MARCH 1, 1999 - TABLE A: THE PIZZA PROBLEM

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<tr>
<th>Timecode</th>
<th>Speaker</th>
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<tr>
<td>00:00:00:23</td>
<td>T/R2</td>
<td>[Robert, Stephanie, and Shelly I. are sitting]...OK, then we didn't meet. And, Amy, I never met you. [Amy-Lynn arrives and sits down.] Hi. [To Amy-Lynn] I'm Gina Kiczek, how are you? [T/R2 sits down.] I'm a, uh, I'm a teacher in Jersey City. I teach at, uh, Ferris High School in Jersey City. And I'm a student of Dr. Maher's and I've been doing a lot of work with Rutgers and, um, I've seen a lot of you over the years in some of the videotapes. [Shelly laughs.] You know, you're famous, but, um, actually, uh, some of what I've been doing, some of my coursework through the years at Rutgers has had to do with things that you've been doing. So I've seen a number of you, so you look familiar to me. Stephanie I've met actually in the flesh [Stephanie laughs], and I've met Robert in the flesh, so I'm glad to meet you. Um, Amy, where are you in school now?</td>
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<td>00:00:48:28</td>
<td>Amy</td>
<td>Um, I'm at Mother Seaton.</td>
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<td>00:00:50:23</td>
<td>T/R2</td>
<td>Oh, OK, OK, and Michelle, Shelly, how about you?</td>
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<td>00:00:53:06</td>
<td>Shelly</td>
<td>...Catholic</td>
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<td>00:00:53:03</td>
<td>T/R2</td>
<td>All right. Well, we are, um, going to be working on these, these next couple of sessions, we're planning on working on some combinatorics tasks, um, for starters. I don't quite know where we're gonna go from there. It depends on where you all take us, actually is what we're going to be doing. And we were hoping that we could revisit some of the things that you had done earlier on, like in fourth grade and fifth grade and sixth grade and so forth. And one of those tasks you might remember having done is the pizza problem. Does that sound familiar to you?</td>
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<td>00:01:27:28</td>
<td>Shelly</td>
<td>No. I remember shirts and pants, but I don't remember anything else.</td>
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<td>00:01:29:24</td>
<td>T/R2</td>
<td>Shirts and pants? That's, that was second grade, so you're really going back. Well, this is the statement of the problem, so I'll just pass that out to you all. [T/R2 hands out papers.] Give you a second to read it, just at least the top of it, see if it</td>
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sounds familiar to you. [Camera focuses on Amy-Lynn's paper, which is partially visible.] Is this familiar? Yeah? Shelly, do you remember this at all?

00:02:13:07 Shelly A little.

00:02:14:13 T/R2 A little bit? A little bit? OK, um, I think, let me think back. I believe you first looked at this or first explored this in the fifth grade. Um, but it's our interest in finding out how you would go about doing this right now, how, uh, how you would like to pursue this and how you could answer at least the first, uh, two questions if not the other ones.

OK, do you want some time to talk about that? Think about that?

00:02:46:13 Shelly Sure. [Stephanie laughs.]

00:02:47:09 T/R2 So do you understand the problem? [To girls] Do you understand the problem? [To Robert]

00:02:49:18 Shelly Yeah.

00:02:49:27 Robert [Nods.] We did this one in the other class.

00:02:51:18 T/R2 OK. OK. Well, that's great. Then I'll leave you...

00:02:52:25 Shelly [To Robert] So you know the answer?

00:02:54:07 Robert Uh-huh.

00:02:54:22 Stephanie Yes!

00:02:55:24 Shelly Yes! We're done. [Stephanie and Shelly laugh.]

00:02:57:23 T/R2 Oh, but now is that all there is to it after all this time of working with Rutgers? Is that all there is to it?

00:03:01:25 Shelly No.

00:03:01:28 T/R2 OK. The answer is 17; I'm done. [Laughs]

00:03:08:21 Stephanie [Unintelligible]

00:03:09:15 T/R2 OK, I'm going to leave you alone.

00:03:12:10 Shelly We just did this in school.

00:03:14:08 Stephanie Did we really?

00:03:15:15 Shelly ...Combinatorics stuff

00:03:17:26 Stephanie Who do you have, Whitehead?

00:03:19:28 Shelly Mackenzie.

00:03:20:01 Stephanie Ooh, forget I said anything. [Laughs]

00:03:25:15 Shelly The grades in there are like A, D, ...

00:03:27:21 Stephanie I heard. [Laughs]

00:03:28:01 Shelly ...A, F. [laughs] [There is a pause while they look at the papers. Camera is focused on Amy-Lynn's paper, which is partially visible.]

00:03:40:10 Shelly This is combination, isn't it? Just...

00:03:44:29 Stephanie Um-hum.

00:03:46:01 Shelly So are we supposed to find an answer? [Laughs]

00:03:49:03 Stephanie I guess, in somewhere.
Shelly [Unintelligible] Um.
Stephanie Um. [Lifts soda can, drinks] You did this already?
Robert [To Robert]
Stephanie Yeah. A couple of days ago. Or a couple weeks ago.
Stephanie So, what was the quickest way to do it? [Laughs] I
mean, what else? I mean, ... [unintelligible]
Shelly Do you want to help us out here?
Stephanie No, I mean, did you guys just kind of like...
Robert I don't know.
Stephanie [Gestures with hand.]
Shelly Gees, we're supposed to remember from fourth
grade...
Stephanie ...And you can't remember two weeks ago. Um.
Robert No, it's more. It's like two months.
Stephanie Oh. Whatever. It doesn't matter. You're useless.
[Stephanie laughs]
Shelly Um. That's a nice way to put it.
Stephanie No. Um, do we just want to, um, plot out the
pizzas, like, how we would do, like with, um, shirts
and pants or towers. Do you know what I'm talking
about? [To Shelly]
Shelly Yeah.
Stephanie Where we just...
Shelly The tree diagram type thing. [Gestures]
Stephanie Yeah, [gestures] kind of like that. Or is there an
easier way to do it that I'm just not thinking of?
Shelly Can't you? Wait. Plain counts as one, right?
Stephanie Yeah, plain.
Shelly So there's...
Stephanie You can order just a plain pizza.
Shelly So there's six choices.
Stephanie Uh-huh.
Shelly So it's combination 6, isn't it? Something like that?
[Whispers] I need my calculator.
Stephanie [Punches calculator buttons] Too bad I don't
remember how to do combinations.
Shelly Well, we just did it.
Stephanie Didn't we do a little bit of it? In algebra two?
Shelly Can I get out my calculator?
Stephanie Yes. Hurry up and do that. [Drinks soda]
Amy-Lynn [Unintelligible] Did you do combinations?
Shelly [Punches calculator buttons] I think we're cheating
this way.
Stephanie I'll take it; they'll probably make us plot it out

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anyway.

00:05:15:22 Shelly No, but see. [Sighs] I'm not doing this right. [Punches calculator buttons]

00:05:18:23 Stephanie Well.

00:05:20:09 Shelly OK, we'll do it their way.

00:05:21:27 Stephanie Yeah. What...

00:05:22:00 Shelly But I'm stuck...

00:05:22:22 Stephanie Why don't we do it the other way and then once we get an answer, we can see if the combination thing...

00:05:27:12 Shelly Is right.

00:05:27:18 Stephanie ...Gives the same answer.

00:05:28:26 Shelly [Unintelligible]

00:05:29:00 Stephanie All right?

00:05:31:28 Shelly ...Confused

00:05:31:23 Stephanie All right. I need a piece of paper here.

00:05:35:17 Shelly Sounds good to me. Want some paper? [To Amy-Lynn] No?

00:05:35:19 Amy-Lynn Yeah, with the...[unintelligible]

00:05:38:10 Stephanie [Unintelligible]...plain pizza

00:05:43:01 Shelly I had a physics test today. I've done enough for today.

00:05:47:06 Amy-Lynn [Unintelligible]...chemistry.

00:05:51:09 Stephanie OK.

00:05:54:25 Stephanie So your combinations are...

00:05:57:24 Stephanie Plain pizza. Now everybody write plain. [Laughs]

00:06:04:05 Amy-Lynn [Unintelligible]...peppers.

00:06:07:12 Shelly This is... There's got to be an easier way.

00:06:09:01 Stephanie See, that's... you know what the thing is. Every time we do one of these, at the end, we find an easier way. And then every time we go back to do the next one, we have no idea how to do it [Girls laugh.] except to start from scratch.

00:06:19:12 Shelly That's good stuff.

00:06:20:03 Stephanie Huh?

00:06:21:26 Shelly OK. P...and peppers. Good stuff, Bobby.

00:06:25:21 Stephanie Then what is Bobby doing? I can't see. What are you doing? I can't see.

00:06:26:27 Shelly Sausage.

00:06:27:00 Robert Yeah, we're doing it.

00:06:27:01 Shelly Mushrooms.

00:06:29:19 Robert Just writing them out. Across.

00:06:32:10 Shelly Oh, pepperoni. That's going to be confusing.

00:06:35:25 Stephanie OK.

00:06:39:04 Shelly Oh you're writing it like that. I should try that.

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00:06:46:07 Stephanie [Unintelligible] All right. So then you can have [camera is focused on Robert's paper, but we can't read what is on it.] I don't know how to organize it.

00:06:46:11 Shelly I don't know what she's doing. Wouldn't it...Oh, OK. Wouldn't it be easier just to go like, plain, and then just go like, peppers, sausage, mushrooms, pepperoni? Or...I know there's an easier way to do this.

00:07:19:09 Stephanie Yeah, but see then when you do peppers...yeah. Then we're just going to have to go back and eliminate it. Cause if you go plain, peppers, and you go, it's the same as just having a peppers pizza. Like, plain with peppers [Robert reaches for a paper.] is the same as a peppers pizza.

00:07:31:26 Shelly OK.

00:07:32:00 Stephanie All right?

00:07:33:01 Shelly Uh-huh.

00:07:33:12 Stephanie So, yeah, you can do it like that, but we're just going to have to go back and like cross things out, when we're done. You know? [To Shelly]

00:07:40:08 Shelly My teacher's gonna kill me because she knows I could do this.

00:07:43:08 Stephanie Like she's ever going to see this. Umm. So yeah, let's just, let's just write it out and then double-check with everybody else's answer.

00:07:56:29 Shelly How are you writing it, though? Are you just putting...peppers?

00:07:58:08 Stephanie I'm just going to write plain, and then under it.

00:08:00:25 Shelly OK. So let's just write this.

00:08:02:16 Stephanie Plain with sausage.

00:08:02:14 Shelly Peppers.

00:08:04:01 Stephanie This is going to take like eight years.

00:08:06:17 Shelly Oh, you're doing it that way. [Camera is focused on Stephanie's paper. She is drawing lines and letters on a tree diagram.]

00:08:23:00 Stephanie Um. [She draws another line.] There's got to be an easier way to do this.

00:08:28:29 Shelly I don't know if it's factorial or combination. I don't know if you would just do like five factorial plus four factorial plus three factorial plus two factorial plus one factorial. Cause, OK, so for...do you have five. Ah.

00:08:42:18 Stephanie Right.

Shelly This is so confusing. I don't know how to explain it. I don't want to do it. Let's go over the easy way.
Peppers [writes on paper]. I can't remember. That was the last section we did. It's so pathetic.
Sausage. [All three girls are writing; the camera is focused on Robert's paper. Robert is drawing a tree diagram.]

Amy-Lynn  Plain. [Crosses out the word mushroom on her paper]
Shelly  Peppers.
Amy-Lynn  [One camera remains focused on Robert who is writing and not discussing anything with the others. The other camera is focused on Amy-Lynn's paper. Across the top is written "Plain Peppers Sausage Mushroom Pepperoni." There are lines and letters underneath Plain and Peppers. Amy-Lynn is drawing lines under sausage and mushrooms and writing letters.]

00:09:59:06  Shelly  [Camera moves to Shelly's paper, where we can see lines and words, but they are not readable.]

00:10:03:11  Shelly  I'm not doing this right.
Amy-Lynn  I think it's a factorial.
Shelly  [Unintelligible]
Amy-Lynn  [Unintelligible] [Shelly looks at Amy-Lynn's paper.]


00:10:22:09  Amy-Lynn  This [unintelligible] already done.

00:10:23:05  Shelly  Let's see how big that answer is. [Punches calculator buttons] OK. So this way you get 153. We'll have to see if that's right. Cause...no wait. That's the same thing. [Sighs]

00:11:17:12  Amy-Lynn  [Camera is focused on Amy-Lynn's paper on which is written "5!+4!+3!+2!+1!=153".] How many of these things do you have on a pizza? Do you just have two-

00:11:27:04  Amy-Lynn  ...Three, four, five.
Shelly  It doesn't matter. You can have one or two or three or four or five. [Sighs] Two, three, two, three, four, [unintelligible]. This is it; it's a factorial.

00:11:54:02  Stephanie  It is a factorial.
00:11:57:01  Shelly  Yeah. OK. Cause if you have...
00:11:57:01  Stephanie  Oh, here. [To someone reaching for soda cans]
00:11:59:20  Shelly  If you have, OK, you have five choices for the first one. Oh no, that's not. It might not be a factorial.

00:12:05:29  Stephanie  [Stephanie flops down on table.] Well, wait. Explain it.

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I'm trying. I'm trying.
Just wait. Explain it to me.
I thought I was right. OK.
Cause I don't....
Hold on. OK, there's plain and let's say you just have two toppings. And you have four choices there, and you have peppers, and you have three choices there, right? [sighs] [Camera focuses on Shelly's paper. She draws S.] Sausage, and then you have two, right? [Shelly draws two lines from the S.] Am I wrong? Plain. One, two, three, four.
And the mushrooms you'd have one choice.
[unintelligible] And then.
Pepperoni none.
Yeah.
Is that right? Mushrooms one.
Yeah, that's right. Cause [unintelligible]
Pepperoni none. And then from here you branch and you have three and then two and then one.
Right? [whispers] There's some way to do this.
OK. I'm trying. I'm trying. [laughs] I hate this.
Calm down. [Shelly laughs] It's going to be OK.
Five, one, two, three, four.
[The girls continue writing.]
[The camera focuses on Stephanie, who is writing and holding her head in her left hand.]
[The camera moves out to focus on all three girls as they continue writing.]
[The camera focuses on Amy-Lynn. Her paper has "PI" with four lines underneath, "Peppers" with three lines underneath, "Saus" with two lines underneath, "Mush" with one line underneath, and "Pepperoni" with no lines." She is writing "PI+Peppers+Sausage+Mushrooms+pepperoni".
Wait a minute. I'm gettin' five and then ten and then ten and then five.
[groans]
So then, then will the next row is just gonna go like that?...How far though...
Yeah... So five, and then ten, then ten, and then five and then one.
Uh-huh.
00:15:34:10 Shelly And that's it. Cause, well, there's no one in the, in the front, right? Cause ... [sighs]

00:15:45:10 Stephanie Wait. What are you trying to know?.
Shelly ...There's gotta be something. Cause you, you...
Stephanie Wait. Say it out loud, cause I...[gestures with left hand]
Shelly You can't have just a pepperoni...oh.
Stephanie Just a pep...
00:15:55:09 Shelly Cause a pepperoni is a plain with pepperoni.
Stephanie Yeah. So...
00:16:01:29 Shelly [shakes head] So that's why there's no one there, right? Right, right. Is that what you're getting? For numbers?
Stephanie I'm doing... I don't know. I'm getting that there's one with four and then there's, uh, three with three and then the next one, there's none with four and there's two with three and then the next one there's none with four and one with three. And the next one I'll have...

00:16:23:26 Shelly I'm so lost with that. [laughs]
00:16:24:25 Stephanie Yeah. It's just like [looks at Shelly's paper], just like what you were doing before. Like here [indicates Shelly's paper].
Shelly Yeah?
00:16:29:05 Stephanie Um. There's...oh, see, you're, you're working like, it would be like doing it backwards. There's ...we can have one pizza with all four toppings.
Shelly Uh-huh.
00:16:38:19 Stephanie And that's it.
Shelly Uh-huh.
00:16:40:01 Stephanie And then...
00:16:42:02 Shelly Oh, see, that's where I'm doing it wrong. See I have it plain and then I have them going there.
00:16:45:10 Stephanie No, you're not, you're just going the opposite way that I'm doing it.
00:16:47:17 Shelly No, because ... OK, one, four,...
Stephanie Like, I'm starting from the...
00:16:50:02 Shelly See these, all of these don't count [indicating her paper], cause that's with plain, that like plain up here. So all of these don't count. [Stephanie nods.] So it's one, and then one, two, three, four, and then one, two, three, four, five, six, and then four and then one.
00:17:02:12 Stephanie Yeah. You see that's...like look.

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Shelly: It's one, four, six, four, ten.
Stephanie: That's what it is. Like, see, there's four and then there's three of those...
Shelly: Yeah.
Stephanie: Well, actually yours is really, I don't know. There's one with that and then if you take it like from the peppers, there's three with peppers and two other toppings.
Shelly: Yeah.
Stephanie: And then there's...[camera is focused on Stephanie's paper, but it is not readable.]
Shelly: [mumbles] P with A. [Camera moves to Shelly's paper, which is also not readable.] Yeah, I think that's it. Five, five, ten, sixteen. It seems kind of small, though. Let me see. That wasn't it. [Shelly crosses out the top line on her paper.]
Shelly: [Camera moves over to Amy-Lynn's paper.]
Shelly: Um. Does that number look familiar to you? Sixteen?
Shelly: OK. I'm done. That's it.
Stephanie: ...I got 17. [Camera is focused on Robert's 's paper, where he is drawing diagrams.]. All right. Hold on.
Shelly: [Continues to write on paper, then counts and writes 16.] Oh, that makes more sense.
Stephanie: All right, wait. Can I... [Stephanie is talking to Shelly. She takes a blank piece of paper from Shelly's pile. No it is not.
Shelly: Feel free.
Stephanie: Well, thank you. [Shelly laughs.] All right, you're going to have to explain it to me, cause I don't understand...
Shelly: OK, so that's plain.
Stephanie: So I've got plain.
Shelly: Uh-hum.
Stephanie: All right. And then I've got one more.
Shelly: And then you have plain.
Shelly: I've got one with peppers.
Stephanie: Yeah. And one with mushrooms.
Stephanie: And one with mushrooms. Oh, wait. [Stephanie writes on her paper.] OK, I've got one with mushrooms.
Shelly: And then one with sausage, and one with
pepperoni is, peppers I mean. Pepperoni.

Yeah. Whatever. [nods her head] Same thing. [Stephanie and Shelly laugh.] OK. Then I have...

Plain with peppers with sausage. [points to her paper with her left hand] and then plain with peppers...

Um, wait

Shelly...

with mushrooms.

You have plain with peppers and sausage. OK, so I have that. [makes marks on her paper] OK.

And then plain with peppers with mushrooms.

Plain with peppers...

And

...and pepperoni.

OK.

OK, and then you have, um, plain with sausage with, yeah. Plain with sausage with mu, with mushrooms.

OK.

Plain, sausage, and pepperoni.

Uh-huh.

Um. Plain, mushrooms, pepperoni.

[marks paper] OK.

OK, and then you have plain, peppers, sausage, mushroom.

Uh-huh.

Plain, pepper, sausage, pepperoni. [Stephanie marks paper.] Plain, peppers, mushrooms, pepperoni. [Stephanie nods.]

OK.

Plain, sausage, mushroom, pepperoni. [Stephanie marks paper.] And you have plain, pepper, sausage, mushrooms, pepperoni, with all five. [Stephanie marks paper.]

This is...the same thing as that [marks paper]. OK! Sixteen.

[leans over her paper and sighs] So we got that. Sixteen. [writes on her paper] It's all possible choices. See tree diagram [as she is writing on her paper] [Stephanie and Shelly laugh.] All right. If you're gonna do it yourself, then I'm just gonna.

That's all your job.

I'm just gonna [unintelligible] Unless they wanted me to copy it. Because I'll.
00:20:20:24  Stephanie  They probably will.
00:20:22:01  Shelly   P-L equals plain.  P-E equals pepper.  [sighs]  M
equals mushrooms.  P equals [voice trails off]
[reads]  Find a way to convince each other that you
have accounted for all possible choices.  What's
that mean?
00:20:54:12  Stephanie  Are you convinced?  [to Shelly]
00:20:55:21  Shelly   Each other.  [laughs]
00:20:56:08  Stephanie  Yes, I think that means each other.  [Shelly laughs.]
I'm not sure.
00:20:59:17  Shelly   All right.  Sixteen.  We're done.  [reads]  Suppose a
fifth topping anchovies were available.
00:21:03:28  Stephanie  You know, there's got to be an easier way to figure
out, you know, than just adding anchovies.
00:21:11:05  Shelly   Did we, we didn't repeat ourselves anywhere,
right?
00:21:13:14  Stephanie  When?  With that?
00:21:15:04  Shelly   Yeah
00:21:15:04  Shelly   No, we're good.  [Shelly punches the keys on the
calculator.]
00:21:25:01  Shelly   No, wait.  [Shelly lifts her work paper and looks at
the paper with the problem statement.]
00:21:45:07  [Camera moves from Shelly's paper to Amy Lynn.]
00:21:49:28  Stephanie  Um.
00:21:56:27  [Camera moves to Amy Lynn's paper.  We can't
see what Amy Lynn is writing.]
00:22:01:11  Shelly   There's got to be some easier way to do that.
00:22:03:16  Stephanie  That's something.
00:22:05:21  Shelly   I just can't go about it.  Cause if you do, OK, so if
you do five plus four plus three plus two plus one, it
just, you get fifteen.  And that's one off.
00:22:13:11  Stephanie  That's off.
00:22:18:28  Shelly   So.  [sighs]
00:22:18:28  Stephanie  Uh.  Um.  All right, you know, let's just add the
anchovies and maybe then we'll see like a pattern.
00:22:24:13  Shelly   OK.
00:22:25:27  Stephanie  You know.  Bobby, do you know anything?  That
you're like...  [Shelly laughs.]
00:22:30:00  Robert   No.  [Robert is writing on his paper.]
00:22:30:26  Stephanie  ...willing to share?  Cause you're like off in your
little world.  [Robert puts down his pen.]
Shelly   [unintelligible]...anchovies.  [unintelligible.  Shelly is
talking to herself as she writes.]
...anchovies...anchovies.
00:22:46:28  Stephanie  Are you done?
Robert: No.
Stephanie: Adding anchovies?
Shelly: Oh, I was going to say, wow, that was quick.
Stephanie: It was like, all right, some one's a super genius. Everybody else is just retarded. [Shelly laughs.]
Shelly: One, four, wait a minute. One four six four one, so the next one will be one. This is the ...
Stephanie: [gestures with right hand.] The triangle.
Shelly: The triangle.
Stephanie: Yeah. [leans over to Shelly's paper.] So The next one is one five ten ten five one.
Shelly: We're done. [laughs]
Stephanie: But what does that mean? [The three girls laugh.]
Shelly: I don't know.
Stephanie: What does that mean to me?
Shelly: I don't know, but that's the answer. [laughs] um.
Stephanie: But what it, like, what does one four six four one.
That means nothing to me.
Shelly: It means nothing to me either, but it's the pattern we saw.
Stephanie: Oh dear lord.. Oh, so we have a pattern, but how do we apply it to getting sixteen pizzas?
Shelly: That, that would be the problem.
Stephanie: Are we even sure that that's the pattern? Like, are we positive that this is the pattern that's happening?
Shelly: Well, if our answers are right, that's the pattern.
Stephanie: OK.
Shelly: If our answer is wrong, then it's not.
Stephanie: So, well, OK, let's figure this is saying that we have one plain pizza.
Shelly: Uh-huh.
Stephanie: And then we have four pizzas with two toppings?
Shelly: With one. Because it's the plain and then with the one topping.
Stephanie: OK. So we have four pizzas with one topping. And we have four pizzas with two toppings. Oh no, we have six pizzas with two toppings, four pizzas with three toppings.
Shelly: And one pizza with four toppings.
Stephanie: OK, so [picks up paper]
Shelly: We have one pizza and how many?
Stephanie: See, but it doesn't add up to sixteen.
Shelly: [points to Stephanie's paper] This. Doesn't? I'm... I just...
Stephanie. [laughs] Yes it does. [Shelly laughs.] OK. So the answer's... Give me a second. I'll add it. Thirty-two?

Shelly Oh, this? [points to Stephanie's paper] Or this? [points to another place on Stephanie's paper]

Stephanie This. [indicating with pen the first place Shelly pointed to] Well, I'm hoping that I got this right the second time when I'm saying this is sixteen.

Shelly Well, if you add this [pointing to Stephanie's paper], it's thirty...

Stephanie ...two.

Shelly Oh, thirty-two. [laughs] I was about to say you must have messed up on the first one and I messed up on the second. Oh god, I'm trying to figure out these problems, I can't even add. It's... OK, so I'll do thirty-two. That would be...

Stephanie Yes.

Shelly Just write that.

Stephanie It's just... yeah. [At this time, the camera is focused on Robert.]

Shelly [After a pause]... OK [Shelly sighs.] There's got to be an easier way to do it.

Stephanie Well no, that was pretty easy. Because look, now we know what all the pizzas are. [Stephanie writes on her paper.] What's it, three one?

Shelly Two one. Right? No. Oh. Is that what it is? I could be wrong.

Stephanie Is it one two one and then one three three... one?

Shelly Yeah.

Stephanie ... one four six four one, and then one five ten ten, five, one... [Stephanie's voice trails off as he continues writing.]

Shelly Yeah. That's it.

Stephanie Yeah. [The camera is focused on Stephanie's paper, where the first five rows of Pascal's Triangle are written.] Well, that works, right? If you have one pizza, you have one pizza.

Shelly Yeah.

Stephanie If you have one pizza with two toppings, you have plain pizza and then pizza with both toppings and then... get two [unintelligible]... all right.

Shelly Looks good to me.

Stephanie Alright, yeah.

Shelly Well. [writes] C, D.

Stephanie [laughs] C D.
Shelly ...Triangle.
Stephanie Whose classroom is this?
Robert Mr. Pantozi's.
Stephanie He's got a big classroom.
00:26:01:24
Robert It used to be a sewing room.
Stephanie Used to be a what room?
00:26:04:03
Robert Sewing.
Stephanie Oh...Sorry. [Shelly laughs.]
00:26:11:21
Shelly So are we done? [laughs]
00:26:13:22
Stephanie I hope so.
00:26:14:05
Shelly I'm just sitting here. I feel bad we're not doing anything.
00:26:17:29
Stephanie Oh wait. Uh oh. and see.
00:26:18:16
Shelly I drew attention to us. [Shelly puts her hand over her face.]
00:26:19:29
Stephanie There you go. [laughs] Should have just acted like you were busy. [T/R2 walks up to Shelly.]
00:26:23:26
T/R2
00:26:35:21
Stephanie Sixteen and thirty-two.
00:26:35:27
Shelly Sixteen and thirty-two.
00:26:37:27
T/R2 Sixteen and thirty-two?
00:26:38:09
Stephanie Yes.
00:26:38:24
T/R2 Let's see. This question says how many different choices for pizza does a customer have and list all possible choices, and your answers are sixteen and thirty-two?
00:26:46:03
Stephanie Yes.
00:26:47:08
Shelly Well, our all possible choices are in that little diagram thing there.
00:26:50:11
T/R2 Ah-hah. And how does that little diagram thing work?
00:26:52:15
Stephanie Go ahead, Shelly
00:26:53:10
Shelly Well, it's a tree diagram, and so it's like you have plain and you have plain with pepperoni. This brings back memories. [Stephanie laughs]. And then you have plain and sausage.
00:27:00:01
T/R2 Now it's coming back.
00:27:00:28
Shelly Yeah, now it's coming back. Now. Then you have plain with mushrooms and plain with, what's that?
00:27:05:15
Stephanie Peppers.
00:27:05:26
Shelly Peppers.
00:27:06:27
Stephanie Or pepper.
00:27:07:28
Shelly Well, that was pepperoni.
00:27:08:16
Stephanie Whatever.
00:27:08:23
Shelly Yeah, so too many p's. [in regular voice] Plain with
peppers and sausage, and that's how it works. You just [gestures with right hand] branch down.

I see. Oh OK, so all your, all your possibilities then, um, if you had to list them all, you would do what? How would that work out? If you actually had to list all of the different choices?

You mean like write?

Write them all out.

Yeah.

Like this is a list and this is a list and [Stephanie takes a paper from her pile and puts it on top and then places both her hands on it.]

[unintelligible]

OK, tell me for a minute how this diagram would help you do that. Like if you were just working from this diagram.

Well, you would put plain.

Uh-huh.

And you would put plain with pepperonis.

OK.

With peppers. That's right. And then you, and then if you just wanted to keep reading down... Then if you just kept read reading down...

You would just kind of build up [gestures to the right]. [Shelly picks up pen and writes on paper.]

OK, so you listed them this way [points to Stephanie's paper].

Yeah.

And Shelly did a tree diagram. .

I kind of went.

Yeah.

The long, retarded route.

She did it the way to make sure...yeah.

Kind of just looked at it said, oh, look.

OK. So you checked it and your answers agree?

That way, we could check.

Yeah, we cross-checked it. [gestures with right hand]...

OK, and you came out with how many combinations?

Sixteen. And, I think Bobby and [looks at Amy-Lynn]...yeah. got 16.

Bobby got that.

Actually, he was the one that said sixteen.
00:28:19:28 Shelly Yeah. So he was checker.
00:28:20:00 Stephanie And we were like, ah.
00:28:27:07 Robert Same way.
00:28:29:23 T/R2 The same way? Well, I see a tree there. Did you make a tree also? OK, so zero equals plain?
00:28:37:06 Robert Yeah, when there are zero toppings, it's plain.
00:28:39:06 T/R2 I see.
00:28:39:24 Robert That's one. And then there's one topping on each. And then just, use the peppers with everything. And then since I used peppers with everything, I just left it out when onto sausage and.... Instead of putting three, like I did here.
00:28:55:00 T/R2 Uh-huh.
00:28:55:10 Robert I only put two, because I already have sausage and peppers.
00:28:57:26 T/R2 I see.
00:28:59:09 Robert And then I just took the remaining two that I didn't use. I just put them in.
00:29:03:28 T/R2 OK, so this P2 stands for what?
00:29:05:21 Robert Oh, pepperoni, because there's two P's, and I didn't feel like writing out pepperoni.
00:29:09:22 T/R2 Oh, that's clever. OK. That's interesting.
00:29:14:00 Robert Peppers.
00:29:15:15 T/R2 OK, so then you, um, I guess you kind of categorized them then? [Robert nods.] Zero toppings and one topping? Is that what this is? And this is?
00:29:24:15 Robert Yeah, two. And that's three toppings.
00:29:25:21 T/R2 Two toppings, three toppings, four toppings. [points to Robert's paper] OK. So there's one with zero toppings and there's...
00:29:33:07 Robert Six. Over here. Oh, one. Zero there's one. One there's four. Two there's six. And three four. And then four [Robert points to his paper.]
00:29:40:15 T/R2 I see. OK. And, Amy-Lynn, how did your, um?
00:29:46:10 Amy-Lynn Exactly the same way.
00:29:47:11 T/R2 Exactly the same way?
00:29:47:29 Amy-Lynn With a tree
00:29:51:10 T/R2 With the one four six four one. What is the...
00:29:53:02 Amy-Lynn Like how it branched out. Where you start with one, yeah, exactly that. [Amy-Lynn gestures toward Stephanie's paper.]
00:29:57:25 T/R2 And what is exactly this? Where did that come
Shelly: See, you have one choice there, and then you have four choices [pointing to the paper], and then six, and then four, and then one.

Amy-Lynn: Four choices, six together.
T/R2: One choice? Or...one
Stephanie: One, you have one pizza.
Stephanie: One pizza with no toppings.
T/R2: Uh-huh.
Stephanie: And then you have four pizzas with two toppings?
Shelly: Two.
Stephanie: Or with one. No, you have four pizzas with one topping. [looks at Shelly]
Shelly: Yeah. Cause plain...
Stephanie: All right. Let me...
Shelly: Plain...doesn't count as a topping.
Stephanie: One pizza with no toppings. That's just plain.
T/R2: OK, which is what you have right?
Stephanie: And then you have...
Stephanie: Yes, and then you have, and then you have... four pizzas with one topping.
T/R2: OK.
Stephanie: Like, just peppers, and then you have...
T/R2: OK. Which is what he has, OK, OK
Stephanie: You have six pizzas with two toppings, like peppers and sausage, or whatever.
T/R2: OK.
Stephanie: And then you have four pizzas with three toppings. And then you have one pizza with all four toppings.
T/R2: OK. All right. And that's what you're talking about, the one four six four one.
Shelly: Uh-huh.
T/R2: OK, so your tree...can I just see your tree for a second? I can't see. [to Amy-Lynn]
Amy-Lynn: I didn't exactly draw it out yet.
T/R2: Oh, OK, but you did...
Amy-Lynn: Yeah, I have an order problem.
T/R2: [Waves right arm in circles.] So you're drawing lines.
Amy-Lynn: Yeah. [Amy-Lynn writes on her paper.]
T/R2: To go from topping to topping.
Amy-Lynn: Yeah. Yeah I have to like write it out more ways and then [unintelligible]...branch long [Amy Lynn
uses hand motions.]

00:31:03:00 T/R2 No, that's all...you don't have to do that. So you're all certain that you have exactly the same pizzas and you know your, your 16 (to Stephanie, who nods) and your 16 (to Shelly) and your 16 (to Amy-Lynn), they're all the same 16. So I guess what I'm interested in is, what's the rest of this here, what are all those numbers?...Were you?

00:31:17:07 Shelly Well, we were just playing with them.
00:31:19:07 Stephanie [Shelly laughs.] Well, we were just making sure. [Stephanie waves her hands.]

00:31:21:08 T/R2 I have a feeling that...
00:31:22:11 Stephanie We were just making sure we were right.
00:31:24:01 T/R2 ...there's a 27 there or something. How did you...
00:31:25:02 Stephanie Well, because then...
00:31:26:11 T/R2 What were you making sure about?
00:31:27:02 Stephanie We got, we got, you know, we didn't want to do the same thing for the anchovies and Shelly recog..., like, we remembered that we had seen it, like the 1 4 6 4 1. And so we just drew the rest of it. And then we, to make sure that the 1 4 6 4 1 was like.

00:31:41:05 T/R2 Where did you see the 1 4 6 4 1?
00:31:43:08 Stephanie Right after the 1 3. [Shelly laughs.] Uh, I know, I know. [Stephanie laughs and waves her arms.]
00:31:48:01 T/R2 [laughs.] I get that part. Where did you remember having seen it before?

00:31:51:00 Stephanie Oh, we did stuff with it before.
00:31:53:29 Shelly Yeah.
00:31:53:39 Stephanie I don't know.
00:31:55:08 Shelly This problem.

00:31:55:08 Stephanie Yeah.

00:31:56:06 T/R1 Can I ask a question? [All turn to look at T/R1.]

00:32:11:23 Shelly Uh-huh.
00:32:12:05 T/R1 When you can select from four toppings, right?

00:32:14:26 Stephanie [Shelly and Stephanie nod.] And this row?
00:32:16:20 Shelly That's the anchovies.
00:32:17:17 Stephanie Those are the anchovy pizzas.
00:32:17:26 T/R1 So tell me what these numbers mean.
00:32:19:27 Stephanie One plain pizza. Uh, five pizzas with one topping.
Ten pizzas with two toppings, ten pizzas with three toppings, five pizzas with four toppings, one pizza with all five. [T/R1 looks toward Amy-Lynn. Shelly nods.]

00:32:32:05 T/R1 Ok. Now my question, this is my question. OK. How did you get this triangle so fast? [T/R1 points to Stephanie's paper.]

00:32:40:27 Shelly Cause we remembered it.

00:32:41:24 Stephanie Well, we didn't like all of a sudden...

00:32:43:04 T/R1 How do you get from one row to the next?

00:32:44:01 Stephanie Oh, we just, oh...

00:32:44:13 T/R1 Like how did you get from the third row to the fourth and the fifth.

00:32:47:27 Stephanie One plus three. You leave the one and the one plus three is four and then three plus three is six and three plus one is four and then the one, then one. [T/R1 looks towards Amy-Lynn while Stephanie talks.]

00:32:54:24 T/R1 OK. Now this is my question. You told me what this meant with pizza and toppings. Right? [Shelly nods.] When you have four toppings to choose from. And you told me what this meant when you have five toppings to choose from. Can you show me, thinking about pizza toppings, why the, for instance, the four plus the six is the ten? You told me what that meant in pizzas, right?

00:33:18:17 Stephanie Uh-huh.

00:33:19:02 T/R1 You could all tell me what that means in pizzas, that 4? [Shelly and Amy-Lynn nod.]

00:33:19:27 Shelly Uh-huh.

00:33:20:02 T/R1 You know the kinds of pizzas they are? [T/R1 points to Stephanie's paper.] And you know the kinds of pizzas these are? [T/R1 points elsewhere on Stephanie's paper.] And you know the kinds of pizzas these are? [T/R1 points to Stephanie's paper.]

00:33:28:02 Stephanie Uh-huh.

00:33:30:02 T/R1 I'd like you to explore why that works with the pizzas, and we're going to leave you alone. Do you understand my question?

00:33:33:10 Shelly Uh-huh.

00:33:33:24 Stephanie Uh-huh.

00:33:34:01 T/R1 OK. [T/R1 touches T/R2's shoulder and leaves. T/R2 leaves.]

00:33:38:03 Shelly I think to explain it we might have to do another
tree diagram. Another.

00:33:40:07 Stephanie    Well, go ahead, Shel.
00:33:43:28 Shelly        OK. [Shelly takes a fresh sheet of paper and
begins writing, then Stephanie, Robert and Amy-
Lynn also begin writing.]
00:34:10:08 -----------    [The camera focuses on Stephanie's paper, where
she has drawn five rows of Pascal's Triangle, with
arrows from the 4 and 6 of the 4th row to the 10 in
the 5th row. Then she writes Plain and Plain next
to the Pascal's Triangle.]
00:34:32:23 Stephanie    Do you know why?
00:34:35:07 Shelly        No, but I have the tree diagram done. [Shelly
laughs.]
00:34:37:24 Stephanie    Oh, OK, I got excited there. I was like, yes!
00:34:39:16 Shelly        Um.
00:34:41:19 Stephanie    Um, well, what if we started, what if we do it from
up here, cause...
00:34:45:02 Shelly        Yeah.
00:34:45:05 Stephanie    ...it's gonna work all the way down. It's gonna be a
lot easier to work with one topping though than
with...
00:34:48:14 Robert        What's the top number? Is that zero toppings or
one topping?
00:34:51:05 Stephanie    That's one plain pie. That's zero..., I guess that's
zero toppings? [Robert nods.]
00:34:55:11 Shelly        Yeah.
00:34:55:25 Stephanie    That's zero toppings. That's a plain pizza. The
next row we have a plain pizza, and then we have
two pizzas with one topping. Right?
00:35:06:04 Shelly        Yeah.
00:35:07:00 Stephanie    And then we have one pizza with both toppings.
00:35:12:10 Shelly        Yeah. OK.
00:35:13:07 Stephanie    Right?
00:35:14:00 Robert        So this is no toppings, one topping, is that how it
goes? [Robert points to Stephanie's paper.]
00:35:20:27 Shelly        Yeah.
00:35:21:20 Robert        But then...
00:35:23:05 Stephanie    Wait, wait. I know what you're..., yeah.
00:35:25:02 Robert        Then I don't think it works.
00:35:27:14 Stephanie    No, it works. We just don't know why. [Shelly
laughs.] It works. Um, yeah, cause this is a plain
pizza. If we had plain with...
00:35:38:12 Amy-Lynn      Plain is zero toppings. And if you had...
00:35:40:20 Shelly        Well, let's see, count, uh, if you count plain as a
topping [Stephanie puts her head on the desk.] or

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you don't count plain as a topping.

00:35:47:10 Stephanie So does that make...

00:35:47:17 Shelly It's two different things, yeah. We have to count it as a topping when we look at it this way.

00:35:51:22 Stephanie Yeah...But it doesn't really matter because even if we counted plain as a topping, here it would still be repeating itself. If we only had one, if this row stands for one topping...

00:36:00:08 Shelly Uh-huh.

00:36:00:19 Stephanie ...this is wrong. Right? [to Robert.]

00:36:04:09 Robert Yeah. So.

00:36:05:09 Stephanie That's what you were thinking. But, like past that, it works. Right?

00:36:11:08 Shelly I think it worked. [Unintelligible; there is laughter.] ...Ignore the top of it.

00:36:11:29 Stephanie It works. The bottom half works.

00:36:12:07 Shelly Ignore the top. The rest of it works.

00:36:15:25 Stephanie No, but if this stands for two toppings, then this works. But does this work for three toppings? Cause this works for f...four. [Stephanie points to paper in front of Shelly.]

00:36:24:13 Shelly Yeah.

00:36:24:29 Stephanie And this works for five.

00:36:27:02 Shelly Yeah. So that's, this is one topping.

00:36:28:00 Stephanie This is five four three two, and this is...

00:36:30:01 Shelly No. Because if that's one topping, then you have one with the topping and one plain.

00:36:33:25 Stephanie OK, let's just ignore this. Ignore the top. Let's work from three to four.

00:36:38:29 Shelly OK.

00:36:39:25 Stephanie I guess we're just gonna have to...

00:36:42:02 Shelly Another diagram!

00:36:42:15 Stephanie Another one.

00:36:42:18 Shelly Fun. [Shelly and Stephanie begin writing, then Amy-Lynn starts to write.]

00:36:54:27 Stephanie One three three one. [Speaking quietly.]

00:36:56:29 Shelly And that's with three toppings.

00:36:57:25 Stephanie But do we understand how the one and the three get to be the four? OK, well, where's the one?

00:37:05:11 Shelly Plain.

00:37:05:27 Stephanie And where's, and these are the three.

00:37:07:15 Shelly Yeah.

00:37:08:05 Stephanie OK, we have to do one for the, the four, the one and the four, too, so you know.

00:37:12:28 Shelly Uh-huh.
00:37:13:11  Stephanie  Cause otherwise we just have one half of the...
00:37:14  Shelly  So wait... I'm just gonna... and then for the next one it's... [Shelly is writing. The camera then moves to Amy Lynn. She is drawing a tree diagram.]
00:37:40  Stephanie  How can you prove how this makes this when you're adding. When you don't have a fourth topping. Do you know what I'm saying though?
00:37:50  Shelly  Yeah.
00:37:51  Stephanie  Like they're saying... It's like you're materializing a topping. Ya know? Like I can't add plain to pepperoni and make sausage all of a sudden and that's how one and three make four. [Shelly laughs.] Ya know? I don't understand.
00:38:15  Shelly  It's the...
00:38:16  Stephanie  I need help. I don't understand.
00:38:17  Shelly  There's gotta be some way to do the top of the triangle. So this was for toppings right? And five choices, four toppings. This was for five toppings. This worked for three. [The camera then moves to Robert. He is writing.]
00:38:33  Stephanie  But this is only five toppings if we count plain as a topping.
00:38:38  Shelly  No, that's if you don't count...
00:38:39  Stephanie  No because look...
00:38:40  Shelly  No look, one, two, three, four, five, and plain doesn't count as a topping.
00:38:47  Stephanie  OK
00:38:48  Shelly  Five, and that's three. So let's try two plain, peppers, sausage, sausage. So that's for two toppings. So why doesn't this work for one topping?
00:39:00  Stephanie  It works if we count plain as zero.
00:39:06  Shelly  Yeah. [The girls laugh.]
00:39:11  Stephanie  That's the only way it works... So we understand why that works, but I still like don't understand how we can like answer the question when you're like bringing a new topping into it.
00:39:22  Shelly  I know.
00:39:23  Stephanie  Cause... I need help. I don't understand. Yeah. Like I understand that one and three is four. [Shelly laughs.] And I understand that this row comes after this row, but I don't understand how like no matter how many times I had ya know pepperoni sausage I'm not gonna get a mushroom pizza. And that's what we're doing here. Ya know what I mean?
00:39:38 Shelly  Yeah. Yeah.
00:39:46 Stephanie Like a completely different pizza. That cannot be.
00:39:48 Shelly Ummm...
00:39:52 Stephanie Bobby, can you help us?
00:39:55 Robert Yeah
00:39:57 Stephanie Good
00:40:05 Shelly It's... This even works... It's
00:40:13 Stephanie Ummm..
00:40:15 Robert Why don't we do it for one topping, then two toppings and see if we see a pattern. And then we can do something based on that pattern.
00:40:23 Stephanie Yeah. Yeah. That's fine.
00:40:26 Robert You can write it though.
00:40:29 Shelly Alright, if you have one topping...
00:40:34 Stephanie But see, I still don't understand the point of the question. Ya know what I'm saying? Because...
00:40:47 Shelly I think I understand the question, but I just can't explain it.
00:40:50 Stephanie I just... yeah... but because... I don't know if it can... ya know... Maybe if it was applied to something else it could be explained.
00:40:56 Shelly Yeah.
00:40:57 Stephanie I think maybe it's... I just can't get past the fact that you can't make a pizza out of other pizzas.
00:41:03 Shelly Yeah.
00:41:04 Stephanie I think maybe if it was applied to something else I could look at it differently. Be like oh... right.
00:41:13 Robert Apply it to towers.
00:41:14 Stephanie I knew someone was gonna say that. I figured it would be them though. All right. No, let's apply it to towers, then. Go ahead, Bobby. [The girls laugh.]
00:41:23 Shelly You're like now I don't think we should [talking to Robert]
00:41:27 Stephanie No alright, wait. Ummm... so this one is just one-the tower.
00:41:31 Shelly Ummm... we need colored pencils.
00:41:38 Stephanie Oh dear lord, don't start. [T/R3 brings colored pencils to the table.] Thank you, Mr. Pantozzi.
00:41:47 T/R3 That's what you asked for.
00:41:50 Stephanie This is going to get us nowhere. I know it.
00:41:53 Shelly It's going to lead us exactly where we were before.
00:41:56 Stephanie We're gonna, yeah, ya know, we're gonna get done and we're gonna be like oh, well now I understand it with towers, but I have no idea how
to do it still.
00:41:58 Shelly We already knew that. Well at least we can have fun with colors.
00:42:03 Stephanie Yeah, but there's no point. Do we think that it's gonna help to build the towers?
00:42:08 Shelly I think it's gonna get us to the same place we are now.
00:42:11 Robert It might be fun though.
00:42:13 Stephanie Well go ahead, Bobby. Here. play with the red pencil.
00:42:14 Robert No. [Robert is busy writing.]
00:42:16 Stephanie But ummm...no...I just don't understand. Ya know? And I can...
00:42:24 Shelly Can see it...
00:42:27 Stephanie I can see it. Ummm...How do you get from one and three to four.
00:42:38 Shelly Add them. [Shelly giggles.]
00:42:39 Stephanie We know four means that we have four pizzas with one topping, right?
00:42:46 Shelly Yeah, well, see, OK, so this is the one...
00:42:49 Stephanie And three...oh...I'm sorry, go ahead.
00:42:51 Shelly So this is the one with two toppings here, right, so you have one with one topping. No, that's with no toppings.
00:42:57 Stephanie Wait. I forget. Are we counting...we're not counting...Ugggh...I'm getting confused.
00:43:03 Shelly How are you guys over there doing?
00:43:04 Stephanie Oh god, OK, we're not counting plain...plain as a topping.
00:43:08 Shelly Yeah. This is...this is with no toppings.
00:43:10 Stephanie Yes.
00:43:11 Shelly And then this is with one topping.
00:43:12 Stephanie Got it.
00:43:14 Shelly And then this is with no topping and then this is with one topping.
00:43:16 Stephanie On topping and this and this are supposed to make this...but all we're doing is adding an extra topping.
00:43:27 Shelly So that's it. You're adding an extra topping.
00:43:29 Stephanie Yeah but this doesn't explain that, like I understand that, but see the problem is there's no like extra topping wondering around up here that just like drops in down, ya know?
00:43:39 Shelly You get the extra topping from one.
00:43:41 Stephanie But one doesn't have an extra topping. Like no, I
know what you're saying.

00:43:55  Shelly  Yeah.

00:43:58  Stephanie  Ya know?

00:44:03  Shelly  I don't know. Unless our diagram is wrong, but I

00:44:10  Stephanie  No. It's not. We're right. That's not the problem.

00:44:28  Shelly  What are you doing over there, Bobby. Just having

00:44:30  Stephanie  I think if Mr. Pantozzi can bring us some markers,

00:44:42  T/R3  What are you doing?

00:44:43  Stephanie  Yes! OK, we had to do this pizza problem, right?

00:44:02  T/R3  [T/R3 nods.] I think, Yes.

00:45:04  Stephanie  OK. And we understand how like this works ya

00:45:24  T/R3  What are the toppings? There are four possible

00:45:28  Stephanie  Yeah. Here, oh...these...There's three toppings

00:45:33  T/R3  Three toppings in that row.

00:45:34  Stephanie  Right. [Looking at her paper in front of her which

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that this makes that. This is just one plain pizza. This is three pizzas with one topping. If you add one plain pizza to three pizzas with one topping, you get like one pizza with one topping, no, no toppings and three pizzas with one topping. Ya don’t, ya know? Like you get four if you’re like using the pattern here, but like in reality you don’t get four. Ya know? So I don’t know how to answer the question.

00:46:43 T/R3 You’re trying to explain what this four means in terms of pizzas. Is that what you’re trying to say?

00:46:48 Stephanie I don’t know. She wants, they want to know how to get, how we got four pizzas out of one pizza and three pizzas.

00:46:53 T/R3 Cause I overheard you saying…

00:46:54 Stephanie I can’t…

00:46:55 T/R3 …that if this wasn’t pizzas then it was possible to do.

00:46:56 Stephanie Yeah. If this wasn’t pizzas I could do it, but it’s not possible with pizzas, so…

00:47:02 T/R3 Well, using the notation that you have here, you’re saying that this one…

00:47:05 Stephanie Plain pizza.

00:47:06 T/R3 One way to get a plain pizza and three ways to get...

00:47:11 Stephanie One topping.

00:47:12 T/R3 A pizza with one topping, choosing from...

00:47:13 Stephanie Three toppings.

00:47:14 T/R3 Three toppings. And then there’s four, using the notation that you’re using here. This means there’s four ways to get...

00:47:23 Stephanie One pizza with one topping out of four toppings.

00:47:28 T/R3 And when you go from one column, one row, to the next, you’re going two toppings, three toppings, to four toppings.

00:47:34 Stephanie Yeah.

00:47:35 Shelly You could say that it works OK cause you’re discarding the plain pizza. So when you add the plain to the one toppings here, that’s like another topping. It’s just like plain was a topping. Ya know what I’m saying?

00:47:46 Stephanie Yeah, I know what you’re saying, but like here that doesn’t stand for a plain pizza.

00:47:43 Shelly I know. I know.

00:47:44 Stephanie Like these four pizzas don’t stand for plain pizzas
at all. None of them are plain.
The only thing that I can really add at this point, I think, is that I used to, my father owned a pizzeria. I never figured out how to toss the things up. My little brother learned how to do that. I couldn't learn how to do that. [The students laugh.] But I mean when I see this, I'm... yeah, but, well, that's true, that's true. Ummm, yeah. I would have to put the toppings on the pizza though. So the only thing I'm seeing, the only thing that you're saying makes me think is that it's like you've done, you've seen what you can do with three toppings and then my dad comes along and says well now I need you to put on a fourth topping, that's just, I don't know. I'm just, if I'm being helpful at all, thinking that about going form here to here seems like adding on another topping in some way.

00:48:47 Stephanie Umm hmmm. Like I think we understand that like we understand that when we're doing the problem we're adding toppings on and that's why this is working out here. [Stephanie points to 14641 in Pascal's Triangle.] And we understand that, but it just doesn't make sense here [Stephanie points to 1331 in Pascal's Triangle.] This doesn't produce that. Even if your dad tells me to put another topping on, throwing that plain pizza in there isn't gonna make a difference. Ya know?

00:49:27 T/R3 Well, what's the new topping in, in the fourth, uh...
00:49:29 Stephanie Uh. Mushrooms. [Stephanie throws up her hands.]

00:49:32 T/R3 So.
00:49:33 Stephanie We had pepperonis, sausages, peppers, and mushrooms.
00:49:35 T/R3 Well, OK, I'm, I'm, you know, since I haven't really thought about this, I'm just, when I look at that, I'm thinking, well now, I, now, me, my dad says well, put pepperoni on. I had a plain pizza, so now I'm putting pepperoni on the plain pizza.

00:49:47 Stephanie Yes.
00:49:49 T/R3 And I'm seeing one there. Maybe this is the pepperoni being added on or something. [Sound fades.]
00:49:57 T/R3 You know, I've exhausted what I could do with three, and now...
00:50:00 Stephanie Yeah. Well, I...
00:50:00 T/R3 Put a pepperoni on top of the plain. Maybe that's one of the four, or something. But I don't know if that explains that. [T/R3 points to Stephanie's paper.]

00:50:06 Stephanie So what you're saying is that this pizza is just the pizza...

00:50:08 Shelly Yeah, like...

00:50:09 Stephanie ...with the topping thrown on it.

00:50:09 Shelly Yeah, like, treat it like a pizza with a topping.

00:50:11 Stephanie OK.

00:50:11 Shelly Because you're discarding the plain, because the plain's here already. [Shelly points to Stephanie's paper.]

00:50:14 Stephanie Uh-huh.

00:50:14 Shelly So treat it like a pizza with a topping. And then just add it to the other toppings.

00:50:18 Stephanie All right.

00:50:23 T/R3 [T/R3 points to Stephanie's paper.] But I'd ask then about, was that, would that explain, like three and three make six, for instance?

00:50:27 Stephanie Well, so this is...

00:50:29 T/R3 I mean, if you, if you come up with something that could explain one of them, maybe that's, as I keep coming up to against in class, it explains one thing [T/R3 gestures towards Robert], but then it doesn't explain something else.

00:50:36 Stephanie OK. So this becomes, if this becomes a pizza with one topping. For this row. Right?

00:50:40 Shelly Uh-huh.

00:50:41 Stephanie Then this becomes a pizza with two toppings.

00:50:41 Shelly Then this is two toppings. [Shelly points to Stephanie's paper.]

00:50:44 Stephanie And three two toppings plus three two toppings equals six two toppings. [Stephanie looks at T/R3.]

00:50:48 Shelly Yeah.

00:50:50 Stephanie That's what, if we were explaining it, how, yeah.

00:50:52 Shelly Yeah.

00:50:53 Stephanie And then this is one pizza with everything. right?

00:50:56 Shelly But then that would become one pizza with two toppings. Right?

00:50:58 Stephanie No. One pizza with...

00:51:00 Shelly No.

00:51:01 Stephanie ...three toppings.

00:51:02 Shelly Yeah.

00:51:02 Stephanie And this would become three pizzas with three
toppings. [Stephanie looks at Shelly.]

00:51:09 Shelly Yeah.
00:51:09 Stephanie For this row. Right?
00:51:12 Shelly I think so.
00:51:16 Stephanie To get four pizzas with three toppings.
00:51:19 Shelly Yeah, so that would become one, or that would become three toppings. [Shelly points to Stephanie's paper.]

00:51:23 Stephanie That's, yeah, that's what I said.
00:51:24 Shelly OK. [Stephanie and Shelly laugh.]
00:51:26 T/R3 So, what did you just say, there's... does that thing I suggested sort of work for this too?
00:51:31 Stephanie Yeah. Uh-huh.
00:51:32 T/R3 So tell me what it means here. [T/R3 points to Stephanie's paper.] Cause when I, now when you went to, this has three toppings, and then it becomes four toppings. What, what happened there?

00:51:37 Stephanie Oh, OK. Well, this, if, this is one pizza with three toppings. [Stephanie points to her paper.] So this becomes three pizzas with three toppings. [Robert points to a paper near T/R3, and T/R3 nods at Robert.] Is that, does he have it better? Because, let him do it.
00:51:50 T/R3 No, no, he's, this is something we did in class that he's, he's working on.

00:51:53 Stephanie Oh.
00:51:54 T/R3 It's OK, I'm sorry, um.
00:51:56 Stephanie So, um. [There is a pause.]
00:51:58 T/R3 I interrupted you.
00:51:59 Stephanie No. Uh, ah, so this becomes three pizzas with three toppings and then three pizzas with three toppings plus one pizza with three toppings is four pizzas with three toppings.

00:52:08 T/R3 [T/R3 points to Stephanie's paper.] This is three pizzas with...
00:52:11 Stephanie Three toppings. For this one. [Stephanie points to her paper.] For this one, right? [Stephanie looks at Shelly.]

00:52:16 Shelly I don't know.
00:52:16 T/R3 I'm just trying to get, I'm just trying to get the language straight.
00:52:18 Stephanie Am I doing this right? Wait, I have to start it all over. I have to go from here, because I'm forgetting what I'm doing. OK, this, to get four
pizzas with one topping, you already have three pizzas with one topping. [Stephanie points at her paper and looks at T/R3.] And the plain pizza becomes the pizza with the new topping.

00:52:34 T/R3 OK.
00:52:35 Stephanie OK, so this becomes, instead of one plain pizza, this is one pizza with one topping. Cause this one's getting like the pepperoni thrown into it.

00:52:43 T/R3 OK. OK.
00:52:43 Stephanie And that produces the one, the four pizzas with one topping.

00:52:48 T/R3 [T/R3 points to Stephanie's paper.] This is four pizzas with one topping. You didn't need to add anything to these, these just sort of became these.

00:52:53 Stephanie Those were, those just got brought down.
00:52:54 T/R3 Those were brought down, OK.
00:52:56 Stephanie Those are the same three pizzas. [Stephanie looks at T/R3.]

00:52:56 T/R3 OK.
00:52:58 Stephanie So then here, um, you have six pizzas with two toppings. Now you already have three pizzas with two toppings. So these three pizzas with one topping get an extra topping added on.

00:53:09 T/R3 OK. [T/R3 nods.]
00:53:10 Stephanie So these become three pizzas with two toppings. And then three pizzas with two toppings plus three pizzas with two toppings equal six pizzas.

00:53:20 T/R3 With, two toppings.
00:53:21 Stephanie With two toppings.
00:53:23 T/R3 But now you're...
00:53:23 Stephanie Right? [Stephanie looks toward Shelly.]
00:53:24 T/R3 You're choosing from four now, right?
00:53:25 Stephanie Yes. So now this is right. I'm not...right? Right? [louder, to Shelly.]

00:53:30 Shelly Yes.
00:53:31 Stephanie OK. [Stephanie and Shelly laugh.]
00:53:33 T/R3 How about the last one. The last one. Now this is three, this is, there is only one pizza that has all three of these toppings.

00:53:38 Stephanie Yes.
00:53:38 T/R3 And then that, how does that one get to here? [T/R3 points to Stephanie's paper.]

00:53:41 Stephanie That just drops down.
00:53:43 T/R3 Oh, it just drops down.
00:53:44 Stephanie That's one pizza.
So it still has...

Yes, so that's the one pizza with three toppings. And then you need, then these become, these all get an extra topping adding on to them. [Stephanie looks at T/R3. T/R3 nods.] Like these are three pizzas with two toppings. So they all get the extra topping. [Stephanie looks at T/R3. T/R3 nods.] That you would have here. Like the pepperoni that is here, or whatever, gets thrown on to these three pizzas that don't have pepperoni but have two other toppings. [T/R3 nods.] So now there are three pizzas with three toppings. You add them to the one pizza with three toppings, and you get your four pizzas with three toppings.

Now I understand all of that. I don't know if that's the answer to the question.

I hope so. Thank you. And here. [Stephanie hands the packet of markers to T/R3.]

[to Robert.] Can I see that again? [Robert gives paper to T/R3.]

That just made me think of the thing you were doing in class, with the adding of the, um...

Oh, yeah, it's kind of, I don't know, I just [unintelligible]...

Because, what, what we were adding, the thing you came up with, you were adding on, what did I ask you to do? Wasn't it something like this? [T/R3 points to Robert's paper.] Was it adding up numbers like this?

Yeah, but it was like a pattern. So I didn't use it, but...

You didn't use what you used in class on this, or...

No. Cause, here...can I show you?

Yeah.

All right. See this is four toppings right here. And one plus four plus six plus four, sixteen. And two to the fourth is sixteen. [Robert points to Stephanie's paper.] And three toppings. One three and eight is eight. Two to the third is eight. And then one two one is four, and two to the second is four. [Stephanie nods.] And there's supposed to be two up here. And one plus one is two, and two to the first is two. And then we got 32 for the next one. And we add that over there. You get 32 and two to the fifth is 32. So this is two to
the number of toppings, how many combinations there are, for pizza toppings.

00:55:47:24 Ralph I wonder whether, like whether the two plays in any role in that, like why all these have two.

00:55:52:17 Robert Yeah, I know

00:55:53:23 Ralph Is that something you thought about

00:55:54:28 Robert I just remembered something with towers that we did, to find total combinations was two to the something

00:56:00:24 Ralph Does that apply here?

00:56:02:05 Robert Yeah, it's the same thing

00:56:02:24 Ralph To pizza toppings. But there's so many different pizza toppings. It's not like there is green and purple or whatever colors you used

00:56:10:25 Stephanie Hmm, hmm.

00:56:16:25 Robert So I guess if you want to find out ten toppings there are, you just do ten two to the tenth and then you got, how many combinations there are

00:56:25:02 Ralph But why does that work? I figure if there are ten toppings, I mean if there's ten different toppings I figure you would have to go through, I don't know if there are ten toppings you want to put on a pizza. Well, I guess there are.

00:56:33:05 Robert I think it is more like, um, it's something like if you have, and then you go to b, and then you have a, and then you keep going by two, this is one and this is two and you keep adding. I forget what it was, but we did it before.

00:56:52:26 Ralph We did...or...

00:56:53:26 Robert Rutgers

00:56:55:06 Ralph With Rutgers

00:56:56:08 Robert and we made like a branch or something and it just kept going by two and that, or something. I forget

00:57:03:20 Amy-Lynn All of the problems went by two.

00:57:04:14 Ralph What was that

00:57:06:00 Amy-Lynn That was with a lot of the problems, they went by two and it had something to do with powers

00:57:07:10 Robert Yeah. We figured out something.

00:57:12:05 Ralph Ok, that is something that 's interesting by itself. This, I thought, I remember, I think I asked you in class, to add these numbers up and how you would figure

00:57:19:09 Robert Yeah, it was like, if it was one four sixteen, or something

00:57:24:04 Ralph Yeah
And it was all multiplied by four, and then well, how would you know what the answer is?

What the total sum would. That came up in

I don't think that works here.

Ok, that doesn't apply to what you're doing. Well, I just wonder, whether, how the two, if the two really does apply here, as it seems to with what he came up with, what that might mean. So you bring me in, and you get another question. I just can't...

All I needed was help

I just had to ask another question. I understand.

Um

Well, you can think about that, I just came in, impromptu. This was not... uhh..

Oh great. Thank you.

Some students in my class think I stage not understanding something. Sometimes I do, but not that time.

Great

So, were you able to answer T/R1's question yet?

Yes, sort of.

But not why

Not why?

I guess. We got four but not three.

What do you mean, four is but not three?

Because three is convinced, but we don't know why.

What are you talking about?

The question was how do you go from these numbers to these numbers, right?

Yes

And were you able to answer that?

Yes.

OK, could you explain it to me, please?

Yes

Ok

Um, ok. [Stephanie draws Pascal's Triangle as she begins her explanation.] This row, has, is a row with pizzas where you can have three possible toppings.
00:58:49:12 Stephanie  Ok? The next row is a row where you can have four toppings on your pizzas. The question was how to get from like one pizza with one topping and three pizzas with two toppings, or I'm sorry, one pizza with no topping I'm sorry, like just a plain pizza and three pizzas with one topping to four pizzas with one topping and we had, well I had trouble because I kept thinking, that we were like materializing this pizza out of nowhere, but then Mr. Pantozzi came over and explained that what happens is, we already have three pizzas with one topping and the three pizzas just kind of get dragged down

00:59:40:15 T/R2 Ok
00:59:41:01 Stephanie because they are still going to be here, we would still have a pepperoni pizza, a mushroom pizza, and a sausage pizza, here, but we are adding peppers. So the plain pizza becomes a pizza with peppers on it

00:59:52:28 T/R2 Ok
00:59:53:06 Stephanie You throw peppers onto the plain pizza, so now you have, this becomes a pizza with one topping and you add that to the three pizzas with one topping and you get the four pizzas with the one topping

01:00:02:15 T/R2 Ok
01:00:04:27 Stephanie Then, to get from there to there, this is six pizzas with two toppings

01:00:09:28 T/R2 And where do they come from?
01:00:11:17 Stephanie You already have three pizzas with two toppings, so these pizzas, like if you are adding on peppers

01:00:17:21 T/R2 Hmm, hmm
01:00:17:29 Stephanie These pizzas are all pizzas without peppers
01:00:20:05 T/R2 Ok
01:00:20:13 Stephanie so you add peppers on to them, and they become three brand new pizzas with two toppings

01:00:25:02 T/R2 Ok
01:00:25:25 Stephanie So if you add the three new pizzas with two toppings with the three old pizzas with two toppings, you get the six new pizzas, the six whatever, the six pizzas with the two toppings

01:00:35:05 T/R2 Ok
01:00:39:08 Stephanie Um, those are two toppings. The four pizzas have three toppings we already have one pizza with three toppings

01:00:48:14 T/R2 Ok
That's the mushrooms, the peppers and the sausage, whatever. Here you're adding the peppers. Pepperoni. So you add the pepperoni to these three pizzas that don't have pepperoni and you get three new pizzas with pepperoni plus the one old pizza and you have the four new pizzas with three toppings.

Ok. Ok, now. How is it that you can take these pizzas and move 'em here and move them there? I don't understand that.

Because you're, ok, these, in this column, aren't changed.

Those are unchanged, ok.

If you are bringing them down to the six

Ok

these three pizzas, remain the same.

Ok

but when you are moving them here, these three pizzas are getting an extra topping

Ok. So these three pizzas then, you have different things that can happen to them,?

they can either remain

the same

the same, or they can get another

topping

topping. So each of those pizzas, what happens to each of those pizzas?

What do you?

When you are moving down a row. For example, what's..could you give me an example of 'em, let's do this one because it's easy

Could you give me an example of one pizza in that category?

In the three pizzas with one topping?

Like she's saying that you literally move it down. So, if you move these down, how can you move those down, if they are already down here?

Because

Isn't that what you are saying?

Oh. Is that what you mean?

Well, I'm... I'm not quite sure if that is what I mean. What I mean, is how come you get to do two things with them?

Because they represent
You get to move them there and there
They are representing different pizzas, like different combinations
Kinds of pizzas in each thing, different combinations
So, can you give me an idea of one pizza in this category
Um, a pizza with peppers
Ok
So you've got. I don't know.
So, here is the pizza with peppers, right
Yes
Ok, to move it down into this. You're doing
Nothing to it
Nothing
It becomes just a pizza with peppers
It's a pizza with peppers
Yes
To move it here, you are
Adding the new toppings
Whatever it happens to be
so it becomes a pizza with peppers and mushrooms
Ok. Ok. So, that's the pizza with peppers and it can go here or here
And the pizza with peppers and mushrooms
Can go here and here
Yes
And so forth
Yes
So each of these pizzas
Has two, like spots
Oh, that's interesting
One, where it has, where it stays the same and one where it gets added a topping
so each one has two new things that happen
Maybe that's where he got the two to the n, I am talking ... isn't that what Bobby said before, maybe that is where the two comes from.
I don't know. What do you think? You had how many pizzas up in this row, all together.
Uh, all together. Um, eight
and how many pizzas in this one
Sixteen
and the next one you said thirty two

You know, it is more of an idea than I had. I don't know
two, two raised to the third is the eight, so...

That was good, that was really good
You remember that, because I'm like the only one
This one only goes here? Does it go here too?
Yeah, it drops down as a plain pizza.
Because this one is the plain, I see. Ok so your drop
down idea is that is stays the same.
It stays the same once Right
and it changes once Ok
Where I guess Amy got the two.
Very interesting. Ok. Do you agree with this?
Hmm, hmm.
ok, um that is really, really interesting. So where um,
did um, I was going to ask you before, where did you
first see that, you said that you remembered
something [inaudible]
yeah, we, they worked on it, we worked on it in
eighth grade
Oh, ok
but like for a different problem I think
Really?
So
Do you remember, what problem that was? What else
We worked with towers. I remember when Dr.
Speiser came and we did something with towers and
Hm
Stuff, we plotted the towers out in the thing, in the
triangle, you know what I mean? There was one
tower, and then there was one, two, and one tower.
Oh...Really?
Yeah. I don't know what the problem was or
anything, but I remember, like, the triangle is like
imprinted on my brain.
Oh, ok.
01:05:11:27  Stephanie  Forever.
01:05:12:14  T/R2    What did the, can you remember what the towers
would look like for this row.
01:05:18:06  Stephanie  Um.
01:05:19:17  T/R2    Do you all remember the towers.?
01:05:21:21  Shelly  I remember being in like the home ec. room and
making little towers
01:05:23:22  Amy-Lynn  yeah, and making the long
01:05:25:15  Stephanie  Yeah, those....But I don't think we had known what it
was then
01:05:29:27  Shelly  No we hadn't
01:05:30:18  Stephanie  We didn't know them.
01:05:31:14  Shelly  That's how we learned that.
01:05:32:16  T/R2    Do you remember, um, what you did with the towers.
What the problem was?
01:05:36:00  Shelly/T/R2/  How many combinations
Amy-Lynn
01:05:38:15  Shelly  It was probably because there were different colors
like for the towers
01:05:42:02  T/R2    And how many colors did you have that you worked
with
01:05:44:06  Shelly  It depended
01:05:44:19  Amy-Lynn  Two
01:05:45:08  Stephanie  We probably had two
01:05:45:11  Shelly  Two or three
01:05:47:22  Stephanie  And then it kept growing and growing and growing.
01:05:51:13  T/R2    That number of colors or
01:05:53:11  Shelly  Yeah, the number of colors like you could, right
01:05:55:00  Stephanie  I think it grew both ways because if it grows with
colors it's got to grow with
01:06:00:25  Shelly  Yeah
01:06:01:01  Stephanie  or you would get stuck, right?
01:06:02:24  Shelly  Yeah
01:06:05:24  T/R2    So, can you remember anything about what these or
can you imagine what these might mean what these
numbers might mean with respect to towers then?
01:06:14:27  Stephanie  I have to think. Um, this is probably, probably I don't
know, this is probably, one with all blue, I don't know,
right? like, if we have blue and red, this is one tower
with all blue, I don't know how high it would have to
think. And this is three with
01:06:33:07  Robert   Two blue one red

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Thank you, two blue one red. and then this is three with two red one blue, oh, and then this is one all red. so, that would, I guess they are three high and there, I don't know. Go ahead Bobby, you seem to know what you are talking about.

They would be three high?

Robert: Yeah

That is how many toppings there were, right. Like for this one there was...

There are three toppings yeah, so now there are four toppings, or four high, with two towers

Robert: Yeah

Shelly: Yeah they were three high

T/R2: Yeah, they were three high, would that work. Oh, what were you doing there?

Oh look, you came prepared.

I was doing this before.

Oh look at this, were you.

Yeah, I was doing that before.

You were doing it before, ok

But I didn't know what it had to do with

He was just playing

So, when you were doing the pizza problem, you started to think of it in terms of towers

And what are these ones and twos here?

Oh, two different colors.

Ok

Different colored crayons.

Ok. And how were you thinking about the pizza problem with in terms with what you were writing here?

I don't know. Because like I just did the one and I saw the three and the three matched up and then I just did it for the next one.

[talking to Stephanie, looking under table, inaudible] I am sorry, say that again.

I don't really know.

Laughing, just talking, and it makes sense

You don't really know, but you spent a lot of time on this so something must be going on. Maybe you can help us out, what do you see here that makes sense to you, that relates to you?

Well what do you mean?
01:08:29:27 T/R2  Well, Robert says that he was working on this.
01:08:32:05 Shelly  Those are the toppings right, for the pizza aren't they?
01:08:35:05 Stephanie  Were you doing it as towers? Or does it
01:08:39:02 Shelly  It is the same thing, so it doesn't really matter
01:08:39:19 Stephanie  Yeah
01:08:41:14 Robert  Yeah, but it only gets like this number like in the middle, I think
01:08:46:06 T/R2  Really
01:08:47:03 Robert  Yeah, I just
01:08:48:24 Stephanie  You never
01:08:49:09 Shelly  Oh, ok, yeah
01:08:51:17 Robert  It didn't work, so we just
01:08:54:01 T/R2  So this one
01:08:54:05 Shelly  That's, ok, I see why, because you are not including like the one, you are just saying with four here you are not saying the ones that three with it, you know what I am saying?
01:09:00:27 Stephanie  hmm, hmm
01:09:03:06 T/R2  I don't know what you are saying.
01:09:04:06 Shelly  Ok, so these are four high, right
01:09:08:15 T/R2  Ok
01:09:09:01 Shelly  But with those four high, you also have, you still have the ones that are three high.
01:09:13:07 Stephanie  hmm, hmm
01:09:13:26 Shelly  You still have those possibilities to make them if you want to, you understand what I am saying, NO.
01:09:20:06 T/R2  I am sorry you don't
01:09:20:07 Stephanie  It's the same thing with the pizzas, right. That is what you are saying?
01:09:23:00 Amy-Lynn  You just bring them down
01:09:23:19 Shelly  yeah
01:09:24:02 Stephanie  Like You are bringing them down, but you are adding one onto it
01:09:25:14 T/R2  So you are bringing these guys down
01:09:26:27 Amy-Lynn/ Stephanie  But you are adding another one onto it
01:09:28:18 Shelly  Yeah, but like his numbers is only the middle one, he only has that one because he didn't like carry those down too
01:09:33:26 T/R2  Oh, thank you. thank you.
01:09:35:29 Shelly  like just as the three
Ok, Ok, if this is in fact the middle one, what does that. I don't know. I am curious to know why you did this? Why, how you saw this as helpful to the pizza problem. Was it helpful to you, when you were doing it? Or not.

I don't know. Um, well not this part. But the towers. [inaudible] I remember how we figured out the total combinations

Hmm, hmm

and that's the same thing. I figured this out.

Hmm

So, I guess that it was helpful in that sense but this wasn't just

I don't need this

I was interested when you grabbed that

That was useless

Well you never know

[Shelly is also talking at the same time] We were just hoping there was an easier way, we were hoping there was a really easy way

Well, I think the way you did it, you started to see some things I think that made it easier, right?

Yeah

I heard you say something about well there is an easier way, and look we can get this right away. And then you started fiddling with these numbers here, so maybe you did find an easier way. I am not quite sure, I think this is very interesting and I think it is interesting that these numbers that you see them reflected over there and I'm just wondering if there is anything, any other relationship between these two. Hmm. Well, you said something before that also interested me. Umm. You said there is supposed to be a two up there or something when you were talking

Oh no.

about what you did in Mr. Pantozzi's class.

I think she left like one thing out. Because like I have a one and then I have two ones cause this is like zero toppings,

Yeah, I was thinking that

Oh with one and one

And then it would actually be

That is why our thing wasn't working

Oh ok.
01:11:22:21 Stephanie Yeah, we were having trouble, yeah Bobby thanks for telling us before.
01:11:26:06 Shelly [Laughing] We are all here, lost, you got anything Bobby. No, I'm just not doing anything.
01:11:29:29 Stephanie We are like, Bobby is like drawing and we are like, why isn't this working?
01:11:33:20 T/R2 Well, he is drawing all kinds of stuff over here, so he is very busy. So, what wasn't working out before, when you were missing that
01:11:38:25 Stephanie Oh, it was just...ummmm.
01:11:41:00 Shelly Let's say this is plain
01:11:41:02 Stephanie Like we would jump from like plain to like two toppings and we had no idea and that wouldn't even work for plain, so we were like
01:11:48:26 Stephanie And we said that wasn't working, of course Bobby had the answer all along and didn't try to share it with us, but we won't hold that against him.
01:11:52:07 Shelly Got anything over there, Bobby, no I don't have anything.
01:11:56:07 T/R2 Well, he wanted to give you the opportunity to think about this
01:11:58:18 Stephanie to look really stupid, great.
01:12:01:29 Shelly So he looks like the smart one.
01:12:03:23 T/R2 I wanted to say this, but I would like to go back to this, if you had, in this problem, uh oh, I lost my problems, ok, um, there is another one, in this problem, we had, let's see, we started out with peppers, sausage, mushrooms and pepperoni, so those were, four toppings and you said there were sixteen pizzas?
01:12:25:26 Robert Yeah
01:12:26:29 T/R2 Ok, and you listed them in various ways. Ok, do you think that you have convinced each other.
01:12:31:29 Rob/Steph/S hel Yeah
01:12:32:06 T/R2 Oh, yeah. Ok, and if you added a fifth topping? you came up with thirty-two.
01:12:36:01 Stephanie Yeah
01:12:38:08 T/R2 Ok, what if we did that ten toppings, what if Mr. Pantozzi's dad had ten toppings to choose from, how many pizzas would there be?
01:12:45:17 Shelly/ Amy-Lynn Two to the tenth.
01:12:48:06 Shelly Where is that calculator?
01:12:49:05  Stephanie  Where is the calculator when you need it?
01:12:49:25  T/R2  What is two to the tenth? Surely you can do two to the tenth in your head, no, well, let's see, if you had ten toppings could you write them out fairly quickly, do you think. I know it would take a long time to do, I shouldn't say it that way, it would take a long time to do that. Could you kind of picture what that would look like? Would that also fit into this, this thing that you did here?

01:13:12:22  Stephanie  Yeah, they would go down, a lot
01:13:14:25  Shelly  Yeah
01:13:16:00  T/R2  Do you have any idea what the beginning of that row would look like? Just the beginning.

01:13:18:27  Steph/Shelly/One
             Amy-Lynn
01:13:19:29  T/R2  And then?
01:13:20:14  Shelly  One, ten [?]
01:13:21:21  T/R2  Ok, why would it be one, what would the one represent

01:13:24:17  Steph/Shelly  The plain
01:13:25:17  T/R2  And what would the ten represent?
01:13:27:13  Shelly  All, with the one topping, because there is ten toppings.

01:13:28:23  T/R2  Ok, ok. all the way on the other end?
01:13:32:19  Stephanie  Ten, one
01:13:33:29  Shelly  ten, one
01:13:35:10  T/R2  The one at the very far right end, what would that be.
01:13:36:24  Steph/Shelle  Is all the toppings.

01:13:38:16  T/R2  And how about the ten before that?
01:13:40:19  Steph/Shelly/nine toppings
             Robert

01:13:42:19  T/R2  Ok, so, you could, I mean it would probably take a long time, sure but you could probably go through all that. So, the ultimate question now is, what if you have n toppings.

01:13:53:12  Steph/Robert  Two to the n?
01:13:54:15  T/R2  Two to the n?
01:13:55:07  Stephanie  Yes, one, n and then whatever.
01:13:59:08  T/R2  One n and whatever, oh that is interesting.
01:14:00:11  Shelly  and then ,n one..
01:14:03:05  T/R2  so you got the beginning and the end, no problem, right? OK, Um, I had a, right out of my head, um, why do you think it is always this two? Why do you think that is two is floating around?
01:14:14 Stephanie Oh. Amy? [Stephanie looks at Amy.]
01:14:15 Amy Cause you're always bringing down, um, the
toppings that you had, like, before. [Amy points to
Stephanie's paper with her pen] that are
unchanged. And then you're bringing down
another set, you add another topping to it.
01:14:23 T/R2 OK. OK. And that's why you keep getting this
doubling effect? [T/R2 gestures with both hands.
Amy nods.] Ok. OK. Very nice OK. Let's see if
T/R1 has, I understand. [T/R2 is speaking to T/R1,
who is outside of camera range.] I understand.
Now we have to see if she understands. I'm, I'm
missing the other two problems, but [unintelligible]
you can have a seat. [T/R1 takes T/R2's seat.]
01:14:39 T/R1 OK. So you can tell me why you add in... [T/R1
points to Stephanie's paper.]
01:14:46 Stephanie Yes
01:14:47 T/R1 Pascal's Triangle?
01:14:49 Stephanie Yes.
01:14:49 T/R1 OK.
01:14:50 Shelly I guess you want us to. [All laugh.]
01:14:51 T/R1 Go ahead, Michelle, tell me.
01:14:52 Shelly Me? Oh. But Stephanie did it so well before.
01:14:54 Amy-Lynn Yeah.
01:14:55 T/R1 We will give you a chance.
01:14:56 Shelly OK. Um, so, oh, wait. I guess I'll just start from
this row, right? [Shelly points to Stephanie's
paper.]
01:15:00 Stephanie Yeah. Do it from that.
01:15:01 Shelly OK, so this is a plain one. [Shelly points to
Stephanie's paper. T/R1 nods.] And this is the
three with one toppings. [T/R1 nods.] So then this
is the plain. And you get this because you give the
plain one topping. [Shelly pauses, and T/R1
nods.] And you just add them to become the four
one topping. Does that, does that make sense?
[T/R1 looks toward Robert.] I can't explain it.
01:15:19 Stephanie No, you did it. That was good.
01:15:21 T/R1 Yeah. It makes sense to me. [T/R1 nods.]
01:15:22 Shelly OK. Then, OK.
01:15:24 Stephanie Go ahead.
01:15:25 Shelly OK. [Shelly and Stephanie laugh.] Um, and then
the, this would be the two toppings. [Shelly points
to Stephanie's paper.] And this was the two
toppings, so you make that two toppings, and then
that two toppings because, like, you're adding, like, pepperonis or whatever on to it. So that becomes the two toppings, too. And then you add them together and you get six.

01:15:41  T/R1  And they're all with two toppings.
01:15:42  Shelly  Cause you're, yeah, cause you add those three original two toppings for up there and then those three with one topping, but you're adding the extra thing on to it. [T/R1 nods.] So that has to become the six, and then you just keep expanding that way.

01:15:52  T/R1  OK, I understand what you said. And then I heard you say something about, this looks familiar to you, Stephanie?

01:15:58  Stephanie  Yeah, we did it...
01:15:58  T/R1  You had...
01:15:59  Stephanie  We did it in eighth grade.
01:16:00  T/R1  Professor Speiser come in.
01:16:01  Stephanie  [Stephanie nods.] Yeah. Um-huh. But we did it with the towers.

01:16:02  T/R1  Did you all know what she was talking about, with the towers?
01:16:07  Amy-Lynn  [unintelligible.]...similar
01:16:08  T/R1  Can you imagine that all in your head, what she was talking about? Cause it's really hard when you don't have it in front of you. Without any pictures or anything. You all can follow what Stephanie was saying with the towers?

01:16:16  Stephanie  Bob, he was drawing it.
01:16:17  T/R1  Were all of you able to follow it? Amy, tell me what she was saying.

01:16:20  Amy-Lynn  What?
01:16:21  T/R1  With the towers. How this works for towers.
01:16:23  Amy-Lynn  Like, we had done it in one of the classes that we had one the sessions. And so we can just kind of picture what it was like. Robert...

01:16:29  T/R1  So, so, so tell me what I'm supposed to picture. Now I'm not thinking of pizzas any more here. [T/R1 points to Stephanie's paper.] I'm thinking of the same rows. Can you tell me what I'm supposed to imagine in my head with towers.

01:16:41  Amy-Lynn  Isn't it like so many high? [Amy-Lynn gestures with both hands.]
01:16:42  T/R1  OK, so.
01:16:43  Amy-Lynn  And then...
01:16:44  T/R1  So, so tell me. [T/R1 points to Stephanie's paper.]
01:16:47  Amy-Lynn  So this would be how high?
            That would be three high, right? [Stephanie nods.]
            Yeah.
01:16:49  T/R1  Three high.
01:16:49  Amy-Lynn  Three high, and then it's how many colors you
            have? [Stephanie nods and turns her paper
            toward Amy-Lynn.]
01:16:53  Stephanie  Except we did it, remember? [Amy-Lynn turns
            Stephanie's paper.]
01:16:56  Amy-Lynn  Yeah.
01:16:56  Stephanie  We did it on the bottom.
01:16:57  T/R1  So...
01:16:58  Stephanie  So if you have blue and red.
01:16:59  Amy-Lynn  And red.
01:16:59  T/R1  OK.
01:17:00  Amy-Lynn  If you had two colors.
01:17:02  T/R1  So what does the one represent? [T/R1 points to
            Stephanie's paper.]
01:17:03  Amy-Lynn  One of all, like, say, blue. [Amy-Lynn gestures with
            her right hand.] Like, three high of all blue cubes.
01:17:09  T/R1  Uh-huh.
01:17:11  Amy-Lynn  And then you have...
01:17:11  T/R1  OK, I could imagine that. And what does the three
            represent?
01:17:14  Amy-Lynn  You can have, like, um, three towers where you
            have, like, two blue and one red. [Amy-Lynn
            gestures with her right hand.] Um. Red and one
            blue.
01:17:20  T/R1  So now we have one blue. Which is it? Does it
            matter? If you were making this all blue. [T/R1
            points to Stephanie's paper.] Right?
01:17:26  Shelly  Hm. Yeah.
01:17:27  T/R1  And you were making this.
01:17:28  Stephanie  Oh, that would probably be, I think that would be
            one. [Stephanie turns to Shelly.] The way they
            would, like it would work out would be one blue?
01:17:34  Shelly  Yeah.
01:17:35  Amy-Lynn  Yeah.
01:17:35  Stephanie  And two reds. [Stephanie turns toward T/R1.] And
            the next one would be two blue, and one red.
01:17:38  T/R1  [T/R1 points to Stephanie's paper.] How many
            blues are here?
01:17:40  Stephanie  All.
01:17:40  Amy-Lynn  All. Three.
01:17:41  T/R1  So this, you're going from three blues to one blue.
01:17:43 Stephanie Oh. I don't know.
01:17:43 Shelly [Shelly points to Stephanie's paper.] Three blues to two blues to one blue to none.
01:17:44 Stephanie [Simultaneously.] To one blue to none.
01:17:46 Shelly Or you can do it the other way around.
01:17:47 T/R1 OK.
01:17:48 Shelly Three reds.
01:17:48 T/R1 No reds.
01:17:49 Shelly Yeah.
01:17:50 Stephanie Uh-huh.
01:17:50 T/R1 To one red.
01:17:51 Stephanie Uh-huh.
01:17:51 T/R1 To two reds.
01:17:51 Stephanie [Simultaneously.] To two reds.
01:17:51 Shelly [Simultaneously.] To two reds.
01:17:52 T/R1 To three reds.
01:17:52 Stephanie [Simultaneously.] To three reds.
01:17:52 Shelly [Simultaneously.] To three reds.
01:17:54 T/R1 OK. I could imagine that in my head. OK. So, let's decide on one. Want to make it, you said the three blues so no reds.

01:18:01 Shelly OK.
01:18:01 T/R1 One red, two reds, three reds.
01:18:06 Shelly Uh-huh.
01:18:07 T/R1 So tell me where the four comes in.
01:18:09 Stephanie OK.
01:18:10 T/R1 Well, let, let Amy do it.
01:18:11 Stephanie OK.
01:18:12 T/R1 Because I'm curious because she hasn't played with it for a long time. [Amy-Lynn laughs and turns her head.] Right?

01:18:15 Amy-Lynn Yeah. [unintelligible.]
01:18:18 T/R1 You too, Michelle, you haven't played with it in a long time so you can help Amy.
01:18:22 Shelly That would be four, like, four blue, three blue, two blue, one blue, no blue. [Shelly points to Stephanie's paper.] Or...
01:18:30 T/R1 OK. Or no red, one red, two red, three red, four red.
01:18:31 Shelly [Simultaneously.] No red, one red, two red, three red, four red.
01:18:33 T/R1 OK, I can imagine that. [T/R1 points to Stephanie's paper.] So why, why does one plus three give you four? You have towers three tall. Now you have towers four tall. [T/R1 gestures with her right
hand.] How does the one plus three give you the four?

01:18:46  T/R1  Why are you adding on from here to here and here to here where this is no reds, right? [T/R1 points to Stephanie's paper.] And this is three with one red, right? [T/R1 looks at Stephanie.] Could you see in your heads the three with one red? Can you imagine those? What do they look like, the three with one red? Can you see them?

01:19:05  Stephanie  Uh-hm. [Shelly nods.]

01:19:06  T/R1  You know there are exactly three. What do you see? I'm curious. What do you see in your heads? [Stephanie, Shelly, and Amy-Lynn laugh.] How do you see the three of them with exactly one red?

01:19:14  Stephanie  Um. One with a red at the top.

01:19:15  Amy-Lynn  Yeah. [Amy-Lynn gestures with her right hand.] One with a red in the middle.

01:19:16  Stephanie  [Simultaneously with Amy-Lynn.] One with the red in the middle. [T/R1 nods.] And one with the red on the bottom. [Amy-Lynn nods.]

01:19:18  T/R1  You all can imagine that. [T/R1 looks toward Robert and nods.] Very impressive. OK. So how do we now get these four with...one red?

01:19:27  Stephanie  Um. Wait, I have to think. [Stephanie moves her paper.]

01:19:33  T/R1  Right?

01:19:34  Stephanie  These are all blue, right? [Stephanie points to her paper.] And these are one blue? Is that what we're saying?

01:19:39  T/R1  These are... [T/R1 moves closer and points to Stephanie's paper.]

01:19:44  Shelly  No red.

01:19:44  Amy-Lynn  Oh yeah, no red.

01:19:45  Stephanie  So these are all blue.

01:19:47  Shelly  Yeah. [Stephanie and Shelly laugh.]

01:19:48  Stephanie  And these are one blue. OK.

01:19:49  T/R1  I have to switch. I'm not as fast as you are, Stephanie.

01:19:51  Stephanie  OK.

01:19:52  T/R1  You're much more expert on these towers than I am without having them in front of me.

01:19:54  Stephanie  Oh, these are all blue, these are two blue. I'm sorry. That was...

01:19:57  T/R1  Right. Or no red and one red.

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Stephanie: Yes. So here, all you're doing is adding one red. [Stephanie looks at T/R1. There is a pause.]

T/R1: OK. [T/R1 nods.] So this has to be a one red. [T/R1 points to Stephanie's paper.]

Stephanie: Yes. See, you already have, um, three with one red. [Stephanie looks at T/R1, who nods.] So here, this becomes the fourth one with, with the one red. Cause here there's no reds. [Stephanie looks at T/R1, who nods.] And each of them get a block like added. [Stephanie forms the shape of a block with her hands.] How do I...OK. [Stephanie pauses and puts her hand under her chin.]

T/R1: I can see my little stack here. [T/R1 gestures as if holding a block.]

Stephanie: [Stephanie makes the same gesture.] Yeah, do you

T/R1: And in this little stack here, there's only one of them, right?

Stephanie: Yes.

T/R1: And it's all blue and no red, right?

Stephanie: Yeah. [To Robert.] Where's your picture? Oh, do you have the picture to that? Or... [Stephanie takes a paper.]

T/R1: You have to help me with this. Why don't you make a picture?

Stephanie: Here we go. [Stephanie draws. T/R1 watches.] OK. So here is...you have the one three high with all blue. And then you have the three with one red, so you have red blue blue, blue red blue, blue blue red. And then these two make...This one is four blues. [Stephanie points elsewhere on her paper and says something unintelligible; Shelly laughs.] OK. And these two together make, um, the one with four, the four with one red. So this one gets a red added on because it's already got three blues. So it can't have any more blues. And then these three all get....

Shelly: A blue added on.

Stephanie: A blue added on [Stephanie turns toward T/R1.] to it.

T/R1: Because they already have one red.

Shelly: Yeah.

Stephanie: They already have one red.

T/R1: [To Robert.] Makes sense?

Robert: Uh-huh. [Stephanie puts down the pen.]
01:21:51 T/R1 OK. That, that helped me a lot. Now my next question is, [T/R1 moves Stephanie's papers around.] ... how many pizzas, how many towers? [T/R1 points to Stephanie's papers.] Are these problems...

01:22:07 Robert The same.

01:22:08 T/R1 The same, or...?

01:22:09 Shelly Yeah.

01:22:10 T/R1 Coincidentally the same answer? How can you convince me that these are the same problem? [There is a pause.]

01:22:15 Shelly [Shelly points to Stephanie's paper.] It's easier to explain...

01:22:18 Stephanie Yeah.

01:22:19 Shelly ... the two thing with this cause there's only two colors.

01:22:21 T/R1 You like that, huh.

01:22:22 Stephanie Uh-huh.

01:22:22 Shelly Yeah.

01:22:23 T/R1 OK, I agree. [Stephanie and Shelly laugh.] I think it's easier to see that with the two colors. I, I think most people...

01:22:27 Stephanie Cause with all those toppings, it throws you off.


01:22:29 Stephanie You expect, like, 800 pizzas.

01:22:30 Shelly Yeah.

01:22:31 T/R1 Is there a way of thinking about the pizzas another way so that the toppings don't...

01:22:38 Robert Yeah.


01:22:39 Robert [Robert points to the paper in front of T/R1.] Toppings is the height. Like four toppings would be a tower four high.


01:22:45 Robert And then the two colors would be with or without toppings. [Unintelligible.]

01:22:50 Stephanie Uh-huh.

01:22:51 T/R1 Say, say more about that now. So if I want to think of this as a pizza. [T/R1 picks up a pen and points to Stephanie's paper.] Right? Let, let's think of these, each of these as a pizza, right. So you're saying four toppings, a tower four tall. That's what you just told me, right?

01:23:08 Robert Yeah.

01:23:09 T/R1 And what would the b mean, for instance, in the
pizza here? If you were thinking of pizzas.

Robert   It's peppers or... oh, no, no, it would be...
Stephanie It would either mean that you had a topping there or you didn't have a topping there.
T/R1      Oh. So b means either put whatever that topping is...
Stephanie Right? Is that what you're trying to say?
Robert    Yeah. It can work cause like...
Stephanie I don't know what is...
Robert    You can have more than one...
Stephanie I think he was...
Shelly    Yeah.
Robert    ...topping.
Stephanie on the right...
T/R1      Well. So...
Stephanie ...track but...
T/R1      So you're saying that the color means that you have the topping or you don't. Let's follow that thinking, right?
Stephanie So...
T/R1      So b will be, means you choose the topping. [T/R1 writes on the paper in front of Robert.] Right? And red will be you don't choose it. Right?
Stephanie OK.
T/R1      Fair enough?
Shelly    Uh-huh. [Shelly nods.]
Stephanie Uh-huh.
T/R1      OK. So you have four toppings. So let's go through our toppings. Right?
Stephanie Uh-huh.
T/R1      We have, what are they?
Stephanie Well, for which...?
Robert    Peppers.
Stephanie For four high?
T/R1      For four high.
Stephanie We have...
T/R1      OK, so this would mean either you choose peppers or you don't. [T/R1 points to Stephanie's paper.]
Stephanie Uh-huh.
Robert    Choose peppers or you don't.
Stephanie You choose mushrooms or you don't. You choose sausage or you don't, and you choose, [Stephanie turns toward T/R1] um...
Shelly    Pepperoni.
Stephanie [Stephanie turns towards Shelly and nods.]
...pepperoni or you don't. Because you're gonna have four of them, and each one's gonna have, like, [Stephanie picks up the pen.] if it was written like that [Stephanie begins to write.] Like here it would be mushrooms, like going all the way across. And each tower [Stephanie turns to T/R1.] you had with the r there meant you didn't have mushrooms.

01:24:32 T/R1 [T/R1 nods.] OK.
01:24:33 Stephanie You know what I mean, so you're gonna have all the towers four high like that. And for each one with an r there, you have mushrooms on it.

[Stephanie turns to T/R1, who nods.] Or you don't have mushrooms on it. You know? And then, here's the peppers. [Stephanie writes.] And if they're, you know. And just like that.

01:24:47 T/R1 The rest of you follow it? [T/R1 looks toward Shelly.] What Stephanie's saying. It's pretty heavy stuff here. [T/R1 turns toward Robert.] What we're talking about. You're really into some very sophisticated thinking. OK. I wonder. I know that you have to leave.

01:24:58 Stephanie Uh-huh.

T/R1 tells the students that she would now like them to pull their ideas together and write up their solutions. She tells them that she would like them to come up with a big idea to share with the other group. The students work on writing up their ideas and notes for the presentation. T/R1 then informs the students that they will share during the next session.