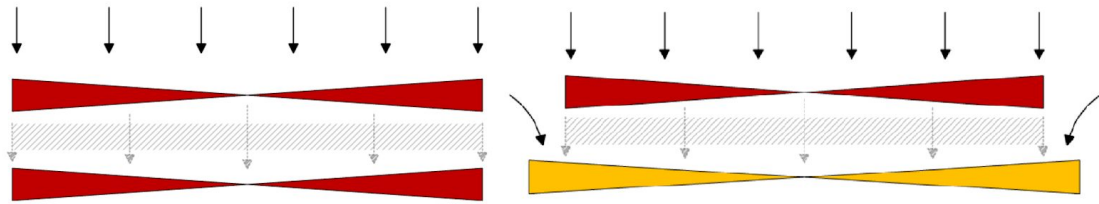
The existing studies focus on characterizing systems of coaxial helicopter rotors, and investigating the thrust and power output losses seen by adding a rotor in the wake of the first rotor. They sought to determine the effects of motor speed, separation distance, and propeller pitch on the efficiency of the system. In this thesis, the thrust losses of systems of propellers will be characterized, and an investigation into whether the thrust losses can be mitigated by varying the diameter of the propellers will be performed.

## **1.2 Theory**

As demonstrated in the literature review, interactions between the flows of multiple rotors result in thrust losses due to the raised velocity of the air entering the second propeller, limiting the second propeller's ability to accelerate the air further. It is also possible that the second rotor and its supporting equipment experience drag



from the airstream, and as both rotors are connected to the same vehicle this will reduce the total thrust capacity of said vehicle. Based on this theory, if the second rotor has a larger diameter than the first, as shown in Figure 4, it will receive stagnant air beyond the flow of the first rotor, and will therefore be able to provide more thrust and mitigate the losses cause by the initial flow.



**Figure 4: A downstream propeller with the same diameter as the first will have all of its airflow accelerated by the first propeller (left). A downstream propeller with a larger diameter will use air from beyond the airflow of the first propeller, producing greater thrust (right).**

As free stream velocity ( $V$ ) into a propeller increases while propeller tip speed ( $\Omega R$ ) stays constant, the advance ratio ( $\mu$ ) increases and the propeller thrust decreases.

$$\text{Advance Ratio: } \mu = V/\Omega R$$

The factors that control the thrust ( $T$ ) and mechanical power output ( $P_{out}$ ) of a propeller are the propeller's shape, Reynolds number, air density, wetted area, and tip speed. Because the motor and support equipment is the same for each propeller in the following experiments, the relationships between these values are more applicable than the aerodynamic equations.

$$\text{Mechanical Power: } P_{out} = Q\Omega = T\kappa\Omega$$

$$\text{Electrical Power: } P_{in} = iv$$

$$\text{Efficiency: } \eta = P_{out}/P_{in}$$

$$\text{Performance: } \Pi = T/P_{in}$$

Performance ( $\Pi$ ) is assigned as the thrust produced by a propeller or a system of propellers per unit of input electrical power. It is the measure of efficiency that is most relevant to this investigation, as the goal is to compare and improve the thrust capacity ( $T$ ) of the systems of propellers. It is also not directly calculated from rotational speed ( $\Omega$ ) of the propeller as efficiency ( $\eta$ ) is, which is important because the second motor will be spinning fast due to drag from incoming air and not because it is producing thrust. In the above equations,  $i$  is electrical current and  $v$  is voltage, and the factor  $\kappa$  is used to represent the slope of linear relationship between thrust and torque for a specific propeller.

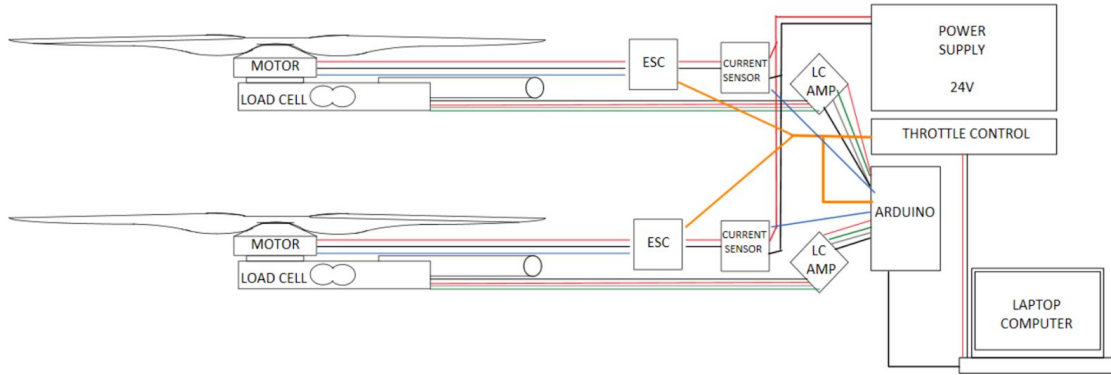
## 2. Experimental Setup

### 2.1 Equipment Details

To investigate the behavior and output of propellers, it is sufficient to test them separately from any aerial vehicle. Instead, the propellers were mounted on aluminum rods, themselves attached to a workbench, which allowed for safe operation of a variety of propellers and stationary load cells to record thrust. Safety measures were taken, including the operation wearing safety glasses and ear protection, as well as a net hung some distance in front of the propeller operating area to catch flying projectiles in case of catastrophic failure.

The propellers used were purchased from T-Motors. Three diameters were used: 10 inches, 13 inches, and 15 inches. Each propeller had an orientation, where it would operate clockwise or counterclockwise. The naming convention used for the propellers is the diameter followed by “L” or “R” for orientation. Therefore, the T-Motor Carbon propellers used in these experiments are as follows: 10R, 10L, 13R, 13L, 15R, and 15L.

Each propeller is mounted on a separate T-Motor MN4006 motor, itself attached to a load cell. The load cell data was acquired and processed using Arduino on a laptop. The diagram for the experimental setup is Figure 5, created by Jonathan Holzager.<sup>14</sup> In addition, the input current to each motor was collected using a multimeter. The power supply was set to provide 22.2 V to each propeller, so that input current could be used to calculate input electrical power.



**Figure 5: A diagram of the experimental equipment setup.**

The current and thrust data were collected and analyzed for the purpose of comparing the thrust achieved by each propeller in a system and the system in whole, as well as the efficiency of each measured as performance.

## 2.2 Calibration

For accurate data collection, the data collection equipment was calibrated at the start of each day of experiments. First, 50 - 100 points of data were collected from each load cell when no load was being applied, and the data averaged for each load cell to determine the offset from zero being returned. All data for that load cell during that run had the offset subtracted from it. Secondly, a mass was weighed on a scale then placed on each load cell, to determine that the code was correctly measuring the output in grams. Finally, during data post-processing, values recorded which were above three standard deviations from the average of that load cell, at that input power, for a specific run were removed from the data, which was then averaged into a single value for thrust.

$$\text{Zero Offset: } T_0 = \text{Average}(X_n \text{ at } P_{in}=0)$$

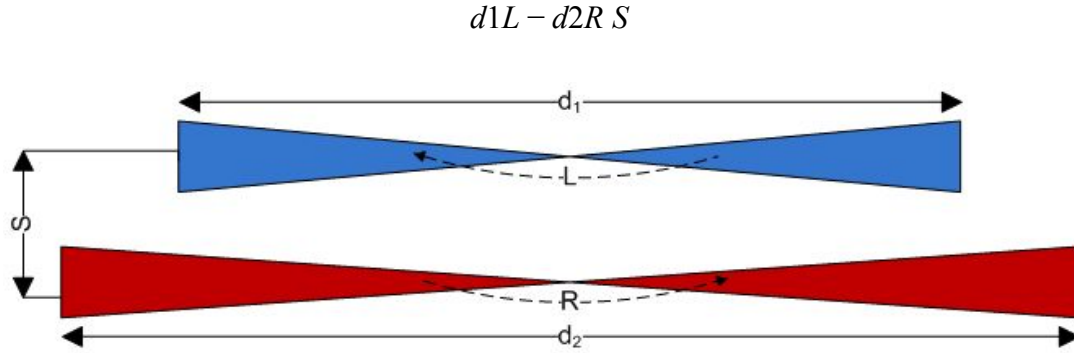
$$\text{Thrust Data Point: } T_i = \text{Average}(X_n \text{ at } P_{in}) - T_0$$

All of the experiments were consistently performed in intervals of increasing input power. To prevent hysteresis error from being a concern during data processing, two experiments were performed both increasing and decreasing power input in 1 amp intervals. In addition, six experiments were paused during the run to allow the motors to cool, then restarted at that input. No hysteresis was observed in the thrust data.

### **2.3 Experimental Procedure**

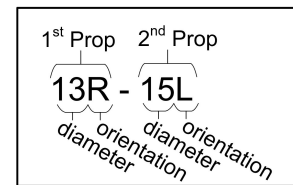
In order to investigate the effect of coaxially rotating multirotor systems, experiments were performed with multiple T-Motor Carbon propellers aligned as to operate coaxially but with separate T-Motor MN4006 motors. The three input system variables were controlled as follows: diameter of each propeller could be 10 inches, 13 inches, or 15 inches; rotation orientation of each propeller could be left or right, denoted by L or R; distance between the propellers could be 2 inches at minimum and 12 inches at maximum, with distance variations limited to 1 inch increments. Practically, inter-propeller distances were 3 inches, 6 inches, 9 inches, and 12 inches.

Each experiment was performed on one to three of these propellers, The naming convention for these experiments lists the propellers in streamwise order, with the front propeller that receives clean air being listed first. This is followed by the number indicating the separation distance between the propellers. Therefore, the name of the system shown in Figure 6 would be:



**Figure 6: A diagram of the naming convention of coaxial rotor combinations.**

For example, a system consisting of two propellers, 10R upstream and 13L downstream, with 3 inches of separation distance, will be called 10R-13L 3. This naming scheme is used throughout this thesis.



The procedure for each experiment was as follows. First, the equipment was set up as explained in section 2.1. Then, after proper safety checks were performed, current was fed to each propeller using a single controller in 1.0 amp increments for the whole system. At each increment, 50 data points on thrust were collected for the purpose of averaging them during data analysis. After the system had reached its maximum input capacity increment, the controller stopped the input current, the propellers came to rest, and the data collection was ceased. The system would then be left to rest and let the motors dissipate heat before a second run was performed.

## 2.4 Uncertainty Analysis

A number of measuring and discretizing devices were used in these experiments, each with a measure of uncertainty to their readings. Each component must be considered to determine the total uncertainty of the measured values.

Thrust measurement begins with the Uxcell load cell. According to its specifications datasheet<sup>15</sup>, the load cell has a zero balance uncertainty of  $\pm 2\%$ F.S., which is an uncertainty of 100 grams for the zero load point. To combat this, data was collected at the beginning of each experimental run and averaged to find the value to establish as the zero load point. The load cell is also sensitive to temperature and has an uncertainty of less than  $0.005\%$ F.S./ $10^{\circ}\text{C}$  with a rating up to five kilograms, causing an uncertainty of 250 milligrams. Using an infrared thermometer, the load cell temperature was measured before and after the experiments, during which it increased by  $3.3^{\circ}\text{C}$ . This would cause an uncertainty of 82.5 milligrams, which is negligible at the operating scale of kilograms seen in the experiments.

The strain gauges in the load cell have resistances that vary according to the magnitude of strain placed upon them. The varying resistance proportionally changes the voltage read by the HX711 load cell amplifier. According to the datasheet<sup>16</sup> on this chip for a gain setting of 128, the typical offset drift is  $0.4\text{mV}$ , noise is  $50\text{ nV}$ , and temperature drift is  $\pm 6\text{nV}/^{\circ}\text{C}$ . The full scale range of the chip is  $\pm 0.5\text{V}$ , or  $1\text{V}$  scaling to raw values on a 24-bit range. The calibration scalar of 0.00285 grams per raw value increment was used so that a single millivolt equated to 47.815 grams of thrust. If these errors stacked during an experiment and the temperature changed with the load cell, the maximum thrust value could be off by 22.5 grams. With maximum thrust values on the order of a kilogram, the error from the load cell amplified could constitute 2.25% uncertainty in this measurement.

Next is the ACS712-30A current sensor module whose datasheet claims a  $\pm 1.5\%$  total output error.<sup>17</sup> Finally, the Fluke clamp meter used for current data collection is accurate to 2% of the measured value.<sup>18</sup>

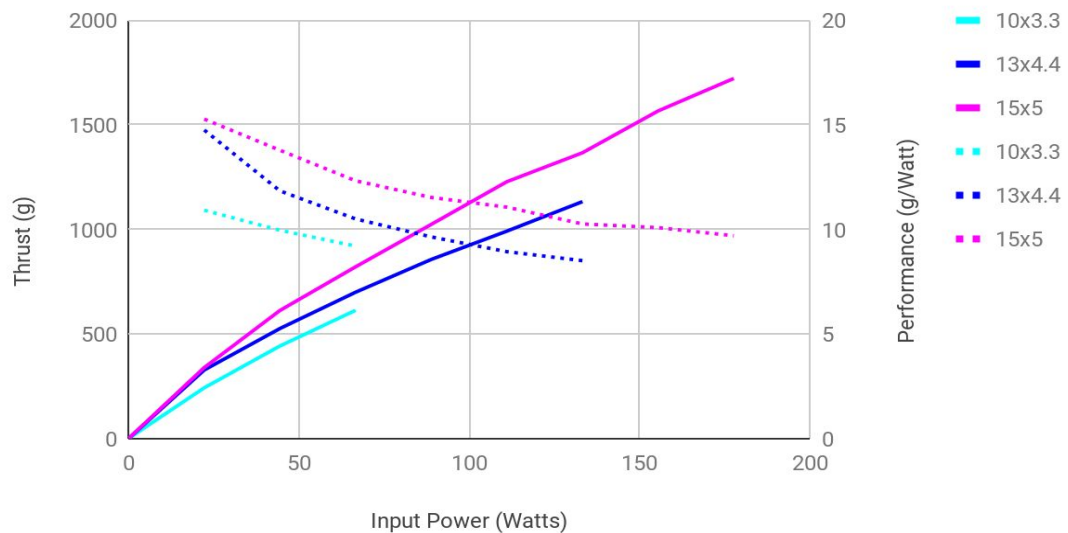
As discussed previously, bias was limited by taking fifty data points to be used as the zero thrust point baseline each time the motor was powered on. Additionally, random error and noise were reduced by taking fifty data points at each step and using the average value of these points in the data analysis. This practice is acceptable as during the motor's operation, the rotor is driven by communication of magnets on the stator. This occurs in pulses, making perfectly smooth operation impossible. Occasionally, clear outliers were measured in thrust without explanation. Outlier data was removed such that the standard deviation of every thrust measurement did not exceed twenty grams.



### 3. Experimental Results

#### 3.1 Single Propeller

In order to properly quantify the effects caused by the addition of propellers, the behavior of each individual propeller had to be studied. The goal of these first experiments was to develop a baseline performance of the 15" propeller, the 13" propeller, and the 10" propeller. This was accomplished by mounting one to the motor situated so that there was minimal interference in the airflow, and then adjusting the electrical power input. The thrust was recorded, and is shown in Figure 7 below, as the solid lines corresponding to the left axis. It was observed that the larger the propeller, the greater its maximum input power, and the greater its maximum thrust output. The 15" diameter propeller can produce 52.0% greater thrust than the 13", and can accept 33.3% greater power input. In turn, the 13" propeller can produce a peak thrust 84.8% greater than the 10", and can accept twice the power input.



**Figure 7: Experimental results of individual propellers, showing thrust (solid) and performance (dotted).**

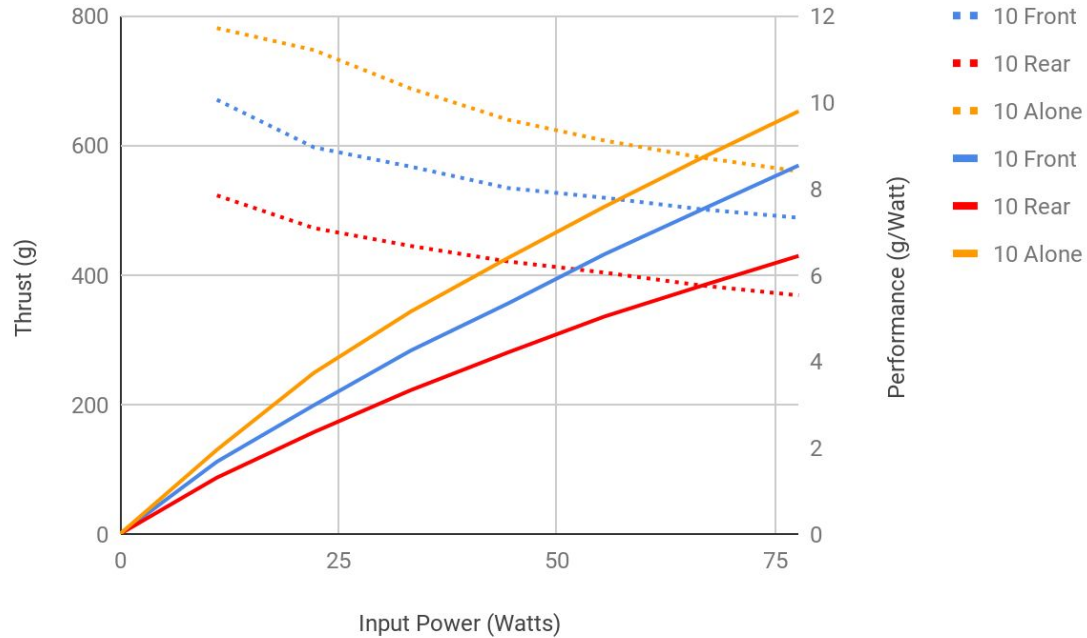
The performance of each propeller was calculated using the methods described in Section 1.3, using the electrical power input and mechanical thrust output. The results are shown in Figure 7 as the dotted lines, corresponding to the right axis. For each propeller, performance is greater at low power, and steadily decreases as power increases. The 15" propeller has the best performance, with a peak performance 18.0% and 39.9% greater than the 13" and 10" propellers, respectively. For the same power input, the 15" propeller has a 24.7% greater performance over the 13" propeller and a 37.4% greater performance over the 10". The trend of increasing diameter resulting in both improved thrust and performance is observed in these experiments, and is therefore expected to be seen in the experiments to follow.

### **3.2 Paired Propellers of Constant Diameter**

The next logical step to take is to analyze the propellers in pairs, maintaining a constant diameter between them. The propellers were each mounted to a motor, then placed so that they rotated coaxially. The electrical power input and thrust output data were gathered, and performance was calculated for each configuration.

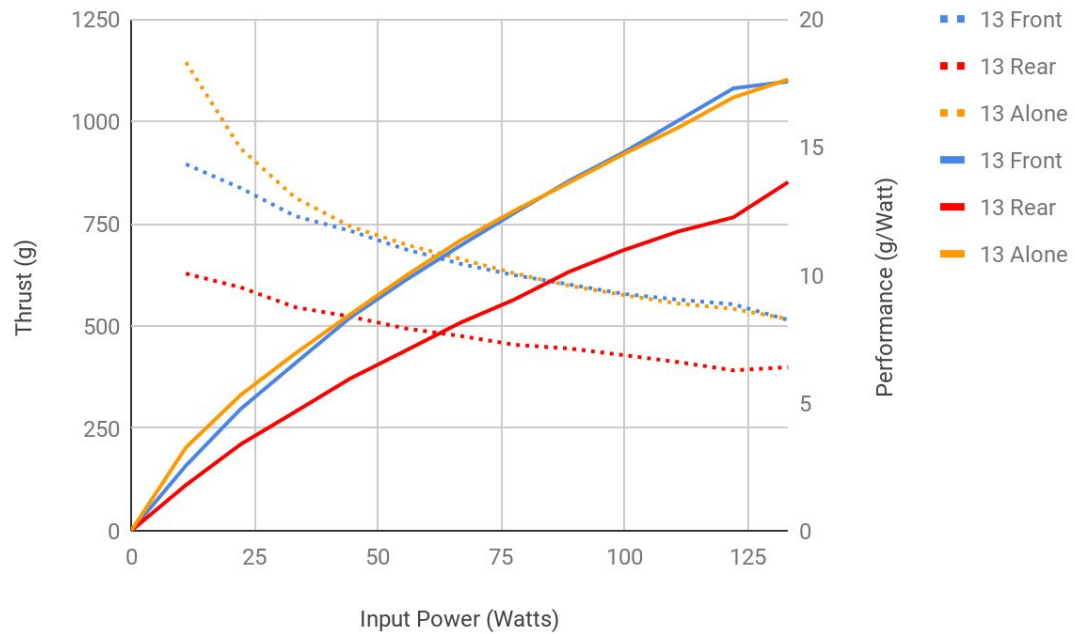
The first step in analyzing the effect of adding a second propeller in the downstream of the first is to compare the behavior of each propeller, front and rear, to the behavior of the same propeller operating alone. This was done by setting up pairs of propellers with the same diameter coaxially contra-rotating with the minimum achievable distance between them. As per the naming convention described above,

these pairs were the 10R-10L 3, the 13R-13L 3, and the 15R-15L 3. Thrust and input power were recorded for each propeller separately and used to calculate performance.



**Figure 8: Experimental results of 10” propellers, showing thrust (solid) and performance (dotted).**

The results of the 10R-10L 3 and the 10R alone system are shown in Figure 8. The thrust curves are represented by solid lines and correspond to the left axis, and the performances are represented by the dotted lines and correspond to the right axis. The shape of the thrust and performance curves for each propeller are the same but offset. It is observed that the both the front and rear propellers interfere with one another at this distance, as neither reach the output of the standalone propeller. When paired, the front 10” propeller produces 15.6% less thrust and performance than the standalone 10” propeller, and the rear 10” propeller produces 34.5% less thrust and performance.

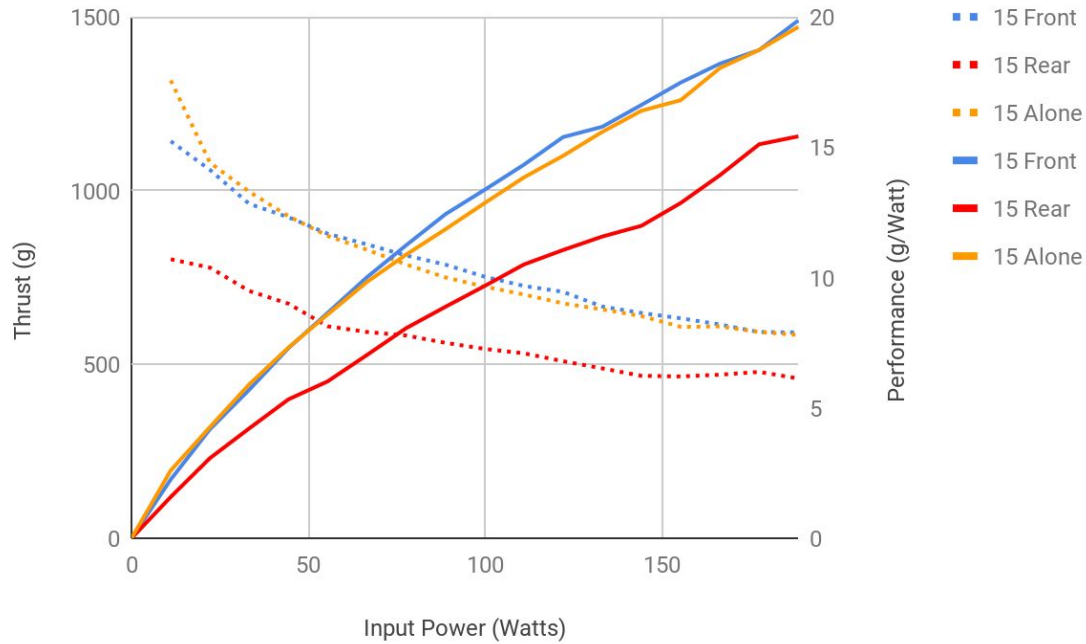


**Figure 9: Experimental results of 13” propellers, showing thrust (solid) and performance (dotted).**

This experiment was repeated for the 13 inch diameter propellers, comparing the thrust and performance of each propeller in the 13R-13L 3 system to the standalone 13R. The behavior is once again similar, but the offset changed. When paired, the front 13 inch propeller produces 3.2% less thrust and performance over the single, and the rear only 29.7% less. Therefore, it can be concluded that the rear 13” propeller experiences less effect from being paired than the 10”. These results are shown in Figure 9, where the thrust curves are represented by solid lines and correspond to the left axis, and the performances are represented by the dotted lines and correspond to the right axis.

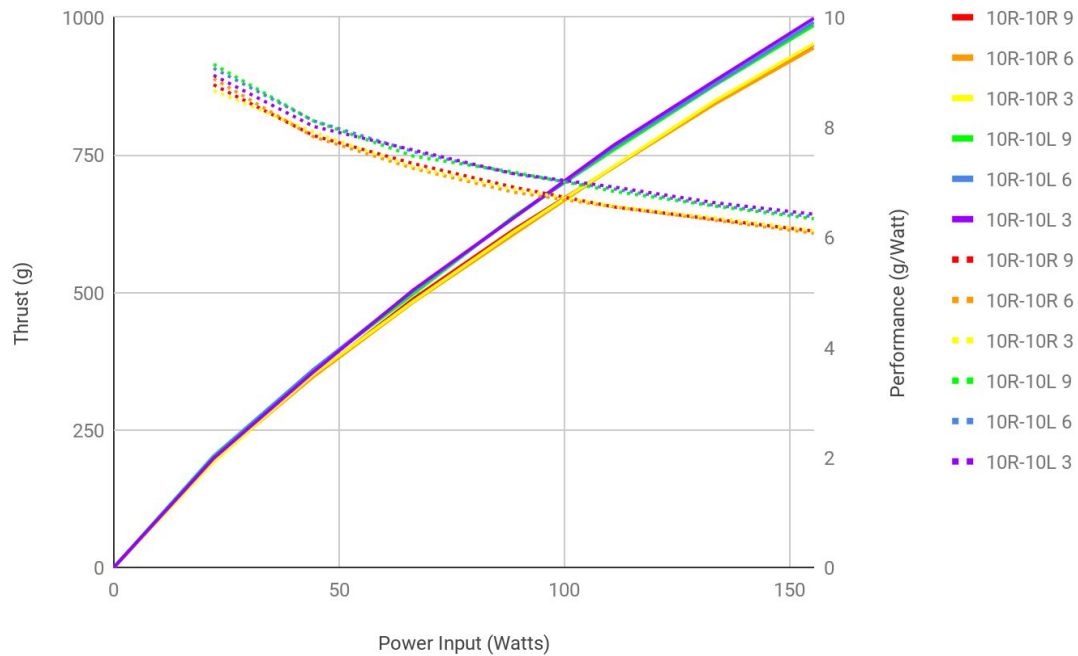
Finally, the results of the 15R-15L 3 are compared to the 15R in Figure 10. The front propeller’s results match to within 1% of the single propeller, leading to the

conclusion that the front 15" propeller is not strongly affected by the rear propeller. The rear propeller is strongly affected by the front propeller's upstream interference, and produces 26.2% less thrust on average than the single propeller.



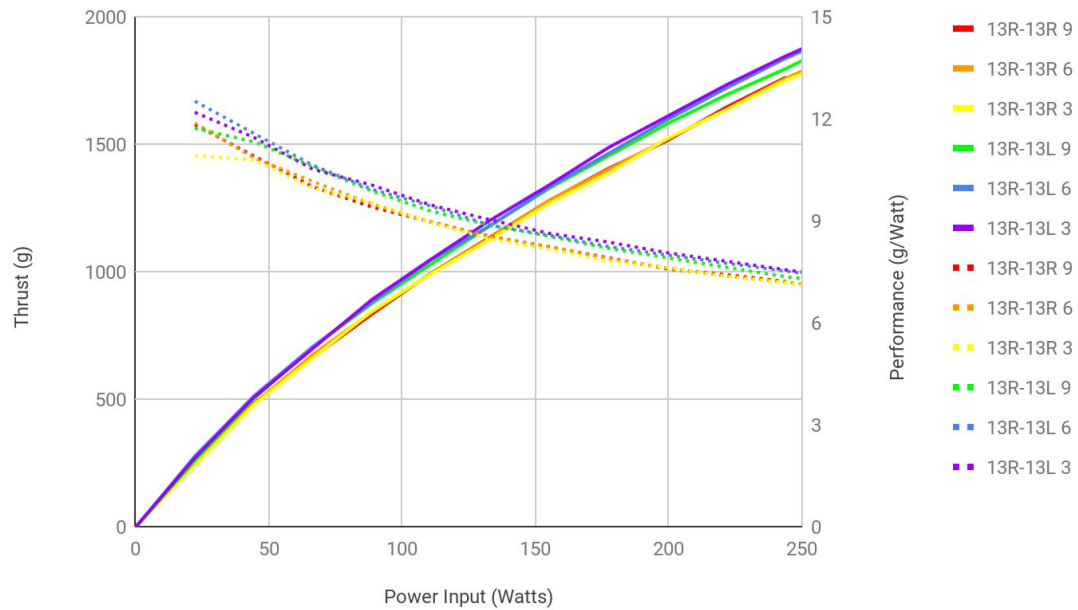
**Figure 10: Experimental results of 15" propellers, showing thrust (solid) and performance (dotted).**

Despite the observation that each propeller is less effective when paired, together they do increase the maximum thrust of the system. The cost of such a system of coaxially rotating propellers is in increased weight and decreased performance, potentially leading to shorter flight times. However, if the peak thrust per area of the system is the requirement, then pairing propellers is a viable solution. The following experiments alter certain variables of the system with the goal of mitigating losses associated with additional coaxial propellers.



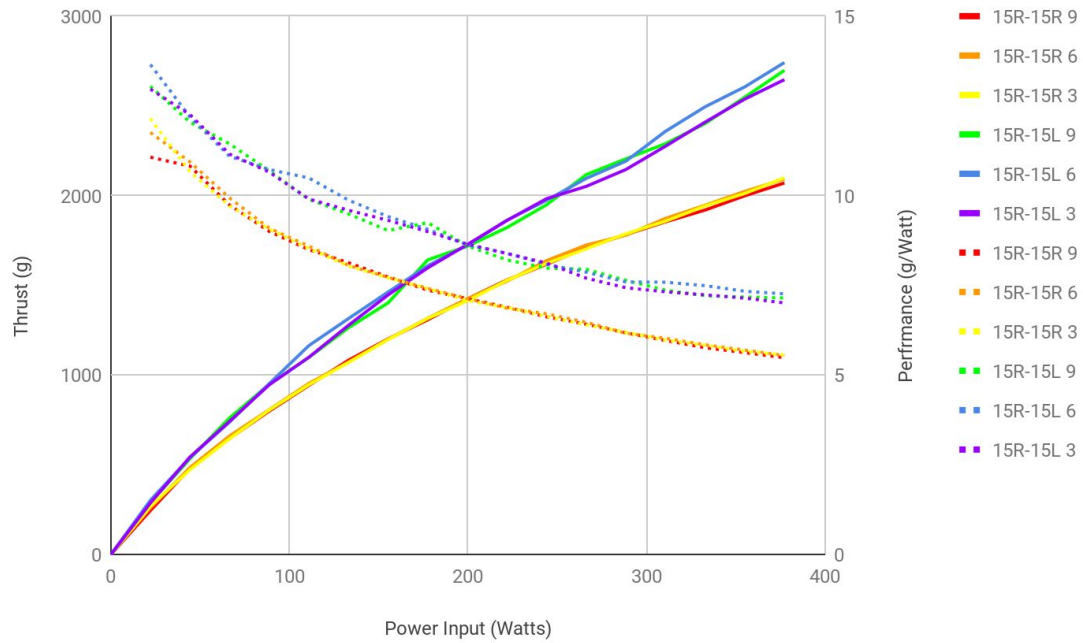
**Figure 11: Experimental results of varying orientation and spacing between 10" propellers, showing thrust (solid) and performance (dotted).**

The results of various combinations of two 10 inch diameter propellers coaxially rotating are shown in Figure 11. Once again, the thrust curves are represented by solid lines and correspond to the left axis, and the performances are represented by the dotted lines and correspond to the right axis. Overall, pairs of 10" propellers have a decrease in performance of 19.2% from a single 10" propeller. The tradeoff to such a loss is that the pairs have a peak thrust 58.7% greater than the single 10". The distance between these propellers does not have any significant effect on the outputs, with less than a 2% difference between the results at 3", 6", and 9" apart. Contra-rotating systems, indicated by 10R-10L in the naming scheme used, produce 3.9% greater thrust and performance over co-rotating systems, 10R-10R.



**Figure 12: Experimental results of varying orientation and spacing between 13” propellers, showing thrust (solid) and performance (dotted).**

Pairs of 13 inch propellers were also analyzed, and the results are shown in Figure 12. Unlike the 10” pairs, the 13” pairs have an average decrease in performance of 3.6% from the single 13” propeller. In addition, the peak thrust of the paired 13” propeller was 67.7% greater than the peak thrust of the solo 13” propeller. Distance between the propellers had no significant effect on the outcome of the experiment. Again, it can be observed that the contra-rotating pairs, named 13R-13L, had 4.9% better thrust and performance over the 13R-13R pairs.



**Figure 13: Experimental results of varying orientation and spacing between 15” propellers, showing thrust (solid) and performance (dotted).**

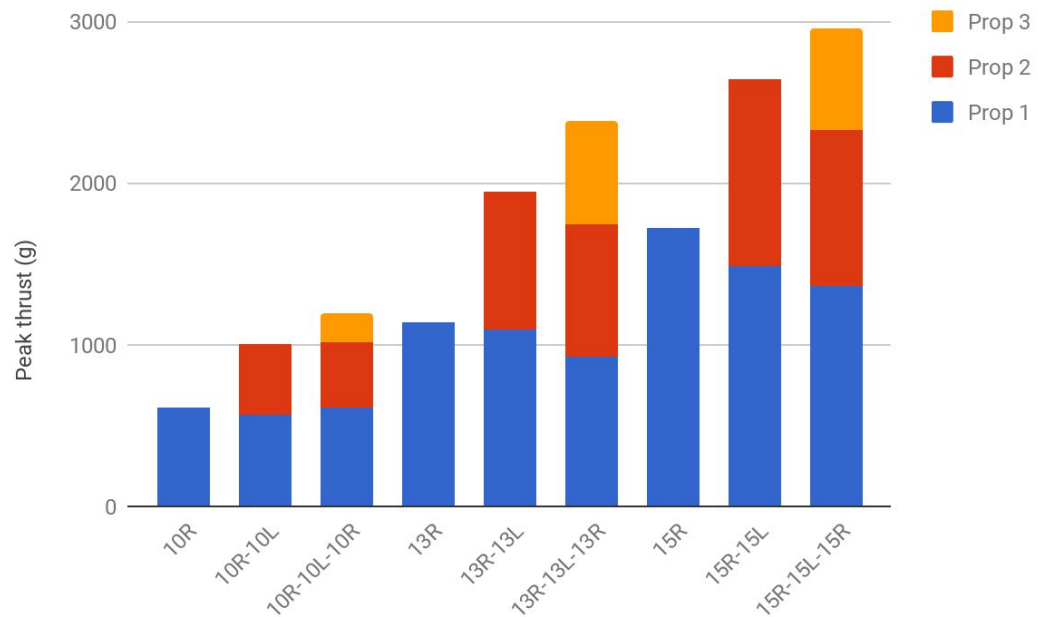
Finally, the performance and thrust outputs for pairs of 15 inch diameter propellers is shown in Figure 13. Overall, the pairs sacrifice 15.6% of the performance of a single 15” propeller for 59.2% greater peak thrust. Once again, the distance between the pairs of coaxially rotating propellers has little effect, less than 2% variation in the thrust and performance at 3”, 6”, and 9”. The advantage of contra-rotating systems is increased in the 15” pairs, with the 15R-15L systems performing 20.9% better than the 15R-15R systems.

In conclusion, if a significant performance loss is an acceptable tradeoff to an increased peak thrust, pairs of coaxially operating propellers should be placed in such an order that they are contra-rotating to one another, at whatever distance is optimal for the geometry of the vehicle.



### 3.3 Trio of Propellers of Constant Diameter

Based on the results of adding a second propeller in the wake of the first, while keeping diameter constant, it can be postulated that adding a third propeller of constant diameter in the wake of the second would have a similar effect on the system. Performance is expected to weaken, while peak thrust is expected to increase. Experiments were run on systems of three propellers of constant diameter with alternating orientation with 3 inch separations between them. The alternating orientation was selected as to take advantage of the previously discussed advantages of contra-rotation. Based on the naming convention discussed in Section 2.3, the system of three 10 inch diameter propellers is called 10R-10L-10R, the system of three 13 inch diameter propellers is called 13R-13L-13R, and the system of three 15 inch diameter propellers is called 15R-15L-15R.



**Figure 14: Thrust capacity of propellers of constant diameter singles, pairs, and trios.**

In Figure 14, the peak thrust results for each system of three propellers is shown in comparison to the results for single and double systems. The peak thrust output for each individual propeller is shown, as well as the total output. For the 10" systems, it can be observed that the front propeller produces a similar peak thrust regardless of how many propellers are operating downstream, suggesting that, at this diameter, the upstream propellers are not strongly affected by the downstream propellers. The average peak thrust variation between the front propellers is 3.1%. This is reinforced by the observation that the second propeller in the 10R-10L system produces only 7.1% greater peak thrust over the second propeller in the 10R-10L-10R system.

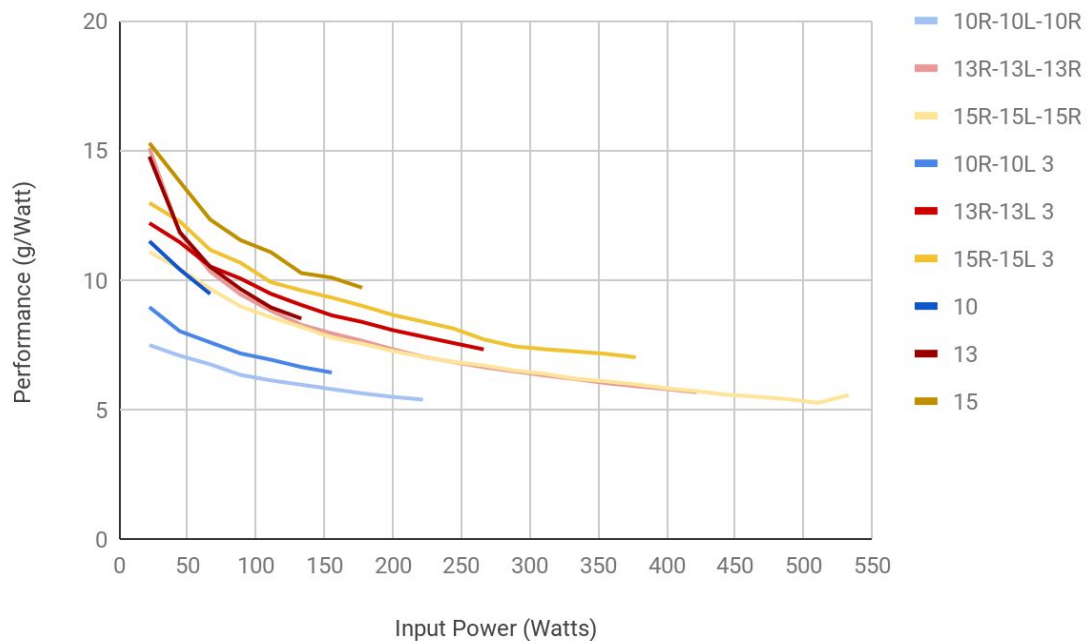
As the diameter of the propeller increases, so do the inter-propeller interference effects increase. For the 13" propeller systems, the peak thrust of the front propeller varies by an average of 7.9%. In particular, the front propeller in the 13R-13L-13R system produces 18.1% less thrust at its peak than the single 13R propeller. The second propeller in the two-propeller system produces 4.3% greater peak thrust than the second propeller in the three-propeller system.

In the systems of 15 inch diameter propellers, the inter-propeller interference effects further increase. The front propeller peak thrust output varies by 8.7%, with the front propeller in the 15R-15L-15R system producing 21.1% less thrust than the single propeller. The second propeller is also strongly affected, with the second propeller in the three-propeller system producing 19.8% less thrust than the second propeller in the 15R-15L system.

A trend of diminishing returns can also be observed in Figure 14 in regards to the addition of additional propellers and the peak thrust of the system. Ideally, the systems of two and three propellers would have a peak thrust 200% and 300% that of the systems of one propeller, respectively. This would be because the thrust output of the first propeller would be equal to the thrust output of the subsequent propellers. In reality, the losses seen by the second and third propellers are significant. For the 10 inch diameter systems, the peak thrust of the second propeller is only 69.3% the peak thrust of the first propeller, and the third propeller has a peak thrust 29.8% the peak thrust of the first. The pair of propellers has a peak thrust 158.5% that of the single propeller, and the 10R-10L-10R has only 189.6% the peak thrust of the single propeller. For the 13 inch diameter systems, the peak thrust of the second propeller is only 80.2% the peak thrust of the first propeller, and the third propeller has a peak thrust 61.9% the peak thrust of the first. The pair of propellers has a peak thrust 172.0% that of the single propeller, and the 13R-13L-13R has only 210.8% the peak thrust of the single propeller. For the 15 inch diameter systems, the peak thrust of the second propeller is only 69.6% the peak thrust of the first propeller, and the third propeller has a peak thrust 41.6% the peak thrust of the first. The pair of propellers has a peak thrust 153.7% that of the single propeller, and the 15R-15L-15R has only 171.8% the peak thrust of the single propeller.

Figure 15 shows the performance of the nine propeller combinations discussed above. For all systems, performance reduces with power. The qualitative behavior of the 10" systems compares well to the 15" systems; the performance of the single

propeller is better than the performance of the double propeller systems, which itself is better than the triple propeller systems. Interestingly, that behavior is not seen the 13" systems. At 22.2 watts of input power, the greatest performance is seen by the 13R-13L-13R system, followed by the 13R and 13R-13L systems, respectively. At greater power, the performance of the 13R and 13R-13L-13R systems matches well, while the 13R-13R system has a greater performance.

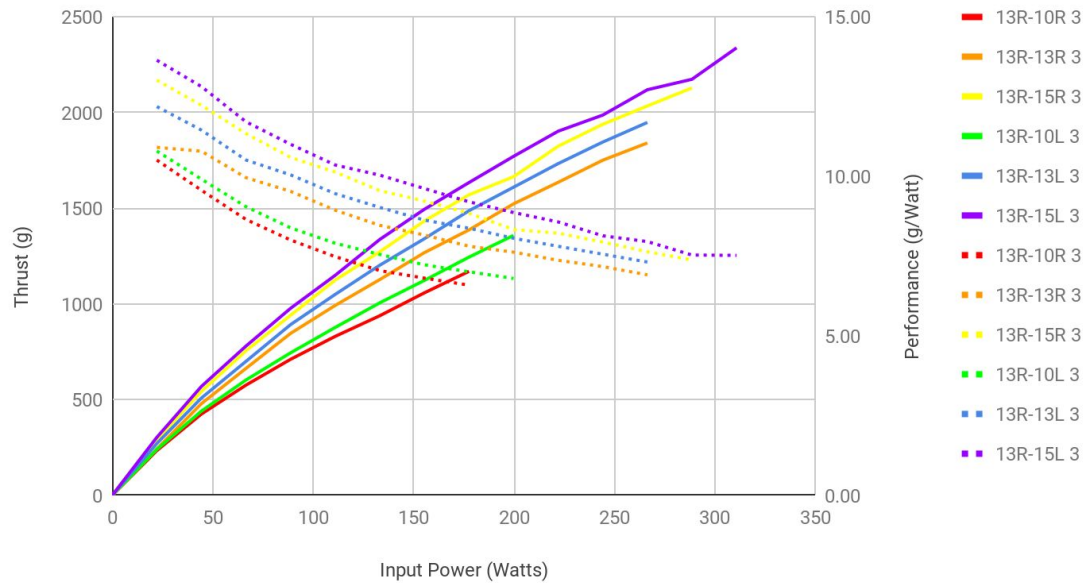


**Figure 15: Thrust output of constant diameter systems, showing thrust (solid).**

### 3.4 Paired Propellers of Varying Diameter

This investigation seeks to investigate the effects caused by varying the propeller diameter within a system of multiple propellers and to see if choosing diameter size and order can help mitigate the losses seen in the multiple-rotor systems of constant diameter. Diameter order can be classified into three trends: increasing diameter,

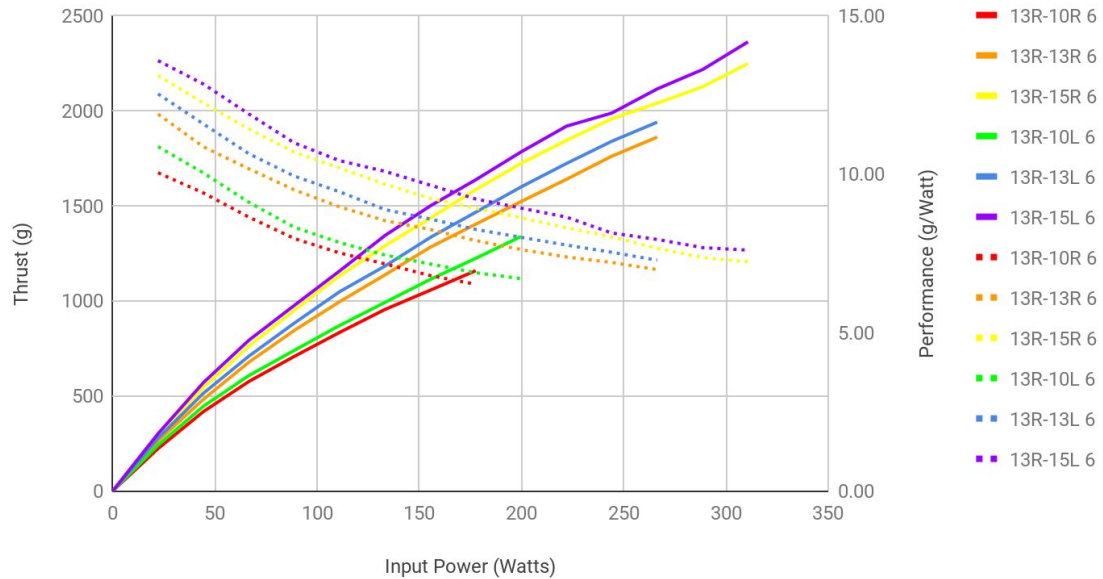
decreasing diameter, and mixed diameter. For pairs of diameters, increasing and decreasing are the only options.



**Figure 16: Experimental results of 13" diameter propeller-led systems at 3" separation, showing thrust (solid) and performance (dotted).**

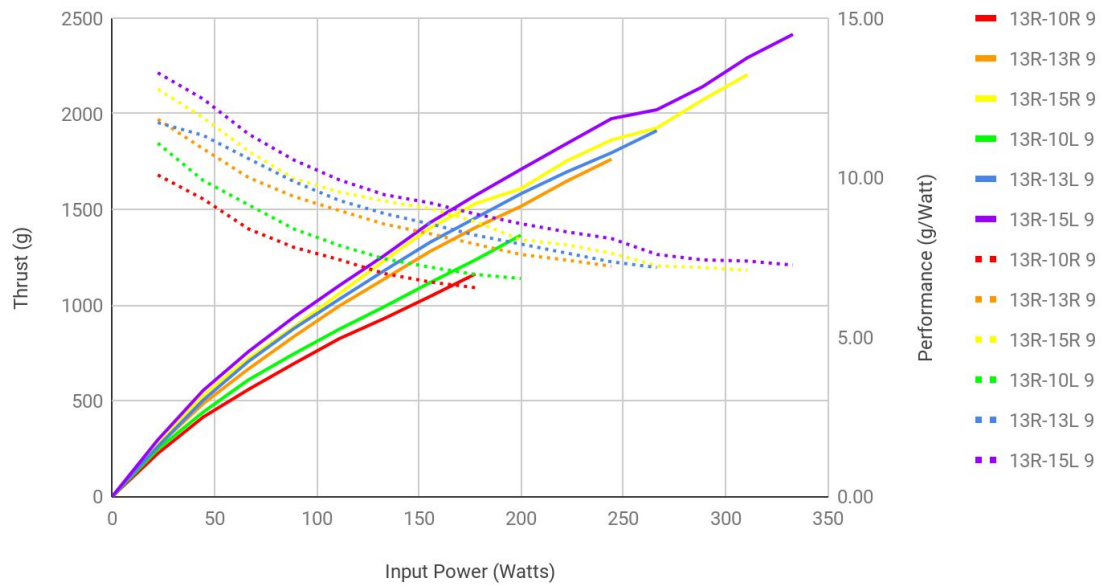
The first step taken was to test the systems of varying diameter for the behaviors established by the systems of constant diameter. The results of testing the 13R propeller with the second propeller of all three available diameters, both rotation orientations, and at separation distances of 3 inches, 6 inches, and 9 inches are shown in Figures 16, 17, and 18. The thrust curves are represented by solid lines and correspond to the left axis, and the performances are represented by the dotted lines and correspond to the right axis. Distances between the propellers has no significant effect, with variations between 3", 6", and 9" separations producing less than 2.5% difference on average. This is expected, as the systems of constant diameter had less than 2% difference for varying separation distances. When comparing the nine

contra-rotating systems to the nine co-rotating systems, holding propeller separation diameters the same, the contra-rotating systems produce 4.8% more thrust than the combinations rotating the same direction. This matches the 13" systems of constant diameter, in which contra-rotation increased output by 4.9%.



**Figure 17: Experimental results of 13" diameter propeller-led systems at 6" separation, showing thrust (solid) and performance (dotted).**

Therefore, to reduce the scope of further testing from 324 possible combination to 9 possible combinations, only contra-rotating combinations at 3 inch separation distances will be tested.

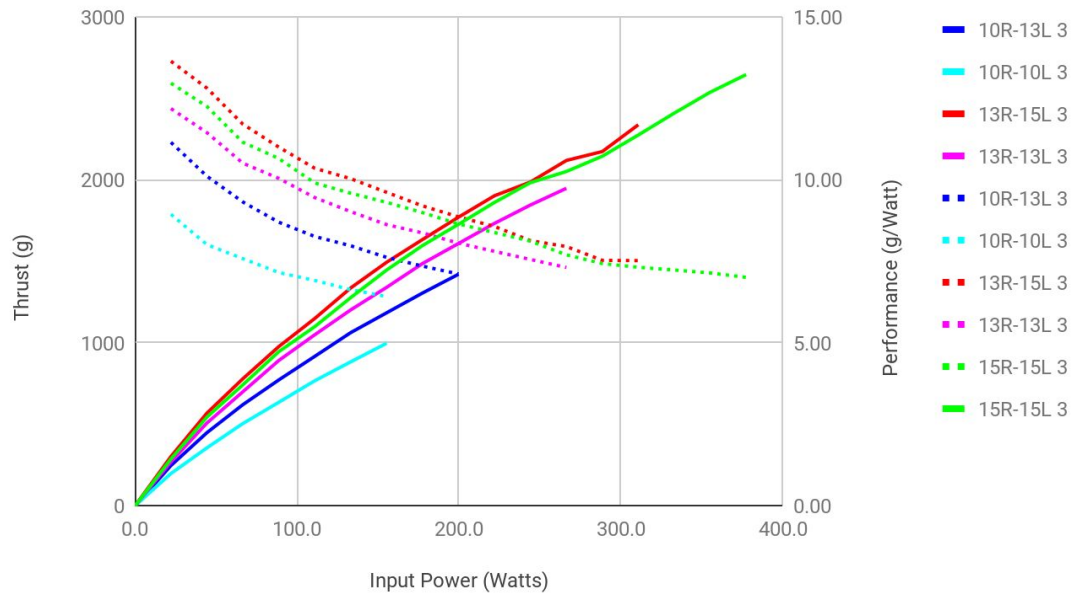


**Figure 18: Experimental results of 13" diameter propeller-led systems at 9" separation, showing thrust (solid) and performance (dotted).**

### 3.5 Paired Propellers of Increasing Diameter

The next set of tests performed were of pairs of contra-rotating propellers at three inch separation distances where the diameter of the first propeller streamwise was smaller than that of the second propeller. These are the systems of increasing diameter: 10R-13L and 13R-15L. As the wake created by rotors widens with distance from the rotor, it is possible that a larger propeller downstream would be able to capture more clean air than a propeller of the same diameter, and therefore pairs of propellers of increasing diameter would perform better than pairs of constant diameter. The thrust and performance results are shown in Figure 19 and compared to systems of constant diameter. The peak thrust of the 10R-13L pair is 42.3% greater than that of the 10R-10L, with only a 28.6% increase in input power. Similarly, the peak thrust of the 13R-15L pair is 20.0% greater than that of the 13R-13L pair, requiring 16.7% higher input power. As shown in Figure 19, the pairs of increasing diameter propellers have greater peak thrust and greater peak performance than that of the pairs of constant diameter propellers. In fact, the increasing diameter pairs have consistently greater thrust and performance than the constant diameter pairs, with 10R-13L having 22.0% better performance on average than the 10R-10L pair and 13R-15L performing 10.3% better on average than 13R-13L.

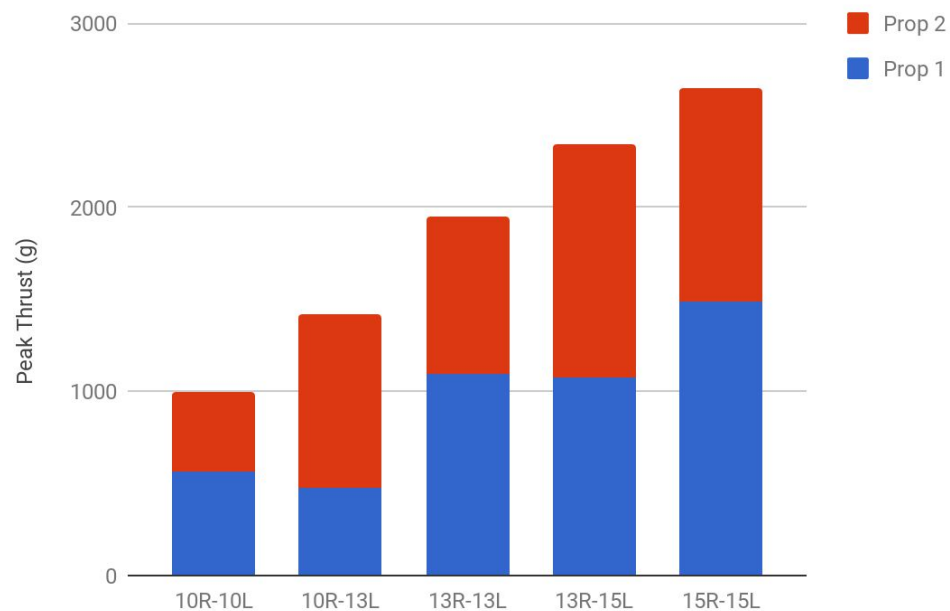




**Figure 19: Experimental results of increasing and constant diameter systems, showing thrust (solid) and performance (dotted).**

Figure 20 shows the thrust of the increasing diameter and constant diameter pairs of propellers at their peak output, as limited by their maximum input power, broken into the thrust produced by each propeller individually. It can be seen that the 10 inch diameter forward propeller is interfered with by the 13 inch rear propeller, producing 16.5% less thrust than the front propeller of the 10R-10L pair and 26.7% less thrust than an individual 10 inch diameter propeller. However, these losses are mitigated by the thrust provided by the 13 inch propeller, which is operating 82.5% the capacity of an individual 13 inch diameter propeller, bringing the total peak thrust to 42.4% greater than the constant diameter pair of 10 inch propellers, as previously discussed. In fact, it can be seen that the rear 13 inch propeller is interfered with less by a front 10 inch propeller than a front 13 inch propeller, as the peak thrust of the rear propeller in 10R-13L is 11.2% greater than the rear propeller of 13R-13L.

The losses seen by the front propeller in the second set of pairs were less, with the front 13 inch diameter propeller in 13R-15L having a peak thrust only 2.0% less than the front 13 inch diameter propeller in 13R-13L. In addition, the rear 15 inch propeller was not strongly interfered with by the front 13 inch propeller, operating with a peak thrust 9.2% better than the rear propeller in 15R-15L, and 26.7% worse than an individual 15 inch propeller.



**Figure 20: Experimental results showing the maximum thrust output of each propeller in pairs.**

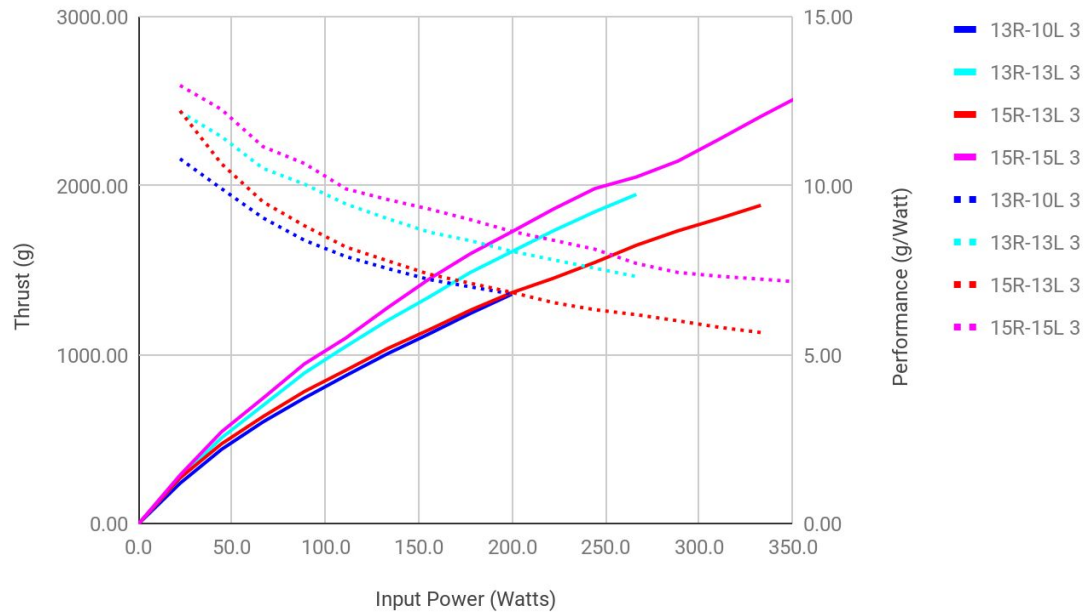
Pairs of increasing diameter have two major requirements over pairs of constant diameter: greater input power and the spatial clearance required to operate the larger propeller. If these constraints can be met, then pairs of propellers of increasing diameter should be used in place of pairs of propellers with diameter equal to the front propeller, as they are both more efficient and have a greater capacity for maximum thrust.

However, pairs of propeller of varying diameter can also be viewed as a smaller propeller in front of a larger one, and therefore must also be compared to pairs of propellers of constant diameter equal to the diameter of the rear propeller. After all, if the spatial clearance and input power can be spared in the system, it may be advantageous to use two of the larger propellers instead of a smaller and larger pair. The comparison will then be made between 10R-13L and 13R-13L, and between 13R-15L and 15R-15L. As shown in Figure 20, the peak thrust of 10R-13L is 27.1% less than the peak thrust of the 13R-13L, and consistently performs worse, with an average performance 11.5% less. Therefore the pair of constant 13 inch diameter propellers should be used instead of the pair of propellers of increasing diameter from 10 inches to 13 inches. Similarly, the peak thrust of the 13R-15L pair is 11.6% less than the peak thrust of the 15R-15L pair. In contrast, however, the 13R-15L pair performs consistently better than the pair of constant diameter, with an average performance 3.2% greater. Therefore, if efficiency is more valuable than peak thrust output, the pair of propeller of varying diameter from 13 to 15 inches should be used instead of the pair of propellers of constant 15 inch diameter.

### **3.6 Paired Propellers of Decreasing Diameter**

The other way in which diameter can vary in pairs is by decreasing, with a front propeller larger than the rear propeller downstream of it. The pairs of propellers arranged like this were 13R-10L and 15R-13L, both at 3 inches of separation between the two propellers. Due to the incoming air stream into the rear rotor coming entirely

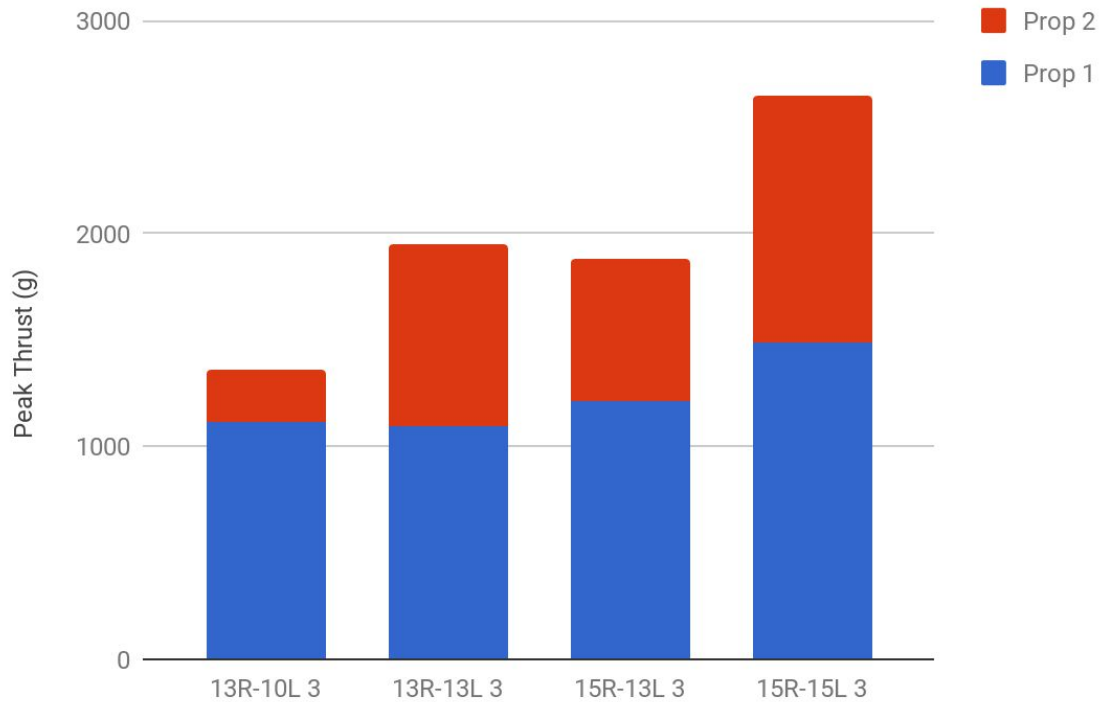
through the front propeller, it is estimated that the performance of pairs of propellers of decreasing diameter would be worse than pairs of constant diameter.



**Figure 21: Experimental results of decreasing and constant diameter systems, showing thrust (solid) and performance (dotted).**

The performance and thrust results of the decreasing diameter pairs of propellers are shown in Figure 21, and compared to the results of pairs of constant diameter propellers. The thrust curves are represented by solid lines and correspond to the left axis, and the performances are represented by the dotted lines and correspond to the right axis. The 13R-10L pair has a peak thrust 30.3% less than that of the 13R-13L pair. It also performs 15.5% worse on average consistently over each input power. The 15R-13L system has a peak thrust 28.8% less than the 15R-15L system, and consistently performs worse, with an average performance 18.3% less than the pair of constant 15 inch diameter propellers.

The peak thrust of each pair is shown in Figure 22, with a delineation between the thrust of the front and rear propellers. The front 13 inch diameter propeller is not strongly affected by the 10 inch or 13 inch propellers downstream, with the front propeller in 13R-10L having a peak thrust 1.2% greater than the front 13 inch propeller in the pair of constant diameter. The rear 10 inch propeller's peak thrust suffered from being put behind the larger 13 inch propeller, and had a peak thrust 42.1% less than the rear propeller in 10R-10L, and 59.4% less than an individual 10 inch diameter propeller. Similarly, the rear 13 inch diameter propeller suffered from being put downstream of the larger 15 inch diameter propeller, with a peak thrust 20.9% less than the rear propeller in the pair of 13 inch propellers, and 40.6% less than a single 13 inch diameter propeller.

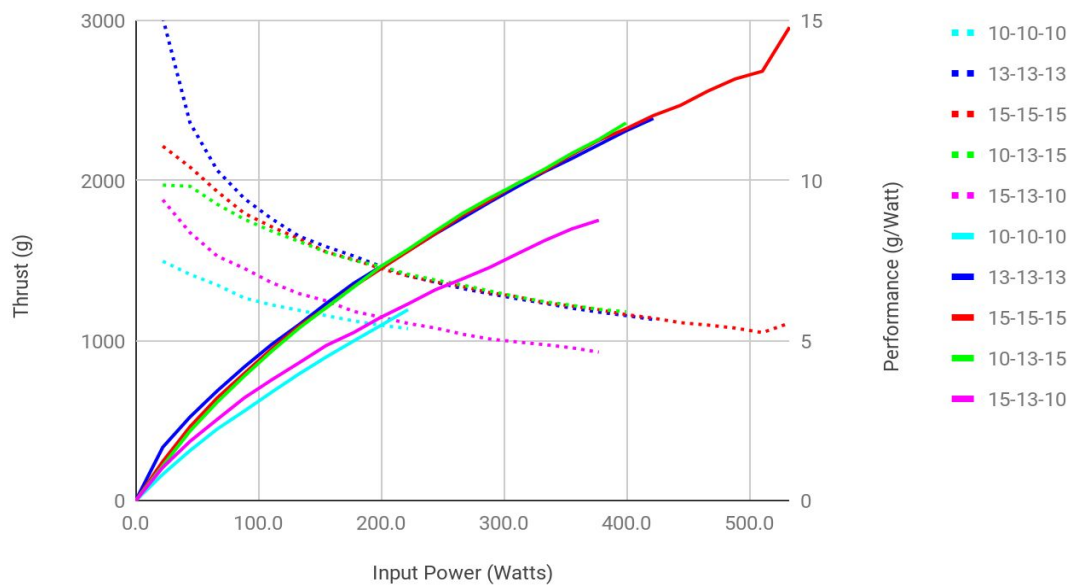


**Figure 22: Experimental results showing the maximum thrust of each propeller.**

Interestingly, the front 15 inch propeller in 15R-13L had a peak thrust 18.7% less than the front propeller in 15R-15L. One possible explanation of this is that the 15R-15L system can accept 44 more watts of input power than the 15R-13L, and therefore the front propeller can use more power at lower performance.

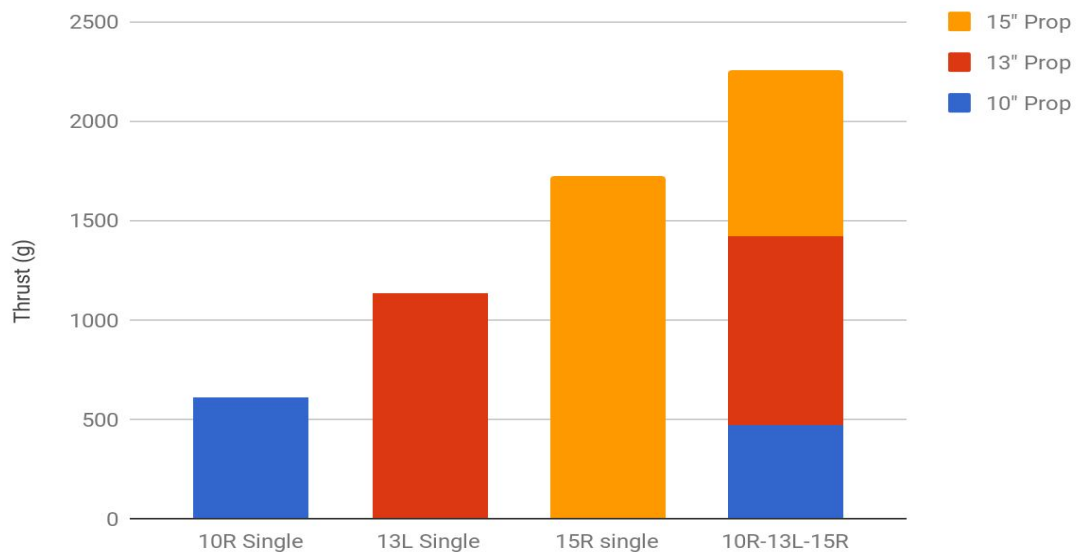
### 3.7 Trio of Propellers of Varying Diameter

As with the previously discussed systems of multiple propellers of constant diameter, the next set of investigations look for the behavioral trends seen in pairs of propellers of varying diameter to be replicated in systems of three varying diameter propellers. With three propeller diameters available, the system of increasing diameters were the 10", 13", and 15" propellers in streamwise order, and the system of decreasing diameters was the 15", 13", and 10" propellers. The systems had 3 inch separations between the propellers, and propellers contra-rotated with respect to the propellers adjacent to them.



**Figure 23: Experimental results of increasing and constant diameter propeller trios, showing thrust (solid) and performance (dotted).**

Figure 23 shows the results of the trios of propellers of varying diameter compared to trios of propellers of constant diameter. Performance for each system is shown with a dotted line and thrust is shown with a solid line. As you can see, the trio of increasing diameter propellers behaved similarly to the trios of 13 inch and 15 inch propellers. Trios of propellers would be used primarily in high thrust requirement applications, and therefore their divergent behavior below 100 watts of input power can be ignored. If a vehicle were to operate for a significant period of time below that input power, a single 13 inch or 15 inch propeller should be used instead of a trio of propellers. Above that power input, the thrust and performance of the 13R-13L-13R, 15R-15L-15R, and 10R-13L-15R vary within 1% of each other on average. Therefore, a trio of increasing diameter propellers is comparable to a trio of propellers of constant diameter equal to the largest diameter, and do not offer any obvious advantage.



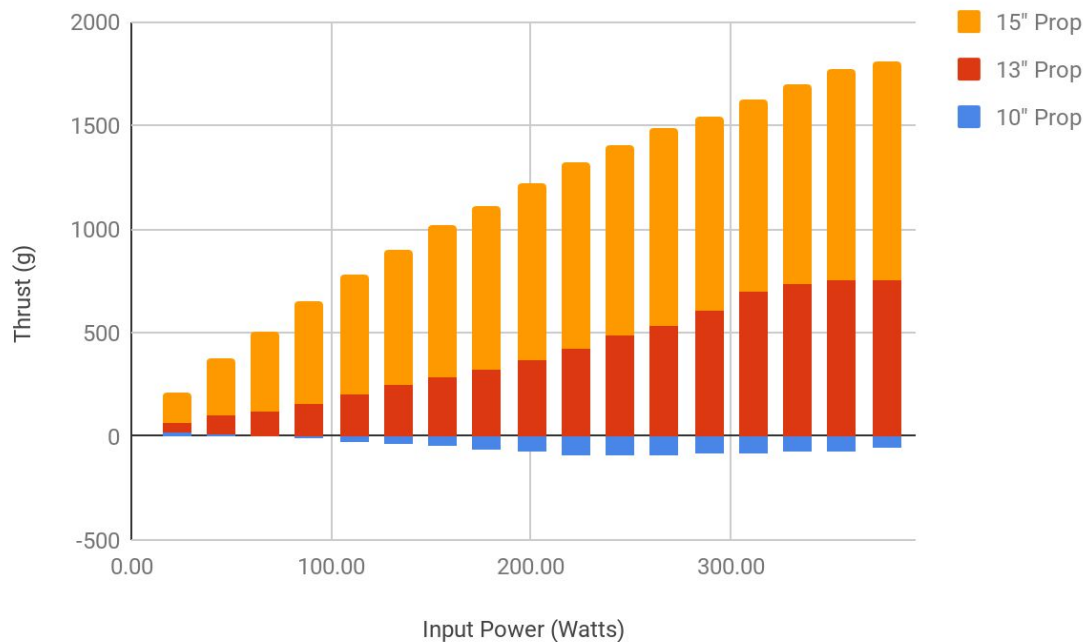
**Figure 24: Maximum thrust of propellers in a coaxial trio of increasing diameter.**

The peak thrust output of the 10R-13L-15R system can be separated into the contributions made by each propeller. The maximum thrust output of the trio of increasing diameter propellers is shown in Figure 24, alongside the maximum thrust output of a single 10", 13" and 15" propeller. As previously discussed, the 10 inch diameter propeller in the 10R-13L system produced 73.3% of the peak thrust capacity of a standalone 10" propeller. Similarly, the 10" propeller of the 10R-13L-15R produces 76.9% the peak thrust of the standalone 10" propeller. There is also agreement in the peak thrust capacity of the 13" propeller downstream of a 10" propeller, with the 13 inch diameter propeller in both the 10R-13L and 10R-13L-15R systems producing 83.5% the maximum thrust capacity of an unimpeded 13" propeller. By the time the airflow reaches the final 15" propeller in the trio of increasing diameter propellers, it has passed through two contra-rotating propellers and is highly turbulent. Therefore, the 15" propeller has a maximum thrust output of only 48.7% of a single 15" propeller, which is 33.6% less thrust than the 15" propeller in the 13R-15L pair. As with the trios of propellers of constant diameter, the maximum thrust of the system follows a law of diminishing returns when adding coaxial propellers.

Given the same hardware to use, the order of the propeller diameters has a great effect on the outputs of the system. As shown in Figure 25, the trio of propellers of decreasing diameter performed much worse than the trios of constant or increasing diameter. The 15R-13L-10R had a peak thrust 74.3% that of the 10R-13L-15R, and performed 19.6% worse on average. The thrust output of each propeller of the trio of



decreasing diameter propellers can be seen in Figure 25. The front 15 inch propeller only produces 61.3% the peak thrust of a single 15” propeller. Similarly to the 15R-13L pair of propellers, in which the 13” propeller produced 59.4% the peak thrust of a single 13” propeller, the 13” propeller in 15R-13L-10R produces 66.6% the maximum thrust output of a single 13” propeller. Due to the momentum imparted to the airflow by the 15” and 13” propellers, the rear propeller was unable to compensate with thrust, and the load cell read that the 10 inch diameter propeller experienced negative thrust. The same conclusion that was drawn with pairs of decreasing diameter propellers must be drawn for trios of decreasing diameter propellers, in that they offer significant disadvantage, and propellers should not be placed in that order when designing a vehicle.



**Figure 25: Experimental thrust results of decreasing diameter trio of propellers.**



### 3.8 Paired Propellers Controlled Separately

The final sets of experiments were performed to determine if controlling each propeller separately in a pair of propellers would offer some advantage over allowing a single controller to operate them both. To determine this, pairs of contra-rotating propellers with 3 inch separation distances were tested, with the rear propeller being 15" in diameter and the front propeller being varying. Each propeller's power input was delivered through a separate controller, so that each propeller could operate at 0%, 25%, 50%, 75%, and 100% of its maximum input power, as determined by the individual results for each propeller shown in Figure 7. The thrust results are shown in Figure 26.

The trends for thrust followed what was previously seen, with thrust increasing as power input increased. The greatest thrust delivered at any given input power was when each propeller was being delivered the same fraction of maximum input power as the other propeller. Therefore, systems of two propellers should be operated with balanced power input to each propeller.

When power is directed to only one propeller, either the front or rear, in 15R-15L, each propeller produces 89% the maximum thrust of a single 15" propeller. However, if only the front propeller was delivered power, the system collectively produced far less power than the system where only the rear propeller was delivered power. This is because, in the front-propeller-driven system, the rear propeller is pushed back by the flow of air from the front propeller and experiences negative thrust. A similar interference pattern is not seen from the rear to the front propellers.

This is why rear-propeller-driven systems, systems where power is slightly off-balance in favor of the rear propeller, produce the most thrust for a given input power.

Total Thrust of System (g)		Rear Prop	15"				
		Input Power (Watts)	0.0	50.0	99.9	149.9	199.8
Front Prop	Input Power (Watts)	% Throttle	0%	25%	50%	75%	100%
15"	0.0	0%	0.00	597.27	935.00	1228.02	1502.90
	50.0	25%	539.17	1044.44	1382.26	1675.45	1953.37
	99.9	50%	834.70	1346.93	1675.00	1985.57	2259.66
	149.9	75%	1116.17	1587.90	1915.78	2204.77	2466.23
	199.8	100%	1336.18	1786.57	2109.76	2404.44	2672.20
13"	0.0	0%	0.00	576.06	943.67	1263.59	1532.51
	33.3	25%	348.70	874.96	1238.84	1561.05	1848.19
	66.6	50%	575.54	1090.70	1468.07	1793.31	2095.89
	99.9	75%	779.31	1282.34	1661.38	1967.15	2259.19
	133.2	100%	952.58	1435.17	1819.69	2127.33	2384.99
10"	0.0	0%	0.00	603.10	1014.77	1289.08	1506.31
	16.7	25%	162.39	747.37	1168.63	1471.60	1710.73
	33.3	50%	284.76	878.88	1298.50	1595.91	1898.13
	50.0	75%	402.42	977.63	1380.35	1685.95	1973.25
	66.6	100%	490.06	1047.63	1459.73	1770.81	2050.40

**Figure 26: Thrust Output of Pairs of Propellers with Separately Controlled Inputs**

The performance results for these experiments are shown in Figure 27. Similarly to previous results, performance of each system is greater at lower input power levels.

Performance also experiences a slight improvement in systems where the rear propeller receives as much or one step more power than the front propeller, as thrust capacity for a given power input is improved in rear-propeller-driven systems. However, balanced systems have the best performance and thrust for a given input power, therefore making separate controllers for each propeller unnecessary.

Performance (g/Watt)		Rear Prop	15"				
		Input Power (Watts)	0.0	50.0	99.9	149.9	199.8
Front Prop	Input Power (Watts)	% Throttle	0%	25%	50%	75%	100%
15"	0.0	0%		11.96	9.36	8.19	7.52
	50.0	25%	10.79	10.45	9.22	8.39	7.82
	99.9	50%	8.36	8.99	8.38	7.95	7.54
	149.9	75%	7.45	7.95	7.67	7.36	7.05
	199.8	100%	6.69	7.15	7.04	6.88	6.69
13"	0.0	0%		11.53	9.45	8.43	7.67
	33.3	25%	10.47	10.51	9.30	8.52	7.93
	66.6	50%	8.64	9.36	8.82	8.29	7.87
	99.9	75%	7.80	8.56	8.32	7.88	7.54
	133.2	100%	7.15	7.84	7.81	7.52	7.16
10"	0.0	0%		12.07	10.16	8.60	7.54
	16.7	25%	9.75	11.22	10.03	8.84	7.90
	33.3	50%	8.55	10.56	9.75	8.71	8.14
	50.0	75%	8.06	9.79	9.21	8.44	7.90
	66.6	100%	7.36	8.99	8.77	8.18	7.70

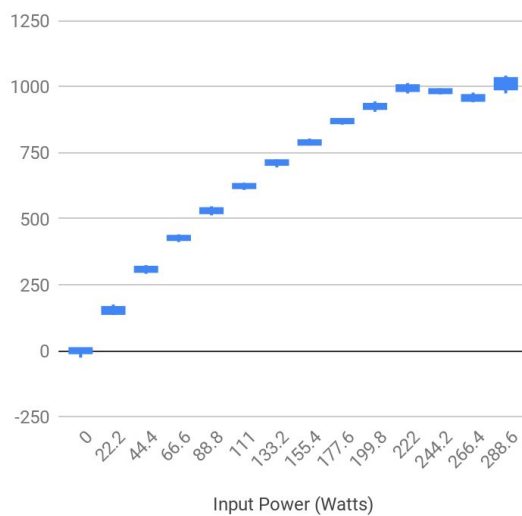
**Figure 27: Performance of Pairs of Propellers with Separately Controlled Inputs**



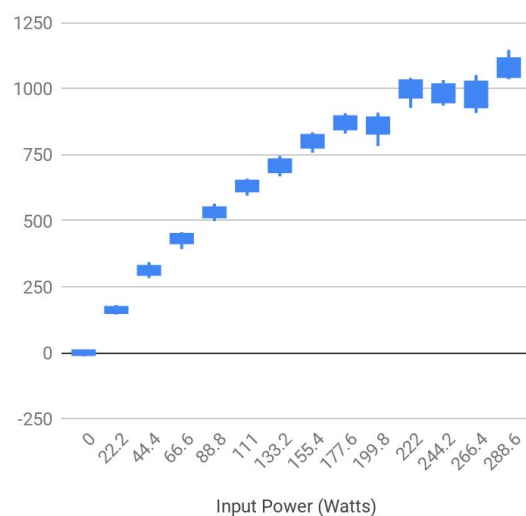
## 4. Analysis and Discussion

### 4.1 Vibration

As observed in some of the tests, the downstream propeller would vibrate back and forth in a violent manner. This due to the front propeller rotating in the air stream, causing the pressure in the airstream to fluctuate with a frequency dependent on the rotational speed of the front propeller.<sup>19, 20</sup> It is assumed that the rear propeller resonated with this frequency, causing the vibration. As the vibration was in the same linear direction, towards and away from the front propeller, as the axis of the load cell, the load measured fluctuated greatly when high vibration occurred. The values used in the data analysis are the average of the collected data points in those conditions. Figures 28 and 29 below shows the data from the rear 15 inch propeller when in the 13R-15R systems with 3 inches and 6 inches of separation at various input powers. The average load value increases as expected, as does the range caused by vibration of the rear propeller for the 13R-15R system at 3 inches of separation.



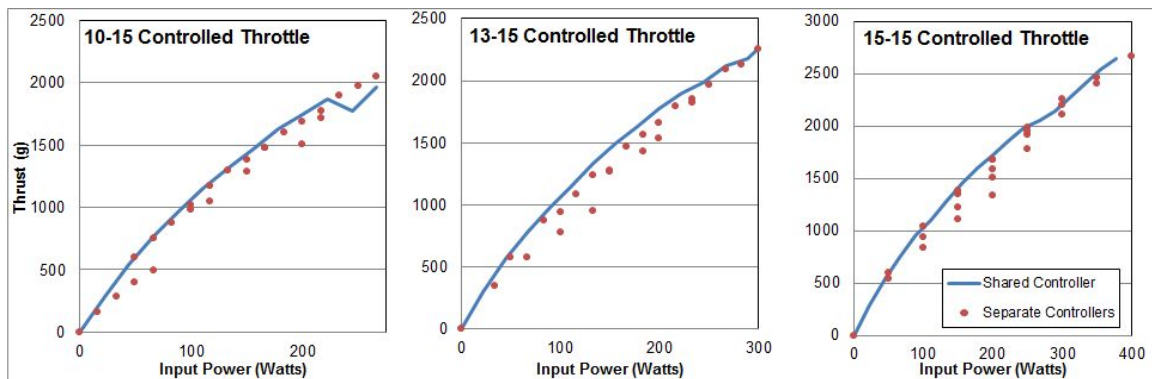
**Figure 28: 13R-15R 6 vibrations.**



**Figure 29: 13R-15R 3 vibrations.**

## 4.2 Input Power Balancing

One concern with the test setup was that the propeller controller was feeding electrical power to all propellers in a system. Therefore, it was possible that it was feeding the power in an imbalanced or inefficient manner. This concern can be eliminated by comparing the results of systems controlled by one controller to those same systems with separate controllers for each propeller. The results of controlling the propellers separately for the 10R-15L, 13R-15L, and 15R-15L systems have been transcribed from Figure 26 and are shown in Figure 30 as red scattered points. The results from those same systems with a single controller for both propellers are also shown in Figure 30 as the blue line. As the blue line is consistently as great or greater than the data from the systems controlled manually, it can be concluded that the controller ideally balanced the power input into each propeller.



**Figure 30: Thrust Output of Various Propeller Configurations with Separate and Shared Input Controls**



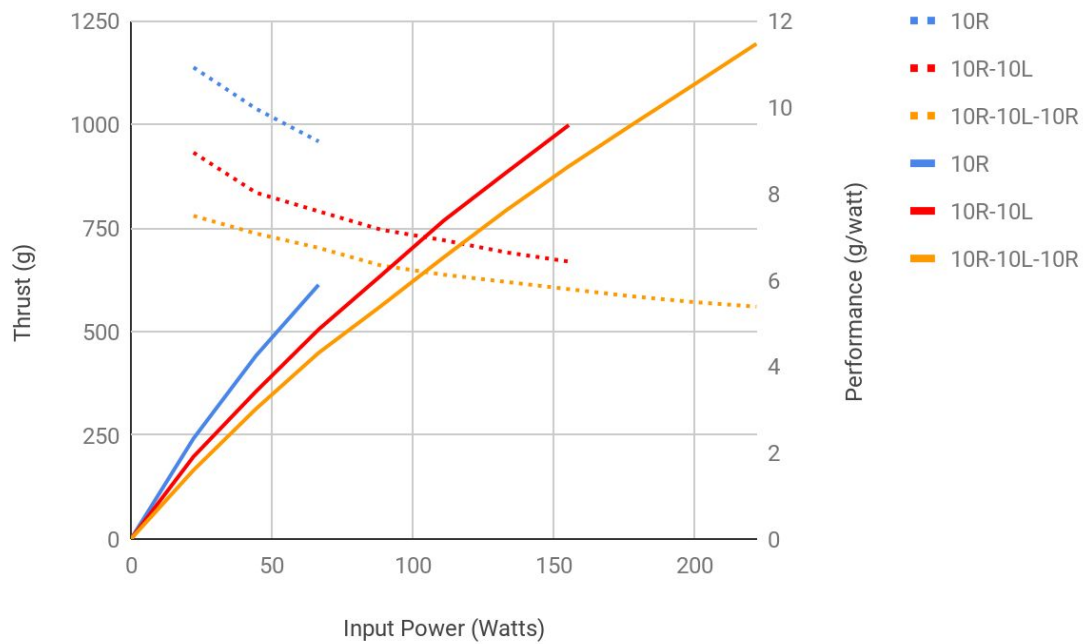
### 4.3 Choosing a System Given Limited Spatial Clearance

When designing an autonomous aerial vehicle, the goals to be achieved by the vehicle must be designed for, along with the constraints of the vehicle. Conclusions drawn from the experiments discussed in Section 3 can be used as part of this design process. The primary goal that can be achieved through the use of multiple coaxially rotating propellers is the increase in maximum thrust output of a system, with a secondary goal being efficiency of the system. Constraints on such a system are spatial limitations for the area in which the propellers can operate, weight, and cost. The following discussion seeks to draw conclusions about systems of propellers about maximizing peak thrust and minimizing performance loss within certain spatial limitations for the propellers. Height is not considered a spatial limitation for this discussion, and separation between propellers is kept at 3 inches. In addition, contra-rotating systems of propellers are used, as they consistently perform better than propellers co-rotating.

In the following figures, the thrust curves are represented by solid lines and correspond to the left axis, and the performances are represented by the dotted lines and correspond to the right axis. The spatial limits are applied such that the maximum propeller diameter usable in a certain application is 10 inches, 13 inches, or 15 inches.

If the application being designed limits the diameter of its propellers to 10 inches or less, the options available that have been investigated in this thesis are shown in Figure 31. A system of two coaxially contra-rotating 10 inch diameter propellers has a 63% increase in maximum thrust output from a single 10 inch diameter propeller

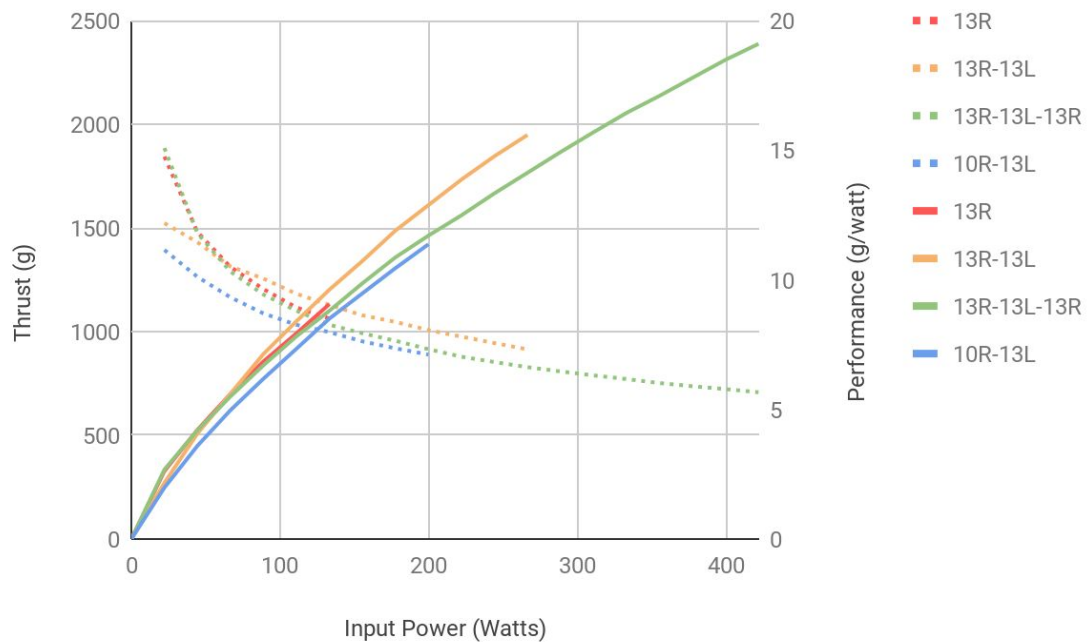
with twice the maximum power consumption. Adding a third 10 inch propeller increases the maximum thrust by 95% from a single propeller, and consumes thrice the total power input. Therefore, pairs and trios of 10 inch propellers should be used in place of a single 10 inch propeller when efficiency can be sacrificed in exchange for increased maximum thrust of the system.



**Figure 31: Experimental results of propeller combinations that fit a spatial limitation of 10", showing thrust (solid) and performance (dotted).**

If the application being designed limits the diameter of its propellers to 13 inches or less, the options available that have been investigated in this thesis are shown in Figure 32. Figure 7 in Section 3.1 shows that the 13 inch diameter propeller has a greater maximum thrust and better performance than a single 10 inch diameter propeller, and therefore if the design of the vehicle allows for the larger propeller to fit, it should be used. Adding a 10 inch diameter propeller in upstream of the 13 inch

propeller increases the peak thrust of the system by 25.5%, but consistently performs worse. Instead, greater maximum thrust than a single 13 inch propeller can provide is needed, a pair of co-axially contra-rotating 13 inch propellers should be used, for an increase to 172.0% the maximum thrust capacity for twice the electrical input, meaning that the efficiency is decreased by only 1.5% for the overlapping input power range. Finally, if even more thrust output is needed, a third 13 inch diameter propeller can be used, for a peak thrust 210.8% greater than a single 13 inch propeller for three times the input power, with a performance 1.0% less than the single 13 inch propeller over the same input power range, and a decreasing performance at greater input power.



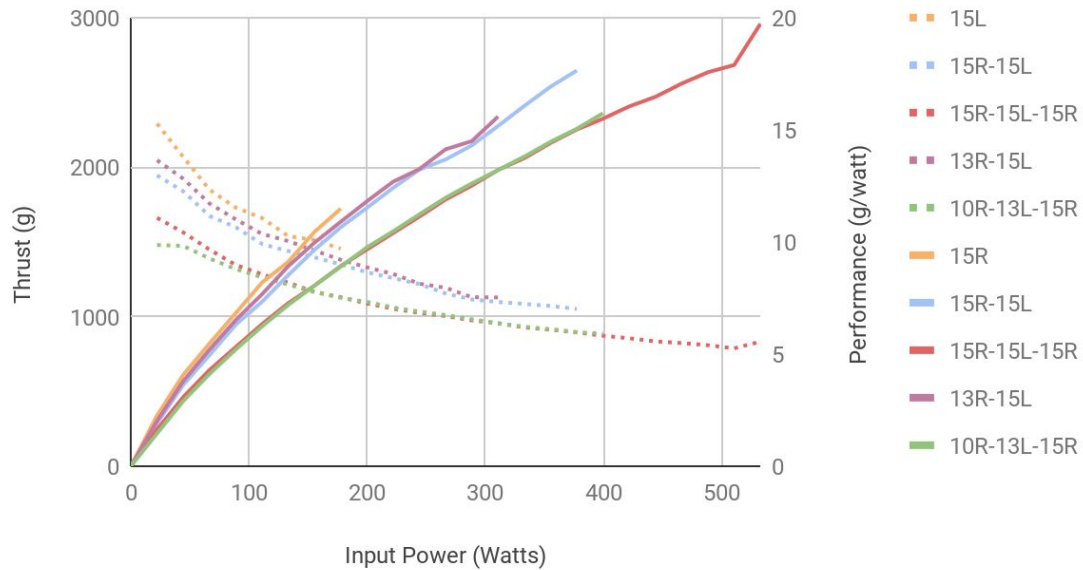
**Figure 32: Experimental results of propeller combinations that fit a spatial limitation of 13", showing thrust (solid) and performance (dotted).**

If the application being designed limits the diameter of its propellers to 15 inches or less, the options available that have been investigated in this thesis are shown in Figure 33. As shown previously in Figure 7, the larger 15 inch single propeller is more efficient and has greater peak thrust than the smaller single propellers, and therefore if the spatial clearance for a 15 inch propeller is available, one should be used. Adding a 13 inch propeller in front of the 15 inch propeller increase the peak thrust by 35.8% but decreases the performance by 5.7%. Adding a 15 inch propeller in front of a 15 inch propeller increases the peak thrust by 53.7% but decreases the performance by 9.4%. Therefore, if greater maximum thrust is the desired design regime, a pair of 15 inch propellers should be used, but if both efficiency and greater peak thrust are needed the increasing diameter system 13R-15L should be used.

Adding the 10 inch diameter propeller in front of the 13-15 pair to create a trio of propellers produces a system with 0.9% greater maximum thrust, but 18.7% worse performance. In practice, the additional weight added to a system to mount and control an additional propeller will further worsen the efficiency, and therefore flight time, of the vehicle. Therefore, the trio of increasing diameter propellers 10R-13L-15R is not practical to use. For maximum thrust regardless of efficiency, three 15 inch propellers can be used coaxially, for a peak thrust 71.8% greater than a single 15 inch propeller but with a performance 23.0% worse within the same input power range. Within the operating regime inaccessible to a single 15 inch diameter propeller and therefore requires additional propellers, the performance of a trio of 15 inch diameter propellers is much worse, requiring three times as much input electrical

power to achieve less than two times the peak thrust. Therefore, systems of multiple constant-diameter propellers should only be used when maximum thrust for a short time is required of the vehicle.

Figure 33: Choosing a System with a Spatial Limit of 15"



**Figure 33: Experimental results of propeller combinations that fit a spatial limitation of 15", showing thrust (solid) and performance (dotted).**

Systems where upstream propellers have a diameter greater than downstream propellers, discussed in this thesis as systems of decreasing diameter propellers perform consistently worse than systems of increasing diameter and systems of constant diameter, and should not be used where other options are available.

#### 4.4 Performance of Obscured Propellers

It is theorized that the thrust capacity for a propeller is proportional to the volumetric flowrate of air into the propeller, and therefore is dependent on the cross-sectional area of the airstream. While downstream propellers operate with the

same diameter and cross-sectional operating area, their effective operating area is reduced by the upstream propeller's operating area. The obscured area can be calculated from the following equations, and the thrust capacity is the peak thrust output of the propeller divided by the maximum thrust output of that same propeller unobscured.

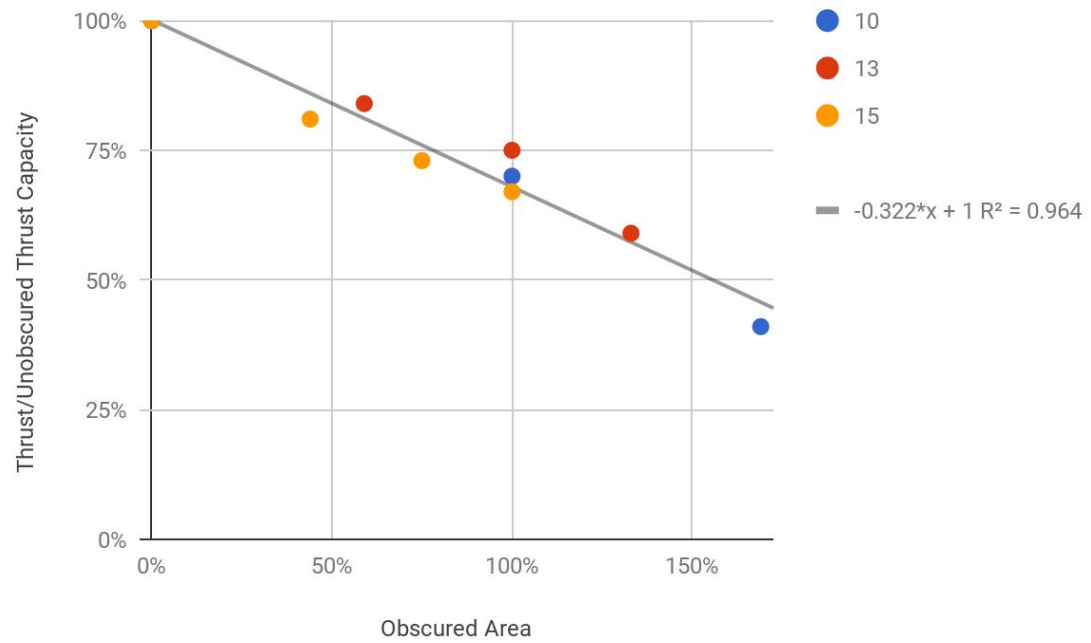
$$A = A_{prop\ 1}/A_{prop\ 2}$$

$$Percent\ Thrust\ Capacity = T_{prop}/T_{single\ prop\ of\ same\ diameter}$$

The results of various levels of obscured operating area of downstream propellers are shown in Figure 34. 0% obscured area propellers are individual propellers and 100% obscured area propellers are propellers downstream of a propeller of the same size. Propellers with other levels of obscured area are studied by pairs of varying diameter propellers, where obscured area between 0% and 100% are pairs of increasing diameter propellers and obscured area above 100% are pairs of decreasing diameter propellers. The trend between percent obscured area and percent maximum thrust capacity is highly linear, with the trendline equation below. The R-squared value is a statistical measure of accuracy between the data and the fitted regression line, and in this case R-squared is very high, at 0.964, and indicates a strong agreement between the fitted trendline and the data.

$$T\ \% \ Capacity = 100\% - 0.322 * A_{\% \ Obscured}$$

Each propeller, 10", 13", and 15" diameters, experiences 100% capacity at 0% obscured area.



**Figure 34: Thrust capacity of a propeller as a function of its obscured operating area.**

## 5. Conclusions

The objective of this study was to explore the advantages and the disadvantages of coaxial propeller systems for a multirotor system. In the literature review section, relevant theory from previous work was summarized and considered for this application. Experiments were done to quantify the thrust and performance losses of systems with combinations of one, two, and three propellers and to examine the behavior when variables such as propeller separation distance, motor speed, rotational direction, propeller diameter, and propeller order were altered. The proposed design from this study is a pair of coaxial contrarotating propellers with a smaller diameter propeller situated upstream and a larger propeller diameter downstream, so as to allow the larger downstream propeller to operate with reduced thrust loss. Such a system of propellers achieves greater thrust over a single propeller and greater performance over a system of constant diameter propellers without increasing vehicle footprint.

Other variables were considered for study, such as the efficiency of the system. Empirical data was examined to determine that the best power management strategy to efficiently reach maximum thrust with two propellers is to advance the rear motor to 100% throttle before advancing the front motor's throttle setting.

Finally, the relationship between the obscured area of a propeller's operating cross-sectional airstream and the thrust capacity of the propeller were quantified by inspection of the empirical data gained from the various combinations of propellers of different diameters discussed in this work.



## 5.1 Future Experiments

For the experiments performed above, repetition would serve to validate the results, and can be limited to experiments with systems of increasing diameter only, as systems of decreasing diameter are demonstrably worse than systems of single, constant, and increasing diameter. In addition, using a wider range of diameters and separation distances would allow for greater validation of the conclusions established above. This could help to determine the end of a propeller wake's field of interaction. Finally, investigating multirotor systems in water and in ducts would allow for a more thorough understanding of the scope and effects of wake interactions, and it would be possible to determine which operations multirotor systems would perform well in.

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