INVESTIGATING A MODEL USING VIDEO STORIES FOR PROFESSIONAL DEVELOPMENT FOR ALGEBRA TEACHERS OF LOW SES MINORITY STUDENTS

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ABSTRACT

This study presents results of the design and implementation of The Teacher's Algebra Workshop (TAW), a professional development intervention. Video stories of urban students' early algebra learning were created by this researcher for this intervention. The video stories feature predominantly black and minority students engaged in mathematical reasoning, making and revising conjectures, and challenging one another. Four stories illustrate 12-year-old boys who participated in an NSF funded Informal Math Learning after school program in an urban school district. The video stories show the boys engaged in persistent problem-solving of "Guess my Rule" problems. as an introduction to linear functions.

The TAW video stories were assembled using the RUAnalytic tool and video data stored in the Video Mosaic Collaborative (VMC), a research library from Rutgers University Libraries.

The intervention was designed to engage teachers (TAW participants) by studying video narratives of student learning from a social constructivist perspective. Five teachers participated in the four-day TAW workshops in August 2016. Each day, guided by specific questions, teachers discussed the mathematics, the pedagogy, and students' engagement in the stories. These discussions were audio-taped, transcribed and analyzed to identify what the teachers noticed in these key areas. The research question that guided the study include(s): Did urban teachers notice the potential benefit of pedagogy that challenges and engages students in their early algebra learning?

Analysis of data indicated rich discussions among the teacher participants. They (the teachers) noticed key aspects of social constructivist pedagogy including: teacher patience, engaging lessons, guiding questions and the recognition of the students' enthusiasm as they persevered in problem-solving. Results suggest (the value of) using the video narratives for teacher professional development as well as conducting a scaled-up study using the artifacts created for this intervention and expanding the content to other mathematical topics.

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CHAPTER 1: INTRODUCTION AND RESEARCH QUESTIONS

Professional Development for Math Teachers

Research suggests that we may be using high-stakes test results to inappropriately label the mathematics achievement of minority and low-income students solely in terms of how they compare to white and upper income students on these tests. Looking primarily at test scores is also allowing a very static view of students to be the dominant picture (race, income, test score) when it may be more appropriate to consider how students perform in many different classes and settings over time (Guiterrez, 2008). Achievement gaps in mathematics performance as exhibited in the National Assessment of Educational Progress (NAEP) in 2015 (NAEP, 2015) generate much news, but Gutierrez points out that "gap gazing" undermines the potential of affected students and "overvalues" the results of a single high-stakes test. Consider that the No Child Left Behind (NCLB) graduation proficiency tests, along with the NAEP and the College Board tests, multiply this phenomenon: Achievement gaps on high-stakes tests become the focus of discussions about mathematics education for minority students. Guiterrez makes this point: Looking at minority and low-income students only with respect to how they compare to others shifts our attention away from simply considering their needs. Guiterrez concludes that we need a revised research agenda that focuses less on narrow statistical analyses of achievement gaps from mathematics tests and more on learning how to better teach low socioeconomic-status (SES) and minority students[1] (Guiterrez, 2008).

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[1] For the American Psychological Association definition and standards for measuring socioeconomic status, see: https://www.apa.org/pi/ses/resources/class/measuring-status.

Research on the education of low-SES, minority students reveals a "gap in cognitive demand" in classes for these students (Anyon, 1980). The College Board (2006-2008) reports that black students generally have lower participation in honors, high-level, and AP math courses than do white or Asian students (Deng & Kobrin, 2007). We can also consider the research of Claude Steele (1997) on "stereotype threat," which tells us that black students, particularly male black students, are vulnerable to performing less well on assessments when they are simply reminded of their status (as black males) because this reminder invokes the social stereotype that labels them "less good" than others at mathematics (Steele, 1997). In light of this research on stereotyped threat, gap gazing may, in fact, help to perpetuate gaps between black and white students on high-stakes mathematics tests. This research validates Guiterrez's (2008) point that it is misleading at best to measure the achievement of minority students by their test results alone.

It is also true that "the gap" is not entirely explained by SES (Lubienski, 2002). In a study of 1996 and 2000 NAEP results (Lubienski, 2002), found that low- and high-SES, black students were much more likely than white students to believe that "mathematics is mostly memorization." Table 1 summarizes some of Lubienski's results.

	% students who agree with the statement: "There is only one correct way to solve a math problem."		% students who agree with the statement: "Learning mathematics is mostly memorizing facts."	
1996	Grade 8	Grade 12	Grade 8	Grade 12
Black Students	12	7	58	55
White Students	6	6	36	29
2000				
Black Students	13	5	55	55
White Students	7	5	31	31

Table 1: Student Mathematical Beliefs by Race (Lubienski, 2002)

Why do more black students believe that mathematics is primarily memorization? This may be due to the instruction they receive (Lubienski, 2002). Research also reveals that low-SES students may respond enthusiastically to a learning environment that encourages active participation and mathematical exploration; students may articulate the positive difference between their "traditional classrooms" and an "active-learning" classroom. For example, in Boaler's study (2016), a sixth grader, Alonzo, said:

> In other classes, , , it used to be so boring doing my work. It used to be so quiet and everything and I used to get frustrated and stuff. [But] right here, we get to do group work, and we get to talk and stuff, and that helps me not be so boring [bored]. (p. 50)

Boaler's (2016) study focused on a five-week summer program for "disaffected" sixth and seventh graders. The summer program participants were 39% Hispanic, 34% White, 11% African American, 10% Asian, 5% Filipino, and 1% Native American. The students had a wide variety of achievement levels in the class they took before coming to this program: 40% had a D or an F, 20% had a C, 40% had an A or a B. The students were diverse ethnically, economically, and also with respect to their grades in math (Boaler, 2016).

Boaler (2016) wrote that many of them "transformed their relationship with math" during the summer program and that they moved from "initial disaffection and low achievement to excitement and high achievement" (p. 14). Boaler made a direct connection between equity for minority students in mathematics and classrooms that have nontraditional instruction, "Students are encouraged to ask questions and share ideas, and to feel safe in doing so, and teachers express a commitment to students' academic and social development" (Boaler, 2016, p. 10).

In view of the evidence that points to mathematics instruction as key to solving the problems facing minority and low-SES students, this study focuses on professional development for mathematics teachers. Focusing on instruction to promote equity for black students and low-SES students raises large questions about mathematics teaching: (1) How do *all* students acquire mathematical knowledge? (2) How do we determine what students know? (3) How should math teachers guide the learning of mathematics? and (4) Do black students (and others) who operate under a burden of stereotyped threat have particular needs that a teacher can address?

This study focuses on a social constructivist theory of teaching mathematics to begin to answer these questions. The vehicle studied for professional development of teachers is in the form of video stories that feature black and minority children engaged in mathematical inquiry. These are the same children who, as research tells us, suffer from stereotype threat and achievement “gap-gazing”.

CHAPTER 2: THEORETICAL FRAMEWORK AND LITERATURE REVIEW

Theoretical Framework: Constructivism

The theoretical framework for this research study is based upon a social constructivist theory of learning and teaching mathematics.

Constructivism has been applied to epistemological theories, theories of human learning and assessment, and theories of teaching. A constructivist view of learning mathematics is that learners construct their knowledge through experience, actions (e.g., problem-solving) and reflection (Maher & Alston, 1990).

According to Maher (1998), a constructivist math teacher creates a learning environment in which: (1) students engage in solving challenging mathematics problems; (2) students create models to help themselves explore and evaluate solutions to the task problems; (3) students test the validity of their models, challenging themselves and others to make sure that their solutions are valid; and (4) students share their solutions and their justifications with others. A theory of constructivist learning of mathematics posits that these are the experiences that students need in order to construct the mathematical knowledge that will serve as a strong foundation for future mathematics learning (Noddings, 1990).

Note that this is not radical constructivism, which posits that each of us constructs our own mathematical knowledge, unique and separate and essentially unknowable from one human being to another. In radical constructivism, we never know if our knowledge is the same as any other individual's knowledge, and we never actually know if our knowledge is true (Goldin, 1990).

In this context, if students are expected to construct their own knowledge, it is reasonable to ask if students need a math teacher at all. Could they work with a textbook, and possibly a computer-based learning system, and learn the math independently? Mathematical knowledge may always be constructed from experience, but is all constructed mathematical "knowledge" guaranteed to be useful, correct, or even partially correct? Research reveals that students lacking sufficient guidance may build mathematical ideas that lead to incorrect mathematical conclusions (Erlwanger, 1973). As described in the case of Benny, who studied mathematics using Individually Prescribed Instruction (IPI) Mathematics software with little guidance, students may construct mathematical ideas that are deeply flawed and actually impede further learning.

Without useful guidance from a math teacher, Benny used the IPI system and created a large set of conflicting and wrong heuristics for arithmetic. These heuristics exploded in complexity over time as Benny sought to support each new observation. This complex set of heuristics was Benny's knowledge base, and this presented a strong barrier to introducing corrections. This knowledge base contained many incorrect rules, and this made it hard for Benny to relearn arithmetic later. Benny used a published software program to learn arithmetic in the presence of a teacher. However, without the guidance of a mathematics teacher, Benny constructed a knowledge base of false conjectures that presented a cognitive barrier to learning arithmetic and moving forward.

Noddings (1990, p. 14) suggested that constructivists should talk about "strong and weak acts of construction." "Strong acts of construction," according to Noddings, would be recognized by mathematicians as mathematically valid. "Weak acts of construction," on the other hand, would be limited as to their mathematical use (but not entirely incorrect). The weakest acts of construction would be invalid conclusions; for example, Benny's rule for setting $\frac{3}{2} = .5$ also works for setting $\frac{2}{3} = .5$. According to Noddings, the main function of the [constructivist] teacher is to “establish a mathematical environment” in which the student (unlike Benny) is always encouraged to look for errors and to correct them (Noddings, 1990).

Is social constructivism the key to learning mathematics?

Constructivism models learning on how individuals construct their own knowledge. Social constructivism involves knowledge construction when social interactions occur, for example, when students collaborate on doing mathematics. Social constructivism posits that individual constructions and collaborative constructions (resulting from social interactions) are both fundamental elements of learning mathematics, and, as such, are of equal importance (Cobb & Bauersfeld, 1995, p. 25).

We may think of individuals constructing mathematical knowledge only through their own actions (engagement with a problem) and thoughts. However, the social interactions that occur when students collaborate (e.g., mathematical problem-solving discussions among students) constitute cognitive behavior that exposes each student to new ways of seeing a problem. Social interactions occurring during collaboration may inspire construction of mathematical ideas that might not have been introduced without the social group. There is evidence that such mathematical discourse deepens student engagement and introduces more ideas, reasoning, and learning (Maher, Mueller, & Yankelewitz, 2012). Note that thinking about a math problem (whether it occurs as an unarticulated thought in the mind of a student, or if it is articulated verbally when the student shares an idea with another student) is always an example of cognitive behavior that makes it possible for the learner to construct ideas in mathematics (Sfard, 1998).

The social interactions in a classroom (between students or between the teacher and students) generate questions, challenges, reflections, and learning. As such, these social interactions fuel the constructive learning process (Yackel, & Cobb, 1996). Students collaborating in groups inspire and challenge one another with the goal of justifying their solution to their mutual satisfaction (Maher et al., 2012). It can be noted that students can participate in groups at many levels; some may be very involved in the group mathematical dialog, and some may speak infrequently. It should not be assumed that students who are quiet are not benefitting from the mathematical dialog in the group; this is an area for further research. However, for the purposes of this study, justifications of mathematical solutions must be articulated by a student and offered for consideration to another person (student or teacher).

If the teacher creates a classroom that encourages students to collaborate and own the process of solving math problems and constructing solutions they can justify, then students may construct mental models of mathematical ideas, problems, and solutions that will grow over time and become more and more useful. How do we teach teachers to create social constructivist classrooms? The proposed research study fits under this question: Will engaging teachers in the study of video narratives of algebra learning help them adopt attributes of the constructivist teacher and begin to create a social constructivist classroom?

What does the social constructivist teacher know and do?

If students construct mathematical knowledge by actively engaging with a problem or question and working collaboratively with others (thereby offering an opportunity to connect new ideas to already existing knowledge, triggered by their exploration of the new problem) then we have to ask: What should teachers know and do to effectively guide students to construct strong mathematical concepts? What is the role of the teacher in a social constructivist classroom? In her 1998 paper, Maher asked if “constructivist teaching” is an oxymoron.

What are the attributes of the constructivist teacher?

The pedagogy of the social constructivist teacher matches the needs of the social constructivist learners s/he guides in the classroom. Decades of research on the nature of student mathematical thinking and learning, from both Davis and Maher, also suggest the attributes of the social constructivist teacher.

In her doctoral dissertation, studying Davis' pedagogy as he worked with sixth graders who were challenged to solve a famous problem called the "Tower of Hanoi,"[2] Mayansky (2007) examined the application of Davis' official pedagogy to his teaching. Mayansky first examined Davis's "philosophy" from his writings, which include discovery teaching, discovery learning, metaphoric thinking, and the role of (mathematical) representations as well as his emphasis on "making sense of mathematical ideas". Mayansky then created a framework for analyzing Davis's teaching style and classroom practices, and, in assessing the level of consistency between his practice and his philosophy of learning and teaching, found "consistency between what he [Davis] wrote about, what his theories were, and how he practiced them in his own teaching" (Mayansky, 2007, p. 159). Furthermore, among the categories she identified with Davis's practice were:

- A strong focus on how each individual student is thinking,
- Listening to students,
- Identifying meaningful representations, and

[2] The Tower of Hanoi is a famous problem involving recursion. There is an ancient story about the problem that hinges on a claim that one has just enough time to solve the problem and then the world will end. The problem is detailed in many places. It can be found at: https://en.wikipedia.org/wiki/Tower_of_Hanoi

- Helping the student build (a synonym for construct) meaning and discover the mathematical properties (Mayansky, 2007).

These categories fit into the attributes of the constructivist teacher that Maher proposed in 1998. Throughout Mayansky's study of Davis's pedagogy and writing, we find "pre-constructivist" statements that illuminate Davis's emerging constructivist views.

In writing about the longitudinal study launched by Maher in 1984 (in partnership with the Kenilworth, New Jersey, school district), Mayansky (2007) wrote that the goals of Davis and Maher were "to create classrooms . . . in which children would be engaged actively in *building mathematical* models, and in which curriculum would be based upon *students' construction* of meaning" (Mayansky, 2007, p. 6). In summarizing Davis's view that learning mathematics requires that student thinking be the central concern, Mayansky wrote, "Davis recommends shifting the responsibility to the learner for discovering or inventing methods for dealing with problems" (Mayansky, 2007, p. 14).

Mayansky (2007) wrote about Davis's view of the weaknesses of traditional math instruction in which "teachers give students abstract definitions." Davis, she said, thought "it important for young children to build on the understanding they already have, not merely applying the understanding that they have. Davis emphasizes the importance of students constructing their own meaning" (Mayansky, 2007, p. 22).

Mayansky's results, after studying Davis's work on the Tower of Hanoi with the sixth graders and contrasting that work with his philosophy, included these statements: "We can observe from Professor Davis's writing and practice the following: . . . cooperative problem-solving, . . . where it is the students' responsibility to invent methods of solution; deliberate creation of mental models, and assimilation paradigms" (Mayansky, 2007, p. 91).

It is not simply that Davis described what students were doing as “building, inventing, or creating,” using synonyms for “construct.” What is more important is that Davis described and demonstrated his pedagogy, showing us how he guided strong constructions of mathematical understanding, using powerful metaphors for learning. Mayansky (2007) described Davis’s “special attention to how metaphors are essential to our own information processing . . . we all have a collection of personal and shared metaphors” (p. 21). An example of a shared metaphor is a “street corner”; we learn and create new metaphors as we learn new ideas. Building on Piaget’s use of the term “assimilation,”[3] Davis’ *assimilation paradigm* is a mental model that we can use to make sense out of a mathematical situation (Mayansky, 2007).

An example of an assimilation paradigm is Davis’s “pebbles in a bag” activity (Davis, 1991): Pebbles in a bag is an activity in which adding or subtracting integers is thought of as adding or removing pebbles from a bag, but the bag starts out with an *unknown number* of pebbles. Consider this an activity to explore operations with integers, particularly negative integers. The idea of performing operations with a “negative number” may be, on its face, confusing to many students. However, given a bag with an unknown number of pebbles inside, we can add six pebbles, then remove 10 pebbles, and we have the result that we now have four fewer ($^{-}4$) than we had before we began. Davis pointed out that the math sentence $6-10={}^{-}4$[4] is abstract and may have little meaning for a student. However, adding six pebbles and then

[3] Piaget used “assimilation” to refer to how an organism can “take in” something without modifying its own structure – like eating food. It is not clear that an assimilation paradigm as described by Davis remains “unchanged” when it is utilized to construct new knowledge.

[4] In a traditional classroom, $^{-}4$, as pictured here, would be written as “negative” 4 or “-4.”

taking out 10 makes the problem clear and gives the student a model to use when adding and subtracting integers (Davis, 1992) in many different situations. It is interesting to realize how powerful this model is. Students might start with a real bag of pebbles, or they might use a visual representation of pebbles in a bag. Once they "play" the pebbles in a bag "game," they share an assimilation paradigm that they can use to discuss problems involving negative numbers and build their understanding. Davis described human understanding using a jigsaw puzzle metaphor:

> One gets the feeling of "understanding" when a new idea can be fitted into a larger framework of previously existing ideas. It is like putting a new piece of a larger puzzle in place. A metaphor that reflects this quite well is the notion that one assembles ideas in one's mind much as one assembles a jigsaw puzzle. (Davis, 1992, p. 228).

Mayansky's research "created a framework for studying how his [Davis's] philosophical view was reflected in his pedagogical practice" (Mayansky, 2007, p. 150-151). The video library of examples of student mathematical learning under the guidance of Robert B. Davis was part of the data analyzed in Mayansky's study and was important to Mayansky's research. Without this repository of research videos, it would have been more difficult to confirm that Davis's philosophy and writings matched his pedagogy.

Davis and Maher collaborated for 10 years, until Davis's death in 1998, on research into how children understand and learn about mathematics. A focus of that collaboration has been captured on the videotapes produced from the longitudinal study that was launched by Maher in 1984. Maher has continued this work into the present day and has produced an additional 10 years of video examples of students who were studied from kindergarten through college and beyond.

One these efforts, the "Informal Mathematics Learning Project" (Informal Mathematics Learning, National Science Foundation Award REC-0309062) was a three- year research project that held an afterschool program in a New Jersey School District, where researchers from the Robert B. Davis Institute of Rutgers University worked with middle-school students (11 to 13 years old) to develop algebra ideas. The selected School District in New Jersey, has been an economically depressed, urban area, whose student population is 98 percent African American and Latino. There were two primary goals of the IML project. One goal involved investigating how middle-school students develop mathematical ideas and reasoning (over time) in an informal, after-school setting. The other goal was to explore how teachers facilitate student learning by focusing on students' ideas and reasoning (Maher, Powell et al. 2006). These afterschool sessions were video-taped and these IML videos are the source of three video stories created for and used in the Teachers' Algebra Workshop (TAW). The children and the teacher/researchers in these videos are as diverse as their home school district.

The videos produced by Maher's work, including the longitudinal study and the IML study, are being made available to all teachers and researchers through the Rutgers Video Mosaic Collaborative (VMC).

The video stories that were created for this study were created using the RUanalytic tool and the rich video library (the VMC) of research results. In these algebra video stories, students can be seen as they construct their own learning. In these video stories, teacher/researchers can be seen as they implement a socially constructivist pedagogy, engaging and empowering students. As such, these video stories provide complex examples to help teachers understand *how* to implement the new pedagogy.

Reflecting on her longitudinal research work, C. A. Maher (1998) pointed out that constructivism is a theory of learning and described attributes of what might be called a constructivist math teacher.

The Teacher:

1. Provides mathematical tasks and experiences that offer accessible and powerful assimilation paradigms. These include models and representations for mathematical ideas that students can visualize, reason with, and use as the basis for discussion.
2. Listens carefully to student mathematical ideas and observes their activities to assess what they may know and what questions they need to answer.
3. Encourages students to create justifications for their ideas and to routinely ask for justifications from others; this means that if students have different viewpoints or "answers," these should be the basis for discussion, exploration, and resolution.
4. Builds a classroom culture that encourages questioning and the exchange of ideas:
    a. Organizes students to work in groups that facilitate social constructivist math discussions.
    b. Provides many opportunities for students to represent mathematical ideas and to discuss these representations.
    c. Revisits key mathematical ideas many times in a course and creates opportunities for students to extend and generalize the math concepts they are constructing.

5. Encourages student-student and student-to-teacher efforts to develop representations and modes of inquiry that may disclose deeper understanding of mathematical ideas (Maher, 1998).

In summary, the theoretical framework for this study is the basis for a professional development model that is guided by a social constructivist approach to learning and in which the teacher takes on characteristics of "constructivist teaching," as outlined above. Two of the video stories that were created/selected from videos in the VMC focused on an early algebra approach developed by Davis when he directed the Madison Project (Davis, 1965) and subsequently used in New Jersey with the students in the Kenilworth district (Mayansky, 2007).

Each of the video stories created for this study documents a story of constructive algebra learning using a set of *guess my rule* problems. The themes of each story include:

- A pedagogy of respect and patience for student thinking.
- *Guess my rule* problems that engaged middle-school students in finding patterns, discovering the nature of linear data and equations, distinguishing linear relationships from proportional relationships, and exploring the properties of linear equations.

These video stories gave the TAW participants the opportunity to consider how the level of student engagement and persistent problem-solving increases when students are encouraged to construct conjectures, find errors, and construct again.

In this study, the TAW participants studied these video stories, discussed the mathematical learning they noticed, the pedagogy they noticed, and the student engagement that they noticed. As such, they constructed their own evidence of constructivist teaching and its impact on student learning. The TAW participants were encouraged to reflect on these episodes in their discussions, and then develop lessons that could be implemented in their classrooms. Finally, in the fall of 2016, the teacher participants met with this researcher (in two small groups) to share the results of their classroom lessons with examples of student work.

The research questions that guided this study are:

(1) What evidence is there that participant teachers noticed social constructivist pedagogical practice in the video stories?
   a. What pedagogical moves did the teachers notice?
   b. What did they describe as changes they planned to make in their own classrooms?
   c. Did participants identify teaching strategies in the video stories that they had done or wanted to use?

(2) What evidence is there that participant teachers noticed student mathematical thinking in the video stories?
   a. What did they notice about the nature and validity of the students' heuristic solutions?
   b. Did they connect the pedagogy they noticed to the students' success or failure to engage in finding a solution?
   c. What did they notice about student engagement in persistent problem solving?
      i. Did they connect persistent problem solving to the pedagogy they observed?
      ii. Did they connect persistent problem solving to mathematics challenges in the video stories?

(3) What evidence is there for possible changes in participant teachers' beliefs about the capabilities of algebra students in the video stories or in their own classrooms?
   a. Note that the majority of the students featured in the video stories are black or minority students; the teacher/researchers featured in the video stories are similarly diverse. This diversity matches the participants in the TAW and their own students.

Literature Review

This study examined how teachers of low-SES algebra students responded to the evidence in the RUanalytics video stories of algebra student learning. In particular, this research investigated how teachers' study of the video stories shaped their discussions of the mathematics, the pedagogy they observed, and the persistent problem solving they noticed. The TAW participants were challenged to discover evidence of constructivist teaching and learning and plan a lesson to engage their own students.

Three areas of literature relevant to this study were examined: (1) professional development on raising cognitive demand in the classroom, (2) the use of video in professional development of mathematics teachers, (3) "stereotype threat" (Steele, 1997) and math education for minority/low-SES students.

Professional development for mathematics teachers is about cognitive demand.

The work of T. Carpenter, E. Fennema, P. Peterson, C. Chiang, and M. Loef (1989) was groundbreaking in researching how knowledge of children's mathematical thinking can change the practice of math teachers: "We hypothesized that the knowledge about differences among [math] problems, children's strategies for solving different problems, and how children's knowledge and skills evolve, would affect directly how and what teachers did in classrooms" (Carpenter et al., 1989, p. 3).

Carpenter et al. studied 20 first-grade teachers who spent four weeks (5 hours/day, 4 days/week, 80 hours in total) in their summer workshop using a workshop model called cognitively guided instruction (CGI). In this workshop, teachers read research on children's mathematical thinking about addition and subtraction and studied videos of children solving problems (These are the primary activities of the CGI workshop.). Their goal was to give teachers an understanding of how children develop the concepts of addition and subtraction and to consider how they might use that new knowledge in the classroom (Carpenter et al., 1989).

Teachers and their students were observed for four week long periods during the school year that spanned the months of November through April. There were pretests and posttests for the students in the classes of teacher participants; these were standardized math achievement tests. When the school year was almost over, teachers were interviewed to collect data on how they believed their students would do on the posttests in terms of the strategies their students would use to solve problems and whether they would get correct answers. In addition, students were also interviewed to further examine their view of the processes they used to solve different problems. Both teachers and students completed questionnaires about their “attitudes and beliefs.” A control group of 20 first-grade teachers was given a different workshop on “non-routine” math problems (not including first-grade addition and subtraction problems) and were also observed for four separate weeks.

Carpenter et al. (1989) collected a comprehensive range of data, including a measure of how well the teacher predictions about student strategies and answers matched the results. The results were powerful in that CGI teachers spent significantly more time on “word problems” and less time on number facts.

Some specific results were: CGI teachers allocated a mean of 54.58% of classroom time to word problems as compared to control group teachers’ spending 36.19% of classroom time on word problems (Carpenter et al., 1989); control group teachers spent 47.20% of classroom time on number fact problems as compared to CGI teachers, who spent only 25.95% of their time on number fact problems (Carpenter et al., 1989).

There were measurable changes in the beliefs of CGI teachers versus control group teachers: CGI teachers increased significantly in their belief that "all components (of math learning) should be taught as interrelated ideas" and control group teachers showed an insignificant decrease in this belief (Carpenter et al., 1989).

The assessment of the students showed that the students of CGI teachers "out-performed students in control group teachers' classes" on "complex addition and subtraction" word problems[5] (Table 8, p. 26) and also showed significant increases in self-reported levels of "confidence" and "understanding" versus no increase in reported levels for the students of the control group teachers.

It is remarkable to consider the impact that the Carpenter et al.'s 1989 study had and yet how recent it really was. Victoria R. Jacobs authored the introductory "Perspective" on this article in the National Council for the Teachers of Mathematics (NCTM) Collection of Classics in Mathematics Education Research. Jacobs points out that it "describes the first major CGI study in which teacher decision making was linked with research-based knowledge of children's mathematical thinking," and that it "began a chain of inquiry" that continues (NCTM Collection of Classics, p. 134). It is startling to realize that specific research linking math teacher decisions to knowledge about children's mathematical thinking is as recent as 30 years ago.

However, there is a distinction, a gap, between learning about student cognition and knowing *how* to use that knowledge to raise cognitive demand in the classroom and strengthen student learning. The work of Boston and Smith (2009) began to address this gap. Boston and

[5] There were some mixed results on the student test results.

Smith report on the components of professional development that help teachers learn to implement tasks that raise cognitive demand. In their 2009 article, the authors presented *a task analysis guide* from (Stein, Smith, Henningsen, & Silver, 2000) that distinguished *high-cognitive demand* tasks from *low-level cognitive demand* tasks (Boston & Smith, 2009, p. 122). High-cognitive demand defies definition to some extent, and in this task analysis guide it was characterized by the idea that students “must engage with conceptual ideas” and that “procedures cannot be followed mindlessly.” “Doing mathematics” is the highest level of cognitive demand in Stein’s Guide (2000), and it referred to “understanding mathematical processes,” “self-regulation of one’s own cognitive processes,” and “involving some level of anxiety for the student due to the unpredictable nature of the solution process” (Boston & Smith, 2009, p, 122).

Boston and Smith (2009) reported on a two-year professional development program they created for middle- and high-school math teachers. They worked with 18 secondary-school math teachers and used observations to evaluate the teachers at the beginning, middle, and end of the project. The observation data captured the level of cognitive demand in the lesson as guided by the teacher; they also documented and coded the level of cognitive demand evident in the student work (for each observed lesson). The results they reported showed statistically significant increases in the cognitive demand in the classroom over the two years of professional development that were implemented for this study.

Boston and Smith (2009) pointed out that raising cognitive demand in a classroom is not simply a matter of giving students (via their teachers) better curricular materials (with problems that rate “high” on the cognitive demand scale) or “telling” teachers what materials to use. They cited an analysis of the 1999 *Trends in International Mathematics and Science Study* (TIMSS)[6] by Heibert and Stigler (2004). This analysis reports that US teachers have been observed eliminating or reducing the cognitive demand even when they are using math problems judged to have "good potential for raising cognitive demand" (Boston & Smith, 2009, p. 120).

Teachers in public school classrooms face many challenges to successful implementation of rich and engaging math problems: Timeframes available in classrooms may be inadequate, behavior in classrooms may be distracting, and administrator expectations may demand a regular pace of curriculum delivery. However, even if all of these challenges are met, *how* do we teach teachers to make good use of a rich and engaging math problem and resist giving students answers that give interfere with the mathematical problem-solving process, possibly reducing the need for thinking time, and thus the need for thinking? How do we teach teachers to guide students without depriving them of the "journey"?

Boston and Smith (2009) addressed this question. They summarized the critical components of their "transformative" professional development for math:

> Professional development studies conducted over the past 20 years have documented that teachers' knowledge, beliefs, and instructional practices can be transformed by challenging teachers' conceptions of mathematics, . . . by introducing teachers to students' ways of thinking and learning , . . . by engaging teachers in analyzing inquiry-based reform-oriented mathematics assisting teachers' implementation of reform-oriented mathematics pedagogy and/or

[6] http://www.timssvideo.com/

> curricula, . . . or by organizing teachers into communities of learners that work together to improve their practice . . . . (p. 126)

The above list seems comprehensive. However, the research proposed here suggests that additional components be added to the model of effective professional development: Challenge and transform teacher perceptions of high-need children with video stories that illustrate, in full detail, the impact of social constructivist guidance on high-need, skills-fragile students.

The Boston and Smith (2009) project, Enhancing Secondary Mathematics Teacher Preparation (ESP), funded by the National Science Foundation (NSF), was designed to mentor middle-/high-school teachers to prepare them for mentoring preservice teachers. The study included 18 mentor teachers, with an average of 8.5 years of experience (range: 3-30); they came from middle and high schools, including both low-income and upper-income schools.

The professional development consisted of three 5/6 day sessions over a two-year period. The focus of the sessions included “ongoing opportunities for teachers to solve mathematical tasks, to assess the cognitive demands of mathematical tasks, and to analyze the implementation of mathematical tasks during instructional episodes” (Boston & Smith, 2009, p. 129). In other words, the teachers who were training to be teacher mentors focused on understanding cognitive demand, how students might meet challenging tasks, and how they would guide the students. These were the drivers for discussion of pedagogical strategies.

Professional development leaders (called ESP facilitators) modeled the instructional strategies that they “intended for project teachers to begin to incorporate into their own classroom” (Boston & Smith, 2009, p. 128). The authors pointed to three main ideas that formed the foundation of ESP:

1. “The importance of building professional development experiences on teachers’ prior knowledge and beliefs.
2. “The assertion that change occurs as new conceptions of mathematics teaching and learning conflict with teachers’ prior knowledge and beliefs, and
3. “The role of social interaction in stimulating and maintaining this type of conflict.” (p. 130)

During each five-day period, the teachers were observed once, and they provided researchers with samples of their students’ work. The information collected included: a log of how each task was used and student-written work for three of the tasks provided in a five-day period. These were used to score teachers on successful implementation of tasks; there were some scores based entirely upon the observation and other scores based upon the level of cognitive demand evidenced in student work.

It is interesting that there was no log of actual teacher activities or how teachers worked together to address disagreements. It is hard to understand the statement that this professional development “provided a supportive and collaborative environment” when there was no qualitative or quantitative data provided on this point and none was described as having been collected. While the principles seem clear, this report did not provide details on the teachers’ discussions, lesson planning, or reflections on the changes they observed in themselves. It is important to address fully the question of exactly how such changes can be implemented.

Boston and Smith (2009) presented powerful evidence for their ESP professional development model, and, although the size of their study was small (n=18 for observation data, but n=12 or fewer for full data), the teachers experienced significant changes in their practice, and the students responded with work that indicated high levels of cognitive demand. What is missing, from this and others models of professional development, is some knowledge of exactly *how* the teachers translated their theoretical understanding of high-cognitive demand into their classroom practice with specific groups of children.

Video is useful in professional development of mathematics teachers

There are many models for using video clips of actual teaching or representations of teaching[7] in professional development for teachers. If the video is of an actual lesson, with teachers and a group of students who are learning in real time, then it is giving us the ability to see what actually happens in a classroom even though we were not actually there. If we videotape our own lessons, we can revisit what happens in our own classrooms and take time to reflect on many of the aspects of a lesson.

The work of Borko, Koellner, Jacobs, and Seago (2011) described two different models of using video in professional development. They pointed out that using video has value as a "medium to provide a shared experience" (Borko et al., 2011, p. 175) for professional development participants. Video allows them all to "be in the same classroom" for learning (Borko et al., 2011, p. 176). One model Borko et al. (2011) described is reflected in the problem-solving cycle (PSC) workshop. In this workshop, teachers would video record their own lessons and share these videos in a lesson-study type discussion with the other teacher participants. The other model described is reflected in the Teaching Geometry (LTG) workshop, which relied upon prepared video recordings of model geometry lessons provided to teachers as examples to learn from.

[7] Some videos use rehearsed presentations of a lesson that was not given to any group of students, but is intended to be used as a model by teachers.

In both types of workshops, participant teachers (1) did the math; (2) analyzed the solutions; (3) prepared lessons; and (4) taught the lessons. The authors concluded that, in both the PSC and LTG workshops, the use of video improved participant "teacher skills for observing and analyzing practice"; however, no measurement reflecting this improvement was described (Borko et al., 2011, p. 185).

Borko et al. (2011) also pointed out drawbacks to each of these particular models. The LTG workshop required a time- and resource-consuming investment in that all the videos had to be created first, and the videos provided did not always meet the needs of the participants. The PSC workshop was more dynamic in that the participant teachers created the videos they used. However, the quality of the video was then dependent upon the experience, abilities, and perspective of the participants in each workshop; this is likely to produce videos of widely varying quality.

This is where the work of Maher, Landis, and Palius (2010) broke new ground: Their professional development model utilized videos of students' mathematical reasoning under the guidance of teacher researchers working as part of two decades of longitudinal and cross-sectional research. The videos are stored in the Video Mosaic Repository (VMC) of Rutgers University and have been collected for over 20 years. These videos were of central importance in the professional development research conducted by Maher and colleagues. The video clips provided real-life examples, showing how teachers/researchers guided children, how children worked together and constructed mathematical ideas, the nature of the math problems that engaged the children, the solutions, and the justifications of those solutions.

The VMC houses videos that "show the reasoning of students from elementary through high-school years, and, in several content strands, where it is possible to search the collection and follow particular students investigating mathematics within and across strands" (Maher, Landis & Palius, 2010, p.2). This means that, over a short period of time in a professional development workshop, teachers have the opportunity to examine a particular student's mathematical thinking in elementary school, then middle school, and high school. As such, it is possible to see how students constructed knowledge as they worked together to solve specific math problems and to also see the impact of the early experiences on the young adult students.

Maher's approach to professional development specified that teachers need "deep knowledge of the underlying mathematics that is taught, of how students learn the mathematics, and how classrooms can be designed to motivate and support children's learning (Maher, Landis, et al., 2010, p. 3). In addition, Maher, Landis, et al. (2010) maintained that solving mathematical problems is, essentially, a social activity and the social component supports learning: "Our research has shown that individual learning manifests itself through the social interactions of others . . . .In the activity of problem-solving, learners build and share ideas and, in so doing, deepen and extend their knowledge" (p. 3).

The Maher professional development model incorporated four primary parts:

1. Teachers do some of the math that they will see the children do on the video.
2. Teachers view and analyze the videos of the children doing the math.
3. Teachers plan lessons that incorporate the math and implement those lessons in their classrooms.
4. Teachers analyze the results in the form of their own students' work.

There are three cycles of professional development in the Maher model, and each cycle repeats steps 1 through 4 above. Across the cycles, tasks became increasingly more complex.

The problems came from a strand of tasks that were used in research on children's reasoning. They were designed to offer opportunities for teachers and their students to make connections between problems of similar structure and, when appropriate, to pose generalizations for the solutions (Maher, Landis, et al., 2010).

In their report, Maher, Landis, et al. (2010) included screen shots of the VMC videos that were used, time frames for the teachers' group problem-solving, and photographs of some of the teacher-created solutions. This level of descriptive detail gives the reader a sense of *how* the teachers in this professional development workshop were learning to implement the new ideas they were receiving. Moreover, the actual videos are available for both the participants in this program as well as any other teacher/researcher to view as many times as needed.

Maher, Landis, et al. (2010) studied the responses of 20 teachers who studied mathematics problems together and then watched videos of students in a constructivist learning environment solving the same problems. They reported:

> In sum, the yearlong intervention engaged teachers and their students in thoughtful mathematical problem solving and reasoning. A question of interest to us was whether their participation in the intervention had an influence on previously held beliefs about what mathematics children are capable of learning and what role a teacher can have in the process. (Maher, Landis, et al., 2010, p. 14)

This research took place in year two of a design research study funded by the National Science Foundation (award DRL-0822204) and directed by Carolyn A. Maher. Twenty New Jersey middle-school classroom and special education teachers participated in their workshop. The teachers were given pretests and posttests to assess their beliefs about how children learn.

Maher and colleagues (2010) reported that over 64% of the posttest items indicated that participating teachers' beliefs shifted significantly after the workshop. These shifts represented growth towards some of these beliefs: "Understanding math concepts is more powerful than memorizing procedures" (p. 15, item 2); "All students are capable of working on complex math tasks" (p. 15, item 3); "Learners generally understand more mathematics than their teachers or parents expect" (p. 16, item 5); "Teachers should intervene as little as possible when students are working on open-ended mathematics problems" (p. 16, item 10).

"It is important that teachers believe that students are capable of thoughtful mathematical reasoning" (Maher, Landis, et al., 2010, p. 3). This is a statement that is easy to agree with but very complex to implement. It is particularly challenging when the students themselves let the teacher know that they are apathetic and do not want to even try. This is where the VMC repository may be particularly helpful: if math teachers can challenge their beliefs about the potential of all students, including students who have been labeled "low level" and "less capable," and actually see how much math such students are capable of, then perhaps they can transmit to these students both a rich experience in math and a new experience of success. If the teacher is confident that the student can be successful, then perhaps the teacher will persist in providing opportunities for the student to begin.

What Do Math Teachers of Minority and Low-SES Students Need to Know?

The research in this study is not focusing on helping teachers to address the needs of children in crisis. Children who suffer from extreme poverty may face health, nutrition, and housing crises on a regular basis, and this study is not about a pedagogy that can reliably distract students from these basic problems of survival. (The suggestion that such huge and basic problems in students' lives can be addressed by any pedagogy must be questioned.)

The research in this study is about helping teachers address the needs of minority and low-SES students, *not* those who are trying to learn while facing basic crises of life and survival, but those who may have particular needs that are related to race and economics, particular needs that their math teachers *can* address. One of the problems that black children in all economic strata share is called stereotype threat.

Steele and Aronson describe stereotype threat as "a social-psychological predicament that can arise from widely known stereotypes about one's group" (Steele & Aronson, 1995, p. 797). They explained the impact of stereotype threat on the performance of black students as resulting from the knowledge that, as a group, they are viewed by the larger society (in the United States) as less competent intellectually, and, therefore, they are expected to do poorly on standardized tests:

> Our reasoning is this: whenever African American students perform an explicitly scholastic or intellectual task, they face a threat of confirming or being judged by a negative societal stereotype—suspicion—about their group's intellectual ability and competence . . . And the self-threat it causes—through a variety of mechanisms—may interfere with the intellectual functioning of these students, particularly during standardized tests (p. 797).

Steele and Aronson (1995) reported on a series of studies designed to identify the impact of tests on black students. Each of their studies compared the performance of black and white students in three situations: A "diagnostic" test, which was presented to the students as a test measuring their intellectual ability, and "non-diagnostic" tests, which were presented to the students as tests to help researchers understand "psychological factors involved in solving verbal problems" (p. 799). In each of the studies, results as shown in Figure 1 below were provided.

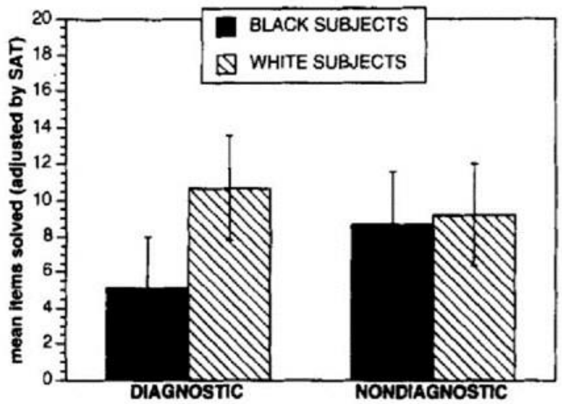


Figure 1: Test Results for Black and White Subjects Vary by Circumstance

Note that the comparisons of student performance on the tests were adjusted for differences in skill level as evidenced by their verbal SAT scores. Therefore, Figure 1 is illustrating that black students performed worse than would be expected alongside their white peers, and this difference was almost entirely erased by changing the context of the exam from “diagnostic” to “non-diagnostic.” Steele and Aronson (1995) parsed out the reasons that might have caused black students to perform worse on tests (than they otherwise might), including the idea that a test which causes frustration triggers stereotype threat (p. 798).

Consider that, since 2002, the results of No Child Left Behind (NCLB) high-school graduation tests have shown a persistent score gap between black and white students in mathematics. Consider that black parents and children have been assaulted with these “facts” for almost two decades, and these data seem well placed to deepen the stereotype threat that black students experience on math tests. As a result of this stress, they may be less likely to participate in honors and advanced placement mathematics, as is noted in research from the College Board (Deng & Kobrin, 2007).

It is important to note that students who live in poverty or low-SES communities are also subjected to a stereotype threat in mathematics because of the known societal expectations of their group (Steele & Aronson, 1995) and also, perhaps, because of the results of NCLB testing, which exposes achievement gaps between economic groups.

Consider also that black students responded well to small changes in test conditions that quieted the impact of stereotype threat (e.g., the “non-diagnostic” test in the Steele and Aronson study (1995)). This suggests that black, minority, and low-income students can be intentionally reassured in math (and other classes) to reduce the anxiety and depressed test scores that are produced by stereotype threat.

Returning then to the question posed by this study: What do math teachers of minority and low-SES students need to know in order to address the specific needs of these students? The social constructivist pedagogy described by Maher (1998) and Davis (1992) emphasized the importance of listening to students and teaching students to value what they think as they try to understand a math problem. This focus, as exhibited in the video stories created for this study, as well as in the large set of research videos stored in the VMC, requires a respectful relationship between the teacher and the students. When a teacher values what a student thinks and seriously considers the mathematics that a student conjectures, that teacher is explicity undermining the societal focus on test results. This may not just be a better way for students to learn mathematics; it may be effective in challenging stereotype threat and bringing social justice into the mathematics classroom. Further research is needed.

CHAPTER 3: RESEARCH METHODOLOGY

Workshop Design

The professional development workshop used in this study was designed using the intervention model described by Maher, Landis, and Palius (2010), which has four key components:

1. Teachers doing mathematics.
2. Teachers studying videos
3. Teachers implementing lessons in classrooms.
4. Teachers analyzing student work.

A complete cycle in the Maher model is a set of one or more meetings that completes all four components in the model. In the 2010 paper, the study they are describing spans one year and three of the cycles (described above) are completed. During one year of professional development the teachers in Maher's study had multiple chances to explore mathematics, learn from the videos, design and deliver lessons and then share the results, including student work, with one another. The study concludes that "Our intervention suggests that a study of carefully selected videos of children doing mathematics can be an effective medium for helping teachers become more aware of the untapped potential of children to build mathematical ideas and ways of reasoning" (Maher, Landis, et al., 2010, p. 21).

The design of this research study builds upon the Maher model to examine how algebra/pre-algebra teachers of students assigned to low-level classes (with > 20% low SES students) learn and apply new pedagogy. This research focuses on the *workshop* component of the Maher intervention.

Another goal of this study was to design a workshop that could generate new and higher teacher expectations for their black and minority students. As such, the IML videos selected to comprise the TAW video stories feature predominantly black and minority student. The teacher/researchers in these video stories are similarly diverse. The students and teachers in the video stories mirrored the diversity of the participants in the TAW and of their own students

This study focuses on the first two components of the Maher model, although it does include elements of three and four. Specifically, the Teacher's Algebra Workshop (TAW) was designed to use the Maher model as follows:

1. Teachers doing mathematics: Algebra

   The teacher participants in this study began each day of the TAW with an algebra problem that was featured in the video scheduled for that day. The daily algebra problems are all *guess my rule* problems, and the solutions are all linear functions. The teachers solved the problems (on paper) and were asked to consider other questions about the math problems. For example: How they would justify their own solution(s)? How might their own students solve the same problem?

2. Teachers studying videos: Video stories

   As in the Maher model, the teacher participants in the TAW view videos of children learning algebra each day. As in Maher, Landis, et al.'s 2010 study, the source of the videos is the Robert B. Davis Institute for Learning Collection of "observational videos" of children learning mathematics. This collection is housed at the Rutgers University Graduate School of Education; it contains the video data that resulted from 24 years of research into the mathematical reasoning of k-12 children. A large part of this collection is available to the public on the Rutgers Video Mosaic Collaborative (VMC); the VMC may be found at https://videomosaic.org/. The VMC is a collection of over 300 "observational videos" of children learning mathematics.

The TAW videos are *video stories* that are created using video clips from the VMC that are assembled into a video analytic using the RUAnalytic Tool (Agnew, Mills, & Maher, 2010); the RUAnalytic Tool is at a link in the VMC web page. Each video story in the TAW is implemented as a video analytic.

The RUAnalytic Tool is a platform for assembling multiple clips (from the same video source or multiple video sources) into a single video analytic; each selected clip is called an "event," and an analytic is a set of these events. An analytic has a title, documentation of its purpose, and annotation for each event (each clip) to explain what should be noticed in that event. Each video analytic designed for the TAW tells a story about a group of children discovering the "rules" and the "secrets"specific to a linear function under the guidance of a teacher/researcher (T/R). As such, each of the TAW video analytics is called a *video algebra story*.

For the purposes of this study, the RUAnalytic Tool was used to create four video algebra stories. Each video algebra story is about a group of children attempting to find the explicit linear function that matches a set of data points that they are given. The stories are closely connected and together they form a bigger story.

- Day 1

A group of sixth- and seventh-grade children, Ariel, Brandon, James, and Yonny, are playing a "game" to find "the rule" (a linear function) that produces the "clues" (data points) that they are given by teacher/researcher #1 (T/R1). They play two games. The first game challenges the boys. They guess, they discuss, they reason, but they do not find the explicit rule. T/R1 does *not* give them the answer. The exploration starts and ends and they do not seem frustrated. Then T/R1gives them an "easier" rule to guess, and they enthusiastically figure out the rule and explain why it works. The children show ownership of the problem-solving process, and T/R1 shows patience and provides guiding questions.

- Day 2

Yonny and Brandon (who were in the video algebra story on Day 1) work with T/R1 (also from Day 1) on finding a new rule. This time they analyze a new set of given "clues," ordered pairs, and they produce several recursive solutions. Brandon explains these solutions in some detail. When T/R1 challenges them to find the "output" value for a very large "input" value, they realize that they cannot use their recursive solution, and they resort to proportional reasoning, which produces an incorrect answer. Yonny is the author of the proportional solution, but he surprises us with the correct answer – a correct y-value at the end of the story. They boys work hard and persist in problem solving. T/R1 shows patience and respect for the boys' thinking.

- Day 3

Ariel and James (from the Day 1 algebra video story) work on a new *guess my rule* problem; they are using rods to build *ladders* and they have to find a rule for the number of rods in a ladder with "n" steps. When Ariel is first introduced to the ladder problem he decides that the solutions are proportional, that a ladder of eight steps must use twice the number of rods of a ladder of four steps. As he generates some counter-examples (i.e constructing the ladders shows that a ladder with four steps uses 14 rods and a ladder with eight steps uses only 26 rods and not 14x2 = 28 rods) he modifies his original solution. Ariel produces a complex heuristic that is based upon a series of corrections to his original (incorrect) proportional solution. Ariel shows extraordinary persistence as he is challenged with questions by another teacher/researcher, T/R2, and he modifies his solution in stages to meet each challenge. T/R2 does not point out errors at each stage, but instead asks questions that help Ariel find the errors himself. Ariel is determined to fix his solution.

There are two video algebra stories on Day 3. The second one shows Ariel as an eighth grader viewing his earlier heuristic solution and laughing at his younger self. He ends up showing us how he understands linear functions, how he now uses the math vocabulary for slope, y-intercept, finite differences, and equations.

- Day 4

A different group of children work with T/R3 to find the "secrets" of the equation that match some "clues" (ordered pairs). These children are also doing a *guess my rule* problem, but they are now trying to find the "secrets" of the rule, and these secrets are the slope and the y-intercept that characterize any linear function. They work together and periodically go to T/R3 to discuss what they notice. T/R3 shows respectful interest in what they find, but does not label their answers *right* or *wrong*. These students are successful and they present their answers.

In summary, each video on each day is a story, and the stories progress, using some of the same ideas, children and teachers. On the first day, we are introduced to students engaged in a *guess my rule* lesson with a very patient teacher/researcher. On the second day, two of the students we met persist in analyzing the *guess my rule* problem in more detail than we saw on Day 1, but recursively. One of them surprises us with an “unexplained” but correct answer. On Day 3, two familiar students (from Day 1) work on a new *guess my rule* problem (the ladder problem); we see further persistence and learning as one student modifies an incorrect answer in many stages. We note that the patient teacher/researcher does not point out mistakes but helps Ariel to find them. On Day 3, we also see a video story confirming that t his persistent student, Ariel, does end up learning all the traditional mathematics about linear equation. Finally, on Day 4, we see a new group of children discussing the nature of scientific secrets, discovering the “secrets” of linear functions (slope and y-intercept). We observe the children desiring to “discover the secret on their own” in preference to just getting the answer. The stories progress each day within the set of common themes: social constructivist pedagogy and *guess my rule* lessons, and so the four days can be thought of as a four-part story.

In the next section, the importance of the four-part set of video algebra stories and the fact that they feature predominantly black and minority children are discussed. The purpose of these elements in the program design is to explore how these elements may have helped the

TAW teacher participants notice the pedagogy (social constructivist), notice the engagement and reasoning of the black children, and form higher expectations for those children.

The TAW workshop was designed around these video algebra stories. Each day, the teacher participants worked on the *guess my rule* problems on paper, they watched the video story (that contains the problem) and then they filled out a questionnaire about what they noticed in the video story. Each daily questionnaire included questions about mathematics, pedagogy, and student engagement. After the TAW participants answered the questions on paper and, also during that process, they discussed what they noticed with one another. (In general, these discussions constituted the largest section of time in each day of the workshop, and the audio-tapes of these discussions are the primary data source for this study.) The analysis of the data on the audio-tapes produced an analysis of what the teacher participants noticed and questioned during the TAW. As such, the effectiveness of the TAW is discussed in terms of these results.

A final component of each TAW day included planning for a *guess my rule* algebra lesson that the TAW participants agreed to implement during the 2016-2017 school year. These discussions raised questions about the practical issues that concern teachers: the size of classes, the limited time allowed for each class, and concerns that not all students would “buy in” to constructivist ideas of students owning their problem-solving process. These concerns are included in the Results Chapter. Teacher participants did leave the workshop with ideas about the promised lessons, but not specific lesson plans. The teachers had a commitment to implement these lessons and meet once with the researcher to share the results of the lesson, including student work.

During the 2016-2017 school year, four out of the five teacher participants delivered a *guess my rule* lesson and did meet with this researcher to discuss the results and share student work. It was not possible for all the teachers to do this, and, furthermore, it was not possible for all the teachers to meet together at a common time. As such, discussion of the lessons and student work was not shared across the group. This study was limited in that there was only one final meeting, and since the student work was shared at this meeting, there was no opportunity to review it and ask questions about it on a subsequent day. A future study that more closely follows the Maher Model would include several opportunities for teachers to give guess my rule lessons, meet and discuss the results and make plans for subsequent lessons.

In summary, the design of the TAW is based upon the Maher model (Maher, Landis, et. al., 2010) with a focus on the first two components on video algebra stories featuring minority children. The discussions that teacher participants created as they considered questions of mathematics, pedagogy, and student engagement produced the data from which the results of this study are derived.

Stories facilitate learning.

Two design elements of the TAW were essential to the goals of this study. That first element is the use of video algebra stories. The connections between the video stories shown over the four days of the TAW are described in the previous section; this creates a larger story that emerges over the four days to generate analysis and reflection. The second essential design element is that predominantly black and minority children are featured in the video algebra stories.

The pedagogical power of stories to help educate professionals is documented across a range of domains; examples of these results include the education of teachers (McDrury & Alterio, 2003) and the education of design engineers (Mcdonnell, Lloyd, & Valkenburg, 2004). The use of *video* stories creates a persistent record of the story that permits broad and repeated access to the same story. In the TAW, teacher participants would routinely return to the video to establish what exactly happened in the story. Finally, the use of video stories created by the RUAnalytic Tool provides *authentication* of the story components. The video clips selected for each analytic come from research videos of actual students and teachers. In this study, the algebra story components are clips of research video taken during the IML afterschool program in a New Jersey school district. (The RUAnaltyic tool and the VMC provide information about the videos that were used in each video story, so that each component of the story can be verified.) As such, the video story is not just a vehicle for effectively conveying ideas, it is a vehicle for showing what it is actually possible to expect from minority and black children in an algebra or pre-algebra classroom. It is a vehicle that shows *how* to engage children to function at a high level of persistent inquiry to find the solution of an algebra problem. It is an authentic and persistent story about how to engage black and minority children in low-income school districts, about how to engage them in mathematical inquiry.

D. Jean Clandinin and F. Michael Connelly suggest that teachers have pedagogical stories that inform and describe their practice. They write: “The narrative context for the ongoing development and expression of teacher knowledge in schools is also of importance” (Clandinin & Connelly, 1996). They present a story metaphor for three different classes of teacher knowledge: Professional knowledge and school system mandates are embodied in “sacred stories,” knowledge of one’s own teaching practice is embodied in “secret stories,” and public presentation of that teaching practice is embodied in “cover stories.” For the purposes of this study, it is not necessary to delve into the precise boundaries between these areas. It is the existence of the gaps between these classes of stories that is of interest. (It can be noted that many professionals[8] will experience gaps between the prevailing theories and mandates of the moment (the sacred stories), their own practice (the secret stories) and the personal-presentation stories that bridge these gaps (the cover stories).)

Consider that the video algebra stories, as a professional development tool, might fall into the category of “sacred” stories in that they are presented as the results of education research. However, they are, by nature, authentic stories of student algebra learning, and, as such, they have been created to become “secret stories” in a teacher’s “zone of proximal development[9].” A teacher inspired to try a social-constructivist *guess my rule* lesson may, in fact, be assisted by these stories. And that same teacher may be inspired by these stories to consider what their black and minority students might be capable of if they were given the social-constructivist classroom with a patient teacher to guide them.

As such, the use of social constructivist video algebra stories that feature minority children are TAW design components that were added to the Maher model for this study.

Selecting and Preparing Data for Analysis

This study produced multiple artifacts.

Once the workshop was complete, in August 2016, the data produced were labelled and stored. Paper records were labelled and stored in a dedicated space, in a lockable filing cabinet. Paper records were also scanned and stored as PDF files on a computer dedicated to this

[8] This author experienced similar dichotomies as a software engineer in the telecommunications industry, 1981–2001.

[9] A zone of proximal development is the term used by Vygotsky to describe the difference between what a student can do with some assistance and what they can do unassisted (Vygotsky/Kozulin, 2011)

research. Audio files were uploaded to this same computer. A preliminary review was completed of these primary data:

1. Two versions of the audio-tape files.

   The entire workshop was recorded using two audio recorders; each was placed at a location near a group of TAW participants. One audio recorder was focused upon participants #2, #3, and #4, and this was the audio file that was transcribed and analyzed as the primary data source for this study. These participants were more vocal in the discussions and this is why this audio file was selected. The other audio recorder was focused upon participants #1 and #5. It should be noted that the one audio file that was selected also picked up discussion contributions from participants #1 and #5.

   The recording devices used are both Philips Voice Tracer digital recorders (DVT800).

2. The First Day Questionnaires

   Each TAW participant completed a short paper survey about their own teaching experience and practice (Appendix F).

3. The Daily Mathematics Worksheet

   Each day of the TAW began with the Mathematics Worksheet. The TAW participants were given a paper statement of the *guess my rule* math problems that the children in the daily video story solve; the teachers were asked to solve the problems so that they would be familiar with the challenges facing the children in the daily video story. They were asked to also consider characteristics of the math problem that related to the children's work in the daily video story.

The work that the teacher participants did to find and discuss their solutions was collected at the end of each TAW day. The Mathematics Worksheets are found in Appendix C.

4. The Daily Questionnaires

Each day the TAW participants filled out a questionnaire that asked about the pedagogy, the mathematics, and the student engagement they noticed in the daily video story. These questionnaires can be found in Appendix C. These questionnaires provided a structure for the discussions that were audio-taped. The paper questionnaires were collected each day.

5. Large Post-its

We did use large Post-it papers to record some of our mathematics when we were exploring ideas in the TAW. Those sheets were saved in the previously described filing cabinet.

6. The *guess my rule* lessons

Additional data was collected in the Fall of 2016 when the TAW participants met with this researcher to share the results of their *guess my rule* lessons.

Only four out of five teachers followed through on the obligation to give a *guess my rule* lesson and meet with this researcher to share the results and students' work. There were two separate meetings because the four TAW participants who did complete the lesson obligation were not all free on the same day. Participants #1, #3, and #5 met with me on Thursday, November 10, 2016.

Participant #2 met with me separately on March 5, 2017, and #4 indicated that s/he gave the lesson but did not schedule a time to meet.

The first meeting was videotaped to reduce the need to take copious notes. This video was also helpful in that it recorded both the voices and the faces of these participants, and thus helped researchers to discern who was speaking on the audio-tapes during the process of transcribing the audio files of the TAW.

All the paper data were scanned into PDF files and stored on a dedicated computer.

The audio data were selected for analysis.

The TAW workshop data were reviewed to examine the quantity and quality of the content in each. The paper questionnaires were filled out, but when this written content was compared with the TAW discussions on the audio files, it was clear that the audio data had more detail. The audio data were, therefore, selected as the source for analyzing the results of this study.

Transcribing the audio files was necessary to permit analysis of the data.

The first step in analyzing the audio data was transcribing the data. The task of transcribing involves listening to short bursts of the audio file and typing what is heard. The speaker of each statement must be identified. It is complex to transcribe an audio file of a discussion that includes multiple speakers. Challenges include:

- Identifying the speakers accurately; different speakers can sound the same or different depending upon the speed that the audio file is played.
- Discerning each individual word in a sentence can be difficult. It was a priority to indicate when a particular word of phrase was not audible.

- Discerning individual voices and statements when speakers interrupt or talk over one another; note that this was a common occurrence in the TAW.
- Transcribing audio data into written text is very time consuming. It is often necessary to replay a small section of the audio file five or more times in order to capture as much information as possible.

In order to support the transcribing process, a tool, *Transcribe*, found at https://transcribe.wreally.com was selected. Transcribe supports speeding up and slowing down sections of the audio in order to better hear each speaker. Some speakers' words are clearer when the pace of their speech is accelerated, and some speakers' words are clearer when the pace of their speech is slowed down. Transcribe provides the expected audio controls (fast forward, rewind) and it also supports speeding up and slowing down the speed at which the speech is played. This speed modification feature is customizable, and the user can accelerate (either speeding up or slowing down) in self-defined increments until the particular section of audio is clear.

The user types the transcript text inside the Transcribe tool and can save the completed document as a Word file. Transcribe saves the document inside the tool while a particular audio file is being worked upon. When each transcript was complete, it was saved as a Word document and stored in a directory outside the Transcribe tool.

The transcripts were validated.

Once the transcripts for all four days of the TAW were complete, they were prepared for validation by two graduate students. The original transcripts were stored in Word files. Using the Find and Replace capability, the names of the TAW participants were replaced with a numeric code. This involved replacing the “speaker name” assigned to each statement and also replacing any reference to a speaker name in the body of the statement.

Using Word’s formatting capability, the statements were numbered. Statement number 1 does not refer to a line number in a document, it refers to the first *set of statements* assigned to the first speaker in the transcript document. When the speaker changes, the statement number increases. This was achieved by separating the verbiage for each speaker into a distinct paragraph. Once the numbers were assigned to each statement and names of the TAW participants were encoded, the Word transcript files were ready for validation.

Two graduate students used Transcribe to validate the transcript files. They used Transcribe to listen to the audio files and identify errors or omissions in the Word transcript files. The errors and omissions were reviewed by this researcher (along with the graduate students), and the Word transcript files were corrected by mutual agreement.

If the correction involved inserting a new statement number, the total numbering that was originally assigned was kept and a “1/2” statement number was inserted between two integers (e.g., statement 8.5 was inserted between statement 8 and statement 9). This type of correction was infrequent.

The more frequent change implemented was to mark a word, phrase or statement *inaudible*. When reviewing the validation results for a particular transcript, agreement between this researcher and the graduate student was the goal. Any disagreement about what was individually "heard" was resolved by recording a statement or phrase as "inaudible." For example, in the Day 2 TAW results, we labelled all or some part of 132 out of 992 (13.3%) statements *inaudible*; 30% of these (40 statements) were labelled *completely or largely inaudible*.

There were cases when the graduate students (the validation team) heard more than this researcher originially heard, and much of this was due to the fact that the Transcribe tool was not in use when transcribing began. At first, the researcher was just listening to the audio files without the benefit of a tool. The validation team used Transcribe for *all* of their work. Using Transcribe with a set of headphones, and the ability to speed up or slow down a particular portion of the audio, provided the clearest evidence for the transcriber.

In a few cases there was some ambiguity about "who the speaker is" for two of the participants. The video of these participants (video of the lesson discussion) was used to help identify/confirm identificiation of "who is speaking." The other three participants, due to diversity of speaking style and accents, were more easily identifiable.

In summary, transcribing discussions involving five different speakers presents many challenges, as has been described. It should also be noted that, during the TAW, the teacher participants talked in a casual register with one another and many statements were not completed or were not completely audible. If the graduate student (who was validating the transcription) found something that this researcher did not, we listened to the audio; if the new finding could *not* be verified, "inaudible" was inserted for that portion of a statement.

When transcript validation was complete, these graduate students returned the paper copies of the Word transcript files they were given, and they deleted any and all electronic copies of the transcripts.

The transcripts were prepared for coding.

Once the transcripts were corrected and verified, they were imported into Excel files to turn them into a database of searchable data. Each transcript was processed as follows:

1. The Word transcript files were first saved as text files. Delimiter characters (#) were inserted between the Statement Number and the Speaker identifier, and between the Speaker and the Statement text. These delimiters were used by Excel to separate data into different columns.
2. The text file version of each transcript was imported into Excel.
    a. Each Statement Number is stored in a single column (Statement No.)
    b. Each Speaker code is stored in a single column (Speaker).
    c. Each statement consists of the sentences said by the Speaker at one time. This set of sentences is stored in a single column (Statement).
3. The Excel versions of the transcript files were created to permit coding of the statements (in one or more added columns) and selection of all the statements coded in a particular way. A column (Code) was added to each Transcript Table.
4. The first step in analyzing the transcript data was to code each statement; the code set was designed to support analysis of the results of the study in terms of what the TAW teacher participants noticed. These codes are described below.

Coding the Data for Analysis

At first there were only three codes: math, pedagogy, and student engagement. Once the transcript data were stored in Excel spreadsheets, there was a spreadsheet containing a Transcript Table for each day of the TAW. The first step of coding the data involved using only three initial codes:

1. M for Mathematics content
2. P for Pedagogical content
3. E for Student Engagement content.

Each statement was coded with one of these codes. As the first attempt to code the transcript proceeded, it was immediately clear that M, P, and E did not comprise a sufficient set to code each statement, and new codes were created to handle other types of statements:

- Wb

  Wb is for statements about the TAW Workshop business (e.g., parking, lunch, technical issues about the viewing of the video stories).

- S

  S is for stories that the Instructor told to illustrate situations encountered in class.

- F

  F is for fragments of statements. This was a frequently used code and it was used, conservatively, whenever speakers said “Yeah” or “Um” or in any case in which it was not completely clear what the speaker was referring to.

- C

  C is for statements about what a student might be thinking. This code was intended to be used when the TAW participants were hypothesizing about what a child in the video story was thinking or what motivated a child; C is for the cases in which the participants did not have direct evidence for their hypotheses. C ended up being a companion code to E or M.

- B

  B was initially intended for statements about student behavior. This code was eliminated because either these statements were recognized to be about student engagement (E) or about Pedagogy (P).

- My

My is for statements about a TAW participant's school.

- Mp

Mp is for statements about a TAW participant's teaching practice.

- A

A is for statements about the video story; video stories are implemented as analytics using the RUAnalytic tool and the video database in the VMC.

- Q

Q is for a statement that poses a question.

Each statement in a transcript was coded, whether or not that statement would be useful in the analysis. For that reason, there are codes for workshop business (Wb) or stories that the instructor told (S). Once the first set of codes was defined, the Day 2 transcript was coded. At this point, transcript statements often had multiple codes (for example MEQ for a statement about mathematics and student engagement in the form of a question).

After the Day 2 transcript was coded, the statements were sorted and a first attempt at extracting useful information was attempted. This attempt was complicated by the number of codes assigned to each statement. For example, if a statement was coded as MEQ, then it would have to be considered in at least two categories for analysis: The Mathematics category and the Student Engagement Category. The code for a question, "Q," was eliminated because it wasn't necessary to know whether an idea was expressed as a statement or a question to select it for analysis in any category. As a result, the Day 2 transcript was recoded to eliminate "Q" and identify a primary category (M, P, or E) wherever possible. Then there were many fewer statements with more than one code.

There were other codes that were briefly introduced and then withdrawn. For example, “X” for a new idea and “R” for restating what another speaker said. These codes are included here to illustrate the process of defining the code set for the TAW audio transcripts. Beginning with three primary codes, the set expanded to include many codes that could have been used to describe individual statements, but that were ultimately not useful. For example, “B” was eliminated because most statements about student behavior related to student engagement (E) or pedagogy (P). “A” was eliminated because there were very few statements that were primarily about the analytic (the video story) and not about specific instances of mathematics, pedagogy, or student engagement in the video story. “Q” was eliminated as described above. My and Mp were collapsed into one code: My.

The final set of codes was:

1. M for Mathematics content
2. P for Pedagogical content
3. E for Student Engagement content
4. Wb for TAW workshop business
5. F for fragment
6. S for story
7. My for a TAW participant’s school or practice (this merged the original My and Mp codes).
8. An alphabetic combination of M, P and/or E, to be used when two or more major topics are included in a statement.

Once the transcribed data were coded, the coded data were verified by a third Rutgers graduate student. This researcher met with the graduate student, explained the code set and how it would be used, and then the graduate student would code the data independently. When coding was complete, the graduate student met with this researcher to discuss and compare their results. When the choices differed, the two interpretations were discussed, and a final choice was agreed upon. Most differences were the result of clarifying the context for certain TAW participant statements.

The coded data were used to analyze the TAW results.

Once the transcript data were coded, Excel was used to extract and group the statements that were coded as “M,” “P,” and “E” for all four days. This was accomplished by using the Excel Sort feature. The transcript data tables in Excel were sorted by Code values, and then it was possible to consider the statements with P, M, E, EM, or MP, or ME separately. Collecting all the statements that were coded to contain a particular type of content, for example, M for mathematics, allowed the researcher to identify major themes within each category.

Analyzing the results of the TAW workshop is framed in terms of answering these questions: What did the TAW participants notice in the daily video algebra story? What did they notice about the mathematics that the children did? What did they notice about the pedagogy in the story? What did they notice about student engagement?

To answer these questions, the Transcript Table for each workshop Day was sorted by the Codes and the statements in each group were analyzed to identify main themes in each group:

I. Pedagogy
   a. Engaging students in a “game” or a discovery process.
   b. The value of patience
   c. Tolerating student excitement/exuberance

d. Teacher/researchers did not "give out answers" or tell the children who was right and who was wrong

e. Guiding students through questions

f. Collegial respect for students

II. Mathematics

a. The challenge of finding explicit solutions (rules or equations) for the *guess my rule* problems.

i. The accuracy and limitations of recursive solutions generated by the children.

ii. The accuracy and limitations of heuristic solutions generated by the children.

iii. Proportional reasoning used in the search for explicit linear functions.

b. Respect for the children's mathematical reasoning

c. Mathematical principles: the TAW participants explored questions about "What is true?" that were inspired by the explorations of the children in the video stories.

III. Student Engagement

a. Behavior and affect that overtly indicate student engagement

b. The children appeared confident

i. They didn't have to be called on and were "not afraid to speak"

ii. They didn't ask, "Am I right?"

iii. They "appeared" enthusiastic

c. Persistent problem solving

d. The differences between the children in the videos and their own students.

i. How do I achieve "this" in a forty-five-minute class with 20 or more students?

Once the themes were identified, supporting statements and sets of statements were collected around each theme. This entailed considering all the statements that were selected in the Transcript Table Excel Sort function for each category and looking at each of these "selected" statements and the conversation that surrounded it. In most cases, it was then necessary to go back to the original transcript to collect all the statements in a "surrounding conversation." Conversations were then collected in a document. When all the selected conversations on pedagogy were collected, they were analyzed to produce the Results Chapter section on Pedagogy; this was repeated for the Results Chapter section on Mathematics and the Results Chapter section on Student Engagement.

The Implementation of the TAW

The Participants

Five teachers from a New Jersey school district participated in this study. These teachers came from the two middle schools in the school district. All of them taught algebra and pre-algebra; four teachers taught honors and non-honors classes. One teacher exclusively taught special education students. Four of the teachers had tenure in their schools, and one was due to get tenure the following school year.

In this group of five, two teachers were black, one was a non-black Hispanic person, and two would be considered white. Two of the teachers were women, and three of them were men. Three of the teachers were under 40 years old, one teacher was between 40 and 50 years old, and one teacher was older than 50. As such, the group of teachers was diverse.

In the Results Chapter, the discussions of the TAW participants are analyzed in terms of what they said to one another when they were analyzing the video stories through the lens of the Daily Questionnaire. (The transcripts of these discussions were *not* analyzed to characterize the individual participants.)

The teacher participants in the TAW were recruited from their school district with a letter (Appendix F) that detailed the nature of the workshop, the time frame (4 half-days in August 2016), and a financial incentive. They were informed that participants who completed all the requirements of the the workshop would get a stipend of $500. The requirements of the workshop included:

- Attendance and participation in all four days of the TAW.
- One *guess my rule* lesson to be given to at least one class during the Fall of 2016.
- At least one meeting with the instructor to report on the lesson, what worked and what didn't work.
- Share student work from that *guess my rule* lesson with the instructor.

Four out of the five participants met all the requirements and received the $500 stipend. One participant did not respond to communications about the Fall 2016 lesson or meet with the instructor, and that participant received a $250 stipend.

The Workshop

The structure of each day in the TAW was as follows:

1. The Math Problem

   Each day the first activity in the workshop was to solve the math problem(s) that were going to be featured in the daily video story. The math problems were printed on a worksheet and included questions about the nature of the problem and/or different approaches that could be taken to find a solution. The math problems are available in Appendix B.

2. The Video Story

   The TAW teacher participants watch the video story. At times they elected to watch all or part of a particular video story more than once. The video stories are described in Appendix A.

3. Daily Questionnaire and Discussion

   As soon as the video story was over, the teacher participants were given the daily questionnaire. The daily questionnaire asked them about what they noticed in the video story, including questions about mathematics, pedagogy, and student engagement. After the TAW participants answered the questions on paper and, also during that process, they discussed what they noticed with one another. The daily questionnaires are available in Appendix C.

The written answers on the math problem sheets and the daily questionnaires were collected each day. All discussions were captured on audio devices. The audio files were the focus of this study.

The discussions (that were audio-taped) were guided by the questionnaire and instructor; however, they did move into tangential areas. For example, the TAW participants began to take interest in the children who were doing mathematics in the videos, and they asked about them (TAW Day 3, Lines 280-286).

The instructor of the TAW attempted to implement a respectful collaborative pedagogy as was modeled in the video stories by teacher/researchers. Note that the teachers/researchers in the video stories are referred to using coded names: The Teacher/Researcher #1 in the Day1 and Day 2 video algebra stories is T/R1, the Teacher/Researcher #2 in Day 3 video algebra story is T/R2, and the Teacher/Researcher #3 is in the Day 4 video algebra story. These code names are also used in this chapter.

The instructor asked questions about what the TAW participants were thinking and referred back to questionnaire questions, but did not directly teach the TAW participants what this researcher hoped they would notice in the video stories. The assumption was that the teachers needed time to discover what there is in the video story and in the mathematics problems they featured and that not everything can be discovered in four half-days. The hope was that the teachers would remember these stories and continue to learn from them over time.

The instructor also reminded the TAW participants that they were *all* peers, all full-time teachers, and all facing many of the same challenges. This was sometimes challenging and the instructor resorted to using personal “stories” about teaching to convey ideas. The instructor also provided a comfortable work space, a break for snacks during the morning (snacks were provided), and a general attempt to make the teacher participants comfortable as they worked.

All of the workshop components are available and/or described in Appendices A, B, C, D, and F.

CHAPTER 4: RESEARCH RESULTS

Research Summary

This study contributes to decades of research on professional development of teachers to promote high-cognitive demand (Davis, Maher, & Noddings, 1990;,Maher, 1998; Stein, Smith, Henningsen, & Silver, 2000) and help teachers motivate students to persevere in mathematical problem solving. The value of using video so that all teachers can share the same experience has been established (Borko, Koellner, Jacobs, & Seago, 2011) as is the Maher model of professional development, which utilizes videos of social constructivist learning (Maher, Landis, et al., 2010).

As described in the Methodology Chapter, the design of the Teacher's Algebra Workshop (TAW) has been based upon the work of these researchers and the Maher model of professional development. The TAW built on this model by utilizing *video stories* that feature *black and minority children* engaged in mathematical inquiry. The choice to create video stories featuring black and minority students was made to give TAW teacher participants rich examples showing students just like their own, persevering in their work to solve challenging mathematics problems.

The value of stories in teaching is established in general by Mcdonnell and Valkenburg (2004), and in particular by Clandinin and Connelly (1996), where the authors reveal that pedagogical knowledge is captured in stories. The five video stories used in the TAW were created with the RUAnalytic tool and the VMC.

The results of this study include a rich discussion among the teacher participants in which they notice the constructionist pedagogy, the mathematical analysis of the children's work, and the student engagement required to persevere in problem solving. Results point to the need for a larger study that would confirm and clarify the use of these video stories in professional development. Teacher participants made many comments that indicated they were interested in and thought highly of the students in the video stories. This may indicate that the choice to use black and minority students in the video stories is useful in helping teachers to learn how to close the achievement gap.

The results of this study indicated that the video stories in this workshop gave TAW participants many examples of how teachers create social constructivist classrooms, and these generated many questions about how the participants could apply these techniques in larger classes with 45-minute class time limits. These discussions generated both concerns and ideas; Results also point to the need for an expanded study with TAW workshops that span one or two school years and support teachers in implementation of these new ideas.

Pedagogy: What the TAW Teacher Participants Noticed About Pedagogy in the Video Stories

The participants in the TAW noticed many aspects of the pedagogy in the video algebra stories they studied. Aspects of pedagogy that the participants discussed included the value of teacher patience in giving the children (in the video stories) the time to explore their own solution paths and encouraging them to take more time to fully understand and justify the solutions they present. The patience that workshop participants noticed is a mix of this patience shown by the teacher/researcher and the patience that the children must learn if they are going to have the ability to engage in a rich mathematical problem-solving process.

The workshop participants also remarked on how exuberant the children became and were candid about how such exuberance can be difficult for some teachers. They remarked on how patient the teacher/researchers were with the behavior of the children in the video stories.

The workshop participants discussed specifically how the teacher/researchers in the video stories resist providing children with efficient paths to a mathematical solution and persist in asking the children questions that turned them back towards their own thinking to justify their solutions or to find their own mistakes. This process took shape differently within each video algebra story; in each video story, the questioning process reflects the participants (teacher/researcher and children) and the algebra problem they tackled. Teacher participants in the workshop noticed this and discussed how to reproduce this engagement and learning in their own classes. Note that this process of using questions that engage the mathematical minds of the children in the video stories is central to the TAW participant discussions in each of the three lenses applied to the TAW transcripts: pedagogy, mathematics, and student engagement.

Finally, the workshop participants noticed the respectful, collegial way with which each of the teacher/researchers in the video algebra stories interacted with the children.

Give the children time to engage in the mathematics.

On Day 1, the teacher participants discussed their perceptions of "how patient" T/R1 was in letting the children (in the Day 1 *guess my rule* video story) take their time and their own paths to a solution.

In the Day 1 video story, T/R1 introduces the math problem as a "a game" in which the children (5 boys) have to discover a rule. The rule takes "in" one number and gives "out" another. The first "point" generated by the rule is (5, 13). Even though T/R1 tries to stop the children (in the video story) from guessing the rule too quickly, one of the children says "+8" (TAW Day 1 Video Story Description, Events #1 and #2). T/R1 asks a child to give him the next number for the rule to process, and Dawud gives him the number 3. T/R1 asks the children what they think the rule will do to "3" and they say "11" (applying the +8). T/R1 shakes his head and says, "My rule takes the number "3" and makes it "7." The boys are looking at a chart (shown below), and they are challenged to guess what the rule will do when the input BOX number is 6. The boys guess and try different answers; then Ariel[10] says, "It has to be more than 13," and he guesses "16" (the correct answer). At this point they have been working on this problem for several minutes and only Ariel seems to have an insight—the others do not indicate

[10] This is a partial, but not a complete description of the mathematical behavior in the Day 1 video story.

whether or not they agree with or understand what Ariel is saying (TAW Day 1 Video Story Description, Event #2, Ariel).

□	Δ
5	13
3	7
6	?

Table 2: Day 1 Guess My Rule Problem 1

The workshop participants noted how patient T/R1 is in waiting for the children to figure out the rule:

Participant #2: *I think that can be difficult at times, when you see that they're not getting it, you want to help them, you want to be like . . . [laughter] . . . . So I thought that, whoever the teacher* [T/R 1] *was, he did a really good job.* (TAW Day 1, Line 18)

Participant #3: *He stood back as much as he could. He let them go. He gave a little guidance and then stood back.* (TAW Day 1, Line 121)

On Day 2 of the TAW, the teacher participants discussed the mathematical reasoning of the children in both the Day 1 and Day 2 *guess my rule* video stories, and they credited T/R1 with giving the students initiative and the confidence to try (TAW Day 1 Lines 7, 76; TAW Day 2, Line 377):

Participant#1: *I wish I was more like the teacher that was doing it—he don't really help out the kids at all—he just lets them score and figure it out. But my class, like the kids, I guess they're so timid and scared to talk about math, to reveal their skills, they don't say* *anything. So, I have to like step in and get the conversation moving along. I admire this guy, what he does.* (TAW Day 2, Line 756)

It is interesting to note that Participant #1 sees his/her own students as “timid and scared to . . . reveal their skills” and admires T/R1. At this point, Participant #1 does not identify *how* T/R1 does “what he does” or speculate about whether his/her own students would appear as timid and scared if they were in T/R1’s class.

The workshop participants do begin to contrast the conditions of the classes in the video stories—where T/R1 is working with no more than six children at a time—with their more traditional classes (including many more children and a very fixed time limit). Participant #3 does suggest, “*I would like to see him in a big class*” (TAW Day 2, Line 757); the other workshop participants signal agreement (TAW Day 2, Line 758, 760) with this sentiment.

The workshop teachers sprinkle their discussions of the workshop questions with comments about their own classrooms and the contrast between the “classroom” conditions in the video stories (the IML afterschool program) and their own. These might have occurred anyway, but they are a natural consequence of the agreement workshop participants made (in order to be part of the TAW) to design and implement a *guess my rule* lesson during the 2016–2017 school year. These discussions led the workshop participants to consider pedagogical changes they could implement to attempt a partial recreation of the conditions in all the video algebra story classrooms (TAW Day 1, Lines 320-329).

On Day 2, the teacher participants summarized their perceptions that T/R1’s patience allowed the children (in the video stories) to engage in the mathematics. After Participant #4 suggested that “*Everyone of us here* (in the workshop) *is already doing something like this* (what was shown in the Day 1 and Day 2 video stories)" (TAW Day 2, Line 928), the conversation returned to the patience and craft of T/R1, led by this same workshop participant:

> Participant #4: *So like here, what I learned is patience*.
>
> Participant #3: *Yeah.*
>
> Participant #4: *You know, this guy* [T/R1] *is very patient; you know, he's not rushing the students, you like looking at the time and saying, you know, 'You have to finish this.' He lets them work at their own pace.*
>
> Instructor: *He lets them think about it.*
>
> Participant #4: *Yes, and then, you know, his input is very minimal.* (TAW Day 2, Lines 932-936)

This conversation focused on teacher patience, but also referenced the contrasting conditions between the video story classroom and their own. Participant #4 noted that T/R1 does not urge the children to "finish," because, in the afterschool program, T/R1 did not have to worry about ensuring that the children finish a problem. In the daytime public-school classroom, Participant #4 (along with other teachers) may feel pressure to make sure that children finish a problem during a single class (TAW Day 2, Lines 778, 784, 934).

The children exhibited patience with themselves.

In the Day 1 *guess my rule* video story, there are two algebra problems. The first problem challenges the children to the extent that they do *not* (in the video story) actually find the underlying rule ($y = 3x - 2$). They work on the problem, but they do not finish it. This problem, like each of the *guess my rule* problems, begins with a growing set of (Box, Triangle) points. The children in the Day 1 video story explore the "clues" they are given, and one of the children, Ariel, figures out one correct “Triangle” (output) answer given a specific “Box” (input) value. Ariel seems to have some insight into the problem that the other children have not yet obtained.

The workshop participants noticed that T/R1 recognized Ariel’s progress, but recognized how he guided Ariel to give the others time to think about the problem and avoided giving Ariel “too much credit”:

> Participant #3: *And then also, the teacher said [to] Ariel to "Don't say anything, be quiet" you know, let the others, you're on to it. Everybody else think.*
>
> Participant #2: . . . *So I thought that, whoever the teacher was, he did a really good job. He was patient, he saw that was a great point that you picked out, he gave little subtle, "Okay you're on to it," but not overboard.* (TAW Day 1, Lines 17-18)

In the Day 4 video story, the children work on finding a “secret” (similar to guessing a rule). As they work, one child is ready to “share the secret,” and another objects. The workshop participants discussed this and suggested that students may want the time to figure out “the secret” themselves—even if the “secret” has been discovered by someone else:

> Instructor: *Okay, so that's what I am asking about . . . he's ready to share and one kid says, "No, I don't want them to share," and . . . so what do you think is going on?*
>
> Participant #4: *Maybe he wasn't done yet . . . maybe he was yet to find the secret, that he didn't want anybody to reveal it yet because he was working on it.*
>
> Participant #3: *He wants the challenge.* (TAW Day 4, Lines 307-309)

The workshop participants noticed the need for patience regarding the time it takes for students to explore, unpack, and experiment as they find mathematical solutions. In this example, they noticed the need for patience regarding the relative time that each student in a classroom may need to find the solutions for him-/herself. This implies that the process matters more than the solution—even to the students themselves. If all I need is the answer, why does one child (in the Day 4 video story) say, “No, I don’t want them to share”? (TAW Day 4 video story description, Event #5, Ankur). He wants “the challenge” of doing the problem, finding “the secret” himself.

The time needed to figure out a solution can cause frustration.

Here patience exhibited by the Teacher/Researcher, T/R2, and the patience requested of the student problem solver is reflected in a discussion (below). The workshop participants talked about Ariel’s perspective. Ariel is a student in the Day 3 video story, in which he is working on “the ladder problem” (TAW Day 3 Video Story Description includes a description of The Ladder Problem). He is challenged by the fact that this problem is linear (f(x) = 3x + 2), and Ariel is making the false assumption that, if f(x) is the underlying rule, then f(2x) = 2f(x). This assumption leads initially to a false answer, and then T/R2 asks Ariel questions to help him discover his error. Under the guise of “I don’t understand” (TAW Day 3 Video Story Description, Events #2 and #4, T/R2), T/R2 repeatedly asks Ariel questions, and Ariel gets a little frustrated as the workshop participants noticed:

> Participant #5: *He* [T/R2] *didn't have as much of a calm demeanor of the first guy. The first guy is like, "Okay, try it." But this guy, he's just like, "Hold on, wait.*”
>
> Participant #2: *He kept on saying to him like, "Wait a minute"; like. . . “show me . . . that's more like a. . . .”*
>
> Participant #4: *Before he was done with one, he would throw another number, . . . and I think it’s because Ariel told him, "Whatever you do, I am leaving here at 3. You'd better hurry up*." (TAW Day 3, Lines 228-230)

Participant #5: *He said "3:30." It's like 3:30.*

Instructor: *Do you really think he just wanted to leave*?

Participant #2: *No, I think he just was, like, "You'd better get it, what I am trying to say to you. . . .*

Instructor: *He was trying to say?*

Participant #2: *Because maybe he was getting frustrated by the fact that the researcher wasn't getting it. He probably was like, you know when you "get something," You think like*. . . . (TAW Day 3, Lines 232-236)

The workshop participants were noticing the patience a student needs to go through an authentic process of mathematical discovery. Ariel's assumption of proportionality creates a challenge, and T/R2's questions coax Ariel to be patient enough to work on the problem. The workshop participants recognized Ariel's possible frustration. They suggested that Ariel wants T/R2 to "understand" as an affirmation of his reasoning and his answer—that he was done and that his solution was correct. As T/R2 keeps asking questions, Ariel continues to have more work to do. He begins to see his own reasoning more clearly (TAW Day 3 Video Story Description).

As the workshop participants continued the discussion of the Day 3 video story, they moved on to consider how they would plan a lesson that allowed their own students to be patient with their own problem solving. They discussed their goals and possibilities for implementation:

Their first concern is expressed by Participant #4 as, "How do you do this in a class of 20 people?" (TAW Day 3, Line # 362). The participants all discussed the need for more "one-on-one" time with students (TAW Day 3, Lines 364-377) and then some ideas for achieving the results they saw in the Day 3 video story. They discussed a few possibilities for using groups that each work on a different problem and/or working with one of the groups to demonstrate the reasoning dialog for the class (TAW Day 3, Lines 417-433).

After these ideas were discussed, Participant #2 returned to talk about the patience that students needed to work as Ariel did in the video story and about how to motivate students to persist in the problem-solving.

Participant #2: *Sometimes when kids see that, they're maybe like . . . I know they are lacking a lot of skills—so doing this—so making it some kind of fun so it's not intimidating, I wish I could think of some way to not frustrate my kids, challenge them, but not frustrate them. So, doing like a game or something like that will make them "okay," get interested*. (TAW Day 3, Line 668)

The theme of structuring lessons and assignments to reduce student frustration returned again and again, particularly for those participants who taught special education students. Participant #2 attempted to address the concern she raised and referred to something she has already tried in her classroom:

> Participant #2: *So one thing I did last year, that I like the idea, is to let them know at the beginning, "Listen, you are not going to talk to me about what the rule is—you can't come to me; you and your coworkers," I call them coworkers, "are going to discuss it. But you are all going to get a chance to come to me, or I am going to each of your stations, and I will talk to you, but for the first something, what was it, five minutes . . .* (TAW Day 3, Line 706)

Participant #2 focused on how to help set student expectations by telling them explicitly that they would have to work on the problem with their "coworkers" for a while before coming to the teacher for help. Commenting on the Day 1 video algebra story, another workshop participant also spoke about the importance of combining this understanding with a motivation to get engaged in the work—by calling it a "game."

> Participant #1: *The other thing that also kinda had it more , , , .getting the students to be more involved and not just say, "Am I right? Am I right?" was the way that it was introduced. It was introduced not as a lesson or a test, it was introduced as a "We're going to play a game." And then from there, it was just like, "OK, let's see what happens."* (TAW Day 1, Line 186)

These ideas, setting student expectations explicitly and describing the problem as a "game," are very concrete pedagogical moves. It is clear how to implement these ideas. They also noticed and discussed the challenging yet engaging, mathematical questioning that was used. Those discussions yielded more diverse opinions among the TAW participants.

Pedagogical choices include problem selection and allowing different solution paths

In the Day 1 video algebra story, T/R1 is working with a group of six boys to "guess a rule" and the first activity is introduced as a "game." The first number that T/R1 used to exercise "the rule" was 5, and he tells the boys that, given 5, the rule gives them 13. (The rule that they *don't* figure out in the video story, is $y = 3x - 2$) On a Box andTriangle chart, T/R1 posts a 5 in the Box column and a 13 in the Triangle column. Then the boys immediately begin guessing the rule. As they are given more "clues," more (Box, Triangle) pairs, they begin trying to guess what the Triangle value (rule output) for a particular Box value (rule input) should be. They struggle to make sense of the clues (the points) they have been given (TAW Day 1 Video Story Description, Events #1- #6).

The second *guess my rule* problem goes much more quickly. The (Box, Triangle) pairs show the pattern: (1, 15), (2, 25), (3, 35), and the boys figure out that the rule in this case is

$y = 10x + 5$. They figure this out in much less time than they used to try (and *fail)* to figure out the first rule (TAW Day 1 Video Story Description, Events #7- #10).

The workshop participants discussed this lesson noting that T/R1, the teacher/researcher, is very patiently allowing the boys to struggle with the first rule. Participant #2 remarked on how teachers struggle to do this, saying, “When you see that they’re not getting it, you want to help them . . . .” (TAW Day 1, Line 18). They note the fact that the children never do find the first rule and so reflect on whether the second problem was selected precisely because they would have the pleasure of finding the rule.

> Participant #4: *Yes, they didn't find* [the first rule].
>
> Participant #2: *And they didn't come up with it.*
>
> Participant #2: *And they did find an explicit function for the second problem. Could that have been predicted by the teacher? I think so because of the number patterns. Right?*
>
> (*“Yeah ,yeah,”* from Participants #3 and #4)
>
> Participant #4: *I think . . . you know like , , , basically, when I was doing it. Was like, you had 5 first and then 3. Look at that. You are used to 1, 2, 3—but in this case they were decreasing. That could have accounted to why it took them so long for them to get the pattern in this one. And then, when they started getting numbers, the three numbers were in random order and it did not help. Not 1 is this, and 2 is this* . (TAW Day 1, Lines 19-22)

The workshop participants discussed this learning process and, in the lines above, reflected on these pedagogical questions:

- Why was the first problem selected?

- Was it harder for the boys because the Box values (the input values) were not provided “in order”?
- Was the second problem selected as an antidote to the frustration of their failure with the first problem?

From this discussion, workshop participants #2, #3, and #4 all seemed to say that T/R1 gave them the second problem because he knew they would find the explicit function for it. This is something that is easy for us to “believe,” seeing the pattern (1, 15), (2, 25), (3, 35). In fact, the boys do not completely avoid struggling to find this rule either. They call out “rules,” and T/R1 asks them to test each one to see if “it works.” In this way, they test their rules, and, after a few tries, they find the answer: $y = 10x + 5$ (TAW Day 1 Video Story Description, Event #10, T/R1). They also indicated that the boys may have found it harder to find the first rule because the input Box values were not in order: “The three numbers were in random order” (TAW Day 1, Line 22).

Why did T/R1 select the first problem and not provide the input values in order? The workshop participants discussed this and concluded that maybe he wanted a rich discussion. They also pointed out that, if they were teaching an honors class (in their own schools), they would see “the result that we saw” (TAW Day 1, Line 30).

The TAW participants discussed this question: Do you think it was specifically planned to happen this way?

> Participant #3: *I can't remember . . . he did give them a starting number and then they threw out* [another] *number. So he probably didn't realize that they were going to pick like all over the place.* [Laughter]. *So could have he predicted the pattern?*

> Participant #4: *Maybe he had given them the first two, ...in giving them the first two, . . . then they could have gone . . . wrong with that. Just give them one. One of the guys said that he already knew* [they all laugh - this is a reference to the +8 suggestion for the first rule].
>
> Participant #2: *I think it maybe he could see the second part: . . . by just giving the one number, he wanted them to come up with these, you know, to figure it out*.
>
> Participant #4: *Yeah, I think if you have experience, you would be able to predict what would happen. I think somebody like him should have had experience in the classroom. So I think he would have seen it already, this is how it's going to make a richer discussion*. (TAW Day 1, Lines 24-27)

As this discussion continued, Participant #3 pointed out how the work that the boys are doing would be what an honor class would be expected to do:

> Participant #3: *Also I think it depends on like—it says if "could you have predicted by the teacher, the struggle for the first one?" So, if you were teaching an honors class and doing this* (Participant #2: [interjects] *Right!), you'd probably have the result that we saw—probably*. (TAW Day 1, Line 30)

Note that, at a break in the workshop, Participant #3 did ask the instructor if the students in the Day 1 video story were selected because they were gifted in some way. The answer is that they were not.

The pedagogical questions raised here are fundamental to teaching: Selecting a good question for the class involves considering what the students know, what they are prepared to learn, and how hard or easy we should make it. For example, how many teachers and textbooks automatically put the x values in order when giving the students points on a line and what are the pros and cons of doing that?

Pedagogy That Engages Students in Mathematical Exploration and Solution Justification

After viewing the Day 1 video story, the participants discussed the pedagogy of T/R1, the teacher/researcher who led the *guess my rule* lesson. in the Day 1 video story. Their comments credit T/R1 with engaging the students in the lesson. Participant #3 says: “In the beginning I thought they were bored. Oh man, but then, pretty quickly he [T/R1] got it going pretty well, I thought” (TAW Day 1, Line 6).

The workshop participants expressed the idea that T/R1 was “in control,” but their conversation indicated that the students were engaged and in charge of their own learning:

> Participant #3: *He was in control; the activity was going the way he wanted it to go. They were engaged*. (TAW Day 1, Line 117)
>
> Participant #3: *He stood back as much as he could. He let them go. He gave a little guidance and then stood back*. (TAW Day 1, Line 121)

As they discuss this idea, there is an attempt to clarify what “in control” means:

> Participant #4: *Is saying that, Particpant #3 means in control —as having a calm demeanor?*
>
> Participant #2: *He is very calm, happy with the kids*.
>
> Participant #4: *He didn’t give them the urge to intervene or . . . you know, he just let them go . . . “let them go*.” (TAW, Day 1, Lines 124-126)
>
> Participant #2: *That's one thing that we talked about in our group. I have a difficult time with that wait time. That's just O - kay-ay* [Laughter]. (TAW, Day 1, Lines 129)

They were discussing how the teacher/researcher let the students take the lead on their work. And Participant #2 reminded us of how difficult it is as a teacher/researcher (for him-/herself) to be patient and wait for students to explore a problem and figure out a solution.

The workshop participants described patience as pedagogy when they discussed how the children in the Day 1 video story "didn't need" teacher validation, didn't ask "Am I right?" and seemed to possess the patience to find solutions that they could justify:

> Participant #4: *They were somehow confident of what they knew. They didn't like . . . . they said . . . the answer—they didn't need the teacher to validate what they knew. They were that confident. When he said, "You should try it out," when they tried it out, they didn't have to ask the teacher "Am I right?" They knew; yes, it works, so I'm right*.
>
> Participant #3: "*Or it doesn't so I am not right*" [said as if it were a quote from one of the children in the Day 1 video story].
>
> Participant #2: *Maybe because the part of the session with the teacher, he never gave them any kind of hints or clues—they were used to him saying "Try it out." So maybe his approach of just "trying it out" made them have that confidence and then they have to prove it*. (TAW, Day 1, Lines 74-76)

The workshop participants were suggesting a connection between T/R1's patience and restraint with the ability of the children in the video story to "try out" their solutions to see if they work.

The questions teachers ask reflect the pedagogy in the classroom.

In the Day 2 video story, Brandon and Yonny work with T/R1 on the *guess my rule* problem with the (undiscovered) explicit rule $y = 2x + 1$. They begin with a chart that shows x and *y*-values in a table:

x	***y***
1	3
2	5
3	7
4	9
5	11
6	
7	

Table 3: Day 2 Guess My Rule Problem 1

The boys, led by Brandon, see that the x values go up by 1 and the y values go up by 2, and they explain this as "the rule" to T/R1. T/R1 listens and then asks the boys to find the y value when $x = 6$, when $x = 20$, and (finally) when $x = 100$. Brandon uses a recursive pattern to find that $y = 13$ when $x = 6$, and he works carefully to find that $y = 41$ when $x = 20$. This last effort required Brandon to find all the y values for x values from 7 to 20 in order to find (20, 41). As a result, in the video story, Brandon and Yonny realize that they need another method when T/R1 asks them for the y value when $x = 100$ (TAW Day 2 Video Algebra Story Description, Event #6, Yonny). The workshop participants discuss why these questions help Brandon and Yonny work on the problem:

> Instructor: *Well, how was he trying to coax them towards this explicit solution, because they come up with the recursive thing fast. They see that pattern*. (TAW Day 2, Lines 324-326)
>
> Participant #4: "*He gives them a large number that they can't do it to. It will be too hard to do it*."
>
> Instructor: *Too hard. Too hard and then what does he say when they [inaudible,] ugh? You know that feeling when kids look at you like, "I can never do it." And what does he say?*

Participant #2: *Doesn't he give them another problem?*

Participant #1: *Does he make a lesser number?* (TAW Day 2, Lines 328,329)

Participant #3: *Is there a shorter way of doing it*? [Describing what T/R1 said]

Participant #1: *Yeah, is there a shorter way to do it*? (TAW Day 2, Lines 331, 332)

Instructor: *And then he said, "Maybe you can find a shorter way?" Suggests it to them.*

Participant #1: *He's forcing them to . . . .[inaudible].*

Instructor: Laugh, *He is trying to motivate, alright? To do something different and um . . . .*

Participant #2: *And I guess he is maybe just trying to get them out of just halving [inaudible] . . . .?* (TAW Day 2, Lines 334-337)

In this discussion, the workshop participants identify how T/R1 motivates Brandon and Yonny to look for an explicit solution. They point out that T/R1 gives them larger $x-values$ for which it may be too difficult ("too hard") for them to use their recursive pattern to find the corresponding $y-values$; T/R1 tells them to find a "shorter way." Participant #2 also suggests that T/R1 is trying to get them to question their assumption of proportionality (get them out of "just halving").

The workshop participants also discussed how they might have guided Brandon and Yonny in the Day 2 video story as they tried to find the $y-value$ when $x=100$. They recognized that a large value for x is a motivator for the students, but they also expressed an interest in making the solution path smoother for the boys:

Participant #1 clearly articulates the value of giving Brandon and Yonny $x=100$:

Participant #1: *The teacher always asks like, for example, what does this come out when x is 100? or how about when x is, like, a large number? They see it as too much and that forces them to find, like, a shortcut.* (TAW Day 2, Line 656)

However, as the discussion continued, the workshop participants articulated how they might have handled the lesson differently from T/R1. Brandon and Yonny figured out that when $x = 20$, then $y = 41$; they did this by using their recursive pattern. (Note that this answer is correct and recall that the underlying rule for these (x, y) values is $y = 2x + 1$). However, when trying to find the $y - value$ when $x = 100$, they made an assumption of direct proportionality. They decided that since $20 \times 5 = 100$, the corresponding, $y - value = 41 \times 5 = 205$ and this is *incorrect*. Participants #2 and #4 discussed what they might have done to help the boys question the proportionality assumption.

Instructor: *And then they're being forced to do what? They do it. And the teacher* [T/R1] *says, "How do you know that works?"*

Participant #2: *Yeah, yeah*.

Instructor: *Does it work? They have to (laughing) try it!*

Instructor: *So umm, so then if you think about that then, why doesn't he* [T/R1] *just say, "You can't do that here, look: 41 x 5 is 205, and he preprinted the chart," Look what it really is, it's not! You can't multiply here," and just say that. Why doesn't he* [T/R1] *do that?*" (TAW Day 2, Lines 674-677)

Participant #4: "*One thing I didn't ask, when did they , , , was it the 5X41, what did I expect him to say was, um, when they did the 21, how they got the 21, they didn't multiply the 2 by 21 to get the 41, that is one thing he should have asked them*.

Instructor: *You would have. That's what you would have said*.

Participant #4: *That's what I would have asked. Now when he found the corresponding way, when x=20, you do multiply your 2 by 21 for the 10 (x=10), did you do that? To get, if had done that you'd have got 42, but you got 41. So I think he should have pointed that out to them*.(TAW Day 2, Lines 679-681)

Note that the instructor asks a direct question about why T/R1 does not just tell the boys that in this problem, "You can't do that"—it isn't proportional and you cannot just multiply 41X5. In response, Participant #4 doesn't say the instructor's question is too directive or fails to let the boys figure out the problem. Instead, Participant #4 suggests that the question should be to compare how they got $y = 41$ when $x = 20$ to how they were getting the $y - value$ when $x = 100$. S/he points out that when $x = 10$, $y = 21$ and that if they used their proportionality assumption, they would get $2 \times 21 = 42 \neq 41$. Participant #4 wants to get Brandon and Yonny to look at their own recursive method more closely and compare the answers they would get if they used their proportional method instead. S/he did not suggest to "just tell them." Perhaps it is difficult for math teachers not to reflect on how they would prefer to conduct a lesson? Perhaps it is difficult for math teachers to consider a large leap in shifting much of the problem-solving process to the student? Nevertheless, TAW participant #4 did not dismiss the "constructivist" pedagogy, but suggested a change that is consistent with that pedagogy: Ask the children to reflect on how they calculated a prior y–value and compare it to their current method.

Both legal and illegal equations can be either true or false.

The Day 4 video story is about T/R3 teaching a larger class of sixth-grade elementary school students using an approach to understanding algebraic equations that encompasses the idea that equations can be "true," but also "illegal." T/R3 uses box and triangle symbols instead of x and y. One of the equations he presents is: $\square + \square + \Delta = 9$. He works with the children in the video story to establish the meaning of true (the math sentence is true) and the meaning of legal (the values placed in the box and the triangle are allowed) (TAW Day 4 Video Algebra Story Description, Event #2, T/R3). For example:

- $2+2+5=9$ is true and legal
- $2+3+4=9$ is true but illegal
- $2+2+3=9$ is false but legal
- $1+2+3=9$ is false and also illegal

The workshop participants discussed the differences between the teacher/researchers in the video stories, and they considered using T/R3's ideas in their own classrooms.

> Participant #1: *I think the difference is, in the other one* [T/R1] *he just gives them, like, the table, alright, "Go find the rule." And this one* [T/R3], *he gives them almost kind of like a "Do Now," you know different skills going, see what skills they need in order to do this*. (TAW Day 4, Line 89)

Participant #1 was pointing out that T/R3's pedagogy includes a model that is familiar—with an introductory "do now." Note also that in the Day 4 video story, T/R3 is working with a larger group of students, a group that is closer to the size of a traditional classroom.

The workshop participants discussed using T/R3's approach. At first, they expressed some reservations, some surprise at the idea of illegal and legal equations:

> Participant #3: *Putting myself back . . .trying to put myself back, you probably all.. . . .going back to that grade . . . thinking . . . I don't know that I would I have liked the legal thing—think of it that.*

Participant #2: *Yeah, I have to think about that, too.*

Participant #3: *Okay, okay, I think I would have to like, "What are you talking about, 'legal*?'"

Participant #5: *Even now, like when I saw the problem, and I saw "legal" I got the true and the false because I've heard that enough, but "legal" and "illegal". . . I'm like. . . . what is this?*

Instructor: *Right, it was not familiar, but once you know what they mean it's not hard . . . .*

Participant #3: *As soon as they caught on . . . .I* .[inaudible] . . . *stopped thinking about it . . .and now I'm pretty good..* (TAW Day 4, Lines 117-122)

As the discussion continued, workshop participants further considered this idea and then the possibility that this approach (the use of "illegal" and "legal") may address a confusion that students have about variable names—how different variable names can have the same value, but if the variable name is the same, they must have the same value.

Participant #4: *I really like the words "legal" and "illegal" . . . I think that sometimes I would use in my algebra class . . . you know? Because it's . . . .*

Instructor*: It's much better than right and wrong.*

Participant #4: *Yeah, yeah.*

Participant #4: *Yeah, so what is legal and illegal?*

Instructor: *It might be true, but I can't use it (it's illegal).*

Participant #4: *Why can't I use it? Why is this not legal*?

Participant #1: *Getting . . . to think about . . . the parameters of an equation, there are certain numbers that you can use and there are some that you can't . . . so . . .* (TAW Day 4, Lines 126 -132)

Participant #5: *I agree . . . with #4 . . . I think I'm definitely going to use it. When we're evaluating expressions and they see "X^2 + X," and they plug the X value into X^2 and then they would leave . . . ."What do I plug into X?" . . . I'm like: "The same value."* (Everybody laughs.) (TAW Day 4, Line 134)

The workshop participants exhibited interest in T/R3’s approach to teaching algebraic equations; first they were surprised, then they experimented, and then they decided to try it. The discussion of this idea from the Day 4 Video Algebra Story, illegal and legal equations, reveals how, through discussion, the workshop participants helped one another to gain a clearer understanding of the pedagogy they were studying.

The “ladder problem” generates perseverance and heuristics

In the Day 3 video story, Ariel and James are working on the ladder problem with T/R2. Ariel’s mathematical thinking is the focus of this video story. The ladder problem is described in the TAW Day 2 Video Story Description, and, briefly, the ladders are modeled with Cuisenaire rods, so that a one-step ladder requires five rods and a two-step ladder requires eight rods as pictured below in Figure 2.

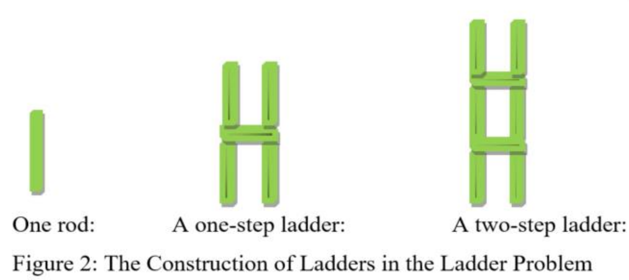


Figure 2: The Construction of Ladders in the Ladder Problem

Following the pattern, each time you add a step to the ladder, three additional rods are needed. As such, the underlying explicit rule for the number of rods is $y = 3x + 2$. T/R2 asks Ariel, “How many rods are needed for a 10-step ladder?” and Ariel approaches the problem by constructing a five-step ladder, counting that he used 17 rods in the five-step ladder. Then, using an assumption of proportionality, Ariel decides that a 10-step ladder has twice the number of steps as a five-step ladder, so it should also use twice the number of rods. Ariel incorrectly concludes that the number of rods in a 10-step ladder is 34 (TAW Day 2 Video Story Description, Event #12, Ariel).

> Participant #1 suggests that Ariel *“didn’t want to build a whole 10-step [ladder] so he just built a five-step, counted how many rods are there and doubled that and that should give . . . ? the number of rods in a 10-step.*” (TAW Day 3, Line 62)

In the video story, after Ariel makes his initial proportional assumption, T/R2 asks a number of questions that move Ariel to find his initial mistake and to, step by step, create a heuristic solution (TAW Day 3 Video Story Description, Events #1 - #6, T/R2).

The workshop participants discussed this process, and it is analyzed further in the Mathematics Results section of this Results Chapter. However, this mathematical process, guided by T/R2’s questions and Ariel’s exploration of the ladder problem, is the backdrop for the discussion that workshop participants held on how they would achieve this level of student engagement and mathematical exploration in their own *guess my rule* lessons.

Can a professional demeanor serve to engage students?

As they discussed the challenge of re-creating the type of math dialog that T/R2 had with Ariel in the Day 3 video story, the workshop participants first expressed concerns about creating this engagement with mathematics in a class of 20 students or more:

Participant #4: *Then, I was going to ask* this [inaudible]. How *do you do this in a class of 20 people?* (TAW Day 3, Line 362)

Participant #4: *Would somebody like, ...are you going to be with one person?*

Participant #2: *Yeah*.

Participant #1: *You can deal with it like one on one*. (TAW Day 3, Lines 364-366)

Participant #5: *Because, after I planned . . . deal with . . . this is double like one on one: teacher to student*.

Participant #1: *This is done like one on one: teacher to student?* (TAW Day 3, Lines 368-369)

The workshop participants asked how they could generate what seems to be a dialog of “one teacher to one student” in a larger class: “*How do you do this in a class of 20 people*?” (TAW Day 3, Line 362).

This conversation recurs as Participants #3 and #4 repeat the concern: “How are you going to be able to dedicate the amount of time to one student?” (TAW Day 3, Line 375). “So how do you use this in your classroom?” (TAW Day 3, Line 377).

The discussion of “how to do it” in a regular classroom reflected practical concerns, and the workshop participants did decide that there were pedagogical strategies they could try:

Participant #3: *Mentioned yesterday was to have a stage, a certain amount of kids doing this, and everybody else watching, maybe talking . . . not ideal, it's a large group. But that's a way of focusing on a few kids and maybe also these other ones who aren't doing it are listening, learning, and say, "Ah." Maybe . . . they're kids . . .* ” (TAW Day 3, Line 433)

> Participant #2: *I understand. Another way that maybe it could work or be a little similar to it, is if you have them all working on the one problem in the groups and stations. But then you have some time at the end where they could present what they thought and how they came up with that answer. Because as they're doing that each of them gets to go; they can start talking about you know: "Oh, I wish I would have thought about it that way," or "I did it this way," or "this is what I did," We need to have an open dialog.* (TAW Day 3, Line 442)

The workshop participants expressed concern about how to create the mathematical dialog they saw in the video stories, but they did discuss how they might innovate to introduce some new ideas into their own classrooms. They considered how they might set up small groups that work together to each explore a math problem and suggested using one group as a "demonstration group" to give the class a better understanding of what they expect. They also considered the idea that this type of pedagogy is what their school administrators are looking for:

> Participant #4: *When they come in and they see something like this, if you're able to . . . manage it well. I know how if my principal comes in and sees something like this and I am able to manage it well, I know he'll be very impressed*. (TAW Day 3, Line 501)

This last statement is important because the teachers will not be interested in learning a pedagogy that they perceive their administrators will not support.

Discovering "secrets" helps create mathematical inquiry.

The workshop participants considered the idea of presenting an algebra problem as a "game" or as "discovering a secret" useful for their students. Their interest in presenting the work as a game was expressed on Day 1 (TAW Day 1, Lines 186, 200) and Day 3, (TAW Day 3

Line 668). They suggested that figuring out “a secret” helped to infuse a lesson with the possibility of fun and excitement.

Participant #3 suggested that, in public school, the “Do Now” is almost like the lesson presented by T/R1 as “I’m going to play a game,” and that this approach makes it more fun for the students (TAW Day 1, Line 200).

As the workshop participants considered how to motivate students to explore mathematics and learn, they considered teacher interventions and student interactions. Participant #5 suggested that his/her own teacher interventions are key. Participant #4 focused on getting students to work together and how sometimes they understand an idea better from a peer than from a teacher:

> Participant #5: *But I think it's a mix though, ’cause when it's like they're trying it and they get stuck and then like there's that teacher intervention, at least for me, they'll do it, they'll get it, they'll be like "Ohhhhh," okay now I see it*.
>
> Instructor: *But you're not actually . . . you're giving them a thing to do . . . and. . . . finds it*?”
>
> Participant #5: *Sometimes I may give an example and then they do it and , , , .*
>
> Instructor: *So the teacher’s a strong component in that?*
>
> Participant #4: *Sometimes in a group, the group by themselves, and somebody in the group who just figured it out, and then they, "I get it, I get it." And then explains to the others. There's really one person that has the "ah ha" moment, and then they say, "Look can't you see?"* [group laughter]
>
> Instructor: *Did they all learn from that person*?

Participant #4: *Most of them do, some of the time; they learn better from that person than learning from somebody trying to [inaudible]. You try and you try, but once one of them gets it, I don't know, maybe it's the language they speak, and that person can really get the others on board and then, "Ohh, I get it! That's what she's talking about?"* (TAW Day 2, Lines 550-556)

The discussion of their own pedagogical models was guided by the questionnaires that workshop participants were answering about how they could reproduce some of the mathematical engagement and mathematical discussion in their own classes. The comments above reflect the beginning of that discussion in which workshop participants summarized some of their own practices and points of success.

When the workshop participants discussed the difference between the pedagogies of the three teachers/researchers in the video stories (T/R1, T/R2, and T/R3), they considered how T/R3 had a very enthusiastic group. They understood that this was a different study where T/R3 worked with a real class, and they appreciated the enthusiasm of the students in what they perceived as a more structured lesson:

Participant #2: *I think we were talking about how the class was, like, so enthusiastic*.

Participant #4: *Yeah*.

Instructor: *But that was true of the other group too.* (TAW Day 4, Lines 85-87)

Participant #1: *I think the difference is in the other one (*T/R1*) he just gives them, like, the table "All right, go find the rule." And this one (*T/R3*), he gives them almost kind of like a "Do Now," you know, different skills going, see what skills they need in order to do this*. (TAW Day 4, Line 89)

Is it possible that what appeared to be the more traditionally structured lesson is more appealing to the teacher participants because it is more familiar? Workshop participants discussed how T/R3 spoke to the children about their ideas in the Day 4 video story:

Participant #2: *He still doesn't say where . . . does he say?* (TAW Day 4, Line 163)
Participant #5: *He does; he says, "That's good*."
Participant #3: *He says, "That's a good thing," or whatever.* (TAW Day 4, Lines 166-167)

Workshop participants noticed that T/R3's restraint applies to behavior (in the video story) that might be considered too noisy or distracting:

Participant #4: *Yeah, . . . when you are teaching*.

Participant #3: *But he let it go. A couple of times he said, you know, "Quiet," but mostly he let it go*.

Participant #4: *Yeah, he let it go*. (TAW Day 4, Lines 176-178

Participant #2: *Also, I guess maybe* [*with*] *you're saying they have been with him such a long time, he may have already known their personalities*.

Participant #3: *Right, and it's going to be okay, it's going crazy.*

Participant #2: *Yeah* [laughs]. (TAW Day 4, Lines 181-183)

Participants discussed the appeal of "finding a secret" to characterize the investigation of a mathematical problem:

Participant #2: *It's kind of nice when they find it and then maybe you can tell them, "Oh that is . . . ."*

Instructor: *You found an important . . . .*

Participant #2: *Yeah, and then they never forget it.* (TAW Day 4, Lines 221-223)

Participant #2: *Especially 'cause of their age . . . it probably was, like a nicer way* (TAW Day 4, Line 228)

TAW participants notice respect and collaboration.

The workshop participants discussed the pedagogy of T/R1 in the Day 2 video story; their discussion moved beyond the importance of patience. After several questions from the instructor, Participant #2 describes T/R1's engaging demeanor:

Instructor: "*There's a . . . No matter what they do, right? He has a certain posture* [T/R1]. (TAW Day 2, Lines 160-166)

Participant #2: *Yeah, he does*.

Instructor: *What is it? How would you describe it?*

Instructor: *He's almost like, not in the same . . . .You know he doesn't seem like he's in the public school. He seems of something else*.

Participant #2: *Yeah*.

Instructor: *What does he seem like he's doing*?

Participant #2: *He seems like he's researching* [laughter], *in a university* [laughter], *yeah.*

These ideas will be revisited in the Mathematics and Student Engagement results in this Results Chapter along with additional detail specific to these lenses on the discussions in the TAW.

Mathematics: What the TAW Participants Noticed About the Mathematics Done in the Video Stories?

In each of the daily video stories studied by the TAW participants, children worked with teachers/researchers to find the missing rule in different *guess my rule* problems. Each of the rules is an undiscovered linear function. The children are given clues in the form of ordered pairs (input, output) or a model for constructing the (input, output) values. The points/clues are sometimes expressed as (Box, Triangle) pairs and sometimes using the standard (x, y) notation. In one example, the children are solving the "ladder problem." In this problem they are given a model for how a ladder is constructed from rods (blocks); as they work, they find ordered pairs when they determine that, for example, a ladder, of eight steps, needs 26 rods. In each daily video story, the children work together with a teacher/researcher to find the rule that generates all the points/clues.

Participants in the TAW noticed many aspects of the mathematics done by the children in search of the missing rules. Their discussions focused on these themes:

1. The challenge of finding explicit solutions (rules or equations) for the *guess my rule* problems.
    a. The accuracy of recursive solutions generated by the children.
    b. The accuracy of heuristic solutions generated by the children.
2. Proportional reasoning used in the search for linear functions.

The TAW participants discussed many details that they noticed in the problem-solving processes that the children employ to discover each "rule." They explored how the children in the video stories use recursive patterns, the challenge of finding explicit solutions, and how the children apply proportional reasoning as they search for the missing rule.

Teacher participants noticed the patterns that the children found and the specific recursive solutions that the children articulated. They identified the progress that the children made: when they were successful in finding an explicit solution, and when they were not successful[11]. They discussed specific heuristic solutions, and whether a particular solution is correct or only partially correct. In one case they stepped through a solution to the "ladder problem" that began with an assumption that $f(2x) = 2f(x)$. This assumption of proportionality is extensively modified by the child in the Day 3 video story; a partially valid heuristic was created. The TAW participants discussed how the heuristic worked, where it was limited in its validity, and whether the child applied the heuristic rule consistently.

TAW participants were respectfully interested in the children's reasoning.

Throughout the mathematics discussions, the TAW participants expressed interest in and respect for the mathematical reasoning of the children in the video stories. This was evident in the time they took to analyze what the children were trying to say, and when they considered whether or not the children were listening to one another. At times, the teacher participants express admiration for the abilities of the children.

TAW participants explored conjectures about cognition.

In contrast to the actual mathematical behaviors observed and described, the TAW participants also considered what the children in the video stories might be thinking when they sought to explain why the children made certain decisions. They sometimes drew upon examples from their own teaching practices to support these explanations. The TAW

[11] These examples are discussed in this section of the Results Chapter, and they are evident in the TAW Video Story Descriptions in Appendix A.

participants sometimes sought to explain why they believed that the children understood something, even when there was little or no direct supporting evidence in the video story.

TAW participants explored mathematical principles.

Discussions that centered on the mathematical reasoning of the children in the video stories sometimes branched out into discussions of more general mathematics questions. For example, they had a discussion about when it may be true that $f(kx) = kf(x)$ if k is a real number.

TAW participants were focused on the mathematics in the daily video stories by the daily workshop questionnaires (Appendix C).

There are recursive and explicit solutions to each guess my rule problem.

In each of the daily video stories, children solve *guess my rule* problems in which they are given "clues" in the form of ordered pairs that the "unknown" rule produces. The ordered pair values are provided in charts, as in the video stories on Days 1, 2, and 4. In the Day 3 video story, the ordered pairs are constructed by the children.

In the Day 1 and Day 4 video stories, input values are called "Box" values and output values are called "Triangle" values. In the Day 2 video x and y are used. In the Day 3 video the children are doing the "ladder problem," in which the number of steps in a ladder (the input) determines the number of rods or building blocks that must be used to build the ladder (the output). In the Day 3 video story, the children construct ladders out of rods and generate the ordered pairs. In the Day 4 video story, the children are given (Box, Triangle) pairs as well as a new understanding of algebraic equations that can be true or false, as well as legal or illegal. The problem contexts and the level of challenge vary, but the theme of discovering a missing rule is part of each video story.

TAW participants pointed out that the children in the Day 1 video story seem to be just guessing (at first), when Participant #3 said, "*Guess and check. They really didn't have a clue, weren't sure what they were supposed to be doing but then I guess eventually some of them finally got a clue"* (TAW Day 1, Line 3). Teacher participants went on to discuss how the students initially seemed a bit bored, but they persisted in guessing and this persistence shows some confidence (TAW Day 1, Lines 2, 4-7). In his/her statement, Participant #3 pointed out that the children persist and progress from the "guess and check" to when "some of them finally got a clue." As the discussion developed, teacher participants noticed ideas and patterns that the children identify:

> Participant #2: *Even his thinking—even when he said, "Imagine you were in a factory"* . . . *so he really got the idea of about a function. I thought that was really good.* (TAW Day 1, Line 11)
>
> Participant #4: *And then, when they first started, it was like they were just throwing out numbers, but at some other point* [I] *could see that they now were making educated guesses because they were seeing a pattern that you see. They see okay, when its 4 it's this, when it's this, so you know when it's 2 you know that 2 has to* . . . *so they say that it was increasing* . . . . (TAW Day 1, Line 16)

In the statements quoted above, Participant #2 is describing how, during the problem solving (in the Day 1 Video story), Ariel tells the other children that a mathematical function, perhaps like their missing rule (TAW Day 1 Video Story Description, Event #4, Ariel) is "like a factory," where something goes in and something else comes out. Participant #4 was describing how the children began to see that the missing rule is an increasing "function." The TAW participants notice that the children persist and that their guesses turn into valid observations about functions and the particular rule they seek.

The TAW participants notice that the children do not find the explicit rule, $y = 3x - 2$, in the Day 1 video story.

Participant #4: *Yes, They didn't find* (the first rule).

Participant #2: *And they didn't come up with it*. (TAW Day 1, Lines 19-20)

Finding the explicit rule is hard.

On the first day of the workshop, the TAW participants tried to explain why the students did *not* come up with $y = 3x - 2$ from the data clues they were given:

> Participant #4: *I think . . . you know like . . . basically, when I was doing it. Was like, you had 5 first and then 3. Look at that. You are used to 1, 2, 3—but in this case they were decreasing. That could have accounted to why it took them so long for them to get the pattern in this one. And then, when they started getting numbers, the three numbers were in random order, and it did not help* . . . . (TAW Day 1, Line 22)

Participant #4 is suggesting that it might have been easier for the children to find the explicit rule if the clues, the (Box, Triangle) points, were given in order (for example: (1, 1), (2, 4), (3, 7), etc.) In fact, T/R1 introduces this *guess my rule* problem as a game and starts with Box = 5, telling them that the rule will give them Triangle = 13. Then the children suggest the next Box value ($Box = 3$) to find out what the rule does. Thus, the children start to find the rule looking at these two clues: (5, 13) and (3, 7). The Box values are not consecutive and not in order (TAW Day 1 Video Story Description, Event #1). Participant #4 considers this a possible reason for the children's failure to find an explicit rule in this case.

In the Day 2 video story, T/R1 is working with two children, Brandon and Yonny, on two new *guess my rule* problems. The points (the clues) *are* in order for both of these problems. The points given to the children are shown in Table 4 and Table 5 below. In spite of having well-ordered data, they still do not find an explicit rule. They devise recursive solutions for each. The first rule they are looking for in the Day 2 video story is $y = 2x + 1$, and the second rule they are looking for is $y = 2x + 5$ (TAW Day 2 Video Story Description, Events #1 and #8).

x	***y***
1	3
2	5
3	7
4	9
5	11
6	

Table 4: $y = 2x + 1$: The Rule for Day 2, Problem 1

x	***y***
1	7
2	9
3	11
4	13
5	15
6	17
7	19
8	21
9	23
10	25

Table 5: $y = 2x + 5$: The Rule for Day 2, Problem 2

The TAW participants discussed how Brandon and Yonny approach calculating the $y-values$ for specific new $x-values$ in the Day 2 video story. They describe (TAW Day 2, Lines 45-65) the recursive solutions that each of the boys articulates.

Participant #1: . . . *one was saying +1, +1, +1, +1* [#1 is quoting Brandon].

Instructor: *What was he talking about with the +1, +1, +1*?

Group: *The x's*.

Participant #2: *Yes*.

Participant #4: *Yes, he did*.

Instructor: *He's talking about the x's*.

Instructor: *What does Yonny say?*

Participant #5: *The y's* [this is what was said in the TAW, but it is not clear on the video story], *you add 1 and then you add 2 . . .*

Instructor: *You add 1 to what*?

Participant #5: *To the x*.

Instructor: *—right—you add 1 to the x, then you add 2 to the x, then you add 3. He was seeing a real different way than Brandon. Were they noticing? Did Brandon notice he was saying something different*? [The Instructor is accepting Participant #5's statement of what Yonny said, but it is not clear on the video story].

Participant #1: *No.*

Group: *No.*

Instructor: *Do you think? They didn't show that. Do you think that they heard each other anyway? And that maybe there was a valuable communication . . . .anyway?*

Participant#3: *Yeah..?*

Participant #5: *Sure.* (TAW Day 2, line 46-61)

Instructor: *So what do you think? Did they actually hear each other so that they could use it later*?

Group: *all mumble, "I don't think so*."

Participant #4: *I . . . but . . . really . . . think when he said +1, then you add 2, then you add 3, I think Yonny eventually got what Brandon was trying to say. Because if look at it, you add, if you're going to get 3..you have to add 1 to 2, you get 3, to get 4 you add the next number and the next number and that is what he was trying to say: +1, +2, +3,+4, +5.* (TAW Day 2, Line 64-66)

There is much to notice in this discussion among the teacher participants in the TAW. The TAW participants described each of the recursive ideas that the boys' expressed. However, what the boys say is not entirely precise, and the TAW participants are also less than precise in their descriptions. In reading the teacher participants' words above, we still cannot be sure exactly what they were referring to. The Instructor asked them to clarify multiple times. This is evident in the first three lines extracted from the transcript above when the instructor asked Participant #5 what the "+1 +1 +1 . . . +1" refers to, and Participant #5 said that Brandon is talking about the $x - values$. However, this is still rather a vague statement. It is the context of the *guess my rule* challenge in the video story, and the values given to the children as shown in Table 3, help us better understand this dialog. Participant #5 was quoting Brandon and Brandon *probably* means that the $x - values$ go up by 1.

Similarly, when Participant #5 described Yonny's expression of the recursive solution, "*the y's, you add 1 and then you add 2 . . . "* (TAW Day 2, Line 53), the Instructor paraphrased what Participant #5 may have been saying. The Instructor also accepted Participant #5's statement that Yonny was talking about the $y-values$. None of these purported facts is clearly true. Participant #5 may have been describing Yonny's solution, indicating that each new $y-value$ is the sum of two consecutive $x-values$ (TAW Day 2, line 56). For example, in Table 3, we have the points (1, 3) and (2, 5), and we can notice that the first $y-value = 3 = 1+2$ and the second $y-value = 5 = 2+3$. Participant #5 may have misheard Yonny or misread what Yonny intended to say. In the video story, Yonny clearly says, ". . . +1, +2, +3, . . . but it is not clear that he is talking about the $y-values$. Yonny might have been talking about the $x-values$ only—and expressing the same idea as Brandon, but in a different form: Starting with $x=1$, add 1 to get $x=2$, add 2 to get $x=3$, etc. We can argue for either possibility, but we cannot be absolutely sure what Yonny meant.

It is important to note that, in the video story, the lack of precision on the part of the children may reflect a lack of formal mathematics language.[12] The Instructor used the context to try to clarify Participant #5's statement, and Participant #5 did not attempt to clarify further. Both were reading specific meaning into each of the boys' statements; they were reaching for more precision than is actually articulated by the boys.

When the Instructor asked the TAW participants if Brandon and Yonny perceived the differences between their explanations (TAW Day 2, Line 56), the group indicated that they did not (TAW Day 2, Lines 57-58). However, the teachers seemed to change their minds when asked if the communication between Brandon and Yonny was "valuable anyway" (TAW Day 2, Line 59). They said that it was (valuable) (TAW Day 2, Lines 60-61). Note that it appears that

[12] The mathematical register.

the teacher participants are changing their minds when they indicate that the communication was valuable “anyway”. This researcher wonders if the teacher participants were trying to please the instructor (also this researcher) by saying that the communication was valuable, or if perhaps they were taking a pedagogical view that communication between students is, a priori, valuable.

Then Participant #4 offered a way to resolve the discussion by suggesting that the children in the video story are really saying the same thing (TAW Day 2, Line 66).

Was this just an attempt to reconcile the discussion in the TAW? There was no pressing need to do this since the TAW participants were not being challenged to reconcile their different responses. Perhaps the discussion was simply thought provoking and this led to Participant #4’s interpretation. Perhaps Participant #4 realized that Yonny might not have been talking about the $y-values$ at all. Consider again what Participant #4 said:

> Participant #4: *I . . . .but . . . really . . . think when he said +1, then you add 2, then you add 3; I think Yonny eventually got what Brandon was trying to say. Because if look at it, you add, if you're going to get 3 . . . you have to add 1 to 2, you get 3; to get 4 you add the next number and the next number and that is what he was trying to say: +1, +2, +3,+4, +5*. (TAW Day 2, Line 66)

Participant #4 may be (agreeing with Participant #5), suggesting that Yonny is giving a (correct) recursive algorithm for finding the $y-values$. If so, it seems wrong to say that Yonny “got what Brandon was trying to say”; everyone had agreed that Brandon was talking only about the difference between the $x-values$. It seems more logical to argue that Yonny was giving another way of generating the $x-values$. (Starting with $x=1$, add 1 to get $x=2$, add 2 to get $x=3$, etc.)

Although #4's interpretation may not correctly describe what Yonny was thinking (something that is impossible to confirm), the interpretation may also demonstrate an effort to find meaning in the student's work and to recognize a possible collaboration between the students. This is valid regardless of which interpretation of the TAW participant's words we accept. This also reflects sustained consideration of the children's mathematical statements.

Respect for the mathematical reasoning of children is evident.

Participant #4's argument suggests a connection between the reasoning of the boys and could be used to support a conjecture that they are listening to each other— even though the TAW participants previously indicated that they did not think so. However, Participant #4 clearly made an effort to reconsider what the children in the video story mean by each of the solutions they describe. This can be considered as demonstrating respect for the mathematics that the children are doing.

Is this the type of respect that produces patience: the patience that TAW participants noticed and attributed to T/R1 (in the video stories on Day 1 and Day 2)? It may be much simpler to respect the work of children doing math in a video story. (This could be because the teacher is free to watch and think without a class to manage. This could be because the teacher is being shown the video story in the context of a workshop that makes it appear important.) Nevertheless, the TAW participants clearly noticed the respect and patience that teacher researchers in all the video stories displayed (Pedagogy Results section in this chapter). Furthermore, they indicated that this was something they connected to student learning and something they aspired to for themselves. As such, it is possible that Participant #4 was showing respect for student mathematics as s/he considered the children in the video stories.

Are They Gifted?

TAW participants' respect for the mathematics done by the children in the video stories was implied when Participant #3 asked if the students in the videos might have been selected for the IML afterschool program (Informal Mathematics Learning, National Science Foundation Award REC-0309062) because they were "gifted" in math.

> Participant #3: *These students who are in the afterschool math program, do you know—could anybody be in that program or do you had to have a little something on the ball?* (TAW Day 2, Line 77)

This statement may seem out of context because it is not about the specific mathematics problem that the boys are exploring. However, it *is* about the mathematical knowledge/talents that the boys appear to have, and which moved Participant #3 to ask if they are not just "any students," but students with some extra "something on the ball?" Consider that this statement emerged early on the second day of the workshop, just after the boys work on the rule: $y = 2x + 1$.

While lack of comment is not conclusive, note that none of the TAW participants challenged this question from Participant #3. The teacher participants in the TAW seemed to notice that the (quite ordinary) students in the video story are doing, perhaps, extra-ordinary work. During the TAW, the workshop Instructor does explain that the boys in the video were *not* selected to be in the afterschool program because they qualified as gifted or talented; the Instructor explained that one boy in the video, Ariel in the Day 1 and Day 2 video stories, is an ESL student. Note that the subject of teacher respect for student mathematical ideas will be addressed again later, in the section on student engagement.

Is recursive reasoning important?

During the TAW, the daily questionnaire[13] guided the discussion and brought the teacher participants back to this question: What do you think the students understand and what don't they understand?

> Instructor: . . . *So let's go back to the . . . from the first page of the question and talk about what you think they know, because we talked about that a little bit already. So, I am interested in what more you came up with. For the y=2x+1 the very first guess my rule problem, Brandon and Yonny*.
>
> Instructor: *What, what do you think they understand; What do you think they don't understand*? (TAW Day 2, Lines 313-314).

In response, the teacher participants pointed out that Yonny and Brandon are *not* finding an explicit rule:

> Participant #1: *They understand, um, the patterns between like going down x, +1 +1, y's are, you know, can see those patterns but . . .*
>
> Instructor: *The recursive thing*.
>
> Participant #1: *Yeah, yeah, yeah—but they just can't really can't get rule. Like: What do I do to x to get the y?*
>
> Instructor: *Do they think about what it is, what do I do to the x to get the y? Did anybody say that*?
>
> Participant #5: *Very little.*

[13] TAW Daily Questionnaires, Appendix C

Participant #4: *They're not really thinking about x and y*. (TAW Day 2, Lines 315 – 321)

While the TAW participants discussed how the children in the video stories were focusing on recursive solutions, they did not return to their earlier interest in making explicit solutions easier "to see" (TAW Day 1, Line 22). They began to discuss what the children are thinking about: They said that the boys are looking for recursive patterns and "not really thinking about x and y" (TAW Day 2, Line 321). Perhaps the TAW participants were starting to think about recursive solutions as a learning phase that requires patience? Perhaps it is not really a matter of putting the x values "in order"?

The children do find explicit solutions with pattern recognition.

The children in the video stories do find explicit solutions to some of the *guess my rule* problems. In the Day 1 video story, the second *guess my rule* problem is given in the form of ordered pairs as clues: (1, 15), (2, 15) and (3, 15). Once these clues are presented, the children quickly know what the rule will produce for Box = 4, Box = 5, etc. The boys don't take too much time figuring out that the rule is "times 10 plus 5" (TAW Day 1 Video Story, Events #7 - #10).

The TAW participants discuss this and raise several points: The children could "see a pattern" in this second problem in contrast to the more elusive pattern of $y = 3x - 2$, and T/R1 knew that they would have success with this challenge. They pointed out that T/R1 may have wanted to give the children a "success" after they failed to find the rule for $y = 3x - 2$.

Participant #2: *I also think that by choosing the right numbers. Or having the right numbers . . . I think maybe the second problem was easier because they conclude they could see a pattern even in the beginning. Yeah even . . . when he started to identify there was a clearer pattern*. (TAW Day 1, Line 14)

Participant #2: *And they did find an explicit function for the second problem. Could that have been predicted by the teacher? I think so because of the number pattern.s Right?* (TAW Day 1, Line 21)

Participant #4: *1 ->15, 2->25, so you could see. The numbers for the* [first] *one were random. So they could have predicted this.* (TAW Day 1, Line 23)

Participant #4*: They did not come up with any rule . . . so maybe he wanted to give them another chance to go and do the second problem. When they do the second one, they might go back . . . sometimes makes it easier*. (TAW Day 1, Line 41)

The TAW participants did not talk about number or raise the possibility that the numbers 15, 25, 35, etc., may have been much more familiar to the children in the video stories. It is possible that this is why the children could "see" the pattern and the explicit rule more quickly.

Discovering secrets is interesting.

In the Day 4 video algebra story, T/R3 talks to the children about scientists discovering "secrets" before they begin their work in algebra. The discussion engages the children in the idea that there are secrets that no one has yet discovered, that scientists are working on. They also discuss the idea even if we know that "someone" has already discovered a secret, we may want to discover it for ourselves.

Then T/R3 shows the children equations written in "Box" and "Triangle" format (without x and y), and together, they create true statements that are "legal" and true statements that are "illegal," and false statements that are "legal" and false statements that are "illegal." The children are being introduced to the semantics of algebraic equations before they are given a *guess my rule* problem (TAW Day 4 Video Story Description, Events #2 and #3).

Then they are given a table of (Box, Triangle) ordered pairs that make a secret rule true and legal. They are given:

$\square$	Δ
0	1
1	3
2	5
3	7
4	9

Table 6: Day 4 Secret Rule 1 is $2\text{box} + 1 = \text{triangle}$

$\square$	Δ
0	1
1	4
2	7
3	10
4	13

Table 7: Day 4 Secret Rule 2 is $3\text{box} + 1 = \text{triangle}$

Unlike the video stories on Days 1, 2, and 3, in this Day 4 video story, the children first use rules and find (Box, Triangle) pairs (points) that make the rules true (and legal).

The children in this video story do find explicit rules and they identify key parts of the equation; they point out that the difference between the "Triangle" values is "the number next to "Box" (the slope) and that the Triangle value that corresponds to "Box" = 0 is the constant in the equation (the y-intercept). These are the secrets the children discover.

The TAW participants did not discuss the explicit solutions that the children in the Day 4 video found. They did not discuss the secrets that the children discovered. They did discuss their view of the model for algebraic equations that is presented in the Day 4 video story, where equations can be true and illegal or false and legal:

> Participant #3: *Putting myself back . . .trying to put myself back. you probably all . . . going back to that grade . . . .thinking . . . .I don't know that I would I have liked the legal thing—think of it that . . .*
>
> Participant #2: *Yeah, I have to think about that, too.*
>
> Participant #3: *Okay, okay, I think I would have to, like, "What are you talking about, "legal?"*
>
> Participant #5: *Even now, like when I saw the problem, and I saw "legal" . . . . I got the true and the false because I've heard that enough, but "legal" and "illegal" . . . I'm like . . . what is this?*
>
> Instructor: *Right, it was not familiar, but once you know what they mean it's not hard . . .*
>
> Participant #3: *As soon as they caught on. . . .I ?? . . . stopped thinking about it . . . and now I'm pretty . . . good.* (TAW Day 4, Lines 118-122)

After discussing their initial encounter with the idea of legal and illegal values in an algebraic equation, the teacher participants concluded that this was a useful model that they would want to adopt in their own classrooms:

> Participant #4: *I really like the words "legal" and "illegal" . . . .I think that sometimes I would use in my algebra class . . . you know? Because it's . . .* (TAW Day 4, Line 126).
>
> Participant #1: *Getting. . . to think about . . . the parameters of an equation, there are*

certain numbers that you can use and there are some that you can't . . ..so . . . . (TAW Day 4, Line 132)

Participant #5: *I agree . . . . with #4. . . . I think I'm definitely going to use it. When we're evaluating expressions and they see "x^2 + x,"and they plug the x value into x^2 and then they would leave . . . "What do I plug into x? " . . . I'm like: "The same value. (Everybody laughs.)* (TAW Day 4, Line 134)

They did discuss the idea of "discovering a secret" as a pedagogical tool for engaging students in exploring the solutions for algebraic equations:

Instructor**:** *This is where Ankur says, "We shouldn't" and Jeff says "we should". . . that's after she does the "zero and one" thing* (TAW Day 4, Line 304).

Instructor***:*** *Okay, so that's what I am asking about . . . he's ready to share and one kid says, "No, I don't want them to share," and . . . so what do you think is going on?*

Participant #4: *Maybe he wasn't done yet . . . maybe he was yet to find the secret, that he didn't want anybody to reveal it yet because he was working on it.*

Participant #3: *He wants the challenge.* (TAW Day 4, Lines 306-308)

The teacher participants also pointed out the advantage of having the students use a "rule" to generate "points" before using points to find the rule:

Participant #2: *I think we were talking about how the class was, like, so enthusiastic*.

Participant #4: *Yeah!*

Instructor: *But that was true of the other group, too*.

Participants #4 and #2: *Yeah*.

Participant #1: "*I think the difference is, in the other one (T/R1) he just gives them, like, the table, alright, "go find the rule." And this one (T/R3), he gives them almost kind of* *like a "Do Now," you know different skills going, see what skills they need in order to do this.* (TAW Day 4 Lines 85 – 89)

In this discussion, the TAW participants are comparing T/R3's approach to T/R1's approach. They refer to the part of the Day 4 video story in which T/R3 discusses some rules with the children, and they practice finding "points" (Box, Triangle) pairs that satisfy a given rule. This is what Participant #1 is calling the "Do Now." The TAW participants pointed out how enthusiastic the children in the Day 4 video are, and in the same discussion they pointed out the value of using a rule before being asked to discover a rule. As Participant #1 said, the children get to "see what skills they need in order to do this" (TAW Day 4, Line 89).

It is interesting that the Day 4 video story did not inspire discussion of the explicit solutions and the secrets of linear functions that the children discovered. Perhaps the fact that this video story has entirely different participants and, even though it is another "guess my rule" story, it is not as connected to the other three stories. Is this shift in the content of video stories the reason that there was less interest in the work the children did? Or perhaps the TAW participants were merely tired by Day 4 of the Workshop

TAW participants consider the choice to use proportional reasoning.

After confirming that the children are thinking primarily about recursive solutions, the TAW participants discussed the "proportionality shortcut" that Brandon and Yonny choose when faced with a large $x-value$. This is interesting because it appeared that the boys had rejected a "doubling rule" earlier:

Participant #5: *But I think you can have, um . . . towards the beginning of . . . what if x = 20? I think even Yonny had an understanding of how to do 2 x 20 or 2 x10 or 2 x6. But . . .*

Instructor: *Why do you think that*?

Participant #5: *Because he said, "You double it." That's where the doubling happened, and Brandon kind of shut that down.*

Instructor: *Before Brandon.*

Participant #5: *Then he said, "Six times 2 is 12."*

Instructor: *Right. He points to the 6 and the 13.*

Participant #5: *Yeah*.

Instructor: "*There's nothing that's doubling," he said, "so it doesn't work here." Did you notice that*? (TAW Day 2, Lines 349-356)

This dialog reflects the attempt of TAW participants to understand what the children did and why. Consider what the TAW participants saw in the video story and what they are providing as evidence[14]. When the children begin working on this first *guess my rule* problem on Day 2, they use the given set of clues/ordered pairs (shown in Table 3: Day 2: Guess My Rule Problem 1). Brandon notices that that the rule is not "multiplying by 2"; he notices that when $x = 6$, $y = 13$. Brandon shows T/R1 how each $y - value$ = sum of the "current" $x - value$ and the "next" $x - value$. He shows T/R1 how $3 = 1 + 2$, $5 = 2 + 3$, $7 = 3 + 4$, etc. But Yonny tells Brandon, "It just doubles by 2" (TAW Day 2 Video Story Description, Event#5, Yonny), possibly seeing that the difference between the $y - values$ is always 2. We also see that Brandon rejects "multiplying by 2" as the "rule," showing Yonny where it doesn't work (Event #4,

[14] Day 2 Video Story Description in Appendix A.

Brandon). Brandon works step by step to find all the $y-values$ for all the $x-values$, from $x=8$ to $x=20$. He finds out that when $x=20$, $y=41$. The TAW participants were considering this discussion as they considered how the boys chose a proportionality assumption to find the $y-value$ when $x=100$ (TAW Day 2, Lines 351-364).

The TAW participants discussed how the children did work on a purely proportional solution for finding the Triangle value when Box = 100. The instructor had described what happens in the video story:

> Instructor: *Yeah, that's kind of interesting, right? They did the strict proportional thing . . . you know? 20 goes into 100 five times, so the answer for 20 should be multiplied by 5 to get, you know, the* [y-value] *. . . .for 100. The triangle value for 20 was 41, so they said, "Multiply 41X5, like you multiply 20 by 5. And then Yonny says, "205, . . . [pause]. . . no 201.*" (TAW Day 2, Line 90)

In the Day 2 video story, Yonny comes up with the proportional explanation first (TAW Day 2 Video Description, Event #6, Yonny). Then Brandon explains their logic to T/R1 while Yonny looks on. At the very end of Brandon's explanation, Yonny says "205, no, 201" (TAW Day 2 Video Description, Event #7, Yonny). The TAW instructor asked the teacher participants, *" . . . what was Yonny doing before he said that*?" Yonny had a clear and logical reason for his (wrong) answer of 205, and Brandon had agreed with him. The TAW instructor asks the participants more than once: What do you think was happening (TAW Day 2, Lines 108, 116, 118)? The answer is expressed by one participant, and then echoed (TAW Day 2, Lines 112,113, 115, 117) by all of them:

> Participant #3 : *He's thinking***.** (TAW Day 2, Lines 112)

In the video story, Yonny has been whistling and banging, and appears to be listening to Brandon recite his own proportional argument explaining that Triangle = 205 when Box = 100 (TAW Day 2 Video Story Description, Event#7, Yonny). Then, Yonny just dismisses the first answer and says that the answer is 201 (Event #7). He does not give any reason for the different answer, but his answer *is* correct. When and how did Yonny figure this out? Yonny's behavior during this episode offers no evidence that he is engaged in thinking about this problem.

The TAW participants suggested that he is thinking about the problem while whistling and watching (TAW Day 2, Lines 38, 189, 481), and we do not know how he comes up with the new answer. They recall Brandon's argument that "nothing is doubling" (TAW Day 2, Lines 356). Participant #4 points out:

> Participant #4: *Well, is that something else that they never brought up. I think maybe if Yonny had persisted in thinking about it, I think maybe he would have come up with why . . . without one like ,um,. . . .* (TAW Day 2, Lines 359)

Participant #4 may have been suggesting a possible connection between the "this is not a doubling rule" discussion and Yonny's final correct answer. Participant #4 may be suggesting that this discussion is the seed of an idea that helps Yonny figure out that the correct Triangle value = 201 (when Box = 100). Is this the "value" in the interaction between Brandon and Yonny on the doubling question?

> Participant #4: *Thinking about 205, there is something about 205 that doesn't seem right, you know, that one isn't 5 times 21. That wasn't the way it was done before, you were not supposed to multiply two times 21 to get 41, you know? So why would you say 5 times 21 to get 201? So, I think he had a lot of things going on in his head.* (TAW Day 2, Line 487)

The TAW participants concluded that Yonny was thinking about the mathematics. Multiple responses indicated agreement that Yonny was thinking about the problem in some way (TAW Day 2, Lines 113, 115, 117). Of course, we do not know what Yonny was actually thinking: Was he thinking while he was whistling? Did Brandon's earlier argument influence him? Did he figure out the new answer, or did he guess? The teacher participants may have concluded that Yonny was thinking because he changed his mind from an incorrect to a correct solution.

We can consider that, with some frequency, mathematical thinking may be happening even when students appear to be distracted. We can also consider the implications for the mathematics dialog in a class where this possibility is always present in the teacher's mind.

Proportional reasoning complicates the ladder problem.

In the Day 3 video story, Ariel is working on the Ladder Problem (TAW Day 3 Video Story Description, Appendix A). He is using rods to construct ladders and he learns that a one-step ladder requires five rods and a two-step ladder requires eight rods:

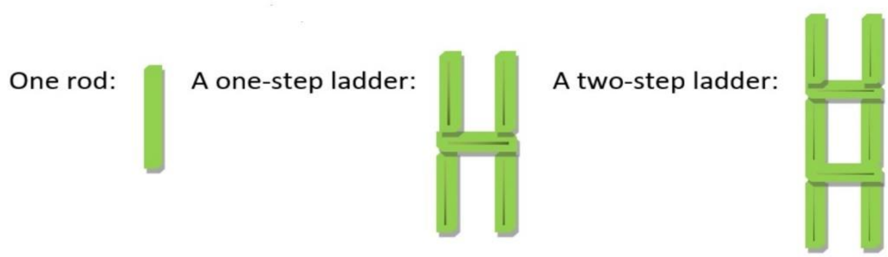


Figure 2: The Construction of Ladders in the Ladder Problem

In the Day 3 video story, T/R2 asks Ariel how many rods he would need to build an 8-step ladder. Ariel decides to build a four-step ladder, count 14 rods and then double the number of rods in the four-step ladder. Ariel starts solving the ladder problem with an assumption of proportionality. He decides that 14 X 2 = 28 rods in an eight-step ladder. After explaining this to T/R2, Ariel actually constructs an eight-step ladder, and realizes that it uses only 26 rods. The TAW participants discuss Ariel's problem-solving process:

> Participant #4: *I think he got, what was it? 14, and so he did the eight steps and he expected to get 28. Now he got 26 and he says "Heck, what*? (TAW Day 3, Line 75)
>
> Participant #3: *And he looks up*.
>
> Participant #1: . . . *wasn't gonna work.*
>
> Participant #3: *At some point* (he) *changed . . . and built the whole thing, right*?
>
> Instructor: *James tries to explain* [it] *to him. But then what does he do*?
>
> Participant #5: *He's going with his doubling theory*. (TAW Day 3, Lines 77-81)

The TAW participants notice that Ariel does not give up on his proportional solution theory when he discovers an error. Instead he "fixes it." On Day 3, there are two video stories. One is the TAW Day 3 video story that is documented in Appendix A. The other is a video story, published and available for viewing on the Rutgers VMC, also about Ariel's work on the ladder problem. In this video story, James (the other child working on the ladder problem) gives Ariel a different and correct way of thinking about the problem by explaining "You add three rods," when you add a step to the ladder, and Ariel appears to hear him when, after a moment of thought, Ariel says, "That's amazing" (Tracing Ariel's Algebraic Problem Solving: A Case Study of Cognitive and Language Growth, Event #3).

However, Participant #5 points out that Ariel does not abandon his pursuit of a proportional solution; he adjusts his answer by "subtracting two" (Tracing Ariel's Algebraic Problem Solving: A Case Study of Cognitive and Language Growth, Event #4, Ariel).

> Participant #5: *He's going with his doubling theory*.

Instructor: *He*'s *doubling, but he sees that it didn't work*

Participant #3: *+2 and -2?*

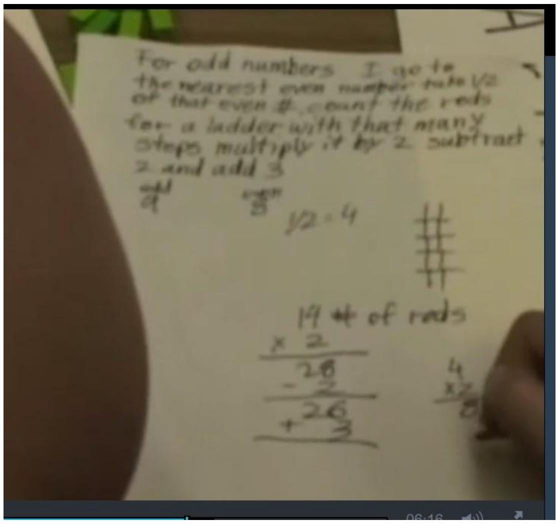
Instructor: *Because it's not 28. So what does he do*?

Participant #3: *modifies* (TAW Day 3, Line 81-85).

This is the first of several discussions of Ariel's evolving heuristic solution. Ariel's original idea for computing the number of rods in a ladder with "n" steps is: take the number of steps, divide by 2, build the ladder, count the rods and double the number. However, Ariel discovers that he needs to subtract 2 in order to get the correct number of rods for the ladder; Participants #3 and #5 recognized this modification. Ariel also learns from James (or by himself) that he can add or subtract three rods to add or subtract one step (TAW Day 3 Video Story, Event #1, Ariel).

The TAW participants discuss how Ariel progressively modifies his solution to deal with new situations. In this first example in the Day 3 video story (TAW Day 3 Video Story, Event #1, Ariel), Ariel explains that, for an odd number, he would use an even number first and then fix it. Ariel applies this rule as he (correctly) calculates the number of rods used in a ladder with nine steps. He explains his process and then he writes it down, as shown below:

- 9-1 = 8 and 8 is even.
- 8/2 = 4.
- 14 rods are used in a 4-step ladder.
- 14 X 2 = 28 and 28-2 = 26. There are 26 rods in an 8-step ladder.
- For 9 steps, add 3. 26 + 3 = 29 rods in a 9-step ladder.



The text in the image at left:

For odd numbers I go to

the nearest even number. Take ½

of that even #, count the rods

for a ladder with that many

steps. Multiply it by 2. Subtract

2 and add 3.

Figure 3: Day 3 Video Story Event #1 at 3:02

The participants discuss Ariel's modification of his solution to account for odd numbers of steps:

Participant #4: *That was . . . I think that was what James was trying to tell him —three— that created the factor of 3.*

Instructor: *Every step, yeah.*

Participant #4: *Every step you reduce by 3.*

Instructor: *That's really different. That's a totally separate rule: if you want to go from one size to another, what do you do? . . .[pause. . . murmuring] . . . .you want to go from 7 to 8 you . . . .*

Participant #4: *You add 3 more rods.*

Instructor: *You want to go from eight to seven[steps], you . . .*

Participants #1 and #5: *Subtract 3.*

Instructor: *So that's like yet another thing. What? Does he, like, come to this all by himself? Does he, you know, he's doubling and does he think, "I'd better check this"*?

Participant #1: *I think James kind of hints.* (TAW Day 3, Lines 103-111)

The discussion above shows that the TAW participants noticed Ariel's emerging algorithm and saw how Ariel probably learned something from James' remark about "adding 3" and incorporated it into his growing solution.

As Ariel continues work with T/R1, he uses his algorithm to calculate the number of rods in a 60-step ladder, a 120-step ladder, and a 125-step ladder. The TAW participants were asked to consider whether or not Ariel is using his algorithm precisely or if he is departing from it.

The dialog below is long and complex, reflecting the complexity of what Ariel does to figure out the number of steps in a 120-step ladder. For that reason, we are considering it in two parts. In part one, the TAW participants recognized that Ariel begins by using his algorithm and divides 120 by 2 to get 60. However, once he has to find the number of steps in a 60-step ladder, he departs from his algorithm, does NOT divide 60 by 2, but takes a shortcut and uses the fact that 60 = 6 X 10.

Instructor: *For this analytic, the first question is, "Explain how Ariel is using his rule in a precise way, in some or part of the work, you know, I mean in ALL or part of the work. So is he doing it all of the time or is he doing it some of the time?" And I think it's helpful to go through what he did so you can , , , all clear on what happened there . . . .*

Participant #5: *It changes there . . . he'll use it for when the numbers are small, but when it gets to 60 or 20, that's when he starts using proportional reasons and try to multiply by 10 instead of sticking to his rule . . . doing it.*

Participant #4: "*I think he actually tried to use his rule. But what happened is that . . . you know, like when he went from 7 to 8, that is only one step*.

Participant #1: *Yeah*.

Participant #4: *So he did his "minus 2." But now when he now did from 60 to 120 there are so many steps in between, so he didn't take that into consideration, so instead of subtracting like 18 or so, he's subtracting just 2 . . . . .*

Instructor: *So let's try to like unpack this a little bit. When he was given 120, he was like "uh, . . ., " so what exactly did he do*?

Participant #1: *He found six steps and then multiplied by 20 . . .*

Participant #3: *He went to 60, and that would be 6 times 10*.

Instructor: *I'll try to write that and you tell me . . . . So he does about 120 steps, that's the problem: how many rods*?

Participant #1: *So half of that is 60. So . . . .*

Instructor: *So he did 120 divided by 2 is 60, so he needed the number of rods for 60* [steps].

Participant #1: *Well he . . . find the number of rods for 6 then*.

Participant #3: *He knew 6*

Instructor: *So he didn't separate this* [the 60] *in half*?

Group: *No, mm,_mm,* (TAW Day 3, Lines 130-144)

The TAW participants were prompted by the Instructor to consider where Ariel is and is not sticking to his algorithms. When Participant #4 said that "from 60 to 120" there are "so many steps" that Ariel didn't consider (TAW Day 3, Line 134), the teacher participant explained the essence of the departure from the halving algorithm, but s/he did not describe Ariel's work precisely. Ariel is being true to his algorithm when he divides 120 by 2 to get 60. It's in the calculation of the number of rods in a 60-step ladder that Ariel deviates from the algorithm.

Consider that the actual number of rods required by a 60-step ladder is 3(60) + 2 = 182 rods (TAW Day 3 Mathematics Problems, Appendix B). When Ariel multiplies the number of rods in a six-step ladder by 10: 20 X 10 = 200, and then 200-2 = 198, he does ignore many required subtractions. Interestingly, although Participant #4 was not precise in characterizing the mistake as being part of "from 60 to 120," s/he was very precise in indicating what should be subtracted from 200: 200-18 = 182. The TAW participants are looking at Ariel's work very carefully. The discussion continues to detail what Ariel did and why:

> Participant #3: '*Cause he knew what 6 was and he already had [not clear] . . . contact again . . . or something.*
>
> Instructor: *That's not his rule . . . But what did he do?*
>
> Participant #4: *It was.*
>
> Instructor: *60 = 6X10, and what did he know? What did he know*?
>
> Participant #4: *Which one of these . . .* ? (TAW Day 3 lines 145-149, 151-158)
>
> Instructor: *He knew 20 rods in six steps.*
>
> Participant #4: *Right.*
>
> Instructor: *What does he do now?*
>
> Participant #4: *Multiply by 10.*
>
> Instructor: *20 X 10 = 200. Then . . .*
>
> Participant #4: *Then he subtracted 2 – that's his rule.*
>
> Instructor: *200-2 = 198.*

Participant #3: *That's a beautiful thing though* . . . . (TAW Day 3 Lines 145-149, 151-158)

The TAW participants recognized precisely what Ariel did to determine (incorrectly) that a 60-step ladder uses 198 rods, and Participant #3 expressed appreciation for Ariel's work in the last statement (TAW Day 3, Line 158). This is worth considering for a moment. Ariel has worked very hard to create an algorithm that did work correctly for him in some cases. Then Ariel is challenged to use the algorithm for a much bigger number. He begins by being faithful to the algorithm, but takes a shortcut that creates an error, and then returns to the algorithm.

The TAW participants continue to discuss Ariel's solution to the ladder problem with 120 steps in the ladder. At this point, they zero in on where Ariel begins to depart from his own algorithm:

Participant #4: *When he multiplied by 2 he subtracted 2. But now he multiplies by 10 and he still subtracts the same "2*."

Instructor: *What should he subtract*?

Participant #4: *When he multiplies by 2, he subtracts 2, right?*

Instructor: *Right*.

Instructor: *Well, what is the answer for 60, we know the formula, right?*

Participant #4: *60 is, ummmm, 182*.

Group: *182* [a small chorus].

Instructor: *It would be 182*.

Participant #4: *So if he had subtracted 18 . . .*

Instructor: *What should he subtract from 200?*

Participant #4: 18.(TAW Day 3, Lines 197-207)

Participant #3: "*Subtract 2 from each one . . .* "

Instructor: *18 in . . . how would we get . . . ?*

Participant #3: *Subtract 2 from each one of 10 and then add 2. He did 10 of, 10 of . . . .* (TAW Day 3, Lines 209-211)

The TAW participants worked to figure out how Ariel did and did not use his own algorithm, and then they laughed and said the work had made them tired and "making my head hurt" (TAW Day 3. Line 220, 223, 225).

Participant #4 revived the detailed discussion to consider how Ariel might have used his heuristic algorithm to get the correct answer, that 182 rods are needed to build a 60-step ladder:

Participant #4: *I am trying to understand something. I hope I will get it. Why the 2 and not 20? I think I get that one . . . .* (TAW Day 3, Line 290)

The TAW participants continued to think about how to justify subtracting 18, and how to do so in the context of Ariel's algorithm (as if we had to justify it to Ariel). Some of the work they did was done on the whiteboard in the room where the workshop was held (TAW Day 3, Lines 124, 126, 315). As they began to understand how to apply Ariel's algorithm, Participant #4 led the discussion:

Participant #4: *Because, for the 20 rods, you already subtracted 2. To get the 20 already, subtracted a 2. Okay, so now you have 10 steps; he's going to subtract 9X2.*

Instructor: *Okay, say that again? I'm not . . . .*

Participant #4: *What I'm saying is this. Look here, we said he subtracted just 2 (possibly pointing to the complex problem we discussed earlier where the answer was 182, he subtracted "just 2" from 200 to get 198 (the output for 60), and we said, earlier, that he* *should have subtracted 18 (which is 9x2). So in my mind, I am thinking, "He should have subtracted 20." But he subtracted . . . .*

Instructor: *He had to subtract 18*.

Participant #4: *Yes, because he already subtracted 2 to get his 20, . . . for the 20 rods.*

Instructor: *But the 20, this is correct . . . . .*

Participant #4: *Yeah, I know, I know. For him to get that, what did he do to get that 20 rods*? (TAW Day 3, Lines 295-301)

The teacher participants worked out a version of Ariel's algorithm that generalized the algorithm by modifying the "subtract 2 when you double" component, to "subtract 2 when you add two ladders." This may or may not be considered a significant modification, but the discussion about subtracting "20 or 18" is logical in the context of adding six steps (with the required 20 rods) 10 times.

It is true that we can justify subtracting 18 if we think about 6 X 10 as repeated addition. A six-step ladder uses 20 rods. Each time we "add six steps," we add 20 rods and subtract 2. Starting with the six-step ladder that uses 20 rods, we only add six steps (and 20 rods) nine more times to produce a 60-step ladder. Each of the nine times that six steps are added, we need to subtract 2. The details of this solution are in Appendix E.

The focus on this heuristic makes it seem like important work. TAW participants noticed that Ariel worked hard on it and that, although it is not "correct" in the narrow sense, it is interesting mathematics. This focus reflects patience on the part of the TAW participants as they worked through the details in two stages: The first stage of inquiry started with a question from the instructor about the video story, but the second, more detailed inquiry, started with a question from Participant #4.

The way the TAW participants adjusted Ariel's algorithm to work for the 60-step ladder indicates interest in Ariel's work and in the mathematics. The small step of changing "subtract 2 on doubling" to "subtract two on adding ladders" made the algorithm work. The TAW participants did not articulate an awareness of this change.

This researcher noticed that one additional, smaller change is necessary for this algorithm to work more completely: it is not necessary to add "congruent" ladders. We could add ladders of any size together and we would only need to add the rods and subtract 2 to get the correct answer (the correct total number of rods for the new ladder). Nevertheless, the work of the TAW participants in understanding and building upon Ariel's heuristic shows interest in Ariel's work that may be considered implicit praise.

The TAW participants did discuss mathematics that the children in the Daily video stories did not do, but which their work inspired. This is one of those discussions. The next discussion was also inspired by the mathematical reasoning of the children in the video stories.

Doing mathematics is figuring out what is true!

The children in the Day 2 video story work on finding the rule $y = 2x = 1$ by using the clues they have been given. Brandon sees a recursive pattern that he explains to T/R1 to justify saying that when $x = 6$, then $y = 13$. But when T/R1 asks Brandon to find the y-value when $x = 20$, he uses recursive patterns for "x" and "y" to find all the points, one by one, from (6, 13) to (20, 41). When the boys are challenged to find the y-value for $x = 100$, they fall back on proportional reasoning: if $x = 100 = 20 \times 5$, then it should be true that $y = 41 \times 5 = 205$.

In addition to what they noticed about the mathematical reasoning of the children in the Day 2 video story, the TAW participants became interested in bigger mathematical questions that the video story inspired:

Why does Brandon's recursive algorithm work?

The TAW participants take a break from analyzing the children in the Day 2 video story and think about Brandon's recursive algorithm: $y =$ current $x - value$ + the next $x - value$. Why does this work? They raised this question during the discussion and someone in the workshop suggested looking at a linear function with a slope of 3:

> Instructor: *Some of you said, somebody said something about it . . . .it really wouldn't work, so I just put an example up there* [on the board]. *There . . . some food for thought, one of these doesn't . . . and those do.*
>
> Participant #1: *Well, if the slope was 3, you mean?*
>
> Instructor: *Yes, I did a 3x+1 table. I didn't want to go that far; I didn't have the room [laughs]. So I used 3 and 9. You know 3's. So I did 3x3 is 9, like here, and then I added the value for 3 and I got 19, not 28. So it clearly doesn't work. You know, what's the thing missing here*?
>
> Participant #1: *It would have worked . . . you wanted . . . you're going by 3 this time.*
>
> Instructor: *Because it's a, it's a slope of 3, I figured I'd do that . . . but*
>
> Participant #1*: It doesn't work, really?* (TAW Day 2, Lines 139-144)

The TAW participants were looking at sample points for the two different rules under discussion. The dialog indicates that they were not quite sure why Brandon's rule works for $y = 2x + 1$, and does not work for $y = 3x + 1$.

x	y = 2x+1
5	11 = 5+6
6	13 = 6+7
7	15 = 7+8

x	y = 3x +1
5	16
6	19
7	22

Table 8: Two Rules Discussed by the TAW Participants

The discussion continued:

Participant #5: *It only works . . . for a multiple of 2?*

Instructor: *Well, come up . . . try it with a multiple of 2 (10?) here . . . instead of a multiple of 3. Since we don't have a precise method there, it's kind of like . . . where we can try different ways, we just aren't sure. Let's do 4 and 2 . . . so, um . . . this would be 3 times 4 is 12, plus 1 is 13. So I want to get 4's value, too. So 2 times 2 equals 4. The value at 2 is 7; the value at 2 equals 7. In this we get . . . the 7.*

Participant #1: *I think it works for the 2 because if you break 20 + 21 down, that's 20 +20 +1, so you have those doubles there*.

Instructor: *You have the two X's in there*.

Participant #1: *Yeah —here you're only having 2 X's when you need 3 X's, so you would have to double the 3*.

Instructor: *You need another x, another 2, and it would work. But you don't have—his rule didn't make a place for that. So, so that works for there . . . that's really, that's cool. So, this is missing one of the x's. One of the . . . 2's. It just doesn't, it doesn't scale up. This other thing, this is the full thing, I'm not sure how he saw it; it does work. It doesn't work for all the . . . [inaudible] of it . . . . So one of the things I . . . kids make them up, they are interesting, they reflect reasoning, you know* . (TAW Day 2, Lines 145-150)

Participant #1 realizes why Brandon's algorithm ($y =$ current $x - value$ + the next $x - value$) works for y = 2x+1, but is still thinking about what an equivalent recursive solution might be for the y = 3x+1 rule. Brandon's rule generated interest and exploration from the TAW participants.

When is it true that $f(kx) = kf(x)$ where $k \in R$?

After exploring the decision that Brandon and Yonny made (if , $x = 100 = 20 \times 5$ then it should be true that $y = 41 \times 5 = 205$), the TAW participants considered the question: when is it true that $f(kx) = kf(x)$? They are focusing on $k = 2$, and the first part of the discussion is about linear functions. They decide that this property holds when the constant (the y-intercept) is zero.

Participant #4: *Umm, when* $f(x) = x$, [then] $f(2x) = 2x$ [and] $2x = 2f(x)$ *And I think. . .*

Instructor: *Does it work in any other situation*?

Participant #4: *Constant, if it is a constant*.

Participant #5: *If there's a* . . . .

Instructor: *Oh if* $f(x) = 6$ *[laughter]* . . . .

Participant #4: Yeah, then . . . . (TAW Day 2, Lines 596-601)

The TAW participants realize that this property does not work for a constant function, but it does work for any direct variation where $f(x) = kx$ and the y-intercept is zero.

Instructor: $f(x)$ *would always be 6, though, it wouldn't work. If* $f(x)$ *is 2 all the time, then* $f(2x)$ *is also 2; it's not going to work there. What about* . . . . *Is there any other place where it works?*

Participant #5: *I think when you just have just the variable, you don't have the constant*. (TAW Day 2, Lines 602-603)

Then the instructor suggests that they think about calculus and the derivative of a function (TAW Day 2, Line 618). The TAW participants consider this idea and try it out by taking the derivatives of familiar functions to demonstrate that $\frac{d(2f(x))}{dx} = 2\frac{d(f(x))}{dx}$ (TAW Day 2, Lines 624-644).

In summary, the TAW participants noticed the mathematical reasoning of the children in all the Daily video stories. They explored the children’s reasoning and displayed interest and respect for their work. Although initial discussions suggested simple interventions to ease the path towards an explicit solution, in further discussions they probed the reasoning in the recursive and heuristic solutions presented by the children.

Similarly, the TAW participants explored the children’s use of proportional reasoning when they needed to deal with a very large input number. This exploration led them to their own mathematical search for functions that do have this proportional property.

The structure of the TAW provided video algebra stories and questionnaires that prompted the discussions in the workshop. As the many examples (provided from the workshop transcripts) indicate, the teacher participants found the algebra stories thought provoking, and they were easily drawn into pedagogical and mathematical discourse on many questions.

Student Engagement: What Do the TAW Participants Notice?

During the TAW, participants noticed many aspects of the engagement behavior of the children in the video stories while they were solving algebra problems. Participants noticed that the children seemed confident (TAW Day 2, Lines 373, 377; Day 1, Lines 7, 76) and that they "didn't have to be called on" (TAW Day 1, Line 105), were not afraid to speak (TAW Day 1, Line 5), appeared "enthusiastic" (TAW Day 1, Line 92), and persisted in their work without asking the teacher/researchers if they were correct (TAW Day 1, Lines 71, 74, 186). The participants pointed out how it appeared that the children in the video stories really cared about their work solving algebra problems:

> Participant #2: *I think it just came across that everybody cares about the work that they are doing. So it kind of makes it, there's a sense of [inaudible] for everybody involved. Even them* [the teacher/researchers], *even in the way that they presented it to them—like, "Today we're doing something different. I need your help with this." I don't know how they might have presented it to them (the kids), 'cause there's so much buy-in from everybody. Everybody's bought in*.
>
> Instructor: *You wish you could buy the "buy-in*."
>
> Participant #2: *Yeah, yeah, yeah*.
>
> Instructor: *That's a real nice way of putting it . . . .*
>
> Participant #2: '*Cause even when they're distracted or whatever they're doing, they do get back to the work. There's a lot of buy-in. They believe in what they're doing, it seems like*.
>
> Participant #3: *They're on task in their own way, in an acceptable way they . . . .*
>
> Participant #2*: Yeah*. (TAW Day 2, Lines 722-728)

It is important to consider that many of the observations made by TAW participants refer to behavior and affect that are overtly recognizable as characteristics of student engagement in mathematical tasks. These include visible aspects of the children's behavior, their affect (mood, level of enthusiasm), and their persistence in finding solutions. TAW participants noticed that the children in the video stories are persisting in their efforts to solve the math problems and that they are willing to share their ideas with little prompting (TAW Day 1, Line 105). The children test out their solutions and use the results of these tests to reconsider or confirm their solutions (TAW Day 1, Lines 74-77).

This section on student engagement will examine what the TAW participants noticed and discussed about these behaviors. This includes how the participants reflected on the difference between the engagement of the children in the stories and their perceptions of their own students. These results also include a discussion of how to engage students in mathematical exploration in a regular public-school classroom of 20 or more students and also meet the expectations of administrators.

The challenge of noticing student engagement is complex. When children in the video stories participate eagerly and show that they care about the work they are doing, the TAW participants noticed and commented on this level of engagement (TAW Day 2, Line 722). Recognizing less overt manifestations of student engagement may be more difficult and more subjective. This is evident in the way the TAW participants first considered one child, Yonny, in the Day 2 video story, to be disengaged in the *guess my rule* problem.

> Participant #1: *That really was the only time they were collaborating? 'Cause even though Yonny was the one that wasn't like really into the problem, right*? (TAW Day 2, Line 38)

When Yonny suddenly emerged from a “disengaged posture” to offer the correct answer, he was reclassified by the TAW participants as having been “thinking” (TAW Day 2, Lines 189, 193).

Participant #2: *I think Yonny was not engaged; he was just going along with it, but then as he started thinking about it.* (TAW Day 2, Line 189)

Participant #3: *He* [Yonny] *was thinking*. (TAW Day 2, Line 193)

This change of opinion serves as a reminder that many judgements about student engagement must be understood as subjective and difficult (if not impossible) to verify.

On Day 2, the algebra video story features Brandon and Yonny solving two *guess my rule* problems under the guidance of T/R1. In this episode of the video story, we see Brandon doing most of the work done on paper and leading the solution explanations to T/R1.

The boys begin with data points created from the rule $y = 2x + 1$ and identify recursive solutions, separate patterns for the $x - values$ and the $y - values$. Brandon finds a recursive solution in which the n^th^ $y - value$ is the sum of the n^th^ $x - value$ and the (n+1)^st^ $x - value$. Brandon uses this method to explain to T/R1 why $y = 13$ when $x = 6$; $y = 6 + 7 = 13$. As they work, T/R1, suggests larger and larger values of x. When they are given $x = 20$, Brandon does the detailed work of figuring out all the $y - values$ for all the integer values from $x = 7$ to $x = 20$; but he does not use the rule he articulated: $y_n = x_n + x_{n+1}$. Instead, Brandon works out each of the $y - values$, one by one (TAW Day 2 Video Story, Event #3, Brandon).

When T/R1 gives the children $x = 100$, Yonny is surprised and says, “You can’t make us do that” to T/R1 (TAW Day 2 Video Story, Event #6, Yonny). Yonny suggests a proportional solution to deal with $x = 100$; this solution is based upon their (correct) answer that $y = 41$ when

$x = 20$. The (incorrect) proportional solution reasons as follows: when $x = 100 = 20 \times 5$ it should be true that $y = 41 \times 5 = 205$ (TAW Day 2 Video Story, Events #6 and #7, Yonny). As Brandon explains this answer to T/R1, Yonny interrupts with the correct answer, $y = 201$, and the TAW participants discussed their "new" view of what may have occurred in this situation:

> Instructor: *201, that's . . . not strictly proportional. But what was Yonny doing before he said that? What was he sitting there doing? Besides the annoying whistle*?
>
> Participant #3: *Flipping all over, he's looking at his numbers, he's looking over*. (TAW Day 2, Lines 108-109)
>
> Instructor: *A difficult kid, but what's he . . . ?*
>
> Participant #3: *He's thinking*.
>
> Group: *He's thinking*. (TAW Day 2, Line 111-113)

During each day of the TAW, the participants watched the video story and then discussed what they noticed. On Day 2 this is also true. They began expressing the view that Yonny was not engaged in the problem solving, and then it was revealed that they did not all hear the moment in the video story when Yonny said, "No – 201." This part of the video is, in fact, quick and hard to hear, so it was replayed for the workshop participants.

> Instructor: *Yeah, that's kind of interesting, right? They did the strict proportional thing . . . you know? 20 goes into 100 five times, so the answer for 20 should be multiplied by 5 to get, you know, the . . . for 100. So, 20 was 41, so they said multiply 41x5, like you multiply 20 by 5. And then Yonny says, "205, no 201*." (TAW Day 2, line 90)
>
> Participant #3: *I didn't catch that. I read it, but I didn't catch it.*
>
> Instructor: *It's hard to hear it, he just kind of says it at the end there*. (TAW Day 2, Lines

92-93)

Analytic Playing and we hear Yonny say "*201*." (TAW Day 2, Line 95)

Instructor: *Did you hear that?*

Participant #2: *Yeah*. (TAW Day 2, Lines 97-98)

We *cannot* know for sure that a student is "dis-engaged" because we cannot know what is happening in his or her mind. Since Yonny's behavior did not indicate engagement with the problem, but suggested, instead, some level of distraction, his engagement with the problem was not initially detectible by the TAW participants. In the Day 2 discussions, the TAW participants said that Yonny was "not engaged." Then they realized that they had been wrong, once they heard Yonny provide a correct answer. In the face of evidence that Yonny figured out the correct answer, the TAW participants changed their view of his level of engagement. This single piece of evidence caused teacher participants to say that instead of just playing around, Yonny "was thinking" (TAW Day 2, Lines 112,113).

This episode in the workshop, when the TAW participants changed their opinion of Yonny's engagement in the algebra problem, raises questions about how teachers do and do not notice student engagement, which are addressed in the Conclusions and Discussion Chapter.

Persistent problem-solving reveals student engagement.

Yonny's behavior, in the Day 2 Video story, did produce many observations. Participant #2 noticed that Yonny may have objected to finding y when $x = 100$, but that he didn't become oppositional or angry:

Participant #2: *And as he* [T/R1] *challenges them, they don't get really frustrated, like they don't lash out at him. They are just like, "Oh, you're going to make us do 100, you know I . . .* (TAW Day 2, Line 181)

As s/he continued to discuss Yonny's level of engagement in the mathematics and the episode where he suddenly produced a correct answer, Participant #2 created a new narrative about Yonny:

> Participant #2: *It seems like at first maybe Yonny was in the lead? and (not?) engaged with it, he just went along with it realizing that something was all wrong.*
>
> Participant #2: *You know at first he was not really that engaged*. (TAW Day 2, Lines 186-187)
>
> Participant #2: *I think Yonny was not engaged, he was just going along with it, but then as he started thinking about it* . . . . (TAW Day 2, Line 189)

As they discussed the children's problem solving in the Day 2 Video story, the TAW participants articulated the level of engagement they saw in Brandon and Yonny. They also discussed the details of the children's problem solving and analyzed their words. Over several pages of discussion (TAW Day 2, lines 46 – 66), they describe the persistent efforts of Brandon and Yonny, just without labeling the efforts of the children as "persistent." The extent of the TAW participant discussion and analysis of the children's problem solving create a picture of the persistent problem-solving efforts of the children in the video stories. An example of a TAW participant analysis of Brandon and Yonny's work to discover the rule $y = 2x + 1$ is shown below. This is an excerpt of a larger discussion that is analyzed more completely in the Mathematics section of this Results Chapter:

> Participant #4: "*I. . . . but . . . really . . . think when he said +1, then you add 2, then you add 3, I think Yonny eventually got what Brandon was trying to say. Because if* [you] *look at it, you add, if you're going to get 3, you have to add 1 to 2, you get 3, to get 4 you*

> *add the next number and the next number and that is what he was trying to say: +1, +2, +3,+4, +5* . (TAW Day 2, Line 66)

The participants did not just talk about examples of the work that Yonny and Brandon did; they analyzed that work and suggested possible meanings for that work, just as is done, above, by Participant #2. The TAW participant discussion and analysis of the children's work in the Day 2 video story spans several pages of the original transcript (TAW Day 2 Transcript, Appendix D).

In the Day 3 video story, Ariel works persistently on his heuristic algorithm for the ladder problem. The TAW participants discussed Ariel's work, and they specifically discussed his persistence. The participants tried to explain why Ariel is so deeply engaged, so persistent in finding a solution that works, without abandoning his first principle of proportionality:

> Participant #4: *The point is, sometimes when you are doing something, and that person has another way of doing . . . . but if you're convinced or have a conviction that what you're doing is going to work, it's very likely that you won't give up on what you are doing. You may not listen to the other person because you feel that "my way is going to work too." I think that was what happened to Ariel and James. Ariel was so focused on using his method to get the right thing. So he wasn't really listening or thinking about what James was saying. And I'm happy to hear James talk, finally.* (TAW Day 3, Line 48)

In the Day 3 video story, Ariel begins his work by letting T/R2 know that he has to "leave by 3:30." This is the first and last mention of his goal to leave at a particular time. Once Ariel begins trying to solve the ladder problem (Tracing Ariel's Algebraic Problem Solving: A Case Study of Cognitive and Language Growth, Event #2, Ariel), he makes an assumption of proportionality that constrains and motivates his work. The underlying rule of the ladder problem is $y = 3x + 2$; the solution is linear and not proportional. Ariel builds a four-step ladder and counts 14 rods in that ladder. Ariel then claims that there must be 28 rods in an eight-step ladder and when he discovers that there are only 26 rods, ($3(8) + 2 = 26$), he creates the first modification of his proportional solution (multiply by 2 and subtract 2) (Tracing Ariel's Algebraic Problem Solving: A Case Study of Cognitive and Language Growth, Event #4, Ariel) . Ariel develops his heuristic solution over time as he responds to questions from T/R2. In the Mathematics section of this Results Chapter, the TAW participant discussion of Ariel's solution is more complete.

As the TAW participants discussed Ariel's work, they considered questions: Did he really want to leave at 3:30 or was he just frustrated that T/R3 didn't understand his solution? Did Ariel really display frustration? He kept working. Ariel did speak about "leaving on time," but then he became focused on perfecting his solution to the ladder problem:

> Participant #4: *Before he* [T/R1] *was done with one, he would throw another number . . . and I think it's because Ariel told him, "Whatever you do, I am leaving here at 3. You'd better hurry up."* (TAW Day 3, Line 230)
>
> Participant #5: *He said "3:30., It's like 3:30 . . .*
>
> Instructor: *Do you really think he just wanted to leave*?
>
> Participant #2: *No, I think he just was, like, "You'd better get it, what I am trying to say to you." [Laughter].*
>
> Instructor: *He was trying to say*?

Participant #2: *Because maybe he was getting frustrated by the fact that the researcher wasn't getting it. He probably was like, you know when you "get something," you think like . . . .*

Instructor: *I told you three times, I'm leaving at 3:30*.

Participant #2: *Yeah, yeah, but he finishes it*.

Participant #4: *You know the times that he said, "I just finished this," or something. It wasn't finished; he said, I will just finish this, I think it was time to go but he still stayed to finish one more. But he just wanted the facilitator, or whoever, to know that "whatever you do, do it quick, because you know I have other things to do."* (TAW Day 3, Lines 232-239)

In this discussion, the TAW participants reviewed how Ariel claims that he has to leave by 3:30, but then they notice that this is followed by his persistence in finding a solution to the ladder problem. He stays to finish his solution. They noticed that Ariel may have wanted to leave, but he also wanted the researcher to understand his solution. They notice Ariel's persistence in finding a solution to the ladder problem appears to be more important to him than his earlier commitment to "leave at 3:30."

These discussions are evidence that the TAW participants noticed that the children in each video story are sufficiently engaged in a math problem to persist in their work. Perhaps persistent problem solving itself is the best evidence of student engagement.

The TAW participant discussions contrasted the persistent problem solving of the children in the video stories with many of their own students who "didn't like math" (TAW Day 1, Line 311) and "gave up" on problem solving easily (TAW Day 2, Lines 853, 855).

Participant #2: *This would be* good [A *guess my rule* lesson] for the opening *week of school—in that beginning part, in that getting to know you part maybe try to do that to keep them engaged in math when you are in there*.

Participant #2: *I hate to see what the kids—how do they feel about math, you know*?

Instructor: *Do you ever ask them? What math is? How do they feel about it?* (TAW Day 1, Lines 310-312)

Instructor: *What do they say?*

Participant #2: *It depends. Sometimes like "I hate math . . . ."* (TAW Day 1, Lines 314-315)

In this dialog, TAW participants are discussing the possibility of generating engagement "in the opening week" with a *guess my rule* problem that invites the students to persist in mathematical problem solving. They are also expressing regret about their own students who say that they "hate math." Participant #2 expresses some uncertainty about the ability to engage his/her own students with a *guess my rule* lesson when s/he says, "Maybe try to do that."

The TAW participants notice confidence and the absence of fear.

On Day 1 of the TAW, the video story featured five children (Ariel, Christian, Yonny, Brandon, and Dawud) who are solving *guess my rule* problems with T/R1. In this video story, the children are given one (Box, Triangle) pair to begin, and then they are asked to "guess" the Triangle value for one Box value at t time. As the game proceeds T/R1 lets them know what the Triangle values are (whether they do or do not "guess" correctly) and they have a growing set of (Box, Triangle) pairs to consider. The underlying rule in the first problem is $y = 3x - 2$.

The TAW participants noticed that the children visibly work to find each correct Triangle value and are not afraid to share ideas that might be wrong. This is interpreted as confidence and an absence of anxiety about whether or not they are correct.

> Participant #2: *One thing I thought was cool was that right away, they weren't afraid to make a mistake. They were saying whatever, so that was good. I guess, however, that class goes because nobody was afraid and no one picked on anyone when they said a crazy answer. They just let them say the crazy answer. But then they thought that maybe that could be a way to get the answer. So I thought that was good.*
>
> Participant #3: *In the beginning I thought they were bored* (*"yeah yeah"* in the background) . . . *Oh man, but then, pretty quickly he (T/R1) got it going pretty well, I thought*.
>
> Participant #2: *They had some kind of confidence that they were gonna be able to do this so maybe it wasn't brand new. Cause it seemed. Remember when he came, he said, "Oh we're going to be guessing a rule," so maybe some of them were familiar a little bit? Maybe it's why they were able, and felt comfortable enough to like "OK we can do this,, take the risk.* (TAW Day 1, Line 7)

The TAW participants discussed how the children seemed to "own their own learning" and seemed to enjoy what they were doing. They began to raise the question about whether the children seemed more engaged in the problem solving because they were in an afterschool program and not in a regular daytime classroom:

> Participant #2: *Now that she told us that it is an afterschool program, it's kinda like they were used to a discussion, maybe*? (TAW Day 1, Line 72)
>
> Participant #4: *They were somehow confident of what they knew. They didn't like . . .*

they said . . . the answer—they didn't need the teacher to validate what they knew. They were that confident. When he said, "You should try it out," when they tried it out, they didn't have to ask the teacher "Am I right?" They knew, yes, it works, "So I'm right."

Participant #3: *Or it doesn't, "So I am not right.*" [continuing the previous thought—if the children tried a solution that did not work.]

Participant #2: *Maybe because the part of the session with the teacher, he never gave them any kind of hints or clues—they were used to him saying "try it out." So maybe his approach of just "trying it out" made them have that confidence and then they have to prove it.*

Participant #4: *And then after a while they actually owned their own learning. It was just there. It was like . . . they almost forgot he was there* ["he" is TR/#1]

Participant #2: *It's true!! laughter.*

Participant #4: *They own their own learning—maybe that's why they didn't think to ask him, to refer back to him . . . in a way they were doing, they were having so much fun with each other that . . . .*

Participant #2: *Yeah. I am still in some amazement (yeah). Lots of laughter*. (TAW Day 1, Lines 74-80)

In the discussion above, the TAW participants consider that the children are perhaps "used to working together and discussing" the math because it is a voluntary afterschool program (TAW Day 1, Line 72). However, then they suggested that the pedagogy of T/R1 may be responsible. T/R1 does not tell them if they are right, s/he merely tells the children to test their answers. The TAW participants considered that the "amazing" (TAW Day 1, Line 80) way the children in the video stories "own their own learning" (TAW Day 1, Line 77). It might be as much a product of the pedagogy as the afterschool setting. They notice that “owning the learning” makes learning math “fun.”

Science, secrets, and discovery are engaging.

On Day 4, the video story features T/R3 and a class approaching a “normal size” (TAW Day 4 Video Story Description, All Events) of 20 students. T/R3 discusses ideas about problem solving with the children before the “lesson” begins. S/he talks to them about scientific research in terms of “finding a secret.” The children appear very comfortable with this theoretical discussion and ask about the difference between a secret that someone knows and one that no one knows. This discussion sets up an algebraic exploration in which the equation and its features (slope and the y-intercept) are the secrets.

When T/R3 begins to discuss equations with the children, s/he uses Box and Triangle notation and reviews this idea: Equations can be true or false; at the same time, they can also be legal or illegal. (TAW Day 4 Video Story Description, Event #2, T/R3) (There are equations that are true, but illegal, and there are equations that are legal, but false.) The children are given a set of (Box, Triangle) points and, as in the previous video stories, they set out to find the “secret” rule. The TAW participants notice that they work together energetically.

Participant #1: *They . . . .geared up, because once they got going . . . they worked together*.

Instructor: *Inquiry-based learning is very familiar to them. They, you know, they go along with it. But, umm so, let's talk about differences in what he's teaching first . . . . Are there substantial differences or is it basically the same*?

Participant #2: *You know it wasn't . . . [inaudible] right*?

Instructor: [To participant #4] *I saw you shaking your head like this—but I didn't hear* *what you were saying when you were watching? You were saying something to participant #2, I think, and shaking your head?*

Participant #4: *Oh oh - oh well*.

Participant #2: *I think we were talking about how the class was, like, so enthusiastic.*

Participant #4: *Yeah!*

Instructor: *But that was true of the other group, too*.

Participants #4 and #2: *Yeah*.

Participant #1: *I think the difference is,, in the other one he (T/R1) just gives them, like, the table all right "go find the rule." And this one (T/R3), he gives them almost kind of like a "Do Now," you know different skills going, see what skills they need in order to do this*. (TAW Day 4, Lines 80-89)

This discussion reveals that the TAW participants noticed the enthusiasm of the children in the Day 4 Video story and commented on the familiarity of the lesson format and number of children who were present. Participant #1 suggested that T/R3 is conducting a lesson that begins with a type of "Do Now" problem. It is reasonable to see these similarities between the "classroom" in the Day 4 Video story and a traditional classroom. At the same time, the participants in the TAW notice the unique pedagogy of T/R3, which does not evoke the traditional classroom image. In the excerpt below, the TAW participants are discussing how T/R3 interacts with the children in the video story as they show him their work.

Participant #5: *He does, he says, "That's good*."

Participant #3: *He says, "That's a good thing," or whatever . . .*

Instructor: *He says several times . . . play again. He sometimes says, "That's worth thinking about*."

Participant #2: *Oh*.

Instructor: *" . . . when somebody has a question. And so they think about it together. And then they show him ideas, and he will say, "That's certainly a good thing." It's not always clear* . . . . (TAW Day 4, Lines 166-170)

The TAW participants noticed a more traditional-sized class and a familiar "do now" to introduce the lesson, but they also noticed a non-traditional approach towards reviewing student work and a non-traditional pedagogy relating to true and false, legal and illegal equations (Pedagogy Results section of this Chapter).

"I want to find the secret myself."

If we consider that persistence in mathematical problem solving is a reliable indicator of student engagement, then persistence in combination with ownership of the learning process is, perhaps, more powerful student engagement. In the Day 4 Video story, one of the children "finds the secret" and another child, Ankur, asks that the secret not be revealed.

Instructor: *So, there was a place that pointed out when, umm, Dr. Davis says, "The secret some of you found out about, maybe it's time to share it?" And, Ankur says, "Don't share it yet," and he says, "Okay, it's not the right time." That's the one I was referring to . . .* (TAW Day 4, Line 299)

The TAW participants searched for the "event" in the video story that shows Ankur asking for more time to find the secret.

Participant #4: *Okay, this is the one.*

Instructor: *This is where Ankur says, "We shouldn't," and Jeff says "We should" . . . that's after she does the "zero and one thing*."

Instructor: *Okay, so that's what I am asking about . . . he's ready to share and one kid says, "No, I don't want them to share," and . . . so what do you think is going on?*

Participant #4: *Maybe he wasn't done yet . . . maybe he was yet to find the secret, that he didn't want anybody to reveal it yet because he was working on it.*

Participant #3: *He wants the challenge.* (TAW Day 4, Lines 305-309)

The TAW participants noticed that Ankur is not just engaged in the problem solving, he wants to “find the secret” even if someone else has found it first. He doesn’t just “want the answer”; he actually wants to find the answer himself. As participant #3 says, “He wants to meet the challenge.” (TAW Day 4, Line 309).

The TAW participants noticed that introducing the lesson as a search for a secret may have generated more engagement in the problem solving. In the following discussion, participants discussed how the children did not have vocabulary for the slope or the y-intercept in a linear equation, but they persisted in communicating what they discovered in their own words:

Instructor: *They were like . . . "this goes here," but they had a hard time saying it because they had no word for this important thing that they had found*.

Participant #2: *It's kind of nice when they find it and then maybe you can tell them, "Oh that is . . . ”*

Instructor: *You found an important . . .*

Participant #2: *Yeah, and then they never forget it* [laughs].

Participant #4: *Yes*.

Instructor: *That could be true*.

Instructor: *I love the secret idea thought*.

Participant #2: *Yeah, especially 'cause of their age, it probably was like a nicer way* . . . . (TAW Day 4, Lines 227-228)

Instructor: *Oh—I was going to show you* . . . .*and I forgot to bring my* . . . ?

Participant #2: *"A secret"* . . . [*laughs*]. (TAW Day 4, Lines 220-230)

The TAW participants noticed that discovering the "slope" and the "y-intercept," even without the vocabulary, would be a strong learning experience; it was labelled "unforgettable" (TAW Day 4, Line 223). Participant #2 is expressing his/her view about the quality of the learning experience of the children in the Day 4 Video story and not just the evidence that they learned about linear equations.

Expectations for teachers may have implications for student engagement.

The TAW participants noticed with some admiration (TAW Day 1, Line 28), the engagement of the children in the video story on Day 1, and at the same time, they expressed a seed of doubt about whether such engagement would be possible in a regular classroom. This theme was raised several times during the four-day workshop (TAW Day 1, Line 80; Day 2, Line 384; Day 3, Line 362). Participants questioned whether the active engagement of the children in the daily video stories would have been possible in a real classroom. TAW participants discussed their own "real" classrooms and the fact that they would be accountable to administrators who have expectations about classroom management and how to properly structure a lesson.

Participant #4: *In our district we have behavior issues. So like, for teachers, they have to deal with all of that. You, as an old teacher you may still have those issues, but when you are a new teacher it becomes overwhelming.*

Instructor: *They are looking for a certain level of quiet?*

Participant #4: *Your classroom management is a big deal. How is your classroom management? Many people don't get their job back because of that. You can't manage your class* [is what they are told].

Participant #4: *When they come to observe you, if you can't manage your class, it's very unlikely for you to do well in the observation*.

Instructor: *This class?* [asking about the class in the video story]. *Would they feel . . . ?*

Participant #4: *No, No, No,—they would love this class* [in the video story] [participant #3 is saying in the background, *seeing engagement] They will look into how you end, how you close? How do you close everything?* (TAW Day 1, Lines 271-276)

In this discussion two possibly orthogonal ideas about "our real classrooms" are presented: One is that the class in the video story showed students engaged and doing mathematics; TAW participant #4 said that the administrators in his/her district would "love the class" (TAW Day 1, Line 276). The other idea is that, under any circumstances, teachers in their district are expected to show good classroom management. The behavior/engagement of the children in the Day 1 video story was *not* considered by the TAW participant to reflect "bad behavior." After denying that the behavior of the children is bad, TAW participants did emphasize the value their district places on classroom management.

There are a couple of times during the TAW that participants complained about the behavior of the children in one of the video story episodes (TAW Day 1, Lines 91-93; Day 2, Line 718; Day 4, Line 171). One example of this, from the Day 2 TAW discussion is provided below. The participants are discussing Yonny's behavior, how it would be viewed in their school and how the TAW participants, themselves, think about the behavior. They point out that some behaviors are not just annoying to teachers but might also bother other students.

Instructor: *Would that be the kind of thing, the whistling, and the [sounds], would that be considered "not managing your class" where you are?*

Participant #4: *It depends*.

Participant #5: *It depends on which* . . . .

Participant #1: *It depends on who's observing you*.

Instructor: *So some people would say it does.*

Participant #5: *It depends on just you as a person, 'cause for me, after some time, I might be like, "Let's settle down." 'Cause it starts getting on my nerves*.

Group: Laughter

Instructor: *You're a person too*.

Participant #4: *Not even just your nerves; some other students are stopped by that—they can't think.*

Participant #2*: Mmhmm, yeah..*

Participant #4: *They say, "Will you stop that already?" They were doing . . . suggesting anything.* (TAW Day 2, Lines 710-720)

The TAW participants were explaining how they sometimes feel about students that behave in distracting ways. On Day 4, one participant makes clear that the children in the video story are "too rowdy" for him/her.

Instructor: *He says several time . . . "play again." He sometimes says, "That's worth thinking about*."

Participant #2: *Oh*.

Instructor: *When somebody has a question. And so they think about it together. And then* *they show him ideas and he will say, "That's certainly a good thing." It's not always clear . . . .*

Participant #4: *It's a little difficult for me to follow. Because I think, I think the kids are rowdy. The kids are too rowdy for me*. (TAW Day 4, Lines 168-171)

Participant #4 brings up the behavior of the children in the Day 4 Video amidst a discussion of the pedagogy in the Day 4 Video story. The TAW participants do notice that T/R3 does not get too concerned about the cacophony of voices in the room:

Participant #3: But he let it go. A couple of times he said, you know, "Quiet," but mostly he let it go. (TAW Day 4, Line 177)

Participant #4 points out (above) that s/he would require a more orderly room than T/R3. It is interesting to consider if our “normal” reactions to student behaviors need to be adjusted in recognition of student needs. This is complicated by the varying expectations of different teachers and different school administratoins. Research is needed to establish healthy standards for behavior that don’t impede learning.

Teachers must meet expectations.

The TAW participants also considered how the “lesson” in the video story would have to change to become acceptable to the teacher evaluators. These changes included concerns about how the class would be received if a “proper closure” wasn’t added to the “lesson.”. In the discussion below, Participant #4 presented a metaphor for the lesson closure, and said that the expected “lesson closure” resembles a talk show host delivering closing remarks on a TV [or radio] program, a professional version of “What *have* we learned today?”

Participant #4: *No, No, No,—they would love this class* [in the video story] [participant #3 is saying in the background, *seeing engagement] They will look into how you end, how you close? How do you close everything?*

Participant #4: *For me the way he ended, it was . . . .he'd have to do a recap.*

Participant #2: *Yeah [others say "yeah!]*

Instructor: *A recap, like "What did we learn today?"*

Participant #4: *It's more like a talk show, you know*?

Group – laughter

Participant #4: *You know, on a talk show, the moderator or whoever, comes and says some things. They want to see that*. (TAW Day 1, Lines 276-282)

The TAW participants indicated that the "part they [administrators] would love," the student engagement, would not be enough for them. The discussion reflected the high value that the TAW participants, as teachers, place on the engagement of the children in the Day 1 Video story. However, it also reflected the experience they have in being critiqued and the anxiety that exists over "packaging" a good lesson so that it appears good to evaluators.

The TAW Participants noted that T/R1 did not attempt to quiet the children in the video story on Day 1. Their surprise at T/R1's ability to work with the "jumping enthusiasm" of the children reveals a particle of concern about behavior:

Participant #2: *You know what else he did? When they did get kind of loud, he never really quieted them. He didn't do that. He didn't do that. He said to sit down. He didn't do like try to like, OK —the noise level is too— he didn't do that . . .*

Participant #3: *But their enthusiasm . . .*

Participant #2: *Even when they were like jumping*. (TAW Day 1, Lines 91-93)

The TAW participants expressed repeated admiration for the patience that the Teacher/Researchers exhibited in the daily video stories (Pedagogy Results in this Chapter). Above, they are noticing that T/R1 is not quieting the children even when they were “enthusiastically” loud or jumping around. Their discussion points to a couple of questions that were not examined during the TAW:

Is student engagement like “this” possible in a regular classroom?

In the following discussion, on Day 1 of the TAW, Participant #3 read a question from the daily questionnaire about student engagement and then began to answer that question:

Participant #3: *Explain how the students did or did not seem engaged*.

Participant #3: *They were guessing or busy, they were trying, express to each member. They didn't need to be called on.*

Participant #2: *Yeah, true.*

Participant #2: *I guess you have to allow them that room to get . . . start talking . . . .they are gonna have disagreements with each other. It’s not only fighting . . . they can reason, challenging*.

Participant #3: *Challenging* . . . (TAW Day 1, Lines 104-108)

Here the TAW participants do suggest that the children need “room” to engage, talk with one another, and challenge each other.

The TAW participants contrasted the engagement of their own middle-school students with the enthusiasm of the children in the video stories:

Instructor: *In your experience, do you do this, any of you? Do you find it hard or easy?*

Participant #1: *Sometimes there are kids that don't say anything. You'd just be like waiting all day. Watching your nails grow*. (TAW Day 1, Lines 127-128)

In a discussion about planning their *guess my rule* lesson, this issue emerged again. The TAW participants were talking about the "do now," or the first activity in the lesson:

> Participant #2: *Maybe—sometimes when I do it, it takes a long time . . . like wait a minute, five minutes* [laughter].
>
> Participant #3: *Yeah . . . . My class—they go on forever sometimes*. (TAW Day 1, Lines 196-197)

The description of "kids that don't say anything" and "waiting . . . watching your nails grow" is vivid. One can imagine the challenge facing a teacher who fears such a response from their class. On Day 1, the TAW participants also discussed the difference in terms of the size of the group and the afterschool setting versus the regular school classroom:

> Participant #4: *One thing—this is a small group. Five or six students. If we get five or six kids in a classroom together, we could get the same thing. Mm-mm.*
>
> Participant #2: *And because it was an afterschool activity, they kinda had* [a] *choice if they wanted to stay or not. It's not like they had to be here for "this" 60 minutes*. (TAW Day 1, Lines 130-131)

This is a recurring theme: the different sized groups and the different setting (afterschool vs. regular school). The TAW instructor made a decision to let concerns emerge (at first) and discuss strategies after the concerns appeared to be all on the table.

On Day 2 of the TAW, admiration for T/R1 was combined with a wish that they could put her/him into a big class and see how s/he does.

> Participant #1: *I wish I was more like the teacher that was doing it— he don't really help out the kids at all—he just lets them score and figure it out. But my class, like the kids, I guess they're so timid and scared to talk about math, to reveal their skills, they don't say*

anything. So I have to like step in and get the conversation moving along. I admire this guy, what he does. (TAW Day 2, Lines 756)

Participant #3: *I would like to see him in a big class*.

Participant #1: *Yeah, I know*.

Instructor: [laughter] *We're going to get him over here*.

Participant #3: . . . *to visit us*. (TAW Day 2, Lines 757-760)

Again, here the implication is that the difference between the small group of children in an afterschool program and the larger group of children in a regular classroom is enormous, and maybe the person they admire would not fare so well in the classroom that the TAW participants know so well.

On Day 3, the TAW participants again discuss how they face bigger challenges with large classes of children who may not be receptive to a change in expectations:

Participant #4: "*But, the issue I have is this . . . like when we have less than 20, at least 20 students, how are you going to be able to dedicate the amount of time to one student, like you know?*

Participant #3: *To make sure*.

Participant #4: *Yes, imagine what they're doing, you're going round and round and there's no way you're going to be able to give that time, unless you have them by yourself. So how do you use this in your classroom?* (TAW Day 3, Lines 376-377)

Notice that this concern is expressed with some detail. Participant #4 is not just saying that it's too challenging to have 20+ students, s/he is saying that s/he lacks the time needed to give each student dedicated attention. It is not surprising that this specific concern comes up on Day 3 of the TAW. On Day 3, the video story is about Ariel solving the ladder problem. Recall that the ladder problem is a *guess my rule* problem that is based upon a type of ladder that is built using green rods. A one-step ladder uses five rods, a two-step ladder uses eight rods, and if y is the number of rods in a ladder with "x" steps, the rule is: $y = 3x + 2$ (Tracing Ariel's Algebraic Problem Solving: A Case Study of Cognitive and Language Growth, Events #1-#4). Ariel works with T/R2 and another child, James. Ariel begins the work with a proportional assumption, and as he realizes that his assumption is not correct, he begins to create a heuristic solution using his (incorrect) assumption of proportionality (Tracing Ariel's Algebraic Problem Solving: A Case Study of Cognitive and Language Growth, Event #2-#3 Ariel).

As T/R2 asks Ariel questions that guide him to find the errors in his solution, Ariel persists and creates a heuristic solution that becomes more and more complex. Ariel persists along his chosen path to solve the problem, and T/R2 also persists in asking Ariel questions that might reveal problems in Ariel's solution. The Day 3 video story shows the dialog between TR/#2 and Ariel; T/R2 needs to pay close attention to what Ariel is doing in order to understand his solution well enough to have this dialog. TAW Participant #4 was talking about "this" when s/he said, "*So how do you use this in your classroom?"* (TAW Day 3, Line 377). On Day 2, they have a discussion about how relevant the pedagogy in the Daily video stories is or is not to the respective teaching practices of the TAW participants and if the children will "try as hard" in a "40-minute class":

> Instructor**:** *So let me just ask something. I don't know if we can answer those questions, but are you seeing them doing something . . . like . . . extraordinary and they wouldn't do it in normal situations? 'Cause they're doing it like because the camera is on and it's important? And somehow they have to do their best.*

Participant #3: *That's what I'm getting at. In my opinion they're trying hard, in their way. They may not try . . . if it was a 40-minute class and the same thing happened, they may just . . . work. Here they are really trying.* (TAW Day 2, Lines 383-384)

The question was raised when the TAW participants considered how they would design a *guess my rule* lesson and when they discussed the contrast between what they saw in the children in the daily videos what they saw in their own classrooms. This question may be considered evidence that the TAW participants were thinking about the challenge of producing enthusiastic, persistent, problem-solving behavior in their own classrooms.

The TAW participants discussed ideas for adapting to the regular classroom.

The TAW participants did consider how they would attempt to create a similar model of small engaged groups of children working together in their larger classrooms over the four-day workshop. These thoughts emerged on Day 1 of the workshop.

Participant #3: *We have groups of five or six kids here, but in a class of 20, you'd need to break out in groups. First, you'd have to have a whole big thing, and then groups have their own kind of a thing*. (TAW Day 1, Line 320)

On Day 2, after the TAW participants laughingly suggested that T/R1 come and teach one of their “real” classes, the instructor made a suggestion that generated some reactions:

Instructor: *Well you know, one way to experiment with this is to create something similar, an afterschool thing and try it out in a small group of kids instead of starting in a bigger group. I will never forget when I did this (chuckling), because they were misbehaving and it was 8th period and they were tired. So I said I would get pizza, and I said . . . .you're gonna get a zero for today because you’re lying down, but if you come this afternoon and you eat the pizza and you work and everybody does at least these three* *problems that they have to do, then, you know, they'll be okay for the day. And a lot of them came. And they acted like little professors with each other (laughter), They were like "so what did you get for your solution?" (sounding falsetto and formal), because they wanted pizza* [laughter]. (TAW Day 2, Line 764)

The TAW participants discussed their own classroom realities: these included time in a class period (TAW Day 2, Lines 777- 789) and the strong emphasis on tests and test results (TAW Day 2, Lines 880-919). They were then challenged to consider how to introduce pedagogical changes that were featured in the video stories and which had advantages that the TAW participants recognized.

Instructor: *Think of a class—you don't have to [inaudible], but think of a class where you would try this. Somehow. Think about what it would be. Think of the first problem you could think of . . . besides that it's 45 minutes and there are 25 of them . . . . Think of like . . . just imagine yourself, you're T/R1 and you're saying, "We're going to play a game*.

Participant #1: *Some students tend to give up if they don't get it right away*.

Instructor: *So they'll try it.*

Participant #1: *Yeah, they'll try it and they'll . . . .[give up]*.

Instructor: *Well, would that be so bad on the first day, if they give up a little? Will everybody give up?*

Participant #1: *No*.

Participant #4: *No. It depends on the group*.

Participant #1: *Yeah.* (TAW Day 2, Lines 852-859)

These lesson-planning discussions raised the kinds of questions that teachers all have to deal with involving class size, administrator expectations, student behavior, and the reality that some students will present as very disengaged and say that they "hate math."

The TAW participants noticed many aspects of pedagogy, mathematics, and student engagement in the daily video stories. As they began to think about planning their own *guess my rule* lessons, they raised significant questions and considered how to transform their own classrooms. In the process they expressed concerns about their own abilities to change the feelings that their own students have had about mathematics.

CHAPTER 5: CONCLUSIONS, QUESTIONS FOR DISCUSSION, AND IDEAS FOR FUTURE RESEARCH

Pedagogy, Mathematics, and Student Engagement

The daily questionnaire helped the TAW teacher participants focus on pedagogy, mathematics, and student engagement in the video algebra stories. The Results Chapter details specifically what the teachers noticed and discussed in each of these three areas. Reviewing these results, it is apparent that TAW participants noticed evidence of social constructivist pedagogy, the mathematical thinking of the children, and perseverance/engagement in finding solutions to the algebra problems.

In summary, the TAW teacher participants noticed that the teacher/researchers (T/Rs) in the video stories employ the following strategies: patience and respect for the children's thinking process, guiding children with questions, asking the children to check if their solutions work (not giving them answers), and engaging the children in a discovery process. They noticed that the T/Rs and the children in the video stories value the problem-solving process more than the "right answer." They commented favorably on additional strategies they observed: turning the lesson into a "game" or a search for "the secret" and allowing students to express some excitement. Teacher participants also noticed how the children in the video stories reason when they try to find each "rule," each linear function, by examining the given "clues" (a set of points).

Over the four days of the TAW, the video stories feature many *guess my rule* problems for which the children find recursive solutions and attempt proportional solutions. In each case, the TAW teacher participants discussed the details of each solution idea, how the children reason, what works, and what does not. This detail was discussed along with explicit comments about how engaged, enthusiastic, and confident the children are. What emerges from this analysis is that the TAW participants recognized the perseverance of the children in the video stories; perseverance in the face of each challenging algebra problem. They noticed that the children do not get frustrated and stay engaged in the problem solving.

It is important to point out that a limitation of this study is that the discussions this researcher had with the TAW participants about their "guess my rule" lessons are not reported on in these results. As described in the methodology chapter, there was one post-lesson meeting that was required of the teacher participants, and only four of the TAW participants attended a meeting. The participants were positive about their lesson experiences and they pointed to the student work they brought with them. However, this researcher did not have access to the student work produced prior to the meeting, and it was therefore not possible to ask detailed questions about the student work during the single meeting. A paper on the student work generated by the TAW participant's guess my rule lessons may be forthcoming. However, a larger study that follows the teacher participants through several cycles of a TAW, with many opportunities to discuss the lessons and the work their students do, would be valuable. Such a study would be valuable inevaluating how the TAW impacts teachers.

Given the quantity of qualitative evidence produced by this study, it is appropriate to propose a larger study to determine the likelihood of teachers responding as did the participants in the TAW. This larger study should follow the teacher participants through the implementation of several *guess my rule* lessons, during at least three full "Maher" cycles of a Teacher's Algebra Workshop (Maher, Landis, et al., 2010). In each cycle, teacher participants would study the math, watch and discuss the video stories, and then, after teaching a *guess my rule* lesson, they would come back together to discuss the results of those lessons with one another. A larger study would enable us to produce qualitative and quantitative data to understand how the participating teachers would be impacted.

In the following sections of this chapter, the value of using video stories featuring black and minority children is discussed, and additional research questions are suggested.

The Use of Stories

Even though the TAW teachers spent a great deal of their time discussing the formal aspects of learning summarized above, they still expressed interest in the children who were featured in the video stories. They expressed interest in the children even though this topic was not present in the questionnaire or initiated by the instructor. One example of this spontaneous expression of interest is when Participant #4 asked about what Ariel was doing “now” (August, 2016). In this discussion, Participant #4 is sure that “grown-up” Ariel has a girlfriend:

Participant #4: *He's in college right now*?

Instructor: *What?*

Participant #4: *Where is Ariel now?*

Instructor: *Oh, I don't know, he's grown up, he went to college, he's probably working . . . .*

Participant #4*: Wow! How many years ago?*

Instructor: *So, he's probably out of college, he's probably in graduate school, maybe he's married, maybe he has a girlfriend?*

Participant #4: *I am sure; I'll bet he has a girlfriend.* (TAW Day 3, Lines 280-286)

On Day 3, the participants discussed why Ariel seemed impervious to James’ reasoning after James explains (correctly) that every new step in a ladder adds three rods to the ladder. During this discussion, Participant #4 explains why s/he thinks Ariel didn’t stop to consider

James' idea and then interjects a note of personal interest (into her own explanation) when she says that she was "happy to hear James talk . . . finally":

> Participant #4: *The point is, sometimes when you are doing something, and that person has another way of doing, but if you're convinced or have a conviction that what you're doing is going to work, it's very likely that you won't give up on what you are doing. You may not listen to the other person because you feel that "my way is going to work, too." I think that was what happened to Ariel and James. Ariel was so focused on using his method to get the right thing. So, he wasn't really listening or thinking about what James was saying. And I'm happy to hear James talk, finally.* (TAW wg 234 Day 3, Line 48)

The interest that teacher participants displayed in the children (above and beyond their interest in the mathematical reasoning that the children were doing) supports the conjecture that the video stories are engaging enough for teachers to think about and remember. The TAW participants were conscientious about discussing the questions they were given each day, but their interest in the children indicates how engaged they were in the stories. Furthermore, their understanding that the video stories were compiled from video of actual research sessions made it clear that the children they were watching are real people. Perhaps this is evidence that the video stories are engaging enough for teacher participants to add these stories to their professional "knowledge base."

According to Maher (Maher, Landis, et al., 2010, p. 21), "carefully selected videos of children doing mathematics" can influence teachers' expectations about what children are capable of. Furthermore, according to Clandinin and Connelly (1996), teachers have professional stories that contain their knowledge of education theory and knowledge of their own practice. They term the former as "sacred" stories and the latter as "secret" stories. Although the video algebra stories could be considered part of educational "theory" because they are introduced in a professional development Teacher's Algebra Workshop; they capture authentic learning and teaching situations. The TAW teacher participants discussed these video stories in such detail while interjecting affection for the children and predictions about their lives. This suggests that these are candidates to become "secret" stories as the teachers begin to inject the pedagogical ideas they learned into their own practice.

As previously mentioned, this study produced quantities of qualitative evidence that the video stories prompted the TAW participants to discuss many aspects of social constructivist pedagogy and the details of the mathematical thinking of the children. We can construct larger studies that follow teacher participants over several "Maher cycles" (Maher, Landis, et al., 2010) and also query the participants specifically on their use and recall of the video stories. (As stated in the first section, a larger study will give the teacher participants several chances to try social constructivist pedagogy, and discuss and refine their efforts.)

Social Constructivism and Stereotype Threat

Can social constructivist pedagogy impact the achievement gap? The use of algebra videos featuring black and minority children doing mathematics was purposeful in terms of raising teacher participant expectations for these children. The TAW participants did recognize how engaged the children were in solving the algebra problems, and, if the video stories are memorable, it may help them raise their own expectations for their minority students. If the video stories are incorporated into their memory of "secret" stories, they may begin to be more sensitive to the needs of their black and minority students, and make sure that these students perceive the respect and patience they are being afforded. As a result, they may generate more engagement from minority students, reinforcing this new "secret story."

Clearly, it is not just a matter of remembering the video stories in order to move from "seeing" what is possible to "doing it." That is another reason for following this study up with a larger study that allows teachers to try implementation of social constructivist pedagogy in their own classrooms, with many opportunities to bring their experience, questions, and concerns back to the study group for support and ideas. If teachers are given access to true stories of engaged mathematical inquiry among a group of black boys in algebra class, and if they are able to discuss what they see and try out the constructivist pedagogy in their classes, their professional views may begin to change. Racism is deeply rooted in our culture, and as pointed out in this study, black children (particularly black boys and men) feel a stereotype threat (Steele, 1997) about their abilities in math. If teachers in the TAW workshop begin to use a social constructivist pedagogy that presumes the importance of each child's thinking and values the exchange of ideas between the students, will that "respect for my thinking" modify the feelings of some black students and reduce the stereotype threat in their classroom?

Of course, the goal is not just that that teachers raise their expectations of minority students and improve their achievement. The goal is that teachers learn social constructivist pedagogy and change the way they teach all children algebra (and all mathematics). My conjecture is that once teachers begin to be successful with social constructivist pedagogy, the change in the classroom learning culture may be of particular help to their minority students. If many black students, who suffer from societal expectations of poor performance in mathematics (Steele, 1997), acquire new confidence in their ability to learn mathematics, will that begin to close the achievement gap in mathematics?

In my own practice, patience and respect for students' thinking (whether their answers or right or wrong) has made a noticeable difference in how many minority students feel about learning math. Over the last 15 years, I have had many "thank you" letters to that effect from my black students. This includes economically/socially disadvantaged students as well as students who are from advantaged families who care very much about their success. I believe that all of my students benefit as well. There are many students who are not classified as suffering from stereotype threat in math who benefit from social constructivist teaching. A majority of students seem to appreciate it when their ideas are respected.

The teacher participants in the TAW did not remark on the racial status of the students, but they did ask if the children in the video stories were different from the mainstream in that they might be "gifted." During a break on Day 1, Participant #3 did ask specifically if the children in the video story were chosen because they were gifted; the answer was that the minority children in the video were not selected because they were gifted or "better" in any way than the mainstream student body.

In summary, this study can be expanded to determine:

- If teachers will make changes in their pedagogy in response to video math stories.
- If teachers' expectations for all children, including minority children, will be raised.
- If, subsequently, minority children exhibit less stress/stereotype threat when they take mathematics tests in these classes.

Perseverence in Problem Solving

The New Jersey Student Learning Standards (NJSLS)[15] for Mathematics is the current New Jersey version of the Common Core Standards (2015), which guides all NJ K-12 public school districts. This document includes the mathematics Practice Standards and the mathematics Content Standards for NJ. The Content Standards cover the specific content areas for each grade level. The Practice Standards are more general and apply to all mathematics; they "describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important 'processes and proficiencies' with longstanding importance in mathematics education" (NJSLS for Mathematics, p. 3).

There are eight enumerated Practice Standards and the first of these, Practice Standard #1, is: *Make sense of problems and persevere in solving them.* The NJSLS for Mathematics goes on at length to explain what this looks like, providing a set of examples. However, the key idea in Practice Standard #1 is better expressed in the section where the NJSLS explains connections between the practice standards and the mathematics content standards:

> The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. (NJSLS for Mathematics, p. 6)

[15] https://www.state.nj.us/education/aps/cccs/math/

The NJSLS recognizes that understanding matters, and without understanding students "may rely on procedures too heavily." Of course, when a student does not understand the mathematics and does not actually expect to understand the mathematics, s/he may just inappropriately use procedures that s/he does not understand. As a teacher, I see many examples of this phenomenon; I have been part of numerous discussions in which math teachers express frustration that a student "did something nonsensical" on a quiz.

As such, it is clear that the NJSLS requires that students understand mathematics and that they persevere in solving math problems to build that understanding. It is interesting to connect this idea and the idea that perseverance in solving a challenging math problem may be the best evidence of student engagement.

In the Results Chapter, there is a summary of what the TAW teacher participants noticed about student engagement in the video stories. There were two types of evidence that the TAW participants noticed student engagement in the video stories. The first were objective statements about engagement behavior; e.g., they were eager, participating, not afraid to voice their ideas, and not asking if their answers were "right." The second were all evidence of persistent problem solving, often the same statements used as evidence that they noticed students' mathematical reasoning. The TAW participants noticed that the students worked on problems that took a long time to solve, and even when they didn't find a solution, they did not exhibit frustration. They noticed that the children seemed to expect to persevere; they exhibited behavior consistent with Mathematics Practice Standard #1.

The results of this study indicate that the teacher participants noticed how patient and respectful the teacher/researchers in the video stories were. They also noticed that the students working with these teacher/researchers were engaged in their work, persevering through many challenges to solve algebra problems. This connection between the socialist constructivist pedagogy and the student's willingness to persevere in problem solving is expressed in Boaler (2016); Maher (1998); Maher, Mueller, and Yankelewitz (2012); and R. B. Davis as described in Mayansk, 2007). Given this strong connection, the results of this study encourage us to consider a larger study that helps to answer these questions about cultivating the expectation of perseverance and understanding in students:

- Since perseverance in problem solving is recognized as an important mathematical practice, if social constructivism develops such perseverance in students, how can we help all teachers learn this pedagogy?
    - How can we help administrators and parents accept this pedagogy?
- Is perseverance in solving a challenging math problem conclusive evidence of student engagement?
- Is such perseverance always visible to teachers?

There was some discussion of the noise and energy level of the children in the video stories; some teacher participants pointed out that they have difficulty with "too much" noise and movement. This is an important issue to consider because students who must put a great deal of effort on "behaving" may not have the energy to also put a great deal of energy into persevering in the face of a challenging math problem. This raises a question: Should we modify our standard for behavior in school classrooms to ensure that students feel respected and have the energy needed to meet challenges?

Consider Yonny, in the Day 2 video story. Yonny appeared to be whistling and banging and basically disengaged. The TAW participants articulated this (TAW Day 2, Line 38). But soon it was recognized that Yonny had somehow figured out the correct answer for y, when x =

100, and the participants admitted that it was possible that Yonny was thinking about the problem when he appeared to be distracted (TAW Day 2, Lines 108-113). If Yonny could solve a problem even as he appears to be disengaged, then, of course, the same is true of many others. (This researcher, for example, often thinks about new math problems when appearing to be just eating dinner.)

This suggests that we may need to rethink our model for appropriate student behavior, albeit in a large classroom, so that more students have the sense that they are respected problem solvers and are enabled to persevere. If teachers (and administrators) understand that perseverance is not always visible, then the dialog about appropriate behavior could, at least, be more respectful.

Further Research Is Necessary

It seems appropriate to propose a larger study of at least three cycles of a Teacher's Algebra Workshop (Maher, Landis, et al., 2010) employing video stories that feature black and minority students. Teacher participants would study the math and the video stories, and then, after teaching several *guess my rule* lessons, they would come back together to discuss the results of those lessons with one another. Then there would be at least two more sets of video algebra stories for which the teachers would complete a full cycle. They would be able to see the video stories whenever they so chose. It would be valuable if the researchers could visit these classrooms of the teacher participants over the three cycles and record what they see. As the teacher participants moved through the three cycles, they would journal changes they do or do not see in their own practice as well as any changes they see in their students. Finally, they would journal any changes they perceive in their relationship with their black and minority students.

Video Story Annotations

The use of video stories created from video analytics using the Rutgers VMCAnalytic tool raises additional questions. These include questions about how much event annotation is useful for teacher audiences and how that annotation may be used. Recall that each event in a video story is a video clip accompanied by annotation. If we are going to use video stories that pair video with text that must be read (the annotation), we will benefit from research-based guidelines on how and if teachers will read the annotations. Will they read the annotation before they watch the video? After they watch the video? Or will they watch the video twice, shifting attention? How much annotation is reasonable? Clearly a few lines are not too much, but what is too much? Are there guidelines for language that should and should not be used? For this study, the annotation was limited to simple descriptions of what is happening in the video; the annotation was there to help the viewer only to understand the actions and statements in the video. It is possible to consider the video story as a stand-alone teaching tool wherein the annotation would point out what the viewer (presumably a teacher) should notice and what questions they should think about.

Finally, even though we have a very rich database of social constructivist examples for many curricular areas for K-12 students, there is a need for more. We can consider other studies that engage, for example, algebra 2 or calculus teachers. The teachers would see the same TAW that algebra teachers see, but their workshop goal would be to create activities that would be similarly challenging and accessible to their own students, and to share these activities with the workshop group. After discussion, they would give these lessons and report back on the results. After three cycles, it is possible that some of the workshop participants would have results worthy of viewing and, possibly, video-taping.

In my own teaching practice, where I do not teach algebra 1 students, but where I do teach calculus, I have spent many years developing calculus activities that build understanding of calculus ideas in a challenging but accessible way. Calculus is a “big idea” class and lends itself to exploration of these ideas. In fact, I have found many different sources of ideas, like “Teaching Calculus in the Middle School” (Barger & McCoy, 2019) that I have used and built upon. I should note that this is not an AP calculus class; my social constructivist approach is not suited to zooming through the curriculum so that we can study for the AP test. During the 2018-2019 school year, I shared my approach with a colleague who began teaching calculus honors for the first time. We worked closely together throughout the year, and it went very well for both of us. From that personal perspective, it would be helpful if we had video stories of teaching and learning in social constructivist classrooms at all levels and for all content areas.

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APPENDIX A: TAW Daily Video Story Descriptions

Day 1: Beginning to Understand Linear Functions: Guess My Rule

Description

The purpose of this analytic is to show how mathematics learning occurs when children are encouraged to "create their own way of understanding" (Davis, 1992). The patience and guidance exhibited by the Teacher/Researcher #1, (T/R1), in this analytic is a pedagogical model for teachers. The way the children explore and struggle with the first *guess my rule* question, and the way they explore with success that excites them on the second problem, provide a rich model of student math learning.

The “game” featured in this analytic is called *guess my rule*. In general, *guess my rule* is played by giving out a few “points” as clues. The points may be expressed as (x,y) values or (Box, Triangle) values or they may be associated with a real situation as in (Fahrenheit, Celsius). The goal of the game is to find an explicit function that takes in a number (an “x,” a “Box,” etc.) and produces the correct “y” or “Triangle” value. (This game was pioneered by Robert B. Davis in his 2003-2006 study, and adapted for use by Dr. Arthur Powell for the Informal Math Learning Study (IML) (Informal Mathematics Learning, Award REC-0309062**).**

In this analytic, T/R1 works with 6 seventh-grade boys on two *guess my rule* problems for linear functions. In the first problem, Ariel, Duwad, Brandon, Christian, Yonny, and James are given (Box, Triangle) points, one at a time; the points correspond to the explicit linear equation/rule $y = 3x - 2$ (which the boys are challenged to discover). T/R1 uses a chart to show each pair of values:

□	Δ
5	13
3	7
6	?

Table 2: Day 1 Guess My Rule Problem

As they work, the boys begin to see a pattern, and Ariel is the student who figures out that if we add 8 to 5 (to get Triangle = 13) and 10 to 6, then we would add 12 to 7 to get Triangle = 19 (not shown in the video) and 14 to 8, etc.

This is a recursive solution that is not clearly articulated or used by all of the boys. Once T/R1 puts zero in the Box column, Ariel appears unsure of his recursive solution. Ariel's prior expression of the solution only went in a positive direction to consider Box = 5, 6, 7, 8, etc. To go "backwards," Ariel would have to subtract 2 from the "number added": add 6 to 4, add 4 to 3, add 2 to 2, add 0 to 1, and add "-2" to 0. It's not clear that Ariel realizes this or if he would (at this time) be able to think about adding negative numbers.

Note that T/R1 always asks the students if they know the rule; he doesn't tell them the answer even though he sees how hard they are working. Instead, after a certain amount of time has passed, T/R1 decides to move on to another rule.

The second rule was discovered by Yonny, one of the six boys. Yonny charts the information and the boys start out by giving Yonny the number 1. Yonny puts (1, 15) on the chart. With this second rule the boys appear energized and quickly call out answers, incomplete and with errors at first. They quickly figure out that the rule is $y = 10x + 5$.

It is notable that the boys first express the rule as a concatenation of symbols: 1-> 15, 2-> 25, 3-> 35. Christian says to take the Box number and “put a 5 at the end” to produce the Triangle value. T/R1 tells them to express the rule as a (mathematical) operation.

The boys in the source videos for this analytic were part of a group of seventh graders in the Frank J. Hubbard Middle School in Plainfield who participated in an afterschool, three-year NSF study called IML (Informal Mathematics Learning, Award REC-0309062**).**

Event #1: Guess My Rule

Description:

T/R1 introduces five 6th graders to an activity, "a game," he calls *guess my rule*.

He explains that he will think of a rule and a number. Then he will tell them the result of using the rule on the number—but he won't tell them the rule, they have to figure it out.

T/R1 starts with the number 5 (since there are 5 of them) and says that for 5, the rule will produce 13.

He then asks the students for another number to use the rule on. Dawud says "3." He asks what they think the rule will do to "3." One student says "11." T/R1 shakes his head and tells them that the rule gives us 7.

****

Source Video: Early algebra, investigating linear functions, series 1 of 7, Guess My Rule introduction and Ariel and James with problems 1-3, Clip 1 of 8: Introduction to Guess My Rule Clip: 00:01-02:34

****

Event #2: Finding Evidence of Student Reasoning

Description:

The students are suggesting the next number they want to use with the rule.

Dawud suggests "six" and the teacher/researcher asks the students what they think "is going to happen to six"

Brandon says "I know!"

Dawud says "It's going to go six to 24" and T/R1 asks if all of them agree.

Brandon says that “six is going to go to 10” and Dawud explains, “We made five with eight, we made seven with four. So for six it will be 24.”

The teacher/researcher asks if anyone else has a guess and Brandon repeats that the answer is 10.

Many boys are talking when Ariel says “It’s going to be more than 13, I know that.”

The T/R1 points to Ariel and says “Hold on, he says it’s going to be more than 13? Why does he think that?”

Ariel says that it’s because the “first one is five and like I think that you keep on adding on, ’cause the number... Like three you added it on four, for five, its matching the five you added on eight. It depends on the number, that’s how much you add on.”

T/R1 shows them that 6 goes to 16 and then Ariel says "I knew it was 10," and explains his reasoning again.

****

Source Video*:* Early algebra, investigating linear functions, series 1 of 7, Guess My Rule introduction and Ariel and James with problems 1-3, Clip 1 of 8: Introduction to Guess My Rule

Clip: 2:54 -4:29

*****

<u>Event #3: Does Ariel Know the Rule?</u>

Description:

T/R1 asks James for another number, and James says "eight."

What will the rule do to number 8?

The students suggest different answers: 18 and then 9.

Then Ariel says, "22."

****

Source Video*:* Early algebra, investigating linear functions, series 1 of 7, Guess My Rule introduction and Ariel and James with problems 1-3, Clip 1 of 8: Introduction to Guess My Rule

Clip: 5:17 – 5:45

****

Event #4: "Box" Is Turned Into "Triangle"

Description:

T/R1 asks everyone to listen to Ariel's explanation of a rule.

Ariel explains, "Alright, so like, the "square"(Box) could be the number you're putting in and it can say like, it can go to like the factory, or something like that and it come out the number in the "Triangle"; square number to Triangle number."

****

Source Video: Early algebra, investigating linear functions, series 1 of 7, Guess My Rule introduction and Ariel and James with problems 1-3, Clip 1 of 8: Introduction to Guess My Rule

Clip: 6:19 – 6:40

****

Event #5: How Is Ariel Doing It?

Description:

(Note that T/R1 made an error in the chart, and shows 18 instead of 16 when the input or Box number is 6.)

T/R1 asks Christian for the next number and Christian chooses "4." Now the question is "What does the rule do to four?" Some of the boys call out answers we have heard before: 24, 9, and 8. Ariel says that it's 10 and explains, "'cause you're going to add 6."

T/R1, "How is Ariel doing it?"

****

Source Video*:* Early algebra, investigating linear functions, series 1 of 7, Guess My Rule introduction and Ariel and James with problems 1-3, Clip 1 of 8: Introduction to Guess My Rule Clip: 7:57-8:56

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Event #6 : Ariel – Don't Tell the Answer

Description:

The T/R1 asks Ariel not to answer this time.

Then he puts "0" in the box column. What does the rule do to zero?

The students all start talking and calling out answers: Brandon, Dawud, and Christian all say that zero goes to zero until Brandon changes his mind and says, "No, don't put zero."

Then Dawud says, "zero," and Yonny says, "two."

T/R1 puts "-2" on the chart.

Yonny and Brandon, who said "two" echo that they thought it was two when "-2" shows up as the answer.

Ariel says, "Zero done messed up my whole thing. Look, this is what I thought: So, for four you added six, for five you added eight, for six you added 10, …I mean (Ariel is looking at the error on the board and frowning).

****

Source Video*:* Early algebra, investigating linear functions, series 1 of 7, Guess My Rule introduction and Ariel and James with problems 1-3, Clip 1 of 8: Introduction to Guess My Rule

Clip: 9:05 – 10:25

****

Event #7: Yonny's Rule

Description:

T/R1 says "lets try another" and Yonny comes up —because it is his rule.

Yonny asks for a number and Brandon says, "Oh, oh! Me, me, me! One!

Yonny puts (1,15) on the chart.

Brandon claims to know the rule ("by" 15) and Yonny says, "Nope."

Yonny asks them to pick another number. Ariel says, "2," and Yonny puts (2, 25) on the chart and they all start talking. We hear them suggest that the next number be 3, no 4, then 3.

T/R1 asks them what they think the answer will be for 3 before Yonny writes it down.

Ariel says "by 10."

****

Source Video*:* Early algebra, investigating linear functions, series 1 of 7, Guess My Rule introduction and Ariel and James with problems 1-3, Clip 2 of 8: Guessing Yonny and Brandon's rules

Clip: 00:00 – 00:51

****

Event #8: Put 5 at the End of Each Number

Description:

They are considering what the new rule does to 4.

The students are all speaking at the same time: we hear a chorus of "45."

Then Ariel explains that we are putting a five at the "end of each number."

****

Source Video*:* Early algebra, investigating linear functions, series 1 of 7, Guess My Rule introduction and Ariel and James with problems 1-3, Clip 2 of 8: Guessing Yonny and Brandon's rules

Clip: 01:06 – 01:16

****

Event #9: I KNOW THE RULE!!!

Description:

Once they see (3, 35) both Brandon and Christian say, "I know the rule!

Christian is jumping up and down with his hand in the air.

Christian sits and says, "The rule is, like, basically you got the same thing. You get, you doing the same numbers, like: 1,1 2,2 3,3 and you just adding five to the same numbers . . . . I think the next number you going to put is 45.

Yonny says “Yes.”

Christian says, "Exactly, I’m too smart."

****

Source Video*:* Early algebra, investigating linear functions, series 1 of 7, Guess My Rule introduction and Ariel and James with problems 1-3, Clip 2 of 8: Guessing Yonny and Brandon's rules

Clip: 1:27 – 1:53

****

Event #10: You Must Use an Operation

Description:

T/R1 says that the rule is using an operation.

Ariel says "He's just putting a five at the end of the number." Christian says the same thing.

Brandon says "times 10! Christian and Brandon say "times 10" several times.

Yonny tells them "That's half of it."

Christian says "Times five."

T/R1 asks, "Does that work?" at this point and after each suggestion: We hear "times 10 times 5," "times 10 divided by 5," and finally "times 10 plus 5."

Guesses repeat and the teacher/researcher keeps telling them to "try it out. see whether or not your rule . . . works. "

Christian repeats "Times ten plus five."

Yonny tells them that this is the rule, and we hear "that was easy."

****

Source Video*:* Early algebra, investigating linear functions, series 1 of 7, Guess My Rule introduction and Ariel and James with problems 1-3, Clip 2 of 8: Guessing Yonny and Brandon's rules

Clip: 2:00 – 2:49

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Day 2: Yonny, Brandon and Ariel Manage Common Cognitive Challenges

Description

The purpose of this analytic is to show some of the paths children's reasoning takes when they are encouraged to "create their own ways of understanding (Davis, 1992) as they tackle *guess my rule* problems. The Teacher/Researchers(T/Rs) who work with these students show interest and patience in the children's thinking, right or wrong. Their questions help guide students to finding explicit rules that are more efficient than the recursive rules they have been using.

In this analytic, the way students cope with three different *guess my rule* questions reveals a common cognitive obstacle.

In the first example, Yonny and Brandon use proportional reasoning to help them use their recursive solution to a linear function rule ($y = 2x+1$). When they are asked to find the y-value when x is 100, Yonny reacts by saying "You can't make us do that" (TAW Day 2 Video Story, Event #6, Yonny). Yonny is pointing out that using their recursive definition would take too long; they would have to work out all of the x and y-values from $x = 20$ and $y = 41$, to $x= 100$. This is the first time the boys encounter the idea that their recursive solution may be inadequate. Then, again in Event #6, T/R1 asks the boys if "there is another way to do it?" The boys embark on using the results they have, results they got using their recursive solution, to find the y-value when $x = 100$. They resort to an (incorrect) assumption of proportionality.

In the second example, where the rule is $y = 2x + 5$, the boys use recursion to find the y-value when $x = 10$ ($y= 25$). Then they create a heuristic solution that also works for the previous rule ($y = 2x+1$); but this heuristic rule only works when the slope of the linear equation is two.

In the third example, Ariel is introduced to the ladder problem and uses proportional reasoning as a shortcut to finding the number of rods in a 10-step ladder. The solution to the ladder problem is linear, and Ariel's assumption, that the number of rods in a 10-step ladder is twice the number of rods in a five-step ladder, is incorrect. However, his work with this assumption starts him on an investigation that shows his perseverance and his willingness to modify his heuristic solution multiple times, as he attempts to keep his initial idea and improve upon it to show that it works.

This example is important for teachers to understand in that it reveals the value of allowing a student the time to explore a solution and build understanding. This example is continued in TAW Day 3 Video Story: Exploring the Ladder Problem and the Development of Algebraic Concepts Over Time. This example is also examined in this analytic: "Tracing Ariel's Algebraic Problem Solving: A Case Study of Cognitive and Language Growth," created by Drs. Robert Sigley and Louise Wilkerson and published on 2-23-15. This published analytic closes the gap by showing how Ariel ended up developing an understanding of linear functions, slope, intercepts, and finite differences. His early explorations did not stop him from building this understanding and may have actually made his understanding of linear functions possible.

This analytic was designed to help teachers to consider why it might valuable to let students explore mistaken assumptions or solutions that are only correct in a limited way. This analytic also gives teachers a range of responses to *guess my rule* linear problems from pre-algebra students in which the students employ recursion and heuristic models on top of recursion. Finally, this analytic gives teachers an example of engaged black and minority students persevering in their efforts to meet a mathematical challenge (NJSLS, 2015, Math Practice Standard #1).

The boys in the source videos for this analytic were part of a group of seventh graders in the Frank J. Hubbard Middle School in Plainfield who participated in an afterschool, three-year NSF study called IML (Informal Mathematics Learning, Award REC-0309062**).**

Event #1: A New *Guess My Rule* Problem

Description:

0->1

1->3

2->5

3->7

4->9

5->11

Given the above *guess my rule* problem, Brandon quickly announces to Yonny, "I finished the problem already, OK? Without your help."

They mumble about the rule being "one, by one, by one".

Then Yonny says, [pointing to the *guess my rule* Problem 1 sheet] "No, so it would be like plus one, plus two, plus three, plus four, see I got it. I am too smart. "

Brandon replies, "Plus five, plus six. See how smart. I am smart."

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Source Video*:* Early algebra, investigating linear functions, series 2 of 7, working on Guess My Rule problems 1-3, Clip 1 of 5: Yonny and Brandon with problem 1

Clip: 00:05 to 00:44
****

Event #2: Explaining the First Solution

Description:

T/R1 asks Brandon, "How'd you come up with the rule?"

Brandon tells him, "I just looked at it and it was easy. It was easy. Just looked at it, you know . . . Plus one, plus two, plus three, plus four, plus five, plus six, plus seven , , , I mean plus six."

T/R1 asks, "Suppose if I gave you the number six, what would it be?"

Brandon says that six goes to 13, and T/R1 asks him how he got that answer.

Brandon explains, "Because, umm, . . . when you got to here, plus six, you have to add six plus seven."

T/R1 says, "Wait, I am not sure if I understand."

Brandon tells him, "You see five, 11? Yeah five plus six equals 11 . . . . And then so you have to add . . . six plus seven because you have to add to get 13."

T/R1: Six plus seven?

Brandon then explains it again more slowly, "OK, look, it's one, zero plus one equals one . . . one plus two equals three, three plus two equals five, three plus four equals seven, four plus five equals nine, and five plus six equals 11 and six plus seven equals . . . 13."

****

Source Video*:* Early algebra, investigating linear functions, series 2 of 7, working on Guess My Rule problems 1-3, Clip 1 of 5: Yonny and Brandon with problem 1

Clip: 1:15 to 2:13

****

<u>Event #3: What Happens if X = 20?</u>

Description:

T/R1 asks the boys, "Suppose I give you 20? What do you think it will be?"

Yonny quickly says that he's not sure, and Brandon starts writing.

T/R1 tries to get Yonny's attention and repeats the problem, "Suppose box = 20, (X = 20)?"

Yonny first says that the number is “too big” and that “Brandon can do it.”

****

Source Video*:* Early algebra, investigating linear functions, series 2 of 7, working on Guess My Rule problems 1-3, Clip 1 of 5: Yonny and Brandon with problem 1

Clip: 2:40 to 3:07

****

Event #4: A Doubling Rule?

Description:

Yonny is working on the problem and announces that he has a solution, “ I know, I know Rule 2. Like, right here it doubles by 2.”

Brandon tells him that he was about to say the same thing, but Yonny doesn’t believe him.

Brandon points with his pen to where X = 6 and says, “The reason that I wasn’t about to say that was because it don’t work right here.”

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Source Video*:* Early algebra, investigating linear functions, series 2 of 7, working on Guess My Rule problems 1-3, Clip 1 of 5: Yonny and Brandon with problem 1.

Clip: 3:35 to 3:50

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Event #5: What is Y when X =20?

Description:

The boys are both working and Yonny is whistling.

Brandon announces that “20 goes to 41” and he sings “41,41, the total would be 41.”

T/R1 asks him: How did you get 20?

Brandon tells him, “On this side, the number goes up by two, so I skipped by two all the way to 20.”

****

Source Video*:* Early algebra, investigating linear functions, series 2 of 7, working on Guess My Rule problems 1-3, Clip 1 of 5: Yonny and Brandon with problem 1

Clip: 4:22 to 4:48

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Event #6: What Happens if X = 100?

Description:

T/R1 asks the boys, “What would y be if x = 100?”

Yonny crinkles his brow and says, “What? You can’t make us do that.”

Brandon says that he doesn’t “feel like going up to 100,” and T/R1 asks if “there is another way to get there?” and “What’s the problem [about the 100]?”

Brandon tells him, “I have to work it out all day.”

Yonny expresses another idea: “Well I think it could be like 41 times five.” He further explains that “20 is a factor of 100. So it multiplies by five . . . so I just multiplied 41 by five.”

T/R1 asks Yonny to think about that, and Yonny agrees.

****

Source Video*:* Early algebra, investigating linear functions, series 2 of 7, working on Guess My Rule problems 1-3, Clip 1 of 5: Yonny and Brandon with problem 1.

Clip: 5:21 to 6:28

****

Event #7: Yonny Says That 100 Goes to 205

Description:

The students are looking at video games on the computer and Yonny is whistling when he just says “205.”.

Brandon says, “So it (the answer) might be 205”.

T/R1 asks, “Do you think it’s 205?” and asks them how they got it.

Brandon explains: “41 times five, because 20 is a factor of 100. 20-> 41, so 41 times five, ’cause 20 times five equals a 100, so we just took the five from the 20, so we took the 41 and multiplied it by five.”

Yonny looks over and says “ . . . (inaudible) it’s 201”

****

Source Video*:* Early algebra, investigating linear functions, series 2 of 7, working on Guess My Rule problems 1-3, Clip 1 of 5: Yonny and Brandon with problem 1.

Clip: 7:30 to 7:52

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Event #8: Another (Familiar) Problem

Description:

The boys get another problem that looks familiar.
0->5
1->7
2->9
3->11
Right away we hear:

Yonny: “Okay, I got the rule already,” and Brandon: “I did too.”

Brandon is writing as he says, “The rule is going up by 1 on the x-side and by 2 on the y-side.”

He repeats this for T/R1.

****

Source Video*:* Early algebra, investigating linear functions, series 2 of 7, working on Guess My Rule problems 1-3, Clip 2 of 5: Yonny and Brandon with problem 1

Clip: 00:01 to 00:57

****

Event #9: Uh Oh, X =20 again!

Description:

T/R1 asks Brandon, “What about 20? (what about x = 20?) Brandon calls Yonny over, and begins to write all the pairs of (x, y) numbers from x = 1 to x = 20. Brandon sighs and says, “He has to write them all.”

T/R1 says, “I wonder if you can find a different way of getting your rule so that you don’t have to write it all out. Why don’t you take a look at the numbers in the table to see whether you can come up with another way of getting it.”

****

Source Video*:* Early algebra, investigating linear functions, series 2 of 7, working on Guess My Rule problems 1-3, Clip 2 of 5: Yonny and Brandon with problem 1 Clip: 1:50 to 2:27

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Event #10: How did you get 20 -> 45?

Description:

Brandon is working and complains about “doing 20 again”.

Yonny suggests that 20->43, and Brandon agrees for a second until he says, “No, its 45.”

After he is asked, Brandon shows T/R1 how he got 45:

10 + 10 =20

10 -> 25

20 + 25 = 45

He says that this worked for the previous rule when 10 -> 21

10 + 10 = 20

10-> 21

20 + 21 = 41

T/R1 asks Brandon where each number comes from and Brandon explains his method carefully.

****

Source Video: Early algebra, investigating linear functions, series 2 of 7, working on Guess My Rule problems 1-3, Clip 2 of 5: Yonny and Brandon with problem 1

Clip: 3:31 to 5:29

****

Event #11: The Ladder Problem

Description

Ariel and James are being introduced to a new problem: How many rods are used to make a ladder?

T/R2 shows them that 5 rods are used to create a one-step ladder. Then he constructs a two-step ladder and asks the boys how many rods it uses. Ariel counts the rods and answers, “Eight.” Then T/R2 tells them that they can build a ladder to be as long as they want. But the question is, “How many rods do you need to build a 10-step ladder?”

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Source Video: Early algebra, investigating linear functions, series 5 of 7, ladder problem, Clip 1 of 7: How many rods for 10 step and 100 step ladders?

Clip: 00:06 to 1:49

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Event #12: Ariel Has a Doubling Idea

Description

Ariel is working on a solution to the “10-step” ladder question. He uses the rods to start building a ladder.

Once he has a three-step ladder, he looks at it and says “Ah, I got an idea.” He carefully counts as he adds more rods to his ladder. James is building a ladder, too.

Ariel says, “I got it,” and tells T/R2 that there are 34 rods in a 10-step ladder. T/R2 says that he doesn’t see 10 steps.

Ariel explains that he only needed to build a five-step ladder. 17 steps are used in a five-step ladder and 17 x 2 = 34. When asked to explain again, Ariel tells T/R2 that 5x2=10 so that’s why the number of rods in a ten-step ladder should be 17x2 = 34.

****

Source Video*:* Early algebra, investigating linear functions, series 5 of 7, ladder problem, Clip 1 of 7: How many rods for 10 step and 100 step ladders?

Clip: 2:05 to 3:45

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Day 3: Exploring the Ladder Problem and the Development of Algebraic Concepts Over Time

Description

This analytic was created to be shown along with the published analytic, "Tracing Ariel's Algebraic Problem Solving: A Case Study of Cognitive and Language Growth," created by Drs. Robert Sigley and Louise Wilkerson and published on 2-23-15. Sigley and Wilkerson's analytic shows Ariel at the beginning of his work on the ladder problem in seventh grade and then again how much he had grown in mathematical understanding 18 months later in eighth grade.

In "Tracing Ariel's Algebraic Problem Solving: A Case Study of Cognitive and Language Growth," we are introduced to the ladder problem. A one-step ladder uses five rods and a two-step ladder uses eight rods as shown below: The ladder problem asks for an explicit function that gives the number of rods used in a ladder with "x" steps. (The answer is 3x + 2).

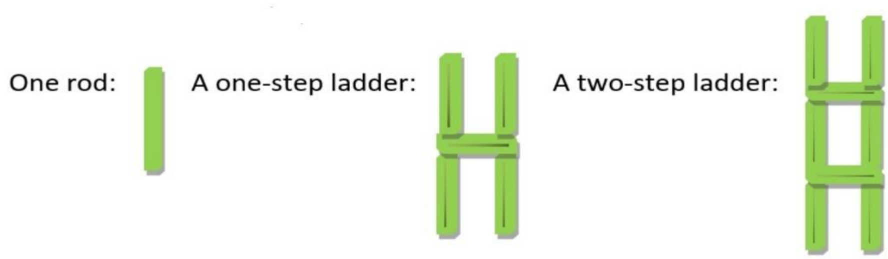

Figure 2: The Construction of Ladders in the Ladder Problem

In "Tracing Ariel's Problem Solving . . . ", seventh graders Ariel and James are asked, "How many rods are in an eight-step ladder?" They begin by building ladders, and Ariel decides that he only has to build a four-step ladder, count the rods (14) and double that number for an eight-step ladder. Note that Ariel isn't using proportional reasoning to avoid doing "a lot" of work; we can observe that building an eight-step ladder isn't that much work. Instead, Ariel is inspired with the idea as he is building the ladder. As he adds rods he says, quietly, "I got an idea." The idea is that the number of rods used to build an eight-step ladder should be twice the number of rods used to build a four-step ladder. Ariel doesn't check this assumption; he counts out 14 rods in the four-step ladder he did build and then says that an eight-step ladder will use 14 X 2 = 28 rods. Soon, he figures out that his original assumption was wrong: A ladder of eight steps does NOT use twice the number of rods that a four-step ladder uses. (A four-step ladder uses 14 rods; An eight-step ladder uses 26, and not 28, rods). James explains why: because there are 14 in a four-step ladder and then, he says, you add three rods for each new step. He explains that there are three rods in each new step, so 3x4=12 new rods for four more steps, so 14 + 12 = 26 (he tells Ariel).

Ariel then modifies his rule to double (the number of rods in a ladder half the requested size) and then subtracts two. This works in the case of an eight-step ladder. He knows a four-step ladder has 14 rods. 2 X 14=28 and 28 -2 =26.

This is his first heuristic, and it works when the ladder in question has an even number of steps. He also extends his heuristic to include ladders with an odd number of steps, by taking the "closest" even number, calculating the number of rods, and then adding (or subtracting) three to get the number of rods in the ladder with an odd number of steps.

This analytic, "Exploring the Ladder Problem and the Development of Algebraic Concepts Over Time," begins to answer the question: How did Ariel change from the student who was just beginning to explore algebraic functions and linear data, to the mature problem solver with formal knowledge of equations for linear functions with constant slope?

We see Ariel attempting to solve ladder problems with 80, 120, and 125 steps. As he attempts to use his rules, we see him incorporate proportional reasoning along with his rules.

For example, calculating the number of rods for a ladder with 80 steps, Ariel does not divide 80 by 2 and work on a 40-step ladder. Instead he claims that since an eight-step ladder has 26 rods, and 8x10=80, he can find the number of rods in an 80 step ladder this way: 10 x 26 =260 and 260 - 2 = 258. This may indicate that Ariel thinks his "even number rule" is for dividing by any number (not just by 2), or it may indicate that he is still using proportional reasoning and his rule is not governing his thinking.

There is also an example of Ariel using his rule when the number of steps is 125. In this case, he takes a nearby even number, 124, and divides it in half (as his rule directs). Then as he is looking at finding the number rods in a 62-step ladder, he decides to add the numbers he already has: A 60-step ladder was calculated (incorrectly) by Ariel to have 198 rods and a two-step ladder has eight rods. He adds these numbers to get 206 rods in a 62-step ladder.

The final event in this analytic deals with this last assumption by Ariel, that the number of steps in ladders can be added. In this last event, Ariel uses his rule successfully by adding the number of steps in each ladder and then subtracting two.

This analytic shows several opportunities for Ariel to explore the ladder problem, to explore when his rules work and when they are difficult to use, and to discover what is true and what is not (about the ladder problem). It provides a small window into the complex cognitive algebra trajectory that Ariel worked through before he emerged as the sophisticated algebra student.

Note that the first analytic that will be shown is Analytic 3a, the published analytic: Tracing Ariel's Algebraic Problem Solving: A Case Study of Cognitive and Language Growth (published by Robert Sigley and Louise Wilkerson).

The video data used in this analytic is taken from the corpus of data collected for Rutgers University under a National Science Foundation (NSF) funded grant entitled Research on Informal Mathematical Learning (IML, REC-0309062) program. The students and teacher/researchers in this analytic participated in the IML program, which included two cohorts of 20 middle-school students attending one urban school in central New Jersey. Ariel, a 13-year-old, bilingual, seventh grader, was a participant in the second cohort.

Event #1: Writing Down the Ladder Problem Rules

Description

T/R2 asks Ariel to write down his rules for the ladder problem.

Ariel expresses reluctance: "Can I just show you?"

T/R2 tells him "but also write it out," and he says that then "they can keep talking about it."

Ariel says, "I have to leave at 3:30," and then he starts writing the rules.

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Source Video: Early algebra, investigating linear functions, series 5 of 7, ladder problem, clip 3 of 7: Recording the procedures for ladders with odd and even numbers of steps.

Clip: 01:13 – 03:13

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Event #2: Writing Down the Ladder Problem Rules

Description

T/R2 has asked Ariel to find the number of rods used in a ladder with 80 steps. Ariel starts by considering an eight- step ladder; he looks back and sees that eight steps = 26 rods.

Then he does this:

26 X 10 = 260

T/R2 asks him, “How did you do it? Which one of the rules did you use?”

Ariel tells him that he used the rule for even numbers.

T/R2 gets more specific, asking Ariel many questions: “What is it? I want to understand. Take that number, divide it by half, right?”

Ariel says “Yeah.”

T/R2 goes on, “Then, make the ladder with that many steps, right? Then multiply the number of rods of that ladder by two and then subtract two. Did you do that for 80?

Ariel: “For who?”

T/R2 reminds him, “For eight zero (for eighty) Did you do that? Eighty is an . . . even number, right?”

Ariel says, “No, because , , , yeah, yeah, and then I subtract two from this. [writing out that 260-2 =258] Two fifty-eight.” "Ka-ching! "I rock!"

****

Source Video: Early algebra, investigating linear functions, series 5 of 7, ladder problem, clip 4 of 7: Predicting the number of rods for ladders with 80 and then 120 steps.

Clip: 00:32 – 1:59

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<u>Event #3: 120 Steps: Will this Question Clarify?</u>

Description

T/R2 asks, “How about for 120 (steps)?”

Ariel sighs and asks “120?” before beginning the problem.

Ariel says, “Six + six, Six, that is 60 times two. I got you!”

Ariel remembers that a six-step ladder uses 10 rods, and he says,

“So 20, that will be six times 10 will be my 60 and 20 times 10 is 200, minus the two, is 198. That will be my 60 times two.”

A summary of the calculations:

Six steps corresponds to 20 rods:

6 X 10 = 60 steps => 20 X 10 = 200 rods

200 – 2 = 198 rods

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Source Video: Early algebra, investigating linear functions, series 5 of 7, ladder problem, clip 4 of 7: Predicting the number of rods for ladders with 80 and then 120 steps.

Clip: 2:27 – 4:18

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Event #4: Did Ariel Use His Rule for 120?

Description

T/R2 is asking Ariel about how he followed his rule. The T/R says, “So if we divide 120 by two, what do you get?

Ariel says “Sixty.”

T/R2 asks, “And then you build that ladder with that many steps?

Ariel says “Yeah.”

T/R2 asks “And then you said multiply by two, right?”

Ariel says “Yeah.”

T/R2 asks, “And then you do what, subtract two? I didn’t see you do that. I don’t see that 60 there. Where is that 60?

Ariel responds “One hundred and ninety-eight.”

T/R2 tells Ariel that he does not see “the 120” on his paper. Ariel explains what he did:

“20 is the 6” (20 rods in a six-step ladder.)

$6 \times 10 = 60$ -> $20 \times 10 = 200$ -> $200 - 2 = 198$

T/R2 asks Ariel what the 198 means, and Ariel tells him, “60 is 198 rods.”

On his paper, Ariel shows the following calculation to find the number of rods in a ladder with 120 steps:

$198 \times 2 = 396 \Rightarrow 396 - 2 = 394$

T/R2 is still trying to understand how Ariel used his rule. He asks him more questions about using his rule for even numbers and using it again.

T/R2 says, “So did you do the same rule of dividing by two?”

Ariel tells him, “I divided 120 by two and got 60.”

T/R2 explains, “No, but how, I am just looking at your rule, you understand what I am saying? I am looking at your rule here, and I am just trying to apply it.”

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Source Video: Early algebra, investigating linear functions, series 5 of 7, ladder problem, clip 5 of 7: Ariel revisits his solution for the 120-step ladder.

Clip: 0:08 – 1:20

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Event #5: Rods for a Ladder with 125 Steps?

Description

Ariel is asked to find the number of rods used in 125-step ladder.

Using his rule for odd numbered steps, Ariel first works on the number of rods in a ladder with 124 steps.

Ariel divides 124/2=62 and then confirms that 62x2=124. He considers how many rods in a 62-step ladder (half of 124), and initially says, “I doubt it" when T/R2 asks him if can figure it out.

T/R2 reminds Ariel that he has a rule he can use.

And Ariel says, “My rule, uh, my perfect rule, I have to set it up for this huge thing.”

But then, Ariel realizes that he already had the number of rods in a 60-step ladder so he starts with 198 rods for a 60-step ladder. He briefly mentions adding 2 to 198, but then counts out eight rods in a two-step ladder.

Then he adds eight to 198 and concludes that there are 206 rods in a 62-step ladder.

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Source Video: Early algebra, investigating linear functions, series 5 of 7, ladder problem, clip 6 of 7: How many rods for a ladder with 125 steps?

Clip: 1:20 – 3:39

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Event #6: Combining Two Ladders

Description:

T/R2 asks Ariel to think an eight-step ladder as being made of a six-step ladder and a two-step ladder.

He asks Ariel how many rods are used in the six-step ladder and suggests that Ariel build the six-step ladder. Ariel does construct the ladder and counts 20 rods in the six-step ladder. Then he says, "Are you happy?"

Ariel does some singing while he constructs ladders and counts.

T/R2 asks Ariel to write down the number of rods used in a six-step ladder and Ariel does so.

T/R2 asks Ariel if he is agreeing that there are eight rods in a two-step ladder.

Ariel says, "Yeah."

T/R2 says, "So you write eight, you get what?"

Ariel says, "Twenty-eight,"

T/R2 says, "So, let's see whether eight has 28.

Ariel [starts to construct an eight-step ladder and then stops] "Wait a minute, I have 26."

T/R2 asks, "So then what we are going to do?"

Ariel says, "Go to my rule, minus two."

Ariel smiles, writes, and says, "Twenty-six."

Ariel counts the rods in the ladder he had constructed: "Here we go, one, two, three, four, five, six, seven, eight, nine, 10,11,12,13,14,15,16,17,18,19, 20, 21,22,23,24,25." [doesn't say 26 here].

Ariel says, "Ooh, Ooh, I am right. I am right, 26."

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Source Video: Early algebra, investigating linear functions, series 5 of 7, ladder problem, clip 7 of 7: Combining a 6-step ladder with a 2-step ladder?

Clip: 1:20 – 3:39

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Video Story #4: Guess My Rule and Its Secrets (Day 4)

Description

This analytic is the fourth video story created for the Teachers Algebra Workshop (TAW). The three analytics that precede this one, use clips of seventhth-grade students from the Informal Mathematics Learning Study; These students had been participating in the IML study for zero or one years prior to the times that these videos were taken.

This fourth analytic is composed of clips of T/R3 working with sixth-grade students at the Harding Elementary school in Kenilworth, New Jersey. The work with these students was part of a longitudinal study of children's mathematical thinking as they work on challenging mathematical tasks. Many of the students that participated in these clips have been part of the longitudinal study since first grade.

T/R3 uses a Box (B) for the independent variable and a Triangle (T) for the dependent variable. This allows him to visually show different values in the box and in the triangle. In event 5, a student does this naturally and puts it on the flip chart to show others. Using this notation, T/R3 first gives the students practice in how to understand the equation. He uses this example first: B + B + T = 9. Using this example, the class creates four types of values that could be places in the equation that are listed below. Note, legal values respect that fact that both "Boxes" have to have the same value. If they don't, the solution is illegal. True solutions will add up to 9, false solutions do not.

legal + true - for example: (4,4,1)

illegal + true -for example: (1,2,6)

legal + false - for example (8, 8, 100)

illegal + false- for example (1, 2, 3)

This model for two-variable equations is a powerful introduction to algebraic ideas. They first exercise their understanding of using an equation (a rule) to build a table of "truth values" — values that are legal and true. Then T/R3 gives them a table of truth values - B,T values and asks them to find the equation. They get several examples like this: a table of truth values, each presenting data to help uncover the secret rule.

As the children search for the equations—the first secrets they seek to find —they notice patterns in the numbers that will help them find the equations. These are the fundamental features of linear equations: the slope which they see as a "first difference" between dependent variable (T) values and the y-intercept (T-intercept), which they see as the T value when B=0. T/R3's model of using questions to guide the students and asking them to show and justify their ideas, allows them to discover the nature of linear equation with an independent variable (B) and a dependent variable (T).

Note that they do not have language for slope or first difference or constant or coefficient. Note that they end up using words like "thingy," which are not helpful to them, and they decide to point to positions in the equation to explain what they mean.

Teachers may want to look at a published analytic on Dr. Robert Davis' work with linear and quadratic equations in terms of box and triangle: Using Questioning to Promote Conceptual Understanding: Robert B. Davis Introduces Algebra Ideas to Sixth Graders.

<u>Event #1: What do Scientists Do?</u>

Description

T/R3 discusses what scientists do and points out that scientists discover secrets (about diseases, nature, etc.) The students know about some scientists, like Albert Einstein.

He tells the children that there are two sides to finding secrets. One is the pride of attaching your name to your discovery. The other is sharing secrets because, "In the long run, no single person could do it all by themselves."

This may suggest that the children are scientists/mathematicians discovering the secrets of the rules (the linear equations).

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Source Video Early algebra ideas involving two variables, Clip 4 of 18: Scientists and the nature of secrets

Clip: 0:05 – 2:00

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<u>Event #2: Creating Truth Sets</u>

Description

T/R3 introduces the sixth-grade students to equations with two variables. Instead of using x and y, T/R3 uses a “Box” and a “Triangle.” This is why he starts out by saying, "Sometimes there is more than one shape."

The children see a drawing of this equation on the flip chart:

Box + Box +Triangle = 9

(note : A triple of values like: (4,4,1) means that 4 goes in the 1st box, 4 goes in the 2nd box, and 1 goes in the triangle.)

When three numbers are "plugged in," the equation can be true or false. True means the sum of the three numbers is 9. False means that the sum is not equal to 9. A set of three numbers produces a legal equation when both boxes have the same number—otherwise it is illegal.)

T/R3 asks, "If I put some number in the (one) box, what do I have to do?"

Brian (a student) explains, "The numbers in the “boxes” have to be the same, but the number in the “triangle” can be different."

Other students agree, "Yeah."

There is a brief discussion on using other shapes (e.g., rectangles),

T/R3 writes: "illegal, true" and asks the students to write something that would be *illegal* but *true*:

Some students select some choices that are *legal* but *untrue* (For example, 8, 8, 2)

T/R3 writes 8, 1, 0 and asks if that is true, since 8+1+0 = 9, but 8 and 1 are both in one of the boxes so it’s illegal.

Then he asks for a statement that is *legal* and *false*. Jeff says "8, 8, 2" and T/R3 confirms that this is *legal,* because 8 is in both boxes and 8 + 8 + 2 does NOT equal 9. T/R3 shows them (1,2,3) and points out that this is *illegal* and *false*.

Michelle suggests (4,4,1) for *legal* and *true.* and Ankur suggests (3,3,3).

There is constant chatter about different solutions. Then T/R3 asks the children to quiet and listen to AmyLynn.

AmyLynn says,"Well, I think that three, three, and three is *legal* because like it’s, it’s *legal* for like the two *Boxes* and then like the *Triangle* is a whole new shape so you can start all over again."

Not everyone heard, so T/R3 asks her to say it again, and she does. Then T/R3 says, "That is certainly true."

****

Source Video: Early algebra ideas involving two variables, Clip 1 of 18: Open sentences that can be made true or false with legal or illegal substitutions

Clip: 0:01 – 3:04

****

Event #3: Truth Sets in a Table

Description:

T/R3 tells the class that instead of using the previous method, he is going to put the truth set in a table (that he draws on the flip chart).

He begins by creating a truth set for 2Box +1 = Triangle. He asks, "If I put zero in the Box, what do I put in the Triangle to make it true?"

Several students say "one" and T/R3 puts (0,1) in the chart,

Jeff asks, "Is there a secret here?" Then Jeff says that "There is no secret" and T/R3 says, "Well there might be."

The students start talking about secrets—e.g. "How can there be a secret if some people know it?"

Then, Stephanie asks, "How do you know there is a secret for everything?"

T/R3 draws everyone's attention to her question and a discussion follows:

Ankur says, "You have to go through the problems and find out."

Stephanie replies, "Yeah, but it could work. You can do like five different problems and it could work and (if) you cannot do this one little problem then it wouldn’t work."

Ankur replies, "Maybe just one. Maybe it just works on everything else but that one.”

And then Stephanie says, "Yeah, but then it doesn’t work then."

T/R3 listens to the student discussion. Then he asks them, "If I put one in the Box, what do I have to put in the Triangle to make it true?"

A Student says "3."

T/R3 asks, "Suppose I put two in the Box, what do I have to put in the Triangle to make it true?

Student, "5.".

Jeff says, "If he knows a secret, it’s not a secret anymore."

T/R3 asks about what happens if Box =3 and the students say 7. A table is drawn on the chart with the truth set they created showing Box values from 0 to 3, and Triangle values from 1 to 7.

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Source Video Early algebra ideas involving two variables, Clip 2 of 18: Truth sets in a table

Clip: 0:01- 2:38

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Event #4: Find the Equation (Guess My Rule)

Description

The students have been given three tables of true values for the missing equations. These are:

The first equation:
(0, 1)
(1,3,)
(2, 5)
(3,7)

The second equation:
(0,5)
(1,7)
(2,9)
(3.11)
(4,13)

The 3rd equation:
(0,1)
(1,4)
(2, 7)
(3, 10)
(4,13)

T/R3 asks the class, "Has somebody got an equation to show me? You have? Come show me. Come show me the equation."

Brian says,"Excuse me. I'm coming through with an answer.

There is a muffled discussion and T/R3 says, "That's certainly an interesting idea. But now what is it that you're going to… What are you going to show me ultimately?

Bobby says, "I got an idea."

T/R3 echoes, "You've got an idea?"

Bobby explains that he notices that the "times number" (the coefficient of Box) is the number

you add to each y-value to get the next y-value.

T/R3 agrees, "That's certainly what you do".

Michelle explains a pattern she sees for one of the problems; she explains that the difference between the x and y values grows by one as the x values go up by 1.

T/R3 says, "Wait, wait. Can everybody sit down for a second? I want to make sure we agree . . . I want to make sure we agree on what we're trying to do here.

What did we do the first time? I gave you an equation, right? The first time I gave you an equation and what did we do? We worked out numbers, pairs of numbers, that would make it true, right?

Now what are we doing now? I've changed it. What are we doing now?

A Student says, "Making equations."

T/R3 says, "Now I'm telling you the numbers that would make it true, I'm telling you the pairs of numbers that would make it true, and you're going to tell me the equation. We probably won't have time for that today."

****

Source Video Early algebra ideas involving two variables, Clip 3 of 18: Introduction to guess my rule

Clip: 1:57 -3:29

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Event #5: Searching for the Rules

Description:

T/R3 is asking Michelle to write her equations on the flip chart:

" Uh, you remember what you wrote on your paper?" Michelle says "no", but T/R3 asks her to put it on the chart anyway.

Michelle come up and asks him where to place the equations, and T/R3 tells her, "Yeah, cause that was sort of neat the way you did that." Michelle writes (box X 2) + 1 = triangle. At first she leaves out one parenthesis, but puts it in, when she is reminded.

Then Michelle places a zero in the box and a one in the triangle. She asks if she should "do more," and T/R3 tells her "Well, that's probably enough."

T/R3 tells the class, " . . . she went down and did that, and you agree that that's what we were doing?"

Students say" Yeah."

T/R3 asks, "Now, what did we do then? We, then we turned the problem around and did something different. Michael what'd we do then? Michelle?"

Michelle starts to say, "We tried to find a secret to it with a pattern like how the numbers . . . ."

Then T/R3 says, "Okay, and some of you did find a very interesting secret and it might be an appropriate one to share, . . . um, no, Ankur says that we shouldn't do that."

Jeff says, "Yes we should."

T/R3, says "Well, okay, well we won't we won't do it just now we will sooner or later. We will sooner or later okay . . . . I gave you the table and what are you supposed to do?"

Romina says, "Find the equation."

****

Source Video Early algebra ideas involving two variables, Clip 5 of 18: Recap of day 1, moving from one to two variables

Clip: 3:51 – 4:42

****

Event #6: Michael and Stephanie Discover Secrets

Description:

The students are working on finding the equations and the secrets. T/R3 asks, "Could I, could I get some idea of how we're coming along here? Um, what's the, how many problems have you people done?"

Several students shout out "I did one, the first one" and "I am on number 4." Ankur tells T/R3 that he is "on # 6." One student says, "We found the secret." T/R3 responds, "You found the secret? Good."

Students are asking each other what the secret is. And T/R3 says, "You, you want to be careful, there might be more than one secret that you might want to think about."

The students continue discussing and thinking. Brian and Romina are working on the table of values for problem 3 (which is 3 X Box + 1 = Triangle). Romina says, " . . . I think it's one this time, isn't it?" Brian says, "It doesn't work." Romina replies, "Oh no . . . How come it doesn't work? 0 times 3 is 0 plus 1equals 1." Brian says, "Oh 1, ok 1."

Milin and Michael are working on problem #2 (2 X Box + 5 = Triangle). Michael thinks he has it, and T/R3 tells him "See if Milin agrees with you".

Michael explains to Milin, "3 times (2 equals) . . . 6 . . . 6 plus 5 is 11 . . . . Yup, I got this one. That was easy! It's easy, you know you take . . . 4 times 2 plus 5 is 13. It works for everything. Is that the secret?

Milin says, "What'd you say?"

Stephanie has found a secret. She shows T/R3. T/R3 asks, "What's the secret?

Stephanie says, “Is the secret that… this number… if you put the top number up here, in this place, it works? Like…”

T/R3 says, “That’s right, that’s a good thing to do… Um, at some point… at some point I want you to say, uh… let me find out how we can get you to say that with the camera.”

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Source Video Early algebra ideas involving two variables, Clip 7 of 18: Michael discovers the secret!

Clip: 0:13 – 4:09

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Event #7: The Secrets are Revealed

Description

Stephanie says, "OK, the secret is that the first number in the triangle row, if you put that in this place (the constant) right before the equal sign, it’ll work, all the time."

T/R3 asks, "Who else has a secret you’re ready to tell (the camera)"?

Michele tells T/R3 about subtracting "seven minus five, and that’s what you add on for each next thing" (thing = next Triangle value).

Ankur explains that this "difference between each two numbers is put in the first column" he points to the coefficient of "BOX."

Michelle explains that the "3" (in the equation 3B +1 = T) came from subtracting the triangle numbers 1 from 4.

T/R3 tells her, "that’s very nice, that’s a very important idea."

Then T/R3 invites AmyLynn and Bobby to explain what they did.

AmyLynn explains, "The ’five’ here we use as a ’plus number’ (for the rule 2B+ 5 = T).........

T/R3 questions Bobby, "OK, now you have another number, how did you find that other number?"

Bobby answers "minus the seven for five.........."(the 2 in the equation above)

T/R3 questions Bobby about his equations and hones in on Bobby’s subtraction: is it 7-5? or 5-7?

After, T/R3 points out that subtracting 7 from 5 gives -2, Bobby corrects his mathematical grammar

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Source Video Early algebra ideas involving two variables, Clip 8 of 18: Sharing secrets for tables 2 through 5

Clip: 00:42 – 2:51

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Tracing Ariel's Algebraic Problem Solving: A Case Study of Cognitive and Language Growth (Day 3)

Description

While research has shown that understanding the concept of a function is essential for success in other areas of mathematics (Carlson, 1998; Rasmussen, 2000) students continue to struggle learning the concept (Vinner & Dreyfus, 1989). Research has revealed that young children, who are engaged in problem-solving activities designed to elicit justifications for their solutions, develop an understanding of fundamental algebraic ideas such as function (Maher, Powell & Uptegrove 2010; Kieran, 1996; Yerushalmy, 2000; Kaput, Carraher, & Blanton, 2008). Davis (1985) advocated the introduction of algebra to elementary school students as young as grade 3. He argued that the idea of function can be built intuitively by students as they engage in explorations of problems requiring identification of increasingly more challenging patterns; further Davis claimed that students can build the conceptual idea before formal notation is introduced. Davis (1985) offered sets of tasks for student exploration, and he video/audio recorder problem children's problem solving as they constructed solutions that can be expressed with linear, quadratic and exponential functions (Giordano, 2008; Mayansky 2007). Extending this work, Bellisio and Maher (1998) studied students who provided verbal expressions of algebraic function prior learning to write the rules in symbolic form.

This analytic extends this earlier work by examining how one student, Ariel, builds an understanding of the linear function concept and represents his understanding of the basic algebra ideas underlying the construction. One focus was to see if students could provide a general solution to the problem. A second focus is on use of the mathematics register, the specialized kind language used in mathematics teaching and learning that is characterized by precision and linguistics in both oral and written language (Silliman & Wilkinson, 2015). This analytic presents a task that requires students to determine how many light green Cuisenaire rods are needed to build a ladder with different number of rungs. The shortest ladder has only one rung and can be built with five light green Cuisenaire rods. A two-rung ladder would be modeled using eight light green rods. The problem was presented as follows: The Ladders Problem: Build a rod model to represent a three-rung ladder. How many rods did you use? How many rods would you need to build a ladder with 10 rungs? How could you represent the number of rods needed if you were to build a ladder with any number of rungs? Justify your solution. This analytic reveals how Ariel first approaches the problem using an arithmetically proportional approach to build a recursive composite function that depends on whether the numbers of rods are even or odd. When he revisits the problem 18-months later his approach changes. He develops a function table, uses first differences, and constructs a general solution to the problem. His gradual adoption of the mathematics register is exemplified in his oral explanation of the meaning of his symbolic notation. This analytic highlights that early, informal open-ended problem solving tasks provide students opportunities to construct their knowledge. These problem-solving tasks are explorations at the heart of developing mathematical understanding, but not as simple follow-up activities to procedural instruction. One implication of this work is that teachers include both time and tasks for students to explore, examine, revisit, and connect ideas and concepts through investigations. In so doing students have authentic opportunities to build strong intuitions of the problem conditions. Students' engagement in activities, such as The Ladders Problem, provide them with the foundation for gaining insights and deeper understandings of mathematics. Ariel used such an opportunity and built his algebra knowledge. His success is revealed in the elegance of his solution, the understanding of his earlier work, and his confidence in offering clear justifications.

Event #1: Making Sense of the Problem

Description:

The researcher, T/R2, introduces the problem to two students, Ariel and James. While introducing the problem, he identifies what the variable (the number of rungs in the ladder) as well as what you are trying to predict (the number of rods in the ladder). The researcher defines vocabulary terms, such as the definition of a rung. Both the problem statement and the vocabulary are clarified by modeling the problem solving in two instances ("where the number of rungs in the ladder are one and two").

****

Source Video: Early algebra, investigating linear functions, series 5 of 7, ladder problem, Clip 1 of 7: How many rods for 10 step and 100 step ladders?

Clip: 00:50-01:51

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Event #2: 34 Rods for 10 rungs based on proportional reasoning

Description

Ariel proposes a solution of 34 rods for a ladder with 10 rungs. He reasons that he built a ladder with five rungs using 17 rods and multiplied it [the number of rods] by two because five times two is 10. At this initial point in his problem solving process, Ariel uses a proportional thinking approach to make a prediction about a ladder with 10 rungs. This is a common problem that is seen in children when moving from pattern recognition to algebra (Kieran, 1992.) In this case, the focus is on the variation in the number of rods, as opposed to the relationship between the number of rods and the number of rungs. Ariel's s oral and written language is neither precise nor complex, although he uses the terms "counted" and "multiplied."

****

Source Video: Early algebra, investigating linear functions, series 5 of 7, ladder problem, Clip 1 of 7: How many rods for 10 step and 100 step ladders?

Clip: 02:51-03:50

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Event #3: Checking initial conjecture for eight rungs

Description

Ariel uses this method to make a conjecture of 28 rods for a ladder with eight rungs. His partner, James, claims there are only 26 rods and challenges Ariel to build a ladder with eight rungs. Ariel builds a ladder with eight rungs and counts up 26 rods, not the 28 he had predicted. James tells Ariel that it is 26 because you add four more (steps) add three more steps for each one. For another step it is three blocks. James focuses on the relationship between the two variables ("the number of rungs and the number of rods") as he points out that as you add a new rung to the ladder there are three new rods [steps] that get added. In response to James' challenge to justify his answer, Ariel notices a discrepancy between his predicted value of 28 and the correct solution of 26.

****

Source Video: Early algebra, investigating linear functions, series 5 of 7, ladder problem, Clip 2 of 7: How many rods for 10 step and 100 step ladders?

Clip: 00:27-01:49

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Event #4: Developing a recursive composite function

Description

A couple minutes after working on a ladder with eight rungs, Ariel offers the solution of 32 for a ladder with 10 rungs. When asked how he solved the problem: Ariel offers a recursive composite function where: For odd numbers, I go to the nearest even number take half of that even #, count the rods for a ladder with that many steps multiply it by 2 subtract 2 and add 3. For even numbers I take half of that number and make a ladder with that many steps. Then I multiply the number of rods of that ladder by 2 then I subtract 2. Ariel tests his rule for four cases and appeared to be satisfied that it addressed the conditions of the problem for those cases. Written using symbols, Ariels formula was: If Odd f(x) = f((x-1)/2)*2-2+3 If Even f(x) = f(x/2)*2-2 which are equivalent. The non-recursive version of the equation is Y = 3x + 2. In Ariel’s equation the -2 compensates for multiplying the constant of +2 by 2 (creating +4 in the equation). The +3 in the odd case is of interest. Since Ariel is going to the nearest even number that in this case meant down, the +3 is equivalent to adding another rung to the ladder. When asked to explain what those numbers meant, Ariel does not provide an explanation of adding 3 or subtracting 2. In this segment of his problem-solving process, Ariel reveals his increasing use of the mathematics register. Both his oral and written linguistic expressions are more precise and more complex. In his formulation of the rules, Ariel uses the technical terms “multiply, even, odd, subtract,” and “number.” Several aspects of his language reveal complexity, including longer sentences, due to his frequent use of elaborated (dense) noun phrases such as the “nearest even number,” “half of that number,” and “the number of rods of that ladder.” Ariel uses a listing strategy with these phrases. After Ariel states the odd number rule and writes it down, he justifies his work: “Because for every new thingy it is three rods, and it will give me 29.”

****

Source Video: Early algebra, investigating linear functions, series 5 of 7, ladder problem, Clip 2 of 7: How many rods for 10 step and 100 step ladders?

Clip: 03:49-07:13

****

Event #5: Using First Difference

Description

Eighteen months later, Ariel was interviewed about the same ladders problem. At this time, Ariel builds a table and provides a justification for his solution to the problem, based on using a recursive approach of first differences. Although Cuisenaire rods were available for Ariel to use in solving the problem, he chose not to use them. He attends to the relationship between the number of rods and the number of rungs, instead of focusing on the recursive relationship. That is exemplified by his reference to the difference between the Y and the variable and his construction of an X-Y table. While constructing the table, Ariel writes the first order difference of 3 between each value. Ariel incorporates more aspects of the mathematics register is this section, including extensive use of precise technical vocabulary such as "difference," "variables", and "linear;" elaborated noun phrases (“linear equation," "x-y table;" "y-variables”); nominalizations (“the first difference”); and uses subordination (“because”).

****

Source Video: Early algebra, investigating linear functions, series 7 of 7, Ariel's 8th grade interview, Clip 2 of 5: Ariel solving the Ladder problem

Clip: 02:48-03:47

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Event #6: Generalizing the Ladders Problem

Description

Ariel is asked to develop a way to represent the number of rods in a ladder with any number of rungs and he quickly offers the solution of Y=3x+2. When asked where his formula came from Ariel references the first differences he earlier constructed and pointed out that the number of rods goes up by three each time and that since the difference is constant, the equation would be linear. He continues by pointing out that the three is related to the X in the equation and that if you just add two you get the answer and that it works every time. Ariel continues to represent his understanding by using the previously referenced elements of the mathematics register. In this excerpt he includes an appositive element ("like if it was the first ladder, second ladder, third ladder"). His statement combines both oral and more literate forms.

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Source Video: Early algebra, investigating linear functions, series 7 of 7, Ariel's 8th grade interview, Clip 2 of 5: Ariel solving the Ladder problem

Clip: 04:51-06:39

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<u>Event #7:</u> Explaining the meaning of the "3" and the "+2"

Description

Ariel is asked to explain what the meaning of the three and the plus two are in his equation. He uses formal academic language in referring to the three as the slope and the two as the y-intercept and points out that his rule works for every one. In explaining his reversing to end up with 2 for the y-intercept he then points to the rods and says to notice that when you take away 3 for each you are left with the 2 bottom rods, making connections between the numbers in his equation and how the ladders are constructed. Events 5, 6, and 7 considered together, reveal that Ariel knows about and uses linguistic complexity to convey his understandings, even though he has not fully realized the mathematics register.

****

Source Video: Early algebra, investigating linear functions, series 7 of 7, Ariel's 8th grade interview, Clip 2 of 5: Ariel solving the Ladder problem

Clip: 06:41-07:52

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<u>Event #8:</u> Comparing solutions

Description

Ariel was given an opportunity to watch a video of himself solving the same problem 15 months earlier. After watching the video, he was asked to reflect on what he did then versus what he did now. Ariel referred to his recursive composite function as "the long way" and commented that he "didn't really know that much about equations.". He also pointed out how his new approach was better and more efficient to obtain the answer since there was only one equation.

In the end he concluded his old approach was a way to check if you got the correct answer and said that both solutions were “effective.” This may suggest that he is open to more than one approach to establish the correctness of his reasoning.

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Source Video: Early algebra, investigating linear functions, series 7 of 7, ladder problem, Clip 3 of 5: How many rods for 10 step and 100 step ladders?
Clip: 06:11-08:47
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APPENDIX B: THE MATHEMATICS QUESTIONNAIRES

The Day 1 Mathematics Problem Worksheet

1. Given the following pairs of values, find the rule that produces the "triangle" value when it is given the "box" value :

□	△
5	13
3	7

2. Given the following pairs of values, find the rule that produces the "triangle" value when it is given the "box" value :

□	△
1	15
2	25

3. Describe the difference, if any, in how you solved these two problems.

The Day 2 Mathematics Problem Worksheet

1. Given the following pairs of values, find the rule that produces the "triangle" value when it is given the box value :

$\square$	$\triangle$
0	1
1	3
2	5
3	7
4	9

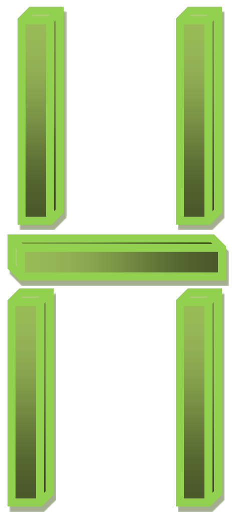
2. Is it always true that $f(2x) = 2f(x)$? Explain and prove or give counterexamples

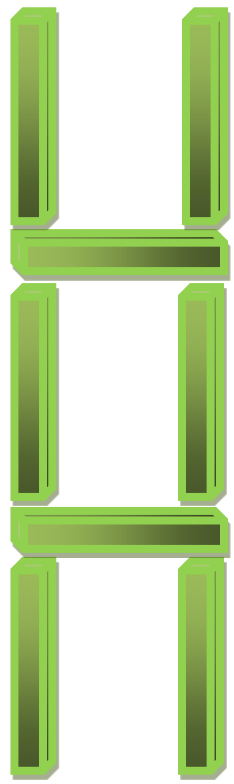
The Day 3 Mathematics Problem Worksheet

One rod: 

1. Ladders are made of

   Rods shown at the right:





   The one-step ladder uses 5

   rods.

   The two-step ladder uses 8

   rods.

a. **How many rods are used in**

a 10-step ladder?

b. **How many rods are used in a ladder with "x" steps?**

The Day 4 Mathematics Problem Worksheet

1. Consider the following equation: $\square + \square + \Delta = 9$

   a. Find a solution to this equation, three numbers that can be placed in the two empty *boxes* and 1 empty *triangle* that is "legal" and "true."

   Find a solution to this equation, three numbers that can be placed in the two empty <u>*boxes*</u> and the empty *triangle* that is "illegal" and "true."

   b. Find a solution to this equation, three numbers that can be placed in the two empty *boxes* and the empty *triangle* that is "legal" and "false."

   c. Find a solution to this equation, three numbers that can be placed in the twoempty *boxes* and the empty *triangle* that is "illegal" and "false".

2. Find a "truth set" for the equation: $2 \boxed{X} \quad +1 = \Delta$

□	Δ

3. Given the following "truth set," find the equation (the rule) that works for the entire truth set.

□	Δ
0	5
1	7
2	9
3	11
4	13

4. Given the following "truth set," find the equation (the rule) that works for the entire truth set.

□	Δ
0	1
1	4
2	7
3	10
4	13

APPENDIX C: DAILY VIDEO STORY QUESTIONNAIRES

Day 1 Video Story Questionnaire

Video Story #1: Understanding Linear Functions – Guess My Rule

In answering these questions, think about the whole analytic as telling a teaching and learning story. Your answers should refer specifically to evidence you find in the video, and/or to your experiences a teacher:

1. Compare the students' approach to finding a solution in the first problem (where the underlying rule was $y = 3x - 2$) with their approach to finding a solution to the second problem (where the rule was $y = 10x + 5$).
   a. Consider how their different problem-solving approaches helped or hindered their efforts.

2. The students didn't find a complete solution for the first problem, and they did find an explicit function for the second problem.
   a. Could that have been predicted by the teacher/researcher?
   b. Do you think that it was specifically planned to happen this way and why?

3. Why do you think the teacher/researcher didn't tell the students "the answer" (the rule for $y = 3x - 2$) before going to the second problem?

4. What did the teacher/researcher do when the students guessed "multiply by 10 and divide by 5" when guessing the second rule?

5. Did the teacher/researcher tell the students "they were right" when they found "multiply by 10 and add 5" for the second problem?

6. Why do you think the teacher/researcher did not, for the most part, label student answers as "right" or "wrong"?

7. Why do you think that the students did not ask the teacher/researcher "Am I right? Am I right?"

8. How did the teacher/researcher get the students to think about different answers?

9. Did the teacher/researcher demonstrate interest in student work? How do you know?

10. What mattered to the teacher/researcher in this "classroom"? How do you know?

11. Explain how the students did or did not seem engaged in the mathematics.

12. Explain why you think that the students did or did not need more "help" from the teacher/researcher?

Day 2 Video Story Questionnaire

Video Story #2: Yonny and Brandon and Ariel Manage Common Cognitive Obstacles

1. There are three different problems that students are solving in this analytic. Please describe how they approached solving the problem, what you think they do understand and what you think they don't understand. Justify your answer with evidence from the analytics.

   i. The $y = 2x + 1$ "Guess My Rule" Problem

   ii. The $y = 2x + 5$ "Guess My Rule" Problem

   iii. The Ladder Problem

2. Consider the solution that Brandon describes in finding what the rule does for X = 20 in the second problem (when the rule is $y = 2x + 5$):

   Brandon shows the teacher/researcher how he found 25 as the y-value , when $x = 10$, by doing the following:

   i. $20 = 2 \times 10$

   ii. Using a chart showing the values, we can see that If $x = 10$, then $y = 25$

X	**Y**
0	5
1	7
2	9
3	11
4	13
5	15
6	17
7	19
8	21
9	23
10	**25**

   iii. $20 + 25 = 45$, so the y-value when $x = 20$, is $y = 45$

   iv. He also showed the teacher researcher that the same thing worked for the previous rule $y = 2x + 1$:

X	**Y**
0	1
1	3
2	5
3	7
4	9
5	11
6	13
7	15
8	17
9	19
10	**21**

   v. $20 = 2 \times 10$

   vi. When , $x = 10$ then $y = 21$ (this was arrived at using the chart, counting y-values by 2)

   vii. $20 + 21 = 41$

INVESTIGATING A MODEL USING VIDEO STORIES FOR PROFESSIONAL DEVELOPMENT

The following questions are about the solution on page 2:

1. Please describe any evidence in the video that Yonny initiated, or collaborated with Brandon on this solution or that he understood it.

2. The algorithm that Brandon carefully explains in the analytic works for these two examples. Why? Is it a mathematical accident or can you justify why it works?

3. Would this solution work for any other values of x? Justify your answer

4. Are there values of x for which this solution does not work? If so, explain why not

5. Try to modify this solution so that it would work for all $x-values$ and justify your solution.

6. Given that we saw Brandon find the y-values for $x=10$ by “adding 2 to the y-values” in both charts, how do you think he and/or Yonny may have come up with this doubling model? (Here I am asking you use your imagination and conjecture how it may have happened. We cannot actually know how it happened.)

Day 3 Video Story Questionnaire

Day 3: Video Story 3a: Tracing Ariel's Algebraic Problem Solving: A Case Study of Cognitive and Language Growth

1. Consider event by event, the emergence of Ariel's problem solving in questions:
    a. How does he approach the ladder problem on his first and each subsequent attempt?

    b. What do you think prompts Ariel to create his first heuristic and each subsequent modification of the heuristic?

    c. What understanding of the ladder problem is reflected in the first heuristic and in each subsequent modification?

2. Describe the difference between the 7th grade Ariel with his heuristic solution to the Ladder Problem and 8th grade Ariel's formal algebraic analysis of the problem.

3. Describe connections between the analysis of the Ladder Problem that Ariel was building in 7th grade and the formal algebra he displays knowledge of in 8th grade.

Day 3: Video Story 3b: Exploring the Ladder Problem and the Development of Algebraic Concepts

This video story features 6 events that show Ariel using his rules for solving the Ladder Problem in different ways. For each event, use evidence from the video to explain your answers to #1, #2, and #3:

1. Explain how Ariel is using his rule in a precise way – in some part or all parts of his work.

2. Explain how Ariel is using his rules in an imprecise way – in some part or all parts of his work.

3. Is his rule helping him to solve some or all parts of the problem or is it presenting a cognitive challenge that he must meet?

4. Once Ariel creates his rule, does he always try to use his rule?

5. As Ariel begins to work on the Ladder Problem (analytic #1) and after he writes his rules for the solution of the ladder problems, as he tries to find the number of rods used in 80, 120, and 125-step ladders, he is not told explicitly when he is correct and when he isn't.

   Explain how different the problem solving might be if Ariel asked "am I right" and was answered each time with an explicit answer, and he was shown what was right and what was wrong.

Day 4 Video Story Questionnaire

Video Story #4: Guess My Rule – and Its Secrets

For each question, please justify your answer with explicit evidence from the video and/or your knowledge of teaching.

1. There are many differences between the Guess My Rule lessons in this last analytic and the Guess My Rule lessons in the first 3 analytics. Please describe these differences in terms of:
   a. Pedagogy
      i. What is being taught
      ii. How it is being taught
   b. Student Participation
      i. Describe how the students are collaborating
      ii. What are the students doing to solve the problems?
      iii. What new ideas are the students exploring?

2. In Event #2, Dr. Davis asks the children to place values in the equation: Box + Box + Triangle = 9 that produce combinations of "illegal" or "legal" in combination with "true" or "false".
    a. Explain why you do or do not think that this is the first time the children have worked with this idea - of plugging numbers into equations to get true or false, illegal or legal?

    b. Why do you think that Dr. Davis asks the children to look at illegal + true, and illegal +false answers? (After all, these are not the answers we will want the children to give later on.)

3. Explain why Dr. Davis repeats "what they were doing" and "what they are doing now". Several events repeat this. Do you think it's random or based on time between classes? Or do you think it's important for another reason?

4. Why do you think Dr. Davis made a point to ask Michelle to put her equation on the chart: Box X

    2 = Triangle, and then to fill in the box and the triangle so that it looked like this:
    $\boxed{0}$ X 2 + 1 = 1

    What is the value of this model for an equation?

5. Why do you think that some of the students don't want to hear "the secret(s)"?

APPENDIX D: THE TAW WORKSHOP CODED TRANSCRIPT DATA

Day 1 TAW **Transcript Data**

Stmt	Speaker	Statement	Code
1	JL	: And the other thing in answering these questions is to think about the whole teaching and learning story. Your answers should be specifically ….to the evidence you find in the video and what your experiences are as a teacher. One, compare students approach to finding a solution in the first problem, where the underlying rule was y=3x-1…., the third approach to finding a solution to the second problem where the rule was y=10x+5. Consider how the different problem approaches helped or hindered their efforts.	
2	3	– Well in just watching the tape with the video, once they got going they all were trying something or guessing pretty much till no one seemed to have a clue. Anyway most were just guessing or repeating what someone else said but yet they were involved. It was good he…inaudible….that was good too. They needed some guidance to get in there.	Wb
3	3	- Guess and check. They really didn't have a clue, weren't sure what they were supposed to be doing but then I guess eventually some of them finally got a clue.	M
4	3	- Just by doing and doing and doing and trying and hearing other kids say this or that and they all were mostly pretty enthusiastic;	E
5	2	- One thing I thought was cool was that right away, they weren't afraid to make a mistake. They were saying whatever, so that was good. I guess, however, that class goes, because nobody was afraid and no one picked on anyone when they said a crazy answer. They just let them say the crazy answer. But then they thought that maybe that could be a way to get the answer. So I thought that was good.	E
6	3	- In the beginning I thought they were bored, (yeah yeah in the background - 2 I think), Oh man, but then, pretty quickly he (Powell) got it going pretty well, I thought.	P
7	2	- They had some kind of confidence that they were gonna be able to do this so maybe it wasn't brand new. Cause it seemed. Remember when he came, he said, "Oh were going to be guessing a rule", so may be some of them were familiar a little	E

		bit? Maybe it's why they were able, and felt comfortable enough to like "OK we can do this", take the risk.	
8	4	– I don't know how objective that would be …inaudible..[this was inserted by Denish and it looks like something JL would say – not 4 – confirm]	Wb
8.5	JL	- don't just trust the audio tapes - don't forget to write some things down.	
9	4	– So one thing I observe again was that by the time…. the first question they tried to get - they didn't really get to guess the rule. But by the time they got to the second one, somebody actually came up with the rule. They got more confident and it was faster for them.	EM
10	3	- Yes, well, the good news…yes, especially the kids that…?...If that's true, then that's fine. If that's 3 and you wanted 5 and hit the rule yet, just do that and he didn't want to know what was the rule.	C
11	2	- Even his thinking - even when he said, "imagine you were in a factory"... so he really got the idea of about a function. I thought that was really good.	M
12	3	- lets write some of this stuff down.	Wb
13	2	- yeah, I know.	F
14	2	– I also think that by choosing the right numbers.. or having the right numbers...I think maybe the second problem was easier because they conclude they could see a pattern even in the beginning. Yeah even (something about "when it made sense" (not clear) and 3 is talking too) when he started to identify there was a clearer pattern.	M
15	3	- clear pattern….good point	M
16	4	– And then, when they first started, it was like they were just throwing out numbers, but at some other point could see that they now were making educated guesses because they were see a pattern that you see. They see okay, when its 4 it's this, when it's this, so you now when it's 2 you know that 2 has to...so they	M

		say that it was increasing. So they wouldn't just throw it out apart from kid saying 20? Laughter.	
17	3	- and then also, the teacher said, Ariel to "don't say anything, be quiet", you know let the others, you're on to it. Everybody else think.	P
18	2	- I think that can be difficult at times, when you see that they're not getting it, you want to help them, you want to be like, (Others interject - yeah, yeah - laughter). So I thought that, whoever the teacher was, he did a really good job. He was patient, he saw that was a great point that you picked out, he gave little subtle - "okay you're on to it" but not overboard.	P
19	4	- Yes, They didn't find (the first rule),	M
20	2	- And they didn't come up with it.	M
21	2	- And they did find an explicit function for the 2nd problem. Could that have been predicted by the teacher. I think so because of the number patterns Right? (Yeah Yeah from 3 and 4).	M
22	4	– I think…you know like…basically, When I was doing it. Was like, You had 5 first and then 3. Look at that. You are used to 1, 2, 3 - but in this case they were decreasing. That could have accounted to why it took them so long for them to get the pattern in this one. And then, when they started getting numbers, the 3 numbers were in random order and it did not help. Not 1 is this, and 2 is this. You can see how 3 is interrupting saying 5, 5, 5,	MP
23	4	- 1 - 15, 2-25, so you could see. The numbers for the one were random. So they could have predicted this.	M
24	3	- I can't remember (should we go back to the…) he did give them a starting a number and then they threw out (another) number. (everyone's talking). So he probably didn't realize that they were going to pick like all over the place. Laughter. So could have he predicted the pattern?	P

25	4	– Maybe he had given them the first two, ..in giving them the first two, (2 -right) then they could have gone ? wrong with that. Just Give them "1". One of the guys said that he already knew (they all laugh - this is a reference to the +8 suggestion for the first rule).	P
26	2	- I think it maybe he could see the second part: (The question is: Do you think it was specifically planned to happen this way?) be by just giving the one number, he wanted them to come up with these, you know, to figure it out.	P
27	4	– Yeah, I think if you have experience, you would be able to predict what would happen. I think somebody like him should have had experience in the classroom. So I think he would have seen it already, this is how it's going to make a richer discussion.	P
28	3	- The second one seemed to go more subtle, once they got used to what they were supposed to be doing, by trial and error on the first, then here's the second - Oh that one makes sense!! So it..I think they are going to do it. Yeah.	M
29	who	yeah	F
30	3	- Also I think it depends on like - it says if "could you have predicted by the teacher, the struggle for the first one?" So if you were teaching an Honors Class and doing this, (2 - right!), you'd probably have the result that we saw - probably (Note - 3 did ask me if these were honors kids when we took a break).	EC
31	3	- In that case, you would try for a harder problem.	P
32	3	- But he was very relaxed.	P
33	2	- Yeah he really was. I thought he did really well. He was a big guy. He did say, "Now OK, sit down". He kept his cool really good.	P
34	3	- He had a student do the second one. The student who got the final solution was up there. He (the student) wanted to throw the other kid out.	P
35	2	- It shows how having student-led work really improves participation because they really got aggressive. Yeah Yeah	P
36	??	We gotta keep going. Take a break?	

37	3	- Why didn't the T/R tell the students the answer, the rule for y = 3x-2 before going to the second problem? (sighs) Why didn’t he tell the answ	M
38	2	did they show the proof?	MQ
39	3	What’s that?	M
40	2	They did not (interjected by 4)	M
41	4	They did not come up with any rule... so maybe he wanted to give them another chance to go and do the second problem. When they do the second one, they might go back….sometimes makes it easier.	P
42	2	Yeah – yeah	F
43	3	Right,..yeah, exactly when you think about the first one.	M
44	2	Maybe when they are getting frustrated, maybe let me give a different approach of making it easier.	M
45	4	Making it easier. I know that somebody already said….for the first one there was -2. Somebody said that and already I know the guy, the number 2 guy said +2, then I think Ariel said -2 and then the guy, question 2 guy said something like 3 …that I found amusing too.	M
46	2	- Take the number, let me write that down.	M
47	4	So what did the teacher…..point “-2”	M
48	2	Because he was saying 2…Somebody said Yonny and Yonny said just add 2.	M
49	2	Yeah…seems he mastered the whole thing. (laughter).	M
50	??	- Obviously they like this	
51	??	- Do you see this exact thing	

52	4	Yonny said something about..something…Younny was talking to	F
53	3	- (reading from the questionnaire "What did the teacher/researcher do when the students guessed "multiply by 10 and divide by 5"	P
54	2	- When the students give a wrong answer. It says here, the teacher keeps telling them to try it out to see whether or not your rule works. So maybe after they said the crazy answer	P
55	??	"just me" inaudible. This is	
56	??	"Ask them"	
57	??	"The teacher asked them to think about whether the answer is right or wrong"	
58	2	- This is my problem as a teacher sometimes, giving the students that wait - for me to have that wait time when they say something wrong, you want to be like "what?" inaudible, then laughter,	P
59	2	- It's just like, say, oh how we did, ... that's right, you know. more chuckling. .... Now that's not right!	P
60	3	- In that case...	F
61	2	- He had a real calm about him as he said, ...motionless...	E
62	3	- inaudible	F
63	2	- Students from Bermuda... laughter	F
64	??	Obvious	
65	JL	- Look at what he says.	
66	2	- I think by doing that it gives them a chance to keep the dialog going rather than saying like "oh that's right" and I think the students will stop. 4 is saying "mm hm" in assent	P
67	3	- Why do you think he didn't label the answers as right or wrong? (quoting the question) To not discourage them, to keep going, freedom.	P

68	2	- and to them to challenge each other. You know? Hm mm.	P
69	3	To get them to challenge each other. Like If you say that "that's right", you know, how do you know? Laughter.	P
70.5	3	- put all of them down.	F
70	JL	- I think you have your answers, but you didn't write them. Yeah, do you want to play it (the analytic)? Someone says no.	
71	3	- why do you think students did not ask "Am I right?" "Am I right?"	E
72	2	- Now that she told us that it is an afterschool program, its kinda like they were used to a discussion, maybe?	E
73	3	- That's a good question	E
74	4	- they were somehow confident of what they knew. They didn't like .... they said....the answer - they didn't need the teacher to validate what they knew. They were that confident. When he said, "You should try it out", when they tried it out, they didn't have to ask the teacher "am I right?" They knew, yes it works, so I'm right …	E
75	3	- or it doesn't so I am not right.	E
76	2	- Maybe because the part of the session with the teacher, he never gave them any kind of hints or clues - they were used to him saying "try it out". So maybe his approach of just "trying it out" made them have that confidence and then they have to prove it.	P
77	4	– And then after a while they actually owned their own learning. It was just there. It was like….. they almost forgot he was there. Laughter.	E
78	2	- Its true!! laughter.	E
79	4	- They own their own learning - maybe that’s why they didn't think to ask him, to refer back to him......in a way they were doing, they were having so much fun with each other that ...	EC
80	2	- Yeah. I am still in some amazement (yeah). Lots of laughter.	EC
81	3	- ...should study more...of the video.	P

82	??	Yeah - turn the video on	Wb
83	JL	- If you want to watch it again - we can; then we hear directions on how to use the software that plays the analytic.	
84	3	- how do you think about......? Laughter.	F
85	3	- There's a simple answer for that one.	Wb
86	3	- If you demonstrate....	Wb
87	3	- .....controlling it	Wb
88	2	- giving out answers, by him participating, does that work? Let them see that he was interested in what they were saying to him? (very muted)	P
89	3	How will you gauge …they were going in the right direction - trying to work? That matter…they were trying making sense and suggestions were making sense to their minds as they were writing	P
90	3	– He didn't need to guide them too much	P
91	2	- you know what else he did? When they did get kind of loud, he never really quieted them. He didn't do that. He didn't do that. He said to sit down. He didn't do like try to like, OK - the noise level is too - he didn't do that.	P
92	3	- But their enthusiasm..	E
93	2	- Even when they were like jumping.	E
94	4	- Also had most of them …you know (interrupted by laughter)	E
95	2	- I never used this Box....	P
96	3	- I have not	P
97	3	– Let’s get together, do that once a month.	Wb
98	2	- Did you have that? I think you said you have them. We can get together. Let’s talk to her.	Wb
99	3	- ......a game once a month?	Wb

100	2	- they said its good in theory but not in practice.	Wb
101	2	- I'm sure she gets frustrated too. ?	
102	2	-Then because it's such a broad range , 6 to the high school like, she needs like an assistant to the assistant.	Wb
103	3	- Yeah, exactly	Wb
104	3	- Explain how students did or did not seem engaged.	E
105	3	- They were guessing or busy, they were trying, express to each member. they didn't need to be called on.	E
106	2	– Yeah, true	M
107	2	- I guess you have to allow them that room to get ..start talking...they are gonna have disagreements with each other. Its not only fighting ......they can reason, challenging.	P
108	3	- challenging.	F
109	2	- Then I guess is one.... they chose to be there	E
110	3	- the parents chose.	F
111	4	– Yes	F
112	3	- strong personalities	F
113	2	- cause when one of them got it right.......	M
114	JL	: It's 10:59, lets discuss...............(we are returning from the break) long pause.	Wb
115	JL	: It seems good to work in little groups as we did, but then you don't hear each other. So lets share what you thought about the teacher - what he was doing and why he was doing it. The same thing about the students.	
116	JL	: So I thought maybe we could start with the group of 3 and maybe you start with one thing you thought about the teacher that was worthy of a remark.	

117	3	- He was in control; the activity was going the way he wanted it to go. They were engaged.	P
118	JL	- Is that something you discussed or is that something you "always lean on" on?	
119	4	- Yeah we talked about it.	P
120	JL	- What makes you – that's the end of that.	
121	3	- He stood back as much as he could. He let them go. He gave a little guidance and then stood back.	P
122	JL	- This is interesting, I heard that word from you 2 also (5 and 1).	
123	JL	- So these 2 things seem different from saying "in control"	
124	4	- Is saying that 3 means in control - as having a calm demeanor.	P
125	2	- He is very calm, happy with the kids.	p
126	4	– He didn't give them the urge to intervene or….you know, he just let them go….."let them go"2	P
127	JL	- In your experience, do you do this any of you? Do find it hard or easy?	P
128	1 or 5	- sometimes there are kids that don't say anything. You'd just be like waiting all day. Watching your nails grow.	EC
129	2	- That's one thing that we talked about in our group. I have a difficult time with that wait time. That's just O - kay-ay. laughter.	P
130	4	- One thing - this is a small group. 5 or 6 students. If we get 5 or 6 kids in a classroom together we could get the same thing. Mm-mm.	P
131	2	- and because it was an after school activity, they kinda had choice if they wanted to stay or not. Its not like they had to be here for this 60 minutes.	E

132	4	- Was it the first drill? Most of them that was like "this", I like the way the teacher drew him in -	P
133	2	- Which one?	F
134	4	- I don't remember who he was.	F
135	JL	- You can use your pictures.	
136	4	isn't sure and is physically displaying how he looked. We finally decide that it's Christian.	Wb
137	4	- The teacher drew him in by asking him to give a number and after that, the boy just went off and you couldn't stop.	P
138	JL	- Did you notice any other specific ways to draw the kids in from the T/R?	
139	JL	- A specific thing? Are there any behaviors - because it's hard to put it into words maybe?	
140	5	- When, first of all, kids...... If the teacher says... He is giving the opportunity to prove. He says "try it out, "see whether or not it will work. (He may be looking at the transcript of the analytic).	P
141	JL	- Yeah, I heard you all talking about that before I asked about it, because I had asked about it. Did that seem weird that he doesn't do that and the kids don't really say (like my students say) "Am I right? Am I right?" So did you find that very surprising?	
142	3	- Surprising that kids didn't ask.	E
143	JL	- So what do you think is the reason?	
144	2	- Because of those questions, they know he is not going to give them that, he is going to just keep saying, “Does that work? Oh,try it out?”	P
145	4	- Maybe because of,.....I don't know if that was the first time they were with that researcher. If you know your teacher, you know what to expect from him or her. Know from your past experience with the person that is teaching. So maybe before	E

		when they ask them "Is that right?" that is what keeps saying. So now they are used to it. So now they are used to it.	
146	2	- agrees.	F
147	3	- don't bother.	F
148	JL	- This is not the first day, I don't know how many times they did this before, but he knows all their names and it's not the first day. He has established this. So what do you think it was like the first day he did it?	
149	2	– He said "Silence"3	F
150	5	- it could have been silent.	F
151	4	- I was supposed to tell you..........	F
152	JL	- I just remember that I am duplicating the group discussion on both tapes. I discuss this for a bit. I guess it doesn't matter. It only matters if the tapes have conflicting things.	
153	JL	- So this is really interesting. I want to capture what you said.	
154	4	- Some of my kids would say, "You are the teacher, you are supposed to tell us if we are right or not."	My
155	JL	- What do you think he (Powell) said?	
156	3	- He might have said "I don't know". Laughter. "I don't know".	P
157	4	- I told them, "You tell me, you tell me what you are thinking".	P
158	JL	- Is there something else? Everyone else can have a different way. There can't be just one way.	
159	2	- Unless he could say, I'll tell you at the end of class. I'll let you know the answer.	P

160	JL	- Each of you has your own style. If you were going to do this, just try it out, how would you handle that first day?	
161	3	- Ummm	F
162	5	- I would ask for their thoughts, very, very. .?.. on the first time. Okay, We have to come up with the results.	P
163	JL	- Do you think that they would do this first time? After the first time they would just stop asking? Would they just come and say, "is this right?" Or would they just tell you, "I found it!" How long do you think its gonna take?	
164	??	Doing one lesson	
165	JL	- How many lessons? (they are pretty quiet) We don't know, I am just asking for what your honest thoughts are.	
166	1	- My honors kids, I think they should be able to get it within a 40 minute period. My other 2 groups would take them a little bit more, less than an hour.	My
167	JL	- Pretty quickly? Anyone think it would take a week? a month? or a year? laughter.	
168	3	- My honors class would do well eventually compared to... my other classes (the other 2) each has 2 or 3 who could've been in honors bur weren't, they are good at this stuff.....so depending upon how they felt that day. But I would .... people would be quiet. I would probably have to be asking questions: what do you think? What do you think? Really try to pull it out of them, kind of a thing. They want to show off. They want to talk. They may not want to raise their hands.	My
169	2	- I think it's important, If you are going to do something like this, I know with my students in this first class, I have to really let them know my intentions. Like I would have to tell them "today we are going to do things differently". I would have to give them that instruction at first so they will know. One way of saying, " I am not doing this for you today, you have to do it yourself today". I think I have to do that with them first and get them comfortable. And then let them go. I will tell them "there are no right or wrong answers" - I think I will say that just to get them comfortable especially on the first day. I will probably let them know at the end what the right answer is. But there is	P

		no right or wrong answers so that they will feel comfortable enough to say "24" however many times.	
170	4	- Umm. The first thing is...instead of saying "I don't know", I think I will ask whether it is to, explain their reasoning to me and to the rest of the class. Ask questions. Like, that's what he did. Ask him, "Did you try it?", "Does it work?" So that way, they are able, without you telling them, to know if the rule works out or not.	P
171	4	–I think that's what happened there. Because they tried it. All they know, "if I divide by 5 I don't get 15 back" so they know that's not wrong, they are going to try something else. So that way more students will be able to get…and be able to participate. Because when you are jumping too fast you lose them and the whole lesson goes and they don't remember. But in this case, I think they will remember for a long time. Because they seem to be having "form"4 fun?	P
172	5	: One of the men says something about starting with the second problem (10x+5) first because it was easier. Inaudible Discussion	M/P
173	5	5 - In the first problem only one student was focusing? (repeated by 2 teachers)	E
174	5	6 - One student understood.	EC
175	5	7 - One student was not giving up. Other students were giving up more.	E
176	JL	- You think? Give me one student that was giving up. Show me where he gave up. What made you think he gave up?	
177	5	: The reason why I think they gave up is because their thoughts are like in the pattern like in a pattern ...of one operation...keep on adding.	M

178	2	8 Adding 8.	M
179	5	- Adding 8.	M
180	2	– Yeah.	F
181	5	- If you start with a problem that's more basic it's more easier, you get more kids. A lot of kids are very like timid with math and a problem like this (easier one) will get more participation kids to tackle, than the first one.	P
182	JL	- Cool, so anybody else want to say something about the teachers? Are we finished with this part of the discussion.	
183	2	: One other thing that I wanted to say that I thought was interesting was that there was no paper given to the students. So that kind of meant they had to discuss more? Cause there was less? discussion when writing down?	P
184	4	- Yeah Yeah.	F
185	JL	: Both times when there just writing then, they're not writing.	
186	1	: The other thing that also kinda had it more ....getting the students to be more involved and not just say "Am I right?, Am I right?" was the way that it was introduced. It was introduced not as a lesson or a test, it was introduced as a "we're going to play a game". And then from there, it was just like "OK, let's see what happens".	P
187	JL	- OK, well umm, what I planned to do for the last part of this .... the idea is to come away from the 4 days with a lesson plan that you will eventually use in your class, even though it won't be an after school class with only 6 kids, ...so there's certain amount of imagination you have to apply to it... So I am not saying that in the next 45 minutes you're going to plan a whole lesson, but maybe to start planning. Start planning a Guess My Rule lesson in a class where it belongs. Which class you'd like to do it in, how would you start that lesson, what would your goal for the first day be? The second day?	
188	JL	realize that this wasn't their first day with him (Powell). So, they might have done other things the first day, but it wasn't like they did it in a day, and they also didn't finish there.	

189	JL	So think about a lesson plan that might be the beginning of maybe a week or two of lessons on this but you're just going to start with this one lesson. I don't have any requirements for lesson formats or objectives in terms of "What will the students learn that day". Eventually create a lesson plan that you'll use, so use whatever makes sense to you.	
190	JL	And if you want do it together as one lesson plan that you'll all do, that's fine or if you want to do it in pairs or individually, that's fine, I have lots of extra paper.	
191	JL	And if you want to ask me what I think, I am happy to tell you, but I'd like you to keep it in your book. You can change it each day and add to it each day. So this will be today's view of what your lesson would be.	
192	3	: I am thinking about what to do before we do it (Guess My Rule)	M
193	4	: Maybe use a shape to represent a number or something. And then have an expression and have them evaluate it.	M
194	3	: That's good. That's a good example (They are looking at something 4 wrote).	M
195	3	: Shapes.	
196	2	: Maybe- sometimes when I do it, it takes a long time ...like wait a minute, 5 minutes, laughter.	My
197	3	: Yeah....My class - they go on forever sometimes.	My
198	JL	: Take a picture of it (collecting data? this is mentioned, but not clearly.)	
199	2	So what next?	
			F
200	3	- (only speaking to 1 or 2 teachers) Next9. In my school, this thing here was like picking a lesson. The Do Now - to me, for this kind of thing is like - the do now is almost like the lesson, so "i'm going to play a game". So this piece here in the lesson be almost mine - for more fun, to keep it funish. For more fun, here's a function, here's a this, here's a that, oh then you lose 'em.	My
201	3	- So this almost goes	F

202	4	- So this parts ..(this Wednesday…a challenge was the instruction part, sorry to say..)10	M
203	3	- The instruction part, right?	M
204	2	- mm-hmm	F
205	2	- I know that with our we have to do that, we have to put, "I do, you do , we do" you know, in our (laughter) lesson plans at school. We have to have it.	M
206	2	- Yeah	F
207	2	- The I do - is the teacher telling me them what the activity is.	M
208	4	- Yes, yes. So that's not really .... Instructions but not .....directions?	M
209	1	- The thing that I do - or like the directions, is explaining...	F
210	JL	- Are you being told that you have to do direct instruction?	
211	2	- They are looking in the lesson plans - Lesson plans are looking for that.	P
212	JL	– Really?	
213	1	- That could even just be explaining, you could even say, use an example and "this is what I am expecting".	P
214	JL	- Oh okay, so they are not saying you have to be the.....	
215	4	- No - you have to be clear with an introduction of what the lesson is about. You have to be clear with the expectations.	P
216	2	- The objective - is part of ......[inaudible]......laughing.	F
217	4	- The students have to know The Objective.	P
218	2	- Yeah Yeah	F
219	4	- [They] come in and ask them	F
220	JL	- You have to put the objective on the board so that kids can read it off the board.	

221	4	- When they come, they.......not telling them to read	F
222	JL	- They want them to know.	
223	4	- They will ask everyone in your class	F
224	2	- Anyone!	F
225	JL	- Before they know how to do it, saying that [what?] is gonna be really weird.	
226	4	- Anyone can come and ask anyone. So you are praying that.	P
227	3	- Please ask....	P
228	4	- Anybody in your class. They look for the kid that looks like he's ..	F
229	2	- Not paying attention!.	E
230	JL	- Who are the people that are coming [into the class]?	
231	4	- the Vice Principal, the Principal, the Supervisor,	Wb
232	2	- State People - anybody.	Wb
233	4	- Any of those 3, Those come in and ask the students who look like...	F
234	3	- ....we still have to have the objective up there,	p
235	JL	- Do the students have to know..	
236	3	- They are supposed to....- I think we get less observed than (teachers from another school)? - But everything's ready for us. Almost everything.	p
237	4	- They have to be in your room at least 2 times.	My
238	JL	- I think that you're going to be allowed to do one lesson without the fear of the police? So maybe just think it they way you think it should be, not the way you have to do it?......... Do what you think is best, you know?	

239	4	- In this case I don't wish we (will do away with the idea because I'm not doing anything..)11	Wb
240	JL	- Lets imagine, for the sake of this workshop, that we live in a world…	
241	3	- "Put a Do Not Observe Sign" on your door	Wb
242	JL	- …..where we can just do it the way we want. They can observe us like 3 or 4 times a year and the rest of the time we can do exactly what we think is best?	
243	JL	- That's I how I view it.	
244	4	- Oh you get observed....	P
245	JL	- How often do you get observed?	
246	4	- Oh like, 3 times a week or 4 times... it depends on...they are called walk-throughs. They give you a write-up.	My
247	JL	– It's a big district, how many people are doing this all day long?	
248	2	- Because it's a Priority School.	My
249	2	- We don't get it that much.	My
250	JL	- So which schools are getting it?	
251	4	- Hubbard [Middle School]	F??
252	4	- There are walk-throughs. They come in, if you are lucky, you get that once a week. If not, you can get it twice a day.	My
253	3	- you can get used to it, I guess	Wb
254	4	- And they look at different aspects and they come in and they look at your binders to see that your lesson plans; you have to have your lesson plans on your desk.	P

255	JL	- (I am giving them ideas from the teacher underground. Oops.)	
256	JL	- Seriously - in math we are lucky - they don't really want to learn the math.....just invite them to sit down and learn the math and they will be out of there so fast....you want have to say anything...	
257	4	- You know why we are not so lucky in my school? My principal used to be a math teacher.	My
258	JL	- Oh no!	
259	2	- .....supervisor in-house...they are ones that come in.	My
260	JL	- You have different math supervisors?	
261	4	- Yes, in-building, we have one in-building because our school is a Priority School.	My
262	JL	- You know something? Your principal and your math supervisor haven't done math in a long time. That's not their job anymore.	
263	4	- Oh well my math supervisor was a teacher until last year.	My
264	JL	- Was she a good teacher?	
265	4	- I don't know.	Wb
266	JL	- You don't know her. Maybe she was good - or not so bad.	
267	4	- She's a good person, she's nice.	Wb
268	JL	- The truth, you know, that we are in this together, to help each other - or we're not. That's my view of it.	
269	4	- So if they come back once or twice and they feel you have it together, they might not bother you too much, but when you think teachers that are - I can imagine it being my first year - so they'd always be on your neck and always have something to say.	P
270	JL	- But that's not a good way to teach people.	

271	4	- In our district we have behavior issues. So like for teachers, they have to deal with all of that. You, as an old teacher you may still have those issues, but when you are a new teacher it becomes overwhelming.	My
272	JL	- They are looking for a certain level of quiet?	
273	4	- Your classroom management is a big deal. How is your classroom management? Many people don't get their job back because of that. You can't manage your class [is what they are told].	P
274	4	- When they come to observe you, if you can't manage your class, its very unlikely for you to do well in the observation	P
275	JL	- This class? Would they feel?	
276	4	– No, No, No, - they would love this class (we saw in the video) (3 - says "seeing engagement") They will look into how you end, how you close? How do you close everything?	P
277	4	- For me the way he ended, it was,……..he'd have to do a recap.	P
278	2	- Yeah (all the others say Yeah!)	F
279	JL	- A recap, like "What did we learn today?"	
280	4	– It's more like a talk show, you know?	P
281	group	laughter – lots	F
282	4	- You know, on a talk show the moderator or whoever, comes and says some things. They want to see that.	P
283	JL	- They want to see the teacher to say something.	
284	4	- Yeah, They want to you do something - bring everything together.	P
285	1	- Also - You can also have like students recap.	P
286	4	- Extra....something	F

287	JL	- So they liked it, but at the end, they want the teacher to say "Now what did we learn today? and the students would raise their hands and say what they learned.	
288	5	- [They want to know] How do you measure what they learned?	P
289	2	- They would ask YOU that?	PQ
290	4	- So you have to have some formalized test	P
291	1	- One of my observations, she said you could even have the students say name "one thing you thought you did very well, one thing you struggled with, one thing you got better understanding with" and just have them say it. And that would be the closing.	P
292	JL	- I just want to know what you think, If the teacher instead of saying, so what did you learn, so right now, what do you think the rule is for that first problem? And they each said what they thought it was - would that be ok for the closing?	
293	4	- Like that first problem. Yeah, use that as an exit ticket. So everybody writes their rule. That way you know if they actually got it.	P
294	JL	- Exit ticket. Ok, but you wouldn't have to prove that they all knew the same thing?	
295	4	- Oh no, (lots of Oh no's) They just show you how far they got. You have the feedback in the exit ticket - that way you know how to structure your next lesson. How did they do? Do I have to go over this? Or do I move on?	P
296	JL	- Ok, so it could work within this structure to do that kind of thing. [01:37:11]	
297	5	- I wouldn't use the exit ticket for the closing because that's not something the observer can observe.	P
298	JL	- Did they say don't use it? Or did they say don't use it when you are being observed?	
299	5	- For the observer.	P?

300	JL	- OK - So you could use it.	
301	JL	- How could you do a proper end of that lesson that we saw or recap?	
302	5	- Use - what is something that you mastered? Or something you need to do a little better or something that you have no clue on?	P
303	JL	- That's a lot of things.	
304	5	- You only need one of those things.	P
305	JL	- Oh - only one of those things.	
306	5	- only one.	P
307	JL	- Do the kids buy into that? Do they do that?	
308	JL	- You could explain it and then you have a recap.	
309	JL	- I am not suggesting that you reproduce this, but how could you do it in your world.	
310	2	-This would be good for the opening week of school - in that beginning part, in that getting to know you part maybe try to do that to keep them engaged in math when you are in there.	P
311	2	- I hate to see what the kids -how do they feel about math, you know?	E
312	JL	- Do you ever ask them? What math is? How do they feel about it?	
313	??	- Yeah, Yeah..	
314	JL	- What do they say?	
315	2	- It depends. Sometimes like "I hate math". Inaudible	C
316	JL	– It depends on the students……	
317	JL	- Put math in a ring of fire and watch it burn	
318	5	- They always want to know how it will help them - how will I use it in life?	C

319	JL	- Yeah, mine don't ask me because I was an engineer and I used it. And I can go on and on and on. So they don't ask.	
320	3	- We have groups of 5 or 6 kids here, but in a class of 20, you'd need to break out in groups. First you'd have to have a whole big thing, and then groups have their own kind of a thing.	P
321	2	– Would each group have a leader?	PQ
322	3	– Oh yeah, they would have to	FY
323	2	- would every group have the same rule?	PQ
324	JL	– No, I think it would be everyone's…?	
325	2	– Can't you think we could do that so that we put some of the same..	F
326	4	– (interrupts)..have them in stations in groups. Move around the groups and hear what they say. We have a class of at least 20 students, so you're not going to be listening to one group. You're going to have to have them in small groups like that. You go around to - I think that would be a nice lesson.	P
327	3	- The first thing I did - show them what you did - some do like that, yeah, yeah - not audible.	P
328	JL	- What's the very very first thing you would do to introduce the idea. They have no idea of equations really, only 3+box = 9. This is a new idea, equations in 2 variables and you are not telling them that, what would be the very first thing you would say or do or show?	
329	5	- I had a DO NOW: Here's a table, here's 3 different (box) values, here's the rule. Find the result. That way they're getting used to ..	M
330	JL	- What's the result then?	MQ
331	5	- ....3x + 7, then the rule for 3, then they have to figure out what is the y-value	M

332	JL	- Oh I get it, you first show them what it means to be a rule; and then they have to find the triangle values from the box values. And then you give them a table of actual values and ask them to figure out the rule.	
333	5	– And then you can say “if I have the x-value of 3 in …3x+7”	M
334	2	– So gives her opinion on the first rule …(her specific words not clear but she is talking about the first rule) [Who is she?]12	M
335	3	- rather than figure out the rule, give them the rule.	M
336	JL	- Only the first day	
337	4	- We have something like that, right?	MQ
338	3	– I think so..	FY
339	2	- We just have the different "fancy symbols".	M
340	3	- Fancy symbols	M
341	1	- See the different types of rules?	MQ
342	JL	- What the first one you would give them?	
343	1	- I would first have the discussion, if I had a discussion if the box is 3 and the triangle is 6, what are the different possible rules, to get them thinking, "I could have more than one rule".	M
344	JL	- right, but here, so here you tell the rule. I see, so first thing you would do is not be to say 5, it would be to give them one value.....	
345	1	- They would say "Oh, +3" or "X 2", or something and I would say ok. And I have this (box, triangle), which rule gives me the answer, satisfies both of them at….	M
346	JL	- You would actually be going for,….. you would be starting with the idea of infinite lines through 1 point, one line through two points. You would be laying the groundwork in that idea.	
347	1	- And then have them in split up into groups and give them the specific rules.	P

348	JL	- Would you start out with a type 2 kind of thing like that?	
349	1	- I think so.	P
350	JL	- So you don't think that the 3x-2 was the first time they tried it? Or you do?	
351	1	- I think that was probably the first time.	P
352	3	- The first time I think for that.	P
353	4	- We did some that are simpler than what he has. What we did, we are trying to establish that the symbols could be represented by numbers. So we have like this "square" (box) is supposed to be a certain number, if triangle is 2times square -2, then we ask them to evaluate the value of the triangle.	M
354	JL	- You did the same idea - give them a rule.	
355	JL	: OK so now, Just before we put it away forever, very different ways of expressing the rules: What do you think this with the plusses and minuses, like here made it just like you are...x and y's. and here you used words they way the children might think about it.	
356	JL	.:Could we talk about that for a minute? do you think its important? It depends upon what you've done before?	
357	JL	- Describe why you did this...oh - you can finish writing.	
358	1	- The reason why I did that, because I know that from the beginning of the year, before we did any diagnostic assessments, or kinda did some basic groundwork, I did like in and out tubes, just to get them used to looking at a table and trying to figure out the rule.	M
359	JL	- So this is the way you expressed it and made it doable for them?	
360	1	- Yeah, and I know that even this past year, I found out that some of the students, when they were trying to find slope, they wouldn't use the slope formula, or anything like that. They were better off finding it from just looking at a table. And just look at the difference. it was just easier to use the table than..	M

361	2	– And then average what you have..	P
362	JL	– You average?	
363	2	– Yeah, yeah	F
364	JL	- So what was the first rule in this one.	
365	4	- So in this one. This is for them to relate symbols to numbers. In this case, all there is, remember how to replace this and how to do the operation.... (plug a value into an equation).	M
366	JL	- Do you think, if you have an honors class, you would need to do this?	
367	4	- Maybe for the math class - we have to.. like the first or second guess.	M
368	JL	- So then the first one that you can guess the rule...	
369	4	- No in this case, not the first one guessing the rule	P
370	JL	#NAME?	
371	4	- This is just the Do Now.	M
372	JL	- I'm sorry, I said something stupid.	
373	4	- And then the objective: To come up with the right rule. I'm going to build on that….	P
374	JL	- if you having trouble, email me and tell me what is happening.	

Day 2 TAW Transcript Data

Stmt	Speaker	Statement	Code
1	JL	: ...All your sideline things are interesting. Put them in there when you're done and we'll save them. Everybody's comfortable answering these from memory? You don't want me to play anything? I see that some of you are using the transcript; I was wondering if anyone would use any of them. I'm glad that they're useful. It took a lot time to produce them; I thought, "no one is gonna want to use these", laughs.	
2	05 or 01?	....and just double it..yeah yeah	M
3	Group	quiet while they write. They have done the math problem and seen the analytic.[00:00:58]	M
4	Time	[00:01:03]	
5	Time	[00:01:06]	
6	Time	[00:01:13]	
7	2	- (mumble) Only just add? [inaudible] just that he does it, always [inaudible] through..its like a	MQ
8	3	- yeah, I wrote addition instead of multiplication, it doesn't connect multiplication and addition..	M
9	2	- Why? Why we do that? I guess he's ....more comfortable with adding?	MQ
10	3	- Adding, yeah. My kids still use their fingers. I've seen them do that.	M
11	2	- RIght...okay	F
12	1	- We've got to ask for a ruler just use...	F
13	5	- [inaudible]	F
14	1	- The teacher did ask for a ruler.......the ladder problem.	M
15	5	- alright	F
16	JL	- We didn't get the Ladder Problem. You're just seeing the first two because there's something common..Tomorrow we'll get deeper into..the ladder problems.	
17	1	- [inaudible]	F
18	JL	- I'm putting the Ice Tea out....my students love that. I get.....it from Cosco.	
19	1	- which pattern did he see?	MQ
20	5	- Because, at first he thought that first one....first [inaudible] ...there's 3 of them, or there's 5 rods and........then 8...respond [inaudible]	M
21	2	- Did he say.....[inaudible]	F
22	5	- ..[inaudible] just double it.	M
23	2	- Did he...say.....I think [inaudible].	F
24	1	- Yeah, okay.	F

25	Group	But then there is silence for a while).	F
26	Time	[00:03:48]	
27	Time	[00:04:05]	
28	JL	- Ahh, ha ha ha, the ladder problem has caught your imagination.	M
29	3	- Yes, ...explain them better	M
30	JL	- I had blocks, but I can't find them. The rods, it's funny - you're all going to be drawing them because you don't have them..the rods..I'm going to ask somebody here if they have them, see if I can get them...[00:04:20]	
31	Time	[00:04:55]	
32	Time	[00:05:24]	
33	Time	[00:05:41]	
34	Time	[00:06:07]	
35	Time	[00:06:42]	
36	Time	[00:07:39]	
37	Time	[00:07:43]	
38	1	- That really was the only time they were collaborating? Cause even though Yonny was the one that wasn't like really into the problem, right?	M
39	JL	- [inaudible]....do that....	
40	Time	[00:08:09]	
41	JL	- We can start at the beginning, it's not that long.	M
42	Analytic Plays		Wb
43	Time	[00:08:49]	
44	Time	[00:09:00]	
45	JL	- Just wait a second, What was different about what they were saying to each other? ....Were not saying the same...did you hear it?	
46	1	- ...one was saying +1, +1, +1, +1	M
47	JL	- What was he talking about with the +1, +1, +1?	
48	Group	- The x's	M
49	2	- Yes	F
50	4	- Yes, he did	M
51	JL	- He's talking about the x's	
52	JL	- What does Yonny say?	
53	5	- the y's, you add 1 and then you add 2,,,	M
54	JL	- You add 1 to what?	
55	5	- To the x.	M

56	JL	- right - you add 1 to the x, then you add 2 too the x, then you add 3. He was seeing a real different way than Brandon. Where they noticing? Did Brandon notice he was saying something different?	
57	1	- No	F
58	Group	- No	F
59	JL	- Do you think? They didn't show that. Do you think that they heard each other anyway? And that maybe there was a valuable communication ..anyway?	
60	3	- yeah..?	M
61	5	- sure	M
62	JL	- It's very hard to know.........just have to....the tracking ball... okay, it's going on.	
63	Time	[00:10:46]	
64	JL	- So what do you think? Did they actually hear each other so that they could use it later?	MQ
65	Group	- all mumble "I don't think so".	M
66	4	- I...but....really...think when he said +1, then you add 2, then you add 3, I think Yonny eventually got what Brandon was trying to say. Because if look at it, you add, if you're going to get 3..you have to add 1 to 2, you get 3, to get 4 you add the next number and the next number and that is what he was trying to say: +1, +2, +3,+4, +5.	M
67	JL	- Okay, let me paraphrase what I think you're saying. You're saying that Brandon was saying "add one, add another one, add another one, because here you added one and then you add one more, and here you added two and then you add one more, add 3 and 2 + 3 is five and then you have add 4, add one to the 3, and 3+4 is 7. So he was seeing 0+1=1, 1+2=3, he's saying this had +1,	M
68	4	- Yeah....	M
69	JL	- and then 2 plus 3 equals 5, there's another plus one in there, 1+1 is 2, 2+1 is 3, you think that was the +1?	
70	4	- yeah	F
71	JL	- and Yonny's just making it clear by saying, "no just think of it as one +2 +"	M
72	4	- Yeah, instead of saying 1, 1, 1, 1,	M
73	3	- He said, "NO", Yonny said, "NO", it's like...they added [inaudible]	M
74	JL	- And we don't know if he's saying "No - you're saying it wrong" or "NO,the math is wrong". We don't know that, but it's interesting, that's a possibility, right? You could think of it that way.	
75	4	- that wasn't ...	F

76	JL	- That's one of my favorite things in teaching - the range of meanings that's behind ... a child who doesn't have precise math language and what he's saying. So it's interesting to consider all of these possibilities are, you know, possible. You know....putting that one on the board......think about it.	
77	3	- These students who are in the after school math program, do you know - could anybody be in that program or do you had to have a little something on the ball?	M
78	JL	- Um, No actually, these were the needier kids, these were not the high performers, because this was not..	
79	3	- [inaudible]	F
80	JL	-This was an enrichment program and I believe it's connected to Abbot status in some school districts. And I believe that, for example, Ariel was a bilingual student...did I tell you that on his?...So he had..he made rapid progress in English, but he had some deficits in math, I believe and that's one of the reasons he was chosen. Or maybe not deficits so much, as just was not necessarily a high performer. They definitely, none of them, none of the one's you'll see at all, even the ones from a totally different school district on the last day, none of them were selected for being high performers in math. They were selected for being ordinary. Laughter. These are students and this is the world and this is who we teach.	
81	JL	- So should I just continue here? You'll see Yonny um working on ....the Brandon thing?	
82	Analytic Plays		Wb
83	Time	[00:14:03]	
84	Time	[00:16:52]	
85	Time	[00:18:11]	
86	JL	- Did you hear that? What he just said? Did you hear what Yonny said...very softly?	
87	1	- Something like....	F
88	JL	- Did I put it on the.... I might've put it on the page [inaudible]	F
89	1	- It's 201	M
90	JL	- Yeah, that's kind of interesting, right? They did the strict proportional thing..you know? 20 goes into 100 five times, so the answer for 20 should be multiplied by 5 to get, you know, the....for 100. So 20 was 41, so they said multiply 41X5, like you multiply 20 by 5. And then Yonny says, "205, no 201".	
91	4	- [inaudible].	F
92	3	- I didn't catch that. I read it, but I didn't catch it.	F
93	JL	- It's hard to hear it, he just kind of says it at the end there.	
94	Time	[00:19:12]	

95	Analytic playing	We hear Yonny say "201`".	M
96	Time	[00:19:55]	
97	JL	- Did you hear that?	MQ
98	2	- Yeah	F
99	JL	- That was a little hard to hear. I wish I could ..ignore everything...I could just...did somebody NOT hear that?	
100	1	- Well, what did he say?	MQ
101	JL	- laughter. Yeah, yeah, so let me um, I just want you to hear that so you're not just believing me..	
102	4	- Yes, I heard it.	M
103	JL	- Yeah, I know, but two people didn't. I'm going to play it, but I'm going to skip until the end of it.	
104	Time	[00:20:24] [00:20:30] [00:21:13]	
105	2	- He said, I think its 201.	M
106	JL	- And what is the answer, for 100 here?	M
107	3	-201	M
108	JL	- 201, that's [inaudible] not strictly proportional. But what was Yonny doing before he said that? What was he sitting there doing? Besides the annoying whistle?	
109	3	- Flipping all over, he's looking at his numbers, he's looking over.	M
110	JL	- He's got some kind of game..[banging sounds]	
111	JL	- A difficult kid, but what's he.....?	F
112	3	- He's thinking.	F
113	Group	- He's thinking.	F
114	JL	- .....thinking about? He's whistling and banging and thinking.	
115	2	- Yeah.	F
116	JL	- ...about a math problem. We're not sure what he got out of it, why he got that right answer, but he disagreed with himself, right?	
117	3	- Yeah.	F
118	JL	- He was the source of the 205 and then somehow, in that moment there, he's just thinking about it and he changes it. It's interesting. What do you think is happening there?	
119	JL	- Okay, so, I guess you can answer your questions about Yonny now.	
120	Time	[00:22:10]	
121	2	- [inaudible] answer [inaudible]	F
122	3	- [inaudible] go out.	F
123	4	- I'm going to do that, I'm going, I'm going to to play this one over...	M

124		1	- [inaudible] second	F
125		4	- I'm going to the [inaudible] last, All the questions about Yonny, Yonny, Yonny, while it's still fresh in my mind.	M
126		4	- I'm going to have ...I don't want to eat....	Wb
127	JL		- It's free....	
128	JL		- So if you want I'll show you how to do it and I'll play it if you want.	
129		1	- Play the videos?	Wb
130	JL		- Play the video. Understand the events from a single event to the whole video.	
131		1	- Oh - no, we're good now.	Wb
132	JL		- Okay, well if you want to I'll show you how to use it at some point.	
133		1	- Okay.	F
134	JL		- It's um...laughter...[00:22:56]	
135		2	- ...oh, this is just explain...	F
136		1	- Yeah. [inaudible] words	F
137		3	- ...explain that..	F
138		1	- very well,....double of the other...	M
139	JL		- Some of you said, somebody said something about it ....???...it really wouldn't work, so I just put an example up there [on the board]. There ...some food for thought, one of this doesn't.....and those do.	
140		1	- Well if the slope was 3 you mean?	M
141	JL		- Yes, I did a 3x+1 table. I didn't want to go that far, I didn't have the room. laughs. So I used 3 and 9. You know 3's. So I did 3x3 is 9, like here and then I added the value for 3 and I got 19 not 28. So it clearly doesn't work. You know, what's the thing missing here?	
142		1	- It would have worked....you wanted....you're going by 3 this time.	M
143	JL		- Because it's a, it's a slope of 3, I figured I'd do that...but	
144		1	- It doesn't work, really?	MQ
145		5	- It only works.....for a multiple of 2..?	M
146	JL		- Well, come up.... try it with a multiple of 2 (10?) here...instead? of a multiple of 3. Since we don't have a precise method there it's kind of like...where we can try different ways, we just aren't sure. Let's do 4 and 2 ....so um....this would be 3 times 4 is 12, plus 1 is 13, So I want to get 4's value too. So 2 times 2 equals 4. The value at 2 is 7; the value at 2 equals 7. In this we get ..the 7.	
147		1	- I think it works for the 2 because if you break 20 + 21 down, that's 20 +20 +1, so you have those doubles there.	M

148	JL	- You have the two X's in there,	
149	1	- Yeah - here you're only having 2 X's when you need 3 X's, so you would have to double the 3.	M
150	JL	- You need another X, another 2 and it would work. But you don't have - his rule didn't make a place for that. So so that works for there..that's really, that's cool. So, this is missing one of the X's. One of the....2's. It just doesn't, it doesn't scale up. This other thing, this is the full thing, I'm not sure how he saw it, it does work. It doesn't work for all the ....[inaudible] of it....So one of the things I....kids make them up, they are interesting, they reflect reasoning, you know . [inaudible] A very important fact....laughs....I think. [00:26:14]	
151	3	- inaudible	F
152	2	- Yes, laughs	F
153	1	- inaudible	F
154	3	- inaudible	F
155	2	- laughing	F
156	3	- [inaudible] Yonny, yonny, yonny.	F
157	JL	- I want to know what's funny.	
158	2	- just thinking....	F
159	3	- What did Yonny [think]....It's a real question.....it requires a real answer.	M
160	JL	- There's a ....No matter what they do, right? He has a certain posture..[Powell]	F
161	2	- Yeah he does.	FY
162	JL	- What is it? How would you describe it?	
163	JL	- He's almost like, not in the same... You know he doesn't seem like he's in the public school. He seems of something else.	
164	2	- Yeah.	F
165	JL	- What does he seem like he's doing?	
166	2	- He seems like he's researching. laughter, in a university, laugh, yeah	F
167	JL	- He's a researcher and he's talking to them like they're researchers too, right?	
168	2	- yeah, yeah.	F
169	JL	- And it doesn't matter...they're kind of rising to it, you know.	
170	3	- It's not like they hear a teacher...	F
171	JL	...They're doing whatever they do, they feel comfortable, but they're thinking about it too.	
172	2	- Yeah.	F

173	3	- If I was doing this, I would, "come on, think about it!" I would say something.....	P
174	2	- Yeah Yeah	F
175	3	- Rather than just watching or seeing them..very intelligent...next to...put it out	P
176	2	- Yeah, yeah, Laughter	F
177	Group	- laughing	F
178	JL	- They complain a little, but they don't tell him to get away, they don't say NO. They say, "oh - you can't make me do that",	
179	3	- Yeah right	F
180	JL	- "but okay" now I'm going to do it. So it's very interesting to watch that.	
181	2	- And as he challenges them they don't get really frustrated, like they don't lash out at him. They are just like, "Oh you're going to make us do 100, you know I... " laughter.	M
182	3	- [inaudible]	F
183	1	- It's too much...	F
184	JL	- Almost like they're on the same research team at Bell Labs, you know? Doing they're thing together.[00:28:17]	
185	Group	writing, murmuring	
186	2	- It seems like at first maybe Yonny was in the lead? and (not?) engaged with it, he just went along with it realizing that something was all wrong.	M
187	2	- You know at first he was not really that engaged.	M
188	4	- Mm hmm	F
189	2	- I think Yonny was not engaged, he was just going along with it, but then as he started thinking about it..	M
190	3	-Yeah that was so..	FY
191	2	- Yeah.	F
192	2	- And maybe he was doing it because when they first started, +1 +1, he was realizing it was not really that..	M
193	3	- He was thinking..	F
194	2	- right.. so maybe he was	F
195	Time	[00:29:23]	
196	Time	[00:29:42]	
197	JL	- ....I am notorious for injuring myself in the classroom; I've broken my arm at school...	
198	1	- Wow!	F
199	JL	- ..my knee problem. I just think and I don't watch where I'm going and I just go hurling into some filing cabinet.	Wb
200	2	- Okay,....inaudible.[00:30:08]	F

201	2	- There's a [inaudible].	F
202	3	- It made sense to him.	F
203	4	- inaudible	F
204	Time	[00:31:16]	
205	2	- Don't punch me...laughing	Wb
206	Time	[00:32:18]	
207	JL	- You guys are busy busy ..[inaudible]..this one ...take off a minute, get something and I'll be right back.	Wb
208	3	- ..saying it works for all x's?	M
209	5	- the only thing is maybe saying [inaudible] the x value.	M
210	2	- He's saying...[inaudible]...going with doubles	M
211	5	- That's what he was starting off here with...doubles by 2.	M
212	1	- Constant.....[inaudible]	F
213	5	- Yeah.	F
214	5	- [inaudible]	F
215	Time	[00:33:10]	
216	2	- going....he said..[inaudible]	F
217	5	- looking at 20, he said, look at 10, multiply by 2, then add 1; he knew that...that was all.	M
218	1	- yeah that was all..	F
219	3	- So are we seeing the work for 15, but he didn't [inaudible]	M
220	5	- We understand that, If you look at 10 and 30, and try to add 25 and 30, you wouldn't get 65, but it would've worked if you used 15, but ...don't . We didn't see 'em...	M
221	2	- oh right...inaudible	F
222	Time	[00:34:35]	
223	2	- Does he ever get to the like plus 1, plus 5 thing? You know? How is he getting ...[inaudible]	MQ
224	5	- 25, I think he was just [inaudible].	F
225	Time	[00:35:28]	
226	4	- I don't know what they're asking here,....pizza.	F
227	JL	- laughs	
228	4	- To me it works, it depends on..	F
229	JL	- I'm going to have to send one of you down	
230	4	- [inaudible] they're doubling.	M
231	JL	- ...when they call because they can't park out there. laughs.	
232	4	- If you double the number, we have 17, you get 35. If you have 34, you add, um, what's it called, 34 to where is that, if you have 34 it should be 69, right? So it works. So it works. What ever number you have it would work, as long as you take the double of it.	M
233	2	- mmhmm	F

234	4	- So his letter box [inaudible] to I don't understand [inaudible], because I already explained myself. I don't understand what this is asking.	M
235	JL	- Okay, so...	F
236	4	- Because here, already said...	F
237	JL	- Okay, so you're saying that there are no values, you're saying that it DOES work.	M
238	4	- As long as they know what he is doing by doubling.	M
239	JL	- Okay, so you're saying that it would work for anybody. I'm just saying "are there values for which it doesn't work?" and you are saying no.	
240	4	- As long as your doubling the number.	M
241	JL	- So I can erase this, right?	
242	2	- mmHmm	F
243	JL	- Cause we have this in our heads?	
244	Time	[00:37:01]	
245	Time	[00:38:16]	
246	2	- See if I can do the problems...[inaudible]	M
247	Time	[00:39:22]	
248	1	- [inaudible]	F
249	4	- So it says you multiply..so whats..	M
250	1	- Whats [inaudible] right?	F
251	3	- [inaudible]	F
252	4	- Try to..	F
253	1	- As long as you're adding, then you're adding.[00:39:30]	M
254	4	- [inaudible] so that it will work for all x.[00:39:36]	M
255	2	- [inaudible]	F
256	4	- Always saw Brandon do [inaudible]	F/M
257	5	- inaudible	F
258	2	- Did you have to recite....them? What's this visual [inaudible]?	F
259	4	- I said it's good... ...it depends on, [inaudible]	F
260	1	- I thought it was was overview, adding a constant.	M
261	2	- Ok, Ok, Ok,	F
262	JL	- Well these are not rods, but use them that way if you want to. You don't have to draw them over and over, like, like you did with the rods. You know, here's a ladder,.....that's one step.	
263	1	- Is that the doubling method, the doubling part - you can see that just from this	M
264	5	- yeah...	F

265	1	- that's how you know...the starting point...that's how you offset the value......you're adding five,....you're always adding..you're doubling, then adding five. When you double, you add 1.	M
266	JL	- And there's two steps...... [00:41:54] and you can build away.	
267	JL	- These are the ones that T/R2 showed on the video. So if you want to test out the algorithm...	M
268	3	- Without ....[inaudible]	F
269	Time	[00:43:51]	
270	4	- I'm going to close?? the ladder problem. The ladder problem. Can you show it to us again?	M
271	JL	- Sure. We're gonna....everything we do tomorrow is Ladder and it's complicated, so I wanted to introduce it today, because the [inaudible] doing anyway. Okay so, The Ladder Problem, the last 2 events.....So this is where T/R2 determines [inaudible]	
272	Time	[00:47:47]	
273	JL	- Okay, ready to do it again?	
274	4	- Ok, that's why they really do this????	MQ
275	JL	- You have a question on your face [laughs]. What, what, what, let's, what's the question?	
276	4	- … It's not a question. I didn't really get what he was trying to do. I knew, oh sorry. I really know he is trying to do what, I don't know. Maybe I just didn't hear anything....maybe I didn't listen well.	M
277	JL	- So, so	
278	4	- I following what he was doing I think that was it	M
279	JL	- Ok. So, so let's just play it again, um but T/R2 shows him the ladder problem,right? Maybe understand he's showing him the one step ladder, constructed a certain way with 5 rods.	
280	4	- mmmhh ah	F
281	JL	- And a 2-step ladder is constructed a certain way and that's 8 rods.	
282	4	- mmmhh	F
283	JL	- And then he says he can make them in any size, and he wants to know how many rods he needs	M
284	4	- For ten	M
285	JL	- For 10 steps and Ariel says "I know" and then what does he do? Does he build a 10-step ladder?	
286	1	- five..	M
287	JL	- He build a five step ladder.	
288	4	- Oh ok.	F

289	JL	- And then he counts, and he says there are seventeen in the five step ladder	
290	4	- OK	F
291	JL	- And then he says thirty-four for the ten step ladder	
292	4	- So he, oh ok. So I see, I see what he did now.	M
293	JL	- Yeah. So lets just…play this last one again while he's saying that.	
294	4	- Ok. So I get what he was…	F
295	JL	- To be clear, I am just paraphrasing what he's doing here. I got a review now [some silence]. Oops! I switched to that, sorry. I want you to watch this.	
296	Analytic Playing		
297	2	- Kindly make it louder.	Wb
298	JL	- Um no no no. But it's…	
299	4	- It's not…That's the loudest it can be.	wb
300	JL	- Yeah I know, I just don't want you to hear the clock. Sometimes the clock starts playing. He's just explaining. He says five times two is ten and seventeen times two is thirty four.	
301	4	- Ok, now I am getting it, I get what he's thinking.	C
302	Analytic Plays		
303	JL	- So Over, laughs	
304	4	- Yeah	F
305	JL	- I will go 4 more seconds	Wb
306	Analytic Plays		
307	JL	- [laughs] So, tomorrow we are, we are gonna go deeper into the ladders	M
308	2	- Oh I am so excited already	F
309	Group	- [talking and laughing]	
310	JL	- So, so, has everyone finished with the questions made for us?	
311	4	- There's just a .. There's just a larger problem. So we discuss it tomorrow, right?	M
312	Group	- talking and laughing	

313	JL	- So, is everybody finished with the questions before us? I'd like you to share [inaudible] . I noticed people writing [inaudible] and that's good. I want to hear everybody's ideas. So let's see, I'm gonna randomly pick, um, hmm. So let's go back to the.. from the first page of the question and talk about what you think they know, because we talked about that a little bit already. So I am interested in what more you came up with. For the y=2x+1 the very first Guess My Rule of problem, Brandon and Yonny.	
314	1	- Yes	F
315	JL	- What, what do you think they understand, what do think they don't understand?	
316	1	- They understand, um, the patterns between like going down X, +1 +1, Y's are you know can see those patterns but…	M
317	JL	- The recursive thing.	
318	1	- Yeah yeah yeah but they just can't really can't get rule. Like: What do I do to X to get the Y?	M
319	JL	- Do they think about what it is, what do I do to the x to get the y? Did anybody say that?	
320	5	- Very little	F
321	4	- They're not really thinking about x and y.	M
322	JL	- Yeah, they're Box and Triangle "ing". They are not talking, there's a language thing. Talk to me about the language thing that you know you may have noticed he is, the teacher is very professorial we say but he is not saying, "what you do to the x to get y?". What is, what instead is he saying?	
323	3	- Giving some tips on how you get 'what's always a different number' that kind of thing, 10 or 20?	M
324	JL	- Well, how was he trying to coax them towards this explicit solution, because they come up with the recursive thing fast. They see that pattern.	
325	4	- He gives them a large number, that they can't do it too. It will be too hard to do it.	M
326	JL	- Too hard. Too hard and then what does he say when they [inaudible] ugh? You know that feeling when kids look at you like "I can never do it". And what does he say?	
327	1	- Um..	F
328	2	- Doesn't he give them another problem?	M
329	1	- Does he make a lesser number?	M
330	JL	- I am gonna play, I am just gonna play that part you didn't see if you have any ideas.	
331	3	- Is there a shorter way of doing it?	MQ
332	1	- Yeah, Is there a shorter way to do it?	MQ

333	JL	- Is there a shorter way?	
334	JL	- And then he said, "Maybe you can find a shorter way?" Suggests it to them.	
335	1	- He's forcing them to...[inaudible].	F
336	JL	- Laughs, He is trying to motivate, alright? To do something different and um…	
337	2	- And I guess he is maybe just trying to get them out of just halving [inaudible] ..?	M
338	JL	- The thing is [inaudible]	
339	Analytic playing		
340	4	-That's the problem	F
341	JL	- He got it	
342	Analytic playing		
343	JL	- He is telling you his thinking, one by one [inaudible]	
344	Analytic playing		
345	JL	- Now the rubber meets the road.	
346	Analytic playing		
347	JL	- So, the first thing he says is what we said, "Is there another way?". And then, Yonny comes up with another way.	
348	4	- mmmhmm	F
349	5	: But I think you can have um.. towards the beginning of…what if X=20?. I think even Yonny had an understanding of how to do 2 X 20 or 2 X 10 or 2 X 6. But..	M
350	JL	- Why do you think that?	
351	5	- Because he said you double it. That's where the doubling happened and Brandon kind of shut that down.	M
352	JL	- Before Brandon	
353	5	-Then he said six times two is twelve..	M
354	JL	- Right. He points to the 6 and the 13	
355	5	- Yeah.	F
356	JL	- There's nothing that's doubling, he said, so it doesn't work here. Did you notice that?	
357	4	- I did.	F
358	JL	Then actually, we could really see them talking in that one spot. So, so there's a doubling aspect but it's not just doubling. Brandon shows us, there's something else.	
359	4	- Well, is that something else that they never brought up. I think maybe if Yonny had persisted in thinking about it, I	M

		think maybe he would have come up with why, without one like um…	
360	JL	Well, remember when they did uh one hundred times five?	
361	4	- mmhhhmm	F
362	JL	- Two hundred and five. They are not coming up [inaudible], have the answer, you know, and then Yonny was [inaudible] 201. Something is going on, but we don't know what that is.	
363	4	- Yeah. [as group mumbles].	F
364	5	- Maybe he realized the pattern is multiply by two and then you add one?	M
365	Time	[00:57:57]	
366	JL	Maybe or maybe it was more like a "wristed"?? but maybe it was more like that..most likely he had to add to different things to get the answer equal...you know,. the 20 and the 21,....the 20 and the 25. So maybe there was a component of that..	
367	3	Power...Also, he was using x's of the same, x and y, ...[inaudible]..,,the same x and y. [inaudible]	M
368	JL	- Yeah, yeah. He was using...	F
369	3	- He was using x and y and they were like..	M
370	JL	- And if they don't..	
371	3	- Using that without.....[inaudible]	F
372	JL	- [inaudible] I noticed they they are, you know, they're not that careful about how they express. Notice the language. He works hard to make them explain and show. But they're not saying "the independent variable" [chuckling] "the dependent variable" and all that. They're talking numbers most of the time.	
373	3	- You think there's some kind of positive influence because they're surrounded by cameras and people and people taking notes, thinking that, giving them confidence that we're .....[JL talks over..] smart enough to	E

374	JL	- [inaudible]..right, that's always a question I ask too...You know I can't KNOW that, the um, if you go into the database, if you go in the VMC, and start looking at all these tapes, you see lots of goofy child behavior. You see, you know they're being regular little boys here.... There are tapes where you know...[laugh], There's one where Kim Morris is trying to interest this boy and literally, you know, the camera is on him, and he's like this [JL acts it out - his head on the table and everybody laughs]. You know they're all..old enough to .... if they were thinking about the camera...they would.....not do that. I don't know. I honestly don't know and I think that's always an important question to ask. But I know there's this other principle, um, and I know it from the world of television and film cause husband is in that, stuff, so so, there is a principle that people can only be tense and performing for the camera for just so long. Then after a time, they can't keep it up. They can't just stand at attention for hours. Children, maybe even less so. Probably sometimes, you know at the beginning they're like " ".	
375	3	- I just flashed to something of my own thing, because thinking when we're getting observed, at first you like, you know, whatever, and then you reflect way back, later, and like, I was just being myself, I forgot they were there, I forgot they were here. You remember, probably 10 minutes later or something, but you do forget and you just become yourself.	P
376	JL	- Did you ever not notice when someone came in cause you were so...? I remember this weird experience I had where these kids were doing this interesting thing in geometry and I got all caught up in like "how did you do that?, show me again". And the assistant principle had walked in, stood there for a while, and everybody was busy, nobody noticed him, nobody! And later he said to me that it was the weirdest experience he ever had, he never walked into a room where everybody was so focused that they didn't even notice him. Usually the kids get nervous and upset when they see him. They didn't see him. Weird. But yes, that's an extreme example. But I don't think they could be acting all the time. So probably, some of the time.	
377	3	- Aside from that, giving them confidence.	
378	2	- Yeah!	F
379	3	- Like, "we're special"	F
380	JL	- That's certainly true, but I wonder if it's the situation as much as the ..	

381	5	- Cause I know sometimes, if I'm like in class or whatever, and they're doing whatever and I'm taking notes, they're like, "what are you writing? what are you writing down?". Is that good? Is that bad? I have some examples where like I'm writing notes while they're working or whatever and they're like trying to figure out "what are you writing?" So I feel like it depends on the student. Cause some students are probably like, "okay". Some students are like, "what are you writing?".	P
382	3	- Yeah.	F
383	JL	- So let me just ask something, I don't know if we can answer those questions, but are you seeing them doing something..like....extraordinary and they wouldn't do it in normal situations? Cause they're doing it like because the camera is on and it's important? And somehow they have to do their best.	
384	3	- That's what I'm getting at. In my opinion they're trying hard, in their way. They may not try...if it was a 40 minute class and the same thing happened, they may just ,....work. Here they are really trying.	M
385	JL	- Right right, so that's how ....there's a valid concern that the pedagogy and the problem aren't enough, that there's something extra about this..It's its, I mean I think its a really reasonable concern for us. We've all gone in the classroom and.....since you've brought it up, I think the biggest thing for teachers, particularly if we're going to do something new or change, is that knowledge of how the classroom can get when you try something very different, when you're not used to it, and they're not used to it. And we all know this, so I will, up front, say, "I get it", laughs, "I've done it. I get it." Um, but I will also say that while I've been doing this, I've been teaching the whole time, you know. I haven't had a moment in the ivory tower by myself. I've been in the classroom the whole time. I'm still in the classroom. I'm going back September 1st. So I don't have a trust fund, I can't not work. And I can tell you that um, you know, I don't work with middle-schoolers. I work with high-schoolers. But they're similar in many ways and many of them are as immature as middle-schoolers. And I will say that that respectful posture and accessible, amazing problems get them all at one point or another. They get it.	

386	JL	And then, sometimes I would say that, you know I have classes that are really unhappy at school, really miserable during the day....Sometimes I make an agreement with them that they come after school, and do it like this. I'll let them put their heads down. Eighth period has problem....[inaudible] you know, but then they come after school and they become little scientists. [chuckle]. Because it's their choice to come and there are snacks, and it's different. So that's my personal, no research base, just taking a chance on an experience. Um, any change you make is scary.	
387	JL	It's hard. Um, but you know, these analytics that I'm showing you , you know are four ...number I could ...[inaudible] like this. ..because there were 20 years of research out of this. 20 years. Dr. Davis and Carolyn Maher devoted ....[inaudible] of their life to it. 20 years. This group only got this for 3 years, so you're seeing kids that are in the first or second year of this program. And I think Ariel is in his first year of the program. Let me see here. The kids I will show you on Friday will have, in 6th grade, have already been in it for 5 years. And they will follow it through college.	
388	JL	There's this huge database of evidence for this type of learning. And we're just exploring some examples of it that were in your school district, with kids just like the kids you work with and I was waiting for the question about how are these children questioned, because this situation...we can say it's not like our classroom situation, but it's not that different: It's school, it's math, it's a teacher, it's kids. And these little changes make you think they're different. You know?, So, ...big pause... so that's really interesting, right?	
389	3	- So - [inaudible] they don't have the effort [inaudible] they don't show..[inaudible] the effort?	E
390	JL	- Oh yeah. I kind of think we all have methods. Some......are unaware of it. That part is a ...mind [inaudible] which doesn't work.	
391	JL	- Okay, Should I - so where were we? We were talking about that first question, about what they know and don't know - which we can't answer authoritatively, you can't just look in his head but what do you think? I thought we should discuss that.	
392	JL	- maybe I'll put um, ..[putting the analytic back on]..start with Brandon. So..the first event, the first couple of events, what do you think he knows ....and he doesn't know? - of course we're not talking about, you know, what he doesn't know in other [inaudible], about this problem.	
393	JL	- Just what you think....what you wrote, there's no um..	

394	1	- He knows the patterns`	M
395	JL	- I don't know for sure...	
396	1	- He knows the patterns...that's a	M
397	JL	- Okay, so he knows, ...so describe , he knows something specific about these patterns. [inaudible] what?	
398	JL	- What does he explain?	
399	1	- By the [inaudible] like it increases..	M
400	JL	- He tells you something about the y-pattern and x-pattern. He explains it in a certain way.	
401	Group	mumbling	F
402	JL	- He says the x goes up by	M
403	Group	- 1	M
404	JL	- and the y goes up by	M
405	Group	- 2.	M
406	JL	- So he's noticing a specific constant change in both.	
407	Group	Rustling. They must be writing.	F
408	JL	- "X goes up by 1, y goes up by 2", that's what he says. Okay?	
409	3	- He doesn't care why/that it happened. It's just that he's been told, asked why did it happen? What was the pattern, he doesn't know a thing about why that happens. If you were to say well, Why ...?	M
410	JL	- Well why does it happen?	
411	3	- He had to..He was forced	F
412	JL	- He was given..	F
413	3	- ,,,,,,,,,to think about it.	F
414	JL	- He was just given it. So he accepts that he	
415	3	- He did.....So the next number will be this and this....	M
416	JL	- Right right, so he was just given it.. So he doesn't question the problem, he doesn't question the situation, which another child might. Why do you think he doesn't question it? ....I.I can imagine some students saying, "what is this for?"	
417	3	- I think so. Recognize...	M
418	JL	- Made sense or.... he seems kind of like......what's his mood? Does he seem like a happy kid, does he seem like he's happy to be there or does he seem like he can't wait to leave? Or does he respect the teacher or does he not?	
419	3	- He respects...	F
420	JL	- He's kind of an easy-going, respectful, "alright, if you want me to do it, I'll do it".	
421	3	- "..[inaudible]....I'll do something".	F
422	JL	- Yeah, you know "we like each other, so I'll do something." It's not a tense, uh....he doesn't think it's a test...	
423	3	- right brain!	F

424	JL	- He's just going to do it.	
425	JL	- Okay, Brandon goes on to come up with that interesting thing. So what do you think he knows there?	
426	3	- [inaudible] He looked at that....	F
427	JL	- the 10, and the 21 and he looked at the 20 and said, "hmmmm", you know he did all the way to 20 once and this thing worked, 20 +21 is 41. So then he went over to the one that had 5, and said, well, for 10 is 25, maybe for 20 is 45? What do you think he understood, ...we can't say for sure, but what do you think?	
428	4	- [INAUDIBLE] The 2 problems, he saw something common.	M
429	JL	- He connected the 2x+1 problem to the 2x+5	
430	4	- said it was the same, he said it was the same thing.	M
431	JL	- 1 to 3, 2 -> 5, 10 goes to 21 and 20 was 41. So he noticed this: This + this = this. (JL points to 10 and 11 in the x column equals the y (21) in the y column for 10.) Now, we don't know why he thinks that's true, but then ...	
432	4	- But in the beginning, didn't he say that ....two are are same?	MQ
433	JL	- Ye-es!	
434	2	- Mmhmmm	
435	4	- It was the same, so if somehow it applies to one, it should apply to the other one.	M
436	JL	- Right! They are the same in a way, right? What's the same?	
437	Group	(mumbles - 2)	M
438	1	- Up by 2	M
439	4	- y is up by 2, and the x increase by 1	M
440	JL	- He's noticing this, he doesn't care where you start, but the difference is 2.	
441	1	- I don't think that's	F
442	JL	- It's almost like, without the word, he's understanding that these problems go together because the slope is 2. Do you think that because he made the connection he just did the same thing?	
443	4	- mmhmm	F
444	3	#NAME?	F
445	2	- mmhmm	F
446	JL	- Okay, you think it was a pattern?	MQ
447	2	- mmhmm	F

448	JL	- Sooo, when he knows, ...identify it.....similarity..... and identify the pattern...I have to ask you something. He said they're the same, when he got it and that was about the 2. Now do you think he thought to himself, "they're the same, so I can do this?"	
449	4	- I don't know what informed him using the method he used in the other one too.	M
450	JL	- Right, he did this, with a value, a [inaudible], plus the number = that.	
451	4	- the same thing..	M
452	JL	- He does it here too, but when he did it, do you think he was thinking that the only reason I can do this is because the slope is 2?	
453	4	- No, he's not thinking about the slope, we are thinking about it. But the difference between two consecutive pairs, two consecutive y's is 2.	M
454	JL	- Right, that is what he knew then. But did that connect to the way he created the pattern or used the pattern again?	
455	JL	- You think so?	
456	3	- He tried it, the 10, the 21..`	M
457	1	- Beautiful.	E/M
458	JL	- Think about what, did you remember, let's look for evidence. Do you remember when he's explaining the 45? The first thing he does is say, "Oh, I did the half of 20 is 10, and 10X2 is 20, and I added the 25" and Arthur is like, "what, what, what? show me what you did". And then he explains himself, he justifies it. What does he do? He doesn't do that one again. Which one does he do?	
459	1	- The other one.	M
460	JL	- Yeah, he goes back, cause he already knows the other one is 41. He says, "look, it worked here." You know he doesn't just come up with it and do it. He says "look, this worked over her and that's what I use." And we don't know, and Powell doesn't know what he's actually doing there, except for the pattern, but we do know that he's thinking about these two problems together somehow.	
461	JL	- What do we think he doesn't know?	
462	5	- He doesn't know the rule.	M
463	4	- He doesn't know, he doesn't know, he doesn't understand how they start.	M
464	JL	- Ahh, so this two different things.	
465	4	- Yes, where they both start.	FY
466	JL	- He doesn't understand the idea of an explicit thing??	
467	5	- Yes, where they both.	FY

468	JL	- Doesn't understand	
469	4	- that's why...	F
470	Group	Lots of mumbling	F
471	JL	- Just think about something though. Remember yesterday? There were 2 examples. They were in there too - Brandon and Yonny. What happened?	
472	JL	- should we take a bathroom break? But think about what happened yesterday because Brandon was involved in it. Brandon was one that got up and was"zoop de doo" with Yonny, right? Think about what he did yesterday in the video.	
473	JL	Then everybody take a break for 10 minutes. I got carried away and didn't give anybody a break. I'm sorry. You can tell me if I...I have very bad ....start getting into it, I am very bad at watching where I am going and I'm falling. Pizza is coming at noon. We are getting a plain pie and half pepperoni and half mushroom from Panico's which is really good.	
474	Group	Just nonsense.[01:16:22]	F
475	JL	We are talking about manipulatives. "I don't have the green rods". I have unifix cubes, cuisinairre rods, but not the green rods.	
476	Time	[01:17:44]	
477	Time	[01:18:00]	
478	Group	They start the discussion casually	F
479	Group	laughter.	F
480	Time	[01:18:35]	
481	2	- Cause you try to think of why they were thinking like this sometimes it's hard you know. Especially when Jonny just whistling, I mean like, [laughter], maybe when he was whistling he was thinking, you know some people cannot just have quiet? Like they have to have music or something...	Wb
482	4	- mmhmmm	F
483	4	- I think that when he said.you., I got the rule, you double it. I think, although Yonny tried to shut him down, I'm sorry - Brandon tried to shut him down, but I think he did it in him mind.	M
484	3	- Yes	F
485	2	- Yes	F
486	1	#NAME?	F
487	4	- ...thinking about 205, there is something about 205 that doesn't seem right, you know, that one isn't 5 times 21. That wasn't the way it was done before, you were not supposed to multiply two times 21 to get 41, you know?, so why would	M

		you say 5 times 21 to get 201? So I think he had a lot of things going on in his head.	
488	2	- yeah.	F
489	4	- But somehow Yonny is sort of distraction to him.	E
490	2	- and I think it may be because when he was talking, Brandon was kind of "overtalking" him.	F
491	4	- Yeah, Brandon solved everything.	FY
492	3	- [inaudible]	F
493	2	- Brandon was like "Oh I know this, ah ah ha". So then maybe he was just like, cause he looked at him one time like...	E
494	Group	laughter	F
495	3	- One thing, at the beginning of one problem he said, "Yonny come here.	F
496	2	- yeah, laughing	FY
497	3	- he said come here, do this now.	F
498	2	- uh huh! so he felt like he knew! A kind of aggressive way.	E
499	4	- If Yonny would just keep it down a little bit,, if Brandon would keep it down maybe Yonny would be able to think.	E
500	2	- Yeah, I think that's what he's sort of like whistling and banging. You know he probably was thinking...	E
501	3	- That was his thinking trying to come out.	M
502	2	- that was his thinking yeah, he kept going like "wah wah wah" [laughter].	E
503	Group	laughter	F
504	5	- Yeah.	F
505	2	- I think that was it.	F
506	1	- [inaudible] pause........ Okay, I think you got it.	F
Stmt	Speaker	Statement	F
508	2	- Yeah, [laughter]	F
509	1	- I'm going to think....[inaudible]	F
510	5	- I'm going to work with what you've got. That's what it seemed like, he was just like, "I'm going to work with what you have, and I'm going to agree and disagree with you."	M
511	Time	[01:20:51]	
512	3	- I forgot my phone in my car and I noticed it about 3/4 of the way here; I was thinking about going back to get it, I had a weird feeling like [motion - not words] calling it and I wanted to see what time it was.	Wb
513	Group	Everyone laughs.	F
514	JL	- Okay are we back from our break?	WbQ

515	2	- Okay, yeah.	F
516	Time	[01:21:19]	
517	JL	- So let's try to finish up some of these. So, what they do yesterday? Did Brandon come up with any explicit formulas yesterday?	
518	Group	Lots of participants talking. We hear fragments, e.g. "first time?"	Wb
519	JL	- They had two problems, the group of 6. The first was hard. The second one was 10x+5	M
520	4	- 10x+5	M
521	2	- 10x+5	M
522	Group	(speaking in unison)	F
523	JL	- Did they come up with 10x+5?	
524	4	- That was one that was easy for them to figure out.	M
525	Group	Lots of background murmuring.	F
526	3	- [background - to another participant] The second one was easier than the first.	M
527	JL	- But they also, they came up with the formula, right?	
528	4	- Yeah, they came up with the formula for that one.	M
529	JL	- they said divide by 10..	M
530	5	- they said 10 times 5, 10 times 10, divide by 5, then they said "times 10 plus 5".	M
531	4	- Yeah	F
532	JL	- Right, and then it was Yonny's problem, so there were, and then they, it kind of, he kept saying "Try it" , doesn't work, doesn't work, and they sort of settled on "times 10 plus 5", and so that's an explicit formula. So he had some experience of it, right?	
533	1	- Well, Christian was the one that said "times 10 plus 5".	M
534	JL	- Yes, and for a couple they were going "yes" and they were calling it out. They certainly don't have experience in a big way with this, but they had a little experience with it. So there is a seed maybe of that, but they certainly are not big in that way, they're not thinking that way, they're not thinking, "what do I do to this guy?". We don't see that, we don't see them say that. But they are getting close. But I do need some other patterns.	
535	JL	- So I guess the reason I brought all that up, cause I'm not sure I want to say he "doesn't understand"? Like if we gave him one [explicit formula] he probably could use it. What is it he's not doing? He's not ...	
536	5	- creating a rule..	M

537	JL	- He doesn't show us that he's using that idea. He's just not using that idea here. We don't know how much of it he understands, right? So he doesn't use ......the explicit formula idea.	
538	3	- When we have a student, and trying, trying, and finally I'll [show it], Ohhhhh, I didn't get that. There is no "ohhh" here.	E
539	JL	- Right right, That's interesting. There is um,....there are these little ones,....not that they	
540	3	- "Oh I got it!"	M
541	JL	- .....leap "Ah ha", not that sound. That's very interesting to notice.You don't hear any "Ah HA's", don't ....don't hear anything..."ah ha". When do you get that "ah ha" in class? Cause I know exactly what you mean, when does that happen?, what usually precedes the "ah ha" in your class.	
542	3	- Trying and trying and trying and	M
543	JL	- So the kid is just working and suddenly goes "Ahhhhh".	M
544	3	- Or someone says something and you say something or whatever..	F
545	JL	- Right...	
546	3	- It could come from anywhere.	F
547	JL	- Those are different, right? When does it mostly happen, when a kid is working on it or when you do something and a kid goes "now I see it"?	
548	3	- I think when I'm teaching.	P
549	5	- Yeah, a more when a teacher ...	F
550	5	- But I think it's a mix though, cause when it's like they're trying it and they get stuck and then like there's that teacher intervention, at least for me, they'll do it, they'll get it, they'll be like "Ohhhhh", okay now I see it.	P
551	JL	- But you're not actually,, you're giving them a thing to do ...and..... finds it?	
552	5	- Sometimes I may give an example and then they do it and	P
553	JL	- So the teacher's a strong component in that?	
554	4	- Sometimes in a group, the group by themselves, and somebody in the group who just figured it out, and then they " I get it, I get it". And then explains to the others. There's really one person that has the "ah ha" moment and then they say, "Look can't you see?" [group laughter];	P
555	JL	- Did they all learn from that person?	

556	4	- Most of them do, some of the time; they learn better from that person than learning from somebody trying to [inaudible]. You try and you try, but once one of them gets it, I don't know, maybe it's the language they speak, and that person can really get the others on board and then "Ohh, I get it! That's what she's talking about?	P
557	2	- Mmhmm, mmhmmm,	F
558	3	- You do this and you do that. In front of the board, you don't go "you do this and you do that...".	P
559	JL	- I made an analytic a little bit about that kind of a thing a long time ago [JLis referring to an analytic called Understanding Understanding] and if we have time, I'm going to show it to you guys, I think you'd like it.	
560	JL	- You [who?] were saying something while OO was talking; it was almost different and I wanted you to share it with everyone. You like when they...	
561	2	- Oh I was just saying that a lot of times I like when they challenge each other like see if they come up with a different way to do it. And then something that maybe that I [the teacher] don't get and then I'm like "I don't get it", and then they're explaining it. I mean it makes perfect sense to them, how they got the answer. And sometimes, someone that's struggling, they can just get it now because whatever way I was teaching it to them they didn't get, "what?". But then their peers were saying something and then what they thought, so I like when they have that exchange.	P
562	JL	- Are you able to do that [inaudible] and let them take the lead, and show the class and stuff.	
563	2	- Yeah!	F
564	JL	- So that's not like counter to your "I", "we", "they".	
565	2	- Oh, no, no, no. Because sometimes in the "you do", that's when they do that.	P
566	Group	laughter	
567	4	- I know when they get it, I notice. My algebra classes, my kids are mainly Spanish [Hispanic], so there's one girl that you know, Amy, she knows?? a mirror - [some pantomime and laughter]. I know this mirror too. You say anything, "mirror, mirror". Watch. So you know why she's saying/singing - see see see! Says, "yes"!	MY
568	JL	- Is it an ESL class?	
569	4	- My ESL class is my Algebra 1 class.	Wb
570	JL	- Are they ESL students, I mean?	
571	4	- No!	F
572	JL	- So they are bilingual.	

573	4	- Yeah but, they are bilingual, but Spanish is the language they spoke first.	MY
574	JL	- Right right right.	
575	4	- So when you get in your own language, there might be some words that you would use that you don't have in English.[01:28:16].	MY
576	JL	- I have a list of [Hatian] creole phrases; I have a lot of Hatian students and they go back and forth, they speak English, no problem, but when they want a special idea, they go into their language. So I have learned. [laughter]. I don't remember it well, but I [inaudible] as soon as I start using it again. Yeah so that presents a big .....challenge.	
577	4	- So that helps me, because, you know. I remember those, believe me, I can't forget her.....But she gets it, she gets it, and you know her group, once she gets it, I know that everybody in her group will get it because, you know, she will say it in the language that really really really understand. [01:28:58]	P
578	JL	- Okay, so let's talk about Ariel now with the ladder problem. You were building ladders there, and it was just the very beginning. You didn't analyze, we're going to analyze that problem when we come in tomorrow. So I don't want to get into too much analysis of the problem yet, it's okay to just have it annoying you a little bit, [group laughter] all afternoon and evening and we'll work on it tomorrow. But what do you think Ariel knows at that point, when he is doing the ladder problem? What are the things he thinks are true? What's his ..his view of the way he's solving this problem? What are the tools he's bringing to it?	
579	1	- proportionality?	MQ
580	JL	- Right, he's got "proportional reasoning" and he's using it.	
581	Time	[01:29:50]	
582	JL	- Is this something that your students come in with? Or is this something that you teach?	
583	3	- I try to teach it. [inaudible]	P
584	JL	- Is it a pre- , it is something you do before you get into the algebra?	
585	2	- You know the 7th grade is where they....[inaudible]	F
586	JL	- And that's before they do linear equations?	
587	2	- yeah yeah.	F
588	4	- Mmhmm	F
589	JL	- So you probably, in your algebra 1 class, get a lot of kids who start with this too, right?	
590	2	- But sometimes if they have a deficiency, ...[inaudible] didn't learn that concept..	P

591	JL	- So they might not even have that to go on. They might not have ...	
592	2	- Most of them have it, because they have practiced with it, but I think that there are some, such as my group, who do..[inaudible].. But they are familiar with it, if you start talking about it..	MY
593	JL	- Right, you might have to review it a little.	
594	2	- Yeah, I have to review a little.	FY
595	JL	- So, in the beginning of today, we talked about this question: When does F(2x)=2f(x) (a proportionality) or, I didn't write "or", f(3x) = 3f(x)? When do you get this proportionality? When does it work and when doesn't it work? And you put up some nice examples. (who is you?) You said it didn't work when f(x) = x^2 and it didn't work for linear equations with a non-zero constant, but it did work for f(x) = x, or ..	
596	4	- Umm when f(x) = x, f(2x) = 2x = 2 times f(2x). And I think...	M
597	JL	- Does it work in any other situation?	
598	4	- Constant, if it is a constant.	M
599	5	- If there's a ..	F
600	JL	- Oh, if f(x) = 6, laughter.	
601	4	- yeah, then ..	F
602	JL	- f(x) would always be 6, though, it wouldn't work. If f(x) is 2 all the time, then f(2x) is also 2, it's not going to work there. What about .....Is there any other place where it works?	
603	5	- I think when you just have just the variable, you don't have the constant..	M
604	JL	- Just one of them..	
605	5	- Yeah.	F
606	4	- f(x) is zero.	M
607	JL	- Then it's always zero.	
608	4	- It's always zero, right.	M
609	JL	- 2x is not, oh, that's nice, That works because you're multiplying by the zero. So that works. But what about, um, what if the f(x) was 5x?	
610	4	- It's gonna work.	M
611	JL	- f(x) is 10x	
612	4	- It's gonna work.	M
613	JL	- Any number of x's.	
614	5	- no constant	M
615	4	- no constant	M
616	5	- There's no constant	M

617	JL	- It's plus zero.................anything else? Think of all the math you know (said silly softly).	
618	JL	- What about the derivative? Is the derivative of 2f(x) = twice the derivative of f(x)?	
619	4	- The derivative of 2 of f(x) ...	M
620	JL	- I mean, you don't have to know this, I was just curious.	
621	JL	- If f(x) is differentiable. [01:33:17], the derivative of 2f(x) = 2 times the derivative of f(x).(written mathematically using dy/dx notation; d(2f(x))/dx=2d(f(x))/dx).You can pull the 2 out. The function is the derivative.	
622	Nothing		
623	Nothing		
624	4	- So wait. Lets say f(x) is 2x (f(x)=2x). The derivative of 2x is 2.	M
625	JL	- Here my "f(x)", my function is the derivative. I am doing calculus. If this is not something..	
626	4	- I'm doing is that since you are taking the derivative of f(x), right? If your f(x) is 2x, mm? then the derivative of 2x is 2.	M
627	JL	- Suppose f(x) is x^2?	
628	1	- Then your derivative is 2x.	M
629	4	- Then your derivative is 2x.	M
630	1	- ....is 4x	M
631	JL	- Right, then the derivative of f(x) is 2x	
632	4	- So that means, for that, is 4x	M
633	JL	- Right so, What is the derivative of 2x^2?	
634	4	- the derivative of 2x^2 is 4x.	M
635	JL	- Which is twice the derivative of x^2. When you learned this way back,	
636	4	- waaay back	F
637	JL	- multiply - out - they had this quality.	
638	4	- What about this? Do this one. f(x) = 2x. Isn't it simple?	M
639	JL	- If f(x) is 2x it works. Any number of x's works. But also derivatives...	
640	4	- What is the derivative of 2x? Isn't that 2?	M
641	1	- 2!	M
642	JL	- 2!	
643	4	- So what is the derivative of 2 times 2x. So that would be also.	M
644	1	- That would work, cause that would be 2 times 2, which is 4. If you do 2 time 2x...	M
645	JL	- It all depended on the limit def if you look in your calculus book, So this proportionality thing is troubling because	
646	4	- Yeah,	F

647	2	- Yeah, laughing	F
648	JL	- ....because it works for a lot of things and it doesn't work for a lot of things. And it's really hard, I think. It's hard for us! It's hard for us to evoke this. We don't make those mistakes of using it randomly, but we also have trouble identifying it. So it seems very reasonable that they use it and have trouble deciding where it doesn't work. So that's kind of an important thing.	
649	JL	- So I want to ask one or two questions before that pizza comes. Then when the pizza comes we can work on our lesson plans. What did do with my book?	
650	2	- Oh, It's over here. yeah.	Wb
651	JL	- So I do in my really......I leave my things all over the room. The kids just pass them back, they know. "Here is your drink, here's your book". So some of the stuff we did discuss. You probably had some strong ideas here. So how much time do we have before the pizza comes, it's supposed to come at 12.	
652	5	- 17, 16 minutes	Wb
653	JL	- Okay we have enough time. So I'd like to think about the common theme from all 3 problems. What was the common idea there? Why did I put those examples together?	
654	3	- The 2x ?	M
655	JL	- The 2x+1, the 2x+5, the ladder problem. They came from different videos. I was watching. I was watching all these videos and I picked those. [01:37:02] What do you think I saw in putting those together?	
656	1	- The teacher always asks like, for example, what does this come out when x is 100? or how about when x is like a large number? They see it as too much and that forces them to find like a shortcut. Like that what..	M
657	JL	- Right, so it's all "guess my rule", it is. All of its Guess My Rule, today, tomorrow, the whole week. But different aspects. Yesterday they were doing something different, they were not dealing as you say with: what do I do if x is 100? what do I do if x is 20? They were guessing as a group. They had a lot of ideas for the 3x-2, but they didn't finish it. And they did great with the 10x + 5. Today, there getting into it some more. Is there a thought process that Yonny, Brandon and Ariel all hit?	
658	5	- the pattern, like x is going up by a certain value and y is going up by a different value.	M
659	JL	- Right, but when they have to do x is 100, what do they do? What's the first thing they do?	

660	5	- They go for an easier...	F
661	3	- They complain.	F
662	JL	- They complain, right, they complain. But then they do something else. They say 100 is equal to ...	
663	2	- [inaudible]	F
664	JL	- They factor because they are using	
665	5	- Proportional reasoning...	M
666	JL	- Yeah, they said, "oh, I gotta divide and conquer here, 100, I'll be here all day, 100 is (good number sense, right?) 20X5 and 20 gets 41, so I'll multiply 41 times 5". And then go "oh, I don't know". They sit there with it. But that's the first thing they do. And what's the first thing Ariel does?	
667	05 and 02	- a 10 step ladder is 17 X 2!	M
668	JL	- He could have build a 10 step ladder. You know? I always wondered about that. Why didn't he just build 10 steps? But he loves that. right?	
669	2	- I thought that that was interesting too, when we were talking, why do they always go to adding? You know, I didn't understand that. But now it kinda makes sense if they're doing it for that reason. Maybe they're looking for the smaller number to start with. Maybe they're more comfortable with that?	M
670	JL	- Well maybe it's a strategy they learned? I always tell my kids, "If you have a complicated problem, simplify", so you might solve that first, that might be a good strategy. So they learned when they did those proportional reasoning problems...	
671	3	- Dividing 10 by 5 might work...	M
672	JL	- Right, right right. So they learned that, so they feel that's a strategy, so they use it.	
673	2	- mmhmmm.	F
674	JL	- And then they're being forced to do what? They do it. And the teacher says, "How do you know that works?"	
675	2	- yeah, yeah.	F
676	JL	- Does it work? They have to (laughing) try it!	
677	JL	- So umm, so then if you think about that then, why doesn't he just say, "You can't do that here, look: 41 X 5 is 205, and he pre-printed the chart, look what it really is, It's not!, you can't multiply here", and just say that. Why doesn't he (Powell) do that?	
678	3	- He wants the other [inaudible].	M
679	4	- One thing I didn't ask, when did they...was it the 5X41, what did I expect him to say was um when they did the 21, how	M

		they got the 21, they didn't multiply the 2 by 21 to get the 41, that is one thing he should have asked them.	
680	JL	- You would have. That's what you would have said.	
681	4	- That's what I would have asked. Now when he found the corresponding way, when x=20, you do multiply your 2 by 21 for the 10 (x=10), did you do that? To get, if had done that you'd have got 42, but you got 41. So I think he should have pointed that out to them.	M
682	JL	- Well he could have done it,.... he could have done that...	
683	4	- Ask them that question.	F
684	JL	- Does everybody hear that question? So, so he does this thing with 41X5, why didn't Dr. Powell say, Well, you did all that work to get 41 for 20, why didn't you just take what 10 had (21) and multiply by 2? And they could have seen that it doesn't work out that way. So then why ...	
685	JL	- Remember the first thing that T/R2 does with the ladder? "This is a one-step ladder. This is a two-step ladder". Right, he goes over it, he asks them, "how many rods in a one-step ladder? How many?	
686	Group	: 5	M
687	JL	- How many in a two step ladder?	
688	Group	: 8	M
689	JL	- If this worked by proportional, what should this be?	
690	4	- [inaudible]	F
691	JL	- Right, it is not. But they don't do that. They don't do it here and they don't do it there. And you think, I thought this too. You think, "point this out". So why don't they?[01:42:25]	
692	3	- Well [inaudible]..But if they were stuck at 205 to get that answer, then maybe they would do that, but they kind of eventually got to [inaudible] (whispered) "wait a minute..", and then, "201".	M
693	JL	- Well that's really [inaudible]. I mean a need a private investigator for the 201. I don't know anything (laughing). Yeah.	
694	3	- They got, Yonny got to that number.	M

695	JL	- Well I'm going to give you a different story because this, I think is a very important question for teachers. This video shows this think....over and over. But it's really hard for us to answer. So when I used to teach HSPA classes, when we used to have that reasonable test instead of the unreasonable test, because a lot of my kids, most of my kids would pass that (earlier) test. I started with this called the Saturday problem and it was because the kids who came to that class were like, "Oh I've died and gone to hell" because now I am in this class, two math classes and they don't even like math. And there they are, and they know that this is the class that you...have to pass the test. So I start with something that doesn't sound unpleasant and it's about them - you know, they like "them". `	
696	2	- mmhmm	F
697	JL	- So they had to take everything they did on a Saturday from Friday midnight to Saturday midnight and they had to write it all out and the amount of time they spent on each thing. And they had to make a pie chart. This is a very complicated problem, they have to take percents and do percents of 360 and make the pie chart. And they've got to think about separating their tasks; all the kids would say, "well, I'm on the phone and I'm watching television and I'm talking to my friends in the room all for 3 hours. And we said, well if you're a data analyst, you have to decide what part of time you did each thing. But, you know, I had many choices about how to teach this and each time I taught it, I did less for them. The first time I was warning them about the percents and 360 and all that. After that, I let them do it wrong. They loved doing their own pie-chart and they would get to ....and say "It's wrong!". I'd say, "how do you know it's wrong?" Because I still have 30 degrees for eating and I can't fit it in here. "Well you have to find the part that's wrong.." "Oh Ms. Leslie, I'm gonna die!" [laughter] And I'd say, "I'm going to give you part credit for that if you want to leave it. But they didn't want to leave it. Because that was their [inaudible]. That's my connection to this "not telling" piece. Because there was an understanding of, combined with the caring that they got to by, because I unscaffolded it, and then I used it as a reference problem. They'd be on a test getting percents and say, "I forgot how to do it, tell me how to do it", and I'd say well just remember the Saturday problem. And they'd say, "oh yeah". And they would get to it. So I don't know, I really don't know, because I know that I for sure wanted to say, "look.....", but maybe all that work in discovering it, is what he (Powell) was going for, I don't know. I made the analytic, but it doesn't	

		come with the researchers thoughts in his head, so I cannot be sure.	
698	3	- [inaudible]	
699	2	- Well you can think of	
700	JL	- RIght, right, well what's going to come next, tomorrow, is - there's actually 2 analytics. I am going to show you Ariel, kind of an overview: where he was and where he ends up a year later; so you kind of know what he got to, which is kind of interesting. And then I'm going to show you some real interesting detail about how he got there. That should be ...I don't know...interesting. Remember, we're doing this together and learning from it, but there's no, I don't have more than you have. I have these; I just have more time with these {analytics}. I've tried this out more than you have. You're going to try it out; it's um something, you know, we work out together.	
701	JL	- Okay, so, let me ask just one more question and then we'll expect pizza. Okay, so um, I ask this over and over. What mattered to the teacher? What matters to Dr. Powell in the first two parts and T/R2 (what is his last name? I never remember) [inaudible]; T/R1 and T/R2, what matters to them?	

702		1	- Prove their answers?	MQ
703	JL		- They didn't actually prove them, right? They explained them, they explained them. Everybody explained them. Does behavior matter to them?	
704		4	- to the teachers? I guess....I don't really think so.	M
705	JL		- Do you think they behaved badly?	
706		4	- they, I don't know	F
707		1	- That's normal	F
708	JL		- Normal kid stuff	
709		3	- still working towards their answer.	
710	JL		- would that be the kid of thing, the whistling, and the [sounds], would that be considered "not managing your class" where you are?	
711		4	- It depends	F
712		5	- It depends on which...	F
713		1	- It depends on who's observing you.	F
714	JL		- So some people would say it does.	F
715		5	- It depends on just you as a person, cause for me, after some time, I might be like, "Lets settle down". Cause it starts getting on my nerves.	P
716	Group		laughter	F
717	JL		- You're a person too.	
718		4	- Not even just your nerves; some other students are stopped by that - they can't think.	E
719		2	- mmhmm, yeah.	F
720		4	- they say, "will you stop that already?" They were doing, ...suggesting anything.	EQ
721	JL		- That doesn't become an issue here because .....they're not packed in. But as we see it, he's not focused too much on that. Besides them justifiying solutions, is there anything else he cares about? Do you see a cultural model for his classroom? Any? T/R1's? T/R1's or T/R2's?	
722		2	- I think it just came across that everybody cares about the work that they are doing. So it kind of makes it, there's a sense of [inaudible] for everybody involved. Even them, even in the way that they presented it to them - like "Today we're doing something different. I need your help with this" I don' t know how they might have presented it to them (the kids), cause there's so much buy-in from everybody. Everybody's bought in.	P
723	JL		- You wish you could buy the "buy-in".	
724		2	- yeah, yeah, yeah.	F

725	JL	- That's really a nice way of putting it. They're all at Bell Labs together, they make good salaries to figure this out together. laughter	
726	2	- Cause even when they're distracted or whatever`they're doing, they do get back to the work. There's a lot of buy-in. They believe in what they're doing, it seems like.	EC
727	3	- They're on task in their own way, in an acceptable way they [inaudible].	F
728	2	- Yeah.	F
729	JL	- interesting. But you must, all teachers do this, you must be thinking sometimes like 04 admitted, "why don't you see this?" (whispered).	
730	2	- Yes!	F
731	JL	- Can you think of other places besides this - other times when you thought, "why don't you just tell them something?"	
732	4	- What I said...	F
733	JL	- I'll bet 05 has one, he looked up when I said that.	
734	5	- What?	MQ
735	JL	- give me one example when you thought, "Why don't you do this cause...." [The phone rings] Oops that's the pizza. You think about that, you're going to tell us.	
736	Group	laughter.	F
737	JL	- Pizza talk - on the phone and about getting the pies.[01:50:45]	Wb
738	JL	- 05 is just going to tell me what he wrote down. laughter.	
739	5	- What are you talking about like when?	F
740	JL	- Any time while they're teaching, what would, there must have been some times when you were thinking, "gee, I would have done something else".	
741	5	- Oh yeah. I have a different way of doing problems..	P
742	JL	- Cause we all have our own way so, so what would you have done? So give me one scenario.	
743	5	- ...I'm trying to think.....I don't know.....some type of word problem.	M
744	JL	- I mean something that really like, when you were saying like, "why don't you just SHOW them its NOT proportional? There's an example there.	
745	5	- You mean I always, you mean like helping out a kid in class?	P
746	JL	- I mean anything, at a point where you would have thought, "I would have done THIS and you did something else".	
747	3	- Sometimes like, "okay, lets let Johnny have a turn". Like "Brandon, hold on".	P

748	JL		- Oh yeah, like "Ariel, you quiet down".	
749		3	- Something like that.....but I think you would like to say something.... but it's dominated by that. "Hold on, I love you're thinking, but hold on. Hold that thought."	EC
750	JL		- He did that though, right? He did that at one point, he said.... Ariel kept giving the answer and then he finally said, "now Ariel", just like you would do, he did, he said, "Ariel, now I want you to be quiet". laughter. He did exactly...what you're saying you'd do exactly what he did.	
751		3	- in that case..	P
752	JL		- You might have done it a little sooner, but he did what you would have done.	
753	JL		- What about um I saw you watching (05) - you would have done something.	
754		5	- I don't know, like, I am trying, I can't think of what I...	F
755	JL		- Okay, well you don't... I'm not going to put you on the spot anymore, I just thought maybe you had something very clear in your mind to say. I think this is all about teaching, observing teaching and I don't think there's any one way to do any one thing. But it's really important to flesh out all the idea that this brings out, to kind of really figure out what you're going to do.	
756		1	- I wish I was more like the teacher that was doing it - he don't really help out the kids at all - he just lets them score and figure it out. But my class, like the kids, I guess they're so timid and scared to talk about math, to reveal their skills, they don't say anything. So I have to like step in and get the conversation moving along. I admire this guy, what he does.	P
757		3	- I would like to see him in a big class.	P
758		1	- Yeah, I know.	FY
759	JL		- laughter - we're going to get him over here..	
760		3	- to visit us.	F
761	JL		- Yeah yeah.	
762		3	- So that would be interesting...what went on.	P
763		1	- These are smaller groups	P

764	JL	- Well you know, one way to experiment with this is to create something similar, an afterschool thing and try it out in a small group of kids instead of starting in a bigger group. I will never forget when I did this (chuckling), because they were misbehaving and it was 8th period and they were tired. So I said I would get pizza, and I said.....you're gonna get a zero for today because your lying down, but if you come this afternoon and you eat the pizza and you work and everybody does at least these 3 problems that they have to do, then, you know, they'll be okay for the day. And a lot of them came. And they acted like little professors with each other (laughter), They were like "so what did you get for your solution?" (sounding falsetto and formal), because they wanted pizza. laughter.	
765	JL		
766	JL	- But they did it. They did it very easily.	
767	3	- All the problems in my school were connected to a ..... period....we're a very long school day. We begin at 8 o'clock and at almost 4 o'clock we're done. Because of the acting, because of the singing and dancing.	MY
768	JL	- Oh well that's .....but you have very highly motivated kids then.	
769	3	- Some, not all.	MY
770	JL	- In some areas.	
771	3	- Yes, yes, certainly not necessarily in math or ... they're there for their passion. They're not there for things that are there also. They're trying to get by and all that but they're long day. There are very few that actually would come......exhausted.	MY
772	JL	- You know, as we move beyond this workshop we can talk on Sakai as you prepare your lesson. We can brainstorm these things and give each other ideas to figure out. Because none of these are trivial questions. We have real constraints in real classrooms. And we have to think it through.	
773	JL	- Alright, how about we take a little break and give out some pizza.[01:55:37]	
774	Time	[01:58:05]	
775	JL	- So, I think we started this discussion with the challenges in implementing the lesson. We haven't finished the lesson plan but we might as well put the challenges on the table: the size of our class, the length of the day, the nature of the student body and how they feel about the day at school. We might as well just put them out there and talk about how we think we could manage it.	
776	Time	[01:59:01]	

777	3	- How long are your periods? [inaudible]	MY
778	2	- Yeah, we're 40 minutes.	MY
779	5	- OO, you're still 40 or 80?	MYQ
780	4	-80	M
781	Group	: Oh!	F
782	4	- I might...	F
783	JL	- 80 minutes? wow.	
784	2	- that's what we feel like, there's not enough time to finish.	P
785	JL	- I miss that, now it's 40 minutes it's hard.	
786	5	- 40 minutes is not enough time.	P
787	2	- You're Do Now	P
788	JL	- 40 minutes is not enough time.	
789	5	- It takes them about 5 minutes to get started.	P
790	JL	- You feel like you have to rush and start too early. I ...hear the bell....start slowly. laughter.	
791	JL	- One year, I had this amazing class, really smart but really oppositional kids and when I started I wouldn't let them take out their pencils. The math was on the board; they weren't allowed to work on it for 5 minutes. They had to take a cup of tea; I had a little tea pot, paper cups, sugar cubes and tea bags. [participant laughter] Most of them wouldn't drink the tea, but they liked making it.	
792	2	- mmhmm	F
793	JL	- They had to take a cup of tea and think about the math. And that's all they were allowed to do for 5 minutes. And it really helped. They were just..... it helped create that kind of "we are all in this together", "we are serious people", we have our tea and we're thinking about the math. laughter.	
794	2	- How did you come up with that? What made you think of that?	P

795	JL	- I don't know. It was probably the videos and, you know, it's like us here, you want to achieve something that seems sort of ridiculous in the context we live in, with real kids, that come to class upset, come to class annoyed that they're in that class, that come to class angry at somebody whose in that class, there's all kind of ...they're people, they come with baggage. How do you redirect them? And it can't be [inaudible], but this thing of, I kept thinking about the way they are, the kids, you know, and we are in this together. I wanted to create this quiet, calmer, respectful thing, and believe me they didn't become different people because they drank a cup of tea. It just started us in a different way so ...day by day they got closer to that. They didn't get there the first time they had tea. They did have 5 minutes of "whoa, what are we doing?" laughter But then they kinda liked it. So two things from it: It was hard, the kids weren't always cooperative, but a lot of times they were. And they were a special group, you don't always get a group like this. They were a group that was incredibly close to ....very strong capabilities. You could just see it below the surface. They almost wanted to do everything. But they screwed around too much and until they slowly started helping each other a lot. They all passed the HSPA - that almost never happens when the whole class passes.	
796	2	is mmhmming throughout the above story.	Wb
797	2	- wow	F
798	JL	- So that created this magical aura and this only happened to me once when I had this magical class and they all passed. And, so then, my director and I created a little mentoring thing, next year. We took, they were seniors the following year, we took them; not all of them could do it, but some of them, and we brought them to the middle school - to the 8th grade class that they were in, that was going to end up taking that HSPA class and the HSPA. And we had them talk about how they felt, why they were in that 8th grade class and how they felt now - how they got out of it. They each told their personal stories. And those were amazing. I learned more from that than from the whole year with them. I learned how they felt in my class. And they kept talking about "I felt this way until Ms. Leslie's class". They kind of attributed it to me when it was really themselves. It was really them with each other, but I didn't realize how it wasn't the tea and it wasn't my attitude. They felt cared for. And because they felt cared for they felt freed up to do it and whatever they did they gave me	

		credit for. One girl, I'm still in touch with because of this bond we created because she did this.	
799	JL	- So I don't know what I'm telling you, I'm not telling you that it's absolutely possible and completely easy, I'm telling you that it absolutely created some amazing situations for me. But it is hard, it is not simple. And everybody has to do their own thing.	
800	JL	- I don't think, for example, that 05 is gonna serve tea. lots of laughter.	
801	2	- laughing loud	F
802	4	- I'm going to try too with my kids.	P
803	Group	Everybody talking	F
804	JL	- That was the rule, I made the hot water, I gave them the sugar lumps. "This is what we're gonna do, you have to trust me".	
805	2	- mmhmm And then did you see that they were more open to discussing the math and everything?	P

806	JL	- Not at once, just over time. It's kind of like, how many of you have children? The first time you tell a child not to touch the hot radiator, they don't learn it the first time. It's this experiential thing, that happens over time period, so... Every day I was trying to get them to do some aspect of this and I was trying to model it and I was trying to be as cool as [inaudible]. Many more things were happening, you know. For example, they would ask me, "Ms. Leslie, I have something from a teacher for just 5 minutes." And I would always just believe them; I knew perfectly well that wasn't the case. I just wrote them a pass and let them go. And they took longer than 5 minutes. But I did this, every time they wanted a pass they got a pass and they left. Then when they were telling their stories, the mentoring stories, I heard about this all the time. I was so mean to Ms. Leslie, half the time I got a pass, I was going to see my friend, I was getting my phone, I was doing this, I was doing that. And still she trusted me. So, I don't think it was bad that I did that. You know, I mean, so - do I always do that? No, some kids could get into terrible trouble. But um, as much as I can, I take this attitude, two things: Whoever you are, you deserve respect, you don't have to earn it first. A lot of kids think you have to earn it first. Nope. You can lose it, but you have it. And, whatever happened today, tomorrow's a new day. I find that helps them work together. I try to get them to do that for each other. I think middle school is a better place perhaps.[02:05:53]	
807	2	- laughs	F
808	JL	- because they're more open.	
809	2	- laughs harder	F
810	JL	- sillier but more open	
811	2	- laughing harder and harder	F
812	JL	- Like 9th graders, are sillier but much more open than 11th graders. [02:05:57]	
813	1	- I made a transition from middle school to high school	Wb
814	JL	- Really, what grade do you teach in HS? 9th grade?	
815	1	- Yeah	F
816	JL	- I think they're much more mature? Just what you like too.	
817	1	- Yeah, it's different.	F
818	2	- mmhmm	F
819	JL	- Great, but not as squirrel-y.	
820	JL	- But 9th graders are much more squirrel-y than 11th and 12th graders.	Wb
821	2	- I'd say that they're a little more serious now that they're in the 9th grade. Right?	Wb

822		1	- Yeah, A lot of them I have (before) cause like I taught 8th grade the year before.	FY
823	JL		- Oh that's really good.	
824		2	- Yeah, yeah.	F
825		1	- A lot of the kids I have I had already.	F
826		2	- mmhmm	F
827	JL		- That's really nice.	
828		1	- I'm looking forward to getting a whole set of new kids. A new batch.	Wb
829	Group		- laughing all talking[02:06:42]	F
830		2	- They're on the way, right?	WbQ
831		4	- Well, you know what? The last 8th graders from [one Middle School] were not bad.	Wb
832		2	- Yeah the same thing from [the other middle school] from us. So you can probably have a nice year.	Wb
833		4	- You will.	F
834	JL		- I am talking about challenges in what I try. I want you talk about the challenges in what you try.	
835		4	- We have a lot of suspensions.	F
836		1	- Nope	F
837		4	- Nope, no problem. But you say the 6th and 7th graders?	F
838		2	- Yeah, yes.	F
839	Group		- all talking at the same time.	F
840		2	- I heard......the 7th graders....the 7th graders are good...	F
841		5	- But I heard that the 6th graders that are now becoming the 7th graders are like angels....love to go to work for..	F
842		3	- Becoming 8th graders...are	F
843		5	- Becoming 8th graders are.	F
844		4	- Our 6th graders are fighters.	F
845		2	- No - it's the 6th graders! Always fighting. Always fighting. Oh my goodness, I don't know 6th graders are come on. I thought maybe it's the, you know. When I go down, I'm like , "Oh they're so cute", you know? laughs	Wb
846		1	- 7th graders....	F
847		2	- 7th grade wing is like, where are we? It's the same school?	F
848		4	- I thought 8th grade was the best...	F
849		2	- Yeah, its..	F
850		4	- behavior wise, 8th grade was the best.	F
851		2	- mmhmm	F

852	JL	- Think of a class - you don't have to [inaudible], but think of a class where you would try this. Somehow. Think about what it would be. Think of the first problem you could think of ...besides that it's 45 minutes and there are 25 of them......Think of like....just imagine yourself, you're Dr. Powell and you're saying, "We're going to play a game". [02:08:41]	
853	1	- Some students tend to give up if they don't get it right away.	P
854	JL	- So they'll try it.	
855	1	- Yeah, they'll try it and they'll ...[give up]	F
856	JL	- Well, would that be so bad on the first day, if they give up a little? Will everybody give up?	
857	1	- No.	F
858	4	- No [inaudible]. It depends on the group.	F
859	1	- Yeah.	F
860	4	- Like the algebra group, you get more work out of them. [02:09:05]	F
861	Group	- all talking and mmhmming	F
862	2	- I think like...	F
863	3	- They have their own culture..	F
864	4	- I have my algebra, [inaudible] said he purposely put my algebra as the last class.	My
865	2	- Of the day?	F
866	JL	- Oh!	
867	4	- Because..he feels at that time of the day, I need students who are easier to manage.	P
868	2	- But at the end of the day?	PQ
869	3	- ....day.	F
870	Group	- exclaiming and laughing.	F
871	JL	- At the end of the day...the calgary.....which group is easier to manage.	
872	4	- Yes, yeah, because they are the algebra.	FY
873	2	- Ohhh, I hear what you're saying...	F
874	4	- As opposed to any of the others..	F
875	JL	- They're more calm..	
876	4	- I had so much fun with my last algebra [class], we used to dance, we used to sing,	P
877	2	- laughing	F

878	4	- ..[inaudible] You know they had this snapchat, so I allowed them because they were quite well behaved so I allowed them to use their phones, you know. And they [inaudible], "oh you don't like us, your algebra ones can use their phones they have so much fun". I say, "Because of the way they behave. They are easy to manage." "When you tell them, 'it's done, it's over', they go back to what they're doing. But Youuuuu?? Oh my goodness. Once you get started, you can't stop.	P
879	JL	- Do you tell them that if they do certain things like during the week, you'll let them try on Friday? Do you give them hope? laughs.	
880	4	- Yes, we do give them, [inaudible] I I spend a lot of money on them, like I buy them jolly [candy???]. But we are a priority school, so we take the assessment tests all the .every unit have to take an assessment test.	P
881	JL	- Every what?	
882	4	- After every unit.	P
883	JL	- Every unit....common assessments.	
884	4	- We have 5 units. So if I...	F
885	JL	- So how often is that? Every month?	
886	4	- No, like maybe every 6 weeks?	P
887	JL	- 6 weeks.	
888	4	- yeah, I know.	FY
889	JL	- What happens as result of this? The grade on the assessment.	
890	4	- The state uses that to do our um SIP - School Improvement Plan - they use that to see where we're at and all of that. So I tell my kids, and this year they did really really good, I had some really good results. So if like, if you get a passing, the passing grade is a C, for the first unit it wasn't so good. I think the best I had was 40-something average. So I told them, the 2nd one, if you do, if you can get a C average, we're going to have um a pizza party for lunch...	P
891	2	- mmhmm, mmhmm, laughs.	F
892	4	- ..for my three groups, you know? They really went for it. They were so excited, I know it's computer based. It's good to watch them, it's called computer based. On one of the pages it says that like, "2 of them do it and do it and do it ", that [inaudible]. Are you sure you checked all your work?" "Yes, I did." "You want me to check or write" I say "yes".	P
893	Group	- laughing and talking	F
894	4	- Oh, [teacher] 04, please come [inaudible] present, but I can't do.....	F
895	Group	- laughing	F

896	4	- [high pitched inaudible] I know this is how almost the second it says, so that must be [inaudible].	F
897	2	- yeah.	F
898	4	- [inaudible]	F
899	Group	- laughing.	F
900	4	- So you know, so it's like a competition.	P
901	Group	- mmhmm	F
902	4	- And then again I'm like, "oh my goodness, I promised to buy you people pizza now"....	Wb
903	Group	- laughter	F
904	4	- Now where is my.....I gotta get the money (laughing).	Wb
905	JL	- [inaudible] testing [inaudible]	
906	4	- A big kit.	F
907	JL	- That's throughout right?	
908	3	- Yeah	F
909	1	- We do the high school. We do those unit tests as well.	P
910	JL	- The same reason you have to ....every 6 months?	
911	4	- mmhmm	F
912	JL	- ....the state needs.....	
913	1	- Well yeah, we don't get, the state does not like, look at it like a rehearsal.	Wb
914	JL	- You can't really look at every....[inaudible] ....look at the sum total of it.	
915	4	- By the time my kids got to the Unit 5, my algebra kids, the average was, I think it was 96 or 97 or so.	P
916	3	- exclamation	F
917	4	- 90 points something. By the time they got to that if anybody should get a low grade, oh my goodness, you feel so bad. "You did What?", "You got WHAT?"` "What were you THINKING?"	P
918	2	- laughing and mmhmm	F
919	4	- You know? Everybody tried to do well. I think I had, the lowest was like 80 or so.	P
920	2	- okay, it was good.	P
921	4	- So you know, so the competition became and, you know, ...give them trophies. My supervisor had trophies for, they had separate events for those that passed. ...Tell them to come after school ....I did well in Unit 1, 6th grade, 7th grade, call their names, you come, [inaudible] they had these little trophies for them, ...	P
922	2	- Oh that's nice.	P
923	4	- and they had their words too.	P
924	2	- That changes the culture about it.	P

925	JL	- So I know your telling us about the way they do and how well they do, but is this, are you saying this like, it would be very hard to switch how you're working and try something like this? Or are you saying, you could but....	
926	4	- It's not going to be too difficult because we're already doing something like this.	P
927	2	#NAME?	F
928	4	- Everyone of us is already doing something like this.	P
929	JL	- So it's not a big deal.	
930	4	- It's not that new. No matter what you are doing, you can still be better, you can learn something.	P
931	JL	- every year.	
932	4	- So like here, what I learned is patience.	P
933	2	- Yeah.	F
934	4	- You know, this guy (Powell) is very patient, you know, he's not rushing the students, you know like looking at the time and saying, you know, "you have to finish this". He lets them work at their own pace.	P
935	JL	- He lets them think about it.	
936	4	- Yes, and then, you know, his input is very minimal.	P
937	2	- mmhmm	F
938	4	- So these are things that I have learned. You know, they always tell us to do things like this. You know? Talk about it. Talk math. This courses, let them challenge one another's reasoning and stuff like that.	P
939	JL	- It's different when you see it.	
940	Group	- mmhmm	F
941	JL	- I learn the way they learn, we learn by experiencing, not so much by somebody telling. So now, to keep to the thing, I'm hearing definitely class size and time, and there's a testing focus. But you could do this, this isn't that different from what you do. It's just there are these aspects of it: allowing the children to go at their pace and being a guide that isn't directing as much. I didn't hear from KH or AK, the practical,......let's put our barriers on the table. laugh	
942	3	- talking about class size and groups, now that's whats not here. Like you said, we'll all be trying it at school [not audible] and we talked about that.	P
943	JL	- Right,.......you have a late day.....	
944	3	- But even so, we all try things, we all do some things and that what it really takes for re-thinking things is for me to have better groups. Choosing groups, who goes with who, kind of a thing. I try to do that, sometimes I don't try it.	P
945	JL	- Ahh	

946	JL	- Well, when they first come, you don't know them yet.	
947	3	- Right, but even when you do, ...you get lazy or something.	P
948	JL	- So what kind of group....like how?	
949	3	- Each group needs a leader. Each group needs someone to be able to be one of "those" kids [kids in the analytics], understands this and that and says, "How did you get that?" Let them talk more. "How do YOU get that?" Not me talking. But they have to be motivated to want to do it and in every class of 15 even, I'm not the honors, the "honors" kids, you're gonna have..there are still 6 kids who watching the clock, no matter what. Trying to get them to ..[inaudible] but still..	P
950	JL	- Well, I don't think you could look at any one of these and say that they look ALL as involved as each other. There were "more involved" and "less involved" in every group. In my whole life, adults too, you know. Some of you are talking all the time and some of you are on your phone a lot.	
951	Group	laughter	F
952	JL	- In every group, there's people are more or less involved. I was the head of my congregation for a while and I can tell you that people have a religious obligation to come to services and contribute. You think everybody's going to do it? You get 5 people who do a lot. You get 10 people who do a little. And everybody else does nothing.	
953	Group	laughter	F
954	JL	- So this is true everywhere. But going back to your (02's) question: what about taking one group and in the room. Everyone else is going to be audience right now. And demonstrate with 4 or 5 kids, that you want to try this with them and you want the class to watch because you want everybody to do it.	
955	3	- That's interesting.	P
956	JL	- Then you could take your "hot leaders" and put them in one [inaudible] and then each one of those could become a leader in the....	
957	3	- So they're allowed to watch rather than....	F
958	JL	- Well, just ...they could write their concerns, you know. If they write they're concerns they get a "I do Jolly Rancher".	
959	JL	- Jolly Ranchers don't temp me, so I have drawer full of Jolly Ranchers and I can bring them out and never eat them up. I never want more than one Jolly Rancher a week. That's my thing.	
960	Group	laughter.	F

961	JL	- I mean that's just a thought. I think we should share...you know,	
962	3	- It's a good thought.	P
963	JL	- I think we can become like a support group strategizing to try. What about you? Challenge! There are a lot. Challenges are big, particularly now in the political climate and the pressures we're under.	
964	5	- Class size and just timing with everything, just making sure that things are planned out accordingly so that you can fit it in.	P
965	JL	- But like specifically, imagine which class you will use, and you're gonna be Arthur and you're gonna come in and you're gonna say, "We're gonna play a game".	
966	5	- Umm, I think I will do it with my Algebra 1 class, um, it's early enough so they'll be able to get into it. Um and I think for me it would just be patience, the wait time, and um just having like that [inaudible] demeanor and not nit-picking on certain things cause that [inaudible].	
967	JL	- So we can all go to yoga class.	
968	Group	- laughter	F
969	2	- I think	F
970	JL	- You have a unique, cause your stuff, you have kids who maybe aren't used to much more than [inaudible].	
971	2	- Yeah I think so, but I keep going back to I think in the way you present this idea to the students. Um, a little similar to this, um, was working with someone else from Rutgers and she had me have the students do something with towers?	P
972	JL	- It must have been Carolyn or Alice..	
973	2	- Alice, Alice, yeah.	F
974	JL	- I love her.	
975	2	- First I was thinking, oh my goodness, ...	F
976	JL	- Building the towers...	
977	2	- ....my kids are gonna be.....like...but guess what? They really.... and think when I first presented it to them, I said, "you know she's from Rutgers", and then she came and she spoke to them they knew I wasn't just making it up, she was a professor, you know. So, based on that, even just,....they were not acting up. They really bought in. So I think that student buy-in is such a rich part. But my challenge will be the wait time, because I am so used to assisting them or I want them - or I don't want them to get frustrated because some of my students, they get frustrated! (laughter Not such an easy...	P
978	JL	- But I think you have to adapt it to your kids.	P
979	2	- Yeah, yeah.	F

980	JL	- So, you know, um, if a student gets upset then we have to do something to help alleviate that tension. So you have to think of how...remember what they did with the towers - so that's going to help you. They can, and I think tomorrow, when you look at the, there's a video in it, you're going to see some interesting things that are going to relate to the frustration issue.	
981	2	- But it did change the culture of my class. You know, I do know that this [inaudible] is possible. And the students will...at first it's a little scary or [inaudible] cause you're like, "oh my goodness, I'm going to be the only one talking", but then, when you step back and just let them, sometimes they really get going.	P
982	JL	- That tower thing - did anybody else do that ever?	
983	3	- I remember the..	F
984	Group	: Yeah	F
985	JL	- It's a combinatorics problem. Two colored blocks. How many towers you can build? If just have a one block, it's either a red tower or a yellow tower. But then it becomes combinatorics...so the kids I....sometimes when they get older in 11th grade they don't want to b treated like babies, they`re very conscious of that. So you bring out yellow and orange blocks, they're like, "I don't think...that". So I had to black and white blocks, cooler colors.	
986	Group	: Laughs.	F
987	JL	- They weren't going to play with multi-colored blocks, I knew it, and I had to apologize, I had to say listen, this is just the best way to do it, it takes too long to draw, so you're gonna humor me. I apologized: "humor me, I want to know how many you can build, it's an important problem".......They got so into it, one kid stole a box of blocks. He didn't get it because he wanted..[inaudible - everyone is laughing]. He stole a box, how he got in there, he got that box. He must have told somebody that he left something in my room, he opened it, he got the box, he takes them home. He figured it out at home, he comes in and he presents.	
988	Group	- Wow! applause!	F
989	JL	- He had not done it in class, he was just looking at it. I said, "What happened?" And then he told me after class, I just didn't know what to do [inaudible] , so I just took em home and I did it. So I can say that sometimes they seem really mean and they care so much.	
990	2	- yeah.	F

991	JL	- Okay this was really great. I think its 12:30 right. You guys can go, I'll clean up. Everybody has the parking pass in the right pass right?	
992	Group	Just cleaning up conversation.	F

Day 3 TAW Transcript Data

Stmt	Spkr	Statement	Code
1	JL	- Well actually, it's very interesting what he [Ariel] did when he didn't have Algebra to.. We should challenge ourselves to write what he really did as an equation - that's not so easy to do [for Ariel at the time]. 'Cause you do divide the "X" you are given by two, so it's really 3(x/2) +2, that's the number of rungs he is working on, right? Cause first he says, "I'm not going to work with 100, I'm going to work with 50". So I do 3 times 50 plus two to get the rungs [152]. And then....then he was adding three....pause.... was multiplying by 2 and adding....	
2	?	Group - lots of murmuring...	
3	4	- I just said 3...	M
4	JL	- For once when we finish	
5	?	Group murmuring	
6	4	– Which is the slope, that 3 that he adds. So the first part is just to get what is before...	M
7	?	Group murmuring	
8	?	We hear the analytic - Ariel talking	
9	?	laughter.	
10	3	– The number of rungs..You just take the number of rungs you multiply with any..next. So you can do both ways..[inaudible].	M
11	JL	- he's not saying its fast, he saying it's the loooong way.	
12	2	- mmHmm	FY
13	JL	- Do you want me to play it again so you can see the "beginning Ariel" and the "end Ariel"? I want you to have those brackets? in your mind before you look at the next analytic.	
14	?	A few murmured yes'.	
15	JL	- Maybe even. we should, umm...we should do....I have questions [on handouts] for this one and questions for the next one,...maybe you should do the yellow questions [explain which type of questionnaire this is] for this [which one?] after you've just seen this one, so they don't get mixed up in our minds.	

16	?	Some-one - mmHmm	
17	JL	- Unless you want me to play the first - he did a few different problems....solving, (you just have to revisit the problem)1, a few different heuristics. In this one, in the beginning and I am asking you to be precise, just like what t4 was doing before, seeing if that algebra equation was correct..and it wasn't actually correct. So I want you to try and capture exactly what he's doing.......So that I can play it again if you want....play it again?	
18	?	murmuring - yes	
19	cmt	Analytic is playing again.... JL - is talking over it a bit (not clear).	
20	JL	- stopped the analytic [it was not finished] and said, "I thought of something,.....I did another version of this for another [group of] teachers and they told me, they wrote in some of the questions, they didn't think Ariel was engaged in the work.	
21	?	Teacher - really?	
22	?	Teacher 2 - really?	
23	4	- Why?	EQ
24	JL	- A couple of times, you know, there were slightly different video components, and he's surprised that the doubling isn't working, and he's struggling with it, and at one point....you know how like when you work very hard sometimes.... he said "I give up"? And then he kept working.	
25	JL	- They quoted that.	
26	cmt	Soft wows from the teachers.	
27	JL	- What do you think is going on here in terms of how interested he is in the problems?	
28	2	- I think he's interested, but he's surprised that his answer is not coming out right.	E
29	3	- He wanted to know why it didn't work.	C
30	2	- He had something that he really thought worked and so I think he just he surprised by	EC
31	?	teacher - challenged...	F
32	JL	- But he doesn't get angry or re-something-ed	
33	2	? and others- loud - NO!	FN
34	JL	- What did he do?	
35	4	- He's go back to start all over again and see what....	M
36	JL	- How would you characterize what he just did? He didn't start all over again...	
37	5	- He tries to like figure...	F
38	1	- modifies	F

39	5	- to modify...	F
40	JL	- He didn't ....you that heuristic thing? He says, okay, I got most of it, fix what I did. He comes up with an algorithm that largely, works...	
41	1 or 5	- ....took into account what the other boy [James] was telling him?	M
42	?	teachers murmuring "yeah", "he did", "quite", "yeah"	F
43	JL	- He didn't quite see it as James...James was lucky to see this groups of three..	
44	Teachers	mmhmms	F
45	JL	- that was really something....he gave a very articulate argument, but Ariel was invested in his own	
46	2	- yeah	F
47	JL	- you know, do you find that kids work together and they hear each other but they also are very invested in their own, so you are seeing that here. He is fixing his own solution.	
48	4	- The point is, sometimes when you are doing something, and that person has another way of doing..but if you're convinced or have a conviction that what you're doing is going to work, it's very likely that you won't give up on what you are doing. You may not listen to the other person because you feel that "my way is going to work too". I think that was what happened to Ariel and James. Ariel was so focused on using his method to get the right thing. So he wasn't really listening or thinking about what James was saying. And I'm happy to hear James talk, finally..	CE
49	?	laughter in the group.	F
50	JL	Note - Nobody pointed out that Ariel did listen to James - James gave him the right answer - and Ariel accepted that answer. He didn't challenge it. So was he convinced? Why?	
51	3	- Same here.	F
52	?	lots of laughter about the James comment	F
53	4	- as good as ....as James...he likes to eat	F
54	?	Much more laughter - inaudible comments	F
55	JL	- Those are all his solutions. So you can play it again if you want to...but you can talk about .... You don't have to remember everything yourself. You don't have the text for this, because he didn't do that; he describes. He doesn't put the quotes in there. And his descriptions are full of opinions so I didn't print them out because its your opinion that matters, not the person who made it. (The author of	

		this analytic didn't provide a transcript? Or I didn't provide the transcript he created.)	
56	JL	- So don't look - you don't do the questions for the other one, because you didn't do the other video yet. Just the first page, I think the first page and there may be one more question back? Yeah the first page and then one question. But I was hoping you guys would talk a little bit about what are the different solutions he has that you are putting....cause he grows, kind of different heuristics, the first one, the second one, the third one?	
57	1	- The first one was proportional right?	MQ
58	JL	- Ask everybody....So could everybody like finish their sentence and be willing to just talk about that? What were the..before we get to the grown area? what were the solutions we saw there? We saw some different things. Pause. Everybody is just too busy writing...	
59	JL	– Before we go to the next page, I think we really talked about this first page, the way he approaches the problem, he keeps trying different ways and what specifically caused him to make the change from the first way of doing it, to the second way of doing it, to the third way of doing it. You could just talk about that and see if you agree? Explain your views? I think that would help before we see the next one.....So somebody be brave and start,	
60	1	- He like initially uses proportional reasoning to figure out the number of rods in a ladder.	M
61	JL	- So, specifically, that very first thing he does? The specific proportion - what does he use?	
62	1	- The gentleman asks him how many rods in a 10-step ladder? And I guess he didn't want to build a whole 10-step so he just built a 5-step, count how many rods are there and doubled that and that should give me the number of rods in a 10-step.	CM
63	JL	- Everybody put a [not clear]...doubling...we saw that yesterday and so we saw it again today.	

64	1	- I think, alot of kids in his grade level, they like proportional reasoning, so that's ...7th grade, 7th grade, yeah 7th grade	M
65	JL	- When we talked about it yesterday, you guys also said you've seen it.	
66	1	- I always noticed it.	M
67	JL	- That's the orientation......	
68	5	- They go to proportional reasoning.	M
69	1	- I guess he notices..um....well he concluded he should have had 34 and I think...	M
70	5	– Did he end up building it?....and found out...	MQ
71	2	- Yes	F
72	4	- Well who knows?	CQ
73	JL	- They don't show that in this...[not clear]..The owner [not clear] is to show that he did 34 but nobody says that he should build 10-step ladder and check it. The next thing they show is what?	
74	?	Lots of voices - ...the 4 steps..[not clear3]...for 8 [steps]	M
75	4	- For what they asked him for the 8 steps and he builds 4 and he got 26.... I think he got, what was it?, 14 and so he did the 8 steps and he expected to get 28. Now he got 26 and he says "heck, what?"	M
76	?	Lots of laughter.	F
77	3	- And he looks up.	F
78	1	- ...wasn't gonna work.	F
79	3	- At some point changed ... and built the whole thing, right?	MQ
80	JL	- James tries to explain to him. But then what does he do?	
81	5	- He's going with his doubling theory.	M
82	JL	- He doubling but he sees that it didn't work	
83	3	-...+2 and -2?	M
84	JL	- ...because its not 28. So what does he do?	
85	3	- modifies	F
86	JL	- So that's like his first change and then what happens?	
87	1	- The doubling rule. Did the doubling rule only work with the even numbers?	M
88	2	- Yeah..mmhmm.	F
89	1	- I think he considered an odd number - he had to do it differently.	M
90	?	- yeah	F
91	5	- He went to the....	F
92	JL	- What he is doing.....what is....something happens.....	

93	4	- first of all, first of all...	F
94	JL	- the teacher asks him to do something..	
95	5	- Look at some ..	F
96	JL	- seven	
97	5	- He looks at 8 and then subtracts 2.	M
98	JL	- two?	
99	4	- and sometimes three.	M
100	5	- its odd -	M
101	1	- Yeah - in the end he subtracts three.	M
102	JL	- Right, so which rule is he using then?	
103	4	- that was ....I think that was what James was trying to tell him - three - that created the factor of 3 -	M
104	JL	- every step, yeah	
105	4	- every step you reduce by 3.	M
106	JL	- Thats really different. That's a totally separate rule: if you want to go from one size to another, what do you do?....pause..murmuring....you want to go from 7 to 8 you..	
107	4	- you add 3 more rods.	M
108	JL	- You want to go from 8 to 7 [steps], you ..	
109	1 and 5	- subtract 3.	M
110	JL	- So that's like yet another thing. What? Does he, like, come to this all by himself? Does he, you know, he's doubling and does he think, "I'd better check this"?	
111	1	- I think James kind of hints..	C
112	JL	- Something outside, kids tell him or the guide says check it. And what about the odd thing - going from 8 to 7. Does he just think, I'd better worry about 7?	
113	5	- It's the researcher who...	F
114	JL	- Yeah, It's critical points where he's directed to find something out.. Okay, ready for the next one? The next one, just to prepare you, Ariel gets complicated here [in the last one] - Boy does he get complicated.	
115	2	- I was just thinking earlier, like wow, like what a difference between 7th and 8th grade, even the way he approached a problem. I was like "What?".	C
116	1	- Your algebra	F
117	3	- ..learned something somewhere along the line...	F
118	cmt	Lots of undescipherable agreement	
119	2	- Yeah, he knew words..	FY*(CM for Joyce)

120	JL	- Well that is very important to think about. This next one is not the 8th grader, this is between the beginning and the end. This is Ariel growing. Because that's our question, how do you grow ......like wow....[*undecipherable*]? laughing	
121	JL	- Sometimes as a parent, I look at my oldest daughter and think, "How did you become so responsible?" When, as a teenager, we thought we would not survive it...	
122	?	We hear laughing as Ariel describes is complex heuristic - for 9 he goes to 8, finds the number of rods in 4, multiplies by 2, subtracts 2, and adds 3.	M
123	?	They are very attentive to Ariel's small remarks ("I have to leave early today") - its like they are getting to know and like him. They seem to be laughing with affection for him and John Franscisco is teacher/researcher.	
124	JL	- Okay, so there was a lot of stuff there. Alot of interesting applications of rules and I don't know if you want to do this but I think that maybe before you answer these questions you might want to use the board to diagram out what he did.	
125	?	Some serious sounding "hmms".	
126	JL	Because I'm asking you to be precise; where was he [Ariel] precise and where was he not precise? And so I think it's easier to just sort of figure out what he did. I wrote down what he did, so it's in the [descriptions]; but I don't say where there is precision or not, I just wrote down what he did. You don't have to look at this again, you could read it; it's up to you, you could play it. But I think that you might want to just put it on the board and ..write them down…the math team to do it. You are going to have to put some....Yeah, you can see some of these rules written. I added the ...sometimes ..we have examples of the student work, so I added some of the interesting things here. What he did with the eight and what he did with the six, and how he wrote it up. So it's up to you, if you want to write it down on the board, or...just discuss it....	
127	1	- Who wants to start?	F
128	4	- What are we supposed to....	F
129	1	- Together, you mean?	F

130	JL	- Well, if you look at the questions and answer? for this analytic, the first question is, "Explain how Ariel is using his rule in a precise way, in some or part of the work, you know, I mean in ALL or part of the work. So is he doing it all of the time or is he doing it some of the time? And I think its helpful to go through what he did so you can ....all clear on what happened there.	
131	5	- It changes there...he'll use it for when the numbers are small, but when it gets to 60 or 20 that's when he starts using proportional reasons and try to multiply by 10 instead of sticking to his rule....doing it..	M
132	4	- I think he actually tried to use his rule. But what happened is that...you know like when he went from 7 to 8 that is only one step.	CM
133	1	- Yeah.	F
134	4	- So he did his "minus 2". But now when he now did from 60 to 120 there are so many steps in between, so he didn't take that into consideration, so instead of subtracting like 18 or so, he's subtracting just 2...	EC
135	JL	- So lets try to like unpack this a little bit. When he was given 120, he was like "uh..", so what exactly did he do?	
136	1	- He found six steps and then multiplied by 20. (DG and t3 are heard echoing this.)	M
137	3	- He went to 60 and that would be 6 times 10.	M
138	JL	- I'll try to write that and you tell me.S..So he does about 120 steps, that's the problem: how many rods?	
139	1	- So half of that is 60. So..	M
140	JL	- So he did 120/2 is 60, so he needed the number of rods for 60 [steps].	
141	1	- Well he....find the number of rods for 6 then.	M
142	3	- He knew 6...	CM(it's about observed math behaviour)
143	JL	- So he didn't separate this (the 60) in half?	
144	Group	no, mm_mm	F(M for J coz it's recognition statement)
145	3	- Cause he knew what 6 was and he already had [not clear]..contact again..or something.	M
146	JL	- That's not his rule....But what did he do?	

147	4	- It was	F
148	JL	- 60 = 6X10 and what did he know? What did he know?	
149	4	- which one of these	F
150	?	all are talking about it - no discernable conversation before:	F
151	JL	- He knew 20 rods in 6 steps.	
152	4	- Right	F
153	JL	- What does he do now?	
154	4	- multiply by 10	M
155	JL	- 20 X 10 = 200 (laugh) then..	
156	4	- then he subtracted 2 - that's his rule..	M
157	JL	- 200 - 2 =198	
158	3	- That's a beautiful thing though..	J
159	JL	- So where is he attempting to use his rule?	
160	2	- First, at the first step...	M
161	JL	- So I'll put checks oh?...this is a rule and then	
162	4	- Then the last.....step	M
163	JL	- And in the middle (laughing) he gets all crazy - more laughter	
164	JL	- But this is kind of a new way of thinking	
165	4	- echoes this	F
166	JL	- This is really interesting, right? He gets right back to it. So I have to ask, and I asked myself this too, when he thinks about the subtracting 2 as a correction, does he think that.... Is he actually thinking that it doesn't matter how many times you	
167	?	Appreciative murmers - hmm - like they are appreciating the question?	F
168	JL	- Or is he has figured it out in this example with 2 and he's forgetting - that'll only work for that example when you multiply by 2, not when you multiply by 10 or 6 or (laughter) 50? Does he really understand the rule in a more general way? That's what I was asking myself when I saw that, or he just forgetting and subtracting two..right here.	
169	1	- Because the rule ...for ......every time [not audible on this recording [00:53:58] look at other recording]	F
170	JL	- Do you think, that he really thinks, in general, that "no matter what, subtract two?"	
171	1	and others - yeah, as long as it's even	M
172	2	- as long as its even	M
173	JL	- It could well be.	
174	JL	- Well, "even"?	

175	1 + another?	- Even when he subtracts 2 every time. Even number of steps at the end, he subtracts 2.	M
176	JL	- Is that true? Should we..	
177	2	- That's what he says..	F
178	1	- Thats a curious thing..	F
179	4	- If it's odd, then it has to....	M
180	1	- If he subracts 2 then he adds 3	M
181	?	lots of unclear voices.	F
182	JL	- He always subtracts 2 doesn't he? When he muliplies by 2, then he subtracts 2.	
183	5	- But he has to first give it....even.	M
184	JL	- Sometimes he has to add 3 at the end.	
185	1	- If it's odd.	M
186	JL	- So that has to do with putting the two together.	
187	4	- Yeah.	F
188	JL	- But maybe he's thinking of putting any more of them together.	
189	4	- In the middle thing he did, in the middle thing he did was he multiplied by 10. But in all the... he didn't have to multiply by a factor like that before he subtracted 2. So because he multiplied ...	M
190	JL	- He multiplied by 2 in the other ones; all the other ones	
191	4	- he only multiplied by 2.	M
192	cmt	lots of unclear voices joining in.	
193	JL	- All the other ones, he multiplied by 2 and subtracted 2.	
194	4	- echoes.	F
195	4	- But here he multiplied by 10 and subtracted 2.	M
196	JL	- echoes	
197	4	- when he multiplied by 2 he subtracted 2. But now he multiplies by 10 and he still subtracts the same "2".	M
198	JL	- What should he subtract?	
199	4	- When he multiplies by 2 he subtracts 2, right?	M
200	JL	- right.	
201	JL	- Well what is the answer for 60, we know the formula, right?	
202	4	- 60 is umm 182	M
203	cmt	a small low chorus of 182	
204	JL	- It would be 182	
205	4	- So if he had subtracted 18	M
206	JL	- What should he subtract from 200?	
207	4	*(subtract)* 18	M
208	JL	- 18, so he should subtract 18.	

209	3	- Subtract 2 from each one...	M
210	JL	- 18 in... how would we get..?	
211	3	- Subtract 2 from each one of 10 and then add 2. He did 10 of, 10 of..	M
212	JL	- Laughing...	
213	3	- I thinking he's thinking...	F/C
214	JL	- It'd be really hard...to figure	
215	3	- Well that doesn't work...I mean it worked…. but it can't..good	F
216	JL	- Right because its really ...two things together with...it's not a multiplicative..really hard to apply it here.	
217	3	- That's what it is (at the same time as the previous statement).	F
218	3	- I think he thought he found a tricky missing step, "Now I got it, I got you, I'm going to do it this way, It's going to work, because.."	C
219	JL	- Right, right. This is, this is really interesting.	
220	3	- ...this is getting late in the day..I'm getting tired.....	Wb
221	JL	- Do you want to take a break before you tackle all these questions? Because this is like a ...makes your head hurt...	
222	?	Laughter	F
223	4	- Makin my head hurt.	Wb
224	2	- Yeah yeah - when he was going over it....I was WHAT?	Wb
225	4	- Its makin my head...	Wb
226	2	- It was tortuous???	Wb
227	?	Lots of laughter.	F
228	5	- He didn't have as much of a calm demeanor of the first guy. The first guy is like "okay, try it". But this guy he's just like, "hold on, wait	P
229	2	- He kept on saying to him like "wait a minute", like..show me..that's more like a ..	P
230	4	- Before he was done with one, he would throw another number.. and I think its because Ariel told him, "whatever you do, I am leaving here at 3". "You'd better hurry up."	P
231	?	Lots of laughter and voices while the speak is talking.	F
232	5	- He said "3:30", It's like 3:30 (interjected).	F
233	JL	- Do you really think he just wanted to leave?	
234	2	- No, I think he just was, like, "You'd better get it, what I am trying to say to you". laughter.	E
235	JL	- He was trying to say?	

236	2	- Because maybe he was getting frustrated by the fact that the researcher wasn't getting it. He probably was like, you know when you "get something", you think like...	E
237	JL	- "I told you 3 times, I'm leaving at 3:30".	
238	2	- Yeah, yeah, but he finished it.	F
239	4	- You know the times that he said, "I just finished this" or something. It wasn't finished, he said, I will just finish this, I think it was time to go but he still stayed to finish one more. But he just wanted the facilitator or whoever, to know that "whatever you do, do it quick, because you know I have other things to do".	E
240	JL	- It's sort of true. There's a bug in the tool that makes the analytics. And umm, I don't know exactly where the bug is and used to be a computer eng. so I think about it, where is the bug? What was happening was when I was making some of these, because of the pattern of the ....when I saved them, it was destroying it. It was multiplying the number of events - I would save 7 of them and I would get like 45 back.	
241	2	- asks a question about food	Wb
242	JL	- there's muffins in one and bagels in the other.	
243	JL	- Anyway, it was messing it up and so I would do all this work and then I would have nothing, because I would have to redo it because it messed it up. So I was getting very very frustrated and when my son was in the room, I said, "I hate this, I am not good at this, I am going to drop this, This tool is terrible". I was sending emails and then I would do it again and then it would break it again.	
244	2	- Oh no.	F
245	JL	- Then I carefully wrote down exactly what I wanted it to be in a document so I wouldn't have to keep thinking it through, what clips and what seconds from what clips, so I was able to just.. So my son says, later on,. I hear him say to my husband, "Oh yeah, she's been droppping that for 6 hours. For 6 hours she's been threatening to throw the whole thing into the garbage." And she keeps doing it. Laughter And I felt like Ariel, saying the same things. He's glued to it, but it's a frustrating process, so he threatening to leave. But I think you're right, t4, he tells to the teacher.....???	
246	3	- ...While he's stumbling into what we're trying to get at, his number sense I think, is terrific....	M
247	JL	- Yeah it empowers him	
248	3	- ..He does it...you know in the 7th grade...still ...	F

249	JL	- Yeah, he's in 7th grade	
250	3	- dum..dum..dum.. ...[not clear]...And whatever it is, he's got it ..[not clear]...	F
251	JL	- That is very impressive	
252	?	There is a lot of simultaneous conversation here - must look at the other recording of this same time frame[01:00:23]	Wb
253	JL	- Later on, in eighth grade, when she asked him about the 75, um, there were 75 steps, and he doesn't have to use a calculator or write 75 X 3. He just does 225 +2 in his head and writes 227. I try to get my [HS]students to do that on HW or in class; on tests I let them use the calculator to be accurate. I try to get them to do it. Sometimes, I race the calculator, so I want to prove to them that it's faster to do it in your head. So, you know, I'll draw something (arithmetic problem) on the board and I'll say, "let's see who can do it faster"? ...They'll say, "you knew it". I'll say, "Well you give me one". Then I'll also do it faster. And some kids'll get the wrong answer on the calculator. I try to prove to them that the calculator should be your "slave" not your "master".	
254	4	- mm_hmm	F
255	5	- It's crazy cause in the summer I work at Ryder as one of the math tutors and all the students have to take a math test without a calculator. Alot of the issues that they have is fractions, decimals, all that stuff that they haven't had to use..	MY
256	JL	- percents	
257	3	- The calculator sometimes, being aware...."how do you fraction this one"? "How do you 100% this one?" You walk them through it....	MQ
258	JL	- Yes, yes. I say, "who's supposed to know fractions, you or the calculator?" You know, that's really valuable..	
259	3	- I say, "I don't know, I don't know, I never ......before, I don't ...supposed...to before, I don't know".	P
260	JL	- I teach some of those things in every class, honors classes, calculus classes, any class, because nobody seems to know all of it. There is a very narrow band of students who know all that. Even in AP classes there are kids that do and kids that don't. Yeah. It's interesting.	
261	3	- That's why I say it’s a "McDonald's Class" [except for] my 8th grade honors..	F
262	3	- very real	F

263	JL	- So we were saying, this is a break time, eat, use the bathroom, don't have to write yet, take a break....need a break	
264	4	- I'm behind in, I'm behind in, I'm not doing those, I'm doing the first one.	F
265	JL	- You need to write? laughter, take a break. Breaks do work for any citizen.	
266	?	weird sounds of paper?? Unwrapping food?	F
267	JL	- We don't have a trash can, put it on the table - we'll clean it later. There's a bag of garbage. I'll fix it...we can use that as a trash can.....................anyone else need it [not clear]...	
268	JL	- I guess its a psychological thing - if you don't have a garbage can in a conference room, nobody can leave - nobody can just stuff, fill the garbage.	
269	4	- Mmm, surprise, my ?? would leave it right there and if you ask them they say "it's not my fault, they didn'nt have a garbage".	Wb
270	4	- It's all a competition.	F
271	JL	- I have to say, my students, particularly my high need kids, they don't make a mess, they respect the rules. My calculus kids leave a mess all over the place.	
272	?	squeaking sound "oops"	F
273	?	muffled comments from JL and t4	F
274	JL	- Wouldn't it be fun if Ariel could come and talk to us now?	
275	3	- yeah!	F
276	JL	- He's probably about 25.	F
277	4	- So tell us in your face! lots of laughter	F
278	3	- And James has been filmed also, we're just watching Ariel mostly, but other people in the room are also being filmed at the same ....because from here.....what's going on over	Wb
279	JL	- Oh yeah, yeah,...............during this research, I don't know if those chapters I did explain that part, they would have multiple cameras in the room focused on different groups of children. They would produce the things in our session. There was 3 hours of video focused on different groups. Yeah I mean, I don't know if we'll have time, but I'd like to show you a lot of different things in the VMC, and maybe we could...... (lots of paper crumpling noise). Maybe I'll just post things on the Sakai site. You know, there's a great one interviewing the older Ariel.	
280	4	- He's in college right now?	F
281	JL	- What?	

282	4	- Where is Ariel now?	F
283	JL	- Oh, I don't know, he's grown up, he went to college, he's probably working....	
284	4	- Wow! How many years ago?	F
285	JL	- This is a while ago. He's 13 or 14 years old here. So this is at least 10 years ago. Ms. Morris looks really young as a teacher there. (laughter) It looks 10 years old. So....I could find out by looking at the original video, So he's probably out college, he's probably in graduate school, maybe he's married, maybe he has a girl friend.	
286	4	- I am sure, I'll bet he has a girl friend..	F
287	JL	You know.....The first one that I showed you? the one that was published? The young man who made that was a child in the study. (lots of interested Hmmms) So he was in the longitudinal study ....these kids were only in for 3 years.....but he was in the longitudinal study through college and then Carolyn hired him to work in her department and he went into a doctoral program for math eduction. And now he's off, he's going to teach at the University of Texas. (Wow!) And It's really interesting, cause we see "little" Robert in the videos when he was 5 and 7 and 10 and its just....its good that he's going to Texas because he's been with Carolyn too long...he needs to be independent and do his own thing. But Its fascinating to talk to ....you know, that's what I am going to do, I am going to show you this analytic I have that's called Understanding Understanding, because I trace some of those kids, little and big, you hear them talking about how they think about math when they're in college. That's fun to see. You start feeling like all these children are YOUR children...watch the videos....old friends over. (laughter).	
288	?	we hear rustling	F
289	?	muffled words	F
290	4	- I am trying to understand something. I hope I will get it. Why the 2 and not 20? I think I get that one..	M
291	JL	- Why is this subtracted, the 2?	
292	4	- Can....either....subtract a 2...times 2?	M
293	4	- Its okay...its in your subtraction I'm not showing it. Would you want me...the 10 steps.	M
294	JL	- yeah?	
295	4	- Because, for the 20 rods, you already subtracted 2. To get the 20 already, subtracted a 2. Okay, So now you have 10 steps, he's going to subtract 9X2.	M

296	JL	- Okay, say that again? I'm not.	
297	4	- What I'm saying is this. Look here, we said he subtracted just 2 (possibly pointing to the complex problem we discussed earler where the answer was 182, he subtracted "just 2" from 200 to get 198 (the output for 60) and we said, earlier that he should have subtracted 18 (which is 9X2)). So in my mind, I am thinking, "he should have subtracted 20". But he subtracted..	M
298	JL	- He had to subtract 18.	
299	4	- Yes, because he already subtracted 2 to get his 20, ...for the 20 rods.	M
300	JL	- But the 20, This is correct...	
301	4	- Yeah, I know, I know.. For him to get that, what did he do to get that 20 rods?	M
302	JL	- Well how about instead doing this like this, how about you do this one step at a time? So 20 + 20 would give you for 12, would give you a 12-step,.....for a 12 step ladder, lets use his algorithm. 20 + 20 - 2, right? 38	
303	4	- That's what I am saying...	M
304	JL	- No no no, I know, I am saying match up what he would have done, step by step.	
305	4	- That's what I'm..	M
306	JL	- This...we'll do it twice...multiply 2, the 12 step ladder; it works, and then you could ummm	
307	?	laughter	F
308	JL	- eventually we want to get 60,right? We want to multiply by 10... We've got, we’ve got 40 of the 200. So know if we do...	
309	3	- 2 of the 10, 2 steps out of 10, multiply by that..	M
310	JL	- I am trying to think of a way, ...What's the best way to do this? I could do this 5 times.	
311	?	There is giggling and laughter throughout.	F
312	2	- What is this?	F
313	JL	- One, two, three, four.....its very hard to think about this.	
314	2	- yeah.	F
315	JL	- and this gives you the 200, right? (There is work on the board we are doing and pointing too?) You have to subtract all those twos.	
316	4	- Thats gonna be just...not gonna give you ..	M
317	JL	- Does this give us the right answer?	
318	4	- No. Cause 8 X 5 = 40, that's what I'm saying, right? 18.	MQ
319	3	- one, two, three, four, five	M

320	5	- but it can't be 18 though.	M
321	JL	Because we have to subtract 2 when we add these, and we then have to subtract 2 when this again...this is where the extra 2's come in. If we add these 2, we get 76 and we have to subtract 2, right?, so that becomes 74. So there's another. And then we add these 2, we get 112 and we have to subtract 2, another one, get 110. And we get these 2, get 148 (74X2) and we subtract 2 and get 146, there it is again, and then we add these 2, 100 and .... 38 (138), so we get 184, we subract 2, and get 182, which is correct, right? So there's 1, 2, 3, 4, 5 for the first set, and then 1, 2, 3, 4 ......9, there they are, the 9 "minus 2"s. That's his rule, but its so hard to use.	
322	?	pause	F
323	JL	- Wow, I never did that before.	
324	?	murmuring - about subtracting 2 --- to get to 120	M
325	?	they are writing?	F
326	JL	- Every time you add.....you have to subtract 2.	
327	?	Laughing	F
328	JL	- What? What?	
329	?	more laughter	F
330	?	we are on break?	F
331	JL	- Every 10 minutes I have to delete 10 political solicitations....	
332	?	I understand the pattern......but I can't do it...	M
333	JL	- Okay then guys, I can't wait to read what you write about this. And remember how it comes out at the end. He comes out a mathematician: "first differences, linear equations, slope, y-intercept".	
334	?	They are still writing	F
335	4	- I'm leaving that,	F
336	?	laughter	F
337	3	- Hope I don't	F
338	4	- ...what to do before that....	F
339	2	- I can't have	F
340	3	- Unrelated thing - If you've looked, ..don't know if you should look or not...Did you get your roster's yet, by any chance? Yeah, I looked. I got an email, but we didn't get them.. .on email so we should have them, but we didn't get them, so it made me look...	MY
341	4	- If we have it ...I hope I'll be surprised. We don't get it until we get there..	MY
342	3	- Somehow ?? [not clear]...	F

343	4	- I hope you'll do it different...	F
344	3	- inaudible I'll look either..	F
345	4	- I would look at my email on the first.......This week I will check my school,.. I think I checked this on Sunday. (laughter in the background)	MY
346	3	– For God's sake...jumbled voices	F
347	4	- I hope I don't end up here .. and find out that they moved me again...	MY
348	2	- Oh yeah, cause not that many..	F
349	2	– Yeah, I don't know (laughter).	F
350	4	- So if Ariel had... if he knew how many [rods] for 60 [steps]	MQ
351	3	- You'd...double.... [not audible]	M
352	4	- Then it would have been easier for him, right?	M
353	5	- You could do for, ..go for 60, break down for 30, and find the steps for 30, and then find it for 60...very long.	M
354	4	- I would find for 6....I went? to 12, 24, 48, then to get to 60 I could have too do 12 steps... I go to 182. I keep using his rule. Double and subtract, double and subtract,	M
355	JL	- mm_hmm	
356	4	- Double and subtract 2, double this, subtract 2, double this subtract 2 would give me this and then 12 steps it's supposed ...subtract 2 I get 36..together...I got that..all these give me 64 [or 60?] steps.	M
357	JL	- Oh...kay	F
358	4	- all this gives me 60 steps.	M
359	2	- mm_hmm	F
360	?	murmuring	F
361	4	- That's all right. We will ....need that.[not clear]...subtract 2, subtract 2, subtract 2	M
362	4	- Then, I was going to ask this??..How do you do this in a class of 20 people?	PQ
363	cmt	laughter	F
364	4	- Would somebody like, ...are you going to be with one person?	PQ
365	2	- Yeah.	F
366	1	- You can deal with it like one on one...	F
367	Group	Yeah, yeah	F
368	5	- Because, after I planned...deal with ….this is double like one on one: teacher to student.	P
369	1or 5	- ??....This is done like one on one: teacher to student?	P

370	JL	- There's a couple of others around..and the clips you're seeing are ...but there is actually, other people do come..	
371	4	- Like the one we saw with, you call her Alice or Allison?	P
372	JL	- Allison	
373	4	- It was in her office - that was only Ariel.	M
374	JL	- That was not part of the IMF session. That was, you're now kind of finished and you know this stuff, I want to interview you about your experience, yes.	
375	4	- But, the issue I have is this...like when we have less than 20, at least 20 students, how are you going to be able to dedicate the amount of time to one student, like you know?	PQ
376	3	– To make sure.	F
377	4	- yes, imagine what they're doing, you're going round and round and there's no way you're going to be able to give that time, unless you have them by yourself. So how do you use this in your classroom?	P
378	JL	- We will make time to discuss it. But why don't we finish answering these questions and (then) we can resume this discussion about you know the challenges and I can tell you what I've tried, or, and we can suggest to each other and think about how to implement. Cause you can't do exactly this..in the classroom. But we can take, I think, the core ideas....pause...There are also expectations on us that are not on researchers.	
379	?	Now they are answering the questions?	F
380	JL	- So....You need a few minutes to finish this, right? I just have to call this dr. I don't want to do it unless you have a few minutes of work...	
381	?	Lots of "OK, mm_hmms"	F
382	JL	- inaudible, but talking in the background	
383	JL	- You can work on it together, you don't have to struggle alone. It's not uh, it's more a group understanding, I think, than an individual... although I am very interested in each individual's views......feel free to help each other form your answers, not just share the answers afterwards.	
384	JL	- I think I am going to... [not clear]...	
385	?	they are still working	F
386	?	murmurming and laughter	F
387	JL	- You want to redesign school. Is there any of you who redesigned school? Which one of you redesigned school?	
388	?	laughter	F

389	JL	- Although, I'll tell you, sometimes, when he had a policy in our district that we were supposed to keep it under, ..I don't know, 12 and under for the real high-need classes. It faded....we ..hS..15, 16....we just didn't have enough teachers but sometimes I only had 5 and sometimes it was not a group of 5 that worked well together...	
390	2	- interjects: Yeah.	F
391	JL	- continuing.. And then there's no other,... no place else for them to work with; They're going to be really unhappy. It's bad group of 5. It seems like 12 is the best, because there'll be some kids that make that work well and we'll get a peaceful class. But I think [5]is too small. Maybe not in middle school but definitely in HS.	
392	4	- Like my algebra 1 class. At the end, well, I end up having 11 kids because some kids dropped out. And it made a lot of difference. I had like 3 groups and, yes, it made a lot of difference.	MY
393	3	- Same here. When I did the honors-ish class, in one class I had 24 and one I had 11. It was like night and day. It was much more fun.	MY
394	4	- interjects "huge", much more fun,	F
395	3	- for everyone	F
396	5	- My first year, I had 21 in my Algebra 1 classs and they, ...and half the kids didn't even belong in there. So that was a struggle within itself. Because it was like extreme differences.	MY
397	4	and others are assenting, laughing	F
398	5	- I think it got up to 15 ....way, way if they grouped.	MY
399	JL	- Even with like...I teach like, I like the bottom and top. The middle, I am not so interested in. I find that they (bottom and top) inform each other really well. But I had these really advanced geometry kids - really smart, I had 32....	
400	?	Whole group - teachers murmur sympathetic sounds.	F
401	JL	- I didn't even get to know some of them. We go so fast, and its such a ..like..we do alot of "this"...but, you know they own a lot of it - they have to get up and present - I had such trouble remembering some of them because they were quieter and 32 of them in there. And then the other class had 22 and we were like a club - we all knew each other. And there are all these unfortunate research (studies) showing class size doesn't matter; and we all know that it does.	
402	4	- It does.	Wb

403	2	- I don't understand....	Wb
404	4	- Most of the time it does.	Wb
405	JL	- Yeah, it does. In my experience, have never met a teacher who said that it doesn't....but umm...okay...why don't we hand them in so I don't forget to collect them and then lets talk practically about doing this.	
406	?	We can hear paper rustllng	F
407	JL	- That's whats so good about the color, I can pass them (out) and I collect the orange....[not clear..- but just about collecting papers]	
408	?	murmuring	F
409	?	laughing	F
410	JL	- You guys are .....you're putting like taking extra math notes? on paper.	
411	2	- yeah.	F
412	3	- somewhat.	F
413	JL	- Keep them in your book, but at the end I'd like to have them. I am trying to add this -- [not clear]...I have like, you know, more data that I can use...you know...so..enjoy the first thing of the [not clear].	
414	?	laughing.	F
415	JL	- I have everything? I have it all?	
416	JL	- OK - so t4 you were the one that raised it today, you were the one that..??.. said, "I have 20 kids in my class; how am I going to do this? So umm, first of all, anybody have thoughts yet about how they'll do/try this?	
417	1	- .. [didn't you say} say, would be a good idea would be do it at the center or something, so I'll have a small group, kids to the group..and just focus on that?	PQ
418	5	- Probably done, like the center..	P
419	JL	- Tell me what that is exactly?	
420	5	- Stations?	F
421	1	- Yeah, stations.	F
422	JL	- Okay so, everybody's at (a station)?	
423	1	- Yeah.	F
424	JL	- And..	
425	1	- So that one station...	F
426	JL	- So one station does it the first time.	
427	1	- Yeah, yeah.	F
428	JL	- So what are the other kids doing when..	
429	1	- There going to be at some type of station..	F
430	JL	- So you'll work with one group at a time.	

431	1	-Yeah that'll be easier..	F
432	JL	- Okay, okay that's one way of doing it.	
433	3	- Mentioned yesterday was to have a stage, a certain amount of kids doing this and every body else watching, maybe talking..not ideal, it's a large group. But that's a way of focusing on a few kids and maybe also these other ones who aren't doing it are listening, learning, and say "Ah". Maybe ....they're kids.....	P
434	JL	- You know, one of the things you are going to see tomorrow, what I show you, is when Bob Davis, the famous guy who "this" is named after, and who lived in my house [laughter] before I had it; He worked with a lot of kids at the same time. He got a whole bunch of them; he started with them when they were really little, so they were used to this model. And ideally that's what we could change our environment to. So, they're all used to being little scientists and it's not any problem in this paper,but we're now in a transition period, so we have to deal with 20 middle schoolers who have not done this before.	
435	3	- They're NOT motivated...cell phones....	F
436	JL	- Certainly its possible, but you know there are different ways. One thing I've done, I have suggested to you. Another thing I've done, I have handled the "do now" problem, like one problem in the class like that. And just, everyone participates - they're all going to do it themselves, but then we have a discussion like this, because its a hard thing, its an introductory thing, its not something...nobody's going to get the answer..they're all going to have ideas. So then, umm, I'll board this. I'll give you an example of one thing I did. I had a, that was actually a college - prep level, a medium level, a senior class in precalculus and if you have taught seniors in high school in spring they are "out of there", they are not interested... I was teaching them about umm...compound interest, which is an annoying..algebraic formula. I put this formula on the board, you know What you accumulate (A) is what you start with, (P) times 1 + interest rate over the number you compound, you know, (1+r/n) "to" the number of times you compound (nt where n is a yearly rate and t is the number of years) (The formula is A=P(1+r/n)^nt).	

437	JL	- So they're calculating, and they're calculating. So I showed them that, you know, if you do it a little more frequently, you get a little more money. (The tone of voice indicates how uninterested the students were.) So then I asked them, "Could you become millionaires doing this, you know, with your $5. Could you compound so many times that you become millionaires?" For whatever reason, that idea grabbed them. "Yes, we can become millionaires!" (laughter) So then I asked, "I said, how many times should we compound?" you know, One kid says, "EVERY DAY", you know, so we do 365...murmuring and you can do it more often than that, then one kid says "how about EVERY HOUR?" you know, they were figuring out how many hours in a year and whatever; they did this, the whole class, they're finally doing it, you know, a million times a year. And they're not making more money. So they're starting to talk...why the hell isn't this working? And they're really talking about it…..why is it that I'm making more money, but it's just getting to be less and less and less more. It’s terrible! They care about money when they're seniors and they got into this. And finally the quietest kid in the room says, "I know why", and they all say what, what, what? - and he's got a big pointer, not a little marker, and he says THAT (the denominator under the interest rate) is competing with THAT (the exponent). And the kids all went Ahhhhhh.	EC
438	JL	- For the first time, they kind of got the idea how "this" makes it tiny (the denominator), you know, the limit as "that" gets big goes to zero? - and how "this" makes it big. The tiny force is competing with the growing force.	

439	JL	- It was so cool, because I didn't really plan that. I just committed to this idea, you know, it was a question, let them run. And so that was it - the whole class - not - they didn't need this constant thing from me. I did, I had to run around the room and say, "you're doing it right", you know and I had to help the figure out how many seconds in a year, and things like that. But you can do it in the large, it doesn't have to be,... it doesn't have to be that dialog. You can't, I suppose, get into one kid's head in this way, in the big class, very often. But even then, sometimes you can. If you can convince your students that it's not only OK to put up a wrong answer, it's brave and important to put up your thinking. If you keep trying and you can't figure out what was wrong but you know - show us what you're thinking and the whole class is invested in figuring out what happened there. And also, then the whole class is making the same mistake, "what are you doing wrong", but its good because they're all getting into the mind of one person and working out that problem that becomes more like that research problem - research group in Bell Laboratories, [better] than, you know, sitting.... “so yeah, it doesn’t work”.	
440	2	- mmmm	F
441	JL	- So yeah, so it - it doesn't always work exactly like this, but there's an essence of this.	
442	2	- I understand. Another way that maybe it could work or be a little similar to it, is if you have them all working on the one problem in the groups and stations. But then you have some time at the end where they could present what they thought and how they came up with that answer. Because as they're doing, that each of them gets to go; they can start talking about you know: "Oh, I wish I would have thought about it that way" or "I did it this way" or "this is what I did" …we need to have an open dialog.	P
443	JL	- Oh yeah, yeah..I think, absolutely. I think you go around the room and figure out who told them, the one that will be the right generator of discussion.	P
444	2	– right, mm_hmm.	F
445	JL	- Also remember, the ones (videos) where the kids didn't say anything are not in the VMC (analytics); Sometimes a kid just is quiet, and that's okay. Sometimes it’s not inspiring, amazing thing. But a lot of times it is, more often than not. So it's not going to be the easiest or the most straightforward thing. But when it works, it's so amazing.	

446	3	- Part of it might, you know, be we're always so "40 minutes, 40 minutes that we have", and we all complain about that...too long or whatever. You need to not worry who's coming in the door watching, who's coming and going a lot7.	MY
447	2	: ooh, oo". yeah yeah, you can't	FY
448	JL	- Right	
449	3	- And again, it's still "forties" in one day. Just try to make it fun and be like play….or something, "think about this tonight".....tomorrow do it, or a 3rd day.	P
450	JL	- That's us being brave...that's so important.	
451	3	- That's right….That's a risk you know, but….	F
452	JL	- I, actually, have never had any push back on that. You know, no one's ever said to me, "Your making them think to much" or "your spending too much time on that problem". Once I had somebody say that to me, just in theory, when I said, "Sometimes I spend 3 days on a problem", and they said "Well that's too long". Somebody would see it. But I've never seen anybody who sees it, say that that doesn't work.	
453	4	and others - all talking at the same time- [not clear]	F
454	3	- ...What does that look like in your plans.	F
455	4	- When they have your lesson plans...	F
456	JL	- Well nobody looks at our plans...our plans..	
457	4	- Yeah, when they come to look at it ….	F
458	2	- They leave notes and stuff like that.10	F
459	cmt	Lots of murmuring11	F
460	JL	- Who looks at your plans?	
461	4	- Oh they look at your plans. You have to have them on the table and when they come in...	MY
462	JL	- Oh you mean when you are observed, but nobody looks every week at every plan.	
463	Group	[not clear]... - Oh yeah. Oh yeah. yeah. (They all disagree with JL)	F
464	3	- Every week. For non-tenured teachers, I am...You have to hand it in. Every Friday you have to hand in your lessons and they sign off on it. Sometimes it's probably just "okay, thank you" and sometimes they are "what's this, you did something just like this before, why are you ya da da?. Or "Did you do this last week?" "Yeah well, I did, but I didn't get to it". "Well you need to tell me that you didn't get to it."	P

465	4	- I have been teaching for 15 years; whether you have been teaching for 50 years, it doesn't matter.....you have to submit your plans by Saturday evening by 5 pm. Your plans have to be in.	MY
466	JL	- Saturday.	
467	4	- Yes	F
468	JL	- Wow, by 5 pm.	
469	JL	- you have the whole week.	
470	4	- Yes, for one week, planning. And they look at it. And sometimes they..	MY
471	JL	- teachers on the... [not clear]...	
472	4	- And they push it. Sometimes, it depends on who your supervisor is, some people, they have been teaching for so long, they have their plans pushed back almost every week.	MY
473	?	laughter	F
474	JL	- They have to…. [not clear]	
475	4	- Yeah, they have to resubmit it. They review it and they have to resubmit.	MY
476	JL	- Wow	
477	4	- And if you don't resubmit by Tuesday, whatever I know, they may write you up. [exclamations] So it was that bad last year. We had a [not clear]12 supervisor, we were lucky, math. We're lucky that we now have a math supervisor, so she is more sympathetic. But the elective teachers....the elective teachers, oh my goodness.	MY
478	JL	- Gym teachers?	
479	?	lots of murmuring [not clear]	F
480	2	- Yeah	F
481	4	- They had their plans pushed back.	F
482	1	- It wasn't done in a certain way. laughter	F
483	4	- Oh my, there were so many things they had to put in it. Okay, the modification, how you're doing it....the one that gets me so upset, is that, like your closing, you're exit, they want to see the specific questions you're going to ask.	P
484	JL	- But you might, I mean, you can write down possibilities, but then when you have the class, you're going to ask questions that make sense..	
485	4	- Exactly!! It depends upon how the class is going; that informs the kind of questions or the specific question you are going to ask.	P

486	JL	- There was a time that I had to do that and I would just write down I will ask one of these (a set) depending on which way they go..I would never say I'm definitely asking one question...but I never had somebody who expected me to..	
487	4	- It makes teaching very very very hard on you.	MY
488	JL	- I mean....Are they going to look at the question you wrote and right there in the room, compare it to the question you asked that you didn't write down...?	
489	4	- They come with their laptop, so they have your lesson plan right in front of them.	MY
490	JL	- THey use teachscape right?	
491	4	- Teachscape? Use Genesis? They have it in front of them so they look at your lesson and see what you're doing.	MY
492	5	- I know like, for us, we're not as bad, I'm sorry.	F
493	cmt	laughter, murmuring	F
494	JL	- This is a building thing? [not clear].....They're all doing differently	
495	5	- Sometimes, like I know for myself, it reads like, I'm getting, "oh great job here", or um "make sure you have answered this here" or "do this" or "do this" are the comments.	MY
496	JL	- Wow. Are all the rest of you tenured?	
496.a	4	- I am not	F
496.b	JL	- (to t2) You're tenured?	
496.c	2	- I am	F
496.d	JL	- to t4, you're tenured?	
497	4	- This is my tenure year.	F
498	2	and others - Oh, congratulations. (warm and excited)	F
499	JL	- So - only t2 is tenured. So that's part of the thing that’s more stressful. They are under so much pressure now. You know, I suspect that if you get the kids to do one quarter of this, that's going to make a big impression on anybody that sees...	
500	2	- (in the background) Oh he'll love it.14	F
501	4	- When they come in and they see something like this, if you are able to ...manage it well. I know how if my principal comes in and sees something like this and I am able to manage it well, I know he'll be very impressed.	P
502	2	- Yeah.	F
503	JL	- Yeah, so..this is ,	

504	4	- Managment of it, managing it so that.... Because they look at the other kids too. They have to see that everybody's in a group, because they pick out ...oh...?...the person at the back. They'll pick out one or 2 kids. When you have like 20 kids...20 kids, its very difficult to have. [more than one teacher talking].	FP
505	JL	- If they see, you know, 12 out of 20 really engaged and the other 8 are, you know, all somewhat engaged and some are watching, they're going to be engaged too. You know how this is doable...in a class has .and the school has to go along with you. I know that um certainly the people I spoke to at the school made it possible for me to do this, they are all in the right place in their thinking......certainly in my district there's different thinking. I had a supervisor, she, I don't know, was an ex-military officer, I think.	
506	4	- Oh my God.	F
507	JL	- She wrote on the back on my observation before I was tenured, that "behavior is a pre-requisite for advanced learning". So, in other words, if they don't behave in a certain way, then you can't give them good problems. So you know, I was..I gave a research report rebuttal...I said, "I'm sorry but I think you've stated this incorrectly." and, I said, "Behavior follows engagement." If students are engaged in learning, their behavior works. If they're not engaged in learning, then they try to distract it, trying to stop it. But if you get them interested, so what if you're going to be part of the problem, they're thinking, they're working. That other stuff is so wrong.....but maybe it wasn't the best thing for me that I did that, but I pushed back. And I think the bottom line is, why are we teaching?	
508	2	- Right. I think it was...our lesson plans. I know in our building, we do have to submit them. But, it will say, I can't cover [not clear].... able to cover everything. Cause they have their own challenges, themselves.	MY
509	JL	- Well sure.	
510	2	- We can write on the lesson plan - we just have to write and say a reason about what happened to them. So when they come in, if they're looking at the lesson plan and it doesn't look like what .....on Tuesday, doing this...I have this written on the lesson plan the day before: I may have to go over, ..you know, then they can, that kinda, they are	P
511	JL	- That seems the best thing to do.	
512	4	- How much do you have to start writing?	PQ
513	?	Lots of murmuring, laughing	F

514	2	- Crazy! Crazy sometimes....laughter	F
515	5	- When I write a reflection, it's like one or two sentences.	PQ
516	JL	– You know what some people teach about documentation is to write a lot. laughter. Cause they're not going to read it. ....not make changes - if they do, they read the whole thing...them all....laughter...	
517	?	lots of murmuring	F
518	1	- So...easier.	F
519	4	- They look at them. They don't say reduce	F
520	2	- I remember I worked at ...easy breeze. Yeah, like the data binder thing is not happening over there. They don't have to worry about that. It's just the high school.	Wb
521	JL	- So how come some schools have to do a partic -	
522	2	- It's just the high school	F
523	2	The others are agreeing	F
524	4	– I am going to the high school.	F
525	JL	- I though we were inundated. You guys are really inundated. Here's just another corporate trick ... that might work, you know, to reduce the pressure. So, you know they give you feedback ...they send you email. So whenever you get an email, you send...."thank you so much for the feedback. Do you have some time for me to talk about this with me, because I really want to understand it and I want to know how you're thinking." They don't have time to do that.	
526	2	- Right, right..laughing	F
527	JL	- They're going to send you fewer comments if you want to meet and discuss/learn it all the time. They're not going to be said that you said that, they're just going to think, "Oh God, I'm not going to ..."	
528	3	- I like that...	F
529	Group	Lots of laughter	F
530	4	- Any time they send me anything, I'm going to try it.	F
531	JL	- That works in the corporate world....you know the guy...."Oh I really want to understand it"....	
532	Group	More laughter	F
533	JL	- Those are my evil moments of the day.	F
534	JL	- So umm, so let's work on a plan now. You started your plan, [not clear]...Let's work on a plan. How about we pick one of these, which one? Somebody volunteer, and we will work together to help you create your plan.	
535	4	- OK, Somebody....the plan or "One of the guys" .	P

535.a	JL	- Who's going to volunteer to be the first, you know? Who’s the brave soul?	
535.b	4	- ...good	F
536	2	– You are right.	F
537	JL	- Obviously we have to do a lot of it asynchronously, after you work that out, you post them, ...comments, help..but...Let's try to get something done....so that first one.	
538	4	- That first..	F
539	JL	- You make progress in planning your lesson.....Whatever format you want.	
540	2	- You just take one ....and add to it...	F
541	JL	- I didn't hear the discussion..	
542	4	- Is this a new plan or the one with Guess My Rule?	MQ
543	1	- Guess my rule	M
544	JL	- That's up to you. It could be a "guess my rule" lesson or you could be thinking now you want to start in a different place and it could be something similar, but not exactly "guess my rule". I mean, this is algebra and "guess my rule" is fundamental too, but you can do it in different ways or for different types of problems. If you have more advanced students, you might do it with quadratics. If you have less advanced students you might do something differently - you might umm use a graph instead of data or... There are different ways of....different problems. But since you have been watching a lot of "guess my rule". And there are easier rules and harder rules. It seems like you could adapt it for every group.	
545	JL	- So is anyone going to volunteer to be the teacher whose lesson we're planning? pause. I was thinking t2 would be interesting because....t2 is laughing....you have these special kids and its a big challenge there and if we could do it..You have so many concerns about these children and if we can help you with that, I think we'll have accomplished something.	
546	2	- Doesn't hurt. Oh - kay	F
547	JL	- If we fail we have to collectively bear responsibility.	
548	JL	- So wanna do a "Guess My Rule" or ...	
549	2	- Yeah, let's stick with the "Guess My Rule", a good place to start.	M
550	JL	- OK, so when you were first thinking about it, where were you thinking you'd actually begin?	

551	2	- Ummm, I think I was beginning with using a....for the Do Now, like a pattern activity I have here, a pattern activity with shapes? So I have an Evaluate, then I have a Triangle + a number/5.	M
552	JL	- So here, Why don't you ....be the teacher? I'll move away [01:58:14]	
553	4	- Oh, Thank God.	F
554	?	Lots of simultaneous comments and laughter.	F
555	JL	- ....I'll see if I have enough batteries...	
556	?	lots of unclear laughing chatter.	F
557	4	- I don't think very well on Wednesdays.	F
558	JL	- Really? You seem very sharp today.	
559	?	laughter	F
560	4	- It's a big group...yesterday? was more than I do it...more often.	F
561	?	laughter	F
562	JL	- Well you know, It's also the heat of summer...	
563	3	– That is the rule.	F
564	JL	- I threw out a check we got in the mail. I recycled it. That's how addled I am. I knew I saw it. My husband's looking for it and he found it in the recycling box.	
565	4	- Well, thank God he found it.	Wb
566	JL	- Well I put all the envelopes and the junk mail in there.	
567	2	...Should I put like the objectives I have and everything?	PQ
568	JL or 4	- do something.	F
569	3	- sure.	F
570	4	- Do something for the next hour...	F
571	3	- We'll help you.	F
572	4	- Nice writing	F
573	JL	- Yes... [not clear]...	
574	JL	- ....send the kids up to write on the board....laughter....horizontal writing....hard to read letters.	
575	JL	- OK, so this is not what you actually write on the board for the students. This is what you write...	
576	5	- We have to write it on the board.	F
577	2	- Yeah, we have to write it..	F
578	3	- Somewhere in the room for me.	F
579	JL	- Really? Students read, "Students use teacher-created materials...",	
580	4	- We have to write...	F
581	JL	- ...alot...	
582	4	- "Our students will be able to..."	P

583	JL	- "Our students will be able to...", we've had that too.	
584	2	- No, It actually should be... "I will be able to..", I'm sorry..That's the..	P
586	4	- No, we use S.	MY
587	5	- We first used S, and the we used the Learn, and then when went to I.	MY
588	4	- They better make up their minds.	F
589	JL	- Wow	F
590	4	- Yes, Its a 3-part objective.	P
591	JL	- You have to actually tell them, before they learn, you have to tell them that its a linear function	
592	2	- And how they're going to..	F
593	JL	- And what percent you expect to get from them on a quiz?	
594	?	They are all talking	F
595	JL	- How do they know if they get...	
596	1	- The teacher plays some, whatever their exit ticket. How do you assess whether...?	P
597	JL	- They use your assessment, they don't have their own.	
598	1	- Their expectation is that once we do it, we spend like a minute or so talking about the objective. A kid reads it out.. and then umm	P
599	JL	You all do it? Its a uniform thing.	
600	4	- mm_hmm.	F
601	JL	- Everybody reads this? But do they...?	
602	1	- They ...what my kids, I tell them 0% like I tell them, what does 80% - I normally do 80% - What does 80% accuracy mean in terms of your ???? And they describe it.	E
603	JL	- And what do they say?	
604	1	- They say like, they kinda get it, it's not like the best, like I could get A+ on everything.	C
605	JL	- Oh, they give a qualitative - they don't say " I get 8 out of 10 answers". They say, "I'm okay with that - I don't get them all right, but I get most of them right".	
606	3	- This is what's strong - The administration stresses the minutae, like this. We all do it.	MY
607	JL	- This is easy to observe, but this doesn't teach them anything.	
608	3	- No.	F
609	1	- The only problem is if you don't have it.	MY
610	3	- Make sure you have it.	MY
611	JL	- You have to do it. You just have to do it.	

612	4	- Not only do you have to do it. When they come into your class, they can pick on any student and ask, "What are you learning today, what is the objective?"	P
613	JL	- And he reads that. The student stays, "Using teacher-created materials, I will be able to discuss linear functions by using "guess my rule"".	
614	cmt	laughter	F
615	4	- If they reading it, it's going to count against you. If you be able to have eye to eye contact with ?? and they be able to tell you...	P
616	JL	- They don't put it in their own words.	
617	4	- So you have to put it in a language, a student-friendly language. And you make your life - one thing I learned from my ???? - we have so many things in the objective - you want one thing - what is the one thing you want students to learn today. You're going to be able to like read a graph, something like that, or be able to write a linear equation. Something very simple. So you just ...one thing. Don't make it...write this - friendly language - so when they ask them, they will be able to express what they're doing. Because when it becomes so many things that you want to achieve in one day - then the kids won't be able to tell them what they are really doing - and they count that against you that when we asked, did you really know what the purpose of what they're doing is? They have to know what is the purpose of what I'm doing. Why am I doing this? Why am I learning this?	P
618	JL	- I wonder what they would say if the child says, "because it's fun"!	
619	?	laughter	F
620	3	- I don't see the word out there. Don't say its fun out there!	F
621	JL	- "I used to - I had fun exploring the powers of my mind! And numbers are a great way to do it!" What if they said that?	
622	4	- I wish they would say that.[02:04:19]	F
623	3	- One of my teachers, teaches ..not home-base, and so she has to teach 3 different classes. So these are printed out on paper then she has to take to each class and write it out. [exclamations!]	P
624	JL	- Carry it around...	
625	3	- Spend more time - non-learning, you know...that kind of teaching..	MY
626	JL	- Yeah, right. So where did you teach before this?	

627	3	- I taught in Plainfield, I taught in NY for a few years and before that I was in business.	Wb
628	JL	- Oh ok - you are like me.	
629	3	- I have been teaching 6, 7 years.	Wb
630	JL	- Oh Okay, You did that..	
631	4	- Where in NY?	Wb
632	3	- For 2 years. I was sent to Barringer HS, they closed the HS. I was one of those; they closed the HS and reinvented itself. But everyone got fired and .....and reapplied for your job. I said okay, that....were given...30 years and reapplied for your job. There was an unemployment service, whole thing I went through was obnoxious and luckily there was an opening here. And the heck with it, I don't need that, I'm not worried about tenure. I'm not, you know, 22 years old or whatever. If I get it, I get it, If I don't, I don't. So to make a long story short, I got a job. I got hired.	Wb
633	JL	- laughing, It's hard, tenure is 4 years right.	
634	1 and 3	- 4 years. fifth, 1st day of the fifth year.	Wb
635	4	- It's 4 years.	Wb
636	JL	- That first day of work - you get tenure. But when I did it, It was only 3.	
637	4	- I started, we ..??...the 4 years. For the first set of ......get me tenure this September. ...mine is not in ...I didn't start on the first day of September, not until September 17th, so I have to wait	Wb
638	JL	- September 17th.	
639	1	- A lot can happen, right?	F
640	4	- A lot...calling out.	F
641	?	laughter	F
642	3	When I was new it was 3 years, that's how recent it was, then I came here and ....restart the clock and I wasn't grandfathered, it was start the clock and now it's 4 years and this is me, I'm beginning my 4th year now.	Wb
643	JL	- Wow	
644	4	- For me, now, I put in 10 years and I was tenured in Newark. I left it to start again here.	Wb
645	1	- Oh wow.	F
646	JL	- Newark has an issue keeping teachers. A teacher in Newark, told me that on any given day in Newark, 100 math classes, its a big district, 100 math classes have subs. Not because people are out, but because they don't have enough math teachers.	

647	4	- Last year, a teacher....came here from East Orange. She was there for 7 years.	Wb
648	1	- why?	F
649	4	- Moodi??. Came to start all over again.	F
650	1	- Why?	F
651	4	- Why? Because they were having problems. So your...For your peace of mind you leave everything...place where you think. You want to start new. Cause my....said, in the morning she would sit in her car and she would be weeping - before she goes in. That was that bad.	Wb
652	1	- I so thankful I am ...in the high school.	Wb
653	JL	- Wow.	
654	4	- Before she goes in...It was not the kids, it was the administration.	Wb
655	JL	- You know I think we should talk about...	
656	?	A cacophany of voices...	F
657	JL	- Hey, hey guys, lets talk about the part thats - not the part that you're stuck with - but the part that's hard and interesting and that we're hoping we can get the kids to do. So also, that's gonna make such a little hole iin the...??...put all the instructions... so you can insert them, but let's talk about how you envision this. So you're gonna put that Do Now up...what are they gonna do? Are they gonna talk to you or are they gonna write it?	
658	2	- They are probably gonna write it. ... Talk to me......and then...like ..[undiscernable]. I can't remember exact wording of the website. I'm going to build something that is like technology based. I think there's like two, I can't remember exact wording of the website - it's either.....Math is fun or ...Math is cool or ?	Wb
659	4	- Math is Fun.	F
660	2	- Yeah, There's like games. I'm sure ..can find a game that is similar to this..that same thing as using.. gameboard	FY
661	JL	- Using this one.. right, what's the idea in starting...I like the idea of Triangle + 5 = 8, it kind of gets you to..	
662	2	- It's easy, yeah, yeah.	FY
663	JL	- Are they going to write the "3" in the Triangle? Or Triangle = 3? how are they going to do it?	
664	2	- They are 8th graders. I wouldn't....do nothing to ..I would think they would start knowing the inverse operations, something like that..	M
665	JL	- So you think they would ....write "-5" on...	

666	2	- I would like to see them come - maybe show me how they got their answer, so I would know. I think they would do it that way. Write Triangle = 3. At the same time, but have them do it on the Promothean Board.	M
667	JL	- Have they done this sort of thing before, where there is a single missing number? Not two variables, but just one.	
668	2	- Yeah, I think why I'm thinking like this, is because I know the 7th graders, I kind of got to see them, I know the 7th graders are very - they're lacking a lot of skills, so I know that this would be a nice way to introduce it. Without it being so overwhelming like a variable, X. Sometimes when kids see that, they're maybe like....I know they are lacking a lot of skills - so doing this - so making it some kind of fun so its not intimidating, I wish I could think of some way to not frustrate my kids, challege them, but not frustrate them. So doing like a game, or something like that will make them "okay" get interested.	M
669	JL	- That almost sounds like mosts of the lesson though.	
670	4	- How long will that take?	F
671	2	- No, no, I don't think that's going to take long. That's not long...	FN
672	JL	- That's fast?	
673	2	- That's fast.	F
674	4	- About how long will this take.	F
675	2	- I think this, both of these, will take 5-7 minutes. I don't think...	F
676	JL	- Really	
677	2	- Cause this is simple. This is not multiplying....this is ...yeah..what he just said.	M
678	JL	- Then you're going to collect like what they did or its in a notebook that you see?	
679	2	- Yeah, you see like..	F
680	JL	- Something like that?	
681	2	- Yeah,....Yeah, that's a little bit more complex.	F
682	JL	- Are you going to be like, "We're going work with, kind of a game, we're gonna call it a rule..and	
683	2	- mm_hmm, mm_hmm, yeah that's what , I don't know how to like..	M
684	JL	- I'm just..that was what Powell suggested.	
685	2	- If we're going to do that, then maybe I"m...	M
686	JL	- So, That's why I am wondering if you.. do you want them to call them linear functions at first or do you want to call them secret rules?	

687	2	- I think they should know what they're...I think they should. Well I don't about using that because they want..	M
688	1	- They want the objectives to be standard.	M
689	2	- Yeah, standard ...??...	MQ
690	JL	- Oh well, then you have to say that. But here you can say...what you think of it..what they need. They're gonna..	
691	2	- A little bit more difficut...and then I'll say, "not today - we're going to do an activity: Guess My Rule". And then the.... I think, for me,...	M
692	JL	- What's the first rule you think they should get?	
693	2	- I think it should be something using 2 operations, like he did, I think at first. it was just	M
694	JL	- Oh Yeah, that was hard that first one. And particularly your kids, you want them to be successful first, so, what would be like?	
695	2	- Although, I don't know, as I was thinking about it, sometimes if you give them something difficult, I think it depends on how it's going to then try to be like, Okay!.	M
696	JL	- You know your kids.	
697	2	- Yeah!	F
698	JL	- What do you think would be a good, these kids are, you know, special need kids, so some of them are gonna have different problems than some of them. Some have learning issues, some have perceptual issues, some have motor issues.	
699	2	- Right.	F
700	JL	- So they're all different. Some are going to be fast and some are going to be slow.	
701	2	- So.. giving them....	F
702	JL	- What would be a good rule? You have to start with something.	
703	1	- I was going to say, because she says she's going to have them in stations. So when do your groups, you put the students that you know that like the challenge, have them do the challenging rule first. And that way the students that get frustrated with the 2 operations, then give them a like simple rule like 1 -15 and 2-25 and have them work with that first and then build up..	M
704	2	- This year, we had someone come in and teach us how to better utilize stations. One thing that I am going to stick with that ....she said, when you are giving the stations out, to prevent the students, in my classroom cause they're a	P

		little needy sometimes, asking "is this right?", "is this right?", so one thing	
705	JL	- They are all needy like that.	
706	2	- So one thing I did last year, that I like the idea, is to let them know at the beginning, "listen, you are not going to talk to me about what the rule is - you can't come to me; you and your co-workers", I call them co-workers, "are going to discuss it". "But you are all going to get a chance to come to me, or I am going to each of your stations, and I will talk to you, but for the first something, what was it, 5 minutes..."	M
707	JL	- You're telling them that it's their job. That's good, okay.	
708	2	- And then, so they don't get freaked out. "I am going to come to you and provide you some feedback and, but not at first."	P
709	JL	- So, so so, I do a multi-level calc class, and while it's not exactly the same thing, it's very similar. And I can't put them in stations and hide this from them. So I make up problems where the first 2 are for my algebraically challenged kids. Everybody does all of them. But my algebraically challenged kids, the first 2 are for them.	
710	2	- OK	F
711	JL	- The other kids do those too fast. And the next 2 are the for the kids who can do the bigger challenge. These kids who take longer to get the other ones and they probably won't get to both. The other kids get to those 2 now. So this way, you have one sheet of problems, but there's a differentiation on whose really going to spend time on what. And the same thing, they're have to work together. So lets come up with 3 rules.	
712	4	- I have something to say. Before they go to their stations, ...you want them to do at the stations. How about you have ...something similar to what they're going to be doing, like a simple rule and the whole class do it together.	M
713	1	- Yeah.	F
714	4	- Before they.. break up. We do parts of it.	M
715	JL	- So the first would not be at the stations, will be a whole class experience.	
716	4	- Yes.	F

717	JL	- And maybe the next day they will get into stations and do it themselves. You have to see how long..You have 80 minutes or you have 40 minutes?	
718	4	- I have 80 minutes, so I could do it the same day.	P
719	JL	- You have 80 minutes, you could definitely do it the same day. You have 40, so you may not be. But you think of this a plan that encompasses 2 days. So what would the rule be for her (t2)'s children - the first one that they do as a class?	
720	4	- I think she should a simple one where you have, something like y = x + 2 or something like that.	M
721	JL	- Or Triangle = Box + 2?	
722	4	- Yeah.	F
723	JL	- Well, but she's going the other way, this is getting them how a rule works, what a rule is? And the other is. I give you the numbers, you tell me the rule.	
724	4	- Yeah	F
725	1	- yeah	F
726	JL	- Maybe she wants to do +1? with the class. You know, you would put, you know, 3 and 4 and 5 and 6 and 7 and 8 and ask them what the rule is?	
727	2	- Okay. I'll give it a try.	M
728	JL	- Maybe you would do 2 of them. Like a really straightforward one they get pretty fast and then maybe you would do a 10x+5 one which they'll also see, but might have more trouble saying. So you might spend some time on how to express that with operations. Then you could put in your goal to express a pattern as a function - when then gets a little bit more into what you're doing that day. Start to get into the obligation??..I always put an objective up for my students - not governed by the district. I give them what I want them to think about today..	
729	4	- mm_hmm chuckles	F
730	?	A small pause. They are writing. We hear the construction of the Rutgers Honors School outside of the GSE windows.	Wb
731	JL	- DId you see what they are doing up there? Did you know that those huge cherry pickers are driven by the guy at the top? There's nobody in the cab! He's using a computerized thing that moves those wheels and gets it exactly where it needs to be.	
732	?	Wow	F
733	JL	- I was watching it yesterday. I could not..	
734	1	- He's caulking the windows..	F

735	JL	- Yeah but he's 40 feet up in the air. But if he makes a mistake on that thing? Bangs himself on that building and flies out! When my son was little, all he wanted to see was construction. we were bored. I would stay at construction sites so he would watch.	
736	JL	- I want to put on the board - what we said, what did we actually say? We said, as a group.. [02:18:39]	
737	1 and 5	talking at the same time ;	F
738	1	- Cynthia Gonzalez, I remember last year. Cynthia Gonzalez. I did Summer school at the Barack Obama charter school and she got kicked out of the high school. And all her sisters. Her, Brianna, all her sisters. All her sisters were in the Charter School.	Wb
739	4	joins in - "Ohhhhh!"	F
740	2	- ....7th graders, you know what I'm saying? so like we could .....Remember you said she was re-vamping?	wb
741	JL	There are 2 conversations happening and I can't understand either one.	
742	JL	- What happened? A very important something? I just want to make sure this is what we said: This might be a whole lesson. There's work here in figuring out the difference of using a rule to find the triangle value, versus using the points, the values, to find the rule.	
743	2	- Yes	F
744	4	- mmm	F
745	1	- You're going to need to know this (using the rule - chart) to check your work on that.	M
746	JL	- Yeah. Right, right. This tells you the whole idea, what you are doing. And this is what you want them to use.	
747	4	- So if I want to do the "we do"...umm	M
748	5 and 2	- "We do" would be the stations..	M
749	5	Cause that's like the guided practice.	M
750	4	- That's the "you do"? The "we do" is the one..	M
751	JL	- Who is the "we "?	
752	1	- "we" is the whole group.	F
753	JL	- The children, or including the teacher?	
754	1	- The teachers, and the whole group.	F
755	JL	- Everybody	
756	4	- The "you" is them.	F
757	1	- That's them.	F
758	4	- The "we do " one that I just talked about. You do it together.	F

759	JL	- "We" dialog about rules, but..we're not doing the same thing, the kids are trying to guess figure the rule out and you're trying to figure out what questions they need that will help them.	
760	4	- What I'm saying is this, what I'm trying to say is this in this lesson we are planning: It isn't going to be the station activity, then what you model in class has to be similar to that. So if we're going to be guessing the rule, then we should do 1 or 2 together, where they guess the rule.	P
761	1	- Yeah, An easy and a medium.	F
762	4	- Yes, Not the ones that you do...	F
763	1	- Not the same exact ones..	F
764	3	- But that's the whole thing..	F
765	JL	- But you can..wouldn't you do that with the whole class first so they understand what they're going to do their stations? So they're going to guess......just like they do in their little group, they're going to do in the big group.	
766	cmt	All were saying "yes" "that's what we do" while JL was talking.	F
767	JL	- It's just not everybody gonna participate in the big group. Not every one of the 20 kids is going to have a chance to raise their hand and talk, right?	
768	4	- What that's the - we do ...	F
769	1	- You should model what...some kids won't understand, what'll we do? They're not going to read the directions if you give them directions. You'll have to model.	P(M torn in between)
770	JL	- You have to model it, sure, I think that's true.	
771	1	- And the modelling thing, that's not that long..	M

772	JL	- It actually could be fun though, and a lot of kids could participate in it, so you want to make allowance for.....because you know, particularly early in the year, you don't know what they know yet, and you don't know what they don't know or what they're worried about and if you get them something like this, those questions can come out. So you want to,.. I don't know if you're flexible, if you're allowed to be that flexible, but if you are allowed to be flexible, you want to, there's an opportunity here for them to say, you know "I never realized that these things could go back and forth", or "I had a problem, could I show you the problem I had and I didn't understand it?" And then you might want to let them ask you an important question. I don't know, but this could to a lot of interesting discovery and you might want to allow time for that. Or, for example, remember the video that we saw where the kids were saying, "the rule is you take the same number and you put 5 next to it"? And then what did Powell say?	
773	1	- The math operations.	F
774	JL	- He said that you gotta explain your rule as a math operation. Now those kids knew just what he meant when he said that. They went and did "times 10" and what not. You're students may not know..you may need to talk about that a lot. There's a syntactic way where you write a number next to a number but in math, we do operations, what are math operations? Multiplication and division and subtraction and addition and you gotta use those things. You know, and that might be more of a discussion, depending on how they do it. I mean, who knows, we all know there are so many things you could talk to them about.	
775	4	- Yeah even this "guess my rule" it doesn't have to be.well...maybe don't guess my rule, we could use this for almost any lesson. It might be a different title, not guess - it might not be a rule, you know?	M
776	JL	- This particular thing is an equation, but it might be something else.	
777	4	- It could be .....for something else.	M

778	JL	- I have these little...a little chant...."roots give you factors and factors give you roots". I give them a little singsong thing. And then I have ,.. I show them on the .....I don't know if you every used the ....I'm forgetting the name of my favorite...the DESMOS Grapher...ever use the Desmos Grapher?....I really like it...so I show them, these are older kids, I show them polynomial graphs - a lot of intersections. And I ask them to guess an equation. And because we have done a lot of graphing equations, they know, they look and see where the x-intercepts are, and they try to guess the sign from the shape and....so it's similar thing. But it's a much more complicated thing.	
779	3	- Solving equations, the students don't come to me, has a huge crew of knowing how to solve equations. A whole teaching thing - one step, two step	M
780	4	- Yeah	F
781	2	I'm not ..from day 1. If I did this on September 25th, that's a litte leap in the sense, I would have to.	M
782	JL	- Well, you want - First of all, you want to do something that is right for your students and you want to do it when it's right for them.	
783	3	- I would say, this is more like	F
784	JL	- I wasn't saying, everyone is doing this on the first day of school.	
785	3	- months and months and in. This is like months in.	F
786	JL	- Yeah, maybe so.	
787	4	- This is .....doing next year. This is like our unit 3.	M
788	JL	- The way I am figuring. I don't know if they would let me come and watch you - I would love to do that. Certainly we can talk together about planning and you are gonna be like....classes. Figure out when its the right time to do it. Because I don't need it to be done in September or November...It would be good if it you could do it before December.	
789	3	- I got the impression I was doing it when....	F

790	JL	- Yeah no...because umm....by the way, I asked about the paperwork - the money. The question I got was, how are we going to work out the sort of things we have to do in the Fall. So my thought was we should just probably put the paperwork in for the whole thing, because it's probably going to take them a month or so to get you the money anyway. But..you're on your honor to finish the thing with me but.. That's what I thought we should do, right? Okay, so, they're going to take time, I don't want to delay it any longer than is necessary. Then, in order for me to be honest with them, we should finish it before January.	
791	2	and others - Yeah.	F
792	JL	- Right. And then maybe we can..??.. Does anybody think they can do it earlier, like October?	
793	5	- Maybe..with....?? (hemming and hawwing)	F
794	JL	- So it would be good to have everybody in on the planning. And when you guys finish your lesson in October, we could all meet a week later and talk about how it went, what didn't work and what did work.	
795	4	- ...building...	F
796	?	Everyone is talking about when they are doing what while JL is saying the above commment.	F
797	5	- I know, like with Algebra 1, the first things that we are doing is writing expressions to represent..	M
798	2	- Oh, so that's good..	F
799	4	- Isn't what we're doing.....something like combustion?? Metric ...that's what we do. Unit 1...Combustion of units? Measurements.	M
800	JL	- Oh conversion of units.	
801	4	- That's we do for...That's our unit 1.	M
802	JL	- Inches into	
803	4	- all those, you know..	F
804	JL	- So you have to do that the first..	
805	4	- Because they're going to be tested. After each unit we get tested. So I bring something that is ...	M

806	JL	- You know, actually, units is an interesting thing to connect to it..you know, because if you think about it, when you do the X's and the Y's, its almost like X's are units. right? Instead of 3 X's, 3 yards or 3 feet, you could have 3 triangles - it might expand their vocabulary, in terms of understanding what the unit is that we're talking about - it also connects to rational numbers if you want to. Sometimes....we have 3 halves, or 3 quarters, or 3 quarters of an inch. The thing that we're talking about, that we're counting, changes, from a real thing, like an inch, to an abstract thing like a triangle, to a formal algebraic thing like an X. I'm just saying, one what to think about connecting it to this, is to anticipate it as you begin...there are connections there. Also because those units sound pretty dull. Laughter... Very important, but .....dry.	
807	?	Lots of laughter and yeah!	F
808	3	- They also might....given this ...around	F
809	JL	- You could also create a unit, something silly, the "candy spot", how many candy spots are in a giga-candy spot? If they create their own units, they'll really understand it.	
810	2	- [undiscernable]	F
811	?	Laughter	F
812	JL	- I'm getting very tired, just thinking.	
813	4	- ..thinking...	F
814	4	- In my first unit 1 - with the testing. Its like boom boom boom boom, test, boom boom boom boom, test.	M
815	5	- That's the other...	F
816	JL	- Who writes the test?	
817	5	- The state.	Wb
818	4	- The state.	Wb
819	JL	- The state writes the test?	
820	5	- The regional..	Wb
821	JL	- So you are teaching it to their test?	
822	2	- Her high school is like that..	MY
823	JL	- Just the high school..?	
824	4	- I don't have enough time to bring in something that is not.. I don't even have enough time to cover the material for the unit.	P
825	JL	- Wow.	

826	5	- And by the time we actually get done with differrent. I know for us, we have that online program - we have an assessment there. And then we have another online program which is like our - support intervention. We have another assessment for that too so. By the time we get done with that, and then we get start information.	MY
827	JL	- When do you get tenure?	
828	5	- Next year.	Wb
829	JL	- Okay, laughter, I was going to say, once September 17th passess, you are going to feel so much better.	
830	?	laughter	F
831	4	- Oh no, everybody's on the ..??..they are all on the.??..I don't really see..???....He knows I am talking about him.	MY
832	JL	- I had an experience last year which really gave me pause. So we also have testing. We have comprehensive tests) after every unit, we have common assessments for this, we have common assessments for that, we have SGOs do you do those?	
833	Group	a cacophany of yesses!	F
834	JL	- At the end of the year, you know, I had my most advanced kids, my brightest kids, they had their chapter test, their common assessment, their final SGO, and then the PARCC, and then....the final exam. So here we are, its the day that we have finish the data for the report cards, (if they're not in that day, they don't get it). And I'm looking at the data there are - we are a very big school, so we have about 500 kids in every grade, and about 100 of them are in this advanced group. We have 5 teachers, no 4 teachers of this advanced group. And I'm looking at the final grades (grades on the final exam); these are kids who get A's and B's - that's the worst, and they FAILED the final and they...horrible, horrible, horrible grades: A couple of A's, mostly C's, lots of D's and F's. And I am saying, is it possible that this bright group of children - they all suddenly got stupid? No. Is it possible that we all became bad teachers in the last quarter? NO. What happened? And I looked at the calendar and I realized: Tests! Tests, Tests, Tests. They're little kids, these are 8th graders - these are not even 9th grade. They come to the high school to do this thing. And they got tired. It was too much already.	

835	JL	So that last day, I had to run around. I wanted our final - Our final counts as 10% of their grade, so it would bring everybody's grade down - I went around, I called the principal, I called the acting supervisor - I said, you got to change the 10% to 5%, this is the reason yadayadayada, we got to DO IT. I was exhausted, I got everybody to agree and then I had to sit...our IT guy is not so good, he disappears, I don't know what he's doing- I had to sit there in his office, waiting for him. "We're changing the 10% to 5% for these 5 classes!". "Really?" "Yes, right now." And he did it because he did not......and he didn't want to explain. But the truth is, that whole adventure gave me alot of pause about test fatigue. Because these are the kids that care the most. Their parents are ready to kill them if they don't do well. So if they didn't do well to the ...test, what is it doing to every other kid?	
836	4	- Your school...	F
837	JL	- Want to say something?	
838	?	Murmuring - not discernable	F
839	2	- No we have some parents who..	F
840	cmt	More murmuring	F
841	JL	- Who has a very different class and needs to do a very different thing?	
842	1	- Use the same setup?	F
843	?	laughter	F
844	JL	- Everybody's going to do the same thing? Well, okay, if that's what you want think.	
845	1	- undiscernable	F
846	JL	- I think tomorrow you need to see something a little different thing and you might get an idea. Okay, so I don't think you need to do any more today, I think you did a lot.	
847	4	- Yeah, I think so too.	F
848	JL	- Have some tea, wrap it up, go home early.	
849	4	- What time is it?	F
850	JL	- You guys are a great group. I can't believe the pressures you are under	
851	?	They are discussing the train schedule	Wb
852	?	Then t3 tells us that he is the only Pre-Algebra teacher in his school.	MY
853	3	- I am the only Pre-Algebra teacher in the school. [everyone is talking]	MY
854	JL	- Everyone knows each other.	

855	3	- There are only 60 or 70 kids in a grade. I don't have like 100's of kids. But still, you're the only one, you can't, there's no "So what are you doing today?", "Let's me see what you did today, what do you think?", "How 'bout you do it today and I'll do it tomorrow?". You know	MY
856	JL	- They don't do that?	
857	3	- No ...there's none of that. - it's just me.	MY
858	JL	- Oh right...	
859	3	- Call up teachers - what you think about this	F
860	JL	- Right, now you can do it with us.	
861	3	- Will you? We talked about this, last year, they were trying to get us together.	Wb
862	JL	- That is really a nice thing.	
863	5	- ....?.... by the first year. I teach Algebra 1; now I teach 8th grade math. The first year I was teaching, I was the only Algebra 1 teacher. So I was doing like..	MY
864	JL	- Couldn't the other teachers help anyway?	
865	5	Yeah, I was picking their brains..but	F
866	3	- definitely	F
867	JL	- We have at least 3 teachers in any subject.	
868	5	- ..to figure out like where you're are, pacing wise	Wb
869	2	- to guage yourself, yeah	Wb
870	5	- I was not able to do that.	Wb
871	2	- But this year you were able to do it.	Wb
872	5	- Yeah, this year I was able to do it.	Wb
873	2	- This year, there's another 8th grade	Wb
874	JL	- It makes so much more sense - we only moved on when they learned something, and then you finish what you finish, and then they won't have to repeat it next year because they know it.	
875	1	- The Algebra 1 kids, because if they didn't master something, and the test would come along, and I would give them the unit test, and but then I'd go back to what I was last working with them, and then like, I would see that they were working too long on this, they're not going to get it, let's move on.	M
876	JL	- Well, right, sometimes you have to leave it and come back, sometimes they get frustrated, they get bored and frustrated, ..	
877	2	- Yeah,	F

878	JL	- Sometimes, you know what I do with kids who struggle, have them make up a game, but they have to make up the problems - I'm not making them. They use file folders. They put a loose-leaf paper or printer paper with the problem and another one with the answer. So it's kind of a matching...It's a game, there's the problem, there's the answer. And they have to figure it out because they want to make a hard problem. So they'll come to you and that's when you have a chance, one on one, to show them. They want to do something, they start it and you help them finish it. They come up with these problems, they get so excited, they jump out of their skins.	
879	?	laughter	F
880	JL	- I just put --- you know that brown art paper? I just staple the folders to the brown paper. we have umm, they have categories like "guess my rule".	
881	2	- I like that	M
882	JL	- For 10, for 20, kind of do it like that. And so they start, They play and when they start getting points and they just, you know, and then they start learning it and you let them play the game over and over. And they learn those problems. And you let them change it a little bit, they start getting really good at those kind of problems. And when it become harder, you know, then instead of X-3, make it 3-X, you know, and you start putting these little things in that they might see on a test, and because it's for a game, they'd cut of their right arm to	
883	2	- yes	F
884	JL	- who is waiting for a doctor to call interrupted by a phone call that is rejected.	
885	3	- I want to observe you, without a pencil and paper, laughter	P
886	JL	- I did it with a 9th grade, like a supplementary class last year and when they played it, people kept coming by, "is there something wrong?" because of all the screaming. [All the teachers are laughing]. So I had a whole crowd, they thought they had to rescue me...but.....so happy, and then when, I guess I had promised a prize to everyone who answered a few questions, and the whole class answered questions, so I pulled out a box of munchkin donuts and they went "AHHHHHH". I think I "killed them" for the day....they just used so much energy.	

887	JL	I can't say every game does that, but that's just one example that sticks in my mind, when they were kind of miserable, but they did the questions. And then, because they knew some of them because they did them, wooo, they were sooo happy. I mean that's just one idea before the unit test. It's miserable going over that, but you want them to do well. Everytime they finish a unit, you could - everyboy make up a problem. I save up those problems - they get moldy really fast.	
888	2	- I like that.	F
889	3	- I like that.	F
890	JL	- There's a workshop here at Rutgers, called the New Teacher Institute, but it's worth going to - I did it when I was a new teacher.	
891	2	- Is it free?	F
892	JL	- Nooooo. It's expensive - you can't pay for it. Your school - But school districts pay for it. I mean it's a really good thing, it's like a 2 week thing.	
893	2	- OK	F
894	JL	- I think they did it..	
895	3	- Like in the summer?	F
896	JL	- In the summer or over a break or spring break or something. I was working here when I did, so I wasn't sensitive to the teaching year yet. But, umm, I'll show you tomorrow, one of the things, I'll do it for you.	
897	2	- OK	F
898	JL	- One of the things, Just a fun thing you can do when they are burnt out or a little angry or don't want to do anything? You do this, and you have them figure out how you did it. And they're like......then they make their own thing and you tell the to go home and do it to their parent.	
899	2	- was aheming and "yeahing" the whole time.	F
900	2	- nice	F
901	JL	- It's fun	F
902	3	- I feel so enthusiastic starting..that's like...ahhhhh....you're killing me..	F
903	2	- Yeah, yeah....	F
904	JL	- It's very exhausting,....nobody gets it, like teachers, how much work we do. Although people do, have you known somebody who says, "oh you work until 3 pm, you have the summers off, you have a cushy life." I just say, "hey, you know, just come try it - I'll let you teach my class for one week."	

905	2	- Well that would be more than....	F
906	5	- When I come, I'm coming home with stuff, I'm coming home with stuff to grade, and they're like "you're always grading!"	Wb
907	3	- "What do you do all day?"	F
908	JL	- What about making up all those materials? That takes time too. It's a legal fiction that we work only 7 hours a day.	
909	2	- Yeah.	F
910	JL	- We work 17 hours a day.	
911	3	- It's really not possible...	F
912	JL	- I remember this one kid. This one class was from my first year at Columbia and I had this class of 5 boys who, I am not exaggerating, I walked in and it looked like they were at a funeral. They looked all "Mr. Cool", you know, with the cell phones and the doo-rags and they looked like somebody died. You know, I'm a mom, and I know when....so I said, "I'm Mrs. Leslie,,I'm here to teach, what's wrong - you all look like somebody died." And you know what, they all told me the same thing in dignified tones: "Well Mrs. Leslie, they warned me, they told me I was ruining my life, and it's ruined". "What do you mean?" "That we're in this class". It's because they were embarrassed, because we had levels,	
913	2	- Ohhhhhh,	F
914	JL	-......and at that time, that was the only senior level class that was labelled "Topics in Algebra". It wasn't a pre-calculus class; and they were thoroughly humiliated by this. I was knew, so I didn't know how rigid schools are, so I said, "well", I'm used to being in businsess you know, I said "Well, I teach the ...called Pre-Calculus, too, so how about I teach that to you. If you pass the midterm, I'll raise your level. Like I have that power?	
915	?	Everybody laughs in appreciation.	F

916	JL	- Well they immediately like, "REALLY?" And it was challenging, but see there were only 5 of them. So I had all these boards, so I had them working at the boards and I sat. And they figured it out and they helped each other. And because there were 5 of them too, we learned and they did better than my level up in pre-calculus. And luckily, just luckily, because I had no idea what's usually involved in it - I had made friends with the lady in data processing because I used to program computers and she used to work for the phone companies too. So Marquitta and I became good friends. And I marched them down to Marquitta and said they're all Pre-Calc snow and she looked at me and said "OK" and just ...in the computer. Everybody was happy because they didn't realize that the paperwork hadn't been followed, nobody had time I guess, and all these kids who had been considered low performers were doing decently in the pre-calc class. And one of them, Ettloyd Celony, I'll never forget him, won the award for "most improved" math student at graduation.	
917	2	- Wow (Many wow's throughout this story).	F
918	JL	- At graduation, I went to see him, and I didn't recognize him because he was wearing a suit, no hat, he looked so handsome and I couldn't... and said "ohhhh" ..get used to this.	
919	2	and others are laughing and saying "wow"	F
920	JL	It was a really great learning year for me because they were so special. They were very talented kids who happened to be misbehaving and got labelled as not engaged when it was really just behavior. And they were so capable. One of them... This is also what happened...	
921	JL	You know, you could tell they just didn't think they had a future, and they didn't want to go to community college because that's "where stupid people go", and that's totally wrong, but I could'nt convince them of that. So I said, we're going to go to Rutgers. We're going to find out where to get the degree you need.	

922	JL	I had worked there, you know, so I called up this woman on the Queens Campus. I got another teacher to help, and during Spring Break we drove them down in my Van - its only 5 of them. And she was amazing. And there all sitting there looking like ....kill, they're not used to this. She gave them each - you know that instant hot with the little marshmallows - she gave them each a hot chocolate. And she's telling them, "Do you know that half the students that graduate from Rutgers come from somewhere else?" She's telling them exactly what I told them, "There's a connection, a legaly mandated connection, between community colleges and Rutgers and students come every year. They pour in and they get the same degree as everybody else.	
923	JL	And they start, ...these are my students and I never hear them ask about curricular requirements. But they start asking, and she gets them all on the computer and they start looking for their majors, and then she gets them matched up with Rutgers students and they take them on a walking tour. I had lunch with them afterwards; I took them to Rafferty's in New Brunswick and the wonder that came out of their mouths, they didn't learn anything I hadn't told them, but there they were in the college, with the college telling them, "this is for YOU". And they couldn't believe it.	
924	JL	And then they graduated, and a year later, not 2 years later cause usually you have to do 2 years in community college, one kid came and showed me his admission to Rutgers.	
925	2	- OH that's sweet!	F
926	JL	- And still... and that makes all the	F
927	?	Everyone is exclaiming	F
928	?	Really cares	F
929	JL	- When you do these things, they know, they feel these things and then they rise to it. So that makes it worth it. We never got that in industry, you know. I got more money, but nobody every came showed me his admission to Rutgers. So its really cool.	
930	JL	- OK guys	
931	2	- That was a great way to end.	Wb
932	4	- Yes	F
933	3	- Very inspirational	F
934	?	Lots of "yeah"	F
935	JL	- See you tomorrow.	
936	2	- Yeah.	F
937	JL	- And then we'll see each other in the Fall.	
938	JL	- Have a great afternoon	

Day 4 TAW Transcript Data

Speaker	Statement	Codes
JL	- Okay.	
JL	- Oh - I am so sorry, I forgot, we spent so much time, we have to do your "orange" questions first. Today is particularly important. Thank you. Chuckling	
JL	- I really didn't get any sleep. laughs. ???....I'm so lucky....Whoever I'm working with tells me.	
Group	- laughter	F
JL	- there's whispering that isn't fully audible.	
JL	- and at school my students ....bring me...water...my pen..	
Group	- more whispering.??? Everyone is working on the problems in the orange sheet	M
JL	- If you are not sure what something means ask me, because there is new terminology in this one.	
JL	- Truth set ....truth..so let me know if that's not clear and I'll create an example. True makes the equation true, but "legal" - what's in the "boxes" has to be the same. You know, because just like "x" values, wherever there's and X, the value has to be the same. So true could be, um, you know, 2+1+6, but that's not legal because you have a 2 in a box and a 1 in a box, and "box" is always the same thing, right? Box is box -not this box is 1 and this box is 6. That's I think, what helps them, you know, learn about variables. I think that's what Bob Davis had in mind here.	
JL	- So you can look at all 4 combinations, so that's clear.	
JL	-They are working - no dialog - whispering and writing sounds.	
JL	- ...??..into the box?.....	
JL	- get a knife......cut a piece. [00:03:49]	
JL	- They are working on the problems for Day 4 and most of what I hear is whispering.	
3	- the third one..in the front side on the third one three numbers are replaced in two empty boxes and the	M

	empty triangle that is legal and false ....like 2 + 2 +5 = 12??	
4	- [yeah, you won't organize]	F
3	- OK.	F
JL	- Inaudible. Maybe they are eating?	
2	- Today there is no..[inaudible]	F
4	- [inaudible] echo	F
2	- Yeah, [inaudible] said that	F
JL	- Oh oh	
2	- Yeah, oh.	F
Group	- Everybody laughs	F
JL	- Thank you	
2	- Your welcome.	F
JL	- One thing to remember is this terminology is ideas about using truth sets and ....guess my rule....originated by Bob Davis....before, maybe a decade before Powell used it. You know, so, these children, Robert Sigley who just got his doctorate and went to Texas, is one of these children.	
2	- Wow.	F
JL	- Oh my goodness... We hear a construction vehicle backing up with pulsing tones.	
JL	- silent working [00:06:35] [00:08:14]	
JL	- OK - Does anybody, another aside, tutor geometry? I got an email for a child, so would you want me to suggest you?	
4	- Where do they live, though? Where is the person?	Wb
JL	- The person is here near Rutgers, somewhere in this area.	
4	- That won't be good.	Wb
JL	- Yeah, this person was getting tutoring in this building from somebody and she's leaving.	
1	- it's a college student?	Wb
JL	- No, its a high school kid - geometry - the mother works here. She got somebody here..she sent around a thing asking for people. I would do it but I have to do this and I can't do that....	
JL	- inaudible, loud scratching noises as if the papers are closer to the audio-taping than the voices.[00:09:17]	

JL	- Okay, if you're done, you can pass the orange things [orange question sheets]. [inaudible]	
JL	- Do I have them all? I think I have them all. Okay.....................long pause	
JL	We hear the 4th analytic, As Bob Davis (in the analytic starts to speak to his students in the video.	
JL	- notice that he has a big class. [return to the analytic] [00:10:10] [00:14:10]	
JL	- [The students in the analytic are creating a legal but false statement: Box + Box + Triangle = 19 (not sure about the number. One students in the analytic says "a million + a million." The teachers are laughing and enjoying this answer.]	
Group	- laughter	F
4	- I wish I had glasses.	Wb
JL	- [referring to the analytic] They like this idea of secret, they are probing it.	
4	- Its okay....laughing.....they're talking about it....	M
Group	- more laughing	F
JL	- [stops the analytic] - I just want to say that, do you remember yesterday, 02 had this same idea in your lesson? Your first, going to have them use the rule, that was very important to you, and I'm just showing it was important to him [Bob Davis] too.	
2	- sighs (a happy sigh?)	F
JL	- Analytic goes back on,	
JL	- (over the analytic) He's given them problems with just points and they have to find the....it's a guess my rule situation, but he calls it a Secret. pause.. Those are the ones they are working on... [00:18:18]	
JL	- Pause	
4	- (over the analytic) They...observe.....they go in your classroom and observe this.....	P
4	- ...they were good....	F
4	- How old is this video?	Wb
JL	- 6th grade..	
4	- How old is the video, How long ago was it?	QWb
JL	- Oh, uh, it's a, in the 80's maybe?	
4	- Yeah, I see by their glasses and their clothing. chuckles	Wb

JL	- Because the kids are in their 30's (now). They were 4 years old.	
4	- Mm_hmm the glasses and the way they are dressed.	Wb
JL	- Yeah - it's very old.	
JL	- They started just a little bit before. These are the first group of the longitudinal study	
4	- Mmmmm	
JL	- They've been in the program years, but they're only 6th graders.	
4	- They look much younger than ...6th graders. I thought they were like in their, fourth graders. They look smaller.	Wb
JL	- See, he's showing, learning the...how she's doing that..	
JL	- [That kid] says share it, and one kid says "no" don't share it (the secret)	
4	#NAME?	F
4	#NAME?	F
JL	- that's the bell [in the analytic]	
JL	- Okay - it always rewinds. So before you do the group of questions, talk to me about the differences between this class and the other group with Dr. Powell. Similar ages, those other kids were the beginning of 7th grade, these kids are at the end of 6th grade. Very similar in age, very different situations. I know the kids in Plainfield ...had..at most one year..working with Dr Powell. These kids [some] have been working with the Rutgers researchers since some of them are in Kindergarten, and many of them, first grade. So some of them are doing this kind of thinking for 4 or 5 years.	
1	- You can tell they know the routine.	Wb
2	- Yes.	F
JL	- And it says he works with a big group because he can.	
4	- Yeah, right.	FY
3	- ..work together.	F
1	- They....??....geared up, because once they got going....they worked together.	P
JL	- Inquiry based learning is very familiar to them. They, you know, they go along with it. But, umm so, Let's talk about differences in what he's teaching first. ....Are there substantial differences or is it basically the same?	

2	-You know it wasn't ....[inaudible] right?	F
JL	- To OO, I saw you shaking your head like this- but I didn't hear what you were saying when you were watching? You were saying something to KH, I think and shaking your head?	
4	- Oh oh - oh well	F
2	- I think we were talking about how the class was, like, so enthusiastic.	P
4	- Yeah!	F
JL	- But that was true of the other group too.	
04 and 02	- Yeah	F
1	- I think the difference is , in the other one (Dr. Powell) he just gives them, like, the table allright "go find the rule". And this one (Dr. Davis) he gives them almost kind of like a "Do Now", you know different skills going, see what skills they need in order to do this.	M
JL	- We are all teachers, right? We know that, you know, if they have no experience with the thing, you can tell how they would do it. Did the children with, the boys working with Dr. Powell - did they seem like they had no idea what he meant by a rule?	
1	- Well they never, they never, they never gave a rule, they just gave the pattern.	M
JL	- For the first one they didn't give a rule, but for ...	
5	- the second..	M
4	- They gave the rule for one.	M
1	- Oh yeah, yeah.	F
JL	- ..second one gave a rule. They did give a rule.	
4	- They gave a rule for one, Christian gave a rule.	M
JL	- And remember? Ariel said something. (I know these much better than anyone.) But Ariel, Dr. Powell says, "I want everyone to listen to what Ariel said". Do you remember what he said?	
Group	– chuckles	F
JL	- He talked about a factory, he said, "you know the Box goes into this factory ..	
2	- Oh yeah.	F
JL	- and the factory does something to it and it comes out a Triangle".	
Group	- "yeah, yeah".	F

JL	- Which was, you know, obviously, it's a function model. So do you think that they...? Here you're seeing some of his "do now" first thing. But, you know, when you get a clip like this, you really aren't, you don't get a firm statement of what came before. And I'm not....you know... Do you think that nothing came before? They never heard about a rule? Or do you think, they must have done something...	
1	- They should've - they're like..	F
JL	- It looked, it seemed like there was a comfort, a familiarity	
4	- They already....	F
1	- ..doing .from memory..	F
JL	- It's just 3x-2 is a really hard rule..	
3	- Talking about a secret. Then he had to talk a little bit about about secrets and scientists.	M
JL	- Right. They weren't at all sure what a secret was. They had a lot of questions about secrets.	
3	- yeah.	F
JL	- But they.they...go in...??.here, right?	
JL	- Fundamentally, Did, um, Is he teaching something different besides the Do Now?	
2	- I guess so, with the "legal" and "illegal", that's ..when he says that, that it makes it kind of different.	M
JL	- What do you think of that? I thought that too. What do you think of that as a piece of this lesson, this idea that ...really there's four combinations of values you can plug in: You can plug in legal values that are true or legal values that are not true. And the there's this interesting set of things: illegal that are true and illegal that are not true.	
3	- Putting myself back ..trying to put myself back. you probably all....going back to that grade...thinking...I don't know that I would I have liked the legal thing - think of it that..	P
2	- Yeah, I have to think about that too.	M
3	- Okay, okay, I think I would have to like, "what are you talking about, "legal" ?"	P
5	- Even now, like when I saw the problem, and I saw "legal"... I got the True and the False because I've heard that enough, but "legal" and "illegal"..I'm like..what is this?	P

JL	- Right, it was not familiar, but once you know what they mean it's not hard...	
3	- As soon as they caught on...I .??..stopped thinking about it..and now I'm pretty..good..	M
JL	- I also thought, exactly as you do, I was thinking about this in positive and negative ways. And I was, you know I think I write a question about it in here. Basically we never want illegal or false answers, we only want legal and true. But sometimes - there's this - you understand the legal and true when it's contrasted with the other. And the other thing that I thought of, it's a possibility - I don't know - um, is that, children who are less quick with algebra....this idea of what an "x" is, what a variable is, ..is really hard to grasp. Having a "Box" that you have to put the number in, and all the Boxes have to hold the same number or it's illegal? It seems like that's an intervention for children who are really confused by what I mean by x+y = 9 ,	
Group	- lots of "yeah"	F
JL	- .....what do I put in there?	
4	- I really like the words "legal" and "illegal" ....I think that sometimes I would use in my algebra class..you know? Because it's...	M
JL	- It's much better than right and wrong.	
4	- Yeah, yeah	F
4	- Yeah, so what is legal and illegal?	M
JL	- "It might be true, but I can't use it (it's illegal)".	M
4	- Why can't I use it? Why is this not legal?	MQ
1	- Getting...to think about ....the parameters of an equation, there are certain numbers that you can use and there are some that you can't..so	M
JL	- Right, right, "Domain",... that naturally extends to the idea of domain and..and kids have a lot of trouble with domain.	
5	- I agree...with OO...I think I'm definitely going to use it. When we're evaluating expressions and they see "X^2 + X" ,and they plug the X value into X^2 and then they would leave ..."what do I plug into X?".. I'm like: "The same value" (Everybody laughs.)	M
JL	- ..Because that's a real abstract idea..that X stands for something else. I think this intervening step,	

	where it's a "BOX" and I fill it with something might be very powerful.	
JL	- If you do it, I can't wait to hear about it.	
4	- I used the BOX a lot...when I do, um, what is it called... Because when they are in elementary school they use BOXes.	P
JL	- Oh okay, Do they use any triangles in elementary school?	
1	- They use any shapes to represent variables..just shapes.	M
JL	- But they use more than one shape?	
1	- Yeah.	F
JL	- Oh OK, so they're familiar with it.	
4	- So like, when you have, when you start doing equations, solving equations, you know...one step..two step...If you write, for some of my kids, if you write 5+X = 10, they're looking at you [not comprehending], but if you write 5 + BOX = 10, they give the answers straight away. I tend to use, you know, until they get used to replace. I just tell them that "instead of the BOX that you used to use in elementary school, now you use a variable. So it's the same thing. So it makes it more familiar to them. So we use shapes too.	P
JL	- OK - cool. Alright, so umm, so let's go to the questions. The last questions. (last day of the workshop.) And you should...	
JL	- This is the last one, so, you know, talk about it, argue about it. I just love watching the Bob Davis tapes because I imagine him in my house,......??....around the steps. He didn't use the kitchen and used the broom closet as a coat closet. As a coat closet.	
2	- Really?	WbQ
JL	- He had a (clothing rod) bar in there and there were jackets hanging up.	
2	- wow!	F
JL	- When I went into the kitchen it was really weird, "nobody cooks in here", 'cause I cook. And he had everything set up for somebody who eats out all the time - or get's take-out from somewhere else. He was a very interesting person.	
2	- laughter	F
JL	- [We hear them murmuring to one another about the questionnaire they are answering.]	

2	- ....any other ones where they have to do like 2 boxes ....just to find one box?	PQ
4	- Right.	F
2	- OK	F
JL	- In that thing, that very first event, he had BOX + BOX + TRIANGLE = 9, ..	
2	- In this one...	M
5	- In this one.	M
JL	- In this one, the very first event, but then the rule later he had 2BOX + 1 = TRIANGLE.	
5	- But the first 3 videos he didn't..the um	F
JL	- The first 3 events..	
5	- yeah..No Dr. Powell, that's ..	F
JL	- Dr Powell didn't write the equation. He put BOX, TRIANGLE in the chart and the kids either did or they didn't. Also, If I recall, Dr. Powell kept switching to X and Y....??>>>>	
2	- He still doesn't say where....[inaudible]..does he say??	M
5	- Who? Him?	MQ
2	- yeah..inaudible	F
5	- He does, he say's "that's good".	M
3	- He says, that's good thing, or whatever. (repeated)	M
JL	- He say's several times... play again. He sometimes says, "that's worth thinking about",	
2	- Oh	F
JL	- .... when somebody has a question. And so they think about it together. And then they show him ideas and he will say, "that's certainly a good thing". It's not always clear...	
4	- It's a little difficult for me to follow. Because I think, I think the kids are rowdy..The kids are too rowdy for me.	E
1	- These kids?	EQ
JL	- The kids all talking at the same time.	
4	- The kids were talking about..	F
JL	- I have, I pulled out the text on the side. We can see it again, but if you want to, you can go on Sakai, I have the link, you can use that transcript I gave you and look at exactly what they are supposed to be saying and think about it that way. It's a little tricky when they are all talking at the same time.	

4	- Yeah,....when you are teaching!	P
3	- But he let it go. A couple of times he said, you know, "quiet", but mostly he let it go.	P
4	- Yeah, he let it go.	P
2	- He had..??..	F
JL	- He does this ..professor...."We're all in this seminar together thing".	
2	- Also, I guess maybe with you're saying they have been with him such a long time, he may have already known their personalities.	P
3	- Right, and it's going to be okay, it's going crazy.	P
2	- yeah, laughs.	F
JL	- I am going to put it on again because I think, umm..,	
4	- Yeah, cause I'm..	F
JL	- this one's softer...lower than the other	
JL	- ....??.. for quadratics with boxes over there..	
4	- yeah I saw the quadratics...and...looking forward to seeing how these kids ..would...	M
JL	- There's there's, I'll show you how to find..there are real clips...I don't have analytics with that..	
4	- ....was really looking forward to seeing..how they would...	
JL	- but they're put somewhere ....when we finish this I'll show you the system..	
JL	- Yeah, yeah, because there's a lot, I just didn't want to expand it [the analytic with Bob Davis] with your kids, I figured they're not going to get to that [quadratics] for a while...	
4	- ..mm_mm, only the algebra..	M
JL	- I have to get to the bottom to get this on..	
4	did...mean by, "start all over again" [referring to a student on the video]..	P
JL	- So these are different styles, right?	
2	- Yeah	F
JL	- You know, how really they ....do that....inaudible..and he does, you know. They do justify though. They're not just saying, "am I right?, am I right?" They're working out the justifying and he has a different style. It seems though that they stated very simple results...	

2	- Yeah, but maybe when he says thats a ..???...maybe he's doing that to generate a discussion with some sort of like NOT arguing with each other but discuss like that's right or wrong, you know?	P
JL	- I think so, but I also think we tend....this is really a question for you, All of us are teachers. When we see an example, we tend to, kind of like, make everything about that example you know is like "holy". Everything about Dr. Powell seemed essential in order to achieve what he did. But I think then there was always that next stage when we make it ours. So we can do it with our group. And then we kind of secretly discover that everything isn't sacred. Only some of the aspacts ...That's the challenge for each of us. How do we get that richer behavior from our students?	
2	- murmurs	F
JL	- They think, and then they're interested and they explain and they find the problem and the they fix. You know? It's probably not a function of "I never say the answer" or "I never say they're right". It's probably much more "what they value". That's why I keep asking, "What do you think is important?"	
2	- murmers yeah.	F
JL	- What does he value? What does Powell value? And then, what are the students taught to value?	
5	- She was saying......the one kid said that the box + box + triangle .... you could put "3 3 3" in it	M
4	- mmmm	F
5	- She was saying ..	F
4	- Oh she was by..with the "trial"??....you start all over again...	M
5	- Yeah..she was basically saying ..with the 3 being in both boxes..it can also be a triangle and the same like - when you go from the box to the triangle it's like a whole 'nother different shape so it's a different variable, it's like starting over..	M
JL	- but it doesn't...right.. she doesn't say this but, they're learning that BOX and TRIANGLE don't have to be different.	
JL	Lots of murmurring sounding like agreement	
JL	...They can be different,.....they don't have to be different. It's just a nice way of saying, "start all over again, start fresh...put anything you want in there."	

1	- Yeah , you know...	F
JL	- That was so cool..	
1	- yeah - just as long as all the boxes have the same value. Boxes....?? to the other boxes have to be	M
JL	- Right, but you realize that language is so important..These kids, they kept saying "this number here, goes over here". laughter. They didn't have that word for y-intercept, they didn't have that word for slope. So they're finding them. And that's another interesting thing, you know. Umm, is it important for them to find it without having that concept? Is it important to just find it and then name it?	
Group	- "yes" and other things murmured in the background.	F
JL	- ....that's a question, I don't know....	
3	- unintelligible	F
JL	- They were like ..."this goes here", but they had a hard time saying it because they had no word for this important thing that they had found.	
2	- It's kind of nice when they find it and then maybe you can tell them "Oh that is..."	E
JL	- "You found an important...."	
2	- Yeah and then they never forget it, laughs	M
4	- Yes	F
JL	- That could be true.	
JL	- I love the secret idea though.[00:48:48]	
2	#NAME?	
2	- Especially 'cause of their age...it probably was like a nicer way..	P
JL	- Oh - I was going to show you...and I forgot to bring my card??	
2	- [overlapping words], "a secret", laughs	P
JL	- Let me see if I can do it while you're doing that (the questionnaire). Even though it's going to be kind of weird ....the cards...	
Group	- all talking is in the background: ?? Do you think it was the ...??	F
Group	- murmuring - barely audible	F
2	- I can't remember what he says before. ... Like when he introduces the idea of legal and illegal, does he say like that this is something new? I can't remember what he says..	P
2	- Like how does that even come apart?	PQ

3	- other people are tr..??	F
5	- Start with something simple...	P
2	- No, not that part, but I mean when he starts doing that "legal" and "illegal", what does he say right before he said it ...I know he writes the words first........right?	PQ
4	- Yeah, he writes the words and then the kids will tell the numbers..	P
2	- But does he tell them what what what makes..?	PQ
1	- I think he kind of puts legal..they probably...maybe they knew.. math...before..already	M
4	- They knew, yes.	M
2	- Right, so they knew that.	M
1	- He didn't really explain what legal means.	M
4	- No	F
2	- Right, yeah, right, okay.	F
2	- inaudible whisper	F
2	- He says that?	PQ
5	- He says that..	F
5	- But then he also says...about some number in the one box, what do I have to do? Somebody says, "the numbers in the boxes have to be the same".	M
2	- Oh, so maybe its in the ...??...okay.. make.....inaudible..	F
5	- Cause he also ...explained...true...false..	P
2	- He does.	M
5	- Yeah.	F
2	- Oh yeah.	F
5	- Some of them	F
2	- yeah, okay	F
5	- [inaudible]	F
2	- Yeah, I saw he did..	F
2	- That's on page 3 in the uh...	F
5	- Yeah, on the, on top..[00:54:05] [00:57:05]	F
5	- ...can't .remember him ...what he was saying they were doing? laughter	F
2	- No, I can't remember that at all.	F
5	- [inaudible] ...Dallas....	F
3	- what they are doing, what they're doing now?	CQ
2	- Yeah, do you remember that?	PQ
JL	- What?	
3	- No. ..[inaudible]...just now to go	F

4	- which one?	F
3	- ...repeats what they were doing,..I don't remember.	F
JL	- You just said that a few times, here lets just...	
5	- The only thing that I'm thinking about is maybe ...is that on page 5? On page 5. What did we do the first time I gave you an equation, right? The first time I gave you an equation, what did we do?..- we worked out numbers..??...problem..??.5...and then we said, Now what are we doing? - we changed it - what are we doing now?	M
1	- [inaudible] yeah	F
JL	-Well they're not doing the same thing, right?	
5	- work....miss [inaudible]	F
JL	- I think this came up when we were talking about....	
2	- Let's look at.event...what is this?..what event is this?..I think it was event 4. If we can find event	4.[00:58:06]
Group	- Participants are heard laughing and talking. Participants and I are laughing about the comments of one of the students in the video (heard saying "it's stupid" alot).	
JL	- This is what Kim did........[inaudible].......you're right	
Group	- JL and participants are talking at the same time	
JL	- This is actually very interesting....it might work for some of them, but not this one? That's a very interesting observation [01:00:11]	
JL	- This is where he's going to say...make sure we remember what we are trying to do here.[01:01:16]	
JL	- Here, he does it again (talking about the analytic).[01:02:41] .....These are different sessions, they're not happening together...so he's reminding them. [01:02:49] [01:08:44]	
JL	- okay.....[inaudible]....I'm sorry.................	
3	- it's wrong	
JL	- yeah, oh no ..??.. oh, I forgot the +1.oh...it's wrong.....thank you for telling me! ....I'm so glad we didn't do too many of those.....look at page 2 of the yellow - that equation is not correct - it's the equation that they have. It should be "BOX times 2 + 1 = TRIANGLE". So please insert the "+1". Thank you..[they are working on the yellow sheets]	

JL	- You know, it's so easy to see what you expect to see..	
4	- Yes.	F
JL	- ..instead of what's there..[more murmuring, but not audible]...	
4	- I going to finish but [inaudible] cause I'm so slow..	F
1	- When did they say they .....get the secrets? Because	M
4	- What?	MQ
1	- the very last questions.....secrets...want to hear...secrets	M
3	- They want to solve it themselves?	MQ
Group	- lots of simultaneous comments "I heard that" and others.	F
1	- when did it occur though? when did it happen?	PQ
5	- Umm, I see something about secrets on page 4? Jeff said that ...were secrets. Was it a secret if some people knew it?	M
4	- I ....[inaudible] had enough of ....secrets?	MQ
JL	- So, there was a place that pointed out when, umm, Dr. Davis says, "the secret some of you found out about, maybe it's time to share it?" And, Ankur says, "Don't share it yet." and he says, "Okay, it's not the right time." That's the one I was referring to...	
5	- Ohhh.	F
JL	- so that's umm, [inaudible] ahhhh (groan).	
4	- laughter, I was feeling it but when you just "ahhhh"	F
JL	- ahh, this is the one (the analytic event).	
4	- Is that Michel?..	MQ
4	- Okay, this is the one..	M
JL	- This is where Ankur says, "we shouldn't" and Jeff says "we should".,.that's after she does the "zero and one thing".	
JL	- Okay, so that's what I am asking about..he's ready to share and one kid says, "no, I don't want them to share", and ...so what do you think is going on?[01:14:26]	
4	- Maybe he wasn't done yet...maybe he was yet to find the secret, that he didn't want anybody to reveal it yet because he was working on it..	M
3	- He wants the challenge..	M

JL	- ....If you finish and you want to ....take a break, go to the bathroom..whatever. I am going to show you a clip of the ....tools?...instead of having to....events....analytic tool.....and I'll show what I put on Sakai....so we (you?) can help each other..[not audible.]	
1	- ...About Sakai, is there a verification code required? I tried a lot of times....I know there's a login ....I never created a password.	Wb
JL	- You have to create a password.	
1	- I created it and it says I needed a verification code to continue.	Wb
JL	- It does?? ..Well everyone.... what we did......yesterday we figured it out. You may just have to ...create new password. I don't know why it would need to verify, I don't know what it means, verification code. Unless it was an error that came up?	
1	- I don't know, I'll try it. again.	Wb
JL	- I don't know... had obviously....misunderstood the password... But there's 2 ways to log in. One is if you have a netid at Rutgers, and the other is if you've been invited to join. I put your emails in there, with your email, and that's how that works for you guys. [01:16:32]	
1	- [inaudible]	F
JL	- You got in?	
1	- ..it sent me verification code.	Wb
JL	- There's, you know......Verification code sounds like you have an error..I can give you one..	
1	- I can see if it works..	Wb
JL	- Yeah, I'm gonna, ...One of you are going to log in to do it here, so we can all see it and then we can post some things and make sure we know how to use it.	
1	- Where would those articles be located?	F
JL	- Under "Resources" on the left..	
1	- I see.........alright, I got it.	Wb
JL	- Ok....we're ...gonna do something together ....just make sure [inaudible].	
JL	- You guys must be tired.	
Group	- tired murmurings	F
1	- You know what?	Wb
4	- You're gonna have another [child?]. The boy's 5 year's old.	Wb
Group	- laughing	F
1	- Chill.	F

4	- [inaudible] get it over with...you know	F
2	- laughs	F
2	- Sometimes it's nice to have a gap.	F
4	- It's better for you to have two growing up together.	Wb
2	- Two what?	F
4	- Two kids growing up at the same time, than just one.	Wb
JL	- Well, it all depends on the kids.	
2	- Right, Yeah, it depends on the kids.	Wb
JL	- I had 2 close, 3 years apart that's all..., my first daughter was okay .....but the second one, "miss needy" you can tell from the beginning. One of the first sentences she had: "I want to tell you something, I want to tell you something!". What? What do you want to tell us? "I want to tell you something". We finally realized, Rachel's always telling us things - she wants to be able to tell us things too.	
2	and others - "Oh!"	P
JL	- On and on and on....So I wanted a third, but I just felt "this kid needed more attention".	
2	- Yeah	F
JL	- A bigger gap.	
4	- I think it's always better for them to have somebody closer to their age to like grow up with.	P
JL	- It is, but for me it wasn't better to have ....well the first two, you... know, it was very ...to have ..."Snap sounds" like that. So they were 3 years apart, but that was okay, until Rachel hit puberty and says "bye Daliah", and that was it for Daliah. Daliah was on her own. And by then we had Adam and Daliah was very good to Adam. She was 6 years older and she was a real Mommy to him. I don't think he realized it - I was their mother too. He thought he had a mom, a dad, a Rachel and a Daliah..he had them all. And I remember telling him once, "I'm her mother too". "Do tell!". laughter It was kind of funny. But she was, Daliah was very intense. I still remember, Adam was a really skinny pathetic looking baby and sitting in a little chair, he was about 3 weeks old and he was looking up at her with his big eyes and his bony face and she starts to cry. "Daliah, what is wrong?" She says, "I was just thinking, baby Moses was like this when she put him in the river!".	

Group	- Everybody laughs.	F
JL	- That's my Daliah..	
??	Somebody - "where did she get that from"?	
JL	- She came up with the most amazing things..[01:20:21]	
Group	- lots of laughter, everybody talking	F
JL	- explains the story of Passover is where she got it from	
Group	- everyone laughs for a while more stories about Daliah (why is JL wasting the time this way?)	Wb
4	- Oh my goodness, worrying about this at 4 years old	Wb
4	- Oh my goodness, I thought I heard it all, but this one, worrying about this at 4 years old, I never heard any child...	F
JL	- It's funny, she's sensitive and thoughtful, but she's ....really deep, she wants to....she barely makes enough money to feed herself, but she's determined to do this. She has created the Hollywood Medical Reporter, which is a blog, on Brainblogger. She got doctors to pay her to write review of shows with medical content; the doctors gave her corrections or improvements in the medical content. She sold this idea to doctors because she said, "maybe they will want you to be consultants in Hollywood". So, the doctors, for a time they paid her, until they figured out that they weren't going to be consultants. (lots of laughing during this story). But she got a lot of things ....out there..while she did this. A lot of things like that.. Now she's working with a group of rap singers, 4 boys, 4 young men. They're rap singers and they're Uber drivers. So I said to her, "Daliah, you can learn from these young men!" "What do you mean?" "They're not just working on their art, they are paying their rent!" They are driving those cars to pay for their bills.	
Group	- Lots of "MM_hmms"	F
JL	- We don't give her money any more...and she's ...	
2	- How old is she.	Wb
JL	- She's 27.	
JL	- So she said, "Yes mom, I'm fine." But she's the intense one.	
4	- She's your 2nd child.	Wb

JL	- She's the second one. The first one is the one with the baby, I'm going to visit them Sunday. The 3rd one is the real baby, Adam, the baby Moses who's no longer a skinny baby. He's 21 and he's the biggest thing in the house.	
4	- The ones I don't know what to do about that are the girls.	Wb
JL	- He comes home to take care of the dog and then goes back to school. Who know's what he'll do...	
4	- What school does he go to?	Wb
JL	- Brandeis in Massachusetts.	
4	- He left home.	Wb
JL	- Yes.	
4	- NJ	F
JL	- They all left home.	
2	- All of them did? Oh gosh! What do you do?	Wb
JL	- Rachel went to Israel before she even went to college. She was also...she was a different kind of difficult. She had a different boyfriend every hour, she was ...get an A in a course she likes and a D in a course she didn't like. She was always leavi	
JL	- She went and she, she needed something hard. She worked, they made her take a year of school. She worked at a job 6 hours, she was in school 6 hours, and barely had time to come home eat and go to sleep. But she did it, and then she went in the army and then they made her an officer, and then she became the first woman that they put in charge of communications with Egypt.	
Group	- lots of "wow" but JL needs to shut up!	Wb
JL	- The language of diplomacy is English, and because she has good language skills, so was good enough in English to do this, and then she would bring all the information back and translate it into perfect Hebrew, and tell all the generals. She said one of two things would happen - yada yada - I am not typing all this junk that I said - I am dominating lunch - just talking about my kids, ugh![01:26:06]	
Group	- everyone is laughing..	F
JL	- I always tell my parents of difficult girls that story because [01:26:39] because I remember how I felt. Your child is so talented and smart and they are behaving so badly and all of that talent and all that intelligence is going to just start to get calmly,	

	purposefully, repackaged into something. You just have to keep her safe until then. Our daughter just needed a big challenge.[01:27:22]	
JL	- [I keep thinking that JL will shut up about the family stuff, but she keeps on going]	
JL	- Now I am talking about Israel - because OO asked. But I can't record all this "Israel is wonderful stuff".	
JL	- This is lunchtime? Is this why JL is going on and on about Israel. [01:32:25] Finally - we return to the workshop! [01:34:48]	
JL	- So umm, What I wanted to do now - I know we've been doing lesson plans, but I thought you probably got far enough .. to continue that on your own, with lots of ideas we can give each other on Sakai. So I thought first we would get on Sakai if one of you ...do...show you...start a discussion about what our goals are for the lesson. And so here,...so who's going to be the first person who logs in? Cause if I log in, it doesn't look like what happens when you log in. I need one of you to log in	
2	- on that computer?	F
JL	- Yeah, on my computer. It's the only one we got in here. I mean we have others, but they're not hooked up to the thing. Okay, so the umm, ...go to this one...	
2	- yes, absolutely	F
JL	- Okay, ...go ahead and log in...and you're going to log in,,, wait wait wait... we have this one....this is the wrong page..I need the ...???... here it is ....email....	
JL	- Alright, the whole purpose of this is just to show you where things are. [01:35:52] And I want us to have a little discussion about ...I'll post other questions, but I thought we could just start with a simple one..Just so...you get used using the system.	
JL	- Okay, so when you get in everybody should see this..and then you come back in.. your using the system...you're gonna get perfect training now. You want to go to the Algebra Teacher's Workshop - click on that one..okay? So now ...notice that it has "new" in Forums now? Over there, you see that?	
2	- mm-hmm	F

JL	- So that's one way to go there. So, before you go to Forums, go to Resources...on the left. Down...All the places you can go are on the left. And Resources is where I post articles. You see there's two articles there that you could download...and..print. I suggest starting with “Understanding Understanding” first. That's Bob Davis..who wrote it.	
JL	- laughs	
JL	- And it's not hard to read....That's a fun problem to work on [in the article] that one in the middle. ...Maybe...you all do it and tell each other if you think you're right..	
Group	- laughter	F
2	- What, that one there...?	F
JL	- That's the problem..that's a math problem....that's an illustration of an interesting, engaging,..... challenging problem. It's not so long.	
2	- [not audible]	F
JL	- Okay, let's get out of this..this for everybody to do on their own. Now go to umm, go to Forums from here. Forums is where we put the discussion. Okay? Notice there are two things here? Click on...I think "General Discussion" - I didn't post that, but put Planning Our Lessons, (that's the one I did post) and look at "View Full Description". That's where the URLs are. So if you go to Planning Our Lessons in Forums and you view Full Description, you find those URLs. I couldn't find a better place to put them, but I might try putting them under resources also... because it's just kind of a weird thing that you have to go here..so they're all here and you can look at them. I can tell you that each time....just like when you teach a course over and over, you learn new things,.... every time I see them I learn new things. Even now, I notice different things.	
4	- mm-mm	F
JL	- So it's worth seeing a few times more. Okay so now, Hide Full Description, and now click on Lesson Plans and Goals and I put...please put goals you have and ideas, and questions about how to achieve these goals. The way you do it, you want to Post, you want to click Reply, you go down to this.....so now you can write down, "I'm so tired at the end of class, I have no ideas, my mind is an empty shell".	

Group	- laughter	F
JL	- So whatever you want to write here, just as a practice..."I am the brave person who is writing this reply..." laughter (a bit).... I think everybody should get in and write just something to make sure they know how to do it.	
2	- ...cause I remember that....inaudible	
JL	- Now you go down to the bottom and you say, "Post". You can Cancel, you can Add an attachment or you can just "Post". There it goes. Now, I think, if I log in, I'll see that it's you who did that, but it, ...oh there it....it has your name...yes, so if we have a discussion, our names will be on our stuff and we can talk to each other.	
4	- okay.	F
JL	- And you can, if you want to start a New Topic, you can do that. Let's see where that is. It's umm, next to ..no no no...ummm, I think I have to go back. You don't want to use those. Click on umm	
2	- Forums?	F
JL	- .Click on[forums?] Click on the Forums at the top. The Algebra Teacher's Workshop Forum. Down in the blue. Oh that's okay. Click "plan your lessons" and now, see "start a new conversation". The Top, Left. So you could do that. And then you put a title and you describe it and people can respond or ignore you..	
JL	- You can...??..bottom if you want. If you are having trouble using it, email me right away and I will try to help you cause at first it's going to be awkward, but then it'll be easy and you'll be able to talk to each other this way. And… All of You should be able to kind of collaboratively develop the lesson plan ..so that.there's a core idea that everybody is using and, then you, you could make it a little bit different for your kids because everybody else will believe it. Plus if the first person doesn't come back and say; 'this was great and this bombs and then everybody will finish it. Although you know I have been having an experience of a lesson that works a million…and that is just for us and just this class is like you know…	
4	: Yeah you know…[yeah]. And sometimes it's [inaudible]….	F

JL	: And the whole class starts to look at you. A lot of it is for kids [yeah] and you know the moment, the time and day and all that stuff.	
3	: As long as it only[inaudible]	F
JL	: This is just nobody else's busi…[AK:so they wanna complain about each other about Plainfield stuff…] I….[laughter from all]. You're right AK. I think it's [inaudible] and I am letting in anybody else in. I created it, I own it. I verified this because I actually sent, um I didn't send to your supervisor because that would violate the positions of my study. If your supervisor is on there, you're not going to say anything. So so, I didn't do that but I did send mine you know to my advisor because I thought she might wanna read stuff. And then I thought, it wouldn't be good thing for her to keep rejecting things until this discussion and let's just be us until she does respond but I think she didn't because she is on vacation. So I am likely to respond later and say, "Carolyn, every time you want, I'll print out the stuff for you to see, but we decided we'd like it to just be us.	
2	: [laughs]	F
5	: [inaudible]	F
JL	: I can't, because that would be um - the data I ..[inaudible].	
Group	- everyone is talking and laughing	F
3	: I think that's right. Alright	F
4	: Yeah	F
2	: Yeah	F
JL	: Ok, so, um, any questions about this?	
3	: We are gonna hurry	F
4	: Yeah	F
JL	: ….sometimes it's easy sometimes it's in here, just that if anything goes wrong but once you log out… and I will show you how to create an analytic. Yes you can, this is a free tool on the internet.	
2	: Oh nice!	F
4	: Wait a second	F
JL	: Ok, um I am gonna go to the um not to that. I am gonna go to the RU analytic tool. I think you end up having to get some kind of ID to do it but if you want to let me know and I will find out the best. I guess that's a community account login and so you will down here to	

	find out about this…..double [not clear] because we do make the tool available…	
2	: Oh can you just say that again? I am sorry I am sorry.	Wb
JL	:Well, that's pretty there. You can login at [AK: Yes] what's called a community account	
2	: Ok ok	F
JL	:…[not clear] reference and if you put them in this I will put them. Um if you open an account, that's membership. And you can put this one community practice but then your mosaic level to make new discoveries of teaching math. So, you can ask for community account login.	
2	: Ok	F
JL	: And you create it and of course it can't exist before any lessons. Put in all the things but um you could do that. I am gonna login because I am Rutgers person. I will do that. Oh it won't work now go back to the beginning. You will see that at the left of the tool. Ok So I am gonna login with my Rutgers ID because that's what I had.	
JL	- Ok, this tool has three places; this in here where you create your analytic, this over here is where you search for the videos you are gonna use to put in an analytic. It's really complicated. I am going to avoid focusing in this[laughs].	
3	: Oh ok	F
JL	: This is not complicated so you gonna do this; Just watch it and then when you try to learn it you're gonna need a document anyway. So this is a little bit complicated but the complications are just annoying at the beginning and then it's easy. Ok? And this is where you will see other analytics either yours or all of them so I have these options here. I can reveal all the analytics,I can look at just mine, or I can look at the published. Now what's "all" if it's not mine and published? All means things that have been shared with the other people who are developing (analytics) and we share them with each other so that I get feedback from somebody else or somebody may use some of my events in there. So, so "all" that means you know, that the ones you have been shared with me, the	

	ones I wrote, and the ones that are published for everybody to see. So I am gonna get my little list here. That's what I intend to be doing , then I have to click this little thing that says "search". This one, the Bob Davis one, is the one is the most recent one that I wrote. So, if I want to edit this, I click on it and I open it. It shows up in the middle here, and this is my workspace.	
2	: Did you type all of the description or was there..	F
JL	: I did!	
2	: Wow!	F
JL	: And plus the editor in here is crappy, so I type it in word and I copy it in.	
2	: Oh ok!	F
JL	: Because if I make a mistake here, it might close and I might lose it and "ugh"…you know,	
2	: Yeah.	F
JL	: So I have a process of my own where I write the text	
2	: mmmhhh	F
JL	: And then I copy it in. But yeah, I mean, it's not fast to make these, but the first thing I do when I create a new one - I will show you when I create a new one but I am just showing you um[someone laughs]. In this tool I am editing the thing and so I can see all my events [mmhh], I can pick and event. It's up there now, and I can close it [mmhhh]. And now if I want I can edit it. Now this is the text, but once I click this crayon here [mmhh],this is the editing menu. And now I can change the title, I can change the description and I can change when it starts and ends. That's actually the biggest thing you create on your own the first time. You pick a video you wanna use and you decide what piece you wanna take out of it: where you wanna start, where you wanna end. And that takes time because sometimes you think you're getting a sentence and you garbled it and have to go back a few seconds. Or sometimes you get too much of the [school] bell, and you need to forward for a few seconds. There's a little bit about the video here: when it's playing, when you wanna change it, you click on this green thing and then it picks the time that's currently on the video . Otherwise I could change this if I just typed zero in there and start from the beginning. And when I finish	

	editing, I have to hit update, it won't prompt me. It's not like word. If I don't hit update, it won't update. So a couple of um many many times I make changes, I did not hit update and then wondered where they were laughs. So there's that and you also have to then save the whole analytic.	
JL	- Let's create the link together, ok? I am gonna close this because I don't wanna change it. Just close. See it didn't say you started to add..[inaudible].. you want so just say you want to close? Fine. And go back here. This is closed, and I am going to say…I am gonna first search for video. So, does everybody want to see more about Davis or more Ariel or something else entirely?	
4	: What do you have that's different from what we've seen already?	Wb
JL	: Oh that's different from what we've seen ?	
4	: Yeah, what do we have?	F
JL	:Oh there's combinatorics, there's um..How about the Binomial theorem? You see those little kids that were doing the guess my rule?	
4	: mmmmhhh	F
JL	: When they are older, they are doing the Binomial theorem. That's zero results. That can't be right. Ok, let's see. Stephanie I think? She is doing Algebra.	
3	: This is all from the collaborative videos?	F
JL	: This is from the VMC, the Video Mosaic Collaboration.	
3	: All from Rutgers?	Wb

JL	: It's started by Rutgers but I believe others can deposit in there, but I still don't know the processes for getting video in there. Here it is! Early Algebra ideas about Binomial Expansion. Stephanie's there. Stephanie was one of those secret discoverers when she was little.	
3	: Alright!	F
JL	: So, let's say I wanna create a new thing. I gonna create a new thing "Old Stephanie does the Binomial dance" and this is a demonstration analytic. And I am gonna pick that video. Notice I can play it here. That's my advisor, Dr. Maher	
3	: mmmm	F
JL	: I am going to stop this here, because what I wanna do to get in the editor I say open. Now it is in my editing space. I found it, I opened it. It will let you play in this space because maybe it's not what you want. Maybe you will go back and look for something else. But, once I decide it's what I want, I open it, and it's here.	
JL	- I am gonna pause and I am gonna create it again. I lost that when I opened. Let me see. "Stephanie does the Binomial dance" demonstration analytic and I am gonna worry about where I want to start. And I am looking at where I am gonna start my [inaudible].	
JL	: Now, see I clicked the "Start" key? I can stop this from going and I add it. And now I have an event here. And if I want to see what the event looks like, I just click here. So it's going to start where I told it to start and it's going to play all the way through.	
3	- So if you are slipping back and forth	F
JL	#NAME?	
2	- Ah ok!	F
JL	: And so, you know, I add lots of description, you know whatever, and then you update it, and then you have it. And then you can pick more from this video or you can say well, you know, I will go back here and find a different video of Stephanie's. So, How about Stephanie and the..... um "the towers problem"? Let's get her when she was little. The 'Gang of four', here we go! So I decide I wanna look at this to see if I wanna use it. [Analytic plays a little]. I definitely want to use this, this is very cute. Let's use this. So I open it	

	up, and now, it's kept the clip from the other place. And I can decide to create a new event [analytic playing].	
JL	- Let me put (0:00) here [a little laughter].	
JL	- Here they were younger than they were when they were solving the 6th grade algebra. She has big glasses, so you can see ....Pick an "end time" and I add it. I have another...	
2	: Do you see Dr. Alston..she still works there????[Yeah yeah]	Wb
JL	: Yeah.That's Dr. Alston. She's retired, but she's on my committee. Yeah, she's active, she does research. She is ....teaching	
2	and others???: Oooooh! I love her!	Wb
JL	- I love her too. She lives in Maine. I'm so jealous.	
JL	- So now, all you have are these events. If you wanna save it as an analytic you have to provide a title: "Stephanie's demo analytic". Now you have the analytic. It's not an analytic until you get the events in there and group them together under this title. They make you, to put this in the library, they make you keep [inaudible] one of those things, and you just click on,…I have pretty long descriptions you will see them in mine, not because I think you want them. I am thinking I'll publish them and I'll do some of the work now; I stuck it in there. So, long and detailed and hopefully clear references, etc. And then once you do that, then you update it. Now you have a new analytic and if you search your workspace it should show up there.	
JL	- So, sometimes there's a little bug here and it doesn't show you what it does. Come on, now it's not showing..did I save that? And now I close it. Now I search. There it is: Stephanie Demo analytic.	
JL	- I don't expect to remember how to do it from this. Just that it's easy, it's really not a lot. You just have to play with the tool and you have to....there's a help document online, and you can e-mail me if you are	

	having trouble with something. I will try to help you and you can make your own analytics.	
JL	- And…I was thinking that, unless you want me to show you something else, then, probably you should just,.....you know those sheets you wrote the scrap paper on? If you could take them out carefully, fold them, put your name on the outside, then I will organize them later. Because I don't wanna have to put your name on..[inaudible] for you. Put all your scrap paper. Just fold it into one packet. I'll be able to scan them into one place if you have your name on it.	
5	: [inaudible]	F
JL	: I am not closed yet but if you want them back, I will give them back to you after scanning them. That's up to you.	
2	: We can keep our lesson plan, right?	P
3	: What about the lesson plans?	PQ
JL	: Um, whatever part that you have, I want it and whatever part you want, let me know because I wanna see what it is now and I can give back to you if you want it .	
3	- Yeah, that's fine	FY
JL	:-That's what I wanted. I will do that.	
JL	- Wow! You did it…do you have any questions for me about...You could always email me or put any questions on Sakai.	

APPENDIX E: An Adaptation of Ariel's Algorithm

Ariel was attempting to use his algorithm for a 60-step ladder. The following shows one way he could have done so accurately. This was discussed by the TAW participants as they worked to understand when Ariel's heuristic solution worked and when it did not.

1. For 12 steps: Starting with 6 steps and 20 rods, add another 6 steps and 20 rods, and subtract 2 to correct for the actual number of rods needed: 20 + 20 – 2 = 38. 38 = 3(12) + 2, which is exactly what the explicit rule produces. Now do the same thing 8 more times
2. For 18 steps: Add 6 steps, 20 more rods, and subtract 2. 38 + 20 -2 = 56 rods. 56 = 3(18) + 2.
3. For 24 steps, Add 6 steps, 20 more rods and subtract 2. 56 + 20 - 2 = 74. 74 = 3(24) + 2.
4. For 30 steps, Add 6 steps, 20 more rods and subtract 2: 74 + 20 – 2 = 92. 92 = 3(20) + 2.
5. For 36 steps, Add 6 steps, 20 more rods and subtract 2: 92+ 20 – 2 = 110. 110 = 3(36) + 2.
6. For 42 steps, Add 6 steps, 20 more rods and subtract 2: 110 + 20 – 2 = 128. 128 = 3(42) + 2.
7. For 48 steps, Add 6 steps, 20 more rods and subtract 2: 128 + 20 – 2 = 146. 146 = 3(48) + 2.
8. For 54 steps, Add 6 steps, 20 more rods and subtract 2: 146 + 20 – 2 = 164. 164 = 3(54) + 2.
9. For 60 steps, Add 6 steps, 20 more rods and subtract 2: 164 + 20 – 2 = 182. 182 = 3(60) + 2.

APPENDIX F: Miscellaneous

An Invitation to Algebra 1 and Pre-Algebra Teachers:

Participate in a Dissertation Research Project

Investigating a Model of Professional Development for Algebra Teachers of Low SES16, Minority Students

Please consider volunteering to participate in workshop that will use video narratives of student learning to support professional development of algebra and pre-algebra teachers. The 4 half-day workshop will be held on August 8th through August 11th.

In each workshop, Teachers will work in small groups to explore a rich algebra problem[17]. Then they will view a video narrative that shows one or more students exploring the same problems. The teachers will then discuss the video narrative and how they might use the same math problem in their own practice.

> The goals of this research are to study math teachers as they (1) explore and respond to a new pedagogy that focuses on engaging students in building mathematical understanding; (2) create lessons that encourage student exploration, collaboration and justification of solutions to problems; and (3) share their reports of the implementation of lessons with each other. The research will also examine the effective use of algebra video narratives as tools to support teachers in raising their expectations of low SES students who are placed in low-level algebra classes.

[16] Low SES refers to Low Socio-Economic Status
[17] These problems are CCSS compliant and the pedagogy in the video stories reflects the practice standards.

The teachers who participate in this study will receive 16 hours of professional development service for their participation and a $500 honorarium from the Robert B. Davis Institute for Learning at Rutgers's Graduate School of Education.

If you would like to participate and/or if you have questions about the study, please contact Joyce Leslie at joyce.leslie@gmail.com. **All inquiries must be received by June 30th**.

First Day Questionnaire

Name____

Instructions: For multiple choice questions circle the number/letter to indicate your choice. For free response questions, use the indicated space; use extra paper if you need more than the space that has been provided.

1. Please indicate which of the following math classes you teach; check the box left of each class you are teaching now. Indicate how many years of experience you have you teaching this class in the box to the right.

		Years of Teaching Experience
	Pre-Algebra	
	Algebra 1	
	Algebra 2	

2. Identify the levels of algebra 1 and/or pre-algebra classes that are given in your school (or explain that all classes are heterogeneous multi-level classes):
   ____
   ____
3. Based on your answer to #2, identify the lowest level of algebra 1 or pre-algebra that you teach:____
4. Identify the textbook(s) and curricular materials you use in the courses you teach:
   ____
   ____
   ____

In questions #5 and #6, you will read a statement of opinion and you are being asked indicate your level of agreement.

5. I enjoy teaching algebra (or pre-algebra).
   I. Strongly Agree
   II. Agree
   III. N/A
   IV. Disagree
   V. Strongly Disagree

6. My students enjoy learning algebra (or pre-algebra).
   I. Strongly Agree
   II. Agree
   III. N/A
   IV. Disagree
   V. Strongly Disagree

7. Many things impact teacher decisions about instruction. Rate the factors listed below from 1 to 4, where "1" is the most important factor, and "4" is the least important factor.
   a. _____Student knowledge and student work
   b. _____Student behavior
   c. _____Textbook/Curricular materials
   d. _____School/State test requirements

8. List questions or concerns you have about teaching Algebra/Pre-Algebra in 2016-17.