A STUDY OF THE EFFECT OF CONTACT RATE AND ASYMPTOMATIC INFECTED IN AN EPIDEMIOLOGY MODEL OF 2019-NCOV

by

JIANKAI HUANG

A thesis submitted to the
Graduate School-Camden
Rutgers, The State University of New Jersey
In partial fulfillment of the requirements
For the degree of Master of Science
Graduate Program in Mathematical Sciences
Written under the direction of
Stephen M. Alessandrini
And approved by

Camden, New Jersey
May 2020
THESIS ABSTRACT

A Study of the Effect of Contact Rate and Asymptomatic Infected in an Epidemiology Model of 2019-nCoV

By JIANKAI HUANG

Thesis Director:
Stephen M. Alessandrini

The 2019-nCoV can be described by an adjusted SEIR model which includes new states comparing to the original compartmental model. To visualize how the virus processes, a numerical method is adopted that converts the differential equations into difference equations and simulations based on these difference equations are run on the MATLAB. In this model, the contact rate and the initial value of the asymptomatic infected are investigated to show how they affect the spreading of the virus.
I want to show my gratitude to my advisor, Dr. Stephen M. Alessandrini who taught me two years of numerical methods and mathematical modelling. He gave me helpful suggestions on the directions, helped with debugging and analyzing for the model and reviewed my paper. Thank you for the time spent on discussing with me through email and writing more than 16 emails to me at this particular pandemic time.

I thank professors of the math department for teaching me a lot of math knowledge: Haydee Herrera-Guzman, Gabor Toth, Siqi Fu, Howard Jacobowitz and Debashis Kushary.

I thank the mathematics department and the graduate school of arts and science for providing many helps for my study on campus of Rutgers-Camden.
Dedication

This dissertation is dedicated to my parents who supported and encouraged me to study abroad and learn mathematics.
1 Introduction

1.1 Literature Review

This paper studies and analyzes the Susceptible-Exposed-Infectious-Recovered (SEIR) model by comparing of results of two models that consider quarantined intervention and the asymptomatic infections.

Since the outbreak of the 2019-nCoV many researchers have tried to study the parameters in the SEIR model and predict the infected cases in Wuhan City. An infection of 191,529 on Feb 4 was predicted in [1] which is a significant overestimation comparing to the infected cases in Wuhan of 8351 in [2] and the accumulated infected cases in Wuhan of 50007 on Mar 31 in [3] with an estimated basic reproduction number of $R_0 = 3.8$. Since this virus has a basic reproduction number significantly higher than one and the effect of the intervention is worth to study.

A modified SEIR model was applied in [4] to investigate the effect of intervention by adding states such as ‘Quarantined Susceptible’ and ‘Quarantined Exposed’ in the original SEIR. Another paper [5] based on [4] studied the quarantine and contact rate more carefully and cancelled the compartment of ‘Asymptomatic Infected’ in the model and also assumed that the transmission ability of an asymptomatic infection is the same as a symptomatic infection.
1.2 SIR Model and SEIR Model

In this section, we will give an overview of the basic Susceptible-Infectious-Recovered (SIR) model and the above mentioned SEIR model. We will also discretized the models and run an example.

1.2.1 SIR Model

The SIR model characterizes individuals in the population as the Susceptible (S), the Infectious (I) and the Recovered (R). It assumes that the total population \( N \) is constant with \( N = S(t) + I(t) + R(t) \) for all \( t \geq 0 \). This model also assumes that the Recovered would gain immunity after recovering from the virus and thus would not become the S and I again in the future.

The equations by Nakul Chitnis [6] are as follows with the initial values \( S(0) = S_0 \geq 0, \ I(0) = I_0 \geq 0 \) and \( R(0) = R_0 \geq 0 \).

\[
\begin{align*}
\frac{dS}{dt} &= -r \beta I \frac{S}{N} \\
\frac{dI}{dt} &= r \beta I \frac{S}{N} - \gamma I \\
\frac{dR}{dt} &= \gamma I 
\end{align*}
\] (1)

The parameter \( r \) stands for the Susceptible met by the Infectious per unit time. \( \beta \) represents the probability of transmitting virus between a susceptible and an infectious individual. So the amount of infectious \( I(t) \) times \( r \) is the people met by the infectious and it times the fraction of susceptible in population \( S/N \) equaling to the amount of susceptible met by the infectious. The probability \( \beta \) multiply this amount is the newly increased cases per unit time.
Since this system of differential equations have no analytical solution [7], what can we learn about it? Note that $S(t) \geq 0$ and $I(t) \geq 0$. Consider the first equation.

$$\frac{dS}{dt} = -r\beta IS/N < 0 \tag{2}$$

Hence $S(t)$ is a strictly monotone-decreasing function.

Consider the second equation.

$$\frac{dI}{dt} = r\beta IS/N - \gamma I \tag{3}$$

When $dI/dt = 0$, $S(t) = \gamma N/r\beta$. At this time the Infectious $I(t)$ achieves its maximum.

The basic reproductive number $R_0$ of SIR [6] is

$$R_0 = r \times \beta \times \frac{1}{\gamma} = \frac{r\beta}{\gamma} \tag{4}$$

It is the product of the number of contacts per unit time and probability of transmission per contact and duration of infection.

Also, the effective reproductive number $R_e$ of SIR describing whether the number of the infectious increase or not, is the product of $R_0$ and the proportion of the Susceptible.

$$R_e(t) = \frac{r\beta S(t)}{\gamma N} \tag{5}$$
Next, we apply the Euler method to the system of differential equations. For a first-order initial value problem

\[ y' = f(x, y), \quad y(a) = y_0, \quad (6) \]

its solution is

\[ y(x) = y_0 + \int_a^x f(s, y(s)) ds. \quad (7) \]

We discretize its interval $[a, b]$ and obtain a mesh

\[ x_{n+1} = x_n + h, \quad a = x_0 < x_1 < \cdots < x_N = b. \quad (8) \]

Here $h = (b - a)/N$. Integrate stepwise

\[ y_{n+1} = y_n + \int_{x_n}^{x_{n+1}} f(s, y(s)) ds. \quad (9) \]

Approximate by the rectangle rule

\[ \int_{x_n}^{x_{n+1}} f(s, y(s)) ds \approx hf(x_n, y_n). \quad (10) \]

Substitute into (9) and we get

\[ y_{n+1} = y_n + hf(x_n, y_n). \quad (11) \]

Notice that

\[ y'(x_n) = f(x_n, y_n) \quad (12) \]
Applying the Taylor expansion

\[ y_{n+1} = y(x_{n+1}) = y(x_n + h) = y(x_n) + y'(x_n)h + O(h^2) \]  

(13)

to (2), we generates the difference equation where \( h = 1 \) is the unit of time in days

\[ S_n = S_{n-1} - r \beta I_{n-1} S_{n-1}/N. \]  

(14)

Similarly we obtain the system of difference equation

\[
\begin{align*}
S_n &= S_{n-1} - r \beta I_{n-1} S_{n-1}/N \\
I_n &= I_{n-1} + r \beta I_{n-1} S_{n-1}/N - \gamma I_{n-1} \\
R_n &= R_{n-1} + \gamma I_{n-1}
\end{align*}
\]  

(15)

However, before running the simulation we have to fix a problem that is the stepsize \( h \). From equation (11), we have

\[ \frac{y_{n+1} - y_n}{h} = f(x_n, y_n) \]  

(16)

Since the global error of the Euler method is \( O(h) \) [8], we have to pick a smaller stepsize \( h \) otherwise it could be unstable. \( h \) is the deltaT in the code shown in the appendix. Hence the system of difference equations (15) now becomes

\[
\begin{align*}
S_n &= S_{n-1} - h(r \beta I_{n-1} S_{n-1}/N) \\
I_n &= I_{n-1} + h(r \beta I_{n-1} S_{n-1}/N - \gamma I_{n-1}) \\
R_n &= R_{n-1} + h(\gamma I_{n-1})
\end{align*}
\]  

(17)
We simulate a SIR model using Table 1 and obtain the result as Figure 1.

<table>
<thead>
<tr>
<th>Compartments &amp; Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>Population</td>
</tr>
<tr>
<td>I</td>
<td>Infectious</td>
</tr>
<tr>
<td>R</td>
<td>Recovered</td>
</tr>
<tr>
<td>S</td>
<td>Susceptible</td>
</tr>
<tr>
<td>$r\beta$</td>
<td>Number of adequate contacts per unit time</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Recovered rate</td>
</tr>
<tr>
<td>$R_0$</td>
<td>Basic reproductive number</td>
</tr>
</tbody>
</table>

Table 1: Initial Values and Parameters of SIR Model
1.2.2 SEIR Model

In the SEIR model, a new component, the Exposed $E$, is introduced into this model. The Exposed would become the Infectious at a rate $\alpha$. But here we do not consider the vital dynamics which is the natural death rate to all the groups and the birth rate. The equations are shown below.

\[
\begin{align*}
\frac{dS}{dt} &= -r\beta \frac{I}{N} S \\
\frac{dE}{dt} &= r\beta \frac{I}{N} S - \alpha E \\
\frac{dI}{dt} &= \alpha E - \gamma I \\
\frac{dR}{dt} &= \gamma I
\end{align*}
\]  

(18)

Applying the Euler’s method as above, we get the following system of difference equations for simulation.

\[
\begin{align*}
S_n &= S_{n-1} - h(r\beta I_{n-1}S_{n-1}/N) \\
E_n &= E_{n-1} + h(r\beta I_{n-1}S_{n-1}/N - \alpha E_{n-1}) \\
I_n &= I_{n-1} + h(\alpha E_{n-1} - \gamma I_{n-1}) \\
R_n &= R_{n-1} + h(\gamma I_{n-1})
\end{align*}
\]  

(19)

Given the initial values and parameters, we can obtain a numerical solution of Figure 2 for the SEIR Model.

This paper will introduce a modified SEIR with the Asymptomatic Infectious in the Chapter 2 and SEIR without the Asymptomatic Infectious in Chapter 3. In Chapter 4 a sensitivity analysis will be conducted on the
<table>
<thead>
<tr>
<th>Compartments &amp; Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N ) Population</td>
<td>10000</td>
</tr>
<tr>
<td>( I ) Infectious</td>
<td>1</td>
</tr>
<tr>
<td>( R ) Recovered</td>
<td>0</td>
</tr>
<tr>
<td>( \beta ) Infectious rate</td>
<td>0.03</td>
</tr>
<tr>
<td>( \gamma ) Recovered rate</td>
<td>0.1</td>
</tr>
<tr>
<td>( \alpha ) Probability of Exposed to Infectious</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Table 2: Initial Values and Parameters of SEIR Model

initial value of the Asymptomatic Infectious. Chapter 5 will analyze the effect of social distancing. Finally, Chapter 6 will summarize the paper and conclusions.

Figure 2: SEIR Model
2 SEIR Model with Asymptomatic Infected

2.1 Assumptions and Data

This paper only apply the model in the Wuhan City, so we will assume that the population of Wuhan remained constant in 120 days from Jan 10, 2020 which was 11,081,000 [9]. It also assume that the Recovered will attain immunity to the virus so that they will not become the Susceptible again.

The authors of [4] introduce four new compartments compared to the original SEIR model. They are $S_q$ for ‘Quarantined Susceptible’, $E_q$ for ‘Quarantined Exposed’, $H$ for ‘Quarantined Infected’ and $A$ for ‘Asymptomatic Infected’. It did not specify the value of parameter of $\theta$, which means that it did not illustrate how the transmission ability of the Asymptomatic Infectious compared to the Symptomatic Infectious. In our simulation will assume that they are the same with $\theta = 1$.

2.2 Parameters and Equations

In our simulation we use the same parameters as in [4] but make the following changes.

We change the $\varrho$ into $a$ which denotes the probability of having symptoms among infected individuals. The contact rate $c$ in [4] is 14.781. We state the parameters in Table 3.

Observe that if the $\gamma_I$ denotes the recovery rate of symptomatic infected individuals who did not get treatment and quarantined in the hospital then it is not accurate for this rate to be higher than the recovery of asymptomatic
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>ratio of transmission ability of the exposed to the infected</td>
<td>1</td>
</tr>
<tr>
<td>$c$</td>
<td>contact rate</td>
<td>2</td>
</tr>
<tr>
<td>$\beta$</td>
<td>probability of transmission per contact</td>
<td>$2.1011 \times 10^{-8}$</td>
</tr>
<tr>
<td>$q$</td>
<td>quarantined rate of exposed individuals</td>
<td>$1.8887 \times 10^{-7}$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>transmission rate of exposed individuals to the infectious</td>
<td>$1/7$</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>rate of $S_q$ to $S$</td>
<td>$1/14$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>probability of having symptoms among infected individuals</td>
<td>0.8683</td>
</tr>
<tr>
<td>$\delta_I$</td>
<td>rate from $I$ to $H$</td>
<td>0.1326</td>
</tr>
<tr>
<td>$\delta_q$</td>
<td>rate from $E_q$ to $H$</td>
<td>0.1259</td>
</tr>
<tr>
<td>$\gamma_I$</td>
<td>recovery rate of symptomatic infected individuals</td>
<td>0.3303</td>
</tr>
<tr>
<td>$\gamma_A$</td>
<td>recovery rate of asymptomatic infected individuals</td>
<td>0.1398</td>
</tr>
<tr>
<td>$\gamma_H$</td>
<td>recovery rate of quarantined infected individuals</td>
<td>0.1162</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>death rate</td>
<td>$1.7826 \times 10^{-5}$</td>
</tr>
</tbody>
</table>

Table 3: Parameters of [4]

class and the $H$ class (quarantined infected). Initial values are shown in Table 4.

<table>
<thead>
<tr>
<th>Compartments</th>
<th>Initial Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S$ Susceptible</td>
<td>11081000</td>
</tr>
<tr>
<td>$E$ Exposed</td>
<td>105.1</td>
</tr>
<tr>
<td>$I$ Infectious</td>
<td>27.679</td>
</tr>
<tr>
<td>$A$ Asymptomatic Infected</td>
<td>53.839</td>
</tr>
<tr>
<td>$S_q$ Quarantined Susceptible</td>
<td>739</td>
</tr>
<tr>
<td>$E_q$ Quarantined Exposed</td>
<td>1.1642</td>
</tr>
<tr>
<td>$H$ Quarantined Infected</td>
<td>1</td>
</tr>
<tr>
<td>$R$ Recovered</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 4: Initial Values of [4]
The quarantined individuals move to the $S_q$ class at a rate of $(1 - \beta)cq$ if they are not infected and move to the $E_q$ class at a rate of $\beta cq$ if they are infected. The infected individuals who are not quarantined move to the $E$ class at a rate of $\beta c(1 - q)$.

The equations are shown in (20).

\[
\begin{align*}
\frac{dS}{dt} &= -(c\beta + cq(1 - \beta))S(I + \theta A) + \lambda S_q \\
\frac{dE}{dt} &= c\beta(1 - q)S(I + \theta A) - \sigma E \\
\frac{dI}{dt} &= \sigma aE - (\gamma_I + \alpha + \delta_I)I \\
\frac{dA}{dt} &= \sigma (1 - a)E - \gamma_A A \\
\frac{dS_q}{dt} &= (1 - \beta)cqS(I + \theta A) - \lambda S_q \\
\frac{dE_q}{dt} &= \beta cqS(I + \theta A) - \delta_q E_q \\
\frac{dH}{dt} &= \delta_I I + \delta_q E_q - (\alpha + \gamma_H)H \\
\frac{dR}{dt} &= \gamma_I I + \gamma_A A + \gamma_H H 
\end{align*}
\]

(20)

2.3 Results

The following Figure 2.1 and 2.2 show the results. We list six maximum values of the compartments in Table 5.

The maximum value of $I$ occurs at $t = 35$ which stands for the Infectious achieves its maximum at the Feb 31, 2020.

Observe that the Asymptomatic Infectious without being quarantined has 64,099 at its peak. This group of individuals may cause the spreading so the contact trace and the 14 day quarantined observing period is very important.

The maximum value of $E$ here is 1.1642. A possible reason is that the $S_q$
Table 5: Maximum Values

<table>
<thead>
<tr>
<th>Compartments</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S(120)$ Susceptible at the 120th  day</td>
<td>7,530,800</td>
</tr>
<tr>
<td>$R$ Recovered</td>
<td>3,144,600</td>
</tr>
<tr>
<td>$I$ Infectious</td>
<td>154,920</td>
</tr>
<tr>
<td>$E$ Exposed</td>
<td>593,610</td>
</tr>
<tr>
<td>$S_q$ Quarantined Susceptible</td>
<td>6,330,300</td>
</tr>
<tr>
<td>$E_q$ Quarantined Exposed</td>
<td>1.1642</td>
</tr>
<tr>
<td>$H$ Quarantined Infected</td>
<td>137,330</td>
</tr>
<tr>
<td>$A$ Asymptomatic Infected</td>
<td>64,099</td>
</tr>
</tbody>
</table>

and $E$ classes contain most of the individuals coming from the Susceptible compartment. But it requires us to study the rate of transferring from the $S$ to the $E_q$, that is, $\beta_{cq}$ more carefully.

Also Figure 4 shows that the Quarantined Susceptible achieve its maximum and the Susceptible achieve its minimum when the Recovered class increased gradually. It indicates that the Quarantined Susceptible is changing back to the Susceptible at a rate of $\lambda = 1/14$. 
Figure 3: Model with Asymptomatic Infected only SEIR
Figure 4: Model with Asymptomatic Infected
3 SEIR Model without Asymptomatic Infected

3.1 Assumptions and Data

The second model [5] also considers the quarantined factor $q$ and hence introduces $S_q$, $E_q$ and $H$ classes. However, it cancels the $A$ compartment since the proportion of this group of asymptomatic infectious is relatively small in the population. We will investigate how the results change with the initial number of asymptomatic in the next chapter.

In this paper we adopt the same data as [5] of Hubei province from Jan 23th to Feb 24th.

3.2 Parameters and Equations

Since the $A$ compartment does not exist in this model, neither does the parameter $\gamma_A$. The compartments $I$ and $E$ now can transmit virus to the Susceptible $S$ compartment. That is the $(I + \theta E)$ in the equation (21).

A new factor is introduced which is $\rho$, the effective contact factor. The constant contact rate $c$ in the first model is replaced by $\rho c$ which is the effective contact rate. By changing the value of $\rho$ we can investigate how isolation, social distancing, remote meeting and personal protection affect the spreading of the virus.

This second paper [5] also changes some value of the parameters and initial values compared to the first one [4] in Table 6 and 7.
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c$</td>
<td>2</td>
</tr>
<tr>
<td>$\gamma_I$</td>
<td>0.007</td>
</tr>
<tr>
<td>$\gamma_H$</td>
<td>0.014</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$2.05 \times 10^{-9}$</td>
</tr>
<tr>
<td>$q$</td>
<td>$1 \times 10^{-6}$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>$2.7 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

Table 6: Parameters of [5]

<table>
<thead>
<tr>
<th>Compartments</th>
<th>Initial Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S$: Susceptible</td>
<td>59170000</td>
</tr>
<tr>
<td>$E$: Exposed</td>
<td>4007</td>
</tr>
<tr>
<td>$I$: Infectious</td>
<td>$524 \times 1.5$</td>
</tr>
<tr>
<td>$S_q$: Quarantined Susceptible</td>
<td>2776</td>
</tr>
<tr>
<td>$E_q$: Quarantined Exposed</td>
<td>400</td>
</tr>
<tr>
<td>$H$: Quarantined Infected</td>
<td>401</td>
</tr>
<tr>
<td>$R$: Recovered</td>
<td>31</td>
</tr>
</tbody>
</table>

Table 7: Initial Values of [5]

The equations are given in (21).

\[
\begin{align*}
\frac{dS}{dt} &= -(cp\beta + cpq(1 - \beta))S(I + \theta E) + \lambda S_q \\
\frac{dE}{dt} &= cp\beta(1 - q)S(I + \theta E) - \sigma E \\
\frac{dI}{dt} &= \sigma a E - (\gamma_I + \alpha + \delta_I)I \\
\frac{dS_q}{dt} &= (1 - \beta)pcqS(I + \theta E) - \lambda S_q \\
\frac{dE_q}{dt} &= \beta pcqS(I + \theta E) - \delta_q E_q \\
\frac{dH}{dt} &= \delta_I I + \delta_q E_q - (\alpha + \gamma_H)H \\
\frac{dR}{dt} &= \gamma_I I + \gamma_H H
\end{align*}
\]  
(21)
<table>
<thead>
<tr>
<th>Compartments</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I$ Infectious</td>
<td>49,152</td>
</tr>
<tr>
<td>$E$ Exposed</td>
<td>52,193</td>
</tr>
<tr>
<td>$S_q$ Quarantined Susceptible</td>
<td>43,061,000</td>
</tr>
<tr>
<td>$E_q$ Quarantined Exposed</td>
<td>400</td>
</tr>
<tr>
<td>$H$ Quarantined Infected</td>
<td>316,400</td>
</tr>
</tbody>
</table>

Table 8: Maximum Values

3.3 Results

Figure 5 shows the results of simulation without the Asymptomatic Infected. Table 8 shows the maximum values of different compartments in about four months. The peak value of the Infectious $I$ occurred at the 36th day which is Mar 1, 2020. It is basically the same as the first model.

Figure 5: Model without Asymptomatic Infected
4 Sensitivity Analysis of the Initial Value of the Asymptomatic Infected

Based on the Chapter 2, we investigate how do the maximum of the Exposed, Infected, Asymptomatic Infectious (A), Quarantined Infectious (H), Recovered and Quarantined Susceptible (Sq) vary as the percentage of initial A in the total population increasing. From Figure 6, we can see that the maximum of the Sq class increases significantly to $8 \times 10^6$ while others do not increase a lot in magnitude.

Figure 6: Max value of E, I, A, H, R, Sq vs Initial A

Figure 7 shows that as the percentage of initial A class increasing, the
maximum of Quarantined Susceptible increases and tends to a steady state when we apply the same contact rate of $c = 14.781$ in [4]. It also illustrates that time at maximum Quarantined Susceptible drops to zero rapidly as the percentage of initial value of A compartment increases.

Figure 7: Max Sq, Time at Max Sq vs Initial A
5 Sensitivity Analysis of the Effective Contact Rate

Section 3.2 replaces the constant contact rate $c$ into the effective contact rate $c\rho$ which is the product of the effective contact factor $\rho$ and the basic contact rate $c$. Hence, the system of differential equations (20) now turns into this new system.

\[
\begin{align*}
\frac{dS}{dt} &= -(c\rho \beta + c\rho q (1 - \beta) + \lambda S_q)S(I + \theta A) + \lambda S_q \\
\frac{dE}{dt} &= c\rho \beta (1 - q)S(I + \theta A) - \sigma E \\
\frac{dI}{dt} &= \sigma a E - (\gamma_I + \alpha + \delta_I)I \\
\frac{dA}{dt} &= \sigma (1 - a) E - \gamma_A A \\
\frac{dS_q}{dt} &= (1 - \beta) c\rho q S(I + \theta A) - \lambda S_q \\
\frac{dE_q}{dt} &= \beta c\rho q S(I + \theta A) - \delta_q E_q \\
\frac{dH}{dt} &= \delta_I I + \delta_q E_q - (\alpha + \gamma_H)H \\
\frac{dR}{dt} &= \gamma_I I + \gamma_A A + \gamma_H H
\end{align*}
\]

When the effective contact factor $\rho$ becomes smaller, it means that the social distancing performs better. Here we first decrease the value of factor $\rho$ starting at day seven by taking $1.0, 0.8, 0.5, 0.3$ respectively. As we decrease the value of $\rho$, the Quarantined Susceptible decreases more sharply from Figure 8 to Figure 11. In addition, the recovered reaches a steady state more rapidly. Note that the vertical lines in all the following graphs indicate the start of the social distancing.
From Figure 12 to Figure 15, the effect of delaying social distancing is investigated by choosing a fixed value of $\rho = 0.5$ and applying it to the model from the first day, after one week, after two weeks and after 30 days, respectively. The graphs show that the number of Quarantined Susceptible increase significantly as the start of social distancing is delayed. It can also be seen that the recovered increases since the exposed and infected increase.

Figure 8: $\rho = 1.0$ starting from day seven of [4]
Figure 9: $\rho = 0.8$ starting from day seven based on [4]
Figure 10: $\rho = 0.5$ starting from day seven based on [4]
Figure 11: $\rho = 0.3$ starting from day seven based on [4]

As shown in the above Figure 8 - 11, starting social distancing at day seven by decreasing contact factor $\rho$ decreases the Quarantined Susceptible rapidly.
Figure 12: $\rho = 0.5$ starting from day zero based on [4]
Figure 13: $\rho = 0.5$ starting from day seven based on [4]
Figure 14: $\rho = 0.5$ starting from day 14 based on [4]
Figure 15: $\rho = 0.5$ starting from day 30 based on [4]

Figure 12 to Figure 15 show that delaying the start of social distancing increases the peak of the Quarantined Susceptible and causes it to decay more slowly.
Appendices

A MATLAB Code for Models

A.1 MATLAB Code for SIR Model

clear;
clc;

N = 1000; %Population

rB = 0.3; % Amount of people contacted
% by an infected * Probability of being infected
y = 0.1; % Probability of recover

deltaT = 0.125;
T = 0.0:deltaT:120.0;
n = length(T);

S = zeros(n,1);
I = zeros(n,1);
R = zeros(n,1);

S(1) = 999;
I(1) = 1;
R(1) = 0;

for i = 1:n-1
    S(i+1) = S(i) - deltaT * (rB*S(i)*I(i)/N);
    I(i+1) = I(i) + deltaT * (rB*S(i)*I(i)/N - y*I(i));
    R(i+1) = R(i) + deltaT * (y*I(i));
end

plot(T, S, T, I, T, R);grid on;
xlabel('Time(days)');ylabel('Number of People')
legend('S','I','R')
A.2 MATLAB Code of SEIR Model

clear;clc;

N = 10000; %Population
E = 0;
I = 1;
S = N - I;
R = 0;

r = 20; % Amount of people contacted by an infected
B = 0.03; % Probability of being infected
a = 0.1; % Probability of the exposed to the infected
y = 0.1; % Probability of recover

deltaT = 0.125;
T = 0.0:deltaT:120.0;
n = length(T);

for idx = 1:n-1
S(idx+1) = S(idx) - deltaT * (r*B*S(idx)*I(idx)/N);
E(idx+1) = E(idx) + deltaT * (r*B*S(idx)*I(idx)/N-a*E(idx));
I(idx+1) = I(idx) + deltaT * (a*E(idx) - y*I(idx));
R(idx+1) = R(idx) + deltaT * (y*I(idx));
end

plot(T,S,T,E,T,I,T,R);grid on;
xlabel('Time(days)');ylabel('Number of People')
legend('S','E','I','R')
A.3 MATLAB Code of SEIR Model with Asymptomatic Infected

% coefficient
deltaI = 0.1326;
deltaQ = 0.1259;
gammaI = 0.3303;
gammaA = 0.1398;
gammaH = 0.1162;

a = 0.8683;
c = 14.78;
beta = 2.1011e-8;
q = 1.8887e-7;
sigma = 1/7;
lambda = 1/14;
theta = 1;
alpha = 1.7826e-5;

% iteration
T = 1 : 120;
n = length(T);

S = zeros(n,1);
E = zeros(n,1);
I = zeros(n,1);
A = zeros(n,1);
Sq = zeros(n,1);
Eq = zeros(n,1);
H = zeros(n,1);
R = zeros(n,1);

% initial value
S(1) = 11081000;
E(1) = 105.1;
I(1) = 27.679;
A(1) = 554050.0; % 53.839;
\[ S(t) = S(t) - (c*\beta + c*q*(1-\beta))*S(t)*I(t)*A(t) + \lambda*S(t) \]
\[ E(t) = E(t) + c*\beta*(1-q)*S(t)*I(t)*A(t) - \sigma*E(t) \]
\[ I(t) = I(t) + \sigma*a*E(t) - (\delta_I + \alpha + \gamma_I)*I(t) \]
\[ A(t) = A(t) + \sigma*(1-a)*E(t) - \gamma_A*A(t) \]
\[ Sq(t) = Sq(t) + c*q*(1-\beta)*S(t)*I(t)*A(t) - \lambda*Sq(t) \]
\[ Eq(t) = Eq(t) + c*\beta*q*S(t)*(I(t)+\theta*A(t)) - \delta_Q*Eq(t) \]
\[ H(t) = H(t) + \delta_I*I(t) + \delta_Q*Eq(t) - (\alpha + \gamma_H)*H(t) \]
\[ R(t) = R(t) + \gamma_I*I(t) + \gamma_A*A(t) + \gamma_H*H(t) \]

end

figure(1);
plot(T, S, T, E, T, I, T, R);
grid on;
xlabel('Time(days)');
ylabel('Number of People');
legend('Susceptible', 'Exposed', 'Infected', 'Recovered', 'Asymptomatic Infected', ...
'Quarantined Susceptible', 'Quarantined Exposed', 'Quarantined Infected', 'Recovered');
title('Adjusted SEIR Model');

A.4 MATLAB Code of SEIR Model without Asymptomatic Infected

%coefficient
\[ \delta_I = 0.1326; \]
\[ \delta_Q = 0.1259; \]
\[
\begin{align*}
\gamma_I &= 0.007; \\
\gamma_A &= 0.1398; \\
\gamma_H &= 0.014; \\
\alpha &= 0.8683; \\
c &= 2; \\
\beta &= 2.05 \times 10^{-9}; \\
q &= 1 \times 10^{-6}; \\
\sigma &= 1/7; \\
\lambda &= 1/14; \\
\theta &= 1; \\
\alpha &= 2.7 \times 10^{-4};
\end{align*}
\]

%Iteration

\[
T = 1 : 120; \\
n = \text{length}(T);
\]

\[
\begin{align*}
S &= \text{zeros}(n,1); \\
E &= \text{zeros}(n,1); \\
I &= \text{zeros}(n,1); \\
A &= \text{zeros}(n,1); \\
S_0 &= \text{zeros}(n,1); \\
E_0 &= \text{zeros}(n,1); \\
H &= \text{zeros}(n,1); \\
R &= \text{zeros}(n,1);
\end{align*}
\]

%Initial value

\[
\begin{align*}
S(1) &= 59170000; \%11081000; \\
E(1) &= 4007; \%105.1; \\
I(1) &= 524 \times 1.5; \%27.679; \\
A(1) &= 53.839; \\
S_0(1) &= 2776; \%739; \\
E_0(1) &= 400; \%1.1642; \\
H(1) &= 401; \%1; \\
R(1) &= 31; \%2;
\end{align*}
\]

\[
\text{for } t = 1:n-1
\]
\[
S(t+1) = S(t) - (c\cdot\beta + c\cdot q\cdot(1-\beta))\cdot S(t)\cdot (I(t) + \theta\cdot E(t)) + \lambda\cdot S_q(t);
\]
\[
E(t+1) = E(t) + c\cdot\beta\cdot(1-q)\cdot S(t)\cdot (I(t) + \theta\cdot E(t)) - \sigma\cdot E(t);
\]
\[
I(t+1) = I(t) + \sigma\cdot E(t) - (\delta_I + \alpha + \gamma_I)\cdot I(t);
\]
\[
\% A(t+1) = A(t) + \sigma\cdot(1-a)\cdot E(t) - \gamma_A\cdot A(t);
\]
\[
S_q(t+1) = S_q(t) + c\cdot q\cdot(1-\beta)\cdot S(t)\cdot (I(t) + \theta\cdot E(t)) - \lambda\cdot S_q(t);
\]
\[
E_q(t+1) = E_q(t) + c\cdot\beta\cdot q\cdot S(t)\cdot (I(t) + \theta\cdot E(t)) - \delta_Q\cdot E_q(t);
\]
\[
H(t+1) = H(t) + \delta_I\cdot I(t) + \delta_Q\cdot E_q(t) - (\alpha + \gamma_H)\cdot H(t);
\]
\[
R(t+1) = R(t) + \gamma_I\cdot I(t) + \gamma_H\cdot H(t);
\]

end

figure(1);

plot(T, S, T, E, T, I, T, S_q, T, E_q, T, H, T, R);

grid on;

xlabel('Time(days)');
ylabel('Number of People');

legend('Susceptible','Exposed','Infected','Quarantined Susceptible','Quarantined Exposed','Quarantined Infected','Recovered');

title('Adjusted SEIR Model');

A.5 MATLAB Code of Sensitivity Analysis of the Initial Value of the Asymptomatic Infectious

close all;
clear;
clc;

totalPopulation = 11081000;

%%coefficient
deltaI = 0.1326;
deltaQ = 0.1259;
gammaI = 0.3303;
gammaA = 0.1398;
gammaH = 0.1162;

a = 0.8683;
c = 14.78;
beta = 2.1011e-8;
q = 1.8887e-7;
sigma = 1/7;
lambda = 1/14;
theta = 1;
alpha = 1.7826e-5;

initialA = 0.0:100.0:500000.0;

nIA = length(initialA);

Emax = zeros(nIA,1);
Amax = zeros(nIA,1);
Imax = zeros(nIA,1);
Hmax = zeros(nIA,1);
Rmax = zeros(nIA,1);
SMin = zeros(nIA,1);
tSMin = zeros(nIA,1);
SqMax = zeros(nIA,1);
tSqMax = zeros(nIA,1);

for k = 1:nIA

deltaT = 0.125;

T = 0.0:deltaT:120.0;

n = length(T);
\[
\begin{align*}
S &= \text{zeros}(n,1); \\
E &= \text{zeros}(n,1); \\
I &= \text{zeros}(n,1); \\
A &= \text{zeros}(n,1); \\
Sq &= \text{zeros}(n,1); \\
Eq &= \text{zeros}(n,1); \\
H &= \text{zeros}(n,1); \\
R &= \text{zeros}(n,1); \\
\end{align*}
\]

%initial value
\[
\begin{align*}
E(1) &= 105.1; \\
I(1) &= 27.679; \\
A(1) &= \text{initialA}(k); \\
Sq(1) &= 739; \\
Eq(1) &= 1.1642; \\
H(1) &= 1; \\
R(1) &= 2; \\
S(1) &= \text{totalPopulation} - A(1) - E(1) - I(1) - Sq(1) - Eq(1) - H(1) - R(1); \\
\end{align*}
\]

for \( t = 1:n-1 \)
\[
\begin{align*}
S(t+1) &= S(t) - \delta T \times ((c*beta*c*q*(1-beta))*S(t)*(I(t)+theta*A(t))+lambda*Sq(t)); \\
E(t+1) &= E(t) + \delta T \times (c*beta*(1-q)*S(t)*(I(t)+theta*A(t))-sigma*E(t)); \\
I(t+1) &= I(t) + \delta T \times (sigma*a*E(t)-(deltaI+alpha+gammaI)*I(t)); \\
A(t+1) &= A(t) + \delta T \times (sigma*(1-a)*E(t)-gammaA*A(t)); \\
Sq(t+1) &= Sq(t) + \delta T \times (c*q*(1-beta)*S(t)*(I(t)+theta*A(t))-lambda*Sq(t)); \\
Eq(t+1) &= Eq(t) + \delta T \times (c*beta*q*S(t)*(I(t)+theta*A(t))-deltaQ*Eq(t)); \\
H(t+1) &= H(t) + \delta T \times (deltaI*I(t)+deltaQ+Eq(t)-(alpha+gammaH)*H(t)); \\
R(t+1) &= R(t) + \delta T \times (gammaI*I(t)+gammaA*A(t)+gammaH*H(t)); \\
\end{align*}
\]

end

\[
\begin{align*}
E_{\max}(k) &= \max(E); \\
A_{\max}(k) &= \max(A); \\
I_{\max}(k) &= \max(I); \\
H_{\max}(k) &= \max(H); \\
R_{\max}(k) &= \max(R); \\
\end{align*}
\]
[SMin(k), kSMin] = min(S);

tSMin(k) = T(kSMin);

[SqMax(k), kSqMax] = max(Sq);

tSqMax(k) = T(kSqMax);

end

percentPop = 100.0 * initialA / totalPopulation;

figure(1);

plot(percentPop, Emax, percentPop, Amax, percentPop, Imax,...
percentPop, Hmax, percentPop, Rmax, percentPop, SqMax);

grid on;

xlabel('Initial A (% Population)');
ylabel('max of E, A, I, H, R, Sq');

legend('E','A','I','H','R','Sq')

figure(2);

subplot(211);

plot(percentPop, SMin);

hold on;

plot(percentPop, SqMax);

grid on;
A.6 MATLAB Code of SEIR Model with Asymptomatic Infected $\rho = 0.3$ from day seven

close all;
clear;
clc;

%coefficient
deltaI = 0.1326;
deltaQ = 0.1259;
gammaI = 0.3303;
gammaA = 0.1398;
gammaH = 0.1162;

rho = 1.0;
a = 0.8683;
c = 2;
beta = 2.1011e-8;
q = 1.8887e-7;
sigma = 1/7;
lambda = 1/14;
theta = 1;
alpha = 1.7826e-5;

%iteration

T = 1 : 120;

n = length(T);

S = zeros(n,1);
E = zeros(n,1);
I = zeros(n,1);
A = zeros(n,1);
Sq = zeros(n,1);
Eq = zeros(n,1);
H = zeros(n,1);
R = zeros(n,1);

%initial value
S(1) = 11081000;
E(1) = 105.1;
I(1) = 27.679;
A(1) = 554050.0;
Sq(1) = 739;
Eq(1) = 1.1642;
H(1) = 1;
R(1) = 2;
for t = 1:n-1
    if t >= 7
        rho = 0.3;
    end
    S(t+1) = S(t) - (c*rho*beta+c*q*(1-beta))*S(t)*(I(t)+theta*A(t))*lambda*Sq(t);
    E(t+1) = E(t) + c*rho*beta*(1-q)*S(t)*(I(t)+theta*A(t))-sigma*E(t);
    I(t+1) = I(t) + sigma*a*E(t) - (deltaI+alpha+gammaI)*I(t);
    A(t+1) = A(t) + sigma*(1-a)*E(t) - gammaA*A(t);
    Sq(t+1) = Sq(t) + c*rho*q*(1-beta)*S(t)*(I(t)+theta*A(t))*lambda*Sq(t);
    Eq(t+1) = Eq(t) + c*rho*beta*q*S(t)*(I(t)+theta*A(t))-deltaQ*Eq(t);
    H(t+1) = H(t) + deltaI*I(t) + deltaQ*Eq(t) - (alpha+gammaH)*H(t);
    R(t+1) = R(t) + gammaI*I(t) + gammaA*A(t) + gammaH*H(t);
end

figure(1);
plot(T, S, T, E, T, I, T, R, T, A, T, Sq, T, Eq, T, H);
grid on;
hold on
plot([7 7],[0, 12e6])
xlabel('Time(days)');
ylabel('Number of People');
legend('Susceptible','Exposed','Infected','Recovered','Asymptomatic Infected',...
'Quarantined Susceptible','Quarantined Exposed','Quarantined Infected');
title('Adjusted SEIR Model');
References


