TRAFFIC CONGESTION AND SHOCKWAVE DAMPING THROUGH
ADVANCED DRIVER ASSISTANCE SYSTEM (ADAS) LONGITUDINAL
VEHICLE CONTROL

By

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ABSTRACT OF THE DISSERTATION

Traffic Congestion and Shockwave Damping Through Advanced Driver Assistance System (ADAS) Longitudinal Vehicle Control

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The Advance Driver Assistance System (ADAS) helps vehicle drivers drive more efficiently, comfortably, and safer. Longitudinal Autonomous Vehicle (AV) control models such as Adaptive Cruise Control (ACC) and Cooperative Adaptive Cruise Control (CACC) models have been intensively studied over the last decade. One limitation identified in this study is the accelerating effect of congestion shockwaves by autonomous vehicle platoons due to the instant response of those systems to preceding stimuli. Stop-and-go waves or traffic shockwaves can be generated naturally by human drivers or roadway bottlenecks such as merging sections. In some cases, if a small perturbation occurs downstream, the traffic shockwave speed could instantly propagate upstream and even lead to congestions with aggressive driving behaviors or badly designed vehicle control models. This case can amplify the traffic instability and deteriorate the traffic flow.

This study designs ADAS longitudinal control models for AVs to dampen the traffic shockwave propagation speed and mitigate the congestion. The rolling horizon control model, also called Model Predictive Control (MPC), is implemented to balance the trade-offs among multiple objectives, including safety, efficiency, and driving comfort. Vehicles models are designed separately depending on their capability of wireless communication. On the one hand, vehicles without wireless communication capability, such as ACC
vehicles, can only obtain the velocity and position of the preceding vehicle through sensors, such that the congestion and shockwave propagation speed can be dampened by implementing the proposed cost function terms. With the proposed cost terms, the controlled vehicle will reduce the spacing with larger by still safe velocity according to the preceding vehicle’s position and velocity. On the other hand, vehicles with Vehicle-to-Vehicle (V2V) wireless communication capability, such as CACC vehicles, enable the ego vehicle to acquire the acceleration, velocity and position of preceding vehicles or even vehicles far downstream. A control model is proposed for vehicles to smoothly decelerate to a target speed to adapt to the detected shockwave downstream.

The proposed framework is applied in both ACC and CACC platoons mixed with manual vehicles, and five different control models are explored. The stability conditions of the proposed models have been derived analytically and analyzed numerically. Three experimental studies, including platoon, ring-road, and traffic simulation studies, indicate promising results of proposed models in reducing shockwave propagation speed and mitigating traffic congestion under different environments compared with existing ACC and CACC models.
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PREFACE

The work conducted in this dissertation has been presented, published, or under-review in several conferences and journals. Below is the list of publications derived from this dissertation:


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1 INTRODUCTION

Shockwaves are generated by a sudden speed rise/drop at the free-flow and congested flow boundary. The propagation speed of shockwaves is around 22 to 24 kilometers per hour based on observations at some locations with queueing flows ranging from about 2,000 to 850 vehicles per hour per lane (Mauch & Cassidy, 2004), with duration usually lasting between 2 to 15 min (Laval & Leclercq, 2010). In traffic flow operated by human drivers, the propagation speed of the shockwave is determined by the average perception-reaction time of drivers. The slow reaction time of human drivers can potentially amplify traffic congestion and trigger stop-and-go waves.

Knowing traffic shockwaves helps researchers and traffic operators better understand congestion patterns and impacts. Traditionally, traffic shockwaves can be detected based on fixed sensors such as loop detectors (Seo, Kusakabe, & Asakura, 2015) or probe vehicles with GPS devices (Herrera et al., 2010; Murakami & Wagner, 1999; Yuan, Van Lint, Wilson, van Wageningen-Kessels, & Hoogendoorn, 2012). Yet emerging wireless communication technologies allow us to obtain traffic conditions, including vehicle trajectories, in real-time. Related studies have been done to detect the shockwaves using vehicle trajectory data. Hegyi et al. (A Hegyi & Hoogendoorn, 2010) identified locations of shockwaves by assuming a max speed $V_{\text{max}}$ and a max volume $q_{\text{max}}$. A shockwave is present if the volume $q_i$ and speed $V_i$ of a defined segment $i$ are smaller than $V_{\text{max}}$ and $q_{\text{max}}$. Izadpanah (Izadpanah, Hellinga, & Fu, 2009) provided an automatic traffic shockwave detection method that identified the first point of shockwaves and grouped different shockwaves using an iterative two-phase piecewise or switching regression. Lu and
Skabardonis (Lu & Skabardonis, 2007) developed a numerical algorithm to estimate the propagation speed of shockwaves based on vehicle trajectory data using a searching algorithm called moving window minimum searching.

Adaptive Cruise Control (ACC) and Cooperative Adaptive Cruise Control (CACC) are longitudinal car-following control models that have been studied in the last decade. While ACC uses Radar or Light Detection and Ranging (LiDAR) measurements to find the distance to the preceding vehicle (PV), CACC vehicles introduce the vehicle-to-vehicle (V2V) communications through DSRC/WAVE technology to acquire information of vehicles in certain ranges. With the wireless communication capability of CACC, vehicles can form a platoon to improve traffic stability by reducing the response delay among vehicles.

1.1 Problem Statement

Previous shockwave damping study shows that shockwaves can be mitigated by using variable speed limit (VSL) (Breton, Hegyi, De Schutter, & Hellendoorn, 2002), which also reduces congestions. The traffic operation center can reduce the shockwave speed by giving a variable speed limit to vehicles through wireless communication.

ACC (Adaptive Cruise Control) assists the drivers in the task of longitudinal control of their vehicle during driving (Marsden, McDonald, & Brackstone, 2001). CACC (Cooperative Adaptive Cruise Control) is a combination of ACC with a cooperative element, such as vehicle-to-vehicle (V2V) and vehicle-to-infrastructure (V2I) communication (S. E.
ACC systems have been adopted in many vehicles coming into the market, yet they appear to be string unstable even at high market penetration rates (S. Shladover, Su, & Lu, 2012). CACC is one of the most tested Connected and Automated Vehicle (CAV) applications forming a platoon to attenuate traffic flow disturbances and improve highway capacity (S. E. Shladover et al., 2015).

Compared with manually-controlled vehicles, ACC and CACC models have shown promising impacts on mitigating traffic congestion due to their ability to shrink the spacing among vehicles. Studies and field tests have shown that the congestion mitigation effects of CACC are caused by a significant reduction in the feasible time headway among vehicles (Ploeg, Serrarens, & Heijenk, 2011; Suzuki & Nakatsuji, 2003). Calvert, Van Den Broek, and van Noort (2011) also pointed out the positive effect of CACC. Due to improving traffic stability, CACC can reduce the congestion shockwaves propagating from a bottleneck location by preventing the occurrence and suppression of exiting shockwaves. Bu et al. proposed an MPC (Model Predictive Control)-based CACC model to mitigate traffic congestion by reducing vehicle time gaps (Bu, Tan, & Huang, 2010). Motamedidehkordi, Margreiter, and Benz (2016) evaluated the effect of inter-vehicle communication on traffic shockwaves. Rabsatt and Gerla (2017) proposed a system named DRIVE-EX to reduce the shockwave impact and uniformly distribute traffic across multiple lanes through V2V communication. In addition to conventional ACC/CACC models, Jam-Absorption Driving models can also control a single vehicle to change its headway to mitigate traffic congestion dynamically (Z. He, Zheng, Song, & Zhu, 2016; Taniguchi, Nishi, Ezaki, & Nishinari, 2015)
One caveat of ACC and CACC control is the potential of amplifying negative impacts of the traffic shockwave speed, given the extremely short perception reaction time among ACC or CACC vehicles (Milanés & Shladover, 2014; Milanés et al., 2013). As proved by Ngoduy’s paper, shorter speed relaxation time (time to adapt to leader’s speed) can lead to shockwaves propagating faster upstream than manual vehicles (MV). As shown in Figure 1, shockwave speed can significantly accelerate under the machine perception-reaction time among ACC/CACC vehicles compared with conventional shockwave speed triggered by human drivers' much slower perception-reaction process. In consequence, the spatial-temporal span of the congestion is expanded. In this paper, the shockwave speed is measured between the trajectory turning point of the first vehicle and the last vehicle entering the congestion area.

![Figure 1. Congestion Propagation of ACC Vehicles and MV](image)

To counter the potential adverse effect of amplifying traffic shockwaves by ACC and CACC, the existing models take three different control strategies: time headway suppression, stability improvement, and mode-switching control based on traffic conditions.
Studies and field tests have shown that the congestion mitigation effects of ACC and CACC models are caused by a significant reduction in the feasible time headway among vehicles (Bu et al., 2010; Motamedidehkordi et al., 2016; Ploeg et al., 2011; Rabsatt & Gerla, 2017; Suzuki & Nakatsuji, 2003). The control system stability improvement has been achieved through local and string stability analysis to ensure system robustness so that the system can recover from small traffic perturbations (Chu, 1974; Darbha & Rajagopal, 2005; Fernandes & Nunes, 2012a; Ali Ghasemi, Kazemi, & Azadi, 2015; Sun, Zheng, & Sun, 2018; D Swaroop & Hedrick, 1996; Darbha Swaroop & Rajagopal, 2001; Talebpour & Mahmassani, 2016; J. Zhou & Peng, 2005). Mode-switching control methods (Kesting, Treiber, Schönhof, & Helbing, 2008; Papacharalampous et al., 2015; Stern et al., 2017) use controllers to estimate the traffic states and classify them into multiple types. Different sets of control parameters are then used for different traffic conditions. The first two strategies indirectly mitigate the shockwave by either increasing the capacity or suppressing traffic perturbations. The third strategy implements variable control modes to actively respond to changing traffic conditions or traffic disturbances. Many parameters designed for different traffic condition scenarios can be difficult to calibrate and may not be transferrable from one location to another.

Several problems are to be addressed for the above congestion mitigation and shockwave damping methods.

1) What is the framework of the vehicle control model?

2) What vehicle dynamic prediction model can be utilized as the basis of the control model?
3) What are the constraints of the control model to be used?

4) How to address both the congestion mitigation and shockwave damping?

5) How to validate and evaluate the proposed control model?

1.2 Research Objectives and Scope of Work

Based on the status of current longitudinal autonomous vehicle control models, the objectives of this research are as follows:

1) Design optimal control models for vehicles with and without wireless communication to mitigate congestion and dampen shockwave propagation speed.

2) Design the constraints of the optimal control model to ensure safety and comfort.

3) Design the multiobjective cost function for proposed control models to restrain the shockwave propagation speed and overall network congestions.

4) Analyze the stability of proposed control models.

5) Evaluate the performance of proposed control models through numerical and microscopic traffic simulation.

6) Evaluate the macroscopic impact of the proposed control model through the traffic flow, shockwave propagation speed and overall congestion area.

Furthermore, the scope of this dissertation is restrained by the following criteria:

1) Focus on the longitudinal car-following models without considering lane changes.

2) Focus on control models that require vehicle position, velocity, and
acceleration of vehicle through detection sensors and vehicle-to-vehicle (V2V) wireless communication.

3) Focus on optimal control problems with multi objectives for the vehicle control model, with .

1.3 Research Contributions

The contributions of this research are as follows:

1) Design optimal control ACC and CACC models with a multi-objective cost function to achieve shockwave propagation speed damping on top of congestion mitigation.

2) Improve the overall traffic efficiency and stability compared with existing models.

3) Derive local stability and string stability conditions, through the Portrigin Maximum Principle (PMP), for all proposed models and validate it through designed platoon simulation.

4) Design experiments to validate the performance of the proposed model for different scenarios.

5) Analyze the sensitivity of model parameters.

1.4 Organization of the Dissertation

The flowchart of this dissertation research is shown in Figure 2. Section 2 conducts a comprehensive literature review of existing ACC or CACC models. Section 3 describes the proposed methodology with the design of the proposed optimal control model with a
detailed framework and stability analysis. Section 4 presents the experiment design to validate and evaluate proposed models through two platforms: MATLAB and VISSIM. And a detailed performance analysis is conducted based on the simulation results. In Section 5, the experiment results are analyzed and compared with existing models. For the last Section, conclusions are drawn, and potential future research directions will be discussed.

![Flowchart of the Dissertation Research](image_url)

**Figure 2. The Flowchart of the Dissertation Research**
2 LITERATURE REVIEW

The autonomous vehicle controls can be categorized into longitudinal control (i.e., car-following, lane-keeping) and lateral control (i.e., lane-change). This paper focuses on the longitudinal control models of autonomous vehicles, including ACC and CACC models. Di and Shi (2021) conducted a thorough literature review on autonomous vehicles control in heterogeneous traffic environments with both manual vehicles and autonomous vehicles. They classify the longitudinal control into four scenarios, including: platooning (Gong, Shen, & Du, 2016; Y. Li, Tang, Peeta, & Wang, 2018; Wei et al., 2017; Y. Zhou, Ahn, Chitturi, & Noyce, 2017), speed harmonization (Arefizadeh & Talebpour, 2018; Ma et al., 2016; Malikopoulos, Hong, Park, Lee, & Ryu, 2018), longitudinal trajectory optimization (X. Li, Ghiasi, Xu, & Qu, 2018; Wei et al., 2017), and eco-approach and departure at signalized intersections (Altan, Wu, Barth, Boriboonsomsin, & Stark, 2017; P. Hao, Wu, Boriboonsomsin, & Barth, 2018; Yao, Cui, Li, Wang, & An, 2018).

2.1 Overview of ACC and CACC Models

ACC and CACC systems assist drivers in longitudinal control during their driving, improving the road capacity significantly after its market penetration reached moderate to high percentages (S. Shladover et al., 2012). The automated system controls the accelerator, engine powertrain, and vehicle brakes to maintain the vehicle ahead's desired time headway or spacing. Related benefit evaluation of ACC/CACC is discussed in (Marsden et al., 2001; Miller, Misener, Godbole, & Deshpande, 1999; Van Arem, Van Driel, & Visser, 2006). ACC/CACC experimental implementations are well-discussed in (G. J. Naus, Vugts, Ploeg,
Most conventional ACC controllers use proportional-derivative (PD) or proportional-integral-derivative (PID) as the controller (Lidstrom et al., 2012; Milanés et al., 2014). And the Model Predictive Control (MPC) approach is another widely used control method in the autonomous car-following model, predicting the model in a rolling time horizon. Such an approach predicts vehicle’s state based on measurements acquired from radar, LiDAR, or other sensors (Bageshwar, Garrard, & Rajamani, 2004; S. Li, Li, Rajamani, & Wang, 2011; Moser, Waschl, Kirchsteiger, Schmied, & del Re, 2015; Stanger & del Re, 2013). Bageshwar et al. (2004) used MPC to compute the spacing-control laws for transitional maneuvers of ACC vehicles in a vehicle cut-in scenario. Moser et al. (2015) developed a conditional linear MPC to predict vehicle states based on radar measurements, V2V, and I2V communication systems.

It should be noted that autonomous vehicles, including ACC vehicles, can monitor other vehicles in their vicinity using sensors continually. Still, there is no direct communication between vehicles and the roadside units. Talebpour, Mahmassani, and Hamdar (2013) presented an analytical and simulation-based investigation that considers modeling three types of vehicles: manual, connected, and autonomous vehicles. According to the simulation results, vehicle automation is more effective than vehicle connectivity alone in
preventing shockwave formation and propagation. Stern et al. conducted a ring-road experiment where an autonomous vehicle is controlled to dampen stop-and-go waves, showing that a single autonomous vehicle can control the flow of at least 20 manual vehicles around it (Stern et al., 2018).

The rest of this subsection describes two significant components: distance control policies and Proportional-Integral-Derivative (PID) Control model. The Constant Distance Headway (CDH) and the Constant Time Headway (CTH) policy are two significant distance control policies of ACC/CACC systems. CDH policies maintain a constant distance between vehicles, while CTH policies maintain a constant time gap. And the PID control model is commonly used in the real-world practices of Autonomous Vehicle deployment for its simplicity and cost-efficiency.

2.1.1 Constant Distance Headway (CDH) Policy
In CDH policy, the distance between the leading vehicle and ego vehicle is independent of the speed of the controlled vehicle, and its implementation requires inter-vehicular communication to ensure string stability (J. Zhou & Peng, 2005). The constant spacing policy is used in situations where a higher flow rate is the main purpose. The constant-spacing can be achieved by:

\[ s_i = x_i - x_{i-1} + L_i \]  

where \( L_i = l_{i-1} + s_{i_{des}} \), with \( l_{i-1} \) being the length of the preceding vehicle and \( s_{i_{des}} \) being the inter-vehicle desired gap; \( x_i \) and \( x_{i-1} \) is the position of the ego vehicle and preceding vehicle, respectively.
Fernandes and Nunes (2012b) designed a constant-spacing model to consider safety and stability, canceling communication delays. Upon emergency occurrences, the platoon’s timely response is ensured by dynamically increasing the weight of the leading vehicle data over the behavior of the following vehicles. Abdorasoul Ghasemi, Kazemi, and Azadi (2013) proposed a model considering the lag of actuators and sensors. A hierarchical platoon controller framework was designed with a two-layer framework based on a constant spacing policy. Guo and Yue (2012) established a novel model comprising the actuator delay and the effect of sensing range limitation. They also added conditions to guarantee string stability and zero steady-state spacing error.

Zheng, Wang, and Li (2020) and J. Wang, Zheng, Xu, Wang, and Li (2020) proposed a control model that uses the deviation from the equilibrium (constant) spacing and equilibrium speed as the objective function of the optimal control problem in a ring-road-single-lane network:

\[ L(X, u) = c_1(s_i(t) - s_c^*)^2 + c_2(v_i(t) - v^*)^2 + c_3u_i^2(t) \]  

(2.2)

where \( c_1, c_2, \) and \( c_3 \) are weights for spacing deviation, speed deviation and acceleration magnitude. \( s_c^* \) is a designing factor and \( v^* \) is the desired traffic speed. \( s_c^* \) and \( v^* \) are the equilibrium traffic states determined based on the ring-road settings like road length and number of vehicles. \( s_c^* \) and \( v^* \) are calibrated together with other reference models under the same settings. In this paper, instead of using closed-loop optimal control, the objective function is implemented in the same open-loop control framework using the iPMP framework, same as other MPC-based models. The corresponding costates and optimal control are:
\[
- \frac{d\lambda_s}{dt} = 2c_1(s_i - s_c^*)
\]
(2.3)
\[
- \frac{d\lambda_v}{dt} = 2c_2(v_i - v^*) - \lambda_s
\]
(2.4)
\[
u^* = -\frac{\lambda_v}{2c_3}
\]
(2.5)

2.1.2 Constant Time Headway (CTH) Policy

The constant time headway (CTH) policy is widely used in the ACC/CACC models. In CTH policy, the desired intervehicle distance is described by \( R_d = R_0 + t_d v_i \), where \( R_0 \) is the standstill distance, \( t_d \) is the time headway, and \( v_i \) is the ego-vehicle speed. A sliding mode controller without using acceleration information can be used to ensure string stability (J. Zhou & Peng, 2005). The desired acceleration is:

\[
a_{i,des} = \frac{\lambda}{t_d} e_i + \frac{\lambda}{t_d} \dot{R}_i
\]
(2.6)

where \( e_i \) is the spacing error, i.e., the difference between the real and desired spacing; \( \lambda \) is the convergence rate of the sliding surface. \( \dot{R}_i \) is the speed difference of the ego and preceding vehicle.

CTH policy can ensure the safety concern in ACC/CACC control laws design. According to (Fernandes & Nunes, 2012b), the desired spacing of CTH policy can also be described by the Intelligent Driver Model (IDM). The IDM was initially proposed by Treiber, Hennecke, and Helbing (2000), formulated as

\[
a_{IDM} = a \left( 1 - \left( \frac{v_i}{v_{max}} \right)^4 - \left( \frac{S(\dot{s}_i v_i)}{s_i} \right)^2 \right)
\]
(2.7)
\[ S(\dot{s}_i, v_i) = s_0 + T_{gap} v_i - \frac{\dot{s}_i v_i}{\sqrt{4ab}} \]  

(2.8)

where \( a \) is the maximum desired acceleration, \( S(\dot{s}, v) \) is the desired spacing, \( s_0 \) is the minimum gap at standstill situation, \( T_{gap} \) is the desired time gap, and \( b \) is the comfortable deceleration. \( S \) increases when speed increases, which also reduces the density. It is inevitable to follow this performance pattern in a platoon of manually driven vehicles because human drivers need a constant time to react but not a constant spacing. However, the original IDM model produces overreactions when the relative velocities and spacings are low. At the same time, real-world human drivers usually assume that PV will not suddenly initiate a full-stop emergency braking without any reason. To characterize this optimistic view of drivers, Kesting, Treiber, and Helbing (2010) proposed the enhanced IDM with constant-acceleration heuristic (CAH) on the safe acceleration:

\[ a_{CAH} = \begin{cases} \frac{v_i^2 \bar{a}_{i-1}}{v_{i-1}^2 - 2s_i \bar{a}_{i-1}} & \text{if } v_{i-1}(v_i - v_{i-1}) \leq -2s_i \bar{a}_{i-1} \\ \bar{a}_{i-1} - \frac{(v_i - v_{i-1})^2 \Theta(v_i - v_{i-1})}{2s_i} & \text{otherwise} \end{cases} \]  

(2.9)

where the effective acceleration \( \bar{a}_{i-1} = \min(a_{i-1}, a) \) is used to avoid outcomes that PV may cause with higher acceleration capabilities. The condition \( v_{i-1}(v_i - v_{i-1}) \leq -2s_i \bar{a}_{i-1} \) is true if the vehicle has stopped when the minimum gap \( s = 0 \) is reached. Otherwise, negative approaching rates do not make sense to the CAH and are therefore eliminated by the Heaviside step function \( \Theta \). An ACC model is then proposed based on both \( a_{IDM} \) and \( a_{CAH} \) to avoid unrealistically high decelerations:

\[ a_{ACC} = \begin{cases} a_{IDM} & \text{if } a_{IDM} \geq a_{CAH} \\ (1 - c)a_{IDM} + c(a_{CAH} + b \tanh\left(\frac{a_{IDM} - a_{CAH}}{b}\right)) & \text{otherwise} \end{cases} \]  

(2.10)

where \( c \) is the coolness factor set in the range of \([0.95, 1.00]\).
Duret, Wang, and Ladino (2019) proposed a CTH-based MPC ACC model for truck platooning. The running cost function is defined as

$$\mathcal{L} = c_1(s_i - s_d)^2 + c_2\Delta v_i^2 + c_3u_i^2$$

(2.11)

where \( s_d = v_i t_d + s_0 \) with \( t_d \) as the constant desired time headway.

M. Wang, Daamen, Hoogendoorn, and van Arem (2014a) designed the two-mode ACC model, including cruising and following modes. A two-regime running cost \( \mathcal{L} \) is proposed as:

$$\mathcal{L} = \begin{cases} 
  c_1 e^{s_0} \Delta v_i^2 \Theta(-\Delta v_i) + c_2(s_d - s_i)^2 + \frac{1}{2} u_i^2, & \text{if } s_i \leq s_f \\
  c_3(v_{free} - v_i), & \text{if } s_i > s_f 
\end{cases}$$

(2.12)

where \( s_f = v_{free} t_{d,m} + s_0 \) is the spacing threshold to determine if the vehicle is in cruising mode \( (s_i > s_f) \) or following mode \( (s_i \leq s_f) \). \( t_{d,m} \) is the user-defined maximum desired time headway, and the \( s_0 \) is the minimum spacing between standstill vehicles. The desired spacing \( s_d = v_i(t) t_d + s_0 \) is determined by the spacing dependent desired time gap:

$$t_d = t_{d,0} + \frac{s}{s_f}(t_{d,m} - t_{d,0})$$

(2.13)

where \( t_{d,0} \) is the minimum desired time headway. And \( \Theta(x) \) is a Heaviside step function defined as:

$$\Theta(x) = \begin{cases} 
  1, & \text{if } x \geq 0 \\
  0, & \text{if } x < 0 
\end{cases}$$

(2.14)

Equation (2.12) consists of three cost terms: safety, efficiency and comfort. The safety cost only occurs when the ego vehicle approaches the PV in the following mode and vanishes in cruising mode. And the safety cost is a monotonic decreasing function of spacing \( s_i \). The shorter the \( s_i \), the larger the safety cost is. The efficiency cost accounts for deviating from
the desired spacing. And the comfort cost represents the penalty of large acceleration or deceleration, which provides comfortable driving behavior.

2.1.3 Proportional-Integral-Derivative (PID) Control Model

The PID control models are linear models that consider several parameters and limited constraints. S. Shladover, VanderWerf, Miller, Kourjanskaia, and Krishnan (2001) proposed an ACC and a CACC model to enhance highway capacity by minimizing time headway and maintaining driver comfort, convenience and safety at the same time. The ACC controller is linear with limits on deceleration and acceleration.

\[ a_i(t) = k_1(v_{i-1}(t) - v_i(t)) + k_2(s_i(t) - s_d(t)) \]  \hspace{1cm} (2.15)

where \( a_{\text{max}} \) is maximum acceleration, \( b_2 \) is the maximum deceleration of the following vehicles. The desired spacing between vehicles \( s_d(t) = v_i(t)t_d \). \( k_1 > 0 \) and \( k_2 > 0 \) are parameters of the PID controller. The proposed CACC vehicle controller has an additional term compared with ACC controller on the PV’s acceleration, defined as follows:

\[ a_i(t) = k_0a_{i-1}(t) + k_1(v_{i-1}(t) - v_i(t)) + k_2(s_i(t) - s_d(t)) \]  \hspace{1cm} (2.16)

where the CACC controller has the same deceleration and acceleration limits as in Equation (2.15). \( k_0 = 1, k_1 > 0 \) and \( k_2 > 0 \) are the same as in ACC controller, while the vehicle spacing is defined by

\[ s_d(t) = \max(s_{\text{safe}}(t), s_{0.5s}(t), s_{\text{min}}) \]  \hspace{1cm} (2.17)

\[ s_{\text{safe}}(t) = \frac{v_{i-1}(t)}{2} \left( \frac{1}{b_{i-1}} - \frac{1}{b_i} \right) + \delta v_{i-1}(t) \]  \hspace{1cm} (2.18)

\[ s_{0.5s}(t) = 0.5v_i(t) \]  \hspace{1cm} (2.19)
where, $s_d$, $s_{safe}$, and $s_{[t]}$ are desired, safety, and spacing with a t-second time gap, $\delta$ is communication delay (20ms), $b_{i-1}$ is the braking capability of the preceding vehicle, $b_i$ is the braking capability of the following vehicle. The time gap is 1.4s when the preceding vehicle is not CACC controlled.

There are some other control models of ACC/CACC, such as the IDM-based model (Y. Li, Li, Wang, Wang, & Xing, 2017; Schakel, Van Arem, & Netten, 2010; Treiber et al., 2000) and gas-kinetic macroscopic model (Ngoduy, 2013), reinforcement learning (Desjardins & Chaib-draa, 2011). Most of the ACC/CACC models discussed above are developed based on different objectives, such as string stability (Kianfar et al., 2012; G. J. Naus et al., 2010a; Xiao & Gao, 2011a; Yi & Horowitz, 2006; J. Zhou & Peng, 2005), safety (Y. Li, Li, et al., 2017; Massera Filho, Terra, & Wolf, 2017; Moon, Moon, & Yi, 2009), fuel economy (Barth, Wu, & Boriboonsomsin, 2015; Stanger & del Re, 2013). And Luo, Liu, Li, and Wang (2010) proposed a multi-objective model considering safety, fuel consumption, comfort, and car-following performance. On the other hand, to improve the user experience, some vehicle control models can be calibrated to meet different drivers’ individual settings and preferences. Bifulco (Bifulco, Simonelli, & Di Pace, 2008) developed a fully-adaptive ACC system with the leaning mode that enables vehicles to learn driving behaviors of different drivers.

2.2 **V2V (Vehicle to Vehicle) Platoon Communication Topologies**

The ACC/CACC controller design is based on the vehicle information flow topology (Dey et al., 2016). The existing topologies can be categorized in predecessor following (PF) (G.
Many existing works focused on analyzing the string stability of CACC systems (Liu, Goldsmith, Mahal, & Hedrick, 2001; Middleton & Braslavsky, 2010; G. J. Naus et al., 2010b; Ploeg, Shukla, Van De Wouw, & Nijmeijer, 2013). It have been proven that 1) with the CDH policy, a PF topology cannot guarantee string stability. Transmitting leader’s or other vehicles’ information to controlled vehicles in the platoon through V2V communications can extend the information flow, and thus ensure string stability. And 2) the CTH policy can be used to ensure string stability, where the vehicle spacing relies on the vehicle velocity, and therefore relax the formation rigidity of the system.

In this study, PF network topology for CACC platoon is considered since the state of the directly preceding vehicle is crucial for safe platoon operation. In other words, the following vehicle reacts merely based on its preceding vehicle’s state.
2.3 Traffic Flow Improvement ACC/ CACC Models

Extensive researches are studying the impacts of ACC and CACC models, especially for their impacts on traffic flow. Studies and field tests have shown that the congestion mitigation effects of CACC are caused by a significant reduction in the feasible time headway among vehicles (Ploeg et al., 2011; Suzuki & Nakatsuji, 2003). However, some of the researchers concluded only through simulation. During their model formulation, there are no discussions regarding the objectives of congestion mitigation and shockwave damping. In other words, the traffic flow improvement is only the side effect of those proposed models because of the reduction in feasible time headways. This section solely focuses on studies having clear objectives on mitigating the congestions or dampening the shockwave propagations during their model formulations.

ACC and CACC car-following models have shown promising impacts on mitigating traffic congestions due to their capability to shrink the distances among vehicles among all vehicles, compared with manually controlled vehicles. The Variable Speed Limit (VSL) control models are another major studied method for congestion mitigation and shockwave damping. In addition to conventional ACC/CACC models, Jam-Absorption Driving models can control a single vehicle to change its headway to mitigate traffic congestion dynamically.

2.3.1 Congestion Mitigation Models

An extensive number of studies illustrate that wireless-communication-enabled vehicles can significantly improve traffic efficiency by suppressing the distance or time headway among vehicles.
Bu et al. proposed an MPC-based CACC model to mitigate traffic congestion by reducing vehicle time gaps (G. J. Naus et al., 2010a). With the information (e.g., speed, throttle/brake command, and gear position of the preceding vehicle) from DSRC communication, the model enhanced the time headway to 0.6 s to 1.1 s, compared to a range of 1.1 s to 2.2 s with the ACC systems, which can significantly improve the highway capacity. Motamedidehkordi et al. evaluated the inter-vehicle communication’s effects on traffic shockwaves (Motamedidehkordi et al., 2016). Rabsatt and Gerla (Rabsatt & Gerla, 2017) proposed a system named DRIVE-EX, which can reduce the impact of shockwaves and uniformly distribute traffic across multiple lanes relying on vehicle communication. Calvert et al. (Calvert et al., 2011) stated the positive effect of CACC: due to the improvement of traffic stability, CACC can reduce the congestion shockwaves propagating from a bottleneck location by both preventing the occurrence and suppression of exiting shockwaves. Accordingly, the congestion mitigation can be achieved by reacting differently according to the change of traffic states.

Traffic State Estimation Model

Papacharalampous et al. proposed Traffic State-Adaptive ACC and CACC models to mitigate congestion by implementing a controller estimating traffic states (Papacharalampous et al., 2015). The proposed model uses an exponential moving average (EMA) of the vehicle speed to determine the traffic states and accordingly choose the desired time headway.

\[
v_{EMA}(t + \Delta t) = \left(1 - e^{-\frac{\Delta t}{\tau_r}}\right)v(t + \Delta t) + e^{-\frac{\Delta t}{\tau_r}}v_{EMA}(t)
\]  \hspace{1cm} (2.20)
where $t$ is the current time step, $\Delta t$ is the control time step length, and $\tau_r$ is the relaxation time that makes the impact of vehicle speeds older than the time instant $t - \tau_r$ decreases exponentially. The simulation showed that this control model produces a 67.7% improvement in the average travel time compared with manual vehicles. However, this model adjusts its preset parameter desired time headway based on the estimated traffic states, which could only handle some simple situations and lead to traffic instability easily.

Similarly, Kesting (Kesting et al., 2008) designed an ACC system that adapts to five different traffic situations to improve both traffic flow and road capacity. The proposed model adjusts the IDM-based ACC model parameters according to estimated traffic states. The parameters represent ACC driving characteristics by the time gap, maximum acceleration, and comfortable deceleration.

The Traffic State Estimation method uses a controller to estimate the traffic states classified into multiple types, where different sets of parameters will be used for different traffic state types. Thus, it requires lots of tests to obtain the right parameters and classification of traffic states. What’s more, the pre-defined parameters and traffic state categories might not be comprehensive enough due to the complexity of traffic patterns.

**Spacing Regulation Model**

In the article from Stern (Stern et al., 2017), the author presented the velocity controllers and demonstrated experimentally that an ACC vehicle implemented with the proposed model could dampen traffic shock waves. When traffic shock waves are present according
to the estimation, the proposed model allows a specific spacing to open up in front of the ACC Vehicle when the PV accelerates, which is then closed when the PV decelerates. An estimate of the average speed required by the controller is obtained by measuring the ACC Vehicle speed over a large enough time horizon.

The general structure of the controllers is as follows. The ACC Vehicle continuously tracks its velocity $v^{AV}$, and measures (at a sampling rate of 30Hz) the spacing $s$ between leading vehicles and ACC vehicles. This signal, suitably smoothed, is used to calculate the velocity difference $\Delta v = \frac{ds}{dt}$ between the PV and the FV. The PV’s velocity is estimated as $v^{PV} = v^{FV} + \Delta v$. Moreover, the desired velocity $U$ is defined, stabilizing the traffic flow when chosen correctly. From the desired velocity $U$, the spacing $s$, and the velocities of the ACC vehicle $v^{FV}$ and preceding vehicle $v^{PV}$, a commanded velocity $v^{cmd}$ is determined to either keep the original desired velocity $U$ or catch up PV with a velocity larger than $U$:

$$v_i^{target} = U + v_i^{catch} \ast \min\left(\max\left(\frac{s_i - s_l}{s_u - s_l}, 0\right), 1\right)$$

(2.21)

$$v_{i,j+1}^{cmd} = \beta_{i,j}(\alpha_{i,j}v_{i,j}^{target} + (1 - \alpha_{i,j})v_{i,j}^{lead}) + (1 - \beta_{i,j})v_{i,j}^{cmd}$$

(2.22)

where $i$ and $j$ denote the vehicle index and time step, respectively. This model ensures the ACC vehicle 1) to drive faster than the average velocity and catch up to the PV when the spacing $s_i$ is larger than the lower threshold $s_l$, and 2) to close the gap by traveling $v_i^{catch}$ above $U$ when the spacing is above the upper threshold $s_u$. The weights $\alpha_{i,j}$ and $\beta_{i,j}$ are determined based on the gap:

$$\beta_{i,j} = 1 - \frac{1}{2}\alpha_{i,j}, \text{ and}$$

$$\alpha_{i,j} = \min\left(\max\left(\frac{s_{i,j} - s_{safe}}{\gamma}, 0\right), 1\right)$$
where $s_{safe}$ is the safety distance, $\gamma$ is a parameter representing the changing rate of $\alpha$.

The control model proposed by Stern et al. regulates the vehicle velocity depending on the spacing between FV and PV. It does not require wireless communication among vehicles, yet it depends on multiple controllers to smooth command velocity and regulate the spacing. And more importantly, the desired speed $U$ is considered the equilibrium speed, which needs to be estimated with data for a period of time. In real-world applications, the desired speed $U$ needs to be estimated from the flow ahead of PV through wireless communication, which might not be achievable in all cases.

2.3.2 Jam-Absorption Driving (JAD) Models

As shown in Figure 3, JAD models mitigate the traffic congestion by controlling a single vehicle to dynamically change vehicles’ headway, which is composed of two steps (Beaty, 1998): 1) “slow-in” step that achieves a large enough headway by keeping at a lower speed compared with preceding vehicles when congestion is detected upstream; 2) “fast-out” step that accelerates quickly when preceding vehicles are exiting the congestion.
Taniguchi et al. (2015) proposed a model that dynamically changes a single car’s headway absorbs the traffic jam. The proposed model takes the spacing, velocity of PV and FV and designs the desired headway (calculated as $d_{jam} + h_{des}v_i$). The vehicle assumes the single controlled vehicle can detect jams of leading vehicles (wireless communication). Additionally, this paper stated that the model might fail to absorb jams when the designed following speed is not chosen correctly.

Nishi et al. (Nishi, Tomoeda, Shimura, & Nishinari, 2013) represented the jam-absorption driving with terms “slow-in” and “fast-out” proposed. The shockwave can be successfully dampened with a single car adjusting its speed. This paper proposed 1) a one-step model that a vehicle instantly decelerates to a slightly slower speed in one single step, and 2) a
two-step model that a vehicle gradually decelerates to the slower speed in two steps. Nishi also uses fundamental diagrams to illustrate the disappearance of the jam. However, the model is over-simplified by omitting the acceleration term and based on strict assumptions. Moreover, there is no detection or estimation method to identify the upstream jam. Additionally, this paper also discussed a little bit on if lane changes are prohibited. If lane changes are not permitted, a “slow-in-and-fast-out” strategy is recommended; if lane changes are enabled, a “fast-out-only” strategy is recommended to avoid potential merging conflicts.

He et al. (Z. He et al., 2016) developed a Newell-model-based strategy by applying slower speed on a single vehicle before capturing traffic oscillations. This paper also developed a search process to identify which vehicle is best controlled to absorb the jam. The proposed 2-step model 1) estimates the ending point of the jam based on Newell’s model and empirical observations and then reacts according to the estimation, and 2) revises the desired speed when the leading vehicle leaves the jam. The proposed model is evaluated through a single oscillation and a series of oscillations. It shows that the model can reduce the number of vehicle stops and travel time when there is only one oscillation while increasing the overall travel time when there is a series of oscillations.

The JAD model controls a single vehicle depending on upstream traffic conditions, yet it requires wireless communication among vehicles and infrastructures. Moreover, there is no analysis on string stability as only a single vehicle is controlled with the JAD model.
2.3.3 Variable Speed Limit (VSL) Based Models

The Variable Speed Limit (VSL) control is a widely used traffic control method for congestion relief. Similar to the “slow-in” step of JAD models, the core of VSL control is to adjust the speed limits upstream of freeway bottlenecks such that potential congestions can be removed due to the change of upstream vehicle speeds. Intensive studies have been conducted on the VSL control (Abdel-Aty, Cunningham, Gayah, & Hsia, 2008; Bertini, Boice, & Bogenberger, 2006; Lin, Kang, & Chang, 2004).

With connected and autonomous technology becoming mature, VSL control is also integrated with CAV technologies. Y. Li, Xu, Xing, and Wang (2017) developed a CACC model combined with the VSL algorithm. The speed limits are calculated according to the downstream traffic data posted on roadside Variable Message Sign (VMS) and to CACC vehicles in upstream segments. The integration of CACC and VSL significantly improves both efficiency and safety: 100% CACC penetration rate reduces the rear-end collision risks by 98% and increases the efficiency by 33%.

A Hegyi, Hoogendoorn, Schreuder, Stoelhorst, and Viti (2008) proposed a speed limit control method to eliminate shockwaves called SPECIALIST (SPEed ControllIng ALgorIthm using Shockwave Theory). The simulation and field test was presented in (A Hegyi & Hoogendoorn, 2010). The SPECIALIST algorithm can resolve jam waves on freeways using roadside technology: detector loops and speed limit gantries. Hegyi et al. also extend the SPECIALIST algorithm with cooperative system technologies and other roadside detectors such as in-car detection, actuation, and video-based monitoring (VBM).
(Andreas Hegyi et al., 2013), through which the speed, flow, and density data are provided. When proposed systems detect the jam, the speed limits upstream of the jam are switched on to dampen shockwaves.

Khondaker and Kattan proposed a VSL control algorithm in a connected vehicle environment through a Model Predictive Control (MPC) approach (Khondaker & Kattan, 2015). The proposed algorithm posts the optimal speed limits to the control sections and adjusts the selected speed limits based on the compliance rate obtained through real-time vehicle trajectories. This CV-based VSL algorithm uses a multi-objective optimization function to find a balanced trade-off among mobility, safety and sustainability, which will be further discussed in the following Section 2.4. However, this method assumes that trajectories of all vehicles in the control section are available all the time.

Yu and Fan (2019) used the extended Cell Transmission Model (CTM) to estimate traffic state and applied Genetic Algorithm (GA) to optimize VSL control. The IDM is implemented as the car-following model of CAVs in this algorithm. The simulation results of this paper showed that the proposed VSL control improves the overall efficiency and reduces the emission rate. The posted speed limit on cell $i$ during time interval $k$ is calculated as

$$u_{i}^{\text{post}}(k) = (1 + \alpha_i(k)) \cdot u_i(k)$$ (2.23)

$$\alpha_i(k) = \frac{v_{i,AV}(k - 1) - u_i(k - 1)}{u_i(k - 1)}$$ (2.24)

where $v_{i,AV}(k - 1)$ is the collected average speed of AVs on cell $i$ during time interval $k - 1$; $u_i^{\text{post}}(k)$ is the posted speed limit on cell $i$ during time interval $k$; $u_i(k)$ is the optimized...
speed limit on cell $i$ during time interval $k$; and $\alpha_i(k)$ is the relative difference between traveling speed and posted speed limit.

In a nutshell, the traditional VSL algorithm posts calculated speed limits for each section separately and recommend vehicles to comply with the speed limit posted on the VMS. The wireless communication capability among vehicles facilitates speed hominization among vehicles and could significantly improve the traffic flow. Nonetheless, the VSL-based connected vehicle models require wireless communication capability among vehicles and roadside infrastructure to be implemented in the real world.

2.4 **Optimal Control Methods**

Optimal control models are widely used to find a control law for a dynamical system over a period of time such that the objective function is optimized. In ACC and CACC models, such algorithms are used to dynamically achieve objectives such as mobility improvement, safety guarantee, fuel efficiency, etc. Due to the almost immediate response needs of autonomous vehicles, selected optimal control models need to satisfy the following criteria at the same time:

1. The computation cost should be low enough for vehicles to respond in time.
2. Specification of all boundary conditions on states and constraints to be satisfied by states and controls is clearly described.
3. Models need to have multiple objectives to meet the requirements of safe, efficient, comfort and economic driving.
4. The needed inputs, such as the leading vehicle’s state, are available through sensors.
or wireless communications.

With the criteria mentioned above, the Linear Quadratic (LQ) optimal control and Model Predictive Control (MPC) is commonly selected for the vehicle control model design.

Table 1 summarizes the differences of the proposed control model with some representative optimal control models of vehicles, including the communication capability, cost function types, control frameworks and solving methods. The details will be introduced in the following section.

Table 1. Comparison of Representative Optimal Vehicle Control Models

<table>
<thead>
<tr>
<th>Model Name</th>
<th>Wireless Communication</th>
<th>Cost Function</th>
<th>Control Framework</th>
<th>Solving Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed Model</td>
<td>Both</td>
<td>Non-quadratic</td>
<td>Open Loop</td>
<td>Iterative PMP</td>
</tr>
<tr>
<td>Distributed MPC (Zheng, Li, Li, Borrelli, &amp; Hedrick, 2016)</td>
<td>Both</td>
<td>Quadratic</td>
<td>Open Loop</td>
<td>Fmincon^1 ‘interior-point’</td>
</tr>
<tr>
<td>Connected Cruise Control LQR (I. G. Jin &amp; G. Orosz, 2016)</td>
<td>Yes</td>
<td>Quadratic</td>
<td>Closed Loop (LQR)</td>
<td>Algebraic Riccati Equation (ARE)</td>
</tr>
<tr>
<td>Weight-free MPC (D. He, Shi, &amp; Song, 2019)</td>
<td>No</td>
<td>Quadratic</td>
<td>Open Loop</td>
<td>Sequential Quadratic Programming (SQP)</td>
</tr>
<tr>
<td>Rolling Horizon MPC (M. Wang et al., 2014a)</td>
<td>No</td>
<td>Non-quadratic</td>
<td>Open Loop</td>
<td>Iterative PMP</td>
</tr>
<tr>
<td>Smart Driving (Kamal, Imura, Hayakawa, Ohata, &amp; Aihara, 2014)</td>
<td>Yes</td>
<td>Quadratic</td>
<td>Open Loop</td>
<td>C/GMRES</td>
</tr>
<tr>
<td>Optimal Shockwave (Strnad, Kramar Fijavž, &amp; Žura, 2016)</td>
<td>Yes</td>
<td>Non-quadratic</td>
<td>Open Loop</td>
<td>Differential Evolution Approach</td>
</tr>
</tbody>
</table>
2.4.1 Linear Quadratic (LQ) Optimal Control

The Linear Quadratic Optimal Control problems have linear states and control inputs, and the cost functions are quadratic. The LQ optimal control is an essential method to solve optimal control problems since it can model many optimal control problems in the real world. The LQ optimal control specifies the cost function to be optimized over an infinite horizon. Furthermore, the LQ problems can reasonably approximate some nonlinear control problems (Yong & Zhou, 1999). Although the LQR method requires quadratic forms of the cost function, the resulting optimal control law has many excellent properties, including closed-loop stability and low computational cost.

The LQ problem is a special case of the optimal control problem, which is of particular importance when the objective function is a quadratic function of $X$ and $u$, and the dynamic equations are linear. This type of LQ control problem is also called the linear quadratic regulator (LQR):

$$ J = \frac{1}{2} X(t_f)^T S_T X(t_f) + \frac{1}{2} \int_{t_0}^{t_f} (X(t)^T Q X(t) + u(t)^T R u(t)) dt $$

(2.25)

where $S_T$ and $Q$ are positive semidefinite matrices, and $R$ is a positive definite matrix.

Jin and Orosz (I. G. Jin & G. J. I. T. o. I. T. S. Orosz, 2016) designed an LQR control model in a CV environment considering both communication delay and driver reaction time. The proposed control model can compensate for the communication delay's negative impacts and provide robust head-to-tail string stability. The state equation is formulated by
a platoon of $n$ vehicles:

$$\dot{X}(t) = AX(t) + BX(t - \tau) + Du(t) + \phi(t)$$

(2.26)

where $X = \begin{bmatrix} x_1 \\ \vdots \\ x_i \\ \vdots \\ x_l \end{bmatrix}$, $\phi = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \phi_i \end{bmatrix}$, $x_i = \begin{bmatrix} N^* s_i - v_i \\ v_{i-1} - v_i \end{bmatrix}$ and $\phi_n = \begin{bmatrix} 0 \\ a_{i-1} \end{bmatrix}$; $N^* = V'(s^*)$ is the derivative of the range policy at the equilibriums; $A$ and $B$ are coefficient matrices taking current and previous states of adjacent vehicles into account; $D$ is the coefficient matrix of the control input $u(t)$. The multi-objective cost function is designed to minimize the fuel economy, active safety and mobility:

$$J = \int_{t_0}^{t_f} (v_i^2 + c_1(N^* s_i - v_i)^2 + c_2(v_{i-1} - v_i)^2) dt$$

(2.27)

Kim (Kim, 2012) proposed an LQ optimal control for both ACC and CACC systems for both urban and rural environments, where vehicles can have a complete stop in urban and cannot stop in a rural environment. A constrained Quadratic Programming (QP) problem was solved to find the optimal vehicle trajectory.

$$\dot{X}(t) = AX(t) + Bu(t)$$

(2.28)

$$Y(t) = CX(t)$$

(2.29)

$$X(t) = [x_{PV} - x(t) \quad v_{PV}(t) \quad v_{FV}(t)]^T, u(t) = [a_{PV}(t) \quad a_{FV}(t)]^T$$

$$A = \begin{bmatrix} 0 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, C = \begin{bmatrix} -1 & 0 & T_{gap} \end{bmatrix}$$

To make $(A, C)$ is observable with minimum modification of the performance index, $C$ is modified as
$$C = \begin{bmatrix} -1 & 0 & T_{gap} \\ 0 & \epsilon_0 & 0 \end{bmatrix}$$

Where the number $\epsilon_0$ is a very small number that can be further reduced depending on the available computational resolution. In addition to Kim’s LQ optimal control models, a Kalman Filter with variable measurement noise covariance was also introduced to reduce the impact of data loss during wireless communication.

### 2.4.2 Model Predictive Control (MPC)

Compared with LQR control models, MPC models have more freedom to design their objective functions (e.g., incorporating complicated nonlinear terms). Yet, they have higher computation costs than LQR control as it optimizes the vehicle dynamics in a receding time horizon. In contrast, the LQR control optimizes on a fixed time horizon (L. Wang, 2009). Thus, MPC control models require a faster processor for real-time applications.

Linear MPC usually solves a multiple-input-multiple-output (MIMO) constrained linear system (Mayne, 2014; James Blake Rawlings & Mayne, 2009), widely used in real-world applications. The Nonlinear MPC (NMPC) deals with nonlinear constraints and dynamics more directly and explicitly.

Although a more generic cost function is preferable, a quadratic positive (semi-)definite running cost makes it easy to ensure closed-loop stability (Rawlings, Angeli, & Bates, 2012), and additional care might be needed at the algorithmic level (Quirynen et al., 2014). However, as proven in (Zanon, Gros, & Diehl, 2014), the feedback control law of any locally stabilizing non-positive-definite NMPC formulation can be approximated up to first
order by an (N)MPC formulation with a positive-definite running cost.

\textit{Constant Time Headway (CTH) MPC}

Many MPC models are implemented on ACC or CACC vehicles trying to reduce the spacing error, relative velocity, and control input of the controlled vehicle. The CTH ($t_d$) is usually used in regulating the spacing error. Bu’s MPC based CACC model uses a quadratic cost function for the optimization problem (G. J. Naus et al., 2010a):

\begin{equation}
J(k) = \sum_{n=1}^{N} \left( \alpha_e e_1^2(k+n) + \alpha_u \Delta u^2(k+n) + \alpha_v \Delta v^2(k+n) \right)
\end{equation} \hspace{1cm} (2.30)

Where $\alpha_e, \alpha_u, \alpha_v$ are weights, $k$ is the current time step, spacing regulation error $e_1 = s_i - v_i t_d$ is the error between current spacing and desired spacing calculated by CTH $t_d$, $\Delta u$ is the change of control input (speed command for FV), and $\Delta v$ is the speed difference between FV and PV. Minimizing this cost function is to 1) reduce the time headway errors, 2) smooth the control input, and 3) speed up the response of FV to the speed change of PV.

Similarly, Duret et al. (2019) proposed an MPC ACC model for truck platooning. The running cost function is defined as

\begin{equation}
\mathcal{L} = c_1 (s_i - s_d)^2 + c_2 \Delta v_i^2 + c_3 u_i^2
\end{equation} \hspace{1cm} (2.31)

where $s_d = v_i t_d + s_0$ with $t_d$ as the constant desired time headway.

\textit{Variable Time Headway (VTH) MPC}

M. Wang et al. (2014a) designed the two-mode ACC model, including cruising and following modes. A two-regime running cost $\mathcal{L}$ is proposed as:
\[ \mathcal{L} = \begin{cases} 
  c_1 e^{\frac{s_0}{e} \Delta v_i} \Theta(-\Delta v_i) + c_2 (s_d - s_i)^2 + \frac{1}{2} u_i^2 , & \text{if } s_i \leq s_f \\
  c_3 (v_{free} - v_i) , & \text{if } s_i > s_f
\end{cases} \]  

(2.32)

where \( s_f = v_{free} t_{d,m} + s_0 \) is the spacing threshold to determine if the vehicle is in cruising mode \((s_i > s_f)\) or following mode \((s_i \leq s_f)\). \( t_{d,m} \) is the user-defined maximum desired time headway, and the \( s_0 \) is the minimum spacing between standstill vehicles. The desired spacing \( s_d = v_i(t) t_d + s_0 \) is determined by the spacing dependent desired time gap:

\[ t_d = t_{d,0} + \frac{s}{s_f} (t_{d,m} - t_{d,0}) \]  

(2.33)

where \( t_{d,0} \) is the minimum desired time headway. And \( \Theta(x) \) is a Heaviside step function defined as:

\[ \Theta(x) = \begin{cases} 
  1 , & \text{if } x \geq 0 \\
  0 , & \text{if } x < 0
\end{cases} \]  

(2.34)

Equation (2.32) consists of three cost terms: safety, efficiency and comfort. The safety cost only occurs when FV approaches the PV in the following mode and vanishes in cruising mode. And the safety cost is a monotonic decreasing function of spacing \( s_i \). The shorter the \( s_i \), the larger the safety cost is. The efficiency cost accounts for deviating from the desired spacing. And the comfort cost represents the penalty of large acceleration or deceleration, which provides comfortable driving behavior.

**Two-Vehicle Smart Driving MPC**

Kamal et al. designed an MPC model to improve the traffic flow and attenuate jamming waves by regulating safe vehicle spacing between the controlled vehicle and the vehicle following the controlled vehicle, based on the predicted downstream traffic (Kamal et al., 2014). The cost function is designed as:
where $c_1, c_2, c_3$ and $c$ are weights of cost function; the first term is the cost of the velocity deviation of the controlled $i$th vehicle $v_i$ from the desired velocity $v_{des}$; the second term is the cost of the control input (acceleration of the controlled vehicle); the third term represents the cost of acceleration of the FV immediately after the control $i$th vehicle; the fourth term is a soft constraint for the safe spacing; $c_4(t)$ is a time-varying weight, which provides a larger penalty when the controlled vehicle is very close to the PV and a smaller penalty when the controlled vehicle is far from the PV:

$$c_4(t) = a_1 e^{-a_2 \tanh(a_3(t_h(t) - t_{hd}))}$$

where $t_h(t) = \frac{(x_{i-1}(t) - x_i(t) - s_0)}{v_i(t) + \alpha}$ is the time headway, and the small constant $\alpha$ is added to avoid singularity at $v_h = 0$; $t_{hd}$ is the desired time headway; $a_1$ and $a_2$ are constant parameters.

**Variable Speed Limits (VSL) MPC**

Khondaker’s VSL-based CV control model (Khondaker & Kattan, 2015) uses the MPC approach to get the optimal speed limit values. Total Travel Time (TTT), Time To Collision (TTC), and Fuel Consumption (FC) was used to measure the mobility, safety and environmental impact. The cost function is designed as:

$$J = c_1 \sum_{t=1}^{N_p} \frac{J_{TTT(t)}}{N_{TTT(t)}} + c_2 \sum_{t=1}^{N_p} \frac{N_{TTC(t)}}{J_{TTC(t)}} + c_3 \sum_{t=1}^{N_p} \frac{J_{FC(t)}}{N_{FC(t)}}$$

where $c_1, c_2, c_3$ are weights; the first term is calculated by summing up each vehicle’s travel time over the prediction time $N_p$; the second and third term is calculated by summing up each vehicle’s ratios of relative speed and relative position and each vehicle’s amount of
fuel consumption time over the prediction time $N_p$, respectively. $N_{TTT}(t)$, $N_{TTC}(t)$ and $N_{FC}(t)$ are normalized values of the corresponding terms.

2.4.3 Numerical Methods Solving Optimal Controls

*Continuous Algebraic Riccati Equation (CARE) Method*

The Continuous Algebraic Riccati Equation (CARE) is commonly used to obtain the solution of linear-quadratic optimal control problems, which is computationally efficient as the resulting optimal control law is obtained as the closed-form feedback gain of the state vector. However, the CARE method requires a quadratic cost function, which lacks the ability to handle highly non-stationary conditions such as collision avoidance when approaching a standstill leader with high speeds (Godbole, Kourjanskaia, Sengupta, & Zandonadi, 1999). Moreover, the CARE method cannot solve non-quadratic terms in the cost function.

The CARE method is described as follows. Given the state-space equation:

$$\dot{X}(t) = AX(t) + Bu(t) \quad (2.38)$$

Where $A$ is the system matrix, and $B$ is the input matrix. And the linear state feedback:

$$u(t) = -K(t)X(t) \quad (2.39)$$

Where the state feedback gain is given by $K = R^{-1}B^TS(t)$ And can be solved by the differential Riccati equation:

$$-S = A^TS + SA - BBR^{-1}B^TS + Q = 0 \quad (2.40)$$

Moreover, if the pair $(A,C)$ is observable, where $C^TC = Q$, then the closed-loop system
\[ \dot{X} = (A - BK)X \] is asymptotically stable, which means the LQR provides a stable solution to the dynamic system.

**Sequential Quadratic Programming (SQP) Method**

The Sequential Quadratic Programming (SQP) approach is another commonly used method to solve NMPC by iteratively solving quadratic approximations of the nonlinear programming until convergence is achieved (Gros, Zanon, Quirynen, Bemporad, & Diehl, 2020; D. He et al., 2019). The nonlinear programming is sequentially approximated by QP, delivering Newton directions for performing steps towards the solution starting from the initial guess. It is powerful enough for real-world applications to handle any degree of non-linearity, including non-linearity in the constraints.

For a nonlinear programming problem:

\[
\begin{align*}
\min_{x} & \quad J(x) \\
\text{s.t.} & \quad h(x) \geq 0 \\
& \quad g(x) = 0
\end{align*}
\] (2.41)

\(J(x)\) is the cost function, \(h(x)\) and \(g(x)\) are constraints of the optimization problem. Critical points of the cost function will also be critical points of the Lagrangian function:

\[
\mathcal{L}(x, \lambda, \mu) = J(x) - \lambda h(x) - \mu g(x)
\] (2.42)

where \(\lambda\) and \(\mu\) are Lagrange multipliers, The SQP simply iterates Newton’s method to find critical points of the Lagrangian function. By applying SQP, the NLP problem becomes quadratic programming subproblem searching with direction \(d_k\) for iteration \(x_k\), which is much easier and tackle than the original problem.
\[
\min_x \ J(x_k) + \nabla J(x_k)^T d + \frac{1}{2} d^T \nabla^2 J(x_k, \lambda_k, \mu_k) d \\
\text{s.t.} \quad h(x) \geq 0 \\
g(x) = 0
\]  

However, the SQP approach involves several derivatives, which need to be pre-calculated before iterating to a solution and making large problems with many variables or constraints cumbersome (Goodman, 2015).

**Pontryagin’s Minimum Principle (PMP) Method**

Wang (M. Wang et al., 2014a) proposed an iterative numerical solution algorithm based on Pontryagin’s Minimum Principle (Boltyanskiy, Gamkrelidze, & Pontryagin, 1961), abbreviated as iPMP, to solve a rolling horizon control problem for CAV systems. The iPMP algorithm streamlines the computation by solving a set of coupled ordinary differential equations instead of partial differential equations used by the SQP method while allowing sophisticated cost functions other than quadratic form. The steps of the iPMP algorithm are as follows:

1. Choose a weight factor \(\alpha(0 < \alpha < 1)\) for smoothly updating the co-state, set the iteration number \(n = 1\), and set the error threshold \(\epsilon_{max}\).
2. Set the initial co-state \(\Lambda(0)(t) = 0\) for \(0 \leq t \leq t_T\).
3. Solve the state dynamic equation:
   \[
   \frac{d}{dt} X^{(n)} = F \left( X^{(n)}, u^* \left( X^{(n)}, \Lambda^{(n-1)} \right) \right) \\
   \text{subject to} \quad X^{(n)}(t_T) = X_0 \text{ forward in time, where } t_T \text{ is the terminal time.}
   \]
4. Solve the co-state dynamic equation:
\[-\frac{d}{dt} \lambda^{(n)} = \frac{\delta e}{\delta x} \left( X^{(n)}, u^* \left( X^{(n)}, \Lambda^{(n-1)} \right) \right)\]

subject to $\lambda^{(n)}(t_T) = \frac{\delta G}{\delta x} \left( X^{(n)}(t_T) \right)$ backward in time, where $G$ is the running cost.

5. Update the co-state $\Lambda^{(n)}$ with the weight factor $\alpha$:

\[\Lambda^{(n)} = (1 - \alpha)\Lambda^{(n-1)} + \alpha \lambda^{(n)}\]

6. If $\epsilon = \|\Lambda^{(n)} - \lambda^{(n)}\| < \epsilon_{\text{max}}$ then stop, otherwise set $n = n + 1$ and back to step 3.

It should be noted that it is essential to choose a right $\alpha$ such that the co-state will converge as fast as possible.

**Continuation and Generalized Minimum Residual (C/GMRES) Method**

The Continuation and Generalized Minimum Residual (C/GMRES) method can also be used to compute the solution of the (N)MPC problem (Kamal et al., 2014; Ohtsuka, 2004). The C/GMRES method does not require iterative searches, making it faster than other iterative methods such as SQP and iPMP. Kamal et al. (Kamal, Mukai, Murata, & Kawabe, 2012) showed that C/GMRES could solve an MPC problem in less than 10 ms. Instead of solving $F(X,u)$ for each iteration, C/GMRES uses derivative of control input $u$ with respect to time such that $F(u(t),X(t),t) = 0$ is satisfied identically. In other words, it chooses $u(0)$ so that $F(X(0),u(0),0) = 0$ and determine $\dot{u}$ so that

\[\dot{F}(u,X,t) = \dot{A}_s F(u,X,t)\]  \hspace{1cm} (2.44)

where $A_s$ is a stable matrix introduced to stabilize $F = 0$. Then the derivative of $u$ is

\[\dot{u} = F_u^{-1} (A_s F - F_X \dot{X} - F_t)\]  \hspace{1cm} (2.45)

where $F_u$ is a nonsingular coefficient matrix to determine $\dot{u}$ for given $u, X, \dot{X}$ and $t$. Thus, the optimal solution $u(t)$ is updated according to $\dot{u}$ without any iterations. However, this
method has several requirements: 1) numerical method is needed to find \( u(0) \) satisfying 
\[ F(X(0), u(0), 0) = 0; \] 2) \( \dot{X} \) must be approximated by finite difference; 3) numerical 
integration of \( \dot{u} \) imposes a specific condition on matrix \( A_s \) to maintain the boundedness of 
the error \( F \).

### 2.5 Stability of ACC/CACC Models

Stability is an essential component in ACC and CACC system design as it plays an essential role in traffic flow improvement, fuel efficiency. There are two types of stability for autonomous systems: local stability and string stability. The local stability is the stability of a single vehicle, whereas the string stability stands for the stability of a platoon of vehicles. The following of this section summarized the concepts and derivation frameworks of stability analysis in existing studies.

#### 2.5.1 Local Stability Analysis

The local stability analysis of the car following model has been conducted for decades as it is relatively simple compared with string stability analysis. Although most recent research focuses on string stability analysis, local stability analysis is still essential because it represents the vehicles’ ability to recover from small traffic perturbations to steady traffic equilibrium. Sun et al. (2018) summarized that the Laplace transform is one of the most used methods to obtain the characteristic equation for local stability analysis.

An Ordinary Differential Equation (ODE) system of deviations from the equilibrium, i.e., 
the gap variation \( y_i(t) \) and the velocity variation \( v_i(t) \), can be defined as
\[
\dot{y}_i(t) = \dot{s}_i(t) = v_{i-1}(t) - v_i(t) \quad (2.46)
\]
\[
\dot{v}_i(t) = (f_s y_i + (f_v - f_{\Delta v}) v_i + f_{\Delta v} v_{i-1})_t \quad (2.47)
\]

where \( f_s, f_v \) and \( f_{\Delta v} \) denote the Taylor expansion coefficients of spacing, velocity, and relative velocity at the equilibrium states. Taking Laplace Transform on Equation (2.46) and (2.47), the following equations can be obtained:

\[
Y(s) = \mathcal{L}\left( \int (v_{i-1} - v_i) \, dt \right) = \frac{1}{s} [\mathcal{L}(v_{i-1}) - \mathcal{L}(v_i)] = \frac{1}{s} [V_{i-1}(s) - V_i(s)] \quad (2.48)
\]
\[
sV(s) = \frac{1}{s} f_s [V_{i-1}(s) - V_i(s)] + f_v V_i(s) + f_{\Delta v} (V_{i-1}(s) - V_i(s)) \quad (2.49)
\]

Accordingly, the relationship between PV and FV in the Laplace frequency domain is obtained

\[
V_i(s) = \frac{s f_{\Delta v} + f_s}{s^2 - s(f_v - f_{\Delta v}) + f_s} V_{i-1}(s) \quad (2.50)
\]

The velocity variation of the FV in the time domain can be obtained by taking the inverse Laplace Transform. According to the characteristics of the inverse Laplace Transform, the denominator \( s^2 - s(f_v - f_{\Delta v}) + f_s = 0 \) can represent the characteristic equation (Y. Li et al., 2013; Wilson & Ward, 2011). The model is locally stable when the real parts of two solutions of the characteristic equation are both negative. Thus, the local stability criterion becomes

\[
(f_v - f_{\Delta v}) < 0 \text{ and } f_s > 0 \quad (2.51)
\]

2.5.2 String Stability Analysis

The term string stability was initially introduced and defined in 1974 (Chu, 1974). The concept of string stability is well illustrated in previous literature (Darbha & Rajagopal, 2005; D Swaroop & Hedrick, 1996; Darbha Swaroop & Rajagopal, 2001), especially
concerning the dynamics of vehicle platoon systems. The concept of string stability is that in a system of coupled components, such as a platoon of vehicles (Eyre, Yanakiev, & Kanellakopoulos, 1997), disturbances should always be attenuated as they propagate through the system, which can lead to a steady-state. Compared with CACC, the ACC system eliminates the capability of wireless communication. Thus, an ACC platoon tends to be unstable. In CDH policy, the distance between the leading and ego vehicle is independent of the speed of the controlled vehicle. Thus its string stability requires inter-vehicular communication to compensate for the traffic oscillations (Fernandes & Nunes, 2012a; Ali Ghasemi et al., 2015; J. Zhou & Peng, 2005). On the contrary, the distance between vehicles of CTH policy is dependent on the vehicle speed, which can is easier to ensure the string stability (Fernandes & Nunes, 2012b).

The string stability requires that the perturbation could attenuate for between two consecutive vehicles, defined as:

\[
\frac{\|e_{i-1}\|_\infty}{\|e_i\|_\infty} < 1
\]  

(2.52)

where \(e_i\) and \(e_{i-1}\) are the magnitude of the perturbations within the vehicle platoon. It should be noted that a platoon of vehicles is string-unstable if any vehicle does not hold the inequality (2.52).

According to Liu et al. (2001), by taking the Laplace Transform, the relationship between two consecutive vehicles in the frequency domain can be obtained:

\[
G(s) = \frac{E_i(s)}{E_{i-1}(s)} < 1
\]  

(2.53)
where $E_i(s)$ is the Laplace Transform of $e_i(t)$. Thus, we have the following equation:

$$G(i\omega) = \frac{V_i(i\omega)}{V_{i-1}(i\omega)} = \frac{f_s + i\omega f_{\Delta v}}{-\omega^2 - i\omega(f_v - f_{\Delta v}) + f_s} \quad (2.54)$$

And the disturbance in a vehicle platoon will dissipate when:

$$|G(i\omega)| = \frac{\sqrt{f_s^2 + \omega^2 f_{\Delta v}^2}}{\sqrt{(f_s - \omega^2)^2 + \omega^2 (f_v - f_{\Delta v})^2}} < 1 \quad (2.55)$$

The string stability condition is derived as:

$$f_v^2 - 2f_s - 2f_v f_{\Delta v} > 0 \quad (2.56)$$

3 METHODOLOGY

This section designs control models that contribute to shockwave damping and congestion mitigation under the MPC optimal control framework. Five proposed control models are described separately with two different platoon topology setups: ACC and CACC topology.

3.1 Vehicle Dynamics and Model Assumptions

3.1.1 ACC Platoon Dynamics

![Figure 4. Vehicle Platoon Topology of ACC](image)

As shown in Figure 4, ACC vehicles follow the predecessor following (PF) topology that PV can obtain the PV's speed $v_{i-1}$ and spacing $s_i$ to PV through sensors such as radar and LiDAR (Line Detection And Ranging). This study proposes a catch-up speed term such that
in free-flow conditions or discharging flow, the ego vehicle will try to reduce the spacing by catching up with PV with fast but still safe speed according to PV’s position and speed.

The proposed model uses the second-order vehicle dynamics model in predicting the vehicle trajectory to avoid expensive computation time as follows,

\[
\begin{bmatrix}
\dot{s}_i(t) \\
\dot{v}_i(t)
\end{bmatrix} =
\begin{bmatrix}
v_{i-1}(t) - v_i(t) \\
u_i(t)
\end{bmatrix}
\]  

(3.1)

where the state vector consists of FV’s spacing \( s_i \), and velocity \( v_i \). \( u_i \) is the control input, i.e., acceleration of FV. In this study, it is assumed that the PV is traveling at a constant speed within the prediction horizon, i.e., \( u_{i-1} = 0 \). And it should be noted that the constant acceleration assumption of the predecessor behavior can also predict the system dynamics (Treiber, Kesting, & Helbing, 2006). Still, the resulting controller is very sensitive to the quality of the measurements of onboard sensors.

Additionally, the second-order vehicle dynamics in Equation (3.1) can be modified as the following second-order vehicle dynamics to involve the jerk into system dynamics and further improve the drivers’ comfort:

\[
\begin{bmatrix}
\dot{s}_i(t) \\
\dot{v}_i(t) \\
\dot{a}_i(t)
\end{bmatrix} =
\begin{bmatrix}
v_{i-1}(t) - v_i(t) \\
a_i(t) \\
u_i(t) - a_i(t) \\
\end{bmatrix}
\]  

\( T_e \)

(3.2)

where the third-order state \( \dot{a}_i(t) \) represents the jerk, with a delay factor \( T_e \) to account for the actuator delay for vehicles to drive from current acceleration \( a_i(t) \) to desired acceleration \( u_i(t) \).
3.1.2 CACC Platoon Dynamics

Figure 5. Vehicle Platoon Topology of CACC

Figure 5 depicts the CACC platoon that follows the PF topology, where vehicles can obtain PV's acceleration, speed, and spacing information from PV through sensors. The CACC vehicle platoon is formed by multiple vehicles with sensing and wireless communication capability (e.g., vehicle 1 to N in the figure).

Furthermore, we formulate the system dynamics of a CACC vehicle platoon with \( N \) vehicles by the following state-space form:

\[
\dot{X} = AX + Bu + C\Phi
\]  

(3.3)

where

\[
X^{2N \times 1} = [s_1 \ v_1 \ s_2 \ v_2 \ \cdots \ s_N \ v_N]^T
\]

\[
A^{2N \times 2N} = \begin{bmatrix}
A_1 & A_2 & \cdots & A_N \\
A_0 & A_1 & & \\
& & \ddots & \\
& & & A_0 & A_N
\end{bmatrix}, \ A_i = \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix}, \ A_0 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix},
\]

\[
B^{2N \times N} = \begin{bmatrix}
B_1 \\
& \ddots \\
& & B_N
\end{bmatrix}
\]

\[
B_i = [0 \ 1]^T, \text{ for } i = 1, 2, \ldots, N
\]

\[
u^{N \times 1} = [u_0 \ \cdots \ u_N]^T
\]

\[
C^{2N \times 1} = [1 \ 0 \ 0 \ \cdots \ 0]^T, \ \Phi = v_0
\]

where \( A \) is the platoon state matrix with \( A_i \) representing the \( i \)th vehicle’s state matrix, \( \tau_{des} \)
represents desired time headway among vehicles, \( \mathbf{B} \) is the platoon input matrix with \( B_i \) representing the \( i \)th vehicle’s input matrix, \( \phi \) is the disturbance caused by the uncontrolled vehicle in front of the CACC vehicle platoon.

With the third-order dynamics in Equation (3.3), the CACC platoon system with \( N \) vehicles is in the following state-space form:

\[
\dot{\mathbf{X}} = \mathbf{A}\mathbf{X} + \mathbf{B}\mathbf{u} + \mathbf{C}\phi
\]  

(3.4)

where

\[
\mathbf{X}^{3N \times 1} = [s_1 \ v_1 \ a_1 \ s_2 \ v_2 \ a_2 \ \cdots \ s_N \ v_N \ a_N]^T
\]

\[
\mathbf{A}^{3N \times 3N} = \begin{bmatrix} A_1 & & \\ & \ddots & \\ & & A_N \end{bmatrix}, \quad A_i = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1/T_e \end{bmatrix}, \quad A_0 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},
\]

\[
\mathbf{B}^{3N \times N} = \begin{bmatrix} B_1 & \cdots & B_N \end{bmatrix}, \quad B_i = [0 \ 0 \ 1/T_e]^T, \text{ for } i = 1, 2, \ldots, N
\]

\[
\mathbf{u}^{N \times 1} = [u_0 \ \cdots \ u_N]^T
\]

\[
\mathbf{C}^{3N \times 1} = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \end{bmatrix}^T, \quad \phi = v_0
\]

### 3.1.3 Model Assumptions and Constraints

The constraints of the designed optimal control problem include the following:

- **Safety Constraints:**

\[
\chi := \{ s > s_0; v \in [0, v_{\text{free}}]\}
\]  

(3.5)

Equation (3.5) is a hard constraint indicating two ranges: controlled vehicles need to be higher than the standstill distance \( s_0 \), ensuring the safety of control models. And the speed should be within the range of \( [0, v_{\text{free}}] \). Limiting the speed within a reachable range. The collision avoidance will be activated by applying the safe control input \( u_{\text{safe}} \) when vehicles
can potentially collide with its preceding vehicle in $t_T$ seconds:

$$u_{safe} = -\frac{2(s_0 - s_i(t) - t_T \dot{s}_i(t))}{t_T^2}$$  \hspace{1cm} (3.6)$$

where $t_T$ is the time with respect to the prediction horizon.

- Driver Comfort Constraints:

Upper and lower boundaries constrain the control inputs to ensure the driver's comfort. And the limitation of the vehicle's acceleration and jerk capability:

$$\mathcal{U} := \{u_i \in [u_{min}, u_{max}]\}, \quad \mathcal{J} := \{j_i \in [j_{min}, j_{max}]\}$$  \hspace{1cm} (3.7)$$

- Sensing and Communication:

  a. The communication delay among vehicles is assumed to be zero.

  b. The onboard sensors can obtain the ego vehicle's spacing, speed, and relative speed to the PV without delay.

  c. CACC vehicles can obtain information from their PV, including the acceleration.

  d. No computational delay is considered during optimization.

  e. The control delay caused by sensing and executing the control output is 0.3 seconds.

This is an average value selected based on the suggested values in the literature of sensor delay between 0.1-0.3 seconds, and the actuator lag in the order of 0.1-0.2 seconds (Ploeg, Van De Wouw, & Nijmeijer, 2013; Rajamani, 2011; Xiao & Gao, 2011b).

- Dynamic speed prediction:

The constant-acceleration heuristic is used to predict PV’s dynamics for ACC vehicles that cannot obtain front vehicles’ acceleration, i.e., accelerations of the considered and leading vehicle will not change in the near future.
3.2 Proposed Control Framework

3.2.1 Optimal Control Framework

The proposed model follows the framework proposed by (M. Wang et al., 2014a) to solve our proposed optimal control problem to achieve both shockwave damping and congestion mitigation. Figure 6 depicts the concept of MPC used by the proposed model. The optimal control problem is solved at each time step \((t, t + 1, t + 2, \ldots)\) according to the predicted cost during the prediction horizon \((t, t + t_T)\), while the calculated control input is scheduled for the control horizon \((t, t + t_C)\). In this study, the control horizon is set as only one step, i.e., only the first optimal control input is selected for the vehicle control at time \(t\).

![MPC Concept of The Proposed Model](image)

Figure 6. MPC Concept of The Proposed Model

3.2.2 Vehicle Control Framework

Figure 7 shows the control framework of the proposed ADAS system. For vehicle \(i\) at time \(t_0\), the controller receives system states \(X(t_0)\), including spacing, velocity (without wireless communication), and acceleration (with wireless communication), through onboard sensors. The MPC controller predicts the system state \(X\) in the prediction horizon \([t_0, t_0 + t_T]\) and the control input \(u^*\) is determined through PMP method based on cost function and
constraints. It should also be noted that the calculated control input \( u^* \), which is the acceleration of vehicle \( i \), will be applied to update the vehicle dynamics without any delay considered.

![Figure 7. Vehicle Control Framework](image)

3.3 **Optimal Control Cost Function Design**

This section designs five MPC Control models with different control objectives to achieve shockwave damping and congestion mitigation. The proposed five models are Catch-Up, Safe-Shock, Elasticity, Catch-Up-Jerk, Safe-Shock-Jerk Model. The corresponding optimal
solutions are obtained through the open-loop optimization PMP method, and the local and string stability conditions are derived based on the analytical solution. It is worth mentioning that the analytical solution derived in the following is to support the model parameter design such that the selected parameters meet the stability condition, as derived in Appendix B.

The proposed method formulates the longitudinal vehicle control problem as a Model Predictive Control (MPC) optimization problem. The objectives of the optimization problem include:

1) Restrain excessive acceleration or deceleration;
2) Improve the flow rate among the controlled vehicles;
3) Damp the shockwave propagation speed when entering the traffic congestion;
4) Reduce the overall spatial-temporal span of the congestion in the road network.

We use the general finite-horizon optimal control framework as follows. At time $t = 0$ with initial state $X(0) = X_0$, the optimal control problem within each prediction horizon can be formulated as:

$$ u^* = \arg \min_{u(t)} \int_0^{t_T} \mathcal{L}(X, u, t) + G(X(t_T)) dt $$

subject to

$$ \dot{x} = f(X(t), u(t)) \quad \text{(3.8)} $$

$$ X(0) = X_0 $$

$$ u(t) \in \mathcal{U}, t \in [0, t_T] $$

where $\mathcal{L}$ is the running cost, $G$ is the terminal cost, $\mathcal{U}$ is the admissible control region. The
terminal cost describes the costs remaining at the end of the prediction horizon $t_T$ (M. Wang et al., 2014a). M. Wang, Daamen, Hoogendoorn, and van Arem (2012) found that when increasing the prediction horizon to 5 seconds or larger, the terminal cost has a negligible impact on the ADAS performance. Thus, the terminal cost $G(x(t_T))$ is deemed as zero by applying a prediction horizon longer than 5 seconds.

The Hamiltonian $\mathcal{H}$ function is widely used to solve optimal control problems of dynamical systems:

$$\mathcal{H}(X, u, \lambda) = \mathcal{L}(X, u) + \lambda^T f(X, u)$$

where $\lambda$ denotes the costates or marginal costs of the state $X$ with the same dimension as state $X$, which can be interpreted as Lagrange multipliers associated with the state equations, and $\mathcal{L}$ is the running costs incurred during the prediction horizon. With the Hamiltonian, the optimal solution is obtained as: $u^* = \arg\min_u \mathcal{H}(X, u, \lambda)$. The followings are the detailed derivation of the optimal control solutions given the proposed control objectives.

3.3.1 Control Model I: Catch-Up Model

With the second-order vehicle dynamics Equation (3.1) in the control problem (3.8), a two-regime running cost function is proposed for $i$th controlled vehicle. The Safety cost term is proposed previously in the existing longitudinal vehicle control model (M. Wang, Daamen, Hoogendoorn, & van Arem, 2014b), and the Comfort term is widely used to restrict the magnitude of control input (Duret et al., 2019; Kamal et al., 2014), while the Control Model $I$ of this paper introduces the Efficiency term:

$$\mathcal{L}(X, u)$$

(3.10)
\[
\begin{align*}
&= c_1 \exp \left( \frac{S_0}{s(t)} \right) \Delta v_i^2(t) \Theta(-\Delta v_i(t)) + c_2 \left( v_{\text{des},i}(t) + v_{\text{add}}(t) - v_i(t) \right)^2 + c_3 u_i^2(t)
\end{align*}
\]

where \(c_1, c_2, c_3\) are weights of cost terms. \(\Theta\) is a Heaviside step function in the following form:

\[
\Theta(x) = \begin{cases} 
1, & \text{if } x \geq 0 \\
0, & \text{if } x < 0 
\end{cases}
\]

\(v_{\text{des},i}(t)\) is the desired speed of ego vehicle, specified as a two-regime function:

\[
v_{\text{des},i}(t) = \begin{cases} 
 v_{\text{free}}, & \text{if } s_i > s_f \\
 s_i(t) - s_0 \tau_{\text{des}}, & \text{if } s_i \leq s_f 
\end{cases}
\]

where \(v_{\text{free}}\) is the free-flow speed, \(s_0\) is the standstill spacing among vehicles, \(\tau_{\text{des}}\) is the desired time headway, and \(s_f = v_{\text{free}} \tau_{\text{des}} + s_0\) is the spacing threshold to determine if the vehicle is in \textit{Cruising} \((s_i > s_f)\) or \textit{Following} \((s_i \leq s_f)\) mode. \(v_{\text{add}}\) is an additional value added up to the desired speed \(v_{\text{des},i}\) for ego vehicle to catch up with PV. \(v_{\text{add}}\) is calculated by evaluating the prevailing spacing \(s_i(t)\) with respect to a pre-determined range of feasible catch-up spacing \([s^-, s^+]\) and a constant value \(v_0\):

\[
v_{\text{add}}(t) = v_0 \ast \max \left( \min \left( \frac{s_i(t) - s^-}{s^+ - s^-}, 1 \right), 0 \right)
\]

The designed \(v_{\text{add}}\) damps the shockwave propagation speed by delaying the ego vehicle’s deceleration when approaching the congestion while keeping a safe spacing. It allows the controlled vehicle to drive faster than the desired speed and catch PV if the spacing is above the lower threshold \(s^-\). Due to the min-max function, the controlled vehicles will drive at the desired speed if at a spacing smaller than \(s^-\). At spacing larger than the upper threshold \(s^+\), controlled vehicles tend to drive at the maximum speed \((v_{\text{des}} + v_0)\).

Equation (3.10) has multiple objectives, including safety, efficiency, and comfort:
• The safety cost term avoids rear-end collisions by providing a large penalty when the ego vehicle approaches PV under an unsafe spacing. In this term, the ego vehicle is more sensitive to the relative speed $\Delta v_i$ and would like to keep the same speed as PV when the spacing $s_i$ is small.

• The efficiency cost term formulates vehicle control dynamics by considering the $v_{des,i}$ and an additional speed $v_{add}$ term that can adapt to different spacing conditions. $v_{add}$ allows the vehicle to drive faster than the desired speed $v_{des,i}$ and catch up with its PV.

• The comfort cost term provides penalties on excessive accelerations and decelerations to provide smoother speed changes and improve driver comfort.

For the sake of brevity, the time instant $t$ is omitted hereinafter. This study uses the PMP method to solve the proposed optimal control problem in an open loop. To derive the optimal control input, the following Hamiltonian $\mathcal{H}$ is defined:

$$\mathcal{H}(X,u,\lambda) = L(X,u) + \lambda^T f(X,u) =$$

$$c_1 \exp\left(\frac{s_0}{s_i}\right) \Delta v_i^2 \Theta(-\Delta v_i) + c_2 \left(v_{des,i} + v_{add} - v_i\right)^2 + c_3 u_i^2 + \lambda \Delta v_i + \lambda_v u_i$$

(3.14)

where $\lambda$ denotes the costate or marginal cost of state $X$. For the Hamiltonian, the costates need to satisfy the following dynamic equation:

$$-\frac{d}{dt} \lambda = \frac{\partial \mathcal{H}}{\partial X} = \frac{\partial L}{\partial X} + \lambda^T \frac{\partial f}{\partial X}$$

(3.15)

subject to the terminal conditions of $\lambda(t_0 + t_T)$. Thus, the costate dynamic of proposed optimal control becomes

$$-\frac{d\lambda_s}{dt} = -\frac{2c_1 s_0}{s_i^3} \exp\left(\frac{s_0}{s_i}\right) \Delta v_i^2 \Theta(-\Delta v_i) + 2c_2 \left(\frac{1}{v_{des}} + \frac{1}{s^* - s_0}\right)\left(v_{des,i} + v_{add} - v_i\right)$$

(3.16)
\[- \frac{d \lambda_v}{dt} = -2c_1 \exp\left(\frac{s_0}{s_i}\right) \Delta v_i \Theta(-\Delta v_i) - 2c_2 (v_{des,i} + v_{add} - v_i) - \lambda_s \]

The optimal control input is then obtained as follows:

\[
\frac{\partial H}{\partial u_i} = 2c_3 u_i + \lambda_v = 0
\]

\[
u^* = -\frac{\lambda_v}{2c_3}
\]

With the dynamics of costates and optimal control input, the proposed optimal control problem can be solved iteratively for each time step.

3.3.2 Control Model II: Safe-Shock Model

The Control Model II introduces the Shockwave penalization term to mitigate the impact of shockwaves. The safety and comfort terms are standard in CTH policy models for longitudinal vehicle control (Kamal et al., 2014; G. J. Naus et al., 2010a).

\[
\mathcal{L}(X, u) = c_1 \left(s_i - s_d\right)^2 + c_2 \exp\left(\frac{\Delta v_i}{\Delta v_{free}}\right) \Delta v_i^2 + c_3 u_i^2
\]

where the desired spacing is determined by \(s_d = v_i \tau_{des} + s_0\), \(\Delta v_{free}\) is the Speed Change Factor to limit the magnitude of the \(\Delta v_i\) and provide reasonable weights to the shockwave term. Equation (3.20) has multiple objectives, including Safety, Shockwave, and Comfort:

- The safety cost term avoids rear-end collisions by providing a large penalty when the spacing deviates from the desired spacing.
- The exponential function in the shockwave term can accommodate different shockwave conditions to either reduce or increase the shockwave speed to reduce congestion. Figure 8. shows the characteristics of the design curve \(W(\Delta v_i) = \exp\left(\frac{\Delta v_i}{\Delta v_{free}}\right) \Delta v_i^2\). When \(\Delta v_i > 0\) , \(W(\Delta v_i)\) increases significantly with the
combination of both positive exponential and square functions. When $\Delta v_i$ is close to zero, the function sinks to around zero. When $\Delta v_i < 0$, the overall $W(\Delta v_i)$ was restricted to within a level. Such characteristics make the design curve a good fit to harmonize shockwaves. When ego vehicles join a queue, their relative speed $\Delta v_i$ becomes negative. Instead of overreacting to large speed differences with the full square term $v_i^2$, the exponential function with a negative $\Delta v_i/\Delta v_{free}$ will damp the increase of $v_i^2$ in the shockwave term and making it less impactful in the optimization process. This mechanism can partially slow down the propagation of backwave traveling queuing shockwave. It should be noted that safety and comfort are still ensured through the other two terms in the objective function. When an ego vehicle is discharged from a queue, the preceding vehicle will have a higher speed then $\Delta v_i$ becomes positive, then the exponential term will positively contribute to the increase of the shockwave term. The optimization result will make the ego vehicle close the gap and speed differences quickly by minimizing the shockwave term with a positive exponential and square term of $\Delta v_i$.

- The comfort cost term penalties excessive acceleration and decelerations to achieve smooth speed changes and improved driver comfort.
Similar to the Catch-Up Speed model, the costates of the Safe-Shock model are obtained as follows:

\[-\frac{d\lambda_s}{dt} = 2c_1(s_i - s_d)\]  \hspace{1cm} (3.21)

\[-\frac{d\lambda_v}{dt} = -2c_1t_{des}(s_i - s_d) - \frac{c_2}{\Delta v_{free}} \exp\left(\frac{\Delta v_i}{\Delta v_{free}}\right)\Delta v_i^2 - 2c_2 \exp\left(\frac{\Delta v_i}{\Delta v_{free}}\right)\Delta v_i - \lambda_s\]  \hspace{1cm} (3.22)

Solving the above models, the optimal control input is as follows:

\[u^* = -\frac{\lambda_v}{2c_3}\]  \hspace{1cm} (3.23)

3.3.3 Control Model III: Elasticity Model

The Elasticity term in Control Model III is modified based on Control Model II to provide the following vehicles with the flexibility of not closing up spacing right away towards desired spacing when the spacing is large. This Elasticity term is proposed based on the
observation that the shockwave damping process, as depicted in Figure 1, tends to have the squeezing effect on spacing after slowing down the shockwaves. Such a squeezing effect may eventually lead to diminished spacing for shockwave damping, and the entire model converges to collision avoidance that can trigger back the stop-and-go waves. A spacing elasticity term is introduced to cope with this effect.

\[
\mathcal{L}(X, u) = \frac{c_1 \exp\left(\frac{s_0}{s_i}\right)}{\text{Elasticity}} (s_i - s_d)^2 + c_2 \exp\left(\frac{\Delta v_i}{\Delta v_{\text{free}}}\right) \Delta v_i^2 \frac{\Delta v_i}{\text{Shockwave}} + c_3 u_i^2 \frac{\Delta v_i}{\text{Comfort}} \tag{3.24}
\]

where the desired spacing is determined by \( s_d = v_i \tau_{\text{des}} + s_0 \), the exponential term will reduce the effect of the original safety term when the spacing \( s_i \) is large.

Equation (3.24) has multiple objectives, including Elasticity, Shockwave, and Comfort:

- As shown in Figure 9, compared with the Safety term in Control Model II, the elasticity cost term provides more spacing “buffer” by delivering a smaller penalty when the spacing is large. Hence, vehicles have more freedom to choose larger spacing than the desired spacing. The elasticity cost term also has safety implications. Rear-end collisions or conflicts can be reduced by providing a large penalty when the ego vehicle is approaching PV under small spacings.

- The shockwave cost term implies that the vehicle is not sensitive to PV’s speed when the relative speed is small (or negative) than the Speed Change Factor \( \Delta v_{\text{free}} \). A typical example of such a scenario is when the ego vehicle approaches the PV.

- The comfort cost term provides penalties on excessive accelerations or decelerations, providing smoother speed changes and improving driver comfort.
The costates of the Elasticity model are obtained through the PMP method:

\[
\frac{d\lambda_s}{dt} = -c_1 s_0 \exp \left( \frac{s_0}{s_i} \right) (s_i - s_d)^2 + 2c_1 \exp \left( \frac{s_0}{s_i} \right) (s_i - s_d)
\]

\[
\frac{d\lambda_v}{dt} = -2c_1 \exp \left( \frac{s_0}{s_i} \right) (s_i - s_d) t_{des} - c_2 \frac{\Delta v_i}{\Delta v_{free}} \exp \left( \frac{\Delta v_i}{\Delta v_{free}} \right) \Delta v_i^2
\]

\[
-2c_2 \exp \left( \frac{\Delta v_i}{\Delta v_{free}} \right) \Delta v_i - \lambda_s
\]

And the optimal control input is as follows:

\[
u^* = -\frac{\lambda_v}{2c_3}
\]

3.3.4 Control Model IV: Catch-Up-Jerk Model

The third-order vehicle dynamics in Equation (3.2) involves the jerk into system dynamics and further improve the drivers’ comfort by minimizing the jerk magnitude:
\( \mathcal{L}(X, u) \)

\[
\begin{align*}
\mathcal{L}(X, u) &= c_1 \exp \left( \frac{s_0}{s_i(t)} \right) \Delta v_i^2(t) \Theta(-\Delta v_i(t)) + c_2 \left( v_{\text{des},i}(t) + v_{\text{add}}(t) - v_i(t) \right)^2 \\
&+ c_3 u_i^2(t) + c_4 a_j^2
\end{align*}
\]

The following Hamiltonian \( \mathcal{H} \) is defined as:

\[
\mathcal{H}(X, u, \lambda) = \mathcal{L}(X, u) + \lambda^T f(X, u) =
\]

\[
\begin{align*}
&c_1 \exp \left( \frac{s_0}{s_i(t)} \right) \Delta v_i^2 \Theta(-\Delta v_i) + c_2 \left( v_{\text{des},i} + v_{\text{add}} - v_i \right)^2 + c_3 u_i^2 + c_4 \frac{(-a_i + u_i)^2}{T_e^2} \\
&+ \lambda_s \Delta v_i + \lambda_v a_i + \frac{\lambda_a (-a_i + u_i)}{T_e}
\end{align*}
\]

Thus, the costate dynamic of optimal control becomes

\[
\begin{align*}
- \frac{d\lambda_s}{dt} &= - \frac{2c_1 s_0}{s_i^2} \exp \left( \frac{s_0}{s_i(t)} \right) \Delta v_i^2 \Theta(-\Delta v_i) + 2c_2 \left( \frac{1}{\tau_{\text{des}}} + \frac{1}{s^+ - s_0} \right) (v_{\text{des},i} + v_{\text{add}} - v_i) \\
- \frac{d\lambda_v}{dt} &= -2c_1 \exp \left( \frac{s_0}{s_i(t)} \right) \Delta v_i \Theta(-\Delta v_i) - 2c_2 (v_{\text{des},i} + v_{\text{add}} - v_i) - \lambda_s \\
- \frac{d\lambda_a}{dt} &= 2c_4 \left( a_i - u_i \right) + \frac{\lambda_v T_e}{T_e + 1}
\end{align*}
\]

The optimal control input is then obtained as follows:

\[
\frac{\partial \mathcal{H}}{\partial u_i} = 2c_3 u_i + \frac{\lambda_v}{T_e} = 0
\]  

\[
u^* = -\frac{\lambda_v}{2c_3 T_e}
\]

3.3.5 Control Model V: Safe-Shock-Jerk Model

With the third-order system dynamics, the cost function of Model II is modified as the following:
\[ L(X, u) = \frac{c_1(s_i - s_d)^2}{\text{Safety}} + \frac{c_2 \exp\left(\frac{\Delta v_i}{\Delta v_{\text{free}}}\right) \Delta v_i^2}{\text{Shockwave}} + \frac{c_3 u_i^2(t)}{\text{Comfort}} + c_4 \dot{a}_i(t) \]  

The costates of the Safe-Shock model are obtained as follows:

\[ -\frac{d\lambda_s}{dt} = 2c_1(s_i - s_d) \]  

\[ -\frac{d\lambda_v}{dt} = -2c_1 t_{\text{des}}(s_i - s_d) - \frac{c_2}{\Delta v_{\text{free}}} \exp\left(\frac{\Delta v_i}{\Delta v_{\text{free}}}\right) \Delta v_i^2 - 2c_2 \exp\left(\frac{\Delta v_i}{\Delta v_{\text{free}}}\right) \Delta v_i - \lambda_s \]  

\[ -\frac{d\lambda_a}{dt} = 2c_4 \frac{(a_i - u_i)}{T_e + 1} + \frac{\lambda_v T_e}{T_e + 1} \]  

Solving the above models, the optimal control input is as follows:

\[ u^* = -\frac{\lambda_v}{2c_3 T_e} \]  

### 3.4 MPC Solution Method

The proposed optimal control problem can be solved efficiently by the iPMP (iterative PMP) framework proposed by (M. Wang et al., 2014b). Since the original iPMP framework uses generic vehicle state and control variables, the following implementation provides a more detailed formula based on the proposed objective functions in this paper. Furthermore, the proposed five models will use the following two solution algorithms depending on the number of vehicle states used for the state dynamic equations.

Algorithm 1 illustrates the implemented control framework with the state dynamics for Model I, II, and III. The major steps of the proposed framework include the initialization, optimal control inputs, and second-order vehicle dynamics update. For each controlled
vehicle, the following steps are taken: 1) set initial guess of costates $\Lambda^{(0)}_i(t)$ to zeros, 2) predict second-order state dynamics forwardly in prediction horizon $[t_0, t_0 + t_T]$ depending on the costates needed, 3) solve costates backwardly with terminal costates $\lambda^{(k)}_i(t_0 + t_T) = 0$, 4) iterate until costates $\Lambda^{(k)}_i$ and $\lambda^{(k)}_i$ converge. The calibration of the control framework includes fine-tuning the weight factor $\alpha$ to achieve a faster convergence rate.

Algorithm 1: Optimal Control Framework Using iterative PMP Method

**DATA:** initial vehicle states $\left[ \dot{s}_i(0), \dot{v}_i(0) \right]$, weight factor $\alpha \in (0, 1)$, error threshold $\epsilon_{\text{max}}$, convergence error $\epsilon_{\text{stop}}$

**RESULTS:** optimal control input: $u^*_i(t)$

**FOR** each time step $t_0$ **DO**

**FOR** each autonomous vehicle $i$ **DO**

Initialization: costates $\Lambda^{(0)}_i(t) = \left[ \Lambda_{s,i}^{(0)}, \Lambda_{v,i}^{(0)} \right] = [0 \ 0]^T \ \forall t \in [t_0, t_0 + t_T]$, open-loop iteration number $k$, initial error $\epsilon = \epsilon_{\text{max}}$

**WHILE** $\epsilon \geq \epsilon_{\text{stop}}$ **DO**

$$u^*_i(k)(t) = \arg \min_{u^*} \mathcal{H} \left( \left[ \dot{s}_i(t), \dot{v}_i(t) \right], u, \lambda \right), u^*_i(k) \text{ is constrained by Eq. (3.5), (3.7)}$$

Solve the state dynamics forwardly based on initial state $X^{(k)}(t_0)$

$$\dot{X}^{(k)}_i(t) = f \left( \left[ \dot{s}_i(t), \dot{v}_i(t) \right], u^*_i \left( \left[ \dot{s}_i(t), \dot{v}_i(t) \right], \Lambda^{(k-1)}_i \right) \right), \ t \in [t_0, t_0 + t_T]$$

Solve the costate backwardly subject to final costates $\lambda^{(k)}_i(t_0 + t_T) = 0$

$$\dot{\lambda}^{(k)}_i(t - 1) = \dot{\lambda}^{(k)}_i(t) - \dot{\lambda}^{(k)}_i(t) = \dot{\lambda}^{(k)}_i(t) -$$

$$\frac{\partial \mathcal{H}}{\partial x} \left( \left[ \dot{s}_i(t), \dot{v}_i(t) \right], u^*_i \left( \left[ \dot{s}_i(t), \dot{v}_i(t) \right], \Lambda^{(k-1)}_i \right) \right), \ t \in [t_0, t_0 + t_T]$$

Update the costate $\Lambda^{(k)}_i(t) = (1 - \alpha) \Lambda^{(k-1)}_i(t) + \alpha \lambda^{(k)}_i(t)$

Update the error $\epsilon = \left\| \Lambda^{(k)}_i(t) - \lambda^{(k)}_i(t) \right\|_2$

**IF** $\epsilon < \epsilon_{\text{max}}$

Continue to next iteration $k = k + 1$

**ELSE**

Fail to converge. Need to tune parameters.

**ENDIF**

**ENDWHILE**

Implement the first timestep ($t_0$) of the optimal solution as the control input

$$a_i(t) = a^{*(k)}_i(t_0)$$

**ENDFOR**
Algorithm 2 uses the third-order vehicle dynamics when updating the vehicle states during prediction. This solution algorithm is used for Control Model IV and V: Catch-Up-Jerk and Safe-Shok-Jerk to consider the jerk evolution.

Algorithm 2: Optimal Control Framework Using iterative PMP Method

**DATA:** initial vehicle states $\left[ \dot{s}_i(0), \dot{v}_i(0), \dot{a}_i(0) \right]$, weight factor $\alpha \in (0,1)$, error threshold $\epsilon_{max}$, convergence error $\epsilon_{stop}$

**RESULTS:** optimal control input: $u_i^*(t)$

**FOR** each time step $t_0$ **DO**

**FOR** each autonomous vehicle $i$ **DO**

Initialization: costates $\Lambda_i^{(0)}(t) = \begin{bmatrix} \Lambda_{s,i}^{(0)} & \Lambda_{v,i}^{(0)} & \Lambda_{a,i}^{(0)} \end{bmatrix} = [0 \ 0 \ 0]^T \ \forall t \in [t_0, t_0 + t_T]$, open-loop iteration number $k$, initial error $\epsilon = \epsilon_{max}$

**WHILE** $\epsilon \geq \epsilon_{stop}$ **DO**

$u_i^{*(k)}(t) = \text{argmin}_u \mathcal{H} \left( \begin{bmatrix} \dot{s}_i(t) \\ \dot{v}_i(t) \\ \dot{a}_i(t) \end{bmatrix} \right), u_i^{(k)}$ is constrained by Eq. (3.5), (3.7)

Solve the state dynamics forwardly based on initial state $X^{(k)}(t_0)$

$$\dot{X}_i^{(k)}(t) = f \left( \begin{bmatrix} \dot{s}_i(t) \\ \dot{v}_i(t) \\ \dot{a}_i(t) \end{bmatrix}^{(k)} \right), u_i^*, t \in [t_0, t_0 + t_T]$$

Solve the costate backwardly subject to final costates $\lambda_i^{(k)}(t_0 + t_T) = 0$

$$\dot{\lambda}_i^{(k)}(t - 1) = \dot{\lambda}_i^{(k)}(t) - \dot{\lambda}_i^{(k)}(t) = \dot{\lambda}_i^{(k)}(t) -$$

Update the costate $\Lambda_i^{(k)}(t) = (1 - \alpha)\Lambda_i^{(k-1)}(t) + \alpha \lambda_i^{(k)}(t)$

Update the error $\epsilon = \left\| \Lambda_i^{(k)}(t) - \lambda_i^{(k)}(t) \right\|_2$
\[ \textbf{IF } \varepsilon < \varepsilon_{\text{max}} \]
\[ \text{Continue to next iteration } k = k + 1 \]
\[ \text{ELSE} \]
\[ \text{Fail to converge. Need to tune parameters.} \]
\[ \text{ENDIF} \]
\[ \text{ENDWHILE} \]

Implement the first timestep \((t_0)\) of the optimal solution as the control input
\[ a_i(t) = a^{*}_{i}(t_0) \]
\[ \text{ENDFOR} \]

Update vehicle dynamics \[
\begin{bmatrix}
\dot{s}_i(t_0 + T) \\
\dot{v}_i(t_0 + T) \\
\dot{a}_i(t_0 + T)
\end{bmatrix}
= f
\begin{bmatrix}
\dot{s}_i(t_0) \\
\dot{v}_i(t_0) \\
\dot{a}_i(t_0)
\end{bmatrix}, u(t_0)
\]
\[ \text{ENDFOR} \]

It should also be noted that different cost functions will have different Hamiltonian \(\mathcal{H}\), in the form of Equation (3.9). The change of Hamiltonian \(\mathcal{H}\) also leads to different optimal control inputs to minimize the designed cost. This paper proposes models with different cost function setups that find the optimal control input \(u_i^*(t)\) corresponding to the designed cost functions.

### 3.5 Stability Analysis

The analytical solution derived in Section 3.3 supports the model parameter design such that the selected parameters meet the stability condition. To further analyze the impacts of parameters of five proposed models and find the needed parameters, this subsection analyzes stability patterns for their parameters.

#### 3.5.1 Control Model I: Catch-Up Model

According to Equation (3.19), we have the optimal control input as follows
\[
u^* = -\frac{\lambda_v}{2c_3} = -\frac{1}{2c_3} \left( \frac{\partial L}{\partial v_i} - \lambda_s \right) = -\frac{1}{2c_3} \left( \frac{\partial L}{\partial v_i} - \frac{\partial L}{\partial s_i} \right)
= -\frac{1}{2c_3} \left( 2c_1 \exp \left( \frac{s_0}{s_i} \right) \Delta v_i \theta(-\Delta v_i) - \frac{2c_1 s_0}{s_i^2} \exp \left( \frac{s_0}{s_i} \right) \Delta v_i^2 \theta(-\Delta v_i) \right) + 2c_2 \left( \frac{1}{\tau_{\text{des}}} + \frac{v_0}{s^+ - s^-} + 1 \right) \left( v_{\text{des},i} + v_{\text{add}} - v_i \right)
\] (3.39)
The solution described in Equation (3.39) can be further used to analyze the string stability conditions of the developed ACC model. When a platoon of vehicles is in the equilibrium state, i.e., vehicles are at the same speed and spacing with zero acceleration, adding perturbations to spacing and speed can give the following first-order Taylor series of the acceleration:

\[ u_i = f_s s_i + f_{\Delta v} \Delta v_i + f_v v_i \]  

(3.40)

where \( f_s \), \( f_{\Delta v} \), and \( f_v \) are partial derivatives of the state dynamics function \( a_i = f(s_i, \Delta v_i, v_i) \) at the equilibrium:

\[ f_s = \frac{\partial f}{\partial s} \bigg|_e = \frac{c_2}{c_3} \left( \frac{1}{\tau_{des}} + \frac{v_0}{s^+ - s^-} \right) \left( \frac{1}{\tau_{des}} + \frac{v_0}{s^+ - s^-} + 1 \right) \]

(3.41)

\[ f_{\Delta v} = \frac{\partial f}{\partial \Delta v} \bigg|_e = \frac{c_1}{c_3} \exp \left( \frac{s_0}{s_e} \right) \]

\[ f_v = \frac{\partial f}{\partial v} \bigg|_e = -\frac{c_2}{c_3} \left( \frac{1}{\tau_{des}} + \frac{v_0}{s^+ - s^-} + 1 \right) \]

According to Sun et al. (2018), the local stability is satisfied when:

\[ f_{\Delta v} - f_v > 0 \quad \text{and} \quad f_s > 0 \]  

(3.42)

the string stability is satisfied when:

\[ f_v^2 - 2f_s - 2f_{\Delta v}f_v > 0 \]  

(3.43)

Combining Equation (3.41) and (3.42), the local stability condition of the proposed model is:

\[ \frac{1}{c_3} \left( c_1 \exp \left( \frac{s_0}{s_e} \right) + c_2 \left( \frac{1}{\tau_{des}} + \frac{v_0}{s^+ - s^-} + 1 \right) \right) > 0 \]

and \[ \frac{c_2}{c_3} \left( \frac{1}{\tau_{des}} + \frac{v_0}{s^+ - s_0} \right) \left( \frac{1}{\tau_{des}} + \frac{v_0}{s^+ - s^-} + 1 \right) > 0 \]

(3.44)

Combining Equation (3.41) and (3.43), the stability condition of the proposed model is:
\[
\left( -\frac{c_2}{c_3} \left( \frac{1}{\tau_{des}} + \frac{v_0}{s^+-s^-} + 1 \right) \right)^2 - \frac{2c_2}{c_3} \left( \frac{1}{\tau_{des}} + \frac{1}{s^+-s^-} \right) \left( \frac{1}{\tau_{des}} + \frac{1}{s^+-s^-} + 1 \right) \\
+ \frac{2c_1c_2}{c_3^2} \exp \left( \frac{s_0}{s_e} \right) \left( \frac{1}{\tau_{des}} + \frac{v_0}{s^+-s^-} + 1 \right) > 0
\]  

(3.45)

The stability condition can be easily checked in the actual implementation using Equations (3.42) and (3.43). Figure 10 depicts the stability region satisfying Equation (3.44) and (3.45) with respect to the equilibrium speed $v_e$ under different $c_1$, $c_2$ and $\tau_{des}$. Other parameter settings are shown in Table 2 and Table 6. The model is both local and string stable within the red areas. It illustrates that larger $c_1$, $c_2$ and $\tau_{des}$ provides a more stable Catch-Up model. To make sure the model is stable, it is better to select a parameter combination that is stable along all possible $v_e$.

Figure 10. Stability Region Analysis of Catch-Up Model
3.5.2 Control Model II: Safe-Shock Model

According to Equation (3.23), we have the optimal control input as follows

\[ u^* = -\frac{1}{2c_3} \left( \frac{\partial L}{\partial v_i} - \frac{\partial L}{\partial s_i} \right) \]

\[ = \frac{1}{2c_3} \left( 2c_1(\tau_{des} - 1)(s_i - s_d) + \frac{c_2}{\Delta v_{free}} \exp \left( \frac{\Delta v_i}{\Delta v_{free}} \right) \Delta v_i^2 \right) \]

\[ + 2c_2 \exp \left( \frac{\Delta v_i}{\Delta v_{free}} \right) \Delta v_i \]  \hspace{1cm} (3.46)

For stability condition analysis, we first derive the partial derivatives \( f_s, f_{\Delta v} \) and \( f_v \) based on Equation (3.48):

\[ f_s = \left. \frac{\partial f}{\partial s} \right|_e = \frac{c_1}{c_3} (\tau_{des} - 1) \]

\[ f_{\Delta v} = \left. \frac{\partial f}{\partial \Delta v} \right|_e = \frac{c_2}{c_3} \]

\[ f_v = \left. \frac{\partial f}{\partial v} \right|_e = \frac{c_1}{c_3} \tau_{des} (1 - \tau_{des}) \]  \hspace{1cm} (3.47)

The local and string stability can be checked directly by inequity (3.42) and (3.43). Similar to Figure 10, Figure 11 depicts the stability region of the Safe-Shock model. With \( \tau_{des} \) as 1.1 second, most area in the analyzed ranges are stable.
3.5.3 Control Model III: Elasticity Model

According to Equation (3.27), we have the optimal control input as follows

\[ u^* = -\frac{1}{2c_3} \left( \frac{\partial L}{\partial v_i} - \frac{\partial L}{\partial s_i} \right) \]

\[ = -\frac{1}{2c_3} \left( \frac{c_1 s_0}{s_i^2} \exp \left( \frac{s_0}{s_i} \right) \left( s_i - s_d \right)^2 \right. \]

\[ - 2c_1 \exp \left( \frac{s_0}{s_i} \right) (\tau_{des} - 1) (s_i - s_d) - \frac{c_2}{\Delta v_{free}} \exp \left( \frac{\Delta v_i}{\Delta v_{free}} \right) \Delta v_i^2 \]

\[ - 2c_2 \exp \left( \frac{\Delta v_i}{\Delta v_{free}} \right) \Delta v_i \]

The partial derivatives \( f_s, f_{\Delta v} \) and \( f_v \) are as follows:

\[ f_s = \frac{\partial f}{\partial s} \bigg|_e = \frac{c_1}{c_3} \exp \left( \frac{s_0}{s_e} \right) (\tau_{des} - 1) \]

\[ (3.48) \]

\[ (3.49) \]
\[ f_{\Delta v} = \frac{\partial f}{\partial \Delta v_e} = \frac{c_2}{c_3} \]

\[ f_v = \frac{\partial f}{\partial v_e} |_{c_3} = \frac{c_1 \tau_{des}}{c_3} \exp \left( \frac{s_0}{s_e} \right) (1 - \tau_{des}) \]

The local and string stability are satisfied by inequity (3.42) and (3.43). Figure 12 shows that the Elasticity model is local and string stable when \( \tau_{des} \) is larger than or equal to 1.1 seconds.

![Figure 12. Stability Region Analysis of Elasticity Model](image)

3.5.4 Control Model IV: Catch-Up-Jerk Model

According to Equation (3.33), we have the optimal control input as follows
\[ u^* = -\frac{\lambda v T_e}{2c_3} = -\frac{T_e}{2c_3} \left( \frac{\partial L}{\partial v_i} - \lambda_s \right) = -\frac{T_e}{2c_3} \frac{\partial L}{\partial v_i} - \frac{\partial L}{\partial s_i} \]

\[ = -\frac{T_e}{2c_3} \left( 2c_1 \exp \left( \frac{s_0}{s_i} \right) \Delta v_i \Theta(-\Delta v_i) - 2c_1 s_0 \frac{\Delta v_i^2}{s_i^2} \exp \left( \frac{s_0}{s_i} \right) \Delta v_i \Theta(-\Delta v_i) \right) \]

\[ + 2c_2 \left( \frac{1}{\tau_{des}} + \frac{v_0}{s^+ - s^-} + 1 \right) (v_{des,i} + v_{add} - v_i) \]

The partial derivatives \( f_s, f_{\Delta v} \) and \( f_v \) are as follows:

\[ f_s = \left. \frac{\partial f}{\partial s} \right|_e = \frac{c_2 T_e}{c_3} \left( \frac{1}{\tau_{des}} + \frac{v_0}{s^+ - s^-} \right) \left( \frac{1}{\tau_{des}} + \frac{v_0}{s^+ - s^-} + 1 \right) \]

\[ f_{\Delta v} = \left. \frac{\partial f}{\partial \Delta v} \right|_e = \frac{c_1 T_e}{c_3} \exp \left( \frac{s_0}{s_e} \right) \]  

\[ f_v = \left. \frac{\partial f}{\partial v} \right|_e = -\frac{c_2 T_e}{c_3} \left( \frac{1}{\tau_{des}} + \frac{v_0}{s^+ - s^-} + 1 \right) \]

The local and string stability are satisfied by inequity (3.42) and (3.43). Figure 13 shows that the Catch-Up-Jerk model requires a larger \( c_1 \) and \( c_2 \) to ensure the stability.
3.5.5 Control Model V: Safe-Shock-Jerk Model

According to Equation (3.38), we have the optimal control input as follows

$$u^* = -\frac{\lambda_v T_e}{2c_3} = \frac{T_e}{2c_3} \left( 2c_1 (\tau_{des} - 1) (s_i - s_d) + \frac{c_2}{\Delta v_{\text{free}}} \exp\left( \frac{\Delta v_i}{\Delta v_{\text{free}}} \right) \Delta v_i^2 \right)$$

$$+ 2c_2 \exp\left( \frac{\Delta v_i}{\Delta v_{\text{free}}} \right) \Delta v_i$$

(3.52)

For stability condition analysis, we first derive the partial derivatives $f_s$, $f_{\Delta v}$ and $f_v$.

$$f_s = \left. \frac{\partial f}{\partial s} \right|_e = \frac{c_1 T_e}{c_3} (\tau_{des} - 1)$$

$$f_{\Delta v} = \left. \frac{\partial f}{\partial \Delta v} \right|_e = \frac{c_2 T_e}{c_3}$$

$$f_v = \left. \frac{\partial f}{\partial v} \right|_e = \frac{c_1 T_e}{c_3} \tau_{des} (1 - \tau_{des})$$

(3.53)

The local and string stability are satisfied by inequity (3.42) and (3.43). With $\tau_{des}$ as 1.1
second, most area in the analyzed ranges are stable, while larger $c_1$ needs a smaller $c_2$ to ensure the stability.

Figure 14. Stability Region Analysis of Safe-Shock-Jerk Model

4 EXPERIMENT DESIGN

4.1 Data Description

This paper uses the vehicle trajectory data on a southbound direction of U.S highway 101 (Hollywood Freeway) in Los Angeles, California, on June 15th, 2005, from the NGSIM project. The entire segment is around 2100 feet, with five main lanes throughout the section and one auxiliary lane. First, the dynamic states of all 34 vehicles on Lane 1 at 8:30 am during peak hour congestion are selected as the initial traffic states on the ring road to reflect realistic “perturbation patterns” to acceleration, velocity, and spacing for stability analysis. Second, the car-following model parameters calibrated based on the same dataset
in a previous study (Jin, Yang, & Ran, 2014) are used for modeling manually-driven vehicles. The proposed models and another model with the best performance in the ring-road test are applied continuously for 8000-time intervals (80 seconds) to evaluate their shockwave patterns.

The proposed model is to control the microscopic vehicles’ longitudinal behavior. Therefore it is crucial to evaluate the proposed model in the road network with lateral movement, i.e., lane-change scenarios need to be considered. Hence, the experiment is also implemented in a VISSIM-based test environment with field data collected between 7 am to 9 am on April 11th, 2012, on the IH-35 corridor in Austin, Texas. This corridor has heavy morning commute traffic coupled with one of the heaviest freight traffic in the U.S. The ending segment of this corridor is a combination of vertical and horizontal curves, which creates a traffic bottleneck. In the VISSIM simulation, we select a weaving section near Woodland Avenue to evaluate the proposed method. The External Drivers’ Model API (Application Programming Interface) of VISSIM is used to implement the proposed control model.

4.2 Model Calibration

This section describes the control parameter calibration of all proposed and reference models. It is important to fine-tune all model parameters under the same constraints to conduct a fair comparison among control models. Equation (4.1) is the objective function to calibrate the parameters of each model under the same constraints summarized in Table 2.
Maximize $f(\mathcal{P}_k)$

$$
= \alpha_1 \frac{\bar{V}}{v_{\text{free}}} - \alpha_2 \frac{\sum N \sum_T (1|v < v^c)}{N \times T} - \alpha_3 \frac{\max\{|j|\}}{J_{\text{max}}} - \alpha_4 \frac{N_{\text{crash}}}{N} 
$$

(4.1)

where $\mathcal{P}_k$ represents the parameter sets of control model $k$; $\alpha_1, \alpha_2, \alpha_3$ and $\alpha_4$ are parameters to adjust the scale of each term to a similar level; The Congestion Area term accounts for the total number of congested steps, lower than the threshold $v^c$, among all $N$ vehicles during the entire simulation steps $T$, normalized by the total number of simulation steps of all vehicles ($N \times T$); The Jerk Magnitude term is the maximum absolute jerk during the simulation, normalized by the maximum jerk $j_{\text{max}}$; The Average Velocity term considers the efficiency with the average velocity $\bar{V}$ of all $N$ vehicles during all steps $T$, normalized by the free-flow speed $v_{\text{free}}$; the Crash Count term is introduced for non-MPC-based models without safety constraints that might lead to crashes, i.e., the Crash Count term of MPC-based models stays at zero thanks to the safety constraint (3.5).

It is worth noting that the calibration is conducted for 60 seconds in the Ring-Road environment, as discussed in Section 4.4.1, with an 88% AV penetration rate (30 of 34 vehicles are AVs) to improve the model performance in mixed traffic.

<table>
<thead>
<tr>
<th>Constraints</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_{\text{min}}$</td>
<td>Minimum Control Input (acceleration)</td>
<td>$-5 \text{ m/s}^2$</td>
</tr>
<tr>
<td>$u_{\text{max}}$</td>
<td>Maximum Control Input (acceleration)</td>
<td>$5 \text{ m/s}^2$</td>
</tr>
<tr>
<td>$j_{\text{max}}$</td>
<td>Maximum Jerk</td>
<td>$1 \text{ m/s}^3$</td>
</tr>
<tr>
<td>$j_{\text{min}}$</td>
<td>Minimum Jerk</td>
<td>$-1 \text{ m/s}^3$</td>
</tr>
<tr>
<td>$s_0$</td>
<td>Stand Still Spacing</td>
<td>$1.5 \text{ m}$</td>
</tr>
<tr>
<td>$\tau_{\text{des}}$</td>
<td>Desired Time Headway</td>
<td>$1.1 \text{ sec}$</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>Costates Update Weight Factor</td>
<td>0.01</td>
</tr>
<tr>
<td>( t_f )</td>
<td>Prediction Horizon</td>
<td>5 sec</td>
</tr>
<tr>
<td>( \epsilon_{stop} )</td>
<td>Convergence Error</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Note: Last three constraints are for MPC-based models only.

The performance of all models is evaluated by designed experiments from different perspectives, as summarized in Table 3. The Computational Time is the average calculation time for each vehicle's decision among the selected evaluation metrics. The Stability is evaluated from the evolutions of vehicle states, the congestion area measures the Congestion reduction, and the Average Velocity determines the Traffic Efficiency.

### Table 3. Experiment Summary

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Settings</th>
<th>Simulation Platform</th>
<th>Evaluation Metrics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Platoon</td>
<td>a fleet of vehicles on one-lane road</td>
<td>MATLAB</td>
<td>Computational Time, Stability</td>
</tr>
<tr>
<td>Ring-Road</td>
<td>a fleet of vehicles on one-lane ring road</td>
<td>MATLAB</td>
<td>Stability, Congestion Reduction, Shockwave Damping, Traffic Efficiency</td>
</tr>
<tr>
<td>Corridor</td>
<td>Real-world highway with real traffic count</td>
<td>VISSIM</td>
<td>Stability under Cut-in Movements, Congestion Reduction, Shockwave Damping, Traffic Efficiency</td>
</tr>
</tbody>
</table>

The proposed models are compared with the selective models, as summarized in Table 4.

### Table 4. Selected Reference Models for Comparison

<table>
<thead>
<tr>
<th>Model Name</th>
<th>Model Type</th>
<th>Equation Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed Models</td>
<td>MPC</td>
<td>/</td>
</tr>
<tr>
<td>IDM</td>
<td>Nonlinear Car Following</td>
<td>(2.31)</td>
</tr>
<tr>
<td>CTH</td>
<td>MPC</td>
<td>(2.32)</td>
</tr>
<tr>
<td>VTH</td>
<td>MPC</td>
<td>(2.8),(2.9),(2.10)</td>
</tr>
<tr>
<td>PID CACC</td>
<td>PID Car Following</td>
<td>(2.16)</td>
</tr>
</tbody>
</table>

### 4.3 Model Validation

The proposed models need validation on their computational time and stability based on the stability analysis in Section 3.5. To validate the derived local and string stability conditions,
a platoon of 5 AVs controlled with the proposed models is tested in Matlab. A leading MV and five following AVs start at the uniform initial speed 20 m/s with spacing 42 m. The leading vehicle decelerates at 2.5 m/s² from 6 to 11 second and accelerate 2.5 m/s² from 31 to 36 second.

4.4 Model Evaluation

The proposed model needs evaluation on two aspects. First, the evaluation for proposed models in single-lane-ring-road mixed traffic should also be conducted to illustrate the traffic stability. Second, the proposed models need to be evaluated under a more complex environment with cut-in movements. Three experiments are developed accordingly, as discussed in the following.

4.4.1 Ring-Road Simulation Experiment

To better understand the interactions between AVs and MVs, the proposed models are evaluated in the ring-road simulation road tests. Because of the well-developed crash avoidance capability of the Gipps model, it is selected as the MV model in this paper. The Gipps model is calibrated based on the NGSIM data whose formulation is as follows (Gipps, 1981):

\[
v_i(t + \tau) = \min \left( v_i(t) + 2.5a_{\text{max}}\tau \left( 1 - \frac{v_i(t)}{V_{\text{des}}} \right) \sqrt{0.025 + \frac{v_i(t)}{V_{\text{des}}}}, \right.
\]

\[
\left. b_{\text{max}}\tau + \sqrt{b_{\text{max}}^2 \tau^2 - b_{\text{max}}^2 \left( 2[s_i(t) - L_{n-1} - s_{\text{safe}}] \right) - \frac{b_{\text{max}}^2 \tau^2 - b_{\text{max}}^2 \left( 2[s_i(t) - L_{n-1} - s_{\text{safe}}] \right)}{b_0}} \right)
\]

where \(\tau\) is the reaction time, representing the MV acceleration update time (1.1 seconds) in the simulation. \(L\) represents the vehicle length. \(a_{\text{max}}\) is the maximum acceleration. \(b_{\text{max}}\) is the most severe deceleration. \(s_{\text{safe}}\) is the safe distance, \(V_{\text{des}}\) is the average desired velocity, and
\( b_0 \) is the average expected deceleration of PV.

The ring-road simulation initiates the vehicle states with disturbance on a circular one-lane ring-road in which vehicles continue running in circular motions. Many researchers have used the ring road environment to calibrate and evaluate their models thanks to its advantages in modeling the traffic system (Cui, Seibold, Stern, & Work, 2017). Zheng et al. (2020) and J. Wang et al. (2020) established an optimal control model of mixed traffic in the ring road settings to smooth the traffic flow. Many existing studies have also used the ring-road numerical simulation to illustrate the dynamic evolution patterns of the initial perturbations to demonstrate the stability or instability of car-following, ACC, or CACC control models. In this experiment, the ring-road simulation uses the vehicle trajectory data on a southbound direction of U.S highway 101 (Hollywood Freeway) in Los Angeles, California, on June 15th, 2005, from the NGSIM project. The entire segment is around 2100 feet (0.61 km), with five through lanes and one auxiliary lane. First, the states of all 34 vehicles on Lane 1 at 8:30 am during peak hour congestion are used as the initial traffic states on the ring road to reflect realistic "perturbation patterns" to the acceleration, speed, and spacing for stability analysis. Second, the car-following model parameters calibrated based on the same dataset in a previous study (Jin et al., 2014) are used for modeling manually-driven vehicles.

As shown in Figure 15a, 34 vehicles are tested in the ring road length with a platoon of ACC/CACC vehicles and a platoon of MVs. Difference ACC/CACC market penetration rates can be tested by changing the number of vehicles in the AV platoon. The proposed
and other reference models are executed continuously for 800 intervals (80 seconds) to evaluate their shockwave patterns. The time step is defined as 0.1 seconds. The MVs update every 1.1 seconds accounting for human reaction time, while the ACC/CACC vehicles update every 0.3 seconds, accounting for sensing, controller, and communication time in the actual systems. A spatial-temporal diagram with color-coded vehicle trajectories is used to visualize the evolution of traffic patterns resulting from different models. Each trajectory point is color-coded by its speed. The concentration and color of vehicle trajectory points clearly illustrate the congestion patterns (see Figure 15b for a sample diagram). Four performance metrics are proposed, including Congestion Area (CA), Average Velocity ($\overline{V}$), Average Absolute Jerk (AAJ), and minimum Time To Collision $TTC_{min}$.

\[
CA = \sum_{i=1}^{K} (A_i|\nabla A_i < v_{cong})
\]

\[
\overline{V}(A) = \frac{d(A)}{t(A)}
\]

\[
AAJ = \frac{\sum_{A_i} |j|}{t(A)/t_0}
\]

\[
TTC_{min} = \min(TTC|TTC > 0)
\]

CA is a metric obtained by calculating the total spatial-temporal area in the congestion contour map for every 10-sec-by-100-ft (30.4m) grid $A_i$ with an average vehicle speed below $v_{cong}$ (set as 45km/h). $\overline{V}(A)$ is calculated by total travel distance (TTD) $d(A)$ over the total travel time $t(A) = N * T$. AAJ is the average of absolute jerk values among the total simulation steps $t(A)/t_0$. $TTC_{min}$ is a safety metrics calculated based on minimum positive $TTC = -s_i/\dot{s}_i$. Additionally, the traffic stability and shockwave propagation speed are other two significant criteria, where models with fewer stop-and-go waves can have better overall fuel efficiency, and slower shockwave propagation speed can reduce the
congestion area. These figures show how the proposed controls work with the designed optimal control problem.

![Image of ring-road experiment setup](image1)

![Image of ring-road vehicle trajectory](image2)

**Figure 15. Ring-road Experiment**

### 4.4.2 Corridor Simulation Experiment

The proposed models focus on the longitudinal control of vehicles. To evaluate its full effect on traffic flow with cut-in movements, ACC models need to be combined with lateral control models like lane-changing in a more realistic road geometric environment. In this experiment, such an integrated model was done in VISSIM. The proposed longitudinal control models are implemented on selected ACC vehicles, and VISSIM handles other aspects of vehicle control like lane changing and routing. The VISSIM model is built and calibrated based on field data collected between 7 am to 8 am on April 11th, 2012, on the IH-35 corridor in Austin, Texas. This corridor has heavy morning commute traffic coupled with heavy freight traffic in the U.S. The end segment has a combination of vertical and horizontal curves, which created a traffic bottleneck. The longitudinal control models are
deployed within a non-merging segment with a length of 845 meters (0.53 miles). As shown in Figure 16, the studied area starts from East Oltorf Street to Woodland Avenue. The congestion starts at 7:15 am and ends at 7:35 am. During this congestion, the speed slows down from 50 km/h to around 17 km/h, and the density rises from 91 vehicles/km to 244 vehicles/km.

![Image](image_url)

Figure 16. VISSIM Studied Area and Baseline Traffic Profile

The default car-following model used in VISSIM is modified through the External Driver Model (EDM) API to implement the proposed ACC control models. The simulation resolution is 0.1 second/step to provide the highest. The lane-changing operations of controlled vehicles are still controlled by the VISSIM model rather than proposed models, i.e., vehicle control is assumed to be deactivated during the lane change. To reduce the computational time of the proposed MPC framework, we shorten the processing time by limiting optimization iterations to 200 times for the inner FOR loop of Algorithm 1, i.e., optimization might be truncated, and a likely-local optimal solution will be used.

Table 5 lists all simulation settings according to empirical studies. Different penetration rates of controlled vehicles (from 5% to 30%) are simulated in VISSIM. The MATLAB-based calibrated model parameters will be used in VISSIM. Besides, the lane change of
controlled vehicles is controlled by the VISSIM model rather than the proposed model, i.e., vehicle control is assumed to be deactivated during the lane change.

Table 5. VISSIM Simulation Model Parameter Settings

<table>
<thead>
<tr>
<th>Desired Speed of Car</th>
<th>96.5 km/h (60mph)</th>
<th>Maximum Acceleration</th>
<th>3.5 m/s²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Desired Speed of HGV</td>
<td>80 km/h (50mph)</td>
<td>Maximum Deceleration</td>
<td>-7.5 m/s²</td>
</tr>
<tr>
<td>Relative Flow of Car</td>
<td>89.5%</td>
<td>Lane Width</td>
<td>3.7m</td>
</tr>
<tr>
<td>Relative Flow of HGV</td>
<td>10.5%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5 EXPERIMENT RESULTS

5.1 Model Calibration Results

Table 6 summarizes the ACC and CACC model parameters and their parameter settings used in three experiments, which are the calibration results discussed at the beginning of this section. Those parameter settings are carefully selected and satisfy the string stability conditions derived in previous sections.

Table 6. Experiments Parameter of Proposed Models

<table>
<thead>
<tr>
<th>Models</th>
<th>Parameters</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Catch-Up</td>
<td>( c_1 )</td>
<td>Safety Cost Weight</td>
<td>0.67</td>
</tr>
<tr>
<td></td>
<td>( c_2 )</td>
<td>Efficiency Cost Weight</td>
<td>0.23</td>
</tr>
<tr>
<td></td>
<td>( c_3 )</td>
<td>Driving Comfort Cost Weight</td>
<td>0.62</td>
</tr>
<tr>
<td></td>
<td>( v_0 )</td>
<td>Catch-up Speed</td>
<td>1.13 m/s</td>
</tr>
<tr>
<td></td>
<td>( s^- )</td>
<td>Lower spacing threshold of ( v_{add} )</td>
<td>6.87 m</td>
</tr>
<tr>
<td></td>
<td>( s^+ )</td>
<td>Upper spacing threshold of ( v_{add} )</td>
<td>28.44 m</td>
</tr>
<tr>
<td>Safe-Shock</td>
<td>( c_1 )</td>
<td>Safety Cost Weight</td>
<td>0.53</td>
</tr>
<tr>
<td></td>
<td>( c_2 )</td>
<td>Efficiency Cost Weight</td>
<td>1.20</td>
</tr>
<tr>
<td></td>
<td>( c_3 )</td>
<td>Driving Comfort Cost Weight</td>
<td>0.54</td>
</tr>
<tr>
<td></td>
<td>( \Delta v_{free} )</td>
<td>Speed Change Factor</td>
<td>10.5 m/s</td>
</tr>
<tr>
<td>Elasticity</td>
<td>( c_1 )</td>
<td>Safety Cost Weight</td>
<td>0.68</td>
</tr>
<tr>
<td></td>
<td>( c_2 )</td>
<td>Efficiency Cost Weight</td>
<td>0.23</td>
</tr>
<tr>
<td></td>
<td>( c_3 )</td>
<td>Driving Comfort Cost Weight</td>
<td>0.51</td>
</tr>
<tr>
<td></td>
<td>( \Delta v_{free} )</td>
<td>Speed Change Factor</td>
<td>14.2 m/s</td>
</tr>
<tr>
<td>Catch-Up-Jerk</td>
<td>( c_1 )</td>
<td>Safety Cost Weight</td>
<td>4.21</td>
</tr>
<tr>
<td></td>
<td>( c_2 )</td>
<td>Efficiency Cost Weight</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>( c_3 )</td>
<td>Driving Acceleration Cost Weight</td>
<td>0.30</td>
</tr>
<tr>
<td></td>
<td>( c_4 )</td>
<td>Driving Jerk Cost Weight</td>
<td>0.30</td>
</tr>
</tbody>
</table>
### Table 2

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu_0$ Catch-up Speed</td>
<td>1.50 m/s</td>
</tr>
<tr>
<td>$s^-$ Lower spacing threshold of $\nu_{add}$</td>
<td>1.35 m</td>
</tr>
<tr>
<td>$s^+$ Upper spacing threshold of $\nu_{add}$</td>
<td>27.65 m</td>
</tr>
<tr>
<td>$c_1$ Safety Cost Weight</td>
<td>7.50</td>
</tr>
<tr>
<td>$c_2$ Efficiency Cost Weight</td>
<td>0.14</td>
</tr>
<tr>
<td>$c_3$ Driving Acceleration Cost Weight</td>
<td>0.52</td>
</tr>
<tr>
<td>$c_4$ Driving Jerk Cost Weight</td>
<td>0.57</td>
</tr>
<tr>
<td>$\Delta v_{free}$ Speed Change Factor</td>
<td>11.2 m/s</td>
</tr>
</tbody>
</table>

Note: other parameters not calibrated are listed in Table 2.

### 5.2 Model Validation Results

The platoon simulation produces a total of 5000 car-following control inputs at the resolution of 0.1 seconds, i.e., 5000 executions of the proposed optimal control model for five controlled vehicles over 100 seconds simulation time. The computer has the processor Intel i7-6700K CPU @4.00GHz. The average computation times of all proposed models per car-following control signal are listed in Table 7, where the control decision is made every 0.1 seconds by each vehicle. Given that the average computational time is far less than the simulation time, the proposed models have fast enough computation time for real-world implementation. The Safe-Shock has the best overall performance. The Catch-Up-Jerk and Safe-Shock-Jerk models have much larger computation time as they involve the third-order jerk dynamic in predicting vehicle movements.

### Table 7. Computation Time Analysis

<table>
<thead>
<tr>
<th>Platoon</th>
<th>Model</th>
<th>Average Computation Time for each control decision (second)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACC</td>
<td>Catch-Up</td>
<td>$1.71 \times 10^{-3}$</td>
</tr>
<tr>
<td></td>
<td>Safe-Shock</td>
<td>$1.23 \times 10^{-3}$</td>
</tr>
<tr>
<td></td>
<td>Elasticity</td>
<td>$1.48 \times 10^{-3}$</td>
</tr>
<tr>
<td></td>
<td>Catch-Up-Jerk</td>
<td>$5.82 \times 10^{-3}$</td>
</tr>
<tr>
<td></td>
<td>Safe-Shock-Jerk</td>
<td>$4.59 \times 10^{-3}$</td>
</tr>
<tr>
<td>CACC</td>
<td>Catch-Up</td>
<td>$1.94 \times 10^{-3}$</td>
</tr>
<tr>
<td></td>
<td>Safe-Shock</td>
<td>$1.57 \times 10^{-3}$</td>
</tr>
<tr>
<td></td>
<td>Elasticity</td>
<td>$1.64 \times 10^{-3}$</td>
</tr>
<tr>
<td></td>
<td>Catch-Up-Jerk</td>
<td>$9.32 \times 10^{-3}$</td>
</tr>
</tbody>
</table>
Figure 17 to Figure 19 indicate the evolution of the acceleration, speed and spacing over time, where each line stands for a vehicle in a platoon.

**Figure 17. Acceleration Evolution of Vehicle Platoon**

Figure 17 compares the acceleration evolution of the proposed models in ACC and CACC platoon. CACC platoons have smoother and more stable acceleration evolution than ACC platoons. Overall, the acceleration evolution of all proposed models looks similar except
that. The Catch-Up model in the top-left figure shows sudden accelerations at the beginning. This sudden acceleration increase is consistent with the Catch-Up cost function design. The following vehicle can drive a bit faster than the desired speed to match the preceding vehicle speed to recover from congestion.

Similarly, the Catch-Up-Jerk model shows such a sudden acceleration increase at the beginning. The Safe-Shock-Jerk model creates the acceleration overshoot when trying to minimize the jerk magnitude, shown and discussed in Figure 19. CTH model creates some unnecessary disturbance when adapting to the leading vehicle’s acceleration. VTH model has a large deceleration at the beginning as the current spacing is much larger than its desired spacing, and it also needs more time to diminish the disturbance. IDM and PID model show string stable and low acceleration magnitude.
Figure 18 compares the speed profile of the proposed ACC or CACC models. The Catch-Up and Catch-Up-Jerk ACC models show overshooting patterns that lead to fluctuation amplification of vehicle speed. In a longer ACC platoon, the speed fluctuation might be amplified, and finally, the speed overshooting patterns will be constrained by Equations (3.5) and (3.7) when ensuring safety and comfort. And all proposed models show better string stability in the CACC platoon.
Similar to the acceleration and speed profiles, Figure 19 shows the jerk profile. All five models show similar jerk magnitudes under a comfort level. In the CACC platoon, the jerk magnitude is further reduced, contributed by the PV’s acceleration information through wireless communication. Same as what is discussed in Figure 17, the Catch-Up model wants to catch its PVs, providing a large yet still tolerable jerk at the beginning. And the
Catch-Up ACC model on the top-left also indicates that the Efficiency cost term makes the controlled vehicles have jerk overshoots when vehicles are accelerating. It is worth noting that the Safe-Shock-Jerk model successfully minimizes the jerk magnitude compared with their original model Catch-Up and Safe-Shock.

5.3 **Ring-Road Experiment Results**

5.3.1 ACC Model Results

The ring-road simulation results are evaluated by using the color-coded space-time diagrams. Each point in the diagram is the trajectory point on the ring road. The color of each point represents the traffic condition based on the vehicle's speed. Blue to red points indicates the free-flow to congested traffic conditions. The diagrams represent the other four models with 30 AVs in a 34-vehicle platoon. The black line indicates the first and the last vehicle in the AV platoon. Purple-dashed lines outline the shockwave propagation speed measured based on the points with the same speed when vehicles are entering the congestion. The slope of the purple-dashed line determines the shockwave damping performance, which is indicated by the number along the purple line. A higher slope value indicates slower shockwave propagation speed in backward-propagating congestion shockwaves and faster recovery shockwave speed in accelerating queue discharging flow.
Figure 20. ACC Models: NGSIM data with 30-Vehicle Platoon

Figure 20 shows the all tested models’ ring-road simulation results with initial vehicle states set to match NGSIM US101 lane 1 data at 8:30 am, June 15, 2005. The initial field vehicle states have large spacing and speed variations, reflecting the “stop-and-go” congestion patterns. Overall, the proposed Catch-Up model shows better capabilities of damping the shockwave than all the other models by dampening the backward propagation speed of the congestion shockwaves. Furthermore, the proposed models can reduce the oscillations and stabilize the traffic flow within congested or free-flow conditions, unlike the results from IDM. The VTH model significantly reduces the CA to 130.76 km*sec and provides a higher Average Velocity of 43.8 km/h thanks to its ability to change the time headway based on current spacing. The IDM model produces the traffic flow stability and generates multiple turbulence bands across the space-time diagram in response to the initial perturbations. In other words, IDM is unstable and did not manage to dissipate...
perturbations caused by stop-and-go waves. The CTH model mitigates the CA to 149.05 km*sec and slows the shockwave propagation speed to -24.57 km/h. It is worth noting that the proposed Catch-Up model significantly dampens the shockwave propagation speed yet sacrifices the Average Velocity compared with other models.

The Elasticity model has a similar performance on shockwave damping compared with the Safe-Shock model as they share similar objective functions. The Catch-Up-Jerk and Safe-Shock-Jerk Model involve the jerk in the objective function and acceleration in the costate to ensure the driver’s comfort. The Catch-Up-Jerk model shows worse CA and Average Velocity than the Catch-Up Model, indicating that the evolvement of jerk reduces traffic efficiency. The Safe-Shock-Jerk model shows smoother speed changes at the (20 seconds, 0.5 km) location, providing a comfortable diving experience.
Figure 21. ACC Models: Congestion Area (km*sec), Average Velocity (km/h), Average Absolute Jerk (m/s^3), minimum TTC (sec)

Figure 21 compares the CA, Average Velocity, AAJ, and minimum TTC performance of three reference models and five proposed models. The horizontal axis is the number of AVs among 34 vehicles in the ring-road, indicating the AV penetration rates. Overall, the Safe-Shock model shows the best overall performance with similar average speed improvement and much smaller congested area, compared with other models, especially in high penetration situations. Although VTH shows higher Average Velocity and lower AAJ at low penetration rates, it is achieved by driving at higher minimum TTC, producing worse safety performance. The CTH model also produces lower AAJ, yet it has a larger CA and slower Average Velocity. The Catch-Up, Catch-Up-Jerk, CTH, and VTH show larger minimum TTC, meaning better safety performance generated. In general, The five proposed models have better performances. Due to the lack of coordination, ACC models still cannot fully eliminate the stop-and-go waves. Still, proposed models can slow down the propagation of hysteresis upstream significantly.

5.3.2 CACC Model Results

The following speed contour figures show the spatial-temporal speed patterns of the proposed CACC model with topology shown in Figure 5. The reference algorithms include VTH, CTH, and PID models. 30 CACC-vehicle are selected in the 34-vehicle platoon.
Figure 22. CACC Models: NGSIM Data with 30-Vehicle Platoon

Figure 22 illustrates that the shockwave propagation speed can be significantly reduced with the proposed Catch-Up-Jerk CACC Model and outperform the other models. The Safe-Shock and Elasticity model show similar patterns, with a CA of 147.52 km*sec and an Average Velocity of 38.1 km/h. Interestingly, due to its instability, the PID CACC model (Shladover et al., 2001) has some minor shockwave bands (congestions propagate upstream quickly). It indicates that the platoon coordination can further improve the congestion mitigation effect of proposed models.

The VTH model significantly reduces the CA to 126.80 km*sec and provides a higher Average Velocity at 44.3 km/h. The CTH model mitigates the CA to 149.05 km*sec and slows the shockwave propagation speed to -25.31 km/h.
The Catch-Up-Jerk model shows a much better performance than all other models, indicating that the additional jerk consideration improves the traffic efficiency with the wireless communication obtaining the PV’s acceleration. The Safe-Shock-Jerk model slows down the shockwave propagation speed yet sacrifices its Average Velocity compared with the Safe-Shock model.

Figure 23 shows that five proposed CACC models have better congestion reduction performance as the penetration rate increases. In the 100% penetration rate, the Catch-Up-Jerk model only produces a Congestion Area of 109.4 km*sec, while PID produces 149.00 km*sec, and CTH produces 145.70 km*sec. The Average Velocity results indicate that all proposed models are better than reference models. The AAJ of VTH is the lowest, but it produces lower speed and larger CA. The AAJ of CTH and PID is not reducing even when
AV penetration goes up because they lack jerk-related constraints or produce unstable traffic. The Catch-Up-Jerk CACC shows worse minimum TTC than compared with Catch-Up-Jerk ACC due to its nature that the CACC platoon provides a smoother adaptation trajectory, as discussed in the platoon simulation results.

5.3.3 Jerk Results

This section discusses the jerk distribution and patterns in the Ring-Road simulation.

Figure 24. Jerk Evolution of ACC Models: NGSIM Data with 30-Vehicle Platoon
Figure 24 and Figure 25 show examples of jerk evolutions in ring road, similar to Figure 15b), where each trajectory point is color-coded by its jerk. Purple and red points represent the large acceleration and deceleration jerks, respectively. The jerk magnitude increases when vehicles reach or leave congestions. As indicated in Catch-Up Model, the first vehicle in the AV platoon decelerates at 2 seconds at 0.02 km when reaching its desired spacing. In some cases, this could force the AV to break the comfort constraints to ensure the driver’s safety.

5.4 Corridor Simulation Results

Figure 26 shows the resulting speed contour maps from VISSIM-based sensitivity analysis on the performance of the first three proposed models for ACC vehicles. Results from four different penetration rates, 10%, 30%, and 50%, are evaluated. Each space-time grid in the
speed contour map is color-coded by the Average Velocity within the grid, with red color indicating congestion and blue color indicating free flow.
Figure 26. Corridor Simulation Results of ACC
The upper-left diagram is the baseline result with all human-driven vehicles and shows a Congested Area (CA) of 0.142 km\(\times\)hour and an Average Speed of 50.4 km/h. The first two columns are reference models CTH and VTH under the MPC framework. The last five columns are proposed five models. Three rows are different penetration rates, up to the bottom, from 10% to 50%. All evaluated models show potentials for congestion mitigation and shockwave damping, improving performance as penetration rates increase. The Average Flow Rate is calculated among all road segments over the simulation time. The traffic flow is directly related to traffic input from the beginning of the simulation network such that the average flow rates are close to each other.

With a 10% ACC penetration rate, both Safe-Shock and Elasticity models reduce the CA to 0.058 km\(\times\)hour. The Safe-Shock model increases the Average Speed to 52.0 km/h while the Elasticity model increases the Average Speed to 51.9 km/h. The proposed Catch-Up model is slightly worse, and only reduces the CA to 0.083 km\(\times\)hour and improves the Average Speed to 52.0 km/h. The Catch-Up-Jerk model generates smaller CA than the Catch-Up model yet still higher than other proposed models. The Safe-Shock-Jerk model provides the lowest CA at 0.050 km\(\times\)hour, with the highest Avg Velocity at 52.5 km/h.

Under 30% penetration rate, the Safe-Shock-Jerk model improves the Average Speed to 53.7 km/h with a much smaller Congestion Area (0.017 km\(\times\)hour) than the baseline and other proposed models. The Safe-Shock-Jerk model also restricted the backward congestion propagation to only 800 m instead of 600 m in the baseline. This illustrates that the Safe-Shock-Jerk model successfully slows down the congestion propagating backward. This slowdown is contributed by the Efficiency term in Equation (3.28)(3.10). The medium to high ACC penetration rate creates some ACC vehicle platoons with sufficient spacing to help stabilize and speed up the vehicle flow. The Safe-Shock model keeps the CA at 0.058
km\(^{-1}\) hour, while the Elasticity model slightly reduces the CA to 0.050 km\(^{-1}\) hour. The Average Speed of the Safe-Shock model increases to 52.2 km/h, which is better than the Average Speed of 52.5 km/h with the Elasticity model.

Starting from the 50\% penetration rate, all models except VTH successfully remove all congestions and bring the average speed to around 56 km/h. The VISSIM simulation results further illustrated the superior performance of the proposed models on congestion reduction and shockwave damping. The Safe-Shock-Jerk model shows the best performance among all models.

6 CONCLUSION

ADAS systems are already implemented on many vehicle models, and the penetration rate of ADAS systems is increasing gradually. However, the existing ADAS systems on the market mostly aim at improving drivers’ safety and comfort. Instead, the models proposed in this study aim at balancing the possible conflicts between drivers’ comfort and road capacity. This study proposes an optimal control structure to dampen the negative effect of shockwaves induced in traffic and mitigate congestion. Proposed models use Model Predictive Control to balance multiple objectives, including safety, efficiency, and driver’s comfort. Five models with different combinations of cost terms are proposed and evaluated.

The proposed models are tested through the platoon, ring-road, and corridor simulation to investigate their stability, congestion mitigation, shockwave damping and safety performance. The calibrated Gipps model is used as the Manual Vehicles’ (MVs) model in the ring-road simulation. Four widely used control models are implemented under the same simulation environment with different AV vehicle fleet sizes to evaluate the proposed models’ performance. The results show that AV platoons implemented with proposed
models perform better on both shockwave damping and congestion mitigation than the other four models, especially when congestion is already formed. Moreover, the proposed model can keep string stability by ensuring the decaying of the spacing and velocity perturbations. On the other hand, the VISSIM simulation model uses the traffic flow data from the morning-peak period on the I-35 corridor in Austin, TX. The VISSIM results show the superior performance of the proposed Safe-Shock-Jerk model in reducing congestion and damping shockwaves than selected reference control models.

One limitation of this paper is that proposed models are only analyzed based on simulation with some assumptions. For instance, it is assumed that all vehicles in the network will maintain manual or autonomous control without switching between AV and MV modes. In reality, drivers may change the mode at any time. Meanwhile, the proposed model assumes zero calculation time, actuator delay or communication delay, which should be compensated through estimation in future works.

Proposed models have potentials to be implemented in realworld for Level 2 or 3 Autonomous Vehicle with an additional control layer to let the vehicle follow the optimal acceleration provided by proposed models, with the throttle and brake actuator considered. Additionally, a mobile application providing recomanded speed or alerts can also be developed to make the proposed control models available to human-control vehicles.

Future work should also focus on following three directions. First, proposed models can be improved to accommodate other road scenarios like arterials with signalized intersections. Second, other V2V communication topologies, such as PLF, should be considered with the communication delay involved. Last, vehicle-to-infrastructure (V2I) communication is another interesting topic to cover as it obtain the traffic congestion from far downstream
such that the control vehicles can react to potential congestion in early stage. The cost function can involve macroscopic traffic values such as traffic flow or density to improve the network performance directly.
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