TEACHER RESPONSES TO STUDENTS' DIFFICULTY WITH MATHEMATICS PROBLEMS AND STUDENT ENGAGEMENT: A STUDY OF THREE URBAN MATHEMATICS CLASSROOMS

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ABSTRACT OF THE DISSERTATION

Research points to students' increasingly negative attitudes towards mathematics throughout the middle and high school years, particularly for students coming from low SES backgrounds. Adverse feelings towards mathematics may be triggered by, among other things, classroom experiences that occur when students encounter difficulty with mathematics problems. Consequently, teacher responses to students in such cases can be instrumental in shaping their attitudes and feelings about mathematics. Teachers in urban, low SES schools often struggle in their efforts both to engage their students in challenging mathematics, and to have the students develop positive feelings towards the discipline. In an effort to better understand this dynamic, this study examined the responses of three urban mathematics teachers to their students' difficulties with mathematics problems and the students' behavioral, cognitive, and affective engagement in mathematics following those responses.

The research questions that guided the study were: 1) What types of responses do the teachers provide when they perceive that their students are experiencing difficulty with mathematics problems? 2) How do the teachers explain their responses in such cases during follow-up interviews? 3) What are the behavioral, cognitive, and affective consequences relating to student engagement in mathematics, following the teachers' responses to students' difficulties with mathematics problems?

To answer these questions, 29 episodes of students' mathematics difficulty during 7 days of video and audio taped class sessions (2-3 days per teacher) were identified and analyzed, along with data from stimulated recall interviews with the teachers and student questionnaire responses. A coding scheme of teachers' responses emerging from the study included five broad categories of response types and between one and six subcodes for each. Participating teachers never provided direct solutions but responded, using various strategies, by encouraging students to address their mathematical challenges. Teachers' retrospective reasons for their responses were likewise coded and most often emphasized mathematical goals and individual students' abilities or skills. Influences of teachers' responses on students' mathematical engagement were student- and context-dependent. Some discrepancies were noted between students' self-reported emotions and emotions inferred from students' observed behaviors on videotape.

This study offers direction for future research examining consequences of teacher interventions on students' mathematical engagement.

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CHAPTER 1: INTRODUCTION AND OVERVIEW

1.1 National Context

There has been concern in recent years regarding the mathematics achievement of students in the United States. The situation is particularly acute for urban students. Current research shows that affect, in addition to cognition, is crucial in students' mathematical thinking. Teachers play an important role in shaping students' affect in relation to mathematics.

Reports of national mathematics assessments show students' mathematics performance in the U.S. has improved over the past two decades. The National Center for Education Statistics (2009) reported the mathematics performance of fourth and eighth grade students across the nation significantly higher in 2009 than in 1990. Results of evaluations, however, point to persistent deficiencies in the mathematics skills of students in the U.S. The 2009 National Assessment of Educational Progress (NAEP), which included nationally representative samples of over 168,000 4th graders, 161,000 8th graders, and 49,000 12th graders, found that only small percentages of these students were proficient in mathematics. Thirty nine percent of 4th graders, 34% of 8th graders, and 26% of 12th graders were proficient in mathematics, suggesting a decline in mathematics performance with increase of grade level. The study also highlighted significant gaps in mathematics scores based on race and income level for both 4th and 8th grade students. White students scored significantly higher than Black and Hispanic students, and students from higher income families scored significantly better than students from lower income families. The scores of 8th grade students attending schools in suburban or rural areas were higher than those of their grade level peers attending schools located in cities (National Center for Education Statistics, 2009).

International mathematics assessments have found that the mathematics performance of students in the U.S. trailed the performance of students in other major industrialized countries.

The Trends in International Mathematics and Science Study (TIMSS) of 2007 reported that in the area of mathematics, 4th grade students in ten countries and 8th grade students in eight countries outperformed their peers in the U.S. (Gonzales et al., 2008). On an assessment of mathematics application skills administered to 15-year-old students by the Program for International Student Assessment (PISA) in 2009, students in the U.S. scored below the international average for their age level counterparts. Fifteen-year-old students in 23 of the other 64 participating jurisdictions scored higher than students in the U.S. (OECD, 2010). While we cannot rely exclusively on quantitative studies of standardized tests as measures of students' mathematics performance, there are indicators that the mathematics performance of U.S. students must be improved.

The central role affect plays in mathematics learning and instruction is receiving increasing recognition in the field of mathematics education research. The study of affect has included students' feelings, attitudes, beliefs, and values in relation to mathematics. Research in this area suggests complex ways in which affect interacts with cognition to influence students' mathematics performance.

Results of the 2007 Trends in International Mathematics and Science Study (TIMSS) indicated relationships between fourth and eighth grade students' self-reported attitudes towards mathematics and self-confidence in learning mathematics and their mathematics achievement, across participating countries (Mullis, Martin, & Foy, 2008). A meta-analysis of 26 studies reported a common population correlation between elementary and secondary school students' mathematics anxiety and achievement in mathematics (Ma, 1999). The importance of affect in children's mathematics learning is reflected in the guidelines for mathematics instruction put forth by The National Research Council (2001). Five strands of mathematical proficiency were proposed by the National Research Council, encompassing goals for successful mathematics learning. Among these strands (along with *conceptual understanding*, *procedural fluency*, *strategic competence*, and *adaptive reasoning*) is one which focuses exclusively on promoting *productive dispositions*.

The reported prevalence of negative mathematical affect for U.S. students, therefore, provides reason for concern. Research studies that followed middle school students throughout the United States have shown that students' attitudes towards mathematics become increasingly more negative throughout middle and high school (Ma & Cartwright, 2003; Wilkins & Ma, 2003). Attitudes towards mathematics of students in low SES schools were found to decline more rapidly than attitudes of students in high SES schools (Ma & Cartwright, 2003). The poverty, city street culture, and peer pressure of students in low SES and urban schools may contribute to their developing negative attitudes towards and lack of motivation to succeed in mathematics, suggesting one possible reason for lower performance by urban students on national mathematics assessments (e.g. Dance, 2002).

Fortunately research suggests that teachers may have strong impacts on students' feelings about mathematics. In their national study of middle and high school students' affect related to mathematics, Wilkins and Ma (2003) found that "perceived encouragement from teachers predicted positive status [regarding attitude toward and beliefs about mathematics] and slower decline in student attitude" (p. 60). Particularly when students have difficulty with mathematics, teachers have opportunities to encourage the development of positive emotions conducive to the learning of mathematics, such as interest or confidence, or negative feelings in relation to mathematics, such as anxiety or embarrassment. In one study college students ascribed their worst mathematics classroom experiences throughout their years of schooling to instructor behaviors, which were often in response to their difficulty in mathematics. These students' negative experiences were often painful to the extent that they left a "lingering and lasting impression" (Jackson & Leffingwell, 1999). The current context points to the importance of studying affect in urban school settings, particularly with regard to the way teachers influence students' affect.

1.2 Purpose of Present Study

The intent of the present study was to further our understanding about the types of responses urban mathematics teachers offer when students have difficulty with mathematics problems, and about how these responses may influence students' cognitive and affective engagement in mathematics. The study examined the responses of three urban mathematics teachers to their students' difficulties with mathematics problems during class, and documented ways in which these responses influenced the students' subsequent mathematical engagement in the moments following the teachers' responses. Its purpose was to help us understand and classify the types of responses teachers offer when students encounter mathematics difficulties, the teachers' reasons for those responses, and the possible consequences of the interactions.

Data for this study were drawn from a larger research project focused on analyzing relationships between urban students' affect and mathematics learning. The broader study, funded by the National Science Foundation and directed by Dr. Gerald Goldin, Dr. Yakov Epstein, and Dr. Roberta Schorr of Rutgers University, was aimed at examining affective and social consequences for urban children learning mathematics and investigating ways in which teachers contribute qualitatively to creating emotionally safe classroom environments for urban students to explore conceptually challenging mathematics. This type of mathematical activity – that is, conceptually challenging mathematics – involves students working to solve non-routine mathematics problems, during which the students build upon or modify their existing mathematical ideas (Goldin, Epstein, & Schorr, 2007). A primary aim of the larger project was to study the powerful mathematical affect students develop during their mathematical activity that encourages determination, perseverance, and eventually success in students' mathematical problem solving.

The present research involved additional data collected during November and December of 2008 for the larger affect project. The following questions guided the investigation:

1) For each of three urban mathematics teachers (Ms. A., Ms. B, and Ms. S.), what types of responses do the teachers offer to their students who experience difficulty with mathematics problems?
2) During retrospective stimulated recall interviews, how do the teachers explain why they responded in the ways they did to students' difficulty with mathematics during the sessions?
3) What does evidence show about students' behavioral, cognitive, and affective engagement in mathematics following the teachers' responses to their difficulties with mathematics problems?

The distinctions among the types of engagement analyzed in this research are discussed in greater detail in Chapter 2. In this research, behavioral engagement refers to whether observable or overt activity during class is on- or off- task, cognitive engagement refers to whether or not thought or reasoning is directed towards the mathematics task and if so, the complexity or depth of that thought or reasoning, and affective engagement refers to whether or not emotions are experienced in relation to the problem and the student's mathematics learning.

This investigation involved the use of qualitative methods. To answer the first and third research questions, a coding scheme was developed for analysis of classroom videotapes of the three junior high school mathematics teachers and their students in the large urban district that participated in the broader project. Stimulated recall interviews with the teachers were conducted and analyzed to address the second research question. To help answer the third research question, emotions questionnaires completed by the students after each mathematics session were examined.

1.3 Findings and Importance

Several themes emerged from this investigation with implications for researchers and practitioners in the field of mathematics education. The findings of the study indicate that the three teachers have preferred strategies they utilize in responding to students' difficulties in mathematics. Teachers' roles were essential in this project, both in providing their reasoning for their responses to students' difficulties and in offering insight into students' behaviors during the class sessions. The study highlighted teacher and student behaviors that may influence students' engagement in mathematics. The importance of using observational data in addition to self-report questionnaires when studying students' affect was emphasized.

Certain patterns were noted regarding the types of responses participating teachers provided to students' mathematics difficulties. The strategies teachers used were classified into five general categories: 1) Having other student(s) help student resolve difficulty, 2) Having student help him/herself resolve difficulty, 3) Helping student directly to resolve difficulty, 4) Helping student indirectly to resolve difficulty, and 5) Not responding/ Responding without offering suggestion for resolving difficulty. The most frequently used type of strategy for all three teachers involved helping students indirectly. The teachers differed, however, in their methods of implementing this strategy. Ms. S. most often offered a simpler problem as an example; Ms. B. most often probed the students. Ms. A. did not use any one method of helping students indirectly more frequently than others. The second most frequent type of response provided by Ms. S. and Ms. B. was having other students help the student resolve the difficulty. For Ms. A., the second most frequent type of response was having the student help him/herself. Highlighting ways in which teachers used particular types of responses provides a resource to mathematics teachers interested in encouraging students' development of conceptual understanding and problem solving skills and creating classroom environments where students feel safe expressing mathematical ideas without fear of embarrassment or loss of face should their ideas be challenged.

Information provided by teachers in the study added an important dimension to this research. Teachers offered insight regarding factors that influenced their choices of response. Factors teachers reported, in decreasing order of frequency, included the students' abilities or skills, the teacher's desire to achieve mathematics goals for students, the students' emotions and attitudes, and time constraints. All the teachers mentioned they try not to provide answers to the students directly in order to encourage student thinking. The three teachers also reported using more directed questions with weaker students to let them feel successful and as Ms. A. explained, "give them hope that it's a process of learning."

Teachers' explanations also offered new lenses through which to understand students' behaviors during the class sessions. Elucidations provided by the teachers could not have been deduced by objective researchers lacking personal knowledge of the students. In one instance, a teacher attributed a student's apparent lack of understanding regarding the mathematics problem to the student's lack of effort due to her negative attitude that day as compared with her usual performance, rather than to the student's difficulty with the problem. Inclusion of interviews with teachers in analyses of students' behaviors based on videotape evidence provides new perspectives in interpreting students' classroom behaviors.

At a surface level, no distinct relationships were detected between types of teacher responses to students' difficulties and the students' subsequent engagement in mathematics. For Ms. A., such relationships could not be tested as her students always demonstrated deep engagement with mathematics across the three domains (behavioral, cognitive, and affective), where information was available to classify their engagement. For Ms. B., a preliminary trend was noted relating her responses of *probing* and *leading* to the students' complexity of thought following those responses. For Ms. B., no relationships were identified between the interventions she provided and her students' subsequent engagement. A deeper level of analysis indicated, however, that student outcomes following teachers' responses to their difficulties were student dependent. In some cases, a particular type of response resulted in students' deep engagement, whereas at other times students showed evidence of lower levels of engagement following the identical response type. Interviews with teachers confirmed that the teachers possessed understanding of their students' abilities and dispositions and chose responses to maximize individual students' engagement in particular contexts.

Connections were noted between students' cognitive and affective engagement in mathematics. Affective engagement in this investigation was believed to potentially consist of either positive or negative emotional feelings. Data from this research showed that students' strong positive feelings were associated with high level mathematical processing and students' negative feelings occurred simultaneously with low level cognitive processing or apparent cognitive disengagement with mathematics. Positive affect was also found to be associated with student behaviors toward each other as they worked on the mathematics problems in groups. The students who demonstrated the most positive affect overall were those in the group in which students most consistently worked together solving the problem and showed respect for one another, patiently providing explanations to each other regarding their mathematical ideas. These students did not tease or belittle each other regarding mathematics or non-mathematics related issues. This finding highlights, as some researchers have suggested, the essential roles of respect and cooperation among students in creating positive affective environments in collaborative learning situations.

In some instances, analysis of student questionnaires demonstrated a lack of correspondence between students' reported emotions during class sessions and the emotions inferred from the students' behaviors during classroom videotapes of those sessions. Where the videotapes seemed to show students experiencing feelings such as embarrassment or inferiority, the students often did not report these emotions on the emotions scales. Particularly with regard to these negative feelings, research involving assessment of students' emotions should include analysis of students' observed behaviors in addition to self-report questionnaires.

The salience of student related affective issues in the classroom underscores the need for professional development aimed at increasing mathematics teachers' awareness of ways in which their interactions contribute to students' engagement and learning.

1.4 Limitations

Since this research is limited to the analysis of one problem solving group in two to three class sessions for each of three urban mathematics teachers, no generalization to other classroom situations is possible. Also, this study presents evidence of students' "in the moment" engagement following teachers' interventions to their difficulties. Influences of teachers' strategies on students' long term engagement were not analyzed. In this work students' emotions and cognitive activity were inferred from their observable behaviors, possibly limiting the accuracy of coding in these areas. In addition, only preliminary reliability on cross check of coding was obtained due to restrictions in the scope of this project. Since only segments of classroom sessions were examined in this research, emotions students reported on their questionnaires may have referred to classroom episodes not analyzed in this study. This may have resulted in some of the discrepancies noted in this work between students' reported emotions and the emotions inferred from their observed behaviors.

1.5 Implications for Further Research

This research suggests influences of teachers' responses on students' engagement may depend on student and contextual variables, such as students' mathematics ability, heterogenous vs. homogenous student grouping, students' daily dispositions, and students' personalities. Future research focusing on influences of teachers' responses on students' engagement should aim to determine whether and how contextual factors such as these may contribute to impacts of teachers' interventions on students' engagement. Impacts of teachers' strategies on student outcomes may have depended on meanings of emotions for particular students in specific situations, or students' meta-affect. Future research examining impacts of teacher interventions on students' engagement should include stimulated recall interviews with students which aim to elicit information regarding ways in which students experienced particular emotions in specific contexts and the implications of those feelings for the students. This may add to current knowledge regarding interactions between teacher strategies, students' affect, and students' cognitive behaviors as students engage in mathematics problem solving tasks.

Future research should also examine influences of teachers' interventions on students' engagement over time. Studies should include students' perspectives to explore whether students feel their behavioral, cognitive, or affective mathematical engagement changes over time as a result of teachers' strategies.

In this study, a pattern was noted for one teacher where responses that were student centered were followed by high level cognitive engagement while responses that were teacher centered were followed by low level cognitive activity. Larger studies researching influences of teachers' responses on students' mathematical engagement should explore the consistency of this pattern. In this research, discrepancies were evidenced between students' reported emotions and emotions inferred from the students' observed behaviors on videotape. More research in therefore necessary which compares students' reports of their emotions to emotions inferred from their behaviors.

CHAPTER 2: REVIEW OF THE LITERATURE

2.1 Conceptual Framework

2.1.1 Student Engagement

Various conceptualizations of student engagement have emerged over the past few decades (Fredricks, Blumenfeld, & Paris, 2004). Researchers' perceptions of engagement range from viewing the state as entirely behavioral (Brophy, 1983; Fisher, et al, 1980; McIntyre, Copenhaver, Byrd, & Norris, 1983), primarily cognitive (Pintrich & De Groot, 1990), and involving interplay of cognition, behavior, and affect (Skinner & Belmont, 1993).

Researchers who associate student engagement with school behavior have used measures such as time on task and compliance with school rules and teacher directions to assess student engagement (Brophy, 1983; Fisher, et al, 1980; McIntyre, Copenhaver, Byrd, & Norris, 1983). Those who perceive student engagement as involving the cognitive domain have viewed engagement as the use of cognitive, metacognitive, and self-regulatory behavior to monitor learning. Student engagement, according to this view, can be inferred from the level of sophistication of cognitive strategies students use and by students' regulation of their behavior to persist with challenging tasks (Pintrich & De Groot, 1990). The third school of thought, which perceives engagement as simultaneous involvement of the cognitive, behavioral, and affective domains, defines student engagement as "intensity and emotional quality of children's involvement in initiating and carrying out learning activities" (Skinner & Belmont, 1993, p. 572). These authors maintain that students engaged in learning tasks display cognitive and behavioral involvement, such as concentration and effort, as well as positive emotions such as enthusiasm and curiosity.

The view of student engagement which frames this study is a modified combination of the three views cited above. In this research, student engagement was perceived as consisting of three discrete but inter-related components: behavioral engagement, cognitive engagement, and affective engagement. This view maintains that engagement involves the behavioral, cognitive, and affective domains. Students' behaviors were used in this study to infer, where possible, cognitive and affective dimensions of students' engagement. Cognitive and affective aspects of students' engagement were considered separately since it is believed that cognitive engagement is not always accompanied by positive affect. A student, for example, can be cognitively engaged in a problem solving activity while at the same time possessing negative affect such as resentment towards the teacher for assigning the difficult problem, fear of being perceived as unintelligent if he/she does not solve the problem correctly, or worried about incurring the displeasure of fellow students for knowing what the other students don't know. Conversely, a student may demonstrate positive affective engagement in a task while displaying little cognitive engagement. A student may be enthusiastic and curious about a learning task but distracted by a fellow student for a few moments and involved in off-task behavior during that time. In order to accurately assess students' cognitive and affective engagement, each construct was considered independently of the other. It was anticipated, however, that there may be relationships between the two components of engagement. In particular classroom situations or with particular students, it was assumed that cognitive engagement may lead to positive affective engagement or vice versa. This study investigated possible relationships such as these between the two constructs.

It has been suggested that while students work on challenging mathematics problems, affect may interact with cognition in particular ways, resulting in characteristic idealized patterns of behavior. These patterns, termed "archetypal affective structures," suggest ways in which affect and cognition may be related (Epstein et al., 2007; Goldin, Epstein, Schorr, 2007; Goldin, Epstein, Schorr, Warner, 2011). Motivating desires these structures suggest may help interpret students' behaviors as they work collaboratively on mathematics tasks.

2.1.2 Affect

McLeod (1989) proposed a structural model of affect which comprises three subdomains: beliefs, attitudes, and emotions. In later work, DeBellis and Goldin suggested a tetrahedral model of affect which includes values as a fourth component (DeBellis & Goldin, 1997; DeBellis & Goldin, 2006; Goldin, 2000). This research focused on the subdomain of emotions included in these affective models. Emotions are defined as rapidly changing states of feeling that are either conscious, preconscious, or unconscious. Emotions are short-term, occur locally, and are context dependent. Attitudes, beliefs, and values, in contrast, are long-term affect, or global affect, and are more stable. They establish context for and can be influenced by local affect (DeBellis & Goldin, 2006).

Mandler (1989) used discrepancy theory to further understanding of emotions as they occur during individuals' mathematical problem solving. According to Mandler, emotions are experienced in response to perceptual or cognitive discrepancies or interruptions of action that occur when particular situations do not conform to one's expectations. These discrepancies may elicit either positive or negative emotions, such as relief after quickly solving a mathematics problem that was predicted to be challenging or disappointment when one's intended strategy for solving a problem is not effective. Mandler suggested that intensity of emotion varies with the extent of discrepancy or interruption. Affective responses during mathematical problem solving are often activated when errors are encountered which contradict one's expectations, such as that mathematics tasks should be solved quickly and easily.

Pekrun, Frenzel, Goetz, and Perry (2007) classified emotions related to academics as achievement emotions. Achievement emotions include feelings associated with learning activities or outcomes, such as boredom during classroom instruction or pride upon receiving a high test score. Achievement emotions often overlap with social emotions in classroom settings, when academic emotions involve others. Pekrun et al. posit that achievement emotions are experienced when one feels in or out of control of achievement activities or outcomes to which one attributes value. The authors note a reciprocal relationship between cognition and emotions, where "Emotions influence cognitive resources, motivation, use of strategies and self-regulation…of learning" (p. 16) and learning processes and achievement outcomes, in turn, influence emotions. This perspective suggests that learning environments contribute to ways in which cognition and emotion impact each other.

2.2 Review of the Research

2.2.1 Introduction

This review provides a description of student-teacher discourse patterns that have been identified in the literature as well as influences of the particular patterns on students' academic engagement. Also noted are documented challenges teachers face in responding effectively to students' mathematical ideas. Studies measuring students' behavioral, cognitive, and affective engagement in mathematics are discussed, emphasizing instruments used to assess students' engagement and findings from the investigations. Findings describing interactions between affect and cognition in individuals' cognitive processing are also reported. Coding systems developed by previous researchers that were utilized in the present study are discussed. Finally, research which underscores the import of studying engagement for minority students in urban, low income communities is presented.

2.2.2 Student-Teacher Discourse Patterns and Students' Engagement

This section discusses types of teacher-student discourse patterns that have been identified in the literature as well as particular ways in which these patterns have been found to relate to students' academic engagement.

IRE Structure

Mehan (1979) described a three-part sequence of teacher-student interaction that is common in classrooms. The first part, "initiation," consists of a teacher's elicitation by asking a question. The second part, "reply," occurs when a student responds to the teacher's question. The third part, "evaluation," sometimes referred to as "feedback," (Sinclair & Coulthard, 1975), consists of the teacher's evaluation of the reply. Mehan explains that once an instructional pattern of interaction has been initiated, interaction continues until the expected reply is offered. In cases where students do not respond, or respond incompletely, incorrectly, or out of turn, the teacher employs one or more strategies to elicit the expected reply from the students. These strategies may include prompting replies, repeating elicitations, or simplifying elicitations. The evaluation provided by the teacher after the expected answer is offered terminates the interaction sequence.

IRE Structure and Students' Engagement

A study that investigated the types of verbal exchanges prospective mathematics teachers had with school students found that the teachers tended to praise students when their answers were correct and supply correct answers when students responded incorrectly. The author of the study points out that in mathematics, praising students when they arrive at correct answers is more harmful to students than in other subjects. In mathematics, the ability to verify and justify one's solution to a problem is a valuable skill that should be encouraged (Crespo, 2002). Although this study was conducted with pre-service teachers, and it is possible that more experienced teachers might have responded differently to students' correct and incorrect answers, it demonstrates that some teacher preparation programs provide limited opportunities for teachers to investigate their discourse in the classroom. Teachers may therefore be likely to enter the classroom unprepared to interact effectively with students.

Teachers' tendencies to praise and/or correct students can negatively influence students' cognitive and affective engagement in mathematics. These types of teacher responses suggest the importance of correct answers as well as the inconsequence of the solution process. By responding in these ways, teachers abort students' exploration of mathematics (Crespo, 2002). Students' affective engagement can be impacted negatively by responses of praise and correction from the teacher as students may begin to fear that their solutions are not valid or may feel discouragement when an idea they offer is pointed out as incorrect by the teacher. A study of 42 fifth- and sixth-grade students in three elementary schools in rural Pennsylvania lends support to this assumption. The authors found that teachers of classrooms in which students reported low involvement in mathematics primarily used Initiation-Response-Evaluation sequences of discourse in their instruction (Turner et al., 1998).

IRFRF Structure

More recently, research has focused on classroom discourse structures that promote student exploration and critical thinking. Mortimer and Scott (2003) proposed a model of teacher-student classroom interaction which encourages constructivist learning. Referred to as the IRFRF structure, the model begins with an initiation by the teacher (I), is followed by a student reply (R), which the teacher follows up (F) to encourage elaboration by the student. This follow-up could be in the form of repetition of the student's comment, an extension of the comment, or a request for the student to elaborate on the comment. The teacher's follow-up then triggers a further response by the student (R), which the teacher again follows up (F) to promote dialogic interaction which supports constructivist learning.

IRFRF Structure and Student Engagement

A study of discourse in a secondary science classroom identified types of feedback teachers offered in the follow-up stage of IRF chains. The study identified two ways in which teachers followed up on correct answers provided by students and two ways in which they responded to incorrect answers. In response to a correct answer, teachers either affirmed and reinforced the answer and continued teaching or accepted the answer and asked a question or series of questions that further developed the idea. When students answered incorrectly, teachers either corrected them and provided an explanation of the correct ideas, or offered evaluative or neutral comments followed by reiteration of the question or challenge with another question. When a student's reply contained both correct and incorrect ideas, the teacher did not offer an evaluation of the response and asked further questions to extend the student's thinking and help the student self-construct understanding. The study found that teacher responses which probed students stimulated their thinking and helped them develop conceptual knowledge (Chin, 2006).

Several studies have assessed impacts of teachers' instructional discourse patterns on students' engagement in mathematics. A study based on video data of 78 middle school mathematics teachers from five school districts found that teachers' responses to students' questions was key in promoting or discouraging rich mathematical discussions. The authors reported that persistent questioning during the videotaped teacher-student interactions led students to deeper conceptual understanding of mathematical ideas. The authors also found that teachers who used a more open-ended questioning approach involving probing or guiding engaged their students in further understanding of the relevant mathematics concepts (Piccolo, Harbaugh, Carter, Capraro, & Capraro, 2008). The study, however, found that teacher, rather than student, talk was dominant in the classrooms investigated. The study did not focus on types of teacher-student discourse that occur when students work on problem solving tasks in collaborative groups.

A study involving 4th through 6th grade classrooms in an ethnically diverse, urban area found that students in mathematics classrooms that focused on “reform-minded” mathematics showed increased motivation and learning in mathematics. These classrooms encouraged students to understand mathematical processes, seek alternate solutions, and take risks. The students showed enhanced conceptual learning and increased positive emotions such as pride in accomplishment and enjoyment of mathematics (Stipek et al., 1998). A study involving 5th and 6th grade classrooms in a predominantly White, rural area found that providing positive, substantive feedback, encouraging persistence, emphasizing learning and understanding rather than memorization, and treating mistakes as learning opportunities can encourage positive affect in mathematics classrooms. Students in classrooms where teachers used instructional practices such as these had higher ratings on questionnaires which measured 12 semantic differential items (e.g., happy–sad, excited–bored) on a 9-point, Likert-type scale (Schweinle, Meyer, & Turner, 2006).

A study by Turner, Meyer, Midgley, and Patrick (2001) highlighted possible influences of offering affective support when responding to students. The study involved observational analysis of teacher discourse and student belief and behavior survey data from two sixth grade classrooms. The authors found that teachers' affective support for students was associated with students' reports of affect and approach vs. avoidance behaviors. The study found that in the two classrooms where students reported differing affect and approach vs. avoidance behaviors, teachers' scaffolding of instruction was similar, but the teachers' emotional support for students differed. In the classroom with higher student self-reports of both negative affect after failure and avoidance behaviors, the teacher offered fewer responses to students that offered positive emotional support and more frequent negative responses.

Researchers suggest that teachers create classroom contexts in which students engaged in solving mathematics problems experience a sequence of emotions which lead to successful problem solving. In these contexts, or emotionally safe environments, students do not feel danger of embarrassment or loss of dignity. Consequently, when the students experience negative emotions while working on mathematics activities, they are able to manage the emotions in ways that lead to positive feelings and successful problem solving (Schorr & Goldin, 2008). Teachers can create emotionally safe environments by focusing on exploration and understanding of mathematics rather than on obtaining correct solutions so that students feel safe taking risks and being unsure of their ideas in the classroom (DeBellis & Goldin, 2006; Hannula, 2006).

2.2.3 Challenges in Responding to Students' Ideas

Although studies have reported that effective teacher follow-ups to student responses do not evaluate but rather use probing and questioning to extend students' thinking, it has been shown that teachers may have difficulty responding in these ways. An analysis of pre-service teachers' experiences during a curriculum and instruction professional development course found that in following up on student responses, teachers intended to learn about students' thinking but ended up leading students to correct answers. In other instances, the teachers had predetermined answers to their questions and therefore judged particular student responses which did not correspond with their thinking as incorrect, when in reality the responses constituted correct answers to their questions. The teachers reported that in order to respond effectively to students, it is important to have an open mind when listening to students' ideas (Nicol, 1999).

2.2.4 Studies Measuring Students' Engagement in Mathematics

Researchers have underscored the need for better instruments aimed at measuring students' engagement due to the complexity involved in students' academic engagement and the ambiguities associated with the concept (Skinner, Furrer, Marchand, & Kindermann, 2008; Zan, Brown, Evans, & Hannula, 2006). Studies measuring students' engagement have utilized various methodological approaches and have focused on the domains of behavior, cognition, and/or affect. Many studies measuring students' engagement have relied primarily on students' self-reports and student interviews rather than classroom observational analysis (Meyer & Turner, 2002; Pekrun, Goetz, Titz, & Perry, 2002; Skinner et al., 2008). This section reports on research focused on studying students' engagement as it occurs in classroom contexts. Measurements utilized for understanding students' engagement as well as findings from the investigations are discussed.

Engagement Studies

A study by Skinner et al. (2008) examined elementary and middle school students' behavioral and emotional academic engagement and disaffection. The participants included 805 predominantly Caucasian, working to middle class fourth through seventh graders in public schools in a rural–suburban school district in upstate New York. Data for the study involved questionnaires students completed in the fall and spring of one school year. On the questionnaires, students completed items pertaining to their behavioral and emotional engagement vs. disaffection in the classroom as well as their perceptions of the support they received from teachers. Results of the study indicated that student reports of teacher support predicted improvements in emotional and behavioral engagement and declines in behavioral and emotional disaffection, specifically, in boredom and frustration, from fall to spring. The study also reported associations between students' emotional and behavioral engagement over time. Students' emotional engagement in the fall predicted improvement in their behavioral engagement and declines in their behavioral disaffection from fall to spring. Conversely, students' emotional disaffection in the fall predicted decreases in their behavioral engagement from fall to spring. The authors suggested the findings' support for the role of positive emotions "as one possible driver of children's effortful involvement in learning activities" (p. 777).

Although the Skinner et al. (2008) study examined various aspects of students' behavioral and emotional engagement, it did not consider students' cognitive engagement in academic activities. The study also relied exclusively on students' reports in assessing students' classroom engagement and did not include observational analysis in addition to the student surveys. Findings of the study also are specific to non-minority, working to middle class students in rural-suburban communities due to the demographics of the research sample. The conceptualization of student engagement and disaffection maintained by the authors of the study assumes 'positive' emotions such as enjoyment and enthusiasm are necessarily indicative of engagement and 'negative' emotions such as boredom and frustration are reflective of disaffection. This assumption has been debated, however (Goldin, 2003; Goldin, Röskin, & Törner, 2009).

Skinner, Furrer, and Kindermann (2009) investigated correlations between students' and teachers' reports of students' engagement and disaffection in the classroom. The authors also examined correlations between student' and teachers' reports of students' engagement and students' behavioral engagement observed on videotape. Questionnaires pertaining to students' behavioral and emotional engagement were completed by 1,018 third through sixth graders in the rural–suburban school district involved in the study by Skinner, et al. (2008), above. Questionnaires pertaining to the students' behavioral and emotional engagement were also completed by the students' 53 teachers. In addition, classroom interactions focused on 56 children were videotaped. Video data were collected primarily during mathematics and English lessons. Students' behaviors were then coded as *On-Task* or *Off-Task* for either *Active Initiative*, *Working*, or *Passive* behavior. The authors found that emotion and behavior were positively correlated for teachers' and students' reports. However, comparisons of teachers' and students' scores indicated that students felt they were more behaviorally engaged than teachers reported them to be. Students and teachers did not differ in their ratings of behavioral disaffection, but they did differ in their reports of students' emotional engagement. Students reported feeling more emotional disaffection than their teachers perceived. Only modest correlations were found between children's observed behavioral engagement and their engagement reported by themselves and their teachers. Correlations were weaker for students' reports than for teachers'. This finding highlights the import of including observational analysis of students' classroom behaviors as well as survey data in research aimed at understanding students' academic engagement.

Pekrun, et al. (2002) developed an instrument focused on measuring students' academic emotions. The Academic Emotions Questionnaire (AEQ) measures 'positive' as well as ‘negative’ emotions related to academics, such as enjoyment, pride, relief, anxiety, shame, and boredom. Several studies conducted by the authors found that university students’ academic emotions, as measured by the AEQ, were related to their learning and achievement (Pekrun, Hochstadt, & Kramer, 1996; Pekrun & Hofmann, 1999; Pekrun, Molfenter, Titz, & Perry, 2000; Titz, 2001; see Relationships between Affect and Cognition, below.) The AEQ assumes, as in other research (Skinner et al., 2008) that emotions can definitively be classified as positive or negative. This assumption does not account for considerations of meta-affect (Goldin, 2003; Goldin, Röskin, & Törner, 2009) in understanding students’ emotions in the classroom.

2.2.5 Affect and Cognition in Mathematical Problem Solving

Researchers have documented interactions between affect and cognition during cognitive processing (Pekrun, 2002; Meinhardt & Pekrun, 2003) and specifically during mathematical activity (McLeod & Adams, 1994). After 10 years of research on motivation as it occurs in the classroom, Meyer and Turner (2002) concluded, “we find emotion, motivation, and cognition inseparable in classroom contexts” (p. 112). This section discusses studies which reported connections between affect and cognition as well as sequences in which emotions typically occur while students engage in problem solving tasks in the classroom.

A study by Meinhardt and Pekrun (2003) investigated influences of experimentally induced emotions on individuals’ task-related processing. Twenty four university students were presented with affective pictures representing positive, negative, or neutral emotions while involved in a discrimination task. The task required participants to monitor counts of either high or low tones from among other presented tones. In a second assessment the researchers had university students imagine emotionally laden life events while performing the same discrimination task. Results indicated that both positive and negative affective conditions led to increases in participants' error rates on both tasks. The researchers attributed the decline in students' performance during the affective conditions to draining of students' task-related processing resources. The authors noted, however, distinctions between intrinsic emotions, those that relate directly to the task, and extrinsic emotions, those relating to the setting, others, or the self. Intrinsic emotions, the authors stated, can preserve cognitive resources and thereby lead to increases in task attentiveness.

In studies focused on analyzing relationships between academic emotions and students' learning and achievement, authors used the Academic Emotions Questionnaire (AEQ) to assess students' emotions and grades as well as performance on written exams as measures of academic achievement. Scales were used to evaluate students' task-irrelevant thinking and questionnaires were used to measure students' learning strategies (Pekrun, Molfenter, Titz, & Perry, 2000; Titz, 2001). The authors found that university students' emotions at the beginning of the semester predicted their cumulative grades and final exam scores at the end the semester. 'Positive' emotions at the beginning of the semester predicted high academic achievement and 'negative' emotions predicted low academic achievement. Results indicated that the 'negative' emotions of boredom and hopelessness were more closely related to achievement than anxiety (Pekrun, Molfenter, Titz, & Perry, 2000). Positive emotions were found to relate positively to metacognitive strategies and critical thinking. Positive emotions, with the exception of relief, were found to correlate negatively with task-irrelevant thinking (Titz, 2001). Titz's latter finding differed from that of Meinhardt and Pekrun (2003), above, possibly since emotions in the Titz study were related to the task, whereas those in the Meinhardt and Pekrun investigation were not.

Sequences of Emotions During Problem Solving

Goldin (2000) described sequences of emotions that typically occur while students engage in mathematical problem solving. Two pathways Goldin described begin similarly, however, at one stage of the proposed pathways, either positive or negative affect can be evoked, and the pathway splits into two routes. The pathways begin with the affective state of curiosity. If the individual is unable to solve the problem quickly, he/she will feel puzzlement, or a sense of unknowing. This feeling, however, is not unpleasant, as is the feeling of bewilderment, the next stage in the path. A sense of bewilderment would involve a feeling of disorientation or confusion.

At this point, a teacher can intervene and the problem solving process may end. If independent problem solving continues, the individual may feel frustration due to perceived lack of progress. Frustration can also have positive effects, as the problem solver may recognize that a particular strategy has not led to success and therefore implement another, effective problem solving heuristic. Encouragement from a teacher can ensure that frustration is channeled appropriately. A student whose frustration acts to aid in the problem solving process may later feel pleasure as progress is perceived, then elation as new insights are discovered, and finally satisfaction with the problem resolved and the understanding gained. Frustration not channeled properly can lead to anxiety, especially when the individual anticipates a negative consequence as a result of failure to solve the problem. Anxiety can lead to feelings of fear and despair (Goldin, 2000).

2.2.6 Meta-Affect

Recently, research has highlighted the value of considering meta-affect when studying students' emotions in the classroom (Goldin, 2003; Goldin, Röskin, & Törner, 2009). Meta-affect, or one's feelings about one's feelings, can determine *how* an emotion is felt and whether it will contribute to students' engagement or disengagement in academic learning. A student, for instance, can experience frustration or anxiety in pleasurable anticipation of success. Learning contexts, including teachers' interventions, can help influence students' meta-affect and therefore, whether the students' feelings lead to academic involvement or disengagement. Issues of meta-affect have implications for research that involves coding of students' emotions as necessarily 'positive' or 'negative.'

2.2.7 Coding Systems Utilized in Present Study

Coding systems identified in previous research (Gresalfi, 2009; Volet, Summers, & Thurman, 2009) were used in the present study to characterize students' behavioral and cognitive engagement in mathematics following instances of teacher responses to students' difficulties. These coding schemes, as well as the investigations in which they were utilized, are described in the sections below.

Coding of Students' Behavioral Engagement

The coding of students' behavioral engagement in the present study followed a system described by Gresalfi (2009). Gresalfi's research focused on understanding students' dispositions during collaborative mathematical problem solving episodes. The study aimed to further knowledge, specifically, about ways in which students worked with content and with others. Through videotape observation of three mathematics sessions for each of four focus students, Gresalfi identified actions students demonstrated that were indicative of on-task and off-task behavior. Specified on-task behaviors included asking for help from group mate; explaining idea to group mate; bent over paper, writing; using calculator; passing out papers; and reading directions aloud. Off-task behaviors included doodling; talking about recess; reading a book; and sleeping. This coding scheme, with slight modifications, was utilized as an original classification system for categorizing students' behaviors as on- or off- task in the present study (see Chapter 3, Methodology).

Coding of Students' Cognitive Engagement

Volet et al. (2009) investigated factors relating to the emergence and sustenance of high-level content-processing episodes in university students' collaborative work. The authors examined video footage of 18 veterinary science students' meetings in which they worked on course assignments in groups of six. The authors initially identified episodes of group engagement in social regulation of learning and coded these instances as involving high- or low-level co-regulation. The authors described low-level content processing episodes as consisting of: acquiring knowledge; considering facts; reading, paraphrasing or referring to materials; checking information; clarifying sources; and help-seeking for details. High level processing episodes included those involving: elaborating; speculating; justifying; drawing inferences or relations; interpreting; reasoning; negotiating; asking thought-provoking questions; building on or linking ideas; explaining in one's own words; or help-seeking for understanding. This coding system was used in the present study to distinguish between students' behaviors that indicated high- and low- level cognitive activity.

2.2.8 Mathematical Engagement among Minority, Low Income, Urban Students

Research has pointed to particular challenges involved in engaging minority students from urban, low income communities in academics (Connell, Spencer, & Aber, 1994; Dance, 2002; National Research Council, 2003). Lleras (2008) examined data from a National Educational Longitudinal Study (NELS) to examine how learning opportunities, student engagement, and mathematics achievement contribute to each other among White and African

American students across school contexts. Data for the study were drawn from a subset of a nationally representative sample of White and African American students. The subset consisted of 6,063 White and 650 African American students in 660 public middle schools. Surveys from the base year (1988) and from the first follow up year (1990), while the students were in 8th and 10th grades, were examined. Results indicated that in 8th and 10th grades, White students were judged by their teachers to have better school engagement than African-American students. Evidence showed that students in low minority schools have better academic engagement as perceived by their teachers than students in high minority schools. In addition, African-American students in high minority, urban schools were found to have lower engagement than African-American students in low minority urban schools. The study also highlighted achievement gaps in mathematics between African-American and White students. These achievement gaps increased slightly over time in both high and low minority schools. Student surveys also showed that African-American students were less likely than White students to have taken advanced mathematics classes such as pre-Algebra by the end of 8th grade. This finding was especially salient in high minority schools.

Utilizing a sample of 614 African American public school youth in Atlanta and New York, Connell et al. (1994) found that youths' perceptions of their families' support contributed to their reports of engagement in school. The students' engagement was found to predict school performance and adjustment. Authors suggested that efforts aimed at improving educational outcomes for disadvantaged African American youth focus on promoting students' engagement.

2.2.9 Summary of Literature Review

This review documented research pertaining to student-teacher discourse patterns and students' academic engagement. Much research has focused on facilitators and consequences of students' engagement in a range of contexts. Multiple conceptualizations of students' engagement as well as varying methodological approaches have been employed in investigations of students' engagement as it occurs in the classroom. Although research has shown that self-reports of students' engagement can differ from engagement inferred from students' observed behaviors, research assessing students' engagement has relied primarily on survey data as opposed to observational classroom analysis. Research in the area has also not targeted students' engagement as it occurs in response to teachers' interventions to students' difficulties, particularly as they work on mathematical problem solving in groups. The present study adds to the knowledge base of student engagement literature by addressing this gap.

CHAPTER 3: RESEARCH QUESTIONS AND METHODOLOGY

This chapter first provides an overview of the larger study, which occurred in two phases. Data for this thesis was based on the second phase. The chapter then lists the research questions for the present study and details the methods used to address those questions.

3.1 Larger Study: First Phase

During the 2006-2007 school year data were collected for a study funded by the National Science Foundation which focused on relations between affect and mathematics learning in the context of urban mathematics classrooms. The research project, directed by Drs. Goldin, Epstein, and Schorr at Rutgers University, involved three middle school classrooms in a small city school district and a large urban school district in economically depressed communities in Eastern United States. Each classroom was observed and videotaped during four cycles of two consecutive days each within the school year. The mathematics teachers who were selected for the project had participated in professional development led by faculty and other researchers at Rutgers University which aimed to help teachers learn to encourage students' development of deep mathematical understanding (Schorr, Warner, Gearhart, & Samuels, 2007). Investigations of the classroom videotapes considered students' social interactions, emotional states, mathematics learning, and teacher interventions.

3.2 Larger Study: Second Phase

During Phase Two of the broader study, additional data were collected in November and December of 2008. A subset of these data form the basis for the present study. Four teachers (Ms. A., Ms. B., Ms. M., and Ms. S.) who taught at public middle schools in the large urban school district involved in Phase One of the broader study participated in Phase Two of the study. Two to three sessions of each the four mathematics teachers were videotaped and observed by the research team. Ms. B. had participated in the first phase of the larger study; Ms. A., Ms. M. and Ms. S. had not. The teachers were selected based on their ability to create a classroom atmosphere conducive to the learning of conceptually challenging mathematics. The teachers volunteered for participation in the project and had been involved for at least one year in a district-wide standards-based professional development project led by Rutgers faculty. During the first of each sequence of class sessions, the students worked on a mathematics problem in groups. During the second class session, the students shared their work from the previous session with the class. In the class taught by Ms. S., students worked on the problem during two sessions and shared their solutions with the class during a third session. The analysis reported in this dissertation is based on a subset of the data collected in this phase of the study.

The Task

The students in each of the class sessions included in this study worked on the following problem, adapted by an eighth grade mathematics teacher involved in the larger affect study (Epstein, Goldin, Schorr, Capraro, Capraro, & Warner, 2010) from a problem entitled *Building Block Dilemma*. A worksheet distributed to the students in the present study contained the following text and diagram:

I was constructing towers as you see below. I noticed that each time I made the tower higher, I had to add more blocks on the sides to stabilize the structure. I would like to know how many cubes I will need to build a 5-block high tower and a 10-block high tower. Generalize, if you can, on how many blocks I will need for any size tower?

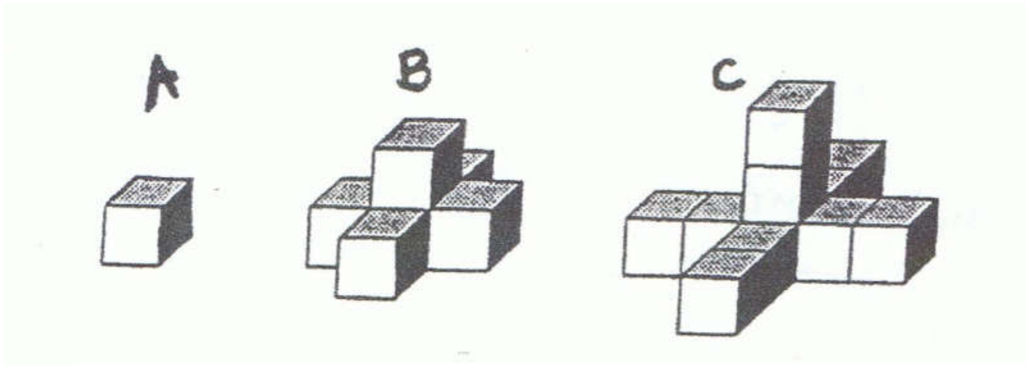

Figure 1: Building Blocks Task Diagram

The task was selected for use in this research because of its potential for high cognitive demand (Stein, Smith, Hennigsen, & Silver, 2000) which was confirmed during the pilot study. The problem was considered conceptually challenging since it encouraged students' development of conceptual understanding, provided opportunities for furthering students' problem solving skills, and allowed for multiple solutions. The task was designed to help students learn to recognize patterns and make connections between patterns and equations. Specifically, the problem was intended to promote students' understanding that many patterns can be represented as equations, that equations, in effect, describe patterns, and that equations can be used to find additional solutions to problems. Possible methods for solving the problem included building towers with interlocking Snap Cubes, drawing sketches of towers, and creating tables to record the number of blocks in various sized towers.

Solutions to the problem included three general types. Solution Type 1 involved noticing the recursive aspect of the problem, where the total number of blocks in each tower is 5 more than in the previous tower, since each tower consists of the previous tower with one block added to each of the four branches and to the height. This solution can be represented as $T_n= T_{n-1}+5$, where *n* represents the height of a tower and T_n represents the total number of blocks used in its construction. Given the number of blocks in the first tower, 1, one can determine the number of blocks in subsequent structures. Students, however, were asked to find a general rule for solving any sized tower.

General solutions to the problem include Solution Types 2 and 3. Solution Type 2 involved noticing that a tower of height *n* could be built by assembling 4 branches and a height, each consisting of *n* blocks, and then removing one block from each of the four branches. The resulting equation is $T_n=5n-4$, where T_n again represents the total number of blocks in a tower of height *n*. Another perception of the Building Blocks pattern which leads to the same equation is that each tower of height *n* consists of 4 branches and a visible height, which all have n-1 blocks, plus the hidden block in the middle of the tower. This view would lend itself to the equation $T_n= 5(n-1) +1$, which simplifies to the equation above, $T_n=5n-4$.

Alternatively, since each of the four branches in every tower has one block less than the blocks in the height, *n*, one may multiply 4 by n-1 to obtain the total number of side blocks and then add *n*, the number of blocks in the tower's height. The numerical representation for this strategy, Solution Type 3, is $T_n=4(n-1)+n$. This equation simplifies to $T_n=5n-4$, in Solution Type 2. The third strategy described here was preferred by all three groups of students analyzed in this research, though not all the groups expressed a formal equation.

A potential obstacle for students as they work on this task is noticing the hidden block at the center of each tower (aside for the first, where the middle block, the only one in the tower, was not hidden). Neglecting to consider the hidden block leads to confusion for students regarding references to the towers. Students who did not notice the center block referred to the 5-block high tower, for instance, as a 4-block high tower. Another typical error is counting the middle, hidden block in each branch of the tower and in the height, instead of counting it only once in the height. This misconception would result in an incorrect general solution of $T_n=5n$.

Data Collection

During each class session, a camera was focused on each group of students. During group presentations, the camera focused on the presenting group was turned towards the front of the classroom. Audio recorders were placed at each group during every class session. Each group's audio recorder was also moved to the front of the classroom for the duration of the group's presentation. An additional camera, placed at the back of the room during each session, captured the full class. Another camera followed the teacher as she moved among the groups. The purpose was to ensure that each student was captured on video at all times, to capture the group's presentation, to ensure the accuracy of audio data collected, and to ensure that all teacher interventions were captured on video during the class sessions.

A questionnaire (see appendix D), developed by Dr. Goldin, Dr. Epstein, and Dr. Schorr of Rutgers University, was distributed to students at the end of each classroom session. The questionnaire provided data directly from the students regarding their affect in relation to mathematics during that day's session. Items on the questionnaire focused on students' emotional feelings, behaviors, and cognitive activity during the class session. One section on the questionnaire asked students about difficulties they may have had that day while working on the mathematics problem. Students were asked whether they had difficulty in mathematics that day and if so, to describe memorable aspects of what happened when they had the difficulty. Data from the student questionnaires were used for comparison with student feelings gathered from observation of student behaviors.

3.3 Research Questions: Present Study

The current study makes use of a subset of data from the second phase of the larger study to focus on three of the participating teachers (see section 3.4). It addresses the following research questions:

1) For each of three urban mathematics teachers, what types of responses do the teachers offer to their students who experience difficulty with mathematics problems?
2) During retrospective stimulated recall interviews, how do the teachers explain why they responded in the ways they did to student difficulty with mathematics during the sessions?
3) What does evidence show about students' behavioral, cognitive, and affective engagement in mathematics following the teachers' responses to their difficulties with mathematics problems?

The data collected during Phase Two of the larger affect study were well-suited to answer these research questions for several reasons. Most importantly, the challenging nature of the problem on which the students worked during the class sessions made it likely that students would encounter difficulty during the class sessions. The students' working in groups allowed their cognitive and affective engagement to be studied as their engagement could be inferred from their dialogue as they worked on the problem. The procedure for data collection permitted the in-depth analysis required for this investigation. The rich data obtained during the larger project provided the level of detail necessary for the examination of teacher responses and student engagement in mathematics that was completed in this study.

The remainder of this chapter describes the detailed methods the current researcher used to complete this study. Sections 3.4 and 3.5 describe the sample of teachers who participated in the study and the instruments utilized in the study, respectively, section 3.6 presents the procedure for conducting teacher interviews, section 3.7 details the methods for data analysis, section 3.8 outlines the steps for completing the pilot study for this project, and section 3.9 explains the use of a qualitative method of analysis in this research.

3.4 Sample of Teachers

The sample in this study included three of the four teachers whose classrooms were videotaped for Phase Two of the larger study and several of their students. Of the four teachers, Ms. A. and Ms. S. were selected for this study since they were the regular classroom teachers of the students. The other two teachers who were videotaped during Phase Two of the larger study, Ms. B. and Ms. M., were the math coaches in their schools and did not regularly teach the students. Ms. B. was nevertheless chosen for inclusion in the analysis because of her participation in Phase One of the larger study as a classroom teacher during the school year of 2006-2007 and to increase the diversity of possible teaching strategies. Videotaped evidence of the first phase of the study had shown that Ms. B. attempted to create an emotionally safe classroom environment where her students would feel safe taking risks in the classroom without fear of embarrassment or loss of face should their ideas be challenged.

The schools in which Ms. A., Ms. B., and Ms. S. taught were classified as low income, with close to 100% of students in each school being members of African American or Hispanic minority groups. During the videotaped class sessions, students worked on the same mathematics problem in groups of three or four. The desks in the classroom were arranged in clusters with group members sitting together. Following is a description of each of the three teachers and their classes.

3.4.1 Ms. A.

Ms. A., who classified herself as Christian Egyptian, had been teaching for four years. Two consecutive sessions of a 7th grade mathematics class Ms. A. considered high track were included in this study. The students worked on the mathematics problem during the first session and part of the second session. They presented their work to the class during the remaining part of the second session. Four groups of students in Ms. A.'s class were videotaped for Phase Two of the larger project. The researcher randomly chose one of these groups for analysis in this research. No instances of student difficulty where the teacher was involved were found on the selected videotape so the researcher chose another group at random. The second selected group included three boys, Emannuel, Eliot, and Juan *(all names of students in this work are pseudonyms)*, and one girl, Amanda.

3.4.2 Ms. B.

Ms. B., who is African American, had eight years of teaching experience. The videotaping of her class sessions took place during her first year as a math coach in a new school. This research analyzed two consecutive mathematics sessions Ms. B. taught to a 7th grade class she considered high track. During the first session, the students worked on the mathematics problem in groups. During the second session, the students continued to work on the problem and then shared their solutions with the class. Four groups of students in Ms. B.'s class were videotaped for the larger study. One of these groups was selected arbitrarily for analysis in this project. The group included two boys, Marcos and Larent, and one girl, Lashanna.

3.4.3 Ms. S.

Ms. S., who is also African American, had been teaching for three years prior to participating in the affect study. The videotapes were gathered during her fourth year as a teacher. Ms. S. considered the 8th grade mathematics class analyzed in this research low track. Three consecutive mathematics sessions of the class were examined. During the first two sessions and the beginning of the third session, the students worked on the mathematics problem in groups. During the remaining part of the third session, the students presented their solutions to the class. Three groups of students in Ms. S.'s class were videotaped by members of the research team. The researcher arbitrarily chose one of the three groups to examine in this study. The selected group consisted of three students: two girls, Ta'keisha and Ordena, and one boy, Leo.

3.5 Instrumentation

This research utilized four instruments: 1) a Student Difficulty Coding Rubric, 2) a Protocol for Verification of Coding, 3) a Protocol for Teacher Interviews, and 4) an Emotions Questionnaire.

The researcher developed an initial Student Difficulty-Teacher Response Coding Rubric *(see Table 3.1)* to facilitate the coding of variables associated with instances of student difficulty with mathematics problems. The rubric, modified during the pilot study for this research, includes a preliminary set of codes for categorizing the type of task with which the student(s) had difficulty, the whole-class vs. small group context of the student-teacher interaction, and the type of teacher response(s). The rubric also contains the initial coding scheme for classifying teachers' explanations for their responses to student difficulty and the contextual factors they report influence their responses. Teacher reports of contextual factors are the circumstances they mentioned when asked during the interview which factors might influence the way they respond to student difficulties. The code 'type of student difficulty' listed as a contextual factor teachers might have mentioned was used for classifying teachers' reasoning for the responses they offer to students; it was not used to code the type of difficulties students had.

Table 3.1: Student Difficulty-Teacher Response Coding Rubric

Student Difficulty-Teacher Response Coding Rubric
Type of Mathematical Task with Which Student(s) Had Difficulty: ***Open- /Closed- Ended:*** 1) Open-ended (variety of correct answers) 2) Closed-Ended (one predetermined correct answer) ***Routine/Non-routine:*** 1) Routine 2) Non-routine
Classroom Context of Student Difficulty: 1) Whole class 2) Small group
Type of Teacher Response: ◆ Saying a student's answer is wrong ◆ Choosing not to respond to an incorrect response ◆ Providing the correct answer/ directly explaining the concept ◆ Leading the student to the correct answer/idea ◆ Asking other students to give their opinions regarding the mathematics issue ◆ Offering a counter example to an incorrect response ◆ Probing- asking the student to explain what he/she means ◆ Offering a simpler problem as an example ◆ Giving the student time to think about the mathematics problem *(cont. on next page)*

(Cont., Table 3.1: Student Difficulty-Teacher Response Coding Rubric)

Teacher Explanation for Particular Response to Student Difficulty: *Teacher explanation for response focuses on the teacher's attempt to:* • Directly explain the mathematics concept(s) and/or procedure(s) to the student (and others) • Encourage the student's mathematical thinking, discovery processes, or problem solving processes • Avoid negative feelings or promote positive feelings in the student   o Possibly: by having other students, as opposed to the teacher, provide feedback • Other reasons
Contextual Factors That Teachers Report Influence Their Responses: • Type of task [open vs. closed ended] • Type of student difficulty (conceptual vs. procedural) • Whole-class vs. small group context of discussion of the student's difficulty • Teacher's perception of the student's ability level in mathematics

A researcher developed protocol for verification of codes (see Appendix E) which described in detail procedures to be followed while verifying data was utilized by the verifier to code types of teacher responses, teachers' reasoning for their responses, students' cognitive engagement, and students' affective engagement.

A researcher developed script (attached in Appendix B) was used during interviews with the teachers. Interview questions target teachers' general philosophies regarding their responses to students' difficulty with mathematics problems and their thoughts concerning their responses to student difficulties that occurred during the class sessions included in this study. For each teacher response to (a) student's(') difficulty with a mathematics problem that was studied in this research, the teacher of the student(s) was asked what she thought the student's(') difficulty was and why she chose to respond in that particular way.

After each class session, students completed an Emotions Questionnaire (see appendix C) developed by Drs. Goldin, Epstein, and Schorr of Rutgers University on which they indicated the thoughts and feelings they had during math class that day. The questionnaire consisted of six parts. In the first part, the students were asked to describe any memorable events that might have occurred during the class session. The second part of the questionnaire listed specific thoughts and feelings the students may have had (e.g., *I worried that I might get in trouble with the teacher*), and students were asked to indicate whether they had the thought *always*, *sometimes*, or *never* in class that day. The third part included a list of feelings and asked the students to rate how much they experienced each one, using *not at all*, *somewhat*, or *very much*. The students were also asked to list any other feelings they may have had. In the fourth section, students were asked to indicate whether they displayed a list of behaviors (e.g., *I gave helpful suggestions*) *hardly ever*, *sometimes*, or *often*. The fifth section listed thoughts students may have had (e.g., *This problem is really interesting*) and asked the students to indicate whether or not they had the thought during the math session that day. Students were also asked to list any thoughts they had that were not on the list. In the last section, the students were asked to note whether they had difficulty with a mathematics problem that day, and if yes, in what classroom context the difficulty occurred and what was memorable about the situation.

3.6 Procedure for Conducting Teacher Interviews

The three teachers of the videotaped classes included in this study were interviewed in order to answer the second research question. Upon completion of transcription and coding of selected videotaped events involving student difficulty (described below), three instances of students' mathematics difficulty per teacher were selected for analysis during the stimulated recall interviews. Instances of student difficulty to be examined during teacher interviews were selected with the criteria of variation in: student(s) who had difficulty and/or teacher response and/or small group vs. whole class setting of the difficulty. Instances of student difficulty to be used during teacher interviews were chosen with these conditions to allow for the possibility of varied teacher explanations for their responses. Selected videotaped instances were clipped for use during teacher interviews. Each video clip included a student's difficulty and the teacher's response to the difficulty.

The researcher conducted one interview, lasting approximately 45 minutes, with each of the three teachers. One interview took place during December 2009; the others were conducted in January 2010. During the interviews, teachers were asked about how they usually respond to their students' difficulty with mathematics problems and why they respond in those ways. Each teacher was then shown a video clip of a selected instance of her student's difficulty, which included her response to the student, and was asked what she thought the student's difficulty was and why she responded in that/those way(s). The teacher was also asked whether she thought the student's mathematics ability influenced the way she responded. The process was then repeated with the other two selected instances for the teacher. Teacher interviews were audio and video taped for purposes of analysis. Minor changes were made to the researcher developed interview protocol after the first teacher interview in order to enhance the clarity of several questions. The final draft of the protocol is included in appendix B.

3.7 Data Analysis

The procedure for data analysis for the present study included three components which involved examination of: classroom videotape data, teacher interview data, and student questionnaires. Analysis of classroom videotape data involved four phases: viewing of videotapes and selection of events, transcription of events, coding of data, and identification of preliminary patterns or themes within the data. In addition, qualitative descriptive analysis of students' cognitive engagement before and after teacher responses was completed for several instances of student difficulty.

3.7.1 Analysis of Classroom Videotape Data

Phase #1: Viewing of Classroom Videotapes and Selection of Events

The initial phase of the videodata analysis for this project involved viewing videotapes of the seven middle school mathematics class sessions included in the study. Instances of student difficulty with mathematics problems were identified and time coded. Student difficulties selected for analysis involved instances where the teacher perceived that a student had some level of difficulty while working on a mathematics problem. The student difficulties analyzed in this project were classified into four types based on the way the difficulties came about. The system for classifying the types of difficulties, which was synthesized during this analysis, is described in detail in section 4.2, below. In some instances difficulties are described as collective, involving several students. In these cases, the students' conversation indicated that all were involved in the same difficulty. A total of 29 instances of student difficulty were identified in the seven videotaped class sessions involved in this investigation. These 29 instances became the data set.

Phase #2: Transcription of Events

Instances of student difficulty were transcribed using videodata and audio data when audio from video was unclear or inaudible. Transcriptions were completed using individual group camera views and teacher or whole class camera views when individual group videodata was unclear. Transcriptions also included recordings of student actions, such as descriptions of their manipulation of blocks and of their use of calculators, during the sessions.

The instances of student difficulty were transcribed as events, where each event included the student difficulty and the teacher response(s). In cases where instances of student difficulty directly succeeded each other, they were transcribed as one event. The 29 identified instances of student difficulty were transcribed within 14 events. Each event included between one and eight instances of student difficulty. Transcriptions were then verified by a graduate student, using audio recordings of the classroom sessions and videotape data when audio recordings were unclear or unavailable.

Phase #3: Coding of Classroom Videotape Data

For each identified instance of student difficulty, qualitative descriptions of the difficulty and of the teacher response(s) were documented. The descriptions included the following elements: 1) how the difficulty came about (for instance, the student was presenting an idea to the class and a classmate asked him/her a question about it), 2) how the student expressed the difficulty (for example, asking the teacher a question or answering “I don’t know” to a teacher’s or classmate’s question), and 3) the mathematics context of the difficulty.

Coding of Type of Difficulty. The type of student difficulty was coded using the Student Difficulty-Teacher Response Coding Rubric included in section 3.4, above. Instances where a student expressed or agreed with an incorrect or insufficient answer or idea regarding a mathematics problem were coded as Incorrect/Insufficient Answer/Idea. Situations where a student arrived at an impasse while working on a mathematics problem were coded as Impasse. Where a student appeared unable to adequately answer a mathematics question posed by a teacher or classmate, the code Unable to Answer was applied. Instances where a student expressed that he/she did not understand a classmate's solution were coded as Does Not Understand Solution.

Coding of Type of Task and Context of Difficulty. The type of mathematical task with which the student(s) had difficulty was categorized in two ways: whether it was open- or closed-ended and whether it was routine or non-routine. Problems to which there were a variety of correct answers were considered open-ended. Closed-ended problems were defined as problems to which there was one predetermined correct answer. Problems for which a standard method of solution is commonly taught were considered routine. Problems for which a standard method of solution is not commonly taught were classified as non-routine. In instances where a student had difficulty with a subtask that was included in a larger task, the researcher coded for the smaller subtask. The small group vs. whole class context of the student's difficulty was also noted for each instance.

Coding of Teacher Responses. The researcher subsequently coded teacher responses using the teacher response section of the Student Difficulty-Teacher Response Coding Rubric described in section 3.4, above and included below.

Type of Teacher Response:

- Saying a student's answer is wrong and correcting the student or inviting other students to respond
- Choosing not to respond to an incorrect response
- Providing the correct answer/ directly explaining the concept
- Leading the student to the correct answer/idea
- Asking other students to give their opinions regarding the mathematics issue
- Offering a counter example to an incorrect response
- Probing- asking the student to explain what he/she means
- Offering a simpler problem as an example
- Giving the student time to think about the mathematics problem

During the process of coding the teacher responses, the researcher modified the list of teacher response codes to include teacher responses that did not correspond with codes on the list. The researcher distinguished between the teacher's initial response to the student(s) and subsequent teacher responses regarding the same mathematics difficulty. The first teacher response to each difficulty was termed an initial response. Any subsequent teacher responses to the difficulty were considered follow-up responses.

The final list of teacher response codes includes 12 types of teacher responses, which are divided into five broader categories. Each type of response includes up to three related responses. The final coding scheme for teacher responses is included in the Findings section of this work.

In several cases, the teacher response code of leading students to correct answers/ideas also involved probing students and/or offering simpler problems. In these situations the more general code of leading students to correct answers/ideas was applied.

After the researcher completed coding the teacher responses, the responses were coded according to response type by an independent researcher with several years of mathematics teaching experience. The independent researcher was guided by an instruction sheet (see Appendix E) and had not seen the coding of teacher responses that was completed previously by the researcher. A cross check was made between the researcher's and the verifier's coding as a cross check of the reliability of the coding scheme. The intent was to refine the coding scheme if there was a discrepancy of 15% or more between the two independent codings of the teachers' initial responses and/or of the teachers' follow-up responses.

Coding of Students' Engagement Following Teacher Responses. After the completion of the coding for teacher responses, type of task, and context of error, *s*tudents' engagement following the teacher responses was analyzed. For each instance of student difficulty, descriptions of observable student behavior related to student engagement were recorded. Descriptions detailed students' behaviors after each instance of difficulty until the conclusion of the classroom session or until the student was involved in another analyzed instance of difficulty. Inferences of the students' emotions based on the behavioral descriptions were then documented.

Based on the descriptions of student behavior following teacher responses and reference to videotapes when necessary, students' behavioral, cognitive, and affective engagement were coded for instances where sufficient information was available regarding students' actions, thoughts, and feelings. Students' behavioral engagement was considered at a global level as being on or off task for the greater portion of time under investigation. Following Gresalfi (2009)'s work (as described in the literature review), students' behavior was categorized as on- or off- task. Behaviors that classified students' actions as being on- or off- task as they worked on mathematics problems in groups, as delineated by the aforementioned research (with slight modifications), are listed in Table 2, below. Several behaviors were added during this research. The complete coding system is included in Chapter 4, Table 4.4.

Table 3.2: Initial Behavioral Engagement Coding Scheme

On- Task Behavior	**Off- Task Behavior**
Asking for help from group mate *Explaining idea to group mate* *Looking at paper* *Writing* *Reading directions aloud* *Using calculator*	*Talking with others about non-math topics* *Sleeping* *Doodling* *Staring into space without noticeably connecting back to work* *Playing with pen/pencil*

A system for classifying students' cognitive activity as high or low level based on their behaviors while they worked in groups was utilized in this analysis (Volet, Summers, & Thurman, 2009) (see literature review). Behaviors that categorized students' cognitive activity as high or low level are specified in Table 3.3, below. Students' cognitive engagement was classified using the code that characterized the highest level of the students' cognitive activity during the time under investigation. During this research three codes were added to the cognitive engagement coding system to describe students' activities not included in the original classification system. The final set of codes for students' cognitive engagement is included in Table 4.5.

Table 3.3: Initial Cognitive Engagement Coding Scheme

Low Level Cognitive Activity	**High Level Cognitive Activity**
Acquiring knowledge *Considering facts* *Reading* *Paraphrasing or referring to materials* *Checking information* *Clarifying sources* *Help-seeking for details*	*Elaborating* *Speculating* *Justifying* *Drawing inferences or relations* *Interpreting* *Reasoning* *Negotiating* *Asking thought-provoking questions* *Building on or linking ideas* *Explaining in one's own words* *Help-seeking for understanding*

To classify students' affective engagement, students' feelings related to mathematics were inferred from the descriptions of their observable behaviors. The students' feelings were then classified using a coding scheme developed during this research. Using the coding scheme, students' feelings related to mathematics were coded as high, medium, or low for positive or negative affect. The descriptors 'high,' 'medium,' and 'low' referred to behaviors that were strong, moderate, or weak, respectively. A students' affective engagement was given the code that described the students' emotions for the majority of the relevant time period. The complete coding system which lists the behaviors indicative of each affective coding category is included in Table 4.6, later in this work.

Students' cognitive and affective engagement following teacher responses were coded by another researcher who had not seen the original coding completed by the present researcher. The intent was to refine the coding scheme for either engagement type if there was a discrepancy of 20% or more between the researcher's and verifier's sets of codes.

Patterns were then sought out within the coded data between types of teacher responses to students' mathematics difficulty and students' subsequent behavioral, cognitive, and affective engagement in mathematics. Statistical data were not used in this study; rather, relationships were sought out via analysis of verified classifications of students' engagement based on qualitative descriptions of student behavior.

Qualitative Analysis of Students' Cognitive Engagement. Students' cognitive engagement preceding and following teachers' responses to their mathematics difficulties were analyzed for each instance of student difficulty selected for investigation via teacher interviews. Qualitative descriptions of these students' cognitive involvement in mathematics after the teachers' responses were culled from Appendix A, which includes descriptions of students' behaviors related to mathematics engagement following each of the 29 instances of difficulty analyzed in this research. Since in Appendix A, descriptions of students' behaviors related to their mathematics engagement for the time period following each instance of difficulty are documented until the next instance of difficulty occurs (or until the end of the class session if there are no further analyzed difficulties), students' behaviors during the time period following one instance of difficulty are the same as their behaviors preceding the subsequent difficulty for that day's session. Therefore, descriptions of students' behaviors related to their mathematics engagement preceding each instance of difficulty were obtained from Appendix A, where the students' behaviors during this time segment were described as the students' behaviors following the preceding difficulty. In cases where the analyzed difficulty was the first that occurred on a particular day (and the students' behaviors preceding the difficulty were therefore not documented in Appendix A), the researcher referred to the videotapes and documented the descriptions.

Overall classifications of the students' cognitive engagement as high, medium, or low both before and after the teacher intervention were also obtained, either from Appendix A, or in cases not included in Appendix A (see previous paragraph), using the cognitive engagement coding system listed in Table 3.3, above. These classifications were based on the qualitative descriptions of the students' behaviors. In-depth analyses of the illustrative cases were conducted which focused on relationships between students' cognitive involvement in the mathematics prior to the teachers' responses, the types of teacher responses, and the students' cognitive engagement following the teachers' intervention.

3.7.2 Analysis of Student Questionnaire Data

The 'Feelings' and 'Mathematics Difficulty' sections of the Emotions Questionnaire for each student involved in this analysis were analyzed. In the 'Feelings' section, students were asked to circle *Not at All*, *Somewhat*, or *Very Much* to indicate the degree to which they experienced each of 22 listed feelings during the class session. In this analysis, students scored 0, 1, or 2 points for each response. For positive feelings, students scored 0 points for a response of *Not at All*, 1 point for a response of *Somewhat*, and 2 points for a response of *Very Much*. For negative feelings, students scored 2 points for *Not at All*, 1 point for *Somewhat*, and 0 points for *Very Much*. Twenty of the 22 listed feelings were classified by the researcher as either positive or negative; the remaining feelings were classified as indicating engagement with impasse. An Overall Positive Emotions Score was calculated and recorded for each student. This score was the sum of the student's scores for his/her responses to the positive and negative feelings.

Since several students did not respond to all the listed feelings and several students added feelings to the list, a Most Positive Score was calculated for each student, indicating the most positive emotions score that was possible for that student. Each student's Most Positive Score was calculated by multiplying the number of positive and negative emotions to which he/she responded by two. The Most Positive Score was recorded along with the Overall Emotions Score for each student. Lists of students' responses of *Not at All*, *Somewhat*, and *Very Much* to the individual feelings were also recorded.

For feelings classified as indicating engagement with impasse, students scored 0 points for a response of *Not at All*, 1 point for a response of *Somewhat*, and 2 points for a response of *Very Much*. An Engagement with Impasse Score was calculated for each student. This score was the sum of the student's scores for his/her responses to the feelings classified as indicating engagement with impasse. Since several students did not respond to both feelings which indicated engagement with impasse, a Most Engaged with Impasse Score was calculated for each student. This score indicated the highest level of engagement with impasse possible for that student, as measured by the feelings section of the questionnaire. The Most Engaged with Impasse score was obtained by multiplying the number of ‘engaged with impasse’ feelings to which the student responded by two. Each student’s responses of *Not at All*, *Somewhat*, or *Very Much* for each of the ‘engagement with impasse’ feelings were also recorded.

Students’ reports of their feelings were compared with the emotions inferred by the researcher from the students’ behaviors observed on videotape. Students’ Overall Positive Emotions Scores obtained from the questionnaires were compared with the affective codes assigned by the researcher to classify the students’ feelings following incidents of student difficulty.

Students’ reports of whether they had difficulty during each mathematics session, and if so, in which setting the difficulty occurred, and their thoughts regarding what was memorable about the situation, were recorded. Students’ reports of whether or not they had difficulty while the worked on the mathematics task were compared with records of instances of student difficulty analyzed in this research.

3.7.3 Analysis of Teacher Interviews

Teacher interviews were transcribed by an independent party using both audio and video tape and were verified by the researcher. Teacher explanations for how they usually respond when students encounter difficulty while they work on mathematics problems were summarized in order to answer the second research question. The teachers’ reports of strategies they typically use when responding to students' difficulties were compared with the types of strategies the teachers were observed using during videotaped analyzed episodes.

Teachers' explanations for why they responded in the ways they did to particular instances of students' difficulty were also summarized. Based on these summaries of teacher explanations for particular responses they offered, the researcher developed a Teacher Reasoning Coding Scheme to classify teachers' primary considerations for their choices of response types when students encounter difficulty with mathematics. The coding scheme included five general categories of teacher intentions when they respond to student difficulties in mathematics. Each general category included between zero and two subcategories. For the nine instances of student difficulty regarding which teachers were interviewed (three instances for each of three teachers), the researcher coded the teacher explanations for the responses they gave using the Teacher Reasoning Coding Scheme. In several instances, more than one teacher intention code was assigned to a particular teacher explanation for a response. A Teacher Reasoning Chart (see Results section) was created to show the frequency for each teacher reasoning code.

The teacher intention codes were then verified by an independent researcher who had not seen the original coding. A cross check was made between the researcher's and the independent researcher's coding as a cross check of the reliability of the coding scheme. The intent was to refine the coding scheme if there was a discrepancy in more than three of the nine episodes between the two independent codings of the teachers' reasoning for their responses.

The teachers' reasoning for their responses were compared with the actual student outcomes following the teachers' responses, focusing on whether the teachers' intended goals for the students were realized.

3.8 Pilot Study

A pilot study was conducted to help frame the questions and refine the methodology for this study. The pilot study involved analysis of two middle school mathematics class sessions that were video and audio taped in June 2008 during the pilot study for Phase Two of the larger study. The students in the June classrooms worked in groups of two or three on the building blocks mathematics problem. The questionnaire students completed at the end of the sessions was slightly modified for the November and December sessions. The classes were located in public schools in the New Jersey city in which the November-December sessions took place and were instructed by female teachers with several years of teaching experience.

As part of the pilot study, seven camera views from one of the June sessions and four camera views from the other June session were transcribed and subsequently analyzed by the researcher and another member of the research team for selection of relevant events. The original intent of this study was to analyze instances of student mathematical error where a teacher was involved. Preliminary results of the pilot study showed that these instances occurred very infrequently during the class sessions. The pilot and main study were therefore expanded to focus on all instances of student difficulty with mathematics problems where a teacher was involved. Events of student difficulty where a teacher was involved were then selected from the set of pilot video data and verified by the researcher and the additional member of the research team. The procedure for analyzing each instance of student difficulty described above was applied to a selected instance of student difficulty from the pilot video data. This process helped refine the method for describing, analyzing, and formatting the data pertaining to each instance of student difficulty examined in the main study.

A pilot interview was conducted with a teacher, presently a doctoral student and member of the research team, whose classes were videotaped for Phase One of the larger affect study. The interview involved several instances of student difficulty identified in one videotaped class session he taught. Each instance was time coded, transcribed, and classified according to teacher response type. The events were then clipped for use during the stimulated recall interview. After the interview, the teacher and present researcher participated in a debriefing session where the teacher reflected on his thoughts regarding the interview. At this time, the teacher mentioned that although the class session under investigation occurred two years prior to the interview, he was able to recall much of what happened during the class session as well as his reasoning for responses he offered to students who encountered difficulty during the session.

3.9 Plan for Qualitative Approach to Data Analysis

In this study a qualitative approach as naturalistic as possible was used to analyze teacher-student interactions and students' engagement in mathematics during class. This approach, which was employed in order to analyze student and teacher behaviors in environments where the social and physical setting were as unchanged as possible, allowed for the observation of teachers and students where their behaviors would not be altered by changes in context (Marshall & Rossman, 1999). The qualitative approach in this research also permitted the study of in-depth relationships between teacher responses and student engagement in mathematics. In addition, since the present research sought to develop understanding about why teachers choose to respond to students in particular ways in specific classroom situations, in-person interviews with teachers were necessary in order to elicit the teachers' thoughts and feelings about their responses. The qualitative approach in this research facilitated the investigation of phenomena related to teacher responses to student difficulty in mathematics and student engagement by allowing for observation in environments as naturalistic as possible, in-depth analysis, and face-to-face interaction with participants.

Data were analyzed using an inductive approach to develop theory "in constant interaction with the data" (Maxwell, 2005, p. 42). Instances of student difficulty with mathematics problems and students' subsequent engagement in mathematics were examined in order to develop theory about teacher responses and student engagement that is grounded in the data from the study.

CHAPTER 4: RESULTS

4.1 Overview

This chapter initially presents a description of the five coding schemes that emerged from this analysis (section 4.2). These involve coding systems for categorizing: 1) Types of Student Difficulties, 2) Types of Teacher Responses, 3) Teacher Intentions for Providing Particular Responses, 4) Students' Behavioral Engagement, and 5) Students' Cognitive Engagement. Also described is a rubric including examples of students' affective engagement, for each of several categories of this construct. Section 4.3 of this chapter reports on the reliability measures for the coding systems.

The following three sections discuss the 29 instances of students' difficulty analyzed in this project. Section 4.4 describes the 4 analyzed instances of students' difficulty during Ms. A.'s class, Section 4.5 discusses the 14 analyzed instances of difficulty during Ms. B.'s class, and Section 4.6 details the 11 examined instances of difficulty during Ms. S.'s class. Each of sections 4.4, 4.5, and 4.6 first outlines the types of difficulties students encountered for the particular teacher and the students involved in each difficulty. The types of responses each teacher provided to her students are then listed along with the frequency for each response. An illustrative case for each type of teacher response is then described in detail. The next portion of each of sections 4.4, 4.5, and 4.6 reports on the reasoning teachers provided when asked during an interview to describe how they usually respond to students' difficulties during mathematics problem solving. A comparison is then made between the teachers' self-reports of ways in which they respond to students' difficulties and the types of responses the teachers actually offered during the analyzed instances of students' difficulty. Next, each teacher's reasoning for the particular responses she offered to students during the instances of difficulty selected for use during stimulated recall interviews with the teachers is presented. These explanations of teachers' reasoning for their interventions are then compared with the students' actual behaviors observed on videotape following the interventions.

An overview of codes describing students' behavioral, affective, and cognitive engagement subsequent to the teacher's intervention during each instance of difficulty is then provided, followed by one detailed example illustrative of each type of engagement. An in-depth, qualitative analysis of students' cognitive engagement before and after the difficulties occur is then presented for each instance of difficulty selected for further investigation via teacher interviews (described above).

The final analysis presented for each teacher involves the relevant students' questionnaire data. Each student's self-reported feelings on the questionnaire are listed, along with Positive Emotions and Engagement with Impasse subscales. Students' self-reported affect is then compared with the affect inferred from that student's behaviors on the videotape of the class session. Students' self-reports on the questionnaire regarding whether or not they experienced difficulty during each math session are also documented and then compared with the frequency and types of difficulties observed on videotape for that student. Each of sections 4.4, 4.5, and 4.6 concludes with a summary of the findings for each teacher. Similarities and differences among teachers in interventions provided to students and subsequent student engagement in mathematics are then highlighted.

The majority of raw data gleaned during this study is included in Appendix A. Data in this appendix is divided into three basic sections: data for Ms. A.'s class, data for Ms. B.'s class, and data for Ms. S.'s class. Data for each teacher's class is divided into 1, 2, or 3 subsections, depending on the number of class sessions included in this study for which there were student difficulties where the teacher was involved. These subsections are referred to as Day 1, Day 2, and Day 3. Data in Appendix A for each day's session includes three components: a) students' self reports from their questionnaires, b) transcripts of events of students' difficulty obtained from classroom videotapes, with codes for associated variables , and c) a comparison of students' self-reports on their questionnaires and data gleaned from the videotaped class sessions. Following is a more detailed description of components a, b, and c.

Students' self-reports from their questionnaires for each day (*a*) include relevant students' reports of feelings and experiences of difficulty for that day's session. Also included are Positive Emotions and Engagement with Impasse subscales for each student. Time coded transcripts of events of student difficulties for each day (*b*) include snapshots of students obtained from videotapes at key moments during their group work. Following each transcribed event of difficulty, 11 elements are documented: 1) a description of each difficulty that occurred during that event, 2) the type of difficulty for each difficulty that occurred, 3) the type of task in which the student(s) was/were involved when each difficulty occurred (i.e. open vs. closed ended, routine/non-routine), 4) the classroom context of each difficulty (i.e. whole/small group), 5) a description of the teacher's response to each difficulty along with the teacher response code, 6) the teacher's reasoning for her responses, along with the code for the teacher's intention, for difficulties selected for use during teacher interviews, 7) a description of the relevant students' behaviors related to their engagement following each difficulty, along with snapshots obtained from the videotape footage at key moments, 8) inferences of the relevant students' emotions following each teacher response, based on the descriptions of the students' behaviors, 9) classifications of the relevant students' behavioral engagement following each teacher response, along with students' behaviors supporting those classifications, 10) classifications of relevant students' cognitive engagement following each teacher response, along with students' behaviors supporting those classifications, and 11) classifications of relevant students' affective engagement following each teacher response, along with students' behaviors supporting those classifications.

Component *c*, the comparison of students' self-reports on their questionnaires and videotaped data, lists each student's Positive Emotions Score and Engagement with Impasse Score, obtained from the students' questionnaires, along with the population means for each of these scores. Also included are the affect codes assigned to the students during analysis of difficulties in which they were involved that day (*element 11 in component b*). A qualitative comparison is then made between each student's self-reported affect and the affect inferred from their behaviors on videotape. A group analysis of students' self-reported affect vs. affect inferred from their behaviors on videotape is also documented. Students' reports on the questionnaire regarding whether or not they experienced difficulty during each math session are then described and compared with the frequency and types of difficulties observed on videotape for that student.

In addition to the material in Appendix A, in describing students' cognitive engagement, the researcher at times referred to the classroom videotapes, since only segments of the videotaped sessions were transcribed. In those instances, a notation is made in the analysis indicating that data was drawn from videotape.

Appendix B contains the protocol the researcher used for stimulated recall interviews with Ms. A., Ms. B., and Ms. S. Appendix C includes the Student Engagement Questionnaire completed by the students after each day's session. Appendix D consists of transcripts of the interviews with the three participating teachers.

4.2 Coding Schemes

This section describes six coding schemes that were developed and/or modified during this research. These include a system for classifying each of the following: 1) Types of Student Difficulties, 2) Types of Teacher Responses, 3) Teacher Intentions for Providing Particular Responses, 4) Students' Behavioral Engagement, and 5) Students' Cognitive Engagement. Also included is a rubric listing examples indicative of several categories of students' affective engagement.

Types of Student Difficulties

Codes for student difficulties based on how the difficulties came about were created during the analysis for this project. Table 4.1 includes a list of the codes, along with their abbreviated notations which were used in the analysis.

Table 4.1: Types of Student Difficulty Coding Scheme

Type of Student Difficulty:
Student expressed or agreed with an incorrect or insufficient answer or idea regarding a mathematics problem *(Incorrect/Insufficient Answer/Idea)*
Student arrived at an impasse while working on a mathematics problem *(Impasse)*
Student appeared unable to adequately answer a mathematics question asked by the teacher or a fellow student *(Unable to Answer)*
Student expressed a lack of understanding of a classmate's solution *(Does Not Understand Solution)*

Types of Teacher Responses

An initial set of codes for classifying teachers' responses was created before beginning the analysis for this study. This set of codes is included in Table 3.1, earlier in this work. During observations of class sessions, this set of codes was modified to include teacher responses that did not fit the existing categories. The teacher response codes were also classified into the following five general response types: Having Other Student(s) Help Resolve Difficulty, Having Student Help Self, Helping Student Directly, Helping Student Indirectly, and Not

Responding/Responding without Offering Suggestion for Resolving Difficulty. Table 4.2 lists the specific teacher response codes included within each general response type. Abbreviations for the teacher response codes, which will be used later in this chapter, are noted. Response types that are highly similar to each other, such as *Asking the student to explain his/her solution* and *Asking the student to prove his/her solution* are listed within one cell.

Table 4.2: Teacher Response Coding Scheme

Type of Teacher Response
Having Other Student(s) Help
Inviting (verbally or non-verbally) (an)other student(s) to respond *(Inviting Another Student)* Suggesting student(s) share difficulty with peer(s) *(Sharing with Peers)* Reminding student about another student's idea *(Using Another's Idea)*
Having Student Help Self (teacher not offering mathematical suggestion/not having others offer mathematical suggestions)
Giving the student time to think about the problem *(Giving Time)*
Helping Student Directly (may also involve student helping him/herself)
Providing the correct answer *(Providing Correct Answer)* Explaining the concept or the mathematics problem/task *(Explaining)*
Helping Student Indirectly (may also involve student helping him/herself)
Leading the student(s) to the correct answer/idea *(Leading)*
Offering a counter example to an incorrect response *(Providing Counter Example)*
Probing- asking the student to explain his/her solution or the reasoning he/she used *(Probing)* Asking the student to prove his/her solution *(Asking for Proof)*
Offering a simpler problem as an example *(Offering Simpler Problem)*
Suggesting a process to use in solving the problem, including following a procedure or constructing a representation *(Suggesting Process)*
Showing the student a diagram/model *(Showing Diagram/Model)*
Not Responding/Responding without Offering Suggestion
Saying a student's answer is wrong *(Saying Answer is Wrong)*
Choosing not to respond to an incorrect response *(Not Responding)*
Acknowledging or repeating the incorrect response *(Acknowledging/Repeating Incorrect Response)* Asking student to repeat his/her insufficient response *(Having Student Repeat)* Asking a question to clarify what the student(s) said without probing for an explanation *(Clarifying)* Repeating the teacher's question *(Repeating Teacher's Question)*

Although teacher response types in several of the five general categories may have involved the students helping themselves in some way, the teacher responses included in the category of Having Student Help Self did not include mathematical suggestions, either from the teacher or other students. Responses classified as Having Student Help Self, as opposed to those in the category of Not Responding/Responding without Offering Suggestion for Resolving Difficulty, conveyed to the students the teacher's confidence that the students can resolve the difficulty on their own.

The response type *Leading Students to the Correct Answer/Idea* included responses where it appeared the teachers' primary aim was to highlight particular ideas or ways of thinking for the students. During these responses, teachers often questioned students as a means of guiding them to the correct answer/idea. The response type *Probing*, in contrast, was used where it appeared the teacher's primary goal in asking questions of the students was to understand, rather than guide, students' thinking.

In some instances, difficulties involved more than one student. In those cases, the response type uses plural form, as in Having Students Help Selves in place of Having Student Help Self.

Teachers' Reasoning for Providing Particular Responses

Teachers were asked during stimulated response interviews to explain why they responded in the ways they did to particular instances of student difficulty examined in this study. The teachers' reasoning for their responses were then coded using a list of teacher reasoning codes developed during this research. The coding chart for teachers' explanations for their responses is included below in Table 4.3.

Table 4.3: Coding Scheme: Teachers' Reasoning for Responses

Teachers' Reasoning Coding Chart
Mathematics Goals (M):
Problem Solving Skills
Conceptual Understanding
Students' Emotions in Regard to Mathematics (E):
Working without Pressure
Saving Face in Front of Classmates
Students' Attitudes (At):
Attitudes regarding participation in mathematics activities
Students' Abilities/Skills (Ab/S):
Mathematics Ability
Organizational Skills
Time Constraints (Time)

In several cases teachers' explanations involved more than one teacher reasoning category and were therefore given more than one code.

Students' Behavioral Engagement

Students' behavioral engagement was considered at a global level as being on or off task for the greater portion of time under investigation. Students' behaviors were categorized as on- or off- task using a modified version of a coding system described by Gresalfi (2009), included in Table 3.2, in Chapter 3 of this work. Several behaviors were added to the original coding system during this research. These behaviors included those involving the usage of manipulatives or other materials, 'Fooling around in other ways,' 'Covering eyes without noticeably connecting back to work,' 'Watching group/class mate(s) explain idea,' 'Expressing agreement/disagreement with group mate's idea,' and 'Redirecting (an)other student(s) to task'. 'Participating in class discussion,' and 'Answering teacher's question' were also added during this investigation as at times students were involved in whole class activities or group discussions, often involving the teacher. The original code of 'Explaining idea to group mate' was modified to 'Explaining idea to group/class mate(s)/teacher.' The complete coding system for students' behavioral engagement is included in Table 4.4.

Table 4.4: Behavioral Engagement Coding Scheme

On- Task Behavior	**Off- Task Behavior**
*Explaining idea to groupmate/*classmate(s)/*teacher* *Answering teacher's question* *Asking for help from group mate* **Watching groupmate/classmate(s) explain idea* **Redirecting (an)other student(s) to task* **Participating in class discussion* *Expressing agreement/disagreement with group mate's idea* *Looking at paper* *Writing* *Reading directions aloud* *Using calculator* **Using manipulatives in way related to task at hand*	*Talking with others about non-math topics* *Sleeping* **Covering eyes without noticeably connecting back to work* *Doodling* *Staring into space without noticeably connecting back to work* *Playing with pen/pencil* **Using manipulatives in way unrelated to task at hand* **Fooling around in other ways*

*Items marked with an asterisk were added to the original coding scheme during this research

Students' Cognitive Engagement

Analysis of students' cognitive engagement in this research began with a system for classifying students' cognitive activity as high or low level based on their observable behaviors while they work on mathematics problems in groups, developed by Volet et al. (2009) and included in Table 3.3, in Chapter 3 of this work. During this research descriptions for students' cognitive activities not included the existing coding system were added, resulting in the coding listed in Table 4.5. Three descriptors for students' cognitive activity were added in this analysis: Restating one's idea without consideration of others' ideas, Using manipulatives (in this case, building towers) without consideration of patterns, and Not Engaged.

The descriptors ‘Restating one’s idea without consideration of others’ ideas’ and ‘Using manipulatives without consideration of patterns’ were added during this study as behaviors involving low level cognitive activity. ‘Restating one’s idea without considering others’ ideas’ was used when a student restated his/her original idea without addressing a challenging argument presented by another student. ‘Building towers without consideration of patterns’ referred to behaviors focused on building towers without consideration of patterns within or among the various sized towers. ‘Not engaged’ was used to describe students’ cognitive engagement when students appeared to be off-task. This included both when students were engaged cognitively with non-task related matters and when students were not noticeably engaged. It should be noted that cognitive behaviors added during this research are classified here according to categories of which they are typically suggestive. Levels of cognitive activity indicated by particular behaviors may vary based upon the context in which the behaviors occur. The final Cognitive Engagement Coding Chart is included in Table 4.5, below.

Table 4.5: Cognitive Engagement Coding Scheme

Low Level Cognitive Activity	**High Level Cognitive Activity**	**No Observable Task-Related Cognitive Activity**
Acquiring knowledge *Considering facts* *Reading* *Paraphrasing or referring to materials* *Checking information* *Clarifying sources* *Help-seeking for details* **Restating one's own idea without consideration of others' ideas* **Using manipulatives (in this case, building towers) without consideration of patterns*	*Elaborating* *Speculating* *Justifying* *Drawing inferences or relations* *Interpreting* *Reasoning* *Negotiating* *Asking thought-provoking questions* *Building on or linking ideas* *Explaining in one's own words* *Help-seeking for understanding*	**Not Engaged*

*Items marked with an asterisk were added to the original coding scheme during this research

Students' Affective Engagement

A classification chart listing examples of students' feelings that correspond with more general categories of students' emotions was adapted from current ideas during this research. The researcher developed the chart by first watching the videotapes included in this research multiple times and transcribing students' behaviors during events of difficulty during the sessions. Students' feelings in relation to mathematics were then inferred from the transcripts. The students' feelings were then categorized as strong, moderate, or mild for positive or negative affect. The affective engagement chart is included below in Table 4.6. It should be noted here that classifying individuals' emotions is complex and difficult. Classifications here are based on inferences from students' behaviors evidenced in videotapes and are necessarily limited by their variability.

Classifications for students' affective engagement offer an overview of the students' emotions. The author does not make the assumption that positive feelings are necessarily indicative of engagement or that negative feelings are evidence of disengagement. The author maintains that negative feelings, in fact, may at times be symbols of engagement and that positive feelings may occur simultaneously with disengagement. A student, for example, may passionately exclaim, "I hate these types of math problems!" while highly engaged with the problem and motivated by the desire to demonstrate to his peers his skill in solving difficult mathematics tasks. Another student may experience excitement regarding a mathematics task, but may at the moment be involved in an animated conversation with her groupmate about her upcoming vacation plans. Affective engagement codes used here are therefore intended to provide information about students' emotions, rather than point to the students' levels of engagement with mathematics.

Table 4.6: Affective Engagement Coding Scheme: Examples of Classifications

	Strong	**Moderate**	**Mild**
Positive Affect	*Excitement regarding mathematics task* *Eagerness to work on mathematics/ Mathematics task* *Pride in work* *Confidence* *High level of interest with no apparent negative feelings* *Satisfaction* *Enjoyment of mathematics task* *Determination to solve mathematics task in anticipation of success*	*Some level of interest with no apparent negative feelings*	*Some level of interest mixed with slight negative feelings such as anxiety, boredom or inferiority to another student*
Negative Affect	*Expressing extremely negative feelings such as anger or hatred towards mathematics/mathematics task* *Defeat* *Inferiority to another student*	*Extreme boredom* *Embarrassment* *Extreme disinterest in mathematics task* *Sense of inadequacy* *Insecurity* *Fear of losing face* *Nervousness* *Expression of dislike of mathematics/ mathematics task* *Anxiety regarding mathematics*	*Disinterest* *Boredom* *Fatigue* *Slight sense of inadequacy*

4.3 Reliability Measures

This section reports on reliability measures for coding in this analysis, including the classification of teachers' responses, teachers' reasoning for their responses, students' cognitive engagement, and students' affective engagement. Verification of codes was completed independently by a former mathematics teacher trained by the present researcher in using the coding systems employed in this investigation.

Reliability: Coding of Teacher Response Types

An independent verifier coded for teachers' initial and follow-up responses to students' difficulty in all 29 cases of teacher responses to students' difficulties examined in this work. Results showed that 28 of the 29 initial teacher responses (96.5%), as coded by the researcher and verifier, were identical. In 4 of the 29 cases, the second coder either added a follow-up teacher response code or omitted a follow-up teacher response code that was assigned by the researcher.

Reliability: Coding of Teachers' Reasoning for Particular Responses

Each of the three teachers was asked during a stimulated recall interview to reflect on the responses she offered to students during three selected episodes of her students' difficulty. In total, teachers provided reasoning for their responses to nine episodes of students' difficulty. One or more of the codes listed in Table 4.3 was applied by the researcher to each teacher's explanation for the way in which she responded during each episode of difficulty. The teachers' explanations for all nine instances were then classified independently by a second coder. In 6 of 9 episodes of teachers' reasoning, classifications assigned by the researcher and second coder were identical. In two cases, the verifier assigned an additional code. These 2 cases were discussed by the researcher and verifier, and an agreement was reached. In one of these cases, the researcher added the additional code to the teacher's reasoning for her response (Code of *Abilities/Skills-Mathematics Ability* added to Ms. A.'s explanation of her reasoning for Difficulty 4). In the other of these cases, the verifier agreed that the additional code did not apply. In the third case where coding was not identical, the verifier omitted one code from an episode of teachers' reasoning.

Reliability: Coding of Students' Cognitive Engagement

Codes applied to describe students' cognitive activity following teachers' responses were verified by an independent coder for four instances of difficulty per teacher. Several difficulties involved more than one student. In these cases, codes assigned to describe each student's cognitive activity were verified. The verifier assigned codes to students' cognitive activity based on descriptions of the students' behaviors in the section of Appendix A entitled "Description of Behavior Related to Student's Engagement Following Response" for each difficulty. The instances of difficulty chosen for verification were selected at random. They included the first four instances of difficulty identified for each teacher. At times, the researcher indicated that there was insufficient information to code for the cognitive activity of one or more students involved in a difficulty. The verifier did not assign codes to these students' cognitive processing in such instances. There were 26 cases of students involved in the 12 instances of students' difficulty selected for verification. In 7 of the 26 cases, there was insufficient information to allow for coding of the students' cognitive activity. This resulted in 19 cases of verification of students' cognitive activity.

Results indicated that codes assigned by the researcher and verifier to describe students' cognitive activity were identical in 18 of the 19 cases (94.7%). The single case where codes were dissimilar resulted in agreement following a discussion, after which the researcher changed a code describing a student's cognitive activity (Difficulty 8, Marcos's cognitive engagement changed from high to low level).

Reliability: Coding of Students' Affective Engagement

Codes applied to describe students' emotional feelings following teachers' responses were verified by an independent coder for four instances of difficulty per teacher. Several difficulties involved more than one student. In these cases, codes assigned to describe each student's emotions were verified. The verifier assigned codes to students' feelings based on descriptions of the students' behaviors in the section of Appendix A entitled "Description of Behavior Related to Student's Engagement Following Response" for each difficulty. The instances of difficulty chosen for verification were selected at random. They included the first four instances of difficulty identified for each teacher. There were 26 cases of students involved in the 12 instances of students' difficulty selected for verification.

Results indicated that codes assigned by the researcher and verifier to describe students' emotions as positive or negative matched in 100% of the 26 cases. In several cases, however, the researcher's and verifier's codes differed with respect to the specific categories indicating the extent to which students experienced positive or negative emotions (i.e., whether the students' feelings were strong, moderate, or mild). Specific codes assigned by the researcher and verifier indicating the extent to which students experienced positive or negative emotions were identical in 20 of the 26 cases (76.9%). After discussion, the verifier agreed with the researcher's coding of students' affective engagement in all instances. Discussions resulted in the researcher adding descriptions of students' behaviors pertaining to their emotions in several cases.

After coding for students' emotions during the selected episodes, the verifier was asked to review the lists of emotions inferred from students' observable behaviors recorded by the researcher, for each of the 26 instances for which students' affective engagement was verified. The verifier was requested to provide feedback regarding agreement or disagreement with the lists of inferred emotions, based on the descriptions of students' behavior in Appendix A. The verifier agreed with the recorded emotions in all instances.

4.4 Teachers' Responses and Students' Engagement: Ms. A.

Two consecutive videotaped sessions of one group of students in Ms. A.'s class were examined in this research. The group included four students: Amanda, Emanuel, Eliot, and Juan. The first day's session contained four instances of student difficulty where Ms. A. was involved. During the second session there were no instances of student difficulty where Ms. A. was involved. For each of the four analyzed instances of difficulty for Ms. A.'s class, Table 4.7 lists the students involved and the type of difficulty.

Table 4.7: Types of Difficulties and Students Involved, Ms. A.

	Students Involved:	*Type of Difficulty:*
Difficulty 1	Amanda	Incorrect Idea
Difficulty 2	Amanda	Incorrect Idea
Difficulty 3	Amanda, Emanuel, Eliot, Juan	Unable to Answer
Difficulty 4	Amanda, Emanuel, Eliot, Juan	Insufficient Answer

Types of Responses

Ms. A.'s four initial responses to her students' difficulties fell into two of the five response categories: *Helping Student Indirectly* and *Having Student Help Self*. Her single follow-up response fell into a third response category: *Having Other Student(s) Help*. Table 4.8 depicts the particular types of responses within each category that Ms. A. provided. Types of responses are listed using short descriptors; refer to table 4.2 for the detailed set of codes.

Table 4.8: Types of Responses, Ms. A.

Numbers preceding a hyphen in the frequency column represent initial responses; numbers after a hyphen refer to follow-up responses.

Type of Teacher Response: Ms. A.	**Frequency**	**Total Frequency**
Having Other Student(s) Help		
Inviting Another Student *Sharing with Peers* *Using Another's Idea*	0-1	1
Having Student Help Self		
Giving Time	1-0	1
Helping Student Directly		
Providing Correct Answer *Explaining*	0-0	0
Helping Student Indirectly		
Leading	1-0	1
Providing Counter Example	1-0	1
Probing *Asking for Proof*	0-0	0
Offering Simpler Problem	0-0	0
Suggesting Process	1-0	1
Showing Diagram/Model	0-0	0
Subtotal: Teacher Helping Student Indirectly	***3-0***	***3***
Not Responding/Responding without Offering Suggestion		
Saying Answer is Wrong	0-0	0
Not Responding	0-0	0
Acknowledging or Repeating Incorrect Response *Having Student Repeat* *Clarifying* *Repeating Teacher's Question*	0-0	0
Subtotal**: **Not Responding/Responding without Offering Suggestion for Resolving Difficulty	*0-0*	*0*
Total Responses	**4-1**	**5**

Following is a detailed analysis of one illustrative instance for each type of response Ms. A. provided to her students.

Helping Students Indirectly: Difficulty 1

Ms. A.'s response to Amanda after Difficulty 1 was classified as *Helping Student Indirectly.* During Difficulty 1, Amanda proposed multiplying the number of blocks in one branch of the 5-block high tower by 2 to find the number of blocks in one branch of the 10-block high tower. The excerpt below begins with Amanda's suggestion.

> Amanda: But I was thinking, right, to get the 10 block, why don't we…couldn't we just multiply the number of blocks to get the 10. Multiply by 2 (*She holds up 2 fingers*)?
> Ms. A.: Why don't you try it? Try it and we'll see if it's going to give you this answer…the same answer.
> Amanda: Ok, ok.
> Ms. A.: So what…what are you going to multiply in this case?
> Juan: 5 times 2
> Amanda: 5 times 2, which is the height, and the height would be 10…and then there would be 8 blocks on each side.
> Ms. A.: So, now, what's the total number of blocks for this one?
> Amanda: (*smiles*) 8, 16, 24, 32…It would be 42, not 46.
> Ms. A.: So, which one is correct?
> Emanuel: So let's just do the pattern that we…
> Amanda: The pattern is right.
> (Appendix #, Event 1, Difficulty 1, lines 44-58)

Ms. A. suggested a process which would allow Amanda to recognize whether her proposed method was valid. After following Ms. A.'s suggestion, Amanda correctly concluded that her group's original method, rather than her proposed "shortcut" method, was correct.

Having Students Help Themselves: Difficulty 3

At the beginning of Event 1, Emanuel, Amanda, and Eliot had described to Ms. A. a pattern they found where the total number of blocks in consecutive towers increases by 5's. While Ms. A. was at the students' table, Emanuel, Amanda, and Eliot discovered another pattern where the total number of side blocks in consecutive towers increases by 4's. Emanuel, Amanda, and Eliot then re-explain the original pattern they found involving the total number of blocks increasing by 5's. Ms. A. asks the students how the two patterns they found relate to each other. The excerpt below begins with Ms. A.'s question regarding how the "increasing by 5's" pattern relates to the "increasing by 4's" pattern.

> Ms. A.: Ok, so now, how is that related to what you're saying increasing 4 on the sides?
> Emanuel: Cuz (*smiles*) cuz…uh…(*leans forward with his hands on his chin. All the other students at the table look at their papers.) (Figure A-D1-11-01-59)*
>
> Figure A-D1-11-01-59
>
> (*Eliot sneezes*.)
> Emanuel: Bless you.
> Eliot: Thank you.
> (*The students are quiet for a while*.)
> Ms. A.: Think of it and I'll come back to you. Ok?
> (*Ms. A. leaves the table*.)
>
> (Event 1, Difficulty 3, lines 159-169)

Difficulty 3 occurs during the excerpt when the students appear unable to answer Ms. A.'s question. Ms. A.'s response to the students was coded as Having Students Help Themselves. The plural form for the code was used in this case since Ms. A.'s response was addressed to the group.

Having Another Student Help Resolve Difficulty: Difficulty 4

At the beginning of Event 2, Amanda, Emanuel, Eliot, and Juan explain to Ms. A. the method they used to solve towers of various heights. The students say that to find the number of blocks in a particular tower, they multiplied 4 by one less than the number of blocks in the height and then added the number of blocks in the height. Difficulty 4 occurs when Ms. A. asks the students to write the general rule they used to solve the 100-block high tower. The students say they multiplied 4 by 99 and then added 100 to solve the 100-block high tower. To assist the students in coming up with the general solution, Ms. A. asks them to label each number in their calculation for solving the 100-block high tower. The students have difficulty describing what each number in their calculation represents. Ms. A. asks the students questions to lead them towards the general solution. Near the end of Ms. A.'s conversation with the students, the discussion concerns the calculation the students used to solve the 5-block high tower, namely, 4 times 4, plus 5. Amanda says that the second '4' in the calculation represents the "amount of cubes added to the side" (Event 2, Difficulty 4, line 179). Emanuel says, "Actually, this is the amount of cubes *in the sides* (with emphasis on 'in the sides')…because it didn't add. Because they didn't add 4 to this side, cuz this was already 3 in the other one" (Event 2, Difficulty 4, lines 183, 186-187). Ms. A. responds by saying to Amanda and Emanuel, "Are you sure you don't want to go back and check? Go back and check in the previous phase. Go back and check in the previous phase and see if what she's saying is right, or what you're saying. Check. Check against each other. Ok?" (Event 2, Difficulty 4, lines 192-195). This follow-up response was coded as Having Another Student Help Resolve Difficulty.

Ms. A.'s Reasoning for Responses

During the retrospective interview, Ms. A. was asked several questions regarding the ways in which she responds when students have difficulty while working on mathematics problems. The section below includes each of those interview questions and a summary of Ms. A.'s responses, along with supporting quotations from the interview transcript (included in Appendix D).

Q1: Can you describe how you usually respond when students have difficulty while they work on mathematics problems in class?

Ms. A. replied that she would pull the student from the group and work individually with him/her. She said she also might relocate the student to a different group which may be a better setting for him/her. Ms. A. added that she has her students work in groups to facilitate peer teaching. She explained, "Sometimes I would say something and they would look at me like I'm talking another language. This same thing…is going to be said by a student in the group and they will 'Oh, now I get it!' So as if it's the magic of being of the same age, they understand each other" (lines 37-40).

Q2: Are there things you try not to do when you respond to student difficulty? (If yes) Can you describe?

Ms. A. answered that she tries not to make the students feel incapable, but rather, tries to "build them with the very little that they have" (lines 47-48). She aims to give weak students hope that they can understand the mathematics concepts and to help them realize that learning is a process.

Q3: Do you respond in different ways in different situations?

Yes

Q4: Can you describe some factors that might determine how you respond?

Ms. A. stated that students' mathematics ability influences the way she responds to students' difficulty. Ms. A. explained that students with high mathematics ability need less teacher intervention as they seek help from each other on their own. Regarding high ability students who have difficulty, Ms. A. said, "If I just give them the road lines and let them explore, they're going to get to the end result" (lines 73-74). Ms. A. maintained that low ability students, however, need specific, step-by-step guidance.

Ms. A. noted that in cases where students appear to be disengaged in mathematics, the students' attitudes towards mathematics and the presence of outside pressures for the students influences the way she responds. Ms. A. remarked, "…the way you handle the student, you're going to be influenced a lot if you know some background" (lines 126-127). Ms. A. reported having "zero tolerance" for a student who understands the mathematics but is simply "fooling around." On the contrary, "a student who you know…is really struggling and is going through a lot of pressures on the outside…coming to a math class is the least of their priorities at this point in their life, you will try to give them the encouragement that they need, the support that they need…" (lines 130-133).

Comparison of Ms. A.'s Self-Description of Response Types to Observed Responses

The responses Ms. A. provided to the students during the analyzed instances of difficulty for her class were consistent with the ways in which she claimed to respond. In one case examined in this research, all the students in the group under investigation, which was part of a high ability class, appeared unable to answer Ms. A.'s question (Difficulty 3). Ms. A. responded by giving the students time to think about the problem without her intervention. This response was concordant with her statement explaining that high ability students require minimal intervention when they encounter mathematics difficulties.

In the other three instances of student difficulty, Ms. A. responded by helping the students indirectly. In one case Ms. A. led the students to the solution, in another instance she offered a counterexample to an incorrect idea, and in the third episode she suggested a process to use in solving the problem. In both cases where only one student was involved in a difficulty (Difficulties 1 and 2), Ms. A. responded without encouraging other students to help the student (Amanda) resolve the difficulty. Even during the two instances where all four students were involved the difficulties (Difficulties 3 and 4), Ms. A. did not actively pull quiet students into the conversation by asking them to share their opinions with the others in the group. These responses were congruent with Ms. A.'s assertion that she works individually with students who have difficulty with mathematics.

Teachers' Reasoning for Particular Responses and Comparison with Actual Outcomes

During the stimulated recall interview, Ms. A. was asked to explain her reasoning for the responses she provided to the students after Difficulties 1, 3, and 4. Ms. A.'s explanations for her response to Difficulties 1 and 3 were given more than one code. The codes for Ms. A.'s explanations for her responses fell into three of the five coding categories for Teachers' Reasoning for their Particular Responses. These categories, in order of the frequency in which they occurred, included Mathematics Goals, Students' Abilities/Skills, and Students' Emotions in Regard to Mathematics. Below are detailed descriptions of Ms. A.'s self-reported reasoning for her responses to Difficulties 1, 3, and 4. Following each description is a comparison of Ms. A.'s reasoning for providing the response to what actually occurred following the response, focusing on whether Ms. A.'s intended goals were realized.

Reasoning for Response to Difficulty 1

Difficulty 1 occurs when Amanda proposes multiplying the number of blocks in one ranch of the 5-block high tower by two to find the number of blocks in one branch of the 10-block high tower. Ms. A. responds by suggesting that Amanda use her proposed "shortcut" method to solve the 10-block high tower and then compare her solution to the one she obtained earlier. When Amanda does so and obtains a solution different from her previous one, Ms. A. asks her which solution is correct.

During the stimulated recall interview, Ms. A. reported that she responded in that way since she wanted the students to make their own decision. She explained:

Because if I just give them the answer they will just take it and that's it, but if

> they spend the time thinking and turning it around, it will stay with them. They'll have ownership of this solution, and they're going to be able to apply it in different problems, similar or non-similar.
>
> (Appendix D, Ms. A. Interview Transcript, lines 213-216)

Ms. A. added that her response aimed to enhance students' confidence in their ability to logically arrive at solutions on their own. Ms. A. reported that the students' mathematics ability influenced her response. She asked the students higher order thinking questions right away since they are bright mathematics students. Ms. A.'s reasoning for her response to Difficulty 1 was categorized using three codes: Mathematics Goals- Problem Solving Skills, Mathematics Goals- Conceptual Understanding, and Students' Abilities/Skills- Mathematics Ability.

As Ms. A. anticipated, following her response, Emanuel and Amanda accurately concluded that the pattern they used previously to solve the 10-block high tower is correct.

Reasoning for Response to Difficulty 3

During Event 1, Ms. A. asks the students how the pattern they found where the total numbers of side blocks in consecutive towers increase by 4's relates to the second pattern they found where the total numbers of blocks in consecutive towers increases by 5's. Difficulty 3 occurs when the students are quiet and appear unable to answer her question. Ms. A. responds by telling the students to think about her question and that she will come back to them. Ms. A. pointed out during the stimulated response interview that she responded in that way since she wanted to give the students time to think without pressure. Ms. A. reported that the students' mathematics ability definitely influenced her choice of response. She explained:

> This was high performing group, so I just gave them the question and left them, and I know when I come back they will be at a different point. They will take that and

> continue. Another group that is not at a high level of performance, they would really struggle with it and they might not be able to move anywhere from there. So I would have needed to stay a little bit longer to make sure I guided them into that step so that they can move on to the next one.
>
> (Appendix D, Ms. A. Interview Transcript, lines 245-250)

The codes assigned to Ms. A.'s reasoning for this response were Students' Emotions in Regard to Mathematics and Students' Abilities/Skills- Mathematics Ability.

After Ms. A. leaves the students' table, the students work diligently on the problem and progress in their mathematical thinking, as Ms. A. intended. The students focus on solving the 100-block high tower. Emanuel finds a pattern where the total numbers of side blocks in consecutive towers are consecutive multiples of four. He says that since the total number of side blocks in the one-block high tower is 4 times 0, the total number of side blocks in the 100-block high tower should be 4 times 99. The students use this inference to solve the 100-block high tower. They then confirm the validity of their method by comparing their solution for the 10-block high tower using their new method to their previous solution for the 10-block high tower. The students demonstrate their excitement and pride at finding the solution to the 100-block high tower. Amanda exclaims, "Yay!" as Juan, Eliot, and Emanuel sit up straight in their chairs and smile. Emanuel says, "We're smart." Emanuel and Amanda wave their hands in the air and call Ms. A. to come to their table.

Reasoning for Response to Difficulty 4

Difficulty 4 occurs when Ms. A. asks Amanda, Emanuel, Eliot, and Juan to label each number in their computation for finding the number of blocks in the 100- block high tower. To obtain the solution for the 100-block high tower, the students have calculated (4 x 99) +100=496. The students have difficulty explaining what each number in the computation represents.

Ms. A. asks the students more specific questions to help them label each number in the computation and thereby express the general solution for finding the number of blocks in any size tower. At one point, Ms. A. asks the students to label each number in their calculation for the 5-block high tower. Emanuel and Amanda disagree about whether the second ‘4’ in their solution for the 5-block high tower represents the amount *in* a side or the amount *added* to a side. Ms. A. tells them to check their answers against each other and then leaves the table.

When questioned during the interview about why she responded this way, Ms. A. stated that she wanted to make sure the students understand the concept completely. She asked questions to encourage the students to think about their answer at a deeper level and to generalize the solution. Ms. A. recounted:

> “At the beginning they thought they got it all…they thought they were done. But once I asked them to explain, and that is the process that would really get them to start comprehending…the concept…Only then they started to struggle and realize that they sort of understand, but they don’t…really know 100%...So they really needed to take time to explain it to themselves, to say it to themselves, ‘What is each number referring to?’ in this answer that they think is the answer to the problem. So that’s really useful, because sometimes students think, “Okay we know the answer.” They write it, but does it really make sense to them? Did they really comprehend what they did and why?…So basically, I just wanted them to really comprehend what they put down…It’s not enough to just understand the concept superficially. I wanted them to have it more stable, to really comprehend it so that when they see it in another problem they will be able to do the same process again.”
>
> (Appendix D, Ms. A. Interview Transcript, lines 255-272, 284-286)

Ms. A. explained that since the students in this group were of high mathematics ability, she used abstract questioning and did not rely on manipulatives to facilitate students’ understanding during her discussion with the group. Ms. A.’s reasoning for her response in this instance was coded as Mathematics Goals- Conceptual Understanding and Students' Abilites/Skills-Mathematics Ability.

The students' mathematical activity after Ms. A. moves to another group is in accordance with Ms. A.'s self-reported intention for providing her response. After Ms. A. leaves the table, Amanda agrees that the second four in their calculation of (4 x 4) +5=21 represents the amount of blocks *in* a side; not the amount of blocks *added* to a side. Amanda writes a verbal explanation for the process they used to solve the various sized towers, which can be applied to any size tower. She says they multiplied "the side" by the number of blocks in a side and then added the height to find the number of blocks in a tower. The other students agree with Amanda's explanation.

Student Engagement

Overall, Emanuel, Amanda, Eliot, and Juan were highly engaged during the analyzed video segments of the classroom session examined in this work. Observations of the videotaped session found the students demonstrated high levels of behavioral, cognitive, and affective engagement during the analyzed episodes. The table below lists the codes assigned to the students' behaviors following Ms. A.'s response to each difficulty in which they were involved, for three domains of engagement: behavioral, cognitive, and affective.

Table 4.9: Teacher Responses and Subsequent Student Engagement Codes, Ms. A.

	Behavioral Engagement	**Cognitive Engagement**	**Affective Engagement**
Difficulty 1 Offering a suggestion of a process to use in solving the problem			
Amanda	On-task	High level cognitive activity	Strong positive feelings
Difficulty 2 Offering a counter example to an incorrect response			
Amanda	On-task	High level cognitive activity	Strong positive feelings
Difficulty 3 Giving the students time to think about the problem			
Amanda	On-task	High level cognitive activity	Strong positive feelings
Emanuel	On-task	High level cognitive activity	Strong positive feelings
Eliot	On-task	High level cognitive activity	Strong positive feelings
Juan	On-task	Insufficient Information	Mild positive feelings
Difficulty 4 Leading the student to the correct answer/idea Suggesting students share difficulty with peers			
Amanda	On-task	High level cognitive activity	Strong positive feelings
Emanuel	On-task	High level cognitive activity	Strong positive feelings
Eliot	On-task	Insufficient Information	Strong positive feelings
Juan	On-task	Insufficient Information	Strong positive feelings

As indicated in the table above, overall, the students were highly engaged in all three dimensions: behavior, cognition, and affect. Following Difficulties 1 through 4, the student(s) involved in each difficulty appeared to be on-task and showed evidence of high levels of cognitive engagement where cognitive behaviors were observable. Aside for one instance involving Juan, the students also demonstrated strong positive affect following each of the difficulties.

Below is an excerpt representative of each type of engagement, illustrating the students' high levels of engagement.

Behavioral and Cognitive Engagement: Following Difficulty #3

Difficulty 3 occurs when Ms. A. asks Emanuel, Amanda, Eliot, and Juan how the pattern they found where the total numbers of side blocks in the towers increase by 4's relates to the pattern they found where the total numbers of blocks increase by 5's. Emanuel leans forward with his hands on his chin. All the other students at the table look at their papers. The students are quiet for a while. Ms. A. tells the students to think about it and says she will come back to them.

After Ms. A. leaves their table, the students work on solving the 100-block high tower. Emanuel writes on his paper while the other students at the table watch him.

Emanuel notices a pattern where the total number of side blocks in each tower is 4 multiplied by one less than the height. He shares his pattern with his group, and the students listen intently. See figure A-D1-11-02-42.

Figure A-D1-11-02-42*

**Photos have been removed from this publication for confidentiality purposes*

Amanda says she thinks she understands what Emanuel is doing but that she's not sure if it would work. Amanda, Eliot, and Juan then chime in, expressing their agreement with Emanuel's idea.

Amanda and Emanuel punch numbers into their calculators. Eliot, who has already found the product with his calculator, says, "396." Emanuel says that 396 plus 100 would be 496. Eliot asks Emanuel to explain the calculation. Emanuel tells Eliot that 396 would be the number of side blocks and 100 would be the height. Eliot expresses his agreement with the solution. Emanuel asks Juan if he understands the calculation. When Juan indicates he does not, Emanuel tells Juan that the side blocks 'times by 4." Juan continues writing. He then punches numbers into his calculator.

Eliot raises his hand to call Ms. A. Juan asks Eliot a question about the process he used to arrive at the solution of 496. Eliot explains how he obtained the solution. Juan writes on his paper and nods. He punches numbers into his calculator. The students check and confirm that the method of multiplying one less than the height by 4, then adding the height, works for the 10-block high tower, which they had solved previously using another method. Amanda says, “Yay!” Juan, Eliot, and Emanuel sit up straight in their chairs and smile. Emanuel says, “we’re smart.” Emanuel and Amanda wave their hands in the air and call Ms. A. to come to their table. See Figure A-D1-11-06-43.

Figure A-D1-11-06-43

When Ms. A. comes over, the students lean towards Emanuel’s paper and watch closely as Emanuel explains their solution to the 100-block high tower. See Figure A-D1-11-06-56.

Figure A-D1-11-06-56

Ms. A. asks the students to check if their method is correct. Emanuel explains to Ms. A. that the solution for an 11-block high tower is the same when calculated by using the group’s method of multiplying one less than the height by 4 and then adding the height, and when obtained by following the “increasing by 5’s” pattern they found earlier.

Difficulty #4 occurs after the scenario above.

The students’ behaviors following Difficulty #3, described here, were on-task and for Amanda, Emanuel, and Eliot, showed evidence of high levels of cognitive activity.

Affective Engagement: Following Difficulty #4

Prior to Difficulty 4, the students obtained the solution for the 100-block high tower by calculating (4x99) +100=496. Difficulty 4 occurs when Ms. A., in an effort to have the students generalize the solution, asks them to label each number in their computation of (4x99) +100=496. The students have difficulty explaining what each number in their computation represents. Ms. A. asks the students more specific questions to help them express the general solution. At one point, Ms. A. asks the students to label each number in their calculation of (4x4)+5=21, their solution to the 5-block high tower. Emanuel and Amanda disagree about what the second ‘4’ represents. Amanda says it is the amount *added* to the side, and Emanuel says it represents the amount *in* a side. Ms. A. tells them to check their answers against each other.

After Ms. A. leaves the table, Amanda and Eliot express their agreement with Emanuel’s idea that the second ‘4’ in their calculation of (4x4)+5=21 represents the amount of blocks in a side. Amanda says to Emanuel, “You *are* right, because you can’t add four.” Emanuel agrees. After a discussion between Emanuel and Amanda about why Emanuel’s idea is correct, Amanda says, “So you’re right. I’m wrong.” Emanuel says “the hardest thing now is explaining it.” “We found the method but we can’t explain it.” Amanda tries to explain her reasoning for agreeing with Emanuel’s method but has difficulty expressing her justification. She writes on her paper then reads from it her explanation of the process the students used to find the total number of blocks in a tower. Reading from her paper, she says that they multiplied “the side” by the number of blocks in a side and added the height to get the number of blocks in a tower. Eliot and Juan say, “That sounds good!” Amanda repeats what she wrote while the other students write. See figure A-D1-11-20-33.

Figure A-D1-11-20-33

Emanuel and Amanda smile. A member of the research team hands the students blank paper. The students say they need to write over their work neatly, in order. Emanuel says, “Yeah, come on. Let’s be the first people that finish.” The students work on rewriting their work and drawing stages of the towers. After the students have been working for a few minutes, Eliot looks up and says to Emanuel, “I thought we didn’t have to draw A, B, and C.” Emanuel says that he changed his mind and traced them. He says it’s up to Eliot whether or not to draw them but that he should get the basics down because they don’t have much time left. Juan keeps looking at Emanuel’s paper as he writes. When Ms. A. tells the class they need to stop, Emanuel sighs, “Oooohhh.” The students keep working until members of the research team collect their papers. During this videotape segment following Difficulty 4, all the students showed evidence of strong positive feelings regarding mathematics. Emanuel demonstrated a high level of interest with no apparent negative emotions, eagerness to complete the task, and pride in his work. Amanda’s behaviors indicated a high level of interest, pride in her work, and only slight frustration. Eliot and Juan showed evidence of a high level of interest with no apparent negative emotions.

Following Difficulty #3

The single analyzed case wherein a student did not show evidence of strong positive emotions following Ms. A.’s response to a difficulty involved Juan following Difficulty 3. Difficulty 3 occurred when Emanuel, Amanda, Eliot, and Juan appeared unable to answer Ms. A.’s question about how two patterns they found relate to each other. Ms. A. suggested the students think about it and said she will come back to them.

After Ms. A. leaves the table, Emanuel, with some participation from Amanda and Eliot, discovers a method for solving a tower of any height. Juan does not participate much in the students’ discussion leading to the solution. When Emanuel asks Juan if he understands the method the group discovered, Juan does not look up from his paper. He shakes his head from side to side, indicating that he does not. Emanuel tells Juan that the side blocks ‘times by 4’ (increase by 4’s for consecutive towers). Juan does not look up from his paper. He continues writing. He then punches numbers into his calculator. After Eliot raises his hand to call Ms. A., Juan looks at Eliot's paper and asks him what he multiplied. Eliot points to his paper and explains to Juan that he multiplied 4 by 99 and got 396 and then added 100 and got 496. Juan writes on his paper and nods. He punches numbers into his calculator. He does not smile.

Juan's affective engagement was coded as slightly positive since he showed evidence of a high level of interest that was at times mixed with slight feelings of inferiority.

Students' Collaborative Effort and Affect

In this group where students demonstrated strong positive affect overall, students showed evidence of working collaboratively in trying to solve the mathematics task and showed respect for each other as they worked. Students in this group did not tease or belittle each other, even when one student had difficulty understanding the problem solving threads being used by the other group members. Three specific incidences highlight the students' collaborative and respectful work ethic.

Incident 1: Group Collaboration

Before Difficulty 1, during the first portion of the class session, the students work on finding the number of blocks in 4- and 5-block high towers. Amanda says the 5-block high tower would have 4 cubes on each side. The students draw diagrams of 4- and 5- block high towers on their papers and count the number of blocks in each tower they drew. Emanuel says the 5-block high tower has 23 blocks and then says he made a mistake and that the tower does not have 23 blocks. The students count the number of blocks in each diagram on the worksheet and record the information. Amanda counts the blocks in a tower she drew and says that Diagram 'E' would have 21 blocks. Juan says he's confused. Amanda says, "He's lost." Emanuel says to Juan, "What are you confused on? We'll help you. This is a group."

Juan then asks Emanuel how many blocks there are in a 4-block high tower and how many blocks there are in each side of the tower.

(Detailed description of this segment below, in Qualitative In-Depth Analysis of Students' Cognitive Engagement, Description of Behavior Related to Students' Cognitive Engagement Preceding Difficulty 1)

Incident 2: Group Collaboration

After Ms. A. leaves the table following Difficulty 3, the students work on solving the 100-block high tower. Emanuel notices that the number of side blocks in any tower is 4 multiplied by one less than the amount of blocks in the height. He says that for the 100-block high tower the number of side blocks would be 4 times 99, or 396. He says that 396 plus 100 would be 496. Eliot asks Emanuel a question about his solution, and Emanuel explains how he obtained the solution of 496. Emanuel then asks Juan, "You get it? You get it?" Juan shakes his head from side to side, indicating that he does not. Emanuel tells Juan that the side blocks "times by 4." Amanda says to Emanuel, "Yay! We're the first group who got it! Give me high five." The students confirm that their method works for the 10-block high tower. Emanuel says, "We're smart." The students call Ms. A. to their table. When she comes over, all the students lean towards Emanuel's paper and watch closely as Emanuel explains their solution. See picture at **11:06:56**.

During this episode, Emanuel demonstrated group responsibility when he checked whether Juan understands the group's method and explained the method to Juan when Juan indicated he did not understand it. Amanda and Emanuel demonstrated collaborative work ethic when they attributed their success to the group, as opposed to particular individuals in the group. Amanda says, "We're the first group who got it," referring to the group rather than to particular students in the group, and Emanuel says, “We’re smart,” complimenting the entire group. When Ms. A. comes over to the group, all the students in the group show interest by watching closely as Emanuel explains the group’s solution.

(This incident is described in Appendix A, Following Difficulty 3, in Description of Behavior Related to Students’ Engagement Following Response)

Incident 3: Group Collaboration

Following Difficulty 4, the students briefly discuss the task. Amanda then waves at the camera, and Juan looks at the camera. Emanuel tells Juan to stop looking at the camera. The students get back to work. Later, when the students get ready to rewrite their work neatly, Emanuel says to his group mates, “Come on, let’s be the first people that finish.”

Emanuel demonstrated group responsibility when he urged Juan to refocus on the task. Emanuel’s encouraging his group members to get to work and complete the task quickly suggested his collaborative group effort in solving the task.

(This incident is described in Appendix A, Following Difficulty 4, in Description of Behavior Related to Students’ Engagement Following Response)

Qualitative In-Depth Analysis of Students’ Cognitive Engagement

In this section, three difficulties of Ms. A.’s students will be analyzed qualitatively, examining the students’ cognitive engagement before and after Ms. A.’s response to the difficulty. Comparisons between students’ cognitive engagement before and after Ms. A.’s response to their difficulties will be made, and possible associations between types of teacher responses and students’ subsequent cognitive engagement will be discussed. Difficulties 1, 3, and 4, analyzed here, are those also used in the researcher’s interview with Ms. A., where Ms. A.

reflected on her reasoning for providing those particular responses. The subheading for each instance of difficulty analyzed below indicates the number of the difficulty and the student(s) involved. The analysis for each difficulty includes: 1) a qualitative description of the student's behavior related to his/her cognitive engagement before the difficulty and the resulting classification of the student's cognitive engagement, 2) a description of the difficulty, 3) a description of the teacher's response, 4) a qualitative description of the student's behavior related to his/her cognitive engagement after the difficulty and the resulting classification of the student's cognitive engagement, and 5) a comparison of the students' cognitive engagement before and after the difficulty.

Difficulty #1: Amanda, Day 1

Description of Behavior Related to Students' Cognitive Engagement Before Difficulty

Time of day: (10:34:04-10:55:30)

(Data drawn from videotape)

Ms. A. begins the class session by introducing the Building Blocks Task. She tells the students they need to figure out how many blocks are in a tower with a height of 5.

Amanda asks her groupmates, "How much did they increase it by?"

All the students agree that the height increases by one in each consecutive tower, and each side also increases by one.

Amanda tells Emanuel that the 4- block high tower would have 3 on each side, so the 5-block high tower would have 4 on each side. Eliot and Emanuel agree.

Amanda says, "Let's draw the 'D' one [*diagram number four, which is not depicted on their worksheets*]." All the students get ready to write on their worksheets.

Emanuel asks Amanda if when they talk about 5 blocks they are referring to the height. Amanda says they are.

All the students write on their papers. Amanda says, "So this is 4 height, and the sides are 3."

Eliot says, "They all increase by 4."

Amanda says the 5-block high tower would have 4 cubes on each side. Emanuel says the hard part is drawing the cubes. All the students write on their papers. Emanuel shows Eliot his drawing. Emanuel looks at Juan's paper and tells him he needs to add another branch in front on his drawing.

Amanda asks Emanuel why you can't just double the number of blocks in each branch when you double the height. She says this method would yield a solution of 8 blocks in each branch of the 10-block high tower. Amanda says that they could then just multiply by 10 to find the solution to the 100-block high tower.

Emanuel agrees with Amanda's method. He counts the blocks in the 5-block high tower he drew and says it has 23 blocks in total. Juan says he has only 18 blocks.

Amanda records a list of consecutive heights of towers beginning from 5 and the corresponding numbers of blocks in their branches. She obtains a solution of 9 blocks in each branch of a 10-block high tower. Amanda tells Emanuel she is having doubts about her multiplying method.

Emanuel says he made a mistake and that the 5-block high tower does not have 23 blocks. Eliot says he's confused about Amanda's and Emanuel's opinions. Emanuel asks Amanda how many cubes there are in total in the 5- block high tower. Amanda says you don't need to figure that out. Emanuel says they do need to. Amanda reads the problem again.

The students count the number of blocks in each diagram on the worksheet and record the information. Amanda says the 3- block high tower has 16 blocks. Amanda counts the number of blocks in the diagram she drew and says Diagram 'E' would have 21 blocks.

Juan says he's confused. Amanda says, "He's lost." Emanuel says, "What are you confused on? We'll help you. This is a group."

Juan asks Emanuel how many blocks were in Diagram D. Emanuel says there are 16 blocks. Juan asks how many blocks there are in each side. Emanuel says there are 3. Eliot checks with his group members that there are 16 blocks in Diagram D. Amanda says, "Let's see if we could find a pattern. It went from 1 to 6, 6 to 16, and 16 to 21." Amanda and Eliot write on their papers.

Emanuel and Juan punch numbers into their calculators. Emanuel says he found a pattern. He says it's going by 5's.

Amanda tells Emanuel that she realized the multiplication method doesn't work because the 10-block high tower has 9 blocks in each branch, not 8.

Eliot checks with Emanuel that the blocks in consecutive towers are "going by 5's."

Amanda determines that the 6-block high tower has 26 blocks and says that the pattern is correct.

The students record the pattern they found on their papers. Amanda says they need to ask Ms. A. about the pattern. Amanda and Emanuel decide they can keep adding 5 to solve the 10-block high tower. Amanda calls over Ms. A. The students tell her they found a pattern where the total number of blocks in consecutive towers increases by five. Amanda tells Ms. A. that a 10-block high tower would have 46 blocks.

Behaviors Pertaining to Student's Cognitive Engagement Before Difficulty and Resulting Classification

High level cognitive activity

Highest level of cognitive engagement involved: Speculating, Drawing relations, Reasoning,

Justifying

Description of Difficulty

Amanda proposes to Ms. A. multiplying the number of blocks in one branch of the 5-block high tower by 2 to find the number of blocks in one branch of the 10-block high tower [lines 28-30, 43-45 of transcript].

Teacher's Response

Ms. A. tells Amanda to try solving for the 10-block high tower using her method to determine if her method is correct. When Amanda obtains a different answer using her method (42 blocks, instead of 46), Ms. A. asks her which answer is correct [lines 46-55 of transcript].

Code: Offering a suggestion of a process to use in solving the problem

Description of Behavior Related to Students' Cognitive Engagement After Difficulty

Time of day: (10:57:24-10:58:16)

Amanda and Emanuel say that the pattern they found is the correct method. Emanuel says the pattern works for the 10-block high tower but now they need to solve the 100-block high tower. The students then get to work trying to solve the 100-block tower. Emanuel writes on his paper while Juan looks on, and Amanda studies her paper. Amanda proposes multiplying the blocks in the 10-block high tower by 10 to find the number of blocks in the 100-block high tower [lines 56-71 of transcript].

Behaviors Pertaining to Student's Cognitive Engagement After Difficulty and Resulting Classification

High level cognitive activity

Highest level of cognitive engagement involved: Speculating

Comparison of Students' Cognitive Engagement Before/After Difficulty 1

Amanda demonstrated higher order thinking before and after this instance of difficulty. Interestingly, though, Amanda did not abandon her "shortcut" multiplying method after she realized on her own, and then during her discussion with Ms. A., that it was incorrect. Even after

Amanda told Ms. A. that the “+5 pattern,” rather than her multiplying method, was correct [line 58 of transcript], she proposed multiplying the number of blocks in the 10-block high tower by 10 to find the number of blocks in the 100-block high tower.

Difficulty #3: Emanuel, Amanda, Eliot, Juan, Day 1

Description of Behavior Related to Students’ Cognitive Engagement Before Difficulty

Time of day: (10:58:38-11:01:48)

After Ms. A. reminds Amanda that her multiplying method won’t work to find the solution to the 100-block high tower, Amanda says she’ll have to keep adding 5’s to solve the 100-block high tower. Amanda then says she thinks she could find a different pattern so she won’t have to add 5’s all the way up to 100.
Emanuel studies his paper and says he thinks if they found a pattern for the side blocks they could figure out how many side blocks are in the 100-block high tower and add those to the height, which is 100. Ms. A. tells Emanuel to apply his idea to the first few stages and see if it works.
Emanuel counts the side blocks in the stage A, B, and C diagrams on his worksheet. He says, “There’s zero sides here. There’s 4 sides…4 side blocks. There’s 1, 2 , 3, 4, 5, 6, 7…Ohhh.”
Amanda, Emanuel, and Eliot explain to Ms. A. the pattern they found where the total number of side blocks in consecutive towers increase by 4’s.
Eliot says, “First it was 5, and now it’s 4.”
Juan says the height was 5.
Emanuel says the height was not 5; the total amount of blocks in each tower is increasing by 5’s. Ms. A. asks if the students can show that to her.
Emanuel points to his paper and tells Ms. A., “Here’s 1. 1 plus 5 is 6, 6 plus 5 is 11, 11 plus 5 is 16, 16 plus 5 is 21.”

Behaviors Pertaining to Student's Cognitive Engagement Before Difficulty and Resulting Classification

Emanuel: High
Highest level of cognitive engagement involved: Speculating, Drawing relations

Amanda: High
Highest level of cognitive engagement involved: Reasoning, Speculating

Eliot: High
Highest level of cognitive engagement involved: Speculating, Drawing relations

Juan: Low
Highest level of cognitive engagement involved: Considering facts

Description of Difficulty

Time of day: (11:01:49-11:02:08)

Ms. A. asks the students how the pattern they found where the total numbers of side blocks in the towers increase by 4's relates to the pattern they found where the total numbers of blocks increase by 5's. Emanuel leans forward with his hands on his chin. All the other students at the table look at their papers. The students are quiet for a while [lines 159-163, 167 of transcript].

Teacher's Response

Ms. A. tells the students to think about it and says she will come back to them [lines 168-169 of transcript].

Code: Giving the student time to think about the problem

Description of Behavior Related to Students' Cognitive Engagement After Difficulty

Time of day: (11:02:09-11:10:52)

After Ms. A. leaves the table, the students work on solving the 100-block high tower. Emanuel writes on his paper while the other students at the table watch him. He says, "4 times 2 is 8, 4 times 3 is 16…" Amanda says, "Wait a minute." Eliot smiles and says, "I'm finding a pattern." Smiling, Amanda says, "Wait, wait, wait…that was my idea." Amanda, Eliot, and Hemmanuel laugh. **See picture at 11:02:33.**

Emanuel notices that the total number of side blocks in the first tower is 4 times 0, in the second tower it is 4 times 1, in the third tower it is 4 times 2, etc. He says that if you multiply it [4] by 9, you would probably get the answer. He shares his pattern with his group, and the students listen intently. **See picture at 11:02:42**.

Amanda says, "I understand what he's doing because you're doing 4 times…" Emanuel says, "But at the same time I'm not sure if that would work." Pointing to Amanda's paper, Eliot says, "Here we have 4, here we have 8, and there we…" Amanda points to her paper while all the students watch. She says, "4 times 1 is 4, 4 times 2 is 8. I think it's like..." Juan says, "4 times 3, 4 times 4, 4 times 5." While all the students watch, Amanda points to her paper and explains that the first diagram on the sheet would be figure one, the second would be figure 2, etc. She says, "So you're off by 1." Emanuel says, "I know." Amanda says, "I think it would be 4 times 99." Emanuel says, "Yeah, yeah. I just said that cuz I was thinking about 10 blocks, I'm sorry." Amanda says, "4 times 99…" She and Emanuel punch numbers into their calculators. Eliot, who has already found the product with his calculator, says, "396." Emanuel says that 396 plus 100 would be 496. Eliot says, "But how…?" Emanuel explains to Eliot that 396 would be the number of side blocks and 100 would be the height. Eliot says, "Yeah. You got it." Emanuel says to Juan, "You get it? You get it?" Juan does not look up from his paper. He shakes his head from side to side, indicating that he does not. Emanuel tells Juan that the side blocks 'times by 4."

Juan does not look up from his paper. He continues writing. He then punches number into his calculator.
Amanda says to Emanuel, "Yay! We're the first group who got it! Give me high five." Eliot raises his hand to call Ms. A. Juan looks at Eliot's paper and asks him what he multiplied. Eliot points to his paper and explains to Juan that he multiplied 4 by 99 and got 396 and then added 100 and got 496. Juan writes on his paper and nods. He punches numbers into his calculator. He does not smile.
The students check to see whether their method works for the 10-block high tower. Emanuel explains to the other students that 9 times 4 is 36, plus 10 is 46, which matches the solution they obtained previously by following the pattern they had found involving the total number of blocks in consecutive towers. Eliot asks Emanuel to repeat his explanation for finding the number of blocks in a 10- block high tower. Emanuel explains to Eliot that the number of side blocks in consecutive towers increases by 4. He says that for the first diagram on the sheet, the number of side blocks is 0, for the second diagram, the number of side blocks is 4, which is 4 times 1. Juan has leaned over to Eliot's desk to listen to Emanuel's explanation. He says, "And right here it's 8 [referring to the number of side blocks in Tower C]."
Amanda says, "Yay!" Juan, Eliot, and Emanuel sit up straight in their chairs and smile. Emanuel says, "we're smart." Emanuel and Amanda wave their hands in the air and call Ms. A. to come to their table (**see picture at 11:06:43**).

Amanda smiles and tells Emanuel to lower his voice when he calls Ms. A. When Ms. A. comes over, the students lean towards Emanuel's paper and watch closely as Emanuel explains their solution to the 100-block high tower. **See picture at 11:06:56.**

Ms. A. asks the students to check if their method is correct. Emanuel explains to Ms. A. that the solution for an 11-block high tower is the same when calculated by using the group's method of multiplying one less than the height by 4, and then adding the height, and when obtained by following the "increasing by 5's" pattern.
Difficulty #4 occurs after the dialogue above.

Behaviors Pertaining to Student's Cognitive Engagement After Difficulty and Resulting Classification

Emanuel: High level cognitive activity

Highest level of cognitive engagement involved: Building on idea, Speculating, Explaining in one's own words

Amanda: High level cognitive activity

Highest level of cognitive engagement involved: Speculating

Eliot: Low level cognitive activity
Highest level of cognitive engagement involved: Considering facts, Checking information

Juan: Low level cognitive activity
Highest level of cognitive engagement involved: Considering facts

Comparison of Students' Cognitive Engagement Before/After Difficulty 3

The students in this group were cognitively engaged at similar levels before and after the present difficulty. Emanuel and Amanda were both highly cognitively engaged before and after the difficulty. Eliot's cognitive engagement before the difficulty was classified as high, while his cognitive engagement after the difficulty was classified as low. The ideas Eliot expressed before the difficulty involving speculation and drawing relations, however, were not entirely his own. At the time Eliot exclaimed together with Emanuel and Amanda that the total amounts of side blocks in the towers are "going by 4's," Emanuel had already indirectly indicated the existence of that pattern. Juan's cognitive engagement was classified as low both before and after the difficulty.

After Ms. A. said she will give the students time to think about her question and left the students' table, the students found the solution to the 100 block- high tower. During her interview, Ms. A. said she left the students to think about the problem on their own since they are a high performing group and they would be able to progress in their thinking without teacher intervention if given time.

After Ms. A. left the table, Emanuel continued building on the idea he had before the difficulty, rather than trying to answer the question Ms. A. told the group members to think about. Before the students' difficulty, Emanuel said he thinks the group could solve the 100-block high tower if they found a pattern in the total amounts of side blocks in the towers. During the students' conversation with Ms. A., Ms. A. told the students to think about how the "increasing by 5's" pattern relates to the "increasing by 4's" pattern the students found. After Ms. A. left the table, however, the students continued to follow their earlier thought process which involved finding a pattern in the total amount of side blocks in the towers. The students then used this pattern to solve the 100-block high tower.

Difficulty #4: Emanuel, Amanda, Eliot, Juan, Day 1

Description of Behavior Related to Students' Cognitive Engagement Before Difficulty

The time period including the students' behaviors prior to Difficulty #4 is the same as the time period including the students' behaviors following Difficulty #3, i.e., from 11:02:09 until 11:10:52 on Day 1. See above for the description of the students' cognitive engagement.

Behaviors Pertaining to Student's Cognitive Engagement Before Difficulty and Resulting Classification

Emanuel: High level cognitive activity
Highest level of cognitive engagement involved: Building on idea, Speculating, Explaining in one's own words

Amanda: High level cognitive activity
Highest level of cognitive engagement involved: Speculating

Eliot: Low level cognitive activity
Highest level of cognitive engagement involved: Considering facts, Checking information

Juan: Low level cognitive activity
Highest level of cognitive engagement involved: Considering facts

Description of Difficulty
Time of day: (11:10:53-11:16:59)

Ms. A. asks the students to label each number in their computation for finding the number of blocks in the 100- block high tower. The students have correctly found the number of blocks in the 100- block high tower, but they have difficulty explaining what each number in their computation represents [lines 5-191 of transcript].

Teacher's Responses

Ms. A. asks the students questions to help them find the general rule. She asks the students what they did to find the number of blocks in the 100- high tower and asks them to label each number in their calculation. When Emanuel and Amanda disagree about what the second '4' represents in their solution for the 5-block high tower, Ms. A. tells them to check their answers against each other [lines 5- 195].

Code: Leading the student to the correct answer/idea
Suggesting students share difficulty with peers

Description of Behavior Related to Students' Cognitive Engagement After Difficulty

Time of day: (11:17:00-11:33:12)

After Ms. A. leaves the table, Amanda says to Emanuel "so you're saying it's the amount in a side. (Eliot and Juan lean forward to listen to Amanda and watch as she points to her paper.) You are right." Eliot says, "he's right." Amanda then waves at the camera, and Juan looks at the camera. Emanuel tells Juan to stop looking at the camera. The students get back to work. Amanda says to Emanuel, "You *are* right, because you can't add four." Emanuel replies, "Yeah, because you're saying you add four, then it would have been…" Amanda says, "5." Emanuel says, "Yeah, 5…wait, wait, not 5." He counts to himself and says, "It would have been 7." Amanda points to her paper and says, "These were original sides." She then motions with her hands in the air showing a stage B tower which has a height and one block on each side. As she does this, she says, "This was the original tower (motioning to the height of the hypothetical tower) and there was one already (motioning to the four sides of the hypothetical tower), so if you added 4, it would have been…" Emanuel says, "7." Amanda says, "So you're right. I'm wrong." Emanuel says "the hardest thing now is explaining it." "We found the method but we can't explain it." Amanda tries to explain her reasoning for agreeing with Emanuel's method but has difficulty expressing her justification. She writes on her paper then reads from it her explanation of the process the students used to find the total number of blocks in a tower. Reading from her paper, she says that they multiplied "the side" by the number of blocks in a side and added the height to get the number of blocks in a tower. Eliot and Juan say, "That sounds good!" Amanda repeats what she wrote while the other students write. **See picture at 11:20:33.**

Emanuel and Amanda smile.
A member of the research team hands the students blank paper. The students say they need to write over their work neatly, in order. Eliot says, "so that they'll (referring to the research team) understand it." Emanuel says, "Yeah, come on. Let's be the first people that finish." The students work on rewriting their work and drawing stages of the towers. Emanuel says they don't need to draw stages A, B, and C. After the students have been working for a few minutes, Eliot looks up and says to Emanuel, "I thought we didn't have to draw A, B, and C." Emanuel

says that he changed his mind and traced them. He says it's up to Eliot whether or not to draw them but that he should get the basics down because they don't have much time left. Juan keeps looking at Emanuel's paper as he writes. When Ms. A. tells the class they need to stop, Emanuel sighs, "Oooohhh." The students keep working until members of the research team collect their papers.

Behaviors Pertaining to Student's Cognitive Engagement After Difficulty and Resulting Classification

Emanuel: High level cognitive activity
Highest level of cognitive engagement involved: Reasoning

Amanda: High level cognitive activity
Highest level of cognitive engagement involved: Reasoning

Eliot: Insufficient information

Juan: Insufficient information

Comparison of Students' Cognitive Engagement Before/After Difficulty #4

Amanda and Emanuel's levels of cognitive engagement were similar before and after the instance of difficulty. Eliot's and Juan's levels of cognitive engagement before and after the difficulty could not be compared for lack of information regarding their cognitive engagement after the difficulty.

Emanuel and Amanda were both engaged in high level cognitive activities before and after the instance of difficulty. Before the difficulty, Emanuel found a method for determining the number of side blocks in any tower and explained his solution to his group members. Emanuel checked that the method works for the 10-block high tower. After Ms. A. left the table, Emanuel explained to Amanda why her answer for the representation of '4' in the general solution was incorrect. Prior to the difficulty, Amanda explained in her own words Emanuel's method of finding the number of side blocks in any tower. After the difficulty, Amanda composed a written explanation of the students' general solution.

During her interview, Ms. A. explained that she asked the students questions when they had difficulty finding the general solution since the students "needed to put more analysis into it…it's not enough to just understand the concept superficially. I wanted them to have it more stable, to really comprehend it so that when they see it in another problem they will be able to do the same process again." After Ms. A. left the table following her response to the students' difficulty, the students found the general solution. Before Ms. A. left the table, she suggested the students discuss with each other their ideas about what the '4' in their solution represents. After Ms. A. left, the students agreed that Emanuel's idea about the '4' represents was correct. Emanuel explained to Amanda why her idea was incorrect.

Analysis of Ms. A.'s students' cognitive engagement before and after the three instances of their mathematics difficulty analyzed here showed that Ms. A. had fine-tuned her interventions to permit the students' engagement to continue.

Analysis of Student Engagement Questionnaire Data

In the present study, two sections of the students' questionnaires were examined. The first section related to students' feelings about mathematics that day; the second asked about whether students experienced difficulty in mathematics that day.

Students' Reports of Emotions

The following chart depicts Ms. A.'s students' scores for each emotion on the Student Engagement Questionnaire as compared with the scores of a population of 83 students in the same urban school district. Scores were measured on a 2-point scale. For positive emotions, students received a score of ‘2’ for a response of ‘Very much,’ a ‘1’ for ‘Somewhat,’ and a ‘0’ for ‘Not at All.’ For negative emotions, the scores were reversed. Means marked with an asterisk include emotions students added to their list of feelings. These additional emotions are noted below the table along with the scores students assigned to these emotions.

Table 4.10: Ms. A. Student Questionnaire Feelings Responses, Day 1

Questionnaire Item	n	Population Mean	Population Standard Deviation	Emanuel	Amanda	Eliot*	Juan
Interested	78	1.58	.570	2.00	2.00	N/A	1.00
Respected	78	1.63	.561	2.00	2.00	N/A	2.00
Proud	79	1.48	.677	1.00	2.00	N/A	2.00
Successful	78	1.56	.636	2.00	2.00	N/A	1.00
Safe	76	1.66	.623	2.00	2.00	N/A	2.00
Excited	75	1.37	.731	2.00	2.00	N/A	1.00
Happy	76	1.46	.720	2.00	2.00	N/A	2.00
Satisfied	77	1.36	.687	2.00	2.00	N/A	1.00
Relieved	78	1.10	.749	2.00	2.00	N/A	0.00
Confident	75	1.41	.737	2.00	2.00	N/A	0.00
Curious	76	.87	.854	2.00	2.00	N/A	2.00
Unhappy (reversed)	76	1.83	.413	2.00	2.00	N/A	2.00
Disappointed (Reversed)	75	1.79	.527	2.00	2.00	N/A	2.00
Worried (Reversed)	76	1.62	.610	2.00	2.00	N/A	1.00
Discouraged (Reversed)	76	1.88	.399	2.00	2.00	N/A	2.00
Angry (Reversed)	76	1.72	.556	2.00	2.00	N/A	2.00
Disrespected (Reversed)	76	1.88	.399	2.00	2.00	N/A	2.00
Bored (Reversed)	76	1.59	.715	2.00	2.00	N/A	1.00
Embarrassed (Reversed)	76	1.79	.549	2.00	2.00	N/A	1.00
Afraid (Reversed)	75	1.89	.352	2.00	2.00	N/A	1.00
Positive/Negative Subscale Means		*1.57*		*1.95*	*1.91***	N/A	*1.33****
Frustrated	76	1.68	.616	0.00	1.00	N/A	0.00
Confused	76	1.55	.641	0.00	1.00	N/A	0.00
Engagement with Impasse Subscale Mean		*1.62*		*0.00*	*1.00*	N/A	*0.00*

*No questionnaire was available for Eliot
**Amanda added the following feelings to her list: Jubilant (Very much; score of 2), Anxious (Somewhat; Score of 1), Nervous (Somewhat; Score of 1)
***Juan added one feeling to his list: Nervous (Very much; Score of 0)

Students' Self-Reported Affect vs. Affect Inferred from Students' Behaviors

To gather information regarding students' affect during the videotaped class sessions, the present researcher initially watched preselected episodes of students' difficulties during the videotaped sessions multiple times. The researcher then created transcriptions of those episodes and the time periods following them which included descriptions of students' behaviors as well as screenshots from the video that were illustrative of students' emotions. The researcher then tried to infer the students' feelings following each instance of difficulty from the transcriptions. When in doubt regarding particular students' feelings, the researcher referred back to the videotapes and in several cases, consulted with a psychologist to determine the feelings that most aptly depicted the constellation of behaviors exhibited by the students.

The emotions inferred by the researcher were then compared with students' self-reported emotions on the questionnaire. Following is a comparison of self-reported and inferred affect for Emanuel, Amanda, and Juan. Eliot's questionnaire was not available, precluding his inclusion in this segment of the analysis.

Emanuel

Emanuel's self-reported emotions for this class session were similar to the emotions inferred from his behaviors during the videotape of the session. Emanuel reported having strong positive feelings during the session, as evidenced by his Positive Emotions Subscale score of 1.95 (of 2.00). As noted in Appendix A, Event 1: Difficulty 3, and Event 2: Difficulty 4, inferences made from Emanuel's behaviors indicated he demonstrated a high level of interest, pride in his work, eagerness in sharing his solution with the teacher, and confidence in his solution.

Amanda

Amanda's reports of strong positive emotions during this class session were also supported by inferences of her behaviors during the session. On her questionnaire, Amanda reported feeling all the listed positive emotions 'Very Much.' She even added the feeling 'jubilant' to her list of emotions. Her Positive Emotions subscale score was 1.91 (of 2.00). As noted in Appendix A , Event 1: Difficulties 1, 2, and 3, and Event 2: Difficulty 4, analysis of the videotaped class session suggested that Amanda showed a high level of interest, determination, excitement, eagerness, pride, and confidence regarding mathematics problem solving that day.

Juan

On his questionnaire, Juan reported feeling somewhat 'worried,' 'bored,' 'embarrassed,' and 'afraid.' Juan added to the list his feeling of very much 'nervous.' His Positive Emotions subscale score was 1.33. As noted in Appendix A, Event 1: Difficulty 3, and Event 2: Difficulty 4, inferences made from Juan's behaviors during this class session showed he demonstrated a high level of interest mixed with feelings of inferiority during a portion of the class session. During most of the session, Juan was quiet and listened to the other students. He copied work from their papers as opposed to initiating courses of action. Perhaps since he did not speak much during the sessions, it was difficult to observe his negative feelings other than 'inferiority.' Although Juan did not demonstrate a high level of understanding and at one point indicated he did not understand the group's solution, he reported feeling 'Not At All' frustrated or confused.

Group Analysis of Students' Self-Reported Affect vs. Affect Inferred from Students' Behaviors

The students who demonstrated the highest level of understanding among the group members, Emanuel and Amanda, reported much more positive affect than Juan, who demonstrated a lower level of understanding.

Self-Reports of Encountering Difficulty and Comparison with Observed Instances of Difficulty

Table 4.11, Students' Reports of Difficulty and Observed Difficulty, Ms. A, Day 1

	Emanuel	**Amanda**	**Juan**
Self-Report of Difficulty			
Had Difficulty?	No	Yes	Yes
Classroom Context	---	Group	Group
What stood out?	---	What other kids said or did because there was a lot of thought in my group."	"Nothing"
Observed Difficulty			
Types of Difficulty and Frequency of Occurrence	Unable to Answer (1) Insufficient Answer (1)	Incorrect Idea (2) Unable to Answer (1) Insufficient Answer (1)	Unable to Answer (1) Insufficient Answer (1)

Emanuel and Juan were both involved in the same types and frequency of analyzed difficulties during this class session. Emanuel, however, reported having no difficulties, while Juan reported having difficulty. Amanda, who was involved in the greatest amount of difficulties during the class session, reported having difficulty.

Summary: Ms. A.

During the four instances of students' difficulty analyzed for Ms. A., Ms. A. used two types of initial responses: *Helping Students Indirectly* and *Having Students Help Themselves*. Ms. A.'s single follow-up response involved *Having Other Students Help*. The most frequent type of response for Ms. A. was *Helping the students indirectly*. Ms. A. explained during her interview that she generally works individually with students who have difficulty with mathematics. She stated that she has her students work in groups to facilitate peer teaching, as students usually understand each other well and can easily grasp concepts when they are explained by peers. Ms. A. noted that her high ability mathematics students require little teacher intervention when they encounter difficulties, as they can explore and resolve their difficulties on their own. Her low ability students, however, require step-by-step teacher intervention to resolve their difficulties. Ms. A. added that she offers encouragement and support to students facing outside pressures. Ms. A.'s responses to her students observed on videotape were concordant with her self-reported classroom practices.

Following Ms. A.'s interventions, the students overall were highly engaged in all three dimensions: behaviorally, cognitively, and affectively and progressed in their mathematical thinking. The students, however, did not always follow the thinking pathways Ms. A. suggested; often, after Ms. A. left their table, the students reverted to their original ways of thinking, whether mathematically correct or not, in order to move forward in their problem solving.

Overall, the students' reports of emotions during the class session were similar to the affect inferred from their behaviors during the classroom videotaped sessions. Students who were involved in the same difficulties during the session, however, had different accounts of whether or not they experienced difficulty.

4.5 Teachers' Responses and Students' Engagement: Ms. B.

Two consecutive videotaped sessions of one group of students in Ms. B.'s class were examined in this research. The group included three students: Marcos, Larent, and Lashanna. The first day's session contained 11 instances of student difficulty where Ms. B. was involved. During the second session there were 3 instances of student difficulty where Ms. B. was involved. For each of the fourteen analyzed instances of difficulty for Ms. B.'s class, Table 4.12 lists the students involved and the type of difficulty.

Table 4.12: Types of Difficulties and Students Involved, Ms. B.

	Students Involved:	*Type of Difficulty:*
Day 1		
Difficulty 5	Marcos, Larent, Lashanna	Impasse
Difficulty 6	Marcos, Lashanna	Incorrect Answer
Difficulty 7	Marcos, Lashanna	Incorrect Answer
Difficulty 8	Marcos	Incorrect Answer
Difficulty 9	Marcos, Lashanna	Incorrect Answer
Difficulty 10	Marcos	Incorrect Answer
Difficulty 11	Larent	Incorrect Answer
Difficulty 12	Larent	Incorrect Answer
Difficulty 13	Lashanna	Incorrect Answer
Difficulty14	Larent	Does not Understand Solution
Difficulty15	Larent	Incorrect Idea
Day 2		
Difficulty16	Marcos	Incorrect Answer
Difficulty17	Lashanna	Unable to Answer
Difficulty18	Marcos	Incorrect Idea

Types of Responses

Ms. B.'s 14 initial responses to her students' difficulties fell into three of the five response categories: *Having Other Student(s) Help Resolve Difficulty, Helping Student Indirectly* and *Not Responding/Responding without Offering Suggestion for Resolving Difficulty.* Her six follow-up responses fell into three response categories: *Having Other Student(s) Help Resolve Difficulty, Helping Student Directly, and Helping Student Indirectly*. Ms. B. most frequently used the response types *Helping Students Indirectly* (Overall total of 9) and *Having Other Students Help Resolve Difficulty* (Overall total of 7). Table 4.6 depicts the particular types of responses within each category that Ms. A. used. Types of responses are listed using short descriptors; refer to table 4.2 for the detailed set of codes.

Table 4.13: Types of Responses, Ms. B.

Numbers preceding a hyphen in the frequency column represent initial responses; numbers after a hyphen refer to follow-up responses.

Type of Teacher Response: Ms. B.	**Frequency**	**Total Frequency**
Having Other Student(s) Help		
Inviting Another Student *Sharing with Peers* *Using Another's Idea*	4-3	7
Having Student Help Self		
Giving Time	0-0	0
Helping Student Directly		
Providing Correct Answer *Explaining*	0-1	1
Helping Student Indirectly		
Leading	0-0	0
Providing Counter Example	0-0	0
Probing *Asking for Proof*	3-1	4
Simpler Problem	1-0	1
Suggesting Process	2-1	3
Showing Diagram/Model	1-0	1
Subtotal: Teacher Helping Student Indirectly	***7 -2***	***9***
Not Responding/Responding without Offering Suggestion		
Saying Answer is Wrong	0-0	0
Not Responding	1-0	1
Acknowledging or Repeating Incorrect Response *Having Student Repeat* *Clarifying* *Repeating Teacher's Question*	2-0	2
Subtotal*: *Not Responding/Responding without Offering Suggestion for Resolving Difficulty	*3-0*	*3*
Total Responses	**14-6**	**20**

Following is a detailed analysis of one illustrative instance for each type of response Ms. B. used.

Helping Students Indirectly: Difficulty 18

In Event 8, Marcos disagrees with Lashanna's solution for the 100-block high tower. Lashanna has arrived at the correct solution of 496 cubes by multiplying 99 by 4 and adding 100. Marcos disagrees, saying he got 460 as the solution by multiplying the number of blocks in a 10-block high tower, 46, by 10 to obtain 460. Ms. B. tells Marcos to pick a number for a tower height that he already solved and have Lashanna solve for that tower using her method. She says Marcos should then compare her answer with his to determine whether Lashanna's method is correct. The excerpt below details this conversation between Ms. B. and the students, Marcos and Lashanna, and the conclusion Marcos reached as a result.

> Ms. B.: (to Marcos) So, you came up with a certain way of figuring it out, right? Why don't you *(pointing to Lashanna)…(pointing to Marcos)* Give her…give her something that you did already and say prove it to me by your…by your way. You give her…pick a number, for how many towers, that you figured out for already.
> Marcos: Um…5
> Ms. B.: *(points to Lashanna)* 5. Ok.
> *(Lashanna removes 5 blocks from the height in her 10- block high tower and begins removing blocks from each branch.)*
> Ms. B.: *(to Marcos)* So you have your answer, right? So now you're gonna see if Lashanna's correct. You came up with what?
> Marcos: 21
> Ms. B.: Let's see. *(Lashanna has almost finished converting her 10- block high tower to a 5- block high tower.) (to Marcos)* What do you think?
> Marcos: *(puts his hands up.* ***See picture at 10:03:25)*** I think…I think she probably won *(smiles).*
>
> Ms. B.: So you think she will? *(to Lashanna)* Ok, so now, can you walk us through your process again?
> Lashanna: Eh…this is 5 *(pointing to the height in her tower)*…so in these 4 *(pointing to the branches of her tower)…*so 4 times 4 is 16, and then you just add the 5, and it would get you 21.
> Marcos: *(smiles at Lashanna)* So you were right. *(to Ms. B.)* The same thing *(smiles).*
> (Appendix #, Event 8, Difficulty 18, lines 19-41)

In this instance, Ms. B. suggested a process by which Marcos could determine on his own whether his or Lashanna's method was correct. By using Ms. B.'s suggested method of testing Lashanna's method with a tower height for which he knew the solution, Marcos correctly concluded that Lashanna's method was correct.

Having Other Student(s) Help: Difficulty 13

Difficulty 13 occurs when Ms. B. asks Lashanna how many blocks there are in a 3-block high tower, and it is implied that Lashanna says "14". Below is the conversation between Ms. B. and the students.

Ms. B.: How many did you get, Lashanna, for 3 towers high? *(Larent turns over his paper. Lashanna raises 4 fingers. Ms. B. turns to Larent.)* Now, it's not a secret. You want to get together with it. So once you come up with an answer…*(she turns over Larent's paper, face up).* She *(pointing to Lashanna)* said 14, you said 11 *(looking at Larent's paper)*, how many do you have *(pointing to Marcos)*?
Marcos: I'm still doing it.
Ms. B.: Ok, you're still doing it. So…
Marcos: *(pointing to diagram C on his paper)* 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11…11. *(Pointing to his paper again)* 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11. *(Writes on his scrap paper)* *After Marcos announced his answer is 11, Lashanna leans over her paper (her face is off camera). She looks up.*
Lashanna: 11
Ms. B.: (*smiles*) Ok. Now, you want to work on the 4th one. And, you want to come up with the same answers. If you have different answers, you have to try and prove that you're right. *(Ms. B. looks at Larent)* Like, you said here, like, 'I'm right.' You turned your paper over. *(She turns his paper over, acting out the situation.* ***See picture below at 10.15.23****.)*

You should have been like, 'Lashanna, how did you get 14? I got 11. Let me show you how I got 11.' Alright? So, I'm gonna come back with the 4, and 5, 6. So keep on going.
(Ms. B. walks away from the table.)

(Appendix #, Event 4, Difficulty 13, lines 117-138, 141)

Ms. B.'s approach of sharing Larent's and Marcos's ideas in order to help Lashanna arrive at the correct solution of 11 blocks for the 3-block high tower was effective. Ms. B.

reminded the students to use the method of sharing answers and trying to come up with one solution agreed upon by all the group mates in the future.

Responding Without Offering Solution: Difficulty 6

In an effort to have Marcos. Lashanna, and Larent generalize their solution to the tower problem, Ms. B. asks them if they could build a 1,000-block high tower. Marcos and Lashanna respond by describing an incorrect method of solving a 500-block high tower (this is Difficulty 6). The transcript below begins with Ms. B.'s question to the students.

Ms. B.: So now, what you have right here, is there a way that you could figure it out without having to build it up because after, say that you did build a 100-high tower with the cubes, so someone might come in and say, now that you did that, can you make a thousand-block high tower? Can you?

Lashanna: It would be 500. 100, 100, 100, 100, 100 (*Pointing to each of the 5 branches of the tower on the desk)*.

Marcos: Oh, yeah. 100, 100 *(Pointing to 2 branches),* that's 200, that's 400 *(pointing to the other 2 branches at the base),* that's 500 *(pointing to the height).* 500 blocks.
Larent gives a big yawn.

Ms. B.: So for, you said, 500 blocks?

Marcos: No, no, no. Hold on. *(Sits up, moving closer to the tower)* This block *(pointing to the middle block of the tower on the desk)* doesn't count for this one *(pointing to a branch),* so the top is 100, this is 99. 99, 99, 99 *(pointing to the other branches).*
(When Marcos begins speaking, Lashanna rests her head on her hand and watches him. She moves her eyes from the tower to which he is pointing to him, and back to the tower.)

Lashanna: There's…(inaudible)…100 in all of them. So it's 500.

(Appendix A, Event 4, Difficulty 6, lines 13-31)

After Ms. B.'s response of asking a question to clarify what Marcos and Lashanna intended to say ("So, for you say, 500 blocks?"), Marcos noticed and corrected his mistake. Lashanna, however, did not become aware of her error and simply repeated her earlier reasoning.

Helping Student Directly: Difficulty 12

The single instance where Ms. B. responded by helping a student directly occurred when Ms. B. explained the towers task to Larent as a follow-up response to Difficulty 12. Difficulty 12 occurs when Ms. B. asks how many blocks there are in a 2-block high tower. Larent says there are 5; Lashanna says there are 6. Ms. B. points out Larent and Lashanna's differing solutions (this initial response was coded as *Reminding student about another student's idea*) . Larent says he said '6' the first time. Ms. B. asks him why he is changing his answer (this follow-up response was coded as *Probing*). Larent says it's because he sees what other people did. After Larent says something inaudible, Ms. B. explains to him that the towers build upon each other. This second follow-up response was coded as *Explaining the mathematics task*, included in the category of *Helping Students Directly*. After Ms. B.'s direct explanation to Larent, Larent correctly states that there are 6 blocks in the 2-block high tower.

Ms. B.'s Reasoning for Responses

During the retrospective interview, Ms. B. was asked several questions regarding the ways in which she responds when students have difficulty while working on mathematics problems. The section below includes each of those interview questions and a summary of Ms. B.'s responses, along with supporting quotations from the interview transcript (included in Appendix D).

Q1: Can you describe how you usually respond when students have difficulty while they work on mathematics problems in class?

Ms. B. stated that she listens to the students and probes them, rather than giving them answers. She said she challenges the higher ability students by asking them to explain their reasoning and provides more guided questions to the lower ability students to let them feel successful so they can continue their problem solving on their own.

Q2: Are there things you try not to do when you respond to student difficulty? (If yes) Can you describe?

Ms. B. answered that she tries not to assume the students have adequate prior knowledge or that the students "know exactly where they should be heading" (line 20).

Q3: Do you respond in different ways in different situations?

Yes

Q4: Can you describe some factors that might determine how you respond?

Ms. B. stated that she would never give students the answers, but would ask different types of question depending on the students' thinking. Ms. B. also mentioned that students' dispositions at times impact the way she responds to their difficulties. She explained:

> It depends on how they come in that day as well. They might come in riled up, it might have been something else that happened at home, in a different class. So of course you can't expect them to answer in the same way they did the day before. So it all depends on them…One student might need a little more time to just get themselves together on their own instead of me questioning and questioning and trying to get them to focus and work harder. They might need a little bit of down time. Or I might just go and like whisper quietly to that student the question. I won't force them into the group at that time but then kind of lead them into the group a little later after I've seen that they calmed down a little bit.
>
> (Appendix D, Ms. B. Interview Transcript, lines 31-41)

When asked whether the type of difficulty or the small group versus whole class setting of the difficulty might influence how she responds, Ms. B. replied that in a whole group setting, for some students, it may be more beneficial to address the difficulty after the students have broken up into small groups. She reasoned:

> Small group is definitely better. Whole group you cannot get to everyone...If it's whole group and I see someone's having a problem…and I throw it out there in a whole group, they might shut down automatically. But in a smaller group it might be less threatening. One on one it might be non threatening at all so again it depends on the student. Some students can handle that whole group instruction but those who are struggling and then if they're not well liked in the classroom already, so then now they'll get teased and so

> they're just going to shut down and not going to answer. So small group, individual is better than whole group.
>
> (Appendix D, Ms. B. Interview Transcript, lines 44-52)

Ms. B. stated that if her students have difficulty while in a whole group setting she may ignore the difficulty until the students break into smaller groups. She asserted, "there's no need to force them to try and think in a whole group especially when you know your students" (Appendix D, Ms. B. Interview Transcript, lines 56-57).

Comparison of Ms. B.'s Self-Description of Response Types to Observed Responses

Ms. B.'s descriptions of the ways in which she typically responds to students' difficulties during mathematics problem solving were consistent with the responses she offered to her students during the analyzed videotaped class sessions. Ms. B. explained that when her students have difficulty, she never gives answers, but rather probes the students. Of the analyzed responses Ms. B. provided to her students, one fifth (4 of 20) involved probing or asking students to explain their reasoning. None of the analyzed responses involved giving students answers. The only response Ms. B. provided that was coded as *Helping Student Directly* involved explaining the towers task to Larent during Difficulty 12, as follow-up response. After Ms. B.'s explanation, Larent arrived at the correct solution on his own.

Teachers' Reasoning for Particular Responses and Comparison with Actual Outcomes

During the stimulated recall interview, Ms. B. was asked to explain her reasoning for the responses she provided to the students after Difficulties 12, 16, and 17. Ms. B.'s explanations for her response to Difficulties 12 and 17 were given more than one code. The codes for Ms. B.'s explanations for her responses fell into three of the five coding categories for Teachers' Reasoning for their Particular Responses. These categories, in order of the frequency in which they occurred, included Students' Abilities/Skills, Students' Attitudes, and Mathematics Goals. Below are detailed descriptions of Ms. B.'s self-reported reasoning for her responses to Difficulties 12, 16, and 17. Following each description is a comparison of Ms. B.'s reasoning for providing the response to what actually occurred following the response, focusing on whether Ms. B.'s intended goals were realized.

Reasoning for Response to Difficulty 12

Difficulty 12 occurs when Ms. B. asks the students how many blocks there are in a 2-block high tower. Larent says there are 5; Lashanna says there are 6. Ms. B. points out Larent and Lashanna's differing solutions (this initial response was coded as *Reminding student about another student's idea*). Larent says he said '6' the first time. Ms. B. asks him why he is changing his answer (this follow-up response was coded as *Probing*). Larent says it's because he sees what other people did. After Larent says something inaudible, Ms. B. explains to him that the towers build upon each other (this second follow-up response was coded as *Explaining the mathematics task*).

During the stimulated response interview, Ms. B. explained that she responded in this way since Larent often has difficulty settling down and working in a group. That day especially, she noted, Larent did not want to be in class due to the videotaping, so Ms. B. spent time with him, trying to get him to think about the problem, even if only at a superficial level. Ms. B. explained:

> …it was kind of like…babying him, pampering him a little bit and giving him that extra time and sitting there. And then just giving him questions where we can just kind of… pull something out of him. Because for him to even just sit there was a good thing. For him to even talk about it a little bit was a great thing. So I was just trying to see… he was thinking a little bit with it and everything. At least he understood what the task was about. His answer is off and everything, but the fact that he just didn't say just one, or just like 'I don't know', that was a good thing. So just trying to like get him on board. Because he does have a difficult time working in a group.
>
> (Appendix D, Ms. B. Interview Transcript, lines 102-110)

When asked if Larent's mathematics ability influenced the way she responded, Ms. B. replied that it did. She explained that Larent had difficulty with organization and sustaining his attention, so she tried to ask him non-challenging questions to encourage him to think about the problem. Ms. B. reflected:

> Larent was one who…organizational skills just in math, like the notebook was all over the place and just him coming and …giving full attention was difficult for him so…for me to be able to sit there and just have a little bit of dialogue with him was good. So it's kind of like they were easy questions right here… it wasn't like giving him the answer but it's just trying to just let him think a little bit and trying to see that one cube that he was missing….he might have just been not thinking more than he had to. Like, 'I see five cubes there, it's five'.
>
> (Appendix D, Ms. B. Interview Transcript, lines 117-124)

Ms. B.'s reasoning for her response to Difficulty 12 was given three codes: Students' Abilities/Skills- Organizational Skills, Students' Abilities/Skills- Mathematics Ability, and Students' Attitudes – regarding participation in mathematics activities.

As Ms. B. intended, after her intervention following Larent's incorrect solution of 5 blocks for the 2-block high tower, Larent correctly concluded that the tower has 6 blocks.

Reasoning for Response to Difficulty 16

This difficulty occurs when Ms. B. asks Marcos how many blocks there are in a 100-block high tower. Marcos says there are 460. Ms. B. asks him how he arrived at that number (this response was coded as *Probing- Asking the student to explain the reasoning he used*). Marcos says he obtained that number by multiplying 46 (the number of blocks in a 10-block high tower) by 10. Ms. B. tells Marcos to write his explanation on a transparency. Ms. B. then asks Lashanna if she agrees with Marcos's solution (this follow-up response was coded as Inviting another student to respond). When Lashanna indicates she does not know if she agrees with Marcos, Ms. B. tells Marcos to convince Lashanna of his answer (this second follow-up response was coded as *Suggesting student share difficulty with peer*).

During the stimulated recall interview, Ms. B. reasoned that she wanted Marcos to commit his thoughts to paper because it would stimulate a class discussion of his answer, which would cause him to think about his answer. Ms. B. said she did not simply tell him his answer was incorrect so that he would realize the error on his own and work through the relevant concepts in a meaningful way. She explained:

> "If I just tell him, automatically he still is not going to understand because he wasn't able to think it out himself…when the students are actually working through the problem and they come up with a clear understanding, conceptually it makes more sense to them. We learn things just procedurally so it's like 'okay just do this because'. There are no questions, we don't even understand why, so when we get the other problem we don't know how to make that connection. So if he knows where he went wrong he won't do that same mistake because he knows, this does not make sense right here, I messed up right here. He won't do it again, hopefully."
>
> (Appendix D, Ms. B. Interview Transcript, lines 172-180)

When asked whether Marcos's mathematics ability influenced the way she responded, Ms. B. replied that it did not. Ms. B.'s reasoning here was assigned one code: Mathematics Goals- Conceptual Understanding. Ms. B. did note that she responded differently to Marcos's difficulty here than to Larent's in Difficulty 12. In this difficulty, Marcos spoke confidently and did not look to Ms. B. for validation of his solution. In Difficulty 12, however, Larent was unsure of his solution and seemed to want confirmation from Ms. B., so Ms. B. probed him to stimulate his thinking about his solution. Marcos apparently recognized at some point prior to his group's presentation that his solution of 460 blocks for the 100-block high tower was incorrect since he did not mention the solution during the presentation.

Reasoning for Response to Difficulty 17

During Difficulty 16, Marcos asserts that there are 460 blocks in the 10-block high tower. When Lashanna indicates she is unsure whether Marcos's solution is correct, Ms. B. tells Marcos to convince Lashanna of his solution. When Ms. B. leaves the table, Lashanna explains to Marcos that she obtained a solution of 496 blocks for the 10-block high tower by subtracting 4 from 500. Difficulty 17 occurs when Ms. B. asks Lashanna how she got her solution for the 100-block high tower. Lashanna does not mention her previous solution of 496 blocks for the 100-block high tower to Ms. B. Lashanna tells Ms. B. instead that Marcos explained the solution for the 100-block high tower to her and then says she cannot explain his solution. Ms. B. tells Lashanna to check whether Marcos's solution is correct, rather than simply accept it. Ms. B.'s response was coded as *Asking the student to prove her solution*.

During the stimulated recall interview, Ms. B. explained that Lashanna's difficulty in this case was attitudinal. She said Lashanna was putting little effort into the mathematics task probably because she was unhappy with the group in which she was placed that day. Ms. B. explained that Lashanna wanted to work in a group with her friends that day, but Ms. B. did not allow her to join that group since she felt Lashanna would likely have been distracted from the mathematics problem in that setting. Ms. B. believed Lashanna therefore responded by seeming to be uninvolved in the task when Ms. B. approached the group. Ms. B. explained that she responded more firmly to Lashanna than to the other students since Lashanna was sitting in her group and did not appear to be trying to work on the mathematics problem at all.

When asked whether Lashanna's mathematics ability influenced the way she responded, Ms. B. replied that it did. She explained:

> I know that she can do more than what she was doing, so I was getting a little frustrated with her, because she wasn't trying. Like I'd seen her in the past where she would… she does have an attitude problem, days that she comes in and it's coming from home probably, so she comes in with an attitude, comes in very like uptight and just ready for any argument or whatever. But when she is calm and we're working through problems she will work and she will ask questions. So for her to sit right there frustrated me because I knew she can do a lot better.
>
> (Appendix D, Ms. B. Interview Transcript, lines 257-263)

Ms. B. added:

> Lashanna is not even one who would just let Mathew be okay with an answer. Like she will get up there and she'll explain the answer. So for her to just be like 'Marcos explained it to me'…that was just her way of like just like, you know, shooing me off, like 'I know it, Mathew explained it to me'. That's it.
>
> (Appendix D, Ms. B. Interview Transcript, lines 270-275)

Ms. B. then reflected on her decision to keep Lashanna in the group despite Lashanna's indications that she wanted to work in another group:

> I can tell by the body language she didn't want to be in that group when she came in. I can tell. You know how you know your students …but she was complying to my request. But then I shut her down. And that could be also like on a teacher's point of view, like okay don't force the groups to work together. But sometimes like you have to put certain people

> together and it's hard sometimes. You know when you have a classroom full of ones that if you put together they're going to just talk and they're not going to get things done, so like she's one that had to be separated. So if I would have maybe just switched her group she might have done better in it. You never know some days it's like a tit for tat and you don't know.
>
> (Appendix D, Ms. B. Interview Transcript, lines 291-299)

Ms. B.'s reasoning for her response was given two codes: Abilities/Skills-Mathematics Ability and Attitudes regarding participation in mathematics activities.

Ms. B. explained that she responded firmly to Lashanna since she is capable of good work and was sitting in her seat without appearing to expend any effort towards solving the mathematics problem. As Ms. B. hoped, after Ms. B. left the table, Lashanna argued with Marcos's solution. Lashanna re-explained her method for solving the 100-block high tower to Marcos. She said that since the 10-block high tower has 10 blocks in the height and 9 blocks in each of its branches, the 100-block high tower will have 100 blocks in the height and 99 blocks in each of the branches. She said that 99 times 4 is 396 and if you then add 100, you get 496. Lashanna then asked Marcos to explain the method he used to arrive at a solution of 460 blocks. Marcos explained that he simply multiplied the number of blocks needed for a 10-block high tower by 10 to get the number of blocks needed for a 100-block high tower. Lashanna asked Larent whether he thinks Marcos's solution or hers is correct. She explained her solution to him, but he did not look up and continued building his tower. Ms. B. has been standing near group 1 and listening while Lashanna explained to Larent how she got 496 cubes. Ms. B. asks Lashanna to generalize her solution for any size tower. Lashanna comes up with a general solution where the number of blocks needed for any size tower is one less than the height of the tower times 4, plus the height of the tower.

Although Lashanna appeared to be upset because she was placed in a group with two boys, and away from her friends, she was able to solve the problem in that setting, as Ms. B. anticipated. Lashanna came up with the solution for the 100-block high tower on her own and was then able to generalize her solution. At the end of the session, during the group's presentation to the class, Lashanna explains her method for finding the total number of cubes in any size tower.

Student Engagement

The table below lists the codes assigned to Ms. B.'s students' behavior following Ms. B.'s responses to their analyzed difficulties. For each student involved in a difficulty, codes are listed for three domains of the students' engagement: behavioral, cognitive, and affective.

Table 4.14: Teacher Responses and Subsequent Student Engagement Codes, Ms. B.

	Behavioral Engagement	Cognitive Engagement	Affective Engagement
Day 1			
Difficulty 5 Suggesting a process to use in solving the problem			
Marcos	On-task	Low level cognitive activity	Moderate positive feelings
Larent	On-task	Low level cognitive activity	Mild negative feelings
Lashanna	On-task	Low level cognitive activity	Moderate positive feelings
Difficulty 6 Asking a question to clarify what the students said			
Marcos	On-task	High level cognitive activity	Moderate positive feelings
Lashanna	On-task	Low level cognitive activity	Moderate positive feelings
Difficulty 7 Leading the students to the correct answer/idea			
Marcos	On-task	Low level cognitive activity	Moderate positive feelings
Lashanna	On-task	Low level cognitive activity	Moderate positive feelings
Difficulty 8 Repeating the teacher's question			
Marcos	On-task	Low level cognitive activity	Moderate positive feelings
Difficulty 9 Asking a student to prove his solution			
Marcos	On-task	High level cognitive activity	Moderate positive feelings
Lashanna	On-task	Insufficient Information	Moderate positive feelings

Difficulty 10 Showing the student a diagram			
Marcos	On-task	High level cognitive activity	Moderate positive feelings
Difficulty 11 Choosing not to respond to an incorrect response Inviting (non-verbally) another student to respond			
Larent	On-task	Low level cognitive activity	Mild positive feelings
Difficulty 12 Reminding student about another student's idea Probing- asking the student to explain his reasoning Explaining the mathematics task			
Larent	On-task	High level cognitive activity	Moderate positive feelings
Difficulty 13 Reminding student about another student's idea Inviting another student to respond			
Lashanna	On-task	High level cognitive activity	Strong positive feelings
Difficulty 14 Inviting another student to respond Suggesting a process to use in solving the problem			
Larent	Off-task	Not engaged	Mild negative feelings
Difficulty 15 Choosing not to respond to an incorrect response			
Larent	Off-task	Not engaged	Mild negative feelings
Day 2			
Difficulty 16 Probing- asking student to explain his solution Inviting another student to respond Suggesting student share difficulty with peer			
Marcos	On-task	Low level cognitive activity	Mild positive feelings
Difficulty 17 Asking student to prove her solution			
Lashanna	On-task	High level cognitive activity	Strong positive feelings
Difficulty 18 Offering suggestion of process to use in resolving difficulty			
Marcos	On-task	High level cognitive activity	Moderate positive feelings

Following is a more in-depth analysis of the students' behavioral, cognitive, and affective engagement.

Students' Behavioral Engagement

Codes used to describe students' behavioral engagement following episodes of difficulty during the two class sessions analyzed for Ms. B. indicate that Marcos, Larent, and Lashanna were on-task overall. In two instances the code 'off-task' was assigned to describe Larent's behavior. According to Ms. B., Larent's off-task behavior was typical for him. Ms. B. mentioned during her interview that Larent often had difficulty focusing and staying on-task during class.

Students' Cognitive Engagement

As noted in table 4.14 above, codes assigned to describe the highest level of cognitive activity students demonstrated during the analyzed segments of videotape included both high and low level cognitive activity. During the two instances in which Larent was off-task, the code 'not engaged' was used. Several trends were noted between Ms. B.'s responses to the students and the complexity of students' subsequent cognitive activity. In all instances where Ms. B.'s response included Asking the student to prove his/her solution (Difficulties 9, 17), all the students involved in the difficulty subsequently demonstrated high levels of cognitive activity, where cognitive behaviors were observable (this excludes Difficulty 9 where there was insufficient information to code for Lashanna's level of cognitive activity). The single instance where Larent demonstrated evidence of high level cognitive activity (of 5 instances of difficulty in which he was involved) occurred following a response from Ms. B. that involved Probing. In all but one instance where Ms. B.'s response included either Suggesting a process to use in solving the problem (Difficulties 5, 14) or Leading the students to the correct answer/idea (Difficulty 7), all the students involved subsequently demonstrated low level cognitive activity or were not cognitively engaged. Ms. B.'s response to Marcos during Difficulty 18 involved Suggesting a process to use in solving the problem; however, Marcos subsequently demonstrated high level cognitive activity.

Students' Affective Engagement

As indicated in table 4.14 above, students' affective engagement following Ms. B.'s interventions included mild, moderate and strong positive feelings as well as slight negative feelings. A preliminary trend was noted relating students' affective and cognitive engagement. Time periods for which a student' feelings were coded as negative were always assigned a code of low level cognitive activity or no apparent cognitive activity related to mathematics. During the two time periods in which strong positive feelings associated with mathematics were evidenced (Difficulties 13, 17), high level cognitive activity was also evidenced.

Following are illustrative examples of Ms. A.'s students' behavioral, cognitive, and affective engagement. These representative instances include episodes involving students' on task and off-task behaviors, students' high and low level cognitive activity as well as cognitive disengagement from the task, and students' positive and negative affect.

Following Difficulty 15 (Day 1):

Off-task Behavioral Engagement, Not Cognitively Engaged, Slight Negative Feelings

Difficulty 15, involving Larent, occurs slightly under one hour after the class session has begun. During Event 5, Ms. B. asks Marcos how many blocks are in a 10-block high tower. Marcos correctly answers that there are 46. Ms. B. asks Marcos how he obtained his solution of 46 blocks. Larent says, "I thought it was 42" (*Appendix A, Event 5, Difficulty 15, line 45*). Ms. B. does not address Larent's comment. Marcos answers Ms. B.'s question. He says he found the answer by "using the blocks and doing the thing…sides" *(he points to a diagram on his task sheet) (Appendix A, Event 5, Difficulty 15, lines 46-47*). Ms. B. tells Marcos to build up from the 6-block high tower, which was the largest tower he had built and solved, so that he can prove if he is correct (Earlier in this event, Marcos had mentioned that he found a pattern where the solutions for consecutive towers end with a 1, then a 6, then a 1, then a 6, etc.). Larent then repeats, "I thought it was 42" *(Appendix A, Event 5, Difficulty 15, line 50*). Ms. B. tells the students to work together and "prove it" *(Appendix A, Event 5, Difficulty 15, line 51*). She walks away from Group 1.

After Ms. B. leaves the table, Larent looks away from his table at another group ***(see picture below at 10:33:35).***

He then leans back and rubs his eyes. He covers his eyes with his hands ***(see picture below at 10:33:52).***

Larent sings and looks at the other students at his table. He does not write or build with the cubes. He yawns ***(see picture below at 10:35:42).***

Marcos and Lashanna build a tower together and record information. Larent looks around blankly and sings.

Larent's engagement following Ms. B.'s response to this difficulty was coded as involving off-task behavior, cognitive disengagement, and slight negative feelings.

Following Difficulty 16 (Day 2):

On-task Behavioral Engagement, Low Level Cognitive Activity, Mild Positive Feelings

Difficulty 16, which involves Marcos, occurs several minutes after the class session begins. The students in Group 1 have not yet begun working. Ms. B. walks over to the students' table and asks them what they came up with so far. Marcos says the 5-block high tower has 21 blocks, the 10-block high tower has 46 blocks, and the 100-block high tower has 460 blocks. Ms. B. asks Marcos how he could find the number of blocks in any size tower. Marcos says, "you could find them because you could see there's a pattern. One's…there's always a 1 or 6. So, like, if you see here 46, the next one's gonna be 51. Then that next one's gonna be 56, and then it would be 61, and so on" (Appendix A, Event 6, Difficulty 16, lines 19-22). Ms. B. asks Marcos what the answer is for a 100-block high tower. Marcos incorrectly replies that it is 460. This was coded as Difficulty 16. Ms. B. asks Marcos how he got that number. Marcos indicates that he did not use the '1, 6, 1, 6' pattern he found. He says he multiplied 10 by 46 (the number of blocks in a 10-block high tower) to get 460. Ms. B. tells Marcos to write his thoughts on a transparency. She then asks Lashanna if she agrees with Marcos. When Lashanna shrugs her shoulders, Ms. B. tells Marcos that he needs to try and convince Lashanna of his solution. Ms. B. then leaves the table.

After Ms. B. leaves, Lashanna tells Marcos that she got 496 as her solution for the 100- block high tower. Marcos keeps repeating that the answer is 460 because "If 10 is 46, 100 of course is gonna be 460." While Marcos speaks, he balances his blank transparency on his marker. **See picture below at 9:43:18**.

Lashanna explains that she got 500 and took away 4. She explains that she took away one for each branch of the tower. She motions with her hands indicating four branches. While

Lashanna speaks, Marcos continues to balance the transparency on his marker. **See picture below at 9:43:37**.

Marcos says that "it's not 500 because it's going by 1, 6, 1, 6. That's the pattern. When a pattern goes it sticks to its pattern." He then continues playing with the transparency, standing it up on his desk, and putting it against his face. **See pictures below at 9:44:14 and 9:44:19**.

He then sings, whistles, and drums his pencil and marker together. Lashannah and Marcos argue about who will write up the presentation. Each says the other should write. Ms. B. comes over to Larent who appears to be sleeping with his head on the back of his chair. When Ms. B. walks to the table, Marcos takes a marker and begins writing.

Marcos's behavior following Ms. B.'s response to his difficulty was coded as on-task and involving low level cognitive activity and mild positive feelings.

Following Difficulty 17 (Day 2):

On-task Behavioral Engagement, High Level Cognitive Activity, Strong Positive Feelings

Difficulty 17 occurs when Ms. B. asks Lashanna how she got the solution for the 100-block high tower. Lashanna says that Marcos explained it to her. She says she agrees with Marcos (*Marcos had obtained an incorrect solution of 460 blocks for the 100-block high tower. Lashanna had previously arrived at the correct solution of 496 blocks for the 100- block high tower and had argued with Marcos, saying that her solution is correct and his solution of 460 blocks is* *incorrect. Lashanna, however, does not mention her own previous solution to Ms. B.*) Ms. B. asks Lashanna if she can explain what Marcos explained to her. Lashanna says, “No, I don’t know” (Appendix A, Event 7, Difficulty 17, line 18). Ms. B. tells Lashanna to try to come up with a way to determine if Marcos’s solution is correct. Ms. B. tells Lashanna that she should not simply agree with Marcos’s solution since he may be right or wrong. While Ms. B. says this, Lashanna leans her head on her hand and looks down. **See picture at 9:48:29**. She does not look up and does not respond to Ms. B.

Ms. B. then checks up on Larent’s work and tells him she will be back in 5 or 10 minutes to see what he has done.

When Ms. B. leaves the table, Lashanna again argues with Marcos’s solution of 460 blocks for the 100- block high tower. Lashanna begins building a 5- block high tower. Lashanna tells Larent that he built his tower incorrectly. She says it should have 5 blocks in the height, not 6, and that the middle block counts in the height. Lashanna and Marcos agree that the 5- block high tower has 21 blocks. When Larent says he got 25 for a 5- block high tower, Lashanna and Marcos show him that he built his tower incorrectly. Lashanna builds a 10- block high tower, then punches numbers into her calculator. She tells Marcos that the 100- block high tower has 496 cubes. When Marcos disagrees, she explains how she got her answer. She says that since the 10-block high tower has 10 blocks in the height and 9 blocks in each of its branches, the 100-block high tower will have 100 blocks in the height and 99 blocks in each of the branches. She says that 99 times 4 is 396 and if you then add 100, you get 496. Lashanna then asks Marcos to explain the method he used to arrive at a solution of 460 blocks. Marcos explains that he simply multiplied the number of blocks needed for a 10-block high tower by 10 to get the number of blocks needed for a 100-block high tower. Lashanna asks Larent whether he thinks Marcos's solution or hers is correct. She explains her solution to him, but he does not look up and continues building his tower. Ms. B. has been standing near group 1 and listening while Lashanna explained to Larent how she got 496 cubes. Ms. B. asks Lashanna to generalize her solution for any size tower. Lashanna comes up with a general solution where the number of blocks needed for any size tower is one less than the height of the tower times 4, plus the height of the tower.

Lashanna's behavior following Ms. B.'s response to her difficulty was coded as on-task and involving high level cognitive activity and strong positive feelings.

Students' Collaborative Effort and Affect

In this group there was little evidence of strong positive feelings (2 codes of strong positive feelings of 19 affective codes). There was also little evidence of group member responsibility towards each other or group cohesiveness. The students in this group often did not appear to work together to try to arrive at solutions/ideas agreed upon by all members. Although Lashanna did at times attempt to explain her solution to Marcos and Larent, Marcos and Larent often did not appear interested in understanding her solution. The following four specific incidences point to a lack of collaboration among the members of this group.

Incident 1: Lack of Collaboration

During Event 4, while Ms. B. is helping Marcos and Lashanna work through their conjecture that there are 500 blocks in the 100-block high tower, Ms. B. asks Lashanna what her solution is for the 3-block high tower. Larent turns over his paper. Ms. B. turns to Larent and says, “Now, it’s not a secret. You want to get together with it” (Appendix A, Event 4, lines 118-119). Before Ms. B. leaves the table, she says to the group members, “You want to come up with the same answers. If you have different answers, you have to try and prove that you’re right” (Appendix A, Event 4, lines 131-133). Ms. B. says to Larent, “You said here, like, ‘I’m right.’ You turned your paper over. You should have been like, ‘Lashanna, how did you get 14? I got 11’” (Appendix A, Event 4, lines 133-137).

Incident 2: Lack of Collaboration

During Event 5, Ms. B. asks the students what they have come up with. Marcos lists the solutions he obtained for towers of heights 1 through 6. Larent points to the task sheet and tells Marcos that he sees towers 1, 2, and 3. He asks Marcos how he got the answers to towers 4, 5, and 6 if those towers are not on the sheet. Marcos says, “We found it with the cubes” (Appendix A, Event 5, line 32). Lashanna adds *(to Larent)*, “While you were busy talking” (Appendix A, Event 5, line 34).

This episode illustrates Larent’s lack of involvement as well Lashanna’s awareness of his lack of involvement, in the group’s problem solving.

Incident 3: Lack of Collaboration

After Ms. B. has left the students’ table at the conclusion of Event 6, Lashanna tells Marcos that she got 496 as her solution for the 100- block high tower. Marcos keeps repeating that the answer is 460 because “If 10 is 46, 100 of course is gonna be 460.” While Marcos speaks, he balances his blank transparency on his marker. **See picture at 9:43:18**.

Lashanna explains that she got 500 and took away 4. She explains that she took one away for each branch of the tower. She motions with her hands indicating four branches. While

Lashanna speaks, Marcos continues to balance the transparency on his marker. **See picture at 9:43:37**.

Marcos says that "it's not 500 because it's going by 1, 6, 1, 6. That's the pattern. When a pattern goes it sticks to its pattern." He then continues playing with the transparency, standing it up on his desk, and putting it against his face. Larent has been leaning back in his chair, covering his eyes. He does not interact with Marcos or Lashanna. **See pictures at 9:44:14 and 9:44:19**.

He then sings, whistles, and drums his pencil and marker together.

While Lashanna explains her solution to Marcos, Marcos shows little evidence of interest in understanding Lashanna's method. He fools around with his transparency while she speaks. Marcos's response to Lashanna suggests he has not been listening to Lashanna's explanation of her solution. Marcos responds to Lashanna by saying the solution is not 500. Lashanna, however, did not say the solution was 500; she had simply mentioned 500 in the explanation of her calculation. Larent had been leaning back in his chair during Marcos and Lashanna's conversations and showed no evidence of participation in solving the mathematics task.

Incident 4: Lack of Collaboration

Following Incident 3 above, Lashanna and Marcos argue about who will write up the group's ideas for their presentation to the class. Both students appear uninterested in taking up the task. Lashanna and Marcos each say the other should write.

This scenario suggests a lack of willingness by Marcos and Lashanna to work collaboratively towards a common goal.

Incident 5: Lack of Collaboration

Following Event 7, Lashanna repeats to Marcos her explanation of her solution of 496 blocks for the 100-block high tower. She says that since the 10-block high tower has 10 blocks in the height and 9 blocks in each of its branches, the 100-block high tower will have 100 blocks in the height and 99 blocks in each of the branches. She says that 99 times 4 is 396 and if you then add 100, you get 496. Lashanna then asks Marcos to explain the method he used to arrive at a solution of 460 blocks. Marcos explains that he simply multiplied the number of blocks needed for a 10-block high tower by 10 to get the number of blocks needed for a 100-block high tower. Lashanna asks Larent whether he thinks Marcos's solution or hers is correct. She explains her solution to him, but he does not look up and continues building his tower.

Qualitative In-Depth Analysis of Students' Cognitive Engagement

In this section, three difficulties of Ms. B.'s students will be analyzed qualitatively, examining the students' cognitive engagement before and after Ms. B.'s response to the difficulty. Comparisons between students' cognitive engagement before and after Ms. B.'s response to their difficulties will be made, and possible associations between types of teacher responses and students' subsequent cognitive engagement will be discussed. Difficulties 12, 16, and 17, analyzed here, are those also used in the researcher's interview with Ms. B., where Ms. B. reflected on her reasoning for providing those particular responses. The subheading for each instance of difficulty analyzed below indicates the number of the difficulty, the student(s) involved, and the day it occurred. The analysis for each difficulty includes: 1) a qualitative description of the student's behavior related to his/her cognitive engagement before the difficulty and the resulting classification of the student's cognitive engagement, 2) a description of the difficulty, 3) a description of the teacher's response, 4) a qualitative description of the student's behavior related to his/her cognitive engagement after the difficulty and the resulting classification of the student's cognitive engagement, and 5) a comparison of the students' cognitive engagement before and after the difficulty.

Difficulty #12: Larent, Day 1

Description of Behavior Related to Student's Cognitive Engagement Before Difficulty

Time of day: (10:13:34-10:13:42)

> Ms. B. asks Larent how he got 6 cubes for a 2- block high tower. Larent points to diagram B on the task sheet and says, "That's 2, 4, and one on the bottom" [lines 57-58 of transcript]. He then says the 2- block high tower has 5 cubes [line 65 of transcript]. (This is Difficulty 11)

Ms. B. does not respond to Larent's explanation. She looks at Marcos [line 59 of transcript]. Marcos gets up, walks around the desk, and points to the diagram on the sheet. He points to the diagram and says "1, 2, 3, 4, 5, 6" [lines 59-61 of transcript]. Larent watches Marcos as he speaks but does not respond to his explanation. As Ms. B. speaks with Lashanna and Marcos, Larent watches and looks around the room periodically. When Ms. B. hands Larent a paper on which to record information, he puts it on Lashanna's desk [lines 75-77 of transcript]. Larent answers Ms. B.'s question of how many cubes there are in a tower that is one-block high [lines 82-86 of transcript].

Behaviors Pertaining to Student's Cognitive Engagement Before Difficulty and Resulting Classification

Low level cognitive activity

Highest level of cognitive activity involved: Considering facts

Description of Difficulty

Ms. B. asks how many cubes there are in a 2- block high tower. Larent says 5, Lashanna says there are 6 [lines 93-96 of transcript].

Teacher's Response

Ms. B. says, "You said 5, you said 6" (pointing to Larent and then to Lashanna) [line 97 of transcript]. When Larent says he said 6 the first time, Ms. B. asks him why he changed his answer [lines 98-100 of transcript]. Larent says he changed his solution because he sees what other people did. After a conversation between Lashanna and Larent which is not clear on the videotape (no audio available for this segment), Ms. B. explains that the task involves towers where each builds on the previous one [lines 102-108 of transcript].

Code: Reminding student about another student's idea
Probing- asking the student to explain his/her solution or the reasoning he/she used
Explaining the mathematics task

Description of Behavior Related to Student's Cognitive Engagement After Difficulty

Time of day: (10:13:42-10:30:26)

When Ms. B. asks Larent if he's changing his answer, Larent says he changed his answer because he sees what other people did [line 101 of transcript]. After Ms. B. explains that the task involves towers which build on each other, she asks Larent how many blocks he thinks there are in a 2-block high tower. Larent says there are 6. Ms. B. points to Larent's paper. Larent, Lashanna, and Marcos record information on their papers [lines 98-111 of transcript].
Ms. B. then asks Lashanna how many blocks she got for the total of the 3-block high tower. Larent turns over his paper, blank side up. Lashanna indicates an incorrect solution for the 3-block high tower (this is Difficulty 13). Ms. B. tells Larent that his solutions should not be kept secret and that he should share them with his group. Ms. B. asks Marcos to share his solution. Marcos shares his correct solution, and Lashanna agrees it is correct. Ms. B. tells the students to work together in coming up with the solution for the 4-block high tower. She then leaves the table. After Ms. B. leaves the table, Larent fools around with the blocks and audio recorder and engages in non-task related discussions with other members of his group. He is not noticeably engaged in the task until Ms. B. returns to the table, approximately 15 minutes after she left. *(Data for the 15 minutes Ms. B. is not at the table were drawn from videotape)*

Behaviors Pertaining to Student's Cognitive Engagement After Difficulty and Resulting Classification

High level cognitive activity

Highest level of cognitive engagement involved: Reasoning

Comparison of Student's Cognitive Engagement Before/After Difficulty 12

Larent appears disengaged from the mathematics task during periods when Ms. B. is not at the students' table and at times, even when she is at the table but not working directly with Larent. When Ms. B. is at the students' table asking questions to Larent, he responds and appears to use some level of critical thinking. After Ms. B. leaves the table following her response to this difficulty, Larent is off-task and engages others in his group in non-task related discussions.

Larent's behavior observed during this segment is consistent with Ms. B.'s description of his classroom behavior. Ms. B. mentioned during her interview that Larent has difficulty focusing and remaining on-task during class.

Difficulty #16: Marcos, Day 2

Description of Behavior Related to Student's Cognitive Engagement Before Difficulty

Time of day: (9:37:30-9:41:37)

During the first four minutes of the class session the students at group 1 sit in their seats but do not work on the mathematics task. Marcos tells Lashanna that she and Larent need to do the writing for the presentation. Ms. B. then walks over to group 1 to check on their progress.

Behaviors Pertaining to Student's Cognitive Engagement Before Difficulty and Resulting Classification

Disengaged

Highest level of cognitive activity involved: Not Applicable

Description of Difficulty

Ms. B. asks the students what they have come up with. Marcos says the students found that the 5-block high tower has 21 blocks and the 10-block high tower has 46 blocks. He then (incorrectly) says the 100-block high tower has 460 blocks [lines 23-24 of transcript].

Teacher's Response

Ms. B. asks Marcos how he got the number 460. When Marcos explains that he multiplied 46 by 10, Ms. B. tells him to write his explanation on a transparency. Ms. B. then asks Lashanna if she agrees with Marcos. When Lashanna indicates that she doesn't know whether she agrees with Marcos, Ms. B. tells Marcos to convince Lashanna of his answer [lines 25-36].

Code: Probing- asking the student to explain his/her solution or the reasoning he/she used

Inviting (verbally or non-verbally) (an)other student(s) to respond

Suggesting student(s) share his/her/their difficulty with peer(s)

Description of Behavior Related to Student's Cognitive Engagement After Difficulty

Time of day: (9:43:11-9:47:50)

After Ms. B. leaves Lashanna tells Marcos that she got 496 as her solution for the 100- block high tower. Marcos keeps repeating that the answer is 460 because "If 10 is 46, 100 of course is gonna be 460." While Marcos speaks, he balances his blank transparency on his marker. **See picture at 9:43:18**.

Lashanna explains that she got 500 and took away 4. She explains that she took away one for each branch of the tower. She motions with her hands indicating four branches. While Lashanna speaks, Marcos continues to balance the transparency on his marker. **See picture at 9:43:37**.

Marcos says that "it's not 500 because it's going by 1, 6, 1, 6. That's the pattern. When a pattern goes it sticks to its pattern." He then continues playing with the transparency, standing it up on his desk, and putting it against his face. **See pictures at 9:44:14 and 9:44:19**.

He then sings, whistles, and drums his pencil and marker together. Lashannah and Marcos argue about who will write up the presentation. Each says the other should write. Ms. B. comes over to Larent who appears to be sleeping with his head on the back of his chair. When Ms. B. walks to the table, Marcos takes a marker and begins writing.

Behaviors Pertaining to Student's Cognitive Engagement After Difficulty and Resulting Classification

Low level cognitive activity

Highest level of cognitive engagement involved: Restating one's own idea without consideration of others' ideas

Comparison of Student's Cognitive Engagement Before/After Difficulty 16

Before Difficulty 16, Marcos is not engaged in the mathematics task and does not appear interested in preparing the group's presentation. After Ms. B.'s response to his difficulty, Marcos is primarily on-task and responds to Lashanna's arguments which support her own solution. Marcos, however, keeps repeating his original solution and does not consider Lashanna's explanation of her correct solution. Marcos continues to appear uninterested in preparing the group's presentation. Marcos's cognitive activity following Ms. B.'s response to his solution was coded as low level.

Difficulty #17: Lashanna, Day 2

Description of Behavior Related to Student's Cognitive Engagement Before Difficulty

Time of day: (9:43:11-9:47:50)

The students' behavior prior to Difficulty 17 is also their behavior during the time period following Difficulty 16, described above.

After Ms. B. leaves the table following Difficulty 16, Lashanna tells Marcos that she got 496 as her solution for the 100- block high tower. Marcos keeps repeating that the answer is 460 because "If 10 is 46, 100 of course is gonna be 460." While Marcos speaks, he balances his blank transparency on his marker. Lashanna explains that she got 500 and took away 4. She explains that she took away one for each branch of the tower. She motions with her hands indicating four branches. While Lashanna speaks, Marcos continues to balance the transparency on his marker.

Marcos says that "it's not 500 because it's going by 1, 6, 1, 6. That's the pattern. When a pattern goes it sticks to its pattern." He then continues playing with the transparency, standing it up on his desk, and putting it against his face. He then sings, whistles, and drums his pencil and marker together. Lashannah and Marcos argue about who will write up the presentation. Each says the other should write. Ms. B. comes over to Larent who appears to be sleeping with his head on the back of his chair. When Ms. B. walks to the table, Marcos takes a marker and begins writing. Ms. B. tells Larent that he needs to begin working. She then turns to Lashanna and tells her that she needs to work as well and check whether Marcos's solutions are correct. Ms. B. then questions Lashanna about her solutions to the 10- and 100- block high towers.

Behaviors Pertaining to Student's Cognitive Engagement Before Difficulty and Resulting Classification

High level cognitive activity

Highest level of cognitive activity involved: Justifying, Reasoning

Description of Difficulty

When Ms. B. asks Lashanna how she got her solution for the 100- block high tower, she says that Marcos explained it to her. When Ms. B. asks Lashanna if she can repeat Marcos's explanation, she says she cannot [lines 14-21].

Teacher's Response

Ms. B. tells Lashanna to try to come up with a way to check whether Marcos's solution is correct. Ms. B. says she should not "just agree with what he says" [lines 22-24 of transcript].

Code: Asking the student to prove his/her solution

Description of Behavior Related to Student's Cognitive Engagement After Difficulty

Time of day: (9:48:38-10:02:20)

When Ms. B. leaves the table, Lashanna again argues with Marcos's solution of 460 blocks for the 100- block high tower. Lashanna begins building a 5- block high tower. Lashanna tells Larent that he built his tower incorrectly. She says it should have 5 blocks in the height, not 6, and that the middle block counts in the height. Lashanna and Marcos agree that the 5- block high tower has 21 blocks. When Larent says he got 25 for a 5- block high tower, Lashanna and Marcos show him that he built his tower incorrectly. Lashanna builds a 10- block high tower, then punches numbers into her calculator. She tells Marcos that the 100- block high tower has 496 cubes. When Marcos disagrees, she explains how she got her answer. She says that since the 10-block high tower has 10 blocks in the height and 9 blocks in each of its branches, the 100-block high tower will have 100 blocks in the height and 99 blocks in each of the branches. She says that 99 times 4 is 396 and if you then add 100, you get 496.

Lashanna then asks Marcos to explain the method he used to arrive at a solution of 460 blocks. Marcos explains that he simply multiplied the number of blocks needed for a 10-block high tower by 10 to get the number of blocks needed for a 100-block high tower. Lashanna asks Larent whether he thinks Marcos's solution or hers is correct. She explains her solution to him, but he does not look up and continues building his tower. Ms. B. has been standing near group 1 and listening while Lashanna explained to Larent how she got 496 cubes. Ms. B. asks Lashanna to generalize her solution for any size tower. Lashanna comes up with a general solution where the number of blocks needed for any size tower is one less than the height of the tower times 4, plus the height of the tower.

Behaviors Pertaining to Student's Cognitive Engagement After Difficulty and Resulting Classification

High level cognitive activity

Highest level of cognitive engagement involved: Justifying, Drawing inferences, Speculating

Comparison of Student's Cognitive Engagement Before/After Difficulty 17

Lashanna is deeply engaged with the mathematics problem, using high level cognitive activity, both before and after her apparent difficulty. It is interesting, therefore, that when Ms. B. questions Lashanna during this event about the number of blocks in the 100- block high tower, Lashanna says she agrees with Marcos's incorrect solution of 460 blocks and does not mention her own solution of 496 blocks. When Ms. B. asks Lashanna if she can explain Marcos's solution, Lashanna says, "No, I don't know" [line 18 of transcript]. Lashanna appears uninterested in the mathematics problem during this apparent difficulty, which is in stark contrast to her high level of cognitive engagement both before and after the difficulty.

Ms. B. addressed this discrepancy in Lashanna's behavior during her stimulated recall interview. She mentioned that Lashanna did not seem to be putting any effort into the mathematics task since she had not wanted to be in the group in which she was placed and preferred to be in a group with her friends. Ms. B. said she did not want Lashanna to join her friends' group because if she did she would likely get distracted from the mathematics problem. According to Ms. B., Lashanna therefore responded by seeming to be uninvolved in the task when Ms. B. came over.

Analysis of Ms. B.'s students' cognitive engagement before and after their three mathematics difficulties analyzed here, combined with Ms. B.'s interview data, shows that Ms. B. possessed an understanding of her students' learning strengths and difficulties. Ms. B. chose responses to her students' difficulties to fit their particular learning needs in order to maximize each student's cognitive engagement with the task.

Analysis of Student Engagement Questionnaire Data

In the present study, two sections of the students' questionnaires were examined. The first section related to students' feelings about mathematics that day; the second asked about whether students experienced difficulty in mathematics that day.

Students' Reports of Emotions

The following tables depict Ms. B.'s students' scores for each emotion on the Student Engagement Questionnaire as compared with the scores of a population of 83 students in the same urban school district. Table 4.15 lists Ms. B.'s students' scores on the questionnaires distributed after the class session on Day 1. Table 4.16 illustrates Ms. B.'s students' scores on the identical questionnaire, distributed after the class session on Day 2. Scores were measured on a 2-point scale. For positive emotions, students received a score of '2' for a response of 'Very much,' a '1' for 'Somewhat,' and a '0' for 'Not at All.' For negative emotions, the scores were reversed. 'N/A' indicates a student's score for a questionnaire item was not available.

Analysis of students' Day 2 questionnaire data does not include Larent's questionnaire responses since Larent was not involved in analyzed instances of difficulty on Day 2.

Table 4.15, Ms. B. Student Questionnaire Feelings Responses, Day 1

Questionnaire Item	**n**	**Population Mean**	**Population Standard Deviation**	**Marcos**	**Larent**	**Lashanna**
Interested	78	1.58	.570	1.00	1.00	1.00
Respected	78	1.63	.561	0.00	2.00	2.00
Proud	79	1.48	.677	1.00	2.00	0.00
Successful	78	1.56	.636	2.00	0.00	1.00
Safe	76	1.66	.623	2.00	2.00	2.00
Excited	75	1.37	.731	0.00	2.00	0.00
Happy	76	1.46	.720	2.00	2.00	1.00
Satisfied	77	1.36	.687	1.00	2.00	0.00
Relieved	78	1.10	.749	1.00	1.00	0.00
Confident	75	1.41	.737	2.00	0.00	0.00
Curious	76	.87	.854	0.00	0.00	1.00
Unhappy (reversed)	76	1.83	.413	2.00	2.00	1.00
Disappointed (Reversed)	75	1.79	.527	2.00	2.00	1.00
Worried (Reversed)	76	1.62	.610	2.00	0.00	2.00
Discouraged (Reversed)	76	1.88	.399	2.00	2.00	2.00
Angry (Reversed)	76	1.72	.556	2.00	2.00	1.00
Disrespected (Reversed)	76	1.88	.399	2.00	2.00	2.00
Bored (Reversed)	76	1.59	.715	2.00	1.00	0.00
Embarrassed (Reversed)	76	1.79	.549	2.00	2.00	2.00
Afraid (Reversed)	75	1.89	.352	2.00	2.00	2.00
Positive/Negative Subscale Means		*1.57*		*1.50*	*1.45*	*1.05*
Frustrated	76	1.68	.616	0.00	0.00	N/A
Confused	76	1.55	.641	0.00	0.00	0.00
Engagement with Impasse Subscale Mean		*1.62*		*0.00*	*0.00*	*0.00*

Table 4.16, Ms. B. Student Questionnaire Feelings Responses, Day 2

Questionnaire Item	**n**	**Population Mean**	**Population Standard Deviation**	**Marcos**	**Lashanna**
Interested	78	1.58	.570	1.00	2.00
Respected	78	1.63	.561	2.00	2.00
Proud	79	1.48	.677	2.00	2.00
Successful	78	1.56	.636	2.00	2.00
Safe	76	1.66	.623	2.00	2.00
Excited	75	1.37	.731	1.00	2.00
Happy	76	1.46	.720	2.00	2.00
Satisfied	77	1.36	.687	2.00	1.00
Relieved	78	1.10	.749	2.00	1.00
Confident	75	1.41	.737	2.00	N/A
Curious	76	.87	.854	0.00	0.00
Unhappy (reversed)	76	1.83	.413	2.00	2.00
Disappointed (Reversed)	75	1.79	.527	2.00	2.00
Worried (Reversed)	76	1.62	.610	2.00	2.00
Discouraged (Reversed)	76	1.88	.399	2.00	2.00
Angry (Reversed)	76	1.72	.556	2.00	2.00
Disrespected (Reversed)	76	1.88	.399	2.00	2.00
Bored (Reversed)	76	1.59	.715	2.00	1.00
Embarrassed (Reversed)	76	1.79	.549	2.00	2.00
Afraid (Reversed)	75	1.89	.352	2.00	2.00
Positive/Negative Subscale Means		*1.57*		*1.80*	*1.65*
Frustrated	76	1.68	.616	0.00	0.00
Confused	76	1.55	.641	0.00	0.00
Engagement with Impasse Subscale Mean		*1.62*		*0.00*	*0.00*

Students' Self-Reported Affect vs. Affect Inferred from Students' Behaviors

To gather information regarding students' affect during the videotaped class sessions, the present researcher initially watched preselected episodes of students' difficulties during the videotaped sessions multiple times. The researcher then created transcriptions of those episodes and the time periods following them which included descriptions of students' behaviors as well as screenshots from the video that were illustrative of students' emotions. The researcher then tried to infer the students' feelings following each instance of difficulty from the transcriptions. When in doubt regarding particular students' feelings, the researcher referred back to the videotapes and in several cases, consulted with a psychologist to determine the feelings that most aptly depicted the constellation of behaviors exhibited by the students.

The emotions inferred by the researcher were then compared with students' self-reported emotions on the questionnaire. Following is a comparison of self-reported and inferred affect for Marcos, Larent, and Lashanna on Day 1 and for Marcos and Lashanna on Day 2 (Larent was not involved in analyzed instances of difficulty on Day 2).

Marcos

Day 1

Marcos's self-reported emotions for this class session were similar to the emotions observed from videotape analysis of the session. As described in Appendix A, Event 3: Difficulty 5, and Event 4, Difficulties 6 through 10, inferences made from Marcos's behaviors during the class session showed Marcos appeared to have some level of interest in the mathematics problem but did not demonstrate strong positive feelings such as excitement, confidence, or pride. On his questionnaire, Marcos reported feeling somewhat interested and proud and not at all excited or curious, although very much confident.

Day 2

Marcos's self-reported emotions for this class session were more positive than the emotions inferred from his observed behaviors. During class, Marcos showed some level of interest in the problem but often appeared bored (described in Appendix A, Event 6: Difficulty 16 and Event 8, Difficulty 18). He did not show evidence of strong positive feelings towards mathematics. On his questionnaire, Marcos reported he was only somewhat interested and not at all curious, but he reported being very proud, successful, and happy. He also reported being not at all bored.

Larent

Day 1

Larent's self-reported emotions for this class session were more positive than the emotions inferred from observations of his behaviors. During much of the class session, Larent appeared disinterested and bored and showed no evidence of strong positive feelings (refer to Appendix A, Event 3: Difficulty 5, Event 4: Difficulties 11 & 12, and Event 5: Difficulties 14 & 15). On his questionnaire, however, Larent reported feeling somewhat interested, not at all bored, and very happy (his happiness may have been unrelated to mathematics).

Lashanna

Day 1

During much of the class session Lashanna appeared interested in the mathematics problem and did not demonstrate strong positive or negative feelings (refer to Appendix A, Event 3: Difficulty 5 and Event 4: Difficulty 6, 7, 9, and 13). Lashanna's 'Positive Emotions

Score" on her questionnaire, however, was low when compared with the mean. Lashanna reported feeling very bored and somewhat angry, disappointed, and unhappy. This discrepancy between Lashanna's observed and reported affect may be due to her negative feelings having been related to the group she was selected to work with as opposed to related to the mathematics problem. During Ms. B.'s interview, Ms. B. mentioned that Lashanna was upset because she had wanted to work with another group.

Day 2

Lashanna's self-reported affect during this class session was similar to the affect inferred from her observed behaviors. During the class session, Lashanna appeared highly interested in the mathematics problem and eager to convince her groupmates that her solution was correct (see Appendix A, Event 7: Difficulty 17). She did not show evidence of any negative feelings. On her questionnaire, Lashanna reported feeling very interested, proud, successful, excited, and happy.

Group Analysis of Students' Self-Reported Affect vs. Affect Inferred from Students' Behaviors

Day 1

Lashanna, who during the next day's session demonstrated the highest level of understanding among her groupmates, reported the most negative affect of the members in her group (and more than 10 points lower than the mean). Larent, who did not work much on the problem during either class session and appeared to have only a superficial understanding of the problem, reported a level of positive affect close to the mean.

Day 2

Marcos and Lashanna both reported considerably more positive affect during this mathematics session than during the previous session. They may have felt more positive regarding mathematics on Day 2 since they came up with a general solution to the problem on that day and presented it to the class. Both students remarked on their questionnaires that they felt smart that day.

Self-Reports of Encountering Difficulty and Comparison with. Observed Instances of Difficulty

Table 4.17, Students' Reports of Difficulty and Observed Difficulty, Ms. B., Day 1

	Marcos	**Larent**	**Lashanna**
Self-Report of Difficulty			
Had Difficulty?	Yes	No	No
Classroom Context	Not during group conversation or whole class discussion	----	----
What stood out?	"I think what kids said or did."	----	"Nothing did not happen I did not teach nothing, I felt board and like a superstar, My teacher just was getting on my nerv, they [other kids] did not do nothing to me."
Observed Difficulty			
Types of Difficulty and Frequency of Occurrence	Impasse (1) Incorrect Answer (5)	Impasse (1) Incorrect Answer/Idea (3) Does not Understand Solution (1)	Impasse (1) Incorrect Answer (4)

Larent and Lashanna reported they did not have difficulty with the mathematics problem. They were each, however, involved in at least 5 analyzed instances of difficulty during the class session.

Table 4.18, Students' Reports of Difficulty and Observed Difficulty, Ms. B., Day 2

	Marcos	**Lashanna**
Self-Report of Difficulty		
Had Difficulty?	No	No
Classroom Context	---	---
What stood out?	"I think it's what I felt because I felt like I was accepted and the kids saw how smart I was today."	"I felt real smart on camer and my math coach and teacher was proud of me."
Observed Difficulty		
Types of Difficulty and Frequency of Occurrence	Incorrect Answer/Idea (2)	Unable to Answer (1)

Information Ms. B. offered during the teacher interview may explain the reason Lashanna reported she did not have difficulty during this session. During the interview, Ms. B. explained that Lashanna's difficulty on Day 2 (Difficulty 17) was attitudinal rather than mathematics-related. Ms. B. said Lashanna was upset she was not permitted to work with another group and therefore was not expending much effort working on the mathematics problem. Videotape evidence following Difficulty 17 showed that Lashanna had, in fact, understood the problem, but probably did not want to discuss her solution with Ms. B.

Summary: Ms. B.

During the fourteen instances of students' difficulty analyzed for Ms. B., three types of initial teacher responses were used: *Having Other Student(s) Help, Helping Student Indirectly* and *Not Responding/Responding without Offering Suggestion for Resolving Difficulty*. Ms. B.'s follow-up responses fell into three response categories: *Having Other Student(s) Help, Helping Student Directly, and Helping Student Indirectly*. Ms. B. most frequently used the response types *Helping Students Indirectly* and *Having Other Students Help Resolve Difficulty*.

During Ms. B.'s interview, she explained that she often respond to students' mathematics difficulties by listening to them and probing to encourage the students' thinking. Ms. B. mentioned that she asks more guided questions of the low ability students when responding to their difficulties. Ms. B. noted that students' dispositions on any given day and their classroom social status impact the ways in which she responds when students encounter mathematics difficulties. Ms. B. added that in order not to compromise a students' social standing, she often waits until her students have divided into smaller groups before she addresses a student's difficulty. Ms. B.'s descriptions of the ways in which she typically responds to students' mathematics difficulties were consistent with the responses she offered to her students during the analyzed class sessions.

Following Ms. B.'s interventions in instances of students' difficulties, the students were on-task overall. Where students were on-task, they showed evidence of both high and low cognitive activity. Several trends were noted between Ms. B.'s responses to the students and the complexity of students' subsequent cognitive activity. In all instances where Ms. B.'s response included Asking the student to prove his/her solution, all the students involved in the difficulty subsequently demonstrated high levels of cognitive activity, where cognitive behaviors were observable. In all but one instance where Ms. B.'s response included either Suggesting a process to use in solving the problem or Leading the students to the correct answer/idea, all the students involved subsequently demonstrated low level cognitive activity or were not cognitively engaged. Students' affective engagement following Ms. B.'s interventions included mild, moderate and strong positive feelings as well as slight negative feelings. A preliminary trend was noted relating Ms. B.'s students' affective and cognitive engagement as observed on videotape following her interventions to their difficulties.

Comparison of students' self-reported affect and affect inferred from their observed behaviors on videotape indicated that students' self-reported feelings were at times more positive or more negative than the feelings inferred from their behaviors. In two instances, students who showed evidence of boredom and did not demonstrate strong positive feelings during the analyzed segments of the class session reported feeling not at all bored and very happy on the questionnaire completed following that day's session. In one instance, a student's questionnaire reported more negative feelings than those inferred from her observed behaviors on videotape during the analyzed segments of the class session that day. Students involved in multiple analyzed instances of difficulty during a class session often reported encountering no difficulties during that day's session.

4.6 Teachers' Responses and Students' Engagement: Ms. S.

Three consecutive videotaped sessions of one group of students in Ms. S.'s class were examined in this research. The group included three students: Ta'keisha, Leo and Ordena. During the first day's session there were 4 instances of student difficulty where Ms. S. was involved. During the second session there were 6 instances of student difficulty where Ms. S. was involved. During the third session there was 1 instance of student difficulty where Ms. S. was involved. For each of the 11 analyzed instances of difficulty for Ms. S.'s class, Table 4.12 lists the students involved and the type of difficulty.

Table 4.19: Types of Difficulties and Students Involved, Ms. S.

	Students Involved:	*Type of Difficulty:*
Day 1		
Difficulty 19	Ordena	Unable to Answer
Difficulty 20	Leo	Incorrect Answer
Difficulty 21	Leo, Ta'keisha, Ordena	Impasse
Difficulty 22	Leo, Ta'keisha, Ordena	Impasse
Day 2		
Difficulty 23	Leo, Ta'keisha, Ordena	Impasse
Difficulty 24	Leo	Incorrect Idea
Difficulty 25	Leo	Does not Understand Solution
Difficulty 26	Leo	Incorrect Answer
Difficulty 27	Leo	Incorrect Answer
Difficulty28	Leo	Incorrect Answer
Day 3		
Difficulty29	Leo	Does not Understand Solution

Types of Responses

Ms. S.'s 11 initial responses to her students' difficulties fell into three of the five response categories: *Having Other Student(s) Help, Helping Student Indirectly* and *Responding without Offering Suggestion for Resolving Difficulty.* Her four follow-up responses fell into two response categories: *Having Other Student(s) Help* and *Helping Student Indirectly*. Ms. S. most frequently used the response types *Helping Students Indirectly* (Overall total of 7) and *Having Other Students Help* (Overall total of 6). Table 4.20 depicts the particular types of responses within each category that Ms. S. used. Types of responses are listed using short descriptors; refer to table 4.2 for the detailed set of codes.

Table 4.20: Types of Responses, Ms. S.

Numbers preceding a hyphen in the frequency column represent initial responses; numbers after a hyphen refer to follow-up responses.

Type of Teacher Response: Ms. S.	**Frequency**	**Total Frequency**
Having Other Student(s) Help		
Inviting Another Student *Sharing with Peers* *Using Another's Idea*	4-2	6
Having Student Help Self		
Giving Time	0-0	0
Helping Student Directly		
Providing Correct Answer *Explaining*	0-0	0
Helping Student Indirectly		
Leading	1-1	2
Providing Counter Example	0-0	0
Probing *Asking for Proof*	1-0	1
Simpler Problem	2-1	3
Suggesting Process	0-0	0
Showing Diagram/Model	1-0	1
Subtotal: Teacher Helping Student Indirectly	***5 -2***	*7*
Not Responding/Responding without Offering Suggestion		
Saying Answer is Wrong	0-0	0
Not Responding	0-0	0
Acknowledging or Repeating Incorrect Response *Having Student Repeat* *Clarifying* *Repeating Teacher's Question*	2-0	2
Subtotal**: **Not Responding/Responding without Offering Suggestion for Resolving Difficulty	*2-0*	*2*
Total Responses	**11-4**	**15**

Following is a detailed analysis of one illustrative instance for each type of response Ms. S. used.

Helping Students Indirectly: Difficulty 23 (Day 2)

Difficulty 23 occurs when Ta'keisha tells Ms. S. that the students at the table disagree about the solution for the 10-block high tower. Ta'keisha says that she and Leo obtained a solution of 46 blocks for the 10-block high tower. She explains that they built the 10- block high tower by adding to the sides and top of a smaller tower. Ta'keisha tells Ms. S. that Ordena obtained a solution of 42 blocks by multiplying the number of blocks in a 5-block high tower, 21, by 2.

Ms. S. responds by asking Ordena why she multiplied by 2 to get the number of blocks in the 10- block high tower:

> Ms. S.: *(to Ordena)* Ok, so tell me what made you want to multiply by 2.
> Ordena: *(pointing to her paper)* Because 21 times 1 is 21, which is 5 blocks, and to get 10 blocks you have to multiply 21 times 2, and that's what I did.
>
> (Appendix A, Event 12, Difficulty 23, lines 28-30)

Ms. S.'s initial response to Ordena's difficulty, above, was coded as *Probing,* included in the category of *Helping Student Indirectly*. After Ordena's explanation of her method (in excerpt above), Ms. S. suggests that Ordena explore her pattern with smaller numbers such as towers of heights 4 and 2, or towers of heights 2 and 1 to determine whether the pattern holds true. This follow-up response was coded as *Offering a Simpler Problem,* also included in the category of *Helping Student Indirectly*. The students shift the conversation back to the 5- and 10- block high towers they built. Ms. S. suggests that the students describe both solutions to the class during their presentation since they are running low on time and haven't worked out their differing solutions yet.

Having Other Students Help: Difficulty 29 (Day 3)

Difficulty 29 occurs during Leo, Ta'keisha, and Ordena's presentation to the class on Day 3 of the students' work on the Towers Problem. Leo explains to the class that he disagreed with Ta'keisha and Ordena's solution of 496 blocks for the 100-block high tower. The following excerpt during the students' presentation pinpoints Leo's difficulty.

> Ta'keisha: For 10 blocks we had 46 cubes.
> (…)
> Ta'keisha: And for 100 we had four hundred and ninety six cubes.
> Leo: So what we did was, we tried to attempt it but we couldn't cuz we didn't have enough room so we just like guessed out like see how many fits on this side cuz she said 99 fit…99 cubes on each…4 sides and 100 is on…on the top.
> Ms. S.: Who said that?
> Ta'keisha: *(Leo points to Ta'keisha. Ta'keisha raises her hand.)* Me
> Ms. S.: Oh, ok. So you believed her?
> Leo: So I disagreed with her because why is it 99 on each side, not 100? And how come there's 100 on each side…uh…on the top.
>
> (Appendix A, Event 14, Difficulty 29, lines 10, 12-21)

Ms. S. intervenes by suggesting to Ta'keisha and Ordena, "…let's see if either Leo can convince you…of his perspective or you can convince Leo or the class, let's see where you guys (the other students in the class) fall on that perspective" (Appendix A, Event 14, Difficulty 29, lines 31-33). This response was coded as *Suggesting student share difficulty with peers,* included in the category of *Having Other Students Help Resolve the Difficulty.*

Ms. S. then facilitates a discussion where Leo and then Ta'keisha provide justification for their solutions to the class. During her explanation, Ta'keisha demonstrates her reasoning for her solution with a model of a 5-block high tower. She then tells the class that the amount of blocks that you can see is the amount of blocks that should be on each side. When Ms. S. asks the students if they agree on that, Leo and Ta'keisha nod. The students in the class then argue about the name for a tower with 4 visible blocks (and one hidden block) in the height. Some students maintain it is a 5-block high tower while others believe it should be called a 4-block high tower. Ms. S. suggests that the class continue with their presentations even though the disagreement has not been resolved. Leo, Ta'keisha, and Ordena return to their seats.

Responding without Offering Suggestion: Difficulty 22 (Day 1)

Difficulty 22 occurs on the first day of the students' work on the Towers Problem. The students in Group 3 disagree about how a 5- block high tower should look. Leo's version of a 5-block high tower is a stage B tower which is missing the middle block (and consists of 5 blocks in total). Ta'keisha has built a tower which is 5 blocks in height but has only 2 branches, resembling an inverted 'T.' The students call Ms. S. to their table to discuss their difficulty.

After Ta'keisha and Leo tell Ms. S. about their differing models of a 5-block high tower, Ms. S. repeats what the students have said and asks if she understood them correctly:

> Ms. S.: So you're…so, I just want to make sure I understand what you're saying. You're *(to Leo)* saying this *(pointing to Leo's stage B tower)* is the five block high *(Leo nods)* but you're *(to Ta'keisha)* saying this *(pointing to Ta'keisha's tower)* is the five block high. *(Ta'keisha nods.)*

(Appendix A, Event 11, Difficulty 22, lines 21-24)

Ms. S.'s initial response to Leo and Ta'keisha's difficulty, above, was coded as *Asking a question to clarify what the student(s) said,* included in the category of *Responding without Offering a Suggestion for Resolving the Difficulty.*

Ms. S. then asks Ordena if she agrees with either Leo or Ta'keisha. When Ordena says she agrees with Leo, who is referring to the stage B tower without the middle block as a 5- block high tower, Ms. S. asks the students questions and reads the directions of the task with them until they understand that the towers need to follow a particular pattern and that each tower builds on the previous one.

Ms. S.'s Reasoning for her Responses

During the retrospective interview, Ms. S. was asked several questions regarding the ways in which she responds when students have difficulty while working on mathematics problems. The section below includes each of those interview questions and a summary of Ms. S.'s responses, along with supporting quotations from the interview transcript (included in Appendix D).

Q1: Can you describe how you usually respond when students have difficulty while they work on mathematics problems in class?

Ms. S. replied that she typically asks her students to discuss their difficulties with their peers. She said she groups her students homogenously in order to encourage all students to think and allow for discussions on various levels. Ms. B. explained her reasoning for homogenous student grouping, "I try not to intimidate my students who struggle with math by putting them next to the student who actually gets it really quickly, because otherwise they tend to not try to think it through for themselves and just default to whatever they think the higher kid is thinking" (Appendix D, Ms. S. Interview Transcript, lines 17-20).

Ms. S. stated that while her students are discussing their difficulties, she listens to their dialogue and asks guiding questions to facilitate their resolution of their difficulties. Ms. S. reported that when stronger students have difficulty, she often provides a resource for them such as a particular spot in their class notes or a mathematics dictionary and returns later to check on their progress. With weaker students, she tends to ask more directed questions and often finds it necessary to provide assistance in clarifying or simplifying the mathematics problem.

Q2: Are there things you try not to do when you respond to student difficulty? (If yes) Can you describe?

Ms. S. stated that when her students encounter difficulties, she tries not to indicate whether or not their answers are correct, in order to encourage her students' development of self-check strategies. Ms. S. explained her approach:

> I try and I… this only after doing the whole Dr Schorr, Dr. Warner thing, I try very hard not to straight out say yes or no, so either validate or negate their response or give them the answer or say 'look this is how you do it.' And my thinking behind that is that… you see it all the time particularly since I deal with sixth grade now. When they come in and they're kind of used to always having their answers either validated or negated, when they get to a test and you tell them check their work, they actually haven't had any practice with kind of assessing their own work so when you tell them to check their work it's almost like just telling them to do nothing. They have no idea how to check their work… all they know how to do then at that point for a lot of them is kind of go to the teacher and wait for you to say yes or no. So one of the skills that I hope to develop in them by not saying either yes this is correct or no this isn't correct just getting them to prove to me like 'yeah Mrs. S., this is right'. Or actually, 'I don't know why it's wrong but I know something is wrong here' so that when they're taking their regular tests…they're able to…self check for themselves so they can catch their own mistakes.
>
> (Appendix D, Ms. S. Interview Transcript, lines 42-56)

Q3: Do you respond in different ways in different situations?

"Definitely."

Q4: Can you describe some factors that might determine how you respond?

Ms. S. stated that her students' mathematics ability influences the way she responds to students' difficulties. Ms. S. explained that weaker math students tend to become frustrated and shut down more quickly than stronger students. She said she uses a more directed approach with the weaker students to ease their confusion. Ms. S. reasoned:

> …because…with them [the weaker students] when I'm more vague it's more frustration and confusion for them. So with them they actually need a little bit more validation, so if they're saying that like 'oh I think this is important because of this,' I'll be like okay…let's focus on these two items. And I'm going to tell you like these are actually the more important items and here's a strategy for figuring out like why they're important.
>
> (Appendix D, Ms. S. Interview Transcript, lines 97-102)

Ms. S. mentioned that at times weaker students become apathetic when they reach frustration, and they therefore stop working until she comes over to assist them. She said she encourages these students to keep working until they receive teacher aid.

When asked whether any other factors influence the way she responds to students' mathematics difficulties, Ms. S. replied that an approaching unit assessment may prompt her to use a more directed approach. Ms. S. explained that since unit assessments affect students' grades and therefore their ability to be accepted at a good high school, "I would be doing them a disservice by not clarifying…kind of all ambiguities out there" (Appendix D, Ms. S. Interview Transcript, lines 125-126).

Ms. S. added that the placement of the lesson within the unit would influence the way she responds to students' difficulties during a lesson. Ms. S. said she would allow for more exploration at the beginning of a unit. Near the end of a unit she said she would use a more directed approach with her students to help them determine which mathematics strategies do or do not work and help her students understand why particular strategies are preferred.

Ms. S. remarked that the number of students in the classroom also determines how she responds to students' difficulties. When there are many students, and therefore many groups, in the classroom, Ms. S. said she tends to use more leading questions since there is less teacher time for each group.

Comparison of Ms. S.'s Self-Description of Response Types to Observed Responses

Ms. S.'s descriptions of the ways in which she typically responds to students' difficulties during mathematics problem solving were consistent with the responses she offered to her students during the analyzed videotaped class sessions. Ms. S. reported that when her students encounter difficulty, she often suggests they discuss the difficulties with their peers. Of the 15 responses Ms. S. offered to her students who encountered difficulties during the analyzed class sessions, 6 were coded as *Having Other Students Help Resolve the Difficulty*.

Ms. S. stated that while her students discuss their difficulties with each other, she listens and asks guiding questions. Ms. S. added that she usually asks her weaker students more directed questions and often finds it necessary to clarify or simplify the problem for them. Ms. S. considered the class analyzed in this research a “low track” mathematics class. Seven of Ms. S.’s 15 observed responses to the students in the class were coded as *Helping the Student(s) Indirectly*. These responses were included in the more specific categories of *Leading the student(s) to the correct answer/idea*, *Probing the student(s)*, *Providing a simpler problem*, and *Showing the Student a Diagram/Model*. None of the responses Ms. S. offered to her students was coded as *Helping the Student Directly* or *Having the Student Help Him/Herself*. These responses were consistent with the way Ms. S. reported she responds when her students, especially those who are weak in mathematics, face difficulties during their problem solving.

Ms. S.’s Reasoning for Particular Responses and Comparison with Actual Outcomes

During the stimulated recall interview, Ms. S. was asked to explain her reasoning for the responses she provided to her students after Difficulties 22, 23, and 29. Each of Ms. S.'s explanations for her responses was assigned two codes for *Teachers' Reasoning for their Particular Responses*. All of Ms. S.’s reports of her reasoning for her responses were assigned the code *Mathematics Goals- Problem Solving Skills*. One additional code of either *Abilities/Skills- Students’ Mathematics Ability*; *Time Constraints*; or *Students’ Emotions in Regard to Mathematics- Saving Face in Front of Classmates* was assigned to each of Ms. S.’s reports of her reasoning for the responses she offered to her students when they encountered difficulty.

Below are detailed descriptions of Ms. S.'s self-reported reasoning for her responses to Difficulties 22, 23, and 29. Following each description is a comparison of Ms. S.'s reasoning for providing the response to what actually occurred following the response, focusing on whether Ms. S.'s intended goals were realized.

Reasoning for Response to Difficulty 22 (Day 1)

Difficulty 22 (also described above, in *Illustrative Examples of Teachers' Responses*) occurs on the first day of the students' work on the Towers Problem. The students in Group 3 disagree about how a 5- block high tower should look. Leo's version of a 5- block high tower is a stage B tower which is missing the middle block (and consists of 5 blocks in total). Ta'keisha has built a tower which is 5 blocks in height but has only 2 branches, resembling an inverted 'T.' The students call Ms. S. to their table to discuss their difficulty.

After Ta'keisha and Leo tell Ms. B. about their differing models of a 5-block high tower, Ms. S. repeats what the students have said and asks if she understood them correctly. Ms. S.'s initial response to Leo and Ta'keisha's difficulty was coded as *Asking a question to clarify what the student(s) said.*

Ms. S. then asks Ordena if she agrees with either Leo or Ta'keisha. This follow-up response was coded as *Inviting another student to respond.* When Ordena says she agrees with Leo, who is referring to the stage B tower without the middle block as a 5- block high tower, Ms. S. asks the students questions and reads the directions of the task with them to help them understand that the towers need to follow a particular pattern and that each tower builds on the previous one (see transcript, Appendix A, Event 11, Difficulty 22). This second follow-up response was coded as *Leading the students to the correct answer/idea*.

During the stimulated response interview, Ms. S. explained that she responded in this way since the students were weak in mathematics and tended to work alone rather than as a group. When they faced impasse, they often struggled individually without using strategies to overcome their difficulties. Ms. S. described the students:

> …they're a group who I would have classified as a struggling group and…because they tend to struggle a little bit more they're less likely to talk to each other. So instead there would be kind of…like silently working in frustration…they had manipulatives out, I think they would be more likely to…just move into a phase of…like moving around the manipulatives to look like they were doing something rather than actively…working to solve the problem.
>
> (Appendix D, Ms. S. Interview Transcript, lines 211-218)

The response offered here, Ms. S. reported, was intended to encourage the students to work together and to guide the students through the problem in a step by step manner. While guiding the students through the problem, Ms. S. said she modeled for the students the deductive thinking that would bring them to the next step at each point. When asked whether her response was influenced by the students' mathematics ability, Ms. S. said it was. Ms. S. explained that Leo is a student who can easily veer off-track in his problem solving without continuous guidance, so she worked with the group, leading them through the difficulty and continuously questioning Leo to redirect him to the correct process. Ms. S. described that her approach was necessary considering Leo's performance in mathematics:

> …Leo…he's one who can easily get kind of like way up in left field. So… a lot of times he can talk himself out of the answer or out of a reasonable solution and his kind of parameters on what's relevant…the filter isn't quite there. So…he has the proper pattern going and if we didn't stop it chunk by chunk and say okay we agree on this, okay we're going to continue down this path…at one point he has the right pattern and then I asked him about the five block high tower and he built up the tower in the middle but didn't add on to the end, and I had to ask him, okay are you continuing the pattern? And he looked

> like, oh okay and then he goes back. But left kind of like unharnessed he would have continued, I think, to just build up the middle. Because he can kind of work himself out of the problem a lot of times that way.
>
> (Appendix D, Ms. S. Interview Transcript, lines 244-254)

Ms. S.'s reasoning for her response to Difficulty 22 was assigned the codes *Mathematics Goals- Problem Solving Skills* and *Students' Abilities/Skills- Mathematics Ability*.

After Ms. S. left the students' table, as she intended, the students worked together at finding solutions to the 5- and 10- block high towers (the students find the correct solution to the 5-block high tower and two different solutions for the 10-block high tower, one correct and one incorrect. At the end of the day's session they have not concluded which solution to the 10-block high tower is correct). The conversation at the group, however, is mainly between Ta'keisha and Ordena. Leo occasionally contributes to the discussion but does so with hesitation and often looks to the other students for confirmation of what he says.

Reasoning for Response to Difficulty 23 (Day 2)

Difficulty 23 occurs when Ta'keisha tells Ms. S. that the students at the table disagree about the solution for the 10-block high tower. Ta'keisha says that she and Leo obtained a solution of 46 blocks for the 10-block high tower. She explains that they built the 10- block high tower by adding to the sides and top of a smaller tower. Ta'keisha tells Ms. S. that Ordena obtained a solution of 42 blocks by multiplying the number of blocks in a 5-block high tower, 21, by 2.

Ms. S. responds by asking Ordena why she multiplied by 2 to get the number of blocks in the 10- block high tower. Ms. S.'s initial response to Ordena's difficulty was coded as *Probing*. After Ordena's explanation of her method, Ms. S. suggests that Ordena explore her pattern with smaller numbers such as towers of heights 4 and 2, or towers of heights 2 and 1 to determine whether the pattern holds true. This follow-up response was coded as *Offering a Simpler Problem*. The students then shift the conversation back to the 5- and 10- block high towers they built. Ms. S. suggests that the students describe both solutions to the class during their presentation since they are running low on time and haven't worked out their differing solutions yet.

During the interview, Ms. S. explained that she tried to give the students a method they could use to determine on their own which solution is correct. She said she did not work with the students until they reached an agreement of which solution is correct because the class was short on time and because the students would present their solutions to the class and would likely reach a conclusion through their discussion with the other students in the class. When asked if her response was influenced by the students' mathematics ability, Ms. S. responded in the negative. Ms. S. stated that she would have responded in the same way had the difficulty involved any of her students.

Ms. S.'s reasoning for her response to Difficulty 23 was assigned codes of *Mathematics Goals-Problem Solving Skills* and *Time Constraints*.

After Ms. S. leaves the students' table Ta'keisha writes on her paper, then continues to work on her own. She then says she found the solution to the 100-block high tower and that it has 496 cubes. Leo disagrees with her solution and says there are 420 blocks in the 100-block high tower. Leo asks Ta'keisha how many blocks were in the 10-block high tower. Ta'keisha says there are 46, the solution she had obtained earlier. Ordena says there are 43 blocks. There is no evidence of the students implementing the strategy Ms. S. recommended of using towers of shorter heights to try to determine if Ordena's doubling method is valid. The students appear to have retained their original solutions after Ms. S. left the table.

Reasoning for Response to Difficulty 29 (Day 3)

Difficulty 29 occurs during Leo, Ta'keisha, and Ordena's presentation to the class on Day 3 of the students' work on the Towers Problem. Leo explains to the class that he disagreed with Ta'keisha and Ordena's solution of 496 blocks for the 100-block high tower. Ms. S. intervenes by suggesting to Ta'keisha and Ordena that they try to convince Leo of their solution or see whether Leo can convince them of his solution. Ms. S. also suggests that the class decide with which solution they agree. Ms. S.'s response was coded as *Suggesting student share difficulty with peers.*

Ms. S. then facilitates a discussion where Leo and then Ta'keisha provide justification for their solutions to the class. During her explanation, Ta'keisha demonstrates her reasoning for her solution with a model of a 5-block high tower. She then tells the class that the amount of blocks that you can see is the amount of blocks that should be on each side. When Ms. S. asks the students if they agree on that, Leo and Ta'keisha nod. The students in the class then argue about the name for a tower with 4 visible blocks (and one hidden block) in the height. Some students maintain it is a 5-block high tower while others believe it should be called a 4-block high tower. Ms. S. suggests that the class continue with their presentations even though the disagreement has not been resolved. Leo, Ta'keisha, and Ordena return to their seats.

During the stimulated recall interview, Ms. S. provided insight regarding Leo's behaviors during this event. Ms. S. explained that she believed Leo was beginning to see that Ta'keisha's solution was correct and his was not. He persisted in defending his solution, however, she said, to save face in front of the other students. Ms. S. explained:

> I think that Leo…and the other students were…starting to see that their idea wasn't going to hold up and that the validity was really in Ta'keisha and Tyshonna and the girls' idea…However, it became this emotional thing of like, I worked on it, it's mine, I'm going to support it even if it doesn't hold up.
>
> (Appendix D, Ms. S. Interview Transcript, lines 293-298)

Ms. S. said she responded to Leo's difficulty by having the students talk with each other instead of directly telling them the correct answer since that response would be more effective for the students both emotionally and cognitively. On an emotional level, Ms. S. wanted the students to recognize that "It's okay to change an idea, like you're not abandoning your idea or your process completely; you're…just editing it" (Appendix D, Ms. S. Interview Transcript, lines 300-302).

Ms. S. cautioned that the method of having students discuss their ideas with each other works better for the students emotionally when they are grouped homogenously. When students share their ideas in a homogenous group, no student is made to feel less capable or less smart than another. In the present situation, Ms. S. said she had the students talk with each other regarding the difficulty even though they were in a whole class setting since the students were working on a non-routine problem with which none of the students felt comfortable.

Ms. S. said that on a cognitive level, having the students talk with each other without much teacher involvement is effective since that way students can explore sincerely as the teacher does not dictate whether their ideas are valid or not. This gives the students experience with articulating their thoughts, listening to others, integrating others' ideas with their own, and questioning other students. Upon reflection during the interview, Ms. S. said she would have liked to have the students talk even more and have herself talk less during the conversation following Leo's disagreement with Ta'keisha's correct solution. Ms. S. said she finds it difficult to initiate student to student dialogue since her students are resistant to speak with each other regarding mathematics. She said she is still working on trying to increase her students' peer to peer conversations regarding mathematics during classroom problem solving.

When asked whether her response was influenced by the students' mathematics ability, Ms. S. replied that it was not. Ms. S.'s reasoning for her response was assigned codes of *Mathematics Goals- Problem Solving Skills* and *Students' Emotions in Regard to Mathematics-Saving Face in Front of Classmates.*

During the beginning of the next group's presentation, Leo says to Ta'keisha, "See, I was right. See, ha. See, ha" (Appendix A, Event 14, Difficulty 29, *Description of Behavior Related to Student's Engagement Following Teacher's Response*). Leo's response appeared to be an expression of trying to save face at a point where he felt defeated, as he recognized Ta'keisha's solution was correct and his was not. On the questionnaire Leo completed that day, Leo reported that he did not have any type of difficulty with the math task. However, when asked on the questionnaire what stands out about the situation (of the difficulty), Leo wrote, "Nothing stands out of my memory

I felt like a mathemetician and smart I thought I was going to be imbarrist."

On another page of the questionnaire (included in Appendix C), where students were asked to describe any thoughts they had that were not on the list of thoughts included in the questionnaire items, Leo wrote, "I felt like I was being bossed around by someone in my group."

Ms. S. intended her response to help Leo save face before his classmates. There was evidence, however, that Leo felt embarrassment when he and his classmates recognized that his solution was not correct. It is possible that the situation in which Leo felt loss of face occurred because Leo was in a group with Ta'keisha, who appeared to be a stronger mathematics student. Ta'keisha found correct solutions to the 5-, 10-, and 100- block high towers, and there were instances during the group's work on the task in which Ta'keisha teased Leo about his confusion regarding the mathematics problem. During her interview, Ms. S. mentioned that she tries to keep the student groups in her class homogenous in order to allow for each student to think on his/her level. The student groups in the class sessions analyzed here were formed at random by the research team; they were not selected based on students' mathematics ability. Leo's feelings during this difficulty may have been different were he placed in a group with other students more similar to him in mathematics ability.

Student Engagement

The table below lists the codes assigned to Ms. S.'s students' behavior following Ms. S.'s responses to their analyzed difficulties. For each student involved in a difficulty, codes are listed for three domains of the students' engagement: behavioral, cognitive, and affective.

Table 4.21: Teacher Responses and Subsequent Student Engagement Codes, Ms. S.

	Behavioral Engagement	Cognitive Engagement	Affective Engagement
Day 1			
Difficulty 19 Inviting other students to respond			
Ordena	On-task	Low level cognitive activity	Moderate positive feelings
Difficulty 20 Acknowledging the incorrect response			
Leo	On-task	Insufficient Information	Moderate positive feelings
Difficulty 21 Leading the students to the correct answer			
Leo	On-task	Insufficient Information	Strong positive feelings
Ta'keisha	On-task	Insufficient Information	Strong positive feelings
Ordena	On-task	Insufficient Information	Mild positive feelings
Difficulty 22 Asking a question to clarify what the students said Inviting another student to respond Leading the students to the correct idea			
Leo	On-task	High level cognitive activity	Mild positive feelings
Ta'keisha	On-task	High level cognitive activity	Strong positive feelings
Ordena	On-task	High level cognitive activity	Moderate positive feelings
Day 2			
Difficulty 23 Probing- asking the student to explain the reasoning she used Offering a simpler problem as an example Suggesting student share her difficulty with peers			
Leo	On-task	Low level cognitive activity	Moderate positive feelings
Ta'keisha	On-task	High level cognitive activity	Strong positive feelings
Ordena	On-task	Insufficient Information	Mild positive feelings
Difficulty 24 Inviting other students to respond			
Leo	On-task	High level cognitive activity	Moderate negative feelings
Difficulty 25 Showing the student a model			
Leo	On-task	Low level cognitive activity	Moderate negative feelings
Difficulty 26 Offering a simpler problem as an example			
Leo	On-task	Low level cognitive activity	Insufficient information
Difficulty 27 Offering a simpler problem as an example			
Leo	On-task	Insufficient information	Mild positive feelings
Difficulty 28 Suggesting student share his difficulty with peers			
Leo	On-task	Low level cognitive activity	Moderate negative feelings

Day 3			
Difficulty 29 Suggesting student share his difficulty with peers			
Leo	On-task	Insufficient information	Strong negative feelings

Following is a more in-depth analysis of the students' behavioral, cognitive, and affective engagement.

As noted in the table above, during analyzed video segments following Ms. S.'s responses to the students' difficulties, the students' behavioral engagement was always coded as on-task. The students' cognitive engagement included both high and low level cognitive activity. The students' affective engagement varied between strong positive and strong negative feelings. Some preliminary patterns were noted between students' affective and cognitive engagement. During the instances where students showed evidence of strong positive feelings, where information was available regarding their cognition, they showed evidence of high level cognitive activity. During two of the three instances where a student showed evidence of negative feelings and information was available regarding his cognition, he showed evidence of low level cognitive activity. No patterns were noted between the types of responses Ms. S. offered to her students and the students' subsequent cognitive activity.

Following are illustrative instances in which students showed evidence of various types of engagement.

Following Difficulty 19 (Day 1):

On-task Behavioral Engagement, Low Level Cognitive Activity, Moderate Positive Feelings

Difficulty 19, which involves Ordena, occurs during Ms. S.'s introduction of the task to the class on the first day of their work on the Towers Problem. Ms. S. asks the students if the stage B diagram is the 5- block tower. Some students say “yes” and some say “no.” She asks a student to reread the last part of the description of the task aloud.

The student (off camera) reads, “I will need to build a 5-block high tower…”

Ms. S. asks the class, “Is letter B a 5- block high *(with emphasis on ‘high’)* tower?” (Appendix A, Event 9, Difficulty 19, line 10). Several students (off camera) answer that it is not. Ms. S. asks why it is not a 5-block high tower. Ordena raises her hand, and Ms. S. calls on her. Ordena smiles and says, “uh…uh, never mind” (Appendix A, Event 9, Difficulty 19, line 14). (Difficulty 19 is Ordena’s apparent inability to answer Ms. S.’s question.) Ms. S. calls on another student who says that perhaps there need to be 5 blocks in the height. Ordena later adds to the discussion by saying she thinks the students are trying to find patterns by building with the blocks. Ms. S. then continues to explain the problem to the class.

When the students in the class begin working, Leo reads the problem aloud with Ta’keisha. Ordena picks up her paper and reads a phrase from it regarding adding blocks to the sides to build the next stage of the tower. Leo explains to Ordena that to build the next stage of the towers, you must add one block to each side. Leo brings blocks to the table and Ordena takes some. She arranges four blocks in a square shape and puts another block on top. Ordena tells Ta’keisha that the stage of the tower she built (stage 2) has five blocks. Ta’keisha tells Ordena that in the picture the blocks are “spaced out” and not “pushed together.” Ordena spreads out the four blocks in the base of the tower she had built but does not place the top block back on the tower.

Following Ms. S.’s response to this difficulty, Ordena showed evidence of on-task behavior, low level cognitive activity, and moderate positive feelings.

Following Difficulty 28 (Day 2):

On-task Behavioral Engagement, Low Level Cognitive Activity, Moderate Negative Feelings

Difficulty 28 occurs during Ms. S.'s conversation with the students in Leo's group after Leo says he disagrees with Ta'keisha's solution for the 100-block high tower. Ta'keisha says there are 99 blocks in each side of the 100-block high tower, but Leo says he does not understand why there are 99 blocks, and not 100 blocks, in each branch. Ms. S. notices that Leo does not recognize there is a hidden block (the first, bottom block in the height of towers of stage 2 and further) in the tower diagrams on the task sheet and in the towers the students built. By asking Leo probing questions, Ms. S. helps Leo to understand that the height in each tower is one more than the number of blocks that can be seen in the height.

Ms. S. then asks Leo, "So then what happens to these *(pointing to the sides of Ta'keisha's 5-block high tower)*? Do these…do your sides end up having as much as the height? (Appendix A, Event 13, Difficulty 28, lines 122-123). Ta'keisha says no. Leo says yes. (This is Difficulty 28.)

Ms. S. tells the students that when they present, Ta'keisha should present her argument and Leo should present the reasons he is not convinced. Ms. S. tells the students to put all their solutions and explanations on paper. She leaves the students' table.

After Ms. S. leaves the table, Leo continues building with the blocks while Ta'keisha prepares for the presentation by writing on a large colored paper. Leo says he's still not convinced about Ta'keisha's solution. Ta'keisha tells Leo that he will need to build the 100-block high tower if he is not convinced of her solution. She says he will make a fool of himself. Leo continues building. He asks Ta'keisha whether the 10-block high tower has 46 cubes. Ta'keisha asks Leo why he agrees with her on that if he said he's not convinced of her method.

Leo responds, “Be happy I agree with you at all.” Ordena says she’s not convinced. Ta’keisha says to her, “How can you agree with a dummy like him (Leo)?” Ta’keisha then says about Leo, “He don’t know what he’s doing.” Leo answers, “I know what I’m doing. Don’t try to make a fool of me.” Ta’keisha asks Leo, “How you get 46 if you don’t agree with me?” Leo says, “Because I did it myself.” Ta’keisha tells Leo to build a 5- block high tower and a 10- block high tower. Leo builds a tower that is 10 blocks in height and has 10 blocks in each of its branches. He says it is a 10- block high tower. He then disassembles the tower. He leans forward and watches Ta’keisha as she writes on the colored paper and on a transparency sheet. Ordena sits with her head resting on her hand. Leo plays with the parts of the 10-block high tower he had built while Ta’keisha continues preparing for the presentation. At approximately 10:33 Ms. S. announces that the students need to stop working.

Following Ms. S.’s response to this difficulty, Leo showed evidence of on-task behavior, low level cognitive activity, and moderate negative feelings.

Following Difficulty 22 (Day 1):

On-task Behavioral Engagement, High Level Cognitive Activity, Mild/Moderate/Strong Positive Feelings

Difficulty 22 involves Ta’keisha, Leo, and Ordena’s disagreement regarding the way a 5-block high tower should be constructed. Following Ms. S.’s response to their difficulty, all the students displayed on-task behavior, high level cognitive activity, and positive feelings. Leo’s positive feelings were coded as mild, Ordena’s were assigned a code of moderate, and Ta’keisha’s were categorized as strong. A more detailed description of the students’ difficulty, Ms. S.'s response to the difficulty, and the students' behavior following Ms. S.'s response is included above, in *Ms. S.'s Reasoning for Response to Difficulty 22*.

Students' Collaborative Effort and Affect

On Day 1 of the students' work on the Towers Problem, all the students in Group 3 showed evidence of positive feelings. No codes of negative feelings were assigned to the students' affect for that day. On Days 2 and 3, however, Leo's affect was coded as involving negative feelings after 4 of the 7 difficulties in which he was involved. On Day 1 of the students' work, the students appeared to work more cohesively and with more respect toward each other than on Days 2 and 3. The students' collaborative effort appeared to be associated with the type of feelings evidenced by members of the group, specifically, by Leo, who appeared to be weakest in mathematics ability.

On Day 1, when students in the group ask questions about the mathematics task, Ta'keisha, who appears to be the strongest in mathematics ability of the group members, explains her reasoning to the other group members, at times using models to explain her ideas. On Day 2, there is also evidence of Ta'keisha explaining her reasoning to Leo and Ordena; however, Ta'keisha also teases and belittles Leo, mocking him regarding his difficulty in understanding the mathematics problem. Following is a description of specific incidents which point to a lack of respect and collaborative effort between Ta'keisha and Leo.

Lack of Collaboration: Incident 1

On Day 2 of the students' work on the Towers Problem, as the students in Group 3 begin their work on the problem, Leo asks Ta'keisha what the group was doing on the previous day. Ta'keisha tells Leo to "Get the blocks, dummy." He asks his question again and she repeats, "Get the blocks." Leo gets up and returns with a bag of red cubes which he puts on the table. Ta'keisha looks up and says, "No, get the other ones." Leo says, "There ain't no other ones." Ta'keisha takes blocks from the bag and begins to build a tower.

Ta'keisha and Leo arrange their blocks into 5 stacks and add blocks to their stacks. At one point, when Ta'keisha has used all the blocks on her desk, Leo drops a block that he had tried to place on his tower. It falls on the desk between Ta'keisha and himself. Leo reaches to pick it up from the desk, but Ta'keisha grabs it and uses it for her tower. **(See picture at 9.52.11).**

Ta'keisha counts the blocks in one of her stacks and says, "Alright, it's 46." Leo counts the blocks in his 5 stacks and says he got 44. Ta'keisha tells Leo that he can't count.

(A complete description of this episode is included below, in *Qualitative In-Depth Analysis of Students' Cognitive Engagement, Description of Behavior Related to Students' Cognitive Engagement Before Difficulty 23)*

Lack of Collaboration: Incident 2

Difficulty 28 involves Leo's difficulty in understanding Ta'keisha's correct solution to the 100-block high tower. After Ms. S. leaves Group 3's table following her response to the difficulty, Leo says he's still not convinced about Ta'keisha's solution. Ta'keisha tells Leo that he will need to build the 100-block high tower if he is not convinced of her solution. She says he will make a fool of himself. Leo continues building. He asks Ta'keisha whether the 10-block high tower has 46 cubes. Ta'keisha asks Leo why he agrees with her on that if he said he's not convinced of her method. Leo responds, "Be happy I agree with you at all." Ordena says she's not convinced. Ta'keisha says to her, "How can you agree with a dummy like him?" Ta'keisha then says, “He don’t know what he’s doing.” Leo answers, “I know what I’m doing. Don’t try to make a fool of me.”

On the first page of the questionnaire Leo completed on Day 2, Leo indicated that a member of his group kept calling him a ‘dummy.’ When asked on the questionnaire whether he encountered any difficulties during the session, Leo reported that he did not. When asked what stands out about the situation, however, Leo responded, “I felt like I was smart and I could get the problem without the person bothering me.”

Lack of Collaboration: Incident 3

When the class session on Day 3 begins, Ta’keisha, Ordena, and Leo continue to work on the Towers Task. They build with the blocks on their table. Ta’keisha builds a 5-block high tower. She says, “I’m doing ten.” Leo says, “I already did ten.” Ta’keisha takes 2 of the stacks Leo built and counts the blocks in each. She asks him why he built 4 tens. She says there are supposed to be 9 blocks in each branch. Ta’keisha takes apart the stacks Leo built and builds 9-block high stacks of yellow blocks. Ta’keisha tells Leo to “stop mixing the colors.” She then directs Leo to give her more yellow blocks. Leo gives Ta’keisha more yellow blocks from the bag. Leo then takes apart the stacks he had built of blue and yellow blocks and gives Ta’keisha the yellow blocks. Ta’keisha completes the 10-block high tower.

The students then decide to build a 100-block high tower. Ordena begins building 10-block high stacks of blocks. Ta’keisha tells Leo to help Ordena. Ta’keisha says she’s tired and that she already built towers. Leo and Ordena build stacks of blocks. Ta’keisha takes 10 stacks of 10 blocks that Ordena built. She says there are 100 blocks. Leo and Ordena then build stacks and give them to Ta’keisha. Ta’keisha counts each stack and assembles a pile of 9-block high stacks. She counts a stack that Leo gave her and tells Leo that he cannot count.

When Group 3 presents to the class, Leo appears to recognize that Ta'keisha's solution is correct. When the students in Group 3 take their seats after their presentation to the class, Leo says to Ta'keisha (apparently, in an effort to save face), "See, I was right. See, ha. See, ha."

On the questionnaire Leo completed after Session 3, in the section where students were asked to describe any thoughts they had that were not on the list of thoughts, Leo wrote, "I felt like I was being bossed around by some one in my group."

Qualitative In-Depth Analysis of Students' Cognitive Engagement

In this section, three difficulties of Ms. S.'s students will be analyzed qualitatively, examining the students' cognitive engagement before and after Ms. S.'s response to the difficulty. Comparisons between students' cognitive engagement before and after Ms. S.'s response to their difficulties will be made, and possible associations between types of teacher responses and students' subsequent cognitive engagement will be discussed. Difficulties 22, 23, and 29, analyzed here, are those also used in the researcher's interview with Ms. S., where Ms. S. reflected on her reasoning for providing those particular responses. The subheading for each instance of difficulty analyzed below indicates the number of the difficulty, the student(s) involved, and the day it occurred. The analysis for each difficulty includes: 1) a qualitative description of the student's behavior related to his/her cognitive engagement before the difficulty and the resulting classification of the student's cognitive engagement, 2) a description of the difficulty, 3) a description of the teacher's response, 4) a qualitative description of the student's behavior related to his/her cognitive engagement after the difficulty and the resulting classification of the student's cognitive engagement, and 5) a comparison of the students' cognitive engagement before and after the difficulty.

Difficulty #22: Leo, Ta'keisha, Ordena, Day 1

Description of Behavior Related to Students' Cognitive Engagement Before Difficulty

Time of day: *(11:40:06-11:53:38)*

After Ms. S. leaves the students' table following her response to their previous difficulty (#21), the students do not talk much to each other. Ta'keisha writes on a paper while Ordena and Leo build towers. Their minimal conversation is task-related.

Behaviors Pertaining to Students' Cognitive Engagement Before Difficulty and Resulting Classification

Leo: Insufficient information
Ta'keisha: Insufficient information
Ordena: Insufficient information

Highest level of cognitive activity involved: Not Applicable

Description of Difficulty

The students disagree about how a 5- block high tower should be constructed. Leo's version of a 5- block high tower is a stage B tower which is missing the middle block. Ta'keisha has built a tower which is 5 blocks in height but has only 2 branches, resembling an inverted 'T.' The students call Ms. S. to their table to discuss their difficulty [lines 1-20 of transcript].

Teacher's Response

Ms. S. repeats what the students have said and asks them if she understood them correctly. She asks Ordena if she agrees with either Leo or Ta'keisha. When Ordena says she agrees with Leo, who is referring to the stage B tower without the middle block as a 5- block tower, Ms. S. asks the students questions and reads the directions of the task with them to help them understand that the towers need to follow a particular pattern and that each tower builds on the previous one [lines 21- 102 of transcript].

Code: Asking a question to clarify what the students said
Inviting another student to respond
Leading the students to the correct idea

Description of Behavior Related to Students' Cognitive Engagement After Difficulty

Time of day: *(11:58:01-12:19:00)*

After Ms. S. leaves the table, the students begin building towers. Ordena says, "Oh, this is easy." Leo says, "easy?" All the students correctly build a 5- block high tower. They count the blocks in their towers and disagree about how many blocks there are. Ta'keisha says there are 21 blocks. Leo and Ordena say there are 25 blocks in total since there are 5 blocks in each side. Ta'keisha tells them that they counted the middle block 4 times. She separates each branch of Ordena's tower from the height **(see picture at 12:01:43)** and shows her that the middle block should only be counted once, with the height, but not as part of each branch.

The students record on their papers that the 5- block high tower has 21 blocks **(see picture at 12:02:36).**

The students then double 21 and get 42 blocks as their solution for the 10- block high tower. They add 21 and get 63 blocks for a 15- block high tower. They try to solve the 100- block tower by considering multiples of 21. Ordena then says that a 5- block high tower should have 25 blocks "because you still have to count the middle one." Ta'keisha builds a stage B tower and counts the blocks, removing each block as she counts it. She shows Ordena that the middle block should be counted only once.

Ta'keisha then builds a 10- block high tower and shows Leo and Ordena that there are 9 blocks in each branch, which would total 36 blocks. She says there are 10 blocks in the height, so there are 46 blocks in total. The students realize this number is not the same as the 42 they had calculated earlier by doubling 21. They decide to rebuild the 5- block high tower. They count the blocks in the 5- high tower they build and again get 21 blocks.

Most of the time, Leo has been watching the other students with a blank expression on his face. He does not smile. He sometimes contributes to the conversation but does so hesitatingly and looks to the other students for confirmation of what he says. He often asks Ta'keisha to repeat what she said. He also makes computational errors in his calculations. Ms. S. announces at approximately 12:19 that the students need to stop working.

Behaviors Pertaining to Student's Cognitive Engagement After Difficulty and Resulting Classification

Leo: High level cognitive activity
Highest level of cognitive activity involved: Justifying, Speculating, Reasoning

Ta'keisha: High level cognitive activity
Highest level of cognitive activity involved Speculating, Justifying, Reasoning, Explaining in one's own words

Ordena: High level cognitive activity
Highest level of cognitive activity involved: Justifying, Speculating, Reasoning

Comparison of Students' Cognitive Engagement Before/After Difficulty 22

Before the difficulty, Leo, Ta'keisha, and Ordena are involved in the task but are working primarily alone. They do not speak much to each other about the task. After Ms. S. leaves the table following their difficulty, the students converse with each other regarding the task. They discuss their ideas and attempt to work through their disagreements. Ta'keisha demonstrates to the other students her reasoning for the validity of her solution.

During Ms. S.'s interview she explained that she responded to the students in the way she did in order to facilitate their collaboration in solving the problem. Ms. S. explained that the students in the group tended to "go through this…silent frustration" and were likely to "move into a phase of just kind of like moving around the manipulatives to look like they were doing something rather than actively…working to solve the problem" (Appendix D., Ms. S. interview, lines 215-219).

Difficulty #23: Leo, Ta'keisha, Ordena, Day 2

Description of Behavior Related to Students' Cognitive Engagement Before Difficulty

Time of day: *(9:47:20-10:00:45)*

The teacher begins the lesson at 9:47:20 by telling the students they will work for another 20 minutes in groups and will then present their solutions to the class. She says they can use transparency paper or large sheets of paper and markers as well as any other materials that were available during class on the previous day. Ta'keisha writes on her paper with her marker. Leo asks Ta'keisha what the group was doing yesterday. She tells him to "Get the blocks, dummy." He asks his question again and she repeats, "Get the blocks." Leo gets up from his seat and walks away. Ta'keisha continues writing on her paper. Leo returns and puts a bag of red cubes on the table. Ta'keisha looks up and says, "No, get the other ones." Leo says, "There ain't no other ones." Ta'keisha takes blocks from the bag and begins to build a tower. Leo then also takes blocks from the bag and begins to build a tower. Ordena sits with her head on her hand and stares downward then to her side with a blank expression on her face. After a few moments Ordena takes blocks from the bag and slowly arranges them into a tower.
Meanwhile, Ta'keisha and Leo have each arranged their blocks into 5 stacks. They add blocks to their stacks. At one point, when Ta'keisha has used all the blocks on her desk, Leo drops a block that he had tried to place on his tower. It falls on the desk between Ta'keisha and himself. Leo

reaches to pick it up from the desk but Ta'keisha grabs it and uses it for her tower. **(See picture at 9.52.11).**

Ta'keisha counts the blocks in one of her stacks and says, "Alright, it's 46." Leo counts the blocks in his 5 stacks and says he got 44. Ta'keisha tells him he can't count. She asks him if there are supposed to be 10 in the middle. Leo says, "alright, then I got 45." Ta'keisha says, "No, you got 46." Leo says, "This is how it's supposed to look," and arranges his 5 stacks into a tower with one vertical branch in the middle and 4 horizontal base branches. Leo counts the blocks in the height of his tower and counts 10. He counts blocks in one branch of his tower and counts 9. Ta'keisha tells him to count them all together and see what he gets. Leo agrees there are 46 blocks. All the students write on their papers. Ta'keisha says now they need to make a 100-block high tower. Ta'keisha and Leo build stacks of blocks. Ta'keisha arranges the stacks into five 20-block stacks. She says it's "for the middle" (of the 100-block high tower). Leo says they need more cubes. He gets up from his seat. Ta'keisha says to Ordena, who has been writing on her paper, "Ordena, why aren't you working?" Ordena says she is. Leo returns with a bag of blocks and builds stacks on his desk.
Ordena says she thinks the 10-block high tower has 42 cubes. Ta'keisha says it has 46 blocks. Ordena repeats that is has 42 and Ta'keisha repeats that it has 46. Ta'keisha asks Leo how many blocks he thinks are in the 10-block high tower. He says there are 46. Ta'keisha points to Ordena's paper and says, "Look, 'cuz that one's 6 (presumably referring to the 2-block high tower), that one's 11 (presumably referring to the 3-block high tower)….6 times 2 is 12…4 (the 4-block high tower) is not 12." (Ordena had multiplied 21, the number of blocks in the 5-block high tower, by 2 to arrive at her solution of 42 for the 10-block high tower. Ta'keisha is arguing with her assumption by showing her method doesn't work for the 2- and 4- block high towers.) Ta'keisha breaks apart the 20-block stacks of blocks on her desk into 10- and 9-block high stacks. She arranges the stacks into a 10-block high tower. While she is doing this, Ordena puts the cap on the marker she has been writing with and scratches her head. She yawns, then stares ahead at Ta'keisha's desk **(See picture at 10.00.34)**. Ms. S. walks by the group's table and Ta'keisha calls her over. Ta'keisha tells Ms. S. that she and Ordena found different solutions for the 10-block high tower.
(Data drawn from videotape)

Behaviors Pertaining to Students' Cognitive Engagement Before Difficulty and Resulting Classification

Leo: High
Highest level of cognitive activity involved: Reasoning

Ta'keisha: High
Highest level of cognitive activity involved: Reasoning, Drawing Inferences

Ordena: High
Highest level of cognitive activity involved: Reasoning

Description of Difficulty

The students found different solutions for the total number of blocks in a ten- block high tower. Ta'keisha and Leo have come up with a solution of 46 blocks, but Ordena arrived at a solution of 42 blocks. Ordena obtained her solution by doubling the number of blocks in a 5- block high tower [lines 2-27 of transcript].

Teacher's Response

Ms. S. asks Ordena why she multiplied by 2 to get the number of blocks in the 10- block high tower. Ordena repeats what she did but does not explain her reasoning. Ms. S. then suggests that Ordena try her pattern with smaller numbers such as 4 and 2, and 2 and 1. The students shift the conversation back to the 5- and 10- block high towers. Ms. S. suggests that the students describe both solutions to the class during their presentation [lines 28-66 of transcript].

Codes: Probing- asking the student to explain the reasoning she used
Offering a simpler problem as an example
Suggesting students share their difficulty with peers

Description of Behavior Related to Students' Cognitive Engagement After Difficulty

Time of day: *(10:04:10- 10:11:59)*

After Ms. S. leaves the students' table, Ta'keisha writes on her paper while Leo builds 10- block high branches and lays them in piles on his desk. Ordena sits with a blank expression on her face. She yawns, looks around the room, plays with her hair, then sits with her elbow on her desk and her head resting on her hand. Ta'keisha punches numbers into her calculator. She says she found the answer for the 100 tower. She says it has 496 cubes. Leo disagrees and says it is 420, the solution they found yesterday. He says, "cuz right now I'm building the legs and I've already got nearly 200." Leo has 2 piles of 10-block branches on his desk. Ta'keisha explains that since the 5- block high tower has 4 blocks on each side and the 10- block high tower has 9 blocks on each side, the 100- block high tower must have 99 blocks on each side.
Ms. S. comes to table 3 with a stack of large colored papers. Ta'keisha takes one and begins writing on it. Ordena writes on her paper while Leo continues building stacks of blocks. Leo asks how many blocks were in the 10- block high tower. Ta'keisha says there were 46. Ordena says there were 43 blocks. Ms. S. then comes to the table and the students explain what they have been doing (This is the next event).

Behaviors Pertaining to Students' Cognitive Engagement After Difficulty and Resulting Classification

Leo: Low level cognitive activity
Highest level of cognitive activity involved: Building towers without consideration of patterns

Ta'keisha: High level cognitive activity
Highest level of cognitive activity involved: Speculating, Justifying

Ordena: Insufficient Information

Comparison of Students' Cognitive Engagement Before/After Difficulty 23

Before the group's difficulty, Leo had shown evidence in one instance of trying to explain reasoning for his solution using a model of a tower. After Ms. S. leaves the table following her response to the students' difficulty, Leo continues building stacks of blocks but does not show evidence of higher order thinking. He says he disagrees with Ta'keisha's solution of 496 for the 100-block high tower but does not provide an argument to try to disprove her solution.

Following Ms. S.'s response to the group's difficulty, Ta'keisha continues working to solve the 100-block high tower. Before Ms. S.'s involvement, Ta'keisha tried to build the 100-block high tower in order to determine the number of cubes it contained. After Ms. S.'s response to the students' difficulty, Ta'keisha works to solve the problem in a generalizable way.

After Ms. S. leaves the table, Ordena does not appear to be very involved in the mathematics problem. Before the difficulty, Ordena had arrived at a solution of 42 blocks for the 10-block high tower. Although Ta'keisha explained her reasoning for her solution of 46 blocks for that tower, Ordena does not appear to have incorporated Ta'keisha's ideas into her thinking. Ordena does not either show evidence of utilizing Ms. S.'s suggestion of using towers of shorter heights to determine if her "doubling" method is valid. She tells Leo during the students' work following their difficulty that the 10-block high tower has 43 blocks.

Ta'keisha, the student who had the highest level of understanding before the difficulty, seems to have increased cognitive involvement in the problem following Ms. S.'s response. The same did not appear to be true for the students who had lower levels of understanding of the problem.

Difficulty #29: Leo, Day 3

Description of Behavior Related to Student's Cognitive Engagement Before Difficulty

Time of day: *(8:59:00-9:50:00)*

The class session on day 3 begins at 8:59 am. Ta'keisha, Ordena, and Leo continue to work on the Towers Task. They build with the blocks on their table. Ta'keisha builds a 5-block high tower. She says, "I'm doing ten." Leo says, "I already did ten." Ta'keisha takes 2 of the stacks Leo built and counts the blocks in each. She asks him why he built 4 tens. She says there are supposed to be 9 blocks in each branch. Ta'keisha takes apart the stacks Leo built and builds 9-block high stacks of yellow blocks. Ta'keisha tells Leo to "stop mixing the colors." She then directs Leo to give her more yellow blocks. Leo gives Ta'keisha more yellow blocks from the bag. Leo then takes apart the stacks he had built of blue and yellow blocks and gives Ta'keisha the yellow blocks. Ta'keisha completes the 10-block high tower.

Leo takes the transparency Ta'keisha had prepared from Ordena's desk and reads it. He returns it to Ordena's desk, takes a marker, and writes on his paper. The students decide to build a 100-block high tower. Ordena begins building 10-block high stacks of blocks. Ta'keisha tells Leo to help Ordena. Ta'keisha says she's tired and that she already built towers. Leo and Ordena build stacks of blocks. Ta'keisha takes 10 stacks of 10 blocks that Ordena built. She says there are 100 blocks. Leo and Ordena then build stacks and give them to Ta'keisha. Ta'keisha counts each stack and assembles a pile of 9-block high stacks. She counts a stack that Leo gave her and tells Leo that he can not count. At 9:14:45 Ms. S. tells the students to stop working. The student groups begin their presentations to the class. Ms. S. arranged the order of the presentations so that students who obtained more advanced solutions to the problem present later than groups who reached impasse earlier in their work. Group 3 is the fourth group to present. During the presentations of the first three groups, Leo is quiet and watches the presentations. He occasionally stares around the classroom with a blank expression on his face. During the first group's presentation, Ms. S. asks the students to raise their hands if they think the middle cube should be counted. Leo, Ta'keisha, and Ordena all raise their hands. During the second group's presentation, the presenting students say Diagram B on the task sheet is a 5-block high tower. Ms. S. asks the students in the class if they agree with that. Ta'keisha, Ordena, and Leo, together with other students in the class, say, "no." Leo says it is a 2-block high tower. At 9:50:00, Ta'keisha, Ordena, and Leo move to the front of the classroom to present their work.
(Data drawn from videotape)

Behaviors Pertaining to Student's Cognitive Engagement Before Difficulty and Resulting Classification

Low level cognitive activity

Highest level of cognitive activity involved: Building towers without consideration of patterns, Referring to Materials, Considering Facts

Description of Difficulty

During the group's presentation, Leo says he disagrees with Ta'keisha's solution of 496 blocks for the 100- block high tower. He says he does not understand why she says there are 99 cubes on each side of the tower and not 100 [lines 20-30 of transcript].

Teacher's Response

Ms. S. tells Ta'keisha and Leo to try to convince each other that their solutions are correct. After Leo and Ta'keisha present their arguments, Ms. S. asks the class if they have any questions. When the class disagrees about whether the 5- block high tower is called a 5-block high or 4- block high tower, Ms. S. says the class will continue with their presentations even though they have not resolved their disagreement. She says that the next group might help them resolve it [lines 31-125 of transcript].

Code: Suggesting student share his difficulty with peers

Description of Behavior Related to Student's Cognitive Engagement After Difficulty

Time of day: *(9:58:00-10:04:38)*

During the beginning of the next group's presentation, Leo says to Ta'keisha, "See, I was right. See, ha. See, ha." Leo watches the next group's presentation. The students demonstrate a method they used to find the number of blocks for the first two stages of the towers: For the stage one tower, they took 5 groups of one block and arranged the blocks into a tower. Then they subtracted 4 blocks by removing one block from each side of the tower. For the stage 2 tower, they took 5 groups of 2 blocks, arranged them into a tower, and then subtracted 4 by removing one block from each side of the tower. The students say they used this method to find the number of blocks in a 100-block high tower: they calculated 5 groups of 100 blocks=500, and subtracted 4=496.

When the presentation is over, Ms. S. asks the class whether the students in the group came up with a general solution and whether the class is convinced of their method. Ta'keisha and Ordena answer "yes" to both questions but Leo does not answer.

Behaviors Pertaining to Student's Cognitive Engagement After Difficulty and Resulting Classification

Insufficient Information

Comparison of Student's Cognitive Engagement Before/After Difficulty 29

Leo presents his difficulty to his classmates but does not speak throughout the remainder of the discussions during his group's presentation. Leo appears to realize his solution is incorrect as the majority of the class agrees with Ta'keisha's solution. On his Day 3 questionnaire, Leo wrote, "I thought I was going to be imbarrist." Leo does not, however, show evidence of a deeper understanding of the problem or of incorporation of Ta'keisha's ideas into his own as he is quiet as the remaining groups present.

Analysis of Ms. S.'s students' cognitive engagement before and after their three mathematics difficulties analyzed here indicated that Ta'keisha, the student who appeared strongest among the group in mathematics ability, showed evidence of deeper understanding of the problem following Ms. S.'s responses. Ordena and Leo, however, who appeared considerably weaker in mathematics ability, did not show evidence of more complex mathematical thinking following Ms. S.'s responses to their difficulties.

Analysis of Student Engagement Questionnaire Data

In the present study, two sections of the students' questionnaires were examined. The first section related to students' feelings about mathematics that day; the second asked about whether students experienced difficulty in mathematics that day.

Students' Reports of Emotions

The following tables depict Ms. S.'s students' scores for each emotion on the Student Engagement Questionnaire as compared with the scores of a population of 83 students in the same urban school district. The Student Engagement Questionnaire was distributed at the end of the class sessions on each day of the students' work. Table 4.22 lists Ms. S.'s students' scores on the questionnaires distributed on Day 1, Table 4.23 lists the students' scores for Day 2, and Table 4.24 depicts the scores for Day 3. Scores were measured on a 2-point scale. For positive emotions, students received a score of '2' for a response of 'Very much,' a '1' for 'Somewhat,' and a ‘0’ for ‘Not at All.’ For negative emotions, the scores were reversed. ‘N/A’ indicates a student’s score for a questionnaire item was not available. Table 4.24, which lists questionnaire results for Day 3, includes only Leo’s responses since only Leo was involved in analyzed instances of difficulty on that day.

Table 4.22, Ms. S. Student Questionnaire Feelings Responses, Day 1

Questionnaire Item	n	Population Mean	Population Standard Deviation	Ta'keisha	Leo	Ordena
Interested	78	1.58	.570	1.00	2.00	2.00
Respected	78	1.63	.561	2.00	1.00	1.00
Proud	79	1.48	.677	1.00	2.00	2.00
Successful	78	1.56	.636	1.00	2.00	2.00
Safe	76	1.66	.623	2.00	2.00	2.00
Excited	75	1.37	.731	2.00	2.00	2.00
Happy	76	1.46	.720	1.00	2.00	2.00
Satisfied	77	1.36	.687	0.00	1.00	1.00
Relieved	78	1.10	.749	0.00	2.00	2.00
Confident	75	1.41	.737	2.00	2.00	2.00
Curious	76	.87	.854	2.00	0.00	2.00
Unhappy (reversed)	76	1.83	.413	2.00	2.00	2.00
Disappointed (Reversed)	75	1.79	.527	2.00	2.00	2.00
Worried (Reversed)	76	1.62	.610	2.00	2.00	1.00
Discouraged (Reversed)	76	1.88	.399	2.00	2.00	2.00
Angry (Reversed)	76	1.72	.556	1.00	2.00	2.00
Disrespected (Reversed)	76	1.88	.399	2.00	2.00	2.00
Bored (Reversed)	76	1.59	.715	1.00	2.00	2.00
Embarrassed (Reversed)	76	1.79	.549	2.00	2.00	2.00
Afraid (Reversed)	75	1.89	.352	2.00	2.00	2.00
Positive/Negative Subscale Means		*1.57*		*1.50*	*1.80*	*1.85*
Frustrated	76	1.68	.616	2.00	0.00	0.00
Confused	76	1.55	.641	1.00	1.00	1.00
Engagement with Impasse Subscale Mean		*1.62*		*1.50*	*0.50*	*0.50*

Table 4.23, Ms. S. Student Questionnaire Feelings Responses, Day 2

Questionnaire Item	n	Population Mean	Population Standard Deviation	Ta'keisha	Leo	Ordena
Interested	78	1.58	.570	1.00	2.00	0.00
Respected	78	1.63	.561	2.00	1.00	N/A
Proud	79	1.48	.677	1.00	1.00	1.00
Successful	78	1.56	.636	2.00	1.00	0.00
Safe	76	1.66	.623	2.00	1.00	1.00
Excited	75	1.37	.731	0.00	1.00	0.00
Happy	76	1.46	.720	1.00	1.00	0.00
Satisfied	77	1.36	.687	1.00	1.00	1.00
Relieved	78	1.10	.749	1.00	1.00	1.00
Confident	75	1.41	.737	2.00	1.00	1.00
Curious	76	.87	.854	1.00	0.00	0.00
Unhappy (reversed)	76	1.83	.413	2.00	1.00	0.00
Disappointed (Reversed)	75	1.79	.527	1.00	2.00	1.00
Worried (Reversed)	76	1.62	.610	2.00	2.00	1.00
Discouraged (Reversed)	76	1.88	.399	2.00	2.00	2.00
Angry (Reversed)	76	1.72	.556	1.00	2.00	0.00
Disrespected (Reversed)	76	1.88	.399	2.00	2.00	1.00
Bored (Reversed)	76	1.59	.715	0.00	2.00	1.00
Embarrassed (Reversed)	76	1.79	.549	2.00	2.00	2.00
Afraid (Reversed)	75	1.89	.352	2.00	2.00	2.00
Positive/Negative Subscale Means		*1.57*		*1.36**	*1.40*	*0.79***
Frustrated	76	1.68	.616	1.00	0.00	2.00
Confused	76	1.55	.641	1.00	0.00	1.00
Engagement with Impasse Subscale Mean		*1.62*		*1.00*	*0.00*	*1.50*

*Ta'keisha added two feelings to her list: Annoyed (Somewhat; score of 1) and Tired (Somewhat; Score of 1). These scores are included in the mean for Ta'keisha.
**Ordena did not respond to the questionnaire item 'Respected.' This item was not included in the mean for Ordena.

Table 4.24, Ms. S. Student Questionnaire Feelings Responses, Day 3

Questionnaire Item	**n**	**Population Mean**	**Population Standard Deviation**	**Leo**
Interested	78	1.58	.570	2.00
Respected	78	1.63	.561	1.00
Proud	79	1.48	.677	1.00
Successful	78	1.56	.636	1.00
Safe	76	1.66	.623	2.00
Excited	75	1.37	.731	1.00
Happy	76	1.46	.720	1.00
Satisfied	77	1.36	.687	1.00
Relieved	78	1.10	.749	1.00
Confident	75	1.41	.737	1.00
Curious	76	.87	.854	0.00
Unhappy (reversed)	76	1.83	.413	2.00
Disappointed (Reversed)	75	1.79	.527	2.00
Worried (Reversed)	76	1.62	.610	2.00
Discouraged (Reversed)	76	1.88	.399	2.00
Angry (Reversed)	76	1.72	.556	2.00
Disrespected (Reversed)	76	1.88	.399	2.00
Bored (Reversed)	76	1.59	.715	2.00
Embarrassed (Reversed)	76	1.79	.549	2.00
Afraid (Reversed)	75	1.89	.352	2.00
Positive/Negative Subscale Means		*1.57*		*1.50*
Frustrated	76	1.68	.616	0.00
Confused	76	1.55	.641	0.00
Engagement with Impasse Subscale Mean		*1.62*		*0.00*

Students' Self-Reported Affect vs. Affect Inferred from Students' Behaviors

To gather information regarding students' affect during the videotaped class sessions, the present researcher initially watched preselected episodes of students' difficulties during the videotaped sessions multiple times. The researcher then created transcriptions of those episodes and the time periods following them which included descriptions of students' behaviors as well as screenshots from the video that were illustrative of students' emotions. The researcher then tried to infer the students' feelings following each instance of difficulty from the transcriptions. When in doubt regarding particular students' feelings, the researcher referred back to the videotapes and in several cases, consulted with a psychologist to determine the feelings that most aptly depicted the constellation of behaviors exhibited by the students.

The emotions inferred by the researcher were then compared with students' self-reported emotions on the questionnaire. Following is a comparison of self-reported and inferred affect for Ta'keisha, Leo, and Ordena on Days 1 and 2 and for Leo on Day 3 (Leo was the only student involved in an analyzed instance of difficulty on Day 3).

Ta'keisha

Day 1

As described in Appendix A, Event 10: Difficulty 21 and Event 11, Difficulty 22, inferences made from Ta'keisha's behaviors during analyzed segments of the class session showed Ta'keisha demonstrated strong positive affect. Ta'keisha appeared confident and very interested in solving the problem. She did not show evidence of negative emotions. On her questionnaire, however, Ta'keisha reported feeling only somewhat interested and happy and not at all satisfied or relieved. She also reported feeling somewhat angry and bored.

Day 2

Ta'keisha was involved in one analyzed instance of difficulty on Day 2. Inferences made from Ta'keisha's behaviors during this videotape segment indicated that Ta'keisha appeared to have highly positive affect (see Appendix A, Event 12, Difficulty 23). Following Difficulty 28, which involved only Leo, however, Ta'keisha appeared impatient and annoyed with Leo when he had difficulty understanding her solution. Her affect at this time was similar to her reported affect on the questionnaire. On her questionnaire, Ta'keisha reported feeling very confident but only somewhat happy, satisfied, and curious. Ta'keisha also reported feeling somewhat disappointed and angry and added the feelings of somewhat 'annoyed' and 'tired' to the list.

Leo

Day 1

Inferences made from Leo's behaviors during the analyzed segments of videotape on Day 1 involving Leo showed that at times Leo displayed much interest and did not appear to have negative feelings. Leo did not, however, show evidence of feelings such as pride, success, excitement, or happiness. During the end of the class session, Leo showed some evidence of feeling inferior to Ta'keisha. He watched the other students at his table and did not smile or contribute much to the conversation (See Appendix A, Event 10, Difficulties 20 & 21 and Event 11, Difficulty 22). On his questionnaire, however, Leo, reported having overall strong positive feelings during the class session. He reported feeling very: proud, successful, excited, and happy and not at all: unhappy, afraid, or worried.

Day 2

Leo's self-reported affect during this class session was more positive than the affect inferred from his observed behaviors. During the class session Leo had difficulty understanding both the mathematics problem and Ta'keisha's solution. Ta'keisha made derogatory remarks towards Leo, calling him a 'dummy' and telling Ordena that Leo does not know what he's doing (see Appendix A, Event 12, Difficulty 23 and Event 13, Difficulties 24-28). On his questionnaire, Leo reported feeling only somewhat successful, safe, and happy, but not at all angry, disrespected, or embarrassed. Leo also reported feeling not at all frustrated or confused.

Day 3

Leo's self-reported emotions on his Day 3 questionnaire were more positive than the emotions inferred from his observable behaviors on Day 3. During the class presentation this session, Leo had difficulty understanding the solution Ta'keisha presented to the class. After the presentation Leo appeared defeated and showed evidence of feeling inferior to Ta'keisha (see Appendix A, Event 14, Difficulty 29). On his questionnaire, however, Leo reported feeling somewhat proud, successful, excited and happy. No evidence was found for these feelings during the session (the complete session of Group 3 on Day 3 was analyzed in this work). Leo also reported feeling not at all unhappy, angry, disrespected, or embarrassed.

Leo did, however, express negative feelings on other portions of his questionnaire. On one page of the questionnaire, where students were asked to describe any thoughts they had that were not on the list of thoughts, Leo wrote, "I felt like I was being bossed around by some one in my group." On the last page of the questionnaire, where students were asked about what stands out in their memory about situations that day involving difficulty, Leo reported that he thought he was going to be embarrassed.

Ordena

Day 1

On her emotions questionnaire, Ordena reported feeling very: proud, happy, and excited. Inferences made from her behaviors during analyzed videotape segments of this session, however, did not provide evidence for these emotions. After Difficulty 19, for instance, Ordena tried to build a stage 2 tower but did so incorrectly. Ta'keisha pointed out her error and Ordena revised her tower, but it was still not structured correctly. During the portion of the class session for which Ordena's affect was coded as Strong positive feelings, Ordena's behavior indicated feelings of interest and confidence, but no evidence was observed for her reported feelings of much pride, happiness, and excitement (Refer to Appendix A, Event 9, Difficulty 19; Event 10, Difficulty 21; and Event 11, Difficulty 22).

Day 2

The affect inferred from Ordena's observed behaviors during the analyzed instance of difficulty on Day 2 in which Ordena was involved was more positive than Ordena's self-reported affect for that day. During class Ordena appeared interested but fatigued (see Appendix A, Event 12, Difficulty 23). On her questionnaire, however, Ordena reported feeling very: unhappy and angry, not at all interested, and somewhat: disappointed, worried, and disrespected. Ordena's behaviors during the analyzed videotape segments of the class session did not provide evidence these feelings.

Ordena explained on her Day 2 questionnaire that she was not feeling well that day. When asked to indicate any thoughts she had that were not on the list of thoughts on the questionnaire, Ordena wrote, "am sick and not concentrating." Ordena's reports of negative feelings on Day 2 may have been attributable to her physical illness on that day.

Group Analysis of Students' Self-Reported Affect vs. Affect Inferred from Students' Behaviors

Day 1

Ta'keisha, who demonstrated the highest level of understanding of the problem among the members of her group, reported the highest level of engagement with impasse (She received an "Engagement with Impasse" subscale score of 1.50). Leo and Ordena, who demonstrated considerably more difficulty with the problem, had "Engagement with Impasse" subscale scores of only .50.

Ta'keisha's Positive/Negative Subscale Mean was the lowest among the scores of the members in her group. The affect inferred from her observed behaviors on videotape, however, was the most positive of all her group members.

Day 2

All students in this group reported more negative affect on Day 2 than on Day 1. The reported affect on Day 2 for each student in the group was also below the Positive/Negative Subscale Mean.

Leo, who appeared to have the most difficulty with the mathematics problem during this session, reported the lowest level of 'Engagement with Impasse' among the members of his group.

Day 3

Only Leo was involved in analyzed difficulty on Day 3 (see above for analysis of his observable vs. self-reported emotions).

Self-Reports of Encountering Difficulty and Comparison with. Observed Instances of Difficulty

Table 4.25, Students' Reports of Difficulty and Observed Difficulty, Ms. S., Day 1

	Ta'keisha	Leo	Ordena
Self-Report of Difficulty			
Had Difficulty?	Yes	Yes	Yes
Classroom Context	Group	Whole Class	Group
What stood out?	----	"I felt like I could do this It does not matter if I was on camera I will still get my work done and focus and cut out all of the things that are happening around the classroom."	"At first I didn't really understand what the problem was trying to say. but teacher explained it carefully and so did my group mate and I understood it."
Observed Difficulty			
Types of Difficulty and Frequency of Occurrence	Impasse (2)	Incorrect Answer (1) Impasse (2)	Unable to Answer (1) Impasse (2)

All students in the group acknowledged they encountered difficulty during this session. Ordena mentioned her teacher's assistance in overcoming her difficulty.

Table 4.26, Students' Reports of Difficulty and Observed Difficulty, Ms. S., Day 2

	Ta'keisha	Leo	Ordena
Self-Report of Difficulty			
Had Difficulty?	Yes	No	Yes
Classroom Context	Group	----	Whole Class and Small Group
What stood out?	----	"I felt like I was smart and I could get the problem without the person bothering me."	"Nothing"
Observed Difficulty			
Types of Difficulty and Frequency of Occurrence	Impasse (1)	Impasse (1) Incorrect Answer/Idea (4) Does Not Understand Solution (1)	Impasse (1)

Leo, who appeared to have the greatest difficulty with the mathematics problem during this session, was the only student in the group who reported having no difficulty.

Table 4.27, Students' Reports of Difficulty and Observed Difficulty, Ms. S., Day 3

	Leo
Self-Report of Difficulty	
Had Difficulty?	No
Classroom Context	----
What stood out (for example, what happened, what you thought, what you felt, your teacher said or did, what other kids said or did)?	"Nothing stands out of my memory I felt like a mathemetician and smart I thought I was going to be imbarrist nothing, nothing"
Observed Difficulty	
Types of Difficulty and Frequency of Occurrence	Does Not Understand Solution (1)

Leo's difficulty on Day 3 occurred during his group's presentation to the class. Leo reported having no difficulty; however, he did indicate he thought he was going to be embarrassed.

Summary: Ms. S.

During the 11 instances of students' difficulty analyzed for Ms. S., Ms. S. used three types of initial teacher responses: *Having Other Student(s) Help, Helping Student Indirectly* and *Responding without Offering Suggestion for Resolving Difficulty*. Ms. S.'s follow-up responses fell into two response categories: *Having Other Student(s) Help* and *Helping Student Indirectly*. Ms. S. most frequently used the response types *Helping Students Indirectly* and *Having Other Students Help*.

During Ms. S.'s interview, she explained that when her students encounter difficulty during mathematics class, she usually suggests they discuss their difficulties with peers. Ms. S. mentioned that she groups her students homogenously to allow for mathematics discussions on all levels. Ms. S. stated that while her students discuss their difficulties, she asks guiding questions to assist them in resolving their difficulties. Ms. S. reported that she uses more directed questions with her weaker students and always tries not to indicate whether students' answers are correct in order to encourage their development of self-check strategies. When asked which factors may contribute to her choices of response to students' difficulties, Ms. S. noted that she uses a more directed approach when unit assessments are approaching and when there are many students in the classroom. Ms. S.'s descriptions of the ways in which she typically responds to students' mathematics difficulties were consistent with the responses she offered to her students during the analyzed class sessions.

Following Ms. S.'s interventions in instances of students' difficulties, the students' behaviors were always coded as on-task. During these time periods, the students showed evidence of both high and low cognitive activity and feelings that ranged from highly negative to highly positive. A preliminary relationship was noted between Ms. S.'s students' affective and cognitive engagement as observed on videotape following her interventions to their difficulties. During the instances where students showed evidence of strong positive feelings, where information was available regarding their cognition, they showed evidence of high level cognitive activity. During two of the three instances where a student showed evidence of negative feelings and information was available regarding his cognition, he showed evidence of low level cognitive activity. No patterns were noted between the types of responses Ms. S. offered to her students and the students' subsequent cognitive activity.

Comparison of Ms. S.'s students' self-reported affect and affect inferred from their observed behaviors on videotape indicated that students' self-reported feelings were at times more positive or more negative than the feelings inferred from their behaviors. The student who appeared weakest in mathematics ability always reported more positive affect than was inferred from his observable behaviors. The student who appeared strongest in mathematics ability reported either similar or more negative affect than was inferred from her observable behaviors. The student who was involved in the most analyzed instances of difficulty on Day 2 and who was the only one involved in an analyzed difficulty on Day 3 reported having no difficulty on both days. The other students, who were involved in analyzed instances of difficulty on Days 1 and 2, did report having difficulty on both days.

4.7 Collective Analysis: Ms. A., Ms. B., Ms. S.

Following is an analysis comparing the findings for Ms. A., Ms. B., and Ms. S.'s classes. This section will discuss similarities as well as discrepancies among the three teachers in: 1) the types of teacher responses offered, 2) the teachers' explanations of ways in which they usually respond to students' mathematics difficulties, 3) the consistency of teachers' reports of ways in which they usually respond to their observed responses 4) the teachers' reasoning for the ways they responded in particular instances of students' difficulty, 5) the students' behavioral, cognitive, and affective engagement following the teachers' interventions to their difficulties, 6) the students' self- reports of their emotions compared with emotions inferred from their behaviors during the videotaped class sessions, and 7) the students' self-reports of whether or not they encountered difficulty during the sessions compared with instances of their involvement in analyzed difficulties.

Types of Teacher Responses Offered

Table 4.28 provides an overview of the types of responses Ms. A., Ms. B., and Ms. S. offered to their students during analyzed instances of students' difficulty.

Table 4.28, Types of Teacher Responses, Ms. A., Ms. B., Ms. S.

Numbers preceding a hyphen in the frequency column represent initial responses; numbers after a hyphen refer to follow-up responses.

Teacher Response	**Ms. A.**	**Ms. B.**	**Ms. S.**	**Total**
Having other student(s) help student resolve difficulty				
Inviting (verbally or non-verbally) (an)other student(s) to respond Suggesting student(s) share his/her/their difficulty with peer(s) Reminding student about another student's idea	0-1	4-3	4-2	***8-6***
Having student help him/herself (teacher not offering suggestion/not having others offer suggestions)				
Giving the student time to think about the problem	1-0	0-0	0-0	**1-0**
Helping student (may also involve student helping him/herself)				
Directly				
Providing the correct answer Explaining the concept or the mathematics problem/task	0-0	0-1	0-0	***0-1***
Indirectly				
Leading the student(s) to the correct answer/idea	1-0	0-0	1-1	2-1
Offering a counter example to an incorrect response	1-0	0-0	0-0	1-0
Probing- asking the student to explain his/her solution or the reasoning he/she used Asking the student to prove his/her solution	0-0	3-1	1-0	4-1
Offering a simpler problem as an example	0-0	1-0	2-1	3-1
Suggesting a process to use in solving the problem	1-0	2-1	0-0	3-1
Showing the student a diagram/model	0-0	1-0	1-0	2-0
Subtotal: Teacher Helping Student Indirectly				*15-4*
Not Responding/Responding without Offering Suggestion for Resolving Difficulty				
Saying a student's answer is wrong	0-0	0-0	0-0	0-0
Choosing not to respond to an incorrect response	0-0	1-0	0-0	1-0
Acknowledging or repeating the incorrect response Asking a question to clarify what the student(s) said Repeating the teacher's question	0-0	2-0	2-0	4-0
Subtotal: Not Responding/Responding without Offering Suggestion				*5-0*

As indicated in the table above, Ms. A., Ms. B., and Ms. S.'s initial responses to students' difficulties fell into four of the five coding categories for teacher responses: *Helping Students Indirectly*, *Having Other Students Help*, *Responding without Offering Suggestion for Resolving Difficulty*, and *Having Students Help Themselves*. None of the teachers' initial responses was coded as *Helping Students Directly*. *Helping Students Indirectly* was the response category most frequently used by all three teachers. Ms. S. and Ms. B. also frequently used responses coded as *Having Other Students Help*.

None of the teachers' responses was coded as *Saying a Student's Answer is Wrong* (included in the response category *Responding Without Offering Suggestion*) or *Providing the Correct Answer* (included in the response category *Helping Students Directly*).

Only Ms. A. offered a response coded as *Having Students Help Themselves*. Ms. S.'s students were in a high ability class and appeared the most motivated of all the student groups analyzed. Perhaps Ms. A. offered this response since she knew the students would be able to resolve their difficulty on their own. During the teacher interviews, all the teachers mentioned they use a less teacher-directed, more exploratory approach when interacting with their higher ability students.

Ms. A. also used the smallest number of follow-up responses relative to the total number of responses she offered (1 of 5, or 20% of her responses were follow-ups). Ms. B. and Ms. S. both used more follow-up responses in proportion to the total number of responses they used (6 of 20, or 30% of Ms. B.'s responses were follow-ups; 4 of 15, or approximately 27% of Ms. S.'s responses were follow-ups). Perhaps since the group in Ms. A.'s class analyzed here included high ability, motivated students, they required less guidance from Ms. A.

Teachers' Reasoning for their Responses and Comparison with Observed Responses

Both Ms. B. and Ms. S. reported listening to their students and asking probing or guiding questions to help them resolve difficulties they encounter during class. They both stated they try not to give their students answers in order to encourage the students' critical thinking. Ms. B. and Ms. S. appeared to respond similarly when their students encountered difficulty during the videotaped class sessions. Ms. B. and Ms. S. often asked their students to discuss their difficulties with each other and asked probing questions to guide their students to deeper mathematical understanding. During analyzed difficulties, Ms. A. also asked her students questions to guide them to resolution of their difficulties and did not simply give her students answers. Ms. A., however, did not often ask her students to share their difficulties with each other. None of Ms. A.'s initial responses was coded as *Having Other Students Help*. Her single follow-up response was assigned that code.

All three teachers reported providing more guidance and asking more directed questions when responding to weaker students' mathematics difficulties. The teachers stated they provide less teacher intervention when higher ability students encounter difficulty. Only Ms. B. mentioned that her students' dispositions on a given day and their classroom social status influence the way she responds to their difficulties.

Ms. A. and Ms. B. mentioned they often address their students' difficulties either one-on-one or in a small group setting to prevent students' embarrassment, especially for those who struggle with mathematics. The single instance in this analysis of a students' loss of face occurred during Ms. S.'s class.

Ms. A. and Ms. S. reported conflicting perspectives regarding students' grouping during classroom problem solving. Ms. A. said she groups her students heterogeneously to encourage peer teaching. Ms. S. noted that she uses homogenous grouping to allow for students' mathematics exploration on all levels.

All three teachers' reports of ways in which they respond to students' mathematics difficulties were consistent with the responses observed during instances of difficulty examined in this research.

Teachers' Reasoning for Particular Responses

Teachers were asked during stimulated recall interviews about their reasoning for providing particular responses during selected episodes of students' difficulty. Each teacher was asked to reflect on three preselected responses she offered to her students. The table below lists the codes assigned to the teachers' explanations and the frequency for each code. Teachers' explanations for individual responses were often assigned more than one code.

Table 4.29, Teachers' Reasoning Codes for Particular Responses, Ms. A., Ms. B., Ms. S.

Teachers' Reasoning Codes for Particular Responses	Ms. A.	Ms. B.	Ms. S.
Mathematics Goals (M):			
Problem Solving Skills			3
Conceptual Understanding	2	1	
Students' Emotions in Regard to Mathematics (E):			
Working without Pressure	1		
Saving Face in Front of Classmates			1
Students' Attitudes (At):			
Attitudes regarding participation in mathematics activities		2	
Students' Abilities/Skills (Ab/S):			
Mathematics Ability	3	2	1
Organizational Skills		1	
Time Constraints (Time)			1

Students' Mathematics Abilities/Skills (the teachers' perceptions of the students' abilities in mathematics or organizational skills) and *Mathematics Goals* were the categories most often used to describe teachers' reasoning for the particular responses they offered when their students encountered difficulty.

Analysis of the students' behaviors following the teachers' interventions indicated that the teachers' intentions for providing the particular responses were realized, overall. All the outcomes Ms. A. and Ms. B. intended for their students (based on the reasoning codes) were actualized (see *Teachers' Reasoning for Particular Responses and Comparison with Actual Outcomes* for Ms. A. and Ms. B., above). Two of Ms. S.'s intentions for her students were not realized. One involved a problem solving goal where the class was short on time (see *Teachers' Reasoning for Particular Responses and Comparison with Actual Outcomes* for Ms. S., Difficulty 23); the other involved a students' saving face where the student presented his difficulty in a whole class setting (see *Teachers' Reasoning for Particular Responses and Comparison with Actual Outcomes* for Ms. S., Difficulty 29)

Student Engagement

Table 4.30 provides an overview of the codes associated with each instance of student difficulty analyzed in this work. For each difficulty, the table indicates: 1) the number of the difficulty, 2) the day the difficulty occurred (Day 1, 2, or 3 of students' work on the Building Blocks Problem), 3) the teacher of the class, 4) the student(s) involved in the difficulty, and codes describing: 5) the type of teacher response(s) and the students' subsequent: 6) behavioral engagement, 7) cognitive engagement, and 8) affective engagement.

Table 4.30, Instances of Student Difficulty Codes: Ms. A., Ms. B., Ms. S.

Difficulty Number, Day	***Teacher***	***Student(s) Involved***	***Teacher's Response***	***Student's Behavioral Engagement***	***Student's Cognitive Engagement***	***Student's Affective Engagement***
1 *(D1)*	*Ms. A.*	Amanda	*Suggesting process*	*On-task*	High level cog activity	Strong pos feelings
2 *(D1)*	*Ms. A.*	Amanda	*Providing counter example*	On-task	High level cog activity	Strong pos feelings
3 *(D1)*	*Ms. A.*	Amanda Emanuel Eliot Juan	*Giving time*	Am: On-task Em: On-task El: On-task J: On-task	Am: High level cog activity Em: High level cog activity El: High level cog activity J: Insufficient Info	Am: Strong pos feelings Em: Strong pos feelings El: Strong pos feelings J: Mild pos feelings
4 *(D1)*	*Ms. A.*	Amanda Emanuel Eliot Juan	*Leading* *Sharing with peers*	Am: On-task Em: On-task El: On-task J: On-task	Am: High level cog activity Em: High level cog activity El: Insufficient Info J: Insufficient Info	Am: Strong pos feelings Em: Strong pos feelings El: Strong pos feelings J: Strong pos feelings
5 *(D1)*	*Ms. B.*	Marcos Larent Lashanna	*Suggesting process*	M: On-task Lr: On-task Ls: On-task	M: Low level cog activity Lr: Low level cog activity Ls: Low level cog activity	M: Moderate pos feelings Lr: Mild neg feelings Ls: Moderate pos feelings
6 *(D1)*	*Ms. B.*	Marcos Lashanna	*Clarifying*	M: On-task Ls: On-task	M: High level cog activity Ls: Low level cog activity	M: Mod pos feelings Ls: Mod pos feelings
7 *(D1)*	*Ms. B.*	Marcos Lashanna	*Leading*	M: On-task Ls: On-task	M: Low level cog activity Ls: Low level cog activity	M: Mod pos feelings Ls: Mod pos feelings
8 *(D1)*	*Ms. B.*	Marcos	*Repeating teacher's question*	On-task	Low level cog activity	Mod pos feelings
9 *(D1)*	*Ms. B.*	Marcos Lashanna	*Asking for proof*	M: On-task Ls: On-task	M: High level cog activity Ls: Insufficient Info	M: Mod pos feelings Ls: Mod pos feelings
10 *(D1)*	*Ms. B.*	Marcos	*Showing diagram*	On-task	High level cog activity	Mod pos feelings
11 *(D1)*	*Ms. B.*	*Larent*	*Not responding* *Inviting another student*	On-task	Low level cog activity	Mild pos feelings
12 *(D1)*	*Ms. B.*	*Larent*	*Using another's idea* *Probing* *Explaining*	On-task	High level cog activity	Mod pos feelings

(*Table 4.30, Instances of Student Difficulty Codes: Ms. A., Ms. B., Ms. S., cont.)*

13 *(D1)*	*Ms. B.*	Lashanna	*Using another's idea* *Inviting another student*	On-task	High level cog activity	Strong pos feelings
14 *(D1)*	*Ms. B.*	*Larent*	*Inviting another student* *Suggesting process*	Off-task	Not Engaged	Mild neg feelings
15 *(D1)*	*Ms. B.*	*Larent*	*Not responding*	Off-task	Not Engaged	Mild neg feelings
16 *(D2)*	*Ms. B.*	*Marcos*	*Probing* *Inviting another student* *Sharing with peers*	On-task	Low level cog activity	Mild pos feelings
17 *(D2)*	*Ms. B.*	*Lashanna*	*Asking for proof*	On-task	High level cog activity	Strong pos feelings
18 *(D2)*	*Ms. B.*	*Marcos*	*Asking for proof*	On-task	High level cog activity	Mod pos feelings
19 *(D1)*	*Ms. S.*	*Ordena*	*Inviting other students*	On-task	Low level cog activity	Mod pos feelings
20 *(D1)*	*Ms. S.*	*Leo*	*Acknowledging incorrect response*	On-task	Insufficient Info	Mod pos feelings
21 *(D1)*	*Ms. S.*	*Ta'keisha* *Leo* *Ordena*	*Leading*	T: On-task L: On-task O: On-task	*T:* Insufficient Info *L:* Insufficient Info *O:* Insufficient Info	*T:* Strong pos feelings *L:* Strong pos feelings *O:* Mild pos feelings
22 *(D1)*	*Ms. S.*	*Ta'keisha* *Leo* *Ordena*	*Clarifying* *Inviting another student* *Leading*	T: On-task L: On-task O: On-task	*T:* High level cog activity *L:* High level cog activity *O:* High level cog activity	*T:* Strong pos feelings *L:* Mild pos feelings *O:* Mod pos feelings
23 *(D2)*	*Ms. S.*	*Ta'keisha* *Leo* *Ordena*	*Probing* *Offering simpler problem* *Sharing with peers*	T: On-task L: On-task O: On-task	*T:* High level cog activity *L:* Low level cog activity *O:* Insufficient Info	*T:* Strong pos feelings *L:* Mod pos feelings *O:* Mild pos feelings
24 *(D2)*	*Ms. S.*	*Leo*	*Inviting others to respond*	On-task	High level cog activity	Mod neg feelings
25 *(D2)*	*Ms. S.*	*Leo*	*Showing model*	On-task	Low level cog activity	Mod neg feelings
26 *(D2)*	*Ms. S.*	*Leo*	*Offering simpler problem*	On-task	Low level cog activity	Insufficient Info
27 *(D2)*	*Ms. S.*	*Leo*	*Offering simpler problem*	On-task	Insufficient Info	Mild pos feelings
28 *(D2)*	*Ms. S.*	*Leo*	*Sharing with peers*	On-task	Low level cog activity	Mod neg feelings
29 *(D3)*	*Ms. S.*	*Leo*	*Sharing with peers*	On-task	Insufficient Info	Strong neg feelings

No patterns were noted between the types of responses Ms. A. offered when her students encountered difficulty and their subsequent mathematical engagement. Following Ms. A.'s responses, her students' engagement was always coded as on-task, as involving high level cognitive activity where information was available regarding the students' cognition, and as involving positive feelings. No patterns were noted, as well, between the responses Ms. S. offered and her students' subsequent mathematical engagement. Ms. S.'s students' behavioral engagement was always coded as on-task following her responses to their difficulties. The students' cognitive engagement, however, varied in complexity of thought and their feelings ranged from highly positive to strongly negative.

Following Ms. B.'s interventions, her students were on-task overall and demonstrated both high and low cognitive activity. Ms. B.'s students' feelings ranged from highly positive to slightly negative. A trend was noted between the responses Ms. B. offered when her students encountered difficulty and the students' subsequent cognitive engagement. In all instances where Ms. B. asked a student to prove his/her solution (*Asking for Proof*), the student subsequently demonstrated high level cognitive activity, where information was available regarding the student's cognition. Following instances where Ms. B. either suggested a process for resolving the difficulty (*Suggesting process*) or led the student to the correct answer/idea (*Leading*), the relevant students showed evidence of low level cognitive activity or were not cognitively engaged.

Trends were found relating students' cognitive and affective mathematical engagement. In instances where a student's affect was coded as involving strong positive feelings, the student's cognitive activity was always coded as high level, where information was available regarding the student's cognition. (There were many instances, however, of students' demonstrating mild or moderate positive feelings and low level cognitive activity.) In all but one instance where a student's affect was coded as involving negative feelings and information was available regarding the student's cognition, the student's cognitive activity was coded as low level or not cognitively engaged.

Detailed analysis of students' cognitive engagement in the mathematics task before and after teachers' responses to their difficulties revealed that students often reverted to their original ways of thinking, whether correct or not, after the teacher left their table. In these instances, students did not follow mathematical pathways suggested by their teachers or classmates, but retained, and in some cases continued to explore, their previous mathematical ideas. Occurrences of this phenomena involved students in all three of the classes examined in this research (see above, *Qualitative In-Depth Analysis of Students' Cognitive Engagement,* Ms. A.: Amanda, Difficulty 1; Amanda, Emanuel, Eliot, Juan, Difficulty 3; Ms. B.: Marcos, Difficulty 16; Ms. S.: Ordena, Difficulty 23).

Resolution of Mathematics Difficulties

The 9 episodes for which students' cognitive engagement was examined in detail prior and subsequent to the teachers' intervention were analyzed to determine whether students correctly resolved their mathematics difficulties by the end of the relevant class session. Analysis of students' mathematical activity revealed that in 5 of these instances, there was evidence that students resolved the difficulties by the end of the day's session. In 3 of the cases, there was no evidence that students resolved the difficulties, although in 2 of those cases (Ms. B., Difficulty 16- Marcos & Ms. S., Difficulty 23- Ordena) the students did not present their incorrect ideas during their groups' presentations, possibly since they realized their ideas were not valid. The third case in which there was no evidence of difficulty resolution involved Leo during his group's presentation to the class in Difficulty 29. In that instance, there was indication that Leo realized his solution was incorrect as he no longer expressed pursuit of his idea after most of his classmates agreed with Ta'keisha's and Ordena's solution. There was no evidence, though, that he understood *why* Ta'keisha's and Ordena's solution was correct. The single case where it appeared students did not resolve the difficulty by the end of the class session involved Difficulty 3 in Ms. A.'s class. The difficulty involved Ms. A.'s question to the students about how two patterns they found related to each other. The students appeared unable to answer her question. After Ms. A. left the table, the students did not find an answer to her question; however, they found the general solution to the Building Blocks Problem using another method.

Students' Collaborative Effort and Affect

The students in Ms. A.'s class analyzed in this project demonstrated collaborative effort while working to solve the Towers Problem. The students' respect for and responsibility towards each other was most evident when a group member needed assistance in understanding the mathematical ideas proposed by (an)other group member(s). In these cases, the students did not belittle or show impatience towards each other, but rather explained their ideas with the goal of ensuring that all group members have shared mathematical understanding. In this group, no codes of 'negative feelings' were assigned to the affective engagement inferred from the students' observed behaviors on videotape.

. For the groups examined in Ms. B.'s and Ms. S.'s classes, a lack of unified effort towards solving the Towers Problem was evidenced. In the group of Ms. S.'s class, the students did work together at times, explaining or discussing their mathematical ideas. One student, however, repeatedly demonstrated a lack of respect for another when he had difficulty understanding the mathematics problem. In both Ms. S.'s and Ms. B.'s classes, codes of 'negative feelings' were assigned to students' affective engagement inferred from the students' observed behaviors on videotape.

Heterogeneous Grouping and Students' Feelings of Inferiority

Several instances were noted where students showed evidence of inferiority to another. One instance involved Juan following Difficulty 3 when he appeared to have difficulty understanding the general solution another group member had found. On the questionnaire Juan completed that day, he reported feeling *Not At All* confident, and *Somewhat* worried, embarrassed, and afraid.

The other cases of evidence of inferiority involved Leo, following Difficulties 24, 25, 28 (Day 2), and 29 (Day 3). On Leo's Day 3 questionnaire, he reported, "I thought I was going to be imbarrist." Leo also wrote on his questionnaire, "I felt like I was being bossed around by some one in my group."

Both students who showed evidence of inferiority were students who appeared to be of weak mathematics ability (Ms. S. also noted in her interview that Leo was a weak mathematics student) and worked in a group with (an)other student(s) of stronger mathematics ability. This difference in mathematics ability among group members may have contributed to the students' feelings of inferiority. During her interview, Ms. B. mentioned risks to students' self-esteem associated with heterogenous group placement. She cautioned:

> I think you run some dangerous ground if you have like heterogeneous grouping where you have the high-low kid, high-low kid going on, because even when they're trying to be nice it can come off as patronizing… I try to keep them [the students] grouped relatively on par with each other so nobody's going to look at the other person and say, 'I'm so stupid'.
>
> (Appendix D, Ms. S. Interview Transcript, lines 338-345)

Ms. S.'s student Leo may have experienced feelings of inferiority despite Ms. S.'s self-reported approach of using homogenous grouping since Ms. S. did not assign the student groups for the sessions analyzed here. Student groups were created through random assignment for purposes of the larger study.

Students' Self-Reported Emotions vs. Emotions Inferred from Their Observed Behaviors

The following table lists the Positive/Negative subscale means for the students in Ms. A., Ms. B., and Ms. S.'s classes. The scores were calculated using the students' self-reports of the extent to which they felt a given set of emotions during that day's mathematics session (see Chapter 3: Methodology). Scores are listed for each day only for students involved in analyzed instances of difficulty on that day.

Table 4.31, Positive/Negative Subscale Means, Ms. A., Ms. B, Ms. S.

	Day 1	***Day 2***	***Day 3***
Emanuel	*1.95*		
Amanda	*1.91*		
Eliot	*N/A*		
Juan	*1.33*		
Pos/Neg Subscale Mean: Ms. A.	***1.73***		
Marcos	*1.50*	*1.80*	
Larent	*1.45*	---	
Lashanna	*1.05*	*1.65*	
Pos/Neg Subscale Mean: Ms. B.	***1.33***	***1.73***	
Ta'keisha	*1.50*	*1.36*	---
Leo	*1.80*	*1.40*	*1.50*
Ordena	*1.85*	*0.79*	---
Pos/Neg Subscale Mean: Ms. S.	***1.72***	***1.18***	***1.50***

Comparison of emotions students reported on their questionnaires and emotions inferred from the students' behaviors observed during analyzed video segments indicated that students' self-reported emotions were at times more positive or negative than the emotions inferred from their observable behaviors. The self-reported emotions of students in Ms. A.'s class were similar to the affect inferred from the students' observed behaviors. For Ms. B.'s and Ms. S.'s classes, however, students' self-reported emotions were often different, and sometimes contradictory to, the feelings inferred from their observed behaviors. The students in Ms. B.'s and Ms. S.'s classes who appeared strongest in mathematics ability among their group members (Lashanna and Ta'keisha, respectively) always reported affect similar to or more negative than the affect inferred from their observed behaviors. The student in Ms. S.'s class who appeared weakest in mathematics ability always reported more positive affect than the affect inferred from his observed behaviors. In two instances involving students in Ms. B.'s class, the students showed evidence of boredom and did not demonstrate strong positive feelings but reported on their questionnaires feeling *Not At All* bored and *Very Much* happy.

Students' Self-Reports of Engagement with Impasse. The following table lists the Engagement with Impasse subscale means for the students in Ms. A., Ms. B., and Ms. S.'s classes. The scores were calculated using the students' self-reports of the extent to which they felt frustrated and confused during that day's mathematics session (see Chapter 3: Methodology). Scores are listed for each day only for students involved in analyzed instances of difficulty on that day.

Table 4.32, Engagement with Impasse Subscale Means, Ms. A., Ms. B, Ms. S.

	Day 1	**Day 2**	**Day 3**
Ms. A.			
Emanuel	*0.00*		
Amanda	*1.00*		
Eliot	*N/A*		
Juan	*0.00*		
Ms. B.			
Marcos	*0.00*	*0.00*	
Larent	*0.00*	*---*	
Lashanna	*0.00*	*0.00*	
Ms. S.			
Ta'keisha	*1.50*	*1.00*	*---*
Leo	*0.50*	*0.00*	*0.00*
Ordena	*0.50*	*1.50*	*---*

As evident in the table above, students who encountered difficulties often reported little or no Engagement with Impasse. Students who appeared to have limited understanding of the mathematics task (specifically, Juan and Leo) reported lower levels of Engagement with Impasse than some students who showed deeper understanding of the task (Amanda and Ta'keisha).

Analysis of the open-ended sections of the students' questionnaires was critical in understanding the students' mathematical engagement. In one instance, Ordena appeared tired and not much interested in the mathematics task. Ordena's report on the questionnaire stating that she was tired and not able to concentrate provided reasoning for her apparent lack of interest in the mathematics problem. On Day 3, Leo showed evidence of loss of face during class. On the closed-ended portion of his questionnaire, however, he reported feeling *Not At All* embarrassed. In an open-ended section, though, Leo indicated that he thought he was going to be embarrassed.

Students' Self-Reports of Encountering Difficulty vs. Observed Difficulties

Analysis of students' self-reports of whether or not they encountered difficulty during each day's session indicated that at times students who were involved in multiple analyzed instances of difficulty on a particular day reported experiencing no difficulty during that day's session.

CHAPTER 5: CONCLUSIONS

5.1 Conclusions

Three research questions framed the present investigation. This chapter presents a summary of the study's major findings by first restating and then addressing each research question.

Research Question #1:

> For each of three urban mathematics teachers (Ms. A., Ms. B, and Ms. S.), what types of responses do the teachers offer to their students who experience difficulty with mathematics problems?

Twenty nine instances of students' difficulty across the three teachers were identified and examined. For each difficulty, the teacher's initial response was coded as well as any follow up responses the teacher provided. Twenty nine initial teacher responses and 11 follow up teacher responses were coded, resulting in a total of 40 coded teacher responses.

Five broad coding categories for teachers' responses emerged from analysis in this study. The categories included: *Having Other Students Help Resolve Difficulty*, *Having Students Help Themselves*, *Helping Students Directly*, *Helping Students Indirectly*, and *Responding without Offering Suggestion for Resolving Difficulty*. Between one and six more specific teacher response codes were included within each coding category.

The five coding categories, in sequence of most frequently to least frequently used, were: *Helping Students Indirectly* (19 events), *Having Other Students Help* (14 events), *Not Responding/Responding Without Offering Suggestion for Resolving Difficulty* (5 events), *Helping Students Directly* (1 event) and *Having Students Help Themselves* (1 event). None of the teachers' initial responses was coded as *Helping Students Directly*. *Helping Students Indirectly* was the response category most frequently used by all three teachers (when examined individually). Ms. S. and Ms. B. also frequently used responses coded as *Having Other Students Help*.

None of the teachers' responses was coded as *Saying a Student's Answer is Wrong* (included in the response category *Responding Without Offering Suggestion*) or *Providing the Correct Answer* (included in the response category *Helping Students Directly*).

Findings indicated that Ms. A., Ms. B., and Ms. S., who were selected for participation in the larger study based on their involvement in professional development aimed at creating emotionally safe classroom environments for students, did not simply provide students with solutions to their difficulties. All the teachers, rather, responded in ways that let students arrive at solutions using varying levels of their own cognitive processing.

Research Question #2:

> During retrospective stimulated recall interviews, how do the teachers explain why they responded in the ways they did to student difficulty with mathematics during the sessions?

Of the 29 instances of students' difficulty described above, nine episodes were investigated further during stimulated recall interviews with the teachers, where the teachers were asked to explain their reasoning for the responses they offered. Five coding categories for teachers' reasoning for their responses emerged from this analysis. These categories included: *Mathematics Goals*, *Students' Emotions in Regard to Mathematics*, *Students' Attitudes*, *Students' Abilities/Skills*, and *Time Constraints*.

The categories most often assigned to describe teachers' reasoning for their responses to students were *Students' Abilities/Skills* (7 episodes assigned this code) and *Mathematics Goals* (6 episodes assigned this code). *Students' Abilities/Skills* applied to situations where a teacher offered a particular response in consideration of a student's mathematics ability or overall organizational skills. *Mathematics Goals* included helping students to develop conceptual understanding of mathematical ideas or helping them to acquire problem solving skills. Two episodes were assigned each of the codes *Students' Emotions in Regard to Mathematics* and *Students' Attitudes*. One episode was assigned the code *Time Constraints*.

Analysis of students' behaviors following the teachers' interventions indicated that in most cases, the teachers' desired results when providing their responses (as explained by the teachers during the interviews, e.g., wanting students to develop conceptual understanding) were achieved. Two exceptions, where teachers' goals were not met, are discussed in Chapter 4.

Comparison of teachers' reports of ways in which they typically respond to students who have difficulty were consistent with the responses the teachers offered during analyzed instances of students' difficulty in this research.

Research Question #3:

> What does evidence show about students' behavioral, cognitive, and affective engagement in mathematics following the teachers' responses to their difficulties with mathematics problems?

At a surface level, no general relationships were noted across all three teachers between the types of teacher responses as coded and the students' subsequent behavioral, cognitive, and affective engagement in mathematics. For Ms. A., these relationships could not be tested since her students always demonstrated deep engagement with mathematics. For Ms. B., a preliminary relationship was noted where her responses coded as *Asking students to prove their solutions* were followed by students' high level cognitive processing (where information was available to code for students' cognitive activity), whereas her responses coded as *Leading students to the correct answer/idea* or *Suggesting a process for resolving the difficulty* were followed by students' showing evidence of low level cognitive processing or no cognitive engagement with mathematics. Ms. B.'s students' cognitive and affective mathematical engagement varied, and no consistent relationships were detected between her interventions and the students' engagement. The relationship noted for Ms. B. suggests that under particular circumstances, responses that are student centered may elicit deeper mathematical engagement than responses that are teacher centered and intend to lead students to particular thinking pathways.

Deeper analysis indicated that student engagement outcomes following teacher responses to their difficulties were student- and context- dependent. In some cases, a student showed evidence of low level cognitive activity following a teacher's response; however, even low level cognitive engagement demonstrated by that student was considered a goal attained for that student (as perceived by the teacher) as the student had difficulty directing and sustaining his attention to tasks. In another instance, a teacher offered one response to all the students in a group who were involved in the same difficulty. The students in the group demonstrated differing types of affective engagement following her response, perhaps since her response was more effective at engaging the high ability students in the group than the student with weaker ability. Interviews with teachers provided insight regarding the effectiveness of particular teacher responses for specific students. As noted above, analysis indicated that teachers' desired outcomes for students were achieved, overall. This finding implies that teachers' strategies when responding to students were effective in most cases. The present study suggests that research aimed at understanding influences of teachers' responses on students' mathematical engagement must reach beyond simply seeking generalizations for types of responses that are most effective in engaging students.

Analysis of codes assigned to describe students' mathematical engagement following teachers' responses to their difficulties revealed connections between students' cognitive and affective mathematical engagement. Students' strong positive feelings were associated with high level cognitive activity. Students' negative feelings were linked to low level cognitive activity or cognitive disengagement from the mathematics task.

Qualitative in-depth analysis of students' cognitive engagement prior and subsequent to episodes of students' difficulty was conducted for the 9 cases regarding which teachers were interviewed. In 3 of the instances (Ms. A., Difficulties 1 & 2 and Ms. S., Difficulty 23), students retained their original pathways of thought after the teachers left their tables following the responses they offered. The students did not follow the suggestions teachers offered as a means to help them resolve the difficulties. In these episodes, following the teachers' interventions, students either simply retained their own original, incorrect ideas or arrived at correct solutions through further exploration of their original ideas, as opposed to the teachers'.

Analysis of students' mathematical activity during these 9 episodes indicated that in 5 of the cases students resolved their mathematics difficulties by the end of the day's session. In the other 4 instances, there was no evidence that the students resolved their difficulties; however, in several of those cases the students discontinued to expressively pursue their mathematical ideas, suggesting they may have recognized their ideas were not correct.

In this research, students' feelings as they worked on mathematics problems were associated with group members' collaborative effort and respect towards each other. In the group where students demonstrated the most positive feelings overall, the students showed responsibility and respect towards each other. There was no evidence of lack of collaboration or disrespect among the group members. The students in the group demonstrated efforts aimed at ensuring that all group members understood the solutions they found and patiently provided explanations to lower ability students without belittling or teasing them. In the single group where a student demonstrated strong negative feelings, there was evidence of disrespect and lack of collaborative effort within the group.

In this research, feelings of inferiority were demonstrated by students in groups where there were notable differences in mathematics ability between group members. The two instances in which feelings of inferiority were evidenced involved students who worked in groups with at least one member who appeared to be of significantly stronger mathematics ability.

Comparison of emotions students reported on their questionnaires and emotions inferred from the students' behaviors observed during analyzed video segments indicated that students' self-reported emotions were at times more positive or more negative than the emotions inferred from their observable behaviors. The self-reported emotions of students in Ms. A.'s class were similar to the feelings inferred from the students' observed behaviors. For Ms. B.'s and Ms. S.'s classes, students' self-reported emotions were often different, and sometimes contradictory to, the feelings inferred from their observed behaviors. The students in Ms. B.'s and Ms. S.'s classes who appeared strongest in mathematics ability among their group members always reported affect similar to or more negative than the affect inferred from their observed behaviors. The student in Ms. S.'s class who appeared weakest in mathematics ability always reported more positive affect than the affect inferred from his observed behaviors.

This research highlights benefits of employing various data collection procedures, specifically, student self-reports, observations of students, and teacher interviews when analyzing students' mathematical engagement. In episodes examined in this study, each of the aforementioned forms of data provided unique perspectives of students' engagement that, when examined as a whole, provided more in-depth, comprehensive understanding of students' engagement. Particular instances are discussed in detail in Chapter 4.

5.2 Discussion

In this study, response types teachers reported they typically use when their students encounter difficulty were similar to the response types they offered to their students during analyzed episodes of students' difficulty. This finding lends support to research that uses teacher reports of their classroom practices as evidence of their actual practices.

In this research, relationships were noted between students' feelings in relation to mathematics and their cognitive engagement in the problem task. These relationships might be better understood through consideration of students' meta-affect (Goldin, 2003; Goldin, Röskin, & Törner, 2009). The notion of affect as a representational system suggests that emotions encode meanings, which in turn, influence students' behavior. One student's feelings of boredom while working on a task he considers easy, for instance, may encode a students' need for further cognitive stimulation, leading to that student's engagement as he searches for a solution to an extension of the problem. Another student's feelings of boredom may reflect her overall apathetic feelings towards mathematics and may lead to her cognitive disengagement from the task. Investigation of meanings encoded by students' emotions in particular situations may provide insight into relationships between students' feelings and cognitive behaviors as they work on mathematics problems, evidenced in this work as well as others' (see McLeod & Adams, 1994).

Meinhardt and Pekrun (2003) described distinctions between influences of intrinsic and extrinsic emotions on cognitive processing. The authors noted that extrinsic emotions, those that relate to the setting, others, or the self, whether negative or positive, drain cognitive resources, while intrinsic emotions, those that relate directly to the task, may increase task attentiveness. Differences in ways extrinsic and intrinsic negative emotions interact with students' cognitive activity were evidenced in this investigation. During Ms. B.'s reflection on an episode involving Lashanna on Day 2 (Difficulty 17), Ms. B. mentioned that Lashanna was upset because of her group placement. Ms. B. said Lashanna had wanted to work in a group with her friends, but Ms. B. had not allowed her to switch groups since according to Ms. B., if Lashanna joined her friends' group, she would likely be distracted from her work. Lashanna's negative feelings that Ms. B. described did not appear to detract from Lashanna's cognitive engagement with the mathematics problem, however, during that day's session, as Lashanna appeared highly interested in solving the problem and arrived at a general solution. Perhaps since Lashanna's negative feelings were extrinsic to task, relating to her group and the teacher as opposed to the mathematics, the feelings did not hinder Lashanna's cognitive involvement with the task. Following Difficulties 25 and 28, Leo (Ms. S.'s class, Day 2) showed evidence of negative feelings associated with mathematics. Leo appeared insecure and embarrassed that he did not understand Ta'keisha's explanation for her solution to the 100- block high tower. During those segments, Leo also showed evidence of low level cognitive activity. Leo's negative emotions possibly detracted from his cognitive engagement with the task since the negative feelings related directly to the mathematics.

Previous research has highlighted students' need to articulate their ideas in order to build meaningful mathematical understanding (Kazemi, 1998; Piccolo, Harbaugh, Carter, Capraro, & Capraro, 2008). Kazemi (1998) found that one teacher who created high "press for conceptual thinking" in her classroom used instances of students' errors as opportunities for students to explore the relevant mathematical ideas and prove that their solutions were correct. A teacher of another classroom, in which low "press for learning" was evidenced, reacted to her students' mathematical errors by providing conceptual reasoning for them. Findings of this project concurred with those of Piccolo et al. and Kazemi. The responses Ms. B. offered in instances of her students' difficulty that were coded as *Asking for proof*, where Ms. B. asked students to justify their solutions, were followed by students' high level cognitive activity (where information was available to permit coding of students' cognitive engagement). Instances where Ms. B. offered responses coded as *Leading* or *Suggesting a process for resolving the difficulty*, where Ms. B. intended to guide students to a particular thought pathway, were followed by students' low level cognitive activity or cognitive disengagement from the task. Analysis of 9 analyzed instances of students' cognitive engagement before and after teachers' interventions to students' difficulties indicated that in 3 cases, students reverted back to their own thinking pathways after teachers left their tables. These students retained or continued to explore their own original ideas and did not follow the teachers' suggestions for resolving their difficulties. This finding illustrates students' difficulty adopting teachers' thought pathways as well as their need to work through their own ideas. Perhaps for this reason students demonstrated higher level thought processing following Ms. B.'s responses of *Asking for proof* than her responses of *Leading or Suggesting a process*. Responses of *Asking for proof* may have elicited students' higher level cognitive activity since those responses encouraged students to articulate and work through their own thinking rather than to adopt the teachers' ideas.

Findings of the study suggested that effectiveness of teachers' responses in engaging students was student- and context- dependent. Evidence pointed to students' mathematics ability, heterogenous vs. homogenous student grouping, students' daily dispositions, and students' personalities as possible contributors to the effectiveness of teacher strategies in fostering students' mathematical engagement. As a teacher of mathematics in both junior high and high school classrooms, the present researcher feels tailoring interventions based on contextual variables is essential in furthering mathematical engagement that leads to students' mathematics success. All three teachers in this study reported using more directed approaches when responding to students with weak mathematics ability. Based on classroom teaching experience, the researcher has found that students with weak mathematics skills, especially those who have had limited problem solving opportunities (where processes necessary to solve mathematics problems have not been prescribed by the teacher, but rather, are left for students to discover), often are easily frustrated when they encounter mathematics difficulties and quickly despair of resolving their dilemmas without direct assistance from the teacher. In these cases, in the researcher's estimation, it is necessary for the teacher to provide guided intervention while gradually withdrawing his/her support over time as the students develop confidence in their problem solving skills. Based on classroom teaching experience, the researcher also feels it essential that teachers consider students' personality traits and social status within the classroom when suggesting students present incorrect mathematical ideas to the class. In the researcher's opinion, students with strong personalities and high social ranking among their peers are more likely than mild mannered students with low social status to experience classmate's rejection of their ideas without loss of face. In this investigation (Difficulty 29), Leo, a reserved student with weak mathematics skills who was bullied verbally by another student while working on the task, showed feelings of embarrassment and inferiority when during his group's presentation, his classmates agreed with his groupmates' solution over his own. These feelings may not have been present had Leo had a more forceful personality or higher social standing within the class.

The researcher has also found through classroom experience that teacher strategies may influence students' long term engagement in mathematics. Teachers' interest in students and high expectations for their success can lead to deeper student engagement in mathematics over time. While this study did not afford opportunities to observe students' long term engagement, the participating teachers' behaviors, such as demonstrating high expectations for students, showing interest in students, and supporting students' autonomy, indicated the possibility of encouraging students' long term engagement.

5.3 Limitations

Due to the methodologies and analytic approach employed in this study, a number of limitations are present. Since this study was qualitative and involved analysis of only three classrooms in three schools in one urban public school district, no generalization to other classroom situations is possible. This investigation documented students' "in the moment" engagement following teachers' interventions. Influences of teachers' strategies on students' engagement over time were not examined.

Only one research assistant was employed to strengthen reliability of coding of teachers' responses and students' behavioral, cognitive, and affective mathematical engagement due to the time involved in analyzing and coding classroom episodes. Also, in this study, students were not interviewed directly regarding their cognition or feelings during class sessions. Students' cognitive engagement was inferred from their written work and their speech. Students' emotional feelings were inferred from their behavioral, facial, and oral expressions of feeling as well as from questionnaires they completed during the class sessions. Students' actual feelings and types of cognitive processing may have been different from the types of cognitive activity and feelings inferred from their observable behaviors.

Emotions students reported having experienced during the class sessions may not have been mathematics related. Also, students completed the Emotions Questionnaires at the end of each class session involved in this analysis. Students reported on the emotions they experienced during the entire class session. It is possible that some of the feelings students reported referred to classroom episodes not analyzed in this study. This may have resulted in some of the discrepancies noted in this work between students' reported emotions and the emotions inferred from their observed behaviors during analyzed episodes of students' difficulty.

During the class sessions involved in this analysis, students were not grouped according to teachers' preferences, but according to random assignment, since other phenomena were being studied for larger project. Students' mathematical engagement while they worked on the mathematics task may therefore have been different than usual.

5.4 Implications

5.4.1 Implications for Further Research

Findings of this investigation indicate that influences of teachers' responses to students' mathematics difficulties on students' engagement are student and context dependent. This study suggests particular factors that may mediate effects of teacher responses on students' engagement, namely, students’ mathematics ability, heterogenous vs. homogenous student grouping, students’ daily dispositions, and students’ personalities. Future research on influences of teachers’ responses should aim to determine contextual factors that contribute to impacts of teachers’ responses on students’ engagement, with particular focus on the aforementioned areas. Research should include interviews with students to ascertain their feelings about teacher interventions that are effective in particular circumstances.

Future research should examine influences of teachers’ interventions on students’ engagement over time. Studies should include students’ perspectives to explore whether students feel their behavioral, cognitive, or affective mathematical engagement changes over time as a result of strategies teachers use when they face mathematical challenges. Investigations should utilize stimulated recall interviews with students so they can reflect on particular interventions teachers offered when they encountered difficulty.

In this study, a pattern was noted for one teacher where responses that were student centered were followed by students’ deeper cognitive engagement than responses that were teacher centered. Larger studies researching influences of teachers’ responses on students’ mathematical engagement should explore whether this pattern is consistent for a larger variety of teachers.

Numerous studies investigating students’ affect have relied primarily on students’ reports of their emotions as indicators of students’ actual feelings (Meyer & Turner, 2002; Pekrun, Goetz, Titz, & Perry, 2002; Skinner et al., 2008). In this study, inconsistencies were evidenced between students’ reported emotions and the emotions inferred from the students’ observed behaviors on videotape, such as when students reported feeling only somewhat interested but demonstrated behaviors suggesting strong interest or reported feeling not at all bored but showed evidence of boredom. More research is therefore necessary which compares students' reports of their emotions to emotions inferred from their behaviors.

5.4.2 Implications for Practice

This project informs professional development aimed at helping teachers create classroom environments conducive to students' engagement in challenging mathematics. Findings of the present study indicated that the participating teachers, who individualized the strategies they implemented during episodes of students' difficulty, were able to maximize their students' engagement in mathematics. This study suggests that teacher training programs emphasize the importance of teachers considering contextual factors when choosing responses to address students' difficulties in mathematics, as students' mathematical engagement following any particular teacher response may vary based upon contextual variables. Among factors teachers should consider when addressing students' difficulties in mathematics are students' mathematics ability, heterogenous vs. homogenous student grouping, students' daily dispositions, and students' personalities.

This investigation also highlights possible academic value of teachers fostering students' positive feelings associated with mathematics, as in this study students' strong positive feelings were associated with high level cognitive processing, while students' negative feelings were associated with low level cognitive activity. In addition, this research underscores the importance of teachers fostering students' collaborative effort and respect toward one another as they work on solving mathematics problems in group settings. As evidenced during this study, students' collaborative work ethic may contribute to group members experiencing positive feelings during their mathematical problem solving.

REFERENCES

Brophy, J. (1983). Conceptualizing student motivation. *Educational Psychologist*, 18, 200-215.

Chin, C. (2006). Classroom interaction in science: Teacher questioning and feedback to student responses. *International Journal of Science Education, 28*(11), 1315-1346.

Crespo, S. (2002). Praising and correcting: Prospective teachers investigate their teacherly talk. *Teaching and Teacher Education*, 18, 739-758.

Connell, J.P., Spencer, M.B., & Aber, J.L. (1994). Educational risk and resilience in African-American youth: Context, self, action, and outcomes in school. *Child Development, 65,* 493–506.

Dance, L.J. (2002). *Tough fronts: The impact of street culture on schooling*. New York: Routledge-Falmer.

DeBellis, V.A., & Goldin, G.A. (1997). The affective domain in mathematical problem solving. In E. Pehkonen (Ed.). *Proceedings of the Twenty-First Annual Meeting of PME, Lahti, Finland: University of Helsinki, 2,* 209-216.

DeBellis, V.A., & Goldin, G.A. (2006). Affect and meta-affect in mathematical problem solving: A representational perspective. *Educational Studies in Mathematics,* 63, 131-147.

Epstein, Y.M., Goldin, G.A., Schorr, R.Y., Capraro, R., Capraro, M.M., & Warner, L.B. (2010). Measuring engagement structures in middle-grades urban mathematics classrooms. *Paper presented at the 2010 annual meeting of AERA*, Denver, CO.

Epstein, Y., Schorr, R.Y., Goldin, G.A., Warner, L., Arias, C., Sanchez, L., et al. (2007). Studying the affective/social dimension of an inner city mathematics class. In T. Lamberg & L.R. Wiest (Eds.), *Proceedings of the 29th Annual Conference of PME-NA,* Lake Tahoe, Nevada: University of Nevada, Reno (pp. 649-656).

Fisher, C., Berliner, D., Filby, N., Marliave, R., Cahen, L., & Dishaw, M. (1980). Teaching behaviours, academic learning time, and student achievement: An overview. In C. Denham & A. Lieberman (Eds.), *Time to Learn*. Washington, D.C.: National Institute of Education.

Fredricks, J.A., Blumenfeld, P.C. & Paris, A.H. (2004). School engagement: Potential of the concept, state of the evidence. *Review of Educational Research, 74*(1), 59-109.

Goldin, G.A. (2000). Affective pathways and representation in mathematical problem solving. *Mathematical Thinking and Learning, 2*(3), 209-219.

Goldin, G.A. (2002). Affect, meta-affect, and mathematical belief structures. In G.C. Leder, E. Pehkonen, & G. Törner (Eds.) *Beliefs: A Hidden Variable in Mathematics Education?* (pp. 59–72). Dordrecht: Kluwer.

Goldin, G.A., Epstein, Y., & Schorr, R.Y. (2007). *Affective pathways and structures in urban students' mathematics learning.* In Pugalee, D. K., Rogerson, A., & Schinck, A., (Eds.). Mathematics Education in a Global Community: Proceedings of the Ninth International Conference of The Mathematics Education into the 21st Century Project, Univ. of North Carolina, Charlotte (pp. 260-265).

Goldin, G.A., Epstein, Y., Schorr, R.Y., & Warner, L.B. (2011). Beliefs and engagement structures: Behind the affective dimension of mathematical learning. *ZDM Mathematics Education, 43*, 547–560.

Goldin, G.A., Röskin, B., & Törner, G. (2009). *Beliefs: No longer a hidden variable in mathematical teaching and learning processes.* In J. Maaas & W. Schlöglmann (Eds.) Beliefs and Attitudes in Mathematics Education: New Research Results. Rotterdam, The Netherlands: Sense Publishers.

Gonzales, P., Williams, T., Jocelyn, L., Roey, S., Kastberg, D., & Brenwald, S. (2008). Highlights from TIMSS 2007: Mathematics and science achievement of U.S. fourth- and eighth-grade students in an international context. National Center for Education Statistics 2009–001, U.S. Department of Education.

Gresalfi, M. (2009). Taking up opportunities to learn: Constructing dispositions in mathematics classrooms. *Journal of the Learning Sciences, 18*(3), 327-369.

Hannula, M.S. (2006). Motivation in mathematics: Goals reflected in emotions. *Educational Studies in Mathematics,* 63, 165-178.

Harkness, S.S., D'ambrosio, B., & Morrone, A.S. (2007). Preservice elementary teachers' voices describe how their teacher motivated them to do mathematics. *Educational Studies in Mathematics, 65*(2), 235-254.

Jackson, C., & Leffingwell, R. (1999). The role of instructors in creating math anxiety in students from kindergarten through college. *Mathematics Teacher, 92*(7), 583-587.

Kazemi, E. (1998). Discourse that promotes conceptual understanding. *Teaching Children Mathematics, 4,* 410–414.

Kloosterman, P. (1988). Self-confidence and motivation in mathematics. *Journal of Educational Psychology, 80*(3), 345-351.

Leder, G., Pehkonen E., & Törner, G, (Eds.) (2002). *Beliefs: A Hidden Variable in Mathematics Education?* Dordrecht: Kluwer.

Lleras, C. (2008). Race, racial concentration, and the dynamics of educational inequality across urban and suburban schools. *American Educational Research Journal, 45,* 886-912.

Lesh, R.A., Hamilton, E., & Kaput, J.J. (Eds.) (2007). *Foundations for the Future in Mathematics Education*. Mahwah, NJ: Lawrence Erlbaum Associates.

Ma, X. (1999). A meta-analysis of the relationship between anxiety toward mathematics and achievement in mathematics. *Journal for Research in Mathematics Education, 30*(5), 520-540.

Ma, X. & Cartwright, F. (2003). A longitudinal analysis of gender differences in affective outcomes in mathematics during middle and high school. *School Effectiveness and School Improvement,14*(4), 413-439.

Malmivuori, M.-L. (2001). *The dynamics of affect, cognition, and social environment in the regulation of personal learning processes: The case of mathematics.* University of Helsinki Dept. of Education Research Report 172. Helsinki, Finland: Helsinki University Press.

Mandler, G. (1989). Affect and learning: Causes and consequences of emotional interactions. In D. McLeod & V. Adams (Eds.), *Affect and mathematical problem solving: A new perspective* (pp. 3-19). New York: Springer-Verlag.

Marshall, C., & Rossman, G. (1999). *Designing qualitative research* (3rd Ed.). Thousand Oaks, CA: Sage.

Maxwell, J.A. (2005). *Qualitative research design* (2nd Ed.). Thousand Oaks, CA: Sage.

McIntyre, D.J., Copenhaver, R.W., Byrd, D.M., & Norris, W.R. (1983). A study of engaged student behaviour within classroom activities during mathematics class. *Journal of Educational Research, 77*(1), 55-59.

McLeod, D. (1989). Beliefs, attitudes, and emotions: New views of affect in mathematics education. In D. McLeod & V. Adams (Eds.), *Affect and mathematical problem solving: A new perspective* (pp. 245-258). New York: Springer-Verlag.

McLeod. D. B. (1992). Research on affect in mathematics education: A reconceptualization. In D. Grouws (Ed.). *Handbook of research on mathematics teaching and learning* (pp. 575-596). NewYork: Macmillan.

McLeod. D. B. (1994). Research on affect and mathematics leaming in the JRME: 1970 to the present. *Journal for Research in Mathematics Education. 25,* 637-647.

Mehan, H. (1979). 'What time is it, Denise?': Asking knowing information questions in classroom discourse. *Theory Into Practice, 18*(4), 285-294.

Meyer, D.K., & Turner, J.C. (2002). Discovering Emotion in Classroom Motivation Research. *Educational Psychologist, 37*(2), 107–114.

Mortimer, E.F., & Scott, P.H. (2003). *Making meaning in secondary science classrooms*. Maidenhead, UK: Open University Press.

Mullis, I.V.S., Martin, M.O., & Foy, P. (with Olson, J.F., Preuschoff, C., Erberber, E., Arora, A., & Galia, J.). (2008). *TIMSS 2007 international mathematics report: Findings from IEA's Trends in International Mathematics and Science Study at the fourth and eighth grades*. Chestnut Hill, MA: TIMSS & PIRLS International Study Center, Boston College.

National Center for Education Statistics. (2009). The nation's report card: Results from the National Assessment of Educational Progress. Retrieved December 28, 2010, from http://nationsreportcard.gov/math_2009/

National Research Council. (2001). *Adding it up: Helping children learn mathematics.* J. Kilpatrick, J. Swafford, & B. Findell, (Eds). Washington, D.C.: The National Academies Press.

National Research Council, Committee on Increasing High School Students' Engagement and Motivation to Learn. (2003). *Engaging schools: Fostering high school students' motivation to learn.* Washington, D.C.: The National Academies Press.

Nicol, C. (1999). Learning to teach mathematics: Questioning, listening, and responding. *Educational Studies in Mathematics, 37*, 45-66.

OECD. (2010). PISA 2009 results: What students know and can do – student performance in reading, mathematics and science, Volume I. Retrieved from http://dx.doi.org/10.1787/9789264091450-en

Pekrun, R. Frenzel, A.C., Goetz, T. & Perry, R.P. (2007). The control-value theory of achievement emotions: An integrative approach to emotions in education. In R. Pekrun, & P.A. Schutz (Eds.), *Emotion in education* (pp. 13-36). San Diego: Academic Press.

Pekrun, R., Goetz, T., Titz, W., & Perry, R. P. (2002). Academic emotions in students' self-regulated learning and achievement: A program of qualitative and quantitative research. *Educational Psychologist, 37*, 91-105.

Pekrun, R., Hochstadt, M., & Kramer, K. (1996). Prüfungsemotionen, Lernen und Leistung [Test emotions, learning, and achievement]. In C. Spiel, U. Kastner-Koller, & P. Deimann (Eds.), *Motivation und Lernen aus der Perspektive lebenslanger Entwicklung* (pp. 151–162). Münster, Germany: Waxmann.

Pekrun, R., & Hofmann, H. (1999). Lern- und Leistungsemotionen: Erste Befunde eines Forschungsprogramms [Emotions in learning and achievement: First results of a program of research]. In R. Pekrun & M. Jerusalem (Eds.), *Emotion, Motivation und Leistung* (pp. 247–267). Göttingen, Germany: Hogrefe.

Pekrun, R., Molfenter, S., Titz, W., & Perry, R. P. (2000). *Emotion, learning, and achievement in university students: Longitudinal studies.*

Paper presented at the annual meeting of the American Educational Research Association, New Orleans, LA.

Piccolo, D.L., Harbaugh, A.P., Carter, T.A., Capraro, M.M., & Capraro, R.M. (2008). Quality of instruction: Examining discourse in middle school mathematics instruction. *Journal of Advanced Academics, 19*(3), 376–410.

Pintrich, P.R., & De Groot, E.V. (1990). Motivational and self-regulated learning components of classroom academic performance. *Journal of Educational Psychology, 82*(1), 33-40.

Schorr, R.Y., & Goldin, G.A. (2008). Students' expression of affect in an inner-city simcalc classroom. *Educational Studies in Mathematics, 68*(2), 131-148. Special Issue, Lesh, R. & Hegedus, S. (Eds.), Democratizing Access to Mathematics through Technology: Issues of Design, Theory, and Implementation – In Memory of Jim Kaput's Work.

Schorr, R., Warner, L., Gearhart, D., & Samuels, M. (2007). Teacher development in a large urban district: The impact on students. In Lesh, Kaput & Hamilton (Eds.), *Real-world models and modeling as a foundation for future mathematics education* (pp. 431-447). Mahwah, NJ: Lawrence Erlbaum Associates.

Schweinle, A., Meyer, D.K., & Turner, J.C. (2006). Striking the right balance: Students' motivation and affect in elementary mathematics. *The Journal of Educational Research, 99*(5), 271-293.

Skinner, E.A., & Belmont, M.J. (1993). Motivation in the classroom: Reciprocal effects of teacher behaviour and student engagement across the school year. *Journal of Educational Psychology, 85*(4), 571-581.

Skinner, E., Furrer, C., Marchand, G., & Kindermann, T. (2008). Engagement and disaffection in the classroom: Part of a larger motivational dynamic? *Journal of Educational Psychology, 100*(4), 765–781.

Skinner, E.A., Kindermann, T.A., & Furrer, C.J. (2009). A motivational perspective on engagement and disaffection: Conceptualization and assessment of children's behavioral and emotional participation in academic activities in the classroom. *Educational and Psychological Measurement, 69*(3), 493–525.

Sinclair, J.M., & Coulthard, R.M. (1975). *Toward an analysis of classroom discourse.* New York: Oxford University Press.

Stein, M., Smith, M., Henningsen, M., & Silver, E. A. (2000). *Implementing standards-based mathematics instruction*. New York: Teachers College.

Stipek, D., Salmon, J.M., Givvin, K.B., Kazemi, E., Saxe, G., & MacGyvers, V.L. (1998). The value (and convergence) of practices suggested by motivation research and promoted by mathematics education reformers. *Journal for Research in Mathematics Education, 29*(4), 465-488.

Titz, W. (2001). *Emotionen von Studierenden in Lernsituationen* [University students' emotions at learning]. Münster, Germany: Waxmann.

Tobias, S. (1993). *Overcoming math anxiety*. New York: W.W. Norton & Company, Inc.

Turner, J.C., Cox, K.E., DiCintio, M., Meyer, D.K., Logan, C., & Thomas, C.T. (1998). Creating contexts for involvement in mathematics. *Journal of Educational Psychology, 90*(4), 730-745.

Turner, J. C., Meyer, D. K., Midgley, C., & Patrick, H. (2001). *The relationship between teachers' affective discourse and students' approach and avoidance beliefs and behaviors in mathematics: What happens in high mastery/high performance classrooms?* Paper presented at the annual meeting of the American Educational Research Association, Seattle,WA.

Volet, S., Summers, M., & Thurman, J. (2009). High-level co-regulation in collaborative learning: How does it emerge and how is it sustained? *Learning & Instruction, 19*(2), 128-143.

Wilkins, J. & Ma, X. (2003). Modeling change in student attitude toward and beliefs about mathematics. *The Journal of Educational Research, 97*(1), 52-63.

Zan, R., Brown, L., Evans, J., & Hannula, M.S. (Eds.) (2006). *Affect in mathematics education.* Special issue of *Educational Studies in Mathematics, 63*(2).

Appendix A: Questionnaire Data and Videotape Transcripts, Descriptions, and Codes Assigned: 29 Analyzed Instances of Students' Difficulty

Ms. A.
Class- 232: Honors Track, Robertino Hervas School, Day 1
Group 2: Emanuel, Amanda, Eliot, Juan*

**Names of all schools and students in this work are pseudonyms. Ethnic origins of pseudonyms are similar to those of actual names.*

Students' Reports on Emotions Questionnaire

Emanuel:

Overall Positive Emotions Score (40=most positive): 39

"Engagement with Impasse" Score (4=most engaged with impasse): 0

Reports of Emotions:

	Very Much:	Somewhat:	Not at All:
Positive/Negative Subscale	*Positive (2 points):* Interested Respected Successful Safe Excited Happy Satisfied Relieved Confident Curious	*Positive (1 point):* Proud	*Positive (0 points):*
	Negative (0 points):	*Negative (1 point):*	*Negative (2 points):* Unhappy Disappointed Worried Discouraged Angry Disrespected Bored Embarrassed Afraid

<table>
<tr><td>Engagement with Impasse Subscale</td><td>(2 points)</td><td>(1 point)</td><td>(0 points)
Frustrated
Confused</td></tr>
<tr><td colspan="4">Reports of Difficulty:

Had difficulty? No.
Classroom context of difficulty: Not applicable
What stands out about the situation? (No response)</td></tr>
</table>

Amanda:

Overall Positive Emotions Score (46=most positive*): 44
**Three feelings added by student*

"Engagement with Impasse" Score (4=most engaged with impasse): 2

Reports of Emotions:

	Very Much:	***Somewhat:***	***Not at All:***
Positive/Negative Subscale ***Feelings added by student*	*Positive (2 points):* Interested Respected Proud Successful Safe Excited Happy Satisfied Relieved Confident Curious **Jubilant	*Positive (1 point):*	*Positive (0 points):*
	Negative (0 points):	*Negative (1 point):* **Anxious **Nervous	*Negative (2 points):* Unhappy Disappointed Worried Discouraged Angry Disrespected Bored Embarrassed Afraid
Engagement with Impasse Subscale	*(2 points)*	*(1 point)* Frustrated Confused	*(0 points)*

Reports of Difficulty:

Had difficulty? Yes.
Classroom context of difficulty: Group.
What stands out about the situation? "What other kids said or did because there was a lot of thought in my group."

Eliot: *[Questionnaire not available]*

Juan:

Overall Positive Emotions Score (42=most positive*): 28
**One feeling added by student*

"Engagement with Impasse" Score (4=most engaged with impasse): 0

Reports of Emotions:

	Very Much:	***Somewhat:***	***Not at All:***
Positive/Negative Subscale ***Feeling added by student*	*Positive (2 points):* Respected Proud Safe Happy Curious	*Positive (1 point):* Interested Successful Excited Satisfied	*Positive (0 points):* Relieved Confident
	Negative (0 points): **Nervous	*Negative (1 point):* Worried Bored Embarrassed Afraid	*Negative (2 points):* Unhappy Disappointed Discouraged Angry Disrespected
Engagement with Impasse Subscale	*(2 points)*	*(1 point)*	*(0 points)* Frustrated Confused

Reports of Difficulty:

Had difficulty? Yes.
Classroom context of difficulty: Group
What stands out about the situation? "Nothing."

Event 1 Difficulty1: Amanda (i.e., the identified difficulty is Amanda's) Difficulty 2: Amanda Difficulty 3: Emanuel, Amanda, Eliot, Juan	
Transcript of Event **Time (of day): (10:55:30-11:02:10)**	The students at table 2 have been working diligently on the task. They have found a pattern where they can add 5 to the total number of blocks in one tower to obtain the total number of blocks in the next tower. They have found the total number of blocks in towers 1 through 10. Approximately 25 minutes into the class session, Amanda raises her hand and calls over Ms. A. to show her the work her group has done. Amanda: Ms. A., we found a pattern. (*Ms. A. walks over to her desk.*) And we can use it, and we can use it, instead of drawing, Emanuel: Instead of drawing… Amanda: Instead of drawing everything until a hundred we just used a pattern. Ms. A.: Ok, so… Amanda: We had, see, cuz… Emanuel: The numbers are increasing by 5. Eliot: Yeah, yeah. You add by 5, yeah. Amanda: The number of blocks you need to build them are increasing by 5. Ms. A.: Ok. Emanuel: So, like, the 6th…if this is the 5th one, it's 21 blocks, so then the 6th one… Amanda: You add 5, it would give you 26, then you add 5 to that, it would be 31, then for the seven it would be… Juan: It goes up by fives Ms. A.: Ok. Emanuel: And then, when we get to the 10 block tower Amanda: It would be…you will need 46 blocks to build a 10 block tower. Emanuel: 10 block tower. Ms. A.: Ok, so…what about the 100 block tower? Emanuel: That we said that we're gonna have to do. Amanda: But I was thinking, right, can't you double…but we tried to double this one (*pointing to her paper*), but this was…cuz if you…if you double it and you make it…it would be…if you multiply this…(*writes on her paper*) Juan: Yeah, just double it. Ms. A.: (*to Amanda*) That's interesting. What are you doing right here? (*points to Amanda's paper*) Juan: Just double the number. Ms. A.: What do you mean by height here equals 5 and sides is 4 (*points to Amanda's paper*)? Emanuel : Cuz the height is 5 and each side… Eliot: (*while Emanuel is talking*) Yeah, cuz the height is 5 and on each sides have 4 blocks.

Amanda: (*completing Emanuel's thought*) has 4 cubes

Juan: It's increasing by one, right?

Ms. A.: (*without looking up from her paper*) Ok.

Amanda: But I was thinking, right, to get the 10 block, why don't we…couldn't we just multiply the number of blocks to get the 10. Multiply by 2 (*She holds up 2 fingers*)?

Ms. A.: Why don't you try it? Try it and we'll see if it's going to give you this answer…the same answer.

Amanda: Ok, ok.

Ms. A.: So what…what are you going to multiply in this case?

Juan: 5 times 2

Amanda: 5 times 2, which is the height, and the height would be 10…and then there would be 8 blocks on each side.

Ms. A.: So, now, what's the total number of blocks for this one?

Amanda: (*smiles*) 8, 16, 24, 32…It would be 42, not 46.

Ms. A.: So, which one is correct?

Emanuel: So let's just do the pattern that we…

Amanda: The pattern is right.

Emanuel: Yeah, the pattern is right, but for the 10 block tower. Now we have to figure out how much blocks we need for the 100 tower. (*Amanda nods*) Cuz the 6 one would be…(*leans forward and writes on his paper*).

(***See picture at 10:57:35. The students are engrossed in their work.***)

Amanda: (*leans her head in her hand and studies her paper*) Can we see…was thinking…10 times 10 is a hundred, right? 100, of course.

Ms. A.: So to find the what? Multiply 10 by 10, what do you get?

Juan, Eliot, Emmanuel: 100

Eliot: 100 blocks

Amanda: Which is…the height, but if we multiply this by 10, it would be…four hundred and sixty (*creases her eyebrows*)?

Ms. A.: For which, eh, tower?

Amanda: For the 100th tower, yeah.

Ms. A.: But did this method work here?

Amanda: No.

Ms. A.: We were able to check with the tower number 5, 5- blocks tower, and the 10th block tower, and it didn't work.

(***While Ms. A. is speaking to Amanda, Emanuel is punching numbers into his calculator, Eliot is writing on his paper, and Juan leans over to listen to the conversation. See picture at 10:58:27.***)

Amanda: So…that's…

Emanuel: Yeah, so it would be…(*licks his lips*)

Amanda: I guess I'm gonna do it all the way up to 100…It's gonna take a long time (*laughs*). (***See picture at 10:58: 43***)

Juan: All the way to 100 will take a long time.

Emanuel: Cuz I was thinking…I was thinking if…(*leans forward and focuses*

on his paper)…increases by…Ms. A.: Do you think that's the only way?
Juan: All the way up to 160.
Amanda: I don't think…I think you could probably find a different pattern to just add…(inaudible)
Emanuel: Hold on, hold on, hold on…Cuz 100 blocks is the height, right? 100. So, 1, 1, 2, 3, 4 , 1, 2, 3. Yeah, each…each…(*leans his head on his hand, studying his paper*) Each time they add 4 blocks, I mean one block on each side…I'm thinking if there's a pattern there also…
Eliot: Yeah…I just…I got a hang-up that right there (*he smiles*).
Ms. A.: Ok, but what's it saying here? What's it saying here?
Emanuel: Like, like, like…um…we already know that the height of 100 is 100 blocks if we find a pattern, like from the sides…then we could figure out how much the side would be for the hundred block and then we add it all up together.
Ms. A.: So why don't you check it on the first few stages? See if it works and then let's see if we can work on it with a bigger number of blocks in height. So, apply what you're saying on, for instance, the 5 blocks…tower. See if it works.
(*All the students begin writing*.)
Emanuel: (*pointing to diagrams on his paper with his pencil*) 1, 2, 3 ,4. 1, 2, 3, 4, 1, 2, 3, 4. There's zero sides here. There's 4 sides…4 side blocks. There's 1, 2 , 3, 4, 5, 6, 7…Ohhh!
Eliot: For some reason
Emanuel: Hold on, hold on…
Eliot: (*continuing*) here it's going by f…
Amanda and Emanuel: It's going (*Eliot chimes in*) by 4's!
Eliot: I knew it.
Eliot and Emanuel: It was going by 4's.
Ms. A.: What's going by 4's?
Amanda: The sides.
Eliot: The sides of the towers are going by 4's.
Emanuel: (*while Eliot is speaking*) The side blocks are going by 4's. You see, here it was zero. And then here it was 1, 2, 3, 4. And here it was…
Amanda: It was 8.
Eliot: First it was 1, then 4, then 8.
Emanuel: (*continuing his thought*) 1, 2, 3, 4, 5, 6, 7, 8, and then here it was 1, 2, 3…
Amanda and Eliot: 12
Emanuel: 12. It was going by 4. And here it would be…
Amanda: And the next one is gonna be 16.
Emanuel: yeah
Eliot: 16
Amanda: So another pattern. Side…
Eliot: It's going by 4's.
Emanuel: So we found out another pattern.
Eliot: First it was 5, then it was 4.

	Amanda: So the sides are going by 4 and then we have… Ms. A.: (*to Eliot*) You said first it was 5 and now it's 4? Eliot: No, no, no… Juan and Emanuel: No, no, no…(*they gesticulate with their hands, showing towers*) Juan: The height was 5, the height was 5. Emanuel: No, no, no, no. Not the height. The height. The total blocks.(Emanuel reaches his hands forward, showing he thinks Juan is wrong. ***See picture at 11:01:16***.) The total amount of blocks of each one are increasing by 5's. Ms. A.: Ok, so the total number of blocks is increasing by 5. Juan: yeah Ms. A.: Can you show that to me? Juan: yeah Emanuel: Yes. Ms. A.: Where does it show? Emanuel: Because, for example… Ms. A.: uh huh. (*She moves closer to Emanuel and looks over his shoulder)* Emanuel: (*pointing to his paper. Eliot leans forward so he can also see Emanuel's paper*) Here's 1. 1 plus 5 is 6. 6 plus 5 is 11. Ms. A.: Ok. Emanuel: 11 plus 5 is 16. Eliot: (*still leaning forward*) 16. Emanuel: 16 plus 5 is Emanuel and Amanda: 21 Amanda: 21 plus 5 is 26. Eliot: 26 Ms. A.: Ok, so now, how is that related to what you're saying increasing 4 on the sides? Emanuel: Cuz (*smiles*) cuz…uh…(*leans forward with his hands on his chin. All the other students at the table look at their papers.* ***See picture at 11:01:59)*** (*Eliot sneezes*.) Emanuel: Bless you. Eliot: Thank you. (*The students are quiet for a while*.) Ms. A.: Think of it and I'll come back to you. Ok? (*Ms. A. leaves the table*.)
Description of Student Difficulty	1) Amanda proposes multiplying the number of blocks in one branch of the 5-block high tower by 2 to find the number of blocks in one branch of the 10-block high tower [lines 30-32, 45-47]. 2) Amanda proposes multiplying the total number of blocks in the 10-block high tower by 10 to find the total number of blocks in the 100-block high tower [lines 64-72].

	3) Ms. A. asks the students how the pattern they found where the total numbers of side blocks in the towers increase by 4's relates to the pattern they found where the total numbers of blocks increase by 5's. Emanuel leans forward with his hands on his chin. All the other students at the table look at their papers. The students are quiet for a while [lines 160-164, 168].
Type of Student Difficulty	1) Incorrect idea 2) Incorrect idea 3) Unable to answer
Type of Task	1) ***Open Ended /Closed:*** Open-ended ***Routine/Non-routine:*** Non-routine 2) ***Open Ended /Closed:*** Open-ended ***Routine/Non-routine:*** Non-routine 3) ***Open Ended /Closed:*** Open-ended ***Routine/Non-routine:*** Non-routine
Context of Student Difficulty	1) Small group 2) Small group 3) Small group
Teacher's Response(s)	1) Ms. A. tells Amanda to try solving for the 10-block high tower using her method to determine if her method is correct. When Amanda obtains a different answer using her method (42 blocks, instead of 46), Ms. A. asks her which answer is correct [lines 48-57]. Code: Offering a suggestion of a process to use in solving the problem 2) Ms. A. reminds Amanda that the multiplying method didn't work for the 5 and 10 block high towers [lines 74-77]. Code: Offering a counter example to an incorrect response 3) Ms. A. tells the students to think about it and says she will come back to them [lines 169-170].

	Code: Giving the students time to think about the problem
Teacher's Explanation for Response	1) Ms. A. wanted the students to make their own decision. After they spent time working on the problem, they would have ownership of the idea, retain it better, and be able to apply it better. Ms. A. also wanted them to develop the ability to check their own solutions. *Response influenced by mathematics ability*? Yes. Ms. A. asked the students higher order thinking questions right away since they are bright mathematics students. Code for teacher intention: M-Problem Solving Skills M-Conceptual Understanding Ab/S-Mathematics Ability 2) N/A Code for teacher intention: N/A 3) Ms. A. wanted to give the students time to think without pressure. *Response influenced by mathematics ability*? Yes. Since they are bright students, Ms. A. gave them time to work on their own. She was confident that they will get to a solution without much teacher intervention. Code for teacher intention: E-Working without Pressure Ab/S- Mathematics Ability
Description of Behavior Related to Student's Engagement Following Response	1) Amanda and Emanuel say that the pattern they found is the correct method. Emanuel says the pattern works for the 10-block high tower but now they need to solve the 100-block high tower. The students then get to work trying to solve the 100-block tower. Emanuel writes on his paper while Juan looks on, and Amanda studies her paper. Amanda proposes multiplying the blocks in the 10-block high tower by 10 to find the number of blocks in the 100-block high tower [lines 58-73]. 2) After Amanda realizes her multiplying method won't work to find the solution to the 100-block high tower, she says she'll have to keep adding 5's to solve the 100-block high tower. Amanda then says she thinks she could find a different pattern so she won't have to add 5's all the way up to 100. Emanuel studies his paper and says he thinks if they found a pattern for the side blocks they could figure out how many side blocks are in the 100-block high tower and add those to the height, which is 100. Ms. A. tells Emanuel to

	apply his idea to the first few stages and see if it works. Emanuel counts the side blocks in the stage A, B, and C diagrams on his worksheet. He says, “There’s zero sides here. There’s 4 sides…4 side blocks. There’s 1, 2 , 3, 4, 5, 6, 7…Ohhh.” Amanda, Emanuel, and Eliot explain to Ms. A. the pattern they found where the total number of side blocks in consecutive towers increase by 4’s. Eliot says, “First it was 5, and now it’s 4.” Juan says the height was 5. Emanuel says the height was not 5; the total amount of blocks in each tower is increasing by 5’s. Ms. A. asks if the students can show that to her. Emanuel points to his paper and tells Ms. A., “Here’s 1. 1 plus 5 is 6, 6 plus 5 is 11, 11 plus 5 is 16, 16 plus 5 is 21.” [lines 81-158]. 3) After Event 1 above, the students work on solving the 100-block high tower. Emanuel writes on his paper while the other students at the table watch him. He says, “4 times 2 is 8, 4 times 3 is 16…” Amanda says, “Wait a minute.” Eliot smiles and says, “I’m finding a pattern.” Smiling, Amanda says, “Wait, wait, wait…that was my idea.” Amanda, Eliot, and Hemmanuel laugh. **See picture at 11:02:33.** Emanuel notices that the total number of side blocks in the first tower is 4 times 0, in the second tower it is 4 times 1, in the third tower it is 4 times 2, etc. He says that if you multiply it [4] by 9, you would probably get the answer. He shares his pattern with his group, and the students listen intently. **See picture at 11:02:42**. Amanda says, “I understand what he’s doing because you’re doing 4 times…” Emanuel says, “But at the same time I’m not sure if that would work.” Pointing to Amanda’s paper, Eliot says, “Here we have 4, here we have 8, and there we…” Amanda points to her paper while all the students watch. She says, “4 times 1 is 4, 4 times 2 is 8. I think it’s like...” Juan says, “4 times 3, 4 times 4, 4 times 5.” While all the students watch, Amanda points to her paper and explains that the first diagram on the sheet would be figure one, the second would be figure 2, etc. She says, “So you’re off by 1.” Emanuel says, “I know.” Amanda says, “I think it would be 4 times 99.” Emanuel says, “Yeah, yeah. I just said that cuz I was thinking about 10 blocks, I’m sorry.” Amanda says, “4 times 99…” She and Emanuel punch numbers into their calculators. Eliot, who has already found the product with his calculator, says, “396.” Emanuel says that 396 plus 100 would be 496. Eliot says, “But how…?” Emanuel explains to Eliot that 396 would be the number of side blocks and 100 would be the height. Eliot says, “Yeah. You got it.” Emanuel says to Juan, “You get it? You get it?” Juan does not look up from his paper. He shakes his head from side to side, indicating that he does not. Emanuel

	tells Juan that the side blocks 'times by 4." Juan does not look up from his paper. He continues writing. He then punches numbers into his calculator. Amanda says to Emanuel, "Yay! We're the first group who got it! Give me high five." Eliot raises his hand to call Ms. A. Juan looks at Eliot's paper and asks him what he multiplied. Eliot points to his paper and explains to Juan that he multiplied 4 by 99 and got 396 and then added 100 and got 496. Juan writes on his paper and nods. He punches numbers into his calculator. He does not smile. The students check to see whether their method works for the 10-block high tower. Emanuel explains to the other students that 9 times 4 is 36, plus 10 is 46, which matches the solution they obtained previously by following the pattern they had found involving the total number of blocks in consecutive towers. Eliot asks Emanuel to repeat his explanation for finding the number of blocks in a 10- block high tower. Emanuel explains to Eliot that the number of side blocks in consecutive towers increases by 4. He says that for the first diagram on the sheet, the number of side blocks is 0, for the second diagram, the number of side blocks is 4, which is 4 times 1. Juan has leaned over to Eliot's desk to listen to Emanuel's explanation. He says, "And right here it's 8 [referring to the number of side blocks in Tower C]." Amanda says, "Yay!" Juan, Eliot, and Emanuel sit up straight in their chairs and smile. Emanuel says, "we're smart." Emanuel and Amanda wave their hands in the air and call Ms. A. to come to their table (**see picture at 11:06:43**). Amanda smiles and tells Emanuel to lower his voice when he calls Ms. A. When Ms. A. comes over, the students lean towards Emanuel's paper and watch closely as Emanuel explains their solution to the 100-block high tower. **See picture at 11:06:56.** Ms. A. asks the students to check if their method is correct. Emanuel explains to Ms. A. that the solution for an 11-block high tower is the same when calculated by using the group's method of multiplying one less than the height by 4, and then adding the height, and when obtained by following the "increasing by 5's" pattern. Difficulty #4 occurs after the dialogue above.
Inference of Student's Emotions Based on Description	1) High level of interest, Determination to solve mathematics task in anticipation of success 2) High level of interest, Determination to solve mathematics task in anticipation of success 3) Em: High level of interest, Excitement at discovering solution, Eagerness to share solution with teacher, Pride in work, Confidence A: High level of interest, Excitement at discovering solution, Eagerness to share solution with teacher, Pride in work, Confidence El: High level of interest, Excitement at discovering solution, Eagerness to share solution with teacher, Pride in work, Confidence J: High level of interest, slight feelings of inferiority

Behaviors Pertaining to Student's *Behavioral* Engagement and Resulting Classification	1) On-task *Answering teacher's question, Explaining idea to group mate, Looking at paper.* 2) On-task *Writing, Explaining idea to group mates and teacher* 3) Em: On-task *Explaining idea to group mates, Watching group mate explain idea, Looking at paper, Writing, Using calculator, Explaining idea to teacher* A: On-task *Explaining idea to group mates, Watching group mate explain idea, Looking at paper, Writing, Using calculator* El: On-task *Watching group mate explain idea, Looking at paper, Writing, Using calculator* J: On-task *Watching group mate explain idea, Looking at paper, Writing, Using calculator*
Behaviors Pertaining to Student's *Cognitive* Engagement and Resulting Classification	1) High level cognitive activity *Highest level of cognitive engagement involved: Speculating* 2) High level cognitive activity *Highest level of cognitive engagement involved: Reasoning, Speculating* 3) Em: High level cognitive activity *Highest level of cognitive engagement involved: Reasoning, Speculating, Justifying, Drawing inferences, Explaining in one's own words* A: High level cognitive activity *Highest level of cognitive engagement involved: Reasoning, Speculating, Justifying, Drawing inferences, Explaining in one's own words* El: High level cognitive activity *Highest level of cognitive engagement involved: Help seeking for understanding, Explaining in one's own words* J: Insufficient information
Emotions Pertaining to Student's *Affective* Engagement and Resulting Classification	1) Strong positive feelings *High level of interest with no apparent negative feelings, Determination to solve mathematics task in anticipation of success* 2) Strong positive feelings *High level of interest with no apparent negative feelings, Determination to solve mathematics task in anticipation of success* 3) Em: Strong positive feelings *High level of interest with no apparent negative feelings, Determination to solve mathematics task in anticipation of success, Excitement regarding mathematics task, Eagerness to share solution with teacher, Pride in work, Confidence* A: Strong positive feelings

	High level of interest with no apparent negative feelings, Determination to solve mathematics task in anticipation of success, Excitement regarding mathematics task, Eagerness to share solution with teacher, Pride in work, Confidence El: Strong positive feelings *High level of interest with no apparent negative feelings, Determination to solve mathematics task in anticipation of success, Excitement regarding mathematics task, Eagerness to share solution with teacher, Pride in work, Confidence* J: Mild positive feelings *High level of interest mixed with slight feelings of inferiority*

Event 2 Difficulty 4: Emanuel, Amanda, Eliot, Juan	
Transcript of Event **Time (of day): (11:10:34-11:16:59)**	The students explain their method of solving the towers to Ms. A. They explain that to find the number of blocks in a particular tower, they multiplied 4 by one less than the number of blocks in the height and then added the number of blocks in the height. Ms. A.: Now, I want you to write the general rule that you can use for the hundred. Juan: On the back? (lifts his paper) Ms. A.: So what did you do with the hundred, again? Amanda: What we did with the hundred was... Emanuel: What we did was 4 times 99. Eliot: 4 times 99 equals 396. Emanuel: Because one of them is zero. Juan: And then we added Eliot: (continuing while Emanuel is speaking) and then we added one hundred and it equals 496. Ms. A.: (to Amanda) Can you label every number here? (She points to Amanda's paper) So what's Amanda: Ok Ms. A.: the 4? What's the 99? And what's the hundred? Amanda: 4 is the…is side…No, right? (looks at Emanuel) Emanuel: No, the 4 is…(smiles and is quiet) Juan: Is the pattern, right? Emanuel: Yeah, the 4 is like…(pointing to his paper) because for 'A' is zero, so there's one less block that's gonna have, uhh… Ms. A.: Why are you multiplying the 4 by the 99? Emanuel: Because if we do it by 100 it would have been the cor…it will not be the correct answer. Ms. A.: Where does it show? Let's go for…eh…for 5. For block 5. (To Amanda, pointing to her paper) This one is 5? Amanda: Uh hmm. Emanuel: (looking at his task sheet) Yes. Ms. A.: Do it. Apply the same thing here. Eliot: 21 blocks. Emanuel: Ok, 4. Ms. A.: Ok. Eliot: 44 Emanuel: 4 times 4 Amanda: 4 times 4 would be 16 Eliot: 16 Emanuel: 16 (Amanda, Emanuel, and Eliot write on their papers) Plus…plus 5 Juan: equals 21 Amanda: is 21

Emanuel: Yeah. And that's the amount of blocks in the…in 'E.'

Amanda: (nods) it takes to build the height

Eliot: For the 5…

Ms. A.: the 5 block tower

Amanda: Yeah. Uh hmm.

Emanuel: For the 5. Yeah. If you count the blocks that's how much there are.

Ms. A.: So in this case, what would be the 4?

Emanuel: The 4 is the…by…

Juan: is the pattern (points to Amanda's paper)…(inaudible)…see…4…

Eliot: (joins in pointing to Amanda's paper) 4

Eliot and Juan: 8…Six…

Eliot: yeah

Emanuel: (while Eliot and Juan are talking) by…by…by how much it's inc…by…4 is by…the amount…4 is the amount that the sides

Amanda: are increasing by

Juan: (while Amanda talks) are going, increasing by

Emanuel: wait, wait, wait, wait, one moment, not ready yet…not…not, not…

Juan: (pointing to Amanda's paper) 4, 8, 12…

Amanda: (to Juan) no, no, no, no….

Emanuel: No, no. It's the total of (inaudible)…(looking downward to concentrate while gesturing with his hands)…(The students smile. They are having difficulty expressing their thoughts. **See picture at 11:12:18**)…I don't know how to explain it, but it's the total of sides…in each thing or…

Amanda: 4 is what…what you multiply it to increase the…

Juan: Sides

Ms. A.: So (pointing to Emanuel. **See picture at 11:12:30**) say it again. Say it again. Say what you said again. It's the total what?

Emanuel: of sides. Like the total of sides.

Ms. A.: Can you show me wh….

Emanuel: Yeah, like here (points to Amanda's paper) 1, 2, 3, 4 cuz there's 1, 2, 3, 4. And here is 1, 2, 3…8.

Ms. A.: Ok. So 4 is the…sides?

Emanuel: Yeah. 4…

Ms. A.: Now, you multiplied by here, by what? To get the 16.

Emanuel: Oh, yeah.

Ms. A.: What did you multiply by?

Amanda: We multiplied by 4.

Eliot: (with Amanda) by 4

Amanda: Cuz it had

Ms. A.: Why 4?

Amanda: Cuz it

Emanuel: (pointing to Amanda's paper) Cuz, look. This is zero cuz there's no sides.

Ms. A.: I'm talking about this phase here. Let's talk about the same thing.

Amanda: You multiply

	Ms. A.: So you multiplied it by 4
	Amanda: Because there was
	Ms. A.: So you said 4, the first 4 is the 4 sides. And then you said you multiplied that by 4. What's the other 4?
	Amanda: The other 4 is how many…um…how many…is…how…
	Emanuel: The other 4 is the total amount of sides are increasing by 4, cuz (pointing to Amanda's paper) this is 4, this is 8, this is 16 (all the students watch closely while Emanuel explains. **See picture at 11:13:22**.)
	Eliot: (with Emanuel) 16
	Amanda: 12 (pointing to her paper). I mean 16.
	Eliot: (pointing to Amanda's paper) That's 12. That's 12. And then this is 16.
	Amanda: Yeah. Sorry. (She erases something on her paper.)
	Ms. A.: Ok, so…
	Emanuel: That's the amount of sides, 16.
	Ms. A.: So, ok. So, can you write the 4 by 4, here, I just want to have it here. (Amanda writes.) So let's do it for this one. (Points to Amanda's paper) Let's see for this one.
	Amanda: Ok, this was 5…
	Ms. A.: And write, write in numbers.
	Amanda : 5 by 4, gave you 20…and then you added the height, which was 26.
	Ms. A.: (pointing to Amanda's paper) Here, what's the 5, what's the 4?
	Amanda: The 5 is the number of…um…sides…the amount of sides
	Emanuel: (while Amanda is speaking) The number of sides…wait, no…5 is the…(He laughs. Amanda hits her head with her hand and says something inaudible)
	Emanuel: 5…5 is the…one minute…5 is the…
	Juan: Isn't it height?
	Ms. A.: What's the 4?
	Emanuel: Ok, 4…
	Eliot: We forgot all about the 5.
	Emanuel: No, no, no, no, no…she's asking because we did 4 times 4 and we're doing 4 times 5. She's asking what is 5.
	Amanda: 4 is like, wait…
	Eliot: Oh, right!
	Amanda: Hold on, hold on. I think I got it. I think I got it.
	Emanuel: Because…you're doing 5 because this is zero so they take, 1 basically is taken away.
	Amanda: Hold on, hold on.
	Emanuel: So…so like…if this is 4 times 1
	Eliot: Cuz there's 1, 2, 3, 4, then increase by 4 more.
	Amanda: But I want to know
	Emanuel: Because there's…one of them is zero. You like have to take one away cuz in order to figure out
	Ms. A.: Let's talk about a specific one
	Emanuel: Yes, yes (nods and leans forward, watching closely where Ms. A. is pointing)

	Ms. A.: (continuing) so that we can really count what we're talking about. So, again, we said here 4 by 4, plus 5. What's the first 4?
	Amanda: The first 4…I think is because in figure 1 it had 4 cubes so what we did was we multiplied each time by f…
	Ms. A.: (pointing to Amanda's paper) We're talking about this particular one, right?
	Emanuel: (while Ms. A. is speaking) in figure 2 (inaudible)…
	Amanda: Yeah, about figure 2
	Emanuel: (with Amanda) Yes
	Ms. A.: Ok
	Amanda: So the 4 would be…um…
	Emanuel: The 4 is…
	Amanda: Oh! The…the…the…the…the, um. (She waves her pencil in the air in 4 directions, showing 4 sides of the tower.)
	Emanuel: Sides. The sides are increasing by 4 each time, so…
	Amanda: 4 would be the number of si…like…this is the building (She motions with her hand, showing a building. **See picture at 11:15:26**)…there's one right there, there's one in front, there's one on the sides, so it would be 4.
	Ms. A.: Ok, so write 'sides' so that you won't forget that this is the 4 for the sides.
	Amanda: (slowly, as she writes) sides
	Ms. A.: And what's the other 4?
	Amanda: Um, how much…was added to each side?
	Ms. A.: So show me that on this particular tower.
	Amanda: See, because this was the original…wait. (She erases something.) You added 4 right here, and 4 right here, and then 4 over there, and 4 over there.
	Emanuel: (while Amanda is speaking) Oh, yeah, yeah, yeah. I…I know what you're talking about. Yeah, yeah, yeah. I see what you're talking about.
	Ms. A.: So…(pointing to Amanda's paper) you have 4, 4, 4, 4. Ok? (Amanda nods) So, this 4 is what?
	Emanuel: (to himself, while Ms. A. is speaking) added in each side, that's the pattern.
	Amanda: The, um…
	Emanuel: What, the second 4? The second 4? It's the amount. The amount that…
	Amanda: (while Emanuel is speaking) The amount added to the side? Amount added to the side.
	Ms. A.: (to Emanuel) You're talking about the amount of what? Be specific.
	Eliot: Cubes.
	Emanuel: The amount of cubes (picks up his paper and shows it to Ms. A.) that here…1, 2.
	Amanda: (while Emanuel is talking) of cubes added to the side. (To herself as she writes) amount of cubes added to the side.
	Ms. A.: Ok, and what about the 5, in this case?

	Amanda: It would be the height. Eliot: 4 sides increase on each tower. Emanuel: Actually, this is the amount of cubes *in the sides* (with emphasis on ‘in the sides’) Amanda: (while Emanuel is speaking) Added to each side Emanuel: No, because it didn’t add. Because they didn’t add 4 to this side, cuz this was already 3 in the other one. They just…this is the amount of s…of… Amanda: of cubes in? Emanuel: of cubes in one side of the… Eliot: of the tower Ms. A.: Are you sure you don’t want to go back and check? Go back and check in the previous phase. Go back and check in the previous phase and see if what she’s saying is right, or what you’re saying. Check. Check against each other. Ok? (Ms. A. leaves the table. Amanda and Emanuel look at each other and smile.)
Description of Student Difficulty	Ms. A. asks the students to label each number in their computation for finding the number of blocks in the 100- block high tower. To obtain the solution for the 100-block high tower, the students have calculated (4 x99) +100=496.The students have difficulty explaining what each number in their computation represents [lines 16-191].
Type of Student Difficulty	Insufficient Answer
Type of Task	***Open Ended /Closed:*** Closed-ended ***Routine/Non-routine:*** Non-routine
Context of Student Difficulty	Small group
Teacher’s Response(s)	Ms. A. asks the students more specific questions to help them express the general solution for finding the number of blocks in any size tower. At one point, Ms. A. asks the students to label each number in their calculation of (4x4) +5=21, their solution to the 5-block high tower. Emanuel and Amanda disagree about what the second ‘4’ represents. Amanda says it is the amount *added* to the side, and Emanuel says it represents the amount *in* a side. Ms. A. tells them to check their answers against each other [lines 25- 195]. Code: Leading the student to the correct answer/idea Suggesting students share difficulty with peers
Teacher’s Explanation for	Ms. A. wanted to make sure the students understand the concept completely.

Response	She asked questions to encourage the students to think about their answer at a deeper level and generalize their solution. *Response influenced by mathematics ability*? Yes. The general type of response, asking questions, would have been the same for weaker mathematics students. However, with weaker students, Ms. A. would have asked more simplified questions and let the students use manipulatives more. Code for teacher intention: M-Conceptual Understanding Ab/S- Mathematics Ability
Description of Behavior Related to Student's Engagement Following Response	After Ms. A. leaves the table, Amanda says to Emanuel "so you're saying it's the amount in a side. (Eliot and Juan lean forward to listen to Amanda and watch as she points to her paper.) You are right." Eliot says, "he's right." Amanda then waves at the camera, and Juan looks at the camera. Emanuel tells Juan to stop looking at the camera. The students get back to work. Amanda says to Emanuel, "You *are* right, because you can't add four." Emanuel replies, "Yeah, because you're saying you add four, then it would have been…" Amanda says, "5." Emanuel says, "Yeah, 5…wait, wait, not 5." He counts to himself and says, "It would have been 7." Amanda points to her paper and says, "These were original sides." She then motions with her hands in the air showing a stage B tower which has a height and one block on each side. As she does this, she says, "This was the original tower (motioning to the height of the hypothetical tower) and there was one already (motioning to the four sides of the hypothetical tower), so if you added 4, it would have been…" Emanuel says, "7." Amanda says, "So you're right. I'm wrong." Emanuel says "the hardest thing now is explaining it." "We found the method but we can't explain it." Amanda tries to explain her reasoning for agreeing with Emanuel's method but has difficulty expressing her justification. She writes on her paper then reads from it her explanation of the process the students used to find the total number of blocks in a tower. Reading from her paper, she says that they multiplied "the side" by the number of blocks in a side and added the height to get the number of blocks in a tower. Eliot and Juan say, "That sounds good!" Amanda repeats what she wrote while the other students write. **See picture at 11:20:33.** Emanuel and Amanda smile. A member of the research team hands the students blank paper. The students say they need to write over their work neatly, in order. Eliot says, "so that they'll (referring to the research team) understand it." Emanuel says, "Yeah, come on. Let's be the first people that finish." The students work on rewriting their work and drawing stages of the towers. Emanuel says they don't need to draw stages A, B, and C. After the students have been working for a few minutes, Eliot looks up and says to Emanuel, "I thought we didn't have to draw A, B, and C." Emanuel says that he changed his mind and traced

	them. He says it's up to Eliot whether or not to draw them but that he should get the basics down because they don't have much time left. Juan keeps looking at Emanuel's paper as he writes. When Ms. A. tells the class they need to stop, Emanuel sighs, "Oooohhh." The students keep working until members of the research team collect their papers.
Inference of Student's Emotions Based on Description	Em: High level of interest, Eagerness to complete the task, Pride in work A: High level of interest, Pride in work, Slight frustration El: High level of interest J: High level of interest
Behaviors Pertaining to Student's *Behavioral* Engagement and Resulting Classification	Em: On-task *Redirecting (an)other student(s) to task, Writing* A: On-task *Expressing agreement with group mate's idea, Explaining idea to group mates, Writing* El: On-task *Watching group mate explain idea, Expressing agreement with group mate's idea, Writing, Asking for help from group mate* J: On-task *Watching group mate explain idea, Expressing agreement with group mate's idea, Writing*
Behaviors Pertaining to Student's *Cognitive* Engagement and Resulting Classification	Em: High level cognitive activity *Highest level of cognitive engagement involved: Reasoning* A: High level cognitive activity *Highest level of cognitive engagement involved: Reasoning* El: Insufficient information J: Insufficient information
Emotions Pertaining to Student's *Affective* Engagement and Resulting Classification	Em: Strong positive feelings *High level of interest with no apparent negative emotions, Eagerness to complete the task, Pride in work* A: Strong positive feelings *High level of interest, Pride in work, Slight frustration* El: Strong positive feelings *High level of interest with no apparent negative emotions* J: Strong positive feelings *High level of interest with no apparent negative emotions*

<table>
<tr><th colspan="5">Comparison of Students' Self-Reports on Questionnaire and Observed Student Behaviors
Ms. A.: Day 1</th></tr>
<tr><td colspan="5">Emanuel</td></tr>
<tr><td rowspan="3">Self-Reported Affect</td><td colspan="2">Positive Emotions Score:</td><td colspan="2">Engagement with Impasse Score:</td></tr>
<tr><td>Emanuel</td><td>Mean</td><td>Emanuel</td><td>Mean</td></tr>
<tr><td>39/40</td><td>31.47/40</td><td>0/4</td><td>.78/4</td></tr>
<tr><td>Affect Inferred from Observed Behaviors</td><td colspan="4">Strong positive feelings (Following Difficulty 3)
Strong positive feelings (Following Difficulty 4)</td></tr>
<tr><td colspan="5">Comparison of Self-Reported vs. Observed Affect
Emanuel's self-reported emotions for this class session were similar to the emotions observed from videotape analysis of the session. Emanuel reported having strong positive feelings during the session. Observations showed Emanuel demonstrated a high level of interest, pride in his work, eagerness in sharing his solution with the teacher, and confidence in his solution.</td></tr>
<tr><td>Self-Report of Difficulty</td><td colspan="4">Had difficulty? No.
Classroom context of difficulty: Not applicable
What stands out about the situation? (No response)</td></tr>
<tr><td>Number of Analyzed Instances of Difficulty</td><td colspan="4">2</td></tr>
<tr><td colspan="5">Comparison of Self-Reported vs. Observed Instances of Difficulty
Reported no difficulty but was involved in:
Difficulty 3- Unable to Answer
Difficulty 4- Insufficient Answer</td></tr>
<tr><td colspan="5">Amanda</td></tr>
<tr><td rowspan="3">Self-Reported Affect</td><td colspan="2">Positive Emotions Score:</td><td colspan="2">Engagement with Impasse Score:</td></tr>
<tr><td>Amanda</td><td>Mean</td><td>Amanda</td><td>Mean</td></tr>
<tr><td>44/46</td><td>31.47/40</td><td>2/4</td><td>.78/4</td></tr>
<tr><td>Affect Inferred from Observed Behaviors</td><td colspan="4">Strong positive feelings (Following Difficulty 1)
Strong positive feelings (Following Difficulty 2)
Strong positive feelings (Following Difficulty 3)
Strong positive feelings (Following Difficulty 4)</td></tr>
<tr><td colspan="5">Comparison of Self-reported vs. Observed Affect:
Amanda's reports of strong positive emotions during this class session were confirmed by observations of the session. On her questionnaire, Amanda reported feeling all the listed positive emotions 'Very Much.' She even added the feeling 'jubilant' to her list of emotions. Observations of the class session confirmed that Amanda showed a high level of interest, determination in anticipation of success, excitement, eagerness, pride, and confidence regarding mathematics problem solving that day.</td></tr>
</table>

<table>
<tr><td rowspan="1">Self-Report of Difficulty</td><td colspan="4">Had difficulty? Yes.
Classroom context of difficulty: Group.
What stands out about the situation? “What other kids said or did because there was a lot of thought in my group.”</td></tr>
<tr><td>Number of Analyzed Instances of Difficulty</td><td colspan="4">4</td></tr>
<tr><td colspan="5">Comparison of Self-Reported vs. Observed Instances of Difficulty
Reported having difficulty and was involved in:
Difficulty 1- Incorrect Idea
Difficulty 2- Incorrect Idea
Difficulty 3- Unable to Answer
Difficulty 4- Insufficient Answer</td></tr>
<tr><td colspan="5">Eliot (Questionnaire not available)</td></tr>
<tr><td colspan="5">Juan</td></tr>
<tr><td rowspan="3">Self-Reported Affect</td><td colspan="2">Positive Emotions Score:</td><td colspan="2">Engagement with Impasse Score:</td></tr>
<tr><td>Juan</td><td>Mean</td><td>Juan</td><td>Mean</td></tr>
<tr><td>28/42</td><td>31.47/40</td><td>0/4</td><td>.78/4</td></tr>
<tr><td>Affect Inferred from Observed Behaviors</td><td colspan="4">Mild positive feelings (Following Difficulty 3)
Strong positive feelings (Following Difficulty 4)</td></tr>
<tr><td colspan="5">Comparison of Self-Reported vs. Observed Affect:
On his questionnaire, Juan reported feeling somewhat ‘worried,’ ‘bored,’ ‘embarrassed,’ and ‘afraid.’ Juan added to the list his feeling of very much ‘nervous.’ Observations of Juan during this class session showed he demonstrated a high level of interest mixed with feelings of inferiority during a portion of the class session. During most of the session, Juan was quiet and listened to the other students. He copied work from their papers as opposed to initiating courses of action. Perhaps since he did not speak much during the sessions, it was difficult to observe his negative feelings other than ‘inferiority.’ Although Juan did not demonstrate a high level of understanding and at one point indicated he did not understand the group’s solution, he reported feeling ‘Not At All’ engaged with impasse.</td></tr>
<tr><td>Self-Report of Difficulty</td><td colspan="4">Had difficulty? Yes.
Classroom context of difficulty: Group
What stands out about the situation? ”Nothing.”</td></tr>
<tr><td>Number of Analyzed Instances of Difficulty</td><td colspan="4">2</td></tr>
<tr><td colspan="5">Comparison of Self-Reported vs. Observed Instances of Difficulty
Reported having difficulty and was involved in:
Difficulty 3- Unable to Answer
Difficulty 4- Insufficient Answer</td></tr>
</table>

Group Analysis of Self-Reported vs. Observed Affect
The students who demonstrated the highest level of understanding among the group members, Emanuel and Amanda, reported much more positive affect than Juan, who demonstrated a lower level of understanding.

Ms. B.
Class- Math 1: Honors Track, Dayfield St. School, Day 1
Group 1: Marcos, Larent, Lashanna

Students' Reports on Emotions Questionnaire

Marcos:

Overall Positive Emotions Score (40=most positive): 30

"Engagement with Impasse" Score (4=most engaged with impasse): 0

Reports of Emotions:

	Very Much:	Somewhat:	Not at All:
Positive/Negative Subscale	*Positive (2 points):* Successful Safe Happy Confident	*Positive (1 point):* Interested Proud Satisfied Relieved	*Positive (0 points):* Respected Excited Curious
	Negative (0 points):	*Negative (1 point):*	*Negative (2 points):* Unhappy Disappointed Worried Discouraged Angry Disrespected Bored Embarrassed Afraid
Engagement with Impasse Subscale	*(2 points)*	*(1 point)*	*(0 points)* Frustrated Confused

Reports of Difficulty:

Had difficulty? Yes.
Classroom context of difficulty: Not during group conversation or whole class discussion.
What stands out about the situation? "I think what kids said or did."

Larent:

Overall Positive Emotions Score (40=most positive): 29

"Engagement with Impasse" Score (4=most engaged with impasse): 0

Reports of Emotions:

	Very Much:	***Somewhat:***	***Not at All:***
Positive/Negative Subscale	*Positive (2 points):* Respected Proud Safe Excited Happy Satisfied	*Positive (1 point):* Interested Relieved	*Positive (0 points):* Successful Confident Curious
	Negative (0 points): Worried	*Negative (1 point):* Bored	*Negative (2 points):* Unhappy Disappointed Discouraged Angry Disrespected Embarrassed Afraid
Engagement with Impasse Subscale	*(2 points)*	*(1 point)*	*(0 points)* Frustrated Confused

Reports of Difficulty:

Had difficulty? No.
Classroom context of difficulty: Not applicable
What stands out about the situation? (No response)

Lashanna:

Overall Positive Emotions Score (40=most positive): 21

"Engagement with Impasse" Score (2=most engaged with impasse*): 0
*No response for 'frustrated'

Reports of Emotions:

	Very Much:	***Somewhat:***	***Not at All:***
Positive/Negative Subscale	*Positive (2 points):* Respected Safe	*Positive (1 point):* Interested Successful Happy Curious	*Positive (0 points):* Excited Satisfied Relieved Confident Proud
	Negative (0 points): Bored	*Negative (1 point):* Angry Disappointed Unhappy	*Negative (2 points):* Worried Discouraged Disrespected Embarrassed Afraid
Engagement with Impasse Subscale	*(2 points)*	*(1 point)*	*(0 points)* Confused

Reports of Difficulty:

Had difficulty? No.
Classroom context of difficulty: Not applicable
What stands out about the situation? "Nothing did not happen I did not teach nothing, I felt board and like a superstar, My teacher just was getting on my nerv, they [other kids] did not do nothing to me."

Event 3 Difficulty 5: Marcos, Larent, Lashanna	
Transcript of Event **Time (of day): (9:46:08-9:47:00)**	Class begins at 9:40 am. Marcos and Larent are sitting at their desks. Lashanna's desk is also in their group but it is vacant. Ms. B. hands out papers and explains the building block dilemma task. Ms. B. says the students may begin working on the problem. At the beginning of this event, Larent is sitting with his chair facing away from the group. Marcos has his head resting on his hand and is staring straight ahead. Ms. B. walks with Lashanna to her seat and places the task sheet and a blank sheet of paper on her desk. Larent is still sitting with his chair facing away from the group and is playing with his pencil. Ms. B. says, "Now, how are we going to start this? Can you show me what it would look like?" She opens the bag of blocks that is on the group's table and pours out some blocks. Marcos takes some blocks. "So let's work on step A, step one. What does that look like? How many cubes?" Marcos says, "One. 'A' is one." Ms. B. says, "Ok, now, you might want to record some information as you're doing this. So, for tower B, or tower 2, can you make this, 'cause we're trying to see how many cubes…(inaudible). Let's go, Lashanna... (inaudible) so that you can have a better understanding (inaudible)." Marcos takes more blocks. Ms. B. puts a handful of blocks on Lashanna's desk and walks away.
Description of Student Difficulty	The students in this group are not involved in the task. They have not started working on the task by the time Ms. B. comes over to their group [lines 5-9].
Type of Student Difficulty	Impasse
Type of Task	***Open Ended /Closed:*** Open-ended ***Routine/Non-routine:*** Non-routine
Context of Student Difficulty	Small group
Teacher's Response(s)	Ms. B. asks the students how many blocks are in the first tower. She gives them blocks and suggests they build tower number 2. She also suggests they record their information [lines 10-20]. Code: Offering a suggestion of a process to use in solving the problem
Teacher's Explanation for	N/A

Response	Code for teacher intention: N/A
Description of Behavior Related to Student's Engagement Following Response	After the event described above, Marcos and Lashanna begin building towers. They work without speaking. Marcos builds a five-high tower. Larent plays with the audio recorder, turning it on and off. Marcos says, "For a 5-block high tower we need 6 blocks." Larent and Lashanna do not respond. Larent fools around while Lashanna is building a tower with blocks. Larent takes blocks and builds a stage 2 tower while looking back and forth between the tower and the task sheet. He plays with the tower while he talks about non-task related topics. Marcos continues working and does not pay much attention to him. Larent looks at Lashanna while he talks. Lashanna responds to some of Larent's comments, but she is working on the task. She has built a 3-high tower with extra blocks between the branches at the base of the tower. Marcos and Lashanna write on their papers. Marcos counts the blocks in his five-high tower and records information on his paper. He then builds a ten-high tower and counts the blocks. Marcos removes blocks from his 10-block high tower and converts it to a 5-block high tower. He does not speak. He then adds blocks to all sides of the tower. Lashanna builds a tower on her desk that is approximately 10-blocks tall. It does not have extra blocks between the branches as in her previous tower. Larent begins building a tower while still chattering about non-task related subjects. He speaks to students at other tables about non-task related topics and plays with markers on his table. He does not write. Marcos has added blocks to his tower so it is again 10 blocks high. Marcos and Lashanna decide to put the towers together to build a 100-block high tower. Lashanna takes apart her tower and together with Marcos, she adds the blocks to his tower to build a tower that is 30 blocks tall. When they use all the blocks on their table, Marcos goes to get more blocks. The blocks he brings do not fit with the blocks in their tower. He puts back the blocks he brought and says there are no more blocks.
Inference of Student's Emotions Based on Description	M: Interest La: Disinterest, Boredom Ls: Interest
Behaviors Pertaining to Student's *Behavioral* Engagement and Resulting Classification	M (Marcos): On-task *Using manipulatives in way related to task at hand* La (Larent): Off-task *Talking with others about non-math topics, fooling around in other ways* Ls (Lashanna): On-task *Using manipulatives in way related to task at hand*
Behaviors Pertaining to Student's *Cognitive* Engagement and Resulting	M: Low level cognitive activity *Highest level of cognitive activity involved building towers without consideration of patterns, checking information* La: Low level cognitive activity *Highest level of cognitive activity involved building towers without consideration of patterns, checking information*

Classification	Ls: Low level cognitive activity *Highest level of cognitive activity involved building towers without consideration of patterns, checking information*
Emotions Pertaining to Student's *Affective* Engagement and Resulting Classification	M: Moderate positive feelings *Some level of interest with no apparent negative feelings, no evidence of strong positive feelings such as excitement or confidence* La: Mild negative feelings *Disinterest, Boredom, no evidence of positive feelings such as interest or confidence* Ls: Moderate positive feelings *Some level of interest with no apparent negative feelings, no evidence of strong positive feelings such as excitement or confidence*

Event 4 Difficulty 6: Marcos, Lashanna Difficulty7: Marcos, Lashanna Difficulty8: Marcos Difficulty 9: Marcos, Lashanna Difficulty10: Marcos Difficulty11: Larent Difficulty12: Larent Difficulty13: Lashanna	
Transcript of Event **Time (of day): (10:09:38-10:15:39)**	Marcos and Lashanna have been working together trying to build a tower that is 100 blocks high. Most of the time, Larent has been playing around with the blocks and talking about non-task-related topics. At the beginning of this event, Ms. B. comes over to check on the students' progress. Ms. B.: So, what did you guys come up with? Marcos: For 5 blocks, we got 21 blocks that we will use. Ms. B.: What do you mean by 5 blocks? Marcos: Like 5 block, you know, 5 block tower. Ms. B.: Ok Marcos: For the 10- block high tower we had to use 46. Ms. B.: So now, what you have right here, is there a way that you could figure it out without having to build it up because after, say that you did build a 100-high tower with the cubes, so someone might come in and say, now that you did that, can you make a thousand-block high tower? Can you? Lashanna: It would be 500. 100, 100, 100, 100, 100 (*Pointing to each of the 5 branches of the tower on the desk*). Marcos: Oh, yeah. 100, 100 *(Pointing to 2 branches),* that's 200, that's 400 *(pointing to the other 2 branches at the base),* that's 500 *(pointing to the height).* 500 blocks. *Larent yawns.* Ms. B.: So for, you said, 500 blocks? Marcos: No, no, no. Hold on. *(Sits up, moving closer to the tower)* This block *(pointing to the middle block of the tower on the desk)* doesn't count for this one *(pointing to a branch),* so the top is 100, this is 99. 99, 99, 99 *(pointing to the other branches).* *(When Marcos begins speaking, Lashanna rests her head on her hand and watches him. She moves her eyes from the tower to which he is pointing to him, and back to the tower.)* Lashanna: There's…(inaudible)…100 in all of them. So it's 500. Ms. B.: So, ok, you said for a 5- block high tower, it was how many cubes? Marcos: 21 Ms. B.: 21. So can you explain that to me?

	Lashanna: (points to each of the 5 branches in the tower on the desk) I think that was, Marcos, it was 25.
	Marcos: Because 5…(points to branches on tower)…25.
	Lashanna: (inaudible) …25.
	Marcos erases something on his paper and writes.
	Ms. B.: Ok, so, for a 2-block high tower…this is a how many block high tower (*points to the task paper*)?
	One of the students at Group 1: One
	Ms. B.: One. How many cubes did you need?
	Marcos, Lashanna, Larent: One
	Ms. B.: For a 2- block high tower, how many cubes is it all together?
	Marcos: 2
	Ms. B.: All together.
	Marcos: All together? 2, 4, 6, 8, 10.
	Lashanna: 10
	Ms. B.: Can you show it to me? Prove that to me.
	Larent: *(points to blocks in diagram B tower, as if counting them***. at 10.11.39.** *He responds promptly*) 6.
	Marcos: *(looks at Larent, then counts on his fingers)* 2, 4, 6….Because 4…times 2… *(He says this haltingly. Lashanna looks at Marcos as he speaks)***.**
	Ms. B.: Look at the picture *(points to diagram B on the task paper.)*
	Larent: 6
	Ms. B.: *(to Larent)* How did you get 6?
	Larent: *(pointing to diagram)* That's 2, 4, and one on the bottom.
	Ms. B. does not respond to Larent's explanation. She looks at Marcos. Marcos gets up, walks around the desk, and points to the diagram on the sheet. Ms. B. watches Marcos ***(see picture 10:12:08)***
	Marcos: *(pointing to the diagram on the sheet)* 1, 2, 3, 4, 5, 6.
	Ms. B.: Six? *(says this slowly)* Do you agree with that, Lashanna? Do you see that? Let me see. Show me. *(She moves the task sheet towards Lashanna.)*
	Larent: No, it's 5.
	Lashanna: (points to the task sheet) 1, 2, 3, 4, 5, and one at the bottom.
	Ms. B.: One at the bottom. So this is for how many towers high *(pointing to diagram B on the task sheet)*?
	Lashanna and Marcos: 2
	Ms. B.: So this is for 2 towers high. Someone might want to…um… keep a record of all this. So you said for 1 tower high, equals how many cubes?
	Marcos: One
	Ms. B.: One. Who's gonna keep track of this?
	Lashanna: Larent
	Ms. B. places a blank sheet of paper on Larent's desk.
	Ms. B.: Actually, all of you can. Make sure you all get it right. Use a marker.
	Larent places the sheet of paper that Ms. B. gave him on Lashanna's desk.
	Ms. B.: Here, both of you, all of you, be responsible. *(Ms. B. takes the sheet of*

paper from Lashanna's desk and puts it back on Larent's desk. She places a marker on the paper.)
Ms. B.: Ok. *(Hands Marcos a paper.)* Here, Marcos, you could write on, too. So we said for 1 tower high… *(Larent had a pile of paper on his desk. He tries to give Marcos some.) (To Larent)* Just write on the stack because the marker bleeds through. So we said for 1 tower high, how many cubes do we need?
Larent: one
Ms. B.: one. *(All the students write on their papers. See **picture below at 10.13.07**.)*
While the students write, Ms. B. looks around the classroom.
Ms. B.: I really like some of the work that I see some of you are doing. You're very focused, I hear some good conversations going on, … *(to a student off camera)* I'll be over to you in a minute. *(Looking at Larent's paper)* Ok, so don't wait for me, so 1 tower high was 1 cube, 2 towers *(points to diagram B),* 2 tower high is how many cubes?
Larent: 5
Lashanna: 6
Ms. B.: You said 5 *(pointing to Larent),* you said 6 *(pointing to Lashanna)*
Larent: I said 6 the first time.
Ms. B.: (*smiling*) And now you're changing? *(Larent nods.)*
Ms. B.: Why?
Larent: I don't know. I see what other people do.
Lashanna points to diagram B and then to branches on the tower on the desk and says something inaudible to Larent.
Larent: But I don't know if they all just put it together and made a (inaudible) of it.
Ms. B.: No, but let's say it keeps building *(points to diagrams on task sheet)*, right, they just keep adding on…they're showing you…So, how many do you think there are?
Larent: 6
Ms. B.: 6. Ok, so the 2- tower high…has 6 cubes…*(Ms. B. points to Larent's paper. The students all record on their papers. Ms. B. looks around the room.)*
Lashanna: (*to Marcos*) You have the first one? *(She holds up 3 fingers. Marcos's face is off the camera.)*
A student from another table (off the camera) asks to use the bathroom. Ms. B. admonishes him for not doing his work. She says she'll answer him when he "gets himself together." Ms. B. turns her head to group 1 again.
Ms. B.: How many did you get, Lashanna, for 3 towers high? *(Larent turns over his paper. Lashanna raises 4 fingers. Ms. B. turns to Larent.)* Now, it's not a secret. You want to get together with it. So once you come up with an answer…*(she turns over Larent's paper, face up).* She *(pointing to Lashanna)* said 14, you said 11 *(looking at Larent's paper)*, how many do you have *(pointing to Marcos)*?
Marcos: I'm still doing it.
Ms. B.: Ok, you're still doing it. So…

	Marcos: *(pointing to diagram C on his paper)* 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11…11. *(Pointing to his paper again)* 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11. *(Writes on his scrap paper)* *After Marcos announced his answer is 11, Lashanna leans over her paper (her face is off camera). She looks up.* Lashanna: 11 Ms. B.: (*smiles*) Ok. Now, you want to work on the 4th one. And, you want to come up with the same answers. If you have different answers, you have to try and prove that you're right. *(Ms. B. looks at Larent)* Like, you said here, like, 'I'm right.' You turned your paper over. *(She turns his paper over, acting out the situation.* ***See picture below at 10.15.23****.)* You should have been like, 'Lashanna, how did you get 14? I got 11. Let me show you how I got 11.' Alright? So, I'm gonna come back with the 4, and 5, 6. So keep on going. *While Ms. B. was talking, Marcos broke off the height of the 30- high tower on the desk and placed it on the desk near the tower.* *Ms. B. walks away from the table.*
Description of Student Difficulty	6) Ms. B. asks the students- even if they could build a 100-block high tower, could they build a 1,000 block high tower [lines 13-17]? Lashanna and Marcos say it would be 500 blocks (presumably referring to the 100- block tower) [lines 18-22]. 7) Ms. B. asks the students to explain how they got their solution of 21 blocks for a 5- block high tower. Marcos and Lashanna say it's 25 [lines 32-39]. 8) Ms. B. asks how many cubes are in a 2- block high tower. Marcos says there are 2 [lines 45-46]. 9) Ms. B. asks how many cubes there are in a 2- block high tower. Marcos and Lashanna say there are 10 [lines 48-49]. 10) Ms. B. asks Marcos to prove his solution of 10 blocks for a 2- block high tower. Larent counts the blocks in the diagram on the sheet and says there are 6, but Marcos counts on his fingers "2, 4, 6" (probably thinking that each branch of the 2- block high tower has 2 blocks). Then he says, "because 4 times 2" (probably referring to 2 blocks in each of the 4 branches at the base) [lines 50- 54]. 11) Ms. B. asks Larent how he got 6 cubes for a 2- block high tower. Larent points to diagram B on the task sheet and says, "That's 2, 4, and one on the bottom" [lines 57-58]. He then says the 2- block high tower has 5 cubes [line 65]. 12) Ms. B. asks how many cubes there are in a 2- block high tower. Larent says 5, Lashanna says there are 6 [lines 93-96]. 13) Ms. B. asks Lashanna how many she got for a 3- block high tower. Lashanna holds up 4 fingers and apparently says "14" [lines 116-117] (Lashanna's answer is not audible on video but is implied by Ms. B.'s later remark, lines 120-121).
Type of Student Difficulty	6) Incorrect Answer 7) Incorrect Answer

	8) Incorrect Answer 9) Incorrect Answer 10) Incorrect Answer 11) Incorrect Answer 12) Incorrect Answer 13) Incorrect Answer
Type of Task	6) ***Open Ended /Closed:*** Closed-ended ***Routine/Non-routine:*** Non-routine 7) ***Open Ended /Closed:*** Open-ended ***Routine/Non-routine:*** Non-routine 8) ***Open Ended /Closed:*** Closed-ended ***Routine/Non-routine:*** Non-routine 9) ***Open Ended /Closed:*** Closed-ended ***Routine/Non-routine:*** Non-routine 10) ***Open Ended /Closed:*** Open-ended ***Routine/Non-routine:*** Non-routine 11) ***Open Ended /Closed:*** Open-ended ***Routine/Non-routine:*** Non-routine 12) ***Open Ended /Closed:*** Closed-ended

	Routine/Non-routine: Non-routine 13) ***Open Ended /Closed:*** Closed-ended ***Routine/Non-routine:*** Non-routine
Context of Student Difficulty	6-13) Small group
Teacher's Response(s)	6) Ms. B. asks if Lashanna and Marcos said 500 blocks [line 24]. Code: Asking a question to clarify what the student(s) said 7) Ms. B. asks how many cubes there are in 1- and 2- block high towers [lines 40-45]. Code: Offering a simpler problem as an example 8) Ms. B. repeats, "All together" [line 47]. Code: Repeating the teacher's question 9) Ms. B. asks Marcos to prove it to her [line 50]. Code: Asking a student to prove his/her solution 10) Ms. B. tells Marcos to look at the picture. She points to diagram B on the task sheet [line 52]. Code: Showing the student a diagram/model 11) Ms. B. does not respond to Larent's explanation. She looks at Marcos [line 59]. Codes: Choosing not to respond to an incorrect response Inviting (verbally or non-verbally) (an)other student(s) to respond 12) Ms. B. says "You said 5, you said 6" (pointing to Larent and then to Lashanna) [line 97]. When Larent says he said 6 the first time, Ms. B. asks him why he changed his answer [lines 98-100]. Larent says he changed his answer because he sees what other people did. After a conversation between Lashanna and Larent which is not clear on the videotape (no audio available for this segment), Ms. B. explains that the task involves towers where each builds on the previous one [lines 102-108]. Code: Reminding student about another student's idea Probing- asking the student to explain his/her solution or the reasoning he/she used Explaining the mathematics problem/task 13) Ms. B. announces Lashanna's and Larent's answers and asks Marcos for his answer [lines 120-122]. Code: Reminding student about another student's idea Inviting (verbally or non-verbally) (an)other student(s) to respond

Teacher's Explanation for Response	6-11) N/A Code for teacher intention: N/A 12) Larent has a difficult time settling down and working in a group. He also didn't want to be there that day because of the videotaping, so Ms. B. was spending time with him, trying to get something from him and make him think about the problem even if only at a superficial level. *Response influenced by mathematics ability*? Yes. Larent has difficulty in math, with organizational skills, and with focusing on his work. So Ms. B. was trying to get him involved in the math problem by giving him easy questions and challenging him a little. Code for teacher intention: Ab/S-Organizational Skills Ab/S-Mathematics Ability At- Attitudes regarding participation in mathematics activities 13) N/A Code for teacher intention: N/A
Description of Behavior Related to Student's Engagement Following Response	6) Marcos sits up, moving closer to the tower, and points to it as he speaks. He corrects himself and points to the middle block, saying it doesn't count for branches in the base. He says that each of the branches in the base would be 99. When Marcos begins speaking, Lashanna rests her head on her hand and watches him. She moves her eyes from the tower to which he is pointing to him, and back to the tower. Lashanna repeats her solution of 500 blocks [lines 24-31]. 7) Marcos, Lashanna, and Larent are all involved in answering Ms. B.'s questions [lines 40-46]. 8) Marcos counts by two's from 2 to 10. Lashanna says it's 10 [lines 48-49]. 9) Larent counts the blocks in the diagram on the sheet and responds promptly that there are 6. Marcos first says, "2, 4, 6" (probably thinking that each branch of the 2- block high tower has 2 blocks). Then he says haltingly, "because 4…times 2…" (probably referring to 2 blocks in each of the 4 branches at the base). Lashanna looks at Marcos as he speaks [lines 51-54]. 10) After Larent repeats that there are 6 blocks and explains it saying, "That's 2, 4, and one on the bottom, Ms. B. looks at him inquisitively. Marcos then gets up, walks around the desk, and points to the diagram on the sheet **(see picture 10:12:08, above)**. While pointing to the diagram, he says, "1, 2, 3, 4, 5, 6" [lines 56-62]. 11) Marcos gets up, walks around the desk, and points to the diagram on the sheet **(see picture 10:12:08, above)**. He points to the diagram and says "1, 2, 3, 4, 5, 6" [lines 59-61]. Larent watches Marcos as he speaks but does not

	respond to his explanation. As Ms. B. speaks with Lashanna and Marcos, Larent watches and looks around the room periodically. When Ms. B. hands Larent a paper on which to record information, he puts it on Lashanna's desk [lines 75-77]. Larent answers Ms. B.'s question of how many cubes there are in a tower that is one-block high [lines 82-86]. 12) When Ms. B. asks Larent if he's changing his answer, Larent says he changed his answer because he sees what other people did [line 101]. After Ms. B. explains that the task involves towers which build on each other, she asks Larent how many blocks he thinks there are in a 2-block high tower. Larent says there are 6. Ms. B. points to Larent's paper. Larent, Lashanna, and Marcos record information on their papers [lines 98-111]. 13) After Marcos announces his answer is 11, Lashanna leans over her paper (her face is off camera). She looks up and says there are 11 blocks [lines 128-130].
Inference of Student's Emotions Based on Description	6) M: Interest Ls: Interest 7) M: Interest Ls: Interest 8) M: Interest 9) M: Interest Ls**:** Interest 10) M: Interest 11) La: Some level of interest, Some level of disinterest and boredom 12) La: Interest 13) Ls: High level of interest
Behaviors Pertaining to Student's *Behavioral* Engagement and Resulting Classification	6) M: On-task *Explaining idea to teacher* Ls: On-task *Watching group mate explain idea* 7) M: On-task *Answering teacher's question* Ls: On-task *Answering teacher's question* 8) M: On-task *Answering teacher's question* 9) M: On-task *Answering teacher's question* Ls**:** On-task *Watching group mate explain idea* 10) M: On-task *Explaining idea to teacher* 11) La: On-task *Watching group mate explain idea, Answering teacher's question* 12) La: On-task *Answering teacher's question*

	13) Ls: On-task *Looking at paper, Answering teacher's question*
Behaviors Pertaining to Student's *Cognitive* Engagement and Resulting Classification	6) M: High level cognitive activity *Highest level of cognitive activity involved: Reasoning* Ls: Low level cognitive activity *Highest level of cognitive activity involved: Restating one's own idea without consideration of other's ideas* 7) M: Low level cognitive activity *Highest level of cognitive activity involved: Considering facts* Ls: Low level cognitive activity *Highest level of cognitive activity involved: Considering facts* 8) M: Low level cognitive activity *Highest level of cognitive activity involved: Considering facts* 9) M: High level cognitive activity *Highest level of cognitive activity involved: Justifying* Ls: Insufficient information 10) M: High level cognitive activity *Highest level of cognitive activity involved: Justifying* 11) La: Low level cognitive activity *Highest level of cognitive activity involved: Considering facts* 12) La: High level cognitive activity *Highest level of cognitive activity involved: Reasoning* 13) Ls: High level cognitive activity *Highest level of cognitive activity involved: Reasoning*
Emotions Pertaining to Student's *Affective* Engagement and Resulting Classification	6) M: Moderate positive feelings *Some level of interest with no apparent negative feelings, no evidence of strong positive feelings such as excitement or confidence* Ls: Moderate positive feelings *Some level of interest with no apparent negative feelings, no evidence of strong positive feelings such as excitement or confidence* 7) M: Moderate positive feelings *Some level of interest with no apparent negative feelings, no evidence of strong positive feelings such as excitement or confidence* Ls: Moderate positive feelings *Some level of interest with no apparent negative feelings, no evidence of strong positive feelings such as excitement or confidence* 8) M: Moderate positive feelings *Some level of interest with no apparent negative feelings, no evidence of strong positive feelings such as excitement or confidence* 9) M: Moderate positive feelings *Some level of interest with no apparent negative feelings, no evidence of strong positive feelings such as excitement or confidence* Ls: Moderate positive feelings *Some level of interest with no apparent negative feelings, no evidence of*

	strong positive feelings such as excitement or confidence 10) M: Moderate positive feelings *Some level of interest with no apparent negative feelings, no evidence of strong positive feelings such as excitement or confidence* 11) La: Mild positive feelings *Some level of interest mixed with slight negative feelings of disinterest and boredom* 12) La: Moderate positive feelings *Some level of interest with no apparent negative feelings, no evidence of strong positive feelings such as excitement or confidence* 13) Ls: Strong positive feelings *High level of interest with no apparent negative feelings*

Event 5
Difficulty 14: Larent
Difficulty 15: Larent

Transcript of Event

Time (of day): (10:30:31-10:33:28)

During the few minutes preceding this event, the students at group 1 were having non-task related discussions and playing with the audio recorder. Ms. B. walks over to group 1 and asks the group what they have come up with. When Marcos picks up the recorder and holds it up towards Larent, Ms. B. admonishes the students for fooling around with the recorder. She then asks the students again what they have come up with.
Ms. B.: What did you come up with?
Marcos: For 1 it's 1, for 2 it's 6, for 3 it's 11, for 4 it's 16.
Larent: *(to Lashanna, while Marcos is talking)* How did you get 4, 5, 6? Show me the paper. *(He laughs.)* Show me the paper, Lashanna *(laughs again while holding the task sheet).*
Ms. B.: Are you listening to another?
Larent: I'm…(inaudible)…4, 5, and 6.
Ms. B.: Marcos is trying to talk. Are you listening to him?
Marcos: For a 1-high tower, it's 1 cube, for a 2- high tower, it's 6 cubes, for a 3- high tower, it's 11 cubes, for a 4- high tower, it's 16 cubes, 5…21, 6…26.
Ms. B.: Ok, now, with what you guys have, are there any kinds of patterns that you see?
Larent: I don't know how they get that. *(He continues speaking while Marcos is describing the pattern he found, but his voice is not heard over Marcos's).*
Marcos: Yeah. Yeah. 1, 6, 1, 6, 1, 6.
Ms. B.: *(to Larent)* Listen to him *(pointing to Marcos). (To Marcos)* He *(Larent)* says he doesn't get it.
Larent: *(to Marcos)* How you get 4, 5, and 6? She didn't even give us the paper yet.
Marcos: 4, 5, and 6. What are you talking about?
Larent: This *(points to the task sheet on the desk).*
Marcos: What do you mean?
Ms. B. picks up the task sheet and gives it to Larent.
Larent: There's 1, 2, and 3, right? She said she was gonna give us the next 3. How you get the answers that…(inaudible)…do yet?
Marcos: We got the answer. We found it with the cubes.
Larent looks at Ms. B..
Lashanna: *(to Larent)* While you were busy talking.
Ms. B.: So you could figure it out yourself if you want to…um…see if they're right or not. You want to see if they're…prove to see if they're right or wrong. *(Ms. B. turns to Marcos)* Now, we want to know how many cubes you need for a 5- block high tower.
Marcos: Oh, for 5 it's…*(he looks at the papers on his desk)*…it's 21.
Ms. B.: Ok, now the next question they ask is that how many blocks do you need for a 10- block high tower? *(While Ms. B. is speaking, Larent covers his*

	eyes with his hand. ***See picture at 10.32.48)*** Marcos: 10? For 10 you need…*(leans forward and looks at his paper)…*46. Ms. B.: How did you get 46? Larent: I thought it was 42. Marcos: Using the blocks and doing the thing…sides *(points to a diagram on his task sheet).* Ms. B.: Can you build up from where you left off with the 6? So you can try and prove yourself to see if you're right? Larent: I thought it was 42. Ms. B.: *(looking at Marcos)* So let's work together and let's prove it. *(She looks away from group 1 to glance at another table.)* Larent: (*to Marcos*) Work together, now. *Ms. B. walks away from group 1.*
Description of Student Difficulty	14) When Marcos announces his solutions of the number of cubes in towers of heights 1 through 6, Larent says he doesn't understand how they got that. He says the task sheet only shows towers 1, 2, and 3, and he is wondering how they got answers for towers 4, 5, and 6 if they didn't get a sheet depicting those towers [lines 8-31]. 15) When Marcos says he got 46 blocks for a 10-block high tower, Larent says he thought it was 42. While Ms. B. is speaking to Marcos, Larent repeats that he thought it was 42 [lines 45, 50].
Type of Student Difficulty	14) Does not understand solution 15) Incorrect idea
Type of Task	14) ***Open Ended /Closed:*** Closed-ended ***Routine/Non-routine:*** Non-routine 15) ***Open Ended /Closed:*** Closed-ended ***Routine/Non-routine:*** Non-routine
Context of Student Difficulty	14) Small group 15) Small group
Teacher's Response(s)	14) Ms. B. tells Larent to listen to Marcos, who is describing a pattern he found for the numbers of cubes in the towers he has solved. She tells Marcos that Larent said he doesn't get it. She then tells him that he should figure it out himself and prove if Marcos's answer is right [lines 24-25, 37-39]. Code: Inviting (verbally or non-verbally) (an)other student(s) to respond Suggesting a process to use in solving the problem

	15) Ms. B. continues her conversation with Marcos and does not comment on Larent's remark. Code: Choosing not to respond to an incorrect response
Teacher's Explanation for Response	N/A (Not Applicable) Code for teacher intention: N/A
Description of Behavior Related to Student's Engagement Following Response	14) After Ms. B. tells Larent to listen to Marcos, Larent leans his head on his hand with his hand covering his eyes [lines 43-44]. He rubs his eyes. He looks at Marcos and and rubs his eyes again. 15) Ms. B. tells the students to work together and prove it. Larent repeats to the other students, "work together, now." (It seems he wants Ms. B. to think he has been working and is taking the task seriously.) When Ms. B. leaves the table, Larent looks away from his table at another group ***(see picture at 10:33:35).*** He then leans back and rubs his eyes. He covers his eyes with his hands ***(see picture at 10:33:52).*** Larent then sings and looks at the other students at his table. He does not write or build with the cubes. He yawns ***(see picture at 10:35:42).*** Marcos and Lashanna build a tower together and record information. Larent looks around blankly and sings.
Inference of Student's Emotions Based	14) Disinterest, Boredom 15) Disinterest, Boredom

on Description	
Behaviors Pertaining to Student's *Behavioral* Engagement and Resulting Classification	14) Off-task *Covering eyes without noticeably connecting back to work* 15) Off-task *Covering eyes without noticeably connecting back to work, Fooling around in other ways*
Behaviors Pertaining to Student's *Cognitive* Engagement and Resulting Classification	14) Not Engaged *Evidence of lack of cognitive activity* 15) Not Engaged *Evidence of lack of cognitive activity*
Emotions Pertaining to Student's *Affective* Engagement and Resulting Classification	14) Mild negative feelings *Disinterest, Boredom, No evidence of positive feelings* 15) Mild negative feelings *Disinterest, Boredom, No evidence of positive feelings*

<table>
<tr><th colspan="5">Comparison of Students' Self-Reports on Questionnaire and Observed Student Behaviors
Ms. B.: Day 1</th></tr>
<tr><td colspan="5">Marcos</td></tr>
<tr><td rowspan="3">Self-Reported Affect</td><td colspan="2">Positive Emotions Score:</td><td colspan="2">Engagement with Impasse Score:</td></tr>
<tr><td>Marcos</td><td>Mean</td><td>Marcos</td><td>Mean</td></tr>
<tr><td>30/40</td><td>31.47/40</td><td>0/4</td><td>.78/4</td></tr>
<tr><td>Affect Inferred from Observed Behaviors</td><td colspan="4">Moderate positive feelings (Following Difficulty 5)
Moderate positive feelings (Following Difficulty 6)
Moderate positive feelings (Following Difficulty 7)
Moderate positive feelings (Following Difficulty 8)
Moderate positive feelings (Following Difficulty 9)
Moderate positive feelings (Following Difficulty 10)</td></tr>
<tr><td colspan="5">Comparison of Self-Reported vs. Observed Affect
Marcos's self-reported emotions for this class session were similar to the emotions observed from videotape analysis of the session. During the class session, Marcos appeared to have some level of interest in the mathematics problem but did not demonstrate strong positive feelings such as excitement, confidence, or pride. On his questionnaire, Marcos reported feeling somewhat interested and proud and not at all excited or curious, although very much confident.</td></tr>
<tr><td>Self-Report of Difficulty</td><td colspan="4">Had difficulty? Yes.
Classroom context of difficulty: Not during group conversation or whole class discussion.
What stands out about the situation? "I think what kids said or did."</td></tr>
<tr><td>Number of Analyzed Instances of Difficulty</td><td colspan="4">6</td></tr>
<tr><td colspan="5">Comparison of Self-Reported vs. Observed Instances of Difficulty
Reported difficulty and was involved in:
Difficulty 5- Impasse
Difficulty 6- Incorrect Answer
Difficulty 7- Incorrect Answer
Difficulty 8- Incorrect Answer
Difficulty 9- Incorrect Answer
Difficulty 10- Incorrect Answer</td></tr>
<tr><td colspan="5">Larent</td></tr>
<tr><td rowspan="3">Self-Reported Affect</td><td colspan="2">Positive Emotions Score:</td><td colspan="2">Engagement with Impasse Score:</td></tr>
<tr><td>Larent</td><td>Mean</td><td>Larent</td><td>Mean</td></tr>
<tr><td>29/40</td><td>31.47/40</td><td>0/4</td><td>.78/4</td></tr>
<tr><td>Affect Inferred from Observed Behaviors</td><td colspan="4">Mild negative feelings (Following Difficulty 5)
Mild positive feelings (Following Difficulty 11)
Moderate positive feelings (Following Difficulty 12)</td></tr>
</table>

	Mild negative feelings (Following Difficulty 14) Mild negative feelings (Following Difficulty 15)

***Comparison of Self-reported vs. Observed Affect**:*
Larent's self-reported emotions for this class session were more positive than the emotions inferred from observations of his behaviors. During much of the class session, Larent appeared disinterested and bored and showed no evidence of strong positive feelings. On his questionnaire, however, Larent reported feeling somewhat interested, not at all bored, and very happy (his happiness may have been unrelated to mathematics).

Self-Report of Difficulty	***Had difficulty?*** No. ***Classroom context of difficulty:*** Not applicable ***What stands out about the situation?*** (No response)
Number of Analyzed Instances of Difficulty	5

Comparison of Self-Reported vs. Observed Instances of Difficulty
Reported having no difficulty and was involved in:
- Difficulty 5- Impasse
- Difficulty 11- Incorrect Answer
- Difficulty 12- Incorrect Answer
- Difficulty 14- Does not Understand Solution
- Difficulty 15- Incorrect Idea

Lashanna

Self-Reported Affect	Positive Emotions Score:		Engagement with Impasse Score:	
	Lashanna	Mean	Lashanna	Mean
	21/40	31.47/40	0/2	.78/4
	Moderate positive feelings (Following Difficulty 5) Moderate positive feelings (Following Difficulty 6) Moderate positive feelings (Following Difficulty 7) Moderate positive feelings (Following Difficulty 9) Strong positive feelings (Following Difficulty 13)			

***Comparison of Self-Reported vs. Observed Affect**:*
During much of the class session Lashanna appeared interested in the mathematics problem and did not demonstrate strong positive or negative feelings. Lashanna's 'Positive Emotions Score" on her questionnaire, however, was low when compared with the mean. Lashanna reported feeling very bored and somewhat angry, disappointed, and unhappy. This discrepancy between observed and reported affect may be due to her negative feelings having been related to the group she was selected to work with as opposed to related to the mathematics problem. During Ms. B.'s interview, Ms. B. mentioned that Lashanna was upset because she had wanted to work with another group.

Self-Report of Difficulty	***Had difficulty?*** No. ***Classroom context of difficulty:*** Not applicable ***What stands out about the situation?*** "Nothing did not happen I did not teach nothing, I felt board and like a superstar, My teacher just was getting on my nerv, they [other kids] did not do nothing to me."
Number of Analyzed Instances of Difficulty	5
Comparison of Self-Reported vs. Observed Instances of Difficulty Reported having no difficulty but was involved in: Difficulty 5- Impasse Difficulty 6- Incorrect Answer Difficulty 7- Incorrect Answer Difficulty 9- Incorrect Answer Difficulty 13- Incorrect Answer	

Group Analysis of Self-Reported vs. Observed Affect
Larent and Lashanna reported they did not have difficulty with the mathematics problem. They were each involved in at least 5 analyzed instances of difficulty during the class session. Lashanna, who during the next day's session demonstrated the highest level of understanding among her groupmates, reported the most negative affect of the members in her group (and more than 10 points lower than the mean). Larent, who did not work much on the problem during either class session and appeared to have only a superficial understanding of the problem, reported a level of positive affect close to the mean.

Ms. B.
Class- Math 1: Honors Track, Dayfield St. School, Day 2
Group 1: Marcos, Larent, Lashanna

Marcos:

Overall Positive Emotions Score (40=most positive): 36

“Engagement with Impasse” Score (4=most engaged with impasse): 0

Reports of Emotions:

	Very Much:	**Somewhat:**	**Not at All:**
Positive/Negative Subscale	*Positive (2 points):* Respected Proud Successful Safe Happy Satisfied Relieved Confident	*Positive (1 point):* Interested Excited	*Positive (0 points):* Curious
	Negative (0 points):	*Negative (1 point):*	*Negative (2 points):* Unhappy Disappointed Worried Discouraged Angry Disrespected Bored Embarrassed Afraid
Engagement with Impasse Subscale	*(2 points)*	*(1 point)*	*(0 points)* Frustrated Confused

Reports of Difficulty:

Had difficulty? No.
Classroom context of difficulty: N/A
What stands out about the situation? “I think it’s what I felt because I felt like I was accepted and the kids saw how smart I was today.”

Lashanna:

Overall Positive Emotions Score (38=most positive*): 33
**No response for 'confident'*

"Engagement with Impasse" Score (4=most engaged with impasse): 0

Reports of Emotions:

	Very Much:	***Somewhat:***	***Not at All:***
Positive/Negative Subscale	*Positive (2 points):* Interested Respected Proud Successful Safe Excited Happy	*Positive (1 point):* Satisfied Relieved	*Positive (0 points):* Curious
	Negative (0 points):	*Negative (1 point):* Bored	*Negative (2 points):* Unhappy Disappointed Worried Discouraged Angry Disrespected Embarrassed Afraid
Engagement with Impasse Subscale	*(2 points)*	*(1 point)*	*(0 points)* Frustrated Confused

Reports of Difficulty:

Had difficulty? No.
Classroom context of difficulty: N/A
What stands out about the situation? "I felt real smart on camer and my math coach and teacher was proud of me."

Event 6 Difficulty 16: Marcos	
Transcript of Event **Time (of day): (9:41:37-9:43:11)**	This event occurs approximately 4 minutes into the class session. The students at group 1 have been sitting in their seats since class began but have not done any work. Marcos tells Lashanna that she and Larent need to do the writing for the presentation. Larent is sitting sideways on his chair and is facing away from the table. Ms. B. walks over to group 1 to check on their progress. Ms. B.: Ok, so what did you guys come up with so far? Marcos: Um, well…we said that for 5 blocks we need twenty…for a 5- block high tower we need 21 blocks. For a 10- block high tower we need 46 blocks. For a hundred block high tower we need 460 blocks. Ms. B.: Ok, so now, how could you find out for any size tower? (*She looks at Larent, who has been completely uninvolved in the task. He is sitting sideways and leaning backwards over his chair.)* Larent. *(Larent looks up. Ms. B. motions to the table.)* Marcos: Oh Ms. B.: *(to Larent)* You need to put some input into this as well. So what's going on? There's the cubes. Now, can you make them? Can you make them? Can you make the towers? *(Ms. B. turns back to Marcos.)* Go ahead. Marcos: Alright. Alright, you could find them because you could see there's a pattern. One's…there's always a 1 or 6. So, like, if you see here 46, the next one's gonna be 51. Then that next one's gonna be 56, and then it would be 61, and so on. Ms. B.: So what's the answer for a 100- block high tower? Marcos: It's…460. Ms. B.: So how did you get that number? Marcos: Oh…I…I didn't do this *(pointing to his paper).* I…I multiplied it. I put 10 multip…10….10 multiply it first um…46 and I got 460. Ms. B.: So can you write that out? Let me give you a transparency. *(She gets a transparency and hands it to Marcos.)* Can you write your thoughts on here, cuz that's…how did you get that. *(To Lashanna)* Do you agree with him? *(Lashanna does not answer.)* Or you don't know *(Ms. B. smiles)*…if you agree with him or not? *(Lashanna shrugs her shoulders.) (To Marcos)* So you need to try and convince her. Because she's not sure. So you need to try to convince her. Ok? Marcos: Uhm hm. Ms. B.: Ok. *(She leaves the table.)*
Description of Student Difficulty	Marcos says there are 460 blocks in the 100- block high tower [lines 23-24].
Type of Student	Incorrect Answer

Difficulty	
Type of Task	***Open Ended /Closed:*** Closed-ended ***Routine/Non-routine:*** Non-routine
Context of Student Difficulty	Small group
Teacher's Response(s)	Ms. B. asks Marcos how he got the number 460. She tells him to write his explanation on a transparency. Ms. B. then asks Lashanna if she agrees with Marcos. When Lashanna indicates that she doesn't know whether she agrees with Marcos, Ms. B. tells Marcos to convince Lashanna of his answer [lines 25-36]. Code: Probing- asking the student to explain his/her solution or the reasoning he/she used Inviting (verbally or non-verbally) (an)other student(s) to respond Suggesting student(s) share his/her/their difficulty with peer(s)
Teacher's Explanation for Response	Ms. B. wanted Marcos to get his thoughts on paper because it would stimulate a class discussion of his answer, and that would cause him to think about his solution. Ms. B. did not tell him the answer because if he would realize the mistake on his own, he would understand the idea more conceptually. *Response influenced by mathematics ability*? No. Code for teacher intention: M- Conceptual Understanding
Description of Behavior Related to Student's Engagement Following Response	After Ms. B. leaves Lashanna tells Marcos that she got 496 as her solution for the 100- block high tower. Marcos keeps repeating that the answer is 460 because "If 10 is 46, 100 of course is gonna be 460." While Marcos speaks, he balances his blank transparency on his marker. **See picture at 9:43:18**. Lashanna explains that she got 500 and took away 4. She explains that she took away one for each branch of the tower. She motions with her hands indicating four branches. While Lashanna speaks, Marcos continues to balance the transparency on his marker. **See picture at 9:43:37**. Marcos says that "it's not 500 because it's going by 1, 6, 1, 6. That's the

	pattern. When a pattern goes it sticks to its pattern." He then continues playing with the transparency, standing it up on his desk, and putting it against his face. **See pictures at 9:44:14 and 9:44:19**. He then sings, whistles, and drums his pencil and marker together. Lashannah and Marcos argue about who will write up the presentation. Each says the other should write. Ms. B. comes over to Larent who appears to be sleeping with his head on the back of his chair. When Ms. B. walks to the table, Marcos takes a marker and begins writing.
Inference of Student's Emotions Based on Description	Some level of interest, Boredom
Behaviors Pertaining to Student's *Behavioral* Engagement and Resulting Classification	On-task *Explaining idea to group mate, Writing*
Behaviors Pertaining to Student's *Cognitive* Engagement and Resulting Classification	Low level cognitive activity *Restating one's own idea without consideration of others' ideas*
Emotions Pertaining to Student's *Affective* Engagement and Resulting Classification	Mild positive feelings *Some level of interest mixed with slight negative feelings of boredom*

Event 7 Difficulty 17: Lashanna	
Transcript of Event **Time (of day): (9:47:50-9:48:38) (no audio this event)**	Ms. B. comes over to group 3 and tells Larent, who has been leaning over the back of his chair with his eyes closed, that he needs to work. After Ms. B. makes certain Larent understands what to do, she turns to Lashanna. Ms. B.: *(to Lashanna)* You also need to start building it because you need to see if what he's putting is correct or not. You can't just sit and let him get the answer. Lashanna: I did…(inaudible)…already Ms. B.: So then how did you get this? *(Ms. B. points to Lashanna's paper)* Lashanna: By building with the blocks. Ms. B.: How did you get 10? *(She points to Lashanna's paper)* Lashanna: 10...(inaudible)…? Ms. B.: Uhm hm Lashanna: I…(inaudible)…build with the blocks 10. With this…(inaudible)… Ms. B.: So then how did you get to 100? Lashanna: Uh…Marcos explained it to me. Ms. B.: So do you agree? *(Lashanna nods.)* So can you explain what he explained, to me? Lashanna: No, I don't know. Ms. B.: So then…so then can you try and come up with a way to see if he really is right? He might be right, he might be wrong. Try and see if he's right or wrong. Don't just agree with what he says. Ok? *(While Ms. B. says this, Lashanna leans her head on her hand and looks down.* ***See picture at 9:48:29****. She does not look up and does not answer Ms. B.)* *(Ms. B. then checks up on Larent's work and tells him she will be back in 5 or 10 minutes to see what he has done.)*
Description of Student Difficulty	When Ms. B. asks Lashanna how she got her solution for the 100- block high tower, she says that Marcos explained it to her. When Ms. B. asks Lashanna if she can repeat Marcos's explanation, she says she cannot [lines 14-21]. *(Note: Lashanna had previously arrived at a solution of 496 blocks for the 100- block high tower and had argued with Marcos, saying that her solution is correct and his solution of 460 blocks is incorrect. It interesting, therefore, that when Ms. B. questions her about the 100- block high tower in this event, she says she agrees with Marcos's solution and does not mention her previous solution of 496. Also, when Ms. B. leaves the table, Lashanna again argues with Marcos's solution of 460 blocks and maintains her previous solution of 496 blocks.)*
Type of Student Difficulty	Unable to Answer
Type of Task	***Open Ended /Closed:*** Open-ended

	Routine/Non-routine: Non-routine
Context of Student Difficulty	Small group
Teacher's Response(s)	Ms. B. tells Lashanna to try to come up with a way to check whether Marcos's solution is correct. Ms. B. says she should not "just agree with what he says" [lines 22-24]. Code: Asking the student to prove his/her solution
Teacher's Explanation for Response	Ms. B. responded more firmly to Lashanna than to the other students because Lashanna was sitting in her group and not working. She did not seem to be putting any effort into the mathematics task. She had not wanted to be in the group in which she was placed and preferred to be in a group with her friends. Ms. B. did not want her join her friends' group because if she did she would likely get distracted from the mathematics problem. According to Ms. B., Lashanna therefore responded by seeming to be uninvolved in the task when Ms. B. came over. *Response influenced by mathematics ability*? Yes. Lashanna is capable of much better work. Some days, she comes into class with an attitude and ready for any argument, which probably stems from her home situation, but other times, she is motivated and becomes involved in the task. Lashanna's sitting in her seat without trying therefore frustrated Ms. B. Code for teacher intention: Ab/S-Mathematics Ability At-Attitudes regarding participation in mathematics activities
Description of Behavior Related to Student's Engagement Following Response	When Ms. B. leaves the table, Lashanna again argues with Marcos's solution of 460 blocks for the 100- block high tower. Lashanna begins building a 5-block high tower. Lashanna tells Larent that he built his tower incorrectly. She says it should have 5 blocks in the height, not 6, and that the middle block counts in the height. Lashanna and Marcos agree that the 5- block high tower has 21 blocks. When Larent says he got 25 for a 5- block high tower, Lashanna and Marcos show him that he built his tower incorrectly. Lashanna builds a 10- block high tower, then punches numbers into her calculator. She tells Marcos that the 100- block high tower has 496 cubes. When Marcos disagrees, she explains how she got her answer. She says that since the 10-block high tower has 10 blocks in the height and 9 blocks in each of its branches, the 100-block high tower will have 100 blocks in the height and 99

	blocks in each of the branches. She says that 99 times 4 is 396 and if you then add 100, you get 496. Lashanna then asks Marcos to explain the method he used to arrive at a solution of 460 blocks. Marcos explains that he simply multiplied the number of blocks needed for a 10-block high tower by 10 to get the number of blocks needed for a 100-block high tower. Lashanna asks Larent whether he thinks Marcos's solution or hers is correct. She explains her solution to him, but he does not look up and continues building his tower. Ms. B. has been standing near group 1 and listening while Lashanna explained to Larent how she got 496 cubes. Ms. B. asks Lashanna to generalize her solution for any size tower. Lashanna comes up with a general solution where the number of blocks needed for any size tower is one less than the height of the tower times 4, plus the height of the tower.
Inference of Student's Emotions Based on Description	High level of interest, Confidence
Behaviors Pertaining to Student's *Behavioral* Engagement and Resulting Classification	On-task *Expressing disagreement with group mate's solution, Explaining idea to group mate, Using calculator, Watching group mate explain solution, Answering teacher's question*
Behaviors Pertaining to Student's *Cognitive* Engagement and Resulting Classification	High level cognitive activity *Justifying, Drawing inferences, Speculating*
Emotions Pertaining to Student's *Affective* Engagement and Resulting Classification	Strong positive feelings *High level of interest with no apparent negative feelings, Confidence in solution*

Event 8 Difficulty 18: Marcos	
Transcript of Event **Time (of day): (10:02:20-10:03:55) (no audio this event)**	When Marcos disagrees with Lashanna's answer of 496 blocks for a 100-block high tower, Lashanna asks Larent if he thinks Marcos's solution or her solution is correct. She tries to explain her method to Larent, but he does not look up and continues building with his blocks without answering her. Ms. B., however, overhears Lashanna's explanation. Lashanna points to the height and then to the branches of her 10- block high tower while she explains that since for the 10- block high tower there are 9 on each side, there would be 99 on each side for the 100- block high tower. She says 99 times 4 is 396, then you add 100 for the height, so the solution would be 496. Ms. B. then asks Lashanna how she would solve a 500- block high tower problem. Lashanna explains that she would multiply 499 by 4 and then add 500. Ms. B. then turns to Marcos. Ms. B.: *(to Marcos)* Now, what are you saying about what she's saying? *(Marcos shrugs his shoulders.)* Does it sound like a whole bunch of just nonsense? Lashanna: (inaudible) Marcos: I said…I said…no, I got 10. 10 blocks, 10 cubes is 46. A 10- high tower is 46 cubes. So 10 to 100…is 100…that's why I put 460. Ms. B.: So, you came up with a certain way of figuring it out, right? Why don't you *(pointing to Lashanna)…(pointing to Marcos)* Give her…give her something that you did already and say prove it to me by your…by your way. You give her…pick a number, for how many towers, that you figured out for already. Marcos: Um…5 Ms. B.: *(points to Lashanna)* 5. Ok. *(Lashanna removes 5 blocks from the height in her 10- block high tower and begins removing blocks from each branch.)* Ms. B.: *(to Marcos)* So you have your answer, right? So now you're gonna see if Lashanna's correct. You came up with what? Marcos: 21 Ms. B.: Let's see. *(Lashanna has almost finished converting her 10- block high tower to a 5- block high tower.) (to Marcos)* What do you think? Marcos: *(puts his hands up.* ***See picture at 10:03:25****)* I think…I think she probably won *(smiles).* Ms. B.: So you think she will? *(to Lashanna)* Ok, so now, can you walk us through your process again? Lashanna: Eh…this is 5 *(pointing to the height in her tower)*…so in these 4 *(pointing to the branches of her tower)…*so 4 times 4 is 16, and then you just add the 5, and it would get you 21. Marcos: *(smiles at Lashanna)* So you were right. *(to Ms. B.)* The same thing *(smiles).* Ms. B.: Can you guys write that up on your transparency?

	43	*(Lashanna takes the transparency from Marcos.)*
Description of Student Difficulty		When Ms. B. asks Marcos what he thinks about Lashanna's method for finding the total number of blocks in a tower of a given height, Marcos shrugs his shoulders. Marcos then describes the multiplying method he used to obtain a solution of 460 for the 100- block high tower [lines 13-18].
Type of Student Difficulty		Incorrect Idea
Type of Task		***Open Ended /Closed:*** Open-ended ***Routine/Non-routine:*** Non-routine
Context of Student Difficulty		Small group
Teacher's Response(s)		Ms. B. tells Marcos to give Lashanna a number of a tower height that he has already solved. She tells Marcos to have Lashanna figure it out her way and then compare her solution to his [lines 19-32]. Code: Offering a suggestion of a process to use in solving the problem
Teacher's Explanation for Response		N/A Code for teacher intention: N/A
Description of Behavior Related to Student's Engagement Following Response		When Lashanna arrives at a solution of 21 blocks for the 5- block high tower using her method, Marcos smiles and says that Lashanna is right [lines 33-41]. After Ms. B. leaves the table, Lashanna writes on the transparency while Marcos builds boxing men out of cubes **(See picture at 10:10:12).** When Ms. B. walks over to the table, Marcos covers the man he built with his hand and pretends to have been working. When the group presentations begin, Marcos continues playing with the blocks while he looking back and forth between the group presenting and his blocks. After about a minute, he stops playing with the blocks and listens to the presentation. During Marcos's group's presentation to the class (they are the second group to present), Lashanna explains her method for finding the total number of cubes for any size tower. Marcos tells the class about the pattern he found where the numbers representing the total number of blocks for consecutive towers switch off between ending in 1and 6. Marcos does not mention his method of multiplying the solution of the 10- block high tower by 10 in order to find the solution of the 100- block high tower. During the third group's presentation, Marcos watches the presentation while he plays with the blocks. Ms. B. tells the students not to play with blocks while students present. She takes the man Marcos built out of blocks away

	from him. Marcos watches the last group's presentation.
Inference of Student's Emotions Based on Description	Interest
Behaviors Pertaining to Student's *Behavioral* Engagement and Resulting Classification	On-task *Expressing agreement with group mate's solution, Watching classmates explain idea, Explaining idea to classmates*
Behaviors Pertaining to Student's *Cognitive* Engagement and Resulting Classification	High level cognitive activity *Explaining idea in one's own words*
Emotions Pertaining to Student's *Affective* Engagement and Resulting Classification	Moderate positive feelings *Some level of interest with no apparent negative or strong positive emotions.*

<table>
<tr><th colspan="5">Comparison of Students' Self-Reports on Questionnaire and Observed Student Behaviors
Ms. B.: Day 2</th></tr>
<tr><td colspan="5">Marcos</td></tr>
<tr><td rowspan="3">Self-Reported Affect</td><td colspan="2">Positive Emotions Score:</td><td colspan="2">Engagement with Impasse Score:</td></tr>
<tr><td>Marcos</td><td>Mean</td><td>Marcos</td><td>Mean</td></tr>
<tr><td>36/40</td><td>31.47/40</td><td>0/4</td><td>.78/4</td></tr>
<tr><td>Affect Inferred from Observed Behaviors</td><td colspan="4">Mild positive feelings (Following Difficulty 16)
Moderate positive feelings (Following Difficulty 18)</td></tr>
<tr><td colspan="5">Comparison of Self-Reported vs. Observed Affect
Marcos's self-reported emotions for this class session were more positive than the emotions inferred from his observed behaviors. During class, Marcos showed some level of interest in the problem but often appeared bored. He did not show evidence of strong positive feelings towards mathematics. On his questionnaire, Marcos reported he was only somewhat interested and not at all curious, but he reported being very proud, successful, and happy. He also reported being not at all bored.</td></tr>
<tr><td>Self-Report of Difficulty</td><td colspan="4">Had difficulty? No.
Classroom context of difficulty: N/A
What stands out about the situation? "I think it's what I felt because I felt like I was accepted and the kids saw how smart I was today."</td></tr>
<tr><td>Number of Analyzed Instances of Difficulty</td><td colspan="4">2</td></tr>
<tr><td colspan="5">Comparison of Self-Reported vs. Observed Instances of Difficulty
Reported no difficulty but was involved in:
Difficulty 16- Incorrect Answer
Difficulty 18- Incorrect Idea</td></tr>
<tr><td colspan="5">Larent</td></tr>
<tr><td colspan="5">Larent was not involved in analyzed instances of difficulty during this class session.</td></tr>
<tr><td colspan="5">Lashanna</td></tr>
<tr><td rowspan="3">Self-Reported Affect</td><td colspan="2">Positive Emotions Score:</td><td colspan="2">Engagement with Impasse Score:</td></tr>
<tr><td>Lashanna</td><td>Mean</td><td>Lashanna</td><td>Mean</td></tr>
<tr><td>33/38</td><td>31.47/40</td><td>0/4</td><td>.78/4</td></tr>
<tr><td>Affect Inferred from Observed Behaviors</td><td colspan="4">Strong positive feelings (Following Difficulty 17)</td></tr>
<tr><td colspan="5">Comparison of Self-Reported vs. Observed Affect:
Lashanna's self-reported affect during this class session was similar to the affect observed from her observed behaviors. During the class session, Lashanna appeared highly interested in the</td></tr>
</table>

<table>
<tr><td colspan="2">mathematics problem and eager to convince her groupmates that her solution was correct. She did not show evidence of any negative feelings. On her questionnaire, Lashanna reported feeling very interested, proud, successful, excited, and happy.</td></tr>
<tr><td>Self-Report of Difficulty</td><td>Had difficulty? No.
Classroom context of difficulty: N/A
What stands out about the situation? “I felt real smart on camer and my math coach and teacher was proud of me.”</td></tr>
<tr><td>Number of Analyzed Instances of Difficulty</td><td>1</td></tr>
<tr><td colspan="2">Comparison of Self-Reported vs. Observed Instances of Difficulty
Reported having no difficulty but was involved in:
Difficulty 17- Unable to Answer

Information Ms. B. offered during the teacher interview may explain the reason Lashanna reported she did not have difficulty during this session. During the interview, Ms. B. explained that Lashanna’s difficulty here (Difficulty 17) was attitudinal rather than mathematics-related. Ms. B. said Lashanna was upset she was not permitted to work with another group and therefore was not expending much effort working on the mathematics problem. Videotape evidence following Difficulty 17 showed that Lashanna had, in fact, understood the problem, but probably did not want to discuss her solution with Ms. B.</td></tr>
</table>

Group Analysis of Self-Reported vs. Observed Affect
Marcos and Lashanna both reported considerably more positive affect during this mathematics session than during the previous session. They may have felt more positive regarding mathematics on Day 2 since they came up with a general solution to the problem on that day and presented it to the class. Both students remarked on their questionnaires that they felt smart that day.

Ms. S.
Class 343: Standard Track, Summit Ave. School, Day 1
Group 3: Ta'keisha, Leo, Ordena

Students' Reports on Emotions Questionnaire

Ta'keisha:

Overall Positive Emotions Score (40=most positive): 30

"Engagement with Impasse" Score (4=most engaged with impasse): 3

Reports of Emotions:

	Very Much:	***Somewhat:***	***Not at All:***
Positive/Negative Subscale	*Positive (2 points):* Respected Safe Excited Confident Curious	*Positive (1 point):* Interested Proud Successful Happy	*Positive (0 points):* Satisfied Relieved
	Negative (0 points):	*Negative (1 point):* Angry Bored	*Negative (2 points):* Unhappy Disappointed Worried Discouraged Disrespected Embarrassed Afraid
Engagement with Impasse Subscale	*(2 points)* Frustrated	*(1 point)* Confused	*(0 points)*

Reports of Difficulty:

Had difficulty? Yes.
Classroom context of difficulty: Group
What stands out about the situation? (No response)

Leo:

Overall Positive Emotions Score (40=most positive): 36

"Engagement with Impasse" Score (4=most engaged with impasse): 1

Reports of Emotions:

	Very Much:	***Somewhat:***	***Not at All:***
Positive/Negative Subscale	*Positive (2 points):* Interested Proud Successful Safe Excited Happy Relieved Confident	*Positive (1 point):* Respected Satisfied	*Positive (0 points):* Curious
	Negative (0 points):	*Negative (1 point):*	*Negative (2 points):* Unhappy Disappointed Worried Discouraged Angry Disrespected Bored Embarrassed Afraid
Engagement with Impasse Subscale	*(2 points)*	*(1 point)* Confused	*(0 points)* Frustrated

Reports of Difficulty:

Had difficulty? Yes.
Classroom context of difficulty: Whole Class
What stands out about the situation? "I felt like I could do this It does not matter if I was on camera I will still get my work done and focus and cut out all of the things that are happening around the classroom."

Ordena:

Overall Positive Emotions Score (40=most positive): 37

"Engagement with Impasse" Score (4=most engaged with impasse): 1

Reports of Emotions:

	Very Much:	***Somewhat:***	***Not at All:***
Positive/Negative Subscale	*Positive (2 points):* Interested Proud Successful Safe Excited Happy Relieved Confident Curious	*Positive (1 point):* Respected Satisfied	*Positive (0 points):*
	Negative (0 points):	*Negative (1 point):* Worried	*Negative (2 points):* Unhappy Disappointed Discouraged Angry Disrespected Bored Embarrassed Afraid
Engagement with Impasse Subscale	*(2 points)*	*(1 point)* Confused	*(0 points)* Frustrated

Reports of Difficulty:

Had difficulty? Yes.
Classroom context of difficulty: Group
What stands out about the situation? "At first I didn't really understand what the problem was trying to say. but teacher explained it carefully and so did my group mate and I understood it."

Event 9 **Difficulty 19: Ordena**	
Transcript of Event **Time (of day): (11:30:27-11:31:20)**	The students in the class are seated at desks which are clustered in groups. Ms. S. introduces the task for the day to class. She says that Leo said something about a 5- block tower and that Tyshonnah said that letter B has 5 blocks. Ms. S. asks the students if that (the stage B diagram) is the 5-block tower. Some students say “yes” and some say “no.” She asks a student to reread the last part of the description of the task aloud. Student (off camera): I will need to build a 5-block high tower, a ten… Ms. S.: Stop there, go ahead, one more time? Student: I will need to build a 5-block high tower… Ms. S.: Is letter B a 5- block high *(with emphasis on ‘high’)* tower? Several students (off camera): no Ms. S.: Why not? You just said it was. *(Ordena raises her hand.)* Ordena, go ahead. Ordena: *(smiling)* uh…uh, never mind *(Ms. S. calls on another student who is not seen on the camera.)* Student: Oh, maybe because the height, like the…in the middle of the height it has to be 5 Ms. S.: Is that true? These are the types of questions… Another student *(off camera)*: I think I know what it’s saying. I think what she [the student who previously spoke] meant is that um…we need um…we have to like figure out how many cubes are made out of the 5 blocks *(Ordena raises her hand again.)* Ordena: I think we’re trying to find different patterns by um making um building the blocks or something
Description of Student Difficulty	Ordena raises her hand to answer the teacher’s question about why the diagram does not represent a 5-block high tower, but then smiles and says “never mind” when the teacher calls on her [lines 10-15].
Type of Student Difficulty	Unable to Answer
Type of Task	***Open Ended /Closed:*** Closed-ended ***Routine/Non-routine:*** Non-routine
Context of Student Difficulty	Whole class
Teacher’s Response(s)	Ms. S. calls on other students [lines 15-22].

	Code: Inviting other students to respond
Teacher's Explanation for Response	---- Code for teacher intention: ----
Description of Behavior Related to Student's Engagement Following Response	After other students speak, Ordena raises her hand and adds to the discussion [lines 23-25]. Ms. S. then continues to explain the problem to the class. When the students in the class begin working, Leo reads the problem aloud with Ta'keisha. Ordena picks up her paper and reads a phrase from it regarding adding blocks to the sides to build the next stage of the tower. Leo explains to Ordena that to build the next stage of the towers, you must add one block to each side. Leo brings blocks to the table and Ordena takes some. She arranges four blocks in a square shape and puts another block on top. Ordena tells Ta'keisha that the stage of the tower she built (stage 2) has five blocks. Ta'keisha tells Ordena that in the picture the blocks are "spaced out" and not "pushed together." Ordena spreads out the four blocks in the base of the tower she had built but does not place the top block back on the tower.
Inference of Student's Emotions Based on Behavioral Description	Interest in participating in class discussion and solving the problem
Behaviors Pertaining to Student's *Behavioral* Engagement and Resulting Classification	On-task *Participating in class discussion, Reading directions aloud, Using manipulatives in way related to task at hand*
Behaviors Pertaining to Student's *Cognitive* Engagement and Resulting Classification	Low level cognitive activity *Highest level of cognitive activity involved: Paraphrasing or referring to materials, Building towers without consideration of patterns*
Emotions Pertaining to Student's *Affective* Engagement and Resulting Classification	Moderate positive feelings *Some level of interest in participating in class discussion and solving the problem. No apparent negative feelings such as disinterest, boredom, or embarrassment. No apparent strong positive feelings such as excitement or confidence.*

Event 10 **Difficulty 20: Leo** **Difficulty 21: Leo, Ta'keisha, Ordena**	
Transcript of Event **Time (of day): (11:37:50-11:40:06)**	Ordena, Leo, and Ta'keisha are building towers with blocks. Leo calls over Ms. S.. Leo: Ms. S., I want to know, you see how this one is, right? *(Pointing to his paper)* These are actually 6 blocks and 11 blocks, cuz the one that's holding it. Ta'keisha: *(with Leo)* That's 6 blocks. Ms. S.: Ok Leo: *(Showing Ms. S. the tower he built which has no middle block)* So what we was trying to do was trying to balance it without the block in the middle. Ms. S.: So, do you think there's a block in the middle on that picture? Leo: yes Ms. S.: So, should you put a block in the middle there? Ta'keisha and Ordena: yeah Ms. S.: Do it. What? You don't need to balance it then, right? *(to Leo)* If these blocks don't work for you, you could take the other ones that don't have the… Leo: Yeah, that's cuz these got the edge Ms. S.: You want the other ones? Leo: Yeah *(He begins putting the blocks on the table in a bag.)* Ms. S.: So while Leo is getting the other one, what question, what's your question. What are you gonna…what are you trying to figure out…why are you guys building towers here? Leo: trying to solve the volume Ms. S.: The what then? Leo: The volume. Ms. S.: the volume Ta'keisha: we need an equation, right? Ms. S.: What makes you think you need an equation? Ta'keisha: It says…um…*(reading from the task sheet)* I would like to know how many cubes I will need to build a five-block high tower, a ten-block and a hundred Ms. S.: Uh hmm. Ta'keisha: (inaudible)...you don't want us to go all the way up to 100 Ms. S.: Ok, well, did you want to go all the way up to 100? I mean, we've got time. Ta'keisha: no Ms. S.: So I guess an equation would be helpful in figuring that out. What are some things that you've done previously that have helped you find equations? When you had scenarios, what are you finding equations from? What did you use to establish an equation. Think back to tests you've had.

	Leo: y=mx+b Ta'keisha: we used tables Ms. S.: We used tables. What else did you use? Leo: graphs Ms. S.: Graphs. Can you construct a table or a graph? It can help you …(inaudible)… Do you have enough information there to do something like that? Ta'keisha: Yeah. Ms. S.: Ok, let's …(inaudible)… and you can start doing it. Or contin… and you can continue building like you were actually doing, but remember to find a way to keep your information organized. Ok, and since Ta'keisha's saying you're looking for an equation, you want to do things that are going to help you get there today. If you decide that that's not what you're looking for, then you're going to have to kind of reach back.
Description of Student Difficulty	20) Ms. S. asks the students what they are trying to figure out in the problem. Leo says they are trying to solve the volume [lines 21-26]. 21) The students are not sure how to proceed in solving the problem [lines 21-49].
Type of Student Difficulty	20) Incorrect Answer 21)Impasse
Type of Task	***Open Ended /Closed:*** 20) Closed-ended 21) Open-ended ***Routine/Non-routine:*** 20-21) Non-routine
Context of Student Difficulty	20-21) Small group
Teacher's Response(s)	20) Ms. S. repeats Leo's incorrect response without commenting on it [line 27]. Code: Acknowledging the incorrect response 21) Ms. S. asks the students specific questions to have them come up with a strategy for solving the problem. She also gives them tips such as remembering to keep their information organized [lines 21-55]. Code: Leading the students to the correct answer
Teacher's Explanation for Response	--------------- Code for teacher intention:
Description of Behavior Related to	20) Leo is involved in answering Ms. S.'s questions [lines 38-42] 21) After the teacher leaves the table, the students do not talk much to each

Students' Engagement Following Response	other. Ta'keisha writes on a paper while Ordena and Leo build towers. Their minimal conversation is task-related. The students work continuously.
Inference of Student's Emotions Based on Behavioral Description	20) Interest in answering the teacher's question 21) L: Interest in solving the problem, seriousness towards task, confidence in their ability to complete the task T: Interest in solving the problem, seriousness towards task, confidence in their ability to complete the task O: Interest in solving the problem, seriousness towards task, confidence in their ability to complete the task
Behaviors Pertaining to Student's *Behavioral* Engagement and Resulting Classification	20) On-task *Answering teacher's question* 21) L: On-task *Using manipulatives in way related to task at hand* T: On-task *Writing* O: On-task *Using manipulatives in way related to task at hand*
Behaviors Pertaining to Student's *Cognitive* Engagement and Resulting Classification	20) Insufficient information 21) L: Insufficient information T: Insufficient information O: Insufficient information
Emotions Pertaining to Student's *Affective* Engagement and Resulting Classification	20) Moderate positive feelings *Some level of interest with no apparent negative feelings* 21) L: Strong positive feelings *High level of interest with no apparent negative feelings, Confidence* T: Strong positive feelings *High level of interest with no apparent negative feelings, Confidence* O: Strong positive feelings *High level of interest with no apparent negative feelings, Confidence*

Event 11 **Difficulty 22: Leo, Ta'keisha, Ordena**	
Transcript of Event **Time (of day): (11:53:39-11:58:00)**	Ta'keisha, Leo, and Ordena are sitting in their group and building towers with their blocks. Leo calls over Ms. S. Ta'keisha: Ms. S., do we have to be, like 5- blocks high? It has to be 5 blocks high? Ms. S.: Um… I don't know. That's what it says in the problem. Leo: That's how…that's what… Ta'keisha: But…that's what we think it means. Ms. S.: Ok…so tell me…what's the difference between what you *(points to Leo)* have and what she *(points to Ta'keisha)* has? Leo: She's laying it down Ta'keisha: No. I had it up. *(The blocks in the height of Ta'keisha's tower were laying flat on her desk. She picks up the blocks and stacks them vertically in the center of her tower as she says this. Ta'keisha's tower is 5 blocks in height but has only 2 branches. It now looks like an inverted 'T.'* ***See picture at 11:54:11****.)* Leo: Well you have more blocks than me. Only you have it up. See how this picture is? Ta'keisha: I had five. Leo: 5 blocks high. Mine's just 1, 2, 3, 4…*(He counts the blocks in his stage B tower, pointing to each block as he counts it.)* Ms. S.: So you're…so, I just want to make sure I understand what you're saying. You're *(to Leo)* saying this *(pointing to Leo's stage B tower)* is the five block high *(Leo nods)* but you're *(referring to Ta'keisha)* saying this *(pointing to Ta'keisha's tower)* is the five block high. *(Ta'keisha nods.)* Ordena, what do you think? Do you agree with either one of them or do you have… Ordena: I have…(inaudible)…cuz that's what he's doing…(inaudible)…on the picture…(*she points to Leo's tower, then to her task paper*) Ms. S.: So in the picture you're saying that he's…that it says that a five block high tower should look like this *(pointing to Leo's tower)*? Ordena: (nodding) yes Ms. S.: *(to Ta'keisha)* She *(Ordena)*'s saying that in the picture it says the five block high tower should look like this. Ta'keisha: But that's *(pointing to the Stage B diagram on her task paper)* not five blocks, that's six. *(Ms. S. looks at Ordena, then at Leo.)* Leo: Take one out…(inaudible)…Take the middle out, see?... (inaudible)… *(He takes the middle block out of his tower as he says this.)* Ta'keisha: But that's not how it look in the picture Ms. S.: Can you take the middle out? Ta'keisha: It is 5.

Leo: I was trying to balance it.

Ms. S.: Ok, so let me ask you this question. Let's read through the problem one more time, right? *(Ms. S. crouches between Ordena and Ta'keisha so that she is at eye level with the students.* ***See picture at 11:55:06****.)* So in the problem it says, 'You're constructing a tower,' right? So we all have towers in front of us. And, um…the towers that you're constructing should look like the towers that they have below, right? Cuz it says, 'I'm constructing towers as you see below.' So can I make up my own towers?

Ordena, Leo: no

Ms. S.: No, right? It has to look like the ones in the picture. Does that make sense?

Ordena, Leo, Ta'keisha: um hm

Ms. S.: Ok. So I noticed that each time I made the tower higher, I added more blocks onto the sides. So what is the very first tower he made? Show me what it looks like, that very first one. *(Leo, Ordena, and Ta'keisha point to their papers.)* Show me with a block. *(Leo and Ta'keisha each pick up one block.)* What it looks like. Ok. When he goes…So Ordena, you show me.

(Ordena points to her paper.)

Ms. S.: Show me with your block. *(Ordena picks up one block.)* So, ok. When he goes to make that second tower, right? Does he start all over again and just make a whole new design?

Ordena, Leo, Ta'keisha: no *(shake their heads 'no')*

Ms. S.: What does he do to get to the second tower?

Leo: He kept adding towers around it.

Ms. S.: Ok, so show me how he adds to make the second tower.

Leo: So, wait. He says that he adds one t…one to the sides…and one to the top *(as he speaks, he adds one block to each side of his stage 'A' tower and one block to the top).*

Ms. S.: So now let me ask you this question. Can the middle be empty?

Students: No.

Leo: Cuz he kept adding. *(Ms. S. smiles and nods)* Oh, ok. I thought it was…so the way he constructed it he just kept adding one more *(he adds one more block to each of the side and to the top of his stage 'B' tower)*

Ms. S.: Ok, now, let me ask you *(looking at Ta'keisha)* a question here. Let's look at Ta'keisha's 5- block tower. Did she follow the instructions? Does she keep adding to the sides?

Leo: Yup. And it not…it actually is goin' upwards. Like a…a 'T' but upside down. It's like little…it's supposed…he has 4 sides…(inaudible)…sides. A cube got 4 sides to them….just can't put…he kept adding once to the sides so with the way he did it, it was like 'x'…em…um… with a stick in the middle. So the way he did it…

Ms. S.: Does that make sense? Are you gonna (inaudible) ? Are we reevaluating now how we kind of constructed them? So, can we now agree

	on a 5- block high tower? Show me what that looks like. Is that 5 blocks? Ordena: But there's one in the middle so it's basically 6 blocks. Leo: (with Ta'keisha) 6 blocks Ms. S.: Ok, so it is 6 blocks. So is that a 5- block *high* tower? *(Leo and Ta'keisha shake their heads 'no.')* Ms. S.: So show me what a 5- block *high* tower….ok, what's this? Describe this to me. This is a what *(she points to Ta'keisha's task sheet)*? Ta'keisha: One Ms. S.: One block *high* tower, right? Cuz it's only one block high. What about this one *(pointing to Ta'keisha's task sheet)*? Leo: 2 blocks high tower Ms. S.: It's a 2- block high tower. What about this one *(pointing to Ta'keisha's task sheet)*? Ordena: Three Ms. S.: So could you show me a five block high tower is? *(She points to Leo's tower)* Did you maintain the pattern with it? *(Leo begins building with blocks on his desk).* Ordena: Ohhh!...*(Ms. S. gets up and begins to walk away from their table.)*…Ohhh! That's what it's trying to say. Let me get some blocks. *(Leo passes a box of blocks to Ordena. Ms. S. walks away from the table).*
Description of Student Difficulty	The students disagree about how a 5- block high tower should be constructed. Leo's version of a 5- block high tower is a stage B tower which is missing the middle block (and consists of 5 blocks in total). Ta'keisha has built a tower which is 5 blocks in height but has only 2 branches, resembling an inverted 'T.' The students call Ms. S. to their table to discuss their difficulty [lines 1-20].
Type of Student Difficulty	Impasse
Type of Task	***Open Ended /Closed:*** Closed-ended ***Routine/Non-routine:*** Non-routine
Context of Student Difficulty	Small group
Teacher's Response(s)	Ms. S. repeats what the students have said and asks them if she understood them correctly. She asks Ordena if she agrees with either Leo or Ta'keisha. When Ordena says she agrees with Leo, who is referring to the stage B tower without the middle block as a 5- block high tower, Ms. S. asks the students questions and reads the directions of the task with them to help them understand that the towers need to follow a particular pattern and that each tower builds on the previous one [lines 21- 102]. Code: Asking a question to clarify what the students said

	Inviting another student to respond Leading the students to the correct idea
Teacher's Explanation for Response	The students were weak in mathematics and tended to work alone rather than as a group. When they faced impasse, they often struggled individually without using strategies to overcome their difficulties. This impeded their ability to move forward with the mathematics problem. The response offered here was intended to encourage the students to work together and to guide the students through the problem in a step by step manner. While guiding the students through the problem, Ms. S. modeled for the students the deductive thinking that would bring them to the next step at each point. *Response influenced by students' mathematics ability?* Yes. Leo is a student who can easily veer off-track in his problem solving without continuous guidance, so Ms. S. worked with the group, leading them through the difficulty and continuously questioning Leo to redirect him to the correct process. Code for teacher intention: M- Problem Solving Skills Ab/S-Mathematics Ability
Description of Behavior Related to Student's Engagement Following Response	After Ms. S. leaves the table, the students begin building towers. Ordena says, "Oh, this is easy." Leo says, "easy?" All the students correctly build a 5- block high tower. They count the blocks in their towers and disagree about how many blocks there are. Ta'keisha says there are 21 blocks. Ta'keisha and Ordena record this on their papers. Ordena counts the blocks in her tower by 5's and says there are 25 blocks. Ta'keisha says there are 21. Leo agrees with Ordena and says there are 25 blocks in total since there are 5 blocks in each side. Ta'keisha tells them that they counted the middle block 4 times. Leo says, "so it's 24." Ta'keisha separates each branch of Ordena's tower from the height **(see picture at 12:01:43)** and shows the other students that the middle block should only be counted once, with the height, but not as part of each branch. Leo counts the blocks in the tower again and says, "yeah, it is 21." The students record on their papers that the 5- block high tower has 21 blocks **(see picture at 12:02:36).** Leo says, "So what about the 10 block high tower? What is it? 42." Ordena says, "So for the 10 block high tower…" Leo says, "42." Ta'keisha says, "double it." They record 42 as their solution for the 10- block high tower. They add 21 and get 63 blocks for a 15- block high tower. They try to solve the 100- block tower by considering multiples of 21. They record multiples of 21 on their papers. Ordena then says that a 5- block high tower should have 25 blocks "because you still have to count the middle one." Ta'keisha

	builds a stage B tower and counts the blocks, removing each block as she counts it. She shows Ordena that the middle block should be counted only once. Ta'keisha then builds a 10- block high tower and shows Leo and Ordena that there are 9 blocks in each branch, which would total 36 blocks. She says there are 10 blocks in the height, so there are 46 blocks in total. The students realize this number is not the same as the 42 they had calculated earlier by doubling 21. They decide to rebuild the 5- block high tower. They count the blocks in the 5- high tower they build and again get 21 blocks. Most of the time, Leo has been watching the other students with a blank expression on his face. He does not smile. He sometimes contributes to the conversation but does so hesitatingly and looks to the other students for confirmation of what he says. He often asks Ta'keisha to repeat what she said. He also makes computational errors in his calculations. Ta'keisha, who has been dominating the conversation in the group, has been speaking mostly to Ordena. Ms. S. announces at approximately 12:19 that the students need to stop working.
Inference of Student's Emotions Based on Behavioral Description	L: Interest, Confusion, Slight feeling of inferiority to another student T: Interest, Confidence O: Interest
Behaviors Pertaining to Student's Behavioral Engagement and Resulting Classification	L: On-task *Using manipulatives in way related to task at hand, Writing, Explaining idea to group mate, asking for help from group mate* T: On-task *Using manipulatives in way related to task at hand, Writing, Explaining idea to group mate* O: On-task *Using manipulatives in way related to task at hand, Writing, Explaining idea to group mate, Asking for help from group mate*
Behaviors Pertaining to Student's *Cognitive* Engagement and Resulting Classification	L: High level cognitive activity *Highest level of cognitive activity involved: Justifying, Speculating, Reasoning* T: High level cognitive activity *Highest level of cognitive activity involved Speculating, Justifying, Reasoning, Explaining in one's own words* O: High level cognitive activity *Highest level of cognitive activity involved: Justifying, Speculating, Reasoning*
Emotions Pertaining to Student's *Affective* Engagement and	L: Mild positive feelings *Some level of interest mixed with slight negative feelings of inferiority to another student* T: Strong positive feelings

Resulting Classification	*High level of interest with no apparent negative feelings, Confidence* O: Moderate positive feelings *Some level of interest with no apparent negative feelings*

<table>
<tr><th colspan="5">Comparison of Students' Self-Reported Affect on Questionnaire and Affect Inferred from Observed Student Behaviors
Ms. S.: Day 1</th></tr>
<tr><td colspan="5">Ta'keisha</td></tr>
<tr><td rowspan="3">Self-Reported Affect</td><td colspan="2">Positive Emotions Score:</td><td colspan="2">Engagement with Impasse Score:</td></tr>
<tr><td>Ta'keisha</td><td>Mean</td><td>Ta'keisha</td><td>Mean</td></tr>
<tr><td>30/40</td><td>31.47/40</td><td>3/4</td><td>.78/4</td></tr>
<tr><td>Affect Inferred from Observed Behaviors</td><td colspan="4">Strong positive feelings (following Difficulty 21)
Strong positive feelings (following Difficulty 22)</td></tr>
<tr><td colspan="5">Comparison of Self-Reported vs. Observed Affect
Inferences made from Ta'keisha's behaviors during analyzed segments of the class session showed Ta'keisha demonstrated strong positive affect. Ta'keisha appeared confident and very interested in solving the problem. She did not show evidence of negative emotions. On her questionnaire, however, Ta'keisha reported feeling only somewhat interested and happy and not at all satisfied or relieved. She also reported feeling somewhat angry and bored.</td></tr>
<tr><td>Self-Report of Difficulty</td><td colspan="4">Had difficulty? Yes.
Classroom context of difficulty: Group
What stands out about the situation? (No response)</td></tr>
<tr><td>Number of Analyzed Instances of Difficulty</td><td colspan="4">2</td></tr>
<tr><td colspan="5">Comparison of Self-Reported vs. Observed Instances of Difficulty
Reported difficulty and was involved in:
Difficulty 21- Impasse
Difficulty 22- Impasse</td></tr>
<tr><td colspan="5">Leo</td></tr>
<tr><td rowspan="3">Self-Reported Affect</td><td colspan="2">Positive Emotions Score:</td><td colspan="2">Engagement with Impasse Score:</td></tr>
<tr><td>Leo</td><td>Mean</td><td>Leo</td><td>Mean</td></tr>
<tr><td>36/40</td><td>31.47/40</td><td>1/4</td><td>.78/4</td></tr>
<tr><td>Affect Inferred from Observed Behaviors</td><td colspan="4">Strong positive feelings (following Difficulty 20)
Strong positive feelings (following Difficulty 21)
Mild positive feelings (following Difficulty 22)</td></tr>
<tr><td colspan="5">Comparison of Self-reported vs. Observed Affect:
Leo reported having overall strong positive feelings during the class session. He reported feeling very: proud, successful, excited, and happy. Leo's behaviors during class, however, did not provide evidence of these feelings. At times Leo displayed much interest with no evidence of negative feelings, but he did not show evidence of feeling proud, successful, excited, or happy. During the end of the class session, Leo showed evidence of feeling inferior to Ta'keisha. He watched the other students at his table and did not smile or contribute much to the conversation. On the questionnaire, Leo reported feeling not at all: unhappy, afraid, or worried.</td></tr>
</table>

<table>
<tr><td>Self-Report of Difficulty</td><td colspan="4">Had difficulty? Yes.
Classroom context of difficulty: Whole Class
What stands out about the situation? “I felt like I could do this It does not matter if I was on camera I will still get my work done and focus and cut out all of the things that are happening around the classroom.”</td></tr>
<tr><td>Number of Analyzed Instances of Difficulty</td><td colspan="4">3</td></tr>
<tr><td colspan="5">Comparison of Self-Reported vs. Observed Instances of Difficulty
Reported difficulty and was involved in:
Difficulty 20- Incorrect Answer
Difficulty 21- Impasse
Difficulty 22- Impasse</td></tr>
<tr><td colspan="5">Ordena</td></tr>
<tr><td rowspan="3">Self-Reported Affect</td><td colspan="2">Positive Emotions Score:</td><td colspan="2">Engagement with Impasse Score:</td></tr>
<tr><td>Ordena</td><td>Mean</td><td>Ordena</td><td>Mean</td></tr>
<tr><td>37/40</td><td>31.47/40</td><td>1/4</td><td>.78/4</td></tr>
<tr><td>Affect Inferred from Observed Behaviors</td><td colspan="4">Moderate positive feelings (following Difficulty 19)
Strong positive feelings (following Difficulty 21)
Moderate positive feelings (following Difficulty 22)</td></tr>
<tr><td colspan="5">Comparison of Self-Reported vs. Observed Affect:
On her emotions questionnaire, Ordena reported feeling very: proud, happy, and excited. No evidence for these emotions was found in her behaviors during class. After Difficulty 19, Ordena tried to build a stage 2 tower but did so incorrectly. Ta’keisha pointed out her error and Ordena revised her tower, but it was still not structured correctly. During the portion of the class session for which Ordena’s affect was coded as Strong positive feelings, Ordena’s behavior indicated feelings of interest and confidence, but no evidence was observed for her reported feelings of much pride, happiness, and excitement.</td></tr>
<tr><td>Self-Report of Difficulty</td><td colspan="4">Had difficulty? Yes.
Classroom context of difficulty: Group
What stands out about the situation? “At first I didn’t really understand what the problem was trying to say. but teacher explained it carefully and so did my group mate and I understood it.”</td></tr>
<tr><td>Number of Analyzed Instances of Difficulty</td><td colspan="4">3</td></tr>
<tr><td colspan="5">Comparison of Self-Reported vs. Observed Instances of Difficulty
Reported difficulty and was involved in:
Difficulty 19- Unable to Answer
Difficulty 21- Impasse
Difficulty 22- Impasse</td></tr>
</table>

Group Analysis of Self-Reported vs. Observed Affect
Ta'keisha, who demonstrated the highest level of understanding of the problem among the members of her group, reported the highest level of engagement with impasse. Leo and Ordena, who displayed considerably more difficulty with the problem, had scores of only 1/4 for "Engagement with Impasse." Ta'keisha's positive emotions score was the lowest among the scores of the members in her group. Her observed affect, however, was the most positive of all her group members.

Ms. S.
Class 343: Standard Track, Summit Ave. School, Day 2
Group 3: Ta'keisha, Leo, Ordena

Ta'keisha:

Overall Positive Emotions Score (44=most positive*): 30
**Two feelings added by student*

"Engagement with Impasse" Score (4=most engaged with impasse): 2

Reports of Emotions:

	Very Much:	**Somewhat:**	**Not at All:**
Positive/Negative Subscale ***Feelings added by student*	*Positive (2 points):* Respected Successful Safe Confident	*Positive (1 point):* Interested Proud Happy Satisfied Relieved Curious	*Positive (0 points):* Excited
	Negative (0 points): Bored	*Negative (1 point):* Disappointed Angry **Annoyed **Tired	*Negative (2 points):* Unhappy Worried Discouraged Disrespected Embarrassed Afraid
Engagement with Impasse Subscale	*(2 points)*	*(1 point)* Frustrated Confused	*(0 points)*

Reports of Difficulty:

Had difficulty? No.
Classroom context of difficulty: N/A
What stands out about the situation? (No response.)

When asked to indicate any thoughts she had that were not on the list of thoughts in the questionnaire, Ta'keisha wrote, "The kids in this class are mad annoy. I can't wait to lunchtime."

Leo:

Overall Positive Emotions Score (40=most positive): 28

"Engagement with Impasse" Score (4=most engaged with impasse): 0

Reports of Emotions:

	Very Much:	***Somewhat:***	***Not at All:***
Positive/Negative Subscale	*Positive (2 points):* Interested	*Positive (1 point):* Respected Proud Successful Safe Excited Happy Satisfied Relieved Confident	*Positive (0 points):* Curious
	Negative (0 points):	*Negative (1 point):* Unhappy	*Negative (2 points):* Disappointed Worried Discouraged Angry Disrespected Bored Embarrassed Afraid
Engagement with Impasse Subscale	*(2 points)*	*(1 point)*	*(0 points)* Frustrated Confused

Reports of Difficulty:

Had difficulty? No.
Classroom context of difficulty: N/A
What stands out about the situation? "I felt like I was smart and I could get the problem without the person bothering me."
(On the first page of the questionnaire, Leo indicated that a member of his group kept calling him a 'dummy'.)

Ordena:

Overall Positive Emotions Score (38=most positive*): 15
*No response for 'respected'

"Engagement with Impasse" Score (4=most engaged with impasse): 3

Reports of Emotions:

	Very Much:	***Somewhat:***	***Not at All:***
Positive/Negative Subscale	*Positive (2 points):*	*Positive (1 point):* Proud Safe Satisfied Relieved Confident	*Positive (0 points):* Interested Successful Excited Happy Curious
	Negative (0 points): Unhappy Angry	*Negative (1 point):* Disappointed Worried Disrespected Bored	*Negative (2 points):* Discouraged Embarrassed Afraid
Engagement with Impasse Subscale	*(2 points)* Frustrated	*(1 point)* Confused	*(0 points)*

Reports of Difficulty:

Had difficulty? Yes.
Classroom context of difficulty: Whole Class and Small Group
What stands out about the situation? "Nothing."

When asked to indicate any thoughts she had that were not on the list of thoughts in the questionnaire, Ordena wrote, "am sick and not concentrating."

Event 12 Difficulty 23: Ta'keisha, Leo, Ordena	
Transcript of Event **Time (teacher camera): (9:59:58-10:03:24)**	At about 13 minutes into the class session, Ms. S. walks over to Group 3. Ta'keisha: Ms. S., um…we got different answers. Ms. S.: Ok, tell me what's going on. Leo: By now we do have…(inaudible)…10…10 tower, and we got 46 cubes. It's not a (inaudible) number. Ms. S.: Talk to me about each strategy. Ta'keisha: She got 15. Ms. S.: How did you, you say you did *(she shows quotation marks with her fingers as she says this)* the 10 towers. Show me what you did. Somebody, explain one strategy at a time. Leo: *(pointing to the 10- block high tower on Ta'keisha's desk, which has 3 branches)* (inaudible)…I kept adding one but we added one to the sides put another one there we kept adding to the sides so we got 10 cubes. Ms. S.: Can you show me? Leo: We started going for the 5 tower. Ta'keisha: And it was 21 blocks. Leo: And then we kept Ms. S.: So you guys all agreed on a 5 tower? *(Ta'keisha and Ordena nod.)* Leo: Then we kept adding Ms. S.: Ok, so start me off with just the 5 tower. Ta'keisha: 5 was 21 blocks, so 10, she multiplied by 2…and got 42, and when we did it this way, we got 46. Ms. S.: *(to Ordena)* Ok, so what made you want to multiply by 2? So you, you, you got the 5 tower- block multiplied by 2, for 10, and you guys *(to Ta'keisha and Leo)* actually built the 10 tower- block. Ta'keisha: *(nods)* Uhm hm. Ms. S.: *(to Ordena)* Ok, so tell me what made you want to multiply by 2. Ordena: *(pointing to her paper)* Because 21 times 1 is 21, which is 5 blocks, and to get 10 blocks you have to multiply 21 times 2, and that's what I did. Ms. S.: Do you want to see if that works for a smaller number? So for example, if you had a height of 2, right? What you're saying is a tower for the height of 4 should be twice as much as a tower for the height of 2. Is that…Am I right in getting…that that's what you're saying? Ordena: Yeah Ms. S.: Ok, see if that works. Or, you could go even smaller. Here you have a tower of the height of what? Ordena: one Ms. S.: One. And here you have a tower of the height of…2, according to what you're saying, right? The tower with the height of 2 should be twice as many blocks as a tower with the height of one. Is that true? Leo: Yes. Cuz it's 2 blocks.

	Ms. S.: Look at this tower *(pointing to the stage A tower on the task sheet).* Now look at this tower *(pointing to stage B on the task sheet)* You're telling me…What's twice the amount of blocks here *(pointing to the stage A tower on the task sheet)*? Leo: 2 Ms. S.: You're telling me there's only 2 blocks here *(pointing to the stage B tower on the task sheet)*? Leo: No (inaudible) Ms. S.: So does that pattern kind of hold true? Leo: Wait one second. This is the 5- block tower. Ms. S.: Ok Leo: So Ta'keisha: (holding a completed 10- block high tower) And this is the 10. Leo: That's the 10. Ms. S.: Ok, so what I would, so here's a rule…cuz we're running…*(Ms. S. looks around the room, then makes an announcement to the class that they should think about whether they want to present on the overhead or on a big sheet of paper.)* Ms. S.: *(to the students at group 3)* Ok, so I would suggest presenting both methods since you didn't kind of work it out, right? Put everything that you kind of come up with…all the strategies that you've come up with to get you where you need to be so far and see if when you're presenting, maybe you could finish working it out. Cuz I don't think anyone here, like, finished yet, you know. *(Ms. S. walks away from group 3)*
Description of Student Difficulty	The students found different solutions for the total number of blocks in a ten-block high tower. Ta'keisha and Leo have come up with a solution of 46 blocks, but Ordena arrived at a solution of 42 blocks. Ordena obtained her solution by doubling the number of blocks in a 5- block high tower [lines 2-27].
Type of Student Difficulty	Impasse
Type of Task	***Open Ended /Closed:*** Closed-ended ***Routine/Non-routine:*** Non-routine
Context of Student Difficulty	Small group
Teacher's Response(s)	Ms. S. asks Ordena why she multiplied by 2 to get the number of blocks in the 10- block high tower. Ordena repeats what she did but does not explain her reasoning. Ms. S. then suggests that Ordena try her pattern with smaller numbers such as 4 and 2, and 2 and 1. The students shift the conversation back to the 5- and 10- block high towers. Ms. S. suggests that the students

	describe both solutions to the class during their presentation [lines 28-66]. Code: Probing- asking the student to explain the reasoning she used Offering a simpler problem as an example Suggesting student share her difficulty with peers
Teacher's Explanation for Response	Ms. S. tried to give the students a method they could use to determine on their own which solution is correct. She did not work with the students until they reached an agreement of which solution is correct because 1) the class was short on time and 2) the students would present their solutions to the class and would likely reach a conclusion through their discussion with the other students in the class. *Response influenced by students' mathematics ability?* No. Code for teacher intention: M- Problem Solving Skills Time
Description of Behavior Related to Student's Engagement Following Response	After Ms. S. leaves the students' table, Ta'keisha writes on her paper while Leo builds 10- block high branches and lays them in piles on his desk. Ordena sits with a blank expression on her face. She yawns, looks around the room, plays with her hair, then sits with her elbow on her desk and her head resting on her hand. Ta'keisha punches numbers into her calculator. She says she found the answer for the 100 tower. She says it has 496 cubes. Leo disagrees and says it is 420, the solution they found yesterday. He says, "cuz right now I'm building the legs and I've already got nearly 200." Leo has 2 piles of 10-block branches on his desk. Ta'keisha explains that since the 5- block high tower has 4 blocks on each side and the 10- block high tower has 9 blocks on each side, the 100- block high tower must have 99 blocks on each side. Ms. S. comes to group 3 with a stack of large colored papers. Ta'keisha takes one and begins writing on it. Ordena writes on her paper while Leo continues building stacks of blocks. Leo asks how many blocks were in the 10- block high tower. Ta'keisha says there were 46. Ordena says there were 43 blocks. Ms. S. then comes to the table and the students explain what they have been doing (This is the next event).
Inference of Student's Emotions Based on Description	T: High level of interest, Confidence L: Interest O: Some level of interest, Fatigue
Behaviors Pertaining to Student's *Behavioral* Engagement and Resulting Classification	T: On-task *Writing, Using calculator, Explaining idea to group mate* L: On-task *Using manipulatives in way related to task at hand, Explaining idea to group mate* O: On-task *Writing*

Behaviors Pertaining to Student's *Cognitive* Engagement and Resulting Classification	T: High level cognitive activity *Highest level of cognitive activity involved: Speculating, Justifying* L: Low level cognitive activity *Highest level of cognitive activity involved: Building towers without consideration of patterns* O: Insufficient Information
Behaviors Pertaining to Student's *Affective* Engagement and Resulting Classification	T: Strong positive feelings *High level of interest with no apparent negative emotions, Confidence* L: Moderate positive feelings *Some level of interest with no apparent negative feelings* O: Mild positive feelings *Some level of interest mixed with slight negative feelings of fatigue*

Event 13
Difficulties 24-28: Leo

Transcript of Event **Time (teacher camera): (10:11:16-10:17:08)**	Leo: *(to Ms. S.)* I almost came up with 420…you know, for the sides?...(inaudible) *(Leo is trying to build a 100-block high tower. He has piles of 10- block high stacks on his desk.)* Ms. S.: Oh, my goodness! *(laughs)* Leo: (inaudible) Ms. S.: Guys, if you feel like it's worth it. I don't know. It's up to you. *(Leo goes to get more blocks.)* Ta'keisha: It's not worth it, we already did it. Ms. S.: *(to Leo)* Ta'keisha said it's not worth it. *(Leo shakes his finger from side to side at Ta'keisha and smiles.)* Ms. S.: *(to Ta'keisha)* It's up to him. Ta'keisha: Ok, he agreed with me, he said (inaudible) what he's doing Leo: I was trying to experiment myself. You said it's four hundred and fifty something. I said it's four hundred and twenty. Ms. S.: *(to Ta'keisha)* So make sure to leave enough space on your sheet of paper to show *all* your work, or all your reasoning or your justifications for what you did. Do you know what I mean? Like I don't…When you go up to present, your answer means nothing to me. I don't know the answer off the top of my head. So if you say 21, we have no idea where that 21 came from, and we really don't know whether it's right, wrong, or in between. So, I mean, if you're gonna show us physically with the cubes, like if that's what you plan on doing, that's fine, but leave room to show, kind of like your mathematical justifications, your symbolic justifications on paper. This stuff, your work. Ta'keisha: We came up with 100 because, um… Ms. S.: Don't tell me, he knows how to…(inaudible) Ta'keisha: I know, but…um Ms. S.: Go ahead Ta'keisha: Cuz it's always a cube less on each side than on the top Ms. S.: Ok Ta'keisha: So…cuz it's gonna be 99 blocks, so we multiplied it by 4 and we added 100 to it and Leo thinking he would do it all the way. Leo: So I could see if it's right or not. You never know. Ms. S.: Cuz he's…I guess he's not convinced by… Leo: yeah Ms. S.: So what made you want to multiply 99 by 4? Ta'keisha: Cuz it's 4 sides Ms. S.: It's 4 sides of what? Ta'keisha: On a…on a…um, a hundred block high tower. It's 4 sides. Ms. S.: One…on one of these…cubes *(she holds up one cube)*. 1, 2, 3, 4 *(she points to each of the 4 sides of the cube)*. That's what you mean?

Ta’keisha: No, I mean like this *(she brings the marker she is holding down on her desk 4 times)*.
Ms. S.: Ok *(nods),* and you counted in 100 because of what now?
Ta’keisha: Because it’s 100 *(she motions up and down in the air with her marker)* um…blocks, …up the top.
Ms. S.: Ok, so what if I wanted you to do it to 1,000 blocks?
(Ms. S. talks to students at another table for approximately 25 seconds, then says to Ta’keisha) So what if I asked you to do it to a thousand block tower?
Ta’keisha: Cuz (inaudible)…cuz…um…the 5- block high tower is 4…is always one number less, so it’s gonna build the blocks.
Ms. S.: I’m with you, I understand what you’re saying. I…(inaudible) *(Ta’keisha laughs) (to Ordena)* Are you with her? Do you understand? *(Ordena nods)* Are you? So you tell me the justification.
Ordena: Um…What she’s trying to say is, the numbers on the sides might be the same as the numbers in the middle , cuz …you could only count…um count the middle number while the number in the middle cube, one time, which is what the, the up *(she motions up and down with her hand)*
Ms. S.: height?
Ordena: yeah
Leo: You see how she’s saying about 99 on each side? But why is it 99, not 100? You’re not counting these? *(He holds up one cube)* If it’s 100 on the top, how come it’s 99 on the sides?
Ta’keisha: Ok, it’s 5 on…the first one that we did was 5 on the top
Ms. S.: Show him…physically show him what a small…(inaudible)
(Ta’keisha builds a 5- high tower. Leo builds with block and doesn’t look up.)
Ordena: What *he’s* trying to say is…*(Laughs. Leo and Ta’keisha laugh too.)*
Ms. S.: Who do you agree with, him *(pointing to Leo)* or her *(pointing to Ta’keisha)*?
Ordena: I don’t know. She…I think she’s right, but there’s 4 sides on the cube, so
Ms. S.: Uh hm.
Ordena: You count this side for this side and that side for that side *(pointing to Ta’keisha’s tower. Leo continues building and does not look up.)*
Ta’keisha: It’s not 5…it’s not 5…on the sides.
Ms. S.: *(to Leo)* Let me ask you a question. Is this…she’s saying this is a 5-block high tower. Is this a 5- block high tower? *(Leo shakes his head ‘no.’)* What is it?
Leo: It’s a 4- block high tower.
Ms. S.: Let me ask you this. How many blocks high is this tower on your paper?
Leo: One
Ms. S.: How many blocks high is this tower on your paper?
Leo: This one?
Ms. S.: yep
Leo: 2 blocks high tower

Ms. S.: This one *(pointing to another tower on Leo's paper)*? Leo: 3 blocks high tower

Ms. S.: But how many do you see?

Leo: 2

Ms. S.: And how many do you see here?

Leo: One

Ms. S.: How many blocks high tower is this one *(pointing to Ta'keisha's tower)*?

Leo: 4

Ms. S.: You're saying this is only a 4- blocks high tower?

Leo: I see 4

Ms. S.: Right. *(Ta'keisha laughs. Leo looks at her and laughs too).* But here you see one block, and how high is the tower that you said?

Leo: Two blocks.

Ms. S.: Here you see four blocks and how high is the tower?

Leo: *(smiles)* 5 blocks

Ms. S.: So is this a 5- block high tower?

Leo: yeah *(He toys with his pen, then puts his hand in the bag of blocks.)*

Ms. S.: Ok. So then, *(to Leo)* you gotta pay attention if you want to come to some kind of consensus. So stop with the blocks for a minute *(Leo stops building with the blocks, leans forward with his chin resting on his hands and looks at the tower to which Ms. S. is pointing) (to Ordena)* Ordena, do you agree with that part so far? Is this a 5- block high tower? *(Ordena nods)* Cuz that's the first thing we gotta agree on. It is? *(Ordena and Ta'keisha nod)* Ok, so now Ta'keisha, go ahead with your explanation, and your justification.

Ta'keisha: *(Lifts the stack of blocks representing the height of her tower)* It's 5…it's 5 right here, and every time one is built up you gotta add one to the sides. *(Pointing to the middle cube, the bottom one in the stack she is holding)* This one don't count, the one that's already in the…on bottom.

Ms. S.: Does it not count, or did you count it already?

Ta'keisha: We already counted it.

Ms. S.: Where did you count it?

Ta'keisha: First

Ms. S.: In the…height *(holds up the stack of blocks which represents the height) (Leo nods blankly.* ***See picture at 10.16.38.****)* Does that make sense? *(Leo nods)* So then what happens to these *(pointing to the sides of the tower)*? Do these…do your sides end up having as much as the height?

Ta'keisha: no

Leo: yes

Ta'keisha: end up having less

Ms. S.: You gotta work that out. You could go, when you go up there to present, you can let Leo present his…Leo you can present why you're not convinced, ok? Ordena, you're in the middle? Or are you…

Ordena: I'm in the middle.

Ms. S.: Ordena's in the middle and Ta'keisha, you can present your argument. But we gonna get them all on the paper. Ok? *(Ta'keisha and Leo nod)*

		(Ms. S. leaves the table.)
Description of Student Difficulty		24) Leo is trying to build a 100-block high tower with cubes. He disagrees with Ta'keisha's solution for the 100- block high tower and says he thinks the total number of cubes in the tower is 420 [lines 1-14]. 25) After Ta'keisha explains her method of multiplying one less than the number in the height of a tower by 4, and adding that to the number in the height, Leo asks why there are 99, and not 100, blocks on each side of the 100- block high tower [lines 61-63]. 26) Ms. S. asks Leo if the tower Ta'keisha built is a 5- block high tower. Leo says it is not; he says it is 4 blocks high [lines 76-79]. 27) After Leo responds saying that the tower where only 1 block in the height is visible is 2- blocks high and that the tower where 2 blocks are visible in the height is 3- blocks high, he again says that Ta'keisha's tower is only 4 blocks high [lines 93-95]. . 28) Ms. S. asks if the sides end up having as many blocks as the height. Ta'keisha says "no." Leo says "yes" [lines 122-126].
Type of Student Difficulty		24) Incorrect idea 25) Does not understand solution 26) Incorrect answer 27) Incorrect answer 28) Incorrect answer
Type of Task		24) ***Open Ended /Closed:*** Closed-ended ***Routine/Non-routine:*** Non-routine 25) ***Open Ended /Closed:*** Closed-ended ***Routine/Non-routine:*** Non-routine 26) ***Open Ended /Closed:*** Closed-ended ***Routine/Non-routine:*** Non-routine 27) ***Open Ended /Closed:***

<table>
<tr><td></td><td>Closed-ended

Routine/Non-routine:
Non-routine

28) Open Ended /Closed:
Closed-ended</td></tr>
<tr><td>Context of Student Difficulty</td><td>24) Small group
25) Small group
26) Small group
27) Small group
28) Small group</td></tr>
<tr><td>Teacher's Response(s)</td><td>24) Ms. S. tells Ta'keisha to write the justification for her solution on her paper. She then asks Ta'keisha to explain her method for calculating the number of cubes in a tower. Ms. S. then asks Ordena if she agrees with Ta'keisha's solution [lines 15-60].

Code: Inviting other students to respond

25) Ta'keisha begins explaining how her method works with a 5- block high tower. Ms. S. tells her to show Leo with the cubes [lines 64-65].

Code: Showing the student a model

26) Ms. S. points to the two- and three- block high towers in the diagrams on the task sheet and for each one, asks Leo how many blocks high it is and how many blocks he can see in its height. When Leo responds correctly to these questions, Ms. S. asks him how many blocks high the tower on Ta'keisha's desk is [lines 80-94].

Code: Offering a simpler problem as an example

27) Ms. S. reminds Leo that he said that in a 2-high tower he can see only 1 block. She then asks him again how high the tower on Ta'keisha's desk is [lines 96-105].

Code: Offering a simpler problem as an example

28) Ms. S. tells the students to put all their solutions and explanations on paper. She tells Leo and Ta'keisha that they should both share their arguments with the class when they present [lines 128-133].

Code: Suggesting student share his difficulty with peers</td></tr>
</table>

Teacher's Explanation for Response	N/A Code for teacher intention: N/A
Description of Behavior Related to Student's Engagement Following Response	24) Ta'keisha explains to Ms. S. her method of multiplying one less than the number in the height of a tower by 4, and adding that to the number in the height. While she speaks, Leo continues building branches of blocks and piles them on his desk. He says he wants to build the 100-high tower to see if Ta'keisha's method is correct because "you never know." Leo does not look at Ta'keisha while she explains her method. Ms. S. asks Ordena if she agrees with Ta'keisha's explanation. Ordena nods. Ms. S. asks Ordena to explain what Ta'keisha was saying. Ordena explains that the amount of blocks on the sides of the towers is not the same as the amount of blocks in the heights because the middle block is counted only once. Leo then asks why there are 99, and not 100, blocks on each side of the 100- block high tower [lines 61-63]. 25) Leo looks up at Ta'keisha when she begins to answer his question but then continues to build branches of blocks without looking at the tower she is building to demonstrate her method. Ordena then joins the conversation and says she wants to explain what Leo is trying to say. She says she thinks Ta'keisha's solution is correct but that she doesn't completely understand why it is correct. Leo builds with the blocks while Ordena speaks. He glances up at her and at the teacher occasionally but continues putting blocks together to build more branches in a hurried manner [lines 64-75]. 26) Leo again says that the tower on Ta'keisha's desk is only 4 blocks high [lines 93- 95]. 27) Leo begins building again with his blocks. Ms. S. tells him to "stop with the blocks" and pay attention. Leo stops building, leans forward, and watches Ms. S. and the other students at his table discuss whether Ta'keisha's tower is 5 blocks high. He then watches Ta'keisha as she explains using her model that the middle cube doesn't count in the branches of the tower. Leo has a blank expression on his face as he watches Ms. S. and Ta'keisha speak [lines 105-121]. 28) After Ms. S. leaves the table, Leo continues building with the blocks while Ta'keisha prepares for the presentation by writing on a large colored paper. Leo says he's still not convinced about Ta'keisha's solution. Ta'keisha tells Leo that he will need to build the 100-block high tower if he is not convinced of her solution. She says he will make a fool of himself. Leo continues building. He asks Ta'keisha whether the 10-block high tower has 46 cubes. Ta'keisha asks Leo why he agrees with her on that if he said he's not convinced of her method. Leo responds, "Be happy I agree with you at all." Ordena says she's not convinced. Ta'keisha says to her, "How can you agree with a dummy like him (Leo)?" Ta'keisha then says about Leo, "He

	don't know what he's doing." Leo answers, "I know what I'm doing. Don't try to make a fool of me." Ta'keisha asks Leo, "How you get 46 if you don't agree with me?" Leo says, "Because I did it myself." Ta'keisha tells Leo to build a 5-block high tower and a 10- block high tower. Leo builds a tower that is 10 blocks in height and has 10 blocks in each of its branches. He says it is a 10- block high tower. He then disassembles the tower. He leans forward and watches Ta'keisha as she writes on the colored paper and on a transparency sheet. Ordena sits with her head resting on her hand. **(See picture at 10:26:33)**. Leo plays with the parts of the 10-block high tower he had built while Ta'keisha continues preparing for the presentation. At approximately 10:33 Ms. S. announces that the students need to stop working.
Inference of Student's Emotions Based on Description	24) Nervous that he does not understand Ta'keisha's explanation, Insecurity, Trying to hide embarrassment 25) Nervous that he does not understand Ta'keisha's explanation, Insecurity, Trying to hide embarrassment 26) Insufficient information 27) Interest, Slight sense of anxiety 28) Frustration, Insecurity, Fear of losing face, Embarrassment, Defensiveness
Behaviors Pertaining to Student's *Behavioral* Engagement and Resulting Classification	24) On-task *Using manipulatives in way related to task at hand, Asking for help from group mate* 25) On-task *Using manipulatives in way related to task at hand* 26) On-task *Answering teacher's question* 27) On-task *Using manipulatives in way related to task at hand, Watching group mate explain idea* 28) On-task *Using manipulatives in way related to task at hand, Expressing disagreement with group mate's solution, Asking for help from group mate*
Behaviors Pertaining to Student's *Cognitive* Engagement and Resulting Classification	24) High level cognitive activity *Highest level of cognitive activity involved: Help seeking for understanding* 25) Low level cognitive activity *Highest level of cognitive activity involved: Building towers without consideration of patterns* 26) Low level cognitive activity *Highest level of cognitive activity involved: Restating one's own idea without consideration of others' ideas* 27) Insufficient information 28) Low level cognitive activity

	Highest level of cognitive activity involved: Building towers without consideration of patterns, Restating one's own idea without consideration of others' ideas, Checking information
Emotions Pertaining to Student's *Affective* Engagement and Resulting Classification	24) Moderate negative feelings *Pretends to be busy while Ta'keisha gives explanation- Nervousness that does not understand Ta'keisha's explanation, Insecurity, Trying to hide embarrassment* 25) Moderate negative feelings *Pretends to be busy while Ta'keisha gives explanation- Nervousness that does not understand Ta'keisha's explanation, Insecurity, Trying to hide embarrassment* 26) Insufficient information 27) Mild positive feelings *Some level of interest mixed with slight negative feelings of anxiety* 28) Moderate negative feelings *Frustration, Insecurity, Fear of losing face, Embarrassment, Defensiveness*

<table>
<tr><th colspan="5">Comparison of Students' Self-Reported Affect on Questionnaire and Affect Inferred from Observed Student Behaviors
Ms. S.: Day 2</th></tr>
<tr><td colspan="5">Ta'keisha</td></tr>
<tr><td rowspan="3">Self-Reported Affect</td><td colspan="2">Positive Emotions Score:</td><td colspan="2">Engagement with Impasse Score:</td></tr>
<tr><td>Ta'keisha</td><td>Mean</td><td>Ta'keisha</td><td>Mean</td></tr>
<tr><td>30/44</td><td>31.47/40</td><td>2/4</td><td>.78/4</td></tr>
<tr><td>Affect Inferred from Observed Behaviors</td><td colspan="4">Strong positive feelings (following Difficulty 23)</td></tr>
<tr><td colspan="5">Comparison of Self-Reported vs. Observed Affect
Ta'keisha's observed affect following Difficulty 23 was coded as very positive. Following Difficulty 28, which involved only Leo, however, Ta'keisha appeared impatient and annoyed with Leo who had difficulty understanding her solution. Her affect at this time was similar to her reported affect on the questionnaire. On her questionnaire, Ta'keisha reported feeling very confident but only somewhat happy, satisfied, and curious. Ta'keisha also reported feeling somewhat disappointed and angry and added the feelings of somewhat 'annoyed' and 'tired' to the list.</td></tr>
<tr><td>Self-Report of Difficulty</td><td colspan="4">Had difficulty? Yes.
Classroom context of difficulty: Group
What stands out about the situation? (No response)</td></tr>
<tr><td>Number of Analyzed Instances of Difficulty</td><td colspan="4">1</td></tr>
<tr><td colspan="5">Comparison of Self-Reported vs. Observed Instances of Difficulty
Reported difficulty and was involved in:
Difficulty 23- Impasse</td></tr>
<tr><td colspan="5">Leo</td></tr>
<tr><td rowspan="3">Self-Reported Affect</td><td colspan="2">Positive Emotions Score:</td><td colspan="2">Engagement with Impasse Score:</td></tr>
<tr><td>Leo</td><td>Mean</td><td>Leo</td><td>Mean</td></tr>
<tr><td>28/40</td><td>31.47/40</td><td>0/4</td><td>.78/4</td></tr>
<tr><td>Affect Inferred from Observed Behaviors</td><td colspan="4">Moderate positive feelings (following Difficulty 23)
Moderate negative feelings (following Difficulty 24)
Moderate negative feelings (following Difficulty25)
Insufficient Information (following Difficulty 26)
Mild positive feelings (following Difficulty 27)
Moderate negative feelings (following Difficulty28)</td></tr>
</table>

Comparison of Self-reported vs. Observed Affect:
Leo's self-reported affect during this class session was more positive than the affect inferred from his observed behaviors. During the class session Leo had difficulty understanding both the mathematics problem and Ta'keisha's solution. Ta'keisha made derogatory remarks towards Leo, calling him a 'dummy' and telling Ordena that he doesn't know what he's doing. On his questionnaire, Leo reported feeling only somewhat successful, safe, and happy, but not at all angry, disrespected, or embarrassed. Leo also reported feeling not at all frustrated or confused.

Self-Report of Difficulty	***Had difficulty?*** No. ***Classroom context of difficulty:*** N/A ***What stands out about the situation?*** "I felt like I was smart and I could get the problem without the person bothering me."
Number of Analyzed Instances of Difficulty	6

Comparison of Self-Reported vs. Observed Instances of Difficulty
Reported no difficulty but was involved in:

- Difficulty 23- Impasse
- Difficulty 24- Incorrect Idea
- Difficulty 25- Does not Understand Solution
- Difficulty 26- Incorrect Answer
- Difficulty 27- Incorrect Answer
- Difficulty 28- Incorrect Answer

Ordena

Self-Reported Affect	Positive Emotions Score:		Engagement with Impasse Score:	
	Ordena	Mean	Ordena	Mean
	15/38	31.47/40	3/4	.78/4
Affect Inferred from Observed Behaviors	Mild positive feelings (following Difficulty 23)			

Comparison of Self-Reported vs. Observed Affect:
The affect inferred from Ordena's observed behaviors was more positive than her self-reported affect. During class Ordena appeared interested but fatigued. On her questionnaire, however, Ordena reported feeling very unhappy and angry and somewhat disappointed, worried, and disrespected. No evidence was observed for these feelings. Ordena also reported feeling not at all interested.
When asked to indicate any thoughts she had that were not on the list of thoughts in the questionnaire, Ordena wrote, "am sick and not concentrating."

Self-Report of Difficulty	***Had difficulty?*** Yes. ***Classroom context of difficulty:*** Whole Class and Small Group ***What stands out about the situation?*** "Nothing."

Number of Analyzed Instances of Difficulty	1
Comparison of Self-Reported vs. Observed Instances of Difficulty Reported difficulty and was involved in: Difficulty 23- Impasse	

Group Analysis of Self-Reported vs. Observed Affect
All students in this group reported more negative affect on Day 2 than on Day 1. The reported affect on Day 2 for each student in the group was also below the mean for 'Positive Emotions Score.' Leo, who appeared to have the most difficulty with the mathematics problem during this session, was the only student in the group who reported having no difficulty. Leo also reported the lowest level of 'Engagement with Impasse' among the members of his group.

Ms. S. Class 343: Standard Track, Summit Ave. School, Day 3 Group 3: Ta'keisha, Leo, Ordena			
Leo: **Overall Positive Emotions Score (40=most positive):** 30 **"Engagement with Impasse" Score (4=most engaged with impasse):** 0 **Reports of Emotions:**			
	Very Much:	**Somewhat:**	**Not at All:**
Positive/Negative Subscale	*Positive (2 points):* Interested Safe	*Positive (1 point):* Respected Proud Successful Excited Happy Satisfied Relieved Confident	*Positive (0 points):* Curious
	Negative (0 points):	*Negative (1 point):*	*Negative (2 points):* Unhappy Disappointed Worried Discouraged Angry Disrespected Bored Embarrassed Afraid
Engagement with Impasse Subscale	*(2 points)*	*(1 point)*	*(0 points)* Frustrated Confused

(Leo Questionnaire, cont.)

Reports of Difficulty:

Had difficulty? No.
Classroom context of difficulty: N/A
What stands out about the situation (for example, what happened, what you thought, what you felt, your teacher said or did, what other kids said or did)? "Nothing stands out of my memory
I felt like a mathemetician and smart I thought I was going to be imbarrist nothing, nothing"

(On one page of the questionnaire, where students were asked to describe any thoughts they had that were not on the list of thoughts, Leo wrote, "I felt like I was being bossed around by some one in my group")

Event 14 Difficulty 29: Leo	
Transcript of Event **Time (of day): (9:51:35-9:58:00)**	The class session on day 3 begins at 8:59 am. The students work on the problem with their group mates for approximately 15 minutes and then begin their presentations to the class. At the beginning of this event, Ta'keisha, Leo, and Ordena set up their presentation at the front of the classroom. They place their transparency on the overhead, hang their task sheet with their written work on the board, and place a 5- block and a 10- block high tower on the teacher's desk. Ordena then begins speaking to the class. Ordena: Ok, um…well, 5 blocks high tower we had 21 cubes and *(to Ta'keisha)* you talk. Ta'keisha: For 10 blocks we had 46 cubes. Ms. S.: One second, one second, move (inaudible) cuz it's hard to see. Ta'keisha: And for 100 we had four hundred and ninety six cubes. Leo: So what we did was, we tried to attempt it but we couldn't cuz we didn't have enough room so we just like guessed out like see how many fits on this side cuz she said 99 fit…99 cubes on each…4 sides and 100 is on…on the top. Ms. S.: Who said that? Ta'keisha: *(Leo points to Ta'keisha. Ta'keisha raises her hand.)* Me Ms. S.: Oh, ok. So you believed her? Leo: So I disagreed with her because why is it 99 on each side, not 100? And how come there's 100 on each side…uh…on the top. So she kind of…(inaudible)…to... Student *(off camera)*: That means she added that middle block Leo:Yeah, but how come…yeah, but…this… Ta'keisha: The…the amount around the sides is not the same as the height Ordena: And then we got to the understanding that we only count the middle blocks one time, just for the…the height. Ms. S.: *(to Leo)* Did you get to that understanding? Cuz Leo still doesn't seem to agree. Ta'keisha: Leo didn't Ms. S.: Oh, but you 2 guys came to that understanding. Ok, well let's see if either Leo can convince you or…of his perspective or you can convince Leo or the class, let's see where you guys fall on that perspective. So Leo is taking the perspective of which group? What Leo…Who's Leo agreeing with? Ta'keisha: This group *(points to a group, off camera)* Ms. S.: Shandrel (inaudible), right? And, uh…Ta'keisha and Ordena are with Tyshonnah and crew, and Nikyah, right? Ok, so let's…let's go ahead and talk about which…what were some of the arguments you made to try to convince…Leo what was the argument that you made to try to convince Ta'keisha and Ordena?

Leo: to…to add another cube on the sides instead of adding it to the top.

Ms. S.: Right, so tell them why you…how…why did you argue you need to do that?

Leo: So it could be equal instead of…cuz she said that it's 21 cubes…um, that it's 496 cubes, but I don't know why you added 99 to each side instead of 100.

Ta'keisha: The middle cube don't get counted cuz…so…that's why, we added 9 on each side for 10 and for 5 we added 4.

Ms. S.: So, I guess what Ta'keisha is saying, right, if I'm hearing this right, is Ta'keisha is saying look back at your smaller examples, forget about 100, right? Right now. Cuz 100 is too hard to build, we couldn't necessarily do it. You agree with that's a 10-tower block right there, in front of you that you built? You guys agreed on that?

Leo: yes

Ta'keisha and Ordena: yeah

Ms. S.: And you guys agreed on the 5- tower one, right? The 5- block high tower or whatever, right? *(Ta'keisha nods.)* So Ta'keisha, use the examples you do have in front of you to try to convince Leo that that middle block only counts for the height.

Ta'keisha: On this paper for example, A is only one block and it's one cube. And then for the B, it's a 2- block high tower which is 6 blocks. It's only one going on each side. It's one more on top of the middle cube. That don't get counted. And then for C, it's 3, it's a 3- block high tower but you only see two. So it's 2 on each side.

Ms. S.: So is she…so, are there any questions? *(To student, off camera)* Go ahead.

Student *(off camera):* Say what you said again.

Ta'keisha: *(smiling)* Again. You know how you don't see the middle cube? I'm saying the amount that you see is the amount on each side.

Students *(off camera):* (inaudible)

Ms. S.: She's saying in any of those examples, right? You can't see that middle cube cuz the sides are blocking it, correct?

Students *(off camera):* yeah

Ms. S.: So she's saying the amount that you *can* see is the number of cubes that should be on your sides. *(To student, off camera)* Yeah.

Student *(off camera):* That's what we said.

Another student *(off camera):* That's what we said.

Ms. S.: Is that what you said?

Student *(off camera):* We didn't say it but like that's what we're trying to say.

Ms. S.: Ok, so, ok, so we agreed on that part, right? *(Ta'keisha and Leo nod.)*

Student *(off camera):* Yeah

Ms. S.: But what she's saying…how would you class…look at that tower that they have right there…that…the little tower. What would you call that?

Student *(off camera):* 4

Ms. S.: What would *you* guys call that?

Student *(off camera):* 4?

	Ms. S.: A 4- block high tower. Ta'keisha, what do you want to call that? Ta'keisha: A 5- block high tower. Ms. S.: There's the dispute. Right? *(Ordena, Ta'keisha, and Leo nod.)* Is that a 4- block, or is that actually 5 blocks? Ta'keisha: If you don't...*(she separates the branches of the 5- block high tower from its height so that the height stands alone.* ***See picture at 9:56:06.)*** It's 5. Ms. S.: How high is that tower, Shandrel? Student *(off camera):* 5 Another student *(off camera):* 5 Ms. S.: So is that a 4- block high tower or a 5- block high tower? Students *(off camera):* 5 Student *(off camera):* She got 4 on top Ms. S.: But you agreed that that's what she should do. You said that the number that you see on the top should be the number on the sides, right? Student *(off camera):* We're trying to say something different. Like, we had 5 on each of our sides, right? Ms. S.: Understood, but listen…look, look at what I'm saying. You said that that…Is that…Is that a valid tower? Student *(off camera):* yeah Student *(off camera):* no Student *(off camera):* yeah Ms. S.: Is…Shandrel says yes, Jimar says no. Student *(off camera)*: No. Cuz she…(inaudible)…cuz she wanted…(inaudible) Student: No. It's 4 around. 4 around and 5 in the middle. Ms. S.: Alright. So...*(Students off camera laugh. Leo and Ta'keisha laugh.)* So we're saying that this is not a valid tower right here. Ta'keisha: Which one? Ms. S.: The little one. Is it valid or it's not*? (Ta'keisha nods.)* It is a valid tower. Raise your hand if you feel like it is a valid tower. *(Ta'keisha raises her hand. Leo and Ordena do not.)* So the majority of the class is saying that it is a valid tower. Student *(off camera):* It's only me. *(Leo and Ta'keisha laugh.)* Ms. S.: That's the majority of the class *(she laughs).* Alright, uh…just for time's sake, what I'm gonna ask you guys to do is…is to go ahead and stick with your presentations. We're still not resolved about this 4- block, 5- block thing, but maybe the next group will help you to resolve it. Thank you guys. *(The students collect their transparency, work papers from the board, and towers and return to their seats.)*
Description of Student Difficulty	Leo says he disagrees with Ta'keisha's solution of 496 blocks for the 100-block high tower. He says he does not understand why she says there are 99 cubes on each side of the tower and not 100 [lines 20-30].
Type of Student Difficulty	Does not understand solution

Type of Task	***Open Ended /Closed:*** Closed-ended ***Routine/Non-routine:*** Non-routine
Context of Student Difficulty	Whole class
Teacher's Response(s)	Ms. S. tells Ta'keisha and Leo to try to convince each other that their solutions are correct. After Leo and Ta'keisha present their arguments, Ms. S. asks the class if they have any questions. When the class disagrees about whether the 5- block high tower is called a 5-block high or 4- block high tower, Ms. S. says the class will continue with their presentations even though they have not resolved their disagreement. She says that the next group might help them resolve it [lines 31-125]. Code: Suggesting student share his difficulty with peers
Teacher's Explanation for Response	Leo was beginning to see that Ta'keisha's solution was correct and his was not. He persisted in defending his solution, however, to save face in front of the other students. Ms. S. responded to Leo's difficulty by having the students talk with each other instead of directly telling them the correct answer since that response would be more effective for the students emotionally and cognitively. On an emotional level, Ms. S. wanted the students to recognize that it is ok to change their ideas. She wanted them to realize that when they change their ideas, they merely modify, rather than abandon, them. Upon reflection during the interview, Ms. S. said she would have liked to have the students talk more and have herself talk less. Ms. S. cautioned that the method of having students discuss their ideas with each other works better for the students emotionally when they are grouped homogenously. When students share their ideas in a homogenous group, no student is made to feel less capable or less smart than another. In the present situation, Ms. S. had the students talk with each other regarding the difficulty even though they were in a whole class setting since the students were working on a non-routine problem with which none of the students felt comfortable. Ms. S. said that on a cognitive level, having the students talk with each other without much teacher involvement is effective since that way students can explore sincerely as the teacher does not dictate whether their ideas are valid or not. This gives the students experience with articulating their thoughts, listening to others, integrating others' ideas with their own, and questioning other students. *Response influenced by mathematics ability*? No. Code for teacher intention: M- Problem Solving Skills

	E- Saving Face in Front of Classmates
Description of Behavior Related to Student's Engagement Following Response	During the beginning of the next group's presentation, Leo says to Ta'keisha, "See, I was right. See, ha. See, ha." Leo watches the next group's presentation. The students demonstrate a method they used to find the number of blocks for the first two stages of the towers: For the stage one tower, they took 5 groups of one block and arranged the blocks into a tower. Then they subtracted 4 blocks by removing one block from each side of the tower. For the stage 2 tower, they took 5 groups of 2 blocks, arranged them into a tower, and then subtracted 4 by removing one block from each side of the tower. The students say they used this method to find the number of blocks in a 100-block high tower: they calculated 5 groups of 100 blocks=500, and subtracted 4=496. When the presentation is over, Ms. S. asks the class whether the students in the group came up with a general solution and whether the class is convinced of their method. Ta'keisha and Ordena answer "yes" to both questions but Leo does not answer.
Inference of Student's Emotions Based on Description	Defeat, Inferiority to another student
Behaviors Pertaining to Student's *Behavioral* Engagement and Resulting Classification	On-task *Watching classmates explain idea*
Behaviors Pertaining to Student's *Cognitive* Engagement and Resulting Classification	Insufficient information
Emotions Pertaining to Student's *Affective* Engagement and Resulting Classification	Strong Negative Feelings *Defeat, Inferiority to another student*

<table>
<tr><th colspan="5">Comparison of Students' Self-Reported Affect on Questionnaire and Affect Inferred from Observed Student Behaviors
Ms. S.: Day 3</th></tr>
<tr><td colspan="5">Leo</td></tr>
<tr><td rowspan="3">Self-Reported Affect</td><td colspan="2">Positive Emotions Score:</td><td colspan="2">Engagement with Impasse Score:</td></tr>
<tr><td>Leo</td><td>Mean</td><td>Leo</td><td>Mean</td></tr>
<tr><td>30/40</td><td>31.47/40</td><td>0/4</td><td>.78/4</td></tr>
<tr><td>Affect Inferred from Observed Behaviors</td><td colspan="4">Strong negative feelings (following Difficulty 29)</td></tr>
<tr><td colspan="5">Comparison of Self-reported vs. Observed Affect:
Leo's self-reported emotions were more positive than the emotions inferred from his observable behaviors. During the class presentation this session, Leo had difficulty understanding the solution Ta'keisha presented to the class. After the presentation Leo appeared defeated and showed evidence of feeling inferior to Ta'keisha. On his questionnaire, however, Leo reported feeling somewhat proud, successful, excited and happy. No evidence was found for these feelings during the session. Leo also reported feeling not at all unhappy, angry, disrespected, or embarrassed.
Leo did, however, express negative feelings on other portions of his questionnaire. On one page of the questionnaire, where students were asked to describe any thoughts they had that were not on the list of thoughts, Leo wrote, "I felt like I was being bossed around by some one in my group." On the last page of the questionnaire, where students were asked about what stands out in their memory about situations that day involving difficulty, Leo reported that he thought he was going to be embarrassed (see cell below).</td></tr>
<tr><td>Self-Report of Difficulty</td><td colspan="4">Had difficulty? No.
Classroom context of difficulty: N/A
What stands out about the situation (for example, what happened, what you thought, what you felt, your teacher said or did, what other kids said or did)?
"Nothing stands out of my memory I felt like a mathemetician and smart I thought I was going to be imbarrist nothing, nothing"</td></tr>
<tr><td>Number of Analyzed Instances of Difficulty</td><td colspan="4">1</td></tr>
<tr><td colspan="5">Comparison of Self-Reported vs. Observed Instances of Difficulty
Reported no difficulty but was involved in:
Difficulty 29- Does not Understand Solution</td></tr>
</table>

Appendix B: Teacher Interview Protocol

Interview Protocol

Thank you for taking the time to reflect on your classroom experiences.
I'm interested in finding out about how teachers respond when students have difficulty while they work on mathematics problems in class and about why the teachers respond in those ways.
First, I'm going to ask you some questions about your thoughts regarding ways in which you respond when students have difficulty while they work on mathematics problems. After that, I will show you video clips of your classes which involve instances of student difficulty and will ask you to reflect on the ways in which you responded.

Part I

Can you describe how you usually respond when students have difficulty while they work on mathematics problems in class?

Are there things you try not to do when you respond to student difficulty? (If yes) Can you describe?

Do you respond in different ways in different situations? [If teacher asks for examples of 'different situations'- student's mathematics ability, student's personality, type of difficulty, whole-class vs. small group setting of difficulty]

[If yes] Can you describe some factors that might determine how you respond?

Part II

I will now show you a few video clips where students in your class encountered difficulty while they worked on the Building Blocks Dilemma task. *[Hand teacher a copy of the task sheet.]*
After each clip, I will ask you why you why you think you chose to respond in that way. I am aware that these class sessions took place a year ago, and you may not remember what you were thinking at the time. Try to answer as well as you can.
After we view this clip, I will ask you about the interactions involving *[name of student(s)]*
[Show clip]
What do you think the student(s') difficulty was?
a)

b)

c)

Why did you respond in that way?

a)

b)

c)

Do you think the student’s(’) mathematics ability influenced the way in which you responded?

a)

b)

c)

Appendix C: Affect Questionnaire

Print Your Name:___________________________ Today's Date:____________________
Your Class:_____________________ People in your group:________________________

THE WAY IT WAS FOR ME IN THIS CLASS TODAY

Please think about the time you spent in your group today. Write <u>one or two sentences</u> for each question.

1. What was the most memorable thing that happened while you were working in the group?

2. Was there any other thing that happened that stands out in your memory?

3. Did anything happen that made you feel especially good (for example, pleasant, happy)? If yes, what?

4. Did anything happen that made you feel especially bad (for example, unpleasant, unhappy)? If yes, what?

5. What stands out in your memory about something your teacher did or said today?

We will read the first few questions to you and then you will read them yourself. After each question, please indicate your answer. For each question, please circle one of the 3 answer choices. The 3 choices are:

0: "It was **never** this way for me in this class **during this lesson.**"
1: "It was like this for me **some of the time** in this class **today.**"
2: "It was like this for me **all of the time** in this class **today.**"

1. I concentrated deeply on today's math problem.

0 never **1** some of the time **2** all of the time

2. I was fascinated by the math today.

0 never **1** some of the time **2** all of the time

3. As I made progress, I became more interested in understanding the math.

0 never **1** some of the time **2** all of the time

4. I was so into my work that I tuned out things going on around me.

0 never **1** some of the time **2** all of the time

5. I felt that learning the math today would benefit me or pay off for me.

0 never **1** some of the time **2** all of the time

6. As I worked on the problem I found it challenging.

0 never **1** some of the time **2** all of the time

7. I realized that if I worked hard at the problem I could figure it out.

0 never **1** some of the time **2** all of the time

8. I got stuck trying to solve a math problem today.

0 never **1** some of the time **2** all of the time

9. I felt it was OK to get frustrated and let the teacher see that I felt frustrated.

0 never **1** some of the time **2** all of the time

10. While working on this problem I thought of myself as a mathematical problem solver.

0 never **1** some of the time **2** all of the time

11 I wanted people to think that I'm smart.

0 never	**1** some of the time	**2** all of the time

12. I tried to impress people with my ideas about the problem.

0 never	**1** some of the time	**2** all of the time

13. People seemed impressed with the ideas I shared about the problem.

0 never	**1** some of the time	**2** all of the time

14. People saw how good I was at the math we did today.

0 never	**1** some of the time	**2** all of the time

15. I felt smart.

0 never	**1** some of the time	**2** all of the time

16. I wanted to teach another student something that I knew that this other student did not know.

0 never	**1** some of the time	**2** all of the time

17. I listened carefully to the ideas of someone I was trying to help.

0 never	**1** some of the time	**2** all of the time

18. I helped someone see how to do the math.

0 never	**1** some of the time	**2** all of the time

19. Others listened carefully to my ideas

0 never	**1** some of the time	**2** all of the time

20. I wanted to show someone that my way was better.

0 never	**1** some of the time	**2** all of the time

21. I argued strongly in support of my ideas.

0 never	**1** some of the time	**2** all of the time

22. I had an unpleasant disagreement.

0 never	**1** some of the time	**2** all of the time

23. My ideas were challenged by others

0 never	**1** some of the time	**2** all of the time

24. Some person or some group of people tried to disrespect me.

0 never	**1** some of the time	**2** all of the time

25. I was not going to let someone disrespect me and get away with it.

0 never	**1** some of the time	**2** all of the time

26. I was worried I might do something that would get me into trouble with one or more students.

0 never	**1** some of the time	**2** all of the time

27. I paid attention to the way others were reacting to me.

0 never	**1** some of the time	**2** all of the time

28. I hoped people would not pay attention to me.

0 never	**1** some of the time	**2** all of the time

29. I cared more about feeling OK than about solving the math problem.

0 never	**1** some of the time	**2** all of the time

30. I wanted to make sure that all the required work was completed.

0 never	**1** some of the time	**2** all of the time

31. The most important thing for me was getting the answer to the problem.

0 never	**1** some of the time	**2** all of the time

32. I worked on getting the answer to the problem.

0 never	**1** some of the time	**2** all of the time

33. I tried to get members of my group to work to get the answer to the problem.

0 never	**1** some of the time	**2** all of the time

34. I wanted the teacher to think I am a good student.

0 never	**1** some of the time	**2** all of the time

35. I felt relieved when all the work was done.

0 never	**1** some of the time	**2** all of the time

36. I felt proud about what I accomplished

0 never	**1** some of the time	**2** all of the time

37. I wanted to look like I was doing work even when I wasn't.

0 never	**1** some of the time	**2** all of the time

38. I worried that I might get in trouble with the teacher.

0 never	**1** some of the time	**2** all of the time

39. I paid attention to things happening in class that had nothing to do with math.

0 never	**1** some of the time	**2** all of the time

40. I enjoyed learning math.

0 never	**1** some of the time	**2** all of the time

41. I achieved a good understanding of the math we worked on today.

0 never	**1** some of the time	**2** all of the time

42. I was a lot better at math than others today.

0 never	**1** some of the time	**2** all of the time

Here is a list of feelings. Please circle how much you felt this way in class today.

01. interested	**0** Not at All	**1** Somewhat	**2** Very Much
02. unhappy	**0** Not at All	**1** Somewhat	**2** Very Much
03. respected	**0** Not at All	**1** Somewhat	**2** Very Much
04. disappointed	**0** Not at All	**1** Somewhat	**2** Very Much
05. proud	**0** Not at All	**1** Somewhat	**2** Very Much
06. worried	**0** Not at All	**1** Somewhat	**2** Very Much
07. successful	**0** Not at All	**1** Somewhat	**2** Very Much
08. discouraged	**0** Not at All	**1** Somewhat	**2** Very Much
09. safe	**0** Not at All	**1** Somewhat	**2** Very Much
10. angry	**0** Not at All	**1** Somewhat	**2** Very Much
11. excited	**0** Not at All	**1** Somewhat	**2** Very Much
12. disrespected	**0** Not at All	**1** Somewhat	**2** Very Much
13. happy	**0** Not at All	**1** Somewhat	**2** Very Much
14. bored	**0** Not at All	**1** Somewhat	**2** Very Much
15. satisfied	**0** Not at All	**1** Somewhat	**2** Very Much
16. embarrassed	**0** Not at All	**1** Somewhat	**2** Very Much
17. relieved	**0** Not at All	**1** Somewhat	**2** Very Much
18. frustrated	**0** Not at All	**1** Somewhat	**2** Very Much
19. confident	**0** Not at All	**1** Somewhat	**2** Very Much
20. afraid	**0** Not at All	**1** Somewhat	**2** Very Much
21. confused	**0** Not at All	**1** Somewhat	**2** Very Much
22. curious	**0** Not at All	**1** Somewhat	**2** Very Much

Please write any other feelings that you had that are not on this list and indicate how much you felt that way:

23. _______________	**0** Not at All	**1** Somewhat	**2** Very Much
24. _______________	**0** Not at All	**1** Somewhat	**2** Very Much
25. _______________	**0** Not at All	**1** Somewhat	**2** Very Much
26. _______________	**0** Not at All	**1** Somewhat	**2** Very Much
27. _______________	**0** Not at All	**1** Somewhat	**2** Very Much

Here are some ways people behaved in their group in class today.. Think about how you behaved. Then tell us how you think you behaved in your group today.

0: "I **hardly ever** behaved this way in my group **today.**"
1: "I **sometimes** behaved this way in my group **today.**"
2: "I **often** behaved this way in my group **today**."

In my group today:

1. I was active ……………………………	**0** hardly ever	**1** sometimes	**2** often
2. I was the leader ………………………	**0** hardly ever	**1** sometimes	**2** often
3. I talked a lot ………………………	**0** hardly ever	**1** sometimes	**2** often
4. I was outgoing ………………………	**0** hardly ever	**1** sometimes	**2** often
5. I gave helpful suggestions ………………	**0** hardly ever	**1** sometimes	**2** often
6. I organized what we should do …………	**0** hardly ever	**1** sometimes	**2** often
7. I was bossy ………………………	**0** hardly ever	**1** sometimes	**2** often
8. I was powerful ………………………	**0** hardly ever	**1** sometimes	**2** often
9. I wanted to show off ……………………	**0** hardly ever	**1** sometimes	**2** often
10. I joked around ……………………	**0** hardly ever	**1** sometimes	**2** often
11. I was sociable ………………………	**0** hardly ever	**1** sometimes	**2** often
12. I was smiling ………………………	**0** hardly ever	**1** sometimes	**2** often
13. I was friendly ………………………	**0** hardly ever	**1** sometimes	**2** often
14. I worked cooperatively ………………	**0** hardly ever	**1** sometimes	**2** often
15. I concentrated on the tasks ……………	**0** hardly ever	**1** sometimes	**2** often
16. I liked to be right ……………………	**0** hardly ever	**1** sometimes	**2** often
17. I was unfriendly ………………………	**0** hardly ever	**1** sometimes	**2** often
18. I was irritable ………………………	**0** hardly ever	**1** sometimes	**2** often
19. I was uncooperative …………………	**0** hardly ever	**1** sometimes	**2** often
20. I showed my feelings …………………	**0** hardly ever	**1** sometimes	**2** often
21. I was likeable ………………………	**0** hardly ever	**1** sometimes	**2** often
22. I was fun to be with ………………	**0** hardly ever	**1** sometimes	**2** often
23. I was trustful ………………………	**0** hardly ever	**1** sometimes	**2** often
24. I was respectful ………………………	**0** hardly ever	**1** sometimes	**2** often
25. I did what I was told to do …………	**0** hardly ever	**1** sometimes	**2** often
26. I acted overburdened, as if I had too much to do ………………………	**0** hardly ever	**1** sometimes	**2** often
27. I was unhappy ………………………	**0** hardly ever	**1** sometimes	**2** often
28. I acted hopeless, as if nothing will work out ………………	**0** hardly ever	**1** sometimes	**2** often
29. I was afraid to try	**0** hardly ever	**1** sometimes	**2** often
30. I doubted my own ability ……………	**0** hardly ever	**1** sometimes	**2** often
31. I was happy to be a member of my group…………………	**0** hardly ever	**1** sometimes	**2** often
32. I was quiet ………………………	**0** hardly ever	**1** sometimes	**2** often

Students have told us about some of the thoughts they have had while working in their math class. Read each of the thoughts they described to us. Circle **yes** next to each thought that you had in class today and circle **no** if you did not have this thought today.

1.	yes no	This problem is really interesting.
2.	yes no	I can't figure this out now, but I want to keep working on it.
3.	yes no	I'm so angry at this person.
4.	yes no	Don't distract me.
5.	yes no	I'm really concentrating on this problem.
6.	yes no	I want you to know just how smart I am.
7.	yes no	I can't wait for this class to end.
8.	yes no	I want you to admit that you were wrong and I was right.
9.	yes no	That kid says things that make me laugh.
10.	yes no	I can't figure this out.
11.	yes no	People think I'm smart.
12.	yes no	This stuff is really boring.
13.	yes no	I like learning math.
14.	yes no	I wish the teacher would call on me, so I can show how much I know.
15.	yes no	I have to get the people in my group to finish the work on the problem.
16.	yes no	I'm excited when I get challenging problems.
17.	yes no	I am pretty sure that I can figure out the work the teacher gave me.
18.	yes no	I hope the teacher doesn't realize that I'm not really working on this problem.
19.	yes no	They're videotaping me so I better be careful about what I say.
20.	yes no	I'm not gonna let you make me look like I don't know what I'm doing.
21.	yes no	Let's see, did I write down all the stuff the teacher said we had to have on our paper?
22.	yes no	I know the answer but I'm not saying anything.
23.	yes no	Finishing the problem is more important than spending a lot of time on any one part.
24.	yes no	I'm good at math.
25.	yes no	I like teaching this person things that I know.

Please write any thoughts that you had that are not on this list:

My Thoughts:

____

____

____

Did you have any type of difficulty with a math task that you worked on today?
____no ____yes

IF YOU HAD DIFFICULTY:
(a) Did the difficulty happen when you talked about it in your group?
____no ____yes

(b) Did it happen when the whole class was talking about the problem?
_____no ____yes

(c) What stands out in your memory about this situation (for example, what happened, what you thought, what you felt, what your teacher said or did, what other kids said or did)?

*****Draft*****

Questionnaire developed by Goldin, G.A., Epstein, Y., Schorr, R., & Warner, L. of Rutgers University

Appendix D: Teacher Interview Transcripts

Ms. A. Interview Transcript

Interviewer: Thank you for taking the time to reflect on your classroom experiences. I'm working on my thesis now and I'm interested in finding out about how teachers respond to student difficulties in mathematics and about why they respond in those ways. So first I have some questions for you about your thoughts regarding ways in which you respond when students have difficulty in mathematics. After that, I'll show you clips of one of your classes where each clip involves an instance of student or students' difficulty and I'll ask you to reflect on the ways in which you responded.

Interviewer: Are there particular ways in which you usually respond when students have difficulty with mathematics?

Ms. A.: Definitely. I try to first of all have in my classroom groups. So I would see that a student or two or three, depending on the group is struggling with math in general. Either because they have not been learning through the years so they lost a lot, and they are all of a sudden in the seventh or eighth grade and they did not learn a lot of skills. Or because they just simply don't get it or they are not interested in putting the effort. So I would put them in groups so that the group would have a student that is strong in the skill and maybe two of them are struggling. Or if it's a group of four it would be two and two so at least there is peer teaching is going on. Also, I…over the years I realized that many times when you work with the student one on one it helps a lot. First of all we don't have the distraction of the class around them and at the same time they are not trying to impress anybody and they are not embarrassed that they do not know. So I always tell them, 'Okay, come in the morning, let's work on it one on one' without the pressures around them. So once they come in the morning we go over the problem together, or the concept together, and they start gaining self confidence, and when they are in the classroom they are willing to put the effort in the work.

Interviewer: So what would you say you do if you find that in middle of class a student has difficulty on a particular problem that the class is working on?

Ms. A.: I would try to address it to the student one on one as the others are working in groups. I would just pull that student maybe and try to work on it with him or her. Or maybe I would relocate him or her from the group. Maybe the group is not the right setting for this particular student, so I will try to put him or her in another group.

Interviewer: And the peer teaching that you said you do, is that…you mean that's in general, not specifically when students have difficulties, but do you say that you…

Ms. A.: No, I put them in groups every day. I mean, we give them maybe twenty minutes of group work. We explore the problem. At the beginning I would launch it to them, and then I let them have time on their own to explore it. I would be of course trying to facilitate the concepts that are being introduced to them. So within the group they learn from each other. Sometimes I would say something and they would look at me like I'm talking another language. This same thing, this same statement is going to be said by a student in the group and they will 'Oh, now I get it!' So as if it's the magic of being of the same age, they understand each other.

Interviewer: Are there things you try not to do when you respond to student difficulty?

Ms. A.: Sorry…

Interviewer: Are there things you try not to do when you respond to student difficulty?

Ms. A.: Definitely, I wouldn't try to make the students feel that he or she is incapable. Even if I realize that they are really, really struggling and they are far, far, far behind, I would try always to give them the hope that it's not that difficult for them to get it. I wouldn't say, 'Oh, you don't get that?' You know what I mean. It's just that…try to build them with the very little that they have, still let them feel that it is enough for them to start and to build on. Just give them hope that it's a process of learning. If you are here and you know it, then you shouldn't be in this classroom. You should be maybe in the next grade. But since you are here then you have the opportunity to learn still.

Interview: Do you respond in different ways in different situations?

Ms. A.: What do mean, like what? What type of situations?

Interviewer: I mean different situations in class. For example, if students…do you respond in different ways if students have different kinds of difficulty, or if students are grouped differently, of if different students have difficulty?

Ms. A.: Well definitely different groups, I don't know. Each class- I see three classes every day. Each class has its mechanism, and each class you have to, to, you know, tailor to their needs. So definitely, it depends on the mechanism of each group. A group could be for instance, very playful. And you have to respond to them in a different way from a group that is really always on task, and they are fast. Sometimes they are faster than you, so you need to move faster with them, and always make sure that they have tasks to do, because some of them will be finished right away, so they cannot be waiting for others to finish. Another group they are playful because they are struggling with the concept and they are sort of trying to deny that they don't know what to do, so they are trying to play. So you have to get them engaged, you have to make it entertaining to them and appealing to them that they are willing to take the risk of trying. So definitely you have to tailor to the students' needs. Every group has its mechanism.

Interviewer: So can you give some examples of how you would respond differently to students' difficulty in math? You were saying that…depending on the group.

Ms. A.: For instance, one of the groups that I have is an advanced group. So, the launching of the problem, for instance does not require me to be very, very detailed and specific and to take them step by step. If I just give them the road lines and let them explore, they're going to get to the end result that they need to be at with this particular question or concept. Whereas another group that is really struggling and they are really trying to put in the effort, in the launch of the problem I would have to be very detailed, very specific, providing a sort of plan of work. For instance, if I'm giving them a problem, I have to provide them the very specific problem. Okay, so you need find the area, number one. To find the area you need for instance to break the shape into regular shapes, because this is not a regular shape. So you need to break it up into shapes, find the area and then you're going to try- one of the problems was, 'How many tiles do we need to buy?' We need to find how many tiles then you have to find the price, so you have to tell them the details of the task, because they just look at the task, they don't know what to do. Whereas for instance the high performing group, I don't need to go into all these details with them. It's just I need to go over the main- 'What do we have here?' They say 'it's the design of the floor.' 'What do we need to find?' Right away they realize we're looking for area. Right away they're going to realize that you need to compare the area of the floor to the area of the tile, how many tiles will fit. And they will just move from one thing to the other. The other group you have to really lead them, one step at a time. They get overwhelmed if you just bombard them with the whole task.

Interviewer: So you spoke about how you give the tasks in a different way to the different groups. I'm wondering if you could give an example of how you might respond differently to difficulties that students have in the different groups, if you think you would. Or do you think you wouldn't respond differently.

Ms. A.: I would of course. Again, I don't know if I'm repeating myself, but the group that is moving fast and they have a higher level of comprehension and understanding of the concept from previous years; they'll be able to move onto the next step. So I would move with them to deeper extensions of the problem, whereas the group that is really struggling with the basic concept, it's not really very appropriate to take them into the extensions before I make sure they understand the main concept they're doing. You know what I mean? For instance if we're talking area and the question is telling them how many tiles do we need to buy, they would for instance tell me look for the perimeter. And I try to now get them to see that we're not talking about the outside of the shape, we're talking about the inside of the shape. Ok?

Interviewer: So, I'm wondering if you think in general you respond… I'm just wondering how this applies to the difficulty the students have. Like what would you do in the stronger class if students have difficulty?

Ms. A.: Again, it's just try to introduce the problem, break it up more for them. Usually the stronger class, they would try, I mean they will not just wait there until somebody comes and helps them. They just keep turning it around until they find a way through, whereas the low…because they already have the skills. It's just they're sharpening it; they're trying to move up, or go a little step further. So they already have a foundation. It's always the weaker group that would really need strong intervention from my side. With the strong group, you know what they would do? If they are struggling in the group, they would send one of them to another group and show them what they are doing and get the feedback from the other group and come back and move from there. Whereas the struggling group would just sit there and they would sort of be intimidated from even going ahead. Or they would be intimidated to try to check with another group. Maybe they think they're not that smart or whatever.

Interviewer: Do you think there are any other factors that determine how you respond in certain situations?

Ms. A.: Are we talking only academically or are we talking also about behavior, included?

Interviewer: No, I'm talking about difficulties in math.

Ms. A.: Okay, it's just so intricate. Because difficulties with math has behavior problems, attitudes that are coming from outside, so of course the way you handle the student, you're going to be influenced a lot if you know some background. So if that is a student that you know that they know what they they're doing, or he or she knows what they're doing but they're just being playful, so it would be zero tolerance. You know, that who knows a lot, a lot is expected from. Whereas a student who you know that he or she is really struggling and is going through a lot of pressures on the outside so for him or her, coming to a math class is the least of their priorities at this point in their life, you will try to give them the encouragement that they need, the support that they need, more than the student that you know has it all but they're just not putting the proper effort in the work.

Interviewer: Okay. Now I'll show you a few video clips where students in your class encountered difficulty while they worked on the building blocks dilemma. This is the problem you were working with. After each clip I'll ask you why you think you chose to respond in that way. I realize these sessions took place around a year ago, so just respond according to the way you think, why you think now that you answered in that way then.

Ms. A.: I can't remember!

Interviewer: Please try as best as you can. Can you see it?

[Showing video clip 1- Difficulty 1: Amanda, Day 1, 10:56:05- 10:57:24]

Interviewer: Were you able to hear?

Ms. A.: Yes.

Interviewer: Do you want to see it again?

Ms. A.: It's okay. I think I can remember. Is this the baby?

Interviewer: Yes.

Ms. A.: How cute!

Interviewer: Thank you…What do think the student's difficulty was?

Ms. A.: They were... I think if I remember from this problem, they were trying to figure out how many blocks they should be adding, if I remember, and they at one point just could not remember that there's a hidden block in there, so they kept adding it twice. And I didn't want just to tell them that right away. I just wanted them to realize that for themselves. I didn't want to give them any hints, because I thought that if they just take a minute to focus that this is a hidden one so they don't need to count it every time, it would at the end get them to see how many they are going to add on each side. And from there they are going to be able to make the general rule that we were looking for at the time.

Interviewer: In this clip that we just saw, let's listen to it again. I'm talking now about what Amanda is saying. She's saying here that she has the amount of blocks that are needed for the five block high tower. And she got five in the height and four on each side, all around. And she's saying that why don't you just double the amount of blocks you need for a five block tower to get the amount of blocks in the ten high tower.

Ms. A.: Okay.

Interviewer: She just wants to multiply by two to get the answer. I'll play it again, that's the difficulty I'm talking about here.

[Showing video clip 1 again]

Ms. A.: I can't hear what I'm saying.

Interviewer: You said, 'try it, see if it's going to work.' …Did you hear that? You said, 'what's the total amount of blocks?'

Ms. A.: I think what I was trying to get them to see- okay if that is true, so from one phase to another, is this what you're doing? So I guess I tried to refer them to the previous stage.

Interviewer: Okay let's hear the rest of it. Now they had already gotten 46 blocks for the ten high tower previously and then Amanda came and said 'can't you double it?'

Ms. A.: I can't remember. So 46 was the right answer for that level?

Interviewer: Yes. 46 was the right answer, and then when you came over Amanda said, 'well can't you just take the answer from the five blocks and double it?' And I think what she wanted to do was take 4 which is the number she got on each side of the five block tower and double it to get eight blocks on each side of the ten high tower, and then double the blocks in the height, which would double five to get ten in the height, and add up all of those. So she did that and then she got 42.

Ms. A.: So it did not coincide with when they did it from one level to the other, right?

Interviewer: Right, so you were saying, ‘try it, see what you get’, and when she got a different number, you said ‘which one is right’? So now what would you say that the difficulty was?

Ms. A.: Actually the difficulty here was that they followed from one phase to another and they got an answer, and then I think she was trying to get the shortcut so that she could do the bigger numbers. So I wanted her to realize that, is this shortcut working, or not? I was sort of trying to tell her, ‘you are sure that the 46 is correct because you moved from one to the other.’ Now is this short cut going to be applicable no matter what, is this a general rule that you can apply to any number of blocks in height, or not. So I was trying to get this group to understand that check before you make the generalization. They used the smaller 1, 2, 3, 4, 5 phase and then they tried to make a generalization right away. Is it working? Try to apply it on the phases that are under your control before you try to make the bigger generalization.

Interviewer: Because she did have up to ten blocks, she did have the answers for up to ten blocks. She was trying to generalize from five to ten.

Ms. A.: But it didn’t work. She realized that this shortcut is not working, so from there they started to move to another technique to try to find the answers with a generalization.

Interviewer: So when you were saying… after Amanda says that she thinks that you can double it, and you said try it, see if it works, and she got her answer, a different answer, and you said, which one is right? So why do you think you responded in that way?

Ms. A.: I wanted them to make their own decision. I mean, okay now you have according to what you came up with, two possibilities for the answer. Which one is right and why? So they needed to go back and check the logic of the work. And I can’t remember what happened after that but I think in this group they realized that the doubling is not the way to go for any of them. So they realized ‘no, it’s not the general rule’. Of course if we did it last year, I would have remembered the exact conversation, but as far as I remember…

Interviewer: Why would you say you wanted them to make their own decision?

Ms. A.: Because if I just give them the answer they will just take it and that’s it, but if they spend the time thinking and turning it around, it will stay with them. They’ll have ownership of this solution, and they’re going to be able to apply it in different problems, similar or non-similar, just having the confidence of the ability to find a logic to an answer.

Interviewer: Do you think the students’ mathematics ability influenced the way you responded?

Ms. A.: Definitely.

Interviewer: How?

Ms. A.: If they have a high level of mathematical abilities and skills you have to ask them the questions that would lead them up into that direction. But if they are really struggling with the basic skill you can’t ask them higher thinking questions, you have to ask them basic questions until they go up to that level and then you can start applying the higher level thinking questions.

Interviewer: This is the second clip…Could you see it?

[Showing video clip 2- Difficulty 3: Emanuel, Amanda, Eliot, Juan; Day 1, 11:01:19-11:02:10]

Ms. A.: So I guess…they were saying five, we're adding five, so I was trying to say, okay you're telling me you're adding four on the sides, how is that related to the five, where does the fifth one come in? So they were not able to answer right away, it took them a few minutes to understand that they're adding one on the top too. I just wanted to make sure they understand what they are adding, and where they are adding it.

Interviewer: So, why did you respond in that way? I think you said, 'think about it and I'll come back.'

Ms. A.: I just wanted to give them the time to be able to understand where they are adding the blocks in the shape. So they were able to see that we're adding four on the sides, 1, 2, 3, and 4. And they really didn't see where the fifth went in. So I just wanted to give them some time to work together. You know, I felt that they got something that they needed to have time on their own, to think without pressure, you know. When somebody's standing with them and trying to get them to get the answer.

Interviewer: Do you think that the students' mathematical ability here influenced the way you responded?

Ms. A.: Definitely. This was high performing group, so I just gave them the question and left them, and I know when I come back they will be at a different point. They will take that and continue. Another group that is not at a high level of performance, they would really struggle with it and they might not be able to move anywhere from there. So I would have needed to stay a little bit longer to make sure I guided them into that step so that they can move on to the next one.

Interviewer: We have one more clip. This clip is a little longer, around six minutes.

[Showing Video Clip 3- Difficulty 4: Emanuel, Amanda, Eliot, Juan, Day 1, 11:10:34-11:17:00]

Interviewer: What do you think the students' difficulty was over there?

Ms. A.: At the beginning they thought they got it all. And they were…they thought they were done. But once I asked them to explain, and that is the process that would really get them to start comprehending really the concept and putting their fingers on everything. You know, really understanding it. When they started, 'What is this for? What is this number? Explain, what is it, what is it?' Only then they started to struggle and realize that they sort of understand, but they don't really, really know 100% sure. And you see that they started conflicting in their answers. One said something, one said the other thing and then Emanuel said, 'I know but I can't really say it in my own words.' So they really needed to take time to explain it to themselves, to say it to themselves, 'What is each number referring to?' in this answer that they think is the answer to the problem. So that's really useful, because sometimes students think, okay we know the answer. They write it, but does it really make sense to them. Did they really comprehend what they did and why? And does it make sense or not? You know, going over it would help them to really establish the concept. Especially in this problem they were trying to come up with a general rule, so they needed to see it for phase one- it applies, phase two- it applies, phase three it will still apply. So yes, it is the general rule that we can carry on. So basically, I just wanted them to really comprehend what they put down, to make sure that they understand. And in this group three of them were very high performing and one was struggling with math, but they really…

Interviewer: Which one was the one that was struggling?

Ms. A.: It was Eliot, the one on the side, the one in the corner. But I remember this boy at the end of the year, he was quite confident, and he really knew how to get himself on the right track. At the beginning of the year he was just completely off. But at the end I realized that he started to gain his confidence and start thinking math. So I guess the group has done a lot.

Interviewer: So…could you tell me why you think you answered in that way? Do you think…I guess you sort of answered this, but do you think the students' mathematics ability here influenced the way you answered?

Ms. A.: Definitely. Because this group I know they have the ability but they needed to put more analysis into it. Because, I mean it's not enough to just understand the concept superficially. I wanted them to have it more stable, to really comprehend it so that when they see it in another problem they will be able to do the same process again, do the same analysis again.

Interviewer: And you mean if they were at a lower level you would have done something different?

Ms. A.: I would have probably… probably they wouldn't be at this point of the problem. They would have been still struggling with, 'what are we adding, where are we adding it?' Definitely, I would in a different group, I would be trying to get them to see what are we adding each time? I wouldn't at this stage be trying to address with them the general rule yet, because they wouldn't be able at this point to come to a general rule. They will be still struggling with the individual phases that they are trying to see, 'What are we adding?' 'Is this the same thing that we're doing every time in the next phase?' 'We're going to do the same thing or not?' It will take them a little bit longer to figure out that it is a pattern; it is a pattern that's going on.

Interviewer: Right, but you mean that you would still have asked them questions to try to lead them to the next step?

Ms. A.: Yes, and I would have maybe let them use more manipulatives.

Interviewer: You mean in a weaker group you would have done that?

Ms. A.: Yes, I would have encouraged them to use the manipulatives more. Everybody used the manipulatives that day, but the high ability group was able to get out of the manipulatives phase faster than the ones that were struggling. They felt more confident by building the exact stage. They wanted to build it, even they were willing to go up to the one hundreds. So they didn't want to come out of that and try to come with the rule. They wanted to actually build it, which was not really practical, but they needed that at the time. Just going back, I would have still ask questions, because there is no point of giving them the answer. It wouldn't have mattered to them if I gave them the answer, they wouldn't have understood better. They would just have taken it and wanted to move to something else, thinking that they understand. Maybe I would ask simpler questions, basic questions so that they try to see that basically, let's play with manipulatives. What are you adding in this phase, how many did you add? And you need to maybe make a table, or move on. How many…how many did you add in the next phase? So I tried to simplify it to them.

Interviewer: Okay, but you're saying the basic way that you responded to their difficulty would have been the same?

Ms. A.: Yes.

Interviewer: Even though their difficulty would have been different, but you're saying the basic way you answered would have been the same.

Ms. A.: More or less. Questions, again.

Interviewer: So would you say that in general the way you answer… the types of answers that you give, you know, would be the same, in the higher group?

Ms. A.: It would be questioning and I'm trying to give them questions that would help them better.

Interviewer: Okay, thank you!

Ms. A.: You're welcome.

Ms. B. Interview Transcript

Interviewer: *Thanks again for your time. I'm interested in finding out about how teachers respond when students have difficulty while they work on math problems in class, and about why teachers respond in those ways. First I'll ask you some questions about your thoughts regarding ways in which you respond when students have difficulty while they work on math problems. After that, I'll show you video clips of your classes which involve instances of student difficulty and I'll ask you to reflect on the ways in which you responded.*

Interviewer: Can you describe how you usually respond when students have difficulty while they work on math problems in class?

Ms. B.: Well, I never give the answer to the students. I try and listen to what's going on and depending on the student, probe them in a different way. Of course you have your higher students where they'll think a little bit more and they like that challenge, so I might just ask them another question, not gearing towards the answer, but just why they got to where they were. For the lower student who you can see who is drifting off I might kind of ask them a question where it would gear towards the answer a little bit to let them have a little bit of a success and then they can begin thinking of the answers on their own.

Interviewer: Are there things you try not to do when you respond to student difficulty?

Ms. B.: I try not to assume that they know… they have prior knowledge. Even though they should, I try not to assume that they know exactly where they should be heading or what they should know already.

Interviewer: You spoke about this a little bit in the last question, but do you respond in different ways in different situations?

Ms. B.: Yes. With the higher and lower students, it depends on the student. And it could be a lower student, but their thinking is there so I would give them the same kind of question as the higher student. Sometimes if I see that they're way off, then I will, I don't know, ask a different type of question, but never give them the answer, so it just depends.

Interviewer: Are there other factors that might determine how you respond in different situations?

Ms. B.: It depends on how they come in that day as well. They might come in riled up, it might have been something else that happened at home, in a different class. So of course you can't expect them to answer in the same way they did the day before. So it all depends on them.

Interviewer: So how would you respond differently based on these different factors?

Ms. B.: One student might need a little more time to just get themselves together on their own instead of me questioning and questioning and trying to get them to focus and work harder. They might need a little bit of down time. Or I might just go and like whisper quietly to that student the question. I won't force them into the group at that time but then kind of lead them into the group a little later after I've seen that they calmed down a little bit.

Interviewer: Do you think the type of difficulty or the small group versus whole class setting of the difficulty might influence how you respond?

Ms. B.: Yes. Small group is definitely better. Whole group you cannot get to everyone. When it's whole group and if I …. If it's whole group and I see someone's having a problem or something like that and I throw it out there in a whole group, they might shut down automatically. But in a smaller group it might be less threatening. One on one it might be non threatening at all so again it depends on the student. Some students can handle that whole group instruction but those who are struggling and then if they're not well liked in the classroom already, so then now they'll get teased and so they're just going to shut down and not going to answer. So small group, individual is better than whole group.
Interviewer: So how might you respond to difficulty in a whole group setting?
Ms. B.: Not put attention all on that student. Ignore it probably, if I know that they're having a difficulty, keep in the back of my mind until we break down into smaller groups. But there's no need to, there's no need to force them to try and think in a whole group especially when you know your students.
Interviewer: Okay, so now I'll show you some clips.
Ms. B.: Okay.
Interviewer: So there are three clips and they're all from one table. It was the table with Larent, Marcos and Lashanna…(*starting up computer*)…So after each clip I'll ask you why you think you chose to respond in that way, and just try to answer as best as you can, you know, as best as you can remember.
Ms. B.: Okay.
Interviewer: The sound, the audio's off, a few seconds off the video.
Ms. B.: Okay.
[Playing video clip 1- Difficulty 12: Larent, Day 1, 10:13:30- 10:14:02]
Interviewer: Do you want to see it again?
Ms. B.: Yes.
Interviewer: If you can… Larent says something, in the middle he says 'well how do I know they didn't just put it all together and make a [something] of it'. I wasn't able to catch that.
Ms. B.: Okay.
Interviewer: If you can listen.
Ms. B.: Okay.
[Playing video clip 1 again]
Ms. B.: Yeah, I heard, but I didn't hear. Yeah, he said…
Interviewer: Should we put it on again?
Ms. B.: Yeah, we can, to see it.
Interviewer: This is the worksheet they had.
Ms. B.: Yes, I do remember it.
[Playing video clip 1 again]
Ms. B.: Okay.
Interviewer: Were you able to catch that?
Ms. B.: No. He just said he just put them all together to build. Is that what he said?
Interviewer: It seems like… right, I don't know what it was but it seemed like you were pointing out that there's a pattern here.
Ms. B.: Okay. Right. With the two and the num…right

Interviewer: At the beginning of this clip Larent said he thinks there are five blocks in the two high tower, and then you said 'you said five, you said six'. And then…

Ms. B.: He said, he said at first he said five and then he changed it to six.

Interviewer: Right

Ms. B.: And then, why did he change it to six?

Interviewer: Right. So someone else had said six before that, and he says five. So then you pointed out that he said five, someone else at this table had said six, and he said he said six the first time. Then you said and now you're changing, why are you changing that? And then he decided that there are six. So what do you think Larent's difficulty was?

Ms. B.: Seeing the center block, the 'A' block. He just kind of got rid of that.

Interviewer: Why did you respond in the way you did?

Ms. B.: Larent, um…he has a difficult time settling down sometimes. He really didn't even want to be in there with the videotaping so it was kind of like I'm going to say babying him, pampering him a little bit and giving him that extra time and sitting there. And then just giving him questions where we can just kind of… I was trying to pull something out of him. Because for him to even just sit there was a good thing. For him to even talk about it a little bit was a great thing. So I was just trying to see… and he has a… he was thinking a little bit with it and everything. At least he understood what the task was about. His answer is off and everything, but the fact that he just didn't say just one, or just like 'I don't know', that was a good thing. So just trying to like get him on board. Because he does have a difficult time working in a group, and that was a small group, and it was only three in that group. But...

Interviewer: Do you think his mathematics ability influenced they way you responded?

Ms. B.: Yes.

Interviewer: How so?

Ms. B.: Larent, he does have a difficult time in math, and learning them because this was like still towards the beginning of me getting to know them because last year was my first year in this school, but quickly learning them, Larent was one who, I don't know, organizational skills just in math, like the notebook was all over the place and just him coming and just giving full attention was difficult for him so just for me to be able to sit there and just have a little bit of dialogue with him was good. So it's kind of like they were easy questions right here it's just, it wasn't like giving him the answer but it's just trying to just let him think a little bit and trying to see that one cube that he was missing. I don't know if he was trying, I don't know if he wasn't trying. You know he might have just been not thinking more than he had to. Like, 'I see five cubes there, it's five'.

Interviewer: How would you describe the method that you were using with him?

Ms. B.: I was just trying to get him to challenge himself and challenge me a little bit. Because I was like, five, six? So I wanted him to really say 'well, no it is five because, or it is six because'. He really didn't give much aggression he just was like there. So I was trying to just… trying to get him excited and trying to get him like to delve in.

Interviewer: I'll show the second clip.

Ms. B.: Okay.

[Playing video clip 2- Difficulty 16: Marcos, Day 2, 9:41:37- 9:43:11]

Interviewer: Do you want to see it again?

Ms. B.: No.

Interviewer: So focusing on Marcos's difficulty here, what do you think the difficulty was?

Ms. B.: He didn't quite see the pattern, I don't think, he just was saying multiply by… can I… yeah I do need to see that again, he said multiply by something.

[Playing video clip 2 again]

Ms. B.: Okay.

Interviewer: What do think the difficulty was?

Ms. B.: He wasn't sure why he was multiplying what. I think he understood an overview of what was going on but he wasn't able to really put it in words. He saw what he was doing, he saw the pattern but he didn't say like, in stage ten or in ten towers high that's why I multiplied, he just said, 'multiply ten'. So I don't know, can he clearly explain what was going on. No?

Interviewer: Okay. Okay, so you were saying he was getting, he saw that there was a pattern, you're saying, but he wasn't explaining what was going on?

Ms. B.: Right, because he was able to throw out the number, so probably just them adding on the blocks, they were able to get their answer. But then I don't think he understood really how to get it mathematically with some kind of an equation or anything. I think they were just adding on, because he didn't give a clear explanation in the end.

Interviewer: And the number that he gave, it was interesting. Because he said that there is a pattern: one, six, one, six. The numbers that you get for the amount of blocks in the towers each end with a one or a six. And the number he gave 460 for the ten block high didn't end with a one or a six, and he didn't…

Ms. B.: Pick that up

Interviewer: …seem to notice that.

Ms. B.: Yeah he didn't, no he didn't. So he does not have a clear understanding.

Interviewer: Why did you respond in the way you did?

Ms. B.: I wanted him to get his thought out whether it be right or wrong because that's good for discussion when they put it on transparency and it becomes a whole class instruction. That might… when he goes to explain it there will be questions, hopefully there will be questions from the other students. So that will help his thinking. And then when they say 'Well, how did you get that answer?', he might get stumped and he might not realize how he got that answer, then he might realize his answer's wrong. He might realize he was on a right track but then somewhere along the line he messed up. So just for him to just get his thought out was okay.

Interviewer: Why would you say that way of responding is better than simply showing him where his mistake was?

Ms. B.: So that he can understand where he went wrong. If I just tell him, automatically he still is not going to understand because he wasn't able to think it out himself. But often times I realize when the students are actually working through the problem and they come up with a clear understanding, conceptually it makes more sense to them. We learn things just procedurally so it's like 'okay just do this because'. There are no questions, we don't even understand why, so when we get the other problem we don't know how to make that connection. So if he knows where he went wrong he won't do that same mistake because he knows, this does not make sense right here, I messed up right here. He won't do it again, hopefully.

Interviewer: Do you think his mathematics ability influenced the way you responded?
Ms. B.: I'm not sure. I'm not sure. He could have just had a thought, and then okay I don't know if he was trying to play off of me and trying to like, see like, was his answer the answer I wanted to hear? So I don't know with him.
Interviewer: You don't know if…
Ms. B.: *inaudible*
Interviewer: Is he, do you know if he was a high or low mathematics ability student?
Ms. B.: He was, he's on the higher end.
Interviewer: You're just not sure if that influenced…
Ms. B.: I'm sorry?
Interviewer: You're not sure if that influenced what you said to him.
Ms. B.: Right, I don't know. Because you know some students you can see like they're looking at you but he really… he seemed pretty like confident, like okay, he was just talking. It wasn't like 'is that the right answer?' like you know he just went on with it. I don't know with him. Larent when we're talking about him he was like talking but then still kind of like looking. But Marcos just went… so I would say no…
Interviewer: No…
Ms. B.: It didn't affect.
Interviewer: You mean… so how would you respond differently if the student is looking at you to see whether his or her answer is right?
Ms. B.: I would ask them, I would ask them. I would say sometimes, depending on the student depending on the mood and everything also, 'are you trying to see how I'm responding to you to see if you're right?' or whatever, you know. I would ask them. Or then I would ask them another kind of question just to see where they're going with this or whatever, because some of them they want you… they want to please you they want the right answer and everything. But you don't want to just let, lead them to believe that they're right. You want them to just be confident that they're right and prove and say yes, 'I am right'. I don't want it to be where they're looking at me and he's like oh, you do get it. I'll never lead them to believe that they got it. 'Okay, write your answer down and prove it to your class', and that's it. So they'll never get it from me and they even say that. 'Ms. Barnes you never give us the answer, you don't even let us know if we're right'. Because I want them to keep on thinking and be able to prove it to all of us.
They're about to do morning announcements so I don't know if you want to stop.
Interviewer: I'll wait.
(After announcement) Again, this is the third clip and after we view this clip I'll ask you about the interactions involving Lashanna.
Ms. B.: Okay.
[Playing video clip 3- Difficulty 17: Lashanna, Day 2, 9:47:50- 9:48:38]
Interviewer: Do you want to see it again?
Ms. B.: Uh huh.
[Playing video clip 3 again]
Interviewer: What do you think Lashanna's difficulty was?
Ms. B.: To me she, when we were watching Larent and Marcos she was just sitting there. So I don't know if she didn't understand the problem, I don't know if she was having a bad day, I don't know if she just didn't want to do the work. She might not have wanted to be in that group, that might have been it right there. So, because she didn't

want to be in that group, she wasn't going to do any work at all. Because I do remember later on she was able to explain what was going on. And it was interesting because she sat there doing absolutely nothing it seemed like, and this is here again where I'm still learning because although she's just sitting that doesn't mean that she's not paying attention. Because later on like if we… like if we were to watch like further along she does get it and she's the one who understood, and I'm like, she sat there the whole time, so she was just probably in a little funky mood.

Interviewer: Interesting. And I did notice on the clip she came into the group later on, a few minutes after the group started. So maybe she didn't want to go to her seat…

Ms. B.: And be probably with the two boys. Because she probably wanted to be with her friends but always, you know being with friends is not the best. So…

Interviewer: So you picked up on all of that.

Ms. B.: Yes

Interviewer: What do you think her… do you think she had a difficulty as you were speaking to her about this problem?

Ms. B.: If… in the beginning, yes. I thought that… like I said, after *(inaudible)* it could have been a numerous amount of things, but it didn't look she was trying at all. It didn't look like she was trying to see what was going on. So where was the difficulty, was the difficulty right in understanding what she had to do or was it just her giving an attitude?

Interviewer: And what did you think after you spoke with her?

Ms. B.: She just wasn't trying. Because like, I don't remember what I was saying right there but I was like saying something a little more stern to her. Like she can't just sit there and just let them put down an answer and be okay with it. You have to try and at least see if his answer is correct, are you okay with this answer? So I was speaking a little bit more firm with her.

Interviewer: Is that how you would explain why you responded in that way?

Ms. B.: Yes.

(Another school announcement)

Interviewer: Do you think Lashanna's mathematics ability influenced how you responded?

Ms. B.: Yes, because I know that she can do more than what she was doing, so I was getting a little frustrated with her, because she wasn't trying. Like I'd seen her in the past where she would… she does have an attitude problem, days that she comes in and it's coming from home probably, so she comes in with an attitude, comes in very like uptight and just ready for any argument or whatever. But when she is calm and we're working through problems she will work and she will ask questions. So for her to sit right there frustrated me because I knew she can do a lot better.

Interviewer: How about when she said that, when you asked her how did you get your answer for the 100 block high tower, and she said Mathew explained it to me. Mathew had, Mathew was using his multiplication strategy, he was just multiplying the number in the amount of blocks that you need for a ten block tower by ten to get the number of blocks you need for a 100 high tower. So do you see that as the difficulty or that was just an overall attitude of hers of…not trying?

Ms. B.: An attitude of hers. Because Lashanna is not even one who would just let Mathew be okay with an answer. Like she will get up there and she'll explain the answer. So for her to just be like 'Marcos explained it to me', Mathew probably didn't even explain it to her because she probably didn't even allow Mathew to explain it to her, you know. So that was just her way of like just like, you know, shooing me off, like 'I know it, Mathew explained it to me'. That's it.

Interviewer: That's interesting. So you don't think that… you don't think Mathew explained it to her and you don't think that she agreed with his answer.

Ms. B.: No.

Interviewer: It wasn't a math difficulty. It's very interesting that you picked up on that because when I was watching the video I saw, you know I was able to see what happened before that and after that, and it's interesting because before this she got an answer of 496 cubes for the 100 block high tower and then when you came over she said 'oh, I just… Mathew explained it, I agreed with him'.

Ms. B.: So when she got that answer does she get it on her own?

Interviewer: Yes.

Ms. B.: Okay.

Interviewer: So you think she just didn't want to work with the others?

Ms. B.: Uh huh.

Interviewer: And she didn't want to tell you, maybe she didn't want to be in that group so she didn't want to tell you that she really got the answer because she didn't want to…

Ms. B.: Well she, I can tell by the body language she didn't want to be in that group when she came in. I can tell. You know how you know your students or anything, she wasn't going to, but she was complying to my request. But then I shut her down. And that could be also like on a teacher's point of view, like okay don't force the groups to work together. But sometimes like you have to put certain people together and it's hard sometimes. You know when you have a classroom full of ones that if you put together they're going to just talk and they're not going to get things done, so like she's one that had to be separated. So if I would have maybe just switched her group she might have done better in it. You never know some days it's like a tit for tat and you don't know.

Interviewer: And it's interesting because after you left the group she also again tried to convince Mathew that her answer was correct, that it's 496 cubes, and she was able to explain why it worked.

Ms. B.: So it was an attitude.

Interviewer: So…

Ms. B.: It was an attitude, clearly. Because like, and knowing the students and everything she's not going to just let Marcos just… she's not.

Interviewer: And she ended up afterwards explaining her solution to the whole class when her group got up to do their presentation.

Ms. B.: And that part I just remember, and that's what made me think. And I spoke with her after and I said, 'I don't understand why you did that because you knew what was going on', but just watching her like this, you would think she's not doing anything. But she was paying attention, she was listening, so it was interesting to me, it was. She was one who stood out to me and I pulled her aside in the end because she got up and she explained her answer, but she sat there and she did absolutely nothing. So that even shows that there's different ways of learning. And she was interesting to me, she was.

Interviewer: And even your questioning didn't get a response until they came up to the… in front of the whole class.

Ms. B.: Uh huh. And that's how I knew, because even with me trying to talk to her and everything, she wasn't really responding to me but then it showed that she did know what was going on.

Interviewer: So if this happened again, would you do something differently?

Ms. B.: I probably would. Seeing her just sitting there and knowing that she can work, I would maybe give her the option of 'okay what group would like to work in?' Okay you go pick your group and just promise me that you're going to work and you're going to try your best over there. So maybe giving her the option she might feel like 'oh I can pick my group'. But then sometimes doing that, that stirs up the rest of the class and it's like 'well I want to move' and 'I want to do this', so you know, I would probably give her an option.

Interviewer: Okay, thank you.

Ms. B.: No problem.

Interviewer: Is there anything else that you want to add?

Ms. B.: No.

Interviewer: Okay, great. Thanks.

Ms. S. Interview Transcript

Interviewer: Thank you for taking the time to reflect on your classroom experiences. I'm interested in finding out about how teachers respond when students have difficulty while they work on mathematics problems in class and about why teachers respond in those ways. First, I'll ask you some questions about your thoughts regarding ways in which you respond when students have difficulty while they work on math problems. After that, I'll show you video clips of your classes which involve instances of student difficulty and I'll ask you to reflect on the ways in which you responded.

Interviewer: Can you describe how you usually respond when students have difficulty while they work on math problems in class?

Ms. S.: I think my general approach is to ask them to speak with each other first, so I try to prompt student to student dialogue. And the way I have my students seated is actually have them grouped homogeneously in their regular seats. And my thinking behind that is that in different cases different students are going to have different strengths and weaknesses, but overall I don't want a student who… because whether or not we tell students where they are, kind of like tracking wise or whatever, they tend to get offended who they think the smart kid is in the class, who they think the not as smart kid is in the class, so I try not to intimidate my students who struggle with math by putting them next to the student who actually gets it really quickly, because otherwise they tend to not try to think it through for themselves and just default to whatever they think the higher kid is thinking. So I ask them to conference with each other first and most of the time while I'm there, I'll sit there listening to the kind of discussion that's going on and I ask them sort of kind of like guiding questions to try to get them to the right answer. And depending on which group of students I'm working with, the level of kind of guidance that I'm giving in those sessions will be different. And for my children who are, who I think are the highest or who have demonstrated to be the strongest in that particular area, a lot of times I have different books like math dictionaries or whatever and I'll just give them the resource, ask them to look back through their notes like point them to a particular spot and give a resource and then come back and ask them what they found, what progress they've made.

Interviewer: And for the weaker students?

Ms. S.: For the weaker students I'll definitely ask kind of more directed questions. So at that point I move to try and chunk the problem for them. A lot of times the difficulties that students encounter in math is… we do Connected Math which is a pretty open curriculum and they get overwhelmed by the problem so instead of doing a bunch of computational problems, initially what Connected Math has to do is it gives them some sort of like real world situations that apply to mathematics. And for the low, for the struggling students what tends to happen is either they get kind of locked in context of the problem and so I may help them kind of decipher what's important, what's relevant. We may talk through what that is. Or they see A, B, C, D and it's already overwhelming for them and so then we talk about ways like chunking the problem, simplifying the problem.

Interviewer: Are there things you try not to do when you respond to student difficulty?

Ms. S.: Yes. I try and I… this only after doing the whole Dr Schorr, Dr. Warner thing, I try very hard not to straight out say yes or no, so either validate or negate their response or give them the answer or say 'look this is how you do it.' And my thinking behind that is that I really… when you see, you see it all the time particularly since I deal with sixth grade now. When they come in and they're kind of used to always having their answers either validated or negated, when they get to a test and you tell them check their work, they actually haven't had any practice with kind of assessing their own work so when you tell them to check their work it's almost like just telling them to do nothing. They have no idea how to check their work. Instead what… all they know how to do then at that point for a lot of them is kind of go to the teacher and wait for you to say yes or no. So one of the skills that I hope to develop in them by not saying either yes this is correct or no this isn't correct just getting them to prove to me like 'yeah Mrs. S., this is right'. Or actually 'I don't know why it's wrong but I know something is wrong here' so that when they're taking their regular tests and they're able to that self check for themselves so they can catch their own mistakes.

Interviewer: Do you respond in different ways in different situations?

Ms. S.: Definitely. If I… it depends on the level of frustration that I see the student kind of working with. This is the first year that I'm teaching inclusion, and so it's been a really interesting year for me learning-wise because… with the inclusion students it really is a different experience particularly for me because I'm not… I mean I've always had special needs students in my classroom but I've never had I guess this many. So now in one class I have eleven special needs students and fourteen kind of regular education students and the difference is very big. And a lot of it is in terms of the students' expectation of the teacher. So a lot of them are coming from self-contained settings where I guess there was a lot more support or what I may consider hand holding and so their level of frustration tends to peak much faster, much quicker and they tend to feel as though… they get emotional about like the struggle and shut down quicker. So I've been trying to find a balance when working with my special needs students of keeping them engaged, keeping them encouraged but at the same token helping them understand that I'm not there to give them the answer. That I have the same expectations for them as I have for my other students. So they do have to work through the problem and like they do have to problem solve and figure it out. So we're still… I mean it's January and it's my first year teaching it and their first, for many of them their first year being in a inclusion class and we're still trying to kind of work that situation out, but it's challenging.

Interviewer: So you say you would give them encouragement.

Ms. S.: Yes, so I may…

Interviewer: Would you ask different questions or just that you're giving encouragement?

Ms. S.: So a lot of times when they're calling me over they're at a point where they're feeling frustration already. So a lot of times what I try to do is like take the emotion out of it, so at that point my first goal kind of when I get to them is to kind of calm them down and be like okay, like alright, not a big deal. So there's two different kinds of ways that they tend to respond. Either they tend to respond by being really anxious and kind of getting upset. Or they tend to be what seems be very apathetic, so it seems that they're not doing anything until I come to them, which is another thing that… a habit that I'm trying to break them out of. Because all together students work you know until I get there, you know, work without me you need to be able to do the same. And so kind of when I go over to them I kind of try to like get the emotions under control; alright let's take a minute, let's look at the problem and then I go into… either some of them are having trouble with just reading a problem so there's literacy issues there because they can't read the words and I'll read it out loud for them. And then we'll kind of work through, ok. I'm still asking questions like 'okay you tell me, what information do you think is important here?'. And then whether the information is important or not, I talk about okay why do ..like let's talk about why you think it's important. And with them I'll honestly be more direct, more direct in my response. And that tends to be because I think what… with them when I'm more vague it's more frustration and confusion for them. So with them they actually need a little bit more validation, so if they're saying that like 'oh I think this is important because of this' I'll be like okay…inaudible…let's focus on these two items. And I'm going to tell you like these are actually the more important items and here's a strategy for figuring out like why they're important. So how about we circle these so that we can remember these are the two places we need to focus. Now what do you think you should do next? So like I kind of like chunk it like that but much more direct for them. And that is only because my experience has been that their level of frustration kind of levels up much faster and they tend to get more confused. So when I leave that opportunity for kind of like explanation they tend to get… not be able to kind of organize that in their minds. So we used a lot of… there's a resource teacher in the room also and like when I'm taking notes on the board, for them I'll use guided notes, and I can give you like an example of that, but basically a lot of my stuff I do off of powerpoint and I just print the powerpoint out for them and I have certain notes kind of like typed on the side. And they do like fill-in-the-blank type notes so that they can pay much more attention to kind of what's going on as opposed to focusing. Because in the beginning of the year we noticed that they would hide behind the writing like instead of like participating in class discussions.

Interviewer: So you mentioned that student ability might have an impact on the way that you respond. Are there any other factors that might influence how you respond to student difficulty in math?

Ms. S.: Student ability. I mean… maybe…the timing. So if we're working, if we've been working on a concept for a while… if we just starting working on a concept, the level of frustration that students get to is much more open. Because I know we're going to keep hitting it. If I know that they're about to be assessed on it, a unit assessment which is something that affects their grades, that affects their ability to get into a good high school and everything like that, I'm much more direct with my approach. Because I feel like there is less room for … I would be doing them a disservice by not clarifying like kind of all ambiguities out there. So at that point I'm much more like 'okay well here's a really efficient way to do it', like let's look at the efficiencies and inefficiencies and let's hone in on some strategies that we want to consistently use.

Interviewer: Would you say that the students' personality or the type of difficulty or the setting of the difficulty would influence how you respond?

Ms. S.: The personality in that some of my students get emotional and like shut down completely after a certain level of frustration? Yes. So in that sense personality does. Absolutely. But to be perfectly honest my experience has been that the students' academic ability and those who tend to be frustrated quickly go hand in hand. The students who feel much more… who have been much more academically successful tend to be much more comfortable with kind of like hanging out there not really knowing, they're much more motivated about trying to figure it out. And I think that's just because they've experienced success before. So they know, they have some sense of conviction in that if I work hard enough, like, I will figure this out. Whereas some of those other students have been experiencing failure for quite some time, or what they perceive as failure, and they haven't. The setting…I actually think after school I'll let students struggle a lot more because it's a smaller setting, it's two or three kids, we have all the time in the world. In the hundred minute block, by the end of the block I know that they need to get the concepts so I'm much more likely for us to delay. A summary and a wrap-up. But then again that's also depending on where we are in the unit. If it's the beginning of the unit we may let those ambiguities kind of hang out there and keep exploring, keep exploring. By the end of the unit we're like okay so like let's review let's go through what strategies work what strategies don't and try fishing, why do we want use them and kind of… we do that in our mathematical reflection section at the end of every unit.

Interviewer: Do you think the small group versus whole class setting of where the difficulty happened would influence how you respond?

Ms. S.: Absolutely. In the small group setting I only have to focus on you so I have much more time to spend here. I'm much more likely to do less leading questions. In the large group setting I'm bouncing from group to group so the level of direction I'd give, because I know you need to get to kind of a point of resolution by the end of a hundred minutes, it's much more kind of like leading and direct. And once again that's all about student ability too. So like for my lower groups I make sure I visit them much more frequently. For the high groups I know that if I give them the resources they'll make some progress and I can go… I can see my struggling groups two or three times before I head back over to the… that challenging group. And a lot of times what we do is…I don't know but you see we have up there is…I'll have the groups at different check points. I normally have a, I have a timer up on the smartboard and so after like ten minutes of working on a problem the kids will put up like where they've gotten so far. So even if one group is like stuck on something they can see, they can look around the room and see what other groups have and like get some idea about strategies that they can pursue afterwards. So a lot of times kids will redirect.

Interviewer: You mean… where do students…

Ms. S.: Oh, so like they have those big chart papers and so they'll just put like, Duke may post over there, Yale will post over there, Princeton's stuff is there. George… the different colleges and I'll just be like alright show your strategies. And they'll show what they have so far. And a lot of times the kids go through three or four before they put their final ones up. So a lot of times, you'll see Duke put up one thing the first time and it may be completely off but then they'll look at what Georgetown put up and the next round when I say fifteen minutes later when I say put it up their strategy looks a lot similar to Georgetown's but it has kind of like their own kick to it. So it may be right, it may be wrong but now we can look at the two at the end and kind of figure out which one works, or if both work.

Interviewer: Okay, so now I'll show you a few clips where students in your class encountered difficulty while they worked on the building blocks dilemma. This was the problem.

Ms. S.: I remember it!

Interviewer: After each clip I'll ask you why you think you chose to respond in that way. I realize these class sessions took over… took place over a year ago, so just answer as best as you can.

[Playing video clip 1- Difficulty 22: Leo, Ta'keisha, Ordena, Day 1, 11:53:39-11:58:00]

Ms. S.: That's so funny because the girl who just came to the door, that was her in the video.

Interviewer: That was…

Ms. S.: The little one, the one who was sitting on the side with the braids.

Interviewer: Ordena?

Ms. S.: Uh huh. That was Ordena.

Interviewer: Oh really, I didn't recognize her.

Ms. S.: Looks completely different, right?

Interviewer: Yeah. Oh, that's cute.

Ms. S.: That was her.

Interviewer: So what do you think the students' difficulty was?

Ms. S.: Interpreting the problem. So I think they were, they were having trouble as a group, deciding kind of what the problem, what were the prerequisites in the problem. what was the given information. So they couldn't continue the pattern until they had come to a consensus on kind of what was, what were the initial, what was the information given. And it's funny because that group would have been one of the groups that I actually would classify as a struggling group and one of the areas in which they struggle, they tend to struggle with…is that in itself. They're all in Read 180 so they struggle with reading comprehension and they have less confidence in their ability, they're… they're more likely to raise their hand to read directions and they'll raise their hand and say, Ms. Samuels what does this mean, even if they have some sense of what it means because they kind of want that validation, they don't trust their own intuition.

Interviewer: Why did you respond in that way?

Ms. S.: I think the reason I responded in that way with them is because once again, they're a group who I would have classified as a struggling group and they're also, the funny thing with them is, because they tend to struggle a little bit more they're less likely to talk to each other. So instead there would be kind of, just kind of like silently working in frustration. They go through this like silent frustration and because there was multiple groups working, they, I think they would, and they had manipulatives out, I think they would be more likely to kind of just move into a phase of just kind of like moving around the manipulatives to look like they were doing something rather than actively like working to solve the problem. So with them kind of, right, it was an example of one of those times when I try to go over and kind of chunk the problem. 'Okay so let's go back and let's read through it together, and let's step by step decide like okay if we agree on what this means. Okay now let's decide if we agree on what this means. Okay now let's decide if we agree on what this means.' And Ordena, which is the same girl who was there, was always very quiet. And you can see even in the video even as they're working through the blocks she still doesn't necessarily… she's not as vocal as the other two are. And her skill level is about on the same level but she just for whatever reason, maybe her accent or whatever, she you know is self conscious of, she doesn't speak as much. And also she was hesitant to even work with the manipulatives, work with blocks. In my

experience when I work with her one on one, she actually tends to get it more when she physically does it, but as she watched the other two do it, she still even when I told her to show me the first block she did it and she didn't do the other steps even though the other two were actively kind of doing it. So I don't know if she was focusing on thinking the problem through or what the case may be. And I didn't want to…I think I didn't want to continue to kind of pressure her into using the manipulatives if she didn't want to use it. Kind of one of the things we were trying to learn there is let the kids pick whatever way they decide to use it then okay. There were a couple of times when she kind of says, "Oh, Oh!" like it's making more sense to her. And so what I was trying to do there is kind of the deduction that they need to be able do on their own like walk them through out loud so like step by step. 'Okay, so if this is true then what do we do next?' And then what's true, and then what's true. That was my attempt there.
Interviewer: You touched on this already but do you think the students' mathematical ability influenced the way you responded?
Ms. S.: It absolutely did. Absolutely did. I think so. I don't know if that's a good or bad thing, but it totally did because I know… I also know, the funny thing is so the other student, the boy in there, Leo, I know that he's one who can easily get kind of like way up in left field. So like one…he made… a lot of times he can talk himself out of the answer or out of a reasonable solution and his kind of parameters on what's relevant…the filter isn't quite there. So he makes… it would have turned into a discussion on … he has the proper pattern going and if we didn't stop it chunk by chunk and say okay we agree on this, okay we're going to continue down this path… As you can see at one point he has the right pattern and then I asked him about the five block high tower and he built up the tower in the middle but didn't add on to the end, and I had to ask him, okay are you continuing the pattern? And he looked like, oh okay and then he goes back. But left kind of like unharnessed he would have continued, I think, to just build up the middle. Because he can kind of work himself out of the problem a lot of times that way.
Interviewer: I'll show you the next clip now.
[Playing video clip 2- Difficulty 23: Leo, Ta'keisha, Ordena, Day 2, 9:59:58- 10:03:24]
Ms. S.: Okay, same group.
Interviewer: Right. What do you think the students' difficulty was there?
Ms. S.: So, there I think that they were, we were moving to generalization and they were approaching it in two different ways. So they were trying to reconcile two different strategies which both seemed logical and reasonable to them. And I think that they were looking for a way to either negate one or prove the other. And what they lacked at that point was either a…inaudible…or a test that was going to give them the ability to that. And so kind of when I went over there, that's what I was trying to help them focus on. Okay, so we have these two strategies and they seem reasonable enough, right? If there's a five block tower, there's a ten block tower, ten is twice as much as five so why not double it. Okay let's see if we can use the strategy of like using a smaller number and see if that pattern holds true. So I was trying to give them the test that they needed to see for themselves whether or not Ordena's strategy was going to hold up. And I don't think that they fully got clarity on it but I think that the reason that I didn't go any further with them was one, we were low on time, and two, they were going to present anyway. So I, what I wanted to do was at least plant the seed, like give them that scenario, have them see that the pattern didn't hold up. So now when they came up to the class to discuss it, one of

the challenging areas or one of the things they were going to say, ‘well we had trouble with this and we’re not sure what happened here. We think it should work but it isn’t working, well we tried this scenario’. And then the class could talk them through the rest. Hopefully.

Interviewer: Do you think the students’ mathematics ability influenced the way you responded?

Ms. S.: Actually in that situation I think I probably would have responded pretty similarly to all of my kids.

Interviewer: Okay, so we have the last clip. It’s also with this group.

Ms. S.: Okay.

[Playing video clip 3- Difficulty 29: Leo, Day 3, 9:51:35- 9:58:00]

Interviewer: What do you think the students’ difficulty was?

Ms. S.: See in that scenario I think their difficulty was… I can’t remember what it’s called but there’s like a name for it, when students come up with a…

Interviewer: It looks like the camera just stopped. Sorry.

Ms. S.: There’s like I think a name for it. When they… when students come up with an idea and I think that they know that their idea isn’t still holding up but because they worked so hard on it and like they’ve invested so much of themselves in it, that it becomes this emotional thing now, and now they’re clinging to this idea. So I think on a very like basic level I think that Leo and Shaheim and the other students were totally starting to see that their idea wasn’t going to hold up and that the validity was really in Ta’keisha and Tyshonna and the girls’ idea. So they had actually done it right and I think that in, within them they knew that there was something not right about it, that it wasn’t going to work. However it became this emotional thing of like, I worked on it, it’s mine, I’m going to support it even if it doesn’t hold up. I’ve got to find a way to make mine seem good and theirs as well. And so you know I guess one of the challenges for me as a teacher at that point would be to learn how to help them see that. It’s okay to change an idea, like you’re not abandoning your idea or your process completely; you’re like just editing it, right. An improved version. Yes, so I think that’s kind of what’s going on there. And I think looking at it now, actually, and reflecting on it, ideally I would have loved to have been able to facilitate that a little better in having myself talk less and have the kind of conversation much more between the students. And that’s I think a difficult thing for me to do. I’m still working on that, trying to get the kids to talk to each other. They’ll talk to me but they’re resistant to speaking to each other, about math at least.

Interviewer: Why do you think it’s better for them to speak to each other than to you?

Ms. S.: Because then they’re going through that process of self-assessing and like doing logic tests, checking if something is reasonable, like that’s all coming from them. When I’m asking the questions and saying is this reasonable, or what do you think about this, or guiding, I’m no… if I could step out and have the other students like peer to peer guide each other, then my students themselves would be kind of… first of all it’s less leading because, you know, there’s no kind of authority or like sense of right or wrong …it’s up to the other kids. So the exploration I think is just more genuine. It’s more like really we don’t know who’s right or who’s wrong and this a sincere question. And it gives the kids practice at like articulating their thoughts because I think that one of the things our children struggle with the most is being able to articulate how they feel, what they’re thinking, their thought process. And so the being able to listen to what someone else is saying or what one of their classmates is saying and process that in their own heads and then see where it aligns, or misaligns with their strategy and then kind of being able to question them through that, would be a great skill for our… I think all of my students to have. And totally help them when they need tests or future classroom projects that they would ever have in the future, and I think it's a life skill that they need to learn. I'm still learning on how to actually as a teacher like appropriately like foster that in them, because they're still much more willing to talk to me, even in small groups. They'll talk to each other somewhat, but the conversation doesn't really start until I get there. And even when I get there, and then now once I'm there instead of talking to each other they all talk to me. So it's like a three way conversation to me, even when I try to redirect to each other.

Interviewer: Do you think that approach of having the students talk to each other works better emotionally?

Ms. S.: Yes. Kind of…sometimes. So I think it depends. I think it depends on the environment you build in your classroom. I think once you've built an environment of where we like respect learning and they like totally know like this is, it's okay to be wrong and the idea is okay, like we're just exploring right now and it's all good, then you get that. I think you run some dangerous ground if you have like heterogeneous grouping where you have the high-low kid, high-low kid going on, because even when they're trying to be nice it can come off as patronizing. There's like a whole…kids create this whole other dynamic in middle school that can get kind of funky, so most of the time I think it is cool, and I think in this scenario the best situation would have been to speak to each other, particularly since I had them in homogeneous groups. When I am going to facilitate something like that I try to keep them grouped relatively on par with each other so nobody's going to look at the other person and say, 'I'm so stupid' because they're all kind of…

Interviewer: But they were… this was in front of the whole class.

Ms. S.: Yes. …(inaudible)… So I mean in this scenario… but the funny thing about this scenario is, with this building block dilemma, is ability almost, I mean it did come into play but they were all new to problems like this. So they all felt just as kind of like intimidated so it kind of leveled the playing field. So this type of problem was great because students who normally at the traditional style kind of feel really shy and like almost know ahead of time that they're going to be struggling, kind of felt like they had just as much footing as the students who are normally really successful, because they were just as lost in the beginning on this problem as the other kids. So a lot of the tone for this kind of… the presentation that happened was that after the cameras were off when the kids left the room after that first day, they were like 'Hey, did you know what you were doing?', and they were like, 'No, I didn't have a clue.' And nobody in the class knew, so even the high kid was like, 'I don't know what's going on'. And so I think they felt much more comfortable with getting up there and like putting themselves out there. It wasn't that 'Oh, that group got it in five minutes and we're just sitting around waiting'. None of the groups had an idea.

Interviewer: Do you think the students' mathematics ability influence the way you responded in that situation?

Ms. S.: No, in that situation, no. I was really proud of that group for coming along as far as they did and for essentially arriving at that answer pretty much on their own, I felt.

And… no, I think if any other groups were up there I would have responded the same way. Like I said, I think at that point the two groups that were still… the boy… because I think Leo got kind of quiet because I think he realized that, ‘Oh, something wasn’t right here.’ And then the one group of boys that were left that were still like, ‘No, no, no, it must be a five block tower’, I think that was more of an emotional response than anything else.

Interviewer: Okay, thank you.

Ms. S.: No problem.

Appendix E: Protocol for Verification of Coding

Instructions for Coding Verification:

Verification of Types of Teacher Responses:

1) For each of the 29 identified difficulties, select the teacher response type from *Table 4.2: Teacher Response Coding Scheme* that best describes the initial response offered by the teacher. Record the initial response types.
2) Indicate the category that best describes any follow- up response(s) the teacher may have offered for each of the 29 student difficulties.

Verification of Codes Pertaining to Teachers' Reasoning for their Responses:

During stimulated recall interviews, Ms. A., Ms. B., and Ms. S. were asked to reflect on 3 particular responses they offered to students. For each of the three responses for Ms. A., Ms. B., and Ms. S.:

1) Read the transcript (Appendix D) of the teacher's explanation for the particular response she offered.
2) Choose the category/categories from *Table 4.3: Teachers' Reasoning Coding Chart* that best describe(s) the teacher's reasoning for providing the response. More than one category may be applied to each instance of teacher's reasoning for providing a response.

Verification of Codes Pertaining to Students' Cognitive and Affective Engagement:

For Difficulties 1-4 (Ms. A.'s class), 5-8 (Ms. B.'s class), and 19-22 (Ms. S.'s class):

1) For each instance of difficulty, please read the section of *Appendix A* entitled 'Description of Behavior Related to Student's Engagement Following (Teacher's) Response'
2) Based on the descriptions of students' behavior, choose and record the classification from *Table 4.5: Cognitive Engagement Coding Chart,* that best describes the cognitive activity of each student involved in the difficulty.
3) Based on the descriptions of students' behavior, choose and record the classification from *Table 4.6: Affective Engagement Coding Chart,* that best describes the emotions of each student involved in the difficulty.
4) For each difficulty, after you have coded for students' cognitive and affective engagement, read the *Inferences of Students' Emotions* section of Appendix A for the relevant difficulty. Please record whether you agree with the emotions inferred from the students' behaviors. If you disagree with any of the emotions listed or feel that any other emotions should be added, please note so.