BLOCKCHAIN ADOPTION AND DESIGN FOR SUPPLY CHAIN MANAGEMENT

by

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ABSTRACT OF THE DISSERTATION

Blockchain Adoption and Design for Supply Chain Management

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In the current business environment, firms are eager to adopt new technologies as they observe more and more successful business applications thereof. One of the disruptive technologies, *Blockchain technology* (BCT), is currently drawing public attention owing to the cryptocurrency phenomenon for which BCT serves as the backbone technology. In view of certain innovative features, BCT holds out the promise of impacting *Supply Chain Management* (SCM), both operationally and financially.

This Blockchain-centered research investigates the impact of BCT on SCM operations as well as its strategic adoption. It provides a holistic treatment and analysis in terms of: (1) Reviewing BCT and comparing it with peer technologies; (2) investigating existing and potential applications of BCT; and (3) identifying business benefits and the impact of BCT, including safety, cost savings, demand growth and yield improvement. Finally, we extract and explore useful managerial insights via rigorous modeling and analysis, and extensive numerical studies.

Combining the strategic decisions on BCT adoption and operational decisions pertaining to supply chains, we develop two mathematical models: (1) a Blockchain-enabled Newsvendor model, and (2) a *Dynamic Programming* (DP) model. With the objective of optimizing total expected profit, a Newsvendor model is developed to study how BCT adoption impacts optimal inventory decisions and ultimately how to determine optimal BCT adoption. The model is illustrated with closed-form solutions for selected demand distributions, specifically, Uniform and Normal distributions.

We construct a DP model underlain by a generic stochastic model, where the firm seeks to maximize the total expected discounted profit by jointly managing (1) *Blockchain design*, (2) *production or ordering decisions*, and (3) *dynamic pricing and selling*. We first show that the deployment of BCT can assist firms in reducing order quantities, lowering selling prices and reducing target-inventory levels. It is also shown that higher volatility in either supply or demand lowers the expected profit as compared to lower-volatility counterparts. Our numerical study produces useful managerial insights. For example, some types of goods (e.g., credence goods and experience goods) greatly benefit from the adoption of BCT, but it may not prove beneficial to leverage BCT for certain other types of goods (e.g., search goods). Finally, considering the lifecycle of typical experience goods, it is recommended to adopt BCT as early as possible.

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Chapter 1

Introduction

Blockchain Technology (BCT) has been widely embraced as a disruptive technology, though it is still in a nascent stage; cf. Babich and Hilary (2018). Indeed, the path to broad Blockchain adoption looks bright and promising. In a most recent survey for the PwC Annual Report 2018, out of 600 executives from 15 territories, 84 percent claim their organizations have had at least some involvement with BCT. To seize on this innovative and disruptive technology, companies have either dabbled in the lab with BCT, or have started to built proofs of concept; cf. PwC (2018). Gartner Inc. projects that Blockchain's value-added business will grow to \$176 billion by 2025, and that BCT will generate an annual business value of more than US \$3 trillion by 2030; cf. Piscini et al. (2017). Optimistically, it can be imagined that 10 percent to 20 percent of the global economic infrastructure will be running on Blockchain-based systems by that same year (*ibid*). Therefore, the ability to deploy BCT to create the next generation of digital supply chain networks and platforms will be a key enabler of business success; cf. Pawczuk (2017).

This disruptive technology and its proliferating implementations have also attracted the attention of academia. As highlighted in Simchi-Levi (2018), it is imperative for the MS/OR community to consider "*the impact of emerging technologies such as Blockchain and the Internet of Things (IoT) on the management of operations and supply chain*". Recently, Babich and Hilary (2018) provides a broad depiction on the research directions of BCT in the *Operations Management* (OM) field. In particular, it is emphasized that OM researchers can apply insights from the literature to quantify the value of the Blockchain technology in operations. This study reaches out to this academic call and the industry need in a timely manner.

1.1 Introduction of Blockchain Technology

Blockchain technology refers to a distributed database that maintains a continuously-growing list of data records in chronological order that are secured from tampering and revision. It consists of blocks holding batches of individual transactions. Each block contains a timestamp and a link to a previous block; cf. Nakamoto (2008), and Kim and Laskowski (2018). Traditional business models maintain the entire history of activities in a single centralized database, which is very vulnerable to cyberattack. BCT distributes databases (ledgers) to all users, which introduces the consensus mechanism concept; since it is very difficult to attack multiple databases simultaneously, the Blockchain system is believed to be relatively secured and transparent. The feature of consensus in Blockchain systems eliminate any concern that a single centralized organization may manipulate transaction information, or demand high fees for indispensable services, etc. Fig. 1.1 depicts a generic Blockchain system process.

Smart Contract is one of the well-known applications enabled by BCT. Smart Contract allows contracts to be automatically enforced and executed immediately in real-time when predefined conditions/terms are met and verified. One potential Smart Contract application is in the art industry. Taking the music industry as an example, a Smart Contract enables royalties to be automatically distributed among artists and songwriters in real time, based on a predefined agreement between the parties. With the aid of Smart Contract implementation, BCT adoption could secure high-quality supply and enhance both the production and yield.

There are two types of Blockchain technology: public (or open) Blockchain and private (or permissioned) Blockchain. The type of Blockchain is defined based on the characteristics of Blockchain users and rights assignment to the users. The user rights can be categorized into three types: reading, writing and validation. The right of reading allows users to view histories of activities; the right of writing allows users to commit activities in the system; the right of validation allows users to verify activities within the system. Public (open) Blockchain is open to the public, which means that everyone is granted the rights of reading, writing, and validation in the public Blockchain. Bitcoin is an example of public Blockchain. Private Blockchain is only accessible to authorized participants, and only authorized participants have the right to read the history of activities; only a subset of those participants may have the right of writing and validation, depending on different business needs. Public Blockchain is referred to as a democratic network without dominating power by using a consensus mechanism; private Blockchain is referred to as a discriminating network with a hierarchical permission system. Babich and Hilary (2018) describes Blockchain design as an art to balance between data verifiability, resource efficiency and the optimal privacy level.

In general, BCT has the following salient features and advantages:

(i) *Transparency*: Because a Blockchain utilizes the concept of distributed consensus, all its users are capable of reading the entire history of activities, which in turn greatly enhances data transparency.

(ii) *Traceability*: Access to timestamped records allows users to effectively and efficiently trace information history.

(iii) *Security*: A distributed ledger greatly increases the difficulty of staging a cyberattack, which significantly strengthens data security.

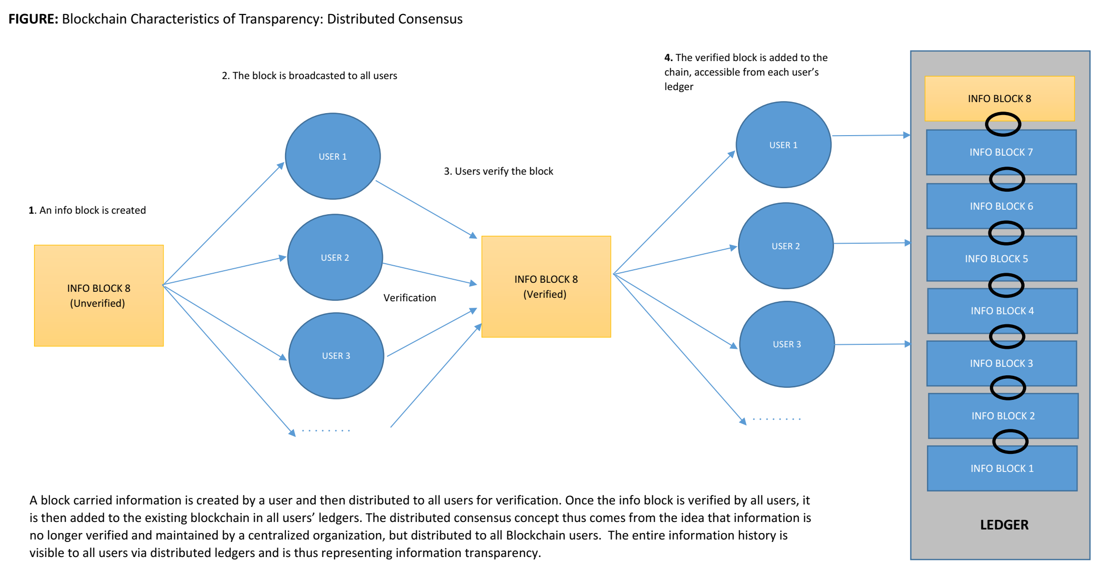
(iv) *Efficiency*: Because BCT obviates the need for a centralized database, disintermediation can be achieved. That is, it is no longer necessary to have a trustworthy intermediary, such as a bank, to maintain the database; hence, both transaction processing time and cost can be significantly reduced.

(v) *Confidentiality*: A Blockchain's decentralized ledger greatly enhances security and transparency; however, it raises other concerns as to confidentiality, since every user on the network can view all activities. As a result, BCT tries to preserve the privacy of users and their data by using pseudonymous addresses and advanced cryptography to hide some aspects of their activities.

(vi) *Immutability*: Once a transaction or activity is validated by a Blockchain system, it can no longer be reversed or amended. In view of this, the integrity of its data can substantially reduce the cost of auditing.

Given the aforementioned characteristics of *transparency, security, efficiency* and *immutability*, BCT remarkably strengthens trust among participants. If effectively applied, it is capable of guaranteeing that all the information accessible by participants is reliable and has not been subject to tampering in any way, which

Figure 1.1: The Blockchain Process



greatly resolves the problem of information asymmetry.[1] It is important to note that stand alone BCT only has the functionality of data storage, and it has to be integrated with other applications, e.g., Internet of Things (IoT), (Radio Frequency Identification) RFID, etc. to extend BCT functionality to acquire data. Therefore, BCT can guarantee the authenticity of information after the information being input in BCT. However, BCT can not prevent information being tempered in the data acquisition phase.

On the other hand, these favorable characteristics of BCT engender some restrictions. Firstly, confidentiality becomes a significant issue. Although some confidentiality techniques are applied, a certain level of privacy is inevitably

[1]World Bank asserts that information asymmetry, as the adjective indicates, refers to situations, in which some agent in a trade possesses information, while other agents involved in the same trade do not.

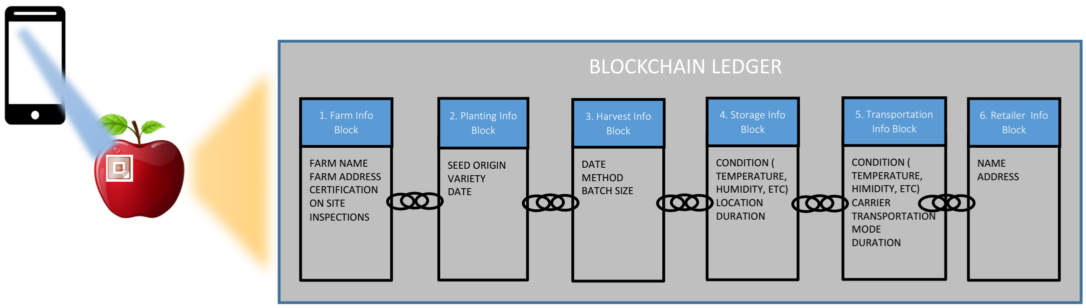
compromised due to the nature of BCT. Distributed ledgers allow all users to view histories of activities; even though users are anonymous, privacy information can still leak in some way.[2] Secondly, although the immutability feature of BCT ensures data integrity, it also brings about some restrictions to business applications. For instance, if BCT is applied to transaction processing, information immutability could cause problems when processing product return and refund. Thirdly, scalability poses another potential issue. The current BCT is very energy intensive and requires much repetitive work in the back end to broadcast the transaction information across the entire Blockchain network. Therefore, when the number of users grows, the Blockchain system encounters a scalability issue. Fourthly, regulatory sectors are struggling to develop an effective system to regulate BCT and its related systems. Fifthly, legacy system integration issues are important. Many, if not all, business have their existing business system, and thus the integration of BCT with the legacy system becomes a major challenge. Last but not least, the passive role of BCT reflects some latent limitations. Data acquisition and disclosure are different from data storage. Although BCT exhibits strong capability in data storage by securing information from tampering and revision, BCT can only passively storage data, instead of actively acquiring and disclosing data. BCT has to integrate with RFID paired with auto-ID or

[2]Zerocoin Electric Coin Co. has developed "Zero knowledge proofs", which claims to enable state-of-art privacy features. But it still requires further assessment.

other equivalent technology (e.g., Internet of Things) to actively capture data. Unable to actively publish information, BCT requires interested individuals (e.g., consumers) to actively access the information from BCT databases. In other words, although BCT can guarantee that the information being carried is secured from tempering and revision, information manipulation might still emerge in the data entry phase. After all, Blockchain can be considered as an information interface or a machine, which passively accepts information input by human beings without auditing. The conventional system requires information auditing at numerous check points (e.g., money transfer between banks, cargo shipment between ports of entry, etc.); the Blockchain system replaces those check points with security and immutability, while still susceptible to data manipulation when information is entered into the system.

The first widely known case of using BCT is in the financial services area, namely, the introduction and proliferation of Bitcoin as a now established cryptocurrency. The aforementioned features of BCT enable Bitcoin to process transactions in a highly secured and efficient way. Furthermore, Bitcoin's legendary (though controversial) success encourages people to think of the possibility of applying it in other contexts, such as in supply chain management. Fig. 1.2 describes an application of BCT for organic foods, such as apples.

FIGURE: Blockchain Characteristics of Traceability – Organic Apple Example



The accessibility of information for organic apples has been theoretically and empirically proven to be critical for market growth. With Blockchain technology, an organic apple may carry traceable and immutable information of the entire history from farm to market place, which can stimulate customers' willingness to pay and boost market growth.

Fig. 3 Bitcoin Price History Chart from 2010 to 2019

In the context of finance, Bitcoin exemplifies a BCT application. Fig.1.3 demonstrates price change of Bitcoin through the recent decade. In May 2010, 10,000

Bitcoins were worth merely 2 pizzas from Papa John's[3]; within 7 years, Bitcoin price reaches its historical high level, $19,783.06 USD in December 2017.[4] There is no other tradable asset in the history that has ever experienced such dramatic value soaring in such a short period of time that has immediately drawn public attention. A growing number of criticisms arise accordingly toward the issues and position of Bitcoin, e.g., investment, currency, or pure entertainment (e.g., gambling). In an attempt to have a better understanding of these questions, it is necessary to review the background of currency evolution. The emergence of government issued currency is a replacement of barter, which is an inconvenient way for trading. While currency, functioning as an indirect exchange for commodities, efficiently resolves the issue of inconvenience of barter, it faces a critical problem, forgery. At the end of American Civil War, one third of American currency was counterfeit. Reportedly, counterfeit money removal from the U.S. market had grown substantially from $61 million in 2005, to $261 million in 2011.[5] In addition to the forgery issue, physical currency suffers from the problems of being stolen, costly monetization and transportation. Therefore, digital currency emerged in an attempt to overcome those shortcomings, and an authorized intermediary, i.e., bank, is required to maintain and manage the

[3]https://techcrunch.com/2016/01/02/why-bitcoin-matters/

[4]http://fortune.com/2017/12/17/bitcoin-record-high-short-of-20000/

[5]http://itsamoneything.com

digital currency system. The current banking system has been fully developed and remained relatively mature for almost a century. However, with the rapid innovation and revolution of technology in recent years, financial institutions have become a major target for cyberattack. In March, 2016, it was reported that Bangladesh's Central Bank lost $101 million in a bank heist that spanned at least four countries.[6] In December 2016, hackers stole 2 billion rubles, equivalent to $31 million, from accounts that banks keep at Russia's central bank [7]. These increasing cybercrimes draw public attention to the existing banking system's vulnerability to cyberattack, and Bitcoin comes to light as a plausible solution for financial security.

In 2008, a research paper, titled "*Bitcoin: A peer-to-peer Electronic Cash System*", was published by an author, who claimed the name Satoshi Nakamoto. On January 3rd, 2009, Bitcoin system was released as an open source project, and in 2010, the first physical product being transacted using Bitcoin was for two pizza pies purchased with an amount of 10,000 Bitcoins.

Bitcoin applies the concept of decentralized public ledgers in BCT in order to remove intermediary, i.e., financial institutions. Bitcoin introduces peer-to-peer networks that timestamp transactions by hashing them into an ongoing chain

[6]http://money.cnn.com/2016/03/15/technology/bangladesh-bank-new-york-fed-bank-robbers-resign/

[7]http://money.cnn.com/2016/12/02/technology/russia-central-bank-hack/

of hash-based proof-of-work, forming a record that cannot be changed without redoing the proof-of-work; cf. Nakamoto (2008). The introduction of Bitcoin directly challenges two existing systems, government and banking. Government controls the supply of the traditional currency, and uses it accordingly as a tool for monetary policy. However, Bitcoin's supply is purely dependent on "coin mining", over which government has no power at all. In other words, if Bitcoin becomes a type of dominant currency, government would lose the critical monetary tool to achieve desired political goals, e.g., lowering interest rates, boosting investment and the economy, etc. Additionally, the functionality of financial institutions would be greatly downplayed (if not entirely replaced) by Bitcoin's decentralized system. The secured and transparent system of Bitcoin obviates the functionalities of transaction verification and processing, prevention of double spending, etc. of centralized banks. However, some of functionalities performed by banks are believed to be very difficult to be replaced by Bitcoin (Harwick (2016)), e.g., loan and credit evaluation, borrowing and loaning channeling, risk management, etc. Thus, financial institutions are exploring opportunities to develop Blockchain protocols for their business models. For example, nine of the world's biggest banks including Barclays and Goldman Sachs havd joined forces with the New York based financial technology firm R3 in September 2015 in order to create a framework for using BCT in the financial market; cf. Crosby et al. (2016). In general, Bitcoin needs to overcome the major opposing power, stemming from government and banking system, to become a prevailing currency.

In addition to those two aforementioned major opposing powers, Bitcoin is facing several issues.

1) *Price volatility issue*. By observing the price history, it is obvious that Bitcoin is characterized with severe price volatility. The price of Bitcoin jumps from $0.06 in 2010 to its historical high level, $19,783.06 USD in December 2017; cf. Fig.1.3. The daily change in the US dollar-bitcoin exchange rate from 2010 to 2015 has reached nearly 50 percent in both directions, and regularly exceeds 10 percent. During the same period of time, the daily change in the US dollar-euro exchange rate never exceeded 2.5 percent in either direction; cf. Harwick (2016). The major reason behind the price volatility is believed to be dramatic demand fluctuation, given relatively stable and limited supply of Bitcoin. It is estimated that 90 percent of Bitcoin transactions are made for speculation (*ibid*). In an attempt to mitigate price volatility, Harwick proposes to link Bitcoin supply to other macroeconomics variables, e.g., unemployment rate, exchange rate, etc. in replacement of stable supply from mining.

2) *Illegal activity issue*. The important feature of Bitcoin, pseudonym, is applied in an attempt to preserve privacy of users and transactions; however, it may be abused for illegal activities, e.g., money laundry, drug dealing, contraband,

etc.

3) *Regulation issue*. The swift development of technology makes regulatory section very difficult to keep up with, and the development of Bitcoin is no exception. Additionally, the facts that Bitcoin is such a currency not backed neither by government nor by a physical commodity such as gold and that Bitcoin is abused in many illegal activities, make many legislative units devoted to monitor and regulate the industry for compliance, and to prevent illegal activities. Furthermore, the regulation must be agile, flexible, active, and balanced against concerns over stifling innovation; cf. Tsukerman (2015).

4) *Scalability issue*. As Bitcoin is backboned by BCT, it is also hindered by scalability concern as mentioned before. In order to ensure security and transparency, the design of Bitcoin requires wasteful replication. The entire transaction history has to be broadcasted to and maintained by all participating nodes, and verification is always repeated; cf. Zohar (2015).

The main benefit of Bitcoin is that it substantially redesigns and reinforces the security of digital banking system; however, it is suffering from a variety of issues such as price volatility, illegal abuse, regulation difficulty, and scalability. Hence, it remains a hot topic, in both industry and academia, regarding the possibility of Bitcoin or other cryptocurrency to become a dominant currency. Extending from its home area of BCT, many efforts have been made to explore the potential application of BCT in *Supply Chain Management* (SCM).

1.3 Blockchain for SCM

Given the aforementioned advantages of BCT, including transparency, traceability, security, efficiency, and immutability, BCT can be applied in *Supply Chain Management* (SCM) in four aspects, including food safety, solution to information asymmetry, transaction cost savings and supply yield improvement.

Firstly, BCT has been embraced as a good way to promote food safety. The recent outbreaks of foodborne illness (e.g., leafy greens in December 2017, Romaine lettuce in December 2018, etc.) inspire people to think about BCT adoption to accelerate food recall. On average, there is one in ten chances of people falling ill because of food contamination every year, and it is very difficult and time consuming to track the contamination origin. For example, it took FDA two months to track the origin of salmonella-tainted papayas back to a Mexican farm in summer of 2017. In response to that need, *Walmart* and *Cargill* have initiated pilot projects to adopt BCT in their food supply chains to mitigate food safety crises. *Walmart* requires all of its suppliers of leafy greens to adopt Blockchain by September 2019. It is reported by *Walmart* that the source tracing time is exponentially reduced from about 7 days to 2.2 seconds with the aid of BCT. *Cargill* adopted Blockchain in 60,000 of its turkeys during Thanksgiving 2017, and expanded to 200,000 turkeys during Thanksgiving 2018. Consumers can access the information of turkey farm location, history, photo, etc., through a simple text or by entering an on-package code at *Cargill* website.[8] The aforementioned advantages of information transparency, traceability and security of BCT are considered beneficial to food safety promotion by mitigating supply chain risk and disruption and accelerating reverse supply chain management.

Next, the salient advantages of information transparency, traceability and security make Blockchain a potential solution to the issue of information asymmetry. It is reported in a survey in 2018 by *Label Insight* and the *Food Marketing Institute* (FMI) that 75 percent of consumers intend to switch to a brand with more in-depth product information, compared with only 39 percent in 2016.[9] Combined with the growing desire for information transparency and the escalating awareness of social responsibility from consumers, BCT is considered as a powerful tool to disclose truthful and secured information to the market. *Martina Spetlova*, a luxury fashion brand, adopted Blockchain to disclose its commitment to environmental sustainability and ethical sourcing. Through a Blockchain platform, *Fuchsia*, a handcrafted shoe maker, broadcasts its promise to ensure fair working conditions of artisans in Pakistan. In other words, Blockchain can be exploited as an innovative marketing tool to promote market growth.

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[8]https://www.supplychaindive.com/news/cargill-expands-blockchain-for-turkeys/541744/

[9]https://www.fooddive.com/news/report-consumers-want-increased-transparency-from-retailers-and-brands/532723/

Additionally, it has been shown, both theoretically and empirically, that the information availability can effectively promote market growth; cf. McCluskey (2000), Giannakas (2002), Rousseau and Vranken (2013). Taking organic food as an example with regard to the credence attribute, consumers are not able to differentiate organic foods from conventional foods, either before or after consumption. In addition, the production cost of organic foods is significantly higher than that of conventional foods. As consumers have no means of differentiating between them, they are only willing to pay the same price for both kinds of products, which in turn provides no incentive for suppliers to produce organic foods, and thus the organic food market would probably fail eventually. In order to stimulate the organic food market, a considerable amount of research proposed that labeling certified by an authorized third party (e.g., FDA and USDA) is the only solution for providing organic product information that will help consumers differentiate. Rousseau and Vranken (2013) show that consumers are willing to pay an approximately 25 percent price premium for labelled organic apples. With the additional provision of information on the actual environmental health effects of organic apple production, the price premium further grows to about 42 percent. This business practice reveals two crucial aspects on price and demand: First, consumers are *willing to pay* extra for organic foods; second, the provision of information would boost demand.

Thirdly, the disintermediation of BCT can greatly reduce transaction costs. In 2016, *Wave* created a Blockchain-based platform to facilitate documentation exchange in the international trade process, and launched a project with *Barclays* to test a transaction of about US$100,000 of cheese and butter. The length of processing time was considerably reduced from 7-10 working days to 4 hours. The entire process includes creation of digital documents (i.e., certificates of origin, certificates of insurance, commercial invoice, bill of landing, etc.) and exchange of cryptographically signed documents between relevant parties, including *Barclays*, the importer and the food retailer. Such lead time saving can be easily translated as cost saving; especially for perishable goods with short shelf lives, any reduction in lead time would bring a tremendous benefit. Financially, disintermediation of BCT improves and facilitates fund circulation, which further cuts down cost (i.e., borrowing, transaction costs, etc).

Fourthly, BCT adoption can help business improve supply yield. In 2016, *AgriLedger*[10] developed a Blockchain-based system to record crop yield of coffee beans with an objective to mitigate crop loss during the transaction process. Recently, it has initiated pilot programs in Kenya, Myanmar and Papua New Guinea. Although a cooperative system has been created in those developing countries to pool small scale farmers to enhance negotiation power with coffee

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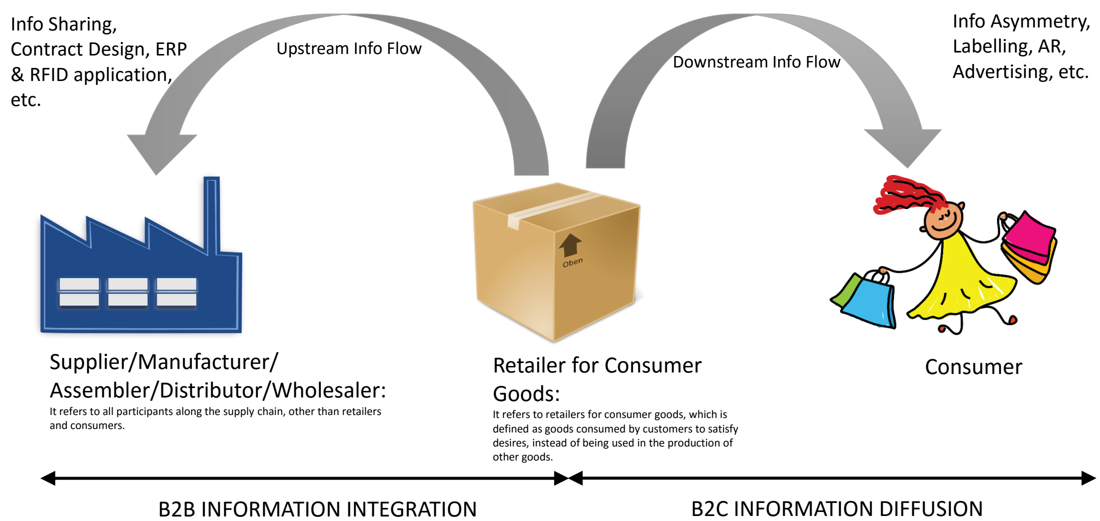
[10]https://www.linkedin.com/pulse/b3-blockchain-interview-founders-agriledgercom-application-lea/

bean traders, it is still very common to have human caused losses of crop due to inefficient and fraudulent paper work, informal verbal agreement, etc. Studies by UN show that up to 50 percent of crop value evaporates between the point of crop harvested and the point of sale. BCT is thus believed to improve yield by providing reliable and transparent data of crop, since any data discrepancy in the transaction process can be easily traced and monitored through the Blockchain system.

To conclude, BCT can achieve improved food safety, market growth, cost savings and supply yield improvement via enhancing traceability along the supply chain, providing secured and truthful information to the consumer market and strengthening business efficiency. Therefore, it is believed that Blockchain adoption would be a viable strategy to promote information transparency along supply chains. However, such assertion would not be convincing without evaluation of alternative information systems. In what follows, we divide the information flow along the supply chain into business to business (B2B) information integration and business to consumer (B2C) information diffusion. Fig.1.4 illustrates that the B2B information integration system represents information sharing with upward supply chain partners (e.g., suppliers, manufacturers, assemblers, distributors, wholesalers, etc.); the B2C information diffusion system represents information disclosure downward to consumer markets. Based on such definitions, we next evaluate alternative B2B information integration system, e.g., ERP (Enterprise Resource Planning) and RFID, and the B2C information diffusion system, e.g., AR (Augmented Reality), labeling advertising, etc.



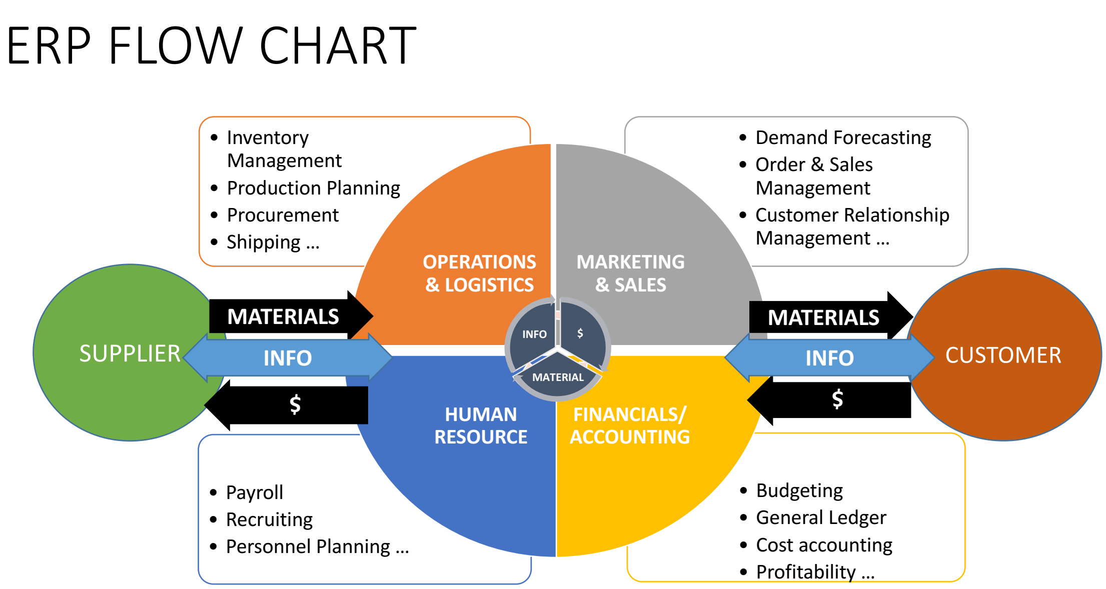
1.3.1 Options for Information Integration

Realizing the benefits of information transparency, ERP (Enterprise Resource Planning) was introduced in early 1990's to meet the needs of integration of business internal information. ERP streamlines the flows of information, materials and financials across the functionalities of operations and logistics, marketing and sales, human resources, financials and accounting, etc. (Fig. 1.5) within a business. Through internal information integration, a business would be able to improve business efficiency by reducing redundancy and waste, promote cross-departmental communication by streamlining business processes, improve information transparency to facilitate strategic decision making, etc. A further enhancement by automatic technology, e.g., barcode, RFID (radio frequency identification system), Internet of Things (IoT), etc., can be incorporated to ERP to further strengthen its capability in inventory management. However, the capability of ERP to achieve business to business (B2B) information integration is unsatisfactory. With the original focus of internal information integration of a business, ERP's capability of extending to incorporate external information (B2B) is possible but very limited. External, cross-company and cross-database information integration of ERP can only be fulfilled by EDI (Electronic Data Interchange), and "the cost of integrating these different elements can be significant and increases as the number of nodes (company/databases) in the network increases" (Babich and Hilary (2019)). So we can say that information integration with business partners along the supply chain is achievable by ERP, but in a very difficult and costly way.

Additionally, the data security level of ERP is limited. Driven by the needs to integrate fragmented information across different departments, ERP digitalizes all the information and maintains it in a centralized database. However, a centralized database is susceptible to cyberattacks as discussed previously. Especially when it comes to B2B information integration, the security issue becomes more important. Different objectives with its partners give a business plausible incentives to manipulate information to maximize its own interests. The requirement of data integrity is especially rigorous when it comes to documentation inspection in the commodity trade. Therefore, vulnerability of ERP in data security makes it an inferior alternative for information integration in supply chain level across different business partners. In other words, the mechanism of distributed ledger (decentralization) makes Blockchain technology an adequate candidate to achieve B2B information integration.

It is also worth noting that the evolution of information integration has been a first centralization and then decentralization process fueled by different needs. Fig.1.6 demonstrates the evolution of supply chain information systems. The development of information technology enables a paper-based documentation system to be replaced with ERP in order to promote information transparency by maintaining information in a centralized database. However, with the growing incidents of data breach, the issue of data security starts to draw public attention. The decentralized, distributed ledger system, Blockchain, is then developed and considered a viable solution to overcome the problem of data integrity, while maintaining a satisfactory level of information transparency. However, the outstanding feature of data security of Blockchain comes with the cost of limited scalability, which makes it inferior to ERP with the respect to handle ever growing internal business data.

In conclusion, ERP is a valid tool to achieve information integration by incorporating information in a centralized database while compromised with the risk of data breach. Given that the internal interest conflicts would be relatively small and hence, it is less risky to suffer from data tempering within a company, we consider ERP a feasible candidate in information integration in the business level. Blockchain, on the other hand, is considered a better alternative for information integration in the supply chain level combined with ERP's limited capability in B2B information integration and Blockchain's salient advantage of information security. Table 1.1 concludes our analysis of information integration



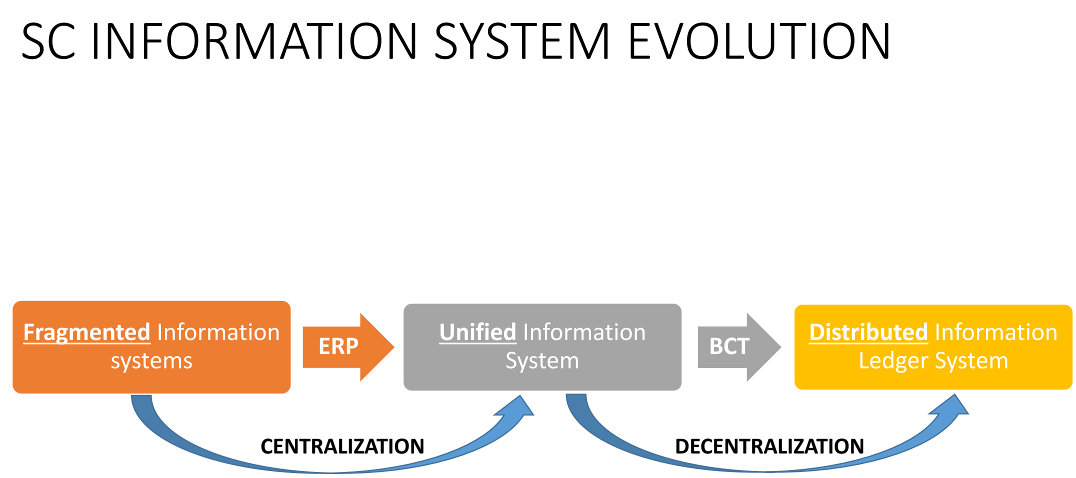


Table 1.1: Summary for information integration

Requirement / Integration Level	Security	Scalability	Technology
Internal	X	✓	ERP
B2B	✓	X	Blockchain

1.3.2 Options for Information Diffusion

According to marketing literature, there are two conventional ways to differentiate consumer product types. First, by different levels of information seeking effort, products can be classified into four categories; cf. Murphy and Enis (1986).

(1) **Convenience product**: Consumers are unwilling to make an effort (e.g., time and money) to purchase convenience products. The risk of making a bad selection is perceived low. Examples of convenience products include laundry detergent, batteries, umbrella, etc.

(2) **Shopping product**: Consumers are willing to spend a certain amount of time and money in research and evaluation of shopping product. Risk associated with bad selection is perceived high. Examples of shopping products include

furniture, clothing, flight tickets, etc.

(3) **Specialty product**: A specialty product exhibits the highest level of selection effort and risk. The monetary price of specialty products is high, and thus consumers are willing to spend a significant amount of time and money to research and evaluate specialty products. Examples of specialty products include luxury cars, professional photographic equipment, etc.

(4) **Unsought product**: Unsought products are those that consumers either are unaware of or do not think about under normal conditions. Examples are new innovative products, life insurance, etc. Compared with the other product types listed above, much more advertising and marketing effort are required for unsought product to draw consumer's attention.

Alternatively, another conventional way to categorize consumer product types is by quality observability; cf. Nelson (1970), Darby and Karni (1973).

(1) **Search goods**: There is perfect information about quality for search goods, which means that consumers can easily differentiate good from bad products before consumption. An example for search goods is computers, which provides detailed product features, e.g., CPU, GPU, RAM, monitor size, etc.

(2) **Experience goods**: It is difficult or costly for consumers to examine the quality of experience goods before consumption; however, consumers would

be able to determine the quality after consumption. Examples for experience goods include cosmetic products, perfume, music, etc.

(3) **Credence goods**: It is difficult or costly for consumers to determine the quality of credence goods even after consumption. Examples of credence goods are organic food, antiques, etc.

We generalize the first consumer product classification method to classify products into "sought-after", including convenience, shopping and specialty products, and "unsought" types of products. For sought-after products, consumers are aware of their needs and actively search for the products to satisfy their needs accordingly. The difference between convenience, shopping and specialty products mainly lies in the varying degree of searching effort consumers are willing to input. Extensive research has been done in the marketing field to devise various strategies according to different levels of consumer searching effort, which is beyond the scope of this study. Therefore, this study simplifies and generalizes the consumer product types by information seeking effort into only two categories, *sought-after* versus *unsought* types. The sought-after type encompasses the products, toward which consumers are very clear about their needs and thus actively (regardless the level of activity) search for the desired products to satisfy their needs. Conversely, for unsought type of products, consumers are unaware of either the products or their needs of the products and thus passively wait for the product information to reach them in a certain way, e.g., via mass media, social network, etc. In other words, given the active role of consumers, the sought-after type of products only requires passive product information providers; given the passive role of consumers, the unsought type of products requires active product information providers to actively reach passive consumers. For example, when innovative products, e.g., *iRobot*, were first introduced to the market, consumers are unaware of such products, and by definition those products are classified as unsought-type of products. Diffusion of Innovation theory (Rogers (2003)) indicates that innovation diffusion processes require some innovators to initiate and then communicate with the participants in the innovators' social networks. In the *iRobot* example, housewife bloggers might take the role of innovators and share unboxing and reviews of *iRobot* with their audience, who might later become consumers (adopters) of *iRobot* (innovation). Adopters then communicate with participants in their own social network, and then innovation starts to diffuse in a ripple effect. The marketing strategy for the unsought type of products is devised on the basis of Diffusion of Innovation theory with the focus on enhancement of communication channels and networks. Once a certain level of innovation diffusion is achieved, the public becomes aware of such innovative products, and then the product type changes from unsought product to sought-after product. Given the nature of a passive information provider, BCT exhibits limitations in the application for unsought products. Only after the transition from unsought to sought-after products is achieved can BCT become effective, the level of which still varies by observability of product quality.

The definition of the sought-after type of products states that consumers are clearly aware of their needs for such type of products and thus the marketing strategy would be different from the one derived from the *Diffusion of Innovation* theory, which strives to enhance market exposure and visibility of products. Instead, the marketing strategy of the sought-after type of products focuses on improvement of product quality observability. As aforementioned, consumer products can be classified into three categories based on the difficulty level of quality observability, including search, experience and credence goods. Different marketing strategies are developed accordingly to overcome such quality information asymmetry. The definition of search goods indicates that consumers are capable of quality differentiation before consumption, and thus textual listing of product details is sufficient to disclose product quality for search goods. One of the examples of search goods is computer products, for which all the quality information, e.g., CPU, RAM, GPU, display, etc. is clear to consumers. Experience goods are those products, which requires customers to try or consume to tell the quality. One of the examples of experience goods is beauty products, which requires customers to try on to feel the product texture, color, etc. The strategy developed to overcome the quality information asymmetry issue of experience goods is to offer free samples or allow customers to try products. The definition of credence goods indicates that it is very difficult or costly to determine product quality even after consumption, and examples include organic food, antiques, wine, high-end seafood, etc. The solution proposed by many researchers for such level of quality information asymmetry is certification or labelling certified by authorities to ensure availability and truthfulness of information. The functionality of certification or labeling in the provision of authentic and truthful information perfectly fits the features of BCT, i.e., transparency, traceability, security, and immutability. Based on those features, we believe that BCT can serve as a perfect alternative for labeling to disclose quality information and to further guarantee information truthfulness and authenticity for credence goods.

With the rapid advance of technology, there has been a dramatic change in business model development. Online retail, which emerged in 1990s with Internet (and now garners 14.3 percent of total retail spending and is growing at a rate of 18 percent in 2018 in the U.S[11]) exacerbates the issue of quality information asymmetry, especially for experience goods. A major purpose of consumption is to satisfy sensory pleasure, including sight (e.g., apparel), hearing (e.g., CD), taste (e.g., desert), smell (e.g., perfume) and touch (e.g., bedding). Online retail enjoys the advantages of virtually infinite shelf space and 24/7/365 service at

[11]https://www.digitalcommerce360.com/article/us-ecommerce-sales

the expense of sensory information loss. It is very difficult (if not impossible) to provide sensory information (other than pictures) to online customers, while the nature of experience goods requires customers to differentiate quality by consumption/use. Considering the high cost of reverse logistics, *Augmented Reality* (AR) is thus applied to overcome the barrier of sight or aesthetic information asymmetry in an attempt to reduce product return. AR is defined as "*the superposition of virtual objects on the real environment of the user*" (Faust et al. (2012), Pg. 1164). In addition to the salient advantage of visual and aesthetic information provision, AR has the advantages of fast information disclosure and customer interaction enhancement. Consumers only have bounded rationality, and are inclined to sacrifice decision accuracy for savings in information processing efforts (Shugan (1980), Bettman et al. (1990), Johnson (1985), Simon (1955)). Combined with the fact that there is a lag for the consequences of inaccurate decision to emerge, consumers are more inclined to "*reduce cognitive effort than improving decision accuracy*" (Haubl and Trifts (2000)). Hence, "*fast information disclosure*" of AR effectively satisfies the needs of efficient information processing by consumers. Yim et al. (2017) indicates that "*AR-based product presentations are generally superior to traditional web-based product presentations in the effect on media novelty, immersion, media enjoyment, usefulness, attitude toward medium, and purchase intention*". Therefore, AR serves as a good solution to compensate for visual and aesthetic information loss stemming from the nature of online retail.

Combined with the fact that AR-base presentation transforms customers from passive product observers to active product experiencers by providing vivid product interaction, which in turn enhances customer shopping experience and enjoyment. Ultimately, AR successfully promotes customer purchase intention. Although BCT can mitigate the information asymmetry issue to some degree for experience goods by providing textual (e.g., customer reviews) and image (e.g., product pictures) information, it lacks of functionality of customer engagement and interaction, which limits its application in quality disclosure for experience goods.

In summary, consumer products can be classified into unsought and sought-after products in terms of information seeking effort, or classified into search, experience and credence goods in terms of observability of quality. When a new (innovative) product is introduced into the market, based on the Diffusion of Innovation Theory it requires some innovators to actively disclose and diffuse the product information to potential consumers through their social network via communication channels. The goal is to make the market aware of such innovative product and realize the needs/desires for such product; once it is successfully achieved, the type of the product transforms from unsought to sought-after. The marketing strategy changes accordingly to focus on quality information disclosure, and the information diffusion effort might change from active to passive. Different quality information disclosure channels are proposed based on the varying levels of quality observability. Textual-based product presentation is considered effective and efficient both for offline and online shopping for search goods, given that consumers are perfectly capable of differentiating search product quality before consumption. Since it requires customers to experience the product to tell the quality of experience goods, it is suggested to offer free samples or try-on for offline retail and adopt AR for online retail to support customer purchase intentions for experience goods. For credence goods, it is very difficult or costly to differentiate the quality. Therefore, it is critical to provide truthful and authentic information to promote customer *willingness to pay* (WTP) and market growth. Although it has been found by many empirical studies (Teisl et al. (2002), Rousseau and Vranken (2013)) that labelling can effectively increases WTP and market share, we believe BCT, equipped with the advantages of transparency, traceability, security, efficiency and immutability, can serve as a superior solution to provide truthful and authentic information than labelling. Table 1.2 provides a summary of information diffusion strategies for different types of consumer products. Incorporating the conclusions made here, we model the Blockchain design problem in numerical experiments by assigning different coefficients to information disclosure levels via BCT; search goods are the least sensitive to information disclosure by BCT, experience goods are next and credence goods are the most sensitive.

	SEARCH	EXPERIENCE	CREDENCE
SOUGHT-AFTER	Textual-based product presentation	Free sampling/try-on or augmented reality (AR)	Labelling or BCT
UNSOUGHT	Objective: To enhance market exposure and visibility of the product. Strategy: To expand communication channel (internet, mass media, etc.) and communication network (social network, etc.)		

1.3.3 Innovations and Limitations of Blockchain-Enabled Information System

Compared with the existing information systems, Blockchain exhibits the following characteristics:

(1) Blockchain is currently the only holistic information system capable of both functionalities of information integration and information diffusion with high information security level. In conventional information systems, B2B information integration and B2C information diffusion are two separate systems with different objectives pursued by different experts. The objective of information integration is to promote transparency within a business unit or among SC participants in order to streamline business processes, improve inventory management, facilitate communication, etc. Typically, it is a focus of SC and IS professionals. On the other hand, the objective of B2C information diffusion is to reach out to markets to disclose product quality to consumers in order to increase market share, promote customer's

WTP, stimulate market growth, etc. Typically, it is a focus of marketing and sales professionals. Blockchain is the only information system that can streamline the end-to-end information flow along the supply chain, from the most upstream raw material suppliers to the most downstream consumer product market while maintaining a high level of information security.

(2) Blockchain provides user interfaces (enabled by barcode or RFID) to consumer product markets, which essentially serve as enablers to successfully combine the functionalities of B2B information integration and B2C information diffusion. The salient feature of consumer accessibility distinguishes Blockchain from its peers (i.e., ERP). Not only is BCT capable of integrating information across organizations within the upstream supply chain, but also accessible by consumers/users to disclose product quality information.

(3) Equipped with the advantages of *transparency, traceability, security*, and *immutability*, it is believed that Blockchain is a viable alternative to labelling to solve information asymmetry issue with credence goods. Not only can BCT provide information to consumers, but it also ensure the truthfulness and authenticity of information, both of which are critical to market sustainability of credence goods.

(4) Blockchain has inherent limitations in information provision and interaction with consumer market. Although Blockchain is equipped with user friendly

interfaces to consumer markets for truthful and reliable information, it is a passive information provider, which requires users to actively access the information. In other words, the application of Blockchain is restricted to only sought-after products, which consumers are aware of and willing to actively access the product information. Additionally, compared with AR, BCT lacks interaction with consumers, because Blockchain can only provide limited sensory and aesthetic information to consumers (e.g., textual information and pictures of products). Based on the definition of experience goods, although experience goods consumers are sensitive to product quality information, Blockchain's lack of interaction with consumers might render unsatisfactory demand growth of experience goods, especially when compared with credence goods markets.

1.3.4 Other Information Technologies Supporting Blockchain Applications in SCM

It is worth noting that the applications of BCT, ERP and AR/VR are different from other information technologies, e.g., RFID, IoT, Cloud, Auto-ID, Big Data, Artificial Intelligence (AI), etc., in terms of functionalities. In particular, Blockchain, ERP and AR/VR are information management systems that facilitate information integration (i.e., ERP for business internal information and BCT for B2B information integration) and B2C information diffusion (i.e., AR/VR for experience goods and BCT for credence goods). In contrast, RFID, IoT, Cloud, Auto-ID, etc., can be applied to acquire data to enhance information management systems; Big Data, AI, etc. are data analytics tools for drawing business insights and facilitating/automating processes of decision makings. RFID and IoT supplement information management systems (Blockchain and ERP) by capturing data, while Auto-ID and Cloud serve as bridges to feed those data into the systems. Automatic data acquisition processes can capture the information of inventory, logistics (e.g., location, shipping company and conditions), etc., and is believed to reduce inventory, improve business efficiency and customer satisfaction, mitigate the Bullwhip effect, etc. Instead of manually creating data in the information management systems, autonomous data acquisition by machines would improve data accuracy, decrease the risk of data breach and reduce the need for data auditing, which further strengthens the information integrity of Blockchain and ERP systems. Any data captured in the data acquisition process or in the information management system can be fed and processed using Big Data analytics to "*draw conclusions by uncovering hidden patterns and correlations, trends and other business valuable information and knowledge in order to increase business benefits, increase operation efficiency, and explore new market and opportunities* " (Tiwari, 2018). AI can be developed to facilitate and automate decision making in the business process, e.g., operations and logistics, marketing and sales, financials and accounting, human resources, etc. One of Blockchain important advantages is data security ensured by a consensus mechanism, but critics have challenged the authenticity of information in the data acquisition stage. Automatic data acquisition enabled by RFID or IoT paired with Auto-ID and Cloud completes the autonomous information management system from input at the origin to output to the end user and thus enhances the data accuracy and reliability. Data analytics systems further explore the potential value of information to allow machines to make informed business decisions by exploiting data captured in the autonomous information management system. Table 1.3 summarizes the *information technology* (IT) applications in SCM covered in this study.

Table 1.3: Summary of IT Applications on SCM

Functionality of IT System	IT Applications
Information Management System	
Information Integration	ERP (Internal)
	Blockchain (B2B)
B2C Information Diffusion	AR/VR (for experience goods)
	Blockchain (for credence goods)
Data Acquisition System	RFID, Auto-ID, IoT, Cloud, etc.
Data Analytics System	Big Data, AI, etc.

Based on the comprehensive analysis of Blockchain application in SCM, including advantages and limitations and comparison with alternatives and compatible technologies, we now draw a conclusion that BCT adoption exhibits the greatest potential in applications for credence, sought-after goods with complicated and costly transaction processes, and data integrity can be further enhanced by autonomous data acquisition systems. In particular, products with the credence feature stemming from the origin of the products would benefit the most from the salient advantage of BCT traceability, such as organic and high-end foods/drinks (e.g., seafood, beef, wine, etc.), diamonds, etc. Businesses requiring complicated and costly transaction processes, such as real estate, international trade, global money transfer, etc., would benefit from the disintermediation of Blockchain technology. According to *Zillow*, on average it roughly takes three months from home listing to closing[12] and the closing cost is 2 percent to 5 percent of house purchase price for buyers and 8 percent to 10 percent for sellers.[13] *Maersk*, the world's largest container shipping company, estimates that shipping flowers overseas require sign-off from 30 distinct organizations and up to 200 communications, and the overall process can take more than a month.[14] For personal international bank transfer, transaction fees range from \$25 to \$85[15] per transfer, and it may take as many as five days to process a transfer[16]. Any simplification of the transaction process and reduction of transaction duration of real estate deals, global trade and international bank transfers can be translated into tremendous

[12]https://www.zillow.com/sellers-guide/costs-to-sell-a-house/

[13]https://www.zillow.com/sellers-guide/closing-costs-for-sellers/

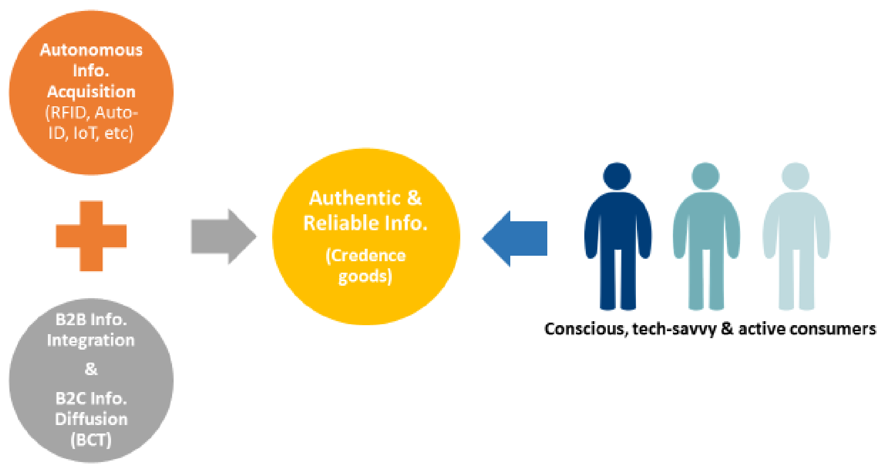
[14]https://www.ibm.com/downloads/cas/VOAPQGWX

[15]https://smartasset.com/checking-account/average-wire-transfer-fee

[16]https://smartasset.com/checking-account/how-long-does-a-wire-transfer-take

cost savings. Blockchain functionalities of disintermediation, *smart contract* and solution to information asymmetry perfectly fit those needs while suffering from passive role of information disclosure, and therefore, we claim that BCT adoption is most promising for sought-after products with the credence feature, which require complicated transaction processes and tech-savvy consumers to actively access the Blockchain information. Fig.1.7 depicts the proposed framework for Blockchain application.

Figure 1.7: Blockchain Application in SCM



Chapter 2

Literature Review

Essentially, BCT provides a platform that enables users to access secure, reliable and tamper-proof information in an efficient manner. Currently, there is little research studying the impact of BCT and related design issues from management perspectives, especially from the perspective of Supply Chain Management; cf. Simchi-Levi et al. (2008). Recently, Babich and Hilary (2018) provided a comprehensive review of the research directions pertaining to BCT in the OM field. They identify five key strengths and five main weaknesses, and point out three research themes for applying BCT to OM. In a companion paper, Babich and Hilary (2019) explore in-depth those three research themes, illustrated through several applications to OM problems. Pun et al. (2018) examine how BCT can be used to combat counterfeiting through the interplay between a manufacturer and a counterfeiter. Besides the aforementioned academic studies, there are numerous technical reports pertaining to BCT, such as Staples et al. (2017), Luu (Jan. 26, 2018), Geer (2018), O'Byrne (Mar. 27, 2018), Hertig (Mar 21, 2018), Pawczuk (2017) Piscini et al. (2017), Brody (2017), Casey and Wong (2017), and many others. In contrast to the above literature, our study aims to investigate the design of BCT for SCM through the development of a stochastic model, where the adoption of BCT impacts both the upstream (suppliers) and the downstream (consumers).

Given that information transparency plays a key role in BCT, we next focus on the literature pertaining to *information sharing* and *information asymmetry*. Table 2.1 displays a side-by-side comparison of our study with extant *Information Sharing*[1] and *Information Asymmetry*[2] literature. It also highlights the contribution our study makes to the extant literature.

2.1 Literature on Information Sharing

The benefits of information sharing within a supply chain have been widely analyzed and discussed. Lee et al. (1997) show that the major cause of the Bullwhip effect is increasing variability of ordering upstream the supply chain, and Srinivasan et al. (1994) propose that information sharing is an effective mitiga-

[1] The information Sharing literature includes but is not limited to Lee et al. (1997), Srinivasan et al. (1994), Lee et al. (2000), Yu et al. (2001), Cui et al. (2015).

[2] The information Asymmetry literature includes but is not limited to Akerlof (1978), Kivetz and Simonson (2000), Rousseau and Vranken (2013), McCluskey (2000).

tion for the Bullwhip effect by reducing order-size fluctuation. Additionally, Lee et al. (2000) build mathematical models, based on a two-level supply chain system comprised of a manufacturer and a retailer, to demonstrate that information sharing can benefit manufacturers by reducing inventory and costs. Yu et al. (2001) carry out a rigorous analysis of supply chain strategic partnerships, and show that an information sharing-based partnership can effectively reduce inventory and costs, and improve the overall performance of a decentralized supply chain. The studies on information sharing mainly focus on partnerships between the retailer and wholesaler (or vendor/manufacturer) and analyze the value of demand information sharing. Most studies draw the conclusion that a wholesaler would reap the benefits of mitigation of order-size variation and reduction of inventory from the demand information shared by a retailer, who faces the end-customers and is thus more familiar with their tastes and demands.

2.2 Literature on Information Asymmetry

Information asymmetry is a prevailing and chronic phenomenon along the entire supply chain. For the upstream supply chain, supply contract design is developed based on the asymmetric information of supplier reliability. Various supply risk management tools are employed as articles of agreement in supply contracts, such as a penalty for shortfalls, backup production (Yang et al. (2009)), outsourcing procurement service (Yang et al. (2012)), monetary subsidy (Babich

(2010)), etc. The objective of supply contract design is to improve buyers' benefit/profit by revealing suppliers' true reliability based on the contract decisions made by suppliers. Yang et al. (2009) studies the asymmetric information of supply disruption between a manufacturer and a supplier, and concludes that "*the quantity received by the manufacturer from the supplier under symmetric information is stochastically larger than the quantity received under asymmetric information*". In other words, information disclosure improves supply. Our study also assumes that the adoption of BCT enhances supply —the greater the level of adoption, the greater the rate, in the stochastic sense.

As for the downstream supply chain, Akerlof (1978) indicates that asymmetric information about product quality could cause market collapse. In a market with asymmetric information about product quality, such as in the used car market, good quality products would be driven out by bad ones, since consumers are not capable of quality differentiation. Numerous empirical studies support the contention that information disclosure by labeling is an effective way to sustain markets having asymmetric information on product quality. For example, Teisl et al. (2002) provide market-based evidence supporting the argument that the dolphin-safe label increased the market share of canned tuna. Rousseau and Vranken (2013) find that consumers are willing to pay a positive price premium of some 33 euro cent per kilogram for labeled organic apples. In addition, truthfulness of information is another critical issue. Giannakas (2002) indicates that labeling alone is not enough to support a market with information asymmetry - consumers' perception toward the authenticity of information provided by labels would be a prerequisite for sustaining markets. If mislabeling is common, consumers will lose faith in the labels, and the market will still fail. In other words, the aforementioned studies support the view that accurate and reliable information (one of the salient features of BCT) plays a crucial role in market success.

2.3 Literature on Blockchain-Based Business Practices

Many professional analysts claim that BCT will be the next technology to revolutionize business and reshape business structures and ecosystems. In recent years, an ample effort has been expanded in academia and industry to explore the potential of Blockchain technology in supply chain applications. Tian (2016) raises the global (especially Chinese) agri-food problems, e.g., a "horse meat scandal" in Europe and toxic milk powder in China, and designs a framework of Blockchain paired with RFID to improve logistics systems of agri-food supply chains. *Walmart* teamed up with *IBM* and launched several pilot projects with their suppliers to track food movements along the supply chain in response to demanding needs of food recall. Korpela et al. (2017) proposes to use Blockchain and Cloud technologies to overcome the obstacles of interoperability along the

Table 2.1: Literature Comparison

	INFO. SHARING	INFO. ASYMMETRY	OUR PAPER
SELLER-BUYER RELATIONSHIP DEFINITION	Business to Business (B2B) Seller: Vendor, supplier and manufacturer Buyer: Retailer	Business to Consumer (B2C) Seller: Retailer Buyer: Consumer	B2B and B2C Any seller and buyer
INFO. FLOW	Buyer to seller	Seller to buyer	Seller to buyer or both
INFO. CONTENT	Demand info	Supply info	Supply info or both
OBJECT OF INFO SHARING	Adjacent upstream partner	Consumer	Any participants in SC
RESEARCH INTEREST	Optimization of decision variables, e.g. lot size, safety stock, inventor level, production cycle, etc. Reduction of Bullwhip effect	Relationship between information availability and demand	Optimization of Blockchain adoption level (λ)
RESEARCH FIELD	Supply Chain Management	Marketing	
RESEARCH METHOD	Modeling	Empirical study and modeling	Modeling
RESEARCH ASSUMPTION	The quality of shared information is truthful		No partnership is required, but each participants have certain level of control toward level of info sharing Consumers have capability to process information
SOLUTION	ERP, vendor managed inventory (VMI)	Brand name, certificate, warranty, etc.	Blockchain
SUGGESTION	A certain partnership within a SC is required	Truthfulness of information is critical	Blockchain fits well to credence, sought-after goods with complex transaction

supply chain among multiple business partners to achieve end-to-end business integration. Abeyratne and Monfared (2016) reviews the characteristics and applications of BCT and uses a business example of a cardboard box product to demonstrate a proposed framework of BCT application.

Some governments consider to exploit BCT to safeguard the digitization of public services. Estonia's government started in 2018 to test Blockchain as a backbone system to secure the development of a digital society. Following the philosophy of "*once only*"[3], all information is required to be entered into the system only once and stored in a chip-ID card, and Estonian citizens are no longer required to prepare for loan application or tax filing. Legislation, voting, education, justice, health care, banking, taxes, and policing have all been digitized and secured by BCT in the back end.

However, most studies focus on Blockchain-driven of efficiency improvement from disintermediation (e.g., digitization of business processes and *e*-government) and B2B information integration to fulfill the needs of food safety; they aim to design a comprehensive framework for Blockchain applications. Few studies address the value of BCT in B2C information diffusion to promote market growth. Kim and Laskowski (2018) pointes out the value of provenance of Blockchain and discusses its application to luxury goods. Hackius and Petersen (2017) exemplifies BCT applications in fighting counterfeit, e.g., drugs

[3]https://www.newyorker.com/magazine/2017/12/18/estonia-the-digital-republic

and diamonds. Francisco and Swanson (2018) identifies the trends of consumer demanding transparency in product information (e.g., fish netting practices, source and authenticity of diamonds, etc.) and discusses why Blockchain's advantage of transparency might be a solution. However, none of those studies clearly specifies the potential of BCT in B2C information diffusion to stimulate demand. In contrast, this study concludes the specific types of products that are good fits to BCT (e.g., sought-after credence goods with complex transaction) by examining their advantages and weaknesses as well as potential alternative technologies.

In today's industry, we observe some effort being expanded to explore BCT's potential in boosting demand by disclosing information to the market. *Martina Spetlova*, a luxury apparel brand committed to sustainability and ethical sourcing, and *Fuchsia*, a handcrafted shoe maker with commitment to environmental responsibility, have tested BCT in hope of disclosing their commitment to social responsibility to the market. *Everledger*, a Blockchain developer company, explores BCT applications to diamonds by recording 40 data points that uniquely identify a diamond by disclosing the source of diamonds to allow buyers to shun "*blood diamonds*" mined in war zones.

2.4 Literature on the Newsvendor Model

The Newsvendor model is one of the most seminal and classic models in inventory management. It considers a single period problem for seasonal or perishable products in the presence of random demand. Qin et al. (2011) review several extensions for the Newsvendor model, including endogenous demand, supplier pricing policies, and buyer risky profiles. Rekik et al. (2008) and Sahin and Dallery (2009) use the Newsvendor model to investigate the economic impact of inventory record inaccuracies in fashion industry (seasonal products) retailers. Rekik et al. (2008) further examine how RFID could improve inventory record inaccuracy problems and derive a closed form solution for the optimal cost of RFID tags. Cohen et al. (2015) use a price-setting Newsvendor model to investigate the impact of demand uncertainty on consumer subsidies for green technology adoption, e.g., electronic vehicles and solar panels, and conclude that consumer subsidies are effective in government and supplier coordination. In the literature, the Newsvendor model is rarely used to tackle new technology adoption problems (e.g., RFID and green technology adoption). In an attempt to examine Blockchain Technology adoption strategies for perishable products (e.g., organic apples), we develop a Newsvendor model that takes into consideration cost savings (e.g., by disintermediation and smart contracts) and demand growth (e.g., by attracting tech-savvy consumers and reducing information asymmetry).

Ultimately, it further treats the BCT design issue via deriving the optimal BCT level of adoption.

2.5 Literature on Inventory Management and Pricing

For upstream supply, the literature on inventory management categorizes supply uncertainty into three types: supply disruption, uncertain supply capacity and uncertain supply yield. The causes of supply disruption may be supplier bankruptcies, labor union strikes, etc. Babich (2010) proposes to subsidize suppliers with financial problems and concludes that a separate policy of inventory management and financial subsidies is optimal for a system with supplier's financial risks under certain conditions. Wang and Gerchak (1996) exemplifies the factors of uncertain capacity, including unexpected machine breakdowns, unscheduled maintenance, uncertain repair duration, reworks of defective items, etc. Using periodic inventory review, Ciarallo et al. (1994) models capacity as a random variable following a given distribution, which is independent of ordering/production quantity. They conclude that an "*order-up-to*" policy is the optimal inventory policy for systems with uncertain capacity. Wang and Gerchak (1996) considers uncertain yield caused by imperfect production processes. In a periodic review system with considerations of uncertain production capacity, random yield and stochastic demand, Wang and Gerchek (1996) built a model to minimize total expected discounted cost. The result of their model which jointly manages uncertain capacity and yield shows that the objective function is quasi-convex, and there exists a unique reorder point for each period, which are similar to the results of models with a consideration of uncertain capacity alone (Ciarallo et al. (1994)) and of random yield alone (Henig and Gerchek (1990)). Different from Wang and Gerchek's work, this study places an emphasis on business strategy by further considering Blockchain design and operational pricing decisions, instead of solely focusing on production processes. Without a consideration of uncertain capacity, our result for ordering quantity echoes Wang and Gerchek's regarding the existence of a reorder point for each period. Yano and Lee (1995) summarizes three ways to model random yield: 1) assuming that the creation of good units is a Bernuelli process; 2) specifying the distribution of the time until the production process becomes "out of control"; 3) specifying the distribution of the fraction of good units. We utilize a dynamic programming model to follow the third method to model supply uncertainty in the form of random yield. Henig and Gerchak (1990) proves that in a periodic review system with stochastically proportional random yield model, there exists a reorder point, and a nonorder-up-to policy is optimal. Our dynamic programming model renders a similar result with random yield literature regarding inventory management decisions, the existence of a unique optimal ordering quantity with an inventory replenishment threshold for each period. Furthermore, it is worth noting that our ultimate goal is Blockchain design while considering operational uncertainties, i.e. random supply and demand. Therefore, we place an emphasis on analyzing the impacts of Blockchain on supply chain performance and eventually obtaining an optimal Blockchain adoption level, which may seem abstract while actually provide helpful insights regarding different product types and product lifecycles.

In terms of methodology and modeling, our work is also related to the extensive literature on joint pricing and inventory management under uncertain demand and supply; cf. Li and Zheng (2006), Roels and Perakis (2006), and Adida and Perakis (2010). Li and Zheng (2006) provides a comprehensive literature review on joint pricing and inventory control. Li and Zheng (2006) were the first to study the joint inventory replenishment and pricing problem with both uncertain demand and supply in multiple periods, and our model is based on theirs with a substantial extension to long-term adoption of BCT, while focusing on Blockchain design for SCM. In the presence of uncertain supply and demand, they show that, given different levels of inventory on hand, there exist an optimal ordering/production quantity and price/demand levels (*ibid.*). Both the optimal price and ordering/production quantity decrease in the inventory level on hand. They further conclude that uncertain supply always results in a higher price and lowers the expected profit of a company. Our study differs from Li and Zheng (2006) in several ways. For example, we focus on the impact of BCT adoption on optimal operational decisions. Importantly, we take it a step further, by considering the design issue for the system.

Lastly, our study is also related to another stream of literature on online learning of demand. This literature includes but is not limited to Agarwal et al. (2014), Agrawal and Devanur (2014), Babich and Tang (2016), Badanidiyuru et al. (2018), Borkar and Jain (2014), Burnetas and Katehakis (1996), Burnetas and Katehakis (1997), Burnetas et al. (2017), Burnetas et al. (2018), and Cowan and Katehakis (2015). Our study assumes that demand distributions are exogenously given; hence we do not consider learning effect.

2.6 Contribution to the Literature

The contribution this study makes to the literature is mainly in the following three aspects.

- First, this study systematically reviews the advantages and disadvantages of Blockchain technology, based on which we further translate them into potential business benefits from supply chain perspective, including food safety, market growth, cost savings and yield improvement.
- Next, this study details the functionalities of BCT (i.e., information integration and diffusion) and investigates and examines alternatives and supplementary technologies. Compared with peer technologies, including

ERP and AR/VR, it is shown that the most favorable applications of BCT are likely to be sought-after, credence goods with complicated transaction processes. In a conventional system, the functionalities of information integration and information diffusion require multiple systems, e.g., ERP for information integration and marketing tools for information diffusion, but those two functionalities can now be integrated into a single Blockchain system. However, we believe that Blockchain is not a panacea, as it suffers from some limitations, including information provision passiveness and lack of interaction with consumers. Given the limitations and advantages of information security, transparency and traceability, we position Blockchain as a promising alternative to replace the ”labelling” business concept. We also suggest Blockchain be paired with autonomous data acquisition systems, e.g., RFID, Auto-ID, IoT, etc. to further strengthen data security.

- Lastly, we incorporate the business benefits of Blockchain adoption, i.e., market growth, cost savings and yield improvement[4], to develop models to investigate optimal BCT adoption levels based on different demand distributions, product types and product lifecycles. In a break from the extant information literature, which typically analyzes B2B information

[4]Blockchain benefits to food safety enhancement are excluded in our model, since food safety is related to the topic of reverse supply chain management, which is beyond the scope of this study.

integration (i.e., information sharing literature) and B2C information diffusion (i.e., information asymmetry literature) separately, this study combines those two research threads to investigate the impacts of information across the entire supply chain (e.g., farm-to-fork transparency). The selection of Blockchain adoption level as a strategic decision variable allows us to draw business insights pertaining to optimal application of BCT by demand distribution, product type and product lifecycle.

A business may benefit from Blockchain adoption via supply cost savings (e.g., disintermediation and *smart contracts*), demand growth (e.g., mitigation of information asymmetry for tech-savvy consumers) and yield improvement (information transparency). We address these impacts (except for food safety[5]) and introduce a novel decision variable, the Blockchain *adoption level*, $\alpha \in [0,\ 1]$, as a fundamental element of our two models: (1) a Newsvendor (a.k.a. Newsboy) model for single-period perishable goods with uncertain demand, and (2) a dynamic programming model for multiple period non-perishable goods with stochastic supply and demand. The models in this study are designed specifically for Blockchain adoption to reflect its thorough impacts on both supply and demand sides. As analyzed in this study, the functionalities of BCT, including B2B information integration and B2C information diffusion, could not be

[5]Our model does not consider the BCT benefit of safety enhancement, which is relevant to supply chain risk management.

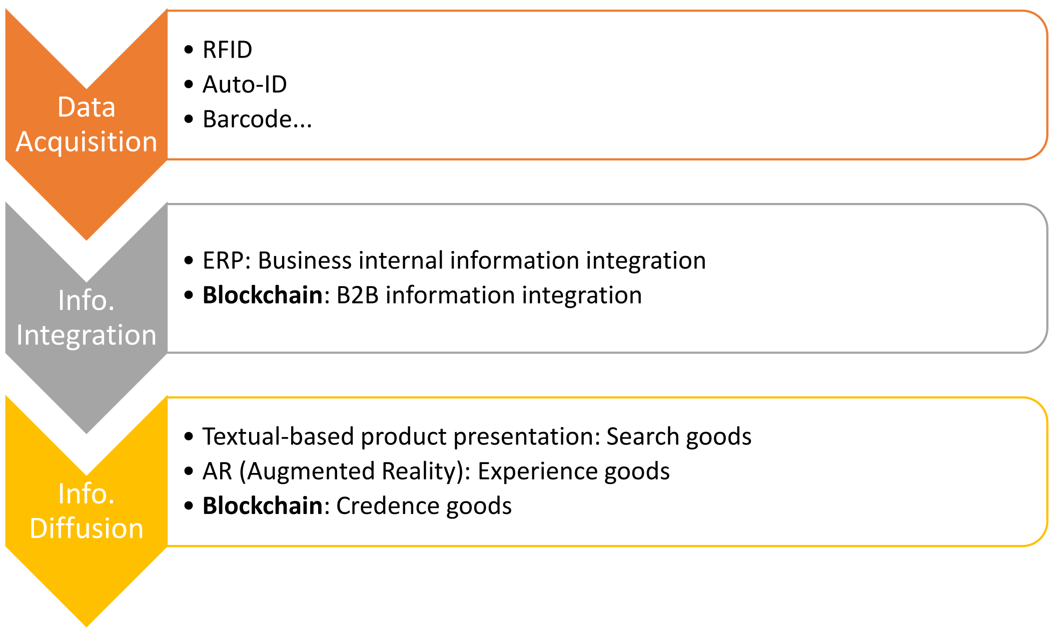
achieved by any other single information technology. Given that we model the impacts of BCT on both information integration and diffusion, one may argue that these can be achieved by two or more technology systems, e.g., ERP and some marketing campaigns. However, replacing BCT with multiple technologies in our models would result in extremely high total adoption costs, or maybe practically infeasible. In this sense, our model captures a more generic setting for BCT that impacts both the supply and demand streams. Fig. 2.1 compares technology applications in SCM.

In our first model, we develop a Blockchain-enabled Newsvendor model for single-period perishable or seasonal products, e.g., agricultural products with short shelf life, fashion products, etc. We incorporate the BCT benefits of cost savings and demand growth by assuming that the higher the BCT adoption level, α, the lower the purchasing costs and the higher the market demand (in the sense of stochastic order). In the presence of demand uncertainty, our goal is to solve the Newsvendor model for the optimal ordering quantity, and ultimately to solve the optimal Blockchain adoption level. For a generic demand distribution, it is shown that increasing the BCT adoption level will increase the critical ratio, as well as the optimal order quantity; it will increase the optimal expected profit if there is no lost-sales penalty. Intuitively, a higher adoption of BCT leads to higher demand and lower ordering costs, each of which would improve profit.

However, we also device some counter examples to show that an increase in the adoption level might lower the optimal order quantity and that it is not always profitable to adopt a higher BCT even when there is no adoption cost. For the selected demand types of Uniform and Normal distributions, we derive a closed-form expression for the optimal decision, based on which useful insights have been developed. Finally, a sequence of numerical studies complements our analytical results with useful insights.

In our second model, we develop a *dynamic programming* (DP) model for non-perishable products. We incorporate the BCT benefits of cost savings, demand growth, and yield improvement to assume that the higher the BCT adoption level, α, the lower the purchasing costs and the higher the market demand and supply yield. We assume that both demand and supply are stochastic and the research objective is to find the optimal ordering quantity and selling price for each period, and ultimately to derive the optimal BCT adoption level. The result of our DP model shows that the adoption of BCT can help a company to reduce ordering quantities, lower selling prices and reduce target inventory levels. Our numerical study further indicates that BCT adoption would benefit credence goods the most, then experience goods, and search goods the least. It is further suggested by our study that a company should adopt BCT as early as possible in the product life cycle.

Figure 2.1: Comparison of SCM technology applications



Chapter 3

Blockchain-Enabled Newsvendor Model

3.1 Mathematical Model

Assume that a firm (e.g., a retailer) operates as a Newsvendor to order and sell a product over a season, and adopts BCT. The operations are managed over an adopted infrastructure of Blockchain Technology, where the level of adoption reflects the percentage of its supply chain information and operations exposed to the Blockchain system. To quantify the information exposure to the public ledger along the Blockchain system, we define $\alpha \in [0,\ 1]$ to be the *adoption level*, where a higher α reflects a higher level of adoption of BCT technology. As a strategic decision variable, α is selected first, subject to a cost function $\psi(\alpha)$. Here, the adoption cost $\psi(\alpha)$ typically covers the set-up cost of the Blockchain infrastructure, partnership management, and information and database management pertinent to the Blockchain, etc. Practically, sharing more information can favor the firm's competitors. In this sense, $\psi(\alpha)$ does also reflect the cost incurred by overexposure of information to competitors along the network; i.e., it is likely to lose competitive advantages by exposing more information; cf. O'Byrne (Mar. 27, 2018), Hertig (Mar 21, 2018) and Luu (Jan. 26, 2018). Typically, the adoption cost $\psi(\alpha)$ is assumed to be convex and increasing in α (with $\psi(0) = 0$) to reflect the fact that the complexity of managing a Blockchain system becomes more significant for each unit of increase in α.[1]

For instance, as the global leader in the Blockchain business, IBM provides a customized service of Blockchain solutions to customers under contractual terms and charges the price according to the scale of adoption level α.[2] In what follows, we shall describe our model. The key notation to be used in the sequel is summarized in Table 3.1.

The adoption of BCT would impact both suppliers and consumers. For upstream suppliers, it can reduce procurement and ordering costs. For example, as a type of BCT application, *Smart Contracts* can be used to find lowest-cost sup-

[1] As a component of BCT, a simple smart contract without complex business logic costs around $7,000. A more advanced contract may cost up to $45,000 and more. It's not uncommon for large organizations with specialized knowledge usually to incur up to $100,000; cf. Problems & Costs of Smart Contract Development, Mar. 25, 2018. https://medium.com/

[2] For example, IBM® Blockchain Platform Enterprise Plan charges the service fee according to the volume of information shared with its peers.
https://console.bluemix.net/docs/services/blockchain/enterprise_plan.html#about-enterprise-plan

pliers in real time. The traceability of BCT can save logistics and transportation costs, which further reduces supply costs. To model procurement savings due to BCT adoption, the ordered unit cost, $c_\alpha \geqslant 0$, is assumed to decrease in α; cf. Geer (2018), Stelmakowich (2016), Aitken (2017) and Brody (2017).

For downstream consumers, let $D_\alpha \in \mathbb{R}^+$ denote the random demand associated with BCT adoption level α, which follows some distributions. For ease of exposition, we shall assume that D_α is a continuous random variable. Let $F_\alpha(x) = \mathbb{P}(D_\alpha \leqslant x)$ and $f_\alpha(\cdot) = F'_\alpha(\cdot)$ denote the *cumulative distribution function* (cdf) and the *probability density function* (pdf) of $D_\alpha,$ respectively. To model the impact of BCT adoption on demand (e.g., considering tech-savvy consumers), we assume that D_α increases in α in the *stochastic order*; *viz.*, $D_{\alpha'} \geqslant_{st} D_\alpha$ if $\alpha' \geqslant \alpha$.

The unit selling price p is exogenous and satisfies $p \geqslant c_\alpha$ for any $\alpha \in [0,\, 1]$. Any unfulfilled demand is lost subject to a penalty of r per unit. To secure a positive profit for a strategic Newsvendor, r is typically not too large. For example, in our later analysis, Lemma 3.1 derives a threshold value of r to avoid the triviality of negative optimal profit. All leftover inventory (if any) will be salvaged (or disposed off) at a constant price (cost) of s per unit. Note that we allow a negative s, in which case s represents a disposal cost per unit, e.g., the unit cost of disposing vehicle tires. Table 3.1 is a glossary of key symbols notations that will be used in our analysis.

Table 3.1: Notation Summary for the Newsvendor Model

α	the Blockchain *adoption level*, $\alpha \in (0,1]$, a strategic decision variable
q	order quantity, an operational decision variable
p	selling price per unit
s	salvage value per unit
r	lost-sales penalty per unit
c_α	ordering cost per unit
ℓ_α	the critical ratio of the Newsvendor model
D_α	random demand, stochastically increasing in α
μ_α	$\triangleq \mathbb{E}\big[D_\alpha\big]$, expected demand, a function of α
$F_\alpha(x)$	*cdf* of demand D_α with pdf $f_\alpha(x) = F'_\alpha(x)$
$V_\alpha(q)$	profit of the Newsvendor, given that BCT has been implemented at adoption level α
$\upsilon_\alpha(q)$	$\triangleq \mathbb{E}\big[V_\alpha(q)\big]$, the expected profit function given an adoption level α and order quantity q
υ^*_α	$\triangleq \upsilon_\alpha(q^*_\alpha)$, the optimal expected profit given an adoption level α
$[x]^+$	$= \max\{x, 0\}$; $[x]^- = \max\{-x, 0\}$

As an important operational decision, let $q \geqslant 0$ denote the order quantity decided by the Newsvendor. With BCT adoption level α, the Newsvendor's profit function is given by

$$
\begin{aligned}
V_\alpha(q) &= p \cdot \min\{q, D_\alpha\} + s \cdot [q - D_\alpha]^+ - r \cdot [q - D_\alpha]^- - c_\alpha q \\
&= p \cdot \Big\{ \overbrace{q - [q - D_\alpha]^+}^{\min\{q, D_\alpha\}} \Big\} + s \cdot [q - D_\alpha]^+ - r \cdot \Big\{ \overbrace{[q - D_\alpha]^+ - (q - D_\alpha)}^{[q - D_\alpha]^-} \Big\} - c_\alpha q \\
&= (p + r - c_\alpha) q - (p + r - s)\, [q - D_\alpha]^+ - r \cdot D_\alpha, \qquad (3.1)
\end{aligned}
$$

where $[a]^+ = \max\{a,\ 0\}$ and $[a]^- = \max\{-a,\ 0\}$ denote the positive and negative parts of a real number a, respectively; the second equality holds by $\min\{a, b\} = a - [a - b]^+$; $[a]^- = [a]^+ - a$. Denote the expected profit by $\upsilon_\alpha(q) \triangleq \mathbb{E}[V_\alpha(q)]$, which can be further written as follows based on Eq. (3.1),

$$\begin{aligned} v_\alpha(q) &= (p+r-c_\alpha)q - (p+r-s)\mathbb{E}[q-D_\alpha]^+ - r\cdot\mu_\alpha \\ &= (p+r-c_\alpha)q - (p+r-s)\int_0^q F_\alpha(z)\,dz - r\cdot\mu_\alpha. \end{aligned} \tag{3.2}$$

In the above, the second equality holds by the rule of *Integration by Parts*,

$$\mathbb{E}[q-D_\alpha]^+ = \int_0^q (q-z)\,dF_\alpha(z) = (q-z)F_\alpha(z)\Big|_{z=0-}^{q} - \int_0^q F_\alpha(z)\,d(q-z) = \int_0^q F_\alpha(z)\,dz,$$

and $\mu_\alpha \triangleq \mathbb{E}[D_\alpha]$ increases in α since D_α increases in α in the *stochastic order*.

It is straightforward to see that $v_\alpha(q)$ as given in Eq. (3.2) is concave in q and $v_\alpha(q) \downarrow -\infty$ as $q \uparrow +\infty$ since $c_\alpha > s$. Hence, there exists a unique q^* that maximizes $v_\alpha(q)$. Accordingly, the optimal order quantity is $q^*_\alpha \triangleq \arg\max_{q\in\mathbb{R}^+} \mathbb{E}[V_\alpha(q)]$. Further, denote the critical ratio of the Newsvendor model by

$$\ell_\alpha \triangleq \frac{p+r-c_\alpha}{p+r-s}. \tag{3.3}$$

Given that $p+r \geqslant c_\alpha > s$, we have $0 \leqslant \ell_\alpha \leqslant 1$, since the numerator in the above equation increases in α owing to the decreasing monotonicity of c_α. Further, ℓ_α is increasing in α. Solving the Newsvendor model to maximize the expected profit as given by Eq. (3.2), via considering the first-order condition of Eq. (3.2), we have the optimal order quantity as the ℓ_α-quantitle:

$$q_\alpha^* = F_\alpha^{-1}(\ell_\alpha), \tag{3.4}$$

where $F_\alpha^{-1}(\cdot)$ is the inverse function of $F_\alpha(\cdot)$. Incorporating BCT adoption, our analysis makes two salient extensions from the classic Newsvendor model. First, due to the adoption of BCT, the critical ratio given by Eq. (3.3) now depends on α. Second, the optimal order quantity given by Eq. (3.4) is determined by α in two intertwined ways, the demand distribution curve $F_\alpha^{-1}(\cdot)$ and the critical ratio ℓ_α. Next, for any α, denote the optimal expected profit by

$$\upsilon_\alpha^* \triangleq \upsilon_\alpha(q_\alpha^*) \triangleq \max_{q \in \mathbb{R}^+} \mathbb{E}[V_\alpha(q)]. \tag{3.5}$$

In what follows, we provide a graphical interpretation of the optimal expected profit.

3.1.1 Graphical Interpretation of the Optimal Profit

Note that the term $p + r - s$ in Eq. (3.2) represents the *relative marginal profit* if one unit of the product can be sold. For ease of exposition, instead of $\upsilon_\alpha(q)$, we shall take the following relative form as an alternative objective function:

$$\bar{v}_\alpha(q) \triangleq v_\alpha(q)/(p+r-s) = \ell_\alpha q - \int_0^q F_\alpha(z)\,dz - r' \cdot \mu_\alpha, \tag{3.6}$$

where

$$r' = r/(p+r-s). \tag{3.7}$$

Since $p+r-s$ is exogenously given, optimizing $v_\alpha(q)$ is equivalent to optimizing $\bar{v}_\alpha(q)$ as given by Eq. (3.6). With the optimal order quantity q^*_α as given by Eq. (3.4), the *optimal relative profit* $\bar{v}^*_\alpha = \max_{q \geqslant 0} \bar{v}_\alpha(q)$ can be written as:

$$\bar{v}^*_\alpha \triangleq \max_{q \geqslant 0} \bar{v}_\alpha(q) = \ell_\alpha q^*_\alpha - \int_0^{q^*_\alpha} F_\alpha(z)\,dz - r' \cdot \mu_\alpha. \tag{3.8}$$

Figure 3.1: Interpretation of the Optimal Relative Profit $\bar{v}^*_\alpha$

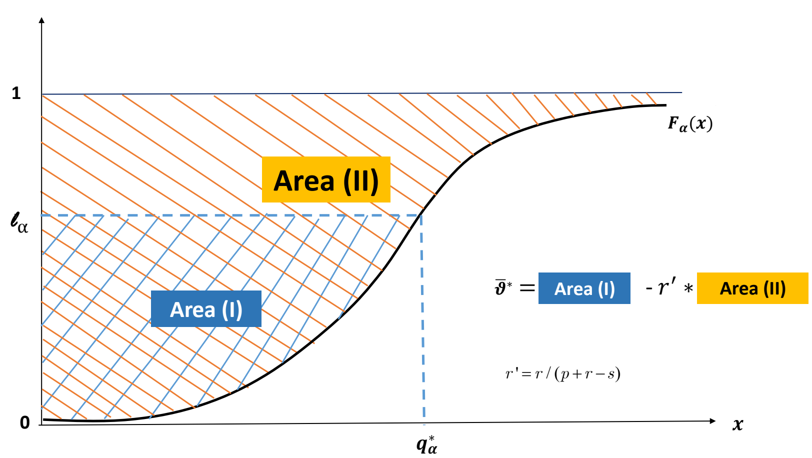
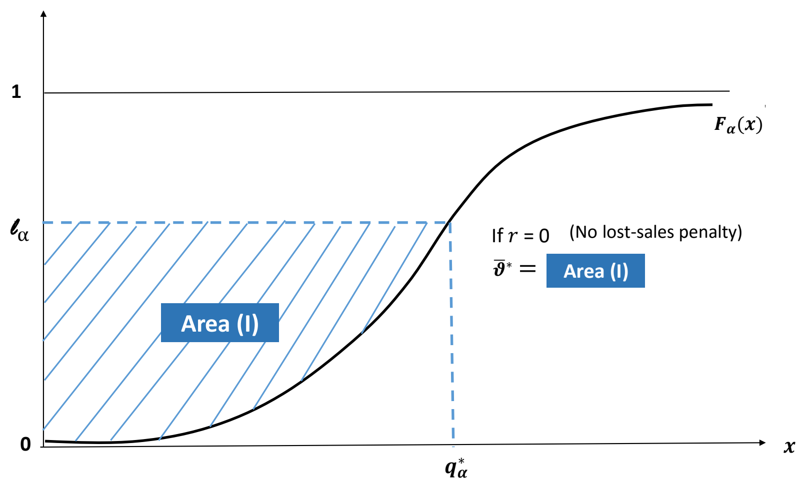


Figure 3.2: Optimal Relative Profit $\bar{\upsilon}^*$ without Lost-sales Penalty ($r = 0$)



To visualize the economic meaning of the optimal profit, Fig. 3.1 provides a graphical interpretation of the *optimal relative profit* $\bar{\upsilon}_\alpha^*$ as:

$$\bar{\upsilon}_\alpha^* = \textbf{Area(I)} - r' \times \textbf{Area(II)}.$$

The value of r' given by Eq. (3.7) plays an important role in the optimal profit. If r' (or equivalently, r) is too high, the optimal profit could become negative. For the case without lost-sales penalties, i.e., $r = 0$, Fig. 3.2 shows that the *optimal relative profit* is just Area (I), which is the shaded area between the dashed line of ℓ_α and the curve of *cdf* $F_\alpha(x)$.

3.1.2 Preliminary Results

The following theorem does a global search and summarizes key results.

Theorem 3.1 *For the optimal solution, the following hold:*

i) ℓ_α increases as α increases;

ii) q^*_α increases as α increases.

iii) If $r = 0$, then υ^*_α increases as α increases.

Proof of Theorem 3.1. For part (i), in view of Eq. (3.3), it is straightforward to see ℓ_α increases in α, since c_α decreases in α.

For part (ii), by reviewing Eq. (3.4), it shows that q^*_α increases in α, because both $F_\alpha^{-1}(x)$ and ℓ_α increase in α. The former holds by the assumption that D_α increases in α in the *stochastic order* and the latter holds by part (i).

To prove part (iii), we compare two settings associated with two different adoption levels, $\alpha_1 \leqslant \alpha_2$, and denote the corresponding optimal order quantity as q^*_1 and as q^*_2, respectively. Since $r = 0$, Eq. (3.6) is simplified as

$$\bar{\upsilon}_\alpha(q) = \ell_\alpha q - \int_0^q F_\alpha(z)\, dz. \tag{3.9}$$

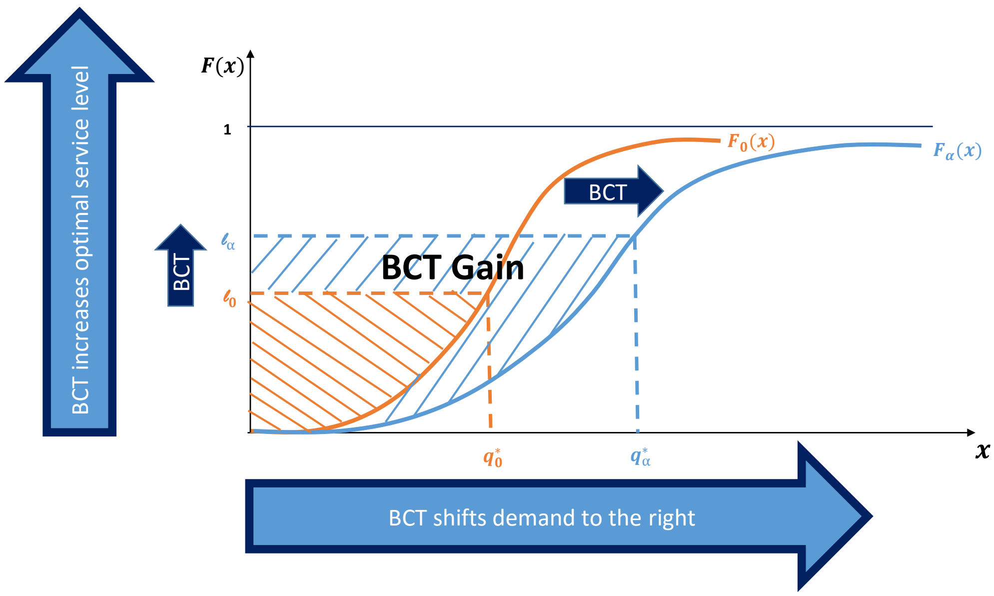
It is sufficient to prove the value function associated with α_2 is larger than that of

α_1. To this end, we take their difference as

$$
\begin{aligned}
& \bar{v}^*_{\alpha_2} - \bar{v}^*_{\alpha_1} \\
= & \left[\ell_{\alpha_2} q_2^* - \int_0^{q_2^*} F_{\alpha_2}(z)\,dz\right] - \left[\ell_{\alpha_1} q_1^* - \int_0^{q_1^*} F_{\alpha_1}(z)\,dz\right] \\
= & \Bigg[\underbrace{\ell_{\alpha_2} q_1^* + \ell_{\alpha_2}(q_2^* - q_1^*)}_{=\ell_{\alpha_2} q_2^*} - \left(\int_0^{q_1^*} F_{\alpha_2}(z)\,dz + \int_{q_1^*}^{q_2^*} F_{\alpha_2}(z)\,dz\right)\Bigg] \\
& \qquad\qquad\qquad\qquad\qquad\qquad - \left[\ell_{\alpha_1} q_1^* - \int_0^{q_1^*} F_{\alpha_1}(z)\,dz\right] \\
= & (\underbrace{\ell_{\alpha_2} - \ell_{\alpha_1}}_{\geqslant 0}) q_1^* + \int_0^{q_1^*} \Big(\underbrace{F_{\alpha_1}(z) - F_{\alpha_2}(z)}_{\geqslant 0}\Big)\,dz + \left[\ell_{\alpha_2}(q_2^* - q_1^*) - \int_{q_1^*}^{q_2^*} F_{\alpha_2}(z)\,dz\right] \\
\geqslant & \int_{q_1^*}^{q_2^*} \Big(\underbrace{\ell_{\alpha_2} - F_{\alpha_2}(z)}_{\geqslant 0}\Big)\,dz \geqslant 0
\end{aligned}
$$

where $\ell_{\alpha_2} \geqslant \ell_{\alpha_1}$ by part (i) of this theorem, and $q_2^* \geqslant q_1^*$ in light of part (ii); $F_{\alpha_1}(z) \geqslant F_{\alpha_2}(z)$ because $D_{\alpha_2} \geqslant_{st} D_{\alpha_1}$ and it follows directly from its definition; and $\ell_{\alpha_2} \geqslant F_{\alpha_2}(z)$ for all $z \leqslant q_2^*$ since q_2^* is the threshold value of the Newsvendor model. The non-negativity of the above proves the increasing monotonicity of $\bar{v}^*_\alpha$, so for v^*_α, which completes the proof. ■

Fig. 3.3 visualizes the impact of a BCT adoption via comparing the case without adoption $\alpha = 0$ with the case with adoption $\alpha > 0$. In particular, the impact is twofold: First, the BCT adoption increases a demand distribution, as depicted in the figure that the demand *cdf* curve $F_\alpha(x)$ moves to the right; Second, the service level (i.e., the critical value) rises up. In this case, the optimal relative profit $\bar{v}_\alpha^*$ is expanded from the area in orange to a wider area, which shows the gain of BCT adoption as highlighted in blue. This explains part (iii) of Theorem 3.1.



Intuitively, one would expect the value function v_α^* given by Eq. (3.5) to increase in α; *viz.* the higher adoption level of BCT, the higher operational profit. The following example illustrates such intuition:

Example 3.1 (Normal Distribution Example). *Consider the special setting with a normal distribution of demand $D_\alpha \sim \mathcal{N}(\mu_\alpha, \sigma^2)$, where μ_α increases in α but σ is a constant. Such setting satisfies the definition of a stochastic increase of D_α.*

Consider two different adoption levels, $\alpha_2 \geqslant \alpha_1$, and denote the difference of the corresponding demand means as $\delta = \mu_{\alpha_2} - \mu_{\alpha_1} \geqslant 0$. Graphically, it can be visualized in

Fig. 3.4 that q_α^* *increases as* α *increases. In what follows, we shall prove that* $\upsilon_{\alpha_2}^* \geqslant \upsilon_{\alpha_1}^*$*. While referring to Eq. (3.2), for any order quantity* q *under* α_1*, we select to order* $q+\delta$ *under* α_2*, then we have the following:*

$$
\begin{aligned}
& \upsilon_{\alpha_2}(q+\delta) \\
= \; & (p+r-c_{\alpha_2})(q+\delta) - (p+r-s)\mathbb{E}[(q+\delta)-D_{\alpha_2}]^+ - r\cdot\mu_{\alpha_2} \\
= \; & (p+r-c_{\alpha_2})q + (p+r-c_{\alpha_2})\delta - (p+r-s)\mathbb{E}[(q+\delta)-D_{\alpha_2}]^+ - r\cdot(\mu_{\alpha_1}+\delta) \\
\geqslant \; & (p+r-c_{\alpha_1})q + [(p+r-c_{\alpha_2})-r]\delta - (p+r-s)\mathbb{E}[q-D_{\alpha_1}]^+ - r\cdot\mu_{\alpha_1} \\
\geqslant \; & (p+r-c_{\alpha_1})q - (p+r-s)\mathbb{E}[q-D_{\alpha_1}]^+ - r\cdot\mu_{\alpha_1} \\
= \; & \upsilon_{\alpha_1}(q),
\end{aligned}
$$

where the first inequality holds by the fact that $c_{\alpha_2} \leqslant c_{\alpha_1}$*, and* $(q+\delta-D_{\alpha_2})$ *and* $(q-D_{\alpha_1})$ *have the identical distribution; the second inequality holds by* $(p-c_{\alpha_2})\delta \geqslant 0$*. Therefore, for any* q*,* $\upsilon_{\alpha_2}(q+\delta) \geqslant \upsilon_{\alpha_1}(q)$ *always. It is straightforward to see* $\upsilon_{\alpha_2}^* \geqslant \upsilon_{\alpha_1}^*$*.*

■

Theorem 3.1 provides a sufficient condition (of $r=0$) for the increasing monotonicity of υ_α^* in α . It is worth noting that the increasing monotonicity of υ_α^* does not hold in general with $r>0$. In other words, υ_α^* might decreases as α increases when there is a lost-sales penalty. To see this, we provide the following counterexample.[3]

Example 3.2 ***(Counterexample).*** *Consider two settings associated with two different BCT adoption levels, $\alpha_1 \leqslant \alpha_2$. Each demand follows a uniform distribution, such that $D_{\alpha_1} \sim \mathcal{U}(0, \frac{1}{2})$ and $D_{\alpha_2} \sim \mathcal{U}(0, 1)$. Clearly, $D_{\alpha_1} \leqslant_{st} D_{\alpha_2}$. Further, set $p = 100$, $r = 50$, $s = 10$ and $c_{\alpha_1} = c_{\alpha_2} = 80$. Hence, $\ell_{\alpha_1} = \ell_{\alpha_2} = \frac{1}{2}$. By Eq. (3.6), one can obtain: $\upsilon^*_{\alpha_1} = -3.75$ ($\bar{\upsilon}^*_{\alpha_1} = -0.0268$) compared with $\upsilon^*_{\alpha_2} = -7.5$ ($\bar{\upsilon}^*_{\alpha_2} = -0.0536$). In this case, $\upsilon^*_{\alpha_2} < \upsilon^*_{\alpha_1}$, which shows the counter-intuitive phenomenon that υ^*_α decreases while α increases.*

However, for this example, under the sufficient condition with a penalty equal to zero, viz. $r = 0$, one has $\upsilon^*_{\alpha_1} = 5.6250$ ($\bar{\upsilon}^*_{\alpha_1} = 0.0625$) compared with $\upsilon^*_{\alpha_2} = 11.250$ ($\bar{\upsilon}^*_{\alpha_2} = 0.1250$). In this case, $\upsilon^*_{\alpha_2} > \upsilon^*_{\alpha_1}$, which exemplifies part (iii) of Theorem 3.1. ■

Theorem 3.2 *For any symmetric distribution of demand D_α, the following hold:*

i) $q^*_\alpha = \mu_\alpha$ *if and only if* $c_\alpha = (p + r + s)/2$*;*

ii) $q^*_\alpha > \mu_\alpha$ *if and only if* $c_\alpha < (p + r + s)/2$*;*

iii) $q^*_\alpha < \mu_\alpha$ *if and only if* $c_\alpha > (p + r + s)/2$*.*

[3]Song (1994) provides a counter example to illustrate a similar observation but for random lead-time demand.

Proof of Theorem 3.2. For a symmetric distribution of demand, we must have $F_\alpha^{-1}(\frac{1}{2}) = \mu_\alpha$. Therefore, we just need to compare $\ell_\alpha \triangleq \frac{p+r-c_\alpha}{p+r-s}$ given by Eq. (3.3) with $1/2$. By applying some basic algebra and the monotonicity of $F_\alpha^{-1}(z)$, the proof readily follows. ■

Theorem 3.2 shows how the ordering cost c_α impacts the optimal order quantity, in terms of a threshold value. The lower the ordering cost, c_α, the higher the optimal order quantity. Further note that c_α is a direct function of α, which shows the one-one mapping relationship between α and c_α. For any symmetric distribution of demand, if the inverse function of c_α exists and is denoted as $c_\alpha^{-1}(\cdot)$, it leads to another interpretation of the ordering policy in terms of a critical value for the adoption level.

Proposition 3.1 *For symmetric distributions of demand, there exists a threshold value of*

$$\bar{\alpha} \triangleq c_\alpha^{-1}\left(\frac{p+r+s}{2}\right),$$

such that the following hold:

i) $q_\alpha^* = \mu_\alpha$ *if and only if* $\alpha = \bar{\alpha}$*;*

ii) $q_\alpha^* > \mu_\alpha$ *if and only if* $\alpha > \bar{\alpha}$*; and*

iii) $q_\alpha^* < \mu_\alpha$ *if and only if* $\alpha < \bar{\alpha}$*.*

Proof of Proposition 3.1. The proof readily follows from the previous discussion.

■

The results presented in Theorem 3.1 are based on the assumption that D_α is stochastically increasing in α. For example, considering two demand distributions of $\mathcal{N}(30, 10^2)$ associated with a small α, $\mathcal{N}(50, 10^2)$ associated with a large α, Figures 3.4 depicts how q^*_α changes from Point P_1 to Point P_2, while α increases from a small value to a large value. In this case, q^*_α increases in α, which complies with part (ii) of Theorem 3.1. However, if the assumption of being *stochastically increasing* is violated, even under the assumption that D_α increases in a *convex order*, the result might not hold. For example, in Fig. 3.5, we consider two demand distributions, which shows that the standard deviation decreases as α increases. For the high critical ratio, q^* decreases while moving from point P_1 to point P_2, as α increases. In contrast, for the low critical ratio, q^* increases while moving from P_1' to P_2', as α increases. In this sense, the impact of α on the optimal order quantity q^*_α becomes opposite. The negative impact can be jointly determined by the settings of the parameters, which further determines ℓ_α as illustrated in Fig. 3.5.

Although the impact of the demand volatility on the optimal order quantity is unclear, we can prove that the volatility of demand diminishes the total profit, which is summarized in the following theorem.

Figure 3.4: Ordering Quantity (Varying Mean)

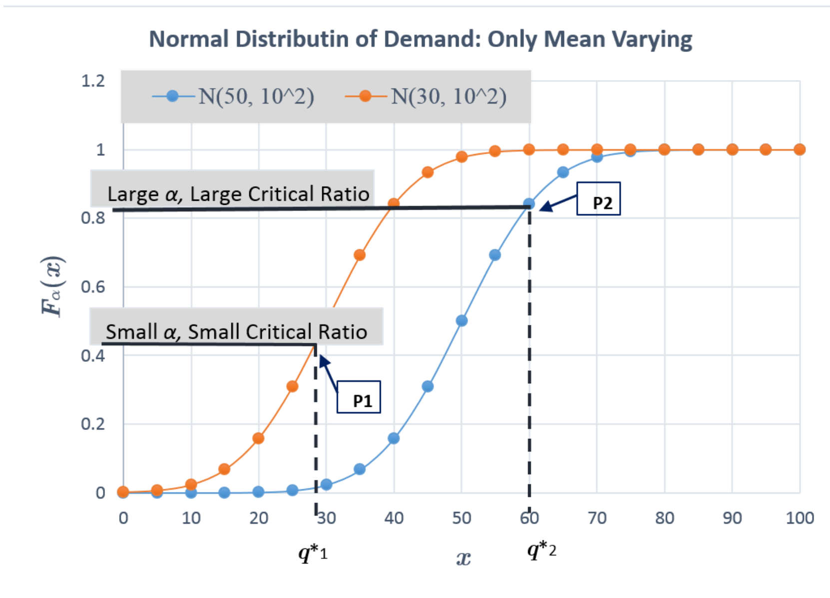
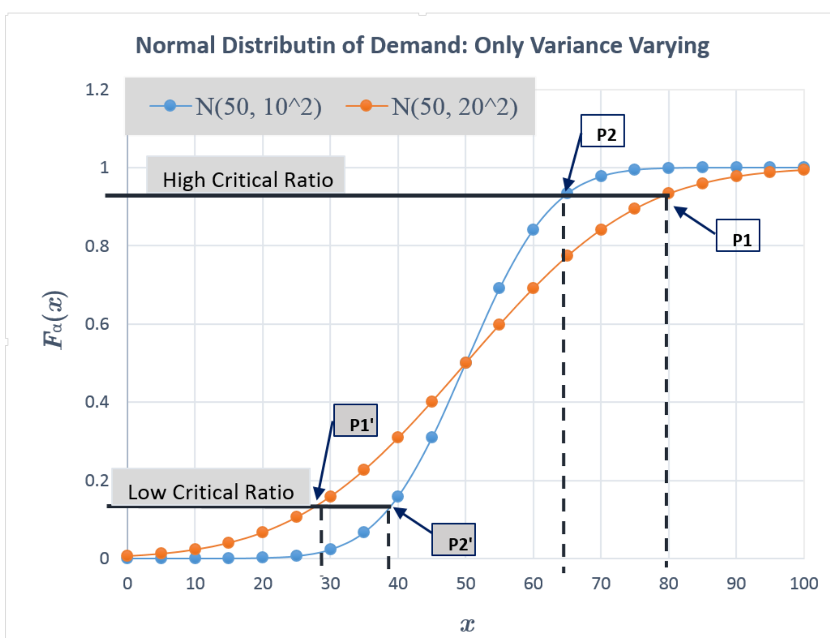


Figure 3.5: Ordering Quantity (Varying Variance)



Theorem 3.3 *For any α, if $\tilde{D}_\alpha \geqslant_{cx} D_\alpha$, then for any order quantity q, then $\tilde{\upsilon}_\alpha(q) \leqslant \upsilon_\alpha(q)$. Furthermore, $\tilde{\upsilon}^*_\alpha \leqslant \upsilon^*_\alpha$.*

Proof of Theorem 3.3. First, revisiting Eq. (3.2), we have

$$\upsilon_\alpha(q) = (p + r - c_\alpha)q - (p + r - s)\mathbb{E}[q - D_\alpha]^+ - r \cdot \mu_\alpha,$$

which is increasingly concave in D_α, since $[q - D_\alpha]^+$ is decreasingly convex. Note that $\tilde{D}_\alpha \leqslant_{cv} D_\alpha$, since $\tilde{D}_\alpha \geqslant_{cx} D_\alpha$, which further implies that $\tilde{D}_\alpha \leqslant_{icv} D_\alpha$ and $\mathbb{E}[\tilde{D}_\alpha] = \mathbb{E}[D_\alpha]$, in light of Lemma 5.6. The proof of $\tilde{\upsilon}_\alpha(q) \leqslant \upsilon_\alpha(q)$ readily follows from the definition of $\tilde{D}_\alpha \leqslant_{icv} D_\alpha$ and Lemma 5.7.

In light of Eq. (3.5), we have

$$\upsilon^*_\alpha = \max_{q \in \mathbb{R}^+} \mathbb{E}[\upsilon_\alpha(q)] \geqslant \max_{q \in \mathbb{R}^+} \mathbb{E}[\tilde{\upsilon}_\alpha(q)] = \tilde{\upsilon}^*_\alpha,$$

where the inequality holds by the first part of the result. Consequently, this completes the proof. ■

3.1.3 Optimal BCT Adoption

Previously, we have discussed the optimal *operational decision* encountered by a Newsvendor. We now proceed to investigate the *strategic decision*, i.e., the optimal BCT adoption level. The optimal α^* can be solved as a *design problem* to decide on the adoption level $\alpha \in [0, 1]$:

$$\alpha^* = \mathrm{argmax}_{\alpha \in [0,1]} \left\{ \upsilon_\alpha^* - \psi(\alpha) \right\}, \tag{3.10}$$

where υ_α^* is given by Eq. (3.5). If υ_α^* and $\psi(\alpha)$ are both smooth and differentiable, then the optimal adoption level α^* can be obtained via solving the first-order condition,

$$\frac{d}{d\alpha}\psi(\alpha) = \frac{d}{d\alpha}\upsilon_\alpha^*.$$

Typically, there are three widely-used functional forms of cost functions, *viz.*, linear, quadratic and cubic forms. The former two can be treated as special cases of the third cubic form. The cubic cost function is consistent with a U-shaped marginal cost curve (e.g., $\frac{d}{d\alpha}\psi(\alpha)$ is quadratic in α). Thus, a cubic form of function has been widely recommended and implemented for costs; cf. Gupta (2011). For example, the well-known Cobb-Douglas production function follows a cubic form. In the sequel, we shall adopt a cubic cost function in our analysis.

Example 3.3 ***(Cubic Cost Function)***: *In what follows, we specify the adoption cost of BCT to be a cubic function as*

$$\psi(\alpha) = b_3\, \alpha^3 + b_2\, \alpha^2 + b_1\, \alpha + b_0 \cdot \mathbf{1}_{\{\alpha > 0\}}, \tag{3.11}$$

where the coefficients $b_i \geqslant 0$*, and the indicator function* $\mathbf{1}_{\{\alpha>0\}} = 1$ *if* $\alpha > 0$*, otherwise 0.*

As explained before, the cubic function incorporates both quadratic and linear forms under special cases, when $b_3 = 0$ and $b_2 = b_3 = 0$, respectively.

3.2 Uniform Distribution of Demand $D_\alpha \sim \mathcal{U}(0,\ K_\alpha)$

For the uniform distribution of demand D_α, denote $D_\alpha \sim \mathcal{U}(0, K_\alpha)$, where K_α increases in α. In this case, $F_\alpha(x) = x/K_\alpha$; hence its inverse function $F_\alpha^{-1}(\ell) = \ell\, K_\alpha$. In view of Eq. (3.2), one has

$$q_\alpha^* = \ell_\alpha\, K_\alpha.$$

Next, inputting the above q_α^* into Eq. (3.2) yields the following:

$$\begin{aligned}
v_\alpha^* \triangleq v_\alpha(q_\alpha^*) &= (p+r-c_\alpha)q_\alpha^* - (p+r-s)\mathbb{E}[q_\alpha^* - D_\alpha]^+ - r\cdot\mu_\alpha \\
&= (p+r-c_\alpha)\ell_\alpha\, K_\alpha - (p+r-s)\mathbb{E}[\ell_\alpha\, K_\alpha - D_\alpha]^+ - r\cdot\mu_\alpha \\
&= (p+r-c_\alpha)\ell_\alpha\, K_\alpha - (p+r-s)\frac{K_\alpha}{2}\,\ell_\alpha^2 - r\cdot\mu_\alpha \\
&= \ell_\alpha\, K_\alpha\left[(p+r-c_\alpha) - (p+r-s)\frac{\ell_\alpha}{2}\right] - r\cdot\mu_\alpha \\
&= \ell_\alpha\, K_\alpha\,\frac{p+r-c_\alpha}{2} - r\cdot\mu_\alpha \\
&= \frac{K_\alpha}{2}\left[\ell_\alpha\,\cdot\,(p+r-c_\alpha) - r\right], \qquad (3.12)
\end{aligned}$$

where the critical ratio ℓ_α is given by Eq. (3.3), the last equality holds by $\mu_\alpha = K_\alpha/2$ for $D_\alpha \sim \mathcal{U}(0, K_\alpha)$, and the third equality holds by,

$$\begin{aligned}
\mathbb{E}[\ell_\alpha\, K_\alpha - D_\alpha]^+ &= \frac{1}{K_\alpha}\int_0^{\ell_\alpha\, K_\alpha} (\ell_\alpha K_\alpha - x)\, dx \\
&= \frac{1}{K_\alpha}\left[\int_0^{\ell_\alpha\, K_\alpha} \ell_\alpha K_\alpha\, dx - \int_0^{\ell_\alpha\, K_\alpha} x\, dx\right] \\
&= \frac{(\ell_\alpha\, K_\alpha)^2}{K_\alpha} - \frac{(\ell_\alpha K_\alpha)^2}{2\, K_\alpha} \\
&= \frac{K_\alpha}{2}\,\ell_\alpha^2.
\end{aligned}$$

Due to the lost-sales penalty r, the optimal profit , υ_α^*, expressed by Eq. (3.12) might be negative under some circumstances, even when the Newsvendor has chosen the optimal order quantity. In light of Eq. (3.12), we derive a threshold value of $\bar{r}_\alpha$ such that $\upsilon_\alpha^* \geqslant 0$ if $r \leqslant \bar{r}_\alpha$. The following lemma shows the threshold value.

Lemma 3.1 *For a uniform demand distribution, $\upsilon_\alpha^* \geqslant 0$ if and only if $r \leqslant \bar{r}_\alpha$, where*

$$\bar{r}_\alpha \triangleq \frac{(p - c_\alpha)^2}{2c_\alpha - (p + s)}. \tag{3.13}$$

In addition, $r \leqslant \bar{r}_\alpha$ is equivalent to $(p + r - c_\alpha)^2 - r\,(p + r - s) \geqslant 0$.

Proof of Lemma 3.1. To prove the result, we set $\upsilon_\alpha^* = 0$ in Eq. (3.12). Hence, we have $\ell_\alpha \cdot (p + r - c_\alpha) - r = 0$ given that $K_\alpha \geqslant 0$. By inputting $\ell_\alpha \triangleq \frac{p+r-c_\alpha}{p+r-s}$

given by Eq. (3.3) into the equation above and applying some algebra, we have $(p+r-c_\alpha)^2 - r\,(p+r-s) \geqslant 0$. Expanding and rearranging the above equation yields Eq. (3.13). This completes the proof. ■

In light of Eq. (3.12), it is straightforward to see υ_α^* is monotonically increasing in α. The following proposition provides a sufficient condition to secure the convexity of υ_α^* in α.

Proposition 3.2 *For demand with a Uniform distribution $D_\alpha \sim \mathcal{U}(0,\, K_\alpha)$, if K_α is convex and c_α is concave in $\alpha \in [0,\, 1]$, then υ_α^* given by Eq. (3.12) is increasingly convex in α.*

Proof of Proposition 3.2. The continuity is obvious as shown by Eq. (3.12). For ease of exposition, we first rewrite Eq. (3.12) as

$$\upsilon_\alpha^* = \frac{1}{2(p+r-s)} K_\alpha \Big[(p+r-c_\alpha)^2 - r\,(p+r-s) \Big]. \tag{3.14}$$

By taking the first-order derivative of υ_α^* as given by Eq. (3.14), we have the following

$$\begin{aligned} \frac{d}{d\alpha}\upsilon_\alpha^* &= \frac{1}{2(p+r-s)} \Big\{ K'_\alpha \big[(p+r-c_\alpha)^2 - r\,(p+r-s) \big] - 2K_\alpha (p+r-c_\alpha) c'_\alpha \Big\} \\ &\geqslant 0. \end{aligned} \tag{3.15}$$

Next, by taking the second-order of υ_α^* based on its first-order derivative given by Eq. (3.15), we further have

$$\begin{aligned}
&\frac{d^2}{d\alpha^2}\upsilon_\alpha^* \\
= \;& \frac{1}{2(p+r-s)}\Big\{K_\alpha''[(p+r-c_\alpha)^2 - r\,(p+r-s)] + K_\alpha'[2(p+r-c_\alpha)(-c_\alpha')] \\
&\qquad -\Big[K_\alpha'[2(p+r-c_\alpha)c_\alpha'] - K_\alpha[2(-c_\alpha')c_\alpha' + 2(p+r-c_\alpha)c_\alpha'']\Big]\Big\} \\
= \;& \frac{1}{2(p+r-s)}\Big\{K_\alpha''[(p+r-c_\alpha)^2 - r\,(p+r-s)] + \{K_\alpha'[2(p+r-c_\alpha)(-c_\alpha')] \\
&\qquad -K_\alpha'[2(p+r-c_\alpha)c_\alpha']\} - K_\alpha[2(-c_\alpha')c_\alpha' + 2(p+r-c_\alpha)c_\alpha'']\Big\} \\
= \;& \frac{1}{2(p+r-s)}\Big\{K_\alpha''\underbrace{[(p+r-c_\alpha)^2 - r\,(p+r-s)]}_{\geqslant 0}\underbrace{-4K_\alpha'[(p+r-c_\alpha)c_\alpha']}_{\geqslant 0} \\
&\qquad +2K_\alpha[(c')^2\underbrace{-(p+r-c_\alpha)c_\alpha''}_{\geqslant 0}]\Big\}.
\end{aligned}$$

Here, the first term on the most hand side of the above is non-negative in light of Lemma 3.1; the second one is non-negative because $c_\alpha' \leqslant 0$; and the third one is non-negative because $c_\alpha'' \geqslant 0$ by the assumption.

Finally, we conclude that $\frac{d^2}{d\alpha^2}\upsilon_\alpha^* \geqslant 0$, which proves the convexity of υ_α^* in α. ■

Proposition 3.2 implies that, when α increases, if the marginal cost savings decrease, whereas the marginal demand growth increases, then the marginal profit from Blockchain adoption increases.

3.2.1 Linear Case

To further illustrate useful insights, we consider a very special case with linear structures of the upper range of a Uniform demand distribution and the ordering cost as below:

$$K_\alpha = k_0 + k_1\alpha; \tag{3.16}$$

$$c_\alpha = c_0 - c_1\alpha, \tag{3.17}$$

where k_0, k_1, c_0, c_1 are positive constants and satisfy $c_0 \geqslant c_1 > 0$ to avoid non-positive ordering costs. In this case, $K'_\alpha = k_1$ and $c'_\alpha = -c_1$. By inputting them into Eq. (3.15), we have the simplified first-order derivative function as below,

$$\begin{aligned} & \frac{d}{d\alpha}\upsilon^*_\alpha \\ = & \frac{1}{2(p+r-s)}\left\{k_1\left[(p+r-c_\alpha)^2 - r\,(p+r-s)\right] + 2K_\alpha(p+r-c_\alpha)c_1\right\} \\ \geqslant & \ 0. \end{aligned} \tag{3.18}$$

It is non-negative because $(p+r-c_\alpha)^2 - r\,(p+r-s) \geqslant 0$, owing to Lemma 3.1.

Proposition 3.3 *For a Uniform distribution of demand $D_\alpha \sim \mathcal{U}(0,\ K_\alpha)$, if both K_α and c_α are linear functions of α, as given by Eqs. (3.16) and (3.17), respectively, then υ^*_α*

given by Eq. (3.12) is a cubic function of α as below:

$$\upsilon_{\alpha}^{*} = \xi_3\,\alpha^3 + \xi_2\,\alpha^2 + \xi_1\,\alpha + \xi_0, \tag{3.19}$$

where the coefficients are all non-negative as defined as

$$\begin{aligned}
\xi_0 &= \frac{k_0}{2}\left(\frac{L^2}{p+r-s} - r\right);\\
\xi_1 &= \frac{1}{2}\left(\frac{L^2 k_1 + 2k_0 L c_1}{p+r-s} - r k_1\right);\\
\xi_2 &= \frac{c_1}{2}\,\frac{k_0 c_1 + 2k_1 L}{p+r-s};\\
\xi_3 &= \frac{k_1 c_1^2}{2(p+r-s)},
\end{aligned}$$

and

$$L \triangleq p + r - c_0. \tag{3.20}$$

Proof of Proposition 3.3. To derive the expression for Eq. (3.24), we input Eqs. (3.16) and (3.17) into Eq. (3.14), and apply some basic algebra, which yields

$$
\begin{aligned}
& v_\alpha^* \\
= \; & \frac{1}{2(p+r-s)}(k_0+k_1\alpha)(p+r-c_0+c_1\alpha)^2 - \frac{r}{2}(k_0+k_1\alpha) \\
= \; & \frac{1}{2(p+r-s)}(k_0+k_1\alpha)(L+c_1\alpha)^2 - \frac{r}{2}(k_0+k_1\alpha) \\
= \; & \frac{1}{2(p+r-s)}(k_0+k_1\alpha)(c_1^2\alpha^2+2Lc_1\alpha+L^2) - \frac{r}{2}(k_0+k_1\alpha) \\
= \; & \frac{1}{2(p+r-s)}\Big[(k_0c_1^2\alpha^2+2k_0Lc_1\alpha+k_0L^2)+(k_1c_1^2\alpha^3+2k_1Lc_1\alpha^2+k_1L^2\alpha)\Big] \\
& \qquad - \frac{r}{2}(k_0+k_1\alpha) \\
= \; & \frac{1}{2(p+r-s)}\Big[k_1c_1^2\alpha^3+(k_0c_1^2+2k_1Lc_1)\alpha^2+(2k_0Lc_1+k_1L^2)\alpha+k_0L^2\Big] \\
& \qquad - \frac{r}{2}(k_0+k_1\alpha) \\
= \; & \underbrace{\frac{k_1c_1^2}{2(p+r-s)}}_{\xi_3\geqslant 0}\alpha^3 + \underbrace{\frac{k_0c_1^2+2k_1Lc_1}{2(p+r-s)}}_{\xi_2\geqslant 0}\alpha^2 + \Bigg(\underbrace{\frac{2k_0Lc_1+k_1L^2}{2(p+r-s)} - \frac{rk_1}{2}}_{\xi_1}\Bigg)\alpha \\
& \qquad + \underbrace{\frac{k_0L^2}{2(p+r-s)} - \frac{r}{2}k_0}_{\xi_0} \\
= \; & \xi_3\alpha^3+\xi_2\alpha^2+\xi_1\alpha+\xi_0.
\end{aligned}
$$

The non-negativities of these coefficients, ξ_i, can be justified as follows: First, it is straightforward to show $\xi_3, \xi_2 \geqslant 0$. To show $\xi_1, \xi_0 \geqslant 0$, first note that

$\xi_1 \geqslant \frac{k_1}{2}(\frac{L^2}{p+r-s} - r)$ and $\xi_0 = \frac{k_0}{2}(\frac{L^2}{p+r-s} - r)$. Therefore, it suffices to prove

$$\frac{L^2}{p+r-s} - r \quad \geqslant \quad 0, \tag{3.21}$$

or equivalently, $L^2 \geqslant r\,(p+r-s)$. To this end, we refer to Lemma 3.1. For $\alpha = 0$, the necessary and sufficient condition becomes $(p+r-c_0)^2 - r\,(p+r-s) \geqslant 0$, which is $L^2 \geqslant r\,(p+r-s)$. This proves $\xi_1, \xi_0 \geqslant 0$. Finally, the whole proof is complete. ■

In light of Proposition 3.3, all the coefficients ξ_i, for $i = 0, 1, 2, 3$ are non-negative, which implies that υ^*_α is increasing and also convex in α. In other words, the marginal profit associated with α gets larger as α increases. This is in agreement with Proposition 3.2. In addition, for zero adoption of BCT, $\alpha = 0$,

$$\upsilon^*_\alpha = \xi_0 = \frac{k_0}{2}\left(\frac{L^2}{p+r-s} - r\right) \geqslant 0, \tag{3.22}$$

which is proportional to k_0, but independent of k_1. Here, the non-negativity follows from Lemma 3.1.

For a special case with $k_0 = 0$ in Proposition 3.3, the cubic function given by

Eq. (3.24) can be further simplified as

$$\upsilon_\alpha^* = \frac{k_1 c_1^2}{2(p+r-s)}\,\alpha^3 + \frac{k_1 c_1 L}{p+r-s}\,\alpha^2 + \frac{k_1}{2}\left(\frac{L^2}{p+r-s} - r\right)\alpha, \tag{3.23}$$

where the coefficients given in Proposition 3.3 are simplified as $\xi_0 = 0$; $\xi_1 = \frac{k_1}{2}\left(\frac{L^2}{p+r-s} - r\right)$; $\xi_2 = \frac{k_1 c_1 L}{p+r-s}$; and $\xi_3 = \frac{k_1 c_1^2}{2(p+r-s)}$. Obviously, for this case, Eq. (3.23) is a linear function of k_1 for any $\alpha \in [0,\, 1]$. In addition, without any adoption of BCT, $\upsilon_\alpha^* = 0$ for $\alpha = 0$. This is true because $D_\alpha = 0$ while $\alpha = 0$. Accordingly, we have the following result.

Proposition 3.4 *For a Uniform distribution of demand $D_\alpha \sim \mathcal{U}(0,\, k_1\alpha)$, if c_α is a linear function of α, as given by Eq. (3.17), then υ_α^* given by Eq. (3.12) is proportional to k_1, such that*

$$\upsilon_\alpha^* = k_1\, g(\alpha), \tag{3.24}$$

where $g(\alpha)$ is a cubic function of α as

$$g(\alpha) \triangleq \frac{c_1^2}{2(p+r-s)}\,\alpha^3 + \frac{c_1 L}{p+r-s}\,\alpha^2 + \frac{1}{2}\left(\frac{L^2}{p+r-s} - r\right)\alpha. \tag{3.25}$$

Proof of Proposition 3.4. The proof readily follows from the previous discussion.

■

In view of Proposition 3.4, it is shown that $v_\alpha^* = 0$ for $\alpha = 0$; namely, zero adoption of BCT leads to zero operational profit. This can be explained by that the demand $D_\alpha = 0$, almost surely, for $\alpha = 0$.

The following proposition provides a closed-form expression for the optimal BCT adoption level.

Proposition 3.5 *For a Uniform distribution of demand $D_\alpha \sim \mathcal{U}(0,\, K_\alpha)$, if both K_α and c_α are linear functions of α, as given by Eqs. (3.16) and (3.17), respectively, and the BCT adoption cost function $\psi(\alpha)$ is cubic as given by Eq. (3.11), satisfying $\xi_1 \geqslant b_1$ and $3(\xi_3 - b_3) + 2(\xi_2 - b_2) + (\xi_1 - b_1) \leqslant 0$, then the optimal adoption level has a closed-form solution,*

$$\alpha^* = \frac{b_2 - \xi_2 - \sqrt{(b_2 - \xi_2)^2 - 3(\xi_3 - b_3)(\xi_1 - b_1)}}{3(\xi_3 - b_3)}. \tag{3.26}$$

Proof of Proposition 3.5. First, we write the objective function as;

$$\pi(\alpha) \triangleq v_\alpha^* - \psi(\alpha). \tag{3.27}$$

By Eqs. (3.24) and (3.11), the Eq. (3.27) can be written as

$$\pi(\alpha) = (\xi_3 - b_3)\alpha^3 + (\xi_2 - b_2)\alpha^2 + (\xi_1 - b_1)\alpha + (\xi_0 - b_0).$$

By taking the first- and the second-order derivatives, we have

$$\pi'(\alpha) = 3(\xi_3 - b_3)\alpha^2 + 2(\xi_2 - b_2)\alpha + (\xi_1 - b_1); \tag{3.28}$$

$$\pi''(\alpha) = 6(\xi_3 - b_3)\alpha + 2(\xi_2 - b_2). \tag{3.29}$$

Then we have the following to justify $\pi'(\alpha) \geqslant 0$ at $\alpha = 0$ while $\pi'(\alpha) \leqslant 0$ at $\alpha = 1$:

$$\begin{aligned} \pi'(0) &= \xi_1 - b_1 \geqslant 0; \\ \pi'(1) &= 3(\xi_3 - b_3) + 2(\xi_2 - b_2) + (\xi_1 - b_1) \leqslant 0. \end{aligned} \tag{3.30}$$

where the inequality holds by the assumption. Therefore, there exists an optimal $\alpha^* \in [0,\ 1]$. Finally, Eq. (3.26) can be obtained via setting $\pi'(\alpha) = 0$ as given in Eq. (3.28) and applying some basic algebra. It can be further justified that α^* given by Eq. (3.26) satisfies $\alpha^* \in [0,\ 1]$. This completes the proof. ■

One typical example of $\pi(\alpha)$ as a cubic function is given in Section §3.2.2. It is worth noting that there typically exist a pair of roots to satisfy the first-order condition $\pi'(\alpha) = 0$. However, one of them,

$$\alpha' = \frac{b_2 - \xi_2 + \sqrt{(b_2 - \xi_2)^2 - 3(\xi_3 - b_3)(\xi_1 - b_1)}}{3(\xi_3 - b_3)}, \tag{3.31}$$

lies beyond the valid interval $[0,\ 1]$, as illustrated in our numerical study later.

3.2.2 Numerical Studies with Uniform Distributions of Demand

In this numerical study, we specify the parameters as follows: $p = 200$; $r = 20$; $s = 10$; $c_0 = 100$; $c_1 = 20$; $k_0 = 100$; $k_1 = 10$. In light of Proposition 3.3, we can easily compute v^*_α by Eq. (3.24). For the adoption cost, we assume it follows a cubic form as given in Eq. (3.11) and set $b_3 = 1000$ and $b_0 = b_1 = b_2 = 0$. Hence, $\psi(\alpha)$ simply takes the form,

$$\psi(\alpha) = 1000\,\alpha^3. \tag{3.32}$$

Accordingly, the objective function $\pi(\alpha)$ given by Eq. (3.27) is

$$\pi(\alpha) = -990.47\alpha^3 + 209.52\alpha^2 + 1385.7\alpha + 2428.6.$$

Fig. 3.6 depicts the functional curve of $\pi(\alpha)$ as given above for the expanded range of $\alpha \in [-2, 2]$. The whole picture shows that $\pi(\alpha)$ might not behave as a concave function in general for a broad interval of α. As a piece of the whole picture confined in the valid interval of α is $[0, 1]$, it is first increasing and then decreasing in α and the optimal adoption level is attained at $\alpha^* = 0.757$.

The impact of BCT adoption is presented in two perspectives: (1) K_α as given by Eq. (3.16) for the demand and (2) c_α as given by Eq. (3.17) for the supply. In what follows, we shall numerically illustrate how those two factors impact the

Figure 3.6: Expanded Structure of $\pi(\alpha)$ with $\alpha \in [-2, 2]$ for Uniform Demand

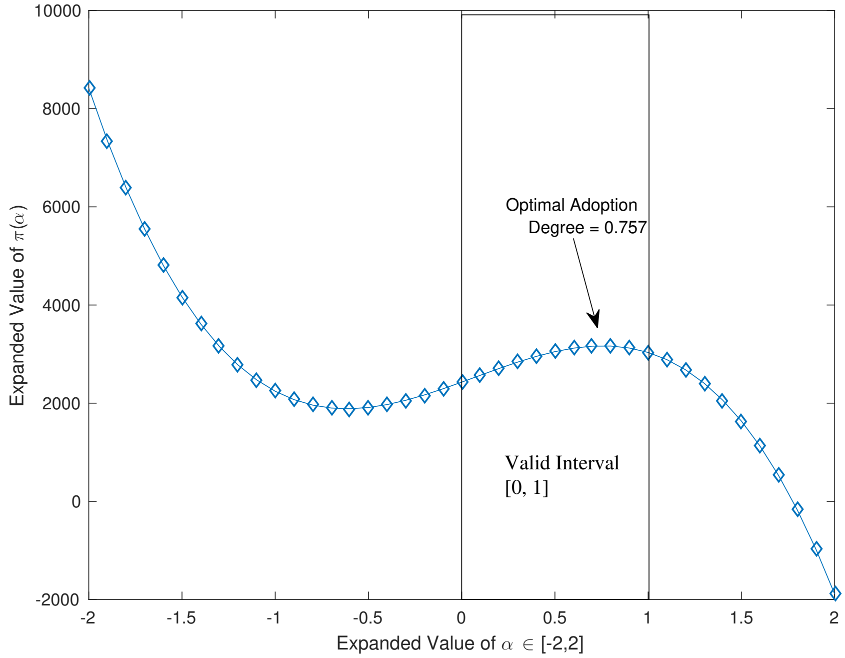
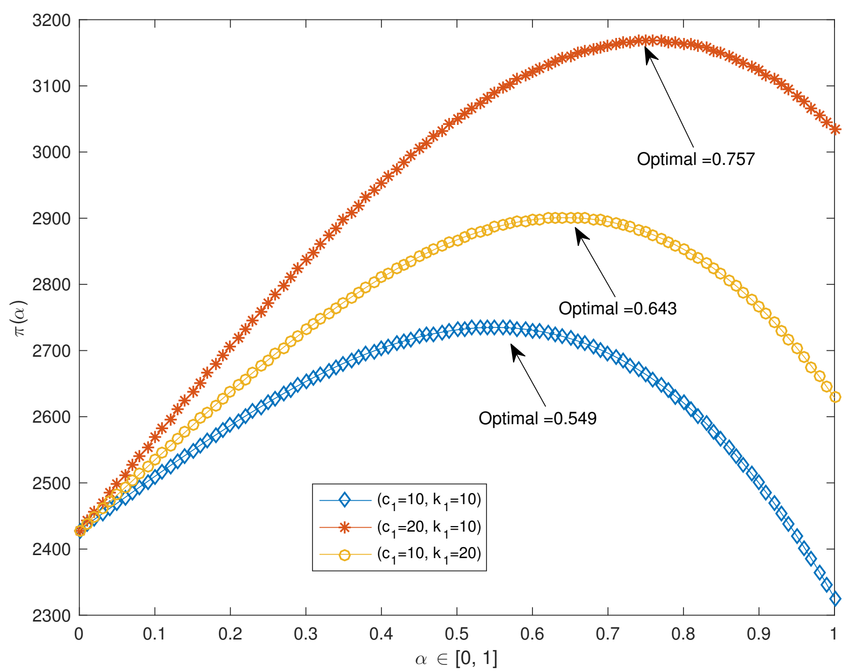


Figure 3.7: Sensitivity of $\pi(\alpha)$ to c_1 and k_1

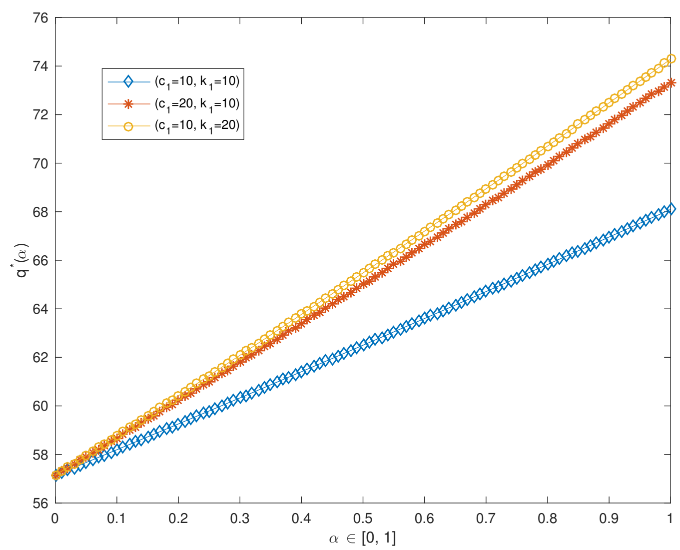


Blockchain performance, and the corresponding results as well. To this end, we consider three settings of c_1 and k_1:

- $c_1 = 10$, $k_1 = 10$;
- $c_1 = 20$, $k_1 = 10$;
- $c_1 = 10$, $k_1 = 20$.

Among the three settings, the first one is taken as a benchmark, while the second setting doubles c_1 and the third doubles k_1. Fig. 3.7 depicts the optimal profit $\pi(\alpha)$ as α increases from 0 to 1, and Fig. 3.8 shows q^*_α in terms of α. Firstly, any increase either in demand or in ordering cost savings will lead to an increase in the total profit for any adoption level α; cf. Fig. 3.7. It also increases the optimal q^*_α; cf. Fig. 3.8. Secondly, the optimal α^* increases as either k_1 or c_1 increases. For example, the $\alpha^* = 0.549$ for the first setting with $c_1 = 10$ and $k_1 = 10$; It is increased to $\alpha^* = 0.643$ while k_1 is doubled, $k_1 = 20$. It is increased to $\alpha^* = 0.757$ while c_1 is doubled, $c_1 = 20$. In other words, if the marginal impact from the BCT adoption on demand becomes higher, then a firm intends to adopt a higher level of BCT. In addition, if the marginal impact from the BCT adoption on ordering cost savings becomes higher, then a firm intends to adopt a higher level of BCT. Thirdly, the optimal order quantity q^*_α increases monotonically in α, roughly in a linear manner. The q^*_α gets bigger while either k_1 or c_1 increases. Interestingly, although the difference of q_α^* between the second and third settings is tiny as shown in Fig. 3.8, the disparity of $\pi(\alpha)$ as showed in Fig. 3.7 is significant.

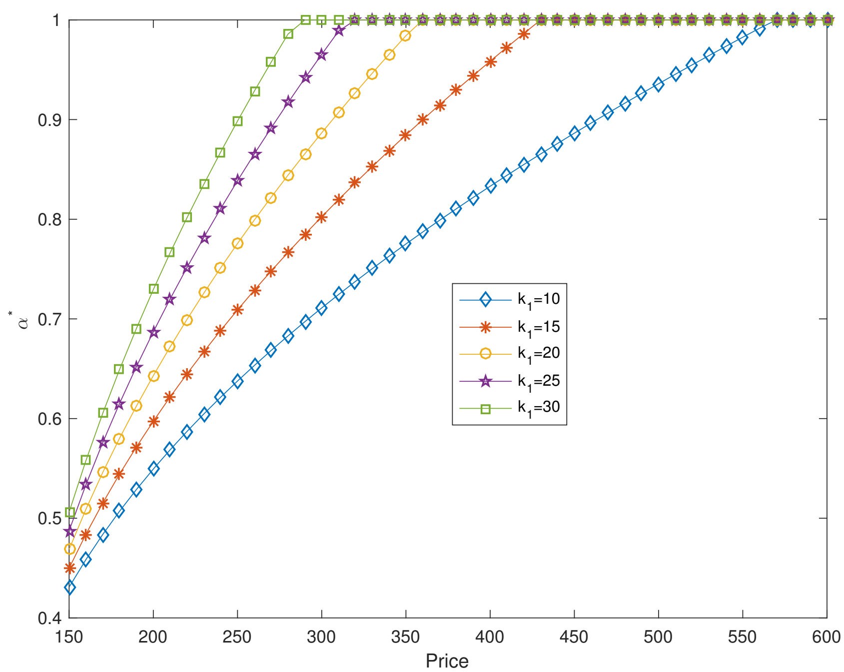
Figure 3.8: Sensitivity of q_α^* to c_1 and k_1



One of the major factors that impacts the optimal BCT adoption level is the selling price p. In our previous analysis, the selling price is assumed to be exogenously fixed. In the following numerical study, we shall explore its impact, in conjunction with other parameters. Fig. 3.9 depicts the optimal adoption level in terms of the selling price and the impact of k_1 varying from 10 to 30, while Fig. 3.10 depicts the same but under the impact of c_1 varying from 10 to 30.

Firstly, it is shown in both figures that α^* increases as p increases; namely, the higher the selling price, the higher the adoption level. In other words, for a more expensive product, a firm is prone to adopt a higher level of BCT. Secondly, for each setting, there exists a threshold value of p, such that when the selling price is higher than this threshold, it is optimal to fully adopt BCT, i.e., $\alpha^* = 1$. It is shown that the threshold of the selling price for a full adoption decreases as k_1 or c_1 get bigger. Thirdly, at a low selling price, the impact of c_1 on α^* is more significant than k_1. For example, for $p = 150$ while k_1 varies from 10 to 30, α^* varies roughly between 0.43 and 0.5; cf. Fig. 3.9. In contrast, while c_1 varies from 10 to 30, α^* varies significantly between 0.4 and 0.85; cf. Fig. 3.10.

Figure 3.9: Optimal Adoption v.s. Price (Variant k_1)



As a disruptive technology in its nascent stage, BCT's adoption cost is relatively high. A firm needs to consider the adoption cost $\psi(\alpha)$ while making an adoption decision. In the following numerical study, we shall illustrate the impact of $\psi(\alpha)$. Fig. 3.11 depicts the total profit in terms of the order quantity and the adoption

Figure 3.10: Optimal Adoption v.s. Price (Variant c_1)

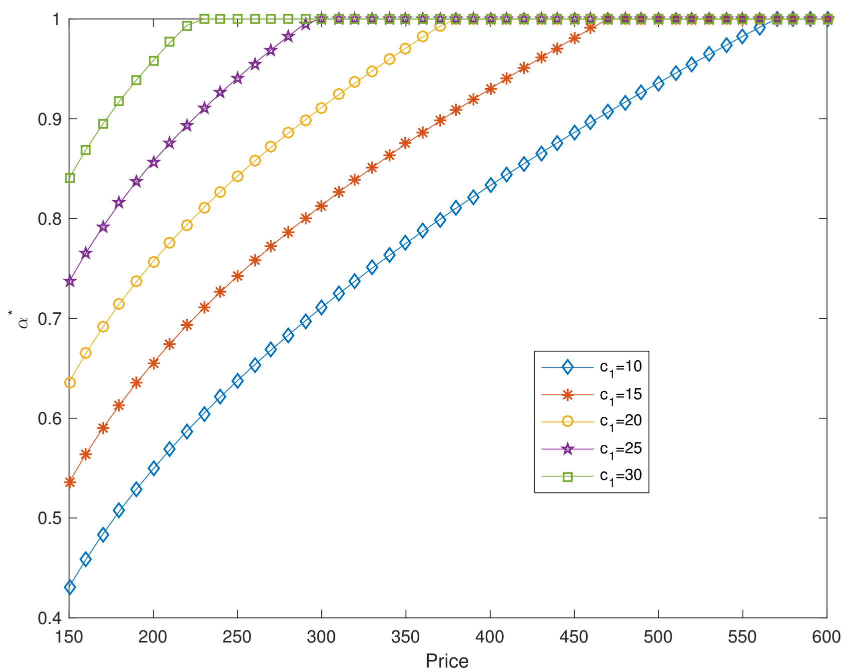


Figure 3.11: Profit v.s. Order Quantity and Adoption Level: Impact of Adoption Costs

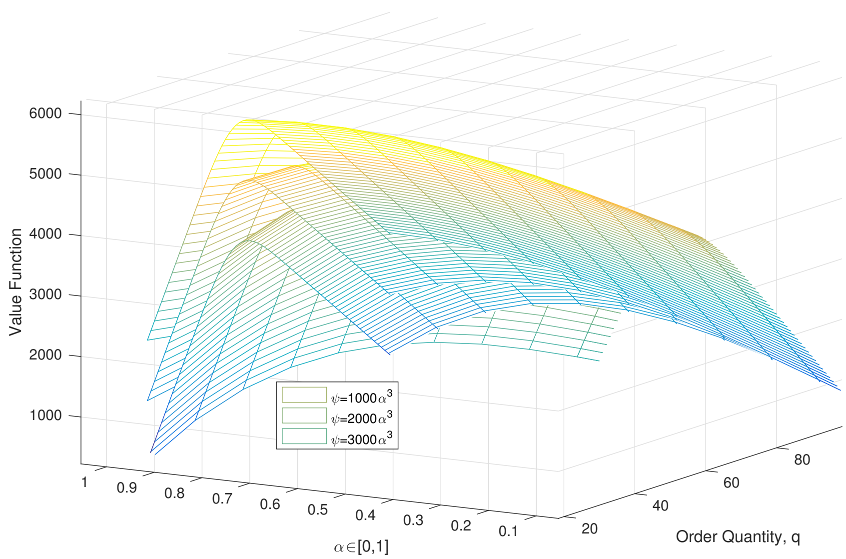
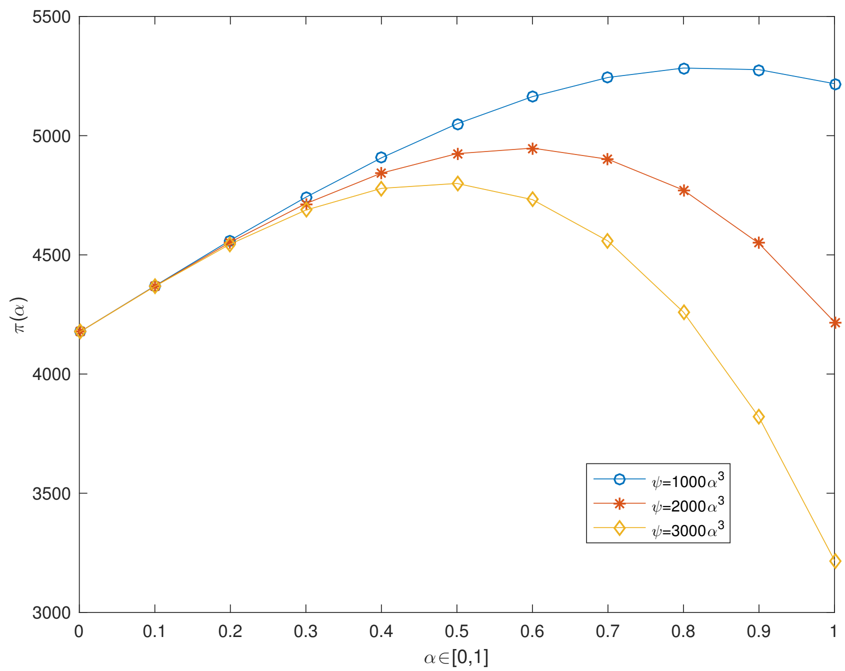


Figure 3.12: Profit v.s. Adoption Level: Impact of Adoption Costs



level. There are three layers, each of which represents a different $\psi(\alpha)$. From the top to the bottom, those layers are associated with $\psi(\alpha) = 1000\alpha^3$, $2000\alpha^3$, and $3000\alpha^3$. Generally speaking, the profit function is jointly concave in (α, q), which further implies the concavity in either α or q.

Fig. 3.12 depicts the total profit in terms of α while the Newsvendor has optimized the order quantity. It provides some important observations. First, the higher the adoption cost, the lower the optimal adoption level and the lower the optimal total profit. In other words, the adoption cost discourages a firm from adopting BCT. Second, $\psi(\alpha)$ imposes a significant impact on the optimal adoption level, while a less significant impact on the total optimal profit. For example, while $\psi(\alpha)$ varies as $\psi(\alpha) = 1000\alpha^3$, $2000\alpha^3$, and $3000\alpha^3$, the optimal adoption level changes as $\alpha^* = 0.8$, 0.6, and 0.5 and their optimal profits are 5200, 4900 and 4800, respectively.

3.3 Normal Distribution of Demand $D_\alpha \sim \mathcal{N}(\mu_\alpha, \sigma_\alpha^2)$

In what follows, we consider the case with a normal distribution of demand, $D_\alpha \sim \mathcal{N}(\mu_\alpha, \sigma_\alpha^2)$.

3.3.1 Analytical Results

In the sequel, we assume that the average demand μ_α is sufficiently large enough such that the probability of $\mathbb{P}(D_\alpha < 0)$ is ignorable.[4] For the standard normal distribution, its *cdf* function is denoted by $\Phi(x)$ and its corresponding inverse function is $\Phi^{-1}(\ell)$. Applying this to the Newsvendor model, we have the following:

$$\begin{aligned} F_\alpha(q_\alpha^*) &= \mathbb{P}(D_\alpha \leqslant q_\alpha^*) \\ &= \mathbb{P}(\frac{D_\alpha - \mu_\alpha}{\sigma_\alpha} \leqslant \frac{q_\alpha^* - \mu_\alpha}{\sigma_\alpha}) \\ &= \Phi(\frac{q_\alpha^* - \mu_\alpha}{\sigma_\alpha}) \\ &= \ell_\alpha \end{aligned} \tag{3.33}$$

[4]For example, per the "3-σ rule of thumb" principle, $\mathbb{P}(\mu - 3\sigma < D < \mu + 3\sigma) \approx 0.9973$. For a real application, one can consider the "*truncated normal distribution*" to avoid any negative value of demand.

Taking the inverse of Eq. (3.33), we have

$$\Phi^{-1}(\ell_\alpha) = \frac{q_\alpha^* - \mu_\alpha}{\sigma_\alpha}.$$

Consequently, we have,

$$q_\alpha^* = \mu_\alpha + \sigma_\alpha \cdot \Phi^{-1}(\ell_\alpha). \tag{3.34}$$

Remarks: i) While σ_α gets larger, i.e., the volatility of demand increases, the order quantity q_α^* gets smaller if $\ell_\alpha \leqslant 0.5$ which implies $\Phi^{-1}(\ell_\alpha) \leqslant 0$; cf. Fig. 3.5.

ii) The optimal order quantity is driven by both the average demand μ_α and the standard deviation intertwined with the critical value ℓ_α. First of all, q_α^* increases as α increases.

$$q_\alpha^* - \mu_\alpha = \sigma_\alpha \cdot \Phi^{-1}(\ell_\alpha) \geqslant 0. \tag{3.35}$$

To facilitate the computation of the optimal result, we introduce the *error function* (a.k.a. the *Gauss error* function). It is a special, sigmoid-shaped function (non-elementary) that occurs in probability, statistics, and partial differential equations describing diffusion. For $x \geqslant 0$, it is defined as

$$\mathrm{erf}(x) \triangleq \frac{1}{\pi} \int_{-x}^{x} e^{-z^2}\, dz = \frac{2}{\pi} \int_{0}^{x} e^{-z^2}\, dz. \tag{3.36}$$

In terms of the error function, the *cdf* function can be expressed as[5]

$$\Phi(x) = \frac{1}{2}\left(1 + \mathrm{erf}(\frac{x}{\sqrt{2}})\right).$$

Hence, its inverse function is

$$\Phi^{-1}(z) = \sqrt{2}\,\mathrm{erf}^{-1}(2\,z - 1),$$

where $\mathrm{erf}^{-1}(z)$ is the inverse function of $\mathrm{erf}(x)$.[6] Accordingly, the optimal order quantity q^*_α given by Eq. (3.34) can be further written as

$$q^*_\alpha = \mu_\alpha + \sqrt{2}\,\sigma_\alpha\,\mathrm{erf}^{-1}(2\ell_\alpha - 1). \tag{3.37}$$

The optimal order quantity given by Eq. (3.37) reveals the impact of α from three perspectives: 1) the average demand μ_α as an isolated term which increases

[5]In statistics and engineering, the well-known Q-function is the tail distribution function of the standard normal distribution. It is $Q(x) = 1 - \Phi(x)$.

[6]Function "$\mathrm{erf}^{-1}(z)$" is a well-developed function in most computational software package. For example, it is "=*erfinv*(z)" in Matlab.

in α; 2) the standard deviation of demand σ_α; and 3) the savings from the ordering cost, c_α, which exclusively vary the critical value ℓ_α as given by Eq. (3.3). Firstly, it exemplifies the results of Theorem 3.2. Note that $2\ell_\alpha = 1$ is equivalent to $c_\alpha = (p + r + s)/2$. Secondly, adopting a high level of BCT may lower the order quantity if the adoption of a high α of BCT could lower the volatility of random demand, *viz.* σ_α decreases while α increases.

Counter intuitively, an increase in α might lead to a lower q^*_α. This is true because σ_α might decrease. If this decreasing impact outperforms the increasing impact from μ_α, then q^*_α can become even smaller.

3.3.2 Numerical Studies with Normal Distribution of Demand

In this numerical study, we follow the same setting of parameters as we did in Section §3.2.2, but replace the Uniform distributions with Normal distributions $D_\alpha \sim \mathcal{N}(\mu_\alpha, \sigma^2_\alpha)$. It is our purpose to show how a demand distribution impacts the expected profit through two dimensions, its mean μ_α and its variance σ^2_α, each of which is driven by the BCT adoption level α.

(1) Impact of Mean: In the first numerical study, we fix $\sigma_\alpha = 10$, but vary the mean μ_α as the following three cases:

- $\mu_\alpha = 50 - 0\alpha$;
- $\mu_\alpha = 50 + 10\alpha$;

- $\mu_\alpha = 50 + 20\alpha$.

Table 3.2 exhibits the optimal q^*, υ^*_α and $\pi(\alpha)$ for selected α for each of the above cases.

Table 3.2: Performance of BCT Adoption Impacted by the Demand Mean $D \sim \mathcal{N}(\mu_\alpha, \sigma^2)$

$\alpha \in [0, 1]$	$\mu_\alpha = 50 + 0\alpha$			$\mu_\alpha = 50 + 10\alpha$			$\mu_\alpha = 50 + 20\alpha$		
	q^*	υ^*_α	$\pi(\alpha)$	q^*	υ^*_α	$\pi(\alpha)$	q^*	υ^*_α	$\pi(\alpha)$
0.0	51.80	4175.69	4175.69	51.80	4175.69	4175.69	51.80	4175.69	4175.69
0.1	52.04	4279.53	4278.53	53.04	4381.53	4380.53	54.04	4483.53	4482.53
0.2	52.29	4383.86	4375.86	54.29	4591.86	4583.86	56.29	4799.86	4791.86
0.3	52.53	4488.68	4461.68	55.53	4806.68	4779.68	58.53	5124.68	5097.68
0.4	52.78	4594.00	4530.00	56.78	5026.00	4962.00	60.78	5457.99	5393.99
0.5	53.03	4699.81	4574.81	58.03	5249.81	5124.81	63.03	5799.80	5674.80
0.6	53.28	4806.12	**4590.12**	59.28	5478.12	5262.12	65.28	6150.12	5934.12
0.7	53.53	4912.93	4569.93	60.53	5710.93	5367.93	67.53	6508.93	6165.93
0.8	53.79	5020.25	4508.25	61.79	5948.25	5436.25	69.79	6876.25	6364.25
0.9	54.05	5128.09	4399.09	63.05	6190.09	**5461.09**	72.05	7252.09	6523.09
1.0	54.31	5236.44	4236.44	64.31	6436.44	5436.44	74.31	7636.44	**6636.44**

As shown in Table 3.2, for the three cases, their corresponding optimal $\alpha^* = 0.6, 0.9, 1.0$ with the optimal profit $\pi(\alpha^*)$ as 4590.12, 5461.09, and 6636.44, respectively. It further provides some important insights. First, for each case, the optimal order quantity increases as α increases, and it is always $q^*_\alpha > \mu_\alpha$ slightly. This observation echoes Theorem 3.2. It is true even for the first case where the BCT adoption does not affect the expected demand at all. For this case, the monotonicity can be explained from the supply side in terms of savings of ordering costs c_α while adopting BCT. Second, the higher the adoption level α, the larger the operational profit υ^*_α, which means an adoption of BCT is operationally lucrative, without considering the BCT adoption cost. Third, for each setting,

$\pi(\alpha)$ first increases and then decreases in α. Finally, the higher the effect of α on $\mathbb{E}[D_\alpha]$, the higher the optimal adoption level α^*, which further yields the higher total profit $\pi(\alpha^*)$. In a loose sense, the sensitivity of expected demand to α imposes a positive impact on the optimal adoption level α^*, as well as on the corresponding optimal total profit.

(2) Impact of Variance: In the second numerical study, we set $\mu_\alpha = 50 + 5\alpha$; meanwhile, we vary the standard deviation σ_α as the following three cases:

- $\sigma_\alpha = 10 - 5\alpha$;
- $\sigma_\alpha = 10 + 0\alpha$;
- $\sigma_\alpha = 10 + 5\alpha$.

Table 3.3: Performance of BCT Adoption Impacted by the Demand Volatility $D \sim \mathcal{N}(\mu_\alpha, \sigma_\alpha^2)$

$\alpha \in [0, 1]$	$\sigma_\alpha = 10 - 5\alpha$			$\sigma_\alpha = 10 - 0\alpha$			$\sigma_\alpha = 10 + 5\alpha$		
	q^*	υ_α^*	$\pi(\alpha)$	q^*	υ_α^*	$\pi(\alpha)$	q^*	υ_α^*	$\pi(\alpha)$
0.0	51.80	4175.69	4175.69	51.80	4175.69	4175.69	51.80	4175.69	4175.69
0.1	52.44	4371.55	4370.55	52.54	4330.53	4329.53	52.65	4289.51	4288.51
0.2	53.06	4569.47	4561.47	53.29	4487.86	4479.86	53.52	4406.27	4398.27
0.3	53.65	4769.38	4742.38	54.03	4647.68	4620.68	54.41	4526.03	4499.03
0.4	54.22	4971.20	4907.20	54.78	4810.00	4746.00	55.34	4648.88	4584.88
0.5	54.77	5174.85	5049.85	55.53	4974.81	4849.81	56.29	4774.92	4649.92
0.6	55.30	5380.28	5164.28	56.28	5142.12	4926.12	57.26	4904.24	4688.24
0.7	55.80	5587.40	5244.40	57.03	5311.93	4968.93	58.27	5036.93	**4693.93**
0.8	56.27	5796.15	**5284.15**	57.79	5484.25	**4972.25**	59.30	5173.11	4661.11
0.9	56.73	6006.45	5277.45	58.55	5659.09	4930.09	60.37	5312.87	4583.87
1.0	57.15	6218.22	5218.22	59.31	5836.44	4836.44	61.46	5456.34	4456.34

As shown in Table 3.3, for the three cases, their corresponding optimal $\alpha^* = 0.8, 0.8, 0.7$ with optimal profit $\pi(\alpha^*)$ as 5284.15, 4972.25, and 4693.93, respectively.

It further provides some important insights. First, for each case, the optimal order quantity increases as α increases, and always $q^*_\alpha > \mu_\alpha$. This can be explained from both supply and demand: for the former by the cost savings c_α while adopting BCT, and for the latter by the demand growth $\mu_\alpha = 50 + 5\alpha$. For the latter reason, although the standard deviation increases in α, the impact of the demand growth μ_α overrides the impact from the standard deviation σ_α. Importantly, for any α, the higher sensitivity of demand volatility, the higher optimal order quantity and the lower operational profit υ^*_α. For example with $\alpha = 0.5$, $q^* = 54.77$, 55.53, 56.29 and their corresponding $\upsilon^*_\alpha = 5174.85$, 4974.81, 4774.92, for the three cases, respectively. Second, the higher the adoption level α, the larger the operational profit υ^*_α, which means an adoption of BCT is operationally lucrative, without considering the BCT adoption cost. Third, for each of the three setting, $\pi(\alpha)$ first increases and then decreases in α. Finally, the higher the effect of α on σ_α, the lower the optimal adoption level α^*, which further yields the lower optimal total profit $\pi(\alpha^*)$. In a loose sense, the sensitivity of demand volatility imposes a negative impact on the optimal adoption level α^*, as well as on the corresponding optimal total levelprofit.

Chapter 4

Dynamic Programming Model

Following the adoption of BCT, a firm orders and sells a product over a horizon $\mathcal{T}$ consisting of T periods where time period t is indexed backward. The supply chain is managed on the adopted infrastructure of a Blockchain, where the level of adoption reflects what percentage information of its supply chain and operations is exposed onto the Blockchain. To quantify the information exposure to the public ledger along the Blockchain, we refer to $\alpha \in [0,\, 1]$ as the *adoption level*. A higher α reflects a higher adoption of the BCT leveraged. As a strategic decision, α will be selected at the very beginning of the time horizon, subject to a cost of $\psi(\alpha)$. Here, the adoption cost $\psi(\alpha)$ covers the setting-up cost of the Blockchain infrastructure, maintenance, managing partnership, and information and database management pertaining to the Blockchain, etc. For instance, it can also reflect the cost caused by overexposure of information to its competitors along the network; i.e., it is likely to lose its competitive advantage by exposing more information; cf. O'Byrne (Mar. 27, 2018), Hertig (Mar 21, 2018) and Luu (Jan. 26, 2018). Typically, it is assumed to be convex and increasing in α (with $\psi(0) = 0$) to reflect the fact that the complexity of managing the Blockchain becomes more significant for each increase in α. For instance, as the global leader in the Blockchain business, IBM provides a customized service of Blockchain solutions to its customers under contractual terms and charges the cost according to the scales of adoption level α.[1] In what follows, we shall introduce our model. The major notation is summarized in Table 4.1.

Table 4.1: Notation Summary for the Multiple-Period Model

$\mathcal{T}$	planning horizon, $\mathcal{T} = \{\, t : t = 1, \ldots, T \,\}$, where time period is indexed backward
α	the Blockchain *adoption level*, $\alpha \in (0, 1]$, a strategical decision variable
I_t	beginning inventory level of period t, where $I_t < 0$ reflects the backorder
q_t	ordering quantity, an operational decision variable
p_t	selling price, an operational decision variable
β_t	discounting factor in period t
$H_t(\cdot)$	inventory cost, i.e., inventory holding cost when $I_t > 0$, backorder penalty when $I_t < 0$
$D_t^\alpha(p_t)$	random demand in period t, depending on α and p_t
$d_t^\alpha(p_t)$	$\triangleq \mathbb{E}\big[D_t^\alpha(p_t)\big]$, expected demand, a function of α and p_t
$R_t^\alpha(q_t)$	the random yield of ordering quantity q_t, a function of α
$V_t^\alpha(I_t)$	the value function at the beginning of period t before ordering decision
$\upsilon_t^\alpha(x_t)$	$\triangleq \mathbb{E}\big[V_{t-1}^\alpha\big(x_t - \omega_t\big)\big]$, the expected profit-to-go function after period t and onward
$W_t^\alpha(y_t)$	the value function in period t before selling decision
$\{x\}^+$	$= \max\{x, 0\}$; $\{x\}^- = \max\{-x, 0\}$

[1]For example, IBM® Blockchain Platform Enterprise Plan charges the service fee according to the information shared with its peers.

4.1 Dynamic Programming Model

The selection of Blockchain adoption level α plays a strategic and critical role impacting the uncertainties from both upstream and downstream of its supply chain. In particular, both customer demand and its supply yield are functions of α. On the demand side, the potential customers are simultaneously Blockchain-savvy and sensitive to α and the selling price p_t. In each period t, the demand distribution follows $D_t^{\alpha}(p) \sim F_p^{\alpha}(\cdot)$, which is in general *stochastically increasing* in α while *stochastically decreasing* in p; *viz.* the higher the adoption level α (or the higher the selling price p_t), the larger (lower) the demand (in the stochastic sense). On the supply side, the material and ordering cost per unit c_t^{α} is reduced thanks to Blockchain technology, which integrates B2B information to streamline transaction process. In this light, we assume that $c_t^{\alpha}(q)$ is, in general, decreasing in α while attempting to produce q units; cf. Geer (2018), Stelmakowich (2016), Aitken (2017) and Brody (2017).

The firm seeks to maximize the total expected discounted profit, by jointly managing (i) the *Blockchain design*, (ii) *production and ordering decisions*, and (iii) *dynamic pricing and selling*. At the beginning of the planning horizon, the firm needs to select α as a strategic decision and to make the investment on its BCT. At this stage, the decision on α is mainly determined by the tradeoff between the setup cost for the Blockchain and the expected discounted profit through the time horizon. In the wake of the adoption of BCT, all subsequent operational decisions on production and pricing through the horizon will be made and executed based on the Blockchain infrastructure.

In terms of operations, the firm orders the production, sells to customers, backorders unmet demand, and holds inventory (if any) over the periods. In particular, at the beginning of the planning horizon, the firm decides on the adoption level of Blockchain via selecting $\alpha \in [0, 1]$. Throughout the operational periods, in each period t, it first reviews the net initial inventory I_t. It then places an order of q_t from its suppliers at the total cost $c_t^\alpha(q_t)$. To manage the demand, the firm needs to decide on the optimal price since the demand is sensitive to price p_t. Taking into account that the consumers are Blockchain-savvy, we assume that the demand $D_t^\alpha(p_t)$ is directly sensitive to both price p_t and α, with the average demand being $d_t^\alpha(p_t) = \mathbb{E}[D_t^\alpha(p_t)]$. On the basis of the selling price and the preselected Blockchain adoption level, the market demand $D_t^\alpha(p_t)$ is then realized. The demand is fulfilled according to the *first-arrived-first-fulfilled* principle, and any unmet demand due to shortage is backlogged; the leftover inventory (if any) is carried over to the next period at a holding cost. For the setting of lost-sales, we shall show that the analysis is still valid in §4.8.1. In a generic setting, we assume the holding and backordering cost function $H_t(I)$ is convex with $H_t(0) = 0$, and its first order derivative $H_t'(\cdot)$ (if it exists) is uniformly bounded. One typical example is $H_t(I) = h_t \cdot I^+ + r_t \cdot I^-$, where for any variable x, its positive part is denoted by $x^+ = \max\{x, 0\}$ and its negative part by $x^- = x^+ - x$; constants $h_t,\ r_t \geqslant 0$ respectively refer to the unit carrying cost of leftover inventory and unit shortage penalty caused by unfulfilled demand. Hence, the initial inventory of the next period $t-1$ is dynamically updated as

$$I_{t-1} = I_t + R_t^\alpha(q_t) - D_t^\alpha(p_t). \tag{4.1}$$

In general, the random demand $D_t^\alpha(p)$ is stochastically increasing in α while stochastically decreasing in p; *viz.* its mean $d_t^\alpha(p) = \mathbb{E}[D_t^\alpha(p)]$ is increasing in α while decreasing in p. The following assumption elaborates on the structure of the demand function.

Assumption 4.1 *The demand is a function of α and p, denoted as $D_t^\alpha(p) = d_t^\alpha(p) + \omega_t$, where the random variable has zero mean, $\mathbb{E}[\omega_t] = 0$.*[2]

For any $\alpha \in [0,1]$, function $d = d_t^\alpha(p)$ has an inverse function, $p = p_t^\alpha(d)$ which is increasing in α but decreasing in d; the expected revenue $\pi_t^\alpha(d) \triangleq d \cdot p_t^\alpha(d)$ is jointly concave in $(\alpha,\ d) \in [0,1] \times \mathbb{R}_+$.[3]

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[2] $D_t^\alpha(p_t)$ is allowed to be negative to reflect the case of customer return of previous sales.

[3] One example is a linear or a power function of $d_t^\alpha(p)$ is, in the form of $d_t^\alpha(p) = f_t(\alpha) - a_t \cdot p^\ell$ with $\ell \geqslant 1$, $a_t > 0$ and $f_t(\alpha)$ is increasing in α. In this case, $d_t^\alpha(p)$ is decreasing and concave in p_t, the inverse of d_t^α is expressed as $p_t^\alpha(d) = (\frac{f_t(\alpha)-d}{a_t})^{1/\ell}$. For such case, $p_t^\alpha(d)$ is increasing in α as well.

In the sequel, under Assumption 4.1, we shall take the mean demand d_t as the decision variable, in lieu of p_t; cf. Li and Zheng (2006). The uncertainty of demand in this situation is mainly caused the whole market volatility that is independent of α and p. In the sequel, we use ω_t to refer to *market factor*, where the larger ω_t, the better market status; the higher variance of ω_t, the more volatility of the market.

From the upstream side of supply chain, BCT can reduce the cost of supply and lower the supply risk by Blockchain-enabled B2B information integration and information discrepancy tracking in the transaction process; cf. Stelmakowich (2016), Aitken (2017). The random supply yield $R_t^\alpha(q)$ is *stochastically increasing* in α and q; cf. Definition 5.2.[4] The procurement cost $c_t^\alpha(q)$ is increasing in q while decreasing in α.[5] Li and Zheng (2006) assume a linear form of q for yield and demand. Our assumption is more generic with nonlinear form of q and also factoring in α. Targeting on a different objective, we aim to derive insightful results pertaining to the adoption level α. Later, a generic framework of $R_t^\alpha(q)$

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[4]One typical example is of the following additive form

$$R_t^\alpha(q_t) = R_t^\alpha(q_t, \varepsilon_t) = \varepsilon_t \cdot q_t + \theta_t^\alpha; \tag{4.2}$$

where the random variable ε_t represents the yields rate such that $\varepsilon_t \in \Omega_{\varepsilon_t}$ with $\Omega_{\varepsilon_t} \subset [0,\ 1]$; θ_t^α refers to the yield improvement due to BCT and it is increasing in α.

[5]One typical example is of the following additive form

$$c_t^\alpha(q_t) = c_t \cdot q_t - \mathcal{L}_t^\alpha; \tag{4.3}$$

where constant $c_t \geqslant 0$ denotes the unit cost; function $\mathcal{L}_t^\alpha$ refers to the cost saving thanks to Blockchain leverage. Hence, $\mathcal{L}_t^\alpha$ is, in general, increasing in α.

and $c_t^\alpha(q)$ will be elaborated in Assumption 4.3.

At the beginning of the planning horizon, the firm faces the *design problem* to decide on the adoption level $\alpha \in [0,1]$:

$$\alpha^*(I_T) = \operatorname{argmax}_{\alpha\in[0,1]}\Big\{V_T^\alpha(I_T) - \psi(\alpha)\Big\}, \tag{4.4}$$

where $V_T^\alpha(I_T)$ is the *total expected discounted operating profit* under Blockchain adoption level α and initial inventory I_T. Given a design selection of α, the operational process consists of two stages in each period $t > 0$: *material procurement* and *dynamic selling*.

(i). During the first stage, *procurement* and *sourcing*, we have:

$$V_t^\alpha(I_t) = \max_{q_t \geqslant 0} \Big\{\mathbb{E}\Big[W_t^\alpha\big(I_t + R_t^\alpha(q_t)\big)\Big] - c_t^\alpha(q_t)\Big\}, \tag{4.5}$$

where the expectation is taken with respect to the random yield R_t^α and $W_t^\alpha(\cdot)$ is the value function derived from the second stage as below.

(ii). During the second stage, *dynamic selling* and *pricing*, we have:

$$W_t^\alpha(y_t) = \max_{d_t \geqslant 0} \Big\{d_t \cdot p_t^\alpha(d_t) - \mathbb{E}\Big[H_t\big(y_t - d_t - \omega_t\big)\Big] + \beta_t \cdot \upsilon_t^\alpha(y_t, d_t)\Big\}; \tag{4.6}$$

where the expectation is taken with respect to ω_t, the discount factor $\beta_t \in$

$(0,\ 1]$ is a constant to reflect the value of time, and the last term in the right hand side of the above is the expected profit-to-go function,

$$\upsilon_t^\alpha(y_t, d_t) \triangleq \mathbb{E}\big[V_{t-1}^\alpha\big(y_t - d_t - \omega_t\big)\big]. \tag{4.7}$$

At the end of the time horizon (i.e., period $t = 0$), $V_0^\alpha(I_0) = s_0 \cdot I_0^+ - r_0 \cdot I_0^-$ where all the leftover inventory (if any) is salvaged at $s_0 \geqslant 0$ per unit, and all accepted by unmet demand incurs a cost $r_0 \geqslant 0$ per unit, with $r_0 \geqslant s_0$ (e.g., via outsourcing from a third party supplier or a spot market).

4.2 Structural Results for Adopted Blockchain

In this section, for an already adopted BCT, i.e., a fixed α, we derive and expound some structural results. The following lemma characterizes the functional properties of the aforementioned model.

Lemma 4.1 *With any selection of $\alpha \in [0, 1]$, for each period $t \in \mathcal{T}$, the following hold:*

(i) $\upsilon_t^\alpha(y_t, d_t)$ *is jointly concave in* $(y_t,\ d_t)$*, and increasing in* y_t *while decreasing in* d_t*;*

(ii) $W_t^\alpha(y_t)$ *is increasing and concave in* y_t*; and*

(iii) $V_t^\alpha(I_t)$ *is increasing and concave in* I_t*.*

Proof of Lemma 4.1.

We use backward induction over t. For $t = 0$, $V_0^\alpha(I_0) = s_0 \cdot I_0^+ - r_0 \cdot I_0^-$ is increasing and concave in $I_0 \in \mathbb{R}$, given that $r_0 \geqslant s$ by assumption. For period $t = 1$, in view of Eq. (4.7), it can be easily justified that $v_1^\alpha(y_1, d_1) = \mathbb{E}\big[V_0^\alpha\big(y_1 - d_1 - \omega_1\big)\big]$ is jointly concave in (y_1, d_1) since $V_0^\alpha(\cdot)$ is concave, the term $y_1 - d_1 - \omega_1$ is linear and expectation reserves the concavity; cf. Lemma 5.1, parts (ii) and (v). Further, it can be shown that $v_1^\alpha(y_1,\, d_1)$ is increasing in y_1 and decreasing in d_1. Hence, those properties trivially hold for period $t = 0$ and part (i) holds for $t = 1$.

To prove part (ii) for period t, we first show $W_t^\alpha(y_t)$ is concave in y_t. In view of Eq. (4.6), the term to be maximized is jointly concave in (y_t, d_t) since the first term is concave in d_t, the second term $-H_t(y_t - d_t - \omega_t)$ and the third term are jointly concave in (y_t, d_t). The concavity is preserved with maximization by Lemma 5.1, part (iii). We next prove that $W_t^\alpha(y_t)$ is increasing in y_t. Toward this end, we consider two inventory levels $y_t' > y_t$, such that $y_t' = y_t + \Delta$ where $\Delta > 0$.

$$
\begin{aligned}
& W_t^{\alpha}(y_t') \\
= \ & \max_{d_t' \geqslant 0} \left\{ d_t' \cdot P_t^{\alpha}(d_t') - \mathbb{E}\Big[H_t\big(y_t' - d_t' - \omega_t\big) \Big] + \beta_t \cdot v_t^{\alpha}(y_t' - d_t') \right\} \\
\geqslant \ & \max_{d_t' \geqslant \Delta} \left\{ d_t' \cdot P_t^{\alpha}(d_t') - \mathbb{E}\Big[H_t\big(y_t' - d_t' - \omega_t\big) \Big] + \beta_t \cdot v_t^{\alpha}(y_t' - d_t') \right\} \\
= \ & \max_{d_t' - \Delta \geqslant 0} \left\{ d_t' \cdot P_t^{\alpha}(d_t') - \mathbb{E}\Big[H_t\big(y_t - (d_t' - \Delta) - \omega_t\big) \Big] + \beta_t \cdot v_t^{\alpha}(y_t + \Delta - d_t') \right\} \\
\geqslant \ & \max_{d_t \geqslant 0} \left\{ d_t \cdot P_t^{\alpha}(d_t) - \mathbb{E}\Big[H_t\big(y_t - d_t - \omega_t\big) \Big] + \beta_t \cdot v_t^{\alpha}(y_t - d_t) \right\} \\
= \ & W_t^{\alpha}(y_t),
\end{aligned}
$$

where the first inequality holds by confining the feasible set of $q_t' \geqslant 0$ to $q_t' \geqslant \Delta$; the second inequality holds by substituting $d_t = d_t' - \Delta$, the first term $d_t' \cdot p_t^{\alpha}(d_t') \geqslant d_t \cdot p_t^{\alpha}(d_t)$ since $d \cdot p_t^{\alpha}(d)$ is increasing in d by assumption and $d_t' \geqslant d_t$, and the last term $v_t^{\alpha}(y_t, d_t' - \Delta) = v_t^{\alpha}(y_t, d_t)$ in light of Eq. (4.7).

In view of Eq. (4.5), it is immediately shown that $V_t^{\alpha}(I_t)$ is increasing in I_t since $W_t^{\alpha}(I_t + R_t^{\alpha} \cdot q_t)$ is increasing in I_t for any realization of R_t^{α} and fixed q_t. This proves Part (iii) for period t.

Finally, we just need to prove Part (i) for period $t+1$. This can be done via revisiting Eq. (4.7), since $V_{t-1}^{\alpha}\big(y_t - d_t - \omega_t\big)$ is jointly concave in (y_t, d_t) for any realization of ω_t thanks to Part (iii), and the joint concavity is preserved with expectation; cf. Lemma 5.1, Part (v). ■

Lemma 4.1 states that more inventory leads to a higher expected discounted profit; however the marginal profit pertaining to inventory gets smaller while inventory level gets higher.

For ease of exposition, we suppress the dependence of the superscript α in the following analysis without causing any confusion. For example, we simply denote the optimum of Eq. (4.5) as $q_t^*(I_t)$, that of of Eq. (4.6) as $d_t^*(y_t)$, and $p_t^*(y_t) \triangleq p_t^\alpha\big(d_t^*(y_t)\big)$. Without considering the factor of adoption level, Li and Zheng (2006) studies a similar setting of the operational process. For completeness, we list some results from Li and Zheng (2006) as below.

Lemma 4.2 *For any selected value of $\alpha \in [0,1]$ and each period $t \in \mathcal{T}$,*

(i) there exists a unique optimal order quantity $q_t^*(I_t)$ which is decreasing in the initial inventory I_t;

(ii) there exists a unique optimal selling price $p_t^*(y_t)$ which is decreasing in the available inventory y_t;

(iii) there exists a unique replenishment threshold $I_t^* \triangleq \inf\{I \in \mathbb{R}^+ : q_t^*(I) = 0\}$ which is independent of the initial inventory I_t, such that it is optimal to order some only if $I_t \leqslant I_t^*$.

Proof of Lemma 4.2.

The proof can be completed via backward induction over t, in conjunction with Lemma 4.1. For some detail, the readers can refer to Li and Zheng (2006). ■

For any selection of α, $p_t^*(y_t)$ is decreasing in y_t; whereas $d_t^*(y_t)$ is increasing in y_t. This implies that while possessing more inventory, it is preferable to set up a lower selling price to induce a higher demand. Under certrain yields, the joint replenishment and pricing problem has been studied by Federgruen and Heching (1999). For such setting, it was showed that the *base-stock-list-price* policy is optimal. That is, there exists an order-up-to level which is independent of the initial inventory, such that, in period t, if $I_t < I_t^c$, then produce up to I_t^c and charge a list price; otherwise, produce nothing and charge a discount price, p_t^c which is lower than the list price. Li and Zheng (2006) extended the study via introducing the yield variability into the model. It is shown that the introduction of random yield causes the solution to lose its produce-up-to feature, and thus there is no list price any more. Both the production quantity and the price are dependent on the initial inventory. Furthermore, with a higher initial inventory, a smaller production quantity should be chosen, and a lower price should be charged.

Theorem 4.1 *Based on the stochastic system in our model, consider a deterministic setting where the fixed yield rate $\bar{r}_t^\alpha(q_t) = \mathbb{E}[R_t^\alpha(q_t)]$ and the fixed demand size $\bar{d}_t^\alpha =$*

$\mathbb{E}[D_t^\alpha]$. *For any function* T_t^α *in the stochastic systems, let* $\bar{T}_t^\alpha$ *denote its counterpart associated with the deterministic system. The following inequality holds:*

$$\bar{V}_t^\alpha(I_t) \geqslant V_t^\alpha(I_t), \quad \textit{and} \quad \bar{W}_t^\alpha(y_t) \geqslant W_t^\alpha(y_t). \tag{4.8}$$

Proof of Theorem 4.1

In light of Lemma 4.1, we can first show that the objective function of Eq. (4.5) before taking expectation is concave in R_t^α and that of Eq. (4.6) is concave in ω_t . Then, the results readily follows from Jensen's inequality. ■

Theorem 4.1 exemplifies the fact that the volatility of random yields and/or uncertain demands negatively affects the value function; cf. Li and Zheng (2006) and Gupta and Cooper (2005). This is true because under uncertain yield, it is more costly to receive each delivery and the uncertainty will reduce the total profit since the marginal profit is decreasing in the inventory level. In contrast to the perception of financial investment with respect to return and risk, this result is counter-intuitive to the conventional wisdom of the risk interpretation, where high risk implies a high profitability. For an efficient operational decision, it calls for the necessary to develop some approaches to mitigate those volatility pertaining to supply and demand uncertainty.

4.3 Impact of Uncertainty

The adoption of BCT will impact both the upstream of supply and the downstream of demand. In this case, we shall investigate the impact of uncertainty of yield and demand. Let $\leqslant_{st}$ refer to usual *stochastic order*, and $\leqslant_{cx}$ *convex order*; see Section §5.2 for a brief review on *Stochastic Comparison*. Then, the following Theorems 4.2 and 4.3 summarize the impact of yield and demand uncertainty on the optimal solution.

Theorem 4.2 *For a fixed α, consider two stochastic systems, where for any function F_t^α in the first stochastic systems, $\tilde{F}_t^\alpha$ denotes its counterpart associated with the second system. Assume that, ceteris paribus, for some period $s \in \mathcal{T}$,*

i) if $R_s^\alpha(q_s) \leqslant_{st} \tilde{R}_s^\alpha(q_s)$ for any q_s, then for any period $t \geqslant s$, $V_t^\alpha(I_t) \leqslant \tilde{V}_t^\alpha(I_t)$, $W_t^\alpha(y_t) \leqslant \tilde{W}_t^\alpha(y_t)$ and $\upsilon_t(\cdot) \leqslant \tilde{\upsilon}_t(\cdot)$ where $\tilde{V}_t(I_t)$, $\tilde{W}_t^\alpha(I_t)$ and $\tilde{\upsilon}_t(\cdot)$ are the counterparts of $V_t(I_t)$, $W_t^\alpha(y_t)$ and $\upsilon_t(\cdot)$ associated with $\{\tilde{R}_s^\alpha\}$, respectively;

ii) if $R_s^\alpha(q_s) \leqslant_{cx} \tilde{R}_s^\alpha(q_s)$ for any q_s, then for any period $t \geqslant s$, $V_t^\alpha(I_t) \geqslant \tilde{V}_t^\alpha(I_t)$, $W_t^\alpha(y_t) \geqslant \tilde{W}_t^\alpha(y_t)$ and $\upsilon_t(\cdot) \geqslant \tilde{\upsilon}_t(\cdot)$ where $\tilde{V}_t^\alpha(I_t)$, $\tilde{W}_t^\alpha(I_t)$ and $\tilde{\upsilon}_t(\cdot)$ are the counterparts of $V_t^\alpha(I_t)$, $W_t^\alpha(y_t)$ and $\upsilon_t(\cdot)$ associated with $\{\tilde{R}_s^\alpha\}$, respectively;

Proof of Theorem 4.2

To prove Part (i), we first show, for period s, $W_s^\alpha\big(I_s + R_s^\alpha(q_s)\big)$ is increasing in R_s^α. This is true because $W_t^\alpha(y_t)$ is increasing in y_t by Lemma 4.1, Part (ii). Therefore, by Lemma 5.5, Part (ii), we have

$$\mathbb{E}[W_s^\alpha\big(I_s + R_s^\alpha(q_s)\big)] \leqslant \mathbb{E}[W_s^\alpha\big(I_s + \tilde{R}_s^\alpha(q_s)\big)].$$

Revisiting Eq. (4.5), it follows that, for any I_s, $V_s^\alpha(I_s) \leqslant \tilde{V}_s^\alpha(I_s)$. Hence, $v_{s+1}(\cdot) \leqslant \tilde{v}_{s+1}(\cdot)$, by Eq. (4.7).

Furthermore, in the objective function of Eq. (4.6), note that the term on its right hand side, $p \cdot D_{s+1}^\alpha(p) - H_{s+1}\big(y - D_{s+1}\big) + \beta_{s+1} \cdot v_{s+1}(y - D_{s+1}) \leqslant p \cdot D_{s+1}^\alpha(p) - H_{s+1}\big(y - D_{s+1}\big) + \beta_{s+1} \cdot \tilde{v}_{s+1}(y - D_{s+1})$. Hence, for any y_{s+1}, $W_{s+1}^\alpha(y_{s+1}) \leqslant \tilde{W}_{s+1}^\alpha(y_{s+1})$.

The induction continues for any period t prior to s, which completes the proof for Part (i).

The proof for Part (ii) readily follows with a similar induction argument as we applied in the proof for Part (i). The major difference is the concavity of $W_t^\alpha(y_t)$ by Lemma 4.1. Since $R_s^\alpha(q_s) \leqslant_{cx} \tilde{R}_s^\alpha(q_s)$, and $-W_s^\alpha(I_s + R_s^\alpha(q_s))$ is convex in $R_s^\alpha(q_s)$, we have

$$-\mathbb{E}[W_s^\alpha\big(I_s + R_s^\alpha(q_s)\big)] \leqslant -\mathbb{E}[W_s^\alpha\big(I_s + \tilde{R}_s^\alpha(q_s)\big)].$$

Therefore, $\mathbb{E}[W_s^\alpha\big(I_s + R_s^\alpha(q_s)\big)] \geqslant \mathbb{E}[W_s^\alpha\big(I_s + \tilde{R}_s^\alpha(q_s)\big)]$. The rest follows, in a similar vein, as in the proof for Part (i). This completes the proof. ■

Theorem 4.2 states any improvement of yield will lead to a higher expected profit; on the contrary, the volatility of yield can diminish the expected profit.

Theorem 4.3 *For a fixed α, consider two stochastic systems, where for any function F_t^α in the first stochastic systems, $\tilde{F}_t^\alpha$ denotes its counterpart associated with the second system. Assume that, ceteris paribus, for some period $s \in \mathcal{T}$,*

i) if $D_s^\alpha(p_s) \leqslant_{st} \tilde{D}_s^\alpha(p_s)$ for any p_s, then for any period $t \geqslant s$, $V_t^\alpha(I_t) \leqslant \tilde{V}_t^\alpha(I_t)$, $W_t^\alpha(y_t) \leqslant \tilde{W}_t^\alpha(y_t)$ and $\upsilon_t(\cdot) \leqslant \tilde{\upsilon}_t(\cdot)$ where $\tilde{V}_t(I_t)$, $\tilde{W}_t^\alpha(I_t)$ and $\tilde{\upsilon}_t(\cdot)$ are the counterparts of $V_t^\alpha(I_t)$, $W_t^\alpha(y_t)$ and $\upsilon_t(\cdot)$ associated with $\{\tilde{D}_s^\alpha\}$, respectively;

ii) if $D_s^\alpha(p_s) \leqslant_{cx} \tilde{D}_s^\alpha(p_s)$ for any p_s, then for any period $t \geqslant s$, $V_t^\alpha(I_t) \geqslant \tilde{V}_t^\alpha(I_t)$, $W_t^\alpha(y_t) \geqslant \tilde{W}_t^\alpha(y_t)$ and $\upsilon_t(\cdot) \geqslant \tilde{\upsilon}_t(\cdot)$ where $\tilde{V}_t(I_t)$, $\tilde{W}_t^\alpha(I_t)$ and $\tilde{\upsilon}_t(\cdot)$ are the counterparts of $V_t^\alpha(I_t)$, $W_t^\alpha(y_t)$ and $\upsilon_t(\cdot)$ associated with $\{\tilde{D}_s^\alpha\}$, respectively;

Proof of Theorem 4.3

To prove Part (i), we first consider period s. In view of Lemma 5.5, Part (iii), since $D_s^\alpha(p_s) \leqslant_{st} \tilde{D}_s^\alpha(p_s)$, there exists a probability space $(\Omega, \mathcal{A}, \mathbb{P})$ and random variable $D(p) = D(p, \omega)$ and $\tilde{D}(p) = \tilde{D}(p, \omega)$ respectively, such that $\tilde{D}(p) = D(p) + \Delta(\epsilon)$ where $\Delta(\epsilon) \geqslant 0$ for any realization of $\epsilon \in \Omega$. Denote $\tilde{p}$ such that $\tilde{D}(\tilde{p}) = D(p)$. Because both $D(\cdot)$ and $\tilde{D}(\cdot)$ are decreasing and $\tilde{D}(p) \geqslant D(p)$, we further have $\tilde{p} \geqslant p$. Revisiting Eq. (4.6), further note that the term on its right hand side satisfies

$$\begin{aligned} & \tilde{p} \cdot \tilde{D}_s(\tilde{p}) - H_s\big(y - \tilde{D}_s(\tilde{p})\big) + \beta_s \cdot V_s(y - \tilde{D}_s(\tilde{p})) \\ = \ & \tilde{p} \cdot D_s(p) - H_s\big(y - D_s(p)\big) + \beta_s \cdot V_s(y - D_s(p)) \\ \geqslant \ & p \cdot D_s(p) - H_s\big(y - D_s(p)\big) + \beta_s \cdot V_s(y - D_s(p)), \end{aligned}$$

where the inequality holds by $\tilde{p} \geqslant p \geqslant 0$. Taking expectation and then maximization on both hand sides of the above, it leads to $\tilde{W}_s^\alpha(y_t) \geqslant W_s^\alpha(y_t)$.

Next, revisiting Eq. (4.5), it follows that, for any I_s, $\tilde{V}_s^\alpha \geqslant V_s^\alpha$, since

$$\mathbb{E}[\tilde{W}_s^\alpha\big(I_s + R_s^\alpha(q_s)\big)] \geqslant \mathbb{E}[W_s^\alpha\big(I_s + R_s^\alpha(q_s)\big)].$$

It is straightforward to show $v_s(\cdot) \leqslant \tilde{v}_s(\cdot)$, by Eq. (4.7).

The induction continues for any period t prior to s, which completes the proof for Part (i).

The proof for Part (ii) follows a similar induction as we applied in the proof for Part (i). The major difference is the concavity of $V_{t-1}^\alpha(y_t - D_t)$ in D_t by Lemma 4.1. This completes the proof. ■

Theorem 4.3 states that any improvement of demand (i.e., higher demand size at the same price and adoption level) will lead to a higher expected profit. This can be simply explained by that the firm can charge an even higher price to deplete the same amount inventory level while facing a better demand market. Similar to the impact from yield, the volatility of demand can diminish the expected profit.

4.4 Randomness of Yield and Demand

To further investigate the impact of volatility of yield or demand, we shall detail their randomness by factoring in random components. The following assumption models the uncertainty factors of supply and demand by $\{\varepsilon_t\}$ and $\{\omega_t\}$, respectively.

Assumption 4.2 *For each period t, the randomness of yield and demand are driven as below:*

(i) The yield is a function of α, q and the random term ε_t, denoted as $R_t^\alpha(q, \varepsilon_t)$, where the yield factor $\varepsilon_t \in \Omega_{\varepsilon_t}$ is independent of α or q.

(ii) The demand is a function of α, p and the random term ω_t, denoted as $D_t^\alpha(p, \omega_t)$, where the market factor $\omega_t \in \Omega_{\omega_t}$ is independent of α or p.

Assumption 4.2 provides relatively generic structures of demand and yield functions. One typical example of demand is given as $D_t^\alpha(p_t, \omega_t) = d_t^\alpha(p_t) + \omega_t$, where $\mathbb{E}[\omega_t] = 0$. One typical example of yield is of the additive form $R_t^\alpha(q_t, \varepsilon_t) =$

$\varepsilon_t \cdot q_t + \theta_t^\alpha$, as given by Eq. (4.2).

In what follows, we investigate the impact of the randomness of $\{\varepsilon_t\}$ and $\{\omega_t\}$ on the optimal decisions. To this end, we denote

$$g_t^\alpha(I_t, q_t, \varepsilon_t) \triangleq W_t^\alpha\big(I_t + R_t^\alpha(q_t, \varepsilon_t)\big) - c_t^\alpha(q_t); \tag{4.9}$$

$$w_t^\alpha(y_t, d_t, \omega_t) \triangleq d_t \cdot p_t^\alpha(d_t) - H_t\big(y_t - d_t - \omega_t\big) + \beta_t \cdot V_{t-1}^\alpha(y_t - d_t - \omega_t) \tag{4.10}$$

Hence, $V_t^\alpha(I_t) = \max\limits_{q_t \geqslant 0} \mathbb{E}_{\varepsilon_t}\big[g_t^\alpha\big(I_t, q_t, \varepsilon_t\big)\big]$. In Eq. (4.5), and

$$W_t^\alpha(y_t) = \max_{d_t \geqslant 0} \mathbb{E}_{\omega_t}\big[w_t^\alpha(y_t, d_t, \omega_t)\big],$$

in Eq. (4.6). We shall mention that there is a disturbance term $\omega_t \cdot P_t^\alpha(d_t)$ in the revenue calculation as given by the first term on the right hand side of Eq. (4.10). However, its expectation over ω_t has been dashed out since $\mathbb{E}[\omega_t] = 0$. For ease of illustration, we further denote the following, while interchanging the decision variable between p_t and d_t,

$$\hat{w}_t^\alpha(y_t, p_t, \omega_t) \triangleq w_t^\alpha(y_t, d_t, \omega_t),$$

which are equivalent by the bijection between price and average demand under

Assumption 4.1. Both can be interchangeably used to present the same profit function in terms of price p_t or average demand d_t.

Lemma 4.3 *For any period t, the following hold:*

(i) If $R_t^\alpha(q_t, \varepsilon_t)$ is concave in ε_t, then $g_t^\alpha(I_t, q_t, \varepsilon_t)$ is also concave in ε_t.

(ii) Function $w_t^\alpha(y_t, d_t, \omega_t)$ is concave in ω_t.

Proof of Lemma 4.3.

To prove Part (i), we first refer to Lemma 4.1, Part (ii), which shows that $W_t^\alpha(y_t)$ is increasing and concave in y_t. Further, $R_t^\alpha(q, \varepsilon)$ is concave in ε. In view of Lemma 5.1, Part (i), it follows that Part (i) holds.

To prove Part (ii), reviewing Eq. (4.10), we see that $-H_t(\cdot)$ is concave since $H_t(\cdot)$ is convex; and $V_{t-1}^\alpha(\cdot)$ is concave, in light of Lemma 4.1, Part (iii) . Further, $y_t - d_t - \omega_t$ is linear of ω_t. Therefore, the concavity of $w_t^\alpha(y_t, d_t, \omega_t)$ in ω_t holds. ■

Theorem 4.4 *For any period t, the following hold:*

(i) If $\varepsilon_s \leqslant_{cv} \tilde{\varepsilon}_s$ for some periods $s \leqslant t$, ceteris paribus, then $V_t^\alpha(I_t) \leqslant \tilde{V}_t^\alpha(I_t)$ for any given I_t, and $W_t^\alpha(y_t) \leqslant \tilde{W}_t^\alpha(y_t)$ for any given y_t.

(ii) If $\omega_s \leqslant_{cv} \tilde{\omega}_s$ for some periods $s \leqslant t$, ceteris paribus, then $V_t^\alpha(I_t) \leqslant \tilde{V}_t^\alpha(I_t)$ for any given I_t, and $W_t^\alpha(y_t) \leqslant \tilde{W}_t^\alpha(y_t)$ for any given y_t.

Proof of Theorem 4.4.

The proof readily follows from Lemma 4.3. ■

Theorem 4.4 states that any volatility, from either the supply side or the demand side, will impact the value function. In this case, under the infrastructure of Blockchain systems, it remains important to develop some strategical solutions to mitigate those inherent volatility, e.g., via more accurate forecasting or enforced contracting.

4.5 Monotonicity Results

To derive some monotonic properties, we introduce the following assumption.

Assumption 4.3 *The yield function $R_t^\alpha(q_t, \varepsilon_t)$ is submodular in (α, q_t) for any realization of ε_t, i.e., $\partial^2 R_t^\alpha(\cdot)/\partial\alpha\partial q_t \leqslant 0$. The cost function $c_t^\alpha(q_t)$ is supermodular in (α, q_t), i.e., $\partial^2 c_t^\alpha(q_t)/\partial\alpha\partial q_t \geqslant 0$.*

Assumption 4.3 reflects the fact that the marginal yield improvement by α decreases in the order quantity q_t. In other words, $\partial R_t^\alpha(q)/\partial\alpha$ decreases in q, i.e., the more ordering, the less marginal yield improvement. One typical example is the additive form as given by Eq. (4.2). In addition, it also implies the fact that the marginal cost per unit of order increases while α increases. In other words, $\partial R_t^\alpha(q)/\partial q$ increases in α, i.e., the higher adoption degree, the higher marginal cost saving. One typical example is the additive form as Eq. (4.3).

Lemma 4.4 *For any period t, under Assumption 4.3, the following hold:*

(i) For any realization of ε_t, $g_t^\alpha(I_t, q_t, \varepsilon_t)$ is submodular in (α, I_t, q_t). Furthermore, function $\mathbb{E}_{\varepsilon_t}[g_t^\alpha(I_t, q_t, \varepsilon_t)]$ is submodular in (α, I_t, q_t); and $V_t^\alpha(I_t)$ is submodular in (α, I_t).

(ii) For any realization of ω_t, $\hat{w}_t^\alpha(y_t, p_t, \omega_t)$ is submodular in (α, y_t, p_t). Furthermore, function $\mathbb{E}_{\omega_t}[\hat{w}_t^\alpha(y_t, p_t, \omega_t)]$ is submodular in (α, y_t, p_t); and $W_t^\alpha(y_t)$ is submodular in (α, y_t).

Proof of Lemma 4.4.

We leverage backward induction over t. First, for the terminal period $t = 0$, $V_0^\alpha(I_0) = s_0 \cdot I_0^+ - r_0 \cdot I_0^-$ is trivially submodular in (α, I_0), since $\partial^2 V_0^\alpha / \partial\alpha\partial I_0 = 0$. Next, assuming the results hold for period $t-1$, it is our objective to show those hold as well for period t. To this end, we first prove Part (ii). For any given ω_t, we show function $w_t^\alpha(y_t, d_t, \omega_t)$ is submodular in (α, y_t), but supermodular in (y_t, p_t)

and (α, p_t), via checking their cross-over derivatives:

$$\frac{\partial^2}{\partial\alpha\partial y_t} w_t^\alpha(y_t, d_t, \omega_t) = \beta_t \underbrace{\frac{\partial^2}{\partial\alpha\partial y} V_{t-1}^\alpha(\cdot)}_{\leqslant 0} \leqslant 0; \tag{4.11}$$

$$\frac{\partial^2}{\partial\alpha\partial d_t} w_t^\alpha(y_t, d_t, \omega_t) = \frac{\partial p_t^\alpha(d_t)}{\partial\alpha} + d_t \underbrace{\frac{\partial^2}{\partial\alpha\partial d_t} p_t^\alpha(d)}_{\geqslant 0} - \beta_t \underbrace{\frac{\partial^2}{\partial\alpha\partial y} V_{t-1}^\alpha(\cdot)}_{\leqslant 0} \geqslant 0; \tag{4.12}$$

$$\frac{\partial^2}{\partial y_t\partial d_t} w_t^\alpha(y_t, d_t, \omega_t) = H_t''(\cdot) - \beta_t \underbrace{\frac{\partial^2}{\partial y^2} V_{t-1}^\alpha(\cdot)}_{\leqslant 0} \geqslant 0, \tag{4.13}$$

The non-negativity in Eq. (4.11) holds because $V_t^\alpha(y)$ is submodular in (α, y) by the hypothesis assumption. The non-negativity in Eq. (4.12) holds because that $p_t^\alpha(d)$ is increasing in α, $p_t^\alpha(d)$ is supermodular by assumption, and $V_t^\alpha(y)$ is submodular in (α, y) by the hypothesis assumption. The non-negativity in Eq. (4.13) holds by the following fact:

If $f(x)$ is concave (convex), then $f(x-y)$ is supermodular (submodular) in (x, y). This is true because $\partial^2 f(x-y)/\partial x\partial y = -f''(x-y) \geqslant (\leqslant) 0$ where the inequality holds by $f''(x) \leqslant (\geqslant) 0$.

Here in Eq. (4.13), $-H_t(\cdot) + \beta_t V_t^\alpha(\cdot)$ is concave. Therefore, the non-negativity holds.

Consequently, it is straightforward to show that $\hat{w}_t^\alpha(y_t, p_t, \omega_t) = w_t^\alpha(y_t, \mu_t^\alpha(p_t), \omega_t)$ is submodular in (α, y_t, p_t), due to the negative bijection between p_t and d_t under Assumption 4.1. Since the submodularity preserves with expectation, we further have $\mathbb{E}_{\omega_t}[\hat{w}_t^\alpha(y_t,p_t,\omega_t)]$ is submodular in (α,y_t,p_t). Finally, $W_t^\alpha(y_t) = \max\limits_{p_t\geqslant 0} \mathbb{E}_{\omega_t}\big[\hat{w}_t^\alpha(y_t,p_t,\omega_t)\big]$ is submodular in (α,y_t) since maximization preserves the submodularity. This completes the proof for Part (ii).

We now proceed to prove Part (i) with the result of Part (ii). For fixed ε_t, we show that function $g_t^\alpha(I_t,q_t,\varepsilon_t)$ given by Eq. (4.9) is submodular in (α,I_t,q_t) in the sense of almost surely, via checking their cross-over derivatives:

$$\frac{\partial^2}{\partial\alpha\partial I_t} g_t^\alpha(I_t,q_t,\varepsilon_t) = \frac{\partial^2}{\partial\alpha\partial I_t} W_t^\alpha(\cdot) \leqslant 0; \tag{4.14}$$

$$\begin{aligned}\frac{\partial^2}{\partial\alpha\partial q_t} g_t^\alpha(I_t,q_t,\varepsilon_t) &= \underbrace{\frac{\partial^2 W_t^\alpha}{\partial\alpha\partial I_t}\frac{\partial R_t^\alpha}{\partial q_t}}_{\leqslant 0} + \underbrace{\frac{\partial^2 W_t^\alpha}{\partial I_t^2}\cdot\frac{\partial R_t^\alpha}{\partial q_t}\cdot\frac{\partial R_t^\alpha}{\partial\alpha}}_{\leqslant 0} + \underbrace{\frac{\partial W_t^\alpha}{\partial I_t}\cdot\frac{\partial^2 R_t^\alpha}{\partial\alpha\partial q_t}}_{\leqslant 0} - \underbrace{\frac{\partial^2 c_t^\alpha(q_t)}{\partial\alpha\partial q_t}}_{\geqslant 0} \\ &\leqslant 0;\end{aligned} \tag{4.15}$$

$$\frac{\partial^2}{\partial I_t\partial q_t} g_t^\alpha(I_t,q_t,\varepsilon_t) = \underbrace{\frac{\partial^2}{\partial I_t^2} W_t^\alpha(\cdot)}_{\leqslant 0}\cdot\underbrace{\frac{\partial}{\partial q_t} R_t^\alpha(q_t)}_{\geqslant 0} \leqslant 0, \tag{4.16}$$

The non-positivaty in Eq. (4.14) holds because $W_t^\alpha(y)$ is submodular in (α,y) by Part (ii). The non-positivity in Eq. (4.15) holds because that $W_t^\alpha(y)$ is submodular in (α,y) by Part (ii); $R_t^\alpha(d)$ is increasing in α and q_t, i.e., $\partial R_t^\alpha/\partial\alpha \geqslant 0$ and $\partial R_t^\alpha/\partial q_t \geqslant 0$; $R_t^\alpha(q_t)$ is submodular by assumption, and $c_t^\alpha(q_t)$ is submodular in (α,q_t) by assumption. The non-negativity in Eq. (4.16) holds by $W_t^\alpha(I_t)$ is concave in I_t by Lemma 4.1, Part (ii); and $\frac{\partial}{\partial q_t}R_t^\alpha(q_t) \geqslant 0$ since $R_t^\alpha(q_t)$ is increasing in q_t by assumption.

Consequently, it is straightforward to show $\mathbb{E}_{\varepsilon_t}[\hat{g}_t^{\alpha}(I_t, q_t, \varepsilon_t)]$ is submodular in (α, I_t, q_t), since the submodularity preserves after taking expectation. Finally,

$$V_t^{\alpha}(I_t) = \max_{q_t \geqslant 0} \mathbb{E}_{\varepsilon_t}\left[\hat{g}_t^{\alpha}(I_t, q_t, \varepsilon_t)\right]$$

is submodular in (α, I_t) since maximization preserves the submodularity. This completes the proof for Part (i) which completes the whole proof. ∎

The proof for Lemma 4.4 is nontrivial, because it involves subtle manipulation of stochastic monotonicity, concavity, and the maximization of submodular objective function. According to Lemma 4.4, we immediately have the following result.

Theorem 4.5 *For any period t, under Assumption 4.3, the following hold:*

(i) $q_t^*(\alpha, I_t)$ *is decreasing in* (α, I_t)*;*

(ii) $p_t^*(\alpha, y_t)$ *is decreasing in* (α, y_t)*; viz.,* $d_t^*(\alpha, y_t)$ *is increasing in* (α, y_t)*;*

(iii) the target inventory after selling, $x_t^*(\alpha, y_t) = y_t - d_t^*(\alpha, y_t)$*, is decreasing in* α*, while increasing in* y_t*.*

Proof of Theorem 4.5.

The proofs of Parts (i) and (ii) readily follow from Lemma 4.4 and Lemma 5.1. To prove Part (iii), we denote $x_t = y_t - d_t$ in Eq. (4.10). Accordingly, it can be rewritten as

$$\begin{aligned}\mathscr{W}_t(\alpha, y_t, x_t, \omega_t) &\triangleq w_t^\alpha(y_t, y_t - x_t, \omega_t)\\ &= (y_t - x_t) \cdot p_t^\alpha(y_t - x_t) - H_t\big(x_t - \omega_t\big) + \beta_t \cdot V_{t-1}^\alpha(x_t - \omega_t).\end{aligned}$$

Therefore, maximizing over d_t is equivalent to maximizing over $x_t \in [0,\ y_t]$; *viz.* $W_t^\alpha(y_t) = \max\limits_{x_t \leqslant y_t} \mathbb{E}_{\omega_t}\big[\mathscr{W}_t(\alpha, y_t, x_t, \omega_t)\big]$. We now proceed to prove that, for any given ω_t, function $\mathscr{W}_t(\alpha, y_t, x_t, \omega_t)$ is supermodular in (y_t, x_t) and (α, y_t), but submodular in (α, x_t). This can be obtained via checking on their cross-over derivatives:

$$\frac{\partial^2}{\partial\alpha\partial y_t}\mathscr{W}_t(\alpha, y_t, x_t, \omega_t) = \underbrace{\frac{\partial p_t^\alpha}{\partial\alpha}}_{\geqslant 0} + (y_t - x_t)\underbrace{\frac{\partial^2 p_t^\alpha}{\partial\alpha\partial y_t}}_{\geqslant 0} \geqslant 0; \tag{4.17}$$

$$\frac{\partial^2}{\partial\alpha\partial x_t}\mathscr{W}_t(\alpha, y_t, x_t, \omega_t) = -\underbrace{\frac{\partial^2 p_t^\alpha}{\partial\alpha\partial d_t}}_{\geqslant 0} - (y_t - x_t)\underbrace{\frac{\partial^2 p_t^\alpha}{\partial\alpha\partial d_t}}_{\geqslant 0} + \beta_t\underbrace{\frac{\partial^2 V_t^\alpha}{\partial\alpha\partial I_t}}_{\leqslant 0} \leqslant 0; \tag{4.18}$$

$$\frac{\partial^2}{\partial x_t\partial y_t}\mathscr{W}_t(\alpha, y_t, x_t, \omega_t) = -2\underbrace{\frac{\partial p_t^\alpha}{\partial d_t}}_{\leqslant 0} - (y_t - x_t)\underbrace{\frac{\partial^2 p_t^\alpha}{\partial d_t^2}}_{\leqslant 0} \geqslant 0. \tag{4.19}$$

In the above analysis, we used the following fact: If $f(x, y)$ is submodular in (x, y) and concave in y, then $g(x, z) = f(x, x - z)$ is supermodular in (x, z). This is true because

$$\partial^2 g(x, z)/\partial x\partial z = -(\partial^2 f/\partial x\partial y + \partial^2 f/\partial y^2) \geqslant 0.$$

The non-negativity in Eq. (4.17) holds by Assumption 4.1, Part (ii). The non-positivity in Eq. (4.18) holds by Assumption 4.1, Part (ii), and the submodularity of V_t^α in view of Lemma 4.4, Part (i). The non-negativity in Eq. (4.19) holds by Assumption 4.1, Part (ii), and $\frac{\partial^2 p_t^\alpha}{\partial d_t^2} \leqslant 0$. The latter holds because the inverse function of a decreasingly convex (concave) function is also convex (concave). To see this, consider a decreasing and concave function $y = f(x)$ and its inverse function $x = g(y)$; see the example pertaining to Assumption 4.1. Then, we have $g(f(x)) = x$. Taking the derivative twice on both sides, we further have $g'' \cdot f'^2 + g' \cdot f'' = 0$. Hence, with $g' = 1/f'$, this yields $g''(y) = -\frac{f''(x)}{[f'(x)]^3} \leqslant 0$, which shows the concavity of $g(y)$.

Finally, in light of Lemma 5.2, we have the result of Part (iii), which completes the whole proof. ∎

Theorem 4.5 asseverates that the leverage of Blockchain can lower the order quantity, induce more sales via lowering the selling price, and reduce the target inventory level that is expected to be carried over to the next period. While facing increasing demand driven by α, it is counter-intuitive to see the ordering quantity and/or target inventory reduce. This can be explained by the improvement from the supply side while increasing α.

Theorem 4.6 *For any period t, both $V_t^\alpha(I_t)$ and $W_t^\alpha(y_t)$ increase in $\alpha \in [0,\ 1]$.*

Proof of Theorem 4.6.

We apply backward induction over t. First for $t = 0$, $V_0^{\alpha}(I_0) = s_0 \cdot I_0^+ - r_0 \cdot I_0^-$ is in dependent of α. Hence, the results hold trivially. Assuming for period $t-1$, both V_{t-1}^{α} and W_{t-1}^{α} increase in α, we shall prove the results hold for period t as well. To this end, we just compare their values with $0 \leqslant \alpha_1 \leqslant \alpha_2 \leqslant 1$. First, in view of Eq. (4.6), we have the following

$$\begin{aligned} W_t^{\alpha_1}(y_t) &= \max_{d_t \geqslant 0} \left\{ d_t \cdot P_t^{\alpha_1}(d_t) - \mathbb{E}\Big[H_t\big(y_t - d_t - \omega_t\big) \Big] + \beta_t \cdot \upsilon_t^{\alpha_1}(y_t,\, d_t) \right\}; \\ &\leqslant \max_{d_t \geqslant 0} \left\{ d_t \cdot P_t^{\alpha_2}(d_t) - \mathbb{E}\Big[H_t\big(y_t - d_t - \omega_t\big) \Big] + \beta_t \cdot \upsilon_t^{\alpha_2}(y_t,\, d_t) \right\}; \\ &= W_t^{\alpha_2}(y_t), \end{aligned}$$

where the inequality holds because $p_t^{\alpha}(d)$ is increasing in α in view of Assumption 4.1, Part (ii); and $\upsilon_t^{\alpha}(y_t,\, d_t)$ increases in α by the hypothesis assumption. Second, in view of Eq. (4.5), we further have

$$\begin{aligned} V_t^{\alpha_1}(I_t) &= \max_{q_t \geqslant 0} \left\{ \mathbb{E}\Big[W_t^{\alpha_1}\big(I_t + R_t^{\alpha_1}(q_t)\big) \Big] - c_t^{\alpha_1}(q_t) \right\} \\ &\leqslant \max_{q_t \geqslant 0} \left\{ \mathbb{E}\Big[W_t^{\alpha_2}\big(I_t + R_t^{\alpha_2}(q_t)\big) \Big] - c_t^{\alpha_2}(q_t) \right\} \\ &= V_t^{\alpha_2}(I_t), \end{aligned}$$

where the inequality holds because: $R_t^\alpha(q, \varepsilon)$ increases in α by assumption, $W_t^\alpha(y_t)$ increases in y_t by Lemma 4.1, Part (ii), and $c_t^\alpha(q_t)$ decreases in α by assumption. This concludes the induction and completes the proof. ∎

Theorem 4.6 states that, from the operational perspective, it is always profitable to adopt higher level of BCT for improving the profit. There are three major drivers for the increasing monotonicity of the profit and value functions. First, it is mainly caused by the demand increase of $D_t^\alpha(p)$ in α, since the whole process is demand driven. While facing increased demand, the firm can alway leverage optimal operational decisions of ordering quantity and selling price which leads to a higher revenue. Second, considering the cost from the supply side, a higher adoption level leads to a higher yield, which typically lowers the operational cost. Third, a higher adoption level yields a lower procurement cost. Those three factors function jointly under the umbrella of BCT and it can improve the profitability significantly.

4.6 Optimal Blockchain Design

Operationally, Theorem 4.6 induces the firm to adopt a high α as possible. However, at the design stage, the firm needs to take account of the adoption cost. Therefore, it is strategically imperative to balance between the operational profit of $V_T^\alpha(I_T)$ and the adoption cost of $\psi(\alpha)$. This section tackles the Blockchain design problem via investigating the optimal decision of Blockchain adoption α^*, as defined by Eq. (4.4).

For optimization purpose, the concavity of the objective function of Eq. (4.4) is not guaranteed in general. Lemma 4.5 provides a sufficient condition for the concavity.

Assumption 4.4 *For any realization of $\varepsilon_t \in \Omega_{\epsilon_t}$, $R_t^\alpha(q_t, \varepsilon_t)$ is jointly concave in $(\alpha,\, q_t)$, while $c_t^\alpha(q_t)$ is jointly convex in $(\alpha,\, q_t)$.*

This assumption reflects the scarcity and expensiveness of supply capacity.[6]

Lemma 4.5 *For each period $t \in \mathcal{T}$, under Assumption 4.4, $v_t^\alpha(y_t,\, d_t)$ is jointly concave in $(\alpha,\, y_t,\, p_t)$; $W_t^\alpha(y_t)$ is jointly concave in $(\alpha,\, y_t)$; and $V_t^\alpha(I_t)$ is jointly concave in $(\alpha,\, I_t)$.*

Proof of Lemma 4.5.

We use backward induction over period t, and for each period its concavity is preserved under maximization; cf. Zipkin (2000). ∎

We shall mention that the result in Lemma 4.5 exposes an extension of our model in another dimension, where the firm can dynamically select α. Typically, α is predetermined at the beginning of the planning horizon and the selected α

[6]The specification is introduced here to secure the concavity and supermodularity of the value function.

is sustained throughout the entire horizon. However, practically some situations (e.g., a low cost for switching) allow to dynamically switch among different adoption level of α over periods. Lemma 4.5 sheds light on such a setting.

In light of Lemma 4.5, one has the following immediate result.

Theorem 4.7 *For any fixed initial inventory I_T at the beginning of time horizon, under Assumption 4.4, there exists a unique optimal $\alpha^* \in [0,\, 1]$ defined by Eq. (4.4) such that the total profit is maximized. In addition, $\alpha^*(I_T)$ decreases in I_T.*

Proof of Theorem 4.7.

In light of defined by Eq. (4.4), the proof readily follows from Lemma 4.5 and the convexity of the adoption cost $\psi(\alpha)$. ■

In view of Theorem 4.7, it is straightforward to see that, the more the initial inventory, the better to leverage a lower α^*. Such negative impact of initial inventory on the optimal α^* will be further observed in a numerical study in §4.7.1.

Typically, it is relatively costly to adopt BCT; cf. Hertig (Mar 21, 2018). The following theorem identifies the impact of $\psi(\alpha)$ on the design decision.

Theorem 4.8 *Considering two different cost settings, $\psi(\alpha)$ and $\hat{\psi}(\alpha)$. If the marginal cost of $\psi(\alpha)$ is no less than that of $\hat{\psi}(\alpha)$, i.e., , $\psi'(\alpha) \geqslant \hat{\psi}'(\alpha)$, for $\alpha \in (0,\, 1)$, then $\alpha^*(I_T) \leqslant \hat{\alpha}^*(I_T)$.*

Proof of Theorem 4.8.

To prove the result, we first note that the optimal α^* is determined by solving the following equation,

$$\psi'(\alpha) = \frac{\partial V_T^{\alpha}(I_t)}{\partial \alpha}, \tag{4.20}$$

where the equality holds only at $\alpha = \alpha^*$ if $\alpha^* \in (0, 1)$. We proceed to prove the result via contradiction. Assume $\alpha^* > \hat{\alpha}^*$. Then, by Eq. (4.20), one has the following

$$\psi'(\alpha^*) = \frac{\partial V_T^{\alpha}(I_t)}{\partial \alpha^*} < \frac{\partial V_T^{\alpha}(I_t)}{\partial \hat{\alpha}^*} = \psi'(\hat{\alpha}^*),$$

which is contradict to the assumption that $\psi'(\alpha) \geqslant \hat{\psi}'(\alpha)$. The inequality above holds by that $V_T^{\alpha}(I)$ is concave in α, according to Lemma 4.5. Therefore, we must have $\alpha^* \leqslant \hat{\alpha}^*$, and this completes the proof. ■

Theorem 4.8 states that the optimal adoption level is mainly determined by its marginal cost in terms of α, rather than the total adoption cost $\psi(\alpha)$. The higher the marginal cost $\psi'(\alpha)$, the lower the optimal adoption level α^*.

4.7 Numerical Experiments

To gain some useful insights, we conduct the numerical study in two dimensions: (1) *vertically*, by considering various types of goods in §4.7.1, and (2) *horizontally*, by considering the stages of a product lifecycle in §4.7.2. Our objective is to dispel misguided notions and myths about BCT being a silver bullet for all businesses. For instance, it will be numerically shown that not all businesses can benefit from BCT adoption.

In the following numerical studies, we assume that the parameters are stationary over periods so we simply suppress the period index t. In particular, we set $h = 2$ for inventory holding cost per unit, $r = 5$ for demand rejection penalty per unit, $\beta = 0.95$. Both yield rate and demand follow uniform distributions of $R^\alpha \sim \mathcal{U}[0.5 + 0.5\alpha,\, 1]$ and $D^\alpha(p) \sim \mathcal{U}[d^\alpha(p) - 50,\, d^\alpha(p) + 50]$, respectively. In the latter setting, the average demand $d^\alpha = E[D^\alpha(p)]$ has a linear form of $E[D^\alpha(p)] = z_0\,\alpha - z_1\,p + z_2$, where constants z_0, z_1 z_2 will be selected later to model customer behavior in terms of tech-savvy and price-sensitivity. The unit ordering cost is $c^\alpha = 50 - 10\,\alpha$ and the adoption cost $\psi(\alpha) = 0.5 \times 10^5\,\alpha^4$. For the terminal condition, we set $s_0 = 10$ for the unit salvage value and $r_0 = 50$ for the lost-sales penalty per unit.

4.7.1 Blockchain-Savvy Buyers for Various Types of Goods

Technology plays a core role in profitability for some businesses, but overhyped reliance on technology investments can soon turn the tables. One such well-known case is that of Nike implementing ERP (i2) in the early 2000's at a cost of $400M, which led to a loss of $100M in sales, a 20 percent dip in share price, and the loss of brand value in the process, which just underlines the failure of ERP. Companies like HP, Target, P&G, and Vodafone had similar nightmares when using IT to replace all other processes, such that these companies had to revert and go back to using old systems.

The purpose of this numerical study is to test for increased profitability when adopting BCT, by considering various products in terms of the varying impacts of the adoption level α. In this numerical study, we follow the setting with the parameters explained before and set the total periods number $T = 10$. To examine the impact of the BCT adoption level and pricing on a consumer's Blockchain-savvy behavior, we compare the following three settings for three types of goods (cf. §4.7.3):

- *Search Goods* with LESS-BLOCKCHAIN-SAVVY BUYERS:

  $\mathbb{E}[D^{\alpha}(p)] = \mathbf{10}\,\alpha - 0.2\,p + 100$;

- *Experience Goods* with MEDIUM-BLOCKCHAIN-SAVVY BUYERS:

$\mathbb{E}[D^{\alpha}(p)] = \mathbf{50}\,\alpha - 0.2\,p + 100$;

- *Credence Goods* with MORE-BLOCKCHAIN-SAVVY BUYERS:

  $\mathbb{E}[D^{\alpha}(p)] = \mathbf{100}\,\alpha - 0.2\,p + 100$.

Therefore, the MEDIUM-BLOCKCHAIN-SAVVY BUYERS create 40α more demand on average than the LESS-BLOCKCHAIN-SAVVY BUYERS; the MORE-BLOCKCHAIN-SAVVY BUYERS bring 50α more demand on average than the MEDIUM-BLOCKCHAIN-SAVVY BUYERS.

For the three types of goods under study, Fig. 4.1 depicts the curves of $V_1^{\alpha}(I_1)$ in terms of α and the initial inventory level I_1, and Fig. 4.2 shows its snapshot for zero initial inventory, i.e., $I_1 = 0$.

One major observation is that it is not always profitable to adopt BCT for some type of goods. In particular, a higher level of adoption is recommended for credence goods (e.g., $\alpha^* = 1$); whereas it might not be wise to adopt BCT for some search goods (e.g., $\alpha^* = 0$). For experience goods, it is imperative to adopt an optimal $\alpha^* \in (0, 1)$. In this case, properly designing the BCT and choosing a proper adoption level of α becomes overwhelmingly important. Furthermore, as shown in Fig. 4.1 of the curve of experience goods, the initial inventory I_0 exposes a negative impact of the optimal $\alpha^*(I_0)$; *viz.* a higher initial inventory level suggests a lower adoption level. Such observation is in agreement with Theorem 4.7.

Figure 4.1: Expected Profit vs Adoption Llevelevel and Initial Inventory for Different Goods

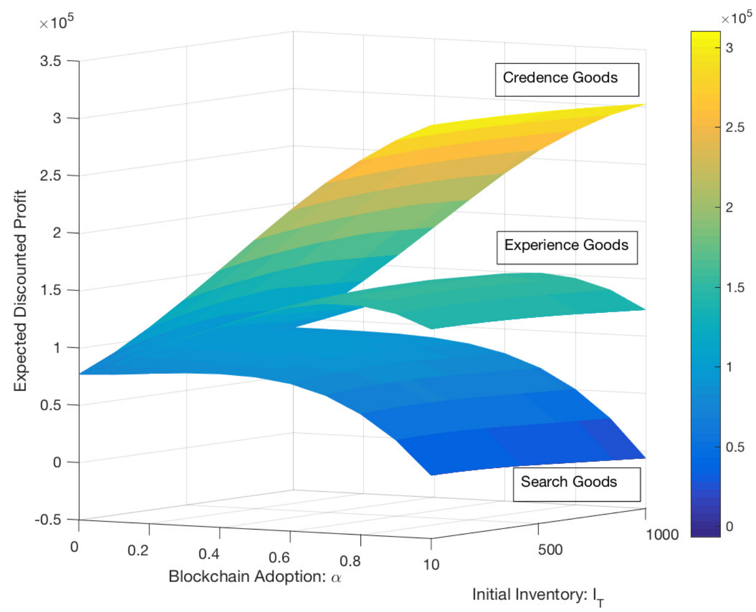
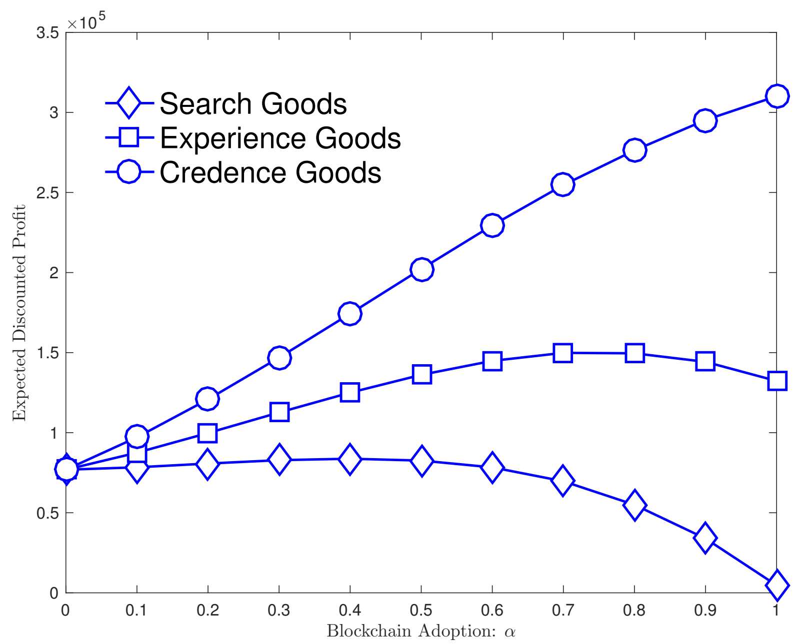


Figure 4.2: Expected Profit vs Adoption Level: Different Goods with Zero Initial Inventory



4.7.2 Blockchain Design Throughout Product Life Cycle

To study the timing issue for adopting BCT, we consider four lifecycle stages: *Introduction*, *Growth*, *Maturity*, and *Decline*. In this case, the number of periods $T = 4$. Considering a typical product (i.e., *experience goods*), to reflect the feature of each stage, we model the demand function as follows:

(i) Introduction: $\mathbb{E}[D^{\alpha}(p)] = \mathbf{60}\,\alpha - 0.2\,p + \mathbf{100}$;

(ii) Growth: $\mathbb{E}[D^{\alpha}(p)] = \mathbf{50}\,\alpha - 0.2\,p + \mathbf{300}$;

(iii) Maturity: $\mathbb{E}[D^{\alpha}(p)] = \mathbf{30}\,\alpha - 0.2\,p + \mathbf{400}$;

(iv) Decline: $\mathbb{E}[D^{\alpha}(p)] = \mathbf{10}\,\alpha - 0.2\,p + \mathbf{100}$.

For each selection of $\alpha \in \{0.0, 0.1, 0.2, \ldots, 1.0\}$, Fig. 4.3 depicts the total expected operational profit $V^{\alpha}(0)$ throughout its lifecycle. First, the expected operational profit increases as α gets highe at each lifecycle stage. Second, the expected operational profit tapers off through the lifecycle stages, for each selected α. From the perspective of operational profit, it is always beneficial to adopt a higher level of BCT, and the earlier the better. In particular, it is shown that the value of adopting BCT can be exaggerated significantly throughout the time horizon. For instance, the operational profit has been increased by over $100K when adopting BCT starting as early as the Introduction stage of the product, compared to merely $6K when adopting BCT during the Decline stage.

Figure 4.3: Expected Operational Profit: Life Cycle and Adoption Levellevel

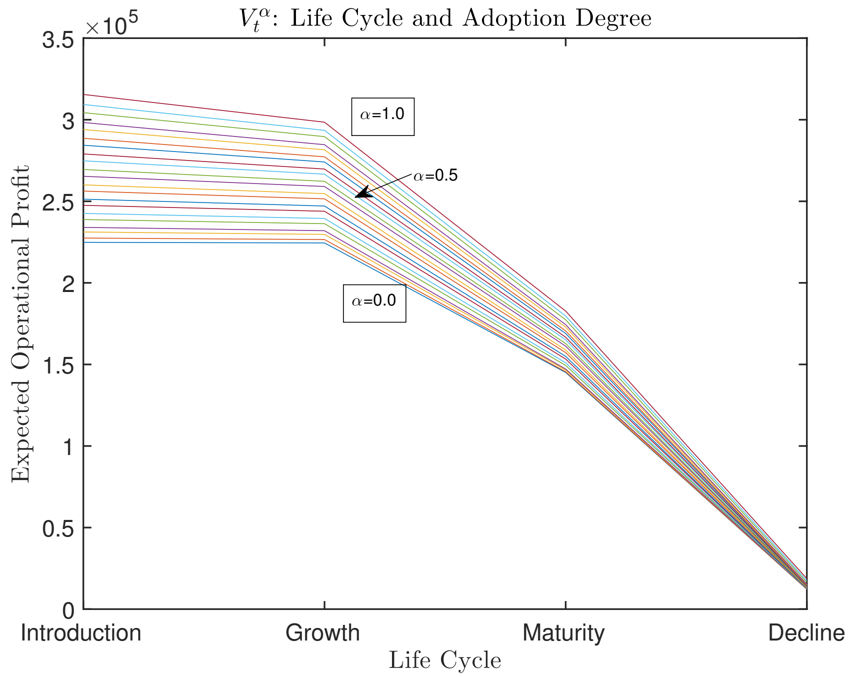


Figure 4.4: Total Expected Profit: Life Cycle and Adoption Levellevel

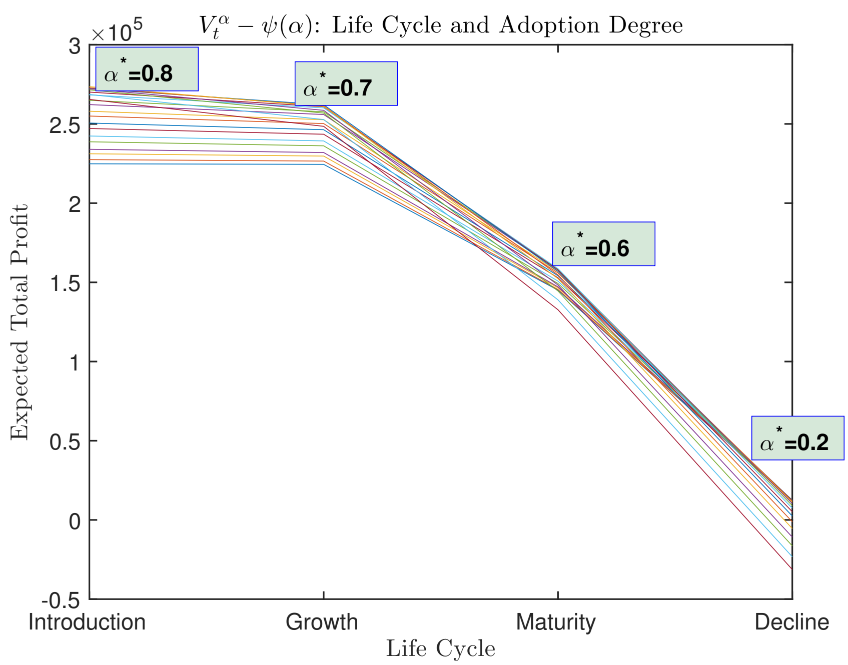


Fig. 4.4 illustrates the expected total profit $V_t^\alpha - \psi(\alpha)$ for each selection of α at each stage. It also highlights the optimal α^* for each stage. For example, at the Introduction stage, $\alpha^* = 0.8$, whereas at the Decline stage $\alpha^* = 0.2$.

(i) **Introduction**: the optimal total profit is \$273524.9, with $\alpha^* = 0.8$; the total profit increases by \$48684.47 when optimally adopting BCT;

(ii) **Growth**: the optimal total profit is \$262102.8, with $\alpha^* = 0.7$; the total profit increases by \$37626.38 of when optimally adopting BCT;

(iii) **Maturity**: the optimal total profit is \$158604.1, with $\alpha^* = 0.7$; the total profit increases by \$13543.35 when optimally adopting BCT;

(iv) **Decline**: the optimal total profit is \$12432.5, with $\alpha^* = 0.2$; the total profit increases by \$270.00 when optimally adopting BCT.

After considering the optimal adoption time of BCT for an experience good, it is found to be always beneficial to adopt BCT as early as possible, as the optimal α^* decreases through the life cycle stages.

4.7.3 Practical Insights with Numerical Studies

Based on previous experience of new technological revolutions (e.g., those involving the Cloud Computing, AI, Big Data, IoT, etc.), there appears to be a prevailing phenomenon in the existing business environment that whenever new technology emerges, many companies irrationally rush to be the first to implement it in hope of exploiting the *first mover advantage*. Naturally, no one wants to be an abandoned loser. However, according to Deloitte, out of 26,000 open-source Blockchain projects created on the software collaboration platform GitHub in 2016, there are only 8 percent remaining active. Babich and Hilary (2018) contribute such phenomenon partly to the fact that "investors and managers had a hard time conceptualizing the strengths and weaknesses of this new paradigm". This study, therefore, becomes of timely value, since it aims to serve as a guideline for determining whether a business is suitable for using BCT, and if so, it then suggests the proper level of adoption. To illustrate some useful managerial insights, we conduct numerical studies in two dimensions: i) Vertically, we consider different types of products; and ii) horizontally, we consider different stages of a product lifecycle.

For the vertical dimension, we look into different goods that can be impacted by different Blockchain-savvy buyers. Based on the observability of quality, goods can be classified into three categories: 1) *search goods*; 2) *experience goods*; and 3) *credence goods*[7]; cf. Nelson (1970), Darby and Karni (1973). As one of the major results, it is revealed that, subject to tech-savvy customer behavior, some types of goods (e.g., *credence goods* and *experience goods*) benefit from the

[7]SEC classification is somewhat subjective, because the capability of evaluation of product quality varies by persons. For example, a technology geek might view PCs as search goods, but others, with limited computer knowledge, might consider PCs as experience goods.

application of BCT, but it may not prove beneficial to leverage BCT for some of the others (e.g., *search goods*).

For the horizontal dimension, we consider the lifecycle of a typical product (e.g., experience goods), comprised of *Introduction*, *Growth*, *Maturity*, and *Decline*. One major insight from this study leads to recommending the adoption of BCT as early as possible and for adopting it to a higher level at an earlier stage.

4.8 Extensions

This section considers some major extensions and shows the robustness of aforementioned results.

4.8.1 Lost Sales

In the previous study, it was assumed that any unmet demand is backordered, subject to backorder penalty. The analysis and results can be easily extended to the lost-sales setting. In this case, the inventory levels over the periods become nonnegative and the inventory flow given in Eq. (4.1) for the backorder case is formulated as

$$I_{t-1} = \left[I_t + R_t^\alpha(q_t) - D_t^\alpha(p_t)\right]^+. \tag{4.21}$$

In case of lost-sale occurrence, there will be a penalty cost γ_t per unit. Let $z_t \triangleq I_t + R_t^\alpha(q_t) - D_t^\alpha(p_t)$ denote the inventory position after demand fulfillment. Then, the inventory cost, in Eq. (4.6), can be expressed as,

$$H_t(z_t) = h_t \cdot z_t^+ + \gamma_t \cdot z_t^-, \tag{4.22}$$

which is a convex function of z_t. In a similar vein, we can show that the previous results for the backorder setting still hold for lost-sales.

4.8.2 Random Capacity

Previously, to model the impact of BCT on the supply side, we assume the supply yield can be stochastically improved if the adoption level α increases. The study can be extended via considering random capacity. In this case, we assume K_t^α as the capacity function of α, which is random and stochastically increasing in $\alpha \in [0, 1]$.

Practically, the ordering decision on q_t can be made with two different epochs: *ex ante* or *ex post* observing the supply capacity. For the setting of *ex ante* capacity realization, at the beginning of each period t, the firm needs to decide on the order quantity q_t, prior to observing the realization of K_t^α. In this case, the supply delivery is expressed as $q_t \wedge K_t^\alpha$, which is stochastically increasing in α and deterministically increasing in q_t. Via specifying

$$R_t^\alpha(q_t) \triangleq q_t \wedge K_t^\alpha, \tag{4.23}$$

in our previous model, i.e., in Eq. (4.5), it is straightforward to show all the analysis still holds.

For the second setting of *ex post* observing the realization of capacity K_t^α, the firm could not order more than K_t^α, i.e., $q_t \leqslant K_t^\alpha$. In this case, the procurement stage as given in Eq. (4.5) can be modeled as

$$V_t^\alpha(I_t) = \mathbb{E}\left[\max_{0 \leqslant q_t \leqslant K_t^\alpha} W_t^\alpha\big(I_t + q_t\big) - c_t^\alpha(q_t)\right], \tag{4.24}$$

where the expectation is taken with respect to the random capacity K_t^α.

For both cases of *ex ante* and *ex post* observing the supply capacity, the following theorem concludes that all the results still hold for random capacity.

Theorem 4.9 *For any period t, if the supply capacity K_t^α is random and stochastically increasing in $\alpha \in [0,\, 1]$, then, for each setting of ordering either ex ante or ex post observing the supply capacity, all the results given by Theorems 4.1 through Theorem 4.8 still hold.*

Proof of Theorem 4.9.

For the setting of *ex ante* to the realization of the random capacity, one can specify $R_t^\alpha(q_t) \triangleq q_t \wedge K_t^\alpha$, as in Eq. (4.23) in our previous analysis. Obviously, the results hold.

For the setting of *ex post*, the proof can be done via backward induction, in a similar vein as the proofs for those theorems. However, through the backward induction, one needs to resort to Lemma 5.4 for concavity, in addition to Lemma 5.1, Part (i). For simplify, we omit the details here for the proof. ∎

Chapter 5

Technical Details and Review

In this chapter, we review the technical details that have been used in our study.

5.1 Brief Review on Concavity and Supermodularity

The following lemma summarizes the properties of convex functions and supermodular functions used in establishing our structural results. Its proof can be found in Boyd and Vandenberghe (2004), Topkis (1998), and Chen et al. (2013).

Lemma 5.1 *The following hold,*

(i) Define $h \circ g(x) = h(g_1(x), \dots, g_m(x))$, with $h: \mathbb{R}^m \to \mathbb{R}$, $g_i: \mathbb{R}^n \to \mathbb{R}$, $i = 1, \dots, n$. Then $h \circ g(x)$ is concave if h is concave and nondecreasing in each argument, and g_i is concave for each i.

(ii) If $h\colon \mathbb{R}^m \to \mathbb{R}$ is a concave function, then $h(Ax + b)$ is also a concave function of

x, where $A \in \mathbb{R}^m \times \mathbb{R}^n$, $x \in \mathbb{R}^n$, and $b \in \mathbb{R}^m$.

(iii) Assume that for any $x \in \mathbb{R}^n$, there is an associated convex set $\mathscr{C}(x) \subset \mathbb{R}^m$ and the set $\{(x, y) : y \in \mathscr{C}(x), x \in \mathbb{R}^n\}$ is a convex set. If $h(x, y)$ is concave and the function $g(x) \triangleq \sup_{y \in \mathscr{C}(x)} h(x, y)$ is well defined, then $g(x)$ is concave over $\mathbb{R}^n$.

(iv) If $f(x)$ and $g(x)$ are concave (supermodular) on X and $\alpha, \beta > 0$, then $\alpha f(x) + \beta g(x)$ is concave (supermodular) on X.

(v) Assume that $F(y)$ is a distribution function on Y. Assume also that $f(x, y)$ is concave (supermodular) in x on a lattice X for each $y \in Y$, and integrable with respect to $F(y)$ for each $x \in X$. Then $g(x) \triangleq \displaystyle\int_Y f(x, y) dF(y)$ is concave (supermodular) in x on X.

(vi) If $\mathscr{X}$ and $\mathscr{Y}$ are lattices, $\mathscr{S}$ is a sublattice of $X \times Y$, $\mathscr{S}_y$ is the section of $\mathscr{S}$ at y in $\mathscr{Y}$, and $f(x, y)$ is supermodular in (x, y) on $\mathscr{S}$, then $\operatorname{argmax}_{x \in \mathscr{S}_y} f(x, y)$ is increasing in y on $\{\, y \in \mathscr{Y} : \operatorname{argmax}_{x \in \mathscr{S}_y} f(x, y) \neq \emptyset \,\}$

(vii) Assume that $g(y, \theta)$ is a supermodular function in (y, θ) on a sublattice $\mathscr{D} \subset \mathbb{R}^{n+1}$ and jointly concave in y for any θ. For every θ, assume that the section $\mathscr{D}_\theta$ is convex. Let $f(I, \theta) \triangleq \max_y \{g(y, \theta) : \sum_{i=1}^n a_i y^i + b\theta = I, (y^1, y^2, \cdots, y^n, \theta) \in \mathscr{D}\}$ and $\mathscr{S} \triangleq \{(\sum_{i=1}^n a_i y^i + b\theta, \theta) : (y, \theta) \in \mathscr{D}\}$, where $a_1, a_2, \cdots, a_n,\ b \geqslant 0$. We have: $f(I, \theta)$ is supermodular on $\mathscr{S}$ and concave in I for any θ.

(viii) For real-valued function $f(\mathbf{x},\ \mathbf{y})$ of $(\mathbf{x}, \mathbf{y}) \in \mathscr{X} \times \mathscr{Y}$,

(viii.1) *if $f(\mathbf{x}, \mathbf{y})$ is supermodular in $(\mathbf{x},\, \mathbf{y})$, then $g(\mathbf{x}) = \max_{\mathbf{y}\in\mathscr{Y}} f(\mathbf{x}, \mathbf{y})$ is supmodular in $\mathbf{x} \in \mathscr{X}$.*

(viii.2) *if $f(\mathbf{x}, \mathbf{y})$ is submodular in $(\mathbf{x},\, \mathbf{y})$, then $g(\mathbf{x}) = \min_{\mathbf{y}\in\mathscr{Y}} f(\mathbf{x}, \mathbf{y})$ is submodular in $\mathbf{x} \in \mathscr{X}$.*

Proof of Lemma 5.1 (Selected Proof for Part (viii)):

Proof for Part (1) : We prove the supermodularity preservation for $f(\mathbf{x}, \mathbf{y})$ via definition. First note that the following holds for $\mathbf{x},\, \hat{\mathbf{x}} \in \mathscr{X}$ and $\mathbf{y},\, \hat{\mathbf{y}} \in \mathscr{Y}$ since $f(\mathbf{x}, \mathbf{y})$ is supermodular:

$$f(\mathbf{x}, \mathbf{y}) + f(\hat{\mathbf{x}}, \hat{\mathbf{y}}) \leqslant f(\mathbf{x} \wedge \hat{\mathbf{x}}, \mathbf{y} \wedge \hat{\mathbf{y}}) + f(\mathbf{x} \vee \hat{\mathbf{x}}, \mathbf{y} \vee \hat{\mathbf{y}}). \tag{5.1}$$

Therefore, we have the following:

$$\begin{aligned}
g(\mathbf{x}) + g(\hat{\mathbf{x}}) &= \max_{\mathbf{y}\in\mathscr{Y}} f(\mathbf{x}, \mathbf{y}) + \max_{\hat{\mathbf{y}}\in\mathscr{Y}} f(\hat{\mathbf{x}}, \hat{\mathbf{y}}) \\
&= \max_{\mathbf{y}, \hat{\mathbf{y}}\in\mathscr{Y}} \Big\{ f(\mathbf{x}, \mathbf{y}) + f(\hat{\mathbf{x}}, \hat{\mathbf{y}}) \Big\} \\
&\leqslant \max_{\mathbf{y}, \hat{\mathbf{y}}\in\mathscr{Y}} \Big\{ f(\mathbf{x} \wedge \hat{\mathbf{x}}, \mathbf{y} \wedge \hat{\mathbf{y}}) + f(\mathbf{x} \vee \hat{\mathbf{x}}, \mathbf{y} \vee \hat{\mathbf{y}}) \Big\} \\
&\leqslant \max_{\mathbf{y}, \hat{\mathbf{y}}\in\mathscr{Y}} f(\mathbf{x} \wedge \hat{\mathbf{x}}, \mathbf{y} \wedge \hat{\mathbf{y}}) + \max_{\mathbf{y}, \hat{\mathbf{y}}\in\mathscr{Y}} f(\mathbf{x} \vee \hat{\mathbf{x}}, \mathbf{y} \vee \hat{\mathbf{y}}) \\
&= \max_{\mathbf{y}\in\mathscr{Y}} f(\mathbf{x} \wedge \hat{\mathbf{x}}, \mathbf{y}) + \max_{\mathbf{y}\in\mathscr{Y}} f(\mathbf{x} \vee \hat{\mathbf{x}}, \mathbf{y}) \\
&= g(\mathbf{x} \wedge \hat{\mathbf{x}}) + g(\mathbf{x} \vee \hat{\mathbf{x}}),
\end{aligned}$$

where the first equality holds by the definition of $g(\mathbf{x}) = \max_{\mathbf{u}\in\mathscr{Y}} f(\mathbf{x}, \mathbf{y})$; the first inequality holds by Eq. (5.2); and the second last equality holds by $\max_{\mathbf{y}\in\mathscr{Y}}\{f_1(\mathbf{y}) + f_2(\mathbf{y})\} \leqslant \max_{\mathbf{y}\in\mathscr{Y}}\{f_1(\mathbf{y}) + \max_{\mathbf{y}\in\mathscr{Y}} f_2(\mathbf{y})$ for any real functions $f_1(\cdot)$ and $f_2(\cdot)$. The above shows that $g(\mathbf{x}) + g(\hat{\mathbf{x}}) \leqslant g(\mathbf{x} \wedge \hat{\mathbf{x}}) + g(\mathbf{x} \vee \hat{\mathbf{x}})$, which proves the supermodularity of $g(\mathbf{x})$.

The proof for part (2) can be done in a similar vein as that for Part (1). ■

Lemma 5.2 *[**Topkis's Theorem** (Topkis 1998)] For differentiable function $f(x, p)$,*

(i) if $f(x,p)$ is supermodular in (x, p), i.e., $\partial f^2/(\partial x \cdot \partial p) > 0$ and set $\mathscr{D}$ is a lattice, then

$$x^*(p) = \arg\max_{x\in\mathscr{D}} f(x,p)$$

is nondecreasing in p;

(ii) if $f(x,p)$ is submodular in (x, p), i.e., $\partial f^2/(\partial x \cdot \partial p) < 0$ and set $\mathscr{D}$ is a lattice, then

$$x^*(p) = \arg\max_{x\in\mathscr{D}} f(x,p)$$

is nonincreasing in p. ■

Lemma 5.3 *If set $\mathscr{X}$ is a sublattice of $\mathbb{R}^n$, $f(\mathbf{x})$ is suprmodular on $\mathscr{X}$, and $h(\mathbf{p}) = sup_{\mathbf{x}\in\mathscr{X}}\{f(\mathbf{x}) - \mathbf{p}^T\mathbf{x}\}$ is finite for each $\mathbf{p} \in \mathbb{R}^n$, then*

(i) $h(\mathbf{p})$ is supermodular in $\mathbf{p}$; cf. Corollary 2.7.3, Topkis (1998).

(ii) $arg\max_{\mathbf{x}\in\mathscr{X}}\{f(\mathbf{x})-\mathbf{p}^T\mathbf{x}\}$ *is decreasing in* $\mathbf{p}$*; cf. Corollary 2.8.2, Topkis (1998).*

Lemma 5.4 *For function* $g:\mathbb{R}^m\times\mathbb{R}^n\to\mathbb{R}$*, assume* $f(\mathbf{y},\mathbf{k})=\max\limits_{\mathbf{x}\leqslant\mathbf{k}} g(\mathbf{x},\mathbf{y})$ *exists.*

(i) If $g(\mathbf{x},\mathbf{y})$ *is jointly concave in* $(\mathbf{x},\mathbf{y})$*, then* $f(\mathbf{k},\mathbf{y})$ *is increasing in* $\mathbf{k}$ *and also jointly concave in* $(\mathbf{y},\mathbf{k})$*;*

(ii) If $g(\mathbf{x},\mathbf{y})$ *is supermodular in* $(\mathbf{x},\mathbf{y})$*, so is* $f(\mathbf{y},\mathbf{k})$*.*

Proof of Lemma 5.4.

Part (i): We prove the concavity of $f(\mathbf{y},\mathbf{k})$ via definition. Since $g(\mathbf{x},\mathbf{y})$ is concave, we have, for any $\alpha\in[0,1]$, $\mathbf{x},\hat{\mathbf{x}}\in\mathbb{R}^m$ and $\mathbf{y},\hat{\mathbf{y}}\in\mathbb{R}^n$,

$$\alpha\cdot g(\mathbf{x},\mathbf{y})+(1-\alpha)\cdot g(\hat{\mathbf{x}},\hat{\mathbf{y}})\leqslant g(\bar{\mathbf{x}},\bar{\mathbf{y}}),$$

where $\bar{\mathbf{x}}=\alpha\mathbf{x}+(1-\alpha)\hat{\mathbf{x}}$ and $\bar{\mathbf{y}}=\alpha\mathbf{y}+(1-\alpha)\hat{\mathbf{y}}$. Therefore, we have the following,

$$\begin{aligned}
\alpha\cdot f(\mathbf{y},\mathbf{k})+(1-\alpha)\cdot f(\hat{\mathbf{y}},\hat{\mathbf{k}}) &= \alpha\cdot\max_{\mathbf{x}\leqslant\mathbf{k}} g(\mathbf{x},\mathbf{y})+(1-\alpha)\cdot\max_{\hat{\mathbf{x}}\leqslant\hat{\mathbf{k}}} g(\hat{\mathbf{x}},\hat{\mathbf{y}})\\
&= \max_{\mathbf{x}\leqslant\mathbf{k};\hat{\mathbf{x}}\leqslant\hat{\mathbf{k}}}\big\{\alpha\cdot g(\mathbf{x},\mathbf{y})+(1-\alpha)\,g(\hat{\mathbf{x}},\hat{\mathbf{y}})\big\},\\
&\leqslant \max_{\bar{\mathbf{x}}\leqslant\bar{\mathbf{k}}}\big\{\alpha\cdot g(\mathbf{x},\mathbf{y})+(1-\alpha)\,g(\hat{\mathbf{x}},\hat{\mathbf{y}})\big\},\\
&\leqslant \max_{\bar{\mathbf{x}}\leqslant\bar{\mathbf{k}}} g(\bar{\mathbf{x}},\bar{\mathbf{y}})=f(\bar{\mathbf{y}},\bar{\mathbf{k}}),
\end{aligned}$$

where the first inequality holds by the constraint relaxation from $\{\mathbf{x} \leqslant \mathbf{k}; \hat{\mathbf{x}} \leqslant \hat{\mathbf{k}}\}$ to $\{\alpha\mathbf{x} + (1-\alpha)\hat{\mathbf{x}} \leqslant \alpha\mathbf{k} + (1-\alpha)\hat{\mathbf{k}}\}$; the second inequality holds by the concavity of $g(\mathbf{x}, \mathbf{y})$. This completes the proof for part (i).

Part (ii): We prove the supermodularity preservation for $f(\mathbf{y}, \mathbf{k})$ via definition. First note that the following holds for $\mathbf{x}$, $\hat{\mathbf{x}} \in \mathbb{R}^m$ and $\mathbf{y}$, $\hat{\mathbf{y}} \in \mathbb{R}^n$ since $g(\mathbf{x}, \mathbf{y})$ is supermodular:

$$g(\mathbf{x}, \mathbf{y}) + g(\hat{\mathbf{x}}, \hat{\mathbf{y}}) \leqslant g(\mathbf{x} \wedge \hat{\mathbf{x}}, \mathbf{y} \wedge \hat{\mathbf{y}}) + g(\mathbf{x} \vee \hat{\mathbf{x}}, \mathbf{y} \vee \hat{\mathbf{y}}). \tag{5.2}$$

It suffices to show that the above inequality holds for function $f(\mathbf{k}, \mathbf{y})$. To this end, we have the following:

$$\begin{aligned}
f(\mathbf{y}, \mathbf{k}) + f(\hat{\mathbf{y}}, \hat{\mathbf{k}}) &= \max_{\mathbf{x} \leqslant \mathbf{k}} g(\mathbf{x}, \mathbf{y}) + \max_{\hat{\mathbf{x}} \leqslant \hat{\mathbf{k}}} g(\hat{\mathbf{x}}, \hat{\mathbf{y}}) \\
&= \max_{\mathbf{x} \leqslant \mathbf{k}; \hat{\mathbf{x}} \leqslant \hat{\mathbf{k}}} \left\{ g(\mathbf{x}, \mathbf{y}) + g(\hat{\mathbf{x}}, \hat{\mathbf{y}}) \right\} \\
&\leqslant \max_{\mathbf{x} \leqslant \mathbf{k}; \hat{\mathbf{x}} \leqslant \hat{\mathbf{k}}} \left\{ g(\mathbf{x} \wedge \hat{\mathbf{x}}, \mathbf{y} \wedge \hat{\mathbf{y}}) + g(\mathbf{x} \vee \hat{\mathbf{x}}, \mathbf{y} \vee \hat{\mathbf{y}}) \right\} \\
&\leqslant \max_{\mathbf{x} \leqslant \mathbf{k}; \hat{\mathbf{x}} \leqslant \hat{\mathbf{k}}} g(\mathbf{x} \wedge \hat{\mathbf{x}}, \mathbf{y} \wedge \hat{\mathbf{y}}) + \max_{\mathbf{x} \leqslant \mathbf{k}; \hat{\mathbf{x}} \leqslant \hat{\mathbf{k}}} g(\mathbf{x} \vee \hat{\mathbf{x}}, \mathbf{y} \vee \hat{\mathbf{y}}) \\
&= \max_{\mathbf{z} \leqslant \mathbf{k} \wedge \hat{\mathbf{k}}} g(\mathbf{z}, \mathbf{y} \wedge \hat{\mathbf{y}}) + \max_{\mathbf{z}' \leqslant \mathbf{k} \vee \hat{\mathbf{k}}} g(\mathbf{z}', \mathbf{y} \vee \hat{\mathbf{y}}) \\
&= f(\mathbf{y} \wedge \hat{\mathbf{y}}, \mathbf{k} \wedge \hat{\mathbf{k}}) + f(\mathbf{y} \vee \hat{\mathbf{y}}, \mathbf{k} \vee \hat{\mathbf{k}}),
\end{aligned}$$

where the first equality holds by the definition of $f(\mathbf{y}, \mathbf{k}) = \max\limits_{\mathbf{x} \leqslant \mathbf{k}} g(\mathbf{x}, \mathbf{y})$; the first inequality holds by Eq. (5.2); and the second last equality holds by

$$\max_{\mathbf{x}\leqslant\mathbf{k};\hat{\mathbf{x}}\leqslant\hat{\mathbf{k}}} g(\mathbf{x}\wedge\hat{\mathbf{x}},\mathbf{y}\wedge\hat{\mathbf{y}}) = \max_{\mathbf{z}\leqslant\mathbf{k}\wedge\hat{\mathbf{k}}} g(\mathbf{z},\mathbf{y}\wedge\hat{\mathbf{y}}), \tag{5.3}$$

$$\max_{\mathbf{x}\leqslant\mathbf{k};\hat{\mathbf{x}}\leqslant\hat{\mathbf{k}}} g(\mathbf{x}\vee\hat{\mathbf{x}},\mathbf{y}\vee\hat{\mathbf{y}}) = \max_{\mathbf{z}'\leqslant\mathbf{k}\vee\hat{\mathbf{k}}} g(\mathbf{z}',\mathbf{y}\vee\hat{\mathbf{y}}). \tag{5.4}$$

Here, the equality in Eq. (5.3) holds by the fact that $\{\mathbf{z}=\mathbf{x}\wedge\hat{\mathbf{x}}:\mathbf{x}\leqslant\mathbf{k},\hat{\mathbf{x}}\leqslant\hat{\mathbf{k}}\}=\{\mathbf{z}:\mathbf{z}\leqslant\mathbf{k}\wedge\hat{\mathbf{k}}\}$ and the equality in Eq. (5.4) holds by the fact that $\{\mathbf{z}'=\mathbf{x}\vee\hat{\mathbf{x}}:\mathbf{x}\leqslant\mathbf{k},\hat{\mathbf{x}}\leqslant\hat{\mathbf{k}}\}=\{\mathbf{z}':\mathbf{z}\leqslant\mathbf{k}\vee\hat{\mathbf{k}}\}$.

This completes the proof for part (ii), and hence concludes the proof. ∎

As an immediate consequence of Lemma 5.4, the following result holds.

Corollary 5.1 *For function $g:\mathbb{R}^m\times\mathbb{R}^n\times\mathbb{R}^m\to\mathbb{R}$, assume $f(\mathbf{y},\mathbf{k})=\max_{\mathbf{x}\leqslant\mathbf{k}} g(\mathbf{x},\mathbf{y},\mathbf{k})$ exists.*

(i) If $g(\mathbf{x},\mathbf{y},\mathbf{k})$ is jointly concave in $(\mathbf{x},\mathbf{y},\mathbf{k})$, then $f(\mathbf{y},\mathbf{k})$ is jointly concave in $(\mathbf{y},\mathbf{k})$;

(ii) If $g(\mathbf{x},\mathbf{y},\mathbf{k})$ is supermodular in $(\mathbf{x},\mathbf{y},\mathbf{k})$, so is $f(\mathbf{y},\mathbf{k})$ in $(\mathbf{y},\mathbf{k})$.

Proof of Corollary 5.1.

The proofs of part (i) and (ii) readily follow from the proofs of parts (i) and (ii) of Lemma 5.4, respectively, via substituting $(\mathbf{x},\mathbf{y})$ with $(\mathbf{x},\mathbf{y},\mathbf{k})$ and noting all the inequalities still hold. ∎

5.2 Brief Review on Stochastic Comparison

The following definition and lemma summarize the stochastic comparison used in establishing our structural results. More details and proofs can be found in Shaked and Shanthikumar (2007), Müller and Stoyan (2002) and Yao and Zheng (2002).

Definition 5.1 *For two random variables X and Y,*

(i) X is smaller than Y with respect to usual stochastic order (written $X \leqslant_{st} Y$), if $\mathbb{P}(X < t) \geqslant \mathbb{P}(Y < t)$ for all real t.

(ii) X is less than Y in convex order (written as $X \leqslant_{cx} Y$), if $\mathbb{E}[f(X)] \leqslant \mathbb{E}[f(Y)]$ for all real convex functions $f(\cdot)$, such that the expectations exist.

(iii) X is less than Y in increasing convex order (written $X \leqslant_{icx} Y$), if $\mathbb{E}[f(X)] \leqslant \mathbb{E}[f(Y)]$ for all real increasing convex functions $f(\cdot)$, such that the expectations exist.

(iv) X is less than Y in increasing concave order (written $X \leqslant_{icv} Y$), if $\mathbb{E}[f(X)] \leqslant \mathbb{E}[f(Y)]$ for all real increasing concave functions $f(\cdot)$, such that the expectations exist.

The stochastic ordering can be extended to a family of random variables parameterized by a real or integer-valued scalar θ, $X(\theta)$. We say $X(\theta)$ is stochastically increasing in θ, if $X(\theta_1) \leqslant_{st} X(\theta_2)$ for any $\theta_1 \leqslant \theta_2$. This is what we mean by texititstochastic monotonicity; cf. Yao and Zheng (2002).

Definition 5.2 *$X(\theta)$ is stochastically increasing in θ, if for any given $\theta_1 \leqslant \theta_2$, there exist, on a common probability space $(\Omega, \mathcal{F}, \mathbb{P})$, two random variables $\hat{X}_1$ and $\hat{X}_2$ that are equal in distribution to $X(\theta_1)$ and $X(\theta_2)$, respectively, and $\hat{X}_1(\omega) \leqslant \hat{X}_2(\omega)$ for all $\omega \in \Omega$.*

$X(\theta)$ is stochastically increasing in θ, if for any increasing function $\phi(\cdot)$, $\mathbb{E}\big[\phi[(X(\theta)]\big]$ (as a deterministic unction) is increasing in θ.

Lemma 5.5 *For random variables X and Y with distribution functions F_X and F_Y, the following statements are equivalent:*

(i) $X \leqslant_{st} Y$;

(ii) the inequality $\mathbb{E}[f(X)] \leqslant \mathbb{E}[f(Y)]$ holds for all increasing functions $f(\cdot)$;

(iii) there is a probability space $(\Omega, \mathcal{A}, \mathbb{P})$ and random variable $\hat{X}$ and $\hat{Y}$ on this space with the distribution functions F_X and F_Y, respectively, such that $\hat{X}(\omega) \leqslant \hat{Y}(\omega)$ for all $\omega \in \Omega$.

(iv) $f(X) \leqslant_{st} f(Y)$ for all increasing real function $f(x)$.

Lemma 5.6 *The following statements are equivalent:*

(i) $X \leqslant_{cx} Y$;

(ii) $X \leqslant_{icx} Y$ *and* $\mathbb{E}[X] = \mathbb{E}[Y]$.

Lemma 5.7 *The following statements are equivalent:*

(i) $X \leqslant_{icx} Y$;

(ii) $-Y \leqslant_{icv} -X$;

(iii) $\mathbb{E}[X-t]^+ \leqslant \mathbb{E}[Y-t]^+$ *for all real* t.

Lemma 5.8 *Let* $\{X_1, X_2, \ldots, X_n\}$ *and* $\{Y_1, Y_2, \ldots, Y_n\}$ *be independent random variables with* $X_i \leqslant_{st} Y_i$, *for* $i = 1, 2, \ldots, n$ *and assume that* $\phi : \mathbb{R}^n \to \mathbb{R}$ *is an increasing function, then*

$$\phi(X_1, X_2, \ldots, X_n) \leqslant_{st} \phi(Y_1, Y_2, \ldots, Y_n). \tag{5.5}$$

Chapter 6

Conclusion

This study provides a comprehensive review of advantages of Blockchain technology via analyzing its salient features of transparency, traceability, security, efficiency, confidentiality and immutability. Those features of BCT have been further translated into several business benefits, i.e., food safety, cost savings, demand growth and yield improvement. Based on those business interpretations, we conduct a thorough analysis with peer technologies of Blockchain, e.g., ERP, AR/VR, etc., functionally supplementary technologies for Blockchain, e.g., RFID, IoT, Big Data, AI, etc., and the existing Blockchain use cases in the industry, e.g., *Walmart* and *IBM*, *Wave* and *Barclays*, *AgriLedger* on coffee bean, *Everledger* on diamond, *Estonian "e-Residency"*, etc. Considering the strengths and limitations of Blockchain technology, our analysis shows that Blockchain is currently the only stand-alone technology capable of information integration and diffusion with high level of information security, based on which we position Blockchain to be the best fit to the application of sought-after, credence goods with complex transaction processes. We further consider Blockchain a promising alternative to replace labeling for credence goods, e.g., organic foods, for which the availability of truthful information is crucial for business sustainability.

6.1 Recapture of the Study

In our mathematical models, a strategic decision variable, Blockchain adoption level, is introduced to capture a concept of the level of product information disclosure via BCT. The business benefits derived from the study, including cost savings, demand growth and yield improvement, are borrowed to serve as assumptions in an attempt to substantiate and explore more business insights from Blockchain adoption. Two models are developed: 1) a Blockchain-enabled Newsvendor model for single-period perishable/seasonal products; 2) a dynamic programming model for multi-period non-perishable products.

In our first model, we develop a Blockchain-enabled Newsvendor model for single period perishable or seasonal products, e.g., agricultural products with short shelf life, fashion products, etc. We incorporate the BCT benefits of cost savings and demand growth to assume that the higher the BCT adoption level, α, the lower the purchasing cost and the higher the market demand (in the stochastic order). In the presence of uncertain demand, our goal is to solve the Newsvendor model for the optimal order quantity, and ultimately to find the optimal Blockchain adoption level. For a generic demand distribution, it is shown that increasing BCT adoption level will increase the critical ratio, as well as the optimal order quantity; it will increase the optimal expected profit if there is no lost-sales penalty. Intuitively, a higher adoption level of BCT leads to higher demand and a lower ordering cost, each of which could improve the operational profit. However, we device some counter examples to show that an increase in adoption level might lower the optimal order quantity, and it is not always profitable to adopt a higher BCT even when there is no adoption cost. For the selected demand types, Uniform and Normal distributions, we derive closed-form expressions for the optimal decisions, based on which useful insights are developed. Finally, a sequence of numerical studies complements our analytical results with more useful insights.

In the second model, we consider a firm that orders from its supplier and sells to its tech-savvy customers. BCT adoption would impact the random supply and demand in a stochastic sense. A firm seeks to maximize the total expected discounted profit, by jointly managing (i) *Blockchain design*, (ii) *production and ordering decision*, and (iii) *dynamic pricing and selling*. It is shown that the deployment of BCT can help firms reduce order quantities, lower selling prices and reduce the target-inventory levels. It is also shown that the volatility of either supply or demand harms the expected profit. The analysis remains robust with some major extensions, e.g., lost-sales of demand and random capacity. Numerically, we show that some types of goods (e.g., *credence goods* and *experience goods*) benefit from the adoption of BCT, but it may not prove beneficial to leverage BCT for some other types of goods (e.g., *search goods*). Considering the lifecycle of experience goods, we recommend the adoption of BCT as early as possible, and it is suggested to adopt a higher level of BCT at an earlier stage.

6.2 Future Research of BCT for SCM

Generally, there are two seemingly conflicting theories concerning a new technology adoption: "*First Mover Advantage*" encourages companies to adopt an innovation as soon as possible to exploit the first-mover advantages, e.g., monopoly power; on the contrary, the other stream of theory claims that a company would benefit from a late adoption by *waiting for an innovation to mature*. The key to these two streams of theories lies in the tradeoff between benefits and risks of innovations. Therefore, by claiming sought-after, credence goods with complex transaction processes is a good fit of Blockchain, we essentially indicate that such businesses may benefit more by first mover advantages; on the other hand, other businesses might be more favorable to the strategy of "wait" (but not abandoning Blockchain). We do believe that Blockchain technology does exhibit a potential to overhaul the existing business ecosystem if properly adopted. Meanwhile, we have to be cautious of the phenomenon of overhypes with new technology. For example, Virtual Reality (VR), a victim of overhypes, is now regaining public attention after years of oblivion[1] for its recent realization of value in job trainings and Augmented Reality (AR), a modification from VR, to promote customer experience.

The hype of Bitcoin, for which Blockchain served as the backbone system, pushes companies to explore possibilities of Blockchain applications. Previous lessons learned from numerous successful cases with new technology adoptions (e.g., Big Data, AI, Cloud, etc.) propel industry to embrace Blockchain. Some media praised BCT as *"the next disruptive technology changing the business world"*, which further ferments an atmosphere of "FOMO" —the *"fear of missing out"*. However, the phenomenon of overhype fads is more and more common, and thus we believe a rational and systematic analysis of Blockchain technology is very important to address the concerns of new technology adoption. The objective of this study is to comprehensively analyze the advantages and disadvantages of Blockchain technology, investigate alternative and supplementary technologies, and conduct profitability analysis by mathematical modeling to ultimately give an objective evaluation of Blockchain. The result indicates that not all businesses are good fits to Blockchain, even when there is no adoption cost.

BCT provides a disruptive and state-of-the-art business solution in a variety

[1] "Why Silicon Valley is betting on VR again?", the cover story of Fortune, July 2019.

of areas, including SCM, especially for those serving tech-savvy consumers. Although we have endeavored to consider the major features of BCT, there are definitely some other compelling factors that need to be considered to enrich the research. As one of potential future research projects, Blockchain technology could significantly shorten the lead times of transactions by speeding up both information processing and paperwork processing times. As mentioned before, BCT could enhance food safety. It might be an interesting direction to study the impact of BCT on the supply chain performance related to quality concerns. From a financial perspective, cash flow could become faster or even immediate by leveraging cryptocurrency ecosystems throughout its supply chain network. In this case, it will be of interest to study the impact of BCT on process acceleration.

In the future, Blockchain solutions from different companies, or even industries, will be able to communicate and share digital assets with each other seamlessly. One related future research is therefore regarding the scalability and expansion of a network, its compatibility and the integration of multiple Blockchains in a value chain; cf. Piscini et al. (2017).

Currently, BCT is still in a nascent stage, so there is very limited business data available. As an important trend with more and more data available, data-driven studies and empirical analyses will be springing up like mushrooms.

In summary, BCT can transform supply chains, industries and ecosystems.

Blockchain is often claimed as a world-changing technology and in many ways, it is. However, it isn't necessarily the cure-all panacea. Definitely, an in-depth transformation of supply chains with an implementation of BCT will not happen instantly. However, supply chains have already started leveraging BCT for small portions of their operations. Optimistically, there is a promising future for the marriage between Supply Chain and Blockchain.

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