ESSAYS ON BANKING AND FINANCIAL FRAGILITY

by

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A dissertation submitted to the

School of Graduate Studies

Rutgers, The State University of New Jersey

In partial fulfillment of the requirements

For the degree of

Doctor of Philosophy

Graduate Program in Economics

Written under the direction of

Todd Keister

And approved by

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New Brunswick, New Jersey

October, 2019

ABSTRACT OF THE DISSERTATION

Essays on Banking and Financial Fragility

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My academic work focuses on banking and financial fragility. A common theme of my research agenda is to study the interaction between the financial crises and the government's policy responses. In particular, I explore how the anticipation of government policy reactions affects the incentives and behavior of bank depositors and other investors. I study this same underlying issue in three distinct settings.

In chapter 1, I study financial fragility in a setting where banking contracts are fixed in nominal terms and all transactions take place using money supplied by the central bank. The existing literature has emphasized that, in such settings, a lender of last resort can effectively prevent self-fulfilling bank runs. I show that, in the event of a crisis, the government will be tempted to intervene ex post to limit the effects of inflation. When depositors anticipate such a reaction, those who have an opportunity to withdraw before the intervention occurs may choose to run on their banks. I show that if the government can commit not to intervene, the efficient allocation will be the unique equilibrium outcome. If the government lacks commitment, however, a self-fulfilling bank run can arise even with nominal banking contracts and a lender of last resort.

In chapter 2, which is joint work with Todd Keister, we ask whether policy makers

should be transparent about their plans for dealing with a future financial crisis or there is a role for ambiguity in an optimal policy. We study a modern version of the Diamond and Dybvig (1983) model in which the regulator is able to bail out banks experiencing a loss on their assets. We show that when the regulator has a perfect regulatory power, the optimal policy is fully transparent in the sense that the regulator specifies the full bailout policy in advance. When the financial institutions experience a real loss on their assets, the regulator wants to provide insurance by using some tax revenue to bail out those financial institutions when the regulation is imperfect. However, the anticipation of such a bailout distorts ex ante incentives and leads financial institutions to become more illiquid than is socially desirable. In this environment, the regulator can sometimes improve welfare by introducing ambiguity about its bailout policy. We consider two distinctive forms of ambiguity: one in which the regulator either bails out all banks or none and the other in which announces what fraction of banks will be bailed out in advance but does not say which specific banks will be included. In both cases, we show that the optimal policy involves providing bailouts with some positive probability. In addition, we show that the policy maker should aim to minimize the amount of aggregate uncertainty created by its ambiguous policy.

In chapter 3, I study how the resolution policy for failed institutions affects welfare and the fragility of the banking system. I again study a modern version of the classic model of Diamond and Dybvig (1983), this time adding partial deposit insurance, fiscal policies and bank-specific fundamental uncertainty about the investment return. I ask how the remaining assets of a failed financial institution should be distributed among different creditors in the process of resolution when the government's deposit insurance fund covers some deposits and, therefore, has a claim on the institution's remaining assets. I compare then two cases: one where claims of the uninsured depositors are subordinated to those of the deposit insurance fund; and the other in which claims on the failed institution are paid so that the deposit insurance funds and the uninsured depositors share losses. The latter allows risk sharing by shifting resources from public consumption to provide consumption for the uninsured depositors, which in turn can raise welfare and lower some depositors' incentive to run on their banks. However, this approach can distort ex ante incentives for the banks to increase their short-term liabilities, and therefore, can increase their depositors' incentives to run. Numerical exercises show that sharing losses between the deposit insurance fund and the uninsured depositors often improves welfare and promotes financial stability. I show that an increase in the deposit insurance coverage can sometimes lower welfare and make the banking system more susceptible to a run by uninsured depositors because any losses will be concentrated among smaller group of uninsured depositors.

Acknowledgements

First and foremost, I would like to express my sincere gratitude my advisor Professor Todd Keister for his continuous support and invaluable guidance. He has been patient and given his knowledge, insight as well as encouragements throughout my graduate study.

In addition, I am grateful to the members of my dissertation committee: Professor Chang, Professor Landon-Lane for their insightful comments and advice on my job market. I am also thankful for the outside member of my committee - Daniel Sanches from the Federal Reserve Bank of Philadelphia for providing valuable feedback.

I would like to express my thanks to the macro-finance group: Yang Li, Yuliyan Mitkov, Ryuichiro Izumi and Yan Xue for exchanging valuable talks and discussions during our meetings. During my time at Rutgers, I have been lucky to have my fellow students, Jieun Choi, Geoffrey Clarke, Jessica Jiang, Ethan Jiang, Maria Pagliari, Weiqi Xiong and Zheng Zhao.

Finally, I want to express my greatest gratitude to my family for their support and encouragement. My special thanks to my beloved husband for his love, sacrifice and support.

Dedication

To my husband and my family.

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Chapter 1

Money, Banking, and Financial Fragility

1.1 Introduction

After the global financial crisis of 2008, many economists have tried to better understand why the financial system appears to be so fragile and how the central bank can play a meaningful role. Despite a substantial and growing literature on this topic, the underlying causes of financial fragility are still not well understood. The situation is particularly puzzling in settings where the central bank is able to provide sufficient money to help banks meet their fixed nominal obligations during times of financial crisis. Existing models suggest that, in such settings, the type of banking panics studied by Diamond and Dybvig (1983) and others should not occur.[1] I revisit this issue and show how financial fragility can indeed arise. The key frictions that create this fragility are sequential service of depositors and the possibility of ex post intervention by a government.

I build on the setup in Allen, Carletti, and Gale (2014), where the transactions between banks, firms and depositors all take place using money provided by the central bank. Banks take deposits from depositors and offer a fixed nominal payment based on the time at which the depositor withdraws. As in Diamond and Dybvig (1983), some depositors receive liquidity shocks, which are private information, that lead them to withdraw early. Money received from the bank is used to purchase consumption in the goods market. Goods are supplied by firms, which borrow from banks to invest in both liquid and illiquid investment projects in the initial period. The price of the real consumption goods in each period adjusts depending on the amount of money withdrawn from banks.

[1] See, for example, Chang and Velasco (2000), Diamond and Rajan (2006), Allen, Carletti, and Gale (2014), Andolfatto, Berentsen, and Martin (2019).

I first show that, in this setting, the equilibrium allocation is unique and self-fulfilling bank runs cannot occur. This result is common in models where banking contract are fixed in nominal terms.[2] The key insight from this literature is that borrowing from a lender of last resort can allow banks to meet their obligations even when withdrawal demand is unusually high. In such events, the amount money in circulation rises which, in turn, drives the price of goods higher. Higher prices in the goods market lowers the real consumption from early withdrawal, making withdrawing early less attractive. To put it differently, nominal contracts with a fully variable money supply tend to create state-contingent consumption profiles. This state-contingency, in turn, removes the strategic complementarity that otherwise generates a self-fulfilling bank-run equilibrium in the Diamond-Dybvig framework. Although no self-fulfilling bank runs can occur, the equilibrium allocation is inefficient. This inefficiency arises because depositors' types are unobservable when they engage in transactions and the market cannot limit their participation. It is well known that such market trading limits a bank's ability to provide risk sharing (Farhi, Golosov, & Tsyvinski, 2009; Jacklin, 1987). As a result, the market equilibrium features on excessively high level of illiquid investment and an inefficient consumption allocation. I assume that the government imposes a liquidity requirement to correct the market inefficiency when investment decisions are made, as in Farhi et al. (2009). This liquidity requirement is able to correct the incentive distortions and implement the efficient allocation as an equilibrium.

I then study what happens when the government can intervene ex post in the market in the form of tax policy. Importantly, I assume that the government cannot pre-commit to its policy, but will instead choose its policy as a best response to the situation at hand. I show that such intervention may introduce the possibility of a self-fulfilling bank run. To see why, suppose that depositors begin to run on all banks. The central bank will act as a lender of last resort, which increases the amount of money in circulation and causes the price of goods to rise. Depositors who have withdrawn will then be facing relatively low levels of consumption. Observing this, the government may be tempted to

[2]See, for example, Diamond and Rajan (2006), Skeie (2008), Sun and Huangfu (2011), Allen et al. (2014), He and Liu (2019).

intervene to prevent the price of goods from rising too much. This intervention takes the form of taxes on future sales for the firms, which provides an incentive to increase the current supply of goods. The ex-post optimal tax policy induces the firms to liquidate some of their long investment, which lowers resources available in the market for those depositors who do not run on their bank. An anticipation of such intervention may lead those depositors who have an opportunity to withdraw before the intervention occurs to run on their banks. In this way, the government's attempt to promote an ex post efficient consumption allocation can undermine the ex ante incentives of depositors and end up destabilizing the banking system.

To summarize, this paper finds that a self-fulfilling bank run can arise in a model of nominal banking contracts and a lender of last resort when depositors are constrained by sequential service and the government is unable to pre-commit to its policy. The key insight is that, once a run is underway, the government will tend to intervene in a way that helps those depositors facing low levels of consumption. I show how the anticipation of this type of intervention can create an incentive for depositors, who have an opportunity to withdraw before the intervention is in effect, to run on their bank.

Related literature: Following Diamond and Dybvig (1983), most of the literature on bank runs considers environments with real contracts and no role for money in the banking system. A few papers do introduce money by having banks offering nominal deposit contracts and having money used to exchange goods in the market. In these papers, self-fulfilling bank runs do not occur and the economy can achieve the efficient allocation of resources. Skeie (2008) and Sun and Huangfu (2011) introduce inside money that can be circulated to exchange goods, whereas Allen et al. (2014) study fiat money. In each of these papers, the banking system provides risk-sharing and the equilibrium is immune to bank runs because of the fully flexible monetary prices. Money circulated in the economy is determined as much as withdrawal demand, affecting the prices for goods. In Diamond and Dybvig (1983), non-contingent demand deposit contracts introduce the possibility of a run. Despite the rigid contracts in banking models with money, prices fully adjust in the market so that the real value of the contract becomes contingent on every state. Chang and Velasco (2000) study an open economy setting under the Diamond-Dybvig framework and show that an elastic money supply by the central bank and a fully floating foreign exchange rate can eliminate the bank run equilibrium. These papers share a key feature that the non-contingent deposit contract and elastic currency prices interact in such a way the equilibrium is run-proof. The key difference between this paper and these papers discussed above is that depositors are not explicitly constrained by the sequential service constraint and there is no ex post government intervention. In this paper, I consider the government's incentive to intervene in the event of a crisis and how the anticipation of the intervention can alter incentives for depositors who choose to withdraw from their bank before the intervention becomes in effect.

There is also a literature on banking crises and monetary policy in infinite horizon economies. For example, Champ, Smith, and Williamson (1996) and Antinolfi, Huybens, and Keister (2001) study an overlapping generations environment with random relocations and show how having an elastic money supply from a lender of last resort can help stabilize equilibrium consumption allocations. Andolfatto et al. (2019), Matsuoka and Watanabe (2017) integrate the Diamond-Dybvig type banks with the Lagos and Wright (2005) model of money. They show that the central bank acting as a lender of last resort can promote stability of the banking system. Relative to these papers, my focus in this paper is on the possibility of self-fulfilling bank runs and how ex-post intervention by the government may end up limiting the benefits of having an elastic currency and flexible prices.

Ennis and Keister (2009, 2010) were the first to study the time inconsistency problem a government faces in responding to a bank run and how this problem can undermine financial stability. They studied a real model in which the policy determines how many goods each depositor receives when withdrawing from the bank and the intervention entails requiring banks to pay out more goods. In my model, depositors always receive the promised amount when withdrawing from their bank; in this sense, banking contracts are never re-negotiated. The time inconsistency problem instead arises in the government's response to inflation, which affects the real value of these nominal withdrawals.

My results show that the implications of a government's inability to pre-commit to a crisis response for financial stability are more pervasive than previously realized.

The rest of this paper is organized as follows. In section 2, I illustrate the framework of the model and characterize the efficient allocation. In addition, the monetary environment is described. In section 3, I study the competitive equilibrium and show that bank runs do not occur and the equilibrium allocation is inefficient, but that a liquidity regulation can correct this market inefficiency. In section 4, I show that a self-fulfilling bank run can arise with ex post intervention. Section 5 contains concluding remarks.

1.2 The model

1.2.1 The real environment

The model is based on a version of Allen et al. (2014) with no fundamental aggregate uncertainty, which allows me to focus on the potential for self-fulfilling bank runs. I extend the model by adding costly liquidation and an explicit sequential service constraint.

There are three periods: $t = 0, 1, 2$. The middle period, $t = 1$, is divided into two subperiods: morning and afternoon. There is a single consumption good in each period.

There is a continuum of ex ante identical depositors in each of a continuum of banks. All individuals are endowed with one unit of good at the beginning of period 0 and nothing afterwards. Each has the utility function

$$u\left(c_1, c_2\;;\;\omega\right) = \frac{\left(c_1 + \omega c_2\right)^{1-\gamma} - 1}{1-\gamma}$$

where c_t is consumption at period $t = 1, 2$. The preference type $\omega \in \{0, 1\}$ is realized and observed privately at the beginning of period 1. With probability λ, each depositor becomes *impatient* and values consumption only in period 1. With probability $(1-\lambda)$, he becomes *patient* and values consumption in both periods. By a law of large numbers, λ is the fraction of impatient depositors in period 1. Assume that all depositors face the same the probability λ of being impatient and that this probability is known to all at $t = 0$. Assume that the coefficient of relative risk aversion, γ, is strictly greater than one, as in Diamond and Dybvig (1983) and many others.

Firms can operate two constant returns to scale technologies that transform goods in period 0 into consumption goods in later periods: short storage and long investment. One unit of good invested in short storage in period t yields one unit of consumption in period $t+1$ for $t = 0, 1$. One unit invested in long investment returns $R > 1$ units of consumption in period 2, however, earns only $r < 1$ units of goods in period 1. There is a continuum of firms that are competitive, risk neutral, and that sell their assets on the goods markets in period 1 and 2 to maximize their profits.

1.2.2 The efficient allocation

Before characterizing the competitive equilibrium where all transactions take place using money, I first study the efficient allocation. Consider the problem of a benevolent planner that can perfectly observe types of all depositors. The planner makes the investment, liquidation, and allocation decisions to maximize the expected utility of depositors subject to the production technologies and the resource constraints. The planner can allocate different amounts of goods depending on a depositor's type. Denote the investment in the long investment as x and the amount of liquidation of this investment in period 1 as l. The planner's problem is then same as in a standard two-asset version of the Diamond-Dybvig model, such as Cooper and Ross (1998):

$$\max_{c_1, c_2, x, l} \quad \lambda u(c_1) + (1-\lambda)u(c_2) \tag{1.1}$$

subject to

$$\begin{cases} 0 \leq x \leq 1, \\ \lambda c_1 \leq 1 - x + rl, \\ (1-\lambda)c_2 \leq R(x-l) + (1 - x + rl - \lambda c_1) \end{cases} \tag{1.2}$$

and nonnegativity constraints. The first condition is a feasibility condition for the investment decision in period 0. The last two constraints state that the total consumption of each type cannot exceed the total supply of goods at $t = 1$ and $t = 2$, respectively.

Let $\{c_1^*, c_2^*, x^*, l^*\}$ denote the solution to the planner's problem. It is well known that the solution satisfies

$$x^* = 1 - \lambda c_1^*, \tag{1.3a}$$

$$l^* = 0, \tag{1.3b}$$

$$\frac{u'(c_1^*)}{u'(c_2^*)} = R, \tag{1.3c}$$

$$\lambda c_1^* + (1 - \lambda)\frac{c_2^*}{R} = 1. \tag{1.3d}$$

As is standard, the solution to the planner's problem has impatient depositors consuming only goods from the short storage at $t = 1$ and patient depositors consuming only from the long investment at $t = 2$. There is no early liquidation of investment. The solution also equates the marginal rate of substitution between consumption at $t = 1$ and $t = 2$ to the marginal rate of transformation R as in (1.3c), which implies that $c_1^* < c_2^*$. Also, $c_1^* > 1$ follows from the assumption that $\gamma > 1$. As is standard in this type of model, the planner would allocate more consumption to patient than impatient depositors, but would also provide liquidity insurance by giving impatient depositors a higher return than that on the short storage.

1.2.3 The monetary environment

In the monetary environment, the central bank can freely print and destroy money that is used as a medium of exchange between banks, firms and depositors. The central bank sets the nominal interest rate I_t for loans to banks and lends whatever quantity of money banks demand at this rate. That is, one unit of money borrowed by banks at t requires repayment of I_t units of money at $t+1$, for $t = 0, 1$. Assume that the central bank charges zero interest rate on within-period loans.

Competitive banks issue loans to firms by borrowing money from the central bank at the beginning of the initial period. They also take deposits from depositors and offer fixed nominal payments D_t where t is the time at which the depositor withdraws. Firms use their borrowed money to purchase goods in period 0 and form a portfolio of short and long investments. For one unit of money borrowed from the banks, firms are required to repay $\{K_1, K_2\}$ where K_t is repayment in period t. Assume that the remaining debt in period 1 can be rolled over and repaid in period 2 if the firm could not meet its nominal repayment obligation. Depositors earn money by selling their endowments in this market and deposit the proceeds with the bank. The flow of money in period 0 is illustrated in Figure 1.1.

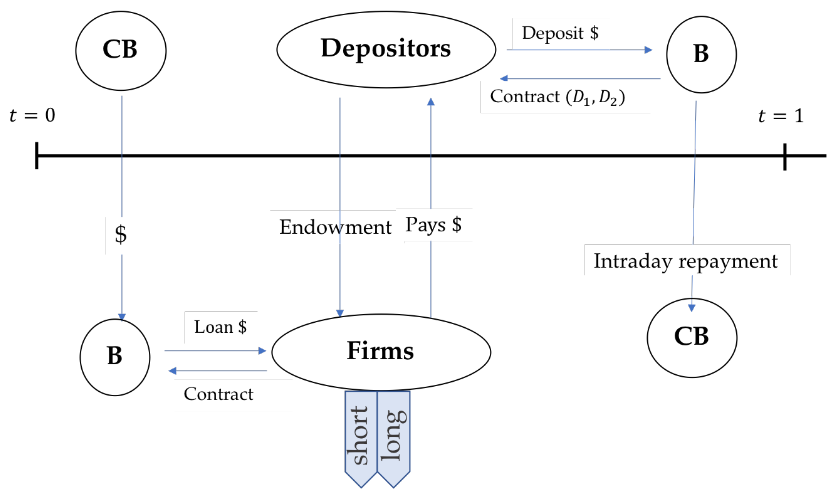

Figure 1.1: Flow of funds in period 0

Without loss of generality, I can normalize the price level in the initial period to one. By the marking clearing condition in period 0, each firm needs to borrow one unit of money from banks to purchase one unit of good to invest. The banks, then, borrow one unit of money from the central bank.

Assume that a continuum of depositors indexed by $i \in [0, 1]$ deposit the income from selling their endowment in each bank. At the beginning of period 1, each depositor learns his own type $\omega \in \{0, 1\}$ and his position in the order in which early withdrawal decisions are made. Without loss of generality, I assume that a depositor's position in this order is given by his index i, meaning that depositor i will have an opportunity to withdraw in period 1 before all depositors with $i' > i$, but after all depositors $i' < i$. Once his type and order of the line are known, each depositor decides to withdraw in period 1 or in period 2 and then use the proceeds to purchase goods in the market. Depositors can also instead withdraw in period 1 and have an option to save until period 2 in the secondary market outside the banking system at rate I_2. Because that the central bank acts as a lender of last resort, banks are able to meet any nominal obligation to depositors.

Period 1 is divided into morning and afternoon and the goods market is open in both subperiods. A depositor who chooses to withdraw in period 1 and is early in line will have access to the morning market to purchase goods supplied by firms. In particular, I assume that the first $\theta \in (0, \lambda)$ fraction of depositors who withdraw enter the goods market in the morning. Depositors who withdraw later in period 1 are able to buy goods in the afternoon market. This is a version of sequential service constraint in the spirit of Wallace (1988, 1990). In my setting, sequential service implies that the price in the goods market only gradually comes to reflect the total demand for purchase of goods in period 1. In particular, some depositors are able to withdraw from the bank and make purchase before the price level reflects total early withdrawal demand.

Firms repay part of their loans to the banks using their revenue earned in the morning and afternoon markets. Banks then repay their loans to the central bank. The same transactions then recur while the goods market opens in period 2. The flows of money in period 1 and 2 are illustrated in the Figures 1.2 and 1.3.

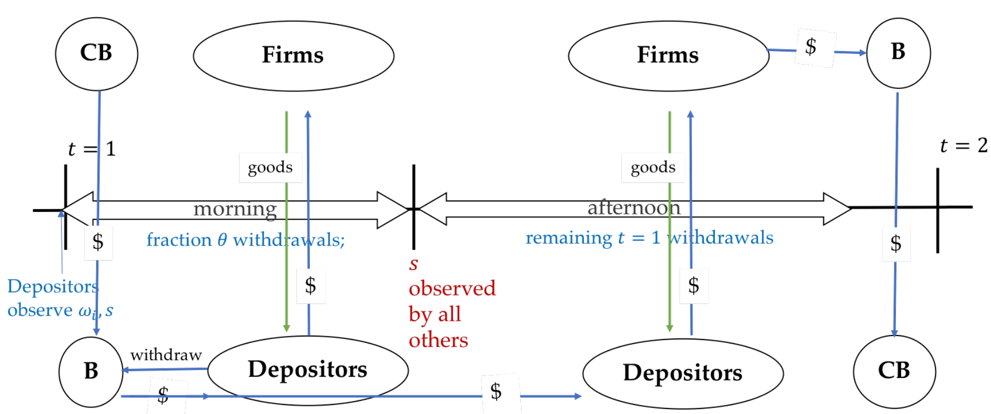

Figure 1.2: Flow of funds in period 1

I also include an extrinsic sunspot signal that is realized at the beginning of period

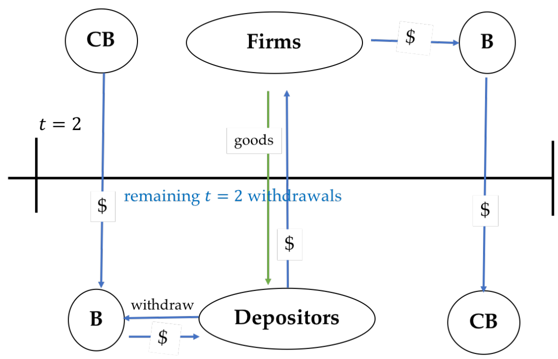

Figure 1.3: Flow of funds in period 2

1 as in Cooper and Ross (1998), Peck and Shell (2003) and others. This signal represents an extrinsic state on which each depositor can potentially condition their actions. Without loss of generality, the extrinsic signal is assumed to be drawn from two possible states $S = \{\alpha, \beta\}$. Assume that the extrinsic signal can be observed by depositors once the state is realized at the beginning of period 1. Therefore, a depositor indexed by his position of the order, i, chooses a withdrawal strategy that specifies an action for each possible combination of his preference type and the sunspot signal. The sunspot state is observed by all agents when the afternoon-market opens.

1.3 Competitive equilibrium

In this section, I study competitive equilibrium where all transactions between banks, depositors and firms take place using money. I show that the equilibrium allocation is inefficient and no run will occur in this economy. The inefficiency can be corrected by introducing liquidity regulation. Under the regulation, there still does not exist a run equilibrium.

1.3.1 Depositor's problem

In period 0, depositors earn money by selling their endowments in this market and deposit the proceeds with the bank. They use the withdrawals to purchase goods in later periods.

At the beginning of period 1, a depositor with index i chooses a strategy that assigns a withdrawal period to each realization of his preference type ω_i and of the state

$$y_i \; : \; \{0,1\} \times S \quad \rightarrow \quad \{1,2\}\,, \tag{1.4}$$

for given market prices in $\mathbb{P} = \mathbb{R}^5_{++}$. A strategy, $y_i = t$, corresponds to a withdrawal by depositor i in period $t = 1, 2$. Let $y = \{y_i\}_{\forall i \in [0,1]}$ denote the profile of all depositors' strategies.

All impatient depositors choose to withdraw and purchase goods in period 1 for any $s \in S$, because they value consumption only in period 1. Impatient depositors who withdraw in the morning can purchase $c_1 = D_1 / P_1^m$ units of goods where D_1 is the amount of money withdrawn from the bank in period 1 and P_1^m is the market price of goods in the morning of period 1. Those who withdraw in the afternoon can consume $c_{1s}^a = D_1 / P_{1s}^a$ units of goods where P_{1s}^a is the market price of goods in the afternoon after the sunspot state is revealed to all agents in the market.

Patient depositors who have an opportunity to withdraw in the morning at $t = 1$ choose to wait until $t = 2$ to withdraw in state s if $\frac{D_1}{P_1^m} \leq \frac{D_2}{P_{2s}}$ where D_2 is the nominal payment by the bank from withdrawal in period 2 and P_{2s} denotes the market price of goods in period 2 for the state s. When patient depositor i is later in the decision order, he decides in state s to withdraw and consume goods at $t = 2$ if $\frac{D_1}{P_{1s}^a} \leq \frac{D_2}{P_{2s}}$.

1.3.2 Firm's problem

Now consider firms' decision problems. Each competitive firm chooses its portfolio of investment in period 0 and the amount of goods that it supplies in both subperiods at $t = 1$ and in period 2 to maximize its profits taking the market prices as given. Assume that the firms do not know the realized sunspot state s at the beginning of period 1 and consider $s = \beta$ as a zero probability event. They learn the state s in the afternoon and can revise their supply decision conditional on the updated information. The firm's problem is solved by working backward, beginning in the afternoon of period 1.

Firm's problem in the afternoon of period 1

In the afternoon at $t = 1$, each firm chooses the quantity to supply immediately, q^a_{1s}, and the quantity to supply in the following period, q_{2s}. This choice determines the amount of early liquidation of long investment, l^a_s. The firm takes market prices as given and aims to maximize profits from the afternoon onwards. The firm earns revenue from selling goods and uses this revenue to repay loans. Assume that the firm has an option of depositing at the bank at $t = 1$, which provides returns $D_{12} = \frac{D_2}{D_1}$ at $t = 2$.

$$V^a_s\left(x, q^m_1, l^m\right) \equiv \max_{\left\{q^a_{1s},\, q_{2s},\, l^a_s\right\}} \quad P^a_{1s} q^a_{1s} + \frac{P_{2s} q_{2s}}{D_{12}} - \left(K_1 + \frac{K_2}{D_{12}}\right) \tag{1.5a}$$

subject to

$$q^a_{1s} \leq 1 - x + r l^m - q^m_1 + r l^a_s, \tag{1.5b}$$

$$q_{2s} \leq R\left(x - l^m - l^a_s\right) + 1 - x + r l^m - q^m_1 + r l^a_s - q^a_{1s}, \tag{1.5c}$$

$$0 \leq l^a_s \leq \left(x - l^m\right), \quad q^a_{1s} \geq 0,\ q_{2s} \geq 0, \tag{1.5d}$$

$$P^m_1 q^m_1 + P^a_{1s} q^a_{1s} + \frac{P_{2s} q_{2s}}{D_{12}} \geq K_1 + \frac{K_2}{D_{12}} \tag{1.5e}$$

where $K_1 + \frac{K_2}{D_{12}}$ is the present valued nominal debt payments to the bank.[3]

Equation (1.5b) is the resource constraint in period 1that the amount of goods that the firms can sell in the afternoon market cannot exceed the amount of goods available from short storage and liquidation of long investment after selling goods in the morning market. The constraint in (1.5c) states the resource constraint in period 2 that the firm can sell goods in period 2 by using its matured long-investment projects that have

[3]The firm is allowed to rollover some of its debt payments for period 1, K_1, to period 2 at rate D_{12}. For example, suppose that the firm repays only $X < K_1$ in period 1 and remaining debts for period 1 are rolled over. Then, the firm needs to repay $\left(K_1 - X\right) D_{12} + K_2$ in period 2. Also, I assume that the firm can choose to prepay at discount rate D_{12} as much as it wants. For example, when the firm wants to prepay Y units of loans from K_2 in period 1, then it pays $K_1 + \frac{Y}{D_{12}}$ in period 1 and $K_2 - Y$ in period 2. In any case, the present-valued total debt payments measured in period 1 is exactly summarized as $K_1 + \frac{K_2}{D_{12}}$.

not been liquidated plus any inventory from the previous period. Equation (1.5d) is another feasibility constraint for early liquidation, including nonnegativity conditions. Equation (1.5e) is the budget constraint that total present-valued revenue earned must be at least as much as debt repayments to the banks.

The firm's choice from the afternoon onwards is summarized in the following vector:

$$\left(q_{1s}^{a}\left(\mathbf{P}, x, q_1^m, l^m\right),\ q_{2s}\left(\mathbf{P}\, x, q_1^m, l^m\right),\ l_s^a\left(\mathbf{P}\, x, q_1^m, l^m\right) \right)_{s\in S} \tag{1.6}$$

where $\mathbf{P} \equiv \left(P_1^m, P_{1s}^a, P_{2s}\right)_{s\in S} \in \mathbb{R}_{++}^5$ is the price vector.

Firm's problem in the morning of period 1

The firm chooses the quantity of goods supplied in the morning, q_1^m, and the liquidation of long investment, l^m, for a given predetermined investment decision and taking market prices as given to solve the following problem. Assume that, while in the morning-market, all firms consider the state $s = \beta$ is realized with zero probability.[4]

$$V_1^m(x) \equiv \max_{\{q_1^m,\, l^m\}} \quad P_1^m q_1^m + V_\alpha^m\left(x, q_1^m, l^m\right) \tag{1.7a}$$

subject to

$$0 \leq\ q_1^m \leq 1 - x + r l^m, \tag{1.7b}$$

$$0 \leq\ l^m \leq x. \tag{1.7c}$$

The first resource constraint states that the supply of goods in the morning-market cannot exceed the available resources from holding storage and liquidating parts of the long investment. The second feasibility condition applies to liquidation of long investment. The solution to the problem (q_1^m, l^m) is summarized as

$$\Big(q_1^m\left(\mathbf{P}, x\right),\ l^m\left(\mathbf{P}, x\right) \Big). \tag{1.8}$$

[4]The assumption that $s = \beta$ occurs with zero probability is commonly used to simplify the analysis. A continuity argument establishes that the same qualitative results obtaion when the bad sunspot state has a positive but sufficiently small probability.

Firm's choice of investment

Each firm makes an investment choice x in period 0 to solve the following maximization problem:

$$\max_{0 \leq x \leq 1} V_1^m(x) \tag{1.9}$$

The optimal investment portfolio depends on the relative market price ratio between goods in the two periods and on the marginal rate of transformation between short and long investments. That is, the firm's decision can be written as a correspondence of the market prices, $x(\mathbf{P})$.

1.3.3 Equilibrium

The competitive equilibrium for the overall economy is introduced in steps. I first define a goods market equilibrium as an equilibrium of a subgame of the overall economy for given nominal interest rates $\{I_t\}_{t=1,2}$, nominal contracts chosen by banks $\{D_t, K_t\}_{t=1,2}$, and the profile of the depositor's strategies y. Second, I define an equilibrium for the withdrawal game by the depositors. Finally, the competitive equilibrium includes the nominal banking contract that is consistent with the equilibrium strategy profile and the goods market equilibrium.

Goods market equilibrium

A *goods market equilibrium* consists of a set of goods prices $\left\{\bar{P}_1^m, \bar{P}_{1s}^a, \bar{P}_{2s}\right\}_{s\in S}$, an investment portfolio $\bar{x}$, early liquidation of long investment $\left\{\bar{l}^m, \bar{l}_s^a\right\}_{s\in S}$, the firm's supply schedule $\{\bar{q}_1^m, \bar{q}_{1s}^a, \bar{q}_{2s}\}_{s\in S}$, and the consumption allocation $\{\bar{c}_1^m, \bar{c}_{1s}^a, \bar{c}_{2s}\}_{s\in S}$ such that, for any given $\{I_t, D_t, K_t, y\}_{t=1,2}$,

i. $\left\{\bar{q}_1^m, \bar{q}_{1s}^a, \bar{q}_{2s}, \bar{l}^m, \bar{l}_s^a, \bar{x}\right\}_{s\in S}$ solves for each firm's optimization problems in (1.5a)-(1.5e), (1.7a)-(1.7c) and (1.9) and,

ii. the goods markets in each period clear for all s as follows:

$$\bar{q}_1^m = \theta \frac{D_1}{\bar{P}_1^m}, \tag{1.10a}$$

$$\bar{q}_{1s}^a = (\lambda_s^y - \theta) \frac{D_1}{\bar{P}_{1s}^a}, \tag{1.10b}$$

$$\bar{q}_{2s} = (1 - \lambda_s^y) \frac{D_2}{\bar{P}_{2s}} \tag{1.10c}$$

where $\bar{c}_1^m = \frac{D_1}{\bar{P}_1^m}$, $\bar{c}_{1s}^a = \frac{D_1}{\bar{P}_{1s}^a}$, $\bar{c}_{2s} = \frac{D_2}{\bar{P}_{2s}}$ are consumption allocations for a depositor in the morning, in the afternoon of period 1, and in period 2, respectively . In addition, $\lambda_s^y = \int_0^1 \mathbb{I}\left(y_i(\cdot, s) = 1\right) di$ denotes the fraction of depositors who choose to withdraw in period 1 for each realization of $s \in S$.

The right-hand-sides of (1.10a)-(1.10c) are the aggregate market demands for goods for the given profile of strategies y. We have $\lambda_s^y \in [\lambda, 1]$ because impatient depositors will always choose to withdraw in period 1.

Lemma 1 For any given y, the following inequalities can never hold in any goods market equilibrium:

$$\max\left\{P_1^m, P_{1\alpha}^a\right\} < \frac{P_{2\alpha}}{D_{12}}, \qquad P_{1\beta}^a < \frac{P_{2\beta}}{D_{12}} \tag{1.11}$$

or

$$\frac{P_{2\alpha}}{D_{12}} < \frac{r}{R} \min\left\{P_1^m, P_{1\alpha}^a\right\}, \qquad \frac{P_{2\beta}}{D_{12}} < \frac{r}{R} P_{1\beta}^a. \tag{1.12}$$

Proof See Appendix A.

The inequalities in (1.11) would imply that for each realized state s, the marginal revenue from selling goods at $t = 1$ is strictly less than the discounted marginal revenue from selling goods at $t = 2$. In this case, the firms would choose to store all of available goods in period 1 between period 1 and 2, then sell everything at $t = 2$. The market clearing condition with nonzero aggregate demand at $t = 1$ implies that the market price at $t = 1$ diverges to infinity. Therefore, (1.11) cannot be satisfied in equilibrium. The inequalities in (1.12) would imply that the marginal revenue from selling goods at $t = 1$ by liquidating long investments exceeds the discounted marginal revenue from selling goods at $t = 2$. If this condition held, the firms would find it optimal to liquidate all long investment projects and sell all goods in the market at $t = 1$, implying $P_{2s} = \infty$. Therefore, (1.12) cannot hold in an equilibrium either.

Lemma 2 For any given y, $\bar{P}_1^m = \bar{P}_{1\alpha}^a = R\frac{\bar{P}_{2\alpha}}{D_{12}}$ must hold in equilibrium.

Proof See Appendix A.

First, note that $\bar{P}_1^m = \bar{P}_{1\alpha}^a$ must hold in any equilibrium. Otherwise, all firms would choose to supply no goods either in the morning or in the afternoon at $t = 1$, violating one of the market clearing conditions in (1.10a) and (1.10b). The second equality is a no-arbitrage condition for investment choice. A firm would find it profitable to invest all its goods in period 0 either in the short storage or in the long investments. If $\bar{P}_{1\alpha}^a < R\frac{\bar{P}_{2\alpha}}{D_{12}}$ held, the firm would invest all in the long investment with early no liquidation. This choice would then violate market clearing conditions in (1.10a) and (1.10b) in state α. If $\bar{P}_{1\alpha}^a > R\frac{\bar{P}_{2\alpha}}{D_{12}}$ held, it would be optimal for firms to invest all in the short storage and sell all goods in period 1, which would violate the market clearing condition in (1.10c).

Equilibrium of the withdrawal game

An *equilibrium of the withdrawal game* for given $\{I_t, D_t, K_t\}_{t=1,2}$ is a profile of all depositor's strategies, $\bar{y}$, such that a set of goods prices $\left\{\bar{P}_1^m, \bar{P}_{1s}^a, \bar{P}_{2s}\right\}_{s\in S}$, and the consumption allocation $\{\bar{c}_1^m, \bar{c}_{1s}^a, \bar{c}_{2s}\}_{s\in S}$ are a goods market equilibrium for the given $\bar{y}$, and $\bar{y}_i(1, s) = 2$ is a best response if $\frac{D_2}{\bar{P}_{2s}} \geq \max\left\{\frac{D_1}{\bar{P}_1^m}, \frac{D_1}{\bar{P}_{1s}^a}\right\}$ for all s and for all i.

The following proposition states that, for any nominal interest rate, patient depositors choose to wait to withdraw in period 2 in an equilibrium.

Proposition 1 For each $\{I_t, D_t, K_t\}_{t=1,2}$, no bank-run is the unique equilibrium of the withdrawal game, that is, all depositors follow the strategy

$$y_i(\omega_i, s) = \omega_i + 1, \quad \forall i, s. \tag{1.13}$$

Proof See Appendix A.

A patient depositor chooses to withdraw at $t = 1$ only if $\frac{D_1}{P_1^m} > \frac{D_2}{P_{2s}}$ or $\frac{D_1}{P_{1s}^a} > \frac{D_2}{P_{2s}}$ for each s. Equation (1.11) in Lemma 1 states that in state α no patient depositor runs on bank at $t = 1$ and purchase goods. Moreover, Lemma 2 implies that in state α consumption at $t = 2$ is strictly greater than consumption both in the morning and in the afternoon of period 1 for all other depositors' strategies. Therefore, withdrawing at $t = 2$ for a patient depositor is a dominant strategy in state α. Patient depositors, who are later in the decision order, always choose to withdraw at $t = 2$ in equilibrium since $P_{1\beta}^a < \frac{P_{2\beta}}{D_{12}}$ from Lemma 1. Those who have an opportunity to withdraw in the morning at $t = 1$ in state β will choose to withdraw early if $\frac{D_1}{P_1^m} > \frac{D_2}{P_{2\beta}}$. Notice that in order for patient depositors to withdraw early the market price of goods in period 2 must be relatively large. Only a fraction $\theta < \lambda$ of depositors, who hold money, can purchase goods in the morning market. Since firms decide in the morning market to supply goods expecting the state to be α, the amount of goods that the depositors in the morning can consume is the same as in state α. In order for patient depositors to withdraw early, the market price in period 2 must be expected to be very large. This happens when the firms' supply of goods in period 2 is decreased because they liquidate at least some of their long investments in period 1. By the firm's optimality condition, the market prices must satisfy $rP_{1\beta}^a \geq R\frac{P_{2\beta}}{D_{12}}$ in order for the firms to liquidate. This, in turn, implies that the market prices for goods in period 2 is very low, making it less attractive for patient depositors to withdraw early.

In a standard Diamond-Dybvig framework, the strategic complementarity among depositors generates a self-fulfilling bank-run. When all depositors withdraw early, the bank has to liquidate its long investment to make the fixed real payments to depositors. The bank then has less resources to pay out in period 2 when more patient depositors withdraw early, which in turn raises the incentive for other patient depositors to run on the bank in period 1. Here, in contrast, the strategic complementarity disappears because the market prices fully adjust depending on the aggregate withdrawal demand. When the withdrawal demand in period 1 is high, the market price of goods in that period rises, meaning lower real consumption from withdrawal at $t = 1$. In this way, there does not arise a bank run in equilibrium.

Proposition 2 The consumption allocation in this equilibrium is $\left\{\bar{c}_1^m, \{\bar{c}_{1s}^a, \bar{c}_{2s}\}_{s \in S}\right\} = \left\{1, \{1, R\}_{s \in S}\right\}$, which is inefficient.

Proof See Appendix A.

As seen in the planner's problem, the planner would provide liquidity insurance by giving impatient depositors a higher return than that on the short storage when depositor's relative risk aversion is greater than one. On the other hand, the no-arbitrage condition in Lemma 2 implies that the market fails to provide insurance for the depositors. Transactions are made in the goods market without revelation of his own depositor's type. It is well known that in an incomplete market risk sharing is limited. As in Jacklin (1987), Allen and Gale (2004), and Farhi et al. (2009), the ability to engage in market transactions without limit on participation undermines the baking system's ability to improve on the risk sharing.

As a result, the equilibrium features on excessively high level of illiquid investment. The real liquidity measured as the short storage in period 1 in equilibrium is $1 - \bar{x} = \lambda < \lambda c_1^*$, which is strictly less than the planner's choice. As shown in Farhi et al. (2009), a liquidity requirement can allow the market equilibrium to achieve efficient allocation. I study an equilibrium with liquidity regulation in Section 1.3.4.

Competitive equilibrium

So far, I have studied the equilibrium for fixed nominal contracts $\{D_t, K_t\}_{t=1,2}$ and nominal interest rate set by the central bank $\{I_t\}_{t=1,2}$. In this section, I define *a competitive equilibrium* as a collection of nominal deposit contracts $\left\{\bar{D}_1, \bar{D}_2\right\}$, and nominal loan contracts $\left\{\bar{K}_1, \bar{K}_2\right\}$ such that (i) $\bar{y}$ is an equilibrium of the withdrawal game for given nominal deposit contracts $\left\{\bar{D}_1, \bar{D}_2\right\}$, (ii) banks offer the best nominal contracts $\left\{\bar{D}_1, \bar{D}_2\right\}$ and $\left\{\bar{K}_1, \bar{K}_2\right\}$ that are feasible for a given nominal interest rate $\{I_t\}$ and for taking market prices as given.

In period 0, the bank borrows money from the central bank to lend to firms and accepts deposits from depositors. Once the central bank sets the nominal interest rates, each bank will offer a nominal deposit contract $\{D_1, D_2\}$ where D_t is the nominal payment for withdrawals made at t. Perfect competition among banks and free entry forces each bank to offer the contract that maximizes its depositors' expected utility, taking market prices as given. Assume that all banks consider the state $s = \beta$ is realized with zero probability. Thus, the equilibrium contract must solve the following optimization problem:

$$\max_{\{D_1,D_2\}} \quad \theta u\left(\frac{D_1}{\bar{P}_1^m}\right) + (\lambda-\theta)u\left(\frac{D_1}{\bar{P}_{1\alpha}^a}\right) + (1-\lambda)\,u\left(\frac{D_2}{\bar{P}_{2\alpha}}\right) \tag{1.14}$$

subject to

$$\frac{\lambda D_1}{I_1} + \frac{(1-\lambda)D_2}{I_1 I_2} \leq 1, \tag{1.15a}$$

$$D_2 \geq D_1 I_2 \tag{1.15b}$$

Equation (1.15a) states that the stream of nominal payments to the depositors can be at most as much as all depositors' savings. The second constraint in (1.15b) is the incentive compatibility constraint that prevents from patient depositors to withdraw in period 1 and save in the secondary market that provides the market return I_2. It can be easily shown that this constraint strictly binds if $I_2 > \bar{P}_{2\alpha} \Big/ \left(\frac{\theta}{\lambda}\left(\bar{P}_1^m\right)^{\gamma-1} + \frac{\lambda-\theta}{\lambda}\left(\bar{P}_{1\alpha}^a\right)^{\gamma-1}\right)^{\frac{1}{\gamma-1}}$. Lemma 2 implies that this condition holds in any equilibrium. That is, banks offer a nominal contract to the depositors with

$$D_1 = I_1, \quad D_2 = I_1 I_2.$$

Finally, the contract between banks and firms for loan repayment satisfies that the stream of repayment by the firms must equal the loans of one unit of money, that is, $\frac{K_1}{I_1} + \frac{K_2}{I_2} \leq 1$. Since the debt can be rolled over and deposit is possible between $t = 1$ and $t = 2$ for return $D_{12} = I_2$, any choice of $\{K_1, K_2\}$ satisfy the condition with equality in equilibrium.

In equilibrium, the quantity theory of money holds. The total money supply depends on the central bank's choice of the nominal interest rate, which affects the bank's nominal contract. It has no effect on the determination of real equilibrium allocation. Any change of the nominal interest rate only changes the nominal payments from the banks and market prices in the same proportion.

1.3.4 Equilibrium with ex ante liquidity regulation

In this subsection, I study an equilibrium when the government can impose a liquidity regulation that aims to improve market allocations and show that there exists a unique equilibrium that implements the efficient allocation and no run occurs.

Based on the results in previous sections, the competitive equilibrium is inefficient because of excessive investment in the long investments by the firms. Excessive illiquidity can be mitigated by the minimum liquidity requirement given by when investment decisions are made. Specifically, the government requires all firms to hold a minimum amount of short storage in period 0, which is set as:

$$1 - x^* = \lambda c_1^* \tag{1.16}$$

where x^* and c_1^* are the solutions to the planners problem that satisfy (1.3a)-(1.3d). With the liquidity regulation, all patient depositors choose to wait to withdraw and consume in period 2 in equilibrium and the competitive equilibrium allocation is efficient.

Proposition 3 There exists a unique equilibrium where the allocation is efficient and no run is possible.

Proof See Appendix A.

As in Allen and Gale (2004) and Farhi et al. (2009), it can be shown that the liquidity requirement improves upon the competitive market allocation. Due to the liquidity requirement, the firms hold more short storage that will be supplied to the market in period 1, which in turn, lowers the market price for goods in period 1 compared to without liquidity regulation. This allows higher consumption for impatient depositors and less consumption for patient depositors, thereby improving risk sharing and thus raising welfare.

The reason why no patient depositor chooses to withdraw early is as discussed? in the previous sections. When withdrawal demand in period 1 is high, the market price of goods in that periods rises, meaning lower real consumption from withdrawal in period 1. Therefore, patient depositors do not withdraw in period 1 in equilibrium.

1.4 Equilibrium with ex-post intervention

In this section, I study an equilibrium when the government can intervene ex post, after depositors' withdrawal decisions have already been made. Importantly, I assume that the government cannot pre-commit to its policy, but will instead choose its policy as a best response to whatever situation the government faces. In this environment, I show that the ex post optimal policy can implement the efficient allocation as an equilibrium, however, may introduce the possibility of a self-fulfilling bank run.

1.4.1 Ex post intervention

Suppose that the government is able to intervene in the afternoon of period 1, after depositors in the morning have already withdrawn and purchased goods from firms. Suppose that a run is underway. Higher withdrawal demand created by a run raises the amount of money in circulation, which increases the market price of goods. This inflation yields lower consumption for those depositors who purchase goods in the afternoon. Then, a benevolent government will be tempted to intervene ex post to limit the effects of inflation.

Specifically, consider a policy that takes the form of a tax on firms' future sales in order to increase firms' incentive to supply more goods in the current period and so lower market prices. Collected tax revenue is transferred back to the firms lump-sum at the end of period 2. By balanced budget constraint for the government, lump-sum transfers in period 2, T_s, equal total taxes collected in period 2. That is, T_s is derived from the choice of τ_s and the following budget constraint,

$$T_s = \tau_s P_{2s} q_{2s}, \quad \text{for each } s \in S. \tag{1.17}$$

For the given tax policy (τ_s, T_s), the firm's objective in (1.5a) can be re-written as follows:

$$V_s^a(\tau_s, T_s) \equiv \max_{\{q_{1s}^a,\, q_{2s},\, l_s^a\}} \quad P_{1s}^a q_{1s}^a + \frac{(1-\tau_s) P_{2s} q_{2s}}{I_2} - \left(K_1 + \frac{K_2}{I_2}\right) + \frac{T_s}{I_2}. \tag{1.18}$$

The firm's choice from the afternoon onwards, then, is summarized as the following vector depending on the given tax policy

$$\left(q_{1s}^a(\tau_s),\ q_{2s}(\tau_s),\ l_s^a(\tau_s)\right)_{s \in S}. \tag{1.19}$$

As discussed in Cooper (1999) and Ennis and Keister (2010), the government chooses its policy τ_s, which is a tax rate on firms' sales in period 2, taking the strategies of depositors and firms as given. At the time when the government learns the state and intervenes in the afternoon, all depositors have already made their own decision and the firms have already sold q_1^m units of goods. The government will then choose the ex post policy that maximizes welfare of the remaining depositors in the banking system taking y, q_1^m and l^m as given and considering that the policy affects the firm's decisions from the afternoon onwards. The government's problem is given as:

$$\max_{c_{1s}^a, c_{2s}, \tau_s} \quad (\lambda_s^y - \theta)\, u(c_{1s}^a) + (1-\lambda_s^y)\, u(c_{2s}) \tag{1.20}$$

subject to

$$(\lambda_s^y - \theta)\, c_{1s}^a \leq \lambda c_1^* + r l^m - q_1^m + r l_s^a(\tau_s), \tag{1.21a}$$

$$(1-\lambda_s^y)\, c_{2s} \leq R\left(1 - \lambda c_1^* - l^m - l_s^a(\tau_s)\right), \tag{1.21b}$$

$$0 \leq l_s^a(\tau_s) \leq 1 - \lambda c_1^* - l^m, \quad c_{1s}^a \geq 0,\ c_{2s} \geq 0. \tag{1.21c}$$

For a given strategy profile, $(\lambda_s^y - \theta)$ fraction of depositors and the remaining $(1 - \lambda_s^y)$ fraction of depositors would want to consume at $t = 1$ and $t = 2$, respectively. The first two constraints are the resources constraints in period 1 and 2 taking the firm's choices of supply of goods in the morning, q_1^m and early liquidation decision made in the morning, l^m.

The optimal ex-post intervention is the solution to above problem, which is summarized as

$$\left\{\tau_s\left(y, q_1^m, l^m\right)\right\}_{s\in S}, \tag{1.22}$$

together with lump-sum transfers, T_s, that satisfies the government's budget balance.

1.4.2 Equilibrium with ex-post intervention

Define that *an equilibrium with ex-post intervention* consists of the depositor's strategy profile $\widehat{y}$, the firm's supply of goods in the morning of period 1, $\widehat{q}_1^m$ liquidation decision in the morning $\widehat{l}^m$ and the policy set $\left\{\widehat{\tau}_s, \widehat{T}_s\right\}_{s\in S}$ such that (i) $\widehat{\tau}_s$ for each s is the solution to the government's problem for given $\left(\widehat{y}, \widehat{q}_1^m, \widehat{l}^m\right)$, (ii) $\left\{\widehat{\tau}_s, \widehat{T}_s\right\}_{s\in S}$ satisfies balanced budget, and (iii) the vector $\left(\widehat{y}, \widehat{q}_1^m, \widehat{l}^m\right)$ is part of a competitive equilibrium.

The following proposition states that there is an equilibrium with ex-post intervention where only impatient depositors withdraw in period 1.

Proposition 4 There exists an equilibrium with ex post intervention in which all patient depositors choose to withdraw at $t = 2$, that is, $y_i(\omega_i, s) = \omega_i + 1$ for all $s \in S$.

As is standard in this type of model, there is always an equilibrium with the ex-post intervention in which the efficient allocation obtains. The resulting consumption allocation is efficient as in the previous section. For given strategy profile in (1.13), $\widehat{q}_1^m = \theta c_1^*$ and $\widehat{l}^m = 0$, it is optimal for the government to set zero sales tax rate for any s. For a zero sales tax rate, the strategy profile in (1.13), $\widehat{q}_1^m = \theta c_1^*$ and $\widehat{l}^m = 0$ are parts of an competitive equilibrium. Notice that the firm's no arbitrage between selling goods in the morning and in the afternoon requires $P_1^m = P_{1\alpha}^a$. Together with the strategy profile (1.13), the market clearing conditions imply that $c_{1\alpha} = c_1^*$ and $c_{2\alpha} = c_2^*$. Indeed, it can be shown that $c_{1s} = c_1^*$ and $c_{2s} = c_2^*$ regardless of the sunspot state. Since $c_2^* > c_1^*$ holds, patient depositors would choose waiting to withdraw until $t = 2$. Therefore,(1.13) is part of a competitive equilibrium. Moreover, in the competitive equilibrium, it holds that $\frac{P_1^m}{P_{2\alpha}/D_{12}} = \frac{c_2^*}{c_1^*} = R^{\frac{1}{\gamma}}$. In this case, $\widehat{l}^m = 0$ is optimal and feasible choice for the firms.

1.4.3 Equilibrium and Financial fragility

In this subsection, I show that there may also exist another equilibrium where a nonzero fraction of patient depositors withdraw and purchase goods in period 1 for some realizations of the sunspot state. To study an existence of a self-fulfilling bank-run equilibrium, I focus on the following partial-run strategy profile for depositors, introduced by Ennis and Keister (2010):

$$\begin{aligned} & y_i(\omega_i, \alpha) = \omega_i + 1, \quad \forall i \text{ and} \\ & y_i(\omega_i, \beta) = \left\{ \begin{array}{c} 1 \\ \omega_i + 1 \end{array} \right\}, \quad \text{for} \quad \left\{ \begin{array}{c} i \leq \theta \\ i > \theta \end{array} \right\}. \end{aligned} \tag{1.23}$$

Impatient depositors always withdraw and purchase goods in period 1 because they value consumption only in period 1. The partial-run strategy profile in (1.23) states that all patient depositors choose to wait until period 2 to withdraw once they observe the state α. In contrast, when the state is β, patient depositors who are among the first θ fraction of investors with an opportunity to withdraw in the morning choose to withdraw at $t = 1$. The run stops once the goods market reopens in the afternoon of period 1. Also, consider the firm's choice $\widehat{q}_1^m = \theta c_1^*$ and $\widehat{l}^m = 0$.

When the government observes the sunspot state is $s = \alpha$, the government knows that no patient depositors run and no intervention is required. The same logic described in the previous subsection that the resulting consumption allocation is efficient. On the other hand, it may be efficient to intervene if the state turns out to be $s = \beta$. Once a run is underway, the excessive withdrawals in the afternoon increase the price of goods in the afternoon too much, putting a larger loss to the impatient depositors. The government may prevent the prices from inflating too much by providing an incentive for firms to supply more goods. When the government imposes taxes on future sales, firms would find it optimal to liquidate some of the long investments and increase the supply of goods in period 1. For the given strategy in (1.23), $\widehat{q}_1^m = \theta c_1^*$ and $\widehat{l}^m = 0$, the optimal tax rate is given as

$$\widehat{\tau}_\beta = 1 - \left(\frac{R}{r}\right)^{\frac{1}{\gamma}-1} \tag{1.24}$$

where the optimal tax policy is derived in Appendix C.

Under the tax policy $\widehat{\tau}_\beta$, the consumption allocation in state β is summarized as

$$c_{1\beta}^a = \frac{\left(\lambda - \theta + r(1-\lambda)R^{\frac{1-\gamma}{\gamma}}\right) c_1^*}{(1-\theta)\left(\lambda + (1-\lambda)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right)}, \quad c_{2\beta} = \left(\frac{R}{r}\right)^{\frac{1}{\gamma}} c_{1\beta}^a, \tag{1.25}$$

and the prices satisfy

$$\frac{P_{1\beta}^a}{P_{2\beta}/I_2} = \frac{c_{2\beta}}{c_{1\beta}^a} = \left(\frac{R}{r}\right)^{\frac{1}{\gamma}}. \tag{1.26}$$

Notice that in this equilibrium, firms choose to liquidate some of their long investment because the after-tax market price ratio they face for the given tax rate in (1.24) is

$$\frac{P_{1\beta}^a}{(1-\tau_\beta)\,P_{2\beta}/I_2} = \frac{R}{r}. \tag{1.27}$$

The patient depositors who have opportunity to withdraw in the morning of period 1 in state β would choose to do so if $\frac{D_1}{P_1^m} > \frac{D_2}{P_{2\beta}}$. That is, the strategy profile in (1.23) is part of a competitive equilibrium if and only if $c_1 > c_{2\beta}$. We know that $c_1 = c_1^*$ from $\widehat{q}_1^m = \theta c_1^*$ and the market clearing condition in (1.10a). It is straightforward then the condition $c_1 > c_{2\beta}$ is equivalent to the following:

$$\left(\frac{R}{r}\right)^{\frac{1}{\gamma}} \frac{\lambda - \theta + (1-\lambda)R^{\frac{1-\gamma}{\gamma}}}{(1-\theta)\left(\lambda + (1-\lambda)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right)} < 1. \tag{1.28}$$

The following proposition summarizes the result.

Proposition 5 For any parameter values R, r, λ, θ, and γ satisfying (1.28), there exists another equilibrium with the ex-post intervention in which depositors with $i \leq \lambda$ run on their banks in state β.

When depositors follow the withdrawal strategy (1.23), the impatient depositors must bear all losses from higher inflation caused by the run without intervention. This is because the excessive early withdrawals raise the amount of money in circulation which, in turn, raises prices. In this case, the government can lead firms to liquidate long investment and supply more goods in period 1 from imposing a tax on the firm's sales in the future. The price adjustment under intervention allows redistribution of consumption between the remaining impatient and patient depositors. Despite providing risk sharing between them, lower consumption for the remaining patient depositors creates an incentive for them to join the run. In such way, the government's attempt to promote ex post efficiency can sometimes undermine the ex ante incentive of depositors and end up destabilizing the banking system.

1.4.4 Numerical example

The financial system is fragile if there is an equilibrium in which depositors follow the partial run strategy profile in (1.23). Such an equilibrium indeed exists under certain parameterizations with intervention. An economy is characterized by the parameters $(R, r, \lambda, \theta, \gamma)$. The black-shaded area in Figure 1.4 illustrates the set of economies that are fragile with the regulations as the parameters R and θ are varied. That is, an economy inside the black-shaded area satisfies the conditions (1.28). Other parameters are given as $(r, \lambda, \gamma) = (0.95,\ \ 0.5,\ \ 8)$.

For a given value of R, the economy tends to be fragile when θ is larger. When the government intervenes more slowly to liquidate long investments, more patient depositors have already withdrawn funds from the banks and converted them into real goods, meaning fewer resources are left in the goods market for the remaining patient depositors. Thereby, the slower reaction by the government, the larger the incentive to join the run. For a fixed θ, as R increases the banking system becomes more prone to

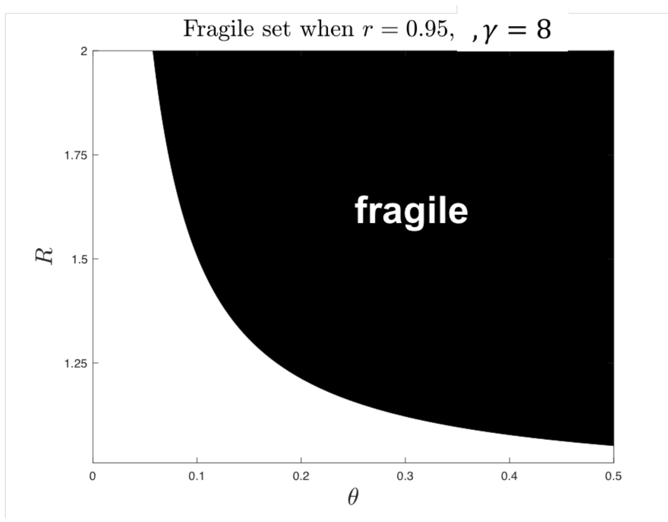

Figure 1.4: The fragile region

runs. A higher rate of return for the long investment allows the planner to allocate more consumption for both at $t = 1$ and $t = 2$, leading the optimal regulation to raise the liquidity requirement ex ante. Withdrawing early becomes more attractive to a patient depositor due to the higher returns from the morning market in period 1. When γ is higher, meaning that the depositors are more risk averse, the government will raise the liquidity requirement ex ante to provide better risk sharing. That is, the banking system will offer relatively higher interest rate for the impatient depositors, yielding more incentive for the patient depositors to join the run.

1.5 Conclusion

It is generally understood from the existing literature that a self-fulfilling bank run cannot occur in equilibrium with nominal banking contracts and a lender of last resort. This idea is based on the flexible price system together with a central bank that varies the money supply, allowing the fixed nominal contracts to generate a state-contingent consumption profile. This fact removes the possibility of a self-fulfilling bank run of the type usually studied in the Diamond-Dybvig framework.

In this paper, I argue that the existing literature did not consider two important frictions - sequential service of depositors and the possibility of ex post intervention by the government - seriously. I put self-fulfilling bank runs back in this literature by considering these frictions. Sequential service of depositors implies that some depositors are able to withdraw their money from their bank and purchase goods before the market prices fully adjust to reflect aggregate withdrawal demand. Once a run is underway, the government may be tempted to intervene to limit the inflation caused by a large withdrawal demand. This intervention may change equilibrium consumption allocations in equilibrium in a way that justifies depositors' initial decision to run.

I assume that the government can announce taxes on the firms' sales in period 2 to provide firms an incentive to liquidate some of their long investments and supply more goods in period 1. This increased supply prevents the market price from increasing too high; in fact, the ex-post optimal tax can implement efficient consumption allocation. However, this intervention lowers the resources available to the remaining patient depositors, making them purchase goods at a higher price. The anticipation to this outcome creates an incentive to join the run for those depositors who have an opportunity before the government acts. In this way, the government's attempt to promote ex post efficient consumption allocation can undermine the ex ante incentives of depositors and end up destabilizing the banking system.

1.6 Appendix A

The Kuhn-Tucker conditions for the firm's problem in (1.5a)-(1.5e):

$$P_{1s}^a - \mu_{1s}^a - \mu_{2s}^a + P_{1s}^a \mu_{4s}^a \leq 0, \tag{1.29a}$$

$$\frac{P_{2s}}{D_{12}} - \mu_{2s}^a + \frac{P_{2s}}{D_{12}} \mu_{4s}^a \leq 0, \tag{1.29b}$$

$$r\mu_{1s}^a + (r - R)\mu_{2s}^a - \mu_{3s}^a \leq 0, \tag{1.29c}$$

$$\mu_{1s}^a \left[1 - x + rl^m - q_1^m + rl_s^a - q_{1s}^a\right] = 0, \tag{1.29d}$$

$$\mu_{2s}^a \left[R\left(x - l^m - l_s^a\right) + 1 - x + rl^m - q_1^m + rl_s^a - q_{1s}^a - q_{2s}\right] = 0, \tag{1.29e}$$

$$\mu_{3s}^a \left[x - l^m - l_s^a\right] = 0, \tag{1.29f}$$

$$\mu_{4s}^a \left[P_1^m q_1^m + P_{1s}^a q_{1s}^a + \frac{P_{2s} q_{2s}}{D_{12}} - K_1 - \frac{K_2}{D_{12}}\right] = 0, \tag{1.29g}$$

where $\mu_{1s}^a, \mu_{2s}^a, \mu_{3s}^a, \mu_{4s}^a$ are nonnegative Lagrange multipliers associated with constraints (1.5b), (1.5c), (1.5d), (1.5e), respectively. The firm's choice also satisfies the constraints in (1.5b)-(1.5e).

The Kuhn-Tucker conditions for the firm's problem in (1.7a)-(1.7c):

$$P_1^m - \mu_{1s}^a - \mu_{2s}^a + P_1^m \mu_{4s}^a - \mu_1^m \leq 0, \tag{1.30a}$$

$$r\mu_{1s}^a + (r - R)\mu_{2s}^a - \mu_{3s}^a + r\mu_1^m - \mu_2^m \leq 0, \tag{1.30b}$$

$$\mu_1^m \left[1 - x + rl^m - q_1^m\right] = 0, \tag{1.30c}$$

$$\mu_2^m \left[x - l^m\right] = 0, \tag{1.30d}$$

where μ_1^m, μ_2^m are nonnegative Lagrange multipliers associated with constraints (1.7b) and (1.7c), respectively. The firm's choice also satisfies the constraints in (1.7b)-(1.7c).

The Kuhn-Tucker condition for the problem (1.9):

$$-\mu_{1\alpha}^a + (R - 1)\,\mu_{2\alpha}^a + \mu_{3\alpha}^a - \mu_1^m - \mu_2^m - \mu^x \leq 0, \tag{1.31a}$$

$$\mu^x \left[1 - x\right] = 0, \tag{1.31b}$$

together with $0 \leq x \leq 1$. μ^x is the nonnegative Lagrange multiplier associated with the constraint $x \leq 1$.

Proof of Lemma 1 Suppose that $\max\{P_1^m, P_{1\alpha}^a\} < \frac{P_{2\alpha}}{D_{12}}$ holds. Then, we know that equations (1.29a) and (1.30a) both hold with strict inequality. Therefore, the firm's best response is to choose $q_1^m = 0$ and $q_{1\alpha}^a = 0$. The market clearing conditions in (1.10a)-(1.10b) implies $P_1^m = \infty$ and $P_{1\alpha}^a = \infty$. Since for any positive market price, we have $\mu_{2s}^a > 0$ from (1.29b) resulting that the constraint (1.5c) binds. With $q_1^m = 0$ and $q_{1\alpha}^a = 0$, we get $q_{2\alpha} > 0$. Thus, $P_{2\alpha}$ is finite from (1.10c), which contradicts to $\infty < \frac{P_{2\alpha}}{D_{12}}$. When $P_{1\beta}^a < \frac{P_{2\beta}}{D_{12}}$, equations (1.29a) holds with strict inequality using (1.29b). Therefore, the firm's choice is to set $q_{1\beta}^a = 0$, implying $P_{1\beta}^a = \infty$ from (1.10b). We get $q_{2\beta} > 0$. Thus, $P_{2\beta}$ is finite from (1.10c), which contradicts to $\infty < \frac{P_{2\beta}}{D_{12}}$.

Suppose now that $\frac{P_{2\alpha}}{D_{12}} < \frac{r}{R}\min\{P_1^m, P_{1\alpha}^a\}$ holds. I show that $q_{2\alpha} = 0$. Assume to the contrary that $q_{2\alpha} > 0$. Then (1.29b) must hold with equality. Combining (1.29a) and (1.29b) with $\frac{P_{2\alpha}}{D_{12}} < \frac{r}{R}P_{1\alpha}^a$, we have $\mu_{3\alpha}^a > 0$. Therefore, we have $l_\alpha^a = x - l^m$. $q_{2\alpha} = 1 - x + rx - q_1^m - q_{1\alpha}^a > 0$ implies nonbinding (1.5b). Using (1.29a) and $\frac{P_{2\alpha}}{D_{12}} < \frac{r}{R}P_{1\alpha}^a$, we get $\frac{P_{2s}}{D_{12}} - \mu_{2s}^a + \frac{P_{2s}}{D_{12}}\mu_{4s}^a > 0$, which is contradiction. The same logic applies to the case for $\frac{P_{2\beta}}{D_{12}} < \frac{r}{R}P_{1\beta}^a$.

Proof of Lemma 2 I first show that $\bar{P}_1^m = \bar{P}_{1\alpha}^a$. If $\bar{P}_1^m > \bar{P}_{1\alpha}^a$, (1.29a) and (1.30a) imply $q_{1\alpha}^a = 0$. The market clearing condition in (1.10b) implies $\bar{P}_{1\alpha}^a = \infty$, which is contradiction. If $\bar{P}_1^m < \bar{P}_{1\alpha}^a$, (1.29a) and (1.30a) imply $q_1^m = 0$. The market clearing condition in (1.10a) implies $\bar{P}_1^m = \infty$, which is contradiction.

In the goods market equilibrium, it must be true that $\bar{q}_1^m > 0$, $\bar{q}_{1\alpha}^a > 0$ $\bar{q}_{2\alpha} > 0$ implying (1.30a), (1.29a), (1.29b) holding with equality at $s = \alpha$. Combining (1.30a) and (1.29a) yields $\mu_1^m = 0$.

Suppose that $\bar{P}_1^m = \bar{P}_{1\alpha}^a < R\frac{\bar{P}_{2\alpha}}{D_{12}}$. Under this condition, we have $\bar{l}^m = 0$ and $\bar{l}_\alpha^a = 0$, implying $\mu_{3\alpha}^a = 0$ and $\mu_2^m = 0$. Hence, (1.31a) is reduced to $-\mu_{1\alpha}^a + (R-1)\mu_{2\alpha}^a - \mu^x \leq 0$. In addition, $\bar{P}_1^m = \bar{P}_{1\alpha}^a < R\frac{\bar{P}_{2\alpha}}{D_{12}}$ implies $\mu_{1\alpha}^a + \mu_{2\alpha}^a < R\mu_{2\alpha}^a$ from (1.29a) and (1.29b). Any choice such that $x < 1$, the condition in (1.31b) requires $\mu^x = 0$ and thus, it violates (1.31a). Therefore, $\bar{P}_1^m = \bar{P}_{1\alpha}^a < R\frac{\bar{P}_{2\alpha}}{D_{12}}$ implies $x = 1$ and $\mu^x > 0$. However, $x = 1$ together with the fact $\bar{l}^m = 0$ implies $q_1^m = 0$, which is contradiction.

Suppose that $\bar{P}_1^m = \bar{P}_{1\alpha}^a > R\frac{\bar{P}_{2\alpha}}{D_{12}}$. This implies from (1.29a) and (1.29b) that $-\mu_{1\alpha}^a + (R-1)\,\mu_{2\alpha}^a < 0$. This fact together with $\mu^x \geq 0$ and (1.31a) requires $\mu_{3\alpha}^a > 0$, and so $x = l^m + l_\alpha^a$ by (1.29f). Binding constraint (1.5c) implies $q_1^m + q_{1\alpha}^a + q_{2\alpha} = 1 - x + rx$. $\bar{P}_1^m = \bar{P}_{1\alpha}^a > R\frac{\bar{P}_{2\alpha}}{D_{12}}$ implies that the firm's profit is maximized by $x = 0$ and $q_{2\alpha} = 0$, violates (1.10c).

Proof of Proposition 1 First of all, $\forall i$, $y_i\,(1, s) = 1$ is a dominant strategy for all s and for any given market prices. Second, for $i > \theta$, depositor i with $\omega_i = 1$ chooses to withdraw if $c_{1s}^a > c_{2s}$ where $c_{1s}^a = \frac{D_1}{P_{1s}^a}$ and $c_{2s} = \frac{D_2}{P_{2s}}$. Lemma 1 implies that it cannot be the case in equilibrium.

Third, for $i \leq \theta$, depositor i with $\omega_i = 1$ chooses to withdraw if $c_1^m > c_{2s}$ where $c_1^m = \frac{D_1}{P_1^m}$. Lemma 1 implies that the condition fails in state α. Finally, if there were runs in state β by the patient depositors, it must be the case that $l_\beta^a > 0$. This requires from firm's problem that $P_{1\beta}^a = \frac{R}{r}\frac{P_{2\beta}}{D_{12}}$. It implies that $c_1 < c_{2\beta}$ for any given λ_β^y. Therefore, no run is possible.

Proof of Proposition 2 The equilibrium consists of the withdrawal strategy profile (1.13), investment and liquidation decisions

$$x = 1 - \lambda,\ l_s^a = l^m = 0,$$

consumption allocation

$$c_1^m = c_{1s}^a = 1,\ c_{2s} = R$$

for both $s \in S$. The consumption allocation is different to c_1^* and c_2^*. The prices in this equilibrium are

$$P_1^m = P_{1s}^a,\quad \frac{P_1^m D_{12}}{P_{2s}} = R.$$

Proof of Proposition 3

First of all, each firm's portfolio in period 0 with minimum requirement $(1-x^*)$ is now by maximizing $V_1^m(x)$ such that $0 \leq x \leq x^*$. The firm would choose $x = x^*$ for $P_{1\alpha}^a \leq R\frac{P_{2\alpha}}{I_2}$.

Fix $\lambda_s^y = \lambda$. The market clearing conditions in (1.10a) - (1.10b) and $P_{1\alpha}^a \leq R\frac{P_{2\alpha}}{I_2}$ imply that consumption allocations are $c_1^m = c_{1s}^a = c_1^*$, $c_{2s} = c_2^*$ for $s \in S$. This implies that the market prices will satisfy that $P_1^m = P_{1s}^a$ and $P_{1s}^a = R^{\frac{1}{\gamma}}\frac{P_{2s}}{I_2} < R\frac{P_{2s}}{I_2}$. The allocation in the goods market equilibrium for $\lambda_s^y = \lambda$ is efficient.

Next, I show that $\lambda_s^y = \lambda$ holds in the equilibrium of withdrawal game. Suppose that $\lambda_s^y > \lambda$, that is, positive mass of patient depositors choose to withdraw in period 1. The market clearing conditions in (1.10a)- (1.10c) and no-arbitrage condition in the morning and afternoon markets imply that

$$\frac{P_1^m I_2}{RP_{2s}} = \frac{1-\lambda c_1^*}{\lambda c_1^*}\frac{\lambda_s^y}{1-\lambda_s^y} = R^{\frac{1-\gamma}{\gamma}}\frac{1-\lambda}{\lambda}\frac{\lambda_s^y}{1-\lambda_s^y} > 1.$$

This implies that $c_{2s} > R^{-\frac{1}{\gamma}}c_{2s} > c_1^m$. Therefore, all patient depositors' must choose to wait to withdraw in period 2, which violates $\lambda_s^y > \lambda$. Therefore, no-run occurs in equilibrium.

1.7 Appendix B

I can show that the market equilibrium coincides with an equilibrium when the central bank chooses nominal interest rates optimally taking the market equilibrium as given. Consider that the central bank sets inter-period nominal interest rates (I_1, I_2) to maximize welfare taking the market prices and the market's optimal behaviors as given. The central bank considers the state $s = \beta$ as zero probability event.

$$\max_{\{I_1,I_2\}} \theta u\left(\frac{I_1}{\bar{P}_1^m}\right) + (\lambda-\theta)u\left(\frac{I_1}{\bar{P}_{1\alpha}^a}\right) + (1-\lambda)u\left(\frac{I_1 I_2}{\bar{P}_{2\alpha}}\right) \tag{1.32}$$

subject to

$$\lambda \frac{I_1}{\bar{P}_1^m} + (1-\lambda)\frac{I_1 I_2}{R\bar{P}_{2\alpha}} = 1, \tag{1.33a}$$

$$\frac{\bar{P}_1^m}{\bar{P}_{2\alpha}/\, I_2} = R. \tag{1.33b}$$

In the market equilibrium, all depositors choose the profile of withdrawal strategies $\bar{y}$, λ fraction of all depositors withdraw and consume at $t = 1$ and the remaining depositors withdraw and consume at $t = 2$. Equation (1.33a) is the resource constraint considering that $1 - \bar{x} = \lambda = \lambda \frac{I_1}{\bar{P}_1^m}$. Any combination of (I_1, I_2) satisfies the constraint (1.33b) and yields the same level of welfare. For any choice of nominal interest rates set by the central bank, the market prices adjust so that the real interest rate between period 1 and period 2 is equalized to the marginal rate of transformation, R. This is common feature in a monetary economy, money is neutral in the sense that the change in money stock from a change in nominal interest rate has no impact on the real variables. For any choice of the nominal interest rate, the price in the goods market will adjust so that the condition (1.33b) is satisfied at the equilibrium.

1.8 Appendix C

Constrained ex-post efficient allocation

Suppose that the planner chooses ex post consumption allocations in the afternoon of period 1 and in period 2 after θ withdrawals have been made in the morning.

$$\max_{c_{1\beta}^a, c_{2\beta}, l_\beta} \quad (1-\theta)\left[\lambda u\left(c_{1\beta}\right) + (1-\lambda) u\left(c_{2\beta}\right)\right] \tag{1.34a}$$

$$\text{subject to} \tag{1.34b}$$

$$\lambda(1-\theta) c_{1\beta}^a \leq (\lambda - \theta)\, c_1^* + r l_\beta^a \tag{1.34c}$$

$$(1-\lambda)(1-\theta) c_{2\beta} \leq R\left(1 - \lambda c_1^* - l_\beta^a\right) \tag{1.34d}$$

$$0 \leq l_\beta^a \leq 1 - \lambda c_1^*, \quad c_{1\beta}^a \geq 0,\ c_{2\beta} \geq 0. \tag{1.34e}$$

The first two constraints are feasibility conditions of goods available for the remaining impatient depositors at $t = 1$ and the remaining patient depositors at $t = 2$. Let $\left\{c_{1\beta}^{a**}, c_{2\beta}^{**}, l_{\beta}^{a**}\right\}$ denote the solution to the above problem. The first order conditions and binding feasibility constraints yields the following:

$$l_{\beta}^{a**} \geq 0, \tag{1.35a}$$

$$\frac{u'\left(c_{1\beta}^{a**}\right)}{u'\left(c_{2\beta}^{**}\right)} = \frac{R}{r} \tag{1.35b}$$

$$(1-\theta)\left(\lambda c_{1\beta}^{a**} + \frac{r}{R}(1-\lambda)c_{2\beta}^{**}\right) = (\lambda-\theta)c_1^* + r(1-\lambda c_1^*). \tag{1.35c}$$

With the CRRA utility function, the closed form solution can be derived. For $\frac{\lambda}{\lambda-\theta} > \left(\frac{1}{r}\right)^{\frac{1}{\gamma}}$, the planner optimally chooses to liquidate parts of long investment and the solutions are:

$$l_{\beta}^{a**} = \left(\lambda - (\lambda-\theta)\, r^{-\frac{1}{\gamma}}\right) \frac{(1-\lambda)R^{\frac{1-\gamma}{\gamma}} c_1^*}{\lambda + (1-\lambda)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}}, \tag{1.36a}$$

$$c_{1\beta}^{a**} = \frac{\left(\lambda - \theta + r(1-\lambda)R^{\frac{1-\gamma}{\gamma}}\right) c_1^*}{(1-\theta)\left(\lambda + (1-\lambda)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right)}, \tag{1.36b}$$

$$c_{2\beta}^{**} = \left(\frac{R}{r}\right)^{\frac{1}{\gamma}} c_{1\beta}^*. \tag{1.36c}$$

Enforcing liquidation in the afternoon so that l_{β}^{**} allows the market to provide insurance between the two periods.

Best responses under the tax policy

Each firm's revised maximization problem under the regulations after the state $s = \beta$ is known can be written as:

$$V_{\beta}^F(\tau_{\beta}, T_{\beta}) = \max_{\left\{q_{1\beta}^a, q_{2\beta}, l_{\beta}\right\}} \quad P_{1\beta}^a q_{1\beta}^a + \frac{(1-\tau_{\beta})\, P_{2\beta} q_{2\beta}}{I_2} - \left(K_1 + \frac{K_2}{I_2}\right) + \frac{T_{\beta}}{I_2} \tag{1.37}$$

subject to

$$q^a_{1\beta} \leq (\lambda - \theta)\, c^*_1 + r l_\beta, \tag{1.38a}$$

$$q_{2\beta} \leq R\,(1 - \lambda c^*_1 - l_\beta) + (\lambda - \theta)\, c^*_1 + r l_\beta - q^a_{1\beta}, \tag{1.38b}$$

$$0 \leq l^a_\beta \leq 1 - \lambda c^*_1,\ q^a_{1\beta} \geq 0,\ q_{2\beta} \geq 0, \tag{1.38c}$$

$$P^m_1 q^m_1 + P^a_{1\beta} q^a_{1\beta} + \frac{(1-\tau_\beta)\, P_{2\beta} q_{2\beta}}{I_2} + \frac{T_\beta}{I_2} \geq K_1 + \frac{K_2}{I_2}. \tag{1.38d}$$

The firm's choice at $s = \beta$ is summarized as the following

$$\left\{ \begin{array}{l} l_\beta > 0 \\ q^a_{1\beta} = (\lambda - \theta) c^*_1 + r l^a_\beta \\ q_{2\beta} = R\left(1 - \lambda c^*_1 - l^a_\beta\right) \end{array} \right\} \quad \text{for} \quad \frac{(1-\tau_\beta)\, P^a_{1\beta} I_2}{P_{2\beta}} = \frac{R}{r}.$$

Optimal tax policy

For a given withdrawal strategy profile (1.23), and pre-determined choices of firms, $\widehat{q}^m_1 = \theta c^*_1$ and $\widehat{l}^m = 0$, the government find optimal to choose τ_β it satisfies $\frac{(1-\tau_\beta) P^a_{1\beta} I_2}{P_{2\beta}} = \frac{R}{r} = \frac{u'\left(c^{a**}_{1\beta}\right)}{u'\left(c^{**}_{2\beta}\right)}$ to implement the ex-post efficient allocation. This implies that

$$(1-\tau_\beta)\, \frac{c_{2\beta}}{c^a_{1\beta}} = \left(\frac{c_{2\beta}}{c^a_{1\beta}}\right)^{\frac{1}{\gamma}} \quad \text{or,} \quad \tau_\beta = 1 - \left(\frac{R}{r}\right)^{\frac{1}{\gamma}-1} \ \in\ (0, 1).$$

Equilibrium under tax policy

The withdrawal strategy profile (1.23), and pre-determined choices of firms, $\widehat{q}^m_1 = \theta c^*_1$ and $\widehat{l}^m = 0$ are best responses to the the given policy $\{\widehat{\tau}_\alpha,\ \widehat{\tau}_\beta\} = \left\{\ 0,\ 1 - \left(\frac{R}{r}\right)^{\frac{1}{\gamma}-1}\ \right\}$.

First of all, the withdrawal strategy profile (1.23) is a best response to the given policy. With $\widehat{q}^m_1 = \theta c^*_1$ and $\widehat{l}^m = 0$, the market prices satisfy that $P^m_1 = P^a_{1\alpha} = I_1 c^*_1$ and $P_{2\alpha} = I_1 I_2 c^*_2$. Consumption allocation in state α is efficient, choosing to withdraw in period 2 is a best response for patient depositors. The tax policy in state β allows the consumption allocation to satisfy that $c_{2\beta} = \left(\frac{R}{r}\right)^{\frac{1}{\gamma}} c^a_{1\beta}$, therefore, patient depositors $i \geq \theta$ chooses to withdraw in period 2. Since the condition (1.28) holds, patient depositors $i < \theta$ would choose to withdraw in period 1.

Notice that in period 1, the firm chooses $\widehat{l}^m = 0$ because the market prices satisfy that $\frac{I_2 P_1^m}{P_{2\alpha}} = R^{\frac{1}{\gamma}} < \frac{R}{r}$. In addition, $\frac{I_2 P_1^m}{P_{2\alpha}} = R^{\frac{1}{\gamma}} > 1$ and $P_1^m = P_{1\alpha}^a$ imply that any choice of $\{q_1^m, q_{1\alpha}^a\}$ that satisfies $q_1^m + q_{1\alpha}^a = \lambda c_1^*$ is optimal. Therefore, $\widehat{q}_1^m = \theta c_1^*$ and $\widehat{l}^m = 0$ are also best responses to the tax policy.

Together with the equilibrium $\widehat{y}$ as (1.23), $\widehat{q}_1^m = \theta c_1^*$ and $\widehat{l}^m = 0$ and the tax policy $\widehat{\tau}_\beta = 1 - \left(\frac{R}{r}\right)^{\frac{1}{\gamma}-1}$, the following lists the competitive equilibrium with the tax policy:

$$c_1^m = c_{1\alpha}^a = c_1^*, \;\; c_{2\alpha} = c_2^*$$

and

$$c_{1\beta}^a = \frac{\left(\lambda - \theta + r(1-\lambda) R^{\frac{1-\gamma}{\gamma}}\right) c_1^*}{(1-\theta)\left(\lambda + (1-\lambda)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right)}, \quad c_{2\beta} = \left(\frac{R}{r}\right)^{\frac{1}{\gamma}} c_{1\beta}^{PLR}$$

supply of goods

$$q_{1\alpha}^a = (\lambda-\theta)c_1^*, \;\; q_{1\beta}^a = \lambda(1-\theta)c_{1\beta}^a, \;\; q_{2\alpha} = (1-\lambda)c_2^*, \;\; q_{2\beta}^a = (1-\lambda)(1-\theta)c_{2\beta},$$

and the market prices

$$P_1^m = P_{1\alpha}^a, \quad \frac{P_1^m D_{12}}{P_{2\alpha}} = R^{\frac{1}{\gamma}}, \quad \text{and} \quad \frac{P_{1\beta}^a D_{12}}{P_{2\beta}} = \left(\frac{R}{r}\right)^{\frac{1}{\gamma}},$$

and

$$P_{1\beta}^a = \frac{1}{\Gamma} P_1^m, \;\; P_{2\beta} = \frac{1}{\Gamma} r^{\frac{1}{\gamma}} P_{2\alpha}$$

where $\Gamma = \frac{\left(\lambda-\theta+r(1-\lambda)R^{\frac{1-\gamma}{\gamma}}\right)}{(1-\theta)\left(\lambda+(1-\lambda)\left(\frac{R}{r}\right)^{\frac{1-\gamma}{\gamma}}\right)}$.

Chapter 2

Transparency vs. Ambiguity in Bailout Policy

2.1 Introduction

Policy makers must often choose how much information to provide about the course of action they will follow in the future or about the way policy would respond to certain contingencies that may arise. There are competing views on what general principles should guide such decisions. One view is that policy makers should aim to be as transparent as possible, that is, they should provide as much detail about the state-contingent policy rule they will follow as is feasible given the circumstances. Others argue, however, that policy makers can sometimes generate better outcomes by being deliberately ambiguous about their future plans. We study whether such *constructive ambiguity* can be a useful tool in the context of bailout policy.

We construct a model in which there is aggregate uncertainty about the value of banks' assets and a policy maker who may choose to bail out banks by using public funds to partially cover their losses. We say that bailout policy is *transparent* if it announces in advance which banks (if any) will be bailed out following a negative shock. We say that policy is *ambiguous* if banks are uncertain about whether or not they will be bailed out and do not learn the policy maker's decision until any bailout payments are actually made. We ask under what conditions the optimal bailout policy is transparent and under what conditions it involves ambiguity.

A fully transparent policy regime offers clear benefits. At the most basic level, transparency reduces agents' uncertainty about the policy maker's future actions and thereby minimizes economic volatility due to policy surprises. In addition, transparency may give policy makers stronger influence over agents' incentives and expectations. For example, a policy maker may aim to encourage financial institutions to act more conservatively by conditioning the assistance offered to an institution in the event of a crisis on the composition of its portfolio or its level of short-term debt. Such a policy is more likely to be effective if institutions know that the policy maker will follow this rule than if they are uncertain whether the criteria will actually be applied.

In spite of these benefits of transparency, some people argue the policy makers can often generate better outcomes by instead withholding information about their future plans. Observers often attribute a policy of deliberate ambiguity, for example, to former Federal Reserve Chairman Alan Greenspan.[1] Two benefits are often claimed for a policy of constructive ambiguity. First, being ambiguous about the policy rule preserves flexibility for dealing with unforeseen contingencies. In other words, a pre-announced rule may end up tying policy makers' hands in unexpected ways. Second, ambiguity may mitigate moral hazard concerns by leaving agents uncertain about precisely what behavior will be rewarded or punished. For example, if a precise set of criteria that qualify an institution for assistance in a crisis is announced, investors may find ways to "game" the system and to meet these criteria without actually becoming less risky.[2]

We show how this second benefit of policy ambiguity can be captured within a standard economic model of financial crises. Our model is based on the classic paper of Diamond and Dybvig (1983), but we do not focus on bank runs. Instead, a financial crisis in our model is an event in which banks suffer a real loss on their assets. We extended the model to include a policy maker with the ability to conduct bailouts as in Keister (2016). Bailouts in this framework offer a risk-sharing benefit: they allow the policy maker to transfer resources to banks in states where private consumption is low. However, the anticipation of being bailed out distorts banks' incentives. In particular, a bank that anticipates being bailed out will have less incentive to provision for bad outcomes and, therefore, will choose to make larger payouts to investors before the state is revealed.[3] The policy maker may be able to mitigate this moral hazard problem through regulation, but we assume its regulatory powers are limited. As a result, in

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[1] Greenspan famously remarked during testimony before the Senate Banking Committee in 1987, "If I seem unduly clear to you, you must have misunderstood what I said."

[2] Put differently, announcing precise rules can open the door to regulatory arbitrage.

[3] The distortionary effects of bailouts have been studied in a wide variety of settings. See, for example, Green (2010), Farhi and Tirole (2012), Bianchi (2016), and Chari and Kehoe (2016).

choosing a bailout policy, the policy maker will aim to balance the risk-sharing benefit against the cost of distorted incentives.

We first study transparent bailout policies in which the policy maker announces in advance that it either will or will not provide bailouts if the bad state occurs. We show that the optimal policy in this class involves providing bailouts if, and only if, the loss suffered by banks in the bad state is sufficiently large. In other words, when banks face only moderate losses, the policy maker should commit not to intervene in order to avoid distorting incentives. If banks suffer a larger loss, however, the risk-sharing benefit of providing bailouts outweighs the cost that comes from distorted incentives and bailouts should be provided.

We then ask whether the policy maker can improve welfare by introducing ambiguity into the bailout policy. We consider two distinct forms of ambiguity in our model. In the first form, the policy maker is restricted to treat all banks equally: it must either bail out all banks or bail out none. However, instead of announcing in advance which of these actions will be taken, the policy maker can act randomly, assigning positive probability to each option. We show that the optimal policy within this class always provides bailouts with positive probability, even when the losses suffered by banks are relatively small. This intuition for this result is straightforward. If the policy maker never provides bailouts, banks' incentives are not distorted. Providing bailouts with a small but positive probability introduces an incentive distortion, but the initial effect of this distortion on welfare is second order. The risk-sharing benefit of providing bailouts, in contrast, is always of first order, which implies that increasing the probability of a bailout above zero necessarily raises welfare. In some cases, the optimal policy will involve bailing out all banks with certainty, while in other cases the optimal policy will involve bailing out all banks with some probability strictly between zero and one, a result that corresponds to constructive ambiguity.

The second form of ambiguity is we study is uncertainty about *who* will be bailed out. In this class of policies, the policy maker announces what fraction of banks will be bailed out in the event of a crisis, but does not specify which banks will be included in this group. We show that policy ambiguity is more attractive within this class of policies. In particular, being ambiguous about who will be bailed out allows the policy maker to use ambiguity to mitigate the incentive distortion without introducing uncertainty about aggregate economic outcomes.

Constructive ambiguity in our model involves playing a mixed strategy, that is, a commitment to random behavior by the policy maker. Such commitments may seem, at first glance, to be difficult to achieve in reality. There are, however, ways in which policy ambiguity arises naturally in practice. Suppose, for example, that the decision of whether or not to bail out some bank(s) will depend on the outcome of inter-agency negotiations or on other factors that require effort to evaluate. A transparent policy would then require negotiating and evaluating these issues in advance, while an ambiguous policy would correspond to waiting until the need arises to make these determinations. Seen this way, ambiguity may be more the norm than an exception in policy decisions. Our analysis identifies situations in which policy makers should or should not undertake the effort required to produce a transparent policy.

Our paper is closely related to two existing works that study the role of policy uncertainty in potentially mitigating moral hazard. In an early contribution, Freixas (1999) studies optimal lender-of-last-resort policy in a reduced-form, microeconomic model of banking. He shows that, under certain conditions, the policy maker will choose to follow a mixed strategy, leaving the banker uncertain about whether it will have access to the lender of last resort. Our results share with Freixas (1999) the feature that ambiguity in the form of a mixed strategy can be a useful tool for influencing banks' incentives and mitigating moral hazard. More recently, Nosal and Ordoez (2016) argue that uncertainty on the part of policy makers can substitute for commitment. In particular, they study an environment in which policy makers are slow to learn the nature of an aggregate shock. This lack of information causes the policy maker to be slow make decisions on bailouts, which in turn gives banks an incentive to avoid being the first to fail. In their setting, the policy maker can commit to being ambiguous by avoiding "ex-ante the implementation of technologies to learn rapidly or to take fast decisions when distress happens." (p. 126) Similarly, ambiguity in our model can be thought of as representing a conscious decision by policy makers to avoid pre-judging how particular situations will be handled.

The remainder of the paper is organized as follows. In the next section, we describe the physical environment of the model and the timing of events. In Section 3, we lay out the game played by banks and the policy maker and some general properties of equilibrium. Section 4 presents a brief analysis of optimal policy when the policy maker can perfectly control banks' actions through regulation. The heart of our analysis is in Section 5, where the study optimal policy with imperfect regulation and illustrate the role of constructive ambiguity. Finally, we offer some concluding remarks in Section 6.

2.2 The Model

Our model builds on that in Keister (2016), which is a version of the Diamond and Dybvig (1983) model augmented to include fiscal policy and a public good. In addition to simplifying the model in some dimensions, we introduce risk in the investment technology, which generates a potential incentive for bailouts even in the absence of bank runs. We also expand the set of bailout policies to include a form of constructive ambiguity.

2.2.1 The environment

There are three time periods, labeled $t = 0,\ 1,\ 2$, and a single consumption good in each period. There is a continuum of locations with measure 1. In each location, there is a continuum of investors, indexed by $i \in [0,1]$. Each investor is endowed with one unit of the good at $t = 0$ and has preferences given by

$$U\left(c_1, c_2, d; \alpha_i\right) = u\left(c_1 + \mathbb{I}_{(\alpha_i=2)} c_2\right) + v\left(d\right),$$

where c_t is consumption of the private good in period t, $\mathbb{I}$ is the indicator function, and d is the level of public good. The functions u and v are both strictly increasing, strictly concave, and satisfy the usual Inada conditions. The preference type of investor i, denoted α_i, is a binomial random variable with support $A = \{1, 2\}$. If $\alpha_i = 1$, investor i is *impatient* and only cares about consumption at $t = 1$, while if $\alpha_i = 2$ she is *patient* and can consume at either $t = 1$ or $t = 2$. An investor's type α_i is realized at the beginning of 1. A fraction λ of investors in each location will be impatient, and λ also represents the individual probability of being impatient for each investor at $t = 0$.

There is a single, constant-returns-to-scale technology for transforming endowments into private consumption in the later periods. A unit of the good invested in period 0 yields one unit of the good in period 1 or $R > 1$ units of the good in period 2 if the investment is sound. There are two aggregate fundamental states, denoted $s \in S \equiv \{G, B\}$. In the good state $(s = G)$, all investments are sound. When $s = B$, in contrast, a fraction $\phi \geq 0$ of the goods invested by each bank become worthless. In other words, a fraction ϕ of the economy's resources are lost in the bad state. Let q_s denote the probability of state $s \in S$.

In each location, there is a representative bank that collects the endowments of all investors in that location and places them into the investment technology. As is standard in models based on Diamond and Dybvig (1983), pooling endowments in a bank provides investors with insurance against the risk of being impatient and needing to consume in period 1, before investment fully matures. Unlike much of the Diamond-Dybvig literature, we assume the preference type α_i of an investor is *observed by her bank*. This assumption implies that there is no possibility of a bank run in this model. Rather than allowing investors to choose when to withdraw, the banking contract will directly assign consumption to impatient investors in period 1 and to patient investors in period 2.

As in Wallace (1988, 1990) and others, banks' ability to provide risk sharing is limited by a sequential service constraint. We study a simplified version of sequential service in which period 1 is divided into two sub-periods, called *morning* and *afternoon*. The fundamental state s is realized at the beginning of the afternoon. A measure $\theta \leq \lambda$ of each bank's impatient investors arrive to withdraw in the morning, before s is known, and receive payments from the bank that cannot depend on the realized state s. The remaining impatient investors arrive in the afternoon and can receive payments from the bank that depend on the realized state s. We place no restrictions on the payments a bank can make to its investors other than those imposed by the information structure and this simplified sequential-service constraint. Competition leads banks to offer the contract that maximizes investors' expected utility.

The public good d is enjoyed simultaneously by all investors in all locations. There is a linear technology for transforming units of the private good into units of the public good in period 1. Without any loss of generality, we assume the transformation rate is one-for-one. This technology is available to all agents, but the fact that both investors and banks are small relative to the overall economy implies that there is no private incentive to provide the public good. Instead, there is a benevolent policy maker who has the the ability to tax banks in period 1 and use the revenue from this tax to produce the public good. The objective of the policy maker is to maximize the equal-weighted sum of all investors' expected utilities.

2.2.2 Policy choices

The policy maker has the ability to tax banks, to provide the public good, and to (partially) regulate the payments banks make to their investors. While all banks are ex ante identical, the policy maker may choose to treat them differently. For example, the policy maker may choose to bail out some banks but not others. To simplify the notation, we assume that there is a finite number J of *types* of banks, and that the policy maker treats all banks of the same type identically. All types are ex ante identical, and there is a measure $1/J$ of each type. Where there is no confusion, we will refer to the banks of type j collectively as "bank j." We think of the number J as being large, so that the policy maker has substantial flexibility to differentiate its policies across banks. We can write the policy maker's objective function as

$$W \equiv \sum_{j=1}^{J} \frac{1}{J} \int_{0}^{1} E\left[U\left(c_1^j(i), c_2^j(i), d; \alpha_i^j\right)\right] di. \tag{2.1}$$

Note that while banks and the policy maker both aim to maximize investor welfare, a key difference is that each bank only cares about its own investors and is small enough that it takes all economy-wide variables (including the level of the public good) as given.[4]

Taxes and bailouts. In the morning of period 1, the policy maker chooses a "baseline" tax τ_G^j for each bank j, which represents the amount of goods the bank will pay if it does not receive a bailout. Note that, because it is chosen in the morning, this tax cannot depend on the realization of the fundamental state s. If the fundamental state is good, each bank pays τ_G^j to the policy maker at the beginning of the afternoon period. If the fundamental state is bad, the policy maker may "bail out" some or all banks by lowering their tax bill. In this case, the policy maker also observes the amount of resources remaining in bank j and chooses a new tax τ_B^j taking into account both the fundamental state and the bank's individual position. The policy maker cannot commit to these new taxes in advance; each τ_B^j is chosen optimally in response to the current situation.

These timing assumptions create a trade-off for the policy maker. If a bank is not bailed out, it pays the tax τ_G^j that was chosen in the morning. This tax will be inefficiently high in the bad fundamental state; the policy maker would like to collect fewer taxes because investors' private consumption is low. By bailing out a bank, the policy maker can make the tax state-contingent, which results in a more efficient division of resources between public and private consumption. At the same time, however, it creates a moral hazard problem. When setting the tax rate τ_B^j in the afternoon, the policy maker will find it optimal (ex post) to give banks that are in worse financial condition a larger bailout. A bank that anticipates being bailed out will, therefore, have an incentive to give larger payments to its withdrawing investors in the morning, since doing so will lead to a larger bailout in the afternoon. The benefit of having state-contingent taxes thus also comes with the cost of distorting banks' incentives in the morning subperiod.

Regulation. Following Keister and Narasiman (2016), we give the policy maker a

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[4] We could instead have a finite number of banks and simply assume that each bank takes all aggregate variables, such as the level of the public good in each state, as given. The only role of our assumption that there is a continuum of banks is to guarantee that this approach is indeed optimal for each individual bank. Dividing the banks into a finite number of types is a technical assumption that simplifies the set of possible bailout policies.

regulatory tool for mitigating this moral hazard problem. In the morning of period 1, the policy maker is able to monitor a fraction $\sigma \in [0,1]$ of the withdrawing investors at each bank and dictate the amount of consumption these investors receive.[5] We interpret funds that are withdrawn in the morning, before s is realized, as representing a bank's short-term liabilities. The activity of monitoring investors and dictating payments represents the various regulatory and supervisory policies that aim to limit such liabilities in practice.[6] The parameter σ in our model represents the policy maker's ability to effectively influence banks' actions through regulation and supervision. When $\sigma = 1$, we say that regulation is perfectly effective: the policy maker can directly determine all payments made by the banking system before the state is realized. We show below that, in this case, the policy maker is able to implement the efficient allocation of resources. Having $\sigma < 1$ represents the (realistic) situation in which the policy maker's regulatory powers are limited or its ability to supervise and enforce the spirit of its regulations is imperfect.

Transparency and ambiguity. In period 0, the policy maker selects a *bailout policy,* which is a specification of which banks will and will not be bailed out if the realized fundamental state is bad. Let $\mathbb{J} = \{1, ..., J\}$ denote the set of all banks. An *ex post* bailout policy is a selection of a set of banks, denoted $\omega \subseteq \mathbb{J}$, that will be bailed out in the bad fundamental state. Let Ω denote the set of possible ex post bailout policies, that is, the set of all subsets of $\mathbb{J}$. An *ex ante* bailout policy is a probability measure on Ω, that is, a function $p : \Omega \to [0,1]$ satisfying

$$\sum_{\omega \in \Omega} p(\omega) = 1.$$

As an example, the policy of bailing out all banks for certain has $p(\mathbb{J}) = 1$ and $p(\omega) = 0$

[5]In Keister and Narasiman (2016), the policy maker does not dictate payments but has the ability confiscate goods from monitored depositors. A bank observes whether or not each depositor will be monitored before she withdraws. In equilibrium, banks will choose to give monitored depositors the maximum amount the policy maker will allow without confiscation. Our approach here of having the policy maker directly dictate the payment leads to the exact same outcome while simplifying the notation, since a bank has no choice to make when a monitored depositor withdraws

[6]Examples of such policies include the Basel III Liquidity Coverage Ratio and Net Stable Funding Ratio. See BCBS (2013, 2014) for detailed discussions of these rules.

for all $\omega \neq \mathbb{J}$. The policy of bailing out all banks with some probability π and otherwise providing no bailouts corresponds to $p(\mathbb{J}) = \pi$, $p(\varnothing) = 1 - \pi$ and $p(\omega) = 0$ for all other sets ω. Let $\mathbb{P}$ denote the set of possible ex ante bailout policies, that is, the set of probability measures on the set Ω.

We say that a bailout policy p is *transparent* if $p(\omega) = 1$ for some $\omega \in \Omega$. In other words, under a transparent policy, everyone knows in advance which banks will and will not be bailed out if the bad fundamental state is realized. If $p(\omega) > 0$ for more than one bailout set ω, in contrast, we say the bailout policy exhibits *ambiguity.* Under such a policy, at least some banks are unsure in the morning of period 1 whether or not they will be bailed out in the afternoon if $s = B$. Our primary interest is in determining the conditions under which the optimal bailout policy is transparent and the conditions under which it exhibits ambiguity.

Timeline. The timing of events in the model is summarized in Figure 2.1, where the key decision points are labelled (a) – (d). The policy maker moves first, choosing the bailout policy at point (a) in period 0. In the morning of period 1, at node (b), banks choose their early payments and the policy maker simultaneously chooses its regulatory policy and the baseline tax rates. At the beginning of the afternoon, both the fundamental state s and the bailout set ω are realized; we refer to the pair (s, ω) as the *state* of the economy. At point (c), if $s = B$, the policy maker chooses new tax rates for those banks in the realized bailout set ω. After taxes have been collected, banks decide at point (d) how to allocate their remaining resources among their remaining $(1 - \theta)$ investors.

2.3 Equilibrium

For a given bailout policy p, the environment described above corresponds to a sequential game in which the players are banks, the policy maker, and nature.[7] The decision

[7]Unlike in Diamond and Dybivg (1983), depositors are not strategic players here because their preference types α_i are not private information. This aspect of our model is similar to Champ, Smith and Williamson (1997) and other papers that focus on the risk-sharing role of banks but not on bank runs.

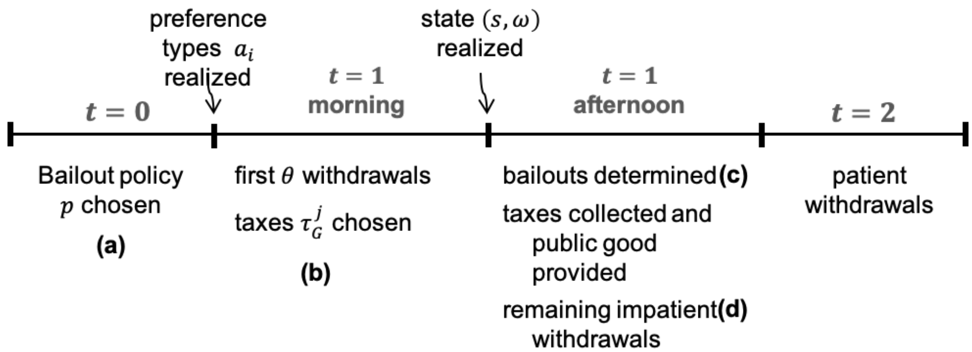

Figure 2.1: Timeline

nodes for banks and the policy maker in this game correspond to the actions labelled $(b)-(d)$ in the timeline in Figure 2.1. Note that there is no asymmetric information in our model; at each of these decision nodes, players are fully informed about the previous moves made by all other players, including nature. As a result, each of these nodes corresponds to the beginning of a proper subgame. Our solution concept for the model is subgame-perfect Nash equilibrium.

In this section, we derive the best responses of agents at each of these decision nodes and the conditions that characterize a subgame-perfect Nash equilibrium of the model.

2.3.1 Actions and objectives

We begin by introducing notation for each of the choices made by banks and the policy maker, as well as for the objective function that each seeks to maximize.

Banks. When impatient investors arrive to withdraw in the morning, the bank will choose to give a common amount $c_1^j \in \mathbb{R}_+$ to each one who is not monitored by the policy maker. In the afternoon of period 1, after the state (s,ω) has been realized and taxes have been collected, each bank decides how to allocate its remaining resources among its remaining $(1-\theta)$ investors. In each state (s,ω), a common payment $c_{1s}^j(\omega)$ will be given to each of the remaining impatient investors in period 1 and a common payment $c_{2s}^j(\omega)$ will be given to each of the bank's patient investors in period 2. All of these payments are chosen with the aim of maximizing the period-0 expected utility of the bank's investors from private consumption,

$$
\begin{aligned}
W^j \equiv\ & \theta\left((1-\sigma)\,u\left(c_1^j\right)+\sigma u\left(\hat{c}_1^j\right)\right) \\
& +\sum_{s\in S} q_s \sum_{\omega\in\Omega} p(\omega)\left((\lambda-\theta)\,u\left(c_{1s}^j(\omega)\right)+(1-\lambda)\,u\left(c_{2s}^j(\omega)\right)\right),
\end{aligned}
\tag{2.2}
$$

subject to the resource constraints

$$
\theta\left((1-\sigma)\,c_1^j+\sigma\hat{c}_1^j\right)+(\lambda-\theta)\,c_{1s}^j(\omega)+(1-\lambda)\frac{c_{2s}^j(\omega)}{R}\leq 1-\tau_s^j(\omega)-\mathbb{I}_{(s=B)}\phi \tag{2.3}
$$

for each $(s,\omega)\in S\times\Omega$. The first term in these constraints corresponds to payments made by the bank during the morning of period 1, with $\hat{c}_1^j$ representing the payment chosen by the policy maker for monitored withdrawals. The second term represents the bank's payments to the remaining impatient investors in the afternoon, while the last term on the left-hand side represents payments made to patient investors in period 2, after investment has earned the return R. The right-hand side of the constraints represents the initial endowment of each investor minus the tax paid by the bank and the loss ϕ suffered in the bad state.

Policy maker. After setting the bailout policy $p\in\mathbb{P}$ at point (a) in the timeline, the policy maker chooses a regulatory policy $\hat{c}_1^j$ and a baseline tax rate τ_G^j for each bank at point (b), and bailout tax rates τ_B^j at point (c). For notational simplicity, we use a single function $\tau_s^j(\omega): S\times\Omega\to\mathbb{R}_+^J$ to represent the tax collected from bank j in each state (s,ω). Feasibility requires that this function satisfy two conditions:

$$
\tau_G^j(\omega)=\tau_G^j \quad \text{for all } \omega\in\Omega \tag{2.4}
$$

and

$$
\tau_B^j(\omega)=\tau_G^j \quad \text{for all } j\notin\omega,\ \text{for all } \omega\in\Omega. \tag{2.5}
$$

The first restriction states that there are no bailouts in the good state, that is, the baseline tax rate τ_G^j is collected when $s=G$. The second restriction says that, in the bad fundamental state, there are no bailouts for those banks that are not in the selected bailout set ω. The policy maker will make all of its choices to maximize total welfare in (2.1). Using (2.2), we can rewrite this expression as

$$\sum_{j=1}^{J} \frac{1}{J} \left(W^j + \sum_{s \in S} q_s \sum_{\omega \in \Omega} p(\omega)\, v(d_s(\omega)) \right), \tag{2.6}$$

where the level of the public good in each state is equal to the total tax revenue collected from all banks,

$$d_s(\omega) = \sum_{j=1}^{J} \frac{1}{J} \tau_s^j(\omega). \tag{2.7}$$

2.3.2 Payments to remaining investors (d)

We begin the analysis of optimal decisions at point (d), after a measure θ of impatient investors have withdrawn from each bank, the state (s, ω) has been realized, and all taxes have been collected. Let $\psi_s^j(\omega)$ denote the quantity of resources available in bank j at this point. The bank will distribute these resources among its remaining $(1-\theta)$ investors to solve

$$V\left(\psi_s^j(\omega)\right) \equiv \max_{\left\{c_{1s}^j(\omega), c_{2s}^j(\omega)\right\}} (\lambda - \theta)\, u\left(c_{1s}^j(\omega)\right) + (1-\lambda)\, u\left(c_{2s}^j(\omega)\right) \tag{2.8}$$

subject to the resource constraint

$$(\lambda - \theta)\, c_{1s}^j(\omega) + (1-\lambda) \frac{c_{2s}^j(\omega)}{R} \leq \psi_s^j(\omega) \tag{2.9}$$

and appropriate non-negativity conditions. In other words, a bank's decision problem at this node corresponds to a standard Diamond-Dybvig allocation problem when there is a measure $\lambda - \theta$ of impatient investors and a measure $1 - \lambda$ of patient investors. Letting μ_s^j denote the multiplier associated with the resource constraint, the solution to this problem is characterized by the usual optimality condition

$$u'\left(c_{1s}^j(\omega)\right) = Ru'\left(c_{2s}^j(\omega)\right) = \mu_s^j(\omega) \tag{2.10}$$

in each state. Together, (2.9) and (2.10) completely characterize bank j's optimal chose of payments $\left(c^j_{1s}(\omega), c^j_{2s}(\omega)\right)$ as a function of the resources available to the bank in each state. Note that there is no strategic interaction at this point; each bank has a unique optimal way of allocating its remaining resources that is independent of the choices made by other banks.

2.3.3 Bailout tax rates (c)

Moving to point (c) in the timeline, the next step is to determine the tax rates $\tau^j_B(\omega)$ that the policy maker will choose in the bad fundamental state for those banks that are in the realized bailout set, that is, for $j \in \omega$. Substituting (2.2), (2.7) and (2.8) into (2.6), and dropping the utilities of the measure θ of investors who have already withdrawn and consumed, we can write the policy maker's objective in choosing these tax rates as

$$\sum_{j=1}^{J} \frac{1}{J} \left\{ V\left(1 - \tau^j_B(\omega) - \theta\left((1-\sigma)\, c^j_1 + \sigma \hat{c}^j_1\right) - \phi\right) + v\left(\sum_{j=1}^{J} \frac{1}{J} \tau^j_s(\omega)\right)\right\}.$$

The first term inside the curly brackets is the utility of bank j's remaining investors from private consumption, recognizing that the bank will allocate its post-tax resources according to (2.8). The second term is the utility these investors receive from the public good.

Keeping in mind the tax rates for all banks with $j \notin \omega$ are fixed at τ^j_G, we can write the first-order condition characterizing the optimal tax rates for banks that are bailed out as

$$\mu^j_B(\omega) = v'\left(d_B(\omega)\right) \quad \text{for all } j \in \omega, \tag{2.11}$$

where $\mu^j_B(\omega)$ equals the marginal utility of private consumption for bank j's remaining impatient investors, as shown in (2.10). It is straightforward to show that the system of equations in (2.11) has a unique solution for each $\omega \in \Omega$ and for any (feasible) combination of $\left(c^j_1, \hat{c}^j_1, \tau^j_G\right)$.

Note that the right-hand side of (2.11) does not depend on j; it is the marginal value of the public good for all agents in the economy. This condition thus implies that, for all $j \in \omega$, the tax rates $\tau_B^j(\omega)$ will be chosen in such a way that the remaining resources $\psi_B^j(\omega)$ are equated across these banks. Looking ahead, the incentive problems caused by this bailout policy are clear: a bank with fewer remaining resources (because it chose a higher value of c_1^j) will be charged less tax (i.e., it will receive a larger bailout). This policy will give all banks that anticipate being bailed out in some states an incentive to set c_1^j higher than they otherwise would.

2.3.4 Morning payments and fiscal policy (b)

We now move to point (b), where banks choose the payment c_1^j they make to non-monitored investors who withdraw in the morning and the policy maker chooses both the payment $\hat{c}_1^j$ on monitored withdrawals and a baseline tax rate τ_G^j for each bank. These choices are made simultaneously, based on each player's anticipation of the current actions of other players, as well as of play in the subgame starting at point (c) in each state.

Looking first at the policy maker, it takes banks' choices of c_1^j as given. It also recognizes that, if the bad state is realized, it will reset the tax rates for all banks that are bailed out. We can, therefore, write the policy maker's decision problem at point (b) as choosing $\left\{\hat{c}_1^j, \tau_G^j\right\}_{j\in\mathbb{J}}$ to maximize

$$\sum_{j=1}^{J}\frac{1}{J}\left\{\theta\sigma u\left(\hat{c}_1^j\right)+\sum_{s\in S}q_s\sum_{\omega\in\Omega}p\left(\omega\right)\left(\begin{array}{c}V\left(\begin{array}{c}1-\tau_s^j\left(\omega\right)-\theta\left(\left(1-\sigma\right)c_1^j+\sigma\hat{c}_1^j\right)\\-\mathbb{I}_{(s=B)}\phi\end{array}\right)\\+v\left(\sum\limits_{j=1}^{J}\frac{1}{J}\tau_s^j\left(\omega\right)\right)\end{array}\right)\right\}, \tag{2.12}$$

subject to the restrictions (2.4) and (2.5) and to the decision rule (2.11) that specifies how the policy maker will choose to set $\tau_B^j(\omega)$ for $j \in \omega$ at the nodes in point (c) where $s = B$.

The first-order condition characterizing the optimal choice $\hat{c}_1^j$ can be written as

$$u'\left(\hat{c}_1^j\right) = \sum_{s \in S} q_s \sum_{\omega \in \Omega} p(\omega)\, \mu_s^j(\omega). \tag{2.13}$$

This equation states that the marginal utility of private consumption for monitored withdrawals in the morning will be set equal to the expected marginal value of private consumption for investors who withdraw from bank j in the afternoon. For the choice of tax rate τ_G^j, the first-order condition is

$$q_G \mu_G^j + q_B \sum_{\omega : j \notin \omega} p(\omega)\, \mu_B^j(\omega) = q_G v'(d_G) + q_B \sum_{\omega : j \notin \omega} p(\omega)\, v'(d_B(\omega)). \tag{2.14}$$

This condition equates the expected marginal utility of private consumption for an investor who withdraws in the afternoon to the expected marginal utility of public consumption, where both expectations are conditional on the event that bank j is not bailed out. Recall that bank j is not bailed out either if $s = G$ or if $s = B$ and $j \notin \omega$.

Turning to bank j's decision problem in making the early payments c_1^j, first note that the bank takes the policy maker's choice of baseline tax rate τ_G^j as given. It recognizes, however, that if it is bailed out, the tax rate $\tau_B^j(\omega)$ chosen in the subgame beginning at point (c) will depend on its choice of c_1^j according to the decision rule (2.11). We can, therefore, write the bank's decision problem at point (b) as maximizing

$$\begin{array}{l} \theta(1-\sigma)\, u\left(c_1^j\right) + q_G V\left(1 - \tau_G^j - \theta\left((1-\sigma)\, c_1^j + \sigma \hat{c}_1^j\right)\right) \\ +q_B \left\{ \begin{array}{l} \quad \sum_{\omega : j \notin \omega} p(\omega)\, V\left(1 - \tau_G^j - \theta\left((1-\sigma)\, c_1^j + \sigma \hat{c}_1^j\right) - \phi\right) \\ + \sum_{\omega : j \in \omega} p(\omega)\, \bar{V}(\omega) \end{array} \right\}. \end{array}$$

The first line of this expression measures the utility of unmonitored investors who withdraw in the morning as well as of all investors who withdraw in the afternoon or in period 2 in the good state. The second line measure that utility of these remaining investors in states where the bank suffers a loss but is not bailed out. The final line corresponds to states where the bank is bailed out, with the $\bar{V}(\omega)$ notation indicating that the consumption these investors does not depend on the bank's choice of early payments c_1^j. Instead, through the bailout policy, these consumption levels will depend instead on the average level of early payments made across all banks in the economy, which each individual bank takes as given. The first-order condition for bank j's choice of c_1^j can be written as

$$u'\left(c_1^j\right) = q_G \mu_G^j + q_B \sum_{\omega : j \notin \omega} p(\omega)\, \mu_B^j(\omega)\,. \tag{2.15}$$

2.3.5 Moral hazard

Comparing equation (2.15) with the condition for the policy maker's choice of $\hat{c}_1^j$ in (2.13) shows how the bailout policy distorts banks' incentives. The policy maker wants to equate the marginal value of consumption for morning withdrawals to the expected marginal value of consumption for afternoon withdrawals. An individual bank, in contrast, only takes into account the value of afternoon consumption in those states in which it is not bailed out. Using the fact that u is strictly increasing (which implies $\mu_s^j(\omega) > 0$ for all ω) and strictly concave, these two equations immediately imply that a bank will set c_1^j higher than the policy maker sets $\hat{c}_1$if, and only if, it will be bailed out with positive probability.

Proposition 6 *In any equilibrium, $c_1^j \geq \hat{c}_1^j$ holds for all j, with strict inequality if $p(\omega) > 0$ for some ω with $j \in \omega$.*

If the bailout policy p is such that bank j is never bailed out, the regulatory policy is redundant in the sense that the bank will choose to set c_1^j equal to the policy maker's choice of $\hat{c}_1^j$. If bank j is bailed out with positive probability, in contrast, it will choose to set c_1^j strictly higher than the policy maker would like and the regulatory parameter σ will have an important effect on the equilibrium allocation.

In the next section, we characterize equilibrium allocations and optimal bailout policy when the policy maker is able to fully correct the moral hazard problem through regulation. In Section 2.5 we then study the more interest case where regulation is imperfect.

2.4 Optimal policy when regulation is perfect $(\sigma = 1)$

In this section, we derive the optimal bailout policy for the special case where $\sigma = 1$, that is, where the policy maker is able to choose the payments given to *all* of the investors who withdraw in the morning of period 1. This case provides a useful benchmark for the analysis in the next section where regulation is imperfect. When $\sigma = 1$, banks' choice of c_1^j is irrelevant because no investors receive this amount. As a result, the moral hazard problem identified in Proposition 6 has no effect on the equilibrium allocation for any bailout policy p. We show that, in this case, the optimal policy is fully transparent and sets $p(\mathbb{J}) = 1$.

When regulation is perfect, we can omit banks' choice of c_1^j from point (b) in the timeline in Figure 1. In the resulting game, the policy maker is the only player who moves at points (b) and (c). While these choices are made sequentially, the fact that no other strategic player moves at these points implies that we can write the policy maker's decision problem as one of choosing $\left\{\hat{c}_1^j, \tau_G^j\right\}_{j\in\mathbb{J}}$ and a state-contingent plan $\left\{\tau_B^j(\omega)\right\}_{j\in\mathbb{J},\omega\in\Omega}$ to maximize

$$\sum_{j=1}^{J}\left\{\begin{array}{c}\theta u\left(\hat{c}_1^j\right) + q_G\left(V\left(1-\tau_G^j-\theta\hat{c}_1^j\right)+v\left(\sum\limits_{j=1}^{J}\tau_G^j\right)\right)\\ +q_B\sum_{\omega\in\Omega}p(\omega)\left(V\left(1-\tau_B^j(\omega)-\theta\hat{c}_1^j-\phi\right)+v\left(\sum\limits_{j=1}^{J}\tau_B^j(\omega)\right)\right)\end{array}\right\} \tag{2.16}$$

subject to the restriction in (2.5) that only banks in the realized bailout set ω can have $\tau_B^j(\omega)$ be different from τ_G^j. As the use of the function V defined in (2.8) indicates, the policy maker anticipates that all banks will efficiently allocate their after-tax resources among their remaining investors at the subgame in point (d) of the timeline.

The policy maker will choose the bailout policy p at point (a) in the timeline to maximize the value of the same objective function in (2.16), anticipating how all payments and tax rates will be set for each possible choice. Our next result shows that the resulting optimal policy is transparent and bails out all banks with probability one.

Proposition 7 *When $\sigma = 1$, the optimal bailout policy sets $p(\mathbb{J}) = 1$.*

The intuition for this result is easy to see. As discussed above, bailouts are part of an efficient risk-sharing arrangement in this environment; they allow the policy maker to effectively allocate resources between policy and private consumption depending on the realized fundamental state. When regulation is perfect, the policy does not need to worry about the anticipation of being bailed out distorting banks' incentives or the allocation of resources. As a result, there is no benefit of restricting the policy maker's ability to make bailouts and the optimal policy always provides bailouts to all banks in the bad state.

In fact, it is straightforward to show that the equilibrium allocation when $\sigma = 1$ and $p(\mathbb{J}) = 1$ is the same as the allocation that would be chosen by a benevolent social planner who could freely allocate resources in the economy subject only to the sequential service constraint. In other words, the equilibrium with perfect regulation is the best allocation that can be achieved in this economy regardless of the policy maker's ability to commit to policies and monitor the actions of agents. As such, it forms a natural benchmark for our analysis that follows. The key takeaway from this benchmark case is that when bailouts do not distort incentives (either because regulation is perfect or because the allocation is chosen by a planner), the optimal bailout policy is transparent. In the next section, we move to the case where regulation is imperfect and ask if there is a role for ambiguity in the optimal policy.

2.5 Optimal policy with imperfect regulation ($\sigma < 1$)

When regulation is imperfect, the moral hazard problem identified in Proposition 6 affects the equilibrium allocation whenever banks anticipate being bailed out with positive probability. In choosing the bailout policy p, the policy maker then faces a tradeoff: bailouts provide efficient risk-sharing, but also distort the payments made to some investors. In this section, we illustrate how this tradeoff shapes the optimal bailout policy.

We begin by restricting the policy maker to choose one of two simple, transparent policies: either all banks will be bailed out or none will. We show that the optimal policy involves bailouts whenever the loss ϕ in the bad state is large enough. We then consider two classes of policies with ambiguity, one in which banks are uncertain about whether or not there will be bailouts and another in which banks know there will be bailouts but are uncertain about who will be bailed out. In each case, we show that ambiguity can be part of the optimal bailout policy, and we highlight the forces that make ambiguity a useful policy tool.

2.5.1 Simple bailout policies

To begin, suppose the policy maker must choose one of the two transparent policies that treat all banks equally: either all banks will be bailed out or none will. In terms of our notation, this restriction requires that the policy maker assign probability 1 to either the set of all banks $\mathbb{J}$ or the empty set $\emptyset$. This simplified choice problem is useful for understanding the tradeoff faced by the policy maker when $\sigma < 1$ and how this tradeoff shapes the optimal policy. Figure 2.2 illustrates the optimal choice as a function of the parameters σ and ϕ.[8]

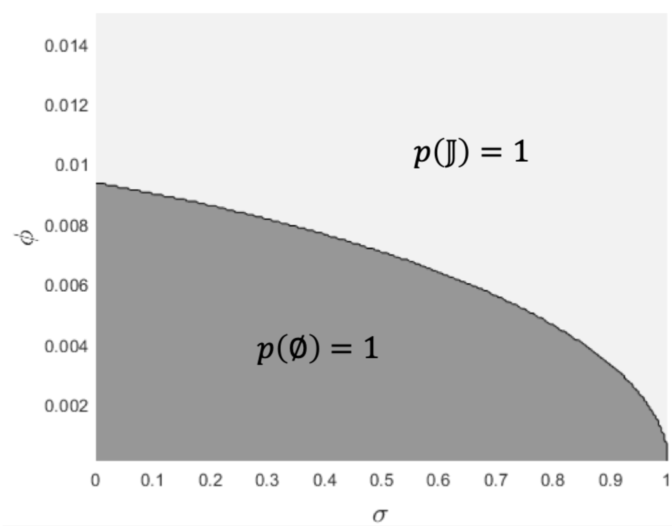

Figure 2.2: Optimal simple bailout policy

The figure shows that when σ and ϕ are both relatively small, the optimal policy is to provide no bailouts $(p(\emptyset) = 1)$. Recall that the parameter ϕ can be interpreted

[8]The examples uses $u(c) = c^{1-\gamma}/(1-\gamma)$ and $v(d) = \delta d^{1-\gamma}/(1-\gamma)$ with $\gamma = 6$ and $\delta = 1$. The other parameter values are $R = 1.1$, $\lambda = 0.5$, $\theta = 0.45$, and $q_B = 4\%$.

as measuring the potential insurance benefit of bailouts. When ϕ is small, banks suffer only a small loss in the bad fundamental state. As a result, the increase in welfare that would result from readjusting the distribution of resources between public and private consumption is modest. The parameter σ measures the fraction of banks' morning payments that are not affected by the incentive distortion. When σ is small, providing bailouts leads to a significant distortion in the allocation of resources. Pairs (σ, ϕ) in the lower-left corner of the figure thus represent economies where the benefits of bailouts are small and the costs are large. In these economies, the optimal simple bailout policy is to bail out no banks. When σ and ϕ are both large, on the other hand, providing bailouts brings significant risk-sharing benefits and, because regulation is very effective, introduces only moderate incentive distortions. In such cases, the figure shows that the optimal simple policy is to bail out all banks.

In between these extremes, the optimal policy has the following features in this example. For any $\sigma < 1$, there exists a cutoff value $\bar{\phi} > 0$ such that the optimal policy provides bailouts if and only if $\phi \geq \bar{\phi}$. This result can be interpreted as saying that policy makers should commit not to intervene when banks' losses are moderate, but should let it be known that they will intervene in extreme events. The cutoff value for intervention to be optimal depends on the effectiveness of regulations in mitigating the incentive distortions associated with bailouts. When regulation is more effective (σ is larger), the cutoff value for intervention $\bar{\phi}$ is smaller.

With these results on the optimal simple bailout policy in hand, we next ask whether introducing ambiguity into the bailout policy can raise welfare. We divide the analysis of ambiguity into two cases. In the first case, we require the policy maker to treat all banks equally, meaning that either everyone is bailed out or no one is bailed out. In addition to reflecting what may be a realistic political-economy constraint,[9] this case cleanly illustrates the benefits of constructive ambiguity and when it is likely to be optimal. In the second case, we allow the policy maker to use more general policies

[9]For example, this case can be thought of as representing the restriction in the Dodd-Frank Act of 2010 that any emergency facilities established by the Federal Reserve must have "broad-based eligibility" and cannot be designed to benefit specific institutions.

that treat banks differently even though they are ex ante identical. We show that constructive ambiguity is often optimal in this case as well, for the same basic reasons as in the first case. We then compare the two different types of ambiguity and show that the optimal policy often involves treating otherwise identical banks differently.

2.5.2 Ambiguity about whether there will be bailouts

Suppose the policy maker is restricted to treat all banks equally in the sense that it can either bail out all banks at $t = 1$ or bail out none of them as before. However, it is now allowed to assign positive probability to both of these outcomes, meaning that banks may be uncertain when choosing their morning payments whether or not they will be bailed out if the bad state occurs. Define

$$\pi_1 \equiv p\left(\mathbb{J}\right),$$

so that π_1 represents the probability that all banks will be bailed out at $t = 1$ and $1 - \pi_1$ represents the probability that no banks will be bailed out. Then the bailout policy is transparent if $\pi_1 = 0$ or $\pi_1 = 1$, while it exhibits ambiguity if $0 < \pi_1 < 1$.

Because bailout policies in this set treat all banks equally, it is straightforward to show that the resulting equilibrium allocation is symmetric across banks. We can, therefore, omit the j superscripts and use $(c_1, \hat{c}_1, \tau_G, \tau_B)$ to denote the equilibrium actions of banks and the policy maker, which are implicitly defined as functions of π_1 by equations (2.11) and (2.13) – (2.15). Taking these functions as given, the policy maker will choose the bailout policy π_1 to maximize

$$\begin{aligned} W_1\left(\pi_1\right) \quad \equiv \quad & \theta\left[(1-\sigma)\,u\left(c_1\right)+\sigma u\left(\hat{c}_1\right)\right]+q_G\left[V\left(1-\theta\bar{c}_1-\tau_G\right)+v\left(\tau_G\right)\right] \\ & +q_B\left(1-\pi_1\right)\left[V\left(1-\theta\bar{c}_1-\tau_G-\phi\right)+v\left(\tau_G\right)\right] \\ & +q_B\pi_1\left[V\left(1-\theta\bar{c}_1-\tau_B-\phi\right)+v\left(\tau_B\right)\right], \end{aligned} \tag{2.17}$$

where we have simplified the expression by defining $\bar{c}_1$ to be the average consumption of investors who withdraw in the morning,

$$\bar{c}_1 \equiv (1-\sigma)\, c_1 + \sigma \hat{c}_1.$$

The first line of (2.17) represents the utility from private consumption of these investors plus the utility from both the private consumption of the remaining investors and the public good in the good fundamental state. The middle line represents these last two objects in the bad fundamental state when there are no bailouts and the tax remains at τ_G, while the last line corresponds to the event where bailouts occur and the tax is lowered to τ_B for all banks.

The marginal effect of an increase in π_1 on welfare is[10]

$$W_1'(\pi_1) = q_B \left\{ \begin{array}{c} V(1-\theta\bar{c}_1-\tau_B-\phi) + v(\tau_B) - [V(1-\theta\bar{c}_1-\tau_G-\phi) + v(\tau_G)] \\ -\frac{dc_1}{d\pi_1}\pi_1\theta(1-\sigma)\, V'(1-\theta\bar{c}_1-\tau_B-\phi) \end{array} \right\} \tag{2.18}$$

This expression illustrates the trade-off the policy maker faces in choosing π_1. The first line of this expression is always positive; for given morning payments $(c_1, \hat{c}_1)$, it represents the gain in welfare from increasing the probability of a bailout. This gain comes from being able to set the tax to τ_B when $s = B$, which maximizes welfare when the loss ϕ occurs, instead of keeping it at τ_G. The second line represents the loss in welfare that comes from the increased distortion of banks' incentives. When the probability π_1 is increased, banks have an incentive to set c_1 higher since it is more likely that they will be bailed out. The increase in the distortion is proportional to the probability π_1 of being bailed out, the measure $\theta(1-\sigma)$ of unmonintored investors in the morning, and the marginal value of resources V' in the state where bailouts occur.

Our first result in this section uses equation (2.18) to show that, as long as banks suffer a non-zero loss in the bad fundamental state, the policy with no bailouts ($\pi_1^* = 0$) is never optimal.

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[10]Because the policy maker will also choose $\hat{c}_1$ and the taxes τ_s to maximize welfare, the effect of π on these variables does not appear in this expression as a result of the envelope theorem.

Proposition 8 *Among bailout policies defined by* π_1, *the optimal policy has* $\pi_1^* > 0$ *for all* $\phi > 0$.

The proof of this result comes from evaluating equation 2.18 at $\pi_1 = 0$, in which case the second line disappears. Given that the first line is always positive, we have $W'(0) > 0$ and the solution must satisfy $\pi_1^* > 0$. Intuitively, when bailouts never occur, the distortion in banks' incentives is completely eliminated. As a result, banks and the policy maker are choosing the same morning payments, that is, $c_1 = \hat{c}_1$. Raising π_1 above zero begins to introduce an incentive distortion, but the effect of this change on welfare is second order. However, the change brings a first-order benefit as indicated by the first line of equation (2.18).

The question of whether the optimal bailout policy within this class is transparent or uses ambiguity thus reduces to whether the policy maker wants to bail out all banks for certain $(\pi_1 = 1)$ or leave some ambiguity $(\pi_1^* < 1)$. The next result shows that either policy can be optimal, depending on parameter values.

Proposition 9 *Among bailout policies defined by* π_1, *the optimal policy has* $\pi_1^* = 1$ *for some parameter values and* $\pi_1^* < 1$ *for others.*

This result is established by the example presented Figure 2.3. This diagram shows the level sets of the optimal bailout policy π_1^* as a function of the regulatory parameter σ and the size of the loss in the fundamental state, ϕ. First note that the general form of the figure is similar to that in Figure 2.2. In particular, when ϕ and ω are sufficiently large, the optimal policy sets $\pi_1^* = 1$. As in the previous section, the policy maker will choose to bail out all banks with certainty when the insurance benefit of bailouts is large and regulation is sufficiently effective. When ϕ and σ are smaller, however, Figure 2.3 shows that there is now a wide region where the optimal policy exhibits constructive ambiguity. In this region, the policy maker optimally balances the insurance benefit of bailouts against the incentive distortion by choosing a bailout probability strictly between zero and one. In fact, it can be shown that π_1^* is a continuous function of parameters at all points in the interior of the figure. In particular, for any $\sigma < 1$, π_1^* takes on every value in $[0, 1]$ as ϕ varies from zero to one. In other words, every possible degree of ambiguity is optimal for some parameter values, including the case where $\pi_1^* = 0.5$ and there is maximal uncertainty about what the policy maker will do.

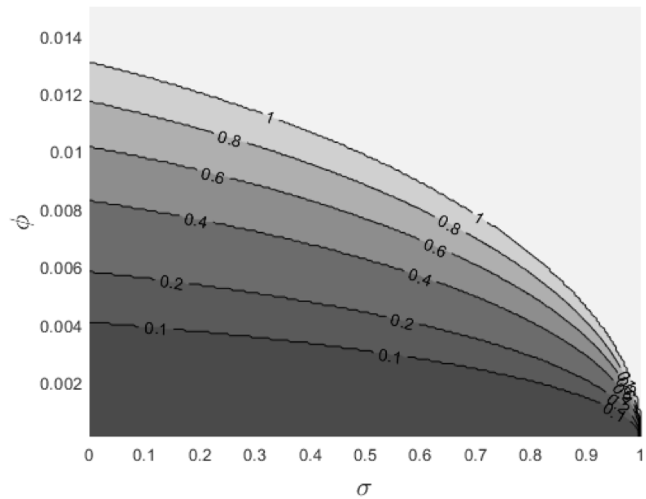


Figure 2.3: The optimal policy π_1^* under equal treatment

The example in Figure 2.3 illustrates the value of constructive ambiguity as a policy tool and how the policy maker will choose to use it. When banks' losses in the bad state (ϕ) are small and regulation (σ) is ineffective, the policy maker will primarily be concerned with mitigating the incentive distortion. In such cases, π_1^* will be small; banks may be bailed out in the bad fundamental state, but this event will be unlikely. As the loss increases and/or regulation improves, the insurance benefit of bailouts becomes relatively more important and π_1^* increases. Along these paths, the policy maker is using ambiguity to optimally balance the desire to provide insurance with the desire to mitigate the resulting incentive distortion. In other words, constructive ambiguity convexifies the policy maker's choice set and generates higher welfare than announcing in advance either that all banks will be bailed out or that none will.

2.5.3 Ambiguity about who will be bailed out

Now suppose we remove the restriction that the policy maker must treat all banks equally. There is then another way to introduce ambiguity into the policy: by randomly choosing to bail out some banks but not others. In this section, we study the class of bailout policies p that place positive probability only on bailout sets ω of the same size, and that place equal probability on all such sets. In other words, the policy maker announces in advance what *fraction* of banks will be bailed out, but provides no information on which banks will be included in this set. For any such policy p, let π_2 denote the fraction of banks that will be bailed out. A policy in this class is transparent if $\pi_2 = 0$ or if $\pi_2 = 1$, and exhibits constructive ambiguity if $0 < \pi_2 < 1$.

To derive the optimal value of π_2, we first note that this class of policies treats all banks equally ex ante and, therefore, the resulting equilibrium allocation will again be symmetric across banks ex ante. Equations (2.11) and (2.13) – (2.15) can be used to define $(c_1, \hat{c}_1, \tau_G, \tau_B)$ as functions of π_2, keeping in mind that these functions are different from those in Section 2.5.2 because the class of policies is different. Taking these functions as given, the policy maker will choose π_2 to maximize

$$\begin{aligned} W_2(\pi_2) \equiv\ & \theta\left[\sigma u(c_1) + (1-\sigma) u(\hat{c}_1)\right] + q_G\left[V(1-\theta\bar{c}_1 - \tau_G) + v(\tau_G)\right] \\ & + q_B \left\{ \begin{array}{c} (1-\pi_2) V(1-\theta\bar{c}_1 - \tau_G - \phi) + \pi_2 V(1-\theta\bar{c}_1 - \tau_B - \phi) \\ v((1-\pi_2)\tau_G + \pi_2 \tau_B) \end{array} \right\}. \end{aligned} \tag{2.19}$$

Comparing this expression with equation (2.17) shows the similarities between the two types of policies and one key difference. First, the two types of policies are equally effective in mitigating moral hazard because both leave an individual bank uncertain about whether or not it will be bailed out. However, the policies in Section 2.5.2 create uncertainty about the level of the public good that will be provided, while the policies in this section do not. This fact has implications for the optimal amount of ambiguity. Working from equation (2.19), the marginal effect of an increase in π_2 on welfare is

$$W_2'(\pi_2) = q_B \left\{ \begin{array}{c} V(1-\theta\bar{c}_1 - \tau_B - \phi) - V(1-\theta\bar{c}_1 - \tau_G - \phi) \\ -(\tau_G - \tau_B) v'((1-\pi_2)\tau_G + \pi_2\tau_B) \\ -\frac{dc_1}{d\pi_2}\pi_2 \theta (1-\sigma) V'(1-\theta\bar{c}_1 - \tau_B - \phi) \end{array} \right\}. \tag{2.20}$$

This expression again illustrates the tradeoff the policy maker faces in choosing π_2. The first two lines captures the benefit of bailing out a larger fraction of banks, holding the morning payments constant. Note that the first-order condition for the choice of τ_B in equation 2.11 can be written for this class of policies as

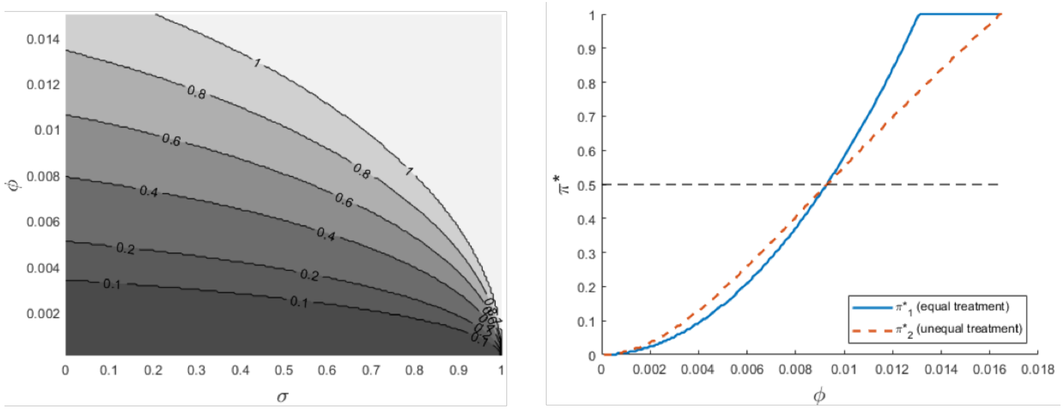
$$V'(1-\theta\bar{c}_1-\tau_B-\phi)=v'((1-\pi_2)\tau_G+\pi_2\tau_B).$$

Using this equation and the strict concavity of V, it is straightforward to show that, taken together, the expression on the first two lines of equation (2.20) are always strictly positive. The third line captures the cost of increasing π_2 that comes from a worsening of banks' incentives in the morning period: an increase in π_2 gives banks an incentive to set c_1 higher. The welfare cost of this change is proportional to the probability π_2 with which each bank is bailed out, the measure $\theta(1-\sigma)$ of investors who receive this higher payment, and the marginal value of funds V' in banks that are bailed out.

If we evaluate this derivative at $\pi_2 = 0$, the third line of 2.20 disappears and the value is necessarily positive. As in the previous section, starting from a policy with no bailouts, the incentive distortion that comes from increasing the probability of bailouts has only a second-order effect on welfare, while the increased insurance brings a first-order benefit. As a result, the only transparent policy that can potentially be optimal is $p(\mathbb{J}) = 1$. Our next proposition formalizes this result and shows that the optimal policy can be either transparent or ambiguous, depending on parameter values.

Proposition 10 *Among bailout policies defined by π_2, the optimal policy has $\pi_2^* > 0$ for all $\phi > 0$. For some parameter values the optimal policy has $\pi_1^* = 1$ and for others it has $\pi_1^* < 1$.*

The second part of Proposition 10 is established by the example presented in Figure 2.4. Panel (a) shows the level sets of the optimal policy π_2^* as a function of (σ, ϕ), using the same parameter values as in Figures 2.2 and 2.3. The patterns are broadly similar to Figure 2.3, with the transparent policy $\pi_2 = 1$ being optimal when σ and ϕ are large, but constructive ambiguity being optimal when σ and/or ϕ are more moderate. This similarity shows not only that constructive ambiguity is often optimal when unequal treatment is allowed, but also that the same basic forces determine the optimal policy in both cases.



(a) Optimal policy π_2^* (b) Comparing π_1^* and π_2^*

Figure 2.4: The optimal policy π_2^* with unequal treatment

One interesting feature of the two figures is that the range of parameter values for which the policy maker uses constructive ambiguity is larger in panel (a) of Figure 2.4 than in Figure 2.3. In fact, the level sets associated with $\pi^* > 0.5$ to are all higher in Figure 2.4, while the level sets associated with $\pi^* < 0.5$ are all lower. To illustrate this point more clearly, panel (b) of the figure plots the optimal policies for each case, π_1^* and π_2^*, as functions of ϕ when $\sigma = 0$. The graph shows that the policy maker is more willing to use constructive ambiguity in the second class of policies: π_2^* lies closer to the point of maximal uncertainty (1/2) for all values of ϕ. As discussed above, when the policy maker is restricted to treat all banks equally, using constructive ambiguity to influence banks' incentives necessarily introduces uncertainly about the level of the public good that will be provided in the bad fundamental state. Because investors are risk-averse, this uncertainty is a cost of using constructive ambiguity. The second class of policies allows the policy maker to introduce uncertainty into each bank's decision problem without creating uncertainty about the level of the public good in the bad fundamental state. As a result, using constructive ambiguity is more attractive with this class of policies, leading to the pattern in panel (b) of figure.

2.5.4 Aggregate vs. idiosyncratic ambiguity

The previous two sections showed how constructive ambiguity can be a useful policy tool using two distinct types of uncertainty. In Section 2.5.2, the policy created aggregate uncertainty within the bad fundamental state by bailing out all banks with probability π_1 and no banks with probability $1-\pi_1$. As discussed above, this policy also creates uncertainty about total tax revenue and the level of the public good when $s = B$. In section 2.5.3, the policy created idiosyncratic uncertainty for banks about whether or not they would be bailed out, but had no aggregate uncertainty about the number of banks that would be bailed out, total tax revenue, or the level of the public good in the bad fundamental state. Is one of these types of policies better than the other, or might the overall optimal policy involve a combination of the two?

To answer this question, we now consider a broader class of bailout policies with the following form: the policy maker first chooses whether there will be bailouts (with probability π_1) or not (with probability $1-\pi_1$). If there are bailouts, the policy maker randomly chooses a fraction π_2 of banks to bail out. This class of policies includes as special cases those studied in Section 2.5.2 (which have $\pi_2 = 1$) and those studied in Section 2.5.3 (which have $\pi_1 = 1$). Our next result shows that, when both types of ambiguity are available, the policy maker will choose to only use the idiosyncratic component.

Proposition 11 *Among bailout policies defined by* (π_1, π_2)*, the optimal policy always sets* $\pi_1^* = 1$*. For some parameter values, it sets* $\pi_2^* = 1$ *and for others it sets* $0 < \pi_2^* < 1$*.*

In other words, when the policy maker has the option to use any combination of aggregate and idiosyncratic ambiguity, the optimal policy is to use idiosyncratic ambiguity only. The intuition for this result has two parts. The first part can be seen by looking at bank j's first-order condition for choosing the payment to give to withdrawing investors in the morning, which is presented in equation (2.15). The bank's incentive depend on the probability that it will be bailed out in the bad fundamental state, but not on the states in which *other* banks are bailed out. As a result, a bank's choice of the morning payment c_1^j depends only on the product $\pi_1\pi_2$ and, hence, can be controlled equally well by either dimension of the policy. The second part is that, as discussed above, setting $\pi_1 < 1$ generates uncertainty about the level of the public good in the bad fundamental state, whereas setting $\pi_2 < 1$ does not. As a result, idiosyncratic ambiguity provides are more efficient way for the policy maker to influence banks' incentives.

Proposition 11 implies that the optimal policy within this broader class is again characterized by the graphs in Figure 2.4. Note that, when $\pi_2^* < 1$, the policy will treat banks differently ex post, bailing out some but not others, even though all banks are initially identical. Such a policy might be criticized by observes for being inconsistent, unfair, or unpredictable. However, it is precisely this unpredictability that allows the policy maker to most efficiently balance the insurance benefit of bailouts against the incentive cost. Moreover, the "unfair" aspect of treating banks differently allows the policy maker to achieve this balance without introducing additional aggregate uncertainty. Requiring the policy maker to be transparent, by announcing in advance which banks will or will not be bailed out, or to treat all banks equally at all times would strictly lower welfare.

2.6 Concluding Remarks

It is widely recognized that the anticipation of being bailed out in the event of a crisis distorts banks' incentives and the allocation of resources. Policy makers typically attempt to correct this distortion through regulation and supervision, but doing so is difficult and such efforts are inevitably imperfect. There has been substantial debate over the past decade about what type of framework should guide bailout policy in the future. Much of this discussion has focused on an essentially binary policy choice, with some observers arguing that bailouts should be prohibited and others arguing that they should be allowed.

We show that when regulation is imperfect, the optimal bailout policy may exhibit a form of constructive ambiguity. In particular, the best way to balance the benefits of bailouts against the cost of the incentive distortions they create is often a form of mixed strategy in which the policy maker is deliberately ambiguous about what bailouts will be made in the event of a crisis. We study two distinct forms of such ambiguity. In one form, the policy maker will either bail out all banks or bail out none, but does not specify in advance which action will be taken. In the second form, the policy maker announces in advance what fraction of banks will be bailed out in the event of a crisis, but does not specific which specific banks will be included. In both cases, we show that when ambiguity is an option, it is never optimal for policy makers to commit to a strict no-bailouts policy; providing bailouts with some positive probability always raises welfare. In addition, in many situations, the optimal policy involves providing bailouts with a probability strictly between zero and one.

During the financial crisis in 2008, some observers criticized policy makers for a lack of predictability. For example, Sorkin (2010) states:

> "it cannot be denied that federal officials … contributed to the market turmoil through a series of inconsistent decisions. They offered a safety net to Bear Stearns and backstopped Fannie Mae and Freddie Mac, but allowed Lehman to fall into Chapter 11, only to rescue AIG soon after. What was the pattern? What were the rules? There didn't appear to be any …" (p. 535)

Following on this type of observation, some prominent policy makers and academics have claimed that future policy should be made as transparent as possible. For example, Lacker (2008) states that "continued ambiguity thus would pose risks to financial stability and the economy" and that policy makers should strive to "establish a credible commitment to following clear, pre-announced rules in times of crisis." Lucas and Stokey (2011) state that "the events of 2008 illustrate the importance of an announced and well-understood policy" and "there is no gain from allowing uncertainty about how the Fed will behave about the safety net." Our results challenge these claims. We show that when regulation is imperfect, ambiguity is often part of the optimal bailout policy. In fact, we show that the optimal policy within a broad class often has precisely the feature that these observes criticize: it randomly chooses to bail out some banks but not others.

One interpretation of these comments is that they reflect *ex post* concerns. Once a crisis occurs, there is no benefit from ambiguity in our model and following the optimal policy is often ex post inefficient. Another possibility, of course, is that policy uncertainty is undesirable for some reason(s) that are missing in our model. What would these reasons be and how could they be introduced into a formal model? We leave these interesting question for future research.

Chapter 3

Deposit Insurance: How Should Uninsured Deposits Be Treated?

3.1 Introduction

Many countries have introduced deposit insurance as a way to protect depositors from failures of financial institutions and to promote financial stability. While these policies provide valuable insurance to depositors and may, in addition, prevent bank runs, they also distort ex ante incentives. One way a regulator can mitigate such distortions is placing limits on deposit insurance. In practice, deposit insurance covers only a fraction of total bank deposits in part because countries put a cap the size of an individual deposit that is eligible for coverage (Demirg-Kunt & Kane, 2002). In the United States, each depositor is insured up to $250,000 per insured bank. Despite the presence of deposit insurance, bank failures are still recurrent: a total of 4,096 banks have failed in the United States since the introduction of federal deposit insurance corporation in 1934 to 2017.[1] Based on the information by the Historical Statistics on Banking (HSOB) from FDIC, failed banks have left about 39 million dollars worth in their assets at the timing of their failure on average. This fact raises a natural question: how should the remaining assets of the failed institution be distributed among the depositors and uninsured creditors in the course of resolution? Or, how should the regulator treat the uninsured deposits when a financial institution fails?

I study an environment where there is a role for the regulator in the process of resolution for a failed financial institution. After a financial institution fails, the regulator steps in and must make payments to the insured depositors who need consumption immediately, using its deposit insurance fund. When the deposit insurance covers only a certain fraction of deposits and there remain some resources in the course of resolution,

[1]The number of failed banks is provided by the FDIC Bank Failures and Assistance Data.

the regulator decides to distribute these resources among the remaining claimants. This paper asks how the regulator should distribute the remaining proceeds of the failed institution between uninsured deposits and deposit insurance fund. In particular, is providing a higher priority on uninsured deposits desirable?

To answer the question, I analyze a model in the traditional Diamond and Dybvig (1983) model with the variation of limited deposit insurance and bank-specific uncertainty.[2] Each bank faces an idiosyncratic fundamental shock on its investment return, which allows me to analyze the resolution of a failed bank. When the investment return is lower than anticipated, the financial institution would not have enough assets to meet its obligations to depositors. In this situation, the regulator steps in to resolve the financial institution.

I first characterize equilibrium outcomes when uninsured deposits are subordinated to the deposit insurance fund, that is they are paid in last order. If a financial institution fails, the regulator will use some tax revenue from the deposit insurance fund to make payments to insured depositors. In the course of resolution, the bank's remaining assets must be distributed to insured deposits first, then to the insurance deposit fund. Any remaining resources are then distributed evenly among the uninsured deposits. Therefore, the uninsured depositors who are late to take out their funds must bear all of the losses from the failed institution. Fundamental bank runs, that is, a run by uninsured patient depositors to banks that are insolvent, can arise. The banking system becomes more susceptible to a run when the deposit insurance coverage expands because losses from failed banks must be concentrated among smaller group of depositors who are uninsured. Therefore, it raises the incentive to withdraw early for those uninsured depositors who have an opportunity to withdraw early.

I then study the case where the regulator treats uninsured deposits equally with the deposit insurance fund when dividing the losses associated with a failed financial institution. In this case, some losses are passed as the burden to the taxpayers where

[2]According to FDIC (2018) annual report, the percentage of insured deposits among insured institutions varies over time. At the time of introducing FDIC in 1934, only about 45% of domestic deposits were insured and it was reported 59.6% in 2018. The substantial percentage of uninsured deposits justifies modeling of partial deposit insurance.

the burden is measured by lower production of public goods in this environment. This second policy provides risk sharing by effectively shifting resources for the public consumption to the private consumption. Allowing this risk sharing can lower some of depositors' incentive to withdraw early. However, it can distort the ex ante incentive of the financial institutions. Anticipating that their depositors will face lower losses, financial institutions will increase their short-term liabilities. This second policy may promote financial stability because the benefit of the risk sharing that yields smaller losses to depositors who wait to withdraw tends to dominate the incentive distortion that would raise payments to early withdrawals. Based on numerical examples, I show that it can often be desirable to adopt the policy where the claims of uninsured depositors and the deposit insurance funds are given equal priority because doing so both raises welfare and makes the economy less susceptible to a run by uninsured depositors.

Literature There are numerous papers that study deposit insurance theoretically and empirically. Demirg-Kunt and Kane (2002) review a large set of countries with deposit insurance. Calomiris (1990) and Allen, Carletti, and Leonello (2011) provide empirical evidence that the deposit insurance historically induces banks to hold more risky assets. Meanwhile, Kareken and Wallace (1978) and Merton (1977) provide theoretical models in which deposit insurance creates a moral hazard problem.

Boyd, Chang, and Smith (2002) and Cooper and Kempf (2016) discuss the welfare impact of deposit insurance, however, their models consider only full deposit insurance. While the model in this paper assumes that all agents are ex ante homogenous, Cooper and Kempf (2016) consider heterogeneous investors and focus on the distributional effect of deposit insurance and the order of liquidation and show that self-fulfilling bank runs can be eliminated under certain conditions. Their results rely on an assumption that the regulator lacks commitment power. Here, I consider a partial deposit insurance system that is chosen ex ante to study different policy rules on the distribution order of consumption by different types of agents.

Many studies have been done to analyze the issue of limited commitment (including among others, Keister (2016); Chari and Kehoe (2016); Cooper and Kempf (2016)). As briefly discussed earlier, setting equal priority on the uninsured depositors with the

DIF has similar flavor of where the regulator has limited commitment despite subtle differences. Government guarantees cannot be fully provided ex-ante because of moral hazard considerations and perhaps (economic and/or political) feasibility. The policy regime where the DIF has higher priority than the uninsured depositors can be considered as the policy with commitment not to provide any bailouts ex-post. The regime where uninsured depositors is equally treated as the DIF allows bailouts in different form by providing more insurance than promised ex ante. Anticipating this outcome creates incentive distortions for rational depositors and financial institutions.

In this paper I assume that deposit insurance covers only a fraction of depositors. Manz (2009) and Dreyfus, Saunders, and Allen (1994) discuss the optimality of placing a cap on deposit insurance when all deposits are insured. The main focus of this paper does not lie on finding the optimal provision of deposit insurance. Instead, this paper is interested in how different resolution policy affect welfare and fragility of an economy for a given partial deposit insurance system.

More recent studies are done by Davila and Goldstein (2016) and Jarrow and Xu (2015). They incorporate a bank run in the welfare function and look for the optimal deposit insurance. These papers focus on the effectiveness of deposit insurance to reduce the likelihood of bank failures. I find in this paper that expansion to the wider range of depositor does not necessarily reduces the probability of failures.

In the next section, the basic framework of the model is discussed. Section 3.3 studies equilibrium outcomes when the uninsured deposits are subordinated in the resolution process, and section 3.4 characterizes equilibrium outcomes when the uninsured deposits are treated equally with the deposit insurance fund. I also compare how the distribution rules affects welfare and fragility of an economy. Finally, it concludes in section 3.5.

3.2 The Model

The model is based on a version of the Diamond and Dybvig (1983) model augmented to include fiscal policy and an idiosyncratic uncertainty. In particualr, each bank faces idiosyncratic uncertainty about the return from its investment, and a low return results in bankruptcy. This feature is intended to highlight the role of deposit insurance and the resolution of failing banks. I begin by describing the basic elements of the model.

3.2.1 The Environment

There are three time periods, $t = 0, 1, 2$, and three types of economic agents - depositors, banks and a regulator. In this economy, there is a continuum of ex-ante identical depositors in each of a continuum of banks.

Each depositor is endowed with one unit of private good and has preferences given by

$$U(c_1, c_2, g; \omega) = u\left(c_1 + \omega c_2\right) + v\left(g\right),$$

where c_t is consumption of the private good in period t and g is the level of a public consumption good, which can only be provided and consumed in period $t = 2$. The utility functions u and v are assumed to satisfy usual conditions - continuously differentiable, strictly increasing, strictly concave, and the Inada conditions. Moreover, assume that $-\frac{u''(c)c}{u'(c)} > 1$ as in Diamond and Dybvig (1983) . A depositor's preference type ω, which is a binomial random variable with support $\Omega = \{0, 1\}$, is realized and revealed privately at the beginning of period $t = 1$. A depositor with $\omega = 0$ is *impatient* and values private consumption only in period $t = 1$. A depositor with $\omega = 1$ is *patient* and values consumption in both periods of $t = 1, 2$. Let π denote the probability with which each depositor turns out to be impatient. By law of large numbers, a fraction π of depositors will be impatient in period 1.

There is a single linear technology that transforms endowments into private consumption in the later periods. A unit of the good invested in period 0 can be transformed into $R > 1$ units in period 2 and 1 unit in period 1. This investment technology is owned by banks in a central location and by the regulator. Suppose that there is one-to-one transformation technology between the private consumption and the public consumption good in period 2, using goods that were invested in period 0.

In period 0, depositors pool their resources to insure individual liquidity risk. A depositor who deposits his/her endowment in period 0 is promised to receive c_1 if withdrawn in period 1 and c_2 if withdrawn in period 2. The payments to depositors are set in period 0 by perfectly competitive banks. Perfect competition in the banking sector implies that banks operate to maximize depositor's expected utility from private consumption.

At the beginning of period 1, each bank faces the idiosyncratic fundamental shock $s \in \{L, H\}$, which determines the rate of return of its investment. For each bank, the state $s = L$ is realized with probability $q \in (0, 1)$ where $R_L < R_H$. Assume no aggregate uncertainty in this economy.

Upon learning his/her preference type and his/her bank's state at the beginning of period 1, each depositor chooses either to withdraw in period 1 or to wait until period 2. Depositors who choose to withdraw in period 1 arrive at their bank one at a time and must consume immediately. Following Ennis and Keister (2010), assume that depositors arrive at their banks in the order indexed by i. That is, a depositor whose position is $i = 0$ knows that s/he would be the first one to withdraw at $t = 1$ when she decides to withdraw. A depositor whose position is $i = 1$ knows that s/he would be the last one to withdraw.

A benevolent regulator can tax deposits in period 0, then use these resources to put them into deposit insurance fund (henceforth, DIF) or to invest until period 2. It operates the technology in period 2 to transform private consumption goods to public goods. The regulator cares about welfare of the economy, which is measured by the equally weighted sum of depositor's expected utilities

$$W = \int_0^1 \mathbb{E} \left[U \left(c_1(i), c_2(i), g \ , \ \omega_i \right) \right] di.$$

3.2.2 Partial Deposit Insurance

Assume that there are two types of depositors in this economy - *insured* and *uninsured*.[3] In this economy, only a fraction $(1 - \phi)$ of depositors are insured. In any circumstances, the insured depositors are guaranteed to obtain the promised payment upon withdrawal.

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[3]Instead of assuming that all depositors are insured only up to a cap on their deposits, I assume that a certain fraction of depositors are born insured as an abstraction.

That is, an insured depositor is paid a promised payment c_1 if withdrawn in period 1 and c_2 if withdrawn in period 2. I further assume that depositor's types - whether s/he is insured or uninsured - are observable, but banks are not allowed to discriminate in providing a deposit contract between them. Each bank is equally likely to take deposits from both types of depositors.

When banks fail, the regulator takes charge of insuring the deposits and resolving the failed banks. The regulator accomplishes payment of insured deposits using its funds in period 1 and period 2. In the resolution process, the regulator is able to collect all information on the assets and liabilities at no cost. Then, it must the distribute available assets to cover liabilities in receivership, including its own.

3.2.3 Timeline

The timeline of this economy is illustrated in Figure 3.1. In period 0, depositors deposit their endowments at the central location. The regulator then collects a fraction τ of deposits as a tax revenue.

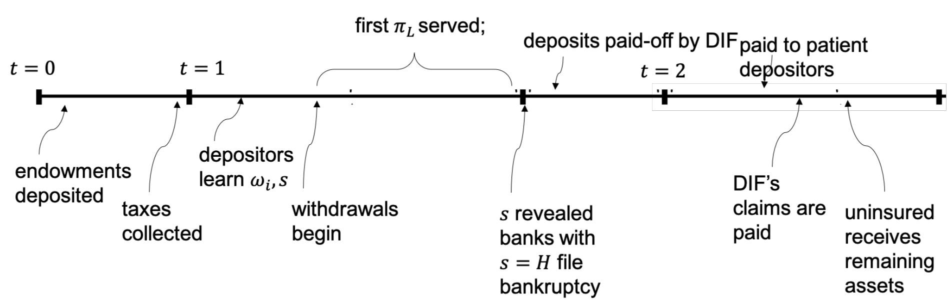

Figure 3.1: Timeline of events

At the beginning of period 1, each depositor learns his/her own preference type ω and the state s of his/her bank, then decides whether to withdraw in period 1 or to wait to withdraw until period 2. After choosing to withdraw in period 1, depositors arrive at their bank one at a time in the order indexed by i. Once s/he receives the payment from the bank, the depositor must consume immediately and is not allowed to re-deposit its consumption.

After serving the first π withdrawals in period 1, banks are able to learn the realization of the state s. Banks with $s = L$ effectively lose some resources so that the total remaining liabilities exceed the assets. These banks must file for bankruptcy and are taken over by the regulator. The regulator immediately can use its DIF to make payments to the insured depositors who choose to withdraw in period 1 and have not yet been served. In period 2, those patient depositors who have not yet withdrawn are paid a promised amount in the first priority. The regulator then divides the remaining assets to claims on receivership by following a stipulated order. In the following sections, I analyze equilibrium of an economy under different policy regimes in terms of the payment ordering. Then, I ask whether one policy regime is more desirable than the other.

For a given policy ϕ, we characterize the equilibrium of the economy by working backward through the timeline in Figure (3.1). I first describe the resolution policies for the failed banks and then study the period 0 deposit contract schedule made by each bank. Finally, the regulator sets the tax rate to maximize welfare.

3.3 Equilibrium with the Uninsured Subordinated

In this section, I study equilibrium under a policy where the uninsured deposits are subordinated to the claim of the DIF when distributing the remaining assets of a failed bank. In the analysis that follows, I study the best responses of banks and the regulator to a particular withdrawal strategy profile of depositors and then analyze whether this withdrawal strategy is best responding to banks, the regulators, and all other depositor's strategies.

3.3.1 Depositor's strategy

Each depositor chooses either to withdraw in period 1 or to wait until period 2 after observing his/her own preference type and the fundamental state s. Denote y_i as the depositor i's strategy, where $y_i = t$ corresponds to withdrawing at t. In the overall game in this economy, an equilibrium consists of a strategy profile for all depositors, together with allocations chosen by banks and the regulator such that all economic agents are best responding to others' strategies.

Consider the following partial-run strategy profile:

$$\begin{aligned} y_i^{\text{Insured}}(\omega_i, s) &= \omega_i + 1, \quad \text{for all } \ i \ \text{ and } \ s, \\ y_i^{\text{Uninsured}}(\omega_i, H) &= \omega_i + 1, \quad \text{for all } \ i \ \text{ and} \\ y_i^{\text{Uninsured}}(\omega_i, L) &= \begin{cases} 1 & \text{for } \ i \leq \pi \\ \omega_i + 1 & \text{for } \ i > \pi. \end{cases} \end{aligned} \tag{3.1}$$

When a depositor is insured, s/he is able to obtain promised payments upon withdrawal. Insured depositors would have no incentive to deviate from withdrawing according to his/her preference type. That is, impatient insured depositors always withdraw in period 1 and patient insured depositors would choose to wait until period 2.

Under this profile, each uninsured patient depositor with $i > \pi$ does not have an opportunity to withdraw early before the bank fails in state L . If an uninsured patient depositor chooses to claim in period 1 after bank's failure, s/he would get nothing. Whereas, s/he may obtain positive payment at the end of resolution if s/he chooses to wait. Therefore, those uninsured patient depositors with $i > \pi$ would choose to wait until period 2 when the state is L. An uninsured patient depositor with $i \leq \pi$ has an opportunity to withdraw early before the bank fails when the state is L. S/he will get less than what was promised if s/he chooses to wait until period 2. Thus, s/he will choose to withdraw early if s/he anticipates lower payments in period 2. Let $\mathbf{y} = \left\{ y_i^{\text{Insured}}(\omega_i, s), y_j^{\text{Uninsured}}(\omega_j, s) \right\}_{\{\forall\ i,j,s,\omega_i,\omega_j\}}$ denote a withdrawal strategy profile for all depositors. With this given strategy profile, the fraction $\phi(1-\pi)^2$ is the number of patient depositors who are uninsured and choose to wait until period 2 at $s = L$.

In the next subsections, I study an equilibrium allocation by first deriving the best responses of banks and the regulator to the above strategy profile.

3.3.2 Resolution of failed banks

In period 1, banks observe their bank-specific state $s \in \{H, L\}$ after π withdrawals are made. Banks with state $s = L$ file for bankruptcy because the total remaining liabilities exceeds total assets due to a lower realization of investment return. The regulator then takes over the bank and pays the promised payment c_1 to the remaining insured depositors who withdraw in period 1. For the given strategy profile (3.1), additional $\pi(1-\pi)\phi$ depositors have chosen to withdraw in period 1 but not yet been served. Among them, the fraction $(1-\phi)$ are insured and impatient. The regulator in period 1 must fulfill the promised payment c_1 to these $(1-\phi)\pi(1-\pi)\phi$ depositors. To minimize any loss to the insurance fund, the regulator charges a gross interest rate R for the funds used to pay these insured depositors. In period 2, the proceeds are distributed in the order of the remaining insured deposits, and liabilities by the DIF. Uninsured deposits are paid last in the order, the remaining resources are distributed evenly among these uninsured depositors if any. Therefore, the amount that a patient uninsured depositor can receive if s/he waits until period 2 is

$$c_{2L} = \max\left\{\frac{R_L\left(1-\tau-\pi c_1\right)-\left(1-\phi\right)\left(1-\pi\right)c_2-R(1-\phi)\phi\left(1-\pi\right)\pi c_1}{\phi\left(1-\pi\right)^2},\ \ 0\right\}. \tag{3.2}$$

The first term in the numerator represents the total remaining assets earned from investment in period 2 in state $s = L$. The fraction $(1-\phi)(1-\pi)$ depositors, who are insured and patient, wait to claim in period 2 and receive a guaranteed payment, c_2. The claims by DIF, $R(1-\phi)\phi\left(1-\pi\right)\pi c_1$, are paid with the second order.

3.3.3 Payment Schedule

Given the tax policy τ and the withdrawal strategy profile (3.1), each bank must choose how much consumption to give to each of depositors who withdraw in period 1 and in period 2. The payment schedule $\{c_1, c_2\}$ will be chosen to maximize

$$U(\tau\,;\mathbf{y}) = \max_{\{c_1,c_2\}} \; (1-q)\left[\pi u(c_1) + (1-\pi)\, u(c_2)\right] \\ +q\left[\begin{array}{l} \pi\left(1+(1-\phi)\phi(1-\pi)\right) u(c_1) + \phi^2 (1-\pi)\,\pi u(0) \\ +(1-\phi)(1-\pi)\, u(c_2) + \phi(1-\pi)^2 u(c_{2L}) \end{array}\right] \tag{3.3}$$

subject to the feasibility constraint

$$\pi c_1 + (1-\pi)\frac{c_2}{R} \leq 1-\tau, \tag{3.4}$$

and (3.2). The first line in (3.3) corresponds to the expected utility when the state is H. The second line represents the first fraction π of depositors who will receive the promised payment c_1 before the bank knows its state, and the $(1-\phi)\pi(1-\pi)\phi$ insured impatient depositors who will be paid the amount c_1 through the DIF. The fraction $\phi^2(1-\pi)\pi$ are uninsured impatient depositors who receive nothing, the fraction $(1-\phi)(1-\pi)$ are the insured patient depositors who are promised to receive c_2, and finally, the uninsured patient depositors will share whatever amounts remains according to equation (3.2). The bank takes the possibility of a failure due to the fundamental shock into account, and therefore considers how its choice of payment to the insured depositors affects the consumption by uninsured depositors.

The solution to the problem is characterized by the first-order-condition

$$\pi_1 u'(c_1) = \begin{cases} \frac{\pi R}{1-\pi}\pi_2 u'(c_2) + \pi q\left[R_L - R(1-\phi)\left(1-(1-\pi)\phi\right)\right] u'(c_{2L}) & \text{for } \; c_{2L} > 0 \\ \frac{R}{1-\pi}\pi_2 u'(d_2) & \text{for } \; c_{2L} = 0 \end{cases} \tag{3.5}$$

where $\pi_1 = \pi\left[(1-q) + q\left(1+(1-\phi)\phi(1-\pi)\right)\right]$ and $\pi_2 \equiv (1-\pi)\left(1-q+q(1-\phi)\right)$ denote the expected number of depositors who withdraw the amounts c_1 and c_2, respectively. This condition together with (3.2) and (3.4) implicitly defines the optimal choice of $\{c_1, c_2\}$ as functions of the tax rate τ. The allocation c_{2L}, then, also can be expressed as a function of the tax rate from equation (3.2).

3.3.4 Choosing the tax rate

The regulator chooses the tax rate in period 0 for the given strategy profile in (3.1) to maximize the welfare of the economy

$$W = \max_{\tau} U(\tau\,; \mathbf{y}) + v(g), \tag{3.6}$$

subject to the budget constraint

$$\begin{aligned} g = \quad & R(\tau - (1-\phi)\phi(1-\pi)\pi c_1) \\ & + q\max\{R(1-\phi)\phi(1-\pi)\pi c_1,\ R_L(1-\tau-\pi c_1) - (1-\phi)(1-\pi)c_2\}. \end{aligned} \tag{3.7}$$

The regulator can use all of its tax revenues collected if it recovers all its deposit insurance funds in period 2, or may suffer losses if the failed banks have fewer resources left to cover all insured deposits, resulting in lower level of public goods. The first-order condition for this problem when the deposit insurance funds are fully repaid is

$$\mu_1 + qR_L u'(c_{2L}) = Rv'(g) \tag{3.8}$$

where μ_1 is the Lagrangian multiplier associated with the feasibility constraint in equation (3.4). The regulator chooses the optimum tax rate at the beginning of period 1 by equating the expected marginal value of private consumption and the marginal value of public consumption, which is the same as the standard Samuelson condition for the optimal provision of public goods.

3.3.5 Equilibrium

The above subsections study the best responses of banks and regulator to the withdrawal strategy profile in (3.1). The best responses are summarized by the allocation vector

$$\mathbf{c}^A = \left\{c_1^A, c_2^A, c_{2L}^A, g^A\right\}$$

that is determined by equations (3.2), (3.4)-(3.5) and (3.7)-(3.8).

Now I need to check whether the strategy profile in (3.1) is part of an equilibrium. Recall that all impatient depositors will always withdraw in period 1 because they value consumption in period 1 only. Moreover, all insured patient depositors will wait until period 2 because they are promised to receive an amount c_2^A, which in equilibrium, is greater than c_1^A. An uninsured patient depositor with $i > \pi$ would choose to wait in state L because s/he can receive $c_{2L}^A \geq 0$ if waiting until period 2, whereas get nothing if withdrawing in period 1. When the state is L, an uninsured patient depositor with $i \leq \pi$ who has opportunity to withdraw early before the bank files a bankruptcy receives c_1^A if s/he runs on the bank and c_{2L} if s/he wait to withdraw later. Therefore, I can construct an equilibrium in which uninsured depositors follow the strategy profile in equation (3.1) if the equilibrium allocation satisfies $c_1^A > c_{2L}^A$. The following proposition states that an equilibrium can sometimes exist.

Proposition 12 *For some parameter values, there exists an equilibrium where some uninsured patient depositors withdraw in period 1.*

I define that an economy is *fragile* if there exists an equilibrium where a nontrivial mass of patient depositors at banks with $s = L$ withdraw in period 1. That is, an economy is fragile when the strategy profile in equation (3.1) is part of an equilibrium. Let Φ denote a set of economies that have (3.1) as part of an equilibrium.

Figure (3.2) depicts the set Φ^A under the policy regime where uninsured deposits are subordinated as the parameters ϕ and R_L are varied, using the utility functions

$$u(x) = \frac{(x + 0.1)^{1-\gamma}}{1 - \gamma} \quad \text{and} \quad v(x) = \delta \frac{(x)^{1-\gamma}}{1 - \gamma}$$

where δ represents depositors' relative preference over private consumption and public consumption. The set of other parameters for this numerical example is given by $(R, \pi, \gamma, \delta, q) = (1.05, 0.5, 2.5, 1, 0.03)$. The dark area plots the region as a function of R_L and Φ^A that satisfies $c_1^A > c_{2L}^A$ in equilibrium.

For a fixed value of ϕ, the economy is more likely to be susceptible to a run by some of uninsured patient depositors when the realized return from investment is smaller.

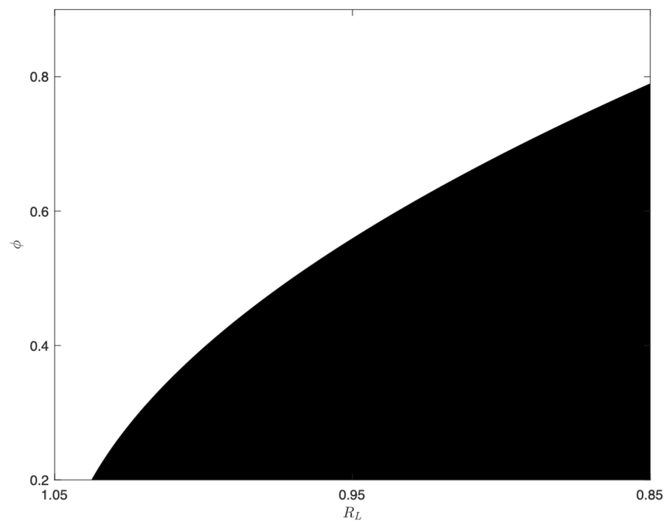

Figure 3.2: The fragile set Φ^A under the policy regime where uninsured deposits are subordinated

When the bank fails, the remaining uninsured depositors, if they choose to wait until period 2, must suffer larger losses as fewer resources are available when R_L decreases. This fact raises the incentive for these depositors to join a run when they have an opportunity to withdraw early.

There are two distinct reasons why the banks are less fragile when there are more uninsured depositors (i.e., higher ϕ). If there are larger group of depositors who are uninsured, the bank's losses can be spread among them. These depositors would then lose less on their deposits. Moreover, higher ϕ implies that more uninsured depositors become impatient and exit the bank, leaving their funds within the banking system, and the remaining uninsured depositors share these funds. To be specific, each uninsured patient depositor gains from unclaimed deposits from impatient depositors by $\phi\pi/(1-\pi)$, which increases in ϕ. Therefore, the banking system is likely to be less fragile as the fraction of uninsured depositors is larger.

The traditional view suggests that expanding deposit insurance coverage may increase or decrease the riskiness of the banking system depending on two competing effects. The system becomes less riskier as the deposit insurance expands because the size of the event would be smaller if there is a run by uninsured depositors. On the other hand, it may elevate the moral hazard problem, creating banks' incentive to raise the short-term liabilities and make the banking system more susceptible to a run.

In this chapter, the model explains another side for the effect of expanding deposit insurance coverage on the fragility in the banking system. When fewer depositors are uninsured, the losses of a failed bank must be concentrated on a smaller group. This fact implies a higher incentive for the uninsured depositors to withdraw early before the bank fails, making the banking system more fragile. That is, an economy may increase the riskiness of the banking system by shrinking deposit insurance.

In the next section, I analyze an equilibrium of the economy where uninsured deposits given equal priority with the DIF and compare whether such a different policy regime improves welfare and promotes stability.

3.4 Equilibrium with the Uninsured Treated Equally

As discussed in the earlier section, upon the bank's failure and paying out the amount of the insured payments, the DIF and uninsured depositors act as the creditors. Consider now that these claimants have equal priority on the residual assets of the failed bank and are paid equally from the matured assets.

3.4.1 Resolution of failed banks

Under the policy regime where uninsured deposits are treated equally to the DIF, they share losses on a pro-rata basis based on their respective percentages of total deposits. Define the payout rate of to be the ratio of total remaining assets to total claims to the failed bank, as follows:

$$\eta \equiv \frac{R_L\left(1-\tau-\pi c_1\right)-\left(1-\phi\right)\left(1-\pi\right)c_2}{R(1-\phi)\phi\left(1-\pi\right)\pi c_1+\phi\left(1-\pi\right)^2 c_2}, \tag{3.9}$$

where the numerator refers to the total remaining assets after fully providing the insured deposits and the denominator denotes total claims by creditors to the failed bank - the DIF and uninsured deposits.

Under this policy regime, each patient depositor who is uninsured receives

$$c_{2L} = \max\left\{\eta c_2, 0\right\}. \tag{3.10}$$

3.4.2 Payment Schedule

Given the tax policy τ and the withdrawal strategy, each bank now must choose the payment schedule $\{c_1, c_2\}$ to maximize (3.3) subject to the feasibility constraint (3.4) and (3.10). The solution to the problem is characterized by the first-order-condition

$$\pi_1 u'\left(c_1\right) = \begin{cases} \frac{R\pi}{1-\pi}\pi_2 u'\left(c_2\right) + q\pi u'\left(c_{2L}\right)\frac{R_L - R(1-\phi)\left(1-\phi(1-\pi)\frac{c_{2L}}{c_2}\frac{1-\tau}{1-\tau-\pi c_1}\right)}{1+\frac{(1-\phi)\pi c_1}{1-\tau-\pi c_1}} & \text{for } c_{2L} > 0 \\ \frac{R}{1-\pi}\pi_2 u'\left(d_2\right) & \text{for } c_{2L} = 0. \end{cases} \tag{3.11}$$

This first-order condition, together with (3.4) and (3.10), implicitly defines the optimal choice of $\{c_1, c_2\}$ as functions of the tax rate τ. The allocation c_{2L} also can be expressed as a function of the tax rate using equation (3.10).

3.4.3 Choosing the tax rate

Under this policy regime, the regulator may not able to fully recover its deposit insurance funds lent to the banks as it receives only η percent of its claims in period 2. The tax burden from providing ex post risk sharing is measured by lower level of provision of the public goods according to the regulator's budget constraint. That is,

$$g = \begin{cases} R\left[\tau - q(1-\phi)\phi\left(1-\pi\right)\pi c_1\left(1-\eta\right)\right] & \text{for } \eta \geq 0 \\ R\left[\tau - q(1-\phi)\phi\left(1-\pi\right)\pi c_1\right] + q\left[R_L\left(1-\tau-\pi c_1\right) - \left(1-\phi\right)\left(1-\pi\right)c_2\right] & \text{for } \eta < 0 \end{cases} \tag{3.12}$$

where q is the total number of banks that failed in period 1, $(1-\eta)$ is the loss rate of DIF's claims with which the amount $R(1-\phi)\left(1-\pi\right)\pi c_1$ are made by paying out to the insured deposits. The regulator must bear all losses if the remaining assets are not enough to cover all insured deposits. The regulator will choose the tax rate τ to maximize equation (3.6) subject to the budget constraint (3.12). The first-order condition is given by

$$\begin{aligned}&\mu_1 + u'\left(c_{2L}\right)\frac{R_L q}{1+\frac{(1-\phi)\pi c_1}{(1-\tau-\pi c_1)}}\\&= Rv'\left(g\right)\left[1 - q(1-\phi)\phi(1-\pi)\pi\left((1-\eta)\frac{dc_1}{d\tau} - c_1\frac{d\eta}{d\tau}\right)\right]\end{aligned}\tag{3.13}$$

where μ_1 is the Lagrangian multiplier associated with the feasibility constraint (3.4). The optimal tax rate τ is chosen by equating marginal value of private consumption and the marginal value of public consumption. The regulator can partially correct the incentive distortion created from the loss-sharing with the uninsured depositors for those banks that fail by affecting banks' choice of c_1 through τ.

3.4.4 Equilibrium and Fragility

The consumption allocation

$$\mathbf{c}^B = \left\{c_1^B, c_2^B, c_{2L}^B, g^B\right\}$$

that satisfies equations (3.4) and (3.10)-(3.13) represents the best responses of banks and the regulator to the strategy profile $\mathbf{y}$ under the policy regime where the uninsured deposits have the same priority as DIF. As in the previous section, the strategy profile (3.1) is part of an equilibrium if the allocation satisfies $c_1^B > c_{2L}^B$. Let Φ^B denote the set of economies for which this condition holds under the policy with equal treatment of between the DIF and uninsured deposits.

The fragile set is demonstrated in Figure (3.3), which adds the set Φ^B to the set Φ^A from Figure (3.2). The relationship between financial fragility and these parameters are similar under both policy regimes . When the investment return decreases sharply, there will be fewer resources left to the remaining depositors, raising the incentive for an uninsured patient depositor to withdraw early in period 1 under both policies. A decrease in the deposit insurance coverage can make the economy less susceptible to a run because losses from a failed bank can be spread among larger group of depositors who are uninsured and withdraw late.

Loss sharing between the DIF and the uninsured deposits affects on depositor's

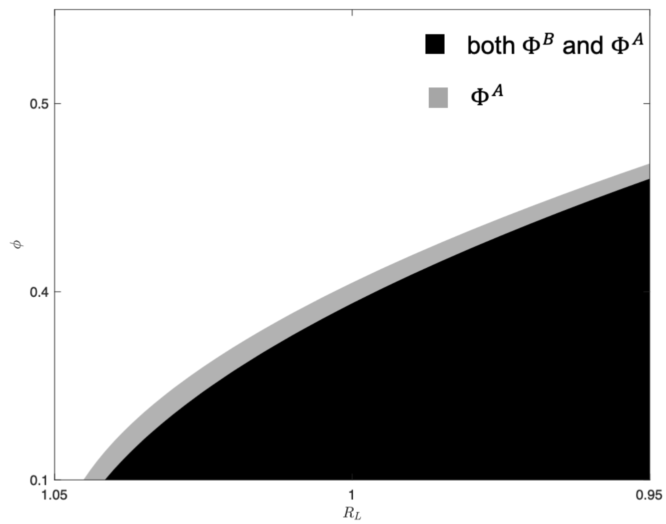

Figure 3.3: Comparing the sets Φ_A and Φ_B

withdrawal incentives. As public funds are available to mitigate losses from a failed bank, the consumption level c_{2L} for the remaining uninsured depositors may not decrease as much as in the case where no public funds share the losses. It would then tend to decrease the incentive for those uninsured patient depositors who have an opportunity to withdraw their funds early. On the other hand, it creates an incentive distortion for the banks. As bad outcomes partially mitigated, the banks would raise their short-term liabilities, which raises the consumption by depositors who withdraw early. When the effect of loss sharing on depositor's withdrawal incentive dominates, a policy where the uninsured deposits are treated equally to the DIF may promote financial stability. Numerical example shows that the fragile set under the policy where the uninsured deposits and the DIF share losses can often be smaller, which implies that such policy can promote financial stability.

3.4.5 Deposit insurance and Welfare

In the previous section, I show that under both policy regimes the banking system can sometimes be fragile. To further evaluate which policy regime is more desirable than the other, the regulator must compare the welfare levels as well conditional on the banking system fragile.

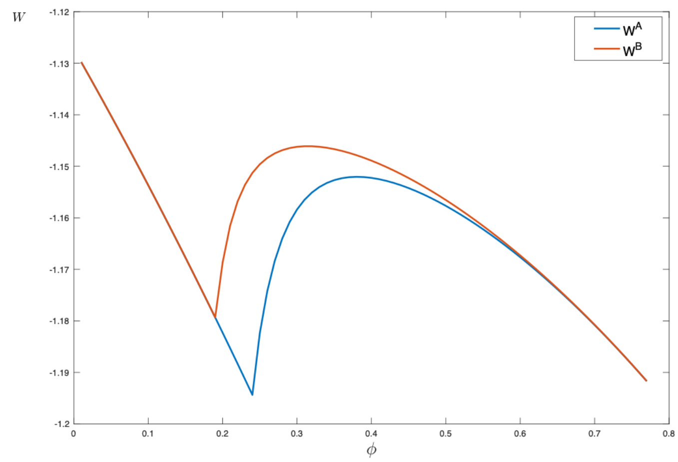

Figure 3.4: Welfares under two policy regimes

Figure 3.4 depicts how welfare varies with the level of deposit insurance coverage under both policies. When ϕ is close to zero, that is, when nearly all deposits are insured, the level of welfare decreases as ϕ increases. When ϕ is very small, all resources of a failed bank will be distributed to insured depositors and the DIF must even bear losses to pay the insured payments to depositors. In this case, there is no gain from increase in the number of uninsured deposits. Depositors who are uninsured are likely to receive no private consumption. An increase in ϕ only raises the fraction of depositors who receive nothing. Although losses for the deposit insurance fund are smaller as it reduces the deposit insurance coverage, the marginal utility losses from zero private consumption by the uninsured depositors are very large. Therefore, increasing ϕ lowers the level of welfare.

When there are moderate number of uninsured depositors, however, a further contraction of the deposit insurance coverage can raise the level of welfare. With the moderate coverage of deposit insurance (ϕ that exceeds from the kink points in Figure 3.4, around 15% on W^B and 22% on W^A, for example), a smaller fraction of resources are promised to be given to the insured depositors en masses and banks become more cautious against worse outcome from larger set of uninsured deposits. This implies that the DIF would bear smaller or no losses and the remaining uninsured depositors can enjoy more proceeds that are left in a failed bank. The marginal utility of the uninsured depositors is initially very large when their payments are relatively small. As ϕ increases, these effects dominate the welfare loss by additional impatient depositors who are uninsured and receives nothing. Therefore, the level of welfare starts to increase as the remaining uninsured patient depositors starts to obtain nontrivial amount of resources. However, as ϕ increases further, the level of welfare decreases because marginal utility gains from the uninsured patient depositors diminishes, on the other hand, higher fraction of impatient depositors who are uninsured still receive zero private consumption.

The levels of welfare are the same under both policy when the remaining patient depositors receive nothing and the DIF bears losses for failed banks. Welfare level starts to differ from one policy to the other as the remaining depositors begins to obtain positive amount of private goods depending on the distribution rule.

In general, the kink point of ϕ at which the welfare starts to increase is smaller under policy where the DIF and uninsured deposits share losses than the other. Clearly, loss-sharing implies that the remaining uninsured patient depositors obtain nonzero consumption at a lower ϕ. Although the risk sharing between private and public consumption under this policy can improve welfare, the rate at which banks become more cautious and lower the short-term liabilities would be slower as ϕ increases because perceived bank's riskiness is mitigated by the risk sharing. That is, benefit of reducing the incentive distortion by a decrease in deposit insurance coverage is smaller when the uninsured deposits have the same priority as the DIF than when their claims are subordinated.

This is demonstrated in Figure (3.4), where the level of welfare W^B begins to fall at a lower value of ϕ and decreases at a faster rate than W^A. In most cases, W^B is at least as large as W^A. Under both policy regimes, there are the same number of uninsured depositors who are impatient. When depositors are risk averse, the welfare gains from risk sharing between private and public goods are higher. The benefit of the policy where losses fall into the uninsured depositors first is that incentive distortion is lower. However, under the policy in which the DIF and the uninsured are treated equally, such distortion can be partially corrected when the regulator affects c_1 through adjusting the tax rate. In most cases, the gains from risk sharing dominates the loss from incentive distortion, implying that welfare is higher when the policy rule adopts the case where the DIF and uninsured deposits equally share any losses from failed banks.

3.5 Conclusion

This chapter studies how welfare and the financial fragility of an economy can differ depending on division rule of remaining assets in a failing bank. To see this, I have described a model in which banks face a fundamental uncertainty of insolvency, the regulator provides deposit insurance to only a fraction of depositors using its funds financed from tax revenues, and the uninsured depositors can potentially withdraw early from their banks when losses are anticipated.

My results indicate that providing loss-sharing between the uninsured deposits and the deposit insurance fund is often more desirable than letting uninsured deposits bear all losses from the failed banks. Despite the resulting incentive distortion, loss sharing between private consumption by uninsured depositors and public consumption can make the banking system less prone to a run and can improve welfare conditional on a fragile banking system. Moreover, an expansion of deposit insurance does not necessarily make the economy more stabile. Rather, it may make the economy more susceptible to a run. An increase in deposit insurance coverage not only creates an incentive distortion by making banks to raise short-term liabilities, but also makes the remaining depositors suffer more concentrated losses, which in turn, increase an incentive for uninsured depositors who have an opportunity to withdraw early to do so.

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