EFFECTS OF CHUNKING INTERVENTION ON ENHANCING GEOMETRY PERFORMANCE IN HIGH SCHOOL STUDENTS WITH MATHEMATICS LEARNING DIFFICULTIES

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ABSTRACT OF THE DISSERTATION

Effects of Chunking Intervention on Enhancing Geometry Performance in High School Students with Mathematics Learning Difficulties

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Geometry education is an important aspect of STEM education and career development, but it is often overlooked in K-12 education in the United States. Although there is some research on teaching geometry to students with learning difficulties at the elementary level, there is a lack of research on teaching advanced geometry skills at high school levels. Chunking is a strategy that can help reduce the cognitive load demanded in the processing of information, and it has been applied as a testing accommodation for high school students with math learning difficulties in geometry assessments. This study aims to expand upon the existing literature by examining the effect of chunking as an intervention, rather than just a testing accommodation, on the geometry performance of high school students with math learning difficulties.

The study used a multiple probe design across participants and address the following research questions: (a) To what extent will the chunking intervention improve the geometry performance of high school students with math learning difficulties as measured by a proximal measure (i.e., geometry problem solving...
probes related to lines and angles)? (b) To what extent will the effects of chunking intervention maintain two weeks after the conclusion of the intervention as measured by the proximal measure? (c) To what extent will the students be able to generalize their geometry knowledge to distal measures, such as geometry proof related to lines and angles, sample items from the NJSILA high school test preparation for geometry, and the KeyMath-3 geometry subtest? (d) What are the perspectives of the high school students with mathematics learning difficulties about the chunking intervention?

Three high school students with difficulties in learning geometry participated in this study during an in-school intervention program at a high school in New Jersey. The intervention was implemented by the interventionist and a trained undergraduate research assistant, and consisted of 6 sessions, divided into three units: (a) a unit on angles and related properties, (b) a unit on parallel lines (c) and a unit about perpendicular lines. The entire intervention, including the baseline and post-test phases, lasted approximately two months, depending on the participants' availability and their sequences of introduction into the intervention. The treatment's fidelity was monitored and reported, and students' perspectives on the chunking intervention was evaluated through a social validity measure at the conclusion of the intervention. The outcomes of visual analysis indicated that participants improved their performance during the intervention phase as compared to the baseline phase in the geometry problem-solving test's proximal measure. Compared to their corresponding scores in the baseline, most of the participants’ scores in the posttest showed significant enhancements on the transfer measures. Participants who underwent the maintenance tests sustained elevated performance levels even two weeks after the conclusion of the intervention. Individual differences in students’ responding to the intervention were
discussed with regard to their pre-requisite skills and conceptual knowledge of the geometry concepts. The researcher then discussed the limitations, contributions to the literature, implications for practice, and future research in teaching geometry, and recommended that chunking strategy should be integrated with, rather than replacing, concept instruction.
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Chapter 1. Introduction

There is a constant need to innovate and improve educational approaches, especially in STEM fields, to enhance learning, drive innovation, and stimulate economic growth (Atkinson, 2012; Naidoo & Kapofu, 2020; Namkung & Bricko, 2021; Stevens et al., 2018). Mathematics education, being a crucial aspect of human life, deserves special attention. Regardless of the schooling level, mathematics is a mandatory subject that students must study throughout their school years in every country (Clarke et al. 2014; Febriyanti et al., 2021; Fischer et al., 2013; Jitendra et al., 2018); and hence many countries allocate significant financial resources to improve mathematics education (Gomides et al. 2021). A strong performance in mathematics is positively associated with success in various areas of life and has a lasting impact on students’ future decisions and overall quality of life (Fuchs et al., 2017; Geary et al. 2020; Jitendra et al., 2021; Lei et al., 2020; Morgan et al., 2009; Namkung & Bricko, 2021). Improving instructional quality and student learning outcomes in mathematics is, therefore, of utmost importance (Stevens et al., 2018).

Despite the efforts made to improve mathematics teaching and learning, mathematics difficulties are prevailing among school learners in the U.S. (Jordan & Levine, 2009; Mulcahy et al., 2016). Approximately 5-10% of students in all elementary or secondary schools experience difficulties in learning mathematics (Schindler et al., 2019). The National Assessment of Educational Progress (NAEP), also known as the nation's report card, reveals that many secondary students struggle with mathematics and perform below the basic level (Jitendra et al., 2021; Myers et al., 2021). In the NAEP 2019, 40% of students scored below the basic level; 4th and 8th graders who scored between the 10th and 25th percentile in NAEP 2019 performed similarly to or worse than their peers in NAEP 1990, indicating no
progress for these lower-performing students over the past 20 years. Furthermore, the percentage of students who scored at or above the proficient level in mathematics performance in the 4th and 8th grades has remained unchanged since 2017 (US NCES, n.d). These results suggest that there is a pressing need to improve mathematics performance among U.S. students.

In this study, SWMLDs refer to students who are failing or at risk of failing in mathematics. These students include, but are not limited to, those who have been officially diagnosed with a learning disability (Nelson et al., 2022). The Individuals with Disabilities Education Act (IDEA, 2004) in the U.S. ensures that students with learning disabilities receive appropriate special educational services. According to IDEA (2004), a specific learning disability is defined as a deficit that impairs a student's ability to speak, spell, read, write, or calculate, due to a variety of reasons (U.S. Department of Education, n.d). Unlike SWMLDs and average-achieving students, students with diagnosed mathematics learning disabilities have disorders or malfunctions in one or more brain operations, such as dyscalculia or dyslexia (Geary et al., 2007). This means that students with learning disabilities, such as dyslexia or dyscalculia, have learning difficulties by definition, but not all students with learning difficulties (i.e., low achievers) are classified as having a learning disability.

It is crucial to address the issue of low mathematics performance among SWMLDs, as it is a national priority to improve the mathematics achievement of U.S. students and provide high-quality mathematics instruction. This is particularly important for SWMLDs, as it is expected to contribute to better future and career opportunities for this group of struggling learners. To achieve this goal, it is necessary to evaluate the effectiveness of existing interventions, procedures, and tools, and to develop innovative solutions that are in line with the latest technological advances and
changing educational requirements (Bokosmaty et al., 2015; Bülbül & Güler, 2021; Geary et al., 2020; Losinski et al., 2019).

**Geometry Learning for SWMLDs**

Geometry is an important domain in the field of mathematics (Ayan & Isiksal Bostan, 2016; Chen et al. 2021; Choo et al., 2021; Giofrè et al., 2014). In many education systems, geometry is introduced as early as elementary school and continues to be taught in middle and high schools. It is essential that students receive proper instruction in geometry so that they can build a strong foundation in the subject and be prepared for future studies in STEM fields (Bailey et al., 2014). Moreover, students who struggle with geometry may develop a negative attitude towards mathematics in general, which may limit their future career choices (Naidoo & Kapofu, 2020).

However, despite the importance of geometry and its real-world applications, students, particularly SWMLDs, struggle in learning the subject (Naidoo & Kapofu, 2020). The poor performance of U.S. students in geometry in international assessments such as TIMSS 2011 highlights the need for effective intervention programs to support SWMLDs in learning geometry (Chen et al., 2021). This low performance is not only when comparing the U.S. students’ scores in geometry with other mathematical domains (e.g., algebra, numeracy, probability), but also compared with other nations (Bailey et al., 2014; Zhang et al., 2021).

Although geometry and teaching geometry have gained attention seeking improving students' geometry performance especially among SWMLDs, the cognitive mechanisms underlying students’ understanding of geometrical concepts has not received sufficient attention from researchers and educational policy makers (Geary, 2005; Giofrè et al., 2014).
Geometry and Working Memory

Working memory is also crucial in understanding and retaining geometric concepts, such as spatial relationships and transformations (Baddeley & Hitch, 1974). Working memory plays a role in learning mathematics in a variety of ways (e.g., numerical, verbal, visuospatial, linguistic) and significantly contributes to students’ mathematical performance (de Mooij et al. 2020; Menon, 2016). In particular, students’ performance in geometry correlates with working memory (Giofrè et al., 2014), including both the visual spatial and verbal working memory (Hawes et al., 2017; Pittalis & Christou, 2010; Schoevers et al., 2020). For learning geometry, students need to store information of specific problems in working memory, retrieve related knowledge in the long-term memory, and process reasoning to integrate information held in working memory and knowledge retrieved from long-term memory. Giofrè, et al. (2013) found that students’ geometry achievement was directly influenced by a complex visual working memory task (i.e., the jigsaw puzzle task), which is not about rote memorization and manipulation of visual images but about the analysis of semantic relations between the puzzle chips (i.e., how to make a watering can with four puzzle chips). Students with lower working memory capacities may struggle in retaining multiple geometric representations and the relationship between them, leading to difficulties in solving geometry problems.

Evidence Based Interventions on Geometry

Evidence based interventions are particularly important for SWMLDs (Kucukalkan et al., 2019). For example, Fuchs et al. (2017) reported that SWMLDs, who initially had low academic performance in early grade level mathematics, succeeded in the later advanced mathematics when evidence-based interventions were implemented appropriately. Improving academic performance of SWMLDs should be
a collective effort made by teachers, parents, schools, districts, and communities (Lein et al., 2020). Multiple meta-analyses examined interventions applied on SWMLDs (e.g., Fischer et al., 2013; Kroesbergen & Van Luit 2003; Lein et al. 2020; Losinski et al. 2019; Jitendra et al. 2018; Methe et al. 2012; Myers et al. 2021; Nguyen et al. 2008; Stevens et al. 2018) and identified beneficial instructional components in enhancing mathematics performance for SWMLDs. The effective instructional components include structured instruction, heuristics, student verbalization, visuals, sequence/range, teacher feedback, teacher feedback plus options, student feedback (and goal setting), cross-age tutoring, and peer-assisted learning within a class.

A major criticism of existing special education interventions for students with learning disabilities or SWMLDs is its historical roots in behaviorism, clinical-based, and a lack of a solid theoretical foundation (Gersten et al., 2009). Researchers call for conducting theory-based interventions that are based on solid educational or psychological theories that have been empirically validated (Michie & Prestwich, 2010). Emerging new educational theories may also inspire educators to re-consider educational practices from a new perspective (Clarke et al., 2014; Dole, 2003; Geary, 2005). Unfortunately, there is a chronic gap between the fast development of psychological lab research and the slow progress in applying these theories to educational practices. This study will develop and examine the effectiveness of an intervention that is based on a solid, well-validated cognitive theory.

Existing special education interventions for SWMLDs on geometry face the same criticism. Firstly, very sparse research existed as to how to teach geometry to SWMLDs. Liu et al., (2021) found only nine published studies or dissertations about geometry interventions to SWMLDs. Secondly, like many special education interventions, these studies lack a solid theoretical foundation. Out of the nine
interventions reviewed in Liu et al., (2021), four incorporated instructional strategies such as direct modeling (Cass et al., 2003; Kozulin & Kazaz, 2017), schema-based instruction (Xin & Hoard, 2020) or manipulative/visual representations (Cass et al., 2003; Kozulin & Kazaz, 2017; Strickland & Maccini, 2013). Four studies (Cihak & Bowlin, 2009; Satsangi et al., 2019; Horner, 1984; Satsangi & Bouck, 2015) successfully delivered geometry instruction through video modeling or computer programs such as virtual manipulatives and LOGO. Lastly, current research on geometry problem solving for SWMLDs has primarily focused on elementary level subjects, such as area, perimeter, and basic spatial imagination skills (Bergstrong & Zhang, 2016), but little research exists on how to help SWMLDs learn advanced geometry at high school levels, when geometry standards become more complex and in-depth.

**Chunking Strategy**

Chunking is a mental strategy that students can use to enhance their learning by leveraging their working memory capacity (Chase & Simon, 1973; Gobet & Simon, 1998). This study aims to develop and evaluate the effectiveness of a chunking intervention for students with SWMLDs in solving high school geometry problems. A chunk is a unit with a conceptual and organized structure, formed by grouping different elements (Chase & Simon, 1973; Isbilen et al., 2020; Thalmann et al., 2019). Each chunk has its own conceptual meaning and relational structure (Chase & Simon, 1973). For instance, a phone number might have chunks that represent the international and regional codes, whereas the word "psychopathology" might have chunks for psychology, pathogens, and -ology (Fonollosa et al., 2015). Chunking helps humans process and integrate information to form larger, relevant pieces (Bor & Seth, 2012; Rabinovich et al., 2014; Thalmann et al., 2019). It is an important
mechanism in cognitive learning (e.g., Cheng & Obaidellah, 2009; Gobet, 2005; Karatzias et al., 2016; Neumann & Kopcha, 2018; Obaidellah & Cheng, 2015; Perlman et al., 2010; Schlaghecken et al., 2000) and can help reduce the load on working memory, which has limited storage capacity (Baddeley, 1986; Chase & Simon, 1973; Miller, 1956; Patterson et al., 2014; Thalmann et al., 2019).

Previous research (Zhang et al., 2012, 2014, 2015, 2021) has shown that highlighting visual chunks is effective in helping elementary and high school SWMLDs to solve geometry problems. Visual schematic chunking is a representation accommodation that presents students with geometry diagrams, in which interactive elements that belong to a schematic chunk are colored with the same color to increase perceptual proximity and facilitate both visual and cognitive integration (Ratwani et al., 2008). This helps students recognize the schematic chunks involved in the geometry problems (Ratwani et al., 2008).

In a single-subject design study, Zhang et al. (2012) investigated the effectiveness of visual chunking in solving intuitive geometry problems for four elementary SWMLDs. They found that the participants demonstrated higher accuracy when the problems were represented with visual chunking representations rather than traditional element representations. Another group study (Zhang et al., 2014) compared the performance of students with and without math difficulties when using visual chunking representation and found that students with geometry difficulties made greater improvements than those without math difficulties, suggesting that the visual chunking representation technique is an effective accommodation for SWMLDs. Zhang et al. (2015) conducted a single-subject design study with SWMLDs to explore the possibility of improving college SWMLD's performance in solving sophisticated geometry problems by connecting visual chunking to the long-
term memory of the SWMLDs. Results showed that all participants improved their accuracy from baseline to posttest.

Zhang et al. (2021) further expanded upon prior research on chunking accommodations for SWMLDs to high school students. They demonstrated that visual-schematic-chunks can effectively improve students’ performance in solving high school complex geometry problems. The study targeted high school geometry problems that required deductive reasoning and the use of theorems, postulates, and properties, and involved sophisticated visual patterns. Results suggested that highlighting the visual patterns or chunks helped SWMLDs to better solve high school geometry problems, especially multi-step problems that involved multiple chunks. To illustrate this, the researchers provided in Figure 1 two examples that contrasted a plain diagram with a diagram that used the chunking accommodation. With the chunking accommodation, SWMLDs could easily recognize the two chunks from other elements.
However, the question remains, if chunking accommodations are not allowed in certain testing conditions, how can SWMLDs help themselves to identify the semantic chunks embedded in complex visual elements in geometric diagrams. This study aims to extend prior research on chunking from testing accommodations to interventions, focusing on helping high school SWMLDs to solve advanced geometry problems by self-identifying semantic visual chunks, particularly those involving multiple chunks.

**Problem Statement**

This study aims to investigate the effectiveness of using chunking as a cognitive intervention to enhance learning geometry among high school students with mathematical learning difficulties (SWMLDs). The hypothesis is that this chunking
intervention will improve the performance of these high school SWMLDs in learning geometry. To evaluate the students' performance, multiple assessment measures will be used, including a proximal measure of students' geometry problem-solving abilities as the main criteria test, and multiple distal measures as generalization tests. These distal measures include (a) a geometry proof probe on related subject matter, (b) selected items from the KeyMath-3 geometry subtest on related subject matter, and (c) selected items from the NJ LSA on the related subject matter. These distal measures will be used to examine whether the SWMLDs will be able to generalize the strategies they have learned to a wider range of problem-solving.

Research Questions

This study aimed to answer a main question:

What is the effectiveness of the chunking strategy as a cognitive intervention on geometry performance in high school SWMLDs?

Sub-questions:

1. What effect will a chunking intervention have on the geometry problem solving performance of high school SWMLDs as measured by a proximal measure (i.e., a geometry problem solving curriculum-based measurement).

2. To what extent do high school SWMLDs maintain their geometry performance two weeks after the conclusion of the intervention as measured by a proximal measure on geometry problem solving (i.e., the geometry problem solving CBM)?

3. To what extent do high school SWMLDs generalize their geometry knowledge to distal measures
a. To what extent do high school SWMLDs generalize the effectiveness of the chunking intervention to their geometry performance as measured with a geometry proof on the same content topics,

b. To what extent will high school SWMLDs generalize the effectiveness of the chunking intervention to their geometry performance on selected items from the state standard test (NJSLA)?

c. To what extent do high school SWMLDs demonstrate generalization of their geometry knowledge to selected items from the KeyMath-3 geometry subtest?

4. What are the attitudes of high school SWMLDs towards implementing the chunking intervention?

**Summary of the Chapter**

Chapter 1 emphasizes the necessity of experimenting with new and improved teaching methods, particularly in the subjects of science and math, and highlights the importance of mastering math skills. It expresses concern about the challenges faced by many U.S. students, especially those labelled as SWMLDs, in their journey to comprehend math. The chapter urges swift action to enhance math proficiency, especially when it comes to understanding geometry.

Furthermore, the chapter explores the common hurdles encountered by U.S. students in math and delves into ongoing initiatives aimed at enhancing math education. It introduces the concept of SWMLDs, encompassing students at risk of struggling with math, including those with learning disabilities. The central theme underscores the essential need for providing support to these students for their future success.
Shifting the focus to geometry, the chapter elucidates its significance in both understanding mathematical concepts and applying them in real-world situations. It points out the challenges, particularly for SWMLDs, in grasping geometric ideas and underscores the crucial role of memory in this learning process. The chapter advocates for a thoughtful consideration of cognitive mechanisms when teaching geometry.

Various approaches to assist SWMLDs are introduced in the chapter, advocating for methods grounded in educational or psychological principles. It discusses the drawbacks of existing special education methods and emphasizes the necessity for approaches aligned with cognitive theories.

Subsequently, the document delves into the "chunking" strategy as a means to assist SWMLDs in tackling high school geometry problems. It defines chunking as a mental strategy utilizing memory and discusses its potential to alleviate mental strain. Previous research on visual schematic chunking as a beneficial technique for SWMLDs is highlighted.

The chapter concludes by presenting the problem statement, research questions, and objectives of the study. The primary aim is to evaluate the effectiveness of the chunking strategy as a tool to help high school SWMLDs learn geometry. The research questions explore the impact of the chunking strategy on geometry problem-solving, its persistence over time, and its applicability to different measures. Additionally, the chapter underscores the importance of assessing the attitudes of high school SWMLDs toward implementing the chunking strategy in this study.
Chapter 2. Literature Review & Theoretical Framework

This chapter reviews the literature on the use of research-based geometry interventions for students with mathematics learning difficulties (SWMLDs). It begins by reviewing the terms used to describe students who have difficulties or disabilities in learning mathematics, as well as the characteristics of these students. Then it focuses on the specific subject of geometry, including the unique difficulties that SWMLDs face in learning this subject and the factors that contribute to their struggles. Finally, it reviews the literature on interventions for students with SWMLDs, with a particular focus on interventions for teaching geometry to these students.

Following the literature review section, the theoretical framework will be introduced. This theoretical framework describes the cognitive load theory and how chunking can help reduce cognitive load in cognitive processing. It provides examples of studies that have successfully applied chunking to enhance learning in different domains, such as language learning and STEM education. Finally, it reviews prior research on applying chunking as a testing accommodation strategy for geometry problem solving in SWMLDs and emphasize the importance of extending this strategy from a testing accommodation to an intervention technique.

Literature Review

Students with Mathematics Learning Difficulties

Mathematics is a subject that requires complex algorithms, deductive reasoning, inductive inference, and abstract analysis based on sophisticated theories. Additionally, it involves non-theoretical activities such as applying mathematical theories to real-life situations, taking into consideration the connected laws in fields like physics, biology, and chemistry (Lai et al., 2015). Mathematics can be
challenging, causing anxieties and frustration in students, parents, and teachers who are unprepared to teach it effectively (Hraste et al., 2018). Given the high reasoning skills and required theoretical complexity, mathematics is often rated as the most disliked subject compared to others (Febriyanti et al., 2021).

Nowadays, almost every classroom in schools includes students with difficulties in learning mathematics, known as Students with Mathematics Learning Difficulties (SWMLDs). According to studies by Febriyanti et al. (2021), Kroesbergen & Van Luit (2003), and Wijaya et al. (2019), these students have IQ scores similar to non-SWMLDs. However, approximately 6% of students in schools have inadequate performance in mathematics, and this percentage may vary based on the strictness of the classification criteria used. If less strict criteria are applied, more students may be considered SWMLDs (Bartelet et al., 2014; Swanson et al., 2018).

There are several terms used by researchers and educators to define, characterize, or identify SWMLDs in the literature (Pappas et al., 2019). In recent years, there has been a shift in special education from a focus on students diagnosed with mathematics learning disabilities to students with mathematics learning difficulties. The Individuals with Disabilities Education Act of 2004 (IDEA) defines learning disabilities as "a disorder in one or more of the basic psychological processes involved in understanding or in using language, spoken or written; the disorder may manifest itself in an imperfect ability to listen, think, speak, read, write, spell, or do mathematical calculations". IDEA (2004) also mandates the provision of free and appropriate public education to students with learning disabilities through an Individualized Educational Program (IEP). However, research that only focuses on students with diagnosed learning disabilities has been criticized for not providing
early enough prevention and for viewing students' difficulties as a deficit (Greenstein & Zhang, 2022).

In contrast, SWMLDs refer to students who generally struggle with mathematics, also known as Mathematics Difficulties (MDs) (Geary, 2003; Geary & Hoard, 2004; Siegler, 2004). These students are also referred to as "mathematics poor learners," "low-achieving math learners," "students struggling in mathematics," and "students at risk for math failure" in the literature. They often have poor performance in class and on standardized tests. Some researchers from the field of cognitive psychology identify SWMLDs primarily based on their mathematics achievement scores that fall below a certain cut-off, which may range from the 5th to the 30th percentile (Geary et al., 2007; Swanson & Hoskyn, 1998). However, this sole-measure approach has been criticized because multiple assessment and evaluation methods, including both standardized assessments and informal assessments, such as observations, teacher ratings, and curriculum-based assessments, should be utilized to determine the extent of a child's difficulties in learning mathematics (Nitko & Brookhart, 2014; Zhang et al., 2022).

This study focuses on SWMLDs, who include, but are not limited to, students with officially diagnosed learning disabilities in mathematics. The implementation of response-to-intervention (RtI) model has accelerated the transition from providing intervention services only to students with identified mathematics learning disabilities to also providing services to at-risk students with learning difficulties in mathematics.

**Characteristics.** SWMLDs are described as a heterogeneous group since they differ in the causes of their mathematics difficulties (Bartelet et al., 2014; Chen et al., 2021). The common features among SWMLDs are not limited to cognitive abilities or social situations, but they might share common difficulties in mathematics-related
aspects. For example, SWMLDs might be able to perform different computational activities that demand numerical operations, but encounter difficulties in interpreting their procedures, justifying their mathematical algorithms, or manipulating ways of answering mathematical problems (Jitendra et al., 2013). As indicated in the literature on mathematics education, SWMLDs have difficulty interpreting or building mathematical proofs, particularly when it comes to proving theorems (e.g., theorems of triangle congruence or similarity) compared to proving ordinary mathematical problems (e.g., proving typical trigonometric identity) (Warli & Rahayu, 2020). It is not easy for students, whether SWMLDs or non-SWMLDs, to understand and analyze mathematical word problems, but SWMLDs encounter difficulties in solving easy, straightforward word problems as well as solving sophisticated problems that demand advanced mathematical skills and different procedural steps (Jitendra et al., 2013; Shantika & Istiyono, 2019). Many STEM courses at university levels highly depend on algebra and algebraic skills, but many SWMLDs struggle with algebra in school and record failing scores, which increase withdrawal rates from schools or advanced mathematics classes, reducing SWMLDs’ opportunities for better future academic tracks or career trajectories (Geary et al., 2020).

Inattention is commonly observed in SWMLDs. It is common for SWMLDs to not pay attention to their mathematics teachers during regular mathematics lessons compared to non-SWMLDs (Geary et al., 2020; Martin et al., 2013). This inattention is common not only because of a high occurrence of ADHD diagnosis and learning disabilities (Zentall, 2005), but also because of regular mathematics curriculums could be too challenging for SWMLDs when special education support is insufficient to meet students’ needs. Indeed, mathematics is an accumulative subject in which new skills are often built upon certain prerequisite skills, and students who lack these skills
can hardly focus on new materials to which they cannot make meaningful connections with their existing knowledge.

SWMLDs often face obstacles in reading, in addition to their struggles in learning mathematics, and these reading difficulties can worsen the situation for SWMLDs (Khanolainen et al., 2020; Viesel-Nordmeyer et al., 2021). In particular, students with diagnosed mathematics learning disabilities, such as dyscalculia or dysgraphia, are highly likely to have co-occurring difficulties in reading or other types of language-related difficulties (Swanson et al., 2018).

As for non-cognitive aspects, SWMLDs often have lower confidence in their personal mathematics performance and higher concerns about their ability to succeed in learning mathematics compared to typical or high mathematics achievers (Lai et al., 2015). In particular, SWMLDs tend to have low self-efficacy in mathematics, especially at higher grade levels where more challenging content is taught and advanced learning skills are required (Geary et al., 2020; Lein et al., 2020). Additionally, SWMLDs are often less motivated or skilled in advocating for themselves to access support or resources compared to non-SWMLDs (Smith et al., 2013).

SWMLDs can also experience various psychological, economic, social, or other issues, which can impact their learning in mathematics (Mananggel, 2020; Morgan et al., 2009). For instance, children from families that struggle to provide financial support may not have sufficient access to learning materials or instructional aids, such as the internet, and are more likely to experience learning difficulties in mathematics (Jordan & Levine, 2009; Martin et al., 2013). Non-genetic environmental factors, such as parental difficulties in mathematics, the home learning environment,
low family income, and inadequate residential facilities, can also play a significant role in the development of mathematics difficulties (Khanolainen et al., 2020).

**Identification and prevention.** Preventing math learning difficulties should begin in kindergarten or early primary school stages (Bryant et al., 2008; Cirino & Berch, 2010; Febriyanti et al., 2021; Morgan et al., 2009; Pappas et al., 2019). Ideally, teachers are expected to be skilled in conducting needs assessments, identifying students’ academic weaknesses, being competent in diagnosing their students’ difficulties and experienced in applying well-designed interventions to help their students learn (Kron et al., 2021). However, mathematics difficulties are not usually diagnosed until third grade when students are required to take a state standardized assessment. Nowadays, there are many different promising intervention-based strategies found in the literature that can be implemented to help educators and teachers diagnose, determine, and treat SWMLDs at these necessary foundation levels (Bryant et al., 2008; Febriyanti et al., 2021). It is essential to diagnose and treat mathematics difficulties starting from the very early stages of learning as research suggests that earlier intervention leads to faster responses and better improvement (Febriyanti et al., 2021). It is difficult for students to correct their mathematical misunderstandings or misconceptions once they get used to making these mistakes (Dole, 2003). SWMLDs face more obstacles as they progress from learning basic skills at kindergarten or elementary stages to acquiring advanced abilities at higher learning stages. The latter demands higher critical thinking, advanced abstract reasoning, intensive accumulated knowledge, and complex problem-solving skills in addition to the ability to leverage and integrate these skills into helpful combinations (Lein et al., 2020).
**Difficulties in Learning Geometry**

Geometry is an important domain in mathematics and is required in K12 math curricula of every nation (Bailey et al. 2014; Choo et al. 2021; Yani & Rosma 2020). It describes characteristics of shapes and explores connections between figure-related properties by applying different related mathematical concepts and theorems (Baranová, & Katreničová, 2018). Therefore, it is essential in learning geometry to master and link other mathematical domains (e.g., algebra, data) and mathematical learning skills (e.g., proving, deducting, inducting) (Bailey et al. 2014). Learning geometry can help us imagine, perceive, visualize, and spatialize the nature of different real-life materials and understand the properties of their constructions (Crompton et al. 2018; Đokić 2018; Schoevers et al. 2020).

Most states in the U.S. require students to complete a geometry course to obtain a high school diploma. Many advanced problem-solving college-level courses, particularly those in STEM fields, require comprehension and application of geometric theorems (e.g., Pythagoras theorem, trigonometric identities). Therefore, passing an advanced high school geometry course is often a prerequisite for enrolling in these college-level courses (Bailey et al., 2014).

Unfortunately, many students struggle to learn geometry (Ayan & Isiksal Bostan, 2016; Naidoo & Kapofu, 2020). Research indicates that many SWMLDs even have difficulties in learning and understanding simple geometric concepts such as arrays, angles, and triangles (Gal & Linchevski, 2010). International competitions have also revealed US students’ underachievement in mathematics, particularly in geometry, among students in comparison to other industrialized nations (Bailey et al., 2014; Mullis et al., 2016). Chen et al. (2021) found that geometry is the weakest domain in mathematics for fourth graders. Reflecting these concerns, the K-12
Common Core State Standards in Mathematics (Common Core State Standards Initiative [CCSS], 2011) have increased the emphasis on transformational geometry (Hughes et al., 2013).

Most of the current research on geometry learning for SWMLD has focused on the elementary level and relatively simple domains, such as area, perimeter, and basic spatial imaginary skills (Bergstrom & Zhang, 2016). However, as students approach high school, geometry standards become more complex and in-depth. According to CCSS (2011), high school students are expected to develop an understanding of the attributes and relationships of geometric objects and apply this knowledge in novel contexts. Students engage in problem-solving (Schoenfeld, 1992) to make connections (Hiebert & Grouws, 2007) across topics such as right triangles, circles, congruence, trigonometry, and geometric measurement. They also formalize their geometry experience by using more precise definitions and careful proofs.

Geometry learning is highly related to students’ skills in other mathematical subjects. Chen et al. (2021) found that students’ performance in geometry is highly associated with their performance in other mathematical domains, such as numeracy, algebra, and probability. Students with the lowest scores in geometry also tend to demonstrate the poorest mathematical scores in other math domains. The results showed that the performance in geometry is highly correlated with the performance in other domains of mathematics. Moreover, the researchers did not find a group of students who only perform poorly in geometry without having low scores in other mathematical domains.

Indeed, like other mathematical domains, success in learning geometry requires a series of mathematical abilities, such as inducting inference, deductive inference, analyzing data statistically, measuring quantities, and solving real-life word
problems, that should be leveraged by creating an integrated combination that eases learning geometry (Bailey et al., 2014). For example, understanding the concept of trigonometric functions demands a set of different mathematical skills, such as an ability to manipulate equations algebraically and apply Pythagoras’ theorem correctly. In addition to the acquisition of those skills, learning geometry requires constructing connections between different related definitions, ideas, characteristics, theories, and so on (Crompton et al., 2018). For example, it is necessary to understand the relationships between right-triangle sides and their opposite angles when dealing with trigonometric ratios (i.e., \(\sin x\), \(\cos x\), \(\tan x\)).

There are numerous factors that impact performance in learning geometry (Bailey et al., 2014; Geary et al., 2020). In addition to the factors that commonly predict students’ achievement in any other academic subject, such as physical health and an active lifestyle (Hraste et al., 2018) and motivation (Bailey et al., 2014), learning geometry is uniquely affected by some factors related to the subject’s characteristics. Geometry is a subject that deals with properties, relationships, and transformations of spatial objects within an interconnected network of concepts and representational systems (Crompton et al., 2018). The unique characteristics of geometry demand that certain cognitive variables are essential. First, spatial reasoning is fundamental to geometry, enabling students to construct and manipulate mental representations of spatial objects (Clements & Battista, 1994). Spatial thinking is also important for cognitive learning in general, regardless of the field of study (Hawes et al., 2017). There is a strong positive relationship between performance in geometry and spatial skills (Baranová, & Katreničová, 2018; Chen et al., 2021; Giofrè et al., 2013; Lourenco & Huttenlocher, 2007; Pittalis & Christou, 2010; Winarti, 2018; Yani & Rosma, 2020). Lacking spatial skills, which are necessary to store, recall, and
analyze visual and pictorial images, may lead to difficulties in learning geometry (Forrest, 2004; Mammarella et al., 2013). However, although it has been well-documented that spatial skills play an important role in learning geometry, there is sparse research investigating improving spatial reasoning in the curriculum (Mulligan et al., 2020).

Secondly, solving geometry problems is a cognitively demanding task. Students must navigate a complex network of concepts and transform multiple representational modes (i.e., verbal, visual, numerical) into an integrated system of representation. Therefore, executive function, including working memory, is crucial for processing information in geometry problem solving. Executive function refers to the set of self-regulatory skills involved in the conscious, goal-directed modulation of thought, emotion, and action (Engle et al., 1999; Jacob & Parkinson, 2015). Executive functions comprise working memory, updating flexibility (shifting from one set of rules to another), and inhibitory control. Struggling students differ from their peers in terms of executive functions (Emslander & Scherer, 2022; Kloow et al. 2022; Toll et al., 2011), particularly in working memory (Giofrè et al., 2013, 2014).

Language is another factor that contributes to students’ difficulties in learning geometry. Learning geometry requires language skills to interpret or restate geometric verbal problems from their worded statements to their corresponding numerical and visual modes. However, many students find it challenging to transfer worded problems into an equivalent abstract mathematical statement due to poor linguistic skills (Lei et al., 2020). Consequently, these students often struggle with solving geometry word problems. Additionally, many SWMLDs lack geometry vocabulary and do not understand the terms used in geometry (Clausen et al., 2021; Cook et al., 2020; Lai et al., 2015; Powell et al., 2009). Therefore, improving the learning of
geometry demands linguistic skills and the ability to convert geometry worded problems to their corresponding correct pure mathematical version.

**Interventions on Teaching Geometry to SWMLDs**

A number of meta-analyses and research syntheses have investigated the effectiveness of special education interventions for students with mathematics learning disabilities or difficulties (e.g., Baker et al., 2002; Codd et al., 2011; Dennis et al., 2022; Fischer et al., 2013; Gersten, Chard et al., 2009; Jacob & Parkinson, 2015; Jitendra et al., 2018; Jitendra et al., 2021; Kroesbergen & Van Luit, 2003; Lein et al., 2020; Losinski et al., 2019; Methe et al., 2012; Myers et al., 2021; Nguyen et al., 2008; Powell et al., 2021; Stevens et al., 2018; Swanson & Hoskyn, 1998; Xin & Jitendra, 1999; Zhang & Xin, 2012). Consistent findings suggest that instructional interventions should be provided in small groups, focusing on meaningful learning of taught concepts and algorithms, and using organized pedagogical strategies to improve the performance of SWMLDs. Researchers have concluded that technology-based interventions can positively influence the learning process for SWMLDs. For example, a meta-analysis by Gersten, Chard et al. (2009) identified several effective instructional components for students with diagnosed learning disabilities, including explicit instruction, heuristics, student verbalization, visuals, sequence/range, teacher feedback, teacher feedback plus options, student feedback (and goal setting), cross-age tutoring, and peer-assisted learning within a class. Dennis et al. (2016) found that interventions that integrate multiple instructional components are more likely to benefit SWMLDs.

Although traditionally, explicit instruction has been one of the most commonly recommended intervention strategies for students diagnosed with learning disabilities or intellectual disabilities, for SWMLDs, more heuristic instructional
strategies are recommended (Foegen & Dougherty, 2017; Gersten, Beckman et al., 2009). Explicit instruction has been criticized for denying students opportunities for strategy development that follows from their own mathematical thinking and reasoning (Gersten, Chard et al., 2009). With the gradual transition of focus from students with diagnosed learning disability to SWMLDs, current explicit instruction often involves an interactive dialogue between the teacher and students, with various levels of practice built into the systematic sequence of instruction (Doabler et al., 2016; Doabler & Fien, 2013; Fuchs, Malone et al., 2016; Fuchs, Schumacher et al., 2016; Liu & Xin, 2017; Powell et al., 2021). As reflected in the 2021 Practices Guide, researchers used the term "systematic instruction" rather than "explicit instruction" when discussing an instructional approach where "materials are designed to develop topics in an incremental and intentional way… [to] support student learning" (Fuchs et al., 2021, p. 5).

Unfortunately, there has been little research on interventions for teaching geometry to SWMLDs. Educational researchers have called for further investigation to help SWMLDs in learning geometry (e.g., Chen et al., 2021; Choo et al., 2021; Crompton et al., 2018). In their meta-analyses of empirical research on teaching SWMLDs, Dennis et al. (2016) urged researchers to pay more attention to the subject of geometry. Bergstrom and Zhang (2016) synthesized 32 intervention studies on geometry interventions for K-12 students with and without disabilities. Five studies examined the effectiveness of new geometry curricula, sixteen studies investigated instructional strategies, and eleven studies explored educational technologies. Although a broad range of geometric subjects were covered for normally achieving students, most of the nine studies for students with special needs (Cass et al., 2003; Hord & Xin, 2014; Kang & Zentall, 2011; Satsangi & Bouck, 2015; Strickland &
Maccini, 2013; Worry, 2011; Xin & Hord, 2013; Zhang et al., 2012; Zhang et al., 2014) primarily focused on very basic geometry skills.

A recent literature review (Liu et al., 2021) identified seven studies on teaching geometry to students with diagnosed learning disabilities. Five of these studies implemented geometry interventions using instructional strategies (Cass et al., 2003; Cihak & Bowlin, 2009; Kozulin & Kazaz, 2017; Strickland & Maccini, 2013; Xin & Hord, 2013), one study (Cass et al., 2003) used manipulatives to instruct students on the concepts of perimeter and area, and three studies used educational technology (Cihak & Bowlin, 2009; Horner, 1984; Satsangi & Bouck, 2015). All of these studies integrated multiple instructional components, such as advance organizers, attributions, control of task difficulty or processing demands, elaboration, explicit practice, large-group learning, novelty in presenting new teaching materials, one-on-one instruction, peer modelling, questioning, reinforcement, sequencing, skill modelling, small-group instruction, strategy cues, supplementing teacher involvement, task reduction, and technology. On average, each study included 12.4 types of instructional components, with a range from 11 to 15. Although six of the seven studies recruited participants from junior and high school levels, the geometry skills taught were at the elementary school level. For example, the studies focused on perimeter and area problems in CCSSM Grade 3, angle recognition in Grade 4, and volume of a rectangular prism in Grade 3.

**Theoretical Framework**

**Chunking**

A chunk in this study refers to a meaningful unit with a unique conceptual structure, and chunking is a way of grouping these units to generate an integrated combination of chunks that form a sophisticated structure (Chase & Simon, 1973;
A chunk can also be called a unit or fragment (Perlman et al., 2010). In general, the word "chunk" indicates a combination of different related smaller parts or a number of items that are considered together as a single structure with a unique meaningful concept (Chase & Simon, 1973; Mathy & Feldman, 2012). For instance, when we write a phone number (e.g., +1 732 *** ****), there is a chunk that indicates an international code (i.e., +1, the international code for the U.S.), and this chunk is built with numbers that together as a whole illustrate a unique conceptual structure. There might also be another chunk that represents a local region or a state. In the phone number +1 732 *** ****, 732 refers to New Jersey, which is one of the U.S. states. Chunking is a process of grouping chunks to create an integrated combination (Fonollosa et al., 2015). Chunking is one of the basic fundamental processes that the brain relies on to function properly, especially when it comes to restoring and retrieving data from memory (Isbilen et al., 2020; Perlman et al., 2010). It has been shown that chunking can be involved in different cognitive models that are constructed to describe the mechanism of a brain process (Perlman et al., 2010).

Chunking is a cognitive process that groups semantically related concepts to form larger meaningful structures (Chase & Simon, 1973). These chunks are cognitively represented in an arrangement in which there are different bonds connecting them together to form a sophisticated structure. Thalmann et al. (2019) demonstrated how chunking helps humans to exceed the limited capacity of working memory. This research found that chunking has a positive impact on the retrieval of both chunked and non-chunked information stored in working memory. The researchers concluded that a chunk reduces the load on the working memory by retrieving a compact chunk representation from the long-term memory that replaces the representations of individual elements of the chunk. They also found that the
working memory capacity is not bounded by a specific number of chunks, regardless of the size of each chunk.

Chunking has received substantial attention from cognitive scientists and psychologists, and is considered essential in several brain functions, including storing and retrieving data, perceiving relationships, operating problem-solving processes, and analyzing data to guide behavior and decision-making (Bor & Seth, 2012; Gobet et al., 2015). Chunking is one of the most beneficial cognitive functions for storing, recalling, analyzing, processing, and producing cognitive information (Bor & Seth, 2012), and it is well-documented that chunking mechanisms do exist (Fonollosa et al., 2015; Rabinovich et al., 2014).

To gain an understanding of chunking mechanisms and their influence on learning and human action, Fonollosa et al. (2015) proposed a dynamical model that incorporates two cognitive processes. According to the researchers, chunking, as a learning process, is a response to two parallel processes: (1) producing shorter patterns from longer sequential ones, and (2) connecting shorter units into longer sequences. Their dynamical model integrates these two processes. By employing sophisticated computer simulations, the researchers observed various patterns that share different descriptions of the chunking process based on their time period and capacity. The researchers suggested that the inability to leverage the chunking process mentally could be linked to mental illnesses that negatively impact an individual's behavior, attention, emotions, intelligence, and so on.

Research has also shown neurological evidence of the effects of chunking. Chunking has been found to enhance the functioning of the prefrontal parietal network in the brain (Bor & Seth, 2012). The inability to apply chunking mechanisms may indicate the presence of various neural system disorders, such as schizophrenia.
and Parkinson's (Fonollosa et al., 2015). In their study on consciousness and the prefrontal parietal network, Bor and Seth (2012) indicated that chunking helps the brain store memorized information in a way that enhances its memory. They concluded that chunking improves the operation of the highly connected prefrontal parietal network, which impacts human behavior, attention, and knowledge. Additionally, they found that chunking supports consciousness by identifying related sequences and relationships in mentally stored data.

**Chunking, working memory and Cognitive Load Theory**

Cognitive load theory deals with the compatibility between the cognitive load necessary for problem-solving and the limited capacity of a cognitive system (Sweller, 1994). If the cognitive loads required for problem-solving surpass cognitive capacity, it can impede one's problem-solving process (Sweller, 1994). Cognitive loads are caused by interacting elements, i.e., elements of information that are logically related and must, therefore, be simultaneously processed to achieve understanding (Sweller, 2010; Sweller et al., 2011), but that have not yet been integrated and stored in the long-term memory as a single chunk or schema (Chen et al., 2015). In geometry problem-solving, students need to understand verbal statements related to a diagram and link them to the elements in the diagram at the same time. The working memory can get overwhelmed because of the high cognitive processes needed for these mental tasks.

Cognitive load can be manipulated by changing the nature of the information or enhancing the learners' expertise (Chen et al., 2015). In the context of learning geometric theorems, the elements of the diagram and statements may be integrated and stored in the long-term memory as a single chunk or schema, resulting in a reduced intrinsic cognitive load (Sweller, 2010). For example, in the case of the angle
bisector theorem, once a student has the knowledge of the constituent elements and properly interpreted the theorem statement, he/she should be able to synthesize those elements into a chunk or schema. As a result, the learner is then able to apply the synthesized chunk as a single element when engaging with problems that require the angle bisector theorem in combination with other elements.

Baddeley (1972) proposed the model that working memory recalls a singular unit of data associated with a limited representation within a few seconds. The human memory system must have the ability to discard or forget certain information due to its constrained capacity. Without this capacity, working memory would struggle to recall or store new information (Baddeley & Paterson, 1971). Baddeley and Hitch (2000) constructed a model illustrating the operation of short-term memory, initially comprising three elements: the central executive, the phonological loop, and the visuospatial sketchpad. Subsequently, Baddeley introduced a fourth component known as the episodic buffer (Baddeley, 2017; Baddeley & Hitch, 2000).

The central executive, supported by the phonological loop and the visuospatial sketchpad, is a concept that centers on the attention-related functions of working memory, traditionally encompassing the visual and verbal domains of short-term memory. As an example, weak central executive might be linked to Alzheimer's dementia, and so the central executive plays a crucial role in human cognitive tasks. The phonological loop's function is not to recall linguistic phrases but to aid in acquiring new words, especially during the early years of language acquisition. The phonological loop's operation is influenced by item length, and its performance can be enhanced through strategies such as mimicking the sounds of newly exposed items to reinforce the structural representation of new words. As an example, exceptional phonological loop capacities may contribute to gifted children's proficiency in
acquiring new languages. The visuospatial sketchpad is characterized as a hierarchical structure related to features and object representations, including color and shape. The episodic buffer serves as a multidimensional store establishing connections that facilitate interactions among the various functions of working memory, long-term memory, and the central executive. Its primary role is to link different data to form a combination of integrated and useful chunks, with its storage capacity that depends on the number of chunks it can process (Baddeley, 2017).

**Chunking Effects on Learning and Teaching**

**Chunking Effects on Language Learning.** The implications of chunking can be observed in various learning domains. Several studies related to language have explored the concept of chunking. Chunking has been examined in research on language development, particularly in comparing the performance of expert and novice users (Gobel, 2005). Theories on speech mechanisms suggest that pronunciation structures are comprised of chunks, and language instruction should utilize the chunking strategy -- words are chunks of letters, each of which are connected to corresponding sounds and meanings (Sevald et al., 1995). Children learn linguistic data by repeating chunking processes in different stages of learning: initially, they chunk phonemes into words, and then they chunk these words to form sentences. Students' listening performance on linguistic tests can be improved when they apply chunking-based strategies to answer listening questions (He, 2016; Isbilen et al., 2020). The listener comprehends the entire spoken sentence by grouping chunks that collectively represent that sentence, with each chunk representing a specific role (e.g., grammar, meaning) in the structure of that sentence. Overall, chunks are present in individual lexical expressions across these studies.
Xu & Padilla (2013) found that chunking strategies improved the learning of Chinese linguistic symbols. In their study, they investigated the effects of an intervention that integrated comprehended perception (i.e., understanding the meaning of the linguistic characters and knowing their historical origins) and chunking (i.e., grouping parts to produce meaningful Chinese characters) on participants’ Chinese learning outcomes. The results suggested that the experimental group, which received the comprehended perception and chunking intervention, significantly outperformed the control group that received the classical pedagogical way of teaching when learning and recalling Chinese linguistic symbols.

Chunking has also been found to improve students' typing skills (Yamaguchi & Logan, 2014, 2016). Novice typists typically begin with a typing method that requires deliberate effort to translate each letter to a key on the keyboard, while skilled typists use touch typing, using all fingers from both hands to navigate the keys without looking. In an earlier study, Yamaguchi and Logan (2014) revealed chunking in perceptual, memory, and motor processes in skilled typewriting. Chunking of letters in words depends on the knowledge of the words. Typists have to process letters in unfamiliar nonwords as separate chunks. Thus, Yamaguchi and Logan (2014, 2016) investigated the development of memory chunks when typists type nonwords and revealed the learning curve by preventing them from utilizing their typing skill.

**Chunking effects on STEM areas.** According to Isbilen et al. (2020), chunking has been found to be an effective approach in learning statistics, particularly when comprehending visualized data. The ability to learn statistical patterns is highly correlated with various high-level cognitive abilities across domains, as demonstrated in their 2022 study. The researchers propose that humans may learn statistical
information through a process similar to the repeating chunking process in language learning. Specifically, they suggest that individuals first identify patterns among the information presented, and statistical sensitivity then leads to chunking based on the statistical characteristics of the data. This approach to learning may help individuals to better comprehend and use statistical information in various domains.

Cheng and Obaidellah (2009) discovered that graphical information is hierarchically organized and that participants used chunking to facilitate rapid learning of visual diagrams. A further analysis of the participants' chunk production sequence revealed that a spatial schema was used to organize the chunks. Cheng and Obaidellah (2009) observed that the participants applied chunking to draw the diagrams after a long period of tracing and copying the diagrams, and that this chunking strategy was common among the participants. The participants considered the diagrams' patterns as chunks and this chunking strategy was found in various mathematics tasks such as writing number sequences (Cheng & Rojas-Anaya, 2005), copying mathematical formulae (Cheng & Rojas-Anaya, 2007), and producing simple geometric figures (Cheng et al., 2001). In order to explore the cognitive processes involved in rebuilding diagrams, Cheng and Obaidellah (2009) asked participants to draw diagrams from memory after a period of time since they last viewed them. The participants who succeeded in drawing a close duplicate followed a spatial schema to build chunks that ultimately led to the final copy. This study suggests that chunking may be a useful strategy in learning geometry, as spatial reasoning is highly important in geometry, which is indicated in various studies in the literature of mathematics education (e.g., Đokić, 2018; Hannafin et al., 2008; Hawes et al., 2017; Mulligan et al., 2020; Pittalis & Christou, 2010; Schoevers et al., 2020).
In a study aimed at investigating the impact of visual chunking as a means of performing spatial thinking in STEM fields, Stieff et al. (2020) divided students into two groups. One group consisting of those with no prior chemistry knowledge and the other consisting of students with general chemistry knowledge. Neither group had learned the material taught in the experiment (i.e., were not chemistry experts) before the study. They found that the ability of both groups to visualize was affected by how the data were visually chunked. They also concluded that students were more precise in identifying color changes within chemistry-related chunks compared to non-chemistry chunks. The researchers believe that their results indicate the important role of chunking the spatial structure of the knowledge when teaching students who have not yet mastered the taught material.

To investigate the Bayesian learning of visualized chunked data, Orban et al. (2008) conducted an experiment that asked participants to identify visual patterns. They concluded that humans do not generate a whole image directly, but rather store chunks from sophisticated visualized organized sequences by precisely developing chunks that form the whole image.

**Reducing the Cognitive Load with Chunking in Geometry Problem Solving**

Cognitive load theory is concerned with the compatibility between an individual's limited cognitive system capacity, the availability of cognitive resources, and the cognitive load required to complete a learning task (Sweller, 1994). When cognitive load surpasses the cognitive capacity and no external cognitive resources are accessible, students often experience difficulties in completing the cognitive task (Sweller, 1994). Cognitive loads arise from information elements that are pertinent to problem-solving and must be cognitively processed to achieve understanding.
(Sweller, 2010; Sweller et al., 2011). Changing the nature of the information or enhancing learners’ expertise can modify cognitive load (Chen et al., 2015).

For example, in order to solve a geometry problem, a student must process information presented in various modes (i.e., verbal statements in the problem prompt, visual information in the pictorial diagram, and numerical information in the numbers involved) and reason about the relations among the complex information. If some elements can be integrated and stored in long-term memory as a single chunk or schema (Chen et al., 2015), then cognitive load may be reduced as the problem solver processes the information at the chunk level. As demonstrated in Zhang et al. (2021), if a student can identify elements related to the angle bisector theorem, group these elements as a chunk, and use the angle bisector theorem to solve the problem ("If a point lies on the bisector of an angle, then it is equidistant from both sides of the angle"), the cognitive load required for solving the problem may be reduced.

Relative to learning geometric theorems, the elements involved in the diagram and statements can be integrated and stored in long-term memory as a single chunk or schema, resulting in a reduced intrinsic cognitive load (Sweller, 2010). In the previous example of the angle bisector theorem, a student who has knowledge of the constituent elements and has interpreted the theorem statement adequately should be able to synthesize those elements into a chunk or a schema. Therefore, the learner is then prepared to tackle problems that rely on the angle bisector theorem by using the synthesized chunk as a single element in interaction with other elements in the posed problem.

**Chunking on Geometry**

Prior research (Zhang et al., 2012, 2014) has used chunking as a testing accommodation technique by highlighting visual chunks and reporting it as helpful
for elementary SWMLDs in solving geometry problems that demand visual imaginary
skills. In a single-subject design study, Zhang et al. (2012) investigated the
effectiveness of visual chunking in helping four elementary SWMLDs to solve
gometry problems that require students to perform geometric transformations (e.g.,
ratios, translations, and reflections) of visual images. All four participants
demonstrated higher accuracy when geometry problems were represented with graphs
of Visual-Chunking-Representation than with traditional-element-representation. A
follow-up group study (Zhang et al., 2014) compared the performance differences
between two groups of SWMLDs and non-SWMLDs when using the visual chunking
representation. They found that SWMLDs made greater improvements than non-
SWMLDs, indicating that the visual chunking representation technique should be
recommended as an effective accommodation for SWMLDs.

Zhang et al. (2021) extended previous research on chunking accommodations
from elementary to high school geometry problems. They investigated the
effectiveness of schematic chunking in improving the performance of SWMLDs and
those at risk of math failure, as well as the moderating effects of problem complexity
and content knowledge on the visual chunking accommodation. Eighteen SWMLDs
and 15 at-risk students were randomly assigned to two groups, one of which was
provided with a "cheat sheet" of related theorems. Both groups completed two
versions of a geometry test: (a) a test with a plain figure representation, and (b) a
parallel test with colors and markings that highlighted the elements of a schematic
chunk. Results showed a main effect of chunking for all participants, with a greater
effect for difficult one-step and multi-step problems than for simple one-step
problems. Providing a cheat sheet increased the chunking effect for solving only the
difficult one-step problems related to low-frequency theorems for SWMLDs.
Although chunking has been shown to be effective as an accommodation for SWMLDs, a question remains: what if the chunking accommodation is not available in real-life testing or instructional situations? It is not best practice for SWMLDs to passively wait for test or curriculum developers to publish all tests or worksheets with geometry diagrams with chunks highlighted. Rather, it is necessary to develop a chunking intervention that teaches SWMLDs to actively recognize the semantic chunks embedded in complex geometry diagrams and then retrieve the related information from the long-term memory to solve the geometry problem.

As a pilot study to explore the effectiveness of chunking strategy as a cognitive intervention for teaching high school geometry to SWMLDs, Zhang (2017) conducted a single-subject design study with college SWMLDs that integrated a working memory training program and the chunking strategy as a direct instructional approach to teach sophisticated geometry problem-solving or proof that involved triangle congruence. The participants were college students who had enrolled in a developmental mathematics remediation program and failed a mathematics screening test (i.e., the geometry subtest of KeyMath-3). The results of the study showed that all participants improved their accuracy from baseline to post-test. However, since the intervention program of this study involved two instructional components (i.e., the visual working memory training and the chunking strategy), it is not clear to what extent the chunking strategy alone would be effective for SWMLDs. It is also unclear to what extent the chunking strategy may be effective when applied to high school SWMLDs.

Summary of the Chapter

This chapter thoroughly explores research aimed at assisting students facing challenges in mathematics, particularly in the context of geometry. It begins by
examining the terminology used to describe students struggling with math and delves into their characteristics. The chapter then addresses the specific difficulties these students encounter in learning geometry and explores the underlying reasons. Extensive attention is given to existing research on various approaches to teaching geometry to these students. The chapter introduces a significant concept called "cognitive load theory and chunking," suggesting that it could enhance the learning experience for them.

A section of the chapter is dedicated specifically to students with math difficulties, discussing the diverse terms used to characterize them. It emphasizes a shift from focusing solely on their problems to finding ways to support them. The chapter highlights the differences in how these students think and feel, addressing issues such as lack of attention, reading challenges, reduced confidence, and external obstacles.

Following this, the chapter delves into the challenges students face when learning geometry, a crucial aspect of mathematics education. It underscores the importance of geometry in various professions and fields. Despite its significance, many students, especially those with math difficulties, struggle to grasp geometric concepts. The chapter explores the reasons behind these difficulties, examining the connections to other math topics and the essential skills required, including spatial reasoning, decision-making, and language skills.

In the final part of the chapter, attention is given to strategies for helping students learn geometry. It synthesizes findings from numerous studies, suggesting that small groups and technology can be effective. The chapter emphasizes the need for more research in this area due to the limited number of existing studies. It
discusses teaching methods, such as using tricks and careful planning, and calls for additional ideas to enhance the learning experience for students with math difficulties.

In summary, the chapter thoroughly investigates the challenges faced by students with math difficulties in learning geometry, drawing insights from existing studies. It introduces a significant concept that could improve their learning and stresses the importance of early identification and support, particularly in the context of geometry.

Chapter 3. Method

Design

This study used a Multiple-Baseline Design across participants, which is a type of Single-Subject Design research. The single-subject research methodology is sometimes described as a timed-sequence, small-group, single-case, within-subject, repeated-measures, dependent-measures, correlated-measures, and quasi-single-subject experiment (Dattilo et al., 2000; Foster, 2010; Gliner et al., 2000; Hawkins et al., 2007). Single-subject design research is widely used in special education to examine intervention effectiveness (Byiers et al., 2012; Bouwmeester & Jongerling 2020; Hembry et al. 2015; Klingbeil et al. 2017; Ledford & Zimmerman, 2023; Pustejovsky et al., 2014). Traditionally, it is recommended that the applying multiple baseline design be applied on at least three participants (Lanovaz & Turgeon, 2020). The single-subject design helps researchers to concentrate on the participants' behaviours or performance actions rather than the group's average work. Therefore, it is suitable for clinical and treatment purposes, particularly because of the possibility to adjust the intervention to create an appropriate environment for different participants in the clinical background (Gierut et al., 2015). In the multiple baseline design, the treatment condition is successively administered to multiple target
participants (Hedges et al. 2013), groups (Hawkins et al. 2007), target outcomes, or target settings in a staggered pattern. Treatment effect is demonstrated by a change in behavior or performance only when the intervention treatment is given.

In this study, an adapted multiple probe design across participants (Horner & Baer, 1978) was used to evaluate the functional relation between the chunking intervention and SWMLDs’ performance improvement in geometry problem solving. With a multiple baseline design across participants, intervention effects can be demonstrated by introducing the intervention to SWMLDs participants at different points in time. Specifically, Participant B continued with the baseline assessment when Participant A had already been introduced to the intervention treatment, and Participant B didn’t start the intervention until Participant A had shown a change in performance. The underlying explanation is that if each baseline changes once the intervention is introduced, the effects can be attributed to the intervention rather than extraneous events, such as history, maturation, and testing (Kazdin, 2011).

Setting

The study was conducted at a high school in New Jersey, where an appropriate room was provided for the researcher and participants to work in. The intervention was conducted as an in-school program, consisting of 1-hour sessions, held 1-2 times per week. Participating SWMLDs worked individually with an instructor during each session.

Participants

Three high school SWMLDs participated in this study. The participants were nominated by their teachers as students having learning difficulties in geometry. These participants were native English language speakers and not identified as English Language Learners (ELLs). The KeyMath-3 geometry subtest was
administered, and participants recorded low scores. Parental consent and student assent forms were collected for these students to participate in the study. As shown in Table 1, participants’ demographic data, including age, gender, and race were collected. All three participants were Hispanic female, 10th graders. And all participants can speak proficient English.

All participants showed poor performance in KeyMath-3 geometry subtest as shown in Table 2. While all three participants scored below 30% percentile rank, their scores were highly differentiated. Specifically, Participant A’s grade equivalent is 7.5 whereas Participant C’s grade equivalent is K.9, indicating the Participant C encountered much greater difficulties in learning mathematics. This achievement difference is consistent with their school teachers’ description when referring participants to the researcher.

Table 1

<table>
<thead>
<tr>
<th>Participant</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grade</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Gender</td>
<td>Female</td>
<td>Female</td>
<td>Female</td>
</tr>
<tr>
<td>Race</td>
<td>Hispanic</td>
<td>Hispanic</td>
<td>Hispanic</td>
</tr>
</tbody>
</table>

Table 2

<table>
<thead>
<tr>
<th>Participants</th>
<th>Scale score</th>
<th>Grade equivalent</th>
<th>Percentile rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>8</td>
<td>7.5</td>
<td>25</td>
</tr>
<tr>
<td>B</td>
<td>5</td>
<td>4.2</td>
<td>5</td>
</tr>
<tr>
<td>C</td>
<td>2</td>
<td>K.9</td>
<td>0.4</td>
</tr>
</tbody>
</table>

Experimenter

The interventions were coordinated by the researcher who is a doctoral student of Educational Psychology and works as a math supervisor in the Ministry of Education in the Sultanate of Oman. The researcher has over 20 years of experience
in mathematics education. One undergraduate research assistant, who has proficient high school mathematics skills, worked as an interventionist after receiving training.

**Measures**

**Geometry problem solving probe (CBM).** The criterion measure of the intervention study was a curriculum-based measurement. Parallel probes were developed to measure students' performance in geometry problem-solving during the baseline phase, intervention phase, post-test phase, and maintenance phase. Each probe included three to five geometry problem-solving problems related to angles and lines, such as parallel lines and their properties, vertically opposite angles, interior consecutive angles, interior alternate angles, and interior corresponding angles. This measure was used to answer Research Questions 1 and 2. A probe can be found in Appendix A.

**Geometry proof test.** This measure was used as the first distal measure to examine the extent to which participants can transfer the learned chunking skills from problem-solving items to proofs on the taught topics (i.e., lines and angles). This measure was administered during the baseline phase and the post-test phase to determine if SWMLDs could transfer their chunking skills from geometry problem-solving to geometry proof on related subjects (i.e., angles, lines) and answer Research Question 3a. This test can be found in Appendix B.

**NJSLA Preparation Test (Geometry).** This test was used as the second distal measure to examine the transfer of the skills learned in the chunking intervention on lines and angles into high school geometry problem solving beyond these two topics. A problem-solving assessment with selected geometry items released from the mathematics portion of the New Jersey Student Learning Assessment (NJSLA) by the New Jersey Department of Education (n.d.) was used.
Geometry problems with content topics adjacent to lines and angles were selected for this intervention, such as a problem involving lines, angles, and a triangle. Items were retrieved from the website of the New Jersey Assessments Resource Center (NJARC). This measure was used during the baseline phase and the post-test phase to determine if the SWMLDs could transfer their chunking skills from geometry problem solving on angles and lines to geometry problem solving on a wider range of topics (e.g., triangles, congruence) to answer Research Question 3b. This test can be found in Appendix C.

**Keymath-3 (Geometry subtest).** This test was used as the third distal measure to examine the effectiveness of the intervention in transferring skills to a broader range of geometry problems. The KeyMath-3 assessment is a validated diagnostic test that measures the mathematics performance of students in specific areas. It assesses students from ages 4 to 21 or grades K-12 across five mathematical domains: numeration, algebra, geometry, measurement, and data analysis and probability. The geometry subtest consists of 36 testing items, and for this study, we selected items related to lines and angles to measure participants’ performance in geometry problem solving, specifically in relation to triangle problem solving (Pearson Assessments, 2022). This measure was used during the baseline and post-test phases to compare the transfer of chunking skills from high school geometry curriculum to more cognitive and general geometry problem solving, as measured by the KeyMath-3 Geometry subtest, to answer Research Question 3c.
Table 3
Research Questions, Dependent Variables, and Measures

<table>
<thead>
<tr>
<th>Research Question</th>
<th>DVs</th>
<th>Measures</th>
<th>Administration time</th>
</tr>
</thead>
<tbody>
<tr>
<td>RQ1. What effect will the chunking intervention have on the geometry performance of high school SWMLDs as measured with a proximal measure (the criterion test)?</td>
<td>Student geometry outcomes on the proximal measure</td>
<td>The Criterion CBM (the geometry problem solving probes)</td>
<td>Baseline, intervention, and posttest phases</td>
</tr>
<tr>
<td>RQ2: To what extent do high school SWMLDs maintain their geometry performance two weeks after the conclusion of the intervention as measured with a proximal measure (the criterion test)?</td>
<td>Geometry outcomes on the proximal measure</td>
<td>The Criterion CBM (the geometry problem solving probes)</td>
<td>Maintenance phase</td>
</tr>
<tr>
<td>RQ3: To what extent do high school SWMLDs generalize their chunking strategies to improve their performance on distal measures (geometry proof, NJ LSA, KeyMath-3)?</td>
<td>Geometry outcomes on three distal measures</td>
<td>KeyMath-3 Geometry subtest, Geometry Proof, NJLSA preparation items</td>
<td>Baseline and Posttest</td>
</tr>
<tr>
<td>RQ4: What are the high school SWMLDs about the chunking intervention?</td>
<td>Student perspective on the intervention</td>
<td>Researcher-developed social validity form.</td>
<td>Posttest</td>
</tr>
</tbody>
</table>

Notes: RQ = research question.

Procedures

After screening the participants, this study followed the sequence of baseline, intervention, posttest, and maintenance phases.

Baseline Phase. All participants completed at least three probes of the criterion test (i.e., geometry problem solving CBM) during the baseline phase. The
three transfer tests were administered. If more than one participant reached stability in baseline probes, the investigator randomly chose one participant to enter the intervention phase. Once Participant A showed the intervention effects, Participant B with a stable baseline was introduced to the intervention. Participant B could not enter the intervention phase until she had reached a stable baseline and Participant A showed an intervention effect.

**Intervention Phase.** This phase consists of 1 unit (consisting of 3 sessions) of instruction on angle-related problems using chunking strategy, and 1 unit (consisting of 3 sessions) of instruction on solving problems about angle relations embedded in parallel lines and perpendicular lines problems with the chunking strategy. Instructors conducted cumulative reviews of the learned chunking strategy for different problem types before the posttests. Throughout the study, calculators were allowed to accommodate participants' lack of proficiency in calculation.

The goal of this intervention was to teach SWMLDs to (a) recognize frequently used visual chunks that are semantically meaningful in a sophisticated geometry diagram, (b) retrieve and utilize related theorems/properties related to the identified visual chunks, and (c) solve the problem with analysing information on the level of the visual chunks with the retrieved theorems/properties. In order to reach these goals, the interventionists introduced and reviewed visual-semantic-chunks that frequently appear in geometry problems (e.g., vertical angles, supplementary angles, etc.), then reviewed its related theorems, postulates, and properties with examples (e.g., Vertical Angles Theorem, Congruent Complements Theorem, Congruent Supplements Theorem, etc.). It was noteworthy that this intervention adopted a structured instruction approach as recommended by the Practice Guide by What Works Clearinghouse (Fuchs et al., 2021) with student-verbalizations, and teacher
prompts and feedback to facilitate students’ problem solving. Appendix D shows an example of a lesson plan that was applied by the interventionist in teaching parallel lines. A lesson was composed of Structured Instruction, Guided Practice, and Independent Practice.

*Structured Interactive Instruction.* The lesson began with Step 1: Identifying the chunks. In the example lesson, the instructor started by reviewing the definitions of parallel lines (i.e., "parallel lines are a set of lines that lie on the same plane but never intersect each other, even if extended infinitely") and related terms such as Corresponding Angles, Alternate Interior Angles, Alternate Exterior Angles, Consecutive Interior Angles, and Vertically Opposite Angles. The instructor used specific items with diagrams to help SWMLDs visualize the chunk of parallel lines with related angles. Next, the instructor provided 2-3 example items and asked participants to identify parallel lines in a complex visual background. It is noteworthy that this structured instruction does not mean it is teacher-centered or teachers simply demonstrating all procedures without student involvement. Instead, teachers prompt and facilitates students to find out the chunks.

Then, the instructor worked with the SWMLDs on Step 2: Recall related information about the identified chunks from long-term memory. "Related information" refers to properties (e.g., parallel lines are always the same distance apart), postulates (e.g., the Corresponding Angles Postulate and its converse), and theorems (e.g., the Alternate Exterior Angles Theorem and the Alternate Interior Angles Theorem, and their converses). Next, the instructor used the 2-3 previous example items to ask the SWMLDs to reason about the relationships among the angles and lines using the above properties, postulates, and theorems.
Step 3 involved problem solving, during which the instructor guided or modelled the SWMLDs to solve geometry problems by analyzing the visual chunks identified in Step 1 and reasoning with the necessary properties, postulates, and theorems from Step 2. The instructor helped the SWMLDs clarify the known conditions and the unknown question to be answered and guided them in figuring out a path to connect the known and unknown. The students were guided to operate on the identified chunks and engage in deductive reasoning with the properties, postulates, and theorems related to those chunks. Throughout each step, prompts and facilitation were provided as needed, and the SWMLDs were asked to explain their responses while receiving feedback.

**Guided Practice.** Following the above structured interactive instruction stage, students worked on another three items on the same topic during the Guided Practice section. During Guided Practice, students were provided with practice opportunities as well as the instructor's facilitations, prompts, and feedback. If students demonstrated difficulty in solving the problem, the instructor prompted them to identify the chunks involved in the problem from the complex diagram and facilitated the recall of the chunk-related properties, postulates, and theorems to solve the problem. This stage took about 10-15 minutes. By checking student performance, the instructor identified student error patterns and provided additional instructions if needed.

**Independent Practice.** In this stage, participants practiced solving geometry problems without guidance from the interventionist. Each participant had 15 minutes to complete three questions. The practice sheet was researcher-developed based on the lesson content, and some items were also from the high school geometry curriculum used in many NJ districts (i.e., the Holt McDougal Larson Geometry Textbook). The
items were also related to angles and lines. Participants worked on solving the problems independently without any prompts or feedback provided. Students’ percent correct on each Independent Practice session were recorded as the data point for the Criterion test (i.e., the geometry problem solving CBM).

**Maintenance Test**

Maintenance probes determined whether the participants could retain the learned concepts and skills for a longer period of time. To assess the maintenance of the intervention, each participant took a criterion test (i.e., a geometry problem-solving CBM probe) during a maintenance test 2 weeks after the completion of the intervention. Each participant completed two CBM probes, so two data points were collected for each participant except Participant A, who was absent at the time of administering the tests. The test administration procedures and conditions for the maintenance tests were identical to those of the baseline.

**Intervention Fidelity**

Intervention fidelity plays a crucial role in achieving the aim of the intervention (Bos et al., 2023; Bova et al., 2017). It shows the level of variation among interventionists in applying the intervention on the participants in order to explore the impact of that variation on the gathered data (Bova et al., 2017; O’Donnell, 2008). In addition, it shows the extent to which the intervention is applied as planned (Nelson et al., 2020). More than 30% of the intervention sessions would be observed to check that the interventions were applied following suitable procedures and data were collected under appropriate conditions (Klingbeil et al., 2017; Ledford et al., 2017). Using observation to check intervention fidelity is the most common strict way to consolidate the correctness of the gathered data, and hence the
confidence of the concluded results (Bos et al., 2023; DeFouw et al., 2019). The fidelity checklist can be found in Appendix F.

To ensure that intervention sessions were applied following appropriate procedures, more than 30% of them should be observed by a different trained observer. In this study, 44% of the 18 intervention sessions were observed by an independent evaluator after receiving a training. The evaluator used a checklist to calculate the intervention fidelity, which was 97.5%. This high percentage indicates that the interventions were applied as planned with the same level of quality among all the sessions.

**Interrater Reliability**

All participants’ answers were graded by the principal researcher based on an answer key (i.e., marking rubric) that had been made as a guide. A different mathematics teacher re-graded 40% of the tests following the same answer key. The agreement between the two raters found to be 88%.

**Social Validity**

Social validity, originally introduced by Wolf (1978), refers to participants' feedback and attitudes toward the objectives, strategies, and impact of receiving an applied intervention (Biggs & Hacker, 2021; Carr, 2005; Elliott, 2017; Hurley, 2012; Larson et al., 2020; Snodgrass et al., 2022). To assess participating SWMLDs' attitudes toward the effectiveness of the chunking intervention, the investigator developed a measure consisting of nine questions on a five-point Likert scale. Participants would score "1" to indicate they strongly disagree with the statement, "2" if they disagree, "3" if they feel neutral, "4" if they agree, and "5" if they strongly agree. After the intervention, students were asked to complete the survey and report
their experience and attitudes regarding the chunking intervention and their geometry performance. The social validity survey can be found in Appendix E.

**Data Analysis Plan**

Both quantitative and qualitative analyses were performed to examine whether and how the chunking strategy makes any differences in improving the geometry problem solving accuracy and facilitating their reasoning.

First, quantitative data analysis, based on students’ accuracy of problem solving, was performed. The investigator can display the participants' results (Kennedy, 2005) visually by graphing data points for each individual participant. Traditionally, visual analysis should be made to conduct comparisons within and between the multiple design phases (Bouwmeester & Jongerling 2020; Ferron et al., 2014; Kratochwill et al., 2010). Visual analysis helps us identify the significant existence and the strength of the relationship between the independent and independent variables (Kratochwill et al., 2010). It is a primary method for evaluating a functional relationship between an independent variable and an outcome variable. Visual data was analysed and interpreted from multiple perspectives, including the level, trend, variability, any latency between the intervention and the effect, overlap, and consistency of data patterns (Ferron et al., 2017; Horner et al., 2005; Kennedy, 2005; Kratochwill et al., 2010; Manolov & Onghena, 2022). The visual analysis requires six measures to investigate within and between the distribution of the phases data: (1) level, as determined by the average of the phase scores, (2) trend, defined as the slope of the best-fitting straight line for the phase outcomes, (3) variability, the variation of the scores about the best-fitting straight line, (4) immediacy of the impact, defined as the difference between the last three data points in a phase and the first three data points of the next phase, (5) overlapping, defined as the percentage of the
outcomes of a phase that overlaps with outcomes from previous phase, and (6) consistency across similar phases (i.e., data patterns within and between the baseline and intervention phases). (Kratochwill et al., 2010).

Effect sizes are numerical descriptors that provide brief indicators that can be used to compare the results of several studies (Hedges et al., 2013). The Percentage of Non-overlapping Data (PND) is a widely used statistic for measuring the Effect Size of single-case experimental designs (Bouwmeester & Jongerling 2020; Manolov & Onghena, 2022; Scruggs & Mastropieri, 1998). PND is calculated by dividing the number of treatment phase scores that exceed the maximum score in the baseline phase by the number of scores in the treatment phase (Scruggs et al., 1987). Typically, a PND below 70% is interpreted as questionable or ineffective, 70-90% as moderately effective, and a PND greater than 90% indicates highly effective treatment (Scruggs & Mastropieri, 1998).

In addition to the quantitative analysis-based students’ problem-solving accuracy from the above six perspectives, I also provided a nuanced qualitative analysis on students’ written solutions to reveal how the chunking strategy facilitated the problem-solving process of SWMLDs. Specifically, I analysed each participant’s worksheets from baseline assessment, through intervention phases and to post-test and maintenance test to provide an in-depth analysis of the participants’ changes of reasoning and problem-solving strategies throughout the intervention. I developed a coding scheme, which includes four aspects of students’ problem solving process: (a) students’ motivation was coded, which is indicated by students’ attempts to solve problems, if they were engaged during problem solving, etc; (b) barriers and hurdles in students’ problem solving, which were indicated by students’ difficulties with understanding the geometry vocabulary, handling the rich information presented,
retrieving needed theorems/postulates, expressing the verbal reasoning, etc., (c) students’ reasoning including mistakes in reasoning, as indicated by students’ written deductive reasoning, problem solving number sentences or equations, etc., and (d) to what extent the student followed the chunking strategy that we taught them to use, which was indicated by whether or not the student followed the prompts to solve problems.

Then I went through the students’ written solutions and coded their worksheets throughout the intervention based on the above coding rubric. Triangulation is provided by having another researcher, who has expertise in SWMLDs, to re-code and interpret selected worksheets. At last, I provided a case-by-case, longitudinal analysis for each of the three SWMLDs to provide an in-depth analysis to reveal their unique experiences, including their struggles in problem solving, hurdles that prevent them from accessing the problem or expressing their solutions, and how the chunking strategy facilitate them during the reasoning process.

**Summary of the Chapter**

This chapter details a research study employing a multiple-baseline design across participants, a widely utilized method in special education for investigating the effectiveness of interventions. This design introduces the treatment to various participants or settings at different times. The primary focus of the study is on high school SWMLDs, and an adapted multiple probe design is employed to evaluate the correlation between a chunking intervention and their enhanced performance in geometry problem-solving.

The research took place in a New Jersey high school, with dedicated sessions occurring 1-2 times weekly in a designated space. Three high school SWMLDs were selected as participants based on teacher nominations indicating their challenges in
learning geometry. Demographic data, including age, gender, and race, were gathered, revealing variations in their performances on the KeyMath-3 geometry subtest.

To assess the impact of the chunking intervention on geometry problem-solving, diverse measures were employed, encompassing curriculum-based measurements, geometry proof tests, NJSLA preparation tests, and the KeyMath-3 geometry subtest. The study adhered to a structured sequence involving baseline, intervention, posttest, and maintenance phases.

The intervention aimed to instruct participants in identifying visual chunks in geometry diagrams, recalling related theorems/properties, and solving problems through the analysis of information at the visual chunk level. Rigorous monitoring of intervention fidelity occurred, with over 30% of sessions observed to ensure accurate implementation, resulting in a reported fidelity of 97.5%. To guarantee scoring accuracy, interrater reliability was verified. Both the principal researcher and another mathematics teacher independently graded randomly selected tests, with an agreement of 88% reported.

Social validity, reflecting participants’ perspectives and attitudes toward the chunking intervention, was evaluated through a survey featuring Likert-scale questions. The data analysis plan incorporated visual analysis and statistical tests, scrutinizing measures such as level, trend, variability, immediacy of impact, overlapping, and consistency of data patterns. Effect sizes, including the Percentage of Non-overlapping Data (PND), were intended to gauge intervention effectiveness. Both visual analysis and statistical tests were considered integral for a thorough interpretation of the study's findings.
Chapter 4. Results

This study aimed to answer a main question: What is the effectiveness of the chunking strategy as a cognitive intervention on geometry performance in high school SWMLDs? Both quantitative and qualitative analyses were performed to examine whether and how the chunking strategy makes any differences in improving the geometry problem solving accuracy and facilitating their reasoning.

Specifically, I investigated data to answer these Sub-questions:

1. What effect will a chunking intervention have on the geometry problem solving performance of high school SWMLDs as measured by a proximal measure (i.e., a geometry problem solving curriculum-based measurement).

2. To what extent do high school SWMLDs maintain their geometry performance two weeks after the conclusion of the intervention as measured by a proximal measure on geometry problem solving (i.e., the geometry problem solving CBM)?

3. To what extent do high school SWMLDs generalize their geometry knowledge to distal measures
   a. To what extent do high school SWMLDs generalize the effectiveness of the chunking intervention to their geometry performance as measured with a geometry proof on the same content topics,
   b. To what extent will high school SWMLDs generalize the effectiveness of the chunking intervention to their geometry performance on selected items from the state standard test (NJSLA)?
c. To what extent do high school SWMLDs demonstrate
generalization of their geometry knowledge to selected items
from the KeyMath-3 geometry subtest?

4. What are the attitudes of high school SWMLDs towards implementing the chunking intervention?

**Quantitative Analysis of Problem-Solving Accuracy**

In this section, I described quantitative visual analyses with the measure of problem-solving accuracy (i.e., percent correct) for this single subject design study. First, to answer Research Questions 1 and 2, I reported findings on students’ accuracy changes on the criteria measure – the Geometry Problem Solving Curriculum Based Measurement (CBM), which was the criteria measure that was applied during every phase of the study, including baseline, intervention, post-test and maintenance test. I analysed the results with the six perspectives as required by visual analysis for single subject designs. Next, I reported findings on the three generalization tests, which answers Research Question 3.

**Improvement of Students’ Problem-Solving Accuracy with the Proximal Measure**

The participants' results were visually represented (Kennedy, 2005) by graphing data points for each individual participant across all four phases (i.e., baseline, intervention, posttest, and maintenance phase). The visual analysis requires six measures to investigate within and between the distribution of the phases data: (1) level (determined by the average of the phase scores), (2) trend (i.e., the slope of the best-fitting straight line for the phase outcomes), (3) variability (i.e., the variation of the scores about the best-fitting straight line), (4) immediacy of the impact (i.e., the difference between the last three data points in a phase and the first three data points
of the next phase), (5) overlapping (i.e., the percentage of the outcomes of a phase that overlaps with outcomes form previous phase), and (6) consistency across similar phases (i.e., data patterns within and between the baseline and intervention phases) (Kratochwill et al., 2010).

**Level**

The Percent of Non-overlapping Data (PND) of each participant was 100%. This indicates that all participants scored in all intervention session above their corresponding scores in the baseline phase. This means that the performance of the participating SWMLDs improved.

On the Geometry problem-solving CBA, all participants scored 0% in the baseline. In contrast, the average scores of their performance in the intervention phase were 86%, 90%, and 70% suggesting a clear improvement in participants geometry CBA as shown in Figure 2.

**Trend**

The slope of the best-fitting line (i.e., trendline) for the data points for every participant in the baseline phase was zero since every participant scored 0% in the baseline probes. As shown in Figure 2, the trendline for every participant in the baseline phase is a horizontal line that lies on the x-axis. Clearly, all participants lack the knowledge that was required to solve CBA geometry problem-solving.

Figure 3 represents the trendline for each participant during the intervention phase only. Although participants showed improvement from baseline to intervention, the trendline during the intervention phase of the scores of each participant has a slightly negative slope. However, the magnitudes of these slopes are approximately zero as shown in Table 4. This indicates that participants’ performance had
approximately a steady trend. The negative slope might be attributed to the increase in the level of complexity of the geometry lessons. The earlier three sessions targeted basic concepts of angles that were easy and straightforward. The later three sessions targeted more complicated concepts (e.g., parallel lines, perpendicular lines, corresponding angles, alternate interior angles, etc.)

**Figure 2**

*A Graph of Data Points for Each Individual Participant*
corresponding angles, etc). In addition, the first session only included geometry problems with one chunk, but the following sessions included problems with multiple chunks (e.g., including a pair of parallel lines and a pair of perpendicular lines). To solve the problems with multiple chunks usually requires multiple problem-solving steps with referring to multiple geometry theorems/postulates/properties, and thus are more challenging than the earlier problems that only require referring to geometry theorem/postulates/properties of one chunk.

**Table 4**
*Trend of the Participants’ Scores*

<table>
<thead>
<tr>
<th>Participant</th>
<th>Baseline</th>
<th>Intervention</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>y = 0x + 0</td>
<td>y = -0.04x + 0.99</td>
</tr>
<tr>
<td>B</td>
<td>y = 0x + 0</td>
<td>y = -0.03x + 1.00</td>
</tr>
<tr>
<td>C</td>
<td>y = 0x + 0</td>
<td>y = -0.04x + 0.81</td>
</tr>
</tbody>
</table>

**Figure 3**
*A Trendline for each Participant during the Intervention Phase.*
**Variability**

Variability is the variation of the scores about the best-fitting straight line within an individual participant. Every participant obtained a 0% score in the baseline CBA, resulting in a SD of 0. Analysing the data variability during the intervention phase, Table 6 illustrates that participants A and B exhibit a narrower range with a lower SD (1.88% for A and 5.66% for B), whereas Participant C displays a broader range with a higher SD. It suggested that Participant A and B demonstrated a coherent pattern of change, specifically, Participant A scored 100%, 96%, 100%, 50%, 71%, and 100% during intervention, Participant B scored 100%, 100%, 88%, 75%, 82%, and 94%. In contrast whereas Participant C scored 47%, 100%, 88%, 71%, and 42%, showing a greater variation of scores achieved and suggesting an unstable pattern of changes.

**Immediacy of Effect**

Immediacy of the impact is the difference between the last three data points in a phase and the first three data points of the next phase (Kratochwill et al., 2010). As shown in Table 5 and Figure 4, the immediacy of the participants A, B, and C were 78%, 99%, and 96%, respectively. Clearly there was a substantial increase in problem solving accuracy of the Geometry CBAs in the first three sessions of the intervention compared to the last three sessions of the baseline phase. This indicates that the intervention has a high immediacy.

Intra-individually, the Participant C has the lowest immediacy compared to Participants A and B. This corresponds to the assessment during baseline session that finds Participant C has a lowest performance in the Key-math geometry sub-test, with a Grade Equivalency of K.2. That is, the size of effectiveness of chunking strategy can differ from a participant to another.
Table 5
*Immediacy of the Effect in the Intervention Phase*

<table>
<thead>
<tr>
<th>Participant</th>
<th>Immediacy (Mean &amp; Range)</th>
<th>Variability (SD)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Baseline</td>
<td>Intervention</td>
</tr>
<tr>
<td>A</td>
<td>0% (0%-0%)</td>
<td>98% (96%-100%)</td>
</tr>
<tr>
<td>B</td>
<td>0% (0%-0%)</td>
<td>96% (88%-100%)</td>
</tr>
<tr>
<td>C</td>
<td>0% (0%-0%)</td>
<td>78% (47%-100%)</td>
</tr>
</tbody>
</table>

Figure 4
*Immediacy of Effects for the Participants on geometry CBA*
**Overlapping**

Overlapping is the percentage of the outcomes of a phase that overlaps with outcomes from previous phase (Kratochwill et al., 2010). As it shown in Figure 5, there is no data point in the baseline that overlaps with a data point in the intervention phase. This means the PND of each participant was 100%, which means that every single data point in the intervention phase was higher than the corresponding maximum data point in the baseline phase.

**Figure 5**
*Overlap data points for geometry CBA.*
**Consistency**

Consistency across similar phases can be assessed by looking for similarities in the data pattern within and between the baseline and intervention phases and for all participants (Kratochwill et al., 2010). All participants got 0% on the geometry CBA in the baseline phase, so they did not show any variation in baseline scores for all three students. Since all three participants demonstrated accuracy in the intervention, variations were observed in all three participants, yet all with at a relatively small amount. The variability of Participant B’s scores was the smallest (SD = 0.09, range = 75% to 100%). The variation of Participant C’s scores had the largest value in the intervention phase (SD = 0.23, range = 42% - 100%).

**Table 6**

<table>
<thead>
<tr>
<th>Participant</th>
<th>Data (Mean &amp; Range)</th>
<th>Variability (SD)</th>
<th>PND</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Baseline</td>
<td>Intervention</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>0% (0%-0%)</td>
<td>86% (50%-100%)</td>
<td>0.19</td>
</tr>
<tr>
<td>B</td>
<td>0% (0%-0%)</td>
<td>90% (75%-100%)</td>
<td>0.09</td>
</tr>
<tr>
<td>C</td>
<td>0% (0%-0%)</td>
<td>70% (42%-100%)</td>
<td>0.23</td>
</tr>
</tbody>
</table>

**Maintenance Effects**

Compared to their performance in the baseline phase (0% correct for every participant), two (i.e., Participant B and C) of the three participants maintained higher performance two weeks after the end of the intervention. As shown in Table 7, Participant B scored 36% and 50%, whereas Participant C scored 24% and 43%. Although their performance in the maintenance phase was higher than their corresponding performance in the baseline tests, they both scored low in the maintenance phase compared to their performance in the intervention phase. Participant A was absent and did not complete the maintenance test.
Table 7
Participants’ Performance in CBA

<table>
<thead>
<tr>
<th>Participant</th>
<th>Baseline</th>
<th>Intervention</th>
<th>Maintenance</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0%</td>
<td>86%</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0%-0%)</td>
<td>(50%-100%)</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>0%</td>
<td>90%</td>
<td>43%</td>
</tr>
<tr>
<td></td>
<td>(0%-0%)</td>
<td>(75%-100%)</td>
<td>(36%-50%)</td>
</tr>
<tr>
<td>C</td>
<td>0%</td>
<td>70%</td>
<td>34%</td>
</tr>
<tr>
<td></td>
<td>(0%-0%)</td>
<td>(42%-100)</td>
<td>(24%-43%)</td>
</tr>
</tbody>
</table>

**Generalization Effects**

As shown in Table 8, all participants in the baseline phase scored 0% in most of the distal measures (i.e., KeyMath-3 geometry subtest, related items selected from NJSILA practice test, and a proof test with items selected from the high school curriculum) of geometry problems-solving. However, in the posttest phase, three participants demonstrated variations in their scores as shown in Figure 2. Participant A improved to 67% on KeyMath-3 Geometry Subtest, 25% on the items selected from NJSILA, but remained 0% on the geometry proof test. In contrast, Participant B scored 40%, 25% and 10% on the three measures KeyMath-3 Geometry Subtest, selected items from NJSILA practice test, and the geometry proof test, respectively. The geometry subtest score of Participant B in Key-Math-3 was 8% less than her score in the baseline phase. Participant C improved from 0% in all the generalization tests to 17%, 25%, and 6% on the three measures, respectively. Interestingly, all participants consistently improved from 12% to 25% on the items selected from NJSILA.

Participants A and C made progress in the Key-math-3 geometry subtest, whereas participants B and C made progress on the geometry proof test. Participant C seemed to be the one who made greatest improvement given her lowest baseline scores, whereas Participant A made the highest performance in the posttest phase. As a conclusion, compared to their corresponding scores in the baseline, most of the
participants’ scores in the posttest showed significant enhancements on the transfer measures as it is shown in Table 8.

<table>
<thead>
<tr>
<th>Table 8 Posttest and Baseline Scores in Transferring Tests</th>
</tr>
</thead>
<tbody>
<tr>
<td>Participant</td>
</tr>
<tr>
<td>--------------</td>
</tr>
<tr>
<td>A</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>B</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>C</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

**Social Validity**

To assess participating SWMLDs' attitudes towards the effectiveness of the chunking intervention, the investigator developed a measure consisting of nine questions on a five-point Likert scale, with 1 indicating “strongly disagree,” 5 indicating “strongly agree”, and 3 indicating “neutral”. Generally, all three participants expressed favourable attitudes towards the intervention, with Participant A showing a median of 4 (agree), Participant B showing a median of 5 (strongly agree), and Participant C showing a median score of 4 (agree), suggesting that all participants gave positive evaluations to the chunking intervention. Students also provided positive comments on the effects of the intervention, for example, participants liked the way geometry was taught in this study and expressed a desire for this strategy to be applied at school.

As it is shown in Table 9, all three participants agreed or strongly agreed on the statement “I feel that I can learn geometry better after I enrolled in this project”. “I like the way of teaching geometry in this project” and on “I wish this way of teaching
would be applied in my school.” Their responses to these statements suggested their positive attitudes to the chunking strategy. However, Participate B disagreed on the statement “I like my participation in this project,” Participant C disagreed on “I like learning geometry after I participated in this project”, and both Participants B and C were unsure or neutral about the statement “I feel my performance in geometry has improved after I joined this project.”

Table 9
Participants’ Responses on the Social Validity Survey

<table>
<thead>
<tr>
<th>Statement</th>
<th>Participant A</th>
<th>Participant B</th>
<th>Participant C</th>
<th>Mean (Median)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I like my participation in this project.</td>
<td>4</td>
<td>1</td>
<td>5</td>
<td>3.33(4)</td>
</tr>
<tr>
<td>I like the way of teaching geometry in this project.</td>
<td>4</td>
<td>5</td>
<td>4</td>
<td>4.33(4)</td>
</tr>
<tr>
<td>I feel that I can learn geometry better after I enrolled in this project.</td>
<td>4</td>
<td>5</td>
<td>5</td>
<td>4.67(5)</td>
</tr>
<tr>
<td>I like learning geometry after I participated in this project.</td>
<td>4</td>
<td>5</td>
<td>1</td>
<td>3.33(4)</td>
</tr>
<tr>
<td>I wish this way of teaching would be applied in my school.</td>
<td>4</td>
<td>5</td>
<td>3</td>
<td>4(4)</td>
</tr>
<tr>
<td>I feel my performance in geometry has improved after I joined this project.</td>
<td>4</td>
<td>3</td>
<td>3</td>
<td>3.33(3)</td>
</tr>
</tbody>
</table>

Qualitative Analysis of Students’ Reasoning

In this section, I presented a nuanced analysis of the learning trajectory of the three participating SWMLDs. I thoroughly looked into participants’ written solutions in their worksheets, particularly the independent practice section that reflected students’ independent problem-solving strategies, error types, and underlying misconceptions. I particularly looked for evidence of students’ learning behaviours in the following four dimensions: motivation, barriers and hurdles in problem solving, mathematical reasoning, and the use of chunking strategy.
Participant A

I identified an interesting reversed bell-shaped curve of motivation change in Participant A. At the early stage of the intervention, Participant A initially demonstrated a high motivation to engage in the learning, as shown in her passions to try everything to solve each problem in Unit 1 (i.e., Angle relations and properties) and she was the one who immediately increased her problem-solving accuracy to 100% and maintained this motivation for all the first three sessions. However, as intervention continued to Session 4 when the Unit 2 (i.e., angle relations embedded in parallel lines and perpendicular lines) started, her motivation waned in subsequent sessions. A further examination of the lessons found that earlier lessons were primarily only cover 1 specific geometry topic. Specifically, Sessions 1-3 covered about relations of angles (e.g., supplementary angles, straight-line angles). Sessions 4-5 were specifically about angle relations involved in parallel lines. Sessions 6 was specifically about angle relations involved in perpendicular lines. Different from Session 1-3, Session 4-6 were a mixed of these problems that include multiple angle relations in parallel lines and perpendicular lines. With that said, the second half of the intervention phase introduced more complex problems, and her declining performance could be attributed to the increased difficulty of these challenges. For instance, in the fifth lesson of the intervention phase, Participant A gave up without any attempts to try on this problem that was seemingly considered as a straightforward task, as depicted in Figure 6a. However, she subsequently answered the more challenging next task which requires the same theorems, illustrated in Figure 6b, correctly. This suggests that her difficulty in answering the question may be more related to a lack of motivation or attention, as was observed.
Figure 6a

*Participant A couldn’t answer this straightforward task*

![Figure 6a](image)

Figure 6b

*Participant A managed to Answer a More Complicated Question*

![Figure 6b](image)

Another line of coding focuses on the challenges and barriers in participants’ problem solving. I found substantial evidence that Participant A were having difficulties the verbally articulate her reasoning. I found that Participant A often omitted explanations that are required to justify her problem-solving steps. Figure 7 illustrates an example that A was able to identify the geometric chunks (i.e., parallel lines, vertical opposite angles) from a sophisticated figure as the first step, however,
in the second step, when asking about related theorems or properties associated with the chunks, the student was not able to verbally articulate any of them. However, in the third step, which requires using the theorems/properties to solve the problem, the student did a good job with using numerical expressions to apply the needed theorems and solve the problems with equations. All this suggested that the student has the geometry concepts, knows the related theorems and properties, however, verbally describing the theorems and properties represented a great barrier to express her mathematical thinking.

**Figure 7a**

*Participant A Preferred Numerical Expressions rather than Verbal Statements*
Another significant barrier to Participant A’s learning is memory, which may explain her lower performance in the posttest test than during the intervention. For instance, Participant A successfully solved the problem presented in Figure 8 during intervention phase. However, she failed answering the same question in the posttest task just a couple of weeks after, as depicted in Figure 7b by showing “I do not know” on the test sheet after making some attempts to solve it and making some annotations on the diagram.

It was observable that Participant A maintained adopted the chunking strategy in the posttest phase, when all prompts were faded. As Figure 8a and 8b illustrated, although the required chunks are clearly shown. In Figure 8a she identified all the required chunks to solve the question (i.e., adjacent angles, straight-line angles, and vertically opposite angles). It is impressive to find that the Participant A re-drew the figure with only copying the upper half of the graph and eliminated the lower half, so that it is easier to see and analyse the three angles that comprised a straight angle.

The same situation occurs while she solved the posttest question that is shown in Figure 8b. Participant A re-drew the graph by eliminating the line that divided one of the vertical opposite angles. From these two examples, we can infer that Participant A adopted the chunk strategy and developed a strategy by herself to modify the diagram
to make the chunk separated from original graph so that she can operate on the chunk without any visual distractions.

**Figure 8a**

*Participant A Re-Drew the Graph to Make the Chunk Clear*

![Figure 8a](image)

**Figure 8b**

*Participant A Re-Drew the Graph to Make the Chunk Clear*

![Figure 8b](image)
Participant B

Similar to Participant A, Participant B also tended to avoid verbally describe the geometric relations with theorems. As shown in Figure 9 and Figure 12, Participant B tended to skip any needed theorem descriptions but to directly use number sentences or equations to present a solution. Additionally, she often omitted additional details about the relevant chunks necessary to solve a problem. For instance, rather than stating "straight-line angles," she simply wrote "straight," and instead of specifying "consecutive interior angles," she wrote "consecutive." Various instances of these occurrences are illustrated in Figure 9. Her focus leaned more towards manipulating mathematical algorithms, neglecting to explain the essential concepts required and failing to organize her response steps systematically.

Participant B seemed to differ in Participant A in making more conceptual mistakes about the theorems/properties with the chunks. For instance, in Figure 9, she adeptly applied the concept of alternate interior angles by stating \( x = 40 \), but erroneously misapplied the concept of consecutive angles by stating that \( X + 70 = 180 \). It suggested that Participant B confused the alternative interior angles and consecutive angles. Another example is in 10a, the Participant B she encountered challenges in pinpointing the second component (consecutive interior angles) during the maintenance test, although she was able to answer this question with complete and independent accuracy during the intervention phase (Figure 10b).
Figure 9

Participant B Misused the Concept of Consecutive Angles

Lesson (4)

Phase III: Independent Practice (15 minutes) – Task (3)

Step 1: Identify the visual chunk(s):

1. Parallel
2. Alternative
3. Consecutive

Step 2: Retrieve the theorem or property related to the identified visual chunk(s):

1. \( 40^\circ = x^\circ \)
2. \( x^\circ + 70^\circ = 180^\circ \)

Step 3: Solve the problem:

1. \( 40^\circ = x^\circ \)
2. \( 40^\circ + 70^\circ = 110^\circ \)

Figure 10a

Participant B couldn’t Solve this Problem in the Maintenance Phase

Find the value of the unknown in the diagram
A similar pair of contrasting scenarios is depicted in Figures 11a and 11b. In Figure 11a, the Participant B was able to identify the three chunks (a pair of parallel lines, a straight angle, and a pair of consecutive interior angles) in the geometry problem during the independent practice section of an intervention session and solved the problem correctly. However, in the maintenance phase, she only identified one chunk (straight angle).
Figure 11a

Participant B identified the three chunks and Solved the Problem in Intervention

Phase III: Independent Practice — Task (2)
Find the value of the unknown in the diagram

Step 1: Identify the visual chunk(s):

\[ \text{Angle } 180^\circ \]

Parallel
Consecutive

Step 2: Retrieve the theorem or property related to the identified visual chunk(s)

\[ 50^\circ + 30^\circ + m_1 = 180^\circ \]
\[ m_1 + x_1 = 180^\circ \]

Step 3: Solve the problem

\[ \begin{align*}
50^\circ + 100^\circ + 30^\circ &= 180^\circ \\
x &= 80^\circ \\
100^\circ + 80^\circ &= 180^\circ
\end{align*} \]

Figure 11b. Participant B Failed the Same Problem in Maintenance Test.
Participant C

In comparison to Participant A and Participant B, Participant C started with greater difficulties in basic geometry knowledge and mathematics skills. In the initial assessment (i.e., baseline phase), Participant C exhibited confusions with basic geometry knowledge, encompassing concepts like angle measures, parallel lines, and perpendicular lines. For example, in the baseline assessment, within the illustrated Figure 1, she inaccurately asserted that the angle measure, with a size of 40°, is equivalent to the length of its adjacent side AE, indicating a lack of conceptual understanding regarding both the magnitude of the angle and its adjacent side.

Throughout various phases, Participant C encountered challenges in performing basic addition and subtraction operations. For example, she incorrectly calculated the sum of 90 and 40, as depicted in Figure 1a. She struggled with subtracting 70 from 120, as illustrated in Figure 1. She also couldn’t calculate (80-20) and (120-40) as shown in Figure 1b. Although the use of calculators was allowed in this project, she did not make use of this accommodation.

Figure 12

Participant C confused an angle magnitude with a side length & made a mistake in calculating 120-70
Participant C couldn’t do basic addition

\[ 90 + 40 + x = 180 \]

\[ 113 + x = 180 \]

\[ x = 67 \]

Participant C demonstrated a tendency to insist employing the concept of a straight angle over alternative approaches, such as considering vertically opposite angles. For example, in Figure 14 a, rather than stating the relationship as \( X + 30 = Y \)
and subsequently determining $Y$ as $80 + 30 = 110$, she opted to rely on the concept of a straight angle. While her answer was accurate, it was observed that she exhibited a preference for utilizing the straight angle concept over other available concepts. Furthermore, she occasionally misapplied this approach, as evidenced in Figure 14b, where she inaccurately applied the idea of a straight angle.

**Figure 14a**

*Participant C utilized of the concept of a straight angle*
During the intervention phase, it was observed that Participant C encountered difficulties both in memorizing and recalling the taught theorems. While she occasionally demonstrated a good understanding of the concept, as illustrated in Figure 15, challenges arose in remembering the verbal statements of the presented theorems, as evident in her responses in Figure 15. Despite receiving instruction on organizing steps, she encountered difficulty articulating the necessary procedures coherently. This led to a lack of proficiency in effectively communicating a
mathematical proof. It is noteworthy that all participants faced challenges in constructing cohesive proof statements.

**Figure 15**

*Participant C recognised the chunks but struggled to verbally justify her problem solving*

Overall, Participant C demonstrated an understanding of the instructional content in the initial lessons, but she encountered challenges in recalling the necessary theorems and properties to solve problems. As the lessons progressed and became more sophisticated, her difficulties in identifying the required elements intensified. While she could grasp the mathematical significance of the taught theorems during instruction, recalling the relevant theorem during testing proved to be a challenge. The lower performance might be attributed to her limited mathematical background, particularly in basic arithmetic operations and conceptual understanding of the geometry concepts, further impeded her overall performance.
Summary of the Chapter

This study investigates a chunking intervention designed to enhance geometry problem-solving skills in high school SWMLDs. The analysis of data involved the visualization of participants' results across four phases: baseline, intervention, posttest, and maintenance, utilizing graphs. Significant improvement during the intervention phase was indicated by the Positive Percentage of Non-Overlapping Data (PND). Participants achieved a 100% PND, signifying that each intervention session's scores surpassed corresponding baseline scores, reflecting an overall enhancement in performance. These results suggest positive outcomes of the chunking intervention, accompanied by individual variations and challenges encountered during and after the intervention.

Trend analysis uncovered a flat baseline with 0% scores. However, intervention phase trendlines displayed negative slopes, possibly attributed to the increasing complexity of geometry lessons. Variability analysis indicated an initial 0% score and zero variability at baseline. During the intervention phase, variability increased among participants, with Participant C exhibiting the widest range. Measuring the immediacy of effect as the difference between the first and last three data points in consecutive phases revealed a significant performance leap in the first three sessions of the intervention phase. Despite general improvements, variations in individual improvement were observed. The absence of overlap between baseline and intervention data points demonstrated improvement for every participant, affirming the effectiveness of the intervention. Consistency across similar phases revealed consistent 0% baseline scores for all participants but diverse responses to the intervention. Participant B excelled during the intervention, while Participant C showed the lowest performance.
Exploration of maintenance effects showed two participants maintained higher performance in the maintenance phase compared to the baseline. However, Participant A was absent during this phase. Generalization effects were observed through distal measures such as KeyMath-3, NJS LA, and proof tests in the posttest phase. Varied progress was noted across participants. The chapter also delved into the social validity of the intervention, with participants expressing positive attitudes toward chunking while also voicing concerns.

Qualitative data provided a more nuanced understanding of individual participant experiences. All three participants demonstrated difficulties with verbally justifying their reasoning with citing related theorems or properties. Participant A's major challenge seems to be related to her motivation issues. In contrast, Participant B and C exhibited confusions with different geometry concepts (chunks) or retrieving the related theorems or properties. Participant C particularly struggled with basic geometry knowledge in the baseline phase and continued with challenges in recalling theorems and basic arithmetic operations.

Chapter 5. Discussion

This study's primary objective was to address the following key inquiry: How does the chunking strategy, when used as a cognitive intervention, impact the geometry performance of high school SWMLDs? In addition, this research endeavours to explore four subsidiary questions: (1) what influence does a chunking intervention have on the problem-solving performance of high school SWMLDs in geometry, assessed through a proximal measure (specifically, a geometry problem-solving curriculum-based measurement); (2) to what extent do high school SWMLDs sustain their geometry performance two weeks after the intervention's completion, as assessed by a proximal measure for geometry problem solving (i.e., the geometry
problem-solving CBM; (3) to what degree do high school SWMLDs generalize their geometry knowledge to more distant assessments, including geometry proofs covering the same content topics, selected items from the state standard test (NJSLA), and selected items from the KeyMath-3 geometry subtest; and (4) what are the perspectives and attitudes of high school SWMLDs regarding the implementation of the chunking intervention?

**Effectiveness of the Chunking Intervention**

The inquiry of the Research Question 1 examines the efficacy of the chunking strategy as a cognitive intervention for enhancing the geometry performance of high school SWMLDs. Results of visual analysis suggest that participants enhanced their performance in the intervention phase compared to the baseline phase on the proximal measure of geometry problem solving test. All participants (A, B, and C) achieved 0% in the baseline phase. In contrast, during the intervention phase, their average performance scores were 86%, 90%, and 70%, respectively. Each participant’s PND was 100%, indicating that every data point in the intervention phase exceeded the maximum data point in the baseline phase.

However, despite this improvement during the intervention phase, each participant's score trendline displayed a slightly negative slope. The magnitudes of these slopes were nearly zero, and these negative slopes may be attributed to the increasing complexity of the geometry lessons. While the earlier lessons focused on basic angle concepts, subsequent lessons introduced more intricate topics, such as parallel lines, alternate interior angles, corresponding angles, and so on. With that said, tasks in later sessions require participants to identify multiple geometry chunks (i.e., angle relations, angles embedded in parallel lines, angles embedded in perpendicular lines) in a sophisticated task. Participants tackled more challenging
tasks in the later lessons compared to the earlier ones. This is consistent with previous research that students showed more struggles in solving problems involving multiple geometry chunks and requires multiple steps (Zhang, et al., 2021).

**Maintenance Effects.** This inquiry of Research Question 3 seeks to assess the ability of high school SWMLDs to retain their geometry skills for a period of two weeks following the conclusion of the intervention. We evaluated this using a proximal measure of geometry problem-solving proficiency, specifically the geometry problem-solving CBM. It is important to note that participant A was unable to participate in the maintenance probe session. However, the remaining two participants (i.e., B and C) displayed improved performance in the maintenance phase compared to the baseline phase. Participant C, for instance, demonstrated a noteworthy improvement from 0% in the baseline phase to 34% in the maintenance phase, while Participant B’s performance increased from 0% to 43% during the maintenance phase. Even though both participants recorded lower scores in the maintenance phase compared to the intervention phase, they still exhibited the retention of certain knowledge and problem-solving skills associated with the CBM probe. Unfortunately, due to constraints imposed by the school, we were unable to conduct additional testing sessions during the maintenance phase, which would have been beneficial in investigating the trend of SWMLDs performance in the maintenance phase.

The chunking strategy has consistently demonstrated its effectiveness in assisting learners with the storage and retrieval of knowledge. This effectiveness is supported by findings from various studies (Isbilen et al., 2020; Perlman et al., 2010; Sweller, 2010). The outcomes of this study align with these findings, indicating that chunking can indeed aid learners in preserving knowledge in their memory. However, the extent to which the implementation of the chunking strategy may result in
improved performance, as compared to traditional teaching and learning methods, remains unanswered.

**Generalization Effects.** Research Question 4 pertains to the extent to which high school SWMLDs apply their geometry knowledge to three distal measures: a geometry proof test, selected items from the NJSLA, and the KeyMath-3 geometry subtest. The items of the three distal measures were selected from two standardized tests and covered the topics (angle relations, parallel lines, and perpendicular lines) taught in the intervention. Results revealed that, during the posttest phase, all participants demonstrated higher performance levels across all these assessments when compared to their baseline phase, except that Participant A, who did not show improvement in the geometry proof test, and Participant B who slightly scored lower in the key-math-3 geometry subtest. The observed enhancements in performance ranged from a 6% to a 25% increase.

It is noteworthy to mention that the types of questions featured in these distal measures (i.e., the geometry proof test, NJSLA, and the KeyMath-3 geometry subtest) differed from those in the proximal assessment (i.e., the geometry CBA). Both the NJSLA and the KeyMath-3 consisted of standardized items that assessed knowledge and skills that were not explicitly taught during the intervention phase. Furthermore, the geometry proof test required the application of mathematical proofing skills that were not explicitly taught in the intervention phase. It indicates that proof problems may require unique reasoning skills different from regular problem solving, as our participants seemed not quite understanding what proof meant and often confused between the known and unknown, or between an argument to be proved and a conjecture they made.
**Student Attitudes.** This social validity survey investigates the attitudes of high school SWMLDs towards implementing the chunking intervention. These attitudes were assessed using a measure consisting of nine questions on a five-point Likert scale. Overall, the participants had positive attitudes towards the intervention, but they also expressed some concerns. For example, two participants liked the way geometry was taught in the study and expressed a desire for this strategy to be applied at school. However, one participant strongly disagreed with this statement. Additionally, while all three participants strongly agreed that they felt they could improve their geometry performance after participating in the project, they did not believe that the project would improve their performance in learning geometry in the long run. Further exploration is crucial to assess the long-term effectiveness of chunking on the performance of SWMLDs. In summary, the SWMLDs in this study exhibited a positive inclination toward using the chunking strategy for solving geometry problems. This optimistic viewpoint highlights the beneficial impact of employing chunking as a cognitive approach.

**Summary:** Results from the single subject design suggest a functional relation between the intervention and the three students’ performance changes. These findings resonate with earlier research studies (e.g., Cheng & Obaidellah, 2009; Gobet, 2005; Karatzias et al., 2016; Neumann & Kopcha, 2018; Obaidellah & Cheng, 2015; Perlman et al., 2010; Schlaghecken et al., 2000) that have explored the efficacy of chunking as a learning strategy. Moreover, they align with previous research by Zhang and colleagues (Zhang et al., 2012, 2014, 2015, 2021), which concluded that highlighting visual chunks is an effective approach in assisting elementary and high school SWMLDs in solving geometry problems.
Common Patterns & Individual Differences

There are a few common patterns identified across the three SWMLDs. First, all SWMLDs significantly increased their accuracy from baseline to intervention, and partially maintained the improvement during the maintenance test. They also showed common patterns in the improvement on the generalization measures, with greater improvement on geometry problem solving tasks of the same topics in two standardized assessments whereas less improvement on the geometry proof of the same topics from the same mathematics curriculum. It indicates that proof problems may require unique reasoning skills different from regular problem solving. Second, all three participants showed a drop in accuracy in Session 2, when they transition from Unit 1 to Unit 2 in which tasks involves a mixture of chunks, yet all three participants improved during Session 5 and Session 6 in the same unit. Third, qualitative data analysis suggested that all three participants struggle with (a) verbally expressing their mathematical reasoning, in particular, citing the related theorems to support or justify their reasoning, (b) organizing their problem-solving steps in written formatting, and (c) maintaining the learned skills over time.

Besides the common patterns, significant individual differences were noticed in both quantitative analysis with the single subject design and the qualitative analysis. First, although all three participants were referred by their school teachers as students with learning difficulties in mathematics and scored below 30% in KeyMath-3 Geometry subtest, they came to the intervention with very different mathematics prerequisite skills. Specifically, Participant A ranked 25% in Geometry Subtest and her major problem, throughout the intervention, seemed to be highly related to her motivation issue. Participant A often gave up when she encountered challenges and was the only one absent from the maintenance test. However, Participant A was the
one who seemed to be most responsive to the intervention, with regard to the immediacy, stability of improvement, level of accuracy scores, and performance on transfer measures. Participant A did not show major issues with identifying the major geometry chunks, or confusing among different geometry concepts. In later sessions of the intervention, she was able to creatively develop a strategy by re-drawing the graph only showing the target chunk by eliminating other distracting parts from the original graph. In contrast, Participant B and C demonstrated greater difficulties in understanding the geometry concepts. Specifically, they often confused different angle pairs and thus wrongly apply the associated theorems/properties. Participant B highly replied on one chunk (right angle) which she was confident in and was reluctant to use other chunks. Participants C, who ranked 0.4% in the Key-Math3 Geometry subtest, was observed to have significant deficiencies in essential mathematics fundamentals and skills. For instance, she made errors in simple arithmetic calculations like subtraction and struggled with solving basic linear equations. As the intervention sessions advanced, these skills became more crucial, contributing to the slight performance decline in participating SWMLDs (Geary et al., 2020; Lein et al., 2020).

**Theoretical Implications**

This study supports the application of chunking as a problem-solving strategy in solving geometry problems for students with mathematics learning difficulties. A "chunk" refers to a meaningful unit with a unique conceptual structure, and chunking is the process of organizing these units into a cohesive combination, leading to a sophisticated overall structure (Chase & Simon, 1973; Miller, 1956). A chunk reduces the load on the working memory by retrieving a compact chunk representation from the long-term memory that replaces the representations of individual elements of the
chunk (Thalmann et al., 2019). The chunking strategy has been identified in various problem-solving domains, including activities like playing chess (Gobet & Simon, 1998) and typing on typewriters (Yamaguchi & Logan, 2016).

Although it has been well-documented (Fonollosa et al., 2015; Rabinovich et al., 2014) that experts use chunking strategy that group elements as patterns, whereas new learners tend to operate on many individual elements without being able to identify the meaningful chunks, there has been rare research further on how we can help these lower achievers to identify and operate on chunk. Given that the literature suggested that many students struggling with mathematics are having working memory issues, it is plausible to hypothesize that chunking strategy would be more effective for SWMLDs than their peers without working memory issues.

We particularly chose the high school geometry problems solving as this is a subject that is quite cognitive demanding. In the realm of geometry problem-solving, students must simultaneously comprehend verbal statements associated with a diagram and correlate them with the elements depicted in the diagram. The demands of these mental tasks, requiring high cognitive processes, can potentially make the working memory overloaded. Based on the cognitive load theory, cognitive load can be manipulated by changing the nature of the information (Chen et al., 2015), for example, operating a few information chunks or many information elements. Using the chunking technique can make it easier for the working memory to process information, thus assisting SWMLDs in solving geometry problems (Baddeley & Hitch, 1974; Hawes et al., 2017; Pittalis & Christou, 2010; Schoevers et al., 2020).

This study advances the chunking theory by showing that we can effectively teach students to identify meaningful chunks and operate on chunks to solve problems. In this study, SWMLDs were observed using the chunking strategy to
approach geometry problems. After a review of geometry concepts that they had already learned in their geometry classes, the three SWMLDs were prompted to retrieve this knowledge and apply the chunking strategy based on appropriate reasoning to construct solutions for the given geometry problems.

This study also documented the challenges or difficulties in the three participants in using the chunking strategy, which helps us to understand under what conditions chunking may not be effective and how to improve that. In this study, we observed that Participant B and C demonstrated greater struggles with the basic geometry concepts, such as the special angle pairs, especially the angle pairs embedded in parallel lines or perpendicular lines. Consequently, the weak conceptual understanding of the geometry concepts hindered Participant B and C from identifying the chunks and retrieving needed theorems or properties to solve the problems. After all, as the literature suggest, expertise is essential for forming chunks (Chase & Simon, 1973). Chunking strategy is based on the assumption that the students have the expertise to form a meaningful chunk, and it is effective in prompting students to actively recognize the meaningful chunks from a sophisticated background; however, if an individual does not have the knowledge to form a meaningful chunk, the effects of this strategy can be compromised.

With that said, the recommending the chunking strategy should not replace a focus on concept instruction. For any chunking strategies, identifying meaningful patterns from many elements needs to base upon expertise in recognizing the underlying meanings and relations among the elements. For example, without expertise in recognizing the underlying patterns in a chess play, prompting a new player to identifying “any patterns” would be meaningless because the new player
simply does not have the expertise to find the patterns or generating any responding strategies even if someone points out the patterns.

**Contributions to the Literature**

This study contributes to the literature by piloting a cognitive strategy for improving geometry performance among high school SWMLDs. Geometry is an important yet under studied area (Bergstrom & Zhang, 2016; Zhang, 2021), and represents the weakest mathematics domain in the U.S. students in international comparison studies (Chen, et al., 2021). Unfortunately, sparse research exists with regard how to help struggling students to learn geometry, and the only seven intervention studies identified in a meta-analysis predominantly focus on the elementary geometry level (Liu et al., 2021).

Many students, particularly at the high school level, find mathematics challenging (Hraste et al., 2018). While elementary and middle school level geometry still focus on basic shapes, areas, perimeters and related properties etc, high school geometry requires advanced reasoning including using theorems, postulates, and properties to perform deductive reasoning to solve problems (CCSSM, 2011). In addition, different from numerical problems, geometry problem solving requires students to effectively coordinate numerical, text, and visual information simultaneously for problem solving (Liu, et al., in press), which makes it particularly changeling for students with working memory issues. This study aims to examine the chunking strategy, which has been well research in basic psychology to support working memory, on helping high school SWMLDs on solving geometry problems. As a pilot study, this project applied a multiple baseline design to examine whether there was a functional relation between the chunking intervention and students’ performance changes in geometry problems solving. Students’ worksheets were also
thoroughly examined to uncover patterns of students’ nuanced barriers, misconceptions, challenges during the intervention.

A key criticism of current interventions in special education for students with learning disabilities or SWMLDs revolves around their historical association with behaviorism and clinical approaches, coupled with a noticeable lack of a robust theoretical foundation (Gersten, Beckmann et al., 2009). Another contribution of this study is its piloting in integrating heuristic pedagogy into structured instruction which was recommended in the new Practice Guidance for teaching mathematics to students with disabilities (Fuchs, et al., 2021). This study integrated the strategy instruction with interactive dialogues between the instructor and students. During one-on-one individualized tutoring, the instructor first reviewed the related concepts (e.g., angle relations, parallel lines, perpendicular lines) together with the students, with an aim to help students to form the “chunks”. In the interactive structured instruction, the teacher prompted the students to recognize the “chunks” in a complicated diagram with referring to the given information from the item text. Cues or prompting questions were provided to facilitate students to identify the “chunk” and relate to the prosperities/theorems that they already learned in previous classes. Participant A was found to develop her own strategy to make the target chunk simplified by re-drawing the diagram and eliminating distracting visuals from the target chunk.

Analysis on students’ common patterns and individual differences throughout the intervention deepens our understanding of how pre-requisite and fundamental mathematics skills affect students’ response to intervention. Learning an advanced mathematics concept builds on a number of pre-requisite skills; and the inadequate preparation with these pre-requisite skills usually hinder students’ progress in learning new materials. In this study, we observed that Participant B and C demonstrated
greater struggles with the basic geometry concepts, such as the special angle pairs, especially the angle pairs embedded in parallel lines or perpendicular lines. Consequently, the weak conceptual understanding of the geometry concepts hindered Participant B and C from identifying the chunks and retrieving needed theorems or properties to solve the problems.

The analysis on the three participants’ barriers, hurdles and challenges encountered in their problem-solving shed lights on our understanding of the difficulties that SWMLDs have to face. The qualitative portion of data analysis revealed the language challenge faced by these SWMLDs, which is consistent with the literature that many SWMLDs struggle with mathematics learning because of limited verbal skills to display their ideas (Chow et al., 2021; van der Walt, 2009). A lack of the needed geometry vocabulary is particularly significant in students with learning difficulties (Driver & Powell, 2017; Hughes et al., 2016; Powell et al., 2019) and how to facilitate students’ verbal expression would be an essential direction for future research and practice. More importantly, this study also documented some students’ lack of conceptual understanding of the geometry concepts, such as confusing among different angle pairs in parallel lines or perpendicular lines, indicating the needs of effective instruction of the concepts.

This study contributes to the literature by adding evidence of applying chunking strategy in the geometry domain for SWMLDs, and this application may be expanded to other mathematics domains for SWMLDs in future research. Although the impact of applying the chunking strategy has been extensively studied and well-documented in psychology fields, only a limited number of studies in the literature explore its advantages in learning mathematics, specifically in the context of geometry. This study expanded the previous research on the use of the chunking
strategy as a testing accommodation (e.g., Zhang et al., 2012, 2014, 2015, 2021), which was passively provided by test designers, to teaching SWMLDs to actively use this strategy in their own problem solving. While the primary emphasis of the study centered on implementing chunking in geometry instruction, its relevance extends across various mathematical domains, including algebra, data analysis, and numerical operations. The documented benefits of the chunking strategy in reducing memory load and facilitating cognitive processing underscore its potential impact. Consequently, this study provides a gateway for additional research into the effects of the chunking strategy in other areas of mathematics.

**Implications for Practice**

The encouraging findings from this study on the utilization of the chunking strategy imply that educators and high school SWMLDs should contemplate incorporating chunking into their teaching and learning approaches. It is noteworthy that the individual differences in intervention effectiveness, possibly due to varying conceptual understanding of the geometry concepts, which may suggest that advocating for the chunking strategy could not replace the mathematics instruction of geometry concepts. However, it is possible to integrate the chunking strategy into the concept instruction in geometry classes. Through structuring lessons based on chunking strategies, educators can facilitate the acquisition of geometry skills among high school SWMLDs. That is, after teaching students with the geometry terms, concepts, and theorems, students may be prompted to look for chunks, which are based on the concepts taught in classes, to solve geometry problems, especially for problems involve multiple concepts or chunks. Based on the results of this study, it is recommended that teachers identify the required chunks for learning the new concept. Learners should be able to retrieve these chunks.
Those educators who develop math curricula may also consider incorporating chunking strategies into the design of geometry-related examples and exercises. When introducing new geometry concepts, educators may present tasks in meaningful chunks and assist students in organizing these chunks to address the given problem. Each lesson plan in geometry textbook or students’ activities book should be designed based on chunking strategy. The learner while reading the textbook should apply chunking strategy that is approved to facilitate learning. With practice, students can develop proficiency in using chunking techniques to solve geometry-related challenges.

Applying beneficial instructional aids is crucial in teaching mathematics, especially for SWMLDs (Marita & Hord, 2017). Furthermore, it is essential to construct these teaching tools based on a cognitive theoretical framework (Anggo & La Arapu, 2018). Consequently, following the findings of this study, it is recommended that teachers develop their instructional aids using the chunking strategy in a manner that facilitates the learning of geometry concepts for SWMLDs. The instructional aid should elucidate the chunks of the represented geometry concepts. For instance, teachers can employ different colors for each chunk to emphasize their presence. Ensuring the incorporation of the chunking strategy into the instructional aid aligns with employing the chunking strategy as a learning method. This alignment can assist SWMLDs in comprehending geometry concepts and tackling geometry problems.

Researchers have asserted that technology-based interventions can facilitate and enhance the learning process for SWMLDs (Doabler et al., 2019; Myers et al. 2021; Ran et al., 2022). Incorporating digitalized interactive components can yield significant benefits in enhancing the understanding of mathematical concepts (Alhadi
et al., 2023). Consequently, educators in general and teachers in particular should integrate the chunking strategy into the implementation of technology-based activities. For instance, self-learning educational video clips designed for students to grasp mathematical concepts should adhere to the chunking strategy in presenting the concepts to learners, especially by visually highlighting or representing the concepts (Orban et al., 2008; Stieff et al., 2020).

In conclusion, the findings of this study carry various implications for educational practice. Broadly, learning activities should be structured such that SWMLDs employ the chunking strategy, aiding in alleviating working memory load and thereby facilitating the learning process. The implications of this study extend beyond the confines of teaching mathematics in classrooms; they encompass other essential domains such as constructing mathematics curricula, conducting learning assessments, and implementing instructional aids, especially with the integration of recent technology.

This study explores the impact of a face-to-face teaching intervention, delving into the effectiveness of implementing the chunking strategy within the regular classroom setting, where a teacher guides a group of learners. Additionally, it prompts further investigation into the application of the chunking strategy as an online intervention, whether for individual learners or groups, given the contemporary importance of technology. Moreover, the study advocates for the examination of incorporating interactive digitalized components when applying the chunking strategy (Alhadi et al., 2023). By doing so, this research contributes to the literature by paving the way for an innovative combination of integrating evidence-based interventions related to chunking, aiming to create a conducive learning environment for high
school students with specific learning disabilities in geometry and other mathematics domains.

**Study Limitations**

A multiple baseline research study serves as a pilot for researchers to assess the effectiveness of an intervention when dealing with a limited number of participants, allowing for concentrated attention on this selected group (Lanovaz & Turgeon, 2020; Gierut et al., 2015). Consequently, multiple baseline designs have gained extensive application, particularly within the realm of special education (Christ, 2007; Hembry et al., 2015; Klingbeil et al., 2017; Ledford & Zimmerman, 2023). However, it is important to acknowledge that multiple baseline designs cannot act as a replacement or complete substitute for large scale, randomized controlled trials (Rhoda et al., 2011). Therefore, it is advisable to pursue further research resembling this study or replicate it using randomized controlled trials with a large sample size of participants.

In this specific study, the intervention phase was restricted to only six sessions, as influenced by the school schedule. It would have been more advantageous to include a greater number of sessions. Due to multiple instances of absence, Participant C’s data points were limited to four in the baseline phase and five in the intervention phase. Furthermore, the study featured only three SWMLDs as participants, and their learning performance might vary due to various potential influencing factors, such as mathematics anxiety, self-motivation, mathematical background, learning attitude, family socioeconomic status (SES), and more. It is advisable to monitor these factors as comprehensively as possible in future research endeavors. Additionally, it is worth noting that this study exclusively concentrated on fundamental geometry concepts (e.g., angles, parallel lines, perpendicular lines, etc.).
Subsequent investigations should consider exploring the impact of the chunking strategy on the acquisition of more advanced geometry concepts (e.g., congruence of triangles, similarity of triangles, circle theorems, etc.).

The research was conducted in the latter part of the second semester, amid time constraints and the school's engagement in various extracurricular activities. Initially, the suggestion was to conduct this research as an after-school program. However, it was observed that students frequently missed these sessions. With cooperation from the school, the sessions were re-scheduled after the lunch break. Nevertheless, it was notable that students sometimes appeared drowsy and less attentive when they attended the sessions immediately after having lunch. Additionally, it was observed that participants did not take the assigned tasks with the same level of seriousness as they did for their regular mathematics class because there is no grade or penalty for not completing the tasks. Therefore, it is recommended to replicate this study, ensuring that participants receive the sessions in their regular mathematics classroom as part of the assigned tasks in the geometry class.

The research employed a traditional face-to-face teaching approach, suggesting a need to explore the effects of implementing the chunking strategy within a classroom setting where a collective of learners is instructed simultaneously. Additionally, many educators and learners presently engage in computer-based or synchronous learning environments. Consequently, the outcomes of this study should be confined to face-to-face learning and teaching settings. Further research may assess the effectiveness of the chunking strategy when implemented in instructive online or computerized instructional contexts, particularly when modern digitalized components have been incorporated as instructional aids, since these tools might be beneficial for SWMLDs (Alhadi et al., 2023; Benavides-Varela et al., 2020).
Summary of the Chapter

This research aimed to evaluate the impact of the chunking strategy, a cognitive intervention, on the geometry performance of high school SWMLDs. The investigation focused on four primary inquiries: the influence of chunking on problem-solving, the sustainability of geometry skills post-intervention, the generalization of knowledge across various assessments, and student perspectives on the chunking intervention. To assess the efficacy of the chunking intervention, the study analyzed its influence on geometry problem-solving performance. Visual analysis revealed a notable improvement during the intervention phase compared to the baseline. However, as the complexity of geometry lessons increased, participants' scores showed a decline. Individual differences surfaced, suggesting the necessity to explore contributing factors such as varying pre-requisite skills and non-cognitive factors such as session timing and participant motivation.

The maintenance effects of the chunking intervention were evaluated two weeks after completion. Despite a decrease in scores compared to the intervention phase, participants exhibited retention of specific knowledge and problem-solving skills related to geometry. The school environment limitations constrained additional testing sessions during the maintenance phase, leaving certain aspects unexplored. The study delved into generalization effects, demonstrating that participants achieved higher performance levels in distal assessments (geometry proof test, NJSLA, KeyMath-3) during the posttest phase compared to the baseline. The distal measures presented different types of questions from the proximal assessment, indicating a broader application of geometry knowledge. A social validity survey probed student attitudes toward the chunking intervention, revealing positive inclinations but also some concerns. While participants expressed positive views about the impact of the
chunking strategy on geometry problem-solving, they questioned its long-term effectiveness.

Theoretical implications were discussed, underscoring the chunking strategy as a valuable approach for SWMLDs to address geometry problems. The study extended existing research by showcasing the effectiveness of teaching students to identify and operate on meaningful chunks to solve problems, contributing to the chunking theory. More importantly, this study documented the challenges and difficulties that SWMLDs experienced when using the chunking strategies, which is informative for educators and researchers to consider any limitations of chunking strategies, and under what conditions the effects of chunking strategy could be compromised; and thus provide improved intervention programs for SWMLDs. Given the important role of expertise in geometry conceptual understanding in forming geometry chunks, we recommend chunking strategy to be integrated with, rather than replace, the geometry concept instruction.

The contributions to the literature were emphasized, highlighting the novelty of the study in focusing on the chunking strategy's effectiveness in geometry for high school SWMLDs. The research incorporates interactive heuristics into a structured instruction for SMLDS, and addressed a gap in the literature by exploring the advantages of chunking in the context of advanced geometry, aligning with educators' calls for increased attention to SWMLDs.

Practical implications were discussed, suggesting that educators integrate chunking into teaching approaches, lesson plans, curricula, and assessment strategies. The study emphasized the potential benefits of the chunking strategy in reducing working memory load and facilitating the learning process for SWMLDs.
The study acknowledged its limitations, including the use of a multiple baseline design with a limited number of participants and sessions. Constraints such as session timing and participant engagement were noted, and the study's applicability to face-to-face learning settings was highlighted, encouraging further research in online or computerized instructional contexts.

In conclusion, the study provided valuable insights into the effectiveness of the chunking strategy as a cognitive intervention for high school SWMLDs in geometry, offering implications for both theoretical understanding and practical application in educational settings.
Appendices

Appendix A. Criterion test: Geometry Problem Solving CBM

1. $\overline{EF} \parallel \overline{CB}$, $\overline{AB} \parallel \overline{CF}$, $\overline{CE} \perp \overline{EF}$ and $\angle ABC = 40^\circ$. Find the $\angle ECB$.

2. $\overline{AF} \parallel \overline{BC}$, $\overline{EF} \parallel \overline{DC}$, and $\angle FAB = 40^\circ$. Find $\angle ABC$ and $\angle CBD$.

3. $\overline{BC} \parallel \overline{AD}$, $\overline{AB} \parallel \overline{DC}$, $\angle BCD = 60^\circ$, $\angle ABD = 70^\circ$. Find $\angle BAD$, $\angle CDF$, $\angle BDA$, and $\angle CDA$. 

\[\text{Diagram of the geometry problem solving CBM.}\]
4. Find the value of \( x \).

5. \( \overline{EA} \parallel \overline{GF} \parallel \overline{CB} \), and \( \overline{BA} \parallel \overline{CD} \). Find two angles with equal sizes.

Two supplementary angles.

Appendix B. Geometry Proof Test

1. \( \overline{AB} \parallel \overline{CD} \parallel \overline{EF} \), \( \overline{AN} \parallel \overline{CM} \parallel \overline{EG} \), and \( \angle A = 20^\circ \). Prove that \( \angle NCM = 20^\circ \).
2. \( \overline{AG} \parallel \overline{CD}, \overline{FB} \parallel \overline{ED} \), and \( \overline{FE} \parallel \overline{GD} \).

Find \( \angle GAB, \angle BGD, \angle EDG, \angle AGB, \angle GBA \).

Prove that \( \overline{FE} \parallel \overline{AC} \).

3. \( ABCD \) is a rectangle, \( \overline{EF} \parallel \overline{BC} \). Prove that \( \overline{EF} \perp \overline{FB} \).

4. \( ABC \) is a right triangle, \( \angle CAB = 50^\circ \), and \( \overline{AB} \parallel \overline{ED} \). Prove that \( \overline{BC} \perp \overline{ED} \).

5. \( \overline{BD} \perp \overline{ED}, \overline{ED} \parallel \overline{FC} \parallel \overline{AB}, \) and \( \overline{FA} \parallel \overline{DB} \). Prove that \( ABCF \) is a rectangle.
Appendix C. Selected items from NJ LSA

Question (1) & (2)

Use the information provided to answer Part A and Part B for question 3.

In the figure shown, \( \overline{CF} \) intersects \( \overline{AD} \) and \( \overline{EH} \) at points \( B \) and \( F \), respectively.

3. Part A

- Given: \( \angle CBD \equiv \angle BFE \)
- Prove: \( \angle ABF \equiv \angle BFE \)

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<tr>
<td>( \angle CBD \equiv \angle BFE )</td>
<td>Given</td>
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<tr>
<td>( \angle CBD \equiv \angle ABF )</td>
<td></td>
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<tr>
<td>( \angle ABF \equiv \angle BFE )</td>
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Which two of the given reasons could be used to correctly complete the proof?

A. Definition of congruent angles
B. Congruence of angles is reflexive
C. Congruence of angles is symmetric
D. Congruence of angles is transitive
E. Vertical angles are congruent
Part B

- Given: $\angle CBD = \angle BFE$
- Prove: $\angle BFE + \angle DBF = 180^\circ$

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<tr>
<td>$\angle CBD = \angle BFE$</td>
<td>Given</td>
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<tr>
<td>$\angle CBD + \angle DBF = 180^\circ$</td>
<td></td>
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<tr>
<td>$\angle BFE + \angle DBF = 180^\circ$</td>
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Which two of the given reasons could be used to correctly complete the

A. Adjacent angles are congruent
B. Adjacent angles are supplementary
C. Linear pairs of angles are supplementary
D. Reflexive property of equality
E. Substitution property of equality
F. Transitive property of equality
Question (3):

The figure shows two perpendicular lines, $s$ and $r$, intersecting at point $P$ in the interior of a trapezoid. Line $r$ is parallel to the bases and bisects both legs of the trapezoid. Line $s$ bisects both bases of the trapezoid.

Which transformation will always carry the figure onto itself?

Select all that apply.

A. a reflection across line $r$
B. a reflection across line $s$
C. a rotation of $90^\circ$ clockwise about point $P$
D. a rotation of $180^\circ$ clockwise about point $P$
E. a rotation of $270^\circ$ clockwise about point $P$
Question (4):

The figure shows lines $r$, $n$, and $p$ intersecting to form angles numbered 1, 2, 3, 4, 5, and 6. All three lines lie in the same plane.

Based on the figure, which of the individual statements would provide enough information to conclude that line $r$ is perpendicular to line $p$?

Select all that apply.

A. $m\angle 2 = 90^\circ$

B. $m\angle 6 = 90^\circ$

C. $m\angle 3 = m\angle 6$

D. $m\angle 1 + m\angle 6 = 90^\circ$

E. $m\angle 3 + m\angle 4 = 90^\circ$

F. $m\angle 4 + m\angle 5 = 90^\circ$
Appendix D. Example Lesson Plans

Each lesson will be divided into three stages: Structured Interactive Instruction, Guided Practice, and Independent Practice. The duration of these stages are 20 minutes, 15 minutes, 15 minutes respectively. In the Structured Interactive Instruction, the interventionist explains to the student the mathematical concepts and theorems. In the Guided Practice, the student should be encouraged to solve geometric problems individually with help from the interventionist as needed. In Independent Practice, the researcher will use a problem-solving probe to measure the student’s performance after receiving the intervention; and students should answer the test independently without any help.

Unit 2. Parallel Lines.

Lesson 1

Review: In this unit, the instructor will first guide the students to review the basic concepts, terms, and theorems.

Terms: Parallel lines, alternate exterior angles, corresponding angles, consecutive interior angles, alternate interior angles,

Activity: Ask students to name the angle pairs in the graph below

![Graph of parallel lines with angle pairs labeled]

Theorems, Postulates and Properties

Corresponding Angles Theorem: If a transversal intersects two parallel lines, the corresponding angles are congruent.
Converse of the Corresponding Angles Theorem: If two lines and a transversal form corresponding angles that are congruent, then the lines are parallel.

Alternate Interior Angles Theorem:
If a transversal intersects two parallel lines, then alternate interior angles are congruent.

Converse of the Alternate Interior Angles Theorem:
If two lines and a transversal form alternate interior angles that are congruent, then the two lines are parallel.

Alternate Exterior Angles Theorem: If a transversal intersects two parallel lines, then alternate exterior angles are congruent.

Converse of the Alternate Exterior Angles Theorem: If two lines and a transversal form alternate exterior angles that are congruent, then the two lines are parallel.

Same-Side Interior Angles Postulate: If a transversal intersects two parallel lines, then the same-side interior angles are supplementary.

Activity: Ask students to find the congruent angles in the parallels above.

Phase I: Structured Interactive Instruction (20 minutes)

(1) The figure at the right shows the front view of a cellular phone tower’s structure. Suppose that lines \( m \) and \( n \) are parallel and that \( m\angle 1 = 46^\circ \) and \( m\angle 6 = (2x)^\circ \). Find the value of \( x \).

Step 1. Identify the chunks.

- Identify the pair of parallel lines (\( m \) and \( n \)) in the graph and ask students to highlight/mark the parallel lines.
Step 2: Retrieve related information about the identified chunks from long term memory.

- Since \( m/n \), then which pairs of angles are congruent?

Step 3. Problem solving

- What is known: \( m/n, m\angle 1 = 46^\circ \)
- What is to be known: \( m\angle 6 = (2x), \text{ what is } x? \)
- Using any of the theorems reviewed in Step 2, which can be inferred from \( \angle 1 \)?
  - \( \angle 5 \text{ and } \angle 1 \text{ are corresponding angles, and thus } \angle 5 = \angle 1 = 46^\circ \)
  - \( \angle 5 + \angle 6 = 180^\circ, \text{ then } \angle 6 = 180^\circ - 46^\circ = 134^\circ \)
  - \( 2x = 134^\circ, \text{ so } x = 67^\circ \)

(2) \( \overline{AC} \parallel \overline{BD} \), and \( \angle 1 = 80^\circ \). Find \( \angle CAB \)

Step 1. Identify the chunks.

- Identify the pair of parallel lines (\( AC \) and \( BD \)) in the graph and ask students to highlight/mark the parallel lines.

Step 2: Retrieve related information about the identified chunks from long term memory.

- Since \( AC \parallel BD \), what postulates/theorems about angle pairs do you know?

Step 3. Problem solving
• What is known: AC//BD, M∠ABD = 80°.

• What is to be known: M∠CAB?

• Using any of the theorems reviewed in Step 2, which can be inferred from M∠ABD
  
  o  M∠ABD + M∠CAB = 180°
  
  o  so, M∠CAB = 180° − M∠ABD = 100°

(3) \( \overline{EA} \parallel \overline{GF} \parallel \overline{CB} \), and \( \overline{BA} \parallel \overline{CD} \). Find

  a) Two angles with equal sizes.
  
  b) Two supplementary angles.

Step 1. Identify the chunks.

• Identify the two sets of parallel lines \( (\overline{EA} \parallel \overline{GF} \parallel \overline{CB}, \text{ and } \overline{BA} \parallel \overline{CD}) \) in the graph and ask students to highlight/mark the parallel lines.

Step 2: Retrieve related information about the identified chunks from long term memory.

• With the parallel theorems and postulates, what can you infer from these two sets of parallel lines?

Step 3. Problem solving
Phase II: Guided practice (15 minutes)

(1) \(BA \parallel DC, M\angle A = 45^\circ\), and \(M\angle C = 45^\circ\). Find \(M\angle AEC\).

(2) \(AF \parallel BC, EF \parallel DC, M\angle FAB = 40^\circ\), and \(\angle FAB = 50^\circ\). Find \(M\angle DCB\).

Phase III: Independent Practice (15 minutes)

In this phase, the student should solve the following problems independently without any prompts or feedback. Students’ responses will be graded as data points of the criterion test (the geometry problem solving CBM) during the intervention phase.

(1) ABC is a right triangle. \(AB \parallel EF\), and \(M\angle ABC = 20^\circ\). Find \(M\angle BCF\), \(M\angle CAB\), and \(M\angle ACE\).

(2) ABC is a right triangle, \(M\angle CAB = 50^\circ\), and \(AB \parallel ED\). Prove that \(BC \perp ED\).
(3) \( AB \parallel CD \).  

(1) Find two angles with equal sizes. 

(2) Find two supplementary angles.
Appendix E. A Social Validity Survey to be filled out by participants.

Dear student:

Thank you for participating in this project. Please fill in the following.

Name: …………………………………………………………………………………

Put ü beside each statement based on your opinion

<table>
<thead>
<tr>
<th>statement</th>
<th>Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strongly disagree</td>
<td>Disagree</td>
</tr>
<tr>
<td>I like my participation in this project.</td>
<td></td>
</tr>
<tr>
<td>I like the way of teaching geometry in this project.</td>
<td></td>
</tr>
<tr>
<td>I feel that I can learn geometry better after I enrolled in this project.</td>
<td></td>
</tr>
<tr>
<td>I like learning geometry after I participated in this project.</td>
<td></td>
</tr>
<tr>
<td>I wish this way of teaching would be applied in my school.</td>
<td></td>
</tr>
<tr>
<td>I feel my performance in geometry has improved after I joined this project.</td>
<td></td>
</tr>
</tbody>
</table>
If you have any comment, please turn this page over and write on the back

Remark:

…………………………………………………………………………………………
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Appendix F. Treatment Fidelity Checking Form.

Fidelity checking form.

<table>
<thead>
<tr>
<th>Observer:</th>
<th>Designation:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lesson:</td>
<td>Date:</td>
</tr>
</tbody>
</table>

Please write in the box the best number that represents your opinion based on your observation. The numbers are ranked from 0 to 3 as follows:

<table>
<thead>
<tr>
<th>Number</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>not existing</td>
</tr>
<tr>
<td>1</td>
<td>poor</td>
</tr>
<tr>
<td>2</td>
<td>good</td>
</tr>
<tr>
<td>3</td>
<td>very good</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Statement</th>
<th>level</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>General view</strong></td>
<td></td>
</tr>
<tr>
<td>The interventionist has written a lesson plan for the observed lesson.</td>
<td></td>
</tr>
<tr>
<td>The objectives of the lesson have been provided to the students clearly.</td>
<td></td>
</tr>
<tr>
<td>Appropriate time has been allocated to teaching this lesson.</td>
<td></td>
</tr>
<tr>
<td>Useful instructional aides have been exploited to teach the lesson.</td>
<td></td>
</tr>
<tr>
<td>The interventionist has raised the student’s motivation towards learning.</td>
<td></td>
</tr>
<tr>
<td>The interventionist has provided a sufficient number of examples and tasks.</td>
<td></td>
</tr>
<tr>
<td>The provided examples and tasks were appropriate.</td>
<td></td>
</tr>
<tr>
<td>The interventionist has engaged the student in the learning process.</td>
<td></td>
</tr>
<tr>
<td><strong>Interactive Structured Instruction</strong></td>
<td></td>
</tr>
<tr>
<td>The interventionist has provided an clear explanation to students.</td>
<td></td>
</tr>
<tr>
<td>The chunking components were clearly explained to the students.</td>
<td></td>
</tr>
<tr>
<td>The interventionist tested the student’s understanding.</td>
<td></td>
</tr>
<tr>
<td><strong>Guided Practice</strong></td>
<td></td>
</tr>
<tr>
<td>The student solves tasks with help from the interventionist.</td>
<td></td>
</tr>
<tr>
<td>The given tasks contained tasks that required chunking concepts.</td>
<td></td>
</tr>
<tr>
<td>The allocated time for solving the tasks was appropriate.</td>
<td></td>
</tr>
<tr>
<td><strong>Measuring performance</strong></td>
<td></td>
</tr>
<tr>
<td>The student received appropriate testing questions to check performance.</td>
<td></td>
</tr>
<tr>
<td>The given tasks contained tasks that required chunking concepts.</td>
<td></td>
</tr>
<tr>
<td>The allocated time for solving the tasks was appropriate.</td>
<td></td>
</tr>
<tr>
<td>The student solved the questions independently.</td>
<td></td>
</tr>
</tbody>
</table>
Appendix G. Rutgers University IRB Approval

Rutgers eIRB: IRB Approval Issued for Study # Pro2022002355 by Moosa Al Hadi

DHHS Federal Wide Assurance Identifier: FWA0003613
IRB Chair Person: Richard Drachman
IRB Assistant Director: Swapnali Chaudhari
Effective Date: 3/29/2023
Approval Date: 3/29/2023

eIRB Notice of Approval for Initial Submission # Pro2022002355

STUDY PROFILE

Study ID: Pro2022002355
Title: Effects of Chunking Intervention on Enhancing Geometry Performance in High School Students with Mathematics Learning Difficulties

<table>
<thead>
<tr>
<th>Principal Investigator: Moosa Al Hadi</th>
<th>Study Coordinator: Dake Zhang</th>
</tr>
</thead>
<tbody>
<tr>
<td>Co-Investigator(s): Dake Zhang</td>
<td></td>
</tr>
<tr>
<td>Sponsor: Department Funded</td>
<td></td>
</tr>
<tr>
<td>Risk Determination: Minimal Risk</td>
<td></td>
</tr>
<tr>
<td>Review Type: Exempt</td>
<td></td>
</tr>
</tbody>
</table>

https://mail.google.com/mail/u/1?ik=163ad2b233&view=pt&search=all&permmsgid=msg-f:1761736900984068810&sinpl=msg-f:1761736900984...
| Subjects: | 5 | Records: | 5 |

**CURRENT SUBMISSION STATUS**

| Submission Type: | Research Protocol/Study | Submission Status: | Approved |
| Approval Date: | 3/29/2023 |

**Vulnerable Population Codes:**
Children 45CFR.46.404 / 21CFR.50.51 (for FDA - Regulated)

**Protocol:** My research protocol Zhang edits 3-28.docx

**Consent:** PARENTAL PERMISSION TO PERMIT CHILD: Zhang edits.docx.pdf
Student ASSENT TO TAKE PART IN A RESEARCH STUDY - Zhang edits.docx.pdf

**Other Materials:** Measuring Tools and example of questions.docx
Example lesson plan Research Protocol Participants and setting.docx
Fidelity check.docx Survey after participating.docx

**Study Performance Sites:**

| Other Rutgers Site | Graduate School of Education |
| Plainfield Academy for the Arts and Advanced Studies (PAAAS) | 1700 W Front St, Plainfield, NJ 07063 |

**ALL APPROVED INVESTIGATOR(S) MUST COMPLY WITH THE FOLLOWING:**

1. Conduct the research in accordance with the protocol, applicable laws and regulations, and the principles of research ethics as set forth in the Belmont Report.

2. Continuing Review: Approval is valid until the protocol expiration date shown above. To avoid lapses in approval, submit a continuation application at least eight weeks before the study expiration date.

3. Expiration of IRB Approval: If IRB approval expires, effective the date of expiration and until the continuing review approval is issued, all research activities must stop unless the IRB finds that it is in the best interest of individual subjects to continue. (This determination shall be based on a separate written request from the PI to the IRB.) No new subjects may be enrolled and no samples/charts/surveys may be collected, reviewed, and/or analyzed.

4. Amendments/Modifications/Revisions: If you wish to change any aspect of this study, including but not limited to, study procedures, consent form(s), investigators, advertisements, the protocol document, investigator drug

brochure, or accrual goals, you are required to obtain IRB review and approval prior to implementation of these changes unless necessary to eliminate apparent immediate hazards to subjects.

5. Unanticipated Problems: Unanticipated problems involving risk to subjects or others must be reported to the IRB Office (45 CFR 46, 21 CFR 312, 812) as required, in the appropriate time as specified in the attachment online at: https://research.rutgers.edu/researcher-support/research-compliance/human-subjects-protection-program-irbs/hssp-guidance-topics

6. Protocol Deviations and Violations: Deviations from violations of the approved study protocol must be reported to the IRB Office (45 CFR 46, 21 CFR 312, 812) as required, in the appropriate time as specified in the attachment online at: https://research.rutgers.edu/researcher-support/research-compliance/human-subjects-protection-program-irbs/hssp-guidance-topics

7. Consent/Assent: The IRB has reviewed and approved the consent and/or assent process, waiver and/or alteration described in this protocol as required by 45 CFR 46 and 21 CFR 50, 56 (if FDA regulated research). Only the versions of the documents included in the approved process may be used to document informed consent and/or assent of study subjects; each subject must receive a copy of the approved form(s), and a copy of each signed form must be filed in a secure place in the subject’s medical/patient/research record.

8. Completion of Study: Notify the IRB when your study has been stopped for any reason. Neither study closure by the sponsor or the investigator removes the obligation for submission of timely continuing review application or final report.

9. The Investigator(s) did not participate in the review, discussion, or vote of this protocol.

10. eCOI: This IRB approval does not infer other approvals which may be required before this study can begin, such as those provided by the Rutgers Conflict of Interest Committee. If your disclosure requires a management plan with any request to change research document(s) (such as consent document(s)), then please submit the revised document(s) via modification to the IRB for review.

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Study PI Name:
Study Co-Investigators:
References


without mathematical learning difficulties. PloS One, 10(6), e0130570–e0130570. https://doi.org/10.1371/journal.pone.0130570


