RECONSTRUCTING THE STAR FORMATION
HISTORIES OF GALAXIES WITH THE DENSE BASIS
METHOD

by
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Written under the direction of
Eric Gawiser
And approved by

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Reconstructing the Star Formation Histories of Galaxies with the Dense Basis Method

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Dissertation Director: Eric Gawiser

The star formation histories (SFHs) of galaxies have long been approximated by simple parametric forms while estimating quantities like stellar masses, star formation rates and ages. However, there is considerable diversity seen among galaxy SFHs in cosmological simulations, and individual SFHs are variable across a range of timescales corresponding to the various physical processes that govern star formation. With higher S/N multiwavelength Spectral Energy Distributions (SEDs) from broadband photometry and sophisticated analysis tools, we develop the Dense Basis SED fitting method (Iyer & Gawiser 2017) to reconstruct the SFHs of individual galaxies with uncertainties. Applying this method to CANDELS data, we reconstruct the SFHs of nearly 50,000 galaxies across a wide range of redshifts. An updated version of the method (Iyer et al. 2019) uses Gaussian Processes to create smooth SFHs that are independent of any choice of functional form, with a flexible number of parameters that recover the maximum amount of information from individual SEDs.

Using this method, we estimate the number and duration of major star formation episodes in a galaxy’s past, in addition to quantifying the evolution of galaxy SFHs with mass, morphology, and redshift. The distribution of SFHs at a particular epoch constrains feedback and wind strengths modeled in simulations of galaxy evolution. (Iyer et al. 2018) uses SFHs as trajectories in SFR-M∗ space to probe the previously inaccessible low-mass, high-redshift regime of the SFR-M∗ correlation. This new technique of SFH reconstruction allows us to probe a wide range of quantities that were previously inaccessible through SED fitting, creating new possibilities of probing galaxy formation and evolution at high redshifts.
Acknowledgements

The culmination of my work over the past few years, this thesis would not exist without the camaraderie, advice, and support from a legion of friends, family, colleagues and mentors. You know who you are, and what you’ve done. I hope you’re happy now.

I am extremely fortunate to have found an advisor in Prof. Eric Gawiser. He took a chance taking me on back when I didn’t know what a Jansky was, and pointed me towards an interesting problem that I’m only slightly closer to solving after four years. His clear explanations of unfamiliar concepts, tolerance for my digressions, and provocative talks at conferences will serve as inspiration for the years to come.

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I’m grateful to the stalwart programmers who built numerous open-source and publicly available python packages including, but not limited to, Numpy, Scipy, Astropy, Scikit-Learn, Matplotlib, Seaborn, Bokeh, Pandas, AstroML, Dynesty, FSPS and Python-FSPS, corner, emcee, sompy, PyTorch, and more. I will walk in your footsteps, at least until I can join your ranks.

Finally, I’m grateful to my friends and family, who kept me sane in the moments between

---

1 and her infectious enthusiasm for Random Forests
2 comrade-in-arms and inspiring researcher to boot
3 who enriched my life with two very different types of chaos
4 sometimes cool. cool cool cool.
5 who taught us all how to dance tree times over.
6 with his sheer amount of energy and optimism
work. Chompa\textsuperscript{7}, Bhoto\textsuperscript{8}, Seccy\textsuperscript{9}, Xlnc\textsuperscript{10}, USA\textsuperscript{11}, Dodo, Pgy, Srinivas, Harini, Tal\textsuperscript{12}, Aisha\textsuperscript{13}, Jan, Rojay, Harrison, Valerie, Nina and everyone else. And to Noopur, thank you. You've always had my back.
Dedication

To potatoes, always and forever.

And to chaos. I’m coming for you.
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Chapter 1

Introduction

There is a theory which states that if ever anyone discovers exactly what the Universe is for and why it is here, it will instantly disappear and be replaced by something even more bizarre and inexplicable. There is another theory which states that this has already happened.

Douglas Adams, *Life, the universe, and everything*

We inhabit the Milky Way, a reasonably massive spiral galaxy. As galaxies go, it is fairly typical, composed of stars, dust, gas and dark matter, with a supermassive black hole in its center.

Current estimates put the mass of the Milky Way at $\approx 0.8 - 1.5 \times 10^{12}$ solar masses (Watkins et al. 2019), with an average radius of 50,000 light years, containing somewhere between 100 to 400 billion stars. While we can make measurements of the mass, size and star formation rate of our own galaxy, it requires knowledge of the broader population of galaxies to put these numbers into context.

The reason this is necessary is because no two galaxies are alike, and they span a large dynamical range in almost every observable property. The Large and Small Magellanic Clouds, two nearby irregular dwarf spiral galaxies that are visible to the naked eye, are roughly a hundredth the mass of the Milky Way. Meanwhile, IC 1101, one of the largest known galaxies, is the brightest in a cluster of galaxies and nearly fifty times more massive than our own. Galaxy morphologies cover a variety of shapes, including spirals and ellipticals, but also complex shapes created by galaxies interacting and merging with each other. Some galaxies are bright in the optical portion of the spectrum while others are extremely dust obscured and only radiate at sub-millimeter wavelengths. Some are rich with pristine gas and rapidly form stars, while others are old, gas-depleted and quiescent. The stellar masses and star formation rates of known galaxies span a range of over six orders in magnitude.

Our knowledge of distant galaxies and their demographics comes from galaxy surveys like SDSS (Eisenstein et al. 2011; Blanton et al. 2017), and CANDELS (Grogin et al. 2011; Koekemoer et al. 2011), which observe galaxies near and far and compile a census of the properties
of different galaxy populations. Using these statistics, we construct and test theories of how galaxies form and evolve, ultimately providing perspective about our own. Since galaxies are a radiant tracer for a significant portion of the universe\(^1\), this in turn plays a central role in understanding the universe and our place in it.

The field of galaxy formation and evolution is relatively nascent, starting with initial observations in the late 1920s of galaxies other than our own (Hubble 1929). Recent advances are enabled by state-of-the-art observations with ground- and space-based telescopes, as well as steadily increasing computing resources that facilitate more realistic simulations of the underlying astrophysical processes responsible.

The current paradigm of galaxy evolution starts with the cosmic web, a filamentary structure of dark matter and gas driven by gravitational collapse as the universe expands with time. At the nodes of the cosmic web are galaxies or clusters of galaxies, gravitationally bound structures of baryons within parent halos of dark matter. To first order, the growth of galaxies is thus tied to the growth of their dark matter haloes. A recent review by Wechsler & Tinker (2018) provides an excellent summary of the current state of our understanding.

However, several questions remain: the relationships between secondary galaxy properties (galaxy colors, sizes and morphologies) and their host halos are still relatively unconstrained, the properties of low-mass halos are poorly understood, and the effects of baryonic feedback beyond nearby galaxies is poorly constrained. To improve our understanding of these issues, we need better observations, as well as better tools and statistical techniques to extract information from the observations and test against predictions from cosmological simulations.

This work chronicles an attempt to develop methods to extract information about the star formation histories (SFHs) of galaxies that are too distant for us to resolve individual stellar populations and infer their ages. This chapter serves as an introduction to the current problems that can be addressed with knowledge of the SFHs of galaxies. Chapters 2 and 3 are methodological developments in extracting galaxy SFHs from their spectral energy distributions (SEDs). Chapter 4 is a detailed application of the method to probing part of a scaling relation that is not directly accessible through current observational methods. Chapter 5 extends the formalism to simultaneously fit for the characteristics of an entire population of galaxies. Chapter 6 compares observational results to those from simulations. Chapter 7 details some of the future projects that are enabled by this work.

\(^1\)Notable exceptions being dark matter halos that do not contain stars, and the intergalactic medium (IGM) - large clouds of hydrogen and helium in the regions between galaxies.
1.1 Galaxy evolution and Physical parameters (Observations)

Most of our understanding of distant galaxies comes from observations in the electromagnetic spectrum\(^2\). These observations cover a wide range of wavelengths, ranging from the radio to the gamma-ray portions of the spectrum.

A spectral energy distribution (SED) is a measurement of the flux density of a galaxy across a range of wavelengths.

Integrated over the entire galaxy, the SED allows us to estimate physical properties like stellar masses \((M_*)\), star formation rates (SFRs), colors, dust attenuation and metallicity\(^3\). The art of extracting these quantities from a galaxy’s SED is further described in Sec. 1.4. Since different astrophysical processes emit radiation at different frequencies, a wide range of wavelengths is crucial to being able to extract information from a galaxy’s SED.

In addition to galaxy SEDs, we can estimate the kinematics, gas properties, environment and nuclear activity from different tracers in resolved multiwavelength observations of galaxies. A brief list of physical parameters and their indicators is given in Table 1.1.

1.1.1 Probing galaxy evolution with distribution functions and scaling relations

In physics, experiments to test a hypothesis are often designed to perturb the system under consideration and record the response. Since this is not feasible in the context of astrophysical systems, we test theories by doing similar experiments using cosmological simulations, and comparing the distributions of physical properties from these simulations to those seen in our observations to determine which of our simulated mock universes best describes reality. By performing rigorous hypothesis testing, we are able to gain a deeper understanding of the different physical processes that govern galaxy formation and evolution. Before going into how these simulations are constructed and what ingredients they incorporate, we will briefly focus on the observables that they are tested against.

---

\(^2\)While recent observations through gravitational waves and neutrinos have widened the scope of future experiments, they are outside the scope of the current work.

\(^3\)In astronomy, a color is the logarithm of the ratio of flux densities at two different wavelengths. Stellar mass refers to the total mass found within stars in a given galaxy, while SFR is the rate at which it is forming new stars. Metallicity refers to the abundance ratio of all elements heavier than helium to that of hydrogen.
<table>
<thead>
<tr>
<th>Property</th>
<th>Notation</th>
<th>Indicator</th>
<th>Example references</th>
</tr>
</thead>
<tbody>
<tr>
<td>Redshift</td>
<td>$z$</td>
<td>SED/emission lines</td>
<td>Brammer et al. 2008</td>
</tr>
<tr>
<td>Stellar Mass</td>
<td>$M_*$</td>
<td>SED (around 1.6µm)</td>
<td>Acquaviva et al. 2011</td>
</tr>
<tr>
<td>Star Formation Rate</td>
<td>SFR</td>
<td>SED (rest-UV, FIR, Hα)</td>
<td>Acquaviva et al. 2011</td>
</tr>
<tr>
<td>Half-mass time</td>
<td>$t_{50}$</td>
<td>SED (esp. rest-optical)</td>
<td>Pacifici et al. 2012</td>
</tr>
<tr>
<td>Dust attenuation</td>
<td>$A_V$</td>
<td>SED (UV+FIR slopes, IRX-β, Hα/Hβ)</td>
<td>Salmon et al. 2015</td>
</tr>
<tr>
<td>Metallicity</td>
<td>$Z_{gas}, Z_*$</td>
<td>SED (absorption, emission lines)</td>
<td>Acquaviva et al. 2011</td>
</tr>
<tr>
<td>Size</td>
<td>(various)</td>
<td>single-band photometry</td>
<td>Barro et al. 2017</td>
</tr>
<tr>
<td>Morphology</td>
<td>(various)</td>
<td>single-band photometry</td>
<td>Conselice 2014</td>
</tr>
<tr>
<td>Cold gas fraction</td>
<td>$f_{gas}$</td>
<td>CO(1-0) lines etc.</td>
<td>Tacconi et al. 2010</td>
</tr>
<tr>
<td>Ordered, random motions</td>
<td>$v_{rot}, \sigma$</td>
<td>kinematics from IFU spectra</td>
<td>Genzel et al. 2011</td>
</tr>
<tr>
<td>Halo mass</td>
<td>$M_h$</td>
<td>abundance matching etc.</td>
<td>Behroozi et al. 2018</td>
</tr>
<tr>
<td>Environment</td>
<td>$\delta$</td>
<td>number counts from galaxy surveys</td>
<td>Darvish et al. 2015</td>
</tr>
<tr>
<td>AGN fraction</td>
<td>$f_{AGN}$</td>
<td>SED (mid-IR, X-ray, radio, nebular emission lines)</td>
<td>Rivera et al. 2016</td>
</tr>
</tbody>
</table>

Table 1.1: A brief summary of different physical parameters used to describe galaxies and their observable tracers.

**Distribution functions**

The distribution function for a given quantity gives the probability of finding a galaxy with a given value of that quantity in a population of galaxies being studied. (Behroozi et al. 2013a, 2018; Yung et al. 2018; Baldry et al. 2012; González et al. 2011).

Astrophysicists study the distributions of both directly observable quantities like luminosities and number counts of galaxies, as well as distributions of derived quantities like stellar masses, halo masses and star formation rates.

Sometimes it is also instructive to study the two-dimensional distribution of quantities like the color-luminosity distribution, which reveals a striking bimodality consisting of a quiescent red sequence and a star forming blue cloud. Sometimes these 2d distributions are extremely tightly constrained to a small part of the available parameter space. This brings us to the next set of observables:

4 or more..
Scaling relations

Scaling relations provide a wealth of information about the physical processes that drive and regulate the formation of stars, redistribution of angular momentum and the change of size across galaxy populations. Assuming that galaxies form an ergodic sample, behaviour seen across a population of galaxies can be used to infer mechanisms that regulate galaxy evolution within individual galaxies. This can then be used to design falsifiable tests for simulations.

Some examples of scaling relations include the Tully-Fisher \((L - V_{\text{rot}})\) (Tully & Fisher 1977) and Faber-Jackson \((L - \sigma)\) (Faber & Jackson 1976) relations, the star forming sequence that correlates the stellar masses of galaxies and the rate at which they form stars (Noeske et al. 2007; Daddi et al. 2007; Elbaz et al. 2007), the size-mass relation (Barro et al. 2017), and the mass-metallicity relation (Tremonti et al. 2004), which finds that massive galaxies are often more metal-rich\(^5\) than their less massive counterparts.

While at first glance these scaling relations do not reveal any causal knowledge, the slope, normalization and scatter of each relation carries important clues to the underlying mechanisms that restrict galaxies to these particular parts of phase space. Teasing out these mechanisms requires robust measurements of individual parameters and a thorough knowledge of systematic uncertainties present in the observations (Conroy et al. 2009; Conroy & Gunn 2010; Dahlen et al. 2013; Mobasher et al. 2015 and Pacifici et al. 2019, in prep.).

1.2 Galaxy evolution and Physical parameters (Theory)

A wide range of physical processes acting over spatial and temporal scales spanning over 20 orders of magnitude play a role in shaping galaxies throughout their lifetimes. These processes include the hierarchical assembly of dark matter halos, the accretion of gas on to halos, its cooling and subsequent collapse into giant molecular clouds that serve as foundries for star formation, feedback from photoionizing sources and exploding stars, the growth of supermassive black holes and the evolution of active galactic nuclei that contain them, the role of galaxy mergers, disk creation and disruption, the chemical evolution of gas in the galaxy due to mergers and supernova explosions, stellar winds that drive gas outside the galaxy whereupon it may exit the gravitational potential well or fall back in, and more. The manner and extent to which these processes the affect the state of the interstellar medium (ISM), the circumgalactic medium (CGM) and the intergalactic medium (IGM) as well as the interplay between them is a subject

\(^5\)metals, in the astrophysical jargon, often denote all elements heavier than Helium
matter of active study and interest, both in the theoretical and observational communities. An informative flowchart summarizing these processes and the order in which we currently understand them to affect galaxies is presented in Figure 1 of Mo et al. (2010). Additionally, a very thorough categorical accounting of each of these processes as well as a thorough summary of simulations can be found in a review by Somerville & Davé (2015).

1.2.1 Simulating galaxy evolution

Owing to the large range in spatio-temporal scales over which different physical processes act, ranging from nuclear fission in stellar cores to millions of years and thousands of light years over which galaxy mergers occur, running a first-principles simulation of galaxy evolution is currently out of the scope of our computing resources. Early gravity-only particle-based simulations (Sellwood & Sparke 1988) considered particles in a box acting solely under the influence of gravity. This led to dark matter-only simulations (Klypin et al. 2011; Rodríguez-Puebla et al. 2016) which were successful at reproducing a wide range of galaxy properties observed in the late-80s and early-90s. From this point, there was a divergence in how simulators approached different open questions.

Cosmological magnetohydrodynamical simulations

The most explicit way to simulate galaxy evolution is to consistently solve equations for gravity, hydrodynamics, electromagnetism and thermodynamics for particles and cells representing dark matter, gas and stars. These simulations are often very large, with the latest generation containing over $10^{10}$ particles in a cosmologically-sized$^6$ box. Even so, the resolution of individual star-particles is limited to $10^4 - 10^7 M_\odot$, and the median spatial resolution of star-forming gas is limited to $10^2 - 10^3$ parsecs, which requires sub-grid recipes for processes that occur on scales smaller than these. Although most state-of-the-art simulations show remarkable consensus in predicting present-day galaxy properties (Schaye et al. 2014; Davé et al. 2019; Pillepich et al. 2017; Weinberger et al. 2018), they often get there in different ways, leading to tensions in comparing statistics at intermediate-redshifts or low masses that are poorly constrained by observational statistics (Hahn et al. 2018).

$^6$usually cubic boxes with $\mathcal{O}(100)$ comoving Mpc to a side and periodic boundary conditions
Zoom-in magnetohydrodynamical simulations

These simulations solve fundamental equations to update the state of individual particles and cells after each timestep, running in a much smaller volume compared to cosmological simulations in order to achieve much higher resolution. This allows these simulations to resolve features such as the effects of turbulence more finely than possible in cosmological simulations. For more detail, see Munshi et al. (2013); Hopkins et al. (2014); Choi et al. (2017). Additionally, zoom-in simulations often take a significantly smaller amount of time to run, allowing for multiple runs where individual prescriptions for processes such as star formation, feedback etc. are varied, in order to better understand their effects. In addition to these models, there are even finer simulations that only consider a small box that approximates a galaxy’s disk or a portion of a star forming region to study the conditions within at extremely high resolutions (Kim & Ostriker 2017; Tonnesen & Bryan 2012).

Semi-analytic models

Procedurally updating the properties of galaxies as they assemble within dark matter merger trees, semi-analytic models (SAMs) use prescriptions for how gas accretes onto halos, cools and turns into stars, how feedback processes regulate star formation, and how mergers transform the galaxy structure, among others. For more detail, readers can refer to Somerville et al. (2008, 2015); Croton et al. (2006). While SAMs may not capture the full effects of turbulence and the effects of baryons on dark matter, they have the huge advantage of being able to run a cosmological volume in a matter of hours, which enables a much more detailed exploration of parameter space than is possible with MHD simulations.

Empirical models

There also exist a class of models - halo occupation distribution (HOD) models (Berlind & Weinberg 2002; Zheng et al. 2005), conditional luminosity function models (Van Den Bosch et al. 2007), and sub-halo abundance matching (Behroozi et al. 2010; Moster et al. 2018) - that use a range of available parameter distributions in conjunction with dark matter merger trees to derive relationships between the observed properties of galaxies and the predicted properties of their dark matter halos. These contain no implementation of physical processes but are useful to test overall consistency between observed data and initial conditions, as well as make data-driven predictions for sparsely sampled regions of parameter space.
1.3 Models of galaxy evolution and SFHs

**SFH:** The *star formation history* of a galaxy is a record of when it formed its stars. This can be thought of as a curve in SFR(t) or M∗(t) vs time, where the stellar mass is the integral of the star formation rate accounting for mass loss and recycling of old stars.

The SFHs of galaxies are closely related to their mass accretion histories - a record of when a galaxy accumulated its stellar mass, different from the SFH in that a merger is counted in the mass accretion history at the time when it entered the galaxy, whereas it contributes to the SFH according to when it formed its stars. The mass accretion histories are in turn closely related to their dark matter accretion histories, which ultimately trace cosmology.

Knowledge of the SFHs of galaxies is more accessible observationally than the mass accretion or DM accretion histories, since we can directly observe the different stellar populations present in a galaxy even if we don’t know whether those stellar populations entered via a merger or were formed in-situ.

SFHs provide the following observables:

- probes to higher redshifts, longitudinal tests that help test theories predicated on cross-sectional statistics of populations of galaxies at different redshift slices. Propagate galaxies backward in time to probe the high-redshift or low-mass regimes of various scaling relations and distribution functions, in regimes where merger rates would not significantly affect the analysis.

- access to quenching timescales, transition through the green valley, merger and morphological transformations, etc. since different physical processes are thought to be responsible for these events, knowledge of the timescales could help constrain the dominant mechanism at a given redshift.

- an indirect (high-redshift) probe of feedback strengths at different redshifts, since the distribution of the SFHs for a population of galaxies is influenced by the processes that regulate star formation.

- a better understanding of galaxy demographics, by correlating SFHs with ancillary observables like redshift, Stellar Mass, SFR, environment, and morphology.
1.4 SED fitting

Spectral Energy Distribution (SED) fitting is the rich field of interpreting the integrated light from galaxies to understand the properties of their constituent stellar populations. This field has seen enormous growth in recent times, thanks to both an increase in high-quality data and the availability of computing resources needed to pose sophisticated inference problems to this data, and is now the leading method to infer the physical properties of large ensembles of galaxies from their integrated light.

The earliest attempts at determining the physical properties of galaxies come from estimating their stellar masses from the total luminosity integrated over the whole galaxy in a single photometric filter (Faber & Jackson 1976), using a 'mass-to-light ratio' as a conversion. As observations improved, and better templates for galaxy spectra were measured, the process of comparing observations to existing templates to understand their properties became more systematized (Bolzonella et al. 2000; Walcher et al. 2011). As of today, stellar population synthesis (SPS) models (Bruzual & Charlot 2003; Conroy et al. 2009) allow us to create mock galaxy spectra corresponding to galaxies with different physical properties (stellar masses, SFRs, ages, metallicities, redshifts, and more) using a combination of theoretical and empirical stellar templates, the relative abundances of different types of stars, and knowledge of how each population of stars evolves with time. Table 1.2 provides a short summary of the key ingredients that go into creating SEDs. This list is by no means exhaustive (see Foreman-Mackey et al. (2014) for a fuller list), but is often enough to recreate the broad features that are needed to fit photometric SEDs.

Equipped with the ability to make mock galaxy spectra with a variety of physical properties, SED fitting methods then consider the inverse problem of inferring physical properties given noisy, sparsely sampled measurements of the SEDs of distant galaxies. A host of recent papers have devised different ways of approaching the problem, adapted to different types of datasets, science questions, computational and statistical methods, modeling assumptions, spectral coverage and noise constraints Bolzonella et al. (2000); Heavens et al. (2000); Reichardt et al. (2001); Tojeiro et al. (2007); Dye (2008); Noll et al. (2009); Acquaviva et al. (2011b); Pacifici et al. (2012); Acquaviva et al. (2011a, 2015); Salmon et al. (2015); Chevallard & Charlot (2016a); Pacifici et al. (2016); Iyer & Gawiser (2017); Lee et al. (2017); Leja et al. (2017); Carnall et al. (2018a). Walcher et al. (2011); Acquaviva et al. (2011a); Gawiser (2009) provide an excellent review of the SED fitting process, with our approach described in more detail in sections 2 and 3.2.
<table>
<thead>
<tr>
<th>Component</th>
<th>Description</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPS model</td>
<td>Stellar population synthesis models are a combination of stellar isochrones and evolutionary tracks that can be used to generate spectra by summing up stellar populations of different spectral classes and ages.</td>
<td>(Tinsley 1980; Bruzual &amp; Charlot 2003; Conroy et al. 2009; Conroy &amp; Gunn 2010)</td>
</tr>
<tr>
<td>IMF</td>
<td>The initial mass function can be thought of as a distribution of stellar masses within a galaxy. This is especially important in computing the overall stellar mass since most of the light is dominated by a small subset of massive bright stars.</td>
<td>(Salpeter 1955; Chabrier 2003; Kroupa 2002)</td>
</tr>
<tr>
<td>SFH</td>
<td>The star formation history of a galaxy is a record of when it formed its stars. Since the spectra of stellar populations change with time the SFH is important in determining the overall shape of the galaxy’s SED.</td>
<td>(Pacifici et al. 2012; Iyer &amp; Gawiser 2017)</td>
</tr>
<tr>
<td>Stellar Metallicity</td>
<td>Changes in metallicity affect the overall SED continuum as well as the strengths of absorption lines. With poor-quality data, this causes what is known as the age-dust-metallicity degeneracy.</td>
<td>(Tojeiro et al. 2007; Pacifici et al. 2012)</td>
</tr>
<tr>
<td>Nebular emission</td>
<td>Light from young stars and AGN heats the hot nebular gas around it, emitting nebular emission lines as well as continuum.</td>
<td>(Allen et al. 2008; Ferland et al. 2013; Byler et al. 2017)</td>
</tr>
<tr>
<td>Dust model</td>
<td>The model used to correct for dust attenuation in the rest-UV to optical portion of the SED, and account for the re-emission of light in the IR. Since the dust can make a large difference in the overall SED while being degenerate with other parameters, it is important to accurately constrain the shape of the attenuation curve.</td>
<td>(Calzetti 2001; Salmon et al. 2015)</td>
</tr>
<tr>
<td>IGM absorption</td>
<td>Absorption of photons by intervening clouds of hydrogen and helium results in absorption of the spectrum blueward of rest-frame Lyα line. This effect is mainly important for high-z galaxies, where the rest-FUV is observed through broadband photometry.</td>
<td>(Madau et al. 1996; Inoue &amp; Iwata 2008)</td>
</tr>
</tbody>
</table>

Table 1.2: A brief summary of the various components that affect a galaxy’s UV-to-NIR SED.
Photometric vs spectroscopic data: The observed SEDs of galaxies are often measured using the integrated flux in broad- or narrow-band filters (photometry), or by dispersing light along a grating at different spectral resolutions (spectroscopy). Obtaining broadband photometry for a large number of galaxies is cheaper in terms of telescope time and thus available for many more galaxies, while spectroscopy contains finer features and thus more information. Medium and narrow band photometry (Domínguez Sánchez et al. 2016; Tomczak et al. 2016; Skelton et al. 2014) can help bridge this gap. Spectroscopically derived physical parameters are also often taken as a ‘gold standard’ and used as a training/testing set for other methods, especially those that employ machine learning (Bolzonella et al. 2000; Masters et al. 2015). This needs to be done with caution, since spectroscopic surveys often have different selection effects compared to photometric samples, may not cover a representative distribution in the physical parameters of interest, and fiber sizes are often \( \ll \) galaxy sizes at low redshifts.

Frequentist vs Bayesian: Early SED fitting routines were mostly template based, finding the single model SED that best matched the observation, usually using a goodness-of-fit metric like the \( \chi^2 \) statistic. Since the SEDs were fit with templates to infer the best one, while estimating a confidence interval for the estimated physical quantities associated with this choice, the question was modeled as frequentist: With what confidence does the observed galaxy have physical parameter \( X \) within bounds \( (X_{\text{low}} < X < X_{\text{high}}) \)? As SED fitting techniques grow more sophisticated, the different ingredients were better understood, as well as the prior probability distributions for the various parameters used to construct SEDs. With this knowledge, Bayesian methods to infer galaxy properties have come to be in vogue, asking the question: Given the observations and prior beliefs, what is the posterior probability distribution for physical parameter \( X \) for a given galaxy? Most questions in SED fitting are well posed as Bayesian inference problems. However, in cases where prior distributions and modeling assumptions are not well understood, it might be advisable to adopt a frequentist approach, but caution must be taken not to mix the two since probabilities are thought of in fundamentally different ways.

1.5 SED fitting and SFHs

The SFH is a key component that needs to be specified in order to generate galaxy SEDs.

SSPs and CSPs: In early times, with only a few datapoints per galaxy, it was assumed that all of the star formation happened instantaneously. These models were called simple stellar populations (SSPs). As the data quality improved, in resolution, wavelength coverage and S/N, \footnote{See https://xkcd.com/1132/ for a more strongly worded summary.}
more complicated models were adopted leading to the term ‘composite stellar populations’ (CSPs) being used to denote contributions from stellar populations of different ages being summed up to create galaxy SEDs. Bruzual & Charlot (2003)

**Simple parametric forms:** Later, while studying old elliptical systems, the SFHs were assumed to have an approximately exponentially declining form. This was popular since it allowed the SFH to be specified completely with only 2-3 parameters, an overall normalization, the timescale of the decline (τ), and an optional third parameter (t₀) specifying the time when star formation ‘turned on’ in the galaxy. While this was not a very realistic model for how we now know galaxy SFHs to look⁸, its simple parametrization was appealing and it continues to be used in a wide range of literature today. In addition to this, in the limit t₀ → ∞, the SFH becomes constant across time, leading to another commonly used SFH parametrization still in use today.

**Complex parametric forms:** To better approximate the complex shapes that real galaxy SFHs could potentially have⁹, models like linear-rise followed by exponential decline (Acquaviva et al. 2011a), lognormal (Dressler et al. 2016) or double power-law (Carnall et al. 2018a) SFHs came to be used in more recent literature.

**Non-parametric SFHs:** Ideally though, it would be optimal to infer the shape of the SFH directly from the observations, which is what nonparametric SFH reconstruction methods attempt to do. Since this requires a lot more information, early methods were developed exclusively to work with spectra (Heavens et al. 2000; Tojeiro et al. 2007). These methods often discretized time into bins with the goal of estimating the SFR in each bin, thus finding a coarse approximation of the galaxy’s SFH bypassing the need for a parametric description. Tojeiro et al. (2007) even made the time bin edges adaptive, allowing for as many bins as the data could support. Later, these methods were generalized to work with broadband photometry as well (Dye 2008; Leja et al. 2018). It is found in these binned nonparametric methods that the results are extremely sensitive to the SFH priors assumed in the fitting, and sometimes need regularization when implemented as a matrix inversion method. Additionally, since the resulting SFHs are often binned logarithmically in lookback time and can be quite coarse, extracting useful information from them can sometimes be challenging.

Pacifici et al. (2012) presented an alternative approach by using a basis of SFHs and chemical enrichment histories from a semi-analytic model to perform SED fitting in a Bayesian framework.

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⁸both through observations of local galaxies and from cosmological simulations

⁹for example, it was known from measurements of the cosmologically averaged SFRs for galaxies at high redshift (3 < z < 8) that these star forming galaxies needed to have rapidly rising SFHs (Papovich et al. 2011).
This method contains almost no parameters apart from an overall normalization and those inherent to the SAM in generating the SFHs. However, this results in an implicit prior imposed through the SAM, which can be quite complex and hard to interpret.

In Iyer & Gawiser (2017) and Iyer et al. (2019) we develop a smooth, nonparametric method to describe galaxy SFHs with a flexible number of parameters. This formalism can be explicitly used in conjunction with any well-motivated prior assumptions to describe SFHs without suffering from most of the disadvantages inherent to other existing methods. It is however important to note that any nonparametric method is only as good as the data being fit, and better SEDs will allow us to better observationally constrain galaxy SFHs.

1.6 Leading open questions in galaxy evolution

Some of the key questions in galaxy evolution at the present time can be framed as:

1. How accurately can we recover the star formation histories of galaxies? Can we link progenitors to their descendants today?

2. What physical processes regulate star formation over different spatial and temporal scales in different galaxy populations? How are they linked to scaling relations observed among galaxy properties?

3. What mechanisms drive galaxy quenching? How are they related to morphological transformations and the growth of central bulges?

4. What are the different populations of galaxies found in the ‘green valley’? How do they evolve, and over what timescales?

This thesis will predominantly address the first question, developing methods to robustly estimate the star formation histories of galaxies from multiwavelength observations (Q1, Ch.2,3). Doing so gives us access to information about the various timescales for astrophysical processes that regulate star formation with galaxies (Q2, Ch.2,3,4,5,6), as well as a longitudinal probe of observed scaling relations (Q2, Ch.4). Correlating SFHs with independent observations of morphology allows us to understand the correlations between different morphologies and coherent features in their SFHs (Q3, Ch.3). Studying the variability of galaxy SFHs over different timescales using both observations (Ch.3,5) and cosmological simulations (Ch.6) allows us to better understand the mechanics of quenching and rejuvenation (Q3,4). The future applications described in (Ch.7) include correlating SFHs with their environment (Q3,4), fitting spatially-resolved observations to better understand the link between their spatial and temporal behavior.
(Q2) and extending the current analysis to determine quenching timescales for a large sample of galaxies (Q3). While the investigations in this thesis attempt to lay a foundation to answer these questions, there is still a long way to go before we definitively understand the physical processes that are responsible for the vast diversity in galaxies we see today.
Chapter 2
Introducing the Dense Basis method

This chapter is reproduced from the paper: Reconstruction of Galaxy Star Formation Histories through SED Fitting: The Dense Basis Approach by Kartheik Iyer and Eric Gawiser, published in the Astrophysical Journal.

https://iopscience.iop.org/article/10.3847/1538-4357/aa63f0/meta
https://arxiv.org/abs/1702.04371

2.1 Introduction

The integrated light of a galaxy offers a vast amount of information. When measured with sufficient precision and suitably analysed, the Spectral Energy Distribution (SED) offers insights about a galaxy’s composition from its birth to its time of observation (Acquaviva et al. 2011a; Conroy & Gunn 2010). This can be used to estimate the galaxy’s star formation rate as a function of time, which traces its evolution and merger history (Heavens et al. 2000; Tojeiro et al. 2007; Chevallard & Charlot 2016b; Leja et al. 2016). Combined with other observations, this provides valuable knowledge of cosmic structure formation.

Existing methods of SED fitting use a variety of sophisticated techniques. These include inversion methods (Heavens et al. 2000), bayesian codes for estimating uncertainties and covariances (Acquaviva et al. 2015; Chevallard & Charlot 2016b), machine learning methods with training sets (Leistedt & Hogg 2016), and template-based models (Bolzonella et al. 2000). To search the large parameter spaces of the variables in consideration, Markov Chain Monte Carlo (MCMC) methods have become increasingly popular.

These advances have been necessitated by the increasing detail provided by theory, and the expanding size of galaxy catalogues available through surveys. A large amount of (spectro)photometric data of unprecedented quality will be generated in upcoming surveys, like LSST (Ivezic et al. 2008), HETDEX (Papovich et al. 2016) and J-PAS (Benitez et al. 2014). SDSS (Eisenstein et al. 2011) has already measured spectrophotometry for $\sim 10^6$ objects. The HET-DEX/SHELA field will cover roughly 600,000 objects with multi-band photometry and fiber
spectroscopy. J-PAS will cover 9,000 square degrees with 59 filters (ugriz+54 narrow-band filters across optical) for $\sim 9 \times 10^7$ galaxies. Large regions covered in the NIR with Euclid (Laureijs et al. 2010) and WFIRST (Green et al. 2012) will overlap with LSST, which leads to SEDs for $\sim 10^8$ objects by 2022, many of which will have panchromatic photometry. In keeping with the large amounts of reduced data generated by these collaborations, it is imperative that advanced methods of analysis are developed in order to gain useful information from the integrated light of the galaxies under consideration.

The star formation history (SFH) of a galaxy can sometimes be poorly constrained through different approaches to SED fitting. Typical methods assume a predetermined parametrization like constant star formation or exponentially declining star formation to estimate physical quantities of interest like the stellar mass, star formation rate (SFR), or the time at which the galaxy started forming stars. A few approaches instead seek to reconstruct the SFH from the data, using methods that include reducing the dimensionality of the parameter space using data compression methods (Heavens et al. 2000), fine-binning the interval that makes the maximum contribution to flux (Tojeiro et al. 2007), mapping the discretized-time photometric fitting to a linear inversion problem (Dye 2008), or comparing against a large basis of realistic model SEDs using a Bayesian method (Pacifici et al. 2012). In the current work, we aim to show that using a well-motivated basis allows us to reconstruct robust star formation histories from galaxy SEDs.

The paper is organized as follows: in §2.2, we introduce the Dense Basis formalism of SED Fitting, and how it can be applied to the specific problem of reconstructing SFHs, including the motivation for a particular choice of basis and the fitting procedure with a particular basis set. We describe the training of the atlas using different sources of realistic SFHs: SAMs, Hydrodynamic simulations, and stochastic SFHs in §2.3. In §2.4 we validate the method on both synthetic SEDs from the SAMs as well as real SEDs from the CANDELS GOODS-S field. We then present results in §2.5 including the number of episodes of star formation in the galaxy’s past and constraints on the timing and duration of star formation activity, quantities that were previously inaccessible through SED fitting. In §2.6, we discuss biases introduced by adopting single parametrizations of SFHs, compare with other SFH reconstruction methods, and mention the application of the Dense Basis method to larger datasets.

2.2 The Dense Basis Formalism

The Dense Basis SED fitting method reconstructs Star Formation Histories (SFHs) of individual galaxies using an atlas comprised of SEDs corresponding to well motivated families of SFHs that
effectively cover the space of all physical SFHs\(^1\). It does so by training the atlas on mock catalogs prior to fitting the full dataset. This allows us to use the reconstructed SFHs to perform novel analyses and to tackle problems that were previously intractable with SED fitting, such as estimating the number and duration of star formation episodes in a galaxy’s past. To avoid any bias due to choice of prior, the method is currently implemented in a frequentist manner. In this section, we briefly describe the Dense Basis methodology and training of the basis set. An overview of the process is described in Figure 2.1.

Figure 2.1: Schematic workflow describing the current implementation of the dense basis method to the reconstruction of Star Formation Histories through photometric SED fitting.

### 2.2.1 A well motivated basis of SFHs

The collection of multiple families of well motivated SFHs and their corresponding SEDs with which we fit galaxies; henceforth atlas of SEDs and SFHs, should be designed to utilise the Dense Basis method to its full potential. The choice of appropriate families of functions to best describe the formation of stars in the galaxy in SFH space (SFR vs \( t \)) determines how the SED-fitting procedure encodes realistic star formation. We employ seven major considerations in the choice of basis that should be satisfied for every functional family under consideration:

\(^1\)While an expansion using an infinite number of polynomials or a Fourier decomposition would provide a true basis in the sense of spanning the space of all possible curves, our basis functions only do so approximately; however, since they can reconstruct any star formation history to the level of precision attainable with spectrophotometric data, they provide an effective basis.
- **Physically or empirically motivated:** The functional form of the SFH needs to be realistic, arising either from statistical analysis of star formation in model galaxies, or deduced from observed galaxies. For the latter, as in Gladders et al. (2013), skewed distributions such as linear rise followed by exponential decline and lognormal arise in physical processes restricted to non-negative domains. In this case, SFHs should also satisfy $SFR|_{t=0} \sim 0$ at the Big Bang.

- **Robustness of reconstruction:** The family of basis SFHs should be chosen such that a good fit in an SED space $(|F_\nu, \lambda|)$ should correspond to a good reconstruction in SFH space $(|SFH(t), t|)$. This correspondence can be tested in various ways and could potentially be different for different datasets since the representative form of the SFH could differ across epochs. It is a useful metric for eliminating SFH families that fit SEDs well but yield biased SFH results, such as exponentially declining SFH parametrizations, which describe star formation reasonably well at recent times, but bias quantities such as Age and $t_{50}$, the lookback time at which the galaxy accumulates 50% of its observed mass, (Pacifici et al. 2012). Analogous to isochrone synthesis and matrix inversion methods, this is possible since the SEDs are piecewise linear in their dependence on the SFH and can be decomposed into multiple representations using different functional families.

- **Dense in SFH space:** To avoid degeneracies and biases, (i.e., to better reveal the local minima of the likelihood surface in parameter space), we need to ensure the basis is sufficiently dense in the space of n-parameter curves spanning $SFR(t)$ in the interval $t \in [0, t_{obs}]$.

- **Minimal number of parameters:** The number of parameters used to describe the functional form of the SFH basis functions will determine the amount of data compression possible in reconstructing the spectrum of the galaxy from its best-fit coordinates in parameter space: $SED(M_*, SFR(t), Z(t), A_v, ...)$). For the present application, we model the star formation history as a sum of star formation basis functions, each needing three parameters to describe each reconstructed episode of star formation, the timing of the peak, the timescale, and the stellar mass formed.

- **Temporally consistent:** The families should be chosen such that they produce consistent results for an SFH, independent of when the galaxy is observed, within uncertainties.

- **Positive definite:** Any functional used to describe the SFH should be positive definite,
since $SFR(t) \geq 0$, $t \in [0, t_{\text{obs}}]$, which allows us to extract physical information from multi-component solutions to the reconstructed SFH, as opposed to methods like PCA (Ferreras et al. 2006) or piecewise-linear matrix inversion Dye (2008), which need regularization to yield physical solutions.

- **Robust to noise**: The atlas spans the space of physically motivated SFHs, but not the space of all possible SEDs. This makes it robust to noise in the sense that distortions due to noise that are not accessible through the physically motivated families of SFHs under consideration do not bias the fits, as described in Appendix.A.4.

We describe a few of the 2-parameter families of curves for the current analysis. An overall normalization corresponding to the stellar mass acts as a third parameter. A visual representation of these families is shown in Figure 2.2.

1. **Top-Hat**: Historically, simple stellar populations (SSPs) assumed that a galaxy’s stellar population formed in a single instantaneous burst (Tinsley 1980). An improvement over that was the extension to constant star formation (CSF) from a start time through the time of observation at a fixed rate. Here we use a two-parameter version of this parametrization,
with a start time and a width\(^2\). This is also useful for comparison with quantities in the literature computed using CSF histories, which correspond to setting \(\tau \geq t_{\text{obs}} - t_0\).

\[
SFR(t, t_0, \tau) = \Theta(t - t_0)(1 - \Theta(t - t_0 - \tau)) \tag{2.1}
\]

where \(\Theta(t)\) denotes the Heaviside function with \(\Theta(t) = 1\) for \(t \geq 0\) and \(\Theta(t) = 0\) for \(t < 0\), \(t_0\) is the time at which star formation starts, and \(\tau\) is the width of the Top-Hat.

2. **ESF**: Exponentially declining star formation rates, a parametrization that performs well for local ellipticals and for comparison with older literature with \(t_0\) the time at which star formation starts and \(\tau\) the rate constant of the exponential decline.

\[
SFR(t, t_0, \tau) = \Theta(t - t_0) \exp\left(-\frac{(t - t_0)}{\tau}\right) \tag{2.2}
\]

3. **Linexp**: The delayed exponential (Lee et al. 2010; Gavazzi et al. 2002; Behroozi et al. 2010) with an additional parametrized start-time \(t_0\) (henceforth Linexp) giving the time at which star formation starts and \(\tau\) setting the width of the episode of star formation.

\[
SFR(t, t_0, \tau) = \Theta(t - t_0)((t - t_0)/\tau)e^{-(t-t_0)/\tau} \tag{2.3}
\]

4. **Gaussian**: A parametrization that is useful for describing symmetric episodes of star formation, where \(t_{\text{peak}}\) is the time at which star formation peaks and \(\tau\) is the standard deviation, which sets the width of the episode of star formation.

\[
SFR(t, t_{\text{peak}}, \tau) = \exp\left(-\frac{(t - t_{\text{peak}})^2}{2\tau^2}\right) \tag{2.4}
\]

5. **Lognormal**: (Gladders et al. 2013; Dressler & Abramson 2014). A two-parameter statistical distribution that appears in many physical processes, \(t_0\) is the time at which star formation starts and \(\tau\) sets the width of the episode of star formation.

\[
SFR(t, t_0, \tau) = \Theta(t - t_0)\frac{1}{t} \exp\left(-\frac{(\ln(t - t_0))^2}{2\tau^2}\right) \tag{2.5}
\]

6. **Besselexp**: Bessel-function rise, followed by exponential decline (henceforth Besselexp).

The order of the Bessel function of the first kind, \(\nu\) determines when the SFR peaks\(^3\), and \(\tau\) sets the width of the episode of star formation.

\[
SFR(t, v, \tau) = J\nu\left(t/\tau\right)e^{-t/\tau} + at \tag{2.6}
\]

---

\(^2\)Since this is a positive definite version of the Top-Hat wavelet, this illustrates the possibility of extending our method to a wavelet basis.

\(^3\)Although there is no closed form expression for this, it can be easily determined from a lookup table for the zeros of \(J\nu'(t/\tau)\) and to linear approximation is \(t_{\text{peak}} \sim 1.5(10^8)\nu \text{ Yr}\)
We add a linear piece such that \( a_{\min} = -\min(J_\nu(t/\tau)e^{-t/\tau}) \), to ensure that the set of functions described by this family remains positive definite, while also satisfying \( SFR(t = 0) = 0 \) at the big bang.

These functions offer the advantages of being able to model short episodes of star formation at specific times (small \( t \)) or long periods of star formation where the rate rises and then falls (e.g., Pacifici et al. (2012); Tomczak et al. (2016)). Figure 2.3 shows a typical star formation history drawn from simulations and fits using the six families of SFHs described above. It can be seen that the standard parametrizations of constant star formation and exponentially declining star formation under and overestimate the stellar mass of the galaxy, while the other families show an improved estimation of the general trend of star formation. Additionally, the expansion of the basis to include all physically motivated combinations of single-component SFHs will allow us to describe SFHs with multiple episodes of star formation separated by periods of relative quiescence in a galaxy’s SFH.

Figure 2.3: Reconstruction of SAM mock star formation history using the six SFH parametrizations being considered as candidates for the Dense Basis method. **Left panel:** Blue curve shows the true spectrum at \( z = 1 \). Red datapoints show the noisified SED obtained by multiplying with filter transmission curves and adding photometric noise realized from a quadrature sum of CANDELS photometric and zero-point uncertainties (10% for the U, CTIO, Ks and IRAC ch1,2 bands, and 3% for the remaining photometric bands: f435w, f606w, f775w, f850lp, f105w, f125w, f160w). Colored circles show the best fit SEDs corresponding to each reconstructed star formation history. **Right panel:** Black dashed curve shows the SAM star formation history. Colored curves indicate SFR at a given lookback time at \( z = 1 \) for SFHs from each family that are best fits to the noisified SED. The top-hat parametrization underestimates the stellar mass of the galaxy by \( \sim 60\% \), while the exponentially declining SFH overestimates the stellar mass by \( \sim 46\% \).
2.2.2 The SED fitting problem: reconstruction of SFHs

For a Simple Stellar Population, which assumes that all of its stars form at a single lookback time (T) and with the same metallicity (Z), the luminosity at a given wavelength (\(\lambda\)) is simply

\[
L_{\lambda} = \int_{t_{bb}=0}^{t_{obs}} dt' L_{\lambda}^{SSP}(t_{obs} - t', Z) \delta(T - t_{obs} + t') = L_{\lambda}^{SSP}(t_{obs} - T, Z) \tag{2.7}
\]

where \(t'\) is the time since the big bang, \(t_{obs} - T\) is the age of the galaxy at \(t_{obs}\), and \(L_{\lambda}^{SSP}(t, Z)\) is the spectrum giving the luminosity of an SSP of metallicity Z at age t since formation. The SSP spectrum contains assumptions for the IMF, stellar tracks, and metallicity, which we hold constant in the current study. Some of the effects of relaxing this assumption are noted in §2.4.3, and are discussed further in §2.6.2.

Generalising from Simple Stellar Populations (SSPs) to Composite Stellar Populations (CSPs), we can then represent the SED for a galaxy with a given star formation history (SFH \(\equiv \psi(t)\)) as an integral over all of the star formation events that occurred at different times from the birth of the universe to the time of observation. Composite stellar populations are written as a sum over a non-orthogonal set of star formation histories that satisfy the constraints outlined in §2.2.1, such that

\[
\psi(t) = \sum_k \epsilon_k \psi_k(t, \{\tau, t_0\}) \tag{2.8}
\]

with \(\epsilon_k \geq 0\) denoting an overall normalization corresponding to the stellar mass formed by the SFH \(\psi_k(t)\). Given a basis of SFHs that spans this space, we can expand this instead as a sum over the parameter space, akin to a Fourier expansion, as,

\[
L_{\lambda} = \sum_k \epsilon_k L_k^\lambda(\psi_k(t, \{\tau, t_0\}, Z) \tag{2.9}
\]

where the contribution to the luminosity from an episode of star formation \(\psi_k(t, \{\tau, t_0\})\) described by a family of curves from eq.(1-6) with the parameters \(\{\tau, t_0\}\) is given by,

\[
L_k^\lambda = \int_{t_{bb}=0}^{t_{obs}} dt' L_{\lambda}^{SSP}(t_{obs} - t', Z) \psi_k(t') \tag{2.10}
\]

Dust reddening and nebular emission lines are then applied to the spectrum as described in §2.2.3, denoted by the notation \(L_{\lambda, R}\). The photometry in passband \(j\) from the \(k^{th}\) basis SFH \(\psi_k(t)\) parametrized by \(\{\tau, t_0\}\), is then given by,

\[
F_j^k = \frac{1}{4\pi d_L^2(1 + z)} \sum_k \left( \int d\lambda T_j(\lambda) \epsilon_k L_{\lambda, R}^i(\psi_k(t; \{\tau, t_0\}, Z)) \right) \tag{2.11}
\]

Using this as a mapping from the basis of SFHs to the space of all physically motivated SEDs, we can then define a \(\chi^2\) surface, which denotes the metric distance in the vector space
of photometry between the observed SED and its closest match in the atlas. Finding the reconstructed SFH in the basis is then reduced to an optimization problem on the likelihood surface. For example, with a surface defined using a $\chi^2$ metric, we get

$$\text{min}(\chi^2) = \text{min} \left[ \sum_j \sum_k \left[ \frac{(4\pi d_j^2(1 + z))^{-1} \int \text{d} \lambda T_j(\lambda) \epsilon_k L_{\lambda,R}^j(\psi_k(t; \tau, t_0), Z) - F_{\text{obs},j}^2}{\sigma_j^2} \right]^2 \right]$$

(2.12)

In the following sections, we train the basis set using different mock datasets for which we can quantify both the goodness-of-fit in SED space, given by $\chi^2$ as well as the goodness-of-reconstruction in SFH space, given by $\Gamma$, defined in §2.3. We choose basis functions that show sufficient correspondence between the optima of these two quantities, which lets us reconstruct SFHs in the presence of model degeneracies, systematics and instrumental noise.

### 2.2.3 Generating the Atlas

In order to implement the dense-basis algorithm, it is necessary to first generate an atlas of template spectral energy distributions (SEDs) and then to use it to fit the observed SEDs. This is done as follows:

1. Basis SFHs belonging to the functional families described in §2.1 are generated on a grid of well-chosen discrete parameter values.

2. SEDs corresponding to these star formation histories are then generated using the isochrone synthesis code BC03. (Bruzual & Charlot 2003), using input parameter ranges as described in Table 2.1.

3. **Nebular emission** is added according to the prescription in Orsi et al. (2014) using MAPPINGS III, a one dimensional shock and photoionization code for modelling nebular line and continuum emission. (Allen et al. 2008). We use in this work the precomputed HII region model grid described in (Kewley et al. 2001), with the incident ionization spectra computed using Staburst99 (Leitherer et al. 1999), at $Z_{\text{cold gas}} = 0.2Z_\odot$, from which we compute the ionization parameter using,

$$q(Z) = 2.8 \times 10^7 \left( \frac{Z_{\text{cold}}}{0.012} \right)^{-1.3}$$

(2.13)

This prescription does not add effective degrees of freedom to the atlas and could be expanded to accommodate more realistic emission in future work with higher S/N SEDs.

4. **Calzetti Dust Attenuation** (Calzetti 2001) is applied to atlas SED spectra with discrete values of $A_v$ to extend parameter space in dust for procedures where dusty SEDs are
fit, using

\[ L_{\lambda,R} = L_{\lambda}10^{-0.4k(\lambda)A_V/R_V} \tag{2.14} \]

er where

\[ k(\lambda) = 2.659(-2.156 + \frac{1.509}{\lambda} - \frac{0.198}{\lambda^2} + \frac{0.0011}{\lambda^3}) + R_V \quad \lambda \in [0.12, 0.63] \]

\[ = 2.659(-1.857 + \frac{1.04}{\lambda}) + R_V \quad \lambda \in [0.63, 2.2] \]

with \( R_V = 4.05 \) and the coefficients adjusted for \( \lambda \) in microns. Since attenuation inferred from nebular emission lines differs from that inferred from the continuum (UV spectral slope), we use \( A_{v,stars} = 0.44 A_{v,gas} \) (Calzetti 2001), where \( A_{v,gas} \) is applied to both UV nebular continuum and nebular emission lines.

5. After nebular emission lines are added to the spectrum, and dust attenuation is applied, the photometry for the basis SEDs \( F_j^k \), where \( j \) denotes the photometric bands, or spectroscopic bins, at a redshift \( z \) is given by,

\[ F_j(\lambda) = \frac{1}{4\pi d_L^2(1+z)} \int d\lambda T_j(\lambda)L_{\lambda/(1+z),R}(\psi(t; \{\tau, t_0\}, Z) \tag{2.15} \]

where \( T_j \) is the transmission curve of passband \( j \) (the spectroscopic equivalent would be the resolution element \( \Delta \lambda \) and throughput at that \( \lambda \)), and \( d_L \) is the luminosity distance (a \( d_L \) of 10 parsecs is assumed when \( z = 0 \), as in BC03). For convenience, the flux densities are obtained as the ratio of the number of photons corresponding to the fluxes (\( \lambda F_\lambda \)) to the number of photons produced by a 1\( \mu Jy \) flat spectrum in passband \( j \). This yields the observations, predictions and uncertainties in identical units. The notation \( L_{\lambda,R} \) indicates that nebular emission and dust reddening have been applied to the spectrum.

2.2.4 Choosing the number of basis functions

In practice, galaxies rarely have sufficiently smooth star-formation histories to be perfectly fit by a functional form, as inferred from our mock datasets as well as Hammer et al. (2005); Kelson (2014); Weisz et al. (2011a); Sparre et al. (2015); Diemer et al. (2017). In addition, considering the errors in the photometry, incomplete empirical knowledge of the mapping from SFH to SED spaces, and degeneracies between the SFH and other factors like dust and metallicity, we need to assess methods of reconstruction using multiple basis SFHs to reconstruct as close to the true SFH as possible given the quality of available data. Considering a solution to the minimization problem in Eqn. 2.12, we can express the Best-Fit SED as

\[ F_{obs}^j = \sum_{k=1}^{Nh_{basis}} \epsilon_k F_j^k(\psi(t)) \approx \sum_{k=1}^{N_p} \epsilon_k F_j^k(\psi(t)) \tag{2.16} \]
where $N_F$ is the number of components determined using the F-test, given by,

$$F(\chi^2_{N_1}, \chi^2_{N_2}) = \frac{(\chi^2_{N_1} - \chi^2_{N_2})/(d_2 - d_1)}{\chi^2_{N_2}/d_2} \quad \text{reject if } p(F, d_1, d_2) < 0.5 \quad (2.17)$$

This is used to determine the number of components in the SFH space that the SED should be fit with. The F-test assesses the null hypothesis that the fit with a larger number of parameters is not a statistical improvement over a fit with a smaller number of parameters$^4$, where $d_2 = N_j - 3N_2, d_1 = N_j - 3N_1$ are degrees of freedom corresponding to the number of components $(N_1, N_2)$ being fit with, with $N_j$ denoting the number of photometric bands.

We then reconstruct the SFH according to the optimal components of the likelihood surface for the chosen $N_f$.

### 2.2.5 Estimating Uncertainties

We estimate uncertainties for the reconstructed SFHs via a fully forward modeled frequentist approach using the likelihood surface of the fit, after rescaling the best-fit $\chi^2$ to correct for artificially low $\chi^2$ obtained for very noisy galaxies and artificially high $\chi^2$ values for the brightest galaxies.

A subsurface of the complete likelihood surface is then obtained by imposing a cutoff using a procedure similar to (Avni 1976). We compare the SFH corresponding to each point in the subsurface to the median SFH and exclude outlier SFHs that have an excursion greater than 1.5 times the maximum value, yielding robust confidence intervals as in (Xie & Singh 2013; Zhao & Ma 2016). The uncertainties in SFR at each point in time are then found using a distribution of the remaining acceptable SFHs. Our tests using the sample of 1200 mock SFHs show that this method robustly estimates the confidence bounds, such that for a formal 68% confidence interval, the true SFH lies within the confidence interval $\sim 79\%$ of the time. We show the $\chi^2$ surface computed using this procedure for a single family in Figure 2.4 showing the best-fit SFH and threshold for uncertainties. In Figure 2.5 we show some representative examples of the uncertainties with the top panel demonstrating the method’s ability to constrain an older episode of star formation and the bottom panel showing the case of uncertainties with multiple episodes of star formation.

$^4$To motivate the choice of $p = 0.5$ as our bounding value, it helps to think of the case with an equal number of degrees of freedom $(N_1 = N_2)$, where a better statistical model has $F > 1$, which corresponds to $p > 0.5$. For the general case of $N_1 \neq N_2$, the $p > 0.5$ cutoff provides a metric where statistical improvement is sufficient to justify the extra degrees of freedom.
Figure 2.4: Plot showing the full $\chi^2$ surface for an individual SAM galaxy computed using the single component basis consisting of the Linexp, Besselexp, Gaussian and Lognormal families of SFHs. Each point represents a single SFH; the SFH corresponding to the global minimum $\chi^2$ is the best-fit SFH and SFHs from all families below a threshold are used in computing the uncertainties on the reconstruction. The curves seen within each family denote $\chi^2$ for different values of $\tau$ with adjoining points differing by $\Delta t_0 \sim 0.1\, \text{dex}$.

Figure 2.5: Representative examples of SFH reconstructions with the uncertainties computed using the outlier-clipped likelihood surface, as described in §2.5.4. The examples show that it is possible to reasonably constrain even older episodes of star formation (top row), as well as to obtain robust uncertainties on multiple episodes (bottom row). Spectra are shown without nebular emission for clarity.

2.2.6 Choice of parameter space and photometric bands

For the initial implementation of the method in this work, we have used the Bruzual and Charlot (2003) library of stellar tracks, with the parameter space as described by Table 2.1. The Dense
Basis formalism can be applied equivalently with any set of free parameters, including the set of SSP models, to use the training and validation steps to help constrain the variable parameters, as discussed in §2.6.2. All three datasets are standardized to contain a sample of 400 galaxies with the same realistic distribution of stellar mass.

Table 2.1: Parameter space for vetting using mock SEDs.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Parameter choice</th>
<th>range</th>
</tr>
</thead>
<tbody>
<tr>
<td>IMF</td>
<td>Chabrier/Salpeter</td>
<td>-</td>
</tr>
<tr>
<td>SED generation</td>
<td>BC03</td>
<td>-</td>
</tr>
<tr>
<td>Bands fit</td>
<td>11</td>
<td>-</td>
</tr>
<tr>
<td>Tracks</td>
<td>Padova’94</td>
<td>-</td>
</tr>
<tr>
<td>Metallicity</td>
<td>0.2Z⊙</td>
<td>-</td>
</tr>
<tr>
<td>Dust law</td>
<td>Calzetti</td>
<td>$A_v \in [0.0, 2.5]$, $R_v = 4.05$</td>
</tr>
<tr>
<td>SFH form</td>
<td>Linexp</td>
<td>$\tau \in [0.014, 138]\text{Gyr}$, $t_0 \in [0.02, 5.9]\text{Gyr}$</td>
</tr>
<tr>
<td></td>
<td>Gaussian</td>
<td>$\tau \in [0.014, 4.36]\text{Gyr}$, $t_{peak} \in [0.02, 5.9]\text{Gyr}$</td>
</tr>
<tr>
<td></td>
<td>Besselexp</td>
<td>$\tau \in [1.38, 4.36]\text{Gyr}$, $t_{peak} \in [0.05, 5.66]\text{Gyr}$</td>
</tr>
<tr>
<td></td>
<td>lognormal</td>
<td>$\tau \in [1.38, 4.36]\text{Gyr}$, $t_0 \in [0.05, 5.66]\text{Gyr}$</td>
</tr>
<tr>
<td></td>
<td>exponential</td>
<td>$\tau \in [0.014, 138]\text{Gyr}$, $t_0 \in [0.02, 5.9]\text{Gyr}$</td>
</tr>
<tr>
<td></td>
<td>tophat</td>
<td>$\tau \in [0.014, 13.8]\text{Gyr}$, $t_0 \in [0.02, 5.9]\text{Gyr}$</td>
</tr>
</tbody>
</table>

For training and validation, we consider fitting 11 of the 17 CANDELS GOODS-S (Guo et al. 2013) bands: [u_ctio, HST/ACS F435w, F606w, F775w, F850lp, HST/WFC3 F105w, F125w, F160w, VLT/HAWK-I Ks, and Spitzer/IRAC 3.6,4.5µm], excluding u_vimos, F814w, F098w, and Isaac Ks for the maximum photometric orthogonality, excluding IRAC 5.8 and 8.0µm since the BC03 tracks do not account for the PAH emission that appear in those bands at z=1. Once the method was tested, we expanded to include the u_vimos, F814w, F098w, and Isaac Ks bands as well, leading to fits using 15-band photometry in §2.4.2. The training is performed on the mock datasets described in §2.3, the validation is performed using both the mocks datasets as well as the CANDELS sample for which SpeedyMC results are available. Finally, the results are compiled using the full CANDELS sample at $1 < z < 1.5$.

We present results at $z=1$ in the current work since it allows us to analyse rest-UV information that comes into the UBV bands as well as the Balmer 4000Å break, while avoiding dust re-emission in the mid-IR. This choice of redshift and filter set is compatible with the BC03 SPS models while providing a moderate S/N regime in which to test the reconstruction of SFHs.
The procedure can be generalized to all redshifts and is discussed in §2.6.3.

2.3 Training the SFH families

To inform the choice of a functional form for the SFH basis, we train and validate the method with three mock datasets.

To inform the choice of a functional form for the SFH basis, we train and validate the method with three mock datasets of 400 galaxies each, drawn from Semi-Analytic models, Hydrodynamical simulations, and stochastic realisations of star formation histories. We work with multiple datasets to minimise the effect of any single training set on our choice of SFH families. Using these three mock catalogs, we look at various families of 2-parameter curves, and their combinations, to find the families that perform best at reconstructing SFHs. The atlas generated using that basis is then used to fit the real catalog. Before we go into the details of the training procedure, we first briefly describe the three datasets being used.

2.3.1 Training with SAMs

The first dataset is drawn from mock catalogs with known realistic star formation histories from state-of-the-art Semi-Analytic Models (Somerville et al. 2015). These simulations use dark matter halo ‘merger trees’ extracted from dissipationless N-body simulations in a ΛCDM universe (Klypin et al. 2011) to determine the masses of dark matter halos collapsing at a given epoch, following which halos merge to form larger structures. In this framework, SAMs use analytic recipes to model the radiative cooling of gas, suppression of gas infall, and cooling due to the presence of a photoionizing background, collapse of cold gas to form a rotationally-supported disk, conversion of cold gas into stars, and feedback and chemical enrichment from massive stars and supernovae. A more recent generation of SAMs also includes prescriptions for the growth of supermassive black holes and the impact of the energy they release on galaxies and their surroundings (Croton et al. 2006; Bower et al. 2006; Somerville et al. 2008). Recent comparisons have shown that SAMs produce similar predictions for fundamental galaxy properties to those of numerical hydrodynamic simulations, perhaps because of the common framework of ΛCDM, which dictates gravitationally driven gas accretion rates (Somerville et al. 2015). However, SAMs require orders of magnitude less computing time for a given volume than hydrodynamical cosmological simulations. The resulting galaxy star formation and enrichment histories are outputs. We then use these SFHs to produce SEDs to train and validate against, using a realistic mass distribution of galaxies with \( M_* \geq 10^9 M_\odot \).
2.3.2 Training with hydrodynamic simulations

We also train the method against a set of SFHs obtained from the MUFASA meshless hydrodynamic simulations (Davé et al. 2016a), which satisfies multiple observational constraints like the stellar mass - halo mass relation (Behroozi et al. 2013a; Munshi et al. 2013), the mass metallicity relation (Steidel et al. 2014; Sanders et al. 2015), and the SFR-M$_*$ relation (Speagle et al. 2014; Kurczynski et al. 2016). The star formation histories are reported as instantaneous star formation events that take place, ranging from $\mathcal{O}(10)$ to $\mathcal{O}(10^5)$ events for different galaxies. We restrict the fits to galaxies with $M_* \geq 10^9 M_\odot$, which have well defined SFHs in the simulation. To ensure the SFHs are not artificially stochastic, we generate the SFHs by convolving the instantaneous star formation events using an Epanechnikov kernel with a width of 100Myr (R.Dave, private comm.) The galaxies in MUFASA follow a realistic mass distribution that we use to sample all three mock datasets. We restrict the fits to galaxies with $M_* \geq 10^9 M_\odot$, which have well defined SFHs in the simulation.

2.3.3 Training with stochastic SFHs

Following the prescription of Kelson (2014), we generate stochastic SFHs with different values for the Hurst parameter $H$, which quantified the autocorrelation of the SFH. A value of $H = 0.5$ corresponds to a random walk in $\Delta SFR/\Delta t$, $H > 0.5$ is correlated and $H < 0.5$ is anti-correlated (Mandelbrot & Van Ness 1968). This provides a sample for testing the other families against to determine possible biases. We generate different SFHs by varying the Hurst exponent, which encodes the long-time correlation of the stochastic SFHs in $H \in [0.5, 1]$, sampling galaxies with the same mass distribution as the SAM and hydrodynamic SFH samples. We exclude from the sample SFHs with a Hurst parameter $H < 0.5$ since these do not correspond to realistic looking SFHs.

2.3.4 Training procedure and results:

We quantify the correspondence between the goodness-of-fit in SED space and the goodness-of-reconstruction in SFH space as a metric to judge the success of each family of basis functions.

For the SED goodness-of-fit, we use $\chi^2_{SED}$, given by,

$$
\chi^2_{SED} = \sum_j \frac{\left( \sum_{k=1}^{N_F} \epsilon_k F_j^k - F_j^{obs} \right)^2}{\sigma_j^2}
$$

(2.18)

where the index k sums over the entire basis of SFHs, with a number of components determined using the F-test as described in §2.2.4 and the index j sums over the photometric bands. The
$\epsilon_k$ is optimized for each basis function, effectively making stellar mass the normalization. The global minima of the $\chi^2$ surface corresponds to the maximum likelihood, given by $\mathcal{L} \propto e^{-\chi^2_{SED}/2}$.

To quantitatively compare how well the families perform at reconstructing the SFHs of the galaxies in the three mock catalogs, we quantify the accuracy of reconstruction of the SFH by computing the $R^2$ statistic, given by,

$$R^2(\psi_{true}, \psi_{rec}) = 1 - \frac{\sum_t (\psi_{true}(t) - \psi_{rec}(t))^2}{\sum_t (\psi_{true}(t) - \langle \psi_{true}(t) \rangle)^2}$$

where $R^2$ (Anderson-Sprecher 1994) quantifies the amount of variance explained by the fit. **We set an ambitious goal for the reconstruction by asking if it does as well as direct fits in SFH space to the true SFHs using polynomials of the same order.** Since the true SFHs exhibit a large amount of stochasticity, the question of good $R^2$ due to overfitting does not usually occur and is handled by the F-test in §2.4.1. We define the $R^2$ statistic in logarithmic time; since the SED is sensitive to changes in the SFR over roughly equal logarithmic intervals of time, this provides a more sensitive estimator. To handle all three datasets on the same footing, since they contain SFHs with differing amounts of structure and stochasticity, we apply a small nonparametric smoothing (Cleveland 1979) to the SFHs. This statistic has proved to be the most robust for the current application, matching the qualitative results with other statistics, as detailed in Appendix A.3. $R^2$ ranges from [0, 1], with the most successful reconstruction given by $R^2 \to 1$.

In the noiseless regime, most galaxies show the expected correspondence between the goodness-of-fit and goodness-of-reconstruction, especially in the regime of high likelihood ($\frac{\chi^2_{SED}}{DoF} < 1$). Since we can access only $\chi^2_{SED}$ observationally, this correspondence is important since it allows us to obtain a good reconstruction for a galaxy whose SED is well fit. In order for an SFH family to be robust, we require that the SFHs for the ensemble of galaxies should be reconstructed as well as possible, comparing with direct fits to the true SFH using a polynomial with the same number of degrees of freedom.

In Figure 2.6, we show the $R^2$ computed for each mock dataset using all six SFH families, showing that the Linexp, Besselexp, Gaussian and Lognormal families perform better overall at SFH reconstruction in comparison to the traditional parametrizations of constant and exponentially declining star formation histories. On the basis of this, we we prune our basis SFH set to retain only the Linexp, Besselexp, Gaussian and lognormal families, hereafter denoted ‘Best4 basis’, to be used in further work.

As an additional step of validating our training statistic we examine the correspondence between $\min(\chi^2_{SED})$ and a related statistic, $\min(\chi^2_{SFH})$, computed using the uncertainties
obtained through the method outlined in §2.2.5 in Appendix E. We also study the possible biases that could arise in the reconstruction with a particular family, and how our choice of basis mitigates them.

Figure 2.6: Boxplots showing the accuracy of reconstruction of each SFH family to each mock dataset using the $R^2$ statistic, with the red line denoting the median and the box denoting the interquartile range. For reference, we also show the $R^2$ from direct polynomial fits to the SFH, with its median forming the horizontal dotted line. We see that on average, fits to the SEDs using the Linexp, Besselexp, Gaussian and Lognormal families perform as well as or better than direct fits to the SFHs with 3rd order polynomials.
2.4 Validation

2.4.1 Validation using three datasets: Hydrodynamic Simulations, Semi-Analytic Models and Stochastic SFHs

Having trained our method to arrive at an optimal basis for the dataset in consideration, we now apply the SED Fitting method to the full sample of 1200 SFHs drawn from the hydrodynamic simulation, Semi-Analytic Model and the stochastic realizations.

Realistic simulated photometric noise has been applied to the mock SEDs to simulate observing conditions. The noise consists of a multiplicative factor corresponding to the zero-point uncertainty in each band: 3% for the space-based bands, (HST/WFC3 and HST/ACS) and 10% for the ground-based bands (U ctio, U vimos, Isaac Ks, HawkI Ks) and IRAC Ch.1-4, as well as a photometric additive factor corresponding to the median errors in each band computed from the CANDELS dataset. With these added in quadrature to yield the $\sigma_i$ for each band $i$, simulated fluxes were drawn from a gaussian distribution $\mathcal{N}(F^i_\nu, \sigma^2_i)$.

We show fits using the Best4 basis that is a combination of Linexp, Besselexp, Gaussian and lognormal families, as was determined through the training step in §2.3.1. The galaxy SEDs have been fit with an atlas consisting of two component basis SFHs. This basis is constructed using all physical combinations of elements from the single episode basis and is seen to have a smaller scatter around the true values, as described in §2.6.1. In Figure 2.7, we show the reconstructed SFHs for two randomly selected galaxies from each mock dataset, illustrating the recovery of both recent episodes of star formation, as well as the overall trend of star formation including periods of relative quiescence.
Figure 2.7: The plots show a randomly drawn sample from the semi-analytic models (Top Rows), Stochastic realizations (Middle Rows), and hydrodynamic Simulations (Bottom Rows) used for the training and validation of the Dense Basis method, showing individual examples from the ensemble results shown in Figure 2.8. (Left:) Plots show the true spectrum (black line) from the mock catalogs, their corresponding noisified photometry (red errorbars) and the best fit SED (blue open circles) using the Dense Basis method. (Right:) Plots show the true SFH (black dashed line) and its reconstruction (blue solid line) with 68% confidence intervals (grey shaded region) computed using the method described in §2.2.5. SAM galaxy 30, is identified as a 1 episode galaxy in the current realisation of noise, constituting a false negative result of the F-test. However, many noisy realizations allow us to currently identify the second episode at $t \sim 6 \times 10^8$ Yr. The episodes of star formation for the Hydro. galaxy 2 is distorted either due to noise or dust. The additional peaks in Stochastic galaxy 106 and Hydro. galaxy 183 require a basis with more components.
Figure 2.8: We show the comparison of reconstructed against true values of the stellar mass ($M_*$), SFR$_{100}$ and $t_{50}$ for three datasets: the stochastic realizations (top row), MUFASA hydrodynamic simulations (middle row), and Semi-Analytic Model SFHs (bottom row), using the 2 episode Best4 basis fit using the Dense Basis method. For each dataset, we use a sample of 400 galaxies drawn from a realistic mass distribution. The shading indicates the likelihood of the fit, with darker shades denoting better fits to the SED.

In Figure 2.8, we illustrate the recovery of four physical quantities: the stellar mass ($M_*$), the star formation rate averaged over the last 100Myr ($SFR_{100}$), the lookback time over which the galaxy accreted 50% of its observed stellar mass($t_{50}$), and dust extinction ($A_v$). The top row depicts the results for the stochastic realizations, the middle row for the hydrodynamic simulations, and the bottom row for the Semi-Analytic Models. All the fits show the bias to now be significantly smaller than the scatter that may occur from using a single family of SFHs, as discussed in §2.6.1. The scatter in stellar mass increases at lower mass, corresponding to more noisy SEDs, while the increased scatter in SFR as compared to the values in §2.6.1 is more due to the presence of dust than noise since fits without dust show a much smaller scatter of $\sim 0.05$ dex. The 0.3 dex scatter in the reconstruction of $t_{50}$ is reasonable. However, the distributions for the SAMs and the hydrodynamic simulations look poor due to the narrow range of true values for these models with the top row being more representative of the method’s
performance with a broader distribution of $t_{50}$. Reconstruction of simulated dust drawn from an exponential distribution is done using an atlas containing 25 values of dust ranging from $A_v = 0$ to $A_v = 2.5$ using the Calzetti dust law, with reasonable scatter of $0.09 - 0.13\text{dex}$ and negligible bias of $\sim 0.01\text{dex}$.

We find that our choice of basis yields comparable good results to all three mock datasets, with the bias in the estimation of these physical quantities derived from the SFH not exceeding $0.05\text{dex}$, as seen in Figure 2.8. This is an important criterion to be met before the method is applied to observational data, since it lets us relax the assumption that the SFHs corresponding to the training SEDs match the actual star formation histories of galaxies at a given epoch, in favor of the slightly weaker assumption that the SFHs are drawn from a similar distribution. The ensemble results show that the reconstruction is nearly unbiased for the physical parameters of interest and can be used to extract a variety of derived quantities from the SED of distant galaxies in a robust manner.

We also present results in Figure 2.9 for the fraction of a sample of galaxies that are reconstructed with a second episode of star formation. For our three datasets, we perform the F-test using Eq.18, with $N_1 = 1, N_2 = 2$, and obtain the fraction of galaxies that are significantly better fit with a second component of star formation. In some cases, the second component has similar peak time and serves only to modify the SFH shape, eg. Gaussian + lognormal, with a single peak; we term these single episode SFHs. We then find the fraction of the mock galaxies that have a distinct second peak to their reconstructed SFH, which can only happen when the reconstruction prefers a second component. We compare this number to the number of galaxies in the sample whose true SFH has two episodes of star formation, computed using a peak finding routine. Since the true SFHs show a large amount of stochasticity, only the most prominent peaks with a separation greater than 100Myr are selected by smoothing over the local variations as in §2.3.1 and finding the lookback times at which the SFH peaks, using $SFR'(t) = 0$ and $SFR''(t) < 0$. The results are summarized as boxplots in Figure 2.9, for two mass bins chosen such that roughly half the sample lies in each mass bin. For the high mass bin, the higher S/N leads to more accurate predictions of the fraction of SEDs with more than one episode of star formation. Since our atlas is restricted to SEDs corresponding to physically motivated SFHs, we generally do not overfit the noise, as is seen by the small number of false positive results\textsuperscript{5}. However, a large amount of noise makes it more difficult for the F-test to detect a statistically significant improvement to the fit. This leads to a systematic underestimation of the fraction

\textsuperscript{5}excluding the SAM (realZ) results, which we discuss further in §2.4.3
of galaxies with more than a single episode of star formation, as shown in our results for the lower-mass galaxies. The decreased S/N also results in the fraction of fits with false negatives being higher in this mass bin.

Figure 2.9: We find the number of galaxies that show a statistical improvement upon being fit with a second component of star formation using the Best4 basis from §2.3.1, determine which of those correspond to a second episode of star formation, and present results for noisy realizations of four datasets of galaxies from the hydrodynamic simulations, stochastic realizations, Semi-Analytic Model, and the realistic metallicity generalization of the Semi-Analytic Model described in §2.4.3. The blue values are obtained directly from the true SFHs using a smoothing algorithm to account for stochasticity and then running a peak finder algorithm, while the black crosses quantify the number of false positive predictions using the F-test. The results are divided into two mass bins, showing that the method is reliable in predicting the number of episodes at the high mass end, due to sufficient S/N.

2.4.2 Validation against SpeedyMC results for CANDELS SEDs

We now apply the method to a sample of 1100 CANDELS galaxies in the GOODS-S field at $1 < z < 1.5$ from Kurczynski et al. (2016), for which we have physical quantities derived using SpeedyMC\(^7\) (Acquaviva et al. 2011a, 2015) for 742 galaxies. This sample provides a good representative redshift to test the method for the recovery of SFHs at moderate S/N, as discussed

\[ \text{false positives} \sim \text{true value} - \text{false negatives} \]

\[^7\] which can be expressed as $f_{fn} \sim f_{true} - f_{rec} - f_{fp}$, where $f_{fn}, f_{fp}$ are the fractions of false negatives and positives, and $f_{true}, f_{rec}$ are the true and reconstructed fractions of galaxies with multiple episodes of star formation.

towards the end of §2.2.6. We perform the fitting with discrete values for \( z \) and \( A_v \), which adds some scatter to the results. For redshift, we choose bin edges at \([1.0, 1.1, 1.2, 1.3, 1.4, 1.5]\), and we let \( A_v \) vary from 0 to 2.5 in increments of 0.1. The purpose of this comparison is to ensure that our SED fitting code developed to implement the dense basis method, which was also used to generate mock SEDs, does not contain circular errors. Additionally, it is a useful test to match the physical quantities that can be recovered through traditional SED fitting before presenting previously inaccessible quantities.

In order to make the comparison as consistent as possible, we match the initial conditions of the fitting procedure to the SpeedyMC parameter space, as summarized in Table 2.2, and limit our SFH basis to single-component Linexp curves. In Figure 2.10, we show the results comparing our fits to the SpeedyMC results for the stellar mass \( (M_\star) \), SFR\(_{100}\) and \( t_{50} \). The slight bias in \( t_{50} \) could be due to the difference in the way the two codes implement nebular emission. The colorbars denote the spectroscopic redshifts corresponding to the observed galaxies.

Figure 2.10: Comparison of physical quantities derived through the Dense Basis fits against the results of the SpeedyMC Markov Chain Monte Carlo code applied to the CANDELS dataset at \( 1 < z < 1.5 \) from Kurczynski et al. (2016), comparing the stellar mass \( (M_\star) \), star formation rates, and the lookback time at which the galaxies accumulated 50% of their observed mass \( (t_{50}) \). The color table indicates the redshifts of the galaxies being fit.

2.4.3 Validation against SAM SEDs with multiple metallicities:

We address a final possible source of systematic bias in the fits: the assumption of a single metallicity \((0.2Z_\odot)\) in building the atlas and performing the fits at \( z \simeq 1 \). To take into account the distribution of metallicities found in real galaxies, we go back to the SAM SFHs and consider the individual metallicity components of the overall SFHs. We generate spectra corresponding to each of these metallicities using six values of metallicity available for the Padova’94 tracks in BC03, given by \( Z = [0.0001, 0.0004, 0.004, 0.008, 0.02, 0.05] \). Using this procedure, we obtain spectra corresponding to the SFH in each metallicity bin and use a weighted sum to obtain
Table 2.2: Comparison of Dense Basis and SpeedyMC parameter spaces used in §2.4.2

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</table>

SEDS corresponding to galaxies with realistic metallicity histories, which we denote as SAM (realZ). We then fit these SEDs with our single-metallicity basis to test how robust our fits are at $z = 1$.

Figure 2.11: Left: Mass-weighted metallicity distribution at present epoch, binned using the BC03 Z range. Solid bars denote the distribution for the sampling in mass used in §2.4.1, while the dashed lines denote the distribution for a uniform sampling in Stellar Mass Right: SED Fits to the ensemble of SAM galaxies taking the range of realistic metallicity values into account. The $t_{50}$ reconstruction has a reasonable scatter of 0.28 dex that looks poor due to the small distribution of true values. The fits are further complicated by dust, but still comparable to the ones with a single metallicity value.

In the left panel of Figure 2.11, we show the distributions of observed metallicities at $z \simeq 1$ in the SAMs, weighted using the dominant contribution to the total mass of the galaxy for realistic sampling in Stellar Mass ($M_*$) used in §2.4.1. Upon examining the SEDs corresponding to a sample of galaxies of different ages generated by combining the spectra corresponding to the star formation histories in each of BC03’s six metallicity bins, we find that older, more massive galaxies are more metal-rich at the observed epoch and thus show a greater deviation from the
template SEDs, which currently assume $Z = 0.2Z_\odot$ for the entire SFH.

In the four panels to the right of Figure 2.11 we show the reconstruction of physical quantities ($M_\star, SFR_{100}, t_{50}$ and $A_v$) for the cumulative SEDs with realistic metallicities. The increased bias in the $t_{50}$ appears to be the result of poorly fitting older galaxies, which have much higher metallicities than those in the atlas. For $t_{50}^{\text{true}} \sim 3\text{Gyr}$ and older, galaxies tend to have $Z > 0.4Z_\odot$ at the time of observation. This effect in addition to the narrow distribution of true $t_{50}$ causes the scatter in $t_{50}$ to appear poor even though it is comparable to the fits in §2.4.1. The recovered SFHs themselves are still representative of the true SFH of the galaxy up to a lookback time of $\sim 3\text{Gyr}$, after which the degeneracies in the $\chi^2$ surface due to the contributions from older stars, dust and differing metallicities impose larger uncertainties on the reconstruction by a factor of $\sim 1.22$.

In Figure 2.9, we now focus on the results in the last columns in each mass bin. The results agree well in the high mass bin, due to roughly equal numbers of false negatives and positives. The net results in both mass bins are still acceptable, as a result of which the method is still valid even in its current simple realisation with a single-metallicity.

2.5 Results

The Dense basis method of SEDfitting allows us to reconstruct the star formation histories of galaxies in a nonparametric fashion, not being restricted to the choice of a particular number or family of basis SFHs, while being able to compress the reconstructed SFHs using a small number of parameters to describe a best fit. We show the results of applying this method to our sample of CANDELS galaxies at $1 < z < 1.5$ and mock SAM galaxies at $z \sim 1$.

2.5.1 Going beyond 'age' and instantaneous SFR

The ‘Age’ of a galaxy, defined as the lookback time at which the galaxy first started forming stars ($\equiv t_0$), is not as meaningful with realistic SFHs as it used to be with simple stellar populations, which formed all their stars at a single lookback time, given by the Age (Tinsley 1980; Bruzual & Charlot 2003). Realistic SFHs as seen in the SAM and the hydrodynamic simulations may maintain a small amount of star formation before ramping up to a major episode of star formation, which results in the true Age for most galaxies approaching the age of the universe. Since the SED of a galaxy is most sensitive to its largest episodes of star formation, with its sensitivity decreasing as we go back in lookback time, the 'Age' recovered through SED fitting methods is not a robust physical quantity. However, if we were to estimate
the lookback time at which the galaxy accumulated the first 10% of its observed stellar mass, we estimate the lookback time at which any major star formation activity in the galaxy started. While the distributions of the Age and $t_{10}$ are similar for a given sample of galaxies, the latter is a more meaningful quantity in terms of studying galaxy growth and evolution and is more robustly estimated through SED fitting. (Pacifici et al. 2015, 2016) This can be seen from the top panels of Figure 2.12, with the right panel showing noiseless reconstructions of the Age, and the left panel showing noiseless reconstructions of $t_{10}$ for all three samples of galaxies used in §2.4.1 using the same basis set and format for the plots. The latter quantity is more robust, as can be seen from the reduced bias and scatter in the estimation of $t_{10}$.

In a similar manner, due to the large amount of stochasticity that realistic SFHs show, it is more robust to estimate the Star Formation Rate (SFR) averaged over the last 100Myr in lookback time, rather than the instantaneous SFR, as shown in the bottom panels of Figure 2.12. The panel on the left denotes $SFR_{100}$, which has less scatter than $SFR_{inst}$, shown on the right. It is widely appreciated that broad-band SED fitting is primarily sensitive to SFR averaged over the past 100Myr\[^8\] (Conroy 2013; Johnson et al. 2013), but SED fitting traditionally reports $SFR_{inst}$ in its chosen parametrization nonetheless. With rapid rises and exponential declines possible, these quantities can differ significantly, leading to the extra scatter in the bottom right panel of Figure 2.12.

2.5.2 The number of episodes of star formation experienced by $1 < z < 1.5$ CANDELS galaxies

It is an important feature of the dense-basis method to be able to recover the number of strong episodes of star formation in a galaxy. Doing so allows us to detect recent bursts of star formation, or a period of relative quiescence between episodes of continuous star formation, with the amount of data that can be extracted depending upon the S/N. This can then be used to infer valuable information about the galaxy’s evolution and merger history.

\[^8\text{However, when nebular emission lines are strong enough to contribute significantly to the broad-band photometry, SED fitting can probe } \sim 10 \text{ Myr timescales.}\]
Figure 2.12: (Top:) Plots showing the ability to extract $t_{10}$, the lookback time at which the galaxy has accumulated 10% of its observed mass, is much more reliably estimated than the ‘Age ($\equiv t_0$)’ of that galaxy with both reduced bias: -0.13dex for $t_{10}$ vs -0.19dex for Age, and reduced scatter: 0.24 dex for $t_{10}$ vs 0.31 dex for Age. (Bottom:) An illustration of a similar robust measure with $SFR_{100}$ showing less scatter than $SFR_{\text{inst}}$. For the $SFR_{100}$, the bias is -0.03dex and the scatter is 0.11dex. For the $SFR_{\text{inst}}$, the bias is -0.04dex and the scatter is 0.37dex. The three different colors show fits to the three different mock datasets, with blue for galaxies from the SAM, yellow for the stochastic galaxies, and red for galaxies from the hydro. simulations, using the same notation as Figure 2.8.

In this paper, we have demonstrated the use of an F-test to detect if the addition of a second component of star formation is a statistically significant improvement to the fit. This is then used to infer the fraction of galaxies whose SFHs contain a second major episode of star formation, and was validated for the mock galaxies in the high S/N regime in §2.4.1. For our current sample of 1100 CANDELS GOODS-S galaxies, we can reliably fit 790 galaxies, with the remaining galaxies either having poor $\chi^2$ or with missing fluxes in multiple filters, preventing robust estimation of the SFH and its uncertainties. The F-test then determines that 134 galaxies out of the sample of 790 galaxies show a statistical improvement upon being fit
Figure 2.13: Plots showing a randomly drawn sample from the 790 CANDELS galaxies at $1 < z < 1.5$ used for the validation and results sections of this paper. 

(Left:) Plots show photometry from CANDELS at $1 < z < 1.5$ (red errorbars) and the best fit SED (blue open circles) using the Dense Basis method. 

(Right:) Plots show the single Linexp SFH fit with SpeedyMC in (Kurczynski et al. 2016) (green line) and the Dense Basis reconstruction (blue solid line) with 68% confidence intervals (grey shaded region) computed using the method described in §2.2.5. The $\chi^2$ of the fit for each of the galaxies is 12.7, 7.0, 30.8, 8.7, 40.7 and 41.5, for fits with 15 of the 17 CANDELS bands, excluding IRAC Ch.3,4 since we have not modeled for PAH emission in our atlas. The spectroscopic redshifts of the various galaxies are 1.0910, 1.1300, 1.2510, 1.3810, 1.0760 and 1.2210 respectively.

with a second component, of which 117 galaxies contain a second episode of star formation. This corresponds to roughly 15% of the sample, similar to the results for the mocks. Figure
2.13 shows six examples of the procedure, showing three galaxies that were fit by a single basis SFH and three with two components.

Additionally, we provide a breakdown of the fraction of galaxies in each mass bin from $[10^8, 10^{10}]M_\odot$ shown in Figure 2.14. This figure reveals a decrease in the fraction of galaxies that are fit with two major episodes of star formation as the stellar mass increases above $10^{9.5}M_\odot$. As seen in §2.4.1, we expect to underestimate the fraction of 2-episode galaxies at lower masses in the CANDELS sample. Hence the increased number of 2-episode galaxies at $M_* < 10^{9.5}M_\odot$ is a robust indication that 2-episode galaxies are more common at lower mass. This discrepancy between the data and simulations is intriguing.

![Figure 2.14: We find the number of galaxies that show a statistical improvement upon being fit with a second component of star formation using the Best4 basis from §2.3.1, and then determine which of those correspond to a second episode of star formation. For the sample of 790 CANDELS GOODS-S galaxies, 15% of the galaxies are fit with multiple episodes of star formation, with the histogram showing the distribution of the fraction of galaxies that are fit with a second episode of star formation across different ranges in stellar mass. The Poisson error bars denote the possible uncertainties due to limited sample size.](image)

2.5.3 Constraints on timing and duration of episodes

Using the reconstructed SFHs for our CANDELS sample, we can obtain constraints on the timing and duration of episodes of star formation. This is possible since the reconstructed SFH using our well motivated basis SFH set captures the general trend of star formation, even if the
Figure 2.15: **Left:** Histogram of the lookback time at which the reconstructed SFH for the sample of CANDELS galaxies peaks. The blue histogram denotes the peak times for the galaxies with a single episode of star formation while the smaller yellow histogram shows the peak times whose reconstructed SFH contains two episodes of star formation. A significant fraction of the galaxies have SFHs that are still rising at the epoch of observation, and represent $\sim 90\%$ of the first bin in the histogram, shown in red. **Middle:** Histogram of the widths of star formation episodes corresponding to the same sample obtained at the FWHM of the reconstructed SFH, with the red histogram representing the portion of the reconstructed SFHs that are still rising, with their widths truncated at the time of observation. **Right:** Histogram of the separation between the two peaks for the SFHs with two episodes of star formation.

Finer stochastic details are lost. For each fit, we obtain the number of episodes of star formation, the lookback time of peak star formation, and the FWHM of that episode, thus obtaining the timescale of star-formation episodes both on a galaxy-by-galaxy as well as an ensemble basis, as shown in Figure 2.15. For $\sim 40\%$ of the galaxies, we find that the SFH is still rising at the time of observation, comparable to $\sim 30\%$ for galaxies from the SAM and Hydrodynamic Simulation. In estimating the width of an episode of star formation, we estimate the width of an episode up to the time of observation, leading to truncated widths for the subsample of galaxies whose SFHs are still rising, shown in red in the histogram. We find that the widths for our sample are smaller by a factor of $\sim 10$ than those for the mock galaxies. This discrepancy bears further investigation, with a similar difference seen in Diemer et al. (2017). Additionally, we can also find the interval between episodes of star formation for the galaxies that are reconstructed with two episodes of star formation.

### 2.5.4 Statistics of $t_{10}, t_{50}, t_{90}$ and uncertainties

The reconstructed SFHs for the CANDELS galaxies computed using the Dense Basis method are used to compute the lookback times at which the galaxy accumulates a certain fraction of
its observed mass. These quantities, defined $t_x$, satisfy the equality,

$$\int_0^{t_x} \psi_k(t') dt' = \frac{x}{100} \int_0^{t_{obs}} \psi_k(t') dt' \implies M_*(t_x) = \frac{x}{100} M_*(t_{obs}) \quad (2.20)$$

Generalising $t_{10}$ from §2.5.1, this lets us follow the mass assembly using the lookback times at which the ensemble of galaxies accumulated a certain fraction of its observed mass. We do this for the CANDELS sample at $1 < z < 1.5$ in Figure 2.16, providing histograms showing the overall lookback times at which the individual galaxies accumulated 10%, 50% and 90% of their observed stellar mass. This allows us to infer the overall growth and evolution of galaxies at that epoch.

![Figure 2.16: Distributions of the timescales at which the galaxies in the CANDELS sample assembled 90%, 50% and 10% of their observed mass, showing the fraction of the sample vs lookback time. The purple histogram shows $t_{90}$, the yellow histogram shows $t_{50}$, and the blue histogram shows $t_{10}$.](image)

2.6 Discussion

2.6.1 Biases from using single SFH parametrizations

The flexibility in the choice of SFH family used for SED fitting makes it possible to quantify the bias introduced in the estimation of physical quantities due to the choice of SFH parametrization used. We briefly list these biases at $z \sim 1$ for the six families of SFHs presented in this work, highlighting the particular families that perform best at the estimation of a particular quantity.

For the seven physical quantities $Q_i$ listed in Tables 2.3 and 2.4 below, we formulate the bias and scatter as the median and standard deviation of the histogram $b(Q) = \{1 - Q_i^{rec}/Q_i^{true}\}$, which gives the scatter after taking the bias into account. This is done using physical quantities computed using the reconstructions of the SFHs of the 1200 mock galaxies from §2.4.1 with a realistic mass distribution, using fits without dust or noise to highlight the bias due to the SFH parametrization. We have included the CSF, Top-Hat and Exponential biases in Table 2.3.
below in an effort to standardize quantities in comparison to older literature, while also listing
the reduced bias and scatter with the full Dense Basis method with up to two components of
basis SFHs from the Linexp, Besselexp, Gaussian and lognormal families. In order to ensure a
fair comparison, all families contain the same number of basis SFHs and are dense enough to
converge, i.e., a denser grid on the parameter space does not change the results significantly.

Table 2.3: Bias in the estimation of physical quantities due to different SFH parametrizations
at z \sim 1. [No dust or noise]

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<th>$M_\star$</th>
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<th>SFR$_{inst}$</th>
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Table 2.4: Scatter in the estimation of physical quantities due to different SFH parametrizations
at z \sim 1. [No dust or noise]

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<td>Linexp</td>
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<td>Dense Basis</td>
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Almost all the families tend to underestimate the stellar mass. However, the scatter in $M_\star$
is generally larger than this bias except for the Top-Hat family. The scatter is even larger
for SFR$_{100}$, and thus the bias doesn’t significantly affect the results except at the low SFR ($SFR_{100} < 10^{-1} M_{\odot} \text{yr}^{-1}$) end, as seen from Figure 2.8. Age has the greatest bias of all the estimated quantities, and it can be seen that it decreases when we estimate $t_{10}$, which is a more robust quantity, as we proposed in §2.5.1. About 40% of the mock galaxies form $< 10\%$ of their mass at $t_{\text{lookback}} > 3 Gyr$. The small contribution to the observed flux from these older stars is dominated by more recent contributions, as a result of which the method does detect these older stars and underestimates the age. Since most of the mock galaxies start forming stars at $t \sim t_{bb}$, the distribution of true ages is extremely narrow and can only be underestimated, since the method does not allow $t_{age} > t_{bb}$. An artifact of this bias is also seen in $t_{10}$, although it is smaller. However, since the Dense Basis method recovers the major episodes of star formation and the bias is largely due to the distribution of the true ages, the reconstructed SFHs are robust. In a similar vein, the bias decreases in considering $t_{50}$ and even further with $t_{90}$. Age has a lower scatter than $t_{10}$, since most galaxies start forming stars at $t_0 \sim t_{bb}$, and this creates a narrow distribution for the true Ages. For SED fitting methods that use Age, setting $Age = t_{bb}$ would lead to a bias of $-23\%$ and a scatter of $32\%$, fully competitive with any of the single families. The scatter in $t_{10}$ for the Dense Basis method is also in part due to expanding to a larger parameter space, which yields a smaller bias at the expense of increased scatter regulated by the F-test. The Top-Hat, exponential and Linexp parametrizations have a large bias in age, and should be accounted for in comparisons of ages in the literature. $t_{90}$ is the most robust of the mass-assembly times, with the Linexp and lognormal families performing best in its estimation. The Dense Basis method offers the least scatter in $M_*$, SFR, $t_{90}$ and is nearly unbiased in these quantities, as well as $t_{50}$.

### 2.6.2 Comparison with other methods

The Dense Basis method offers an SED fitting approach that minimizes the bias and scatter introduced due to traditional SFH parametrizations. In this section, we consider comparisons with existing methods of SFH reconstruction. MOPED (Heavens et al. 2000) fixes logarithmic time bins and finds the SFR in each bin with a piecewise constant SFH using fitting with data compression, giving more weight to those pixels in the spectrum that carry most information about a given parameter. VESPA (Tojeiro et al. 2007) adaptively bins the lookback time, i.e., the $t_i$ in Eq.11, provided there are enough free parameters to avoid overfitting. Dye’s (2008) method adopts a similar approach with photometry, but uses regularization in order to the make the SFR in each bin positive, which might bias the likelihood surface and is computationally more expensive. None of these methods reconstructs smooth SFHs; the fits do not provide us
with SFHs that allow us to analyse the number of episodes of star formation or to analyse the peak times and widths of star formation episodes. The method introduced here uses a physically motivated functional form of SFHs that requires a smaller number of free parameters to fit the SFH, thus obtaining smooth SFHs with multiple components through photometric SED fitting, comparable to what was previously accessible with spectroscopy or CMD reconstruction (Weisz et al. 2011a). Another advantage is the ability to use real SEDs to test functional forms for a best match against star-formation mechanisms at a given redshift. The usage of well-motivated parametrized functional forms instead of time bins with variable heights allows us to obtain a smooth reconstruction of the SFH with a smaller number of parameters without the need for regularization, since the basis SFHs are smooth and positive definite.

2.6.3 Possible extensions of the Dense Basis method

In addition to the two parameter families described in §2.2.2, it is possible to extend the approach to a larger parameter space by using families of curves including the 4-parameter families described in Simha et al. (2014) and the Exponential+ power law (Behroozi et al. 2013a),

$$f(t, t_0, \tau, \alpha) = \Theta(t - t_0)(t - t_0)/\tau)\alpha e^{-(t-t_0)/\tau}$$

(2.21)

where $\tau, \alpha \in \mathbb{R}_+$, and $t_0$ indexes the time at which star formation begins.

Currently, however, we restrict our attention to the two parameter families since we also consider combinations of curves from these families, which let us model a much more versatile set of trajectories in SFH space.

An advantage of our method is that it will recover only as many SFH basis components as are needed to produce a good fit to the SED, thus enabling us to extend the procedure to reconstructing metallicity histories, and to use multiple dust extinction models. It is also possible to extend the code to additional SPS models, which is naturally incorporated with the Conroy FSPS models (Conroy & Gunn 2010) that contain the BaSTI and Padova isochrone sets. Model dependency due to the choice of tracks and IMF is also an issue that could be incorporated into future versions, which will have more data available that can be used to address degeneracies between different sets of isochrone synthesis models, stellar evolution tracks, and IMF choices.

Additionally, the superposition of ‘stochastic’ bursts on top of these smooth functional forms has been better shown to reproduce the observed spectroscopic properties of individual galaxies (Kauffmann et al. 2003; Brinchmann et al. 2004). This can be explored in future applications of the dense basis method to spectroscopic data, using realistic stochastic SFHs as in the approach of (Pacifici et al. 2015), or the theoretical stochastic SFHs from (Kelson 2014).
The current formulation is frequentist, and the training and validation produce parameter uncertainty estimates consistent with this approach. A Bayesian formulation of the method is certainly possible, but since the priors on SFHs are poorly known, significant care would be required.

2.6.4 Handling Big Data

A large amount of data will be generated from the upcoming generation of surveys including LSST (Ivezic et al. 2008), HETDEX/SHELA (Papovich et al. 2016) and J-PAS (Benitez et al. 2014), which will yield a mixture of broad-band photometry and spectroscopy for \( N \sim O(10^8) \) galaxies.

Methods for analysing these galaxies using SED Fitting techniques need to be both computationally efficient as well as capable of handling and storing large volumes of data in a memory-efficient manner.

The dense basis method was designed with these two requirements in mind. It takes \( O(0.1)s \) for a single run on a 2.9 GHz laptop, albeit with large memory requirements for storing the 2-component basis, which runs to \( O(200Gb) \) with 18 values of \( \tau \) and 99 values of \( t_0 \). After the initial generation of the atlas, the fits themselves can be stored simply by saving the index of the best-fit SED and the normalization for each component, leading to efficient storage of the fits as \((N_{\text{component}} \times 3)\) coefficients for each reconstructed SFH.

2.6.5 Broader Data science applications

This method can be used to solve problems of the general type

\[
d = \sum_i \int \mathbf{m}_i(t)dt = \sum_i \sum_j a_j \mathbf{m}_{ij}
\]

where \( d \) represents a vector of observables, i.e., galaxy SEDs in the current work, and the functionals \( m_i \) represent possible SFHs. The index \( i \) sums over basis functions and \( j \) refers to multiple photometric bands. We adopt functionals that can be shifted by varying \( t_0 \) and scaled by varying \( \tau \) because this is reasonable for the underlying physics of star formation. This is not a requirement for solving Eq (2.22) and additional constraints upon the functionals will depend upon the problem being considered. Upon generalization, this formulation is particularly useful for the class of problems where constrained observed data is used to recover quantities in an otherwise inaccessible parameter space, such as single-epoch observations of historical processes. In the absence of a known analytic mapping from the parameter space \( \{m_i(t)\} \) to the space
of observables \( \{d_j\} \) and the lack of a definite correlation between the goodness of estimation in these two spaces, traditional methods like Monte-Carlo estimation through the parameter space need not lead to accurate estimation of the \( m_i \), since a good fit need not correspond to an accurate reconstruction of the functional. Methods like Principal Component analysis may be used in the parameter space, but the principal components do not always correspond to physical representations of the observables. Such situations can frequently arise due to the presence of noise and degeneracies between different parameters that affect the observables.

In such cases, it is possible to apply the training method described in the current work, based on pruning a training atlas from a large space of informed estimates from empirical observations and statistical motivations, leading to an oversampled nonorthogonal ‘Dense Basis’. This lets us perform any subsequent fitting to the data in a subset of parameter space where the correspondence between the goodness-of-fit and goodness-of-reconstruction exists and is well defined. Since the functionals in the parameter space are well motivated, they do not span the space of all observables and are robust to noise that would correspond to ‘unphysical’ results. In the current framing, the method is readily applicable to timeseries problems, where the observables are integrated quantities depending on the overall nature of the timeseries.

In an expanding arsenal of data-science tools, the Dense Basis method provides a convenient formalism to solve the above class of problems in a tractable manner, and to train and implement a solution finding method. The advantages of using this method include not having the constraints of regularization imposed by matrix inversion methods or suffering from the lack of correlation between observables and principal vectors in solution space that techniques like PCA exhibit, while also being robust to noise.

### 2.7 Conclusions

The standard assumption of a simple parametric form for galaxy Star Formation Histories (SFHs) during Spectral Energy Distribution (SED) fitting biases estimations of physical quantities and underestimates their true uncertainties. In this paper, we introduce the Dense Basis method, which offers a general approach when a vector of observed data points \( d \) can be modelled as a sum of positive-definite, continuous functionals \( m_i \) obeying \( d = \sum_i \int_t m_i(t) dt \). Here we apply it to the case where \( d \) represents a galaxy SED and the functionals are possible SFHs.

We train the method using SFHs from mock catalogs at \( z \sim 1 \) from three different sources: a Semi Analytic Model (SAM), meshless hydrodynamic simulations, and stochastic realizations. We do this to ensure that the method can successfully reconstruct a wide variety of SFHs.
allowing us to relax the assumption that our training SFHs are perfectly representative of the true SFHs of galaxies at that epoch. The training step allows us to compare the goodness-of-fit in SED space to the goodness-of-reconstruction in SFH space. We use this comparison to eliminate SFH families that provide poor or biased reconstructions, leading us to drop the Top-Hat and Exponential families from our basis, while keeping the Linexp, Besselexp, Gaussian and Lognormal families.

A basis consisting of these four families and their combinations is then used to apply the Dense Basis method to the broad-band CANDELS photometry of a sample of galaxies at $1 < z < 1.5$. The method allows us to accurately estimate physical quantities of interest that explicitly depend on the SFH, notably Stellar Mass ($M_\ast$), and SFR$_{100}$, which we note is more robust than SFR$_{\text{inst}}$ and dust attenuation. The method also allows us to estimate previously inaccessible quantities, including the number and duration of star formation episodes in a galaxy’s past, and the lookback times at which the galaxy accumulates 10, 50, 90% of its observed mass, which are more robust quantities than the Age of a galaxy, and allow us to track the galaxy’s growth and evolution as a function of lookback time. The current frequentist implementation of the method allows us to estimate confidence intervals for these quantities. We quantify the bias and scatter in these quantities due to various SFH parametrizations including the traditional parametrizations of constant and exponentially declining SFHs.

The method can be expected to have broad data science applications, and can be scaled and applied to high S/N spectrophotometry from upcoming surveys across all redshift ranges to reconstruct the SFHs of individual galaxies, as well as to infer the growth and evolution of the ensemble of galaxies at various epochs.
Chapter 3

Full independence from parametric forms: SFH reconstruction with Gaussian Processes

This chapter is reproduced from the paper: Non-parametric Star Formation History Reconstruction with Gaussian Processes I: Counting Major Episodes of Star Formation by Kartheik Iyer, Eric Gawiser, Sandra M. Faber, Henry C. Ferguson, Anton M. Koekemoer, Camilla Pacifici, Rachel Somerville, and Jeyhan Kartaltepe, accepted for publication in the Astrophysical Journal.

https://arxiv.org/abs/1901.02877

3.1 Introduction

Galaxies are massive, turbulent systems, shaped by physical processes that regulate star formation across many orders of magnitude in spatial and temporal scales, (White & Rees (e.g., 1978); Searle et al. (e.g., 1973); Hopkins et al. (e.g., 2014); Genel et al. (e.g., 2018), also see the reviews by Somerville & Davé (2015); Naab & Ostriker (2017)). Despite this apparent chaos, observations of ensembles of galaxies across cosmic time reveal several correlations, such as the Tully-Fisher relation (Tully & Fisher 1977), the SFR-M* correlation (Noeske et al. 2007; Daddi et al. 2007; Elbaz et al. 2007), the black hole mass-velocity dispersion correlation (Gebhardt et al. 2000), and the mass-metallicity correlation (Tremonti et al. 2004). Median trends constructed using these scaling relations indicate an equilibrium mode of galaxy growth through baryon cycling, punctuated by mergers and followed by eventual quiescence (Peng et al. 2010; Davé 2008; Tacchella et al. 2016; Behroozi et al. 2018). However, these trends can be equivalently recovered through stochastic evolution (Kelson 2014) or simple parametric models with minimal physics (Abramson et al. 2016). Given information about the present state of a galaxy, a lot can be said about its evolutionary history from this state, since the gas inflow rate depends to first order on the halo formation rate, which is driven by gravity and evolves over ~ a Hubble time. It is an important question to determine the extent to which its evolution is driven by evolving physical processes such as baryon cycling and star formation suppression,
which depend on the physical conditions of galaxies such as their size, morphology and stellar mass; as opposed to stochastic processes governing halo and galaxy mergers and the creation and destruction of molecular clouds that regulate in-situ star formation, which remains broadly invariant across many orders of magnitude as inferred from the SFR-M$_*$ correlation extending from galaxy-wide scales (Whitaker et al. 2014; Kurczynski et al. 2016) down to kpc scales in resolved observations (Hsieh et al. 2017).

A key observable that correlates the present state of a galaxy with its evolutionary history is its star formation history (SFH) - a record of when a galaxy formed its stars. The SFHs of galaxies bear imprints from all the physical processes that shape galaxy growth by regulating star formation. This includes inflows and outflows of gas, mergers between galaxies, and feedback due to supernovae and Active Galactic Nuclei, which leave imprints on the SFH on timescales ranging from $< 1$ Myr to $> 10$ Gyr (Somerville et al. 2008, 2015; Sparre et al. 2015; Inutsuka et al. 2015; Torrey et al. 2017; Behroozi et al. 2018; Matthee & Schaye 2018; Weinberger et al. 2018).

Summary statistics of the SFH allow us to calculate traditionally estimated quantities like the stellar masses, star formation rates at the epoch of observation, and mass- and light-weighted ages of individual galaxies (Bell et al. 2007). Additional information about the shape of galaxy SFHs allows us to infer whether the galaxy is actively forming stars, or if it formed most of its stars in the distant past (Kauffmann et al. 2003; Brinchmann et al. 2004). Non-parametric estimates of the median SFH for a sample of galaxies let us better understand the width and strength of major episodes of star formation (Heavens et al. 2000; Tojeiro et al. 2007; Pacifici et al. 2016; Iyer & Gawiser 2017), the origin and evolution of scaling relations (Iyer et al. 2018; Torrey et al. 2018; Matthee & Schaye 2018), mass functions (Pacifici et al. 2019, in prep.) and the cosmic star formation rate density (Leja et al. 2018).

With the advent of large surveys and the impending arrival of the next generation of surveys with JWST, developing more sophisticated non-parametric techniques of recovering galaxy SFHs will allow us to estimate galaxy properties out to higher redshifts. More importantly, more precise multiwavelength data allows us to go beyond traditional parametric forms and estimate finer features, such as SFHs with multiple strong episodes of star formation (Iyer & Gawiser 2017; Morishita et al. 2018) that can be caused by violent events like mergers or smoother events like stripping followed by inflow of pristine gas (Kelson 2014; Torrey et al. 2018; Tacchella et al. 2018; Boogaard et al. 2018). For example, Morishita et al. (2018) find that even old, quiescent galaxies sometimes show evidence for multiple episodes of star formation. Correlating these features of the SFH with other observables such as the metallicity of the galaxy, evidence of recent
mergers, morphological and environmental conditions will allow us to test different models that can help explain the diversity seen in SFHs at a particular epoch.

In the observational domain, the integrated light from distant galaxies contains a host of information about the physical processes that shape them during their formative phases (Tinsley 1968; Bruzual & Charlot 2003; Conroy et al. 2009; Conroy & Gunn 2010). Since stellar populations of different ages have distinct spectral characteristics, careful analysis of multiwavelength spectral energy distributions (SEDs) allows us in principle to disentangle these different populations (Heavens et al. 2000; Reichardt et al. 2001; Tojeiro et al. 2007; Dye 2008; Acquaviva et al. 2011a; Pacifici et al. 2012; Smith & Hayward 2015; Pacifici et al. 2016; Iyer & Gawiser 2017; Domínguez Sánchez et al. 2016; Lee et al. 2017; Ciesla et al. 2017; Carnall et al. 2018a; Leja et al. 2018). SED fitting allows us to estimate the SFHs for a much larger population of galaxies, but requires much more sophisticated analysis to avoid biases. Assumptions of simple parametric forms for SFHs lead to biases, as shown in Iyer & Gawiser (2017); Ciesla et al. (2017); Lee et al. (2017); Carnall et al. (2018b). On the other hand, more complicated parametric forms, as well as methods that estimate the SFR in time bins require us to estimate a much larger number of parameters. Therefore, we are in a situation where we would like to estimate as much information about the SFH as possible, encoded in the smallest possible number of estimated variables, while attempting to be as non-parametric as possible to avoid biases. As Ivezić et al. (2014) states, non-parametric does not imply an absence of parameters, but an independence of model structure such as a functional form for the SFH. Ideally, the nature and number of parameters that specify the shape of a galaxy’s SFH are flexible and determined by the data. A non-parametric method in the context of star formation histories should avoid assuming a simple parametric form (e.g., exponentially declining or lognormal SFHs), be flexible enough to describe any function in SFH space, and infer the number of parameters from the quality of the data being analyzed.

In Iyer & Gawiser (2017) we introduced the Dense Basis method, which uses a basis of SFHs comprised of four different functional families and all of their combinations, determining the optimal number of SFH components using a statistical test. While this approach produces a basis of SFHs that is effectively dense in SED space and was shown to minimize the bias and scatter due to SFH parametrization, it still retains a minor dependence on the functional families under consideration. Additionally, it can not be flexibly incorporated into a MCMC or nested sampling framework, and becomes computationally expensive as we go to large numbers of SFH components - making it inefficient at extracting all the SFH information present in high quality spectrophotometric data.
In this work, we introduce an improved version of the Dense Basis method that uses non-parametric SFHs constructed using Gaussian Process Regression (GPR) (also called kriging or Wigner-Kolmogorov prediction (Chiles & Delfiner 2009; Wackernagel 2013; Krivoruchko 2011)), using the lookback times at which a galaxy assembled certain quantiles of its overall stellar mass. Gaussian Processes (Rasmussen & Williams 2006) extend the Gaussian probability distribution to the space of functions, allowing us to describe posteriors in SFH space without the need for parametric forms or bins in time. Historically, GPR developed as a method to interpolate between noisy data points. In our case, we have two sets of noisy data, the noisy SEDs themselves, and the noisy estimates of physical properties such as the SFH estimated using them. While Leistedt & Hogg (2016) uses the former, in this paper we use the GPR exclusively to perform interpolation in SFH space to create smooth curves from sparse constraints on the SFH determined through SED fitting. Compared to traditional interpolation methods that require us to specify a smoothing scale or bandwidth, Gaussian Processes allow us to specify a covariance function, also called the kernel, which defines how SFRs separated by a time $\Delta t$ are related. In combination to constraints on the SFR at certain times from the observable SEDs, the Gaussian process routine is used to compute the likelihood of the SFR at every point in lookback time. While retaining all the advantages of the previous method, this has the additional advantages of being completely independent of any choice of functional form, scaling linearly with the number of SFH parameters, and providing a modular framework capable of being incorporated into any existing SED fitting routine. We establish the robustness of the method using Semi-Analytic Models and Hydrodynamical Simulations for which the true SFHs are known, and apply it to galaxies at $0.5 < z < 3.0$ across the five CANDELS fields to study the evolution of galaxies around cosmic noon using their SFHs.

This paper is structured as follows: Section 3.2 describes the methodology, including the formalism used for constructing smooth, non-parametric SFHs and incorporating them into a SED-fitting framework. Section 3.3 contains a suite of validation tests for the methodology developed and applied in this work. In section 3.4, we introduce the CANDELS dataset that we use for the current analysis. In section 3.5, we describe the SFHs reconstructed from the CANDELS sample, including the fraction of galaxies with multiple strong episodes of star formation, the evolution of this fraction with time, implications for the timescales of quenching followed by rejuvenation as well as for the morphological transformation of galaxies as they approach quiescence. Section 3.6 considers caveats and future directions for applying the Dense Basis method, and Section 3.7 concludes. Throughout this paper magnitudes are in the AB system; we use a standard $\Lambda$CDM cosmology, with $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$ and $H_0 = 70$ km
Figure 3.1: Applying Gaussian Process regression to create smooth approximations of a single SFH from the Santa Cruz semi-analytic model (Somerville et al. 2008; Porter et al. 2014) at different redshifts of interest using the parametrization described in Sec. 3.2.1. The nine panels show how versatile the method is at capturing details of the SFH using a varying number of parameters, which can be tuned based on the quality of the observations such as rest-frame wavelength coverage or S/N. In SFH space, this demonstrates the versatility of the method at describing arbitrary SFH shapes, with the accuracy of the approximation increasing with the number of parameters. An advantage of the Gaussian process formalism is that it is possible to describe an SFH with multiple episodes with as little as 3 parameters.

\[ \text{Mpc}^{-1} \text{s}^{-1}. \]

3.2 Methodology

3.2.1 Star Formation Histories

The main thrust of the Dense Basis method is to encode the maximum amount of information about the SFHs of galaxies using a minimal number of parameters. In this respect, the formalism used to describe SFHs in this work can be used as a module in existing sophisticated inference methods developed in public SED fitting codes like Bagpipes (Carnall et al. 2018a), Beagle (Chevallard & Charlot 2016a), CIGALE (Noll et al. 2009), and Prospector (Leja et al. 2017) to flexibly describe SFHs. As shown in our validation tests (§3.3), this description minimizes the bias in estimating SFHs at all lookback times, compared to both existing parametric and non-parametric methods (Iyer & Gawiser 2017; Lee et al. 2010; Ciesla et al. 2017; Carnall et al. 2018b; Leja et al. 2018).

This subsection describes the methodology for creating SFHs using this formalism for a given number of parameters, and the following subsections handle the construction of the SED fitting machinery needed to infer the optimal number of SFH parameters corresponding to the
amount of information available in individual galaxy SEDs.

We define a SFH by the tuple: \((M_*, SFR, \{t_X\})\), where \(M_*\) is the stellar mass, \(SFR\) is the star formation rate at the epoch of observation, and the set \(\{t_X\}\) are \(N\) ‘shape’ parameters that describe the SFH. The shape parameters \(\{t_X\}\) parameters are \(N\) lookback times at which the galaxy formed equally spaced quantiles of its total mass (Pacifici et al. 2016; Behroozi et al. 2018). For the first few values of \(N\), we can write

\[
\begin{align*}
N = 1 & \quad P = \{t_{50}\} \\
N = 2 & \quad P = \{t_{33}, t_{67}\} \\
N = 3 & \quad P = \{t_{25}, t_{50}, t_{75}\} \\
N = 4 & \quad P = \{t_{20}, t_{40}, t_{60}, t_{80}\} \\
\end{align*}
\]

... 

This can be seen graphically in Figure 3.1, where we show the \(\{t_x\}\) parameters with \(N = 2, 4, 9\) (vertical dashed black lines) for a mock SFH (blue line), along with our construction of the SFH using these parameters (black solid lines). As expected, the shape of the mock SFH is better captured as \(N\) increases, with multiple episodes captured using four parameters. In practice, we find that it is possible to recover multiple episodes of star formation with as little as \(3\) \(\{t_x\}\) parameters. Together with the stellar mass and SFR, this tuple describes a set of integral constraints that describe the shape and overall normalization of the SFH. For a galaxy at redshift \(z\), when the universe was \(t_z\) Gyr old, the constraints are:

\[
M_*(t_z) = \int_{t=0}^{t_z} SFH(t) f_{ret}(t - t_z, Z) dt
\]

\[
\left\{ \frac{iM_{*,tot}(t_z)}{N + 1} = \int_{t=0}^{t_z, i} SFH(t) dt \right\} \forall i \in N
\]

\[
M_* = \frac{\int_{t=0}^{t_z} SFH(t) f_{ret}(t - t_z, Z) dt}{\int_{t=0}^{t_z} SFH(t) dt} M_{*,tot}
\]

The second line is a set of \(N\) equations, one for each parameter in the set \(\{t_x\}\) that requires that the galaxy form ‘\(x\)’ fraction of its total mass by time \(t_x\).

This description of a galaxy’s SFH already offers several advantages over methods found in the literature, a few of which are summarized below:

1. Not being restricted to a particular functional form minimizes bias due to SFH parameterization (Iyer et al. 2018).

2. Describing an SFH using \(\{t_X\}\) reduces the discrepancy in S/N per parameter in comparison to methods that determine the SFR in bins of lookback time, since here for example
$t_{20}$ might be less well constrained compared to $t_{80}$, but the overall signal depends on the shape of the SFH. e.g. the parametrization will not try to constrain the SFR in the first year after the big bang unless enough stars were formed that early to provide a discernible signal in the SED. This can be compared to methods that adaptively choose time bins eg. VESPA (Tojeiro et al. 2007).

3. This provides a novel framework for compressing the amount of information present in an SFH to a small set of numbers given a way to reconstruct an SFH from a tuple, and hence for comparing SFHs across different simulations and observations on the same footing.

4. The distribution of different $\{t_X\}$ among galaxies at a given epoch within a simulation can be extremely useful in defining and checking the physical assumptions of the SFH priors during SED fitting.

Reconstructing an SFH from the tuple $(M_*, SFR, \{t_X\})$ can be done in multiple ways, but we seek to minimize the information lost in doing so, while remaining computationally inexpensive. As with all compression methods, we approach the true SFH as the number of parameters $N$ in the set $\{t_X\} \rightarrow \infty$, but we would like to minimize the loss even with a relatively small number of parameters.

Reconstructing an SFH requires quantifying the integral constraints as points on a fractional mass $(M_*,t_{tot}(t))$ - cosmic time $(t)$ plane and drawing a piecewise smooth curve passing through these points. Rescaling the mass axis and differentiating this cumulative curve would then yield the SFH as a star formation rate at each lookback time. The simplest approximation would be to connect each point such that the resulting SFH is piecewise linear, and while this provides an acceptable solution, it is not very physical in the sense that taking the derivative yields a SFR with jump discontinuities. While generalizing to polynomials for the interpolation causes problems with the derivative going negative in parts of the SFH, methods such as tensioned cubic splines and piecewise-cubic Hermite polynomial interpolation (PCHIP) (De Boor et al. 1978; Butt & Brodlie 1993) provide more sophisticated solutions to this problem.

In this work, we use Gaussian Process Regression (Rasmussen & Williams 2006; Leistedt & Hogg 2016) implemented through the george python package (Foreman-Mackey 2015; Ambikasaran et al. 2014) to create a smooth SFH along with uncertainties following a physically motivated covariance function (kernel) for a given SFH tuple. The Gaussian Process framework uses a set of constraints, given by the set of equations 3.1, along with a covariance function (or kernel) to estimate the probability of SFH$(t)$ at a given time $t$. We use a Matern32 kernel
(Seeger 2004) in the present application, given by:

\[ k(r^2) = \left(1 + \sqrt{3}r^2\right) \exp \left(-\sqrt{3}r^2\right) \]  

(3.3)

where the covariance function contains a scale length hyperparameter \( r^2 \propto (\Delta t)^2 \) that sets how much the SFR(t) can vary from the SFR(t+Δt) separated by a time interval Δt. The hyperparameter in the kernels essentially encode the amount of stochasticity in the SFHs and is described further in section 3.3.5. In practice, this can be thought of as setting the tension in a string that passes through all the constraints in cumulative mass-cosmic time space, and therefore affects the shape and amount of ringing that can happen between two quantiles. While using a spline to reconstruct the SFH, this ringing can sometimes cause the SFR to be negative. Our choice of physically motivated kernel minimizes this behaviour and limits it to extreme SFH tuples (e.g. \( t_{25} = 0.1 \) Myr, \( t_{50} = 10 \) Gyr, \( t_{75} = 10.1 \) Gyr, i.e., the SFH formed the first and third quarters of its mass in bursts of 0.1 Myr, separated by a long period of relative quiescence) which account for < 3% across our basis. In these cases we set the relevant portion (i.e., \( SFR(t) < 0 \)) to 0 and check that the overall error in stellar mass due to this is < 5%. By doing this, we ensure that \( SFR \geq 0 \) at all times across our entire basis. Setting the SFR to 0 in cases like this is physically motivated - stretches with \( SFR = 0 \) might be found in nature as well, as seen in SFH reconstructions of local dwarfs (Weisz et al. 2011a) or in high-resolution simulations (Wright et al. 2019), for galaxies with very bursty SFHs. Examples of approximating SFHs using this formalism are shown in Figure 3.1. The advantage of this method is that both parameters (M*, SFR, \( \{t_x\} \)) and uncertainties on these parameters can be passed as arguments while constructing the SFH posterior.

This is tested using SFHs from the Santa Cruz semi-analytic models (Somerville et al. 2008; Porter et al. 2014) and the MUFASA Davé et al. (2016b) simulation to minimize loss during the reconstruction of SFHs, finding that in both cases the error in approximating the true SFH decreases as we increase the number of parameters. The Santa Cruz semi-analytic model uses dark matter halo merger trees from the Bolshoi-Planck simulation (Klypin et al. 2011) in combination with analytic recipes for the radiative cooling of gas, collapse of cold gas to form rotationally supported disks, star formation, feedback and chemical enrichment. The MUFASA hydrodynamical simulation uses the GIZMO (Hopkins 2015) meshless code including prescriptions for \( H_2 \)-based star formation, chemical evolution and kinetic outflows following scalings from the FIRE (Hopkins et al. 2014) simulation as well as an evolving halo-mass based quenching mechanism. Both simulations reproduce a wide range of present-day observables including \( z \sim 0 \) mass functions and scaling relations, with further details in (Somerville et al.

### 3.2.2 SED fitting

While we are primarily interested in the star formation history, to determine this from the galaxy’s SED we need to account for several other factors such as the chemical enrichment, stellar initial mass function (IMF), dust attenuation model, and absorption by the inter-galactic medium (IGM). We then formulate the SFH estimation as an inference problem given by,

$$P(SFH, A_V, Z, z...) = \frac{P(F_{\nu,j}^{obs})}{P(F_{\nu,j}^{obs} | SFH, A_V, Z, z...)} \frac{P(SFH, A_V, Z, z...)}{P(F_{\nu,j}^{obs})}$$

The term $P(F_{\nu,j}^{obs} | SFH, A_V, Z, z...)$ is the likelihood, given by $L \propto \exp(-\chi^2/2)$, where

$$\chi^2 = \sum_{j=1}^{N_{Iters}} \left( \frac{F_{\nu,j}^{obs} - F_{\nu,j}^{model}(SFH, A_V, Z, z)}{\sigma_j} \right)^2$$

The term $P(SFH, A_V, Z, z...)$ denotes the prior distribution for the model. If we assume uncorrelated priors for all the parameters, this can be written as $P(SFH)P(A_V)P(Z)P(z)...$. $F_{\nu,j}^{obs}$ is the observed photometry being fit, in the $j^{th}$ photometric filter. $SFH$ denotes the star formation history tuple $(M_*, SFR, \{t_x\})$, $A_V$ is the dust model, $Z$ the stellar metallicity and $z$ is the redshift. In addition to this, we need to consider the stellar population synthesis models, stellar initial-mass function, absorption by the intergalactic medium, and a self-consistent implementation of nebular emission lines using CLOUDY through FSPS. We adopt the Calzetti attenuation law for the dust attenuation (Calzetti 2001) with 1 free parameter since the Calzetti law couples the birth-cloud attenuation to that from older stars and one for stellar metallicity, a Chabrier initial mass function (Chabrier 2003) with no free parameters, and define our SFH parametrization in section 3.2.1 with $N+2$ parameters, where $N$ is the number of SFH percentiles, given by the set $\{t_x\}$. We use the Flexible Stellar Population Synthesis FSPS code (Conroy et al. 2009; Conroy & Gunn 2010) to generate spectra corresponding to the Basel stellar tracks and the Padova isochrones. With $N$ SFH quantiles, the model then has $N + 5$ $(M_*, SFR, A_V, Z, z)$ free parameters that need to be determined from the data. We construct the method in a way that $N$ itself is a variable that is tuned to extract the maximum amount of information present in a galaxy’s SED. It is important to keep in mind that these modeling choices can impose an implicit prior on our SED fits. The effects of testing our model assumptions and priors are further explored in §3.3, where we find that our models and priors are suitably robust considering the S/N and wavelength coverage of our dataset.
An illustrative example of the SED fitting method, applied to mock noisy photometry for a galaxy from the Somerville et al. (2008); Porter et al. (2014) semi-analytic model with more than one major episode of star formation, with the photometry simulated using the GOODS-S filter set. The **top-right** panel shows the simulated photometry (blue errorbars) and the spectrum (solid black line) corresponding to the median parameter values. The corresponding reconstructed SFH (solid black line) is shown in the **middle-right** panel below it, with the true SFH (solid blue line) from the SAM shown for comparison. The thin black lines in both panels show draws from the posterior distribution for comparison. This particular SED was determined to contain enough information to estimate 6 correlated SFH parameters using the BIC model selection criterion, as shown in the inset panel (**bottom right**). The corner plot (**left**) shows the posteriors for each parameter using our brute-force bayesian approach, with the blue lines representing the true values used to generate the mock noisy photometry.

Although redshift is formally a free parameter, we fit galaxies at the \( (z_{best} \pm 0.05) \) from Kodra et al. (in prep.), finding that within this small dynamic range the redshift posterior is effectively flat, as expected. Dashed black lines for each histogram show the median and 16-84\(^{th}\) percentile range.
Figure 3.3: The prior distributions of SFHs in $t_{25}, t_{50}, t_{75}$ space for SFHs of galaxies at $z \sim 1$ from the Santa Cruz semi-analytic model, and the MUFASA hydrodynamical simulation, in addition to the basis assembled using the Dirichlet prior we adopt.

We implement the posterior computation numerically using a brute-force bayesian approach similar to Pacifici et al. (2012, 2016); Da Cunha et al. (2008) using a large pre-grid of model SEDs constructed through random draws from the prior distributions corresponding to each free parameter in Eqn. 3.5. To ensure that the pre-grid samples the priors finely enough and is effectively dense in SED space, we perform fits to a sample of 1000 galaxies while varying the size of our pregrid. Using this, we estimate the optimal size of the pregrid as the point where the improvement in median $\chi^2$ for the sample as a function of pregrid size is negligible, leading to a pre-grid with $\sim 900,000$ SEDs.

To construct the pre-grid, we draw random values from our prior distributions for stellar metallicity, dust attenuation, and SFH parameters ($M_*, SFR, \{t_x\}$). For metallicity, we adopt a flat prior on $\log Z/Z_\odot$ (Pacifici et al. 2016; Carnall et al. 2018a; Leja et al. 2018), an exponential prior on dust attenuation (Iyer & Gawiser 2017), and a Dirichlet prior for the lookback times $\{t_x\}$ that specify the shape of the SFH. The Dirichlet prior is a generalization of the Beta distribution to N variables, such that a random draw yields N random numbers $x_i$ that satisfy $\sum_i x_i = 1$. More details about this prior can be found in Leja et al. (2017, 2018). In practice, for a galaxy SFH at redshift $z$, we generate the set $\{t_x\}$ by performing a random draw multiplied by the age of the universe at that redshift, giving the set of lookback times at which a galaxy formed various quantiles of its stellar mass. The dirichlet prior has a single tunable parameter $\alpha$ that specifies how correlated the values are. In our case, values of this parameter $\alpha < 1$ results in values that can be arbitrarily close, leading to extremely spiky SFHs since galaxies have to assemble a significant fraction of their mass in a very short period of time, and $\alpha > 1$ leads to smoother SFHs, with more evenly spaced values that nevertheless have considerable diversity. In practice, we use a value of $\alpha = 5$, which leads to a distribution of parameters that is similar to what we find in the SAM and MUFASA. This can be seen in Figure 3.3.
For each galaxy in our pre-grid, we turn the corresponding \((M_*, SFR, \{t_x\})\) tuple into an SFH using our Gaussian process routine assuming fiducial 1% uncertainties. Using this SFH, in combination with the values for metallicity, dust and redshift, we generate a spectrum through the FSPS stellar population synthesis routine. Multiplying this spectrum with the appropriate filter transmission curves gives the SED corresponding to each tuple in the pre-grid.

SFH uncertainties are computed after the fit is performed, using the posterior for each parameter describing the tuple \((M_*, SFR, \{t_x\})\). For each galaxy, 100 self-consistent random draws are performed from the posterior with the covariances between parameters taken into account, and corresponding SFH realizations are constructed using the Gaussian Process routine. For a given realization, if the set of parameters already exist in the pre-grid, the corresponding precomputed SFH is used to decrease the computational cost. Figure 3.2 shows 20 draws from the SFH posterior for that galaxy using this approach as thin black lines in the SFH inset panel. 68% confidence intervals are then constructed by taking the 16\(^{th}\) to 84\(^{th}\) percentile of the SFR distribution at each point in lookback time.

### 3.2.3 Non-parametric SFH reconstruction

The Gaussian Process based SFH (GP-SFH) formalism allows us to gain independence from having to make a choice of functional form for the shape of the SFH, without having to bin SFHs in lookback time. This results in smooth, effectively non-parametric SFHs that minimize the bias and scatter in SFH reconstruction, as seen in Sec. 3.3. However, since we would like the method to be truly non-parametric, we require that the number of \(\{t_x\}\) parameters be optimized for the amount of information present in a given noisy SED. To implement this in practice, we generate pre-grids for SFHs with the set \(\{t_x\}\) ranging from 3 to 9 parameters, since it is difficult to specify the shape of complex SFHs with less than three parameters, and it is impractical to recover more than nine from broadband SEDs. We then fit each observed SED with all 7 pre-grids, and obtain the most appropriate number of parameters using an appropriate model selection criterion. Ideally the Bayesian evidence would be used for this model selection step (Liddle 2007). However, in practice, the numerical computation of the evidence is expensive due to the need for a nested sampler, and can not be completed for the number of galaxies typically found in large photometric surveys. Having tested the properties of the likelihood surfaces using this method, however, we find that while they may be multimodal, they generally do not contain pathological features that necessitate this numerically expensive procedure. In light of this, we perform our model selection using an approximation of the evidence, given by the
Bayesian Information Criterion (Schwarz et al. 1978; Liddle 2007), defined as

\[ BIC = k \ln(n) - 2 \ln \mathcal{L}_{\text{max}} \]  

where \( n \) is the number of photometric datapoints, \( k \) is the number of parameters in the SFH tuple, and \( \mathcal{L} \) is the maximized value of the likelihood function given by Eqn. 3.5. The latter term in this equation is a measure of how well the model describes the data, and the former term is a penalty for an increased number of parameters. The results from the two model selection criteria are equivalent when all the parameters are independent (Kass & Wasserman 1995; Szydlowski et al. 2015). However, if parameters are strongly covariant, the volume will reflect this (since the intrinsic dimensionality is lower than the number of dimensions describing the likelihood) while the BIC does not. This can lead to the optimal number of parameters estimated with the BIC ≤ the optimal number of parameters estimated using the evidence. Our validation tests show that using the BIC does not cause any significant issues in fitting an ensemble of SEDs to recover their SFHs. By finding the minima in the BIC, we find the number of SFH parameters that can be robustly extracted from the SED being fit. This leads to a truly non-parametric description of the SFH, based on the amount of information about the different stellar populations encoded in the galaxy’s SED. An example of this for a single galaxy is shown in the bottom right plot of Figure 3.2. Iyer & Gawiser (2017) used a similar non-parametric estimate of the number of SFH components using a F-test, but was computationally expensive to implement as the number of SFH components grew. Tojeiro et al. (2007) and Dye (2008) also used non-parametric methods to estimate the optimal number of time bins during SFH reconstruction, although this is prone to bin edge effects, as described in Leja et al. (2018).

Our Gaussian process SFH reconstruction method can be summarized as follows:

1. We introduce a formalism based on Bayesian statistics to describe the SFH, which we will express not as a functional form but rather as N-tuples, plus \( M_\ast \) and SFR at the time of observation. To generate a curve \( \text{SFR}(t) \) from the tuples, the GPR needs a covariance function or kernel, which determines how SFR distributions at \( t \) and \( t+\Delta t \) are related, i.e., how smooth the SFHs are.

2. The Bayesian likelihood is computed for each of a sufficiently large number of model SFHs realized using GPR, having known values of the n-tuples, \( M_\ast \) and SFR-today (the pre-grid) drawn from our chosen prior. By estimating the likelihood of each basis N-tuple, we can compute posteriors for the N-tuples (and \( M_\ast \) and SFR-today) together with their credible intervals and covariances.
3. The GPR is now used to interpolate between these (noisy) estimates of the n-tuples to obtain draws from the posterior distribution. Each of these draws is a model SFH. The distribution of these draws at each point time gives the uncertainty in the SFH at that time.

4. This procedure is repeated as we vary the number of parameters in the N-tuples, using a Bayesian model selection criterion to evaluate the optimal number of parameters.

3.2.4 Quantifying the number of Major Episodes of Star Formation in a SFH

Unlike simple SFH parametrizations commonly used in SED fitting, the Gaussian Process based star formation histories can have multiple maxima, even with four or fewer $t_x$ parameters, as seen in Figure 3.1. Hence, it is possible to analyze them using a peak finding algorithm to quantify the number of major episodes of star formation in a galaxy’s past. For this particular analysis, we use the best-fit SFH for each galaxy, since the median SFH for each galaxy is biased towards smooth SFHs. This is an effect due to our choice of kernel, which prefers smooth solutions when the $\{t_x\}$ parameters are uncertain, as seen in the left column of Figure 3.1. With tighter constraints on $\{t_x\}$ from higher S/N data this problem is alleviated. As a result, while the number of major episodes ($N_{ep}$) estimated using this method for individual galaxies is susceptible to noise, the overall distribution is seen to be recovered without any significant bias, as shown in §3.3.

For each SFH, we quantify the number of episodes as follows: We first find the number of peaks in an SFH as the set of points that satisfy $dSFH/dt = 0$ and $d^2SFH/dt^2 < 0$. To separate multiple peaks within an overall episode of star formation from different episodes, we impose a peak prominence criterion by requiring that

$$\left(\log SFR_{\text{peak}} - \log SFR_{\text{min, local}}\right) > 1.5 + \frac{1.5}{4} \log \frac{M_*}{10^8 M_\odot}$$

(3.7)

where $SFR_{\text{min, local}}$ is the minima between two peaks in the SFH. This condition is shown in §3.3 to minimize the type-1 (overestimating $N_{ep}$) and type-2 (underestimating $N_{ep}$) errors in our validation sample. It arises because the sensitivity to star formation drops approximately logarithmically with time (Ocvirk et al. 2006). Since more massive galaxies tend to have older stellar populations, we found that a mass-independent peak-prominence criterion caused a mass-dependent bias in our estimates of the number of episodes. We correct for this effect by requiring a more (less) stringent dip in the SFH for a massive (low-mass) galaxy, and while this does not
improve the result for every galaxy, it accurately recovers the distribution, which is the quantity that we are interested in.

3.3 Validation tests

At each state of the method development, we perform validation using an ensemble of galaxies from two cosmological simulations: the Santa Cruz semi-analytic models (Somerville et al. 2008; Porter et al. 2014) and the MUFASA hydrodynamical simulation (Davé et al. 2016b). Using two different simulations allows us to alleviate the potential biases induced by testing our method against a single simulation. For the tests described in this section, we created a mock catalog with 10,000 galaxies sampling randomly from both simulations at 0.5 < z < 3.0, matching our analysis sample.

For each galaxy in our mock catalog, we draw a random redshift $z_{\text{mock}}$ between 0.5 and 3. We then create synthetic spectra using FSPS with the corresponding star formation history and mass-weighted metallicity, with a Calzetti dust attenuation $A_V$ sampled from an exponential distribution as in Iyer & Gawiser (2017). We then multiply these spectra by the appropriate filter transmission curves corresponding to one of the five CANDELS fields and perturb the photometry in each band by adding realistic noise derived from the median photometric uncertainties for the CANDELS catalog in the redshift range $[z_{\text{mock}} - 0.1, z_{\text{mock}} + 0.1]$.

Figure 3.2 shows an example following this procedure, using a galaxy with a SFH that is not well approximated by a simple parametric form currently used in the literature, but is recovered well with the GP-SFH approach. Using our mock catalog, we then perform a series of validation experiments: In §3.3.1 we consider the robustness of the reconstructed SFHs. In §3.3.2 we quantify the bias and scatter in estimating stellar masses, star formation rates, dust attenuation and stellar metallicities, and in §3.3.3 we consider the robustness of the uncertainties on the reconstructed SFHs. In §3.3.4 we test the recovery of the number of major episodes of star formation for our mock sample, find a mass-dependent bias and correct for it. In §3.3.5 we test our choice of Gaussian process kernel, and in §3.3.6 we test the information loss and how well the method compares to existing non-parametric SFH descriptions.

3.3.1 SFH robustness

To quantify possible biases in our SFH reconstruction, we plot the difference between the true and recovered SFH for our sample of validation galaxies in Figure 3.4. The two panels show the linear and logarithmic differences between the true and the recovered SFHs, which represent the
Figure 3.4: Comparing the ensemble of true SFHs from cosmological simulations to SFHs reconstructed from SEDs using the Gaussian Process Dense Basis method. Thin black lines show the difference in log SFR for individual galaxies, with the pointwise median in time shown as a solid blue line and the shaded blue region denoting the 16th to 84th percentile. The left panel shows the difference in terms of $\Delta \log SFR$, while the right panel shows the difference in terms of $\Delta SFR$ at each point in lookback time.
Figure 3.5: Results of estimating stellar masses, star formation rates, dust attenuation and metallicities for the ensemble of mock galaxies described in §3.3. Each parameter is shown as a log-scale heatmap, with the adjacent colorbar detailing the number of galaxies in a given bin. Bins with no galaxies are shown in black. The dashed white line shows the 1:1 relation, and each plot title contains the overall bias and scatter around the mean for the sample.

difference and ratio of the true vs predicted SFR(t) respectively. This difference is smaller than 0.3 dex for most of lookback time. While the reconstructed SFHs can not recover every short episode of star formation, as seen in the errors for individual galaxies (thin black lines), the overall SFH for the ensemble of galaxies is unbiased out to nearly $\sim 8\, \text{Gyr}$. In the $\Delta \log SFR$ error plot, the bias blows up as we approach the big bang. This is because the SFHs in the mocks can abruptly fall to 0 close to the big bang while the Gaussian Process SFHs smoothly decline to 0 at $t = 0$, leading to a one sided error. However, the arithmetic $\Delta SFR$ plot shows that this is in fact a very small difference, exaggerated by the fact that $\log SFR_{\text{true}} \to -\infty$ as $SFR_{\text{true}} \to 0$ close to the big bang.

3.3.2 Parameter robustness

In addition to the star formation histories, Figure 3.5 shows the results of estimating traditional SED fit parameters - stellar mass, star formation rate averaged over the last 100 Myr, dust attenuation and stellar metallicity for the validation catalog. The stellar masses are constrained better than the traditional scatter around the mean of $\sim 0.14$ dex (Mobasher et al. 2015). Star formation rates have a larger scatter (0.29 dex) due to degeneracies with dust attenuation (0.13 dex) and metallicity (0.32 dex). At low stellar masses, we are prone to overestimating the overall mass due to our choices of SFH prior dominating the fit for low S/N SEDs. Restricting the fits to galaxies with $H < 25$ is found to largely exclude these low S/N objects, leading to more robust estimates of these parameters.
Figure 3.6: Validation performed to check the robustness of SFH uncertainties estimated from using the posterior from SED fitting. For our sample of 10,000 mock galaxies, we compute the fraction of the true SFHs that lie within the 10-95% credible intervals, shown in the left panel. For robust uncertainties, we expect a 1:1 correspondence between the fraction of truth that lies within a credible interval and the interval itself. The middle and right panels show two examples of this computation for two galaxies in our sample, one where the majority of the true SFH lies within the uncertainties (middle) and the other where a sharp peak is not recovered by the SFH reconstruction due to photometric noise (right).

### 3.3.3 SFH uncertainties

As described in Sec. 3.2, SFH uncertainties are computed at each point in lookback time for various percentiles, ranging from the 10 (55−65th percentile bounds) to the 95th (2.5−97.5th) percentile credible intervals of SFHs constructed from 100 draws from the SFH posterior $(M_*, SFR, \{t_x\})$ using the Gaussian Process routine. To check whether these represent true 68% uncertainties, we consider the uncertainties estimated for each of the 10,000 galaxies in the mock catalog that we fit and compute the fraction of the true SFH that lies within the uncertainties for each galaxy. This is similar to the concept of a ‘coverage probability’ (Levasseur et al. 2017) used to evaluate the accuracy and precision of uncertainty estimates. The distribution of this fraction is given in the left panel of Figure 3.6, which shows the fraction of the truth that lies on average within a given credible interval. For an ideal method we expect this to lie along the 1:1 line, while our method is extremely close to this across the entire evaluated range. This indicates that our uncertainties are robust, expanding on the tests in Iyer & Gawiser (2017).
Figure 3.7: Validation tests recovering the number of major episodes of star formation in a galaxy’s past by fitting the ensemble of mock noisy SEDs described in App. 3.3. The plots show the results of our analysis for the same redshift bins and mass range used for the main CANDELS sample. The solid lines show a sliding median within $\pm 0.25$ dex in stellar mass for each quantity, the shaded regions show the uncertainty for the estimates assuming Poisson noise.

which solely checked the 68% uncertainties. Figure 3.6 shows two examples, one where the uncertainties are representative of the truth, and another where both the reconstruction and the uncertainties miss a relatively short ($< 0.5$ Gyr) episode of star formation that follows a relatively quiescent period, due to the smoother reconstructed SFH producing a comparable SED given the photometric noise.

### 3.3.4 Number of episode estimation

For each galaxy in the mock sample, we compute the number of major episodes of star formation ($N_{ep}$) using the peak-finding algorithm described in Sec. 3.2. For the true SFHs, we compute the number of episodes requiring a dip of $\log SFR_{\text{peak}} - \log SFR_{\text{min,local}} \geq 0.3$ dex, to model the scenario of a galaxy on the star-forming sequence dropping below the sequence before rejoining it. The distribution of galaxies with multiple strong episodes of star formation for the mock sample is then given by the thick dashed black lines in Figure 3.7. We then estimate the
number of episodes for the reconstructed SFHs with the same criterion, and find a flatter trend
than observed in the mocks. This is a combination of two effects: (1) The S/N contributed to
the overall SED by a stellar population of a certain age decreases roughly logarithmically with
lookback time (Ocvirk et al. 2006) leading to poorer constraints on intermediate and older stellar
populations. (2) There is a mass-dependent correlation towards older stellar populations as
galaxies grow more massive. As a result of this, galaxies that formed most of their mass, followed
by an extended period of quiescence can be erroneously classified as having multiple episodes
since when the SFR at the time of observation is low, it is easier to have a fluctuation of $\geq 0.3$
dex with a small variation in SFR. We find that introducing a stellar mass dependent threshold
to determine the number of peaks given by equation 3.7 accounts for these mass-dependent
effects on the distribution, allowing us to better reproduce the true trends in $N_{ep}$ with stellar
mass and redshift, as seen in Figure 3.7. The thin lines also show the number of overestimates
and underestimates, such that $N_{ep,true} = N_{ep,rec} -$ underestimates $+$ overestimates. Since we
are using the best-fit SFHs for this computation, we find that our results are quite sensitive
to noise for individual galaxies. However, the distributions are estimated robustly, across a
range of stellar masses and redshifts. Additionally, the differing behaviour of the observational
sample from our mock catalog is indicative of the fact that our mass-dependent threshold does
not impose an artificial trend on the distribution. Reconciling the differences between the two
distributions is an interesting topic for further study.

3.3.5 Effect of varying GP kernel and hyperparameters

To generate a curve in SFR vs lookback time space, we use Gaussian Process Regression (GPR)
to perform interpolation in cumulative mass ($\int SFR(t) dt$) as a function of time, conditioned
on constraints given by the N-tuples (SFR, $M_*$, $\{t_X\}$), as described in §3.2. The choice of
kernel can make a significant difference to this interpolation, since it determines the extent
to which the cumulative mass (and hence the SFR) at two times (t and $t'$) are correlated
Rasmussen & Williams (2006); Foreman-Mackey (2015). In the current implementation, we
choose to focus on kernels that are 1-dimensional and stationary, which means that the kernel
only depends on the separation between two times, and not the times themselves. This choice
of kernel is physically motivated in the sense that SFRs over short timescales are thought to be
strongly correlated, with this correlation dropping off as the time separation increases, similar
to the models explored by (Kelson 2014; Caplar & Tacchella 2019). To determine the most
appropriate kernel from those widely used in the literature, we consider four stationary kernels
implemented in the george python package: the squared-exponential or Radial Basis kernel, the
Figure 3.8: The effect of varying the Gaussian process kernel, showing the SFH posterior constructed using the (a) Exponential-Squared (also called RBF), (b) Matern32 (used in this work), (c) Matern52, and (d) Rational Quadratic kernels. For each choice of kernel, we take the same input SFH (blue), compute the \( (M_* , \text{SFR}, \{ t_X \}) \) tuple, and then reconstruct an SFH using GPR with different kernel choices. For most well motivated radial kernels, the choice of kernel only contributes to percent-level changes in the reconstructed SFH, which is a much smaller effect than the \( \sim 0.2 - 0.3 \) dex uncertainties due to modeling systematics found in our validation tests (Figure 3.5).
Figure 3.9: Varying the kernel hyperparameter controlling the scale length (amount of covariance between the cumulative SFH ($\int SFH(t)dt$) at two times ($t$ and $t'$) does not significantly influence the reconstructed SFH obtained by sampling from the predictive posterior distribution conditioned on the tuple ($M_*, SFR, t_{25}, t_{50}, t_{75}$). The left panels show the covariance matrix between the cumulative SFH ($\int SFH(t)dt$) at different points in time.
Matern32 and Matern52 kernels from the Matern class of kernels (Seeger 2004) and the Rational Quadratic kernel, which can be thought of as adding together many Radial Basis kernels with differing length scales. We test all of these different kernel functions using the SFHs from our mock catalog. An example of the reconstruction for a single SFH using different kernel choices is shown in Figure 3.9. In general, we find that the choice of kernel only makes a percent-level difference in computing the median SFH, although the uncertainties can be significantly different. We choose the Matern32 kernel as it gives the best reconstructions and most accurate uncertainties, as explored in §3.3.3. The Matern32 kernel as implemented in the george python package is given by the equation,

$$k(r^2) = \left(1 + \sqrt{3}r^2\right) \exp\left(-\sqrt{3}r^2\right)$$

where $r^2$ is related to the separation between two times. Since our kernel is 1-dimensional and stationary (independent of choice of origin), this simplifies to $r^2 \propto (t - t')^2$, where $r^2$ is related to the separation by a scale length. This scale length is determined directly within the george package by minimizing the log-likelihood of the data under the Gaussian process model. While the scale length varies with the number of SFH percentiles (N) since there is more information to condition the likelihood with for greater N, it is fixed across the pre-grid for a given choice of N. In Figure 3.9 we test the effect of increasing and reducing the scale length used in the Gaussian Process kernel. This causes the covariance matrix to become more (less) coupled to SFRs separated by a given time interval. While this does not have a major effect on the reconstructed SFH, it can cause a decrease (increase) in the uncertainties estimated using random draws from the SFH posterior. Taken in conjunction with our validation of the SFH credible intervals in §3.3.3, we find that our choice of kernel and hyperparameters robustly reconstruct SFHs with accurate uncertainties.

In addition to the basic kernels described in this section, it is possible to design more physically motivated kernels by computing the covariance function of SFHs from cosmological simulations. However, since this is subject to a wide variety of systematics depending on resolution, simulation size and differing physical prescriptions, such an analysis requires a detailed comparison of different simulations. Although this out of the scope of the current paper, it is being actively investigated in a follow-up paper.

### 3.3.6 Minimizing loss of information

We test the effectiveness of the Gaussian process based SFH reconstruction method in compressing data about the SFH and encoding the information in a small number of parameters. Since
Figure 3.10: Comparison between the Gaussian Process based method to other commonly used data compression methods, including a polynomial fit to SFHs and approximating an SFH using linear bins in time with constant SFR in each bin (Heavens et al. 2000; Tojeiro et al. 2007; Dye 2008). An example of this for a single SFH and N=7 is shown in the top panel. We use a sample of 1000 randomly selected SFHs from our mock catalogs for this test, where we approximate the known ‘true’ SFH for each galaxy with three different methods, and quantify the error in the approximation using the mean-squared error (MSE) metric (middle panel) as well as the error in the SFR at the time of observation (lower panel) vs the number of parameters. While the error reduces with increasing # parameters for all three methods, the GP-SFH method consistently performs better in both metrics, yielding better approximations of the true SFH for a fixed number of parameters.
Figure 3.11: The distribution of galaxies across the five CANDELS fields in HST/WFC3 F160w magnitude (left) and redshift (right). Thin black lines show the full distribution for each field and colored lines show the sample used in the current analysis after the selection described in 3.2. While most redshifts are photometric, the sample contains \( \approx 7,000 \) galaxies with spectroscopic redshifts. Grey regions show parts of the sample that we exclude in the current analysis.

the SFH percentiles store information about the shape of the SFHs, the accuracy of describing an arbitrary SFH should decrease as we increase the number of parameters (N). We see this to be the case in Figure 3.10, where the error in describing an ensemble of randomly drawn SFHs from our mock catalog decreases with increasing N. This analysis is done purely in SFH space as a test of the loss of information in going from the full SFH to the GP-SFH tuple.

In addition to this, we compare the method with existing non-parametric techniques of approximating an arbitrary function using 1. Polynomial coefficients, and 2. Binning the SFH linearly in time such that the SFR in any given bin is a constant. The latter approach and its variants such as binning in logarithmic lookback time, or using adaptive bins, are seen across literature that explore non-parametric SFH reconstruction (Heavens et al. 2000; Tojeiro et al. 2007; Dye 2008; Da Cunha et al. 2008; Leja et al. 2017). While these are generally effective, they have the disadvantages of introducing unphysical discontinuities at bin edges. Repeating the test with these two methods, we find that their error approaches the GP-SFH error for large N, with the GP-SFH performing better for any given N. Tuning the number of parameters during SED fitting therefore optimally estimates the amount of SFH information from each SED being fit.
<table>
<thead>
<tr>
<th>Field</th>
<th>Filter set</th>
</tr>
</thead>
<tbody>
<tr>
<td>GOODS-S (Guo et al. 2013)</td>
<td>Blanco/CTIO U, VLT/VIMOS U, HST/ACS f435w, f606w, f775w, f814w, f850lp, HST/WFC3 f098m, f105w, f125w, f160w, VLT/ISAAC Ks, VLT/Hawk-I Ks, Spitzer/Irac 3.6µm, 4.5µm, 5.8µm, 8.0µm</td>
</tr>
<tr>
<td>GOODS-N (Barro et al., in prep.)</td>
<td>KPNO U, LBC U, HST/ACS f435w, f606w, f775w, f814w, f850lp, HST/WFC3 f105w, f125w, f140w, f160w, f275w, MOIRCS K, CFHT Ks, Spitzer/Irac 3.6µm, 4.5µm, 5.8µm, 8.0µm</td>
</tr>
<tr>
<td>UDS (Galametz et al. 2013)</td>
<td>CFHT/MegaCam u, Subaru/Suprime-Cam B, V, Rc, i', z', HST/ACS f606w, f814w, HST/WFC3 f125w, f160w, VLT/Hawk-I Y, Ks, WFCAM/UKIRT J, H, K, Spitzer/Irac 3.6µm, 4.5µm, 5.8µm, 8.0µm</td>
</tr>
<tr>
<td>EGS (Stefanon et al. 2017)</td>
<td>CFHT/MegaCam U*, g', r', i', z', HST/ACS f606w, f814w, HST/WFC3 f125w, f140w, f160w, Mayall/NEWFIRM J1, J2, J3, H1, H2, K, CFHT/WIRCAM J, H, Ks, Spitzer/Irac 3.6µm, 4.5µm, 5.8µm, 8.0µm</td>
</tr>
<tr>
<td>COSMOS (Nayyeri et al. 2017)</td>
<td>CFHT/MegaCam u*, g*, r*, i*, z*, Subaru/Suprime-Cam B, g+, V, r+, i+, z+, HST/ACS f606w, f814w, HST/WFC3 f125w, f160w, Subaru/Suprime-cam IA484, IA527, IA624, IA679, IA738, IA767, IB427, IB464, IB505, IB574, IB709, IB827, NB711, NB816, VLT/VISTA Y, J, H, Ks, Mayall/NEWFIRM J1, J2, J3, H1, H2, K, Spitzer/Irac 3.6µm, 4.5µm, 5.8µm, 8.0µm</td>
</tr>
</tbody>
</table>

Table 3.1: Collected measurements comprising the UV-to-NIR SEDs of galaxies across the five CANDELS fields

### 3.4 Data

In the current analysis we use a sample of galaxies from the HST/F160w selected catalogs for the five Cosmic Assembly Near-infrared Deep Extragalactic Legacy Survey (CANDELS) (Grogin...
et al. 2011; Koekemoer et al. 2011) fields covering a total area of \( \sim 800 \) arcmin\(^2\): GOODS-S (Guo et al. 2013), GOODS-N (Barro et al., in prep.), COSMOS (Nayyeri et al. 2017), EGS (Stefanon et al. 2017) and UDS (Galametz et al. 2013).

The GOODS-South (Guo et al. 2013) field contains 34,930 objects and covers an area of \( \sim 170 \) arcmin\(^2\), with a 5\( \sigma \) limiting depth of 27.4, 28.2, and 29.7 AB magnitudes in the three overlapping survey regions (CANDELS wide, deep, and HUDF regions). The GOODS-North field (Barro et al., in prep.) contains 35,445 objects over a similar area, with a 5\( \sigma \) limiting depth of 27.5 AB mag (Pacifici et al. 2016). The UKIRT Infrared Deep Sky Survey (UKIDSS) Ultra-Deep Survey (UDS) catalog (Galametz et al. 2013) contains 35,932 sources over an area of 201.7 arcmin\(^2\) with a 5\( \sigma \) limiting depth of 27.45 AB magnitudes. The Extended Groth Strip (EGS) catalog (Stefanon et al. 2017) contains 41,457 objects over an area of \( \approx 206 \) arcmin\(^2\) reaching a depth of 26.62 AB mag. The COSMOS field (Nayyeri et al. 2017) contains 38,671 objects covering an area of \( \approx 216 \) arcmin\(^2\) with a limiting depth of 27.6 AB mag. The catalogs select objects via SExtractor in dual-image mode using F160w as the detection band. The dual image mode (Galametz et al. 2013) is optimized to detect both faint, small galaxies in ‘hot’ mode without over de-blending large, resolved galaxies detected in ‘cold’ mode. The HST (ACS and WFC3) bands were point spread function (PSF) matched to measure photometry, and TFIT (Laidler et al. 2007) was used to measure the photometry of ground based and IRAC bands using the HST WFC3 imaging as a template. The SEDs in the five fields include a wide range of UV-to-NIR measurements, with the flux measured in 17, 18, 19, 23, and 43 photometric bands in GOODS-S, GOODS-N, UDS, EGS and COSMOS respectively. The photometric filters used for each field are detailed in Table 3.1.

In this paper, we focus on galaxies at \( 0.5 < z < 3.0 \), since we do not have multiple reliable rest-UV measurements from the HST bands at \( z < 0.45 \), leading to larger uncertainties in SFR and cannot accurately constrain the 1.6\( \mu \)m bump at redshifts \( z > 3.06 \) which is important for robust estimation of stellar masses. In addition to estimating robust stellar masses and star formation rates, it is necessary to ensure robust S/N in the rest-optical portion of the SED, which is most sensitive to variations in the SFH. As a proxy to the total S/N, we restrict our sample to galaxies with \( H < 25 \), where H is the HST/WFC3 F160w band. In addition to restricting our sample to those with good S/N, this cut also helps alleviate inhomogenities in the depths of the different fields, especially GOODS-N and GOODS-S, which have a wedding-cake structure. After implementing these selection cuts, we are left with a total of 48,791 galaxies. The effect of each selection effect and the total number of galaxies used for the analysis is given in Table 3.2 and Figure 3.11. To perform our fits, we use an updated CANDELS photometric redshift
catalog by Kodra et al. (in prep.) containing an increased number of spectroscopic redshift measurements as well as photometric redshifts with Bayesian combined uncertainties estimated by comparing the redshift probability distributions of four different SED fitting methods. We perform our fits using their $z_{\text{best}}$ binned to the resolution of our pre-grid, with $\delta z = 0.01$.

<table>
<thead>
<tr>
<th>Field</th>
<th>All Objects</th>
<th>Good Flags$^a$</th>
<th>$0.5 &lt; z &lt; 3$</th>
<th>$H &lt; 25^b$</th>
<th>$\chi^2_{\text{red}} &lt; 10$</th>
<th>Final samp</th>
</tr>
</thead>
<tbody>
<tr>
<td>GOODS-S (Guo et al. 2013)</td>
<td>34,930</td>
<td>31,273</td>
<td>22,713</td>
<td>8,520</td>
<td>8,299</td>
<td>8,299</td>
</tr>
<tr>
<td>GOODS-N (Barro et al., in prep.)</td>
<td>35,445</td>
<td>34,693</td>
<td>26,838</td>
<td>9,551</td>
<td>9,206</td>
<td>9,206</td>
</tr>
<tr>
<td>UDS (Galametz et al. 2013)</td>
<td>35,932</td>
<td>26,917</td>
<td>21,263</td>
<td>10,234</td>
<td>10,176</td>
<td>10,176</td>
</tr>
<tr>
<td>EGS (Stefanon et al. 2017)</td>
<td>41,457</td>
<td>31,714</td>
<td>24,444</td>
<td>10,554</td>
<td>10,261</td>
<td>10,261</td>
</tr>
<tr>
<td>COSMOS (Nayyeri et al. 2017)</td>
<td>38,671</td>
<td>30,070</td>
<td>22,092</td>
<td>10,883</td>
<td>10,849</td>
<td>10,849</td>
</tr>
</tbody>
</table>

Table 3.2: The number of galaxies used in our analysis, and the effect of each step in the selection process. $^a$: Galaxies with flags indicating no contamination by nearby objects, halos or star spikes, as well as objects with a low stellarity classification given by the SExtractor CLASS STAR output. $^b$: We select galaxies brighter than 25 mag in the HST/WFC3 F160W band, to ensure enough S/N in the rest-optical regime of the SED necessary for accurate SFH reconstruction. $^c$: This column gives the number of galaxies with confirmed spectroscopic redshifts in each field for our final analysis sample.

3.5 Results

3.5.1 The star formation histories of galaxies at $0.5 < z < 3$

We now apply the Dense Basis SED fitting method as described in Sec. 3.2 to the sample of CANDELS galaxies at $0.5 < z < 3.0$ to reconstruct the SFH of each galaxy with uncertainties. Figure 3.12 shows the distribution of the number of SFH parameters that individual galaxies are best fit with in each of the five fields, with a slight trend towards increasing amounts of SFH information recovered with more photometric bands.

In Figure 3.13 we show the distributions of reconstructed SFHs in the five CANDELS fields and how they evolve with mass and redshift. The median and interquartile range are computed at each point in time as in Pacifici et al. (2016), who performed a similar calculation for quiescent galaxies using a basis of SFHs derived from a semi-analytic model (De Lucia & Blaizot 2007). In the current analysis, we have used a redshift bin width of $\delta z = \pm 0.1$, and a mass bin width of $\delta M_* = \pm 0.25$ dex, such that the $M_* \sim 10^{10} M_\odot$ bin includes all galaxies with $10^{9.75} < M_* < 10^{10.25} M_\odot$. Dashed black lines in each panel show the mean SFR ($\equiv M_*/t_{\text{univ}}$) assuming
Figure 3.12: **Top:** Distributions of the number of SFH parameters estimated while fitting the SEDs of galaxies at $0.5 < z < 3.0$ in the five CANDELS fields, with the vertical dashed lines showing the mean value of the distribution. **Middle:** The distribution of total S/N $= \sqrt{\sum_j (F_{\nu,j}/\sigma_{\nu,j})^2}$ for the five fields, with the vertical dashed lines showing the medians of each distribution. **Bottom:** The distribution of the number of photometric bands used in fitting the SEDs in each field. The bimodality in the EGS observations is due to partial coverage of the field with the six NEWFIRM bands ($J1,J2,J3,H1,H2,K$).
Figure 3.13: The median star formation histories (SFHs) of galaxies in the five CANDELS fields in bins of stellar mass (horizontal) and redshift (vertical), showing how galaxies evolve with cosmic time and as they grow in stellar mass. Solid colored lines show the median SFH for each field separately, showing a remarkable similarity across the different fields in the majority of the bins. The shaded regions show the $16^{th} - 84^{th}$ percentile in the SFHs, highlighting the diversity in SFHs as a function of stellar mass and epoch. The dashed black line shows the mean SFR ($\equiv \frac{M_\star}{t_{\text{univ}}}$) assuming constant SFR for that redshift and mass bin, and the $f_{\text{samp}}$ at the top-left corner of each panel shows the fraction of all galaxies in our sample at that redshift that fall in a particular mass bin. In good agreement with cosmological simulations, semi-analytical and empirical models, galaxy SFHs tend to rise with time at low masses and high redshifts, starting to turn over at high masses, with the turnover mass decreasing as we go to lower redshifts. However, in addition to the average SFH behaviour, we also have access to the individual SFH for each galaxy, which now opens up the possibility of repeating this analysis to trace the evolution of SFHs with quantities like $t_{50}$, metallicity, morphology, central density, size, environment, and other probes of galaxy evolution.
constant SFR for that redshift and mass bin, and the $f_{\text{samp}}$ at the top-left corner of each panel shows the fraction of all galaxies in our sample at that redshift that fall in a particular mass bin. Since there can be a few galaxies that fall outside the plotted mass range, this may not sum to 1.

We see that the median SFH across the five CANDELS fields are remarkably similar, as expected. There are discrepancies in a few of the bins, most notably at $2 < z < 3$ and $M_* \sim 10^{10} M_\odot$ for UDS, which could be the result of correlated photometric noise, or smearing effects in the photo-z that was used for the calculation. In the highest redshift bin, the portion of the SED with wavelengths greater than rest-frame 1.6 $\mu$m are not well sampled, leading to poorer constraints on the SFHs in the the last row.

In general, SFHs tend to rise at high redshifts and low stellar masses, similar to those from cosmological simulations, with a turnover and subsequent decline as we go to higher masses and lower redshifts. In agreement with Pacifici et al. (2012); Behroozi et al. (2018), massive galaxies tend to peak earlier in their SFHs, and galaxies on average tend to move towards quiescence at lower masses as the universe grows older, with the threshold changing from nearly $10^{11.5} M_\odot$ at $z \sim 3$ to $10^{10} M_\odot$ at $z \sim 0.5$, without the need for any implicit assumptions about the SFHs, such as Behroozi et al. (2018)'s well motivated assumption that earlier forming halos get lower SFRs. The additional advantage of the Dense Basis method is that in addition to the average SFHs, the individual SFHs of galaxies reconstructed from the observations allow us to explore the additional factors that drive the diversity of SFHs at a given mass and epoch. This allows us to extend our analysis beyond the physics encoded in the stellar mass-halo mass relation, which gives a constraint upon the first order behaviour of SFHs.

3.5.2 The number of major episodes of star formation experienced by galaxies

Most studies of galaxy SFHs focus on the overall rise and fall of an ensemble of SFHs (Leitner 2012; Pacifici et al. 2016; Ciesla et al. 2017; Leja et al. 2017), which has led to a well-constrained understanding of the overall behaviour seen in Figure 3.13. However, with smooth, non-parametric SFHs it is now possible to ask questions about the second order statistics of an ensemble of SFHs, analyzing the departures from this overall behaviour in the form of periods of relative quiescence between episodes of star formation.

In Figure 3.14 we show the fraction of galaxies with different numbers of major episodes of star formation in a galaxy’s past at redshifts $0.5 < z < 3.0$. Since the number of episodes are
Figure 3.14: The fraction of galaxies which show multiple major episodes of star formation in their SFHs as a function of stellar mass at various redshifts for galaxies in the five CANDELS fields. The solid line shows a running median within a bin of \( \pm 0.25 \) dex in stellar mass, and the shaded regions show the uncertainty for the estimates assuming Poisson noise.
a discrete quantity, Poisson noise dominates the formal uncertainties in an individual galaxy’s number of episodes while calculating functions of the sample, as discussed in §3.3. The different fields (colored lines), are in good agreement with each other and the median of the full sample (solid black line).

We find that at low redshifts, the fraction of galaxies with multiple major episodes of star formation decreases as we go up in stellar mass above $10^{10.5} M_\odot$, in agreement with Iyer & Gawiser (2017). In addition to this, we find a slight decrease in the overall fraction of galaxies with multiple episodes with increasing redshift at any given mass, with the notable exception of $M_* \approx 10^{10.5} M_\odot$, with does not show a noticeable evolution with redshift. Although we have accounted for S/N variations, the decrease at lower masses could be at least partially due to insufficient S/N to resolve multiple episodes of star formation as we go to lower masses and higher redshifts. A few explanations are possible for the behavior at high masses: AGN feedback quenching galaxies (Weinberger et al. 2018) could lead to SFHs that form most of their stars at $z \sim 3$, which could look like a single early episode of star formation without the S/N in the SEDs to resolve the older populations at $z \sim 1$. This is made more probable by the fact that while most galaxies with multiple episodes are found to lie on the SFR-M$_*$ correlation, the greatest number of galaxies with low SFRs at the time of observation and multiple episodes occurs at masses close to $10^{10.5} M_\odot$. Another reason could be the central limit theorem (Kelson 2014): massive galaxies that grow primarily through mergers (Brinchmann et al. 2004; Pérez-González et al. 2008; Bundy et al. 2005) at early times could be composed of multiple progenitors. As the number of progenitors grows with mass, by the central limit theorem their star formation histories should look smoother than those for less massive galaxies. Low mass galaxies in comparison should have more stochastic star formation histories since they are growing most of their mass in-situ, which would be in close agreement with the findings of Guo et al. (2016); Matthee & Schaye (2018); Emami et al. (2018); Shivaei et al. (2015); Broussard et al. (2019).

If a galaxy is found to have multiple strong episodes of star formation in its lifetime, an interesting question would be whether the galaxy was actively forming stars and the star formation was temporarily suppressed by a quenching attempt (short time interval between peaks), as opposed to a galaxy that was on its way to quiescence but restarted star formation due to an inflow of pristine gas or merger. This is especially interesting within the context of rejuvenation of galaxy SFRs (Fang et al. 2012), since it sets timescales for how long a galaxy spends off the star-forming sequence when it makes such an excursion. To quantify this, we measure the time interval between multiple peaks for the subsample of galaxies with $N_{ep} > 1$. We plot
this as a function of redshift and mass in Figure 3.15, finding that although the separation
doesn’t vary strongly with mass, it does show a strong trend with redshift. However, upon nor-
malizing by the age of the universe at different redshifts, this trend is significantly decreased,
leaving us with a roughly constant timescale across which galaxy rejuvenation occurs, given by
\[ t_{\Delta\text{peak}} \sim 0.42^{+0.15}_{-0.10} t_{\text{univ}} \text{ Gyr}, \]
where \( t_{\text{univ}} \) is the age of the universe at the redshift of observation.

This is similar to the result by Abramson et al. (2016), which found the transit time through
the green valley (i.e. half the time between two peaks for a rejuvenating SFH) to be \( \sim 0.2 t_{\text{univ}} \)
Gyr roughly independently of redshift (see also Pandya et al. 2017, which studies the transition
timescales in massive CANDELS galaxies using a statistical analysis of their number densities).

This is also related to the result by Pacifici et al. (2016) that found that the width of the SFH
for quiescent galaxies is roughly constant across stellar mass and redshift when the age of the
universe is factored out, and with Muzzin et al. (2014), who find that the post-starburst spectra
of galaxies at \( z \sim 1 \) are well fit with a quenching timescale of \( 0.4^{+0.4}_{-0.3} \) Gyr. Fang et al. (2012)
identify a subsample of galaxies at \( z \sim 0.1 \) that could linger in the green valley for \( \mathcal{O}(\text{Gyr}) \).
The astute reader may wonder how significant it is to find that two episodes are typically separated
by roughly half the age of the universe at the time of observation, as this also corresponds to
the median period of a generic sine wave possessing two peaks without that interval. Given the
present data quality, it is difficult to test this further by investigating the separation between
the earliest two star formation episodes in galaxies whose SFHs show 3-5 major episodes of
star formation, but that should be done with higher S/N spectrophotometry. At present, we
can compare the fit for separation between episodes against the predictions of galaxy formation
models, finding similar trends albeit with a slightly smaller value of \( \approx 0.3 \pm 0.15 t_{\text{univ}} \) Gyr. The
consistency of our result with a variety of similar results across a range of redshifts summarized
above is an additional reassuring check. In Behroozi et al. (2018), galaxy rejuvenation is a
generic feature of a population, with the timescale depending on the mode by which a halo is
accreting mass: through mergers, accretion from another halo or infall. In this scenario, reju-
venation occurs more often when the quenched population evolves more slowly than the halo
dynamical time, during which it can switch between modes of accretion, increasing or decreasing
the SFR as a result. This results in estimates of the fraction of galaxies with past rejuvenation
as a function of mass, decreasing as redshift increases and showing a trend in stellar mass that
is consistent with our results. Our result seems to indicate that the rejuvenation timescales
remain relatively constant over cosmic time. It is important to note that the scatter in this
quantity is quite large, and the evolution in the quenched fraction happens most rapidly at
redshifts \( 0 < z < 0.5 \) (Muzzin et al. 2013; Behroozi et al. 2018; Hahn et al. 2018; Donnari et al.
Figure 3.15: The separation between multiple peaks of star formation, as a function of mass and redshift for the subsample of galaxies that have $N_{ep} > 1$. The redshift bins are the same as fig 3.14 and the solid line and shaded region show the median and 16-84th percentiles respectively. The top panel shows the distribution across stellar mass at different redshifts. The bottom shows the same, but divided by the age of the universe at that epoch.

2018), so the extrapolation to that regime needs to be tested with further data.

3.5.3 The different demographics of galaxies

In keeping with Pacifici et al. (2016)’s finding that the widths of the SFHs of passive galaxies are roughly constant upon factoring out the age of the universe and our similar finding for the time interval between two peaks of star formation, we consider galaxy SFHs binned in $t_{50}$. We bin galaxies in four bins, from $0.1t_{univ} < t_{50} < 0.3t_{univ}$, $0.3t_{univ} < t_{50} < 0.5t_{univ}$, $0.5t_{univ} < t_{50} < 0.7t_{univ}$, and $0.7t_{univ} < t_{50} < 0.9t_{univ}$ at different redshifts. We show the results in Figure 3.16. As in Figure 3.13, each panel lists the fraction of the sample in a particular bin. However in this case, the fractions are no longer tracing the mass function of galaxies. Instead, the four bins in time serve as proxies for galaxy SFHs in different stages of their lifetimes. This enables us to identify different populations of galaxies, including starbursting
galaxies, late bloomers (Dressler et al. 2016), star-forming galaxies, post-starburst or green-valley galaxies (Fang et al. 2012) and quiescent galaxies using different redshift-dependent $t_{50}$ cuts, either independently or in combination with other factors like $t_{25}, t_{75}$, size and morphology. To further interpret these SFHs, Figure 3.17 looks at the positions of the galaxies within each panel in Figure 3.16 on the SFR-$M_*$ plane. We find that the left (right) columns in both figures select star forming (quiescent) galaxies that lie on (off) the star-forming sequence. At intermediate values of $t_{50}$, the populations are a combination of star forming and quiescent galaxies, with the selection gradually shifting from star-forming to quiescent galaxies. The intermediate $t_{50}$ panels also show an excess of galaxies with multiple strong episodes of star formation ($N_{ep} > 1$). While this is an intuitive result, since galaxies that have assembled most of their mass recently or those that have long since shut off their star formation are not very likely to contain multiple episodes of star formation, it has important implications for analyses that assume that galaxies evolve along smooth SFH trajectories (Leitner 2012) or are described by simple parametric forms (Dressler & Abramson 2014; Ciesla et al. 2017; Lee et al. 2017).

3.5.4 Correlation with morphology

The morphologies of galaxies are seen to strongly correlate with their stellar masses and redshifts (Conselice 2014), as well as sSFR (Whitaker et al. 2015). While this is a combined effect of the different processes that regulate star formation within galaxies, including mergers, gas accretion through inflows, stellar and AGN feedback (Hopkins et al. 2008, 2014; Anglés-Alcázar et al. 2017; Weinberger et al. 2018; Boselli & Gavazzi 2006), it is difficult to observationally disentangle the relative strengths of these effects. However, the different timescales that these processes act on enable us to discriminate between the relative effects of these processes if we can observationally constrain the timescales on which morphological transformation occurs across a population of galaxies, as was done for groups in eg. Kovač et al. (2010). The reconstructed SFHs of galaxies provide a direct probe of these timescales by correlating the morphologies of galaxies with their SFHs as compared to indirect measurements of timescales through the frequencies of different morphologies, which are subject to a variety of systematics and selection effects.

We use the CANDELS wide morphology catalogs by Kartaltepe et al. (2015) to study the star formation histories of galaxies with different morphological features at $0.5 < z < 1.0$. We limit our redshift range to avoid the effects of small number statistics of classifications as we go to higher redshifts. The morphology catalogs contain visual classifications for over 50,000 objects spanning $0 < z < 4$ with $f_{160w} < 24.5$, which gives a large overlap with our sample. The catalogs contain flags for main morphology class (disk, spheroid, peculiar/irregular, point
Figure 3.16: SFHs split into linearly increasing bins of $t_{50}$, the lookback time at which a galaxy assembled 50% of its stellar mass, from $0.1t_{\text{univ}} < t_{50} < 0.3t_{\text{univ}}$ (formed recently), $0.3t_{\text{univ}} < t_{50} < 0.5t_{\text{univ}}$, $0.5t_{\text{univ}} < t_{50} < 0.7t_{\text{univ}}$, and $0.7t_{\text{univ}} < t_{50} < 0.9t_{\text{univ}}$ (formed earliest) in four bins of redshift. The plotting scheme and colors are the same as Figure 3.13.

In each bin, the SFHs are normalized to the same mass since we are most interested in the diversity of SFH shapes for the entire demographic. Vertical dashed black lines show the $t_{50}$ bounds for each panel. We see that the SFHs in a bin broadly tend to describe one of four demographics of galaxies: starbursting galaxies at high redshifts and late bloomers at $z \sim 0.7$ (Dressler et al. 2016) can be found in the first column from the left, star forming galaxies contribute to the median SFH in columns 1-3, post-starburst or green-valley galaxies (Fang et al. 2012) in columns 2-3 and quiescent galaxies in columns 3-4. The $f_{\text{samp}}$ at the top left of each panel shows the fraction of galaxies at each redshift that fall into each demographic. In addition to the UVJ diagram and position on the SFR-$M_*$ plane, the SFHs of galaxies allow for additional diagnostics regarding its evolutionary phase.
Figure 3.17: Positions on the SFR-M∗ plane for the galaxies shown in each panel of Figure 3.16 above. The underlying grey heatmap shows the full sample at a given redshift, and blue points show all galaxies satisfying $0.1t_{\text{univ}} < t < 0.3t_{\text{univ}}$ (formed recently), $0.3t_{\text{univ}} < t < 0.5t_{\text{univ}}$, $0.5t_{\text{univ}} < t < 0.7t_{\text{univ}}$, and $0.7t_{\text{univ}} < t < 0.9t_{\text{univ}}$ (formed earliest) within a redshift bin. The black crosses are a subset of the blue points that are identified as having more than one major episode of star formation during their lifetimes ($N_{\text{ep}} > 1$), with this fraction of galaxies given in the bottom right corner. Black contours are used where there are more than 100 galaxies with $N_{\text{ep}} > 1$ in a given panel.
Figure 3.18: The star formation histories of galaxies at $0.5 < z < 1.0$ with different morphological features identified in Kartaltepe et al. (2015). For each class, the top panel shows the position of the galaxies under consideration (colored points) in the SFR-$M_*$ plane, with the full population shown as black dots. The bottom panel shows the median SFH (blue solid line) and diversity ($16^{th}$ to $84^{th}$ percentile) shown as a shaded blue region, of all the galaxies with $M_* \in [10^{10}, 10^{10.5}] M_\odot$ that satisfy a particular morphological criterion, except for the last bin, where we have chosen a lower mass bin due to insufficient statistics. The second and third panels show galaxies that have both a disk and bulge component, which are then broken down into disk-dominated as opposed to bulge-dominated galaxies.
source/compact, and unclassifiable), a class for mergers and other interactions and structure flags for bars, tidal features, spiral arms and more. For each class and flag, the catalog reports the fraction of classifiers who were confident about the existence of that feature.

We use this to analyse the SFHs of six classes of galaxies, described as follows:

- **Disk:** \((f_{\text{disk}} > 0.9)\) AND \((f_{\text{sph}}, f_{\text{irreg}} < 0.1)\). This includes the set of all galaxies classified as disky galaxies.

- **Disk dominated galaxies:** \((f_{\text{disk dom}} > 0.9)\). Disks with a central bulge where the disk dominates the structure.

- **Bulge dominated galaxies:** \((f_{\text{bulge dom}} > 0.9)\). Disks with a central bulge where the bulge dominates the structure.

- **Spheroid:** \((f_{\text{sphk}} > 0.9)\) AND \((f_{\text{disk}}, f_{\text{irreg}} < 0.1)\). This includes the set of all galaxies classified as spheroidal galaxies.

- **Galaxies with spiral arms:** \((f_{\text{arms}} > 0.9)\).

- **Mergers and interactions:** galaxies that are either appear to have undergone a merger as evidenced by tidal features, structures such as loops or highly irregular outer isophotes \((f_{\text{merger}} > 0.9)\) or are interacting with a companion galaxy within the segmentation map from SExtractor \((f_{\text{int1}} > 0.9)\).

The results of this analysis are shown in Figure 3.18. The Figure contains six sets of panels, one for each subsample of galaxies. The top panel shows where the galaxies lie on the SFR-\(M_*\) correlation, and the bottom panel shows the median SFH for the subsample of these galaxies at \(M_* \in [10^{10}, 10^{10.5}]M_\odot\). This is useful to test feedback driven models of quenching that posit a correlation between bulge-total ratios and SFH shape (Zolotov et al. 2015; Tacchella et al. 2016; Belfiore et al. 2016; Abramson et al. 2018). While the SFHs of our pure disk population at \(M_* \sim 10^{10.25}M_\odot\) seem to actively form stars throughout their lifetime, with the maximal peak in their SFHs maximum SFR close to the time of observation, the galaxies containing a disk and bulge component seem to show a downward trend in their median SFHs, with disk dominated galaxies peaking earlier on average than pure disks without a bulge, followed by a decline in SFR. This trend continues to bulge dominated galaxies and spheroids, showing an evolution in timescales that can be tested with simulations implementing different models for quiescence. For each population, we consider the time since the SFH of each galaxy peaked \((t_{\text{fall}})\) and use this distribution to quantify the timescale on which galaxy SFHs began their decline. For each morphologically distinct population, we find the median and 68% of the \(t_{\text{fall}}\) distribution for galaxies at \(0.5 < z < 1.0\) and \(10^{10} < M_* < 10^{10.5}M_\odot\) given by:
• Mergers: \( t_{fall} : 0.00^{-0.00}_{+0.39} \) Gyrs

• Galaxies with spiral arms: \( t_{fall} : 0.60^{-0.54}_{+1.54} \) Gyrs

• Disks: \( t_{fall} = 0.81^{+0.80}_{-2.50} \) Gyrs

• Disk-dominated galaxies: \( t_{fall} : 0.70^{-0.38}_{+2.73} \) Gyrs

• Bulge-dominated galaxies: \( t_{fall} : 2.15^{-1.55}_{+3.07} \) Gyrs

• Spheroids: \( t_{fall} : 2.50^{-1.60}_{+2.25} \) Gyrs

In the absence of a bulge component, we also see that galaxies with spiral arms inhabit the high-stellar mass, high SFR portion of the SFR-\( M_\ast \) plane, continuing to actively form stars till they lose rotational support or start forming bulges. We also see that mergers and interactions show a noticeable increase in recent SFR, over timescales within the last \( \sim 0.5 \) Gyr of their SFHs. The timescales for these morphological transformations can be further constrained by determining the resolved SFHs of individual galaxies using IFU surveys like SDSS-IV MaNGA and CALIFA (Delgado et al. 2014; Belfiore et al. 2016).

3.6 Discussion

3.6.1 Improvements from better datasets, models, and priors

Since non-parametric methods make no explicit assumption about the form of SFHs, they are only as good as the data being used for SED fitting. In this regard, there are three main avenues for data-driven improvement: better wavelength resolution, better wavelength coverage, and better S/N. Spectroscopy contains more information about stellar populations of different ages and metallicity as compared to broadband photometry, but often suffers from wavelength dependent flux calibration issues that need to be accounted for prior to fitting. Panchromatic SEDs allow us to test models of dust attenuation and re-emission to better constrain dust effects while estimating the SFHs of galaxies.

The SFH reconstructions we obtain are also subject to several modeling uncertainties: Stellar Population Synthesis models can introduce systematics into the mapping between physical parameters and observed photometry (Conroy & Gunn 2010; Han & Han 2018). Differences in dust models can introduce systematics into the measurement of recent SFR, which would then propagate into differences in the SFH. Pacifici et al. (2019) in prep. compares the results from 14 different SED fitting codes applied to the same sample of CANDELS/GOODS-S galaxies at \( z \sim \)
1. This allows us to examine the effects of inter-code variability and model assumptions for dust, IMF, SFH, and SPS models including the effects of binary populations. Additionally, Han & Han (2018) tested multiple SPS models, SFH assumptions and dust models using a comprehensive bayesian formalism that allowed them to estimate the bayesian evidence in comparing different models.

Finally, the choices of prior assumed during SED fitting are extremely influential in the estimates of physical parameters and their covariances. While we have tried to be agnostic about the priors in this work, it is important to note that an informative prior could be especially useful while fitting noisy, low S/N data with limited wavelength coverage. Predictive checks could be put in place to ensure that the priors do not introduce significant biases into the estimates, or cause artificially tight correlations due to regression to the mean. These informative priors could be developed by studying the distributions of physical quantities at a particular epoch from a small subset of high S/N observations, scaling relations and mass functions, as well as semi-analytic or empirical models that encode the physics that lead to these observables, explicitly quantifying the covariance between star formation, chemical attenuation and dust enrichment and destruction histories.

3.6.2 SFHs as a probe to higher redshifts

The SFHs of galaxies allow us to probe the behavior of mass functions and scaling relations out to higher redshifts than is currently possible. At low to intermediate redshifts, this can be used as a consistency check, to ensure that the reconstructed SFHs are not biased due to noise or prior assumptions. At high redshifts, this can be a powerful tool to increase observational statistics and push measurements out to higher redshifts than those directly accessible through observations. Since we can only measure the SFH summed over all progenitors, it is important to consider the effects of mergers while propagating galaxies backwards in time for periods longer than the typical merger timescale (Mantha et al. 2017; Duncan et al. 2019) at a given redshift and stellar mass.

Iyer et al. (2018) propagated galaxies backwards in time along their SFHs in the form of trajectories in SFR-M∗ space to probe the high-redshift low stellar-mass regime of the SFR-M∗ correlation, finding that the projected correlation at intermediate redshifts matches the observed distribution well, and extending it by nearly two orders of magnitude out to z ∼ 6 where observations are extremely faint. Pacifici et al. (2019), in prep. implements a validation test by reconstructing the stellar mass function using galaxies at lower redshifts and comparing
them to the stellar mass function obtained through direct fits. Leja et al. (2018) use star formation histories to probe the cosmic SFRD using galaxies at low redshifts, finding that the apparent mismatch between the mass functions and star formation rate functions is alleviated using non-parametric SFHs.

3.6.3 Galaxy evolution studies enabled by SFH reconstruction

The smooth, non-parametric star formation histories obtained with the improved Dense Basis method offer a window into the pasts of different galaxy populations. While interesting itself, this has the potential to be combined with a variety of ancillary data to probe a wide range of previously inaccessible quantities, some of which we briefly describe below:

- Higher S/N observations or spectrophotometric data would help obtain better SFH constraints, allowing us to better constrain the number of major episodes of star formation and the timescales on which rejuvenation, starbursts, and quiescence occur at different epochs. While current observations studying the separation between multiple major episodes or transition timescales (this work, Abramson et al. (2016); Pandya et al. (2017)) show a flat trend with mass and a linear one with the age of the universe, it is an interesting problem to understand the physical mechanisms responsible for this trend and the dispersion of \( \sim 0.25t_{\text{univ}} \) Gyr using simulations.

- Spatially resolved SFHs computed using IFU data from surveys like SDSS-IV MaNGA and CALIFA allow us to better understand the correlation between the SFH and morphology and discriminate between inside-out vs outside-in scenarios for galaxy growth and quenching (Goddard et al. 2016), better examine the connection between the physical properties of individual regions within galaxies and their SFHs (Rowlands et al. 2018) and test scaling relations at different regimes (Hsieh et al. 2017). Care needs to be exercised in interpreting these results since we only see where the stellar populations are today. Additional kinematic information would help alleviate this problem to a certain extent.

- Correlating SFHs with environment, size, kinematics, central density and morphology in addition to stellar mass, SFR and redshift could help build a unified picture of how galaxies evolve, with the SFHs providing a link between galaxy populations of different types and their earlier progenitors. Although this approach is similar to empirical models (Behroozi et al. 2018; Moster et al. 2018), it has the advantage of much richer observational constraints from the individual SFHs of galaxies. A comparison of SFH distributions
between simulations and observations would allow us to qualify additional factors that are not directly accessible.

- The Gaussian Process based parametrization can also be used as a general compression method for compressing and storing PDFs from all kinds of codes, similar to Malz et al. (2018).

3.6.4 Caveats in SED fitting and SFH reconstruction

While we have performed an extensive range of validation tests (§3.3) to ensure that all the quantities reported in this work are robust, there are some caveats to keep in mind while extending the SED fitting to different datasets or the analysis beyond what is performed here.

- **SFHs are not mass accretion histories:** The SFH is a record of when the stars present in a galaxy at the time of observation were formed, as opposed to the mass accretion history, which is a record of when those stars entered the galaxy. These two quantities are the same for stars formed in-situ, but are differ when the stars were brought in through mergers. This needs to be taken into account in certain kinds of analysis, for example by using a mass- and redshift-dependent merger fraction that to correct for mass functions calculated by propagating galaxies backwards in time along their SFHs.

- **Lack of sensitivity to the shortest timescales:** The smooth SFHs reconstructed using SED fitting in this work can not capture starbursts that can happen on extremely short timescales of $\sim \mathcal{O}(10) \text{ Myr}$. While fitting galaxies from the semi-analytic model that contain such starbursts, we generally find that the overall stellar mass is well recovered, but the starburst is smeared out over larger timescales, depending on when it occurred. This needs to be accounted for in the uncertainty budget for example, while calculating the scatter along the SFR-$M_*$ correlation using galaxies propagated backwards in time along their SFR-$M_*$ trajectories.

- **Non-uniform sensitivity to variations in SFH:** SED fitting is more sensitive to recent star formation than it is to star formation older than a few Gyr. As we go back in time, our SFH reconstruction transitions from being likelihood dominated to being prior dominated, and while we show that this does not cause biases in our SFH reconstruction in §3.3, it does mean that we are less sensitive to sharp variations in SFH at large lookback times compared to closer to the time of observation.
• **Correlation with chemical enrichment histories:** While we have considered the problem of estimating the star formation histories of galaxies in this work, in practice they are highly correlated with the chemical enrichment histories of galaxies. While metallicity is poorly constrained in our current observations, while working with higher S/N data or spectra the analysis should include a joint model for SFR(t) and Z(t), which can be achieved through joint priors on the metallicity given by Z(M_*, \{t_x\}) informed using simulations.

### 3.7 Conclusions

Studying the star formation histories (SFHs) of galaxies lets us better understand the timescales on which different physical processes shape galaxy growth. High S/N multiwavelength observations from current and upcoming galaxy surveys make it possible to reconstruct the SFHs for large ensembles of galaxies with suitably sophisticated analysis techniques.

We update the Dense Basis Spectral Energy Distribution (SED)-fitting method (Iyer & Gawiser 2017) using a flexible SFH parametrization described by the tuple (M_*, SFR, \{t_x\}) where M_* is the stellar mass, SFR is the star formation rate averaged over the past 100 Myr, and the set \{t_x\} contains the lookback times at which a galaxy formed N equally spaced quantiles of its stellar mass. These parameters represent a set of integral constraints and SFHs corresponding to a particular tuple are constructed using Gaussian Process regression in cumulative mass vs time, which creates smooth curves that satisfies these constraints and is completely independent of the choice of a functional form. We reconstruct the SFHs of galaxies with uncertainties using a brute-force Bayesian approach with a large pre-grid of model SEDs. To make the method fully non-parametric, we determine N on an SED to SED basis using a Bayesian Information Criterion (BIC) based selection. Using the reconstructed SFHs and a peak finding algorithm, we determine the number of major episodes of star formation in a galaxy’s past.

The method provides the following advantages:

- The method encodes the maximal amount of SFH information in a minimal number of parameters.

- Being independent of the choice of a functional form, it does not suffer from the traditional biases associated with simple parametric assumptions for the SFH shape.

- The method also circumvents the pitfalls associated with the traditional non-parametric approach of describing SFHs as fixed bins in lookback time with constant SFR within a
bin such as artifacts due to bin edges and reduces uneven S/N distribution across different parameters.

- The parameters used to describe SFHs are physically interpretable, and allow easy comparison between different datasets from observations and simulations.

- Informative priors can be constructed by studying these parameters in cosmological simulations, which can be used while fitting low S/N data or SEDs with partial wavelength coverage.

- The method is computationally fast, able to fit \( \sim 34 \) galaxies /minute/core on a 2.9 GHz Intel processor, and capable of being adapted to most data compression problems.

We apply the method to a sample of 48,791 galaxies across the five CANDELS fields with HST/WFC3 \( F160W < 25 \) and \( 0.5 < z < 3.0 \).

We use the reconstructed SFHs to study galaxy evolution across stellar mass and redshift, and quantify the fraction of galaxies at each epoch that have multiple strong episodes of star formation. For the galaxies that show multiple strong episodes of star formation, we find that timescale separating two peaks in the SFH is roughly constant with mass, and increases linearly with the age of the universe as \( t_{\text{peak-to-peak}} \sim 0.42^{+0.15}_{-0.10} t_{\text{univ}} \) Gyr. We also find that classifying galaxies by \( t_{50} \) is a robust way of selecting for star forming galaxies at a given epoch.

Using the Kartaltepe et al. (2015) morphology catalog, we can examine the SFHs for subsets of galaxies with particular morphological features, finding the expected correlation between the SFHs of galaxies and morphological features. In addition, we quantify the timescale on which the SFH declines as a function of morphology, finding that this increases from \( \sim 0.60^{+0.54}_{-0.54} \) Gyr for galaxies with spiral arms to \( 2.50^{+1.60}_{-2.25} \) Gyr for spheroids.

The SFH formalism presented here is broad in scope and can be incorporated into any SED fitting code, can be used to compress and store SFHs in simulations, and can be used as a common parametrization to compare SFHs across different observations and simulations.
Chapter 4

Galaxy evolution with SFHs: the low-mass high-redshift star forming sequence


https://iopscience.iop.org/article/10.3847/1538-4357/aae0fa
https://arxiv.org/abs/1809.04099

4.1 Introduction

The SFR-M∗ correlation couples a galaxy’s Star Formation Rate (SFR), an effectively instantaneous quantity, to its stellar mass (M∗), accumulated over its lifetime (Noeske et al. 2007; Daddi et al. 2007; Elbaz et al. 2007; Salim et al. 2007). The correlation persists across a wide range of stellar masses and SFRs and over a range of redshifts (Whitaker et al. 2012; Speagle et al. 2014; Salmon et al. 2015; Tasca et al. 2015; Schreiber et al. 2015; Kurczynski et al. 2016; Johnston et al. 2015; Santini et al. 2017). This has led to speculations about its origin, with theories suggesting this is controlled by halo mass accretion (Dutton et al. 2010; Forbes et al. 2014; Rodríguez-Puebla et al. 2015), or the regulation of gas infall and feedback (Tacchella et al. 2016; Mitra et al. 2016), or that the observed correlation is simply a cross-section of a more fundamental SFR-M∗-Z relation, with the evolution explained by increasing metallicities with cosmic time (Mannucci et al. 2010; Lilly et al. 2013; Torrey et al. 2017).

Galaxy formation models predict the evolution of individual star-forming galaxies comprising the SFR-M∗ correlation. Testing these predictions has been difficult, with observations up to now unable to reveal if individual galaxies evolve along the correlation, as assumed by the ‘Main Sequence Integration’ technique (Leitner 2012; Muñoz & Peeples 2015) or make significant excursions above and below it (Pacifici et al. 2012; Tacchella et al. 2016).
One of the most common ways of probing the SFR-M_* relation is estimating the stellar mass and SFR through SED fitting, with modern techniques capable of handling large quantities of data and extracting high-fidelity information. A key improvement in obtaining the stellar masses and SFRs comes from relaxing the assumption that a galaxy’s Star Formation History (SFH) be described by a single parametric form such as exponentially declining or constant star formation (Iyer & Gawiser 2017; Pacifici et al. 2012; Acquaviva et al. 2011a; Lee et al. 2017; Ciesla et al. 2017). Although the stellar masses and SFRs obtained through SED fitting offer a probe of the SFR-M_* correlation to extremely high redshifts, it is sensitive to the systematic assumptions inherent in SED fitting, as well as the decreasing S/N as we approach dimmer objects and higher redshifts. To probe the correlation in this regime, we thus need to probe beyond these traditionally estimated SED-fit quantities.

In this paper, we apply the Dense Basis SED fitting method developed in Iyer & Gawiser (2017), which reconstructs a galaxy’s Star Formation History using the best-fit from among multiple families of smooth SFHs. We use these SFHs to construct SFR-M_* trajectories, along which individual galaxies are propagated backwards in time to reveal the SFR-M_* diagram at higher redshifts. This method allows us to gain statistical power from the large number of low-redshift galaxies that can contribute to the SFR-M_* correlation at higher redshifts. In contrast to direct fits, these trajectories possess the advantage of probing lower masses as we extend the method to higher redshifts. Currently, the lowest stellar masses the correlation has been probed at involve using galaxies observed in the Hubble Ultra Deep Field to go down to $10^7 M_\odot$ at $z \sim 1.5$ (Kurczynski et al. 2016), or using gravitationally lensed galaxies in the HST Frontier Fields to go down to $10^{8.8} M_\odot$ at $z \sim 6$ (Santini et al. 2017). In this paper, we show that our technique of reconstructing SFR-M_* trajectories allows us to recover the SFR-M_* correlation down to $10^6 M_\odot$ at $1 < z < 6$.

The paper is structured as follows; in §4.2, we specify the choice of dataset, followed by the details of our analysis in §4.3. In §4.4 we present our main results, describe the validation tests we performed in §4.5, and discuss the implications and caveats in §4.6. Throughout this paper magnitudes are in the AB system; we use a standard ΛCDM cosmology, with $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$ and $H_0 = 70$ km Mpc$^{-1}$ s$^{-1}$. 
Figure 4.1: Redshift distribution of the CANDELS/GOODS-S galaxies at $0.5 < z < 6.0$ (blue line) and for the sample we use in our analysis (solid green histogram) excluding bad fits and galaxies with F160w magnitudes $< 25.9$, 26.6 and 28.1 in the wide, deep and HUDF regions, respectively.

4.2 Dataset

We fit 17-band photometric SEDs\(^1\) spanning the rest-frame UV through near-IR (IRAC) from the Cosmic Assembly Near-Infrared Deep Extragalactic Legacy Survey (CANDELS; Grogin et al. (2011); Koekemoer et al. (2011); Nonino et al. (2009); Retzlaff et al. (2010); Fontana et al. (2014); Ashby et al. (2015)). The catalog selects objects in the GOODS-S field via SExtractor in dual-image mode using F160w as the detection band. The dual image mode (Galametz et al. 2013) is optimized to detect both faint, small galaxies in ‘hot’ mode without over de-blending large, resolved galaxies in ‘cold’ mode. The HST (ACS and WFC3) bands were point spread function (PSF) matched to measure photometry, and TFIT (Laidler et al. 2007) was used to measure the photometry of ground based and IRAC bands using the HST WFC3 imaging as a template. We consider galaxies in the GOODS-S field (Guo et al. 2013) at redshifts $0.5 < z < 6$. The three different depths (wide, deep, HUDF) allow us to probe a population of galaxies across a wide range of masses to construct SFR-M\(^*\) trajectories. To reduce the effects of incompleteness for the sample while accounting for the different depths, we require our sample to be brighter than a F160w magnitude limit of 25.9, 26.6 and 28.1 in the wide, deep and HUDF regions, respectively (Guo et al. 2013).

In performing our fits, we exclude objects that are marked as stars or X-ray detected AGN (Hsu et al. 2014) in the Santini et al. (2015) mass catalog (487 objects), as well as poor fits in our SED fitting routine ($\chi^2 > 50$, 595 objects). After these cuts, our analysis includes

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\(^1\)The photometric bands used are: U/(CTIO), U/(VIMOS), HST/ACS F435w, F606w, F775w, F814w, F850lp, HST/WFC3 F098w, F105w, F125w, F160w, VLT/HAWK-I Ks, VLT/Isaac Ks, and Spitzer/IRAC 3.6, 4.5, 5.8, 8µm. (Guo et al. 2013)
∼ 17,873 galaxies (94.7% of our parent sample). To perform our fits, we use an updated photometric redshift catalog by Kodra et al (in prep.) containing an increased number of spec-z measurements as well as photometric redshifts with Bayesian combined uncertainties estimated by comparing the redshift probability distributions of four different SED fitting methods. Figure 4.1 shows the redshift distribution of our analysis sample. We perform our fits using their $z_{\text{best}}$ binned to the resolution of our pre-grid, with $\delta z = 0.01$. The $z_{\text{best}}$ in our redshift range includes ∼ 1917 spectroscopic and ∼ 384 grism redshifts, in addition to photometric redshifts.

4.3 Methodology

4.3.1 SED fitting

The Dense Basis method described in Iyer & Gawiser (2017) uses a physically motivated basis of Star Formation Histories to generate an atlas of template SEDs. The best-fit SFH, dust and metallicity values for each observed galaxy SED are computed using standard $\chi^2$ minimization over the entire atlas. To generate spectra corresponding to a galaxy with a given basis SFH, we use the Flexible Stellar Population Synthesis (FSPS) model (Conroy et al. 2009; Conroy & Gunn 2010; Foreman-Mackey et al. 2014). We use a Chabrier IMF (Chabrier 2003), Calzetti dust reddening (Calzetti 2001), and IGM absorption according to the Madau et al. (1996) prescription. Star Formation Histories are drawn from the Linexp (linear rise followed by exponential decline, sometimes called Delayed-$\tau$ models), Gaussian and Lognormal families of curves, excluding SFHs that are extremely similar in shape to reduce the size of the basis, with a slight modification of Iyer & Gawiser (2017), where we also considered Bessel function rise followed by exponential decline (Bessel-exp), Top-hat (Exp) and Constant Star Formation (CSF) histories. In this work, we do not consider these three families because Bessel-exp SFHs are extremely similar in shape to the SFHs we already consider in our basis, and Exponential and CSF were shown in Iyer & Gawiser (2017) to lead to biased estimates of galaxy properties. The parameter ranges for the various families are given in Table 4.1. Defining the slope as a tangent to the log SFR-log $M_*$ trajectory at a given point, the range of SFH shapes lead to trajectories that can have a wide range of slopes at low masses ∼ [0,16] as well as flat and negative slopes ∼ [−34,52] at high masses, as galaxies enter a quiescent phase. Figure 4.2 (a,b) shows examples of SFHs from each of the three families, as well as their corresponding trajectories in SFR-$M_*$ space. Insets in panels (c,d) of the same figure show examples of mock SFHs from the MUFASA hydrodynamic simulation (panel c, Davé et al. (2016b)) and a Semi-Analytic Model (panel d, Somerville et al. (2015)) and their reconstructed best-fit SFHs with
uncertainties. The mock SEDs fitted to reconstruct the SFHs were generated using the same filters as the CANDELS/GOODS-S catalog, with realistic photometric noise and dust, as in Iyer & Gawiser (2017). While the method does not recover short stochastic episodes of star formation, it does well approximate the overall trend of the galaxy’s SFH, thus allowing us to construct robust trajectories in SFR-M$_*$ space. Since we fit galaxies with a single episode of star formation, we find that our reconstructions can sometimes fail in cases where the true SFH of the galaxy contains multiple strong episodes of star formation. However, in Iyer & Gawiser (2017) we find that only about 15% of galaxies at z~1 support fits with two episodes of star formation. Fits to the galaxy SEDs at different redshifts provide us with both the Stellar Masses and effectively instantaneous Star Formation Rates at the epoch of observation, referred to as ‘Direct Fits’ for the rest of this work. In addition to this, the reconstructed Star Formation Histories are then used to construct SFR-M$_*$ trajectories, which allow us to infer the Stellar Masses and Star Formation Rates at higher redshifts of interest.

Table 4.1: Parameter ranges for SFH families, in Gyr, adapted for 0.5 < z < 6 from Iyer & Gawiser (2017). In addition to these, there is a normalization corresponding to the stellar mass, which can be considered a third free parameter in specifying the SFH.

<table>
<thead>
<tr>
<th>SFH</th>
<th>param 1</th>
<th>param 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linexp</td>
<td>$\tau \in [0.05, 10]$</td>
<td>$t_0 \in [0, t_{univ}]$</td>
</tr>
<tr>
<td>Gaussian</td>
<td>$\mu \in [0, t_{univ}]$</td>
<td>$\sigma \in [0.1, 10]$</td>
</tr>
<tr>
<td>Lognormal</td>
<td>$\mu \in [0, t_{univ}]$</td>
<td>$\sigma \in [0.1, 2]$</td>
</tr>
</tbody>
</table>

4.3.2 SFH uncertainties:

We compute uncertainties on the reconstructed SFH using the full $\chi^2$ surface over the multidimensional space spanning SFH, dust and metallicity, generated through the fitting procedure, following a method similar to Iyer & Gawiser (2017). Using the cumulative histogram of the $\chi^2$ values, we select the 100 SFHs corresponding to the set of lowest $\chi^2$ values among the set and compute the median, which we hereafter refer to as the median SFH. We find that changing this threshold of 100 SFHs does not substantially affect the computed uncertainties for this analysis. We then prune the set of good SFHs, removing those that are simply bad fits ($\chi^2 > 10 \times min(\chi^2)$) or outliers in SFH space ($max(|SFH_i - SFH_{median}|)/\langle SFH_{median} \rangle > 5$).
similar to any robust algorithm that is insensitive to outliers. We use the distribution of remaining SFHs to derive pointwise 68% confidence intervals in time for the reconstructed SFH, as well as a robust median SFH. We use the stellar masses and SFRs of the full set of good fits to derive confidence intervals for those quantities at any lookback time along the trajectory. This technique is also used to find the uncertainties on SFR and Stellar Mass at lookback times corresponding to the redshifts of interest in Sec. 4.4. To check that our uncertainties are robust at all lookback times as a function of rest-frame wavelength coverage (depending on \( z_{obs} \)) or SED S/N, we use mock SEDs to analyze possible biases in estimating SFR and \( M_* \) as a function of lookback time for different \( z_{obs} \). This is detailed further in Appendix B.1, where we find that our median uncertainties are conservative and increase accordingly at lookback times where the Stellar Mass or SFR is poorly constrained.

4.3.3 SFR-M\(_*\) trajectories:

The best-fit reconstructed Star Formation Histories are curves of SFR(t) against time. At any instant in time, the Stellar Mass is given by

\[
M_*(t) = \int_0^t SFR(t') f_{ret}(t' - t, Z) dt
\]  

(4.1)

where \( f_{ret}(t' - t, Z) \) is a metallicity dependent fraction of the mass of formed stars that is retained as stars or stellar remnants at the time of observation obtained from FSPS (Conroy et al. 2009), which is typically between 0.6-1.0. Using this relation and the best-fit SFH, we can construct a parametrized curve corresponding to \([SFR(t), M_*(t), t]\), which provides a trajectory in SFR-M\(_*\) space. Observing this trajectory at any redshift gives the \([SFR(z), M_*(z)]\), allowing us to extend trajectories to higher redshifts and fill in the SFR-M\(_*\) correlation using previously inaccessible data from earlier periods in a galaxy’s lifetime. Panel (b) of Figure 4.2 shows examples of trajectories corresponding to each of the basis SFHs shown in panel (a). Panels (c,d) of Figure 4.2 show a couple of examples of reconstructed SFR-M\(_*\) trajectories corresponding to a couple of SFHs at \( z = 1 \) from a hydrodynamical simulation (MUFASA, Davé et al. (2016b)) and a Semi-Analytic Model (Somerville et al. 2015, 2008). As seen in the figure, the true trajectory is well approximated by the smooth reconstruction and its corresponding uncertainties, matching the observations not only at the epoch of observation (\( z = 1 \)), but also at earlier epochs (\( z = 2, 3, 4 \)). While the galaxy in panel (c) has a SFH that can be traced back to very high redshifts, this is not in general true for most observed galaxies. Galaxy trajectories may fail to contribute meaningfully at higher redshifts either because they drop off the plot, i.e., they formed most of their mass at more recent epochs, or because their uncertainties grow
extremely large. The latter case is illustrated through the example galaxy in panel (d), which can be reliably traced back to \( z \sim 2 \), but has large uncertainties beyond that. In our analysis, we exclude such trajectories at redshifts where they have large uncertainties.

Figure 4.3 shows a randomly selected sample galaxies at \( 1 < z < 3 \), that are propagated backwards in time along their trajectories to infer the SFR-\( M_\ast \) correlation at \( z \sim 3 \). Black lines denote trajectories for galaxies that can be propagated backwards to \( z = 3 \), while blue dotted lines denote trajectories for galaxies whose trajectories do not reach back to \( z = 3 \). Since we choose a random subsample of galaxies to plot at each redshift, the F160w selection threshold results in the appearance of a rising lower limit in stellar mass as we go to higher redshifts - this doesn’t imply that galaxies are more massive at \( z \sim 2.6 \), but that observationally selected galaxies, of which we pick a random sample, tend to be the more massive ones. The average amount of time a galaxy is propagated backwards in time along its trajectories shows a mild increase as we go to higher redshifts but remains much smaller than the amount of time between \( z = 0.5 \) and the redshift of interest, as shown in appendix B.1.

For the rest of this work, while considering a sample of galaxies propagated backwards in time along their SFR-\( M_\ast \) trajectories, we restrict ourselves to the sub-sample of galaxies with low uncertainties \( \sqrt{\sigma_{SFR}^2 + \sigma_{M_\ast}^2} < 1 \) dex) to minimize the effects of possible biases. This is explored in detail in Appendix B.1, where we fit mock SEDs corresponding to SFHs from simulations to assess the robustness of SFR and Stellar Mass as we propagate galaxies backwards in time along their trajectories. In doing so, we find that the uncertainties closely trace possible biases, incorporating effects due to factors like S/N and the rest-frame wavelength coverage during SED fitting. This allows us to isolate a subsample with minimal bias that we use for the analysis in this paper.

4.4 Results: The SFR-\( M_\ast \) correlation from direct fits and trajectories

In Figure 4.4, we present the SFR-\( M_\ast \) correlation at \( z = [1,2,3,4,5,6] \), including estimates from galaxies observed at those epochs (henceforth direct fits), as well as from galaxies observed at later epochs propagated backwards in time along their trajectories (henceforth trajectories). The direct fits are shown as contours and as individual datapoints at high redshifts where the number of galaxies are small. The contribution from trajectories at each redshift is shown as a coloured heatmap.

We find that the locus of the direct fits and trajectories broadly agree with each other. To quantify this statistically, we perform a KS test comparing the two datasets at each redshift.
of interest, as shown in Table 4.2. Since we do not want to compare outlier distributions, due to starbursts or quenched galaxies, we impose a cutoff, excluding galaxies that are at a distance $\geq 0.4$ dex from the best-fit SFR-$M_*$ correlation. In Appendix B.2 we also compare distributions using a variable threshold based on the observed scatter and find that the results do not change. The number of galaxies that contribute to the SFR-$M_*$ correlation from direct fits and trajectories are given in the table. The p-values for all these comparisons are $> \alpha$, indicating that the two distributions are not statistically different. The significance level for each test is $\alpha' = (0.05/6) \approx 0.0083$, where we apply a Bonferroni correction (Goeman & Solari 2014) to control for false positives since we are performing a family of tests to evaluate a single hypothesis. Since our results remain consistent across this broad range of tests, we can not reject the null hypothesis that the two samples are drawn from a common underlying distribution at $> 95\%$ confidence. While this does not completely rule out the possibility that the two distributions are different, this agreement justifies the usage of a combined sample to obtain our primary results. We further explore the comparison between the two distributions in Appendix B.2.

We plot the best-fit line to the combined dataset of direct fits and trajectories in Figure 4.4, determined using an iterative robust fitting routine that excludes outliers (Holland & Welsch 1977). To compare trajectories and direct fits on the same footing, the direct fits are analyzed at $z=[1,2,3,4,5,6]$ in bins of $\Delta z = 0.1$. The uncertainties are determined using 1000 Monte Carlo realizations of the data perturbed within the $M_*$ and SFR error estimates obtained through the Dense Basis SED fitting routine. We find that the SFR-$M_*$ correlation extends to redshifts as high as $z \simeq 6$ (Steinhardt et al. 2014), and remains linear down to masses as low as $\log M_*/M_\odot \sim 7$, which is a factor of 10 below current estimates from direct fits. To test the linearity of the correlation, we fit the combined data at each redshift with polynomials of order 1 (linear) and 2 (quadratic) and see if the corresponding improvement in the goodness-of-fit is statistically significant using an F-test. At all redshifts, we find that the linear fit is preferred, at $> 90\%$ confidence, with values at individual redshifts given in Table 4.4. From the reconstructed SFHs, $\sim 92\%$ of the galaxies in the sample have $t_{10} \leq 3\text{Gyr}$ and $\sim 70\%$ of the galaxies in the sample have $\text{age} \leq 3\text{Gyr}$, where $t_{10}$ is the lookback time at which they formed the first 10% of their observed stellar mass. This implies that most of the galaxies that contribute to the SFR-$M_*$ diagram form the majority of their stellar mass in $\leq 3$ Gyr in the redshift range we consider, with 70% entering the observable SFR-$M_*$ range within that time. To account for the fact that the F-test need not guarantee that the correlation is indeed linear, we also perform non-parametric regression in Appendix B.4. Using this, we see that the nonparametric methods
closely approximate the best-fit line as we go to low stellar masses at high redshifts.
Figure 4.2: (a) Examples of basis Star Formation Histories belonging to the three functional families used for SED fitting in the paper, normalized to the same stellar mass of $10^{10} M_\odot$, with similar SFRs. Inset panel shows corresponding SEDs observed at $z=1$ with reference spectra plotted vs observed wavelength. (b) Trajectories in the SFR-$M_*$ plane corresponding to the four SFHs shown above, with black circles illustrating their locations when lookback time equals zero. The dashed black line shows the Speagle et al. (2014) SFR-$M_*$ relation at $z=1$ and the solid grey line shows $\log SFR = \log[M_*/10^9 M_\odot]$ for reference. (c,d) Examples of SFH reconstructions of individual $z=1$ MUFASA (panel c) and SAM (panel d) galaxies with their uncertainties, extended to trajectories in SFR-$M_*$ space. Inset figures show the simulated SFH (true SFH, blue) and the SFH reconstructed through SED fitting the noisy simulated photometry (DB fit, orange). The main panels show their corresponding trajectories in SFR-$M_*$ space. Coloured circles and triangles show SFR, $M_*$ estimates at $z=1,2,3,4$ (darker to lighter colors), and dashed lines show the 68% confidence interval, corresponding to the grey shaded region in the inset. Example galaxy 2 (panel d) can be reliably propagated back to $z \sim 2$, but not beyond that. In our analysis, we exclude such trajectories at redshifts where they have large uncertainties.
Figure 4.3: The SFR-M\_* correlation at $z = 3$ (black points) constructed by propagating a randomly chosen subset of galaxies at redshifts $z = [1.0, 1.4, 1.8, 2.2, 2.6]$ (colored points) backwards in time along their best-fit SFR-M\_* trajectories (black lines are galaxies whose trajectories go back to $z=3$, blue dotted lines are galaxies whose trajectories drop off the plot at lower redshifts). Trajectories that go beyond $z = 3$ are truncated at $z = 3$ for clarity. Three orientations of the figure are shown along different viewing angles. The plots allow us to see that the correlation evolves along a relatively narrow phase space, although individual galaxies can make significant excursions above and below it. This allows us to probe the correlation at high redshifts using higher S/N SED fits at lower redshifts. It also allows us to probe the correlation down to lower masses than previously possible.
Table 4.2: Comparing distributions from Direct fits vs Reconstructed Trajectories. For both methods (galaxies observed at a particular epoch vs those observed at lower redshifts and propagated along their trajectories), we compute the distribution of distances of individual galaxies from the combined best-fit SFR-$M_*$ correlation at $z=1,2,3,4,5,6$.

These distributions are compared using a KS test, to test the hypothesis that they are consistent with being drawn from the same distribution. We reject this hypothesis if the p-value $> \alpha (=0.0083)$. To arrive at this value we apply a Bonferroni correction (Goeman & Solari 2014) to control for false positives since we are performing a family of tests to evaluate a single hypothesis. Since we do not want to include starburst and quiescent galaxies, we exclude galaxies that lie at a distance $>0.4$ dex from the correlation. In Table B.2 we also give p-values for the case where we exclude galaxies that lie farther than $1 \times$ the observed scatter at each redshift. The last two columns show the number of galaxies from each dataset used in comparing the two distributions. The test shows that the direct fits and trajectories are consistent with being drawn from the same distribution at all redshifts as evinced by small values of the KS statistic, which measures the maximum distance between the CDF of the two distributions. The larger KS distance in the $z \sim 5$ redshift bin is due to the small number of points from direct fits.

<table>
<thead>
<tr>
<th>redshift</th>
<th>p-value</th>
<th>KS-statistic</th>
<th>cutoff [dex]</th>
<th>#traj</th>
<th>#direct</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0 (0.9 &lt; z &lt; 1.1)</td>
<td>0.85</td>
<td>0.03</td>
<td>0.4</td>
<td>947</td>
<td>1667</td>
</tr>
<tr>
<td>2.0 (1.9 &lt; z &lt; 2.1)</td>
<td>0.10</td>
<td>0.06</td>
<td>0.4</td>
<td>2950</td>
<td>1003</td>
</tr>
<tr>
<td>3.0 (2.9 &lt; z &lt; 3.1)</td>
<td>0.02</td>
<td>0.10</td>
<td>0.4</td>
<td>3394</td>
<td>419</td>
</tr>
<tr>
<td>4.0 (3.9 &lt; z &lt; 4.1)</td>
<td>0.67</td>
<td>0.08</td>
<td>0.4</td>
<td>3019</td>
<td>162</td>
</tr>
<tr>
<td>5.0 (4.5 &lt; z &lt; 5.0)</td>
<td>0.20</td>
<td>0.13</td>
<td>0.4</td>
<td>2396</td>
<td>183</td>
</tr>
<tr>
<td>6.0 (5.5 &lt; z &lt; 6.0)</td>
<td>0.98</td>
<td>0.09</td>
<td>0.4</td>
<td>2101</td>
<td>81</td>
</tr>
</tbody>
</table>

To study the redshift evolution of the slope and normalization of the SFR-$M_*$ correlation, we analyze the direct fits and trajectories separately and present the results in Table 4.4 and graphically in Figure 4.5. The slope from both approaches are consistent within uncertainties, and roughly match the published meta-analysis of Speagle et al. (2014) in figure 4.5, shown as a solid purple line. This result is reassuring considering the Speagle et al. (2014) relation was calibrated at stellar masses above $10^9 M_\odot$. In comparison to the Speagle et al. (2014) relation, however, we find the slope to be consistent with little to no evolution with redshift. The normalization for the trajectories shows the same trend in redshift as the direct fits and the Speagle et al. (2014) relation, albeit being systematically lower at high redshifts. For better
comparison with literature, we also considered a set of estimates for the slope and normalization
where we use an additional UVJ selection criterion to select star forming galaxies for the direct
fits dataset. Details of the UVJ selection can be found in Appendix B.3, where we find that
the results don’t vary much due the robust fitting algorithm we use. We do not consider such
a criterion for trajectories since galaxies that are quiescent at the epoch of observation can
be traced back to epochs when they were star forming and thus contribute to the SFR-M∗
correlation at higher redshifts. Observed scatter is computed using the same procedure as
Kurczynski et al. (2016), finding the standard deviation of the distribution of ΔSFR from the
best-fit correlation excluding points beyond 1 dex. We find that the scatter is close to 0.3 dex
at all epochs with the possibility of being higher at low redshifts as seen for the direct fits. The
observed scatter contains contributions from noise that needs to be deconvolved to estimate the
intrinsic scatter (Kurczynski et al. 2016) and while our scatter for direct fits and trajectories
are consistent within uncertainties, it is possible that we underestimate the intrinsic scatter
measured with trajectories since we do not consider short-timescale excursions from the smooth
best-fit SFH for individual galaxies (Matthee & Schaye 2018). The published Kurczynski et al.
(2016) and Salmon et al. (2015) values for slope, normalization and scatter are also shown as
blue triangles at 0.5 < z < 2.5 and purple stars at 3.5 < z < 6.5. Whitaker et al. (2014)
and Schreiber et al. (2015) report a nonlinear SFR-M∗ correlation due to a turnover at high
stellar mass (M∗ > 10^{10.5}M⊙). Since most of our galaxies fall below this mass range, we fit
their reported correlation near 10^{9}M⊙ with a line to find the effective low mass slope and
normalization shown in Figure 4.5.

Fitting the slope and normalization of the best-fit SFR-M∗ correlation as a function of cosmic
time, we find that the relation is well described by a linear fit, given by

\[
\log \text{SFR} = (0.80 \pm 0.029 - 0.017 \pm 0.010 \times t_{\text{univ}}) \log M^* - (6.487 \pm 0.282 - 0.039 \pm 0.008 \times t_{\text{univ}})
\] (4.2)

where \(t_{\text{univ}}\) is the age of the universe in Gyr at a given redshift. This relation is shown as
the grey shaded region in Figure 4.5.

In estimating the slope and normalization of the SFR-M∗ correlation, we use all galaxies
that satisfy the selection criterion described in method (c) in Appendix B.1. In Table 4.3, we
estimate the minimum well-sampled mass at each redshift, below which the statistics may be
insufficient to confirm that the values for slope and normalization of the SFR-M∗ correlation still
apply. This is based on the distribution in Stellar Mass and SFR for direct fits and trajectories,
conservatively quantified as the 10th percentile of the respective distributions. We find that we can probe the SFR-M$_*$ correlation to $\sim 1$ dex lower than possible with just direct fits. This is possible since we are no longer limited by selection effects such as the F160w detection threshold, which does not allow us to detect the faint, low mass galaxies at high redshifts that we would see at lower redshifts. Using trajectories is thus a useful tool to go deeper in SFR and M$_*$ at high redshifts.

Considering the trajectories allows us to effectively increase the survey volume obtained by propagating galaxies observed at later epochs backwards in time. We estimate this effective increase in the volume of the survey by comparing the ratio of the number of galaxies from just direct fits vs direct fits + trajectories in a comparable mass range. The comparable mass range is obtained by requiring that the median of M$_*$ for the direct fits and trajectories in this mass range be separated by $< 0.1$ dex. In theory, this can be applied to any survey that probes a wide variety of galaxy types to allow us to further extend our SED fitting results using SFR-M$_*$ trajectories. Although it is beyond the scope of this work, it is important to include corrections to the effective volume on an individual galaxy basis (based on the amount of time they have been extrapolated backwards along their trajectory) while considering problems such as calculating luminosity functions or number densities using the combined trajectories + direct fits datasets.

<table>
<thead>
<tr>
<th>redshift</th>
<th>log M$_*$ (direct) (10th percentile)</th>
<th>log M$_*$ (traj.) (10th percentile)</th>
<th>log SFR (direct) (10th percentile)</th>
<th>log SFR (traj.) (10th percentile)</th>
<th>Eff. Volume (direct + traj.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>7.99</td>
<td>7.62</td>
<td>-1.19</td>
<td>-1.82</td>
<td>1.29x</td>
</tr>
<tr>
<td>2.0</td>
<td>7.91</td>
<td>7.74</td>
<td>-0.85</td>
<td>-1.16</td>
<td>3.25x</td>
</tr>
<tr>
<td>3.0</td>
<td>8.59</td>
<td>7.57</td>
<td>-0.50</td>
<td>-1.05</td>
<td>5.49x</td>
</tr>
<tr>
<td>4.0</td>
<td>8.59</td>
<td>7.59</td>
<td>-0.40</td>
<td>-1.03</td>
<td>11.74x</td>
</tr>
<tr>
<td>5.0</td>
<td>8.67</td>
<td>7.44</td>
<td>-1.13</td>
<td>-1.15</td>
<td>8.61x</td>
</tr>
<tr>
<td>6.0</td>
<td>8.34</td>
<td>7.38</td>
<td>-0.13</td>
<td>-1.2</td>
<td>13.74x</td>
</tr>
</tbody>
</table>

Table 4.3: 10th percentile of Stellar Mass and SFR probed using direct fits and trajectories at different redshifts. We see that using trajectories allows us to probe the SFR-M$_*$ correlation to nearly 1 dex deeper at high redshifts as compared to simply using direct fits. The last column estimates the increase in the effective volume of the survey, using the increased number of galaxies at a particular redshift obtained by adding trajectories in a mass range where the direct fits and trajectories are comparable.
While simulations don’t yet make predictions for the slope and normalization of the correlation at the lowest masses, our results help put strong constraints on the models. When comparing $M_\star$, SFR distributions obtained through trajectories with simulations, it is important to compare SFR-$M_\star$ correlations between our results and the simulations on the same footing, it is important that the correlation be compiled at any redshift using galaxies summed over all progenitors at the redshift of observation - for example, to compare accurately to reconstructed trajectories at $z \sim 6$, a simulation should be allowed to run to at least 3 Gyr in the future to about $z \sim 4$, then traced back to $z \sim 6$. This imposes resolution requirements at both the lower redshift, for the discovery of galaxies and at the redshift of interest, to be able to distinguish progenitors that contribute to the trajectories. The similarity of the two distributions when compared using the KS test indicate that this difference is not a major one for our observed sample of CANDELS/GOODS-S galaxies.

<table>
<thead>
<tr>
<th>z</th>
<th>$m_{\text{direct}}$</th>
<th>$m_{\text{traj}}$</th>
<th>$m_{\text{total}}$</th>
<th>$c_9_{\text{direct}}$</th>
<th>$c_9_{\text{traj}}$</th>
<th>$c_9_{\text{total}}$</th>
<th>$\sigma_{\text{obs,direct}}$</th>
<th>$\sigma_{\text{obs,traj}}$</th>
<th>$\sigma_{\text{obs,total}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.65±0.06</td>
<td>0.69±0.09</td>
<td>0.69±0.08</td>
<td>0.03±0.04</td>
<td>0.06±0.08</td>
<td>0.05±0.06</td>
<td>0.56±0.14</td>
<td>0.33±0.12</td>
<td>0.38±0.12</td>
</tr>
<tr>
<td>2</td>
<td>0.79±0.07</td>
<td>0.75±0.07</td>
<td>0.76±0.07</td>
<td>0.39±0.05</td>
<td>0.30±0.06</td>
<td>0.32±0.06</td>
<td>0.44±0.15</td>
<td>0.26±0.11</td>
<td>0.29±0.11</td>
</tr>
<tr>
<td>3</td>
<td>0.79±0.10</td>
<td>0.80±0.08</td>
<td>0.80±0.07</td>
<td>0.53±0.08</td>
<td>0.48±0.08</td>
<td>0.49±0.06</td>
<td>0.33±0.11</td>
<td>0.27±0.11</td>
<td>0.28±0.11</td>
</tr>
<tr>
<td>4</td>
<td>0.77±0.18</td>
<td>0.76±0.09</td>
<td>0.77±0.09</td>
<td>0.86±0.12</td>
<td>0.51±0.09</td>
<td>0.55±0.09</td>
<td>0.25±0.08</td>
<td>0.27±0.10</td>
<td>0.27±0.10</td>
</tr>
<tr>
<td>5</td>
<td>0.70±0.26</td>
<td>0.74±0.11</td>
<td>0.75±0.10</td>
<td>0.73±0.26</td>
<td>0.55±0.11</td>
<td>0.56±0.10</td>
<td>0.39±0.11</td>
<td>0.27±0.10</td>
<td>0.28±0.10</td>
</tr>
<tr>
<td>6</td>
<td>0.82±0.47</td>
<td>0.78±0.11</td>
<td>0.78±0.11</td>
<td>0.44±0.47</td>
<td>0.57±0.12</td>
<td>0.57±0.12</td>
<td>0.41±0.13</td>
<td>0.27±0.10</td>
<td>0.27±0.10</td>
</tr>
</tbody>
</table>

Table 4.4: Results: The slope, normalization at $10^9 M_\odot$ and observed scatter of the best-fit to the SFR-$M_\star$ correlation at different redshifts as shown in Figure 4.5 for direct fits to galaxies observed at each epoch (direct; orange points in Figure 4.5), galaxies observed at later epochs propagated backwards along their SFR-$M_\star$ trajectories (traj; blue points in Figure 4.5) and the combined sample (total; black points in Figure 4.5). Including the low mass data in our fits, we observe a milder evolution of the slope and normalization with time in comparison to Speagle et al. (2014), finding that the evolving correlation is best described by Eqn. 4.2: \( \log SFR = (0.80 \pm 0.029 - 0.017 \pm 0.010 \times t_{\text{univ}}) \log M_\star - (6.487 \pm 0.282 - 0.039 \pm 0.008 \times t_{\text{univ}}) \), where $t_{\text{univ}}$ is the age of the universe at a given redshift. The last column details the confidence levels (1−p-value) from the F-test to check the hypothesis that a linear fit to the SFR-$M_\star$ is favoured over a quadratic fit.
4.5 Validation:

SED fitting allows us to estimate $M_*$ and SFRs at the epoch of observation. In addition to this, we estimate the Stellar Masses and SFRs at previous epochs during which the galaxy was forming stars, by propagating galaxies backwards in time along their reconstructed SFR-$M_*$ trajectories. In Appendix B.1 we verify the robustness of our trajectories, and restrict our analysis to the subsample of galaxies whose trajectories have low uncertainties in SFR and $M_*$ at a given redshift of interest. A reassuring check of the robustness of our method come from the similarity between the distributions around the SFR-$M_*$ correlation from direct fits and trajectories in Table 4.2.

<table>
<thead>
<tr>
<th>Validation test</th>
<th>$m_{orig}$</th>
<th>$m_{fit}$</th>
<th>$c_{9,orig}$</th>
<th>$c_{9,fit}$</th>
<th>$\sigma_{true}$</th>
<th>$\sigma_{fit}$</th>
<th>$\alpha_{orig}$</th>
<th>$\alpha_{fit}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unchanged</td>
<td>0.886</td>
<td>0.896 ± 0.067</td>
<td>-0.212</td>
<td>-0.180 ± 0.041</td>
<td>0.238</td>
<td>0.231</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Increased slope (z=1)</td>
<td>1.208</td>
<td>1.189 ± 0.038</td>
<td>-0.135</td>
<td>-0.091 ± 0.040</td>
<td>0.198</td>
<td>0.213</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Decreased slope (z=1)</td>
<td>0.478</td>
<td>0.461 ± 0.072</td>
<td>-0.302</td>
<td>-0.288 ± 0.038</td>
<td>0.231</td>
<td>0.217</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Changed shape (z=1)†</td>
<td>-0.075</td>
<td>-0.086 ± 0.093</td>
<td>-0.215</td>
<td>-0.228 ± 0.142</td>
<td>0.607</td>
<td>0.629</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Increased slope (z=2)</td>
<td>1.049</td>
<td>1.028 ± 0.015</td>
<td>-0.158</td>
<td>0.082 ± 0.054</td>
<td>0.249</td>
<td>0.403</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Decreased slope (z=2)</td>
<td>0.663</td>
<td>0.532 ± 0.055</td>
<td>0.165</td>
<td>0.268 ± 0.027</td>
<td>0.059</td>
<td>0.181</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Changed shape (z=2)†</td>
<td>0.459</td>
<td>0.495 ± 0.049</td>
<td>-0.421</td>
<td>-0.139 ± 0.067</td>
<td>0.640</td>
<td>0.389</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 4.5: Validation: estimating the sensitivity of the fits to slope, normalization and shape of the SFR-$M_*$ correlation. $m$ is the slope of the linear correlation, $c$ is the normalization, and $\alpha$ denotes the degree of the polynomial that the correlation is best fit with. The $m_{orig}$ and $c_{9,orig}$ are the linear coefficients obtained from the best-fit to the SFR-$M_*$ correlation generated directly using SED fitting with the modified SFHs for each test case. †Although the slope and normalization are reported, the correlation in this case is better fit with a quadratic, and is not well described by the linear coefficients. For the $z = 1$ case, the true correlation is described by $\log SFR = -0.364(\log M_* - 9)^2 + 0.515(\log M_* - 9) + 0.053$. For the $z = 2$ case, the coefficients are $\log SFR = -0.199(\log M_* - 9)^2 + 0.342(\log M_* - 9) + 0.271$.

To further ensure that we do not get an artificially linear correlation due to our fitting method, we use a sample of mock SFHs from the MUFASA hydrodynamic simulations (Davé et al. 2016b) to run a series of validation tests by changing the slope, normalization and shape of the simulated SFR-$M_*$ correlation to see if our fits can recover these changes. The results of are reported in Table 4.5. The first six columns report the results of a linear fit, to $\log SFR =$
$m \log [M_\star / 10^9 M_\odot] + c_9$, reporting the slope ($m$), normalization ($c_9$) and the observed scatter ($\sigma$). The last two columns compare the goodness of fit of a first order (linear) and second order (quadratic) polynomial fit using an F-test, to determine if the improvement upon fitting with a second-order curve is statistically significant. This test allows us to test the hypothesis that the correlation is linear. For all tests, the simulated galaxies are fit at $z = 1$.

The first row of Table 4.5 evaluates that the SFR-$M_\star$ correlation is robustly recovered for a randomly selected sample of galaxies at the epoch of observation ($z = 1$, direct fits) with no modifications to their SFHs. This represents the control case for our validation tests. The next two rows change the slope of the correlation to see if our method can recover the artificially high or low slope. This is done using a Gaussian envelope to modify the SFH in a way that the overall galaxy mass remains constant. We find that the recovered slope and normalization match the truth within uncertainties, as seen in columns 2-5 of Table.4.5. In the fourth row we check that the recovered correlation is sensitive to the linearity of the underlying correlation by curving the correlation such that it is better fit by a quadratic curve. This is achieved by adding a single Gaussian component to a randomly drawn MUFASA SFH while varying lookback time at which the SFH peaks. This results in a curved SFR-$M_\star$ correlation. Although there is always an improvement to the fit with an additional degree of freedom, we use an F-test with a threshold p-value of 0.1 to see if the quadratic fit provides a statistically significant improvement in describing the variance of the data and find that the order of our recovered correlation matches the input, shown in columns 8-9 of Table.4.5.

To check the robustness of the SFR-$M_\star$ correlation recovered from galaxies propagated backwards in time along their trajectories, we also perform the same tests using a randomly selected sample of galaxies that are fit at $z = 1$ and analyzed at $z = 2$. We choose $z = 2$ to test the trajectories, since we find that most galaxies contributing to the SFR-$M_\star$ correlation through trajectories at different redshifts form the bulk of their stellar mass at $\leq 3$ Gyr from the epoch of interest. We repeat the same tests for the trajectories as we did for the direct fits, while attempting to keep the $z=1$ correlation the same. Since achieving such changes through modifications to the MUFASA SFHs was difficult, we use a Monte Carlo method generating random gaussian contributions to the SFH until the resulting SFH satisfies a ‘normal’ MS slope at $z = 1$ and a modified slope at $z = 2$ significantly higher or lower than $m \sim 0.89$. We then generate SEDs corresponding to these SFHs and fit them, finding that the fitting method is reasonably sensitive to changes in the slope of the correlation at higher redshifts. A small number of quiescent galaxies at $z = 1$ also contribute to the correlation at higher redshifts, which helps increase the sensitivity of our approach. Additionally, we repeated the test where
the shape of the correlation is changed to being better described by a quadratic rather than a linear curve, finding that this too is robustly recovered by the fits.

4.6 Discussion

The tight correlation between the SFRs and Stellar Mass of star forming galaxies has been extensively studied, with simulations matching observations at $z \sim 0$ and high redshifts (Sparre et al. 2015; Salmon et al. 2015), but with some tension at intermediate redshifts around $z \sim 2$ (Sparre et al. 2015). The Dense Basis method allows us to extend the dynamic range across which we fit the SFR-$M_*$ correlation to estimate the slope and normalization, helping provide more robust estimates of these quantities. We find sub-linear slopes at all redshifts consistent with mild evolution, similar to Speagle et al. (2014); Salmon et al. (2015) and Kurczynski et al. (2016) at $z > 1$. This is in contrast (about 2.5 $\sigma$) to Schreiber et al. (2015) and Whitaker et al. (2012), who find a slope closer to 1. Speagle et al. (2014) attributes the slope to a steady, environment driven mode of star formation, where a slope slightly below unity occurs due to feedback, leading to the growth of hot halos around higher mass galaxies and slows down gas accretion (Finlator et al. 2006; Davé 2008). Salmon et al. (2015) argues that gas accretion onto dark-matter halos at high-$z$ is smooth over large timescales (Cattaneo et al. 2011; Finlator et al. 2011) assuming a power-law form of the SFH, which controls the scaling of both the SFR and stellar mass (Stark et al. 2009; González et al. 2011; Papovich et al. 2011). Results from our more versatile SFHs appear to extend this interpretation across a wider range of redshifts. A sub-linear slope to the SFR-$M_*$ correlation is also relevant in the context of Abramson et al. (2016), which considers how the growth of a bulge adds $M_*$ but not SFR. It would be an interesting analysis to further study how the scatter around the SFR-$M_*$ correlation correlates with explicit SFH parameters like $t_{10}$ (the lookback time when the galaxy forms the first 10% of its stellar mass) and morphological quantities like the bulge/disk ratio (Abramson et al. 2014). The evolution of the normalization, which could be related to changing cosmological gas accretion rates with redshift (Dutton et al. 2010) generally agrees with the literature, while being $\sim 0.2 - 0.4$ dex lower than the Speagle et al. (2014) meta-analysis at all redshifts and $\sim 0.5$ dex lower than Salmon et al. (2015) at $z \sim 6$, albeit with larger uncertainties.

SFR-$M_*$ trajectories obtained through SED fitting provide a valuable tool to extract information about where galaxies lie on the Stellar Mass - SFR plane at multiple epochs, allowing us to probe the low-mass portion of the SFR-$M_*$ correlation as we go to higher redshifts, with greater numbers than previously available. However, in interpreting the results we obtain, it
is important to keep in mind the limitations of the observational data, as well as the current implementation of the Dense Basis method. Since our method estimates the smooth overall trend of star formation in a galaxy’s past, it does not recover stochastic ‘short timescale’ star formation events that contribute to the intrinsic scatter of the SFR-$M_\star$ correlation. However, we find that only $\sim5\%$ of a sample of galaxies from SAMs (Somerville et al. 2015) and $\sim7\%$ of galaxies from MUFASA (Davé et al. 2016b) show a short burst where $\text{SFR}_{10\text{Myr}} / \text{SFR}_{\text{life}} > 10$, where $\text{SFR}_{10\text{Myr}}$ is the SFR averaged over the last 10 Myr lookback time, and $\text{SFR}_{\text{life}}$ is the SFR of a galaxy averaged over its lifetime. This is not significant enough to bias estimations of the slope/normalization, or to alter the results from our validation tests.

In hierarchical cosmology, the process of galaxy evolution includes major and minor mergers along with gradual mass growth due to infall. When SED fitting yields information about an observed galaxy’s star formation history, even a summary statistic such as the age of its stellar population, this information is about the sum of all stars in that galaxy’s progenitors. Hence the trajectories that result from our SFH reconstruction represent trajectories of the summed stellar masses and star formation rates of each observed galaxy’s progenitors, rather than those of its most massive progenitor at each epoch. However, observable samples of galaxies at e.g., $z = 4$, will contain the most massive progenitors of observable galaxies at e.g., $z = 2$, along with a few additional progenitors that are massive enough to be detected. Looking backwards in time, a minor merger with 10:1 or 3:1 mass ratio causes only a 0.04 or 0.12 dex offset, respectively, in mass between the sum-of-progenitors and the most massive progenitor, with the maximum offset of 0.3 dex coming from a 1:1 major merger. Such major mergers are predicted to be rare (Kaviraj et al. 2015; Rodriguez-Gomez et al. 2015; Ventou et al. 2017), even at high redshift, for galaxies massive enough to be detected at $z \sim 1$. Nonetheless, the KS test described in §4.4 found the distribution of trajectory values about the inferred SFR-$M_\star$ correlation to match that of the observed values from direct fits well enough that the hypothesis of these being drawn from a single underlying population is not ruled out at higher than 99% confidence for any of the samples.

Recent studies (Hsieh et al. 2017) indicate that a strong correlation exists between the Star Formation Rate Density ($\Sigma_{SFR}$) and the Surface Mass Density ($\Sigma_{M_\star}$) in star forming galaxies at kpc scales. This indicates that the SFR-$M_\star$ correlation may extend to much lower scales than currently measured, with the method described in this work providing a unique bridge to intermediate scales.
4.7 Conclusions

SFH reconstruction through SED fitting yields the trajectories of galaxies that evolve through SFR-\(M_\ast\) space and is thus uniquely suited to probe the low-mass end of the SFR-\(M_\ast\) correlation as we go to higher redshifts. In this paper, we used the Dense Basis method (Iyer & Gawiser 2017) to fit a sample of \(\sim 17,800\) galaxies in the CANDELS GOODS-S field at redshifts \(0.5 < z < 6.0\). We used the reconstructed SFHs to obtain the stellar masses and star formation rates of galaxies at \(z = 1, 2, 3, 4, 5, 6\). Using the combined dataset from galaxies observed at the epochs of interest (direct fits) and galaxies observed at lower epochs propagated backwards in time along their SFR-\(M_\ast\) trajectories, we find that the SFR-\(M_\ast\) correlation is linear to \(\sim 10^6 M_\odot\) at high redshifts.

This allows us to study the nature and evolution of the SFR-\(M_\ast\) correlation in greater detail than allowed by previous approaches like Main Sequence Integration (Leitner 2012) which assumes that star forming galaxies stay on the correlation throughout their lifetimes. We find that the overall trend of the evolution of the slope of the SFR-\(M_\ast\) correlation with redshift is roughly consistent with the evolving Speagle et al. (2014) relation, while the normalization seems systematically lower by a factor of \(\sim 0.2 \text{ dex}\).

Thus new approach provides a probe of the correlation at much lower masses than previously possible (Salmon et al. 2015; Kurczynski et al. 2016) since stellar masses decrease as we propagate galaxies backwards along their SFR-\(M_\ast\) trajectories. This is more important in view of the selection effects in the direction of increasing stellar mass as we go to higher redshift in galaxy surveys like CANDELS. It also allows for a closer comparison between observations and simulations, by providing constraints for simulations through the comparison of the predicted SFR-\(M_\ast\) distributions to the reconstructed ones down to much lower masses.
Figure 4.4: The SFR-M\_ correlation at $z = [1, 2, 3, 4, 5, 6]$. Galaxies observed at the epochs of interest (direct fits) are shown as red contours for $z < 5$, and as red points with error bars for $z = 5, 6$ where there are insufficient points to yield representative contours. Galaxies observed at later epochs and propagated backwards in time along their SFR-M\_ trajectories are shown as the colored heatmap, with the colorbars denoting the number of galaxies in a particular pixel. The black solid line shows our best-fit to the combined dataset, with uncertainties denoted by the dashed black lines. The shaded black region shows the uncertainties + observed scatter around the best-fit. Dotted blue lines show the $10^{th}$ percentile in stellar mass for trajectories. Additional non-parametric fits to the correlation are shown in Appendix B.4. We see that the SFR-M\_ correlation is consistent with being linear out to very low masses and high redshifts.
Figure 4.5: Evolution of the slope ($m$), normalization ($c_9$, the intercept at log $M_\ast = 9$) and observed scatter of the SFR-$M_\ast$ correlation. Black circles with errorbars show our results for the combined (direct fits + trajectories) dataset, and the grey shaded region is generated using the evolving relation defined in Eqn.4.2. We find that the slope from the direct fits (orange circles with error bars) and trajectories (blue diamonds with error bars) are roughly consistent within uncertainties with each other at all times, with some disagreement in normalization at $2 < z < 4$. Our estimates for slope agree well with Speagle et al. (2014) at low redshifts and Salmon et al. (2015) at high redshifts, while being consistently sub-linear, in comparison to Whitaker et al. (2014); Schreiber et al. (2015). Our normalization is closer to Schreiber et al. (2015) than Speagle et al. (2014) in value, but has a rate of evolution more consistent with the latter. Our measurement of observed scatter agrees well with the observed scatter reported in Kurczynski et al. (2016); Salmon et al. (2015). Measurements from Whitaker et al. (2014); Schreiber et al. (2015) are shown for comparison, using a local slope at $10^9M_\odot$ for non-linear reported correlations.
Chapter 5

Galaxy evolution with SFHs: characteristic timescale of star formation stochasticity

5.1 The role of short timescales

Physical processes in galaxies occur over a range of timescales. On long timescales (> Gyr), the overall conditions across the galaxy are dictated by its environment and the depth of its gravitational potential well, proportional to the mass of its host dark matter halo (Behroozi et al. 2018). A deeper potential well will result in the accretion of more gas and mergers, which would naively result in more star formation. However, baryonic processes can significantly alter this state, with feedback from supernovae or photoionisation from bright stars significantly inhibiting the efficiency though which giant molecular clouds are created and destroyed (Brooks & Zolotov 2014; Hopkins et al. 2014). In some cases, simulations have even shown that baryonic processes can affect star formation and the distribution of matter in dwarf galaxies to such an extent that even their dark matter distributions are altered as a result (Brooks et al. 2013; El-Badry et al. 2016).

These baryonic processes occurring on short timescales are complex, turbulent interactions that are not well understood at present. While simulations like FIRE-2 and Romulus provide indications of how they could affect star formation efficiencies and thus the growth of galaxies, the estimated stochasticity in star formation remains in many cases to be observationally verified. (Weisz et al. 2011b; Guo et al. 2016; Broussard et al. 2019; Emami et al. 2018; Caplar & Tacchella 2019) explore the short timescale stochasticity using star formation probed by indicators that are sensitive to different timescales such as SFR$_{UV}$, which is sensitive to SFR over the past 70-100 Myr and SFR$_{H\alpha}$, which is sensitive to the past 4-10 Myr. Doing so allows us to measure the average star formation over two timescales, and use its ratio (and the distribution of this ratio, in some cases) to estimate the stochasticity of star formation for a population of galaxies. Such analyses are subject to multiple systematics (the effects of dust modeling, metallicity, IMF stochasticity and more) as well as large uncertainties.

In this analysis, we propose a formalism to estimate the short timescale stochasticity of
galaxies through star formation histories (SFHs) estimated through SED fitting. Since the full SED of the galaxy contains more information than individual SFR indicators (or pairs thereof), leveraging this information through SED fitting can provide us with better constraints. In addition, the nonparametric formalism we use (Iyer et al. 2019) is ideally suited to simultaneously analyzing large populations of galaxies with heterogeneous observations to provide constraints on the stochasticity on different timescales.

5.2 Methodology: Probing Star Formation Timescales with Dense Basis SED Fitting

5.2.1 SED fitting and SFH reconstruction

We adopt the improved Dense Basis SED fitting method outlined in Chapter 3. Briefly, we describe an SFH using the tuple \((M_*, \text{SFR}, \{t_X\})\). Using Gaussian Processes (Rasmussen & Williams 2006; Foreman-Mackey 2015), we then realize smooth, nonparametric curves in SFR(t) space for each tuple. We construct a basis of these SFHs by making repeated draws from our prior distribution for each parameter, using a Dirichlet prior for \(\{t_X\}\). Adding random draws for redshift, dust attenuation and metallicity, we then generate SEDs corresponding to each \((\text{SFH}, A_v, Z, z)\) set using the FSPS package (Conroy et al. 2009; Conroy & Gunn 2010; Foreman-Mackey et al. 2014). We then fit each observed SED using the basis and use the resulting likelihood surface to determine the posterior distribution for each parameter.

5.2.2 Varying Burstiness using the Matern Kernel

One key ingredient in realizing SFHs using the Gaussian Process routine is the kernel - a function that specifies the autocorrelation between star formation rates separated by an interval \((t - t' = \Delta t)\). By varying the kernel, it is possible to control how rapidly the SFR varies in draws from an SFH posterior. In practice, we accomplish this by varying the parameter \(\nu\) in the Gaussian Process kernel, which we so far had held constant \((\nu = 3/2)\). This parameter controls how differentiable the draws from the kernel are, so \(\nu \to \infty\) leads to infinitely differentiable curves, while \(\nu \to 0\) leads to increasingly choppy curves that approximate the kind of burstiness we expect from star formation histories. An example of this procedure for a single galaxy SFH can be seen in Figure 5.1.
Figure 5.1: The effect of varying the hyperparameter $\nu$ on individual draws from the SFH posterior (thin black lines) using the Gaussian Process with a Matern kernel for a single CANDELS/GOODS-S galaxy. The solid black line shows the median SFH. The draws are self consistent, taking into account both the posteriors for the individual SFH parameters as well as their covariances. In Sec. 3.3.5, we see that while individual draws from the posterior change dramatically while varying the kernel, doing so only causes percent-level changes in the overall median SFH. As $\nu$ decreases, the SFH draws get increasingly stochastic around the median.
5.2.3 Refitting a population of galaxies: finding \( \nu \)

To find the optimal \( \nu \) for a sample of galaxies, we refit the sample with SEDs generated using realizations from the posterior SFH while varying the hyperparameter \( \nu \) that controls how bursty the SFH realizations are. For each observed SED and value of \( \nu \), we generate 10 mock SEDs using realizations from the SFH posterior. In practice, these constitute only percent-level changes in the final SED, and thus we do not see any significant change in the likelihood for any given realization for individual galaxies. For larger samples of galaxies, however, the cumulative residuals begin to show a preference for the burstiness that best approximates the behaviour of the population. In this manner, we obtain a burstiness posterior for our sample as we vary \( \nu \).

5.2.4 From burstiness to a timescale: excursions around the median

To subsequently quantify the timescales of the burstiness, we then consider the median time interval \( (\Delta t) = \lambda/2 \) between when an SFH draw makes an excursion from the smooth median and rejoins it. Considering this behaviour to be roughly periodic makes this time interval analogous to half the wavelength of a oscillation timescale around the smooth mean. This quantity can then be computed for a given \( \nu \), translating it from an overall stochasticity metric to a timescale on which this stochasticity is most prominent.

In practice, this is done by computing the difference between the median SFH and any individual draw \( (\Delta SFH = \langle SFH \rangle - SFH_i) \). Computing the number of excursions from the median is then equivalent to computing the number of times the \( \Delta SFH \) changes sign. The average excursion is given by the total number of times the sign changed within the total time of observation.

\[
t_{burst} = t_z / \sum (|\text{sign}(\Delta SFH(t_i)) - \text{sign}(\Delta SFH(t_{i-1})|)/2) \quad (5.1)
\]

where \( t_z \) is the age of the universe at the redshift of observation, and the sign operator returns (+1) if the input is positive, (0) if the input is 0, and (-1) if it is negative. For each individual draw, we compute an average burstiness timescale, leading to a distribution of timescales associated with a given value of \( \nu \). This is shown in Figure 5.2.

5.3 Data

We perform our initial analysis using a sample of bright \( (H < 27) \) galaxies at \( 0.95 < z < 1.05 \) in the CANDELS/GOODS-S field. To confirm that the results are not sensitive to the choice of sample, we repeat the analysis on an equivalent sample from the CANDELS/GOODS-N and
Figure 5.2: Distributions of the burstiness timescale ($t_{burst}$) for different values of the Gaussian Process hyperparameter $\nu$ for a single SFH, although repeating this procedure with different SFHs gives consistent results. For each value of $\nu$, we draw 1000 realizations from the SFH posterior. Using this, we compute a value of $t_{burst}$ for each draw, leading to a distribution of values for $t_{burst}$ for each value of $\nu$. The distributions are all tightly clustered about their median values, providing a translation from $\nu$ to a characteristic star-formation timescale.

CANDELS/COSMOS fields with the same selection criteria, finding consistent results. The relevant photometric bands for each field are reported in Chapter 3.

5.4 Results

We show the results of fitting an ensemble of galaxies at $z \sim 1$ in Figure 5.3. The top and middle rows show the median $\chi^2$ and relative likelihood, given by $\mathcal{L}(\nu|F_{obs})/\mathcal{L}(\nu_{best}|F_{obs}) = \exp(-\frac{(\chi^2_{median,\nu_{best}} - \chi^2_{median,\nu})}{2})$ where $\chi^2_{median,\nu}$ is the median $\chi^2$ at a given value of $\nu$ and $\chi^2_{median,\nu_{best}}$ is the lowest value of the median $\chi^2$ for any value of $\nu$ for that dataset. Since we fit each galaxy 10 times with SFHs sampled from the posterior for each value of $\nu$, we obtain a much larger set of $\chi^2$ values for each galaxy. This is shown in the bottom row of the figure, where we find that both the original and larger resampled distributions are well behaved and centered around $\chi^2/\text{DoF} \sim 1$ for all datasets. The individual columns show the effects of repeating the experiment on different datasets, from the CANDELS COSMOS, GOODS-N and GOODS-S fields, as well as for a combined sample. These datasets differ in both signal-to-noise distributions as well as which photometric filters are used to measure the galaxy SEDs, and provide a useful validation check to ensure that our results do not depend on either. While there is some spread in the preferred values of $\nu$ for different datasets, GOODS-S finds the optimal
Figure 5.3: Plots showing the median $\chi^2/\text{DoF}$ for fits to galaxies at $z \sim 1$ while varying the hyperparameter $\nu$ using the procedure described in Sec. 5.2. The first three columns show the results for three CANDELS fields, and the last shows the result of combining the likelihood from all three fields. The top row shows the median $\chi^2/\text{DoF}$, the middle row uses it to calculate the likelihood ratio to the best $\chi^2/\text{DoF}$ in the sample. The bottom row shows the distributions of $\chi^2$ values for the individual galaxies. The hatched histogram uses galaxies fit with the SED fitting method in Chapter 3, with a default value of $\nu = 1.5$. The much larger blue distribution contains the $\chi^2$ values for each galaxy fit at varying values of $\nu$ using samples from the GP-SFH posterior distribution.

The value of $\nu \approx 1.9 \pm 0.2$, with GOODS-S as well as the combined dataset finding a slightly higher value of $\nu \sim 2.1 \pm 0.2$ and COSMOS yielding overall looser constraints with $\nu \sim 1.7 \pm 0.4$.

Figure 5.4 shows the results of converting the relative likelihood in $\nu$ into a burstiness timescale by computing the time between excursions from the smooth median SFH at different values of $\nu$. The figure shows the effects of performing this computation using a randomly selected subsample of 1, 10, 100 and 1000 galaxies from the full dataset, showing increasing signal as we increase the sample size. Expressed as a burstiness timescale, we find the optimum to be $\approx 39.8^{+39}_{-4.3}$ Myr. While this timescale is longer than the characteristic timescales for SFR$_{H\alpha}$ (Broussard et al. 2019), it is shorter than the timescales probed by the rest-UV, as well as the reported timescale of $\approx 200$ Myr for $M_* \sim 10^{10}M_\odot$ galaxies at $z \sim 0$ reported by Caplar & Tacchella (2019).
Figure 5.4: It is possible to get increased constraints on the relative likelihood of a given timescale by fitting multiple galaxy SEDs simultaneously and utilizing the increased information content. While individual galaxies show only percent-level change in the relative likelihood for a given timescale, an ensemble of galaxies reveal a preferred timescale as seen through the likelihood ratio. The panels are similar to the middle row of Figure 5.3 with the exception of having expressed the $\nu$ hyperparameter as a burstiness timescale using the procedure described in Sec.5.2.
5.5 Discussion

Cosmological simulations of galaxy formation exhibit fluctuations in SFR over a wide range of timescales, depending on the resolution and prescriptions adopted for various the feedback mechanisms that regulate star formation within galaxies (Hopkins et al. 2014; Brooks et al. 2013; Munshi et al. 2013). This can even be seen in competing theories for explaining the origin of the scatter around the SFR-M∗ correlation, ranging from mergers (Mitra et al. 2016) to AGN and stellar feedback (Matthee & Schaye 2018; Katsianis et al. 2019; Nelson et al. 2019), which manifest as SFR fluctuations over a range of timescales. Observationally, the frequency and strength of SFR fluctuations have been estimated via short timescale tracers and the distributions of their ratios (Weisz et al. 2011b; Guo et al. 2016; Broussard et al. 2019; Emami et al. 2018; Caplar & Tacchella 2019). The current analysis opens this up to a range of timescales, utilizing the greater amount of information available in galaxy SEDs. It would be instructive to compare our findings to those from other simulations and observations, the latter to quantify possible selection effects and biases, and the former to understand the physical mechanisms responsible for these fluctuations, and ultimately explain the scatter in the SFR-M∗ correlation.

For greater robustness, it would be advantageous to repeat the current analysis using the greater information content of spectrophotometry, and with resolved observations in the future. Additional validation tests to quantify possible degeneracies with other SED fitting parameters like metallicity and dust attenuation using simulated galaxies from semi-analytic models (Somerville et al. 2015) or catalogs such as the EAGLE+SKIRT catalog (Trayford et al. 2017) that performs full radiative transfer can help better inform the scales to which the current analysis is sensitive to. Although the effects of additional factors like binaries and stochastic IMF sampling are not considered in the current analysis, larger datasets can allow us to jointly model for these parameters as well.

An additional caveat is that our current analysis relies on two assumptions: (1) that the stochasticity stays constant within a population of galaxies e.g., at a fixed redshift, and that (2) the stochasticity stays constant for a given galaxy throughout its lifetime. Testing the regimes of validity for this assumption using SFHs from cosmological simulations will allow us to better refine our Gaussian Process SFH model to constrain the timescales for SFH fluctuations.
5.6 Conclusions

The SED fitting method described in (Iyer et al. 2019) allows us to reconstruct SFH posteriors from observed SEDs using Gaussian Processes. The Gaussian Process performs regression in cumulative mass-time space, satisfying observational constraints on when a galaxy formed equally spaced quantiles of its total mass. In order to perform an interpolation between these constraints, the Gaussian Process uses a kernel that determines how two points separated by a given distance are correlated. In (Iyer et al. 2019), we used a Matern kernel with a fixed value for a kernel hyperparameter $\nu = 3/2$, that controls how differentiable draws from the SFH posterior are. Increasing $\nu$ results in smoother, less bursty SFHs, while decreasing $\nu$ has the opposite effect. While varying this hyperparameter can change the draws from the SFH posterior considerably, it only causes percent-level changes on the overall median SFH, and thus the SED. Varying the hyperparameter while simultaneously fitting an ensemble of SEDs, however, allows us to obtain the relative likelihood for different values of $\nu$. For GOODS-S, we find that the optimal value of $\nu \approx 1.9 \pm 0.2$. Translating $\nu$ to a burstiness timescale then allows us to observationally determine this sub-temporal-resolution timescale using an ensemble of SFHs. For an analysis conducted using CANDELS/GOODS-S SEDs at $z \sim 1$, we find this timescale to be $39.8^{+39}_{-43}$ Myr.
Chapter 6

Harnessing the power of cosmological simulations

This work was initiated as part of the Kavli Summer Program in Astrophysics (KSPA) 2018, with Sandro Tacchella, Lars Hernquist, Chris Hayward and Shy Genel.

6.1 Introduction

Explaining the diversity of galaxies observed in the universe today is one of the key challenges facing theories of how galaxies form and evolve. The broad features of galaxy assembly have been found to correlate with the assembly of their dark matter haloes (Wechsler & Tinker 2018). Understanding finer features, however, requires us to understand the effects of baryonic physics acting across a range of spatial and temporal scales. These include AGN, supernova feedback and stellar winds, as well as mergers, inflows and outflows of gas. Observationally, these effects manifest in terms of scaling relations such as the observed correlation between the star formation rates (SFR) and stellar masses ($M_*$) of galaxies across a wide range of stellar masses and redshifts (Noeske et al. 2007; Elbaz et al. 2007; Daddi et al. 2007). The star formation histories (SFHs) of galaxies within state-of-the-art cosmological simulations allow us to test the emergence of this correlation (Davé 2008; Dutton et al. 2010; Torrey et al. 2017; Sparre et al. 2015; Matthee & Schaye 2018; Katsianis et al. 2019), as well as contrasting theories about the role of internal vs environmental factors that determine when and how galaxies quench (Weinberger et al. 2018; Teimoorinia et al. 2016).

This analysis is possible because the star formation histories (SFHs) of galaxies contain imprints from the various processes that regulate star formation on different timescales, ranging from dark matter accretion and halo mergers on the longest timescales, to baryon cycling and supernova feedback on shorter timescales. Analyzing the SFHs of large ensembles of galaxies using both observations and simulations allows us to better understand the timescales associated with these processes, and therefore the relative strengths of the mechanisms by which galaxies
6.2 Dataset

We consider the star formation histories from a wide range of cosmological simulations, ranging from hydrodynamical codes (Illustris, IllustrisTNG, MUFASA, SIMBA), a semi-analytic model (Santa-Cruz SAM), an empirical model tuned to match observations across a range of simulations (UniverseMachine), and a zoom-in simulation with a higher resolution and more explicit prescriptions for stellar feedback (FIRE-2). While the SFHs from these simulations show broad consensus for intermediate mass galaxies ($M_\ast \sim 10^{10} - 10^{11} M_\odot$), there is disagreement on when galaxies at the highest and lowest masses formed their stars, due to differences in the choices of subgrid physics for star formation and feedback, as well as their numerical implementation. An additional source of discrepancy comes from the lack of observational constraints as we go to low masses due to the systematics involved in constraining the physical parameters of faint galaxies, and the small number statistics for the highest-mass galaxies due to the limited volumes that cosmological simulations are run in. Figure 6.1 shows the distributions of stellar masses for the galaxies from each simulation at $z \sim 0$, where we perform the current analysis. The FIRE-2 zoom-in simulation (Hopkins et al. 2014) has a much small sample of 22 galaxies spanning a range of stellar masses from $\sim 10^6 M_\odot$ to $\sim 10^{11} M_\odot$. While these simulations allow us to probe SFH fluctuations to shorter timescales, the lack of a statistically-large ensemble of galaxies restricts us from generalizing these results. Each simulation is described briefly below, with references to relevant papers containing more detailed descriptions. Table 6.1 contains a summary of the resolution, box size and number of galaxies from each simulation.

- **Illustris** (Vogelsberger et al. 2014a; Genel et al. 2014)

  The Illustris project is a large-scale hydrodynamical simulation of galaxy formation. The model includes recipes for primordial and metal-line cooling, stellar evolution and feedback, gas recycling, chemical enrichment, supermassive black hole growth and AGN feedback.

- **IllustrisTNG** (Pillepich et al. 2017; Weinberger et al. 2016)

  A significantly updated version of the original Illustris project, IllustrisTNG carries over recipes for star formation and evolution, chemical enrichment, cooling, feedback with outflows, growth and multi-mode feedback from Illustris. In addition to this, it incorporates new black hole driven kinetic feedback at low accretion rates, magnetohydrodynamics and
Figure 6.1: The stellar mass distribution of $z\sim0$ galaxies from cosmological simulations and models we consider, normalized to a common volume: Illustris (Vogelsberger et al. 2014b; Genel et al. 2014), IllustrisTNG (Pillepich et al. 2017; Weinberger et al. 2016), EAGLE (Schaye et al. 2014), MUFASA (Davé et al. 2016b), SIMBA (Davé et al. 2019), the Santa Cruz SAM (Somerville et al. 2008; Porter et al. 2014), and UniverseMachine (Behroozi et al. 2018).
improvements to the numerical scheme. In addition to the regular TNG100 run, a number of 25Mpc$^3$ small box simulations vary the different physics models to quantify the effects of feedback and winds on the SFHs of galaxies. While the statistics of these SFHs may be subject to some cosmic variance in comparison with TNG100, the small-volume runs can be compared self-consistently with each other.

- **EAGLE (Schaye et al. 2014)**
  The Evolution and Assembly of GaLaxies and their Environments (EAGLE) is a cosmological hydrodynamic simulation of galaxy formation, resolving down to 0.7 kpc for gravity which allows for accurate modeling of the warm gas within galaxies. For processes on smaller scales, the simulation includes subgrid recipes for stellar evolution, cooling and heating of gas due to stars and other emission, metal enrichment of ISM gas and energy injection due supernovae, and the formation, accretion and feedback due to AGN.

- **MUFASA (Davé et al. 2016b)**
  The MUFASA meshless hydrodynamic simulations include prescriptions for cooling and heating with Grackle, star formation and feedback from massive stars using scalings from FIRE (Hopkins et al. 2014) producing outflows that are tightly correlated with mass, almost independent of redshift. The model also includes feedback from long lived and AGB stars, and an implementation of quenching feedback that quenches galaxies by heating all the gas in massive halos (except gas that is self-shielded). At $z \sim 0$, their quenching mass is $M_h \sim 10^{12} M_\odot$.

- **SIMBA (Davé et al. 2019)**
  The SIMBA cosmological galaxy formation simulations are built on the MUFASA simulations including black hole growth and feedback. The simulation uses a torque limited BH accretion model, along with BH feedback that operates on two modes depending on the Eddington rate (Anglés-Alcázar et al. 2017).

- **Santa-Cruz SAM (Somerville et al. 2008, 2015; Yung et al. 2018; Brennan et al. 2016)**
  The Santa-Cruz Semi-Analytic Model contains a number of well motivated sub-grid prescriptions that are used in conjunction with the Bolshoi-Planck dark matter N-body simulations’ merger trees to construct populations of galaxies that are tuned to match observations at $z = 0$. The model implements two modes of star formation: a ‘normal’ mode following the Schmidt-Kennicutt relation along with exploding supernovae which drive
outflows with recycling that occurs in isolated discs, and a ‘starburst’ mode that occurs as a result of a merger or internal disc instability.

- **UniverseMachine (Behroozi et al. 2013b, 2018)**

  The UniverseMachine is an empirical model that determines the star formation rates of galaxies as a function of their host haloes’ potential well depths, assembly histories, and redshifts. The model uses Bolshoi-Planck DM merger trees and a variety of observational constraints including observed stellar mass functions, SFRs, quenched fractions, UV luminosity functions, UV-stellar mass relations, autocorrelation functions, and quenching dependence on environment to constrain its free parameters.

- **FIRE-2 (Hopkins et al. 2014)**

  The Feedback In Realistic Environments simulations consider a fully explicit treatment of multi-phase ISM, stellar feedback. This is important since the $M_\ast-M_{\text{halo}}$ relation is sensitive to feedback physics. Supernova feedback alone is not enough, radiative feedback (photo-heating and radiation pressure) is needed to destroy GMCs and enable efficient coupling of later SNe to gas. Feedback also produces reservoirs of gas for extended SF at late times. Gas particles in the simulation follow an ionized+atomic+molecular cooling curve, and star formation is only allowed in dense, molecular, self-gravitating regions above a density threshold. Stellar feedback includes contributions from radiation pressure, supernovae, stellar winds, photo-ionization and photo-electric heating.

### 6.3 The variability of SFHs at different timescales

To compare the star formation histories from different simulations on the same footing and understand the contribution to an SFH arising from SFR fluctuations on different timescales, we compute the power spectral densities (PSDs) of SFHs. The PSD corresponding to the SFH for an individual galaxy reports a phase-averaged estimate of the strength of SFR fluctuations at a given frequency. For a sinusoidal signal with a frequency $\nu$, the corresponding PSD is a single spike at $\nu$. Generalized to more complicated timeseries, the PSD therefore provides a way to disentangle and interpret the strength of the fluctuations on different timescales, as previously done in studies of AGN variability and theoretically with SFHs (Caplar et al. 2017; Caplar & Tacchella 2019; Sartori et al. 2018). Figure 6.2 shows the technique applied to SFHs,

---

1 or, inverting it, at a given timescale
Figure 6.2: Power spectral densities (PSDs) for three galaxies from IllustrisTNG. (Left:) The individual galaxy SFHs, obtained by binning mass-weighted star particles in 100 Myr bins. (Middle:) Isolating the contributions from fourier modes corresponding to different timescales - the red curves show the power arising due to the long timescale ($\lambda \sim 13$ Gyr) mode, green curves show the power contribution from intermediate timescales ($\sim 1.3$ Gyr) and blue from relatively shorter timescales ($\sim 410$ Myr). (Right:) The integral under the curve of the contribution for each timescale gives the strength of the PSD at that timescale. The black line shows the full PSD, while the three colored points correspond to the individual timescales shown in the middle panel. As expected, overall trends in the SFH can be described using the first few fourier modes which correspond to the longest timescales. However, depending on the shape of the SFH, the distribution of power on shorter timescales can change significantly.
<table>
<thead>
<tr>
<th>Simulation Name</th>
<th>Type</th>
<th>Box Length</th>
<th>$m_{DM}$</th>
<th>$m_*$</th>
<th>$n_{galaxies}$</th>
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</thead>
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<tr>
<td>Illustris</td>
<td>Hydro</td>
<td>106.5 cMpc</td>
<td>6.26 $10^6 M_\odot$</td>
<td>1.26 $10^6 M_\odot$</td>
<td>19354</td>
</tr>
<tr>
<td>IllustrisTNG</td>
<td>Hydro</td>
<td>110.7 cMpc</td>
<td>7.5 $10^6 M_\odot$</td>
<td>1.4 $10^6 M_\odot$</td>
<td>12220</td>
</tr>
<tr>
<td>Mufasa</td>
<td>Hydro</td>
<td>50 cMpc</td>
<td>96 $10^6 M_\odot$</td>
<td>48 $10^6 M_\odot$</td>
<td>3042</td>
</tr>
<tr>
<td>Simba</td>
<td>Hydro</td>
<td>100 cMpc</td>
<td>96 $10^6 M_\odot$</td>
<td>18 $10^6 M_\odot$</td>
<td>11300</td>
</tr>
<tr>
<td>Eagle</td>
<td>Hydro</td>
<td>100 cMpc</td>
<td>9.7 $10^6 M_\odot$</td>
<td>1.81 $10^6 M_\odot$</td>
<td>7482</td>
</tr>
<tr>
<td>Santa-Cruz SAM</td>
<td>SAM</td>
<td>100 cMpc</td>
<td>203.7 $10^6 M_\odot$</td>
<td>N/A</td>
<td>12821</td>
</tr>
<tr>
<td>UniverseMachine</td>
<td>Empirical</td>
<td>47.8 cMpc</td>
<td>203.7 $10^6 M_\odot$</td>
<td>N/A</td>
<td>7361</td>
</tr>
<tr>
<td>FIRE-2</td>
<td>Zoom-in</td>
<td>N/A</td>
<td>8.9($10^2$) to 1.8($10^2$) to 22</td>
<td>1.58($10^6$) $[M_\odot]$ 0.26($10^6$) $[M_\odot]$</td>
<td></td>
</tr>
</tbody>
</table>

Table 6.1: Details of the various models compared in this chapter. The box length for UniverseMachine denotes the subset of the full $250/h\,h^{-1}Mpc$ box used in the current analysis. The number of galaxies reported is the subset of central galaxies with stellar masses $> 10^9 M_\odot$.

yielding complex PSDs with features spread over a broad range of timescales. Unlike the case for the sine wave, the power in these PSDs is spread over a large dynamic range, indicative of the wide range of timescales over which physical processes in galaxies induce fluctuations in the star formation rates. With a thorough understanding of a galaxy’s evolution and merger history, it might be possible to interpret its individual PSD. However, in the current work we focus on studying the broader trends in a sample of galaxy SFHs and their corresponding PSDs as a way to compare different models for galaxy evolution on the same footing. In doing so, we examine the variability of galaxy SFHs on medium to long timescales, and correlate them with the observed diversity in galaxy SFHs. In future work, we will narrow our focus to analyze the PSDs of galaxies within individual simulations as we vary the relative strengths of the different processes regulating star formation.

### 6.3.1 Power Spectral Density

For a continuous time series $\psi(t)$, the PSD is defined in terms of the Fourier transform $f(k) = \int dt \ e^{-ikt} \psi(t)$ as $PSD(k) = |f(k)|^2$.

We compute SFHs for each galaxy in the hydrodynamical simulations under consideration (Illustris, IllustrisTNG, MUFASA, SIMBA, EAGLE, FIRE-2) by performing a mass-weighted binning of the star particles with $\Delta t = 100 \, \text{Myr}$. For each SFH, we then compute the PSD using Welch’s method (Welch 1967), implemented in the scipy.signal.welch module. Since the
Figure 6.3: The median star formation histories (SFHs; left) and corresponding power spectral densities (PSDs; right) of galaxies from different cosmological hydrodynamical simulations, shown here in 0.5 dex bins of stellar mass. PSDs are computed from individual SFHs prior to being averaged, with the shaded regions showing the 16th-84th percentile of the distribution in a given mass bin at each point in time (left) and fluctuation timescale (right). Dashed lines indicate regions where shot noise due to discrete star particles may contaminate the PSDs according to our validation tests (see Section D.2). While the SFHs of galaxies in the same mass bins look broadly consistent across the different models, there are notable differences in the shapes of their PSDs, which indicates different physical processes shaping these galaxies.
Figure 6.4: Figure 6.3 continued.
SFHs span a large dynamic range, we work in log SFR space in order to be able to better quantify the relative strengths of SFR fluctuations. This is also motivated by physical considerations, since the SFRs of star forming galaxies are often found to be distributed normally in log SFR space, with a tail towards low SFRs due to galaxies that are experiencing episodes of quiescence (Feldmann 2017; Hahn et al. 2018). Analyzing the SFHs in linear SFR space effectively amounts to a different weighting scheme. In choosing the $\Delta t$ for our time bins, we need to consider the effects of the discreteness of individual star particles, since the SFR will be zero in bins that do not contain any star particles. This effect is particularly important for low-mass galaxies, where the number of star particles is $O(10^2 - 10^3)$. If not accounted for, these bins lead to shot noise in log SFR space, biasing the computed PSDs. We avoid this by increasing the size of the time bins until the fraction of our data with $SFR = 0$ is significantly reduced. In practice, we find that with time bins of 100 Myr, the percentage of bins where $SFR = 0$ is $\leq 3\%$. We then set values of $SFR = 0$ to $SFR = SFR_{min} = 10^{-3} M_\odot yr^{-1}$ for a given simulation to avoid values of $-\infty$ in the PSD computation. We test that this does not significantly affect the PSDs of quenched galaxies, finding that even for quenched galaxies the SFR is $\geq 10^{-3} M_\odot yr^{-1}$ throughout the lifetimes of most of the galaxies in our sample. An example of this can be seen in the bottom row of Figure 6.2, which shows the SFH for a single quenched galaxy from IllustrisTNG.

6.3.2 Shot noise due to discrete star particles

In the hydrodynamical simulations we consider, gas is turned into a star particle probabilistically, depending on whether certain temperature and density conditions are met. This introduces a $O(1)$ fluctuation in a given time bin (width $\Delta t$) based on whether the N+1th star particle is created. In log SFR space, this creates large fluctuations when the SFR is low, i.e., there are only a few star particles in a given time bin. To avoid this, we only consider the portion of the PSD on timescales ($\Delta t > \Delta t_{min}$) that are large enough than there are enough star particles in a bin to minimize the effects of discrete star particles.

For a 5% tolerance in the power spectrum due to the white noise at a given timescale, the constraint on the shot noise follows from the definition of the PSD

$$\left| \sum_n SFR(t_n) e^{-i f_n \Delta t} \right|^2 - \left| \sum_n (SFR(t_n) \pm O(1)) e^{-i f_n \Delta t} \right|^2 < 0.05 \frac{\tau_H}{(\Delta t)^2}$$

(6.1)

Given a constant SFR and a uniform probability for a star particle to be created at any
timestep, this simplifies to

$$\Delta t > 0.05 \cdot \frac{4\tau_H}{\langle SFR(t) \rangle}$$

(6.2)

i.e., as the average SFR (or stellar mass) for a given galaxy decreases, we are limited to increasingly longer timescales for which we can probe the PSD accurately.

Appendix D.2 models the effects due to discrete star particles of different masses in more detail with realistic SFHs, to determine the shortest timescales that can be probed at a given epoch given a certain mass for the star particles in the simulation.

### 6.3.3 SFHs and PSDs across different cosmological simulations

In Figures 6.3 and 6.4 we show the star formation histories and corresponding PSDs for galaxies binned in intervals of stellar mass. We see that there is a large amount of diversity in both the star formation histories and the PSDs of galaxies from different simulations, although some broad trends can be observed. In all cases, the PSDs generally rise towards longer timescales, indicating the increasing contribution to the overall SFH from fluctuations on the longest timescales where the hierarchical merging of dark matter halos dominates (Wechsler & Tinker 2018). The slope of the PSDs change as a function of stellar mass, with a slight rise in the overall normalization as galaxies grow more massive. This is because the total area under the PSD is proportional to the total power of the signal, which grows as the median SFR increases with galaxy mass.

Apart from these overall similarities, the PSDs and corresponding SFHs display a lot of variety across different models, with MUFASA and SIMBA showing greater variability on short timescales compared to Illustris, IllustrisTNG, Santa Cruz SAM and UniverseMachine. To examine if this is in part due to resolution effects, since MUFASA and SIMBA have much higher-mass star particles compared with Illustris, IllustrisTNG and EAGLE, we consider the PSDs of three different IllustrisTNG runs with varying resolution in Section 6.3.6.

From the SFHs, we see that galaxies tend to peak in SFR at earlier times as they grow massive. As seen with an overlapping set of simulations in (Hahn et al. 2018), lower-mass galaxies are mostly star forming, with the fraction of quenched galaxies increasing as we go to higher masses. The location SFR peak and the width of the SFHs as a function of mass changes strongly depending on the implementation of stellar and AGN feedback across different simulations. This is also reflected in the PSDs of the highest-stellar mass bins. Observational constraints on the strength and slope of the PSD at the 300 Myr to 1 Gyr regime will help put strong constraints on the mechanisms responsible for the quenching of galaxies.
6.3.4 SFHs and PSDs of zoom-in simulations

In addition to the cosmological models, the zoom-in simulations allow us to test specific parts of the PSD parameter space that are not accessible at present with cosmological simulations. In Figure 6.5, we probe the PSDs of galaxies down to very short timescales using galaxies from the FIRE-2 simulations, with star particles of $10^2 - 10^6 M_{\odot}$ (Hopkins et al. 2014). This is especially relevant since observations of star formation rates estimated from $H\alpha$ and the UV can allow us to constrain the slope of the PSD in this regime. On the longest timescales, the FIRE-2 dwarfs have less power since they do not currently contain an implementation for AGN feedback and thus don’t experience episodes of periodic quenching due to it. On shorter timescales, the PSDs continue to display the same power-law behaviour as the cosmological simulations, albeit with a steepening towards shorter timescales. This indicates the processes regulating star formation on short timescales are even more heavily correlated than that on longer timescales, possibly due to the feedback and spatial turbulence in these high-resolution simulations.
Figure 6.5: The SFHs and corresponding PSDs of individual galaxies from the FIRE-2 high-resolution cosmological zoom-in simulation. The simulation self-consistently models explicit supernova feedback and resolves the turbulent ISM, allowing us to probe the PSDs to much lower timescales than possible with the large cosmological simulations (shaded grey box). For ease in reading the figure both SFHs and PSDs have been smoothed, with a representative un-smoothed curve shown in both panels (rapidly fluctuating lines) along with its smoothed counterparts (thick solid lines). Individual PSDs are computed by re-binning SFHs such that < 10% of the time bins have 0 star formation, minimizing the effects of shot noise. Individual galaxies are colored by stellar mass, with darker, bluer colors denoting more massive galaxies. There is a notable break in the PSD at \( \sim 10^{2.5}\) Myr, with a steepening going to shorter timescales.

6.3.5 The PSDs of Dark Matter Accretion Histories

To better understand the relation between the PSDs of galaxy SFHs and their parent halos, we compute the PSDs for a sample of Dark Matter accretion histories, defined as \( \Delta M_{\text{halo}} \) from one timestep to another with the same \( \Delta t = 100\) Myr bin width. We find that the PSDs show a remarkable self-similarity, with a slight increase at the longest timescales corresponding to the overall normalisation of the halo mass.
6.3.6 Resolution effects in computing PSDs

We consider three realizations of the TNG100 simulation, which are identical except for resolution. These are described in further detail in Pillepich et al. (2017), and contain star particles with initial masses of $9.44 \times 10^5 M_\odot$, $7.55 \times 10^6 M_\odot$, and $6.04 \times 10^7 M_\odot$ respectively in TNG100-1,2 and 3. We show the SFHs and corresponding PSDs for these runs in Figure 6.7.

In SFH space, we see that the different resolutions have a large impact on SFHs across all masses, especially in the portions with low SFRs. For the three simulations, with our adopted 100 Myr time bins, the lowest SFRs we can probe is $\approx 10^{-2.02}$, $10^{-1.12}$ and $10^{0.68} M_\odot yr^{-1}$ neglecting mass loss. In practice, mass loss decreases the star particle mass somewhat, which allows us to probe SFRs to $\sim 0.22$ dex lower at later times. We see this in effect as the median SFHs for low-mass galaxies grow increasingly dominated by shot-noise and in the case of TNG100-3, completely drop off the plot. This resolution effect also affects high-mass quenched galaxies, which leads to the apparent more rapid quenching of the median SFHs in the highest-mass bins. This is simply because the SFRs can only drop to their minimum value from the quantum of SFR given the resolution, leading to a steeper apparent drop in the SFHs.

In PSD space, we see that the effect of lower resolution is to increase the amount of white-noise in the PSDs, which manifests as a flattening to spectral slopes of 0 towards shorter timescales. The affected areas are shown in the figure with dotted lines. In addition to affecting the PSDs to higher masses, the white noise also increases in magnitude proportional to the mass of the star particles, leading to contamination at longer timescales in a given stellar mass bin.
6.4 What sets the diversity of SFH shapes?

In order to quantify the diversity in galaxy SFHs, we look at three key quantities, the specific star formation rate (sSFR), the lookback time since the galaxy accumulated half of its mass \((t_{50})\), and the time interval between when it accumulated its first quarter and before it accumulated its last quarter of stellar mass \((t_{75} - t_{25})\). The sSFR is defined as the ratio of a galaxy’s SFR to its stellar mass. For a linear SFR-\(M_\star\) correlation (Whitaker et al. 2014; Schreiber et al. 2015), the sSFR therefore measures a galaxy’s excursion from the correlation, which can be defined as the mode of the sSFR distribution. The sSFR can then be used as a tracer of whether a galaxy is actively star forming or quiescent. Since the existence of the SFR-\(M_\star\) correlation implies a relation between a galaxy’s current star formation rate and its built-up mass, this correlates with the overall shapes of a galaxy’s SFH. In cases where the SFR-\(M_\star\) correlation is sub-linear (Speagle et al. 2014; Iyer et al. 2018), the reliability of this tracer is reduced, although still valid for most of the models we consider (Hahn et al. 2018), and pairing it with other indicators of SFH shape help alleviate this issue. Similarly, the \(t_{50}\) indicates the lookback time when a galaxy accumulated half its total mass. Since most galaxy SFHs are found to have one major episode of star formation (Dressler et al. 2016; Iyer et al. 2019), the \(t_{50}\) indicates the time at which the galaxy was most actively star forming. Since the galaxy formed equal amounts of its mass before and after its \(t_{50}\), it acts as a good tracer of whether the galaxy formed more of its mass at early or late times. For galaxies with small \(t_{50}\), we expect their SFHs to be actively rising, while galaxies with \(t_{50}\) closer to the age of the universe are expected to have SFHs that are quenched, since they finished forming the first half of their mass at an earlier time. Finally, the quantity \((t_{75} - t_{25})\) provides a measure of the overall width of the SFH, since it traces the time it took to accumulate the median 50% of its total mass. In practice, we find that this quantity is roughly normally distributed, with a tail towards older, more massive quenched galaxies that have shorter overall widths over which they formed most of their mass. Distributions for these quantities are shown in Figure 6.8. In addition, the distributions of these quantities along with their mutual covariances and covariances with stellar mass and star formation rate for each simulation are shown in Appendix D.1.

By correlating how galaxy PSDs vary with these quantities, we gain an insight into how SFH fluctuations on different timescales contribute in creating a diversity in these quantities. Considering the amount of information in the full PSDs, it is a daunting task to understand the full correlation between every timescale probed by the PSD and these tracers of SFH diversity. We are helped by the fact that the PSDs for any ensemble of galaxies are relatively smooth, as
Figure 6.7: Exploring the effects of lowering the resolution (increasing the star particle masses) on using IllustrisTNG.
Figure 6.8: Distributions of the specific star formation rate, half-mass lookback time and SFH width for our sample of cosmological simulations. The shaded region indicates the location of the SFR-$M_*$ correlation.
seen in Figures 6.3 and 6.4. We therefore fit the PSD for each galaxy to find its spectral slope at shorter (1 – 3 Gyr) and longer (3 – 10 Gyr) timescales and show plots of how they vary as a function of sSFR, $t_{50}$, and $(t_{75} – t_{25})$ for each simulation in Figure 6.9.

The spectral slope ($\alpha$), defined as the power-law slope of the PSD, is an important quantity in characterising how power is transferred from longer timescales (lower frequencies) to shorter timescales (higher frequencies). For turbulent systems, this slope characterises the energy cascade from longer to shorter scales, and is known to have a slope of 5/3 for classical Kolmogorov turbulence (Zakharov et al. 2012). In the context of time-series, a slope of 0 indicates an equal amount of power on all timescales and is called white noise, where star formation is an essentially stochastic process and the SFRs at two times are not related to each other. Brownian motion (or brown noise, as it is referred to in signal processing), has a spectral slope of 2 and is the integral of white noise. Brownian motion in the context of star formation has been previously explored (Kelson 2014) as a potential explanation for the SFR-M$_*$ correlation, which emerges naturally if the SFR at a given time is correlated with star formation rates at previous times. Although this might be too simplistic, it provides a useful framework to analyse deviations from brownian motion due to different physical processes.

In addition to measuring the spectral slope itself, it is also instructive to analyze any breaks that may occur in the slope from one timescale to another. This is the rationale between measuring the slope on two timescales - both the slope itself and the difference in the slope from one reference timescale to the next can be correlated with the SFHs. In Figure 6.9, we see that the PSD slopes for the longer (3-10 Gyr) timescale range between 0 and 4. With the exception of UniverseMachine, most models show an anticorrelation between sSFR and PSD slope. This could point towards star formation in galaxies with high sSFR being more correlated across a range of timescales. Low sSFR galaxies display a higher slope on long timescales, presumably due to the physical processes responsible for quenching galaxies. For $t_{50}$, the longer timescale slope is $\sim 2$ for most models, with the exception of the Santa-Cruz SAM. However, there is an increasing break between the longer and shorter timescale PSD slopes for galaxies with smaller $t_{50}$, which tend to be younger and have rising SFHs. This indicates that there is more variability on shorter timescales for these galaxies, with the recent star formation being uncorrelated on smaller timescales with the previous overall trend of star formation. The origin and strength of this break, however, seem to be highly model dependent, with a sharp break in Illustris and EAGLE, and little-to-no-break with UniverseMachine, and need observational constraints for determining which model is most representative of reality. Finally, there is a consistent anticorrelation across the models with respect to the SFH width, with the PSD slope decreasing
Figure 6.9: Correlation of the short (1-3 Gyr; dashed lines) and long (3-10 Gyr; solid lines) spectral slope of the PSD as a function of quantities that trace the diversity of SFHs in different simulations. Solid lines indicate the spectral slopes for brownian motion ($\alpha = 2$) and white noise ($\alpha = 0$) respectively.

smoothly as the widths of the SFHs increase. This is consistent with the rest of the results, since the SFHs with the narrowest widths tend to be the oldest and most massive quenched galaxies, which are known to have high long-timescale slopes from the sSFR analysis. There is an interesting break between the longer and shorter timescale slopes for intermediate widths, which corresponds to galaxies that might be undergoing periods of rejuvenation that might be related to the strength of baryon cycling in the different models.

The diversity in SFHs therefore seems to be a result of the different strengths at which power from the star formation over the shortest timescales feeds galaxy growth over longer timescales.

6.5 Discussion

The SFHs of galaxies across different cosmological simulations show a considerable amount of diversity. This is a result of (1) different model assumptions and prescriptions for physical processes such as black hole growth, wind speeds, and stellar and AGN feedback (2) differences in resolution, numerical implementation, and degree of sub-grid vs explicit hydrodynamical...
modeling across these simulations. Studying the PSDs of these SFHs across different masses provides a representation to better understand these effects. Comparing runs of IllustrisTNG-100 with different resolutions allows us to quantify the effects of resolution on the PSDs, which manifest as a ‘white-noise’ floor that contaminate the PSDs at shorter timescales and lower masses.

Although averaged over all phases, the PSDs of galaxy SFHs contain a considerable amount of information about their overall shapes and behaviours. This may due to the fact that different physical processes that induce fluctuations on different timescales are also correlated with their phase, leading to a direct correlation between the PSD and the SFH shape.

To first order, the accretion rates of the dark matter halos that host individual galaxies are thought to influence the shapes of their star formation histories (Wechsler & Tinker 2018), and have long been used as a cornerstone for empirical models of galaxy formation (Behroozi et al. 2013b; Moster et al. 2018; Behroozi et al. 2018). These halos have self-similar PSDs as a result of hierarchical growth, with most of their power on long timescales ($> 1Gyr$). We can therefore consider the amount of SFH diversity set by these long timescales to be correlated with the DM accretion histories, as seen through the cross-power spectra in Figure 6.10.

By correlating the SFHs of galaxies with the slopes of their PSDs on long and short timescales, we see that there is a considerable difference across simulations in the extent to which the coupling between fluctuations on different timescales (i.e., PSD slope) influences the sSFR, $t_{50}$ and $(t_{75} - t_{25})$ (which describes the shape of each SFH), beyond the white-noise regime. This is partly due to differences in baryonic physics (between MUFASA and SIMBA due to a new torque-driven accretion mode for supermassive black holes (Davé et al. 2019),
and between Illustris and IllustrisTNG (Pillepich et al. 2017; Weinberger et al. 2018), due to improved stellar winds and black hole feedback models) that influence how galaxies build up mass on short timescales (Hopkins et al. 2014), and how they quench (Terrazas et al. 2019). Additionally, both UniverseMachine and the semi-analytic model have little contributions from short timescales, since they do not explicitly model the turbulent hydrodynamics of galaxies. As a result of this, the variance in star formation rates can be explained in different ways, although these simulations are all calibrated to reproduce the same observables such as luminosity and mass functions, and scaling relations like the SFR-M∗ correlation at z ∼ 0.

Observational constraints, both from the SFHs of galaxies through techniques like SED fitting (Pacifici et al. 2012; Leja et al. 2017; Iyer et al. 2019) and from observed distributions of SFR over different timescales (Broussard et al. 2019; Emami et al. 2018; Caplar & Tacchella 2019) can help constrain the PSDs, and therefore the relative strengths of different physical processes that regulate star formation within galaxies. Additional analysis using the IllustrisTNG 25Mpc^3 runs that vary different physical prescriptions can be used to obtain better constraints on which feedback models need to be refined.

6.6 Conclusions

A range of physical processes acting over different timescales regulate star formation within galaxies. The resulting process of galaxy growth is therefore diverse, and understanding the impact of the underlying processes helps explain this diversity. On long timescales (> 3 Gyr), galaxy SFHs are mostly regulated by the behaviour of their host dark matter halos and processes that quench galaxies, while on shorter timescales (300 Myr to 3 Gyr) baryonic processes dominate. Studying the power spectral densities (PSDs) of galaxy SFHs allows us to better understand the contribution to galaxy growth from processes that cause SFR fluctuations on different timescales.

We find that most galaxies are characterised by PSDs that can be described as a piecewise power-law, with the most power on the longest timescales (shortest frequencies). The slope of the power-law at a given timescale is related to how tightly coupled star formation is at that timescale. Binned by stellar mass, there is considerable diversity between PSDs of galaxies at the same average mass from different simulations. Linking the power-law index to different processes using simulations can allow us to better understand the effects of different baryonic processes involved in regulating star formation on different timescales. Comparing the PSDs of SFHs to those of dark matter accretion histories from IllustrisTNG allows us to explain the power-law
behaviour to first order. While contamination to the PSD due to discrete star particles (i.e., resolution effects) prevents us from probing the shorter timescales for cosmological simulations, the high-resolution FIRE-2 simulations allow us to probe the PSDs for a small number of galaxies to much shorter timescales, finding a similar power-law behaviour. Correlating the PSD slopes on shorter (1-3 Gyr) and longer (3-10 Gyr) timescales with different tracers of SFH diversity, like $\text{sSFR}$, $t_{50}$ and $(t_{75} - t_{25})$ reveals correlations between the PSD slope and quenched galaxies, as well as breaks in the PSD slope for galaxies of intermediate SFH width ($t_{75} - t_{25}$). Further refining these models with observational constraints will allow us to better understand the complex multiscale physical processes involved in baryon cycling within galaxies.
Chapter 7
Conclusions, and looking forward

It is a mistake to think you can solve any major problems just with potatoes.

Douglas Adams

7.1 Summary

Interpreting the observed SEDs of galaxies allows us to gain an understanding of their physical properties. By looking at the spectral energy distributions (SEDs) of galaxies either in observational spaces (color-magnitude, UVJ, BzK diagrams etc.) or in spaces of derived physical parameters (SFR-M_*, M_*-Z, etc.) it is possible to identify and study the different demographics of the overall galaxy population at a given redshift.

Section 1.6 started this thesis with the aim of establishing a methodological foundation, based on which current open questions in galaxy evolution can be answered. At the summary of this work, we have answered the first (can we accurately estimate the SFHs of galaxies from observations?), with a clear direction along which to proceed to make further progress. This section briefly summarizes the conclusions of the preceding chapters, followed by short descriptions of motivating future investigations.

In keeping with better broadband SEDs from current and upcoming surveys (higher resolution/more filters, increased wavelength coverage, higher S/N), SED fitting techniques have been improving to extract the maximum amount of information from the observations. One such methodological improvement has been developing methods that allow us to relax the traditional assumption of simple parametric forms for a galaxy’s star formation history (SFH). With the Dense Basis method, we do this by developing a novel basis space to express the SFH in, first by considering all the parametric forms found in the literature and their combinations (Chapter 2), and subsequently via a formalism that describes the SFH using an N-tuple consisting of (M_*, SFR, \{t_X\}) (Chapter 3). Doing so allows us to construct robust, non-parametric descriptions of a galaxy’s SFH, which results in minimizing the bias and scatter in deriving estimates of physical properties that depend on the SFH.
Inferring robust nonparametric SFHs with uncertainties for individual galaxies also allows us to estimate quantities that were previously inaccessible through SED fitting, such as the number and duration of major episodes of star formation in a galaxy’s past (Chapters 2 and 3). It also allows us to quantify timescales for episodes of rejuvenation (Ch.3) and quenching, as well as the timescales on which morphological transformations and mergers impact the SFH by studying the correlation with independently derived morphological classifications (Ch.3).

The existence of robust SFHs with uncertainties allows us to place better observational constraints on current theories of galaxy evolution. Propagating galaxies backwards in time along their SFR-M$_*$ trajectories allows us to probe the low-stellar mass, high-redshift regime of the SFR-M$_*$ correlation, finding that it continues its power-law form in this previously unexplored parameter space (Chapter 4). Studying an ensemble of galaxies simultaneously allows us to estimate the characteristic timescales of star formation stochasticity through their broadband SEDs (Chapter 5). Finally, by studying the power spectral densities (PSDs) of the SFHs corresponding to galaxies in cosmological simulations, it is possible to better understand the influences of different physical processes that affect galaxy growth on different temporal scales (Chapter 6) and to formulate better SFH priors for usage during SED fitting.

The current implementation of the method is extremely fast (capable of fitting $\sim 30$ galaxies/core/sec for galaxies in the CANDELS/GOODS-S field), and the SFH formalism itself can be incorporated into any sophisticated SED fitting code. In addition, it has applications as an independent data compression method.

7.2 Further Developments in SED fitting

7.2.1 Extending to fitting spectroscopy and spectrophotometric data

Most of the data used in the analysis so far has been broadband photometry, due to the availability of photometric SEDs for large numbers of high-redshift galaxies from surveys like CANDELS (Grogin et al. 2011; Koekemoer et al. 2011) and 3D-HST (Skelton et al. 2014). However, future surveys including HETDEX (Papovich et al. 2016; Hill et al. 2008), SUBARU PFS (Takada et al. 2014) and those conducted with the James Webb Space Telescope (JWST; Williams et al. 2018) will observe high-quality spectra for $O(10^6)$ galaxies across a range of redshifts. The rest-optical portions of these spectroscopic SEDs contain key information that will help constrain the SFHs of galaxies to higher accuracy. However, modeling these SEDs requires more precision than currently accounted for by our templates, with additional parameters needed to account
for any wavelength-dependent flux calibration of the spectra. Efforts to extend the Dense Basis method to fitting spectroscopic data, as well as jointly fitting spectrophotometric data are currently underway. Once completed, this will allow us to take full advantage of current and future spectroscopic datasets to better constrain the high-uncertainty regimes in the SFHs of distant galaxies.

7.2.2 Resolved SED fitting

High-resolution spectroscopic observations of nearby galaxies using integrated field unit (IFU) spectroscopy with surveys like CALIFA (Delgado et al. 2014) and SDSS-IV MaNGA (Rowlands et al. 2018; Goddard et al. 2016) position rows of fibers over galaxies, allowing for spatially resolved spectra that can be analyzed to obtain resolved maps of galaxy properties\(^1\).

Fitting these IFU observations to recover spatial distributions of galaxy properties is key to understanding and testing how galaxies build central bulges, what drives morphological transformations, and the roles of mergers, as well as structures such as spiral arms and bars in regulating star formation. Multiple studies have looked at the radial profiles of star formation within galaxies (Delgado et al. 2014), as well as the spatially resolved SFR-M\(_*\) correlation (Hsieh et al. 2017). In conjunction with kinematic analyses, these observations help better correlate processes that occur over dynamical timescales in galaxies to global properties like stellar masses and chemical abundances.

Reconstructing the spatially resolved star formation histories from these observations unlocks the possibility of recovering timescale information about each of the processes described above, testing theories from cosmological simulations about bulge-growth, environmental (outside-in) vs morphological (inside-out) quenching, merging timescales and delayed bursts of star formation, the role of feedback on spatially resolved scaling relations within galaxies, and more. Currently, resolved SEDs are still fit in individual bins or using simple parametric models, an area where the Dense Basis method would be a definite improvement. Additionally, spectra are either fit individually (on a spaxel-by-spaxel basis ignoring covariances between adjacent spaxels) or using a spatial binning technique called Voronoi tesselations that bin SEDs in terms of similarity before fitting. Forward modeling the fitting process using techniques like Gaussian Processes will allow us to take advantage of the information in these datacubes and obtain tighter constraints than currently possible.

The 15th data release of SDSS MaNGA (Bundy et al. 2014) contains publicly available

\(^1\)https://jwst-docs.stsci.edu/display/JPP/Introduction+to+IFU+Spectroscopy gives an excellent introduction to the topic.
datacubes for nearly 5000 galaxies. Preliminary spectroscopic fits to two galaxies from this dataset are shown in Figure 7.1, illustrating a small subset of the available physical properties that can be probed using this process. Upcoming NIRSpec-IFU observations with JWST\(^2\) will allow us to extend these techniques to much higher-redshift galaxies, making the development of such techniques timely.

### 7.2.3 Better models for metallicity histories and differential dust attenuation

In addition to the stellar masses (\(M_\star\)) and star formation rates (SFRs) of galaxies, the metallicity (\(Z\)) is an important tracer of galaxy evolution, probing galaxy growth at intermediate timescales. The addition of metallicity histories can conclusively ascertain whether galaxies evolve along a ‘fundamental’ SFR-M\(_\star\)-Z surface and, if so, connect it to the self-regulation of star formation and enrichment of the ISM within galaxies through a combination of feedback and baryon cycling. The scatter in this relation can be used to provide constraints on the relative strengths of these processes, as explored in the Shark (Lagos et al. 2018) and UniverseMachine (Behroozi et al. 2018) models or within cosmological simulations such as IllustrisTNG (Pillepich et al. 2017; Weinberger et al. 2016), EAGLE (Schaye et al. 2014) and MUFASA (Davé et al. 2016b).

Constraining metallicity and metallicity histories from broadband photometry is a hard problem. This is why it is important to develop a physically motivated model to describe the metallicity evolution of individual galaxies and to formulate informative priors based on cosmological simulations that help constrain the free parameters.

Since leaving the \(Z(t)\) free increases the number of free parameters, one can exploit the fact that the \(M_\star(t)\) and \(Z(t)\) are not independent of each other (Tremonti et al. 2004; Mitra et al. 2016; Hunt et al. 2016) to build prior distributions for galaxies at various epochs and stellar masses. Studying the differences between simulations will allow us to quantify the model dependence of these priors. In the simulations, star formation histories are compiled from individual star particle formation times, with enrichment for each star tracked for metals from Type II and Ia supernovae and AGB stars. Using these, we obtain histories that represent the distribution of metallicities of stars formed at each epoch, which are closely related to gas-phase metallicities that one would measure observationally from nebular emission lines at the same epoch. This makes our proposed results appropriate for comparison with the M\(_\star\)-Z and SFR-M\(_\star\)-Z correlations, since they use gas-phase rather than stellar metallicity. We can then

\[^2\]https://jwst-docs.stsci.edu/display/JTI/NIRSpec+IFU+Spectroscopy
Figure 7.1: These plots show the results of applying the Dense Basis method to fitting two galaxies from SDSS-IV MaNGA. The figures for each galaxy have six panels: (a) the MapSpec view showing the galaxy itself as seen by MaNGA, (b) Stellar mass surface density (M_sun/spaxel), (c) SFR surface density (M_sun/yr/spaxel), (d) t50 at every point for the galaxy, (e) radially varying SFHs in increasing annuli (radial colormap shown in inset), and (f) resolved vs integrated comparison between SFHs obtained by either fitting every spaxel and then summing up the SFHs, or summing up the SEDs and fitting to get a single SFH estimate for the entire galaxy.
adopt the basis of physically motivated SFHs from (Iyer et al. 2019) in conjunction with priors developed using multiple state-of-the-art cosmological simulations (Davé et al. 2016a, 2019; Pillepich et al. 2017; Weinberger et al. 2016) to assign corresponding metallicity histories for each SFH in the basis. An illustration of this procedure is shown in Figure 7.2.

There are various ways of sampling from these prior distributions to create a large basis of \((\text{SFH}(t), Z(t))\) that can then be used for SED fitting. We find that in practice even a polynomial regression-based method serves to associate a metallicity history to an input SFH. As seen in Figure 7.2, the scatter around the median of this relation is usually small, hinting that the metallicity distribution corresponding to an SFH of a given shape does not vary significantly. At every stage of method development, validation tests can be designed using mock galaxy catalogs from semi-analytic models (Somerville et al. 2015) whose true SFH\((t)\) and Z\((t)\) are known. Figure 7.3 shows the reconstruction of \([\text{SFH}(t), Z(t)]\) for two representative galaxies from mock catalogs as well as two representative galaxies from the 3D-HST catalog using of this proposed procedure.

7.2.4 Machine learning methods

Machine learning techniques are applicable at every stage of SED fitting, from generating templates, visualization and clustering high-dimensional SEDs, understanding the mapping between observables and theoretical properties of interest, and the subsequent analysis of large datasets of derived physical properties. Wadadekar 2004 (see also Gerdes et al. 2010; Collister & Lahav 2004; Masters et al. 2015; Teimoorinia et al. 2016; Leistedt & Hogg 2016; Hemmati et al. 2019; Calderon & Berlind 2019; Lovell et al. 2019; Wu & Boada 2019) describe some of the current applications of machine learning methods like support vector machines, boosted decision trees, Gaussian processes and various architectures of neural networks to deriving photometric redshifts, stellar masses, SFHs and other properties from SEDs, visualising the complex SED space in a 2-dimensional representation for applications such as checking overlap between different datasets, as well as determining the most important physical properties responsible for galaxy quenching after estimating them from observations.

This is also true of the current work. In Chapter 3, Gaussian Processes were used to create smooth, nonparametric curves in SFH space that satisfy a set of observational constraints. Chapter 6 uses random forests to identify the portions of the power spectra of galaxy SFHs that contribute the most diversity. Additionally, in Appendix A, we discuss some more possibilities where the usage of a machine learning method greatly improves the scope of the analysis.
Figure 7.2: **Top Left:** A single basis SFH and the 100 closest MUFASA SFHs in the sample to it, using mean integrated squared error (MISE) as a goodness-of-fit metric. **Bottom Left:** Corresponding metallicity histories, Z(t) for each MUFASA SFH. Thick lines denote the median and 68% range of the histories, showing only modest diversity for metallicity histories with a similar SFH shape. **Bottom Right:** Metallicity histories recast as a function of stellar mass $Z(M_* (t))$, with the solid blue line showing the fifth order polynomial fit to the median Z(t) (solid black line). Green lines show polynomial fits corresponding to some other SFHs in the basis, illustrating the diversity in the metallicity histories, which results in a broad range in $M_* - Z$ space spanned by our basis. **Top Right:** We can then use the mass history for the basis SFH along with the polynomial relation for $Z(M_* (t))$ to obtain its corresponding Z(t).
Figure 7.3: Reconstructions of SFH i.e., SFR(t) and Z(t) for two representative SEDs from the mocks for which the true star formation and metallicity histories are known (top), and two from the 3D-HST catalog (bottom) [Skelton+14]. True values (spectra, SFR(t), Z(t)) are shown as black dotted lines where available. For each galaxy: Top: Observed (red circles with errorbars) and their best-fit (blue dots) SEDs. Middle: Reconstructed SFHs with 68% confidence intervals. Bottom: Metallicity histories, Z(t), as reconstructed using the expanded dense basis method. It should be noted that the metallicity for the younger SED (bottom left) is largely unconstrained before and during the peak of star formation. With higher S/N, however, both the SFR(t) and Z(t) are better constrained, as seen on the bottom right. This information regarding the galaxy’s SFH and metallicity history has been obtained directly from its SED.
Figure 7.4: Median SFH in bins of different environmental overdensities for CANDELS/GOODS-S at $z \sim 1$. Shaded regions show the 68% range in SFHs at each point in time in a given density bin.

7.3 Correlating SFHs with ancillary data

In addition to SED-determined quantities like dust attenuation and metallicities, the stellar masses and SFRs of galaxies have been shown to correlate with a range of independently measured quantities, including morphology (Whitaker et al. 2015), size (Barro et al. 2017), bulge/total ratio (Diemer et al. 2017), environment (Darvish et al. 2015) and more. Understanding these relations and explaining the observed scatter is possible now that we have estimates of the SFHs of individual galaxies. Figure 3.18 examined the correlation between independent estimates of morphological features for a sample of galaxies and their star formation histories. Additionally, Figure 7.4 shows the correlation between galaxy SFHs and the environments they live in. Existing ancillary data for CANDELS galaxies includes tidal features, bulge/disk decompositions, AGN activity, and more. Correlating SFHs with these variables allows us to estimate the timescales along which statistical samples evolve in each of these spaces, and to gain a more unified understanding of how galaxies evolve with time.

7.4 Comparing SFHs from different methods

Assumptions made while interpreting the integrated light can crucially change their final results. An apparent factor of two tension between the integral of the cosmic SFH and the stellar mass currently detected (Hopkins & Beacom 2006; Borch et al. 2006) assuming a Salpeter IMF. Switching to a Chabrier or Kroupa IMF could mitigate this issue (Davé 2008), so could using...
better star formation history (SFH) models that account for older stellar mass (Leja et al. 2018). There is also a tension between the simulated and observed sSFR distributions at intermediate redshifts ($0.5 < z < 2.0$) across a wide variety of simulations and observations, that is resolved by decoupling star formation and feedback from dark matter evolution in the simulations and better SFH modeling in the observations (Sparre et al. 2015; Katsianis et al. 2015). It is therefore important to accurately measure these quantities and their uncertainties for a representative sample of galaxies.

We briefly describe the SED fitting methods used to infer the SFHs of individual galaxies in this work, but would like to refer readers to the release papers for these codes (Pacifici et al. 2012; Iyer et al. 2019; Carnall et al. 2018a) for further details, including validation tests performed to ensure that SFHs are reconstructed accurately.

- **Pacifici+12:**
  Pacifici et al. (2012) uses a large basis of star formation and metallicity histories from semi-analytic models to model and fit observed spectrophotometric data. In addition to this, the SFR at the time of observation is modeled as an additional free parameter. The fitting is performed using a brute-force bayesian method. Modeling assumptions are given in Table 7.1.

- **Dense Basis:**
  Uses a basis of SFHs following the ($M_*, SFR, \{t_x\}$) parametrization from Iyer et al. (2019). The basis is built through random sampling from the prior distributions, and subsequent fitting is performed using a brute-force bayesian method. The method assumes a Dirichlet prior for the $\{t_x\}$ parameters governing SFH shapes, with the modeling assumptions given in Table 7.1.

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Table 7.1: Priors/ model assumptions from the different methods [enter values here]

<table>
<thead>
<tr>
<th>Method</th>
<th>SPS</th>
<th>SFH</th>
<th>Dust</th>
<th>IMF</th>
<th>IGM</th>
<th>Z(t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pacifici+12</td>
<td>BC03 (2011)</td>
<td>SAM based</td>
<td>CF00</td>
<td>Chabrier</td>
<td>Madau’95</td>
<td>SAM based,  free, constant</td>
</tr>
<tr>
<td>Dense Basis</td>
<td>FSPS</td>
<td>GP-SFH</td>
<td>Calzetti</td>
<td>Chabrier</td>
<td>Madau’95</td>
<td>(free, constant)?</td>
</tr>
<tr>
<td>Bagpipes</td>
<td>BC03 (2016)</td>
<td>Double power-law</td>
<td>Calzetti</td>
<td>Kroupa</td>
<td>Inoue’14</td>
<td></td>
</tr>
</tbody>
</table>
Figure 7.5: Examples of reconstructed SFHs from the different SED fitting codes. (a) shows the three SFHs in our sample with highest consensus, according to our comparison metric. (b) shows the three SFHs in our sample with lowest consensus, and (c) randomly chosen examples.

- **Bagpipes:**

  BAGPIPES (Bayesian Analysis of Galaxies for Physical Inference and Parameter Estimation), (Carnall et al. 2018a) is a state-of-the-art publicly available spectrophotometric fitting tool. The method uses an efficient bayesian framework incorporating nested sampling to compute posteriors on the parameters being fit. For the current use case, the method uses a double power-law parametrization to describe SFHs, with the other modeling assumptions being given in Table. 7.1.

For the current comparison, we use the reduced $\chi^2$ at each point in linear lookback time, defined as:

$$\chi^2_{m1,m2} = \frac{1}{N} \sum_{i=0}^{N} \frac{((SFH_{m1}(t_i) - SFH_{m2}(t_i))^2)}{\sigma_{SFH_{m1}(t_i)}^2 + \sigma_{SFH_{m2}(t_i)}^2}$$

(7.1)
Figure 7.6: Distributions of the reduced $\chi^2$ for pairs of codes (colored lines) and all codes (black line).

where $m_1$ and $m_2$ denote the two methods between which we perform the SFH comparison, summed over the discretized lookback time ($t_i$) from the present to the big bang. Lower values of mw-MSE indicate better agreement between the SFHs. Since we consider SFHs over a discrete grid, this statistic is defined in terms of a sum over a time grid as opposed to an integral over time. Figure 7.6 shows the distribution of mw-MSE values for our sample of galaxies for each pair of methods as well as a combined consensus metric, given by

$$\log \chi^2_{all} = \text{mean}(\{\log \chi^2_{m_i,m_j}\})$$ (7.2)

We compute the consensus of reconstructed SFHs for each galaxy in our sample, quantified using the mw-MSE metric. Figure 7.5 shows examples of the best and worst consensus between SFHs in our sample, as well as a randomly chosen subset of galaxies that exhibit intermediate values of consensus. In general, we find that galaxies with rising SFHs that are actively star forming at the time of observation show overall agreement in terms of the different methods, while galaxies with more complicated star formation histories or periods of quiescence are more subject to modeling assumptions and SFH parametrizations. Starbursting galaxies are also subject to large disagreements, which could be due to the sensitivity of recent SFR to the choice of nebular emission model adopted in fitting the galaxies.
A.1 Consistency across filter curves

It is possible to perform fits to the mock galaxies observed at different redshifts and ensure that the reconstructed SEDs yield physical quantities that are robust independent of the redshift. This analysis can be extended to determine the redshift range across which a given atlas is robust, since the amount of information contained within the filters changes with redshift.

Since we restrict our mock dataset to $z = 1$ and the observed dataset to $1 < z < 1.5$ in current work, we perform this consistency check fitting the same mock galaxies whose rest frame spectra are computed considering them to be at $z = 1$ and $z = 2$. We perform Dense Basis fitting on the galaxies, and compare the derived quantities $t_{10}, t_{50}, t_{90}$ and find that the estimation of these quantities remains robust within uncertainties.

A.2 Dot-product SED fitting as a computational speedup

We present an additional approach to finding the optimal reconstruction given an atlas of SEDs using a non-orthogonal dot-product, i.e., a projection product, that might prove to be a useful computational speedup for dealing with large datasets.

Since the projection product is done in a non-orthogonal basis, reconstruction of the original vector using the dot-product coefficients is more involved than the procedure in the case of the inner product in an orthogonal space. Various methods have been tested for this reconstruction, including iteratively refitting the residuals as long as they remain above the noise level, constructing a reduced orthogonal space by projecting out components of vectors along a principal component, and constructing an expanded basis of linear combinations of the basis functionals. This method is expected to operate on the timescale of $O(N \times M)$ operations, where $N$ is the size of the basis and $M$ is the number of bands of the photometry in consideration.
The best-fit is estimated through a non-orthogonal equivalent of a dot product in the photometric vector space, through a mapping given by,

\[ \phi(F_{ij}, F_{ij}^{\text{obs}}) = \sum_j F_{ij}^1 F_{ij}^{\text{obs}} \left\| F_{ij} \right\| \left\| F_{ij}^{\text{obs}} \right\| = a_i, \] (A.1)

where \( \phi \) is a mapping such that \( \phi : \mathbb{R}^{N_{\text{filt},+}} \times \mathbb{R}^{N_{\text{filt},+}} \rightarrow \mathbb{R}[0,1] \).

For an equivalent orthogonal basis, the dot product coefficient is given by the same mapping, with an additional constraint imposed due to orthogonality, which is,

\[ \phi(F_{ij}, F_{ij}) = 0 \] (A.2)

which allows us to reconstruct the original vector simply using

\[ F_{ij}^{\text{obs}} = \sum_i a_i F_{ij} \] (A.3)

However, in the absence of orthogonality, we turn to more involved methods of reconstruction, bounded by both computational costs and error margins on the photometry, which could lead to overfitting if not accounted for.

Other factors held constant, the coefficient of the photometric dot-product indicates the projection of the true SFH of the galaxy on to the basis SFH. Therefore, without any degeneracies in the basis SEDs, a higher coefficient would mean that the SFH is closer to the true SFH of the galaxy, with \( a_i = 1 \) denoting a perfect match with basis vector \( i \). The procedure returns similar results to the \( \chi^2 \) fitting procedure described in §2.4, with a slight computational speedup requiring \( \sim 1/3^{rd} \) of the time for fitting an SED, which might be helpful in fitting large datasets of SEDs from upcoming surveys.

### A.3 Alternative methods of defining goodness-of-reconstruction:

Given that the SFHs are not a directly measurable quantity, care must be taken in comparing the reconstructed SFHs to the true ones, accounting for the unequal sensitivity of the SEDs to the same interval of time at different epochs, as well as the large amount of stochasticity present in the simulated SFHs (SAM, Hydro., Stochastic). We outline some of the methods viable for this as alternatives to be considered in other applications of the Dense Basis method. These statistics, while useful for comparing how well a given reconstruction approximates the true SFH, are significantly affected by stochasticity. Since we are only interested in the relative performance of the families of curves in current work, we choose to compare the goodness-of-reconstruction to that of a polynomial fit with the same number of degrees of freedom as the parametrizations under consideration.
1. $R^2$ and $R^2$ adjusted:

The coefficient of determination is among the simplest ways to compare two sets of points, comparing how well the reconstructed SFH approximates the true one. This gives the first indication of the fact that some families of SFHs may be more useful than others for a given dataset at SFH reconstruction.

The $R^2$ statistic is given by,

$$R^2 = 1 - \frac{\sum_t (\psi_{\text{rec}}(\log(t)) - \psi_{\text{true}}(\log(t)))^2}{\sum_t (\psi_{\text{true}}(\log(t)) - \langle \psi_{\text{true}}(\log(t)) \rangle)^2}$$  \hspace{1cm} (A.4)

which quantifies the amount of variance explained by the fit. Since the stochasticity of the different mocks differs, the median $R^2$ for fits to the three datasets can vary widely, with the SAM galaxies doing the best and the MUFASA galaxies doing the worst. It is possible to adjust this by smoothing the true SFHs using a nonparametric method until they all exhibit an equal level of stochasticity, or simply by rebinning the SFH with a time interval of the order of the least stochastic sample. Another improvement, as implemented in the current work, is to compare the $R^2$ of the reconstruction with a reference $R^2$ with the same number of degrees of freedom, such as a fit using a polynomial.

2. The Pearson correlation coefficient ($\rho_p$)

Since the standard implementation of $R^2$ as a goodness-of-reconstruction metric fails to account for the different amounts of stochasticity present in the different mock datasets, we consider the Pearson correlation coefficient, which accounts for the inherent stochasticity of an SFH through a normalization. As an alternative to the previous method, we can present the results for the training step as likelihood vs the Pearson correlation coefficient, written as

$$\rho_{\text{true},\text{rec}} = \frac{\text{cov}(\text{true},\text{rec})}{\sigma_{\text{true}}\sigma_{\text{rec}}} = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_i (x_i - \bar{x})^2} \sqrt{\sum_i (y_i - \bar{y})^2}}$$  \hspace{1cm} (A.5)

since this could better provide an estimate of the goodness-of-reconstruction for highly stochastic SFHs without the need for an additional $R^2$ adjustment step. However, the coefficient in this form assumes Gaussian statistics which are not always applicable for our datasets.

3. Spearman’s correlation coefficient ($\rho_s$)

Pearson’s correlation coefficient assumes a Gaussian distribution of noise around a linear relation, and finds the degree of correlation around it. However, the relation we seek, to compare two time-ordered sets of curves, needs to be more robust. Therefore we considered
the Spearman coefficient, which compares two monotonic functions using ranks in order to find the degree of correlation between them. For distinct ranks, the coefficient is given by \( r_s = 1 - 6 \sum \frac{d_i^2}{n(n^2 - 1)} \), where \( d_i \) is the difference between the two ranks of each observation. This, however, does not work very well at describing the fit for young galaxies, where a significant fraction of the two star formation histories is tied at the same rank due to long periods of vanishing SFR at early times.

4. MISE

The mean integrated square error given by \( MISE = \sum_t \psi_{rec}(\log(t)) - \psi_{true}(\log(t)) \) also provides a method to quantify the goodness-of-reconstruction. However, it does have a well-defined range to compare different quantities, and provides no accounting for the varying amounts of stochasticity of the different datasets. The concept of minimum distance estimation that this method implements can also be generalized to the Kolmogorov-Smirnov statistic, which depends on the maximum absolute difference between the true and estimated cumulative SFHs, but does not provide sufficiently sensitive results to make a distinction between the different families using a correspondence between the statistic and the goodness-of-fit.

A.4 Robustness to noise

The Dense Basis method performs SED fitting using an atlas of SEDs corresponding to well-motivated basis SFHs that satisfy the conditions in §2.2.2. Although the mapping from SFH space to the observed photometry is theoretically bijective, an SED at a given noise level for a given set of photometric bands is degenerate in SFH space to the extent that all the SFHs that produce the same SED within error limits are an acceptable fit. Our formulation then ensures that the basis is effectively dense in SFH space, allowing us to reconstruct the overall trend of star formation even if it doesn’t capture the finer stochastic details. However, even though the basis is effectively dense in SFH space, it is not dense in SED space, since a large region of the photometry space is not accessible through any physically motivated SFH. This allows our method to ignore all noise that is ‘unphysical’ while performing the SED-Fitting step. Even though this yields worse \( \chi^2 \) and the noise biases the reconstruction to an extent documented in Figure 2.8, it does not overfit the SED by fitting for any noise that does not correspond to a physically motivated SFH. This allows our method to be robust to a large fraction of the noise, as is seen in Figure A.1, where we show an example of 1000 noisy realisations to a SAM spectrum in red and the corresponding reconstruction in blue, which successfully ignores major
Figure A.1: **Left:** Fits using the Dense Basis method (blue circles) to 1000 noisy realisations (red circles) of the photometry corresponding to a galaxy with true spectrum (black dotted line) showing the robustness of the method to noise that corresponds to unphysical regions of the SFH space. Spectra are shown without nebular emission for clarity. **Right:** The pointwise 68% intervals of the reconstructed SFHs for each noisy realisation (blue shaded region) compared to the true SFH (red solid line) showing that the reconstructions are also largely robust to the noise.

Outliers in fitting the SED. Extended to the entire ensemble of 1200 mock galaxies, we find that the ratio of the residuals to the noise is \( \sim 45\% \), with a standard deviation of \( \sim 9\% \), i.e.,

\[
\frac{|F^k_j - F^{true}_j|}{|F^{obs}_j - F^{true}_j|} \approx N(0.45, 0.09) \quad (A.6)
\]

The decomposition of this quantity into the sensitivity to noise in individual bands in Figure A.2 shows that the F160w is the most sensitive to noise, with the method being remarkably robust to the noise in the ground-based bands. The maximum deviation due to noise is computed and found to be in the bounding bands (u ctio and IRAC 4.5\( \mu \)m). This is expected, since the endpoints are the most unconstrained in the fitting process.

### A.5 Examining the \( \chi^2_{SED} - \chi^2_{SFH} \) correspondence for individual galaxies

We examine the correspondence between \( \chi^2_{SED} \) and \( \chi^2_{SFH} \) for individual galaxies in greater detail. We provide examples for three randomly chosen galaxies from each mock catalog as examples in Figure A.3, and discuss possible biases and how they should be minimised.
Figure A.2: Boxplots showing the results of fitting the ensemble of mocks with multiple noisy realisations and comparing the residuals of the fits to the noise.
Figure A.3: Plot of the correspondence between $\chi^2_{SED}$ and $\chi^2_{SFH}$ for three randomly selected galaxies from each mock dataset, using all six families as the basis. The top three rows are galaxies drawn from Semi-Analytic Models (galaxy id = 204,278,243), the middle three rows from the Hydrodynamical simulations (galaxy id = 270,5,228) and the last three rows from stochastic realisations (galaxy id = 74,109,60).
We encounter two types of biases in the $\chi^2$ plots, summarised as follows:

- **Degenerate $\chi^2_{SED}$**: If, in addition to the correspondence, some good fits to the SED ($\chi^2_{SED}/\text{DoF} < 1$) correspond to bad reconstructions of the SFH ($\chi^2_{SFH}/\text{DoF} > 1$), the SFH reconstruction may be biased. However, these are often removed as outliers in the procedure used to compute uncertainties, as described in §2.2.5.

- **Sub-optimal $\chi^2_{SFH}$**: The best fit to the SED corresponds to a significantly worse reconstruction than the best possible reconstruction of the SFH with that basis. However, like the true SFH, the best possible reconstruction is generally within our reported uncertainties around the best-fit determined via $\chi^2_{SED}$.

For the first point, we quantify the two kinds of biases using the $\chi^2$ surface generated for each galaxy in the ensemble of 1200 galaxies using each SFH family. An example of the two kinds of bias is shown in Figure A.4, showing the $\chi^2_{SED} - \chi^2_{SFH}$ plot for a single galaxy with a single SFH family. Since there is a certain amount of degeneracy introduced in SED fitting due to noise, we consider the set of all good fits ($\chi^2_{SED}/\text{DoF} < 1$) instead of the best fit $\min(\chi^2_{SED}/\text{DoF})$. As shown in Figure A.3, we see that there is generally a good correspondence between $\chi^2_{SED}$ and $\chi^2_{SFH}$ in the regime of good fits. We then find the families that minimise the two types of biases in SFH reconstruction.

We quantify the degenerate $\chi^2_{SED}$ bias by examining the histogram of the set $S = \{\chi^2_{SFH,i} | \chi^2_{SED,i} < 1\}$. Since this is the set of good fits, we then say that a galaxy has a type 1 bias if it has multiple peaks in this histogram, separated by a minimum distance of 1 dex in $\chi^2_{SFH}$. This is the more common type of bias, and the probability that it will bias the fit towards a poorer reconstruction depends on the ratio of the areas under the two peaks. We show the number of occurrences of this type of bias for each family in Table A.1, finding that the exponential and CSF families have the highest occurrence of this behaviour. While these biases are more common than sub-optimal $\chi^2_{SFH}$ biases, they only indicate the possibility of a bias due to noise, and are usually much milder than the example shown.

For sub-optimal $\chi^2_{SFH}$ bias, we find the distance $d = (\chi^2_{SFH}|_{\min(\chi^2_{SFH})} - \chi^2_{SFH}|_{\min(\chi^2_{SED})})$ for each galaxy with each SFH family. This distance denotes the difference between the best $\chi^2_{SFH}$ possible in the basis and the $\chi^2_{SFH}$ corresponding to the best-fit SED in the basis. If the latter quantity is much worse than the former, we say that a galaxy has a bias due to sub-optimal $\chi^2_{SFH}$. We find that this is best quantified by the condition $d > 0.4\text{dex}$. We show the number of these biases for each family in Table A.1, finding much lower rates of occurrence and that the Top-Hat family shows the highest occurrence of this behaviour.
Figure A.4: Plot of the correspondence between $\chi^2_{SED}$ and $\chi^2_{SFH}$ for an individual galaxy with the Top-Hat family of SFHs, computed using noiseless fits to the SED. This illustrates the biases that can occur in SFH reconstruction through SED fitting, which we try to minimise through the training procedure.

Table A.1: Comparison of the Goodness of Fit to the Goodness Of Reconstruction for different samples of mock SFHs

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</tbody>
</table>
Appendix B
Appendices from Chapter 3

B.1 Validation of trajectory robustness

In Sec. 4.5, we showed that our SED fitting technique is robustly able to recover the SFR-$M_\ast$ correlation corresponding to a mock dataset from the MUFASA simulation. This is true both for the case of direct fits at $z \sim 1$, and for stellar masses and star formation rates recovered from the SFR-$M_\ast$ trajectories at $z \sim 2$.

Here we check for biases in the reconstruction of individual trajectories in SFR-$M_\ast$ space at different redshifts. If the estimated uncertainty from our SED fit is bigger than the bias, this should not affect our analysis since the corresponding uncertainties will re-weight these points when we fit for the slope and normalization. We also examine possible ways where we can reduce any possible ensemble bias using tracers that are sensitive to factors like the rest-frame wavelength coverage, and SED S/N.

Using a sample of mock galaxies at different redshifts and stellar masses, we span a range in S/N and wavelength coverage similar to what we see in the CANDELS data. For all of these galaxies we reconstruct the SFHs through SED fitting, along with uncertainties on log SFR(t) and log $M_\ast(t)$ at each point in lookback time. We find that these estimated uncertainties at each point in lookback time trace of the bias in both log $M_\ast$ and log SFR, and allow us to restrict ourselves to a minimally biased subset of galaxies for fits at any redshift. This is seen in Figure B.1.

We consider a few different options for reconstructing the high-z SFR-$M_\ast$ relation based on trajectory data from galaxies observed at later epochs (lower redshifts).

- Method (a): Use all available galaxies at lower redshifts / later epochs.
- Method (b): Use a subset of available galaxies with bounds on how far back in lookback time the galaxies are propagated, using SFHs from simulations to find the bounds.
- Method (c): Using the simulations, we find that bias in estimating quantities like SFR, $M_\ast$ are traced closely by the pointwise uncertainties on these quantities estimated during
SED fitting. These uncertainties depend on factors such as the S/N of the SED, rest frame wavelength coverage, and degeneracies of the SFH with dust and metallicity. Since this criterion is based on the uncertainties in log $SFR$ and log $M_*$ and not their actual values, this does not correspond to a selection in log sSFR. We can thus reduce the bias by limiting our analysis to a 'high confidence subsample that avoids significant biases. To ensure that the bias on $M_*$ and SFR are less than 0.2 and 0.3 dex respectively, we use a threshold of 1 dex on the combined uncertainties \( \sqrt{\sigma_{\log M_*(t)}^2 + \sigma_{\log SFR(t)}^2} \) to select galaxies for analysis at different epochs. We find that varying the threshold in the range of 0.5 to 2 dex does not significantly impact the analysis. This selection is possible for both the direct fits and trajectories since the SED fitting procedure estimates uncertainties on $M_*$ and SFR at each point in lookback time. For example, at $z = 1$ we consider the uncertainties for all galaxies that are directly fit at $z \sim 1$ and exclude those with $\sqrt{\sigma_{\log M_*(t)}^2 + \sigma_{\log SFR(t)}^2} > 1$ dex. Similarly, for the trajectories we consider all the galaxies at $z < 1$, and compute the uncertainties on their $M_*$ and $SFR$ after propagating them backwards in time to $z = 1$ before applying our selection criterion.

These three approaches are highlighted in Figure B.1, where we show the error in estimating Stellar Mass and SFR for a sample from the simulations at $z \sim 1$ as a function of lookback time (i.e., trajectory run-time). In Figure B.2 and Figure B.3 we show this test run at a range of redshifts. We find that the dependence of the reconstruction on SED coverage or S/N is captured by the uncertainties, and thus using them as a tracer of the bias in our fits is an effective method that accomplishes both objectives: propagating robust SFR-$M_*$ trajectories farther back in time while avoiding samples with large uncertainties due to bad SED coverage, low S/N, or bad fits. In Table B.1 we show the median amounts of time that galaxies in our actual analysis are propagated backwards along their trajectories to be included in the analysis at a given redshift of interest. While this increases as we go to higher redshifts, most galaxies are propagated backwards by only about 15 – 30% of their full SFHs at any epoch.

B.2 Robustness of trajectory - direct fit comparison to sample selection

We use a KS test to compare the distributions of the distances from the best-fit SFR-$M_*$ correlation that we get from the direct fits to the distances we get from galaxies observed at later epochs and propagated backwards in time along their trajectories. The similarity of the two distributions in addition to our previous validation suggests that the reconstructions are...
Table B.1: Median amounts of time that sub-samples of low-uncertainty galaxies (observed at $z_{\text{obs}}$) are propagated along their redshifts to reach desired redshifts at which we have performed our analysis ($z_{\text{analysis}}$). We see that the amount of time grows as we go to higher redshifts, but is much less than the time interval between the lowest redshift and the redshifts of interest (the maximum amount of time a galaxy can be propagated in our current analysis). This indicates that most galaxies analyzed at a given redshift come from the vicinity of that redshift, rather than being propagated backwards all the way from $z \sim 1$.

robust and that the effects due to mergers do not significantly affect our analysis of quantities like the slope and normalization of the SFR-M$_*$ correlation. To ensure that the KS test is not affected by possible systematics arising from sample selection, we perform the test on a few different samples at each redshift:

- The full distribution of distances from both direct fits and trajectories, out to a distance of 0.4 dex from the best-fit correlation.
- The distribution of distances corresponding to the sample of galaxies with uncertainties < 1 dex, out to a distance of 0.4 dex from the best-fit correlation.
- The distribution of distances corresponding to the sample of galaxies with uncertainties < 1 dex, out to $1 \times$ the observed scatter from the best-fit correlation.
- The distribution of distances corresponding to the sample of galaxies with uncertainties < 1 dex restricted to the mass range where the direct fits have good statistics, out to $1 \times$ the observed scatter from the best-fit correlation. To find the lower mass threshold, we find the minimum mass at for which the $\text{median}(M_*^{\text{direct fits}}) - \text{median}(M_*^{\text{trajectories}}) < 0.1$ dex.
Table B.2: P-values corresponding to the KS test comparing the distributions of distances from the best-fit SFR=M_∗ correlation for the direct fits and trajectories. The first column (full) shows the p-values comparing the distribution across all M_∗ within 0.4 dex of the best-fit line. The second column (low uncert.) repeats this analysis for the subsample of galaxies used for trajectories that have low uncertainties at the redshift of interest. The third column (low uncert., 1×scatter) uses a threshold of 1×the observed scatter at each redshift instead of a fixed threshold of 0.4 dex. The fourth column (low uncert., 1×scatter, M_∗ threshold) performs the KS test on a further reduced dataset restricted to the mass range where the direct fits have good statistics so that stellar mass effects don’t enter into our comparison. The p-values for all these comparisons are > α, indicating that the two distributions are not different to a statistically significant level.

The results of our KS test are consistent at all redshifts (p-value > α), as summarized in table B.2. The significance level for each test is α = 0.05. However, since we are performing a family of comparisons to test a single hypothesis, we need to control for the increased probability of false positives. To this end, we use adjusted significance levels of α’ = α/N = 0.05/6 = 0.0083 using a Bonferroni correction (Goeman & Solari 2014), or α’ = 1−(1−α)^1/N = 1−(1−0.05)^1/6 ≈ 0.0085 using the more conservative Sidak correction (Šidák 1967). This choice of correction does not affect our results since our lowest p-value is 0.02. Since our results remain consistent across this broad range of tests, we can not reject the null hypothesis that the two distributions are the same. While this does not completely rule out the possibility that the two distributions are different, the probability of this being the case is lower than α = 5%.
B.3 Effects of UVJ pre-selection

The Star Formation Rates of actively star forming galaxies are tightly correlated with their Stellar Masses across a range of redshifts, with $\geq 68\%$ of such galaxies found within a narrow range of a single best-fit line. However, when galaxies enter periods of quiescence or undergo starbursts, they make excursions from this correlation. Improperly taking these galaxies into account (both by failing to exclude them or by being too rigorous in excluding them, thereby excluding some star forming galaxies as well) could lead to biases in our estimates of slope and normalization for the SFR-M$_*$ correlation. To mitigate this issue, we used an iterative robust fitting routine that excludes outliers (Holland & Welsch 1977), which effectively re-calibrates to the data at each redshift slice to identify which points could be outliers. This avoids the use of pre-determined conditions for when galaxies are quiescent, since we find that our star-formation histories allow us to robustly distinguish between galaxies with low SFRs throughout their lifetime and galaxies that are experiencing a rapid fall in their SFR.

However, in order to better compare our results to literature that uses a pre-selection step to select star forming galaxies (see for example Schreiber et al. (2015), where they use an optical selection criterion since even quiescent galaxies could still show residual IR emission due to a warm ISM), we adopt the UVJ selection criterion from Williams et al. (2009). We use the rest-frame U-V and V-J colors derived by Pacifici et al. (2016). In brief, Pacifici et al. (2016) use a large library of model SEDs to fit all available photometric data and derive median values and uncertainties of the rest-frame colors for each galaxy in the sample. The library is generated by combining the output of a semi-analytical model of galaxy formation with models of the stellar and gas emission and the attenuation by dust (see Pacifici et al. (2012) for more details). The $z \sim 1$ sample with the selection criteria is shown in Figure B.4.

Refitting the correlation to determine the slope and normalization does not significantly change our results, since most of the points excluded by the selection criterion would be classified as outliers by our algorithm. The fractional changes to slope and normalization at different redshifts are: -1.48 $\%$, -0.05 $\%$, 0.1 $\%$, 0.78 $\%$, 0.4 $\%$, and 0.02 $\%$, while the fractional changes to normalization are: 0.01, 0.011, 0.025, 0.033, 0.011, and 0 dex at $z=1,2,3,4,5,6$ respectively.
B.4 Using nonparameteric regression methods to quantify the SFR-\(M_*\) correlation

In our analysis, we assume a linear relation between log SFR and log Stellar Mass and fit for its slope and normalization. However, it is not necessary that the correlation be linear. Indeed, Schreiber et al. (2015) and Whitaker et al. (2014) find that the relation flattens out at the high mass end. Using an F-test, we checked to see if a quadratic fit is statistically preferred over a linear one, and found this not to be the case.

However, this is not enough to prove the linearity of the relationship. While some studies quantify the correlation between SFR and Stellar Mass by finding the effective SFR in bins of Stellar Mass, or even by binning perpendicular to the correlation, methods involving binning potentially suffer from effects due to bin size and the locations of bin centers. Here, we use LOWESS (Cleveland 1979) a non-parametric regression technique that uses local weighting to create a smooth non-parametric estimate of the correlation. We also create a similar set of plots using Gaussian Process Regression (GPR) (Rasmussen & Williams 2006) to estimate the correlation at each point in \(M_*\). From Figure B.5, we can see that the relation is indeed linear at the low mass end, closely matching our best-fit. While this alone isn’t enough to state that the correlation is linear, it is certainly consistent within uncertainties with our best-fit linear relation.

From Figure B.5 we also see that the direct fits and trajectories agree extremely well in the high-mass regime at redshifts where both sets have significant statistics. However, the high mass end has a slightly higher slope than the low-mass end at \(z \sim 1 – 3\). From this, we conclude that when we fit the direct fits and trajectories, possible discrepancies may arise due to the different mass ranges over which they are being fit.
Figure B.1: Reconstructed SFHs for an ensemble of mock galaxies at $z \sim 1$ using the Dense Basis SED fitting method. Top panel shows the estimated bias (model - SED fitting estimate) and uncertainties (from SED fitting) in stellar mass as a function of lookback time, and the bottom panel shows the same for SFR.

In each plot, the **blue solid line** is pointwise median for all galaxies in the sample. This generally grows with time since our smooth SFHs can go to 0 while the mock SFHs generally go to some small nonzero value, which leads to a one-sided bias. However, since this bias would affect both SFR and $M_*$ identically, it simply shifts points along the diagonal, and shouldn’t affect our estimates of slope and normalization. The **orange line** is pointwise median for the subsample of ‘good fits’ (method a) and the **green line** is pointwise median excluding contributions from galaxies which have large uncertainties ($\sigma \log(SFR(t), M_*(t)) > 1$ dex). (method c, using the uncertainties to avoid SFHs with biases).

The white horizontal line shows the average uncertainties on SFR ($\sim$0.3 dex) and $M_*$ ($\sim$0.2 dex), for comparison. The **white vertical lines** indicate redshifts instead of lookback times. (method b would be truncating our trajectories to where the orange line hits our tolerance bias of the white horizontal lines, here around $\Delta t \sim 2.5 \text{Gyr}$.) The **blue shaded region + white solid line** shows the uncertainty on SFR-$M_*$ estimates, showing that the uncertainties closely follow the bias. Additionally, we see that for the full ensemble of galaxies (sample (a)), the uncertainties are larger than the bias and thus should not affect a statistical analysis that takes the uncertainties into account.
Figure B.2: Same as figure B.1 for Stellar Mass, repeated across a range of redshifts.
Figure B.3: Same as figure B.1 for Star Formation Rate, repeated across a range of redshifts.

Figure B.4: UVJ diagrams for the sample at $z \sim 1$. Rest frame U-V and V-J colors are computed through SED fitting, using the best-fit model template as a prior. The diagram shows the expected correlations with SFR and dust.
Figure B.5: Non-parametric regression using Locally Weighted Scatterplot Smoothing (LOWESS) and Gaussian Process Regression (GPR) performed on the direct fits (blue lines) and trajectories (green lines) datasets at different redshifts. The curves are limited to the 5th to 95th percentile in Stellar Mass for each dataset with GPR, and from the 1st to 95th percentile for LOWESS. The curves corresponding to direct fits at \(z > 4\) are not reliable due to the small number of points available at those redshifts (see Figure 4.4. Dotted blue lines show the 10th percentile of the \(M_*\) distributions at different epochs as reported in Table. 4.3.)
Appendix C
Appendices from Chapter 4

C.1 Quantifying information-rich parts of galaxy SEDs with Extremely Randomized Trees

It has been a question of interest to determine the portions of a galaxy’s spectrum that contain the maximum information regarding its physical properties. Early SED fitting methods like MOPED (Heavens et al. 2000) used information compression techniques since fitting the entire spectrum was too expensive given the computational resources at that time. In recent times, studies like Masters et al. (2015); Hemmati et al. (2019) attempt to reduce the dimensionality of the high-dimensional SED or pairwise color space to better understand the relation between SED shape and physical properties like redshift and stellar mass. Doing so helps us to better understand systematics, eg. that catastrophic failure modes in redshift estimation can be caused by the degeneracy between the Lyman and Balmer breaks due to how similar their SED shapes look at very different redshifts. (Lovell et al. 2019) uses convolutional neural networks to predict the SFHs of galaxies from their spectra.

Using stellar population synthesis models, it is now possible to understand the effects of changing the stellar masses, SFRs, dust attenuation and metallicity on a galaxy SED. However, most of these changes are extremely nonlinear, sometimes localized to part of the spectrum, and degenerate with other transformations to a galaxy’s SED. Due to this, sometimes even the trained eye can not distinguish between the many different factors that contribute to an individual galaxy’s SED.

Understanding the portions of the spectrum that contain the most information about these individual quantities has twofold advantages: (1) it helps in the planning of future surveys that wish to optimize measurements for a particular physical property, and (2) by considering the different properties of stellar populations, nebular emission, etc. it is possible to relate signatures of a given physical quantity with the stellar populations responsible for that signal.

To this end, we use Extremely Randomized Trees (ERTs; Geurts et al. (2006)), implemented using via scipy.ensemble.ExtraTreesRegressor similar to the application in §6.3.4. ERTs
use an ensemble of decision trees to perform regression, with each tree being built using a subset of the available data. Decision trees are generally built using features (elements of the input, in this case, the flux density at a given wavelength) along which it is most advantageous to split the data in order to predict the quantity of interest. Random forests go one step further and take the best split from among a subset of the features, which often results in a slight increase in bias at the expense of lowering the variance. ERTs go one step further in the randomization process, by implementing random thresholds for each candidate feature and picking the best threshold as the splitting rule.

To generate the training data, we use recovered parameters for SFH ($M_*$, SFR, \{t_X\}) along with the Gaussian Process routine described in Sec. 3.2), dust attenuation, metallicity and redshift estimated from the SED fitting of galaxies in the CANDELS GOODS-S field at $0.9 < z < 1.1$. By running these parameters through the Flexible Stellar Population Synthesis (FSPS, Conroy et al. (2009); Conroy & Gunn (2010); Foreman-Mackey et al. (2014)) code, we are able to generate rest-frame spectra for each galaxy. For training, we then train the algorithm using the spectra expressed in log $F_\nu$ (in $\mu$Jy) as the inputs (features) along with the different physical quantities of interest (labels).

After training the ERT to predict a quantity of interest based on rest-frame galaxy spectra, we can then extract information about the importance of different spectral features (Louppe 2014) by looking at the relative rank of each feature with respect to predicting the target parameter. Features near the top of the tree contribute to the final prediction for a larger fraction of galaxies. In Figures C.1, C.2 and C.3 we show the importance of different spectral features in predicting the Stellar Mass, SFR, $t_{50}$, the full SFH n-tuple, and the number of SFH episodes, as well as the feature importance if we change the spectral coverage and resolution.

The current results are reliable for $M_*$ ($R^2_{\text{test}} = 0.99$), SFR ($R^2_{\text{test}} = 0.97$) and SFH ($R^2_{\text{test}} = 0.84$), and marginally reliable for $t_{50}$ ($R^2_{\text{test}} = 0.7$) and the number of episodes ($R^2_{\text{test}} = 0.56$), as compared with a baseline estimated using random gaussian data ($\mathcal{N}(0,1)$) as input ($R^2_{\text{test}} = 0.06$) Future implementations of the feature selection method using methods such as Gradient Boosting Regression Trees (GBRT) might provide better performance and discriminatory power in this context.
Figure C.1: Studying the importance of different spectral features in determining the various physical properties that determine a galaxy’s SFH, determined using CANDELS/GOODS-S galaxies at $0.9 < z < 1.1$. Black lines show the median feature importance, and the grey shaded region shows the 68% confidence intervals. We find that the stellar mass is dominated by the portion of the SED that contains the maximum contribution from cool stars. The SFR is constrained by the Lyα line and dust re-emission in the IR, and the rest-UV to a lesser extent. The $t_{50}$ is constrained by a range of features including those of SFR and $M_\star$ (since they are correlated) as well as the region in the rest-optical near the 4000 angstrom break.
Figure C.2: Same as Figure C.1, but for more involved SFH features. The full SFH is a combination of all of the preceding factors, showing the need for full SED analysis in accurately constraining the SFH. Finally, the number of major episodes of star formation in a galaxy’s SFH is spread over the entire spectrum, with a slight amount of information from the Hα and the 4000 angstrom break, presumably used to constrain the short-vs-long-timescale burstiness in deciding whether multiple episodes occur.
Figure C.3: Similar to Figure C.1, but considering various conditions for determining SFR. The top panel uses the full spectrum from FSPS between $100 \text{nm} < \lambda < 10000 \text{nm}$ to calculate the importance of different spectral features, finding that the Ly$\alpha$ and dust re-emission in the IR are the major constraining factors. The two panels below show the effects of excluding each of these features in determining SFRs, while the last panel shows the importance of the rest-UV that we expect in SED space, by coarsening the entire spectrum into $\sim 20$ points.
Appendix D

Appendices from Chapter 6

D.1 SFH diversity across models

Star formation histories in individual simulations show a wide range of trajectories that are influenced by a multitude of factors like the dark matter accretion history and mergers, environment, inflows and outflows, and stellar and AGN feedback. In recent times, this has also been quantified by observations, such as the SFHs of dwarfs in Weisz et al. (2011a), ensemble studies like Pacifici et al. (2016), or the resolved SFHs from Hsieh et al. (2017).

In Sec. 6.4 we examine the diversity of SFHs using the distributions of sSFR, $t_{50}$ and $t_{25} - t_{75}$, which denote the star-forming vs quenched natures, peaks and widths of the SFHs respectively. To better understand how these correlate with traditionally studied quantities like stellar masses and star formation rates, as well as to examine their covariances within a simulation, we analyze the distributions of these quantities and show corner plots (Foreman-Mackey 2016) for each simulation in Figures D.1, D.2, D.3, D.4, D.5, D.6, and D.7.

D.2 Finding the shortest timescales we can probe

Both cosmological (Illustris, TNG, MUFASA, SIMBA) and zoom-in simulations (FIRE-2) have limits on the lowest SFR possible in any given time bin that is set by the size of the star particles they use. All the simulations listed above turn gas into a star particle probabilistically depending on whether certain temperature and density conditions are met. In practice, this introduces portions in the SFH where the $SFR = 0$, punctuated by small spikes which contain $O(1)$ star particles. The effect of this on the power spectrum is to introduce white noise on the timescales where the SFR is probabilistically populated by discrete star particles. Looking at the PSD of individual galaxies, we see the effects of this effectively Poisson-distributed ‘shot noise’ as a flattening as we approach short timescales. This depends on the amount of time the SFH spends in the vicinity of the minimum SFR threshold, set by
Figure D.1: Priors on SFH parameters at $z=0$ for IllustrisTNG. The five histograms of the corner (Foreman-Mackey 2016) plot show the distribution for log Stellar Mass ($M_*$), log star formation rate (SFR), log specific star formation rate (sSFR), the half-mass time ($t_{50}$), and the width of the galaxy’s star forming period ($t_{25} - t_{75}$). The remaining panels show the covariances between the different quantities. The first panel in the second row shows the SFR-$M_*$ correlation.
Figure D.2: Same as Figure D.1, but for Illustris.
Figure D.3: Same as Figure D.1, but for Eagle.
Figure D.4: Same as Figure D.1, but for Mufasa.
Figure D.5: Same as Figure D.1, but for Simba.
Figure D.6: Same as Figure D.1, but for the Santa-Cruz SAM.
Figure D.7: Same as Figure D.1, but for UniverseMachine.
\[
\langle SFR_{\text{min}} \rangle = \langle M_{*, \text{sp}} \rangle / t_{PSD}
\]  

where \( M_{*, \text{sp}} \) is the average stellar mass of the star particles in the simulation, and \( t_{PSD} \) is the timescale being probed. From this relation, we see that the effects of shot noise on the PSD are greater on short timescales, as well as for simulations that have more massive star particles. However, finding the amount of time SFHs at a given stellar mass spend below \( SFR_{\text{min}} \) is a nontrivial task, depending on the shape of the SFH itself, and the number of the fluctuations around the median shape that could take it below \( SFR_{\text{min}} \).

In the simplest case, given an SFH that is simply a constant \( SFR_{\text{const}} = \psi_{\text{mean}} + \text{stochastic fluctuations } SFR_{fluct} = (N(0, \psi_{\sigma})) \), the distribution of SFR(t) at any given time is simply given by a Gaussian \( N(\psi_{\text{mean}}, \psi_{\sigma}) \). \( \psi_{\text{mean}} = M_*/\tau_H \) is set by the stellar mass of the galaxy, where \( \tau_H \) is the age of the universe at the epoch of interest. The amount of time any SFH at a given stellar mass spends below a threshold SFR is then given by

\[
t(SFR < SFR_{\text{min}} | M_*, z) = \tau_H \int_{-\infty}^{SFR_{\text{min}}} \exp \left( - \frac{(SFR - \psi_{\text{mean}})^2}{(\psi_{\sigma})^2} \right) dSFR
\]

\[
= \tau_H \psi_{\sigma} \sqrt{\frac{\pi}{2}} \left(1 + erf \left( \frac{M_{*, \text{sp}} / t_{PSD} - M_*/\tau_H}{\psi_{\sigma}} \right) \right)
\]

Using this, we can set a threshold on the amount of shot noise, and limit our analysis to timescales above that. We then consider two more complicated cases,

- Where the SFH has a power spectrum with a nontrivial power law \( (PSD(f) \propto f^{-2}) \) as seen in the simulations at long timescales, and postulated in Kelson (2014), and

- Where the median SFH is not stationary, i.e. Evolves with time following the SFR-M* correlation (Behroozi et al. 2018; Moster et al. 2018).

Since deriving the average time an SFH spends at the shot noise limit analytically is involved for these cases, we perform a series of numerical experiments by generating mock SFHs that satisfy these criteria and modeling the effects of discrete star particles in the same way as the simulations. To do this, we discretize the mock SFH by rounding the SFR in each time bin to its nearest number of star particles, and consider the excess as the gas probability that a star particle will be formed in that time bin.

We then compute the power spectra for these SFHs before and after the discretization procedure, and quantify the timescale at which the divergence from the original PSD exceeds a
certain threshold (here 0.2 dex). We also tried fitting the PSD corresponding to the discretized SFH with a broken power-law to quantify the timescale at which the transition from $\alpha = 2$ to white noise ($\alpha = 0$) happens, and find that our results do not significantly change. Based on these numerical experiments, figure D.9 shows the thresholds for the case where we consider the case with the a constant SFR with $\alpha = 2$, and figure D.8 for the case where we consider an ensemble of SFHs that yield the Schreiber et al. (2015) SFR-M* correlation at different epochs. Results of repeating the analysis using log SFR as opposed to linear SFR are shown in Appendix D.2. For all cases, the figures can be read in two ways:

- Read horizontally, the figures give the minimum timescale to which we can study the PSDs for galaxies in a given stellar mass bin at a particular epoch.
- Read vertically, the figures give the minimum SFR (and therefore the minimum stellar mass) needed to probe a certain timescale or regime of the PSDs of galaxies.

Below these stellar masses (and timescales) the effects of discretization of the star particles begins to dominate the SFRs, and thus the PSDs. While the solid black lines are the results of the experiment for SFHs that produce an SFR-M* correlation with $\sim 0.3$ dex scatter, the figures also contain two additional curves for each star particle mass, showing the sensitivity curves for the case with $\sim 0.1$ dex and 0.5 dex scatter. We find that the sensitivity improves with an increase in the scatter, since it corresponds to more power in the PSD at all timescales - this results in a larger portion of the PSD above the shot noise threshold. Since the analysis with log SFR effectively re-weights the SFHs such that lower SFRs (and thus the shot noise due to discrete star particles) is on a more comparable footing to the regions of the SFH with high SFR, the sensitivity to short timescales is much worse for this case, as seen in Appendix D.2.

D.3 Probing variability on different timescales within IllustrisTNG

We use the IllustrisTNG 25 Mpc$^3$ small volume runs to quantify the effects of changing the strengths of feedback and winds, as well as the way they are implemented. This allows us to see the effects on the PSDs in different stellar mass bins, and better understand the timescales on which the feedback and wind prescriptions influence the SFHs of individual galaxies.

From Figure.D.10, we see that TNG with the slower Illustris winds (as compared to TNG), cause a mass-dependent deficit in the PSD on timescales of 2-4 Gyr. This can also be explicitly seen for the case of slower winds in TNG itself. Corresponding to the slower winds case, stronger winds result in a mass-dependent increase in the PSD on $\sim 1Gyr$ timescales. In comparison to
Figure D.8: Quantifying the lowest timescales we can probe at different stellar masses for galaxies that follow the SFR-Mₖ correlation from Schreiber et al. (2015) with perturbations that add different amounts of scatter to the relation (between 0.1 to 0.5 dex) for different masses for the star particles following the procedure described in sec. D.2.
Figure D.9: Quantifying the lowest timescales we can probe at different stellar masses for galaxies with a constant SFH + perturbations that add different amounts of scatter to the relation (between 0.1 to 0.5 dex) for different masses for the star particles following the procedure described in sec. D.2.
Figure D.10: The power spectral densities of galaxies in a series of small-box IllustrisTNG runs with different conditions. The lines and shaded regions show the median and 68 percentile distributions at each point in time. The PSDs are truncated at the point below which shot noise contamination begins to flatten them out. The different models allow us to observe the pasts of the PSD that are affected by different physical conditions, allowing us to understand the contributions at different timescales.

Other effects like AGN, the effects of winds manifest across a wide range of effects in the PSD. For the highest mass bin, faster winds seem to reduce the variability at longer $\sim 3-4\text{Gyr}$ and shorter $<100\text{Myr}$ timescales and redistribute it on intermediate $\sim 250\text{Myr}$ timescales.

The lack of AGN kinetic feedback (no BH kin fdbk) seems to lower the PSD on the longest timescales ($>10\text{ Gyr}$), which might be due to less efficient quenching since the lack of kinetic feedback implies that not as much gas is being blown out of the galaxy by AGN driven winds.
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